1 $(\sin y + y\sin x + 1/x)dx + (x\cos y - \cos x - 1/y)dy = 0$

$$\begin{cases} u'_x = \sin y + y\sin x + 1/x \\ u'_y = x\cos y - \cos x - 1/y \end{cases}$$

$$u = \int (\sin y + y\sin x + 1/x)dx = x\sin y - y\cos x + \ln|x| + g(y)$$

$$u'_y = x\cos y - \cos x - 1/y$$

$$x\cos y - \cos x + g'(y) = x\cos y - \cos x - 1/y$$

$$g'(y) = -1/y$$

$$g(y) = -\ln|y|$$

$$u = x\sin y - y\cos x + \ln|x| - \ln|y|$$

$$C = x\sin y - y\cos x + \ln|x| - \ln|y|$$

2 $\dfrac{2xdx}{y^3} + \dfrac{(y^2 - 3x^2)dy}{y^4} = 0$

$$\begin{cases} u'_x = \dfrac{2x}{y^3} \\ u'_y = \dfrac{y^2 - 3x^2}{y^4} \end{cases}$$

$$u = \int \frac{2x}{y^3}dx = \frac{x^2}{y^3} + g(y)$$

$$u'_y = \frac{y^2 - 3x^2}{y^4}$$

$$\frac{-3x^2}{y^4} + g'(y) = \frac{y^2 - 3x^2}{y^4}$$

$$g'(y) = \frac{1}{y^2}$$

$$g(y) = \frac{-1}{y}$$

$$u = \frac{x^2}{y^3} - \frac{1}{y}$$

$$C = \frac{x^2}{y^3} - \frac{1}{y}$$

3 $(1 - x^2y)dx + x^2(y - x)dy = 0 \quad \mu = \phi(x)$

Случай $x \equiv 0$ подходит.

$$\underbrace{\mu'_y}_{0}(1 - x^2y) + x^2(y - x)\mu'_x = ((x^2(y - x))'_x - (1 - x^2y)'_y)\mu$$

$$x^2(y - x)\mu'_x = (-2xy - 3x^2 + x^2)\mu$$

$$x^2y\mu' - x^3\mu' = -2xy\mu - 2x^2\mu$$

$$\begin{cases} x^2\mu' = -2x\mu \\ -x^3\mu' = -2x^2\mu \end{cases}$$

$$\frac{\mu'}{\mu} = -\frac{2}{x}$$

$$\ln|\mu| = -2\ln|x|$$

$$|\mu| = |x|^{-2}$$

$$\mu = \pm|x|^{-2}$$

$$\mu = \pm\frac{1}{x^2}$$

$$\left(\frac{1}{x^2} - y\right)dx + (y - x)dy = 0$$

$$\begin{cases} u'_x = \frac{1}{x^2} - y \\ u'_y = y - x \end{cases}$$

$$u = \int(y - x)dy = \frac{y^2}{2} - xy + g(x)$$

$$u'_x = \frac{1}{x^2} - y$$

$$-y + g'(x) = \frac{1}{x^2} - y$$

$$g'(x) = \frac{1}{x^2}$$

$$g(x) = -\frac{1}{x}$$

$$u = \frac{y^2}{2} - xy - \frac{1}{x}$$

$$C = \frac{y^2}{2} - xy - \frac{1}{x}$$

Ответ: $x \equiv 0$ или $C = \frac{y^2}{2} - xy - \frac{1}{x}$

4 $(2xy^2 - 3y^3)dx + (7 - 3xy^2)dy = 0 \quad \mu = \phi(y)$

Случай $y \equiv 0$ подходит.

$$\mu'_y(2xy^2 - 3y^3) + (7 - 3xy^2)\underbrace{\mu'_x}_{0} = ((7 - 3xy^2)'_x - (2xy^2 - 3y^3)'_y)\mu$$

