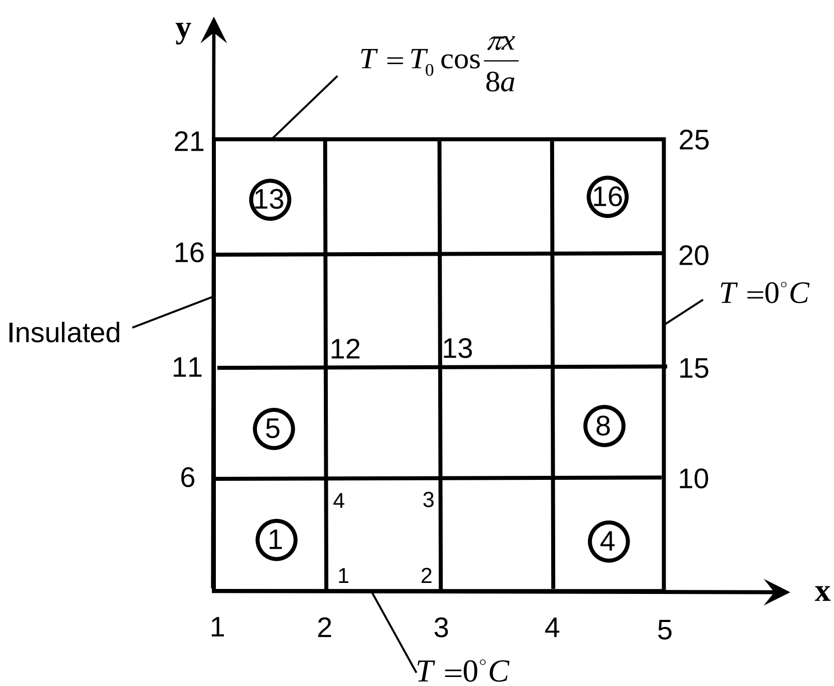
Project 2

Consider steady-state heat conduction in an isotropic rectangular region of dimension $4a$ by $4a$ (see Fig. 1). The origin of the x and y coordinates is taken at the lower left corner. The boundary $x=0$ is insulated, the boundaries $y=0$ and $x=4a$ are maintained at zero temperature. The boundary $y=4a$ is maintained at $T=T_0\cos(\pi x/8a)$. The heat transfer is governed by

$$-k\nabla^2 T = f$$

with zero internal heat generation ($f=0$).

1. Develop the finite element model of the equation above.
2. Identify the element coefficient matrices for linear rectangular and right triangular elements.
3. Write the finite element equations associated with nodes 13, 16, and 19 for the rectangular elements shown in Fig. 1a and triangular elements shown in Fig. 1b.
4. Write the specified primary and secondary variables at all boundary nodes shown in Fig. 1a and Fig. 1b.
5. Develop a general computer code using the finite element method with linear rectangular elements and right triangular elements and run the code for the meshes shown in Figs. 1a and 1b to determine the temperature distribution.
6. Compare the nodal temperatures $T(x,y)/T_0$ obtained using rectangular elements and triangular elements shown in Figs. 1a and 1b in a table with the analytical solution
$$T(x,y) = \frac{T_0 \sinh(\pi y/8a)\cos(\pi x/8a)}{\sinh(\pi/2)}$$
7. Develop a computer code using the finite element method with arbitrary unstructured linear triangular elements, as it is shown in Fig. 1c, run the code with the mesh shown in Fig. 1b first to check the code and to see whether you obtain the same results as those in Step 5. Then run the code with the mesh shown in Fig. 1c (attached data file). Plot the results with temperature contour lines for the meshes shown in Figs. 1a, 1b and 1c.



$f = 0 W/m^3$

$k = 25 W/(m \bullet {}^\circ C)$

$a = 1 m$

$T_0 = 100^\circ C$

Fig. 1a

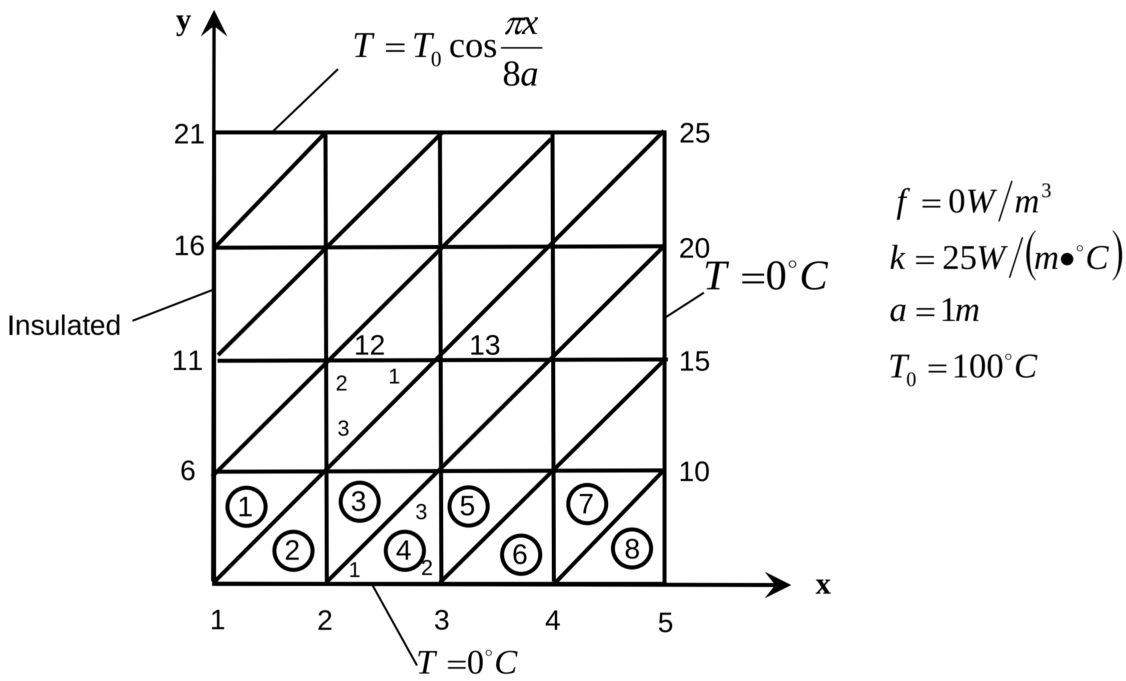

y
$T = T_0 \cos\frac{\pi x}{8a}$
21
25
16
20
$T = 0^\circ C$
Insulated
11
12
13
15
6
10
x
1
2
3
4
5
$T = 0^\circ C$
$f = 0 W/m^3$
$k = 25 W/(m \bullet {}^\circ C)$
$a = 1m$
$T_0 = 100^\circ C$

Fig. 1b

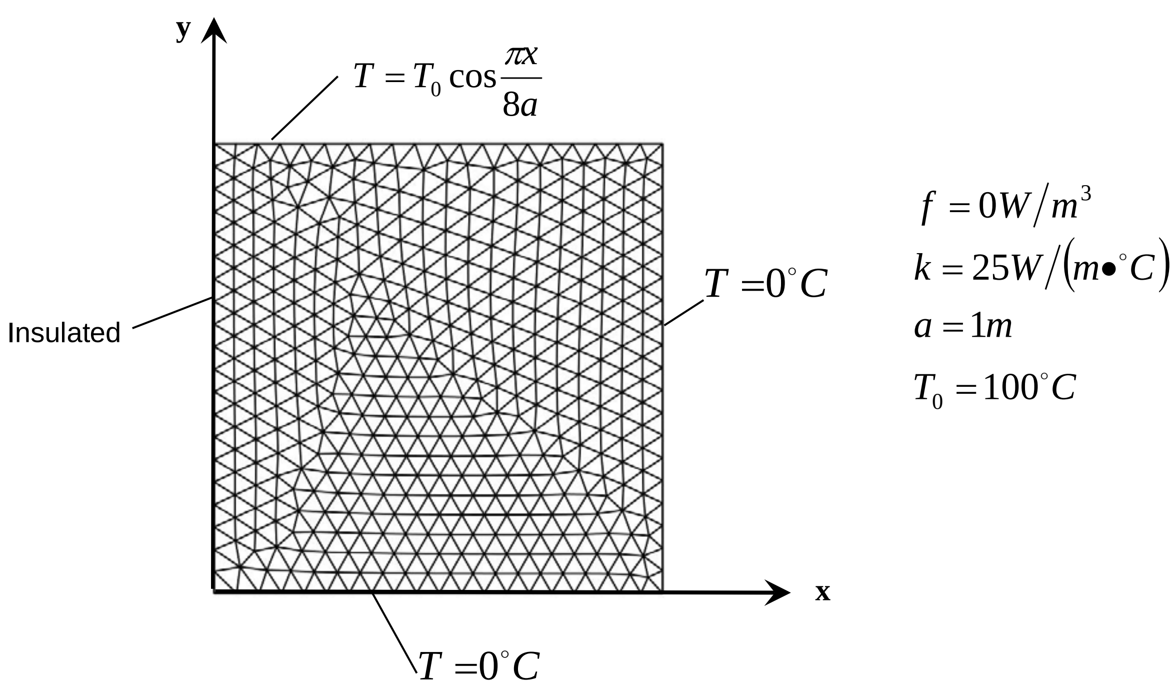

y
$T = T_0 \cos\frac{\pi x}{8a}$
$f = 0 W/m^3$
$k = 25 W/(m \bullet {}^\circ C)$
$T = 0^\circ C$
$a = 1m$
Insulated
$T_0 = 100^\circ C$
x
$T = 0^\circ C$

Fig. 1c

Notes:

Please follow the instructions and develop three codes for the following three types of elements:

1. Rectangular elements
2. Right triangular elements
3. Unstructured triangular elements

Your codes should be general. For the rectangular and right triangular elements, you can assume number of grid points in X and Y as *nx* and *ny*, respectively, and obtain the total number of nodes and elements for a given type of elements in terms of *nx* and *ny* (You have *nx* = 5, and *ny* = 5 for the elements shown in Figs. 1a-1b). You need to define the connectivity matrices for the rectangular elements and right triangular elements, respectively, in a similar numbering manner as what shown in Figs. 1a-1b with respect to local and global nodes numbering as well as element numbering. For the unstructured triangular elements, the number of nodes and elements, the connectivity matrix, and coordinates of the nodes are given in the file domain.data.

Run your codes for three given type of elements presented in Figs. 1a-1c, respectively, and analyze the results.

The file domain.data is the input data file required for the unstructured triangular elements in the Step 7 described above. The format of the input data file is as follows:

Card 1: Title Line
Card 2:

- Number of elements: **nelem** **(integer)**
- Number of node points: **npoin** **(integer)**

Card 3: Title Line
Cards 4.1 – **4.nelem:**

- Element number: **ielem** **(integer)**
- Global nodal points corresponding to each element: **intmat(1:3)** **(3* integer)**

Card 5: Title Line
Card 6.1 – 6.npoin:

- Node number: **ipoin** **(integer)**
- Coordinates of nodal point: **coord(1:2)** **(2*real)**

Please note that the attached data file has one additional real number and one additional integer number in each line after the coordinates. Please just ignore these two numbers when you write a routine to read the input data file.

To validate your code for the unstructured triangular elements, you can prepare an input data file for the mesh shown in Fig. 1b using the same format as that in data file domain.data. You should obtain the same results from your unstructured triangular elements code as that from your right triangular element code. You can then run your unstructured triangular elements code with the data file domain.data, so that you can get the results for the mesh shown in Fig. 1c.

Please prepare a report in the format of a scientific paper and cover all steps described in the project. Please use tables to compare the temperatures obtained for the mesh shown in Figs. 1a-1b with the analytical solution at all nodes. Please plot the temperature contours for analytical solution and the three types of meshes shown in Figs. 1a-1c.