$$\mu'(2xy^2 - 3y^3) = (-3y^2 - 4xy + 9y^2)\mu$$

$$\mu'(2xy^2 - 3y^3) = (6y^2 - 4xy)\mu$$

$$\mu'(2xy - 3y^2) = (6y - 4x)\mu$$

$$\begin{cases} 2y\mu' = -4\mu \\ -3y^2\mu' = 6y\mu \end{cases}$$

$$\frac{\mu'}{\mu} = -\frac{2}{y}$$

$$\mu = \pm y^{-2}$$

$$(2x - 3y)dx + (\frac{7}{y^2} - 3x)dy = 0$$

$$u = \int(2x - 3y)dx + g(y) = x^2 - 3xy + g(y)$$

$$-3x + g'(y) = \frac{7}{y^2} - 3x$$

$$g'(y) = \frac{7}{y^2}$$

$$g(y) = -\frac{7}{y}$$

$$x^2 - 3xy - \frac{7}{y} = C$$

Ответ: $y \equiv 0$ или $x^2 - 3xy - \frac{7}{y} = C$

5 $(3y^2 - x)dx + (2y^3 - 6xy)dy = 0$

$$3y^2 - x + (2y^3 - 6xy)y' = 0$$

$$t := y^2 \quad y' = \frac{t'}{2\sqrt{t}}$$

$$3t - x + \frac{2t^{3/2} - 6x\sqrt{t}}{2\sqrt{t}}t' = 0$$

$$3t - x + (t - 3x)t' = 0$$
$$a := \frac{t}{x} \quad t' = xa' + a$$
$$3ax - x + (ax - 3x)(xa' + a) = 0$$
$$3ax - x + ax^2a' + a^2x - 3x^2a' - 3xa = 0$$
$$-x + ax^2a' + a^2x - 3x^2a' = 0$$
$$-1 + axa' + a^2 - 3xa' = 0$$
$$a'(ax - 3x) = 1 - a^2$$
$$\frac{a'(a-3)}{1-a^2} = \frac{1}{x}$$
$$\int \frac{a-3}{1-a^2} da = \ln|x| + C$$
$$\int \frac{1}{a-1} da - 2\int \frac{1}{a+1} da = \ln|x| + C$$
$$\ln|a-1| - 2\ln|a+1| = \ln|x| + C$$
$$\ln|\frac{y^2}{x} - 1| - 2\ln|\frac{y^2}{x} + 1| = \ln|x| + C$$

6 $xdx + ydy + x(xdy - ydx) = 0$

$$(x - xy)dx + (y + x^2)dy = 0$$
$$(\mu(x - xy))'_y = (\mu(y + x^2))'_x$$
$$\mu'_y(x - xy) - x\mu = \mu'_x(y + x^2) + 2x\mu$$
$$\mu'_y(x - xy) = \mu'_x(y + x^2) + 3x\mu$$

Рассмотрим случай $\mu'_y \equiv 0$, т.е. μ — функция только от x.

$$-\mu'_x(y + x^2) = 3x\mu$$

$$\nexists \mu$$

Рассмотрим случай $\mu'_x \equiv 0$, т.е. μ — функция только от y.

$$\mu'_y(x - xy) = 3x\mu$$

$$\mu'_y(1 - y) = 3\mu$$
$$\frac{\mu'_y}{\mu} = \frac{3}{1-y}$$
$$\ln|\mu| = -3\ln|1-y|$$

$$\mu = \pm|1-y|^{-3}$$
$$\mu = \pm(1-y)^{-3}$$
$$\frac{x-xy}{(1-y)^3}dx + \frac{y+x^2}{(1-y)^3}dy = 0$$
$$\begin{cases} u'_x = \dfrac{x-xy}{(1-y)^3} = \dfrac{x}{(1-y)^2} \\ \\ u'_y = \dfrac{y+x^2}{(1-y)^3} \end{cases}$$
$$u = \int \frac{x}{(1-y)^2}dx + g(y) = \frac{x^2}{2(1-y)^2} + g(y)$$
$$-\left(\frac{x^2}{2(1-y)^2}\right)' + g'(y) = \frac{y+x^2}{(1-y)^3}$$
$$\frac{x^2}{(1-y)^3} + g'(y) = \frac{y+x^2}{(1-y)^3}$$
$$g'(y) = \frac{y}{(1-y)^3}$$
$$g(y) = \frac{1+2y}{2(1-y)^2}$$
$$C = \frac{1}{(1-y)^2}\left(\frac{1+2y+x^2}{2}\right)$$

7 $y' = y^2 - 2ye^x + e^{2x} + e^x$

Это уравнение Рикатти.

$$y' = (y-e^x)^2 + e^x$$

Частное решение $y = e^x$

Найдем общее решение заменой $y = z + e^x \quad y' = z' + e^x$

$$z' + e^x = (z + e^x - e^x)^2 + e^x$$
$$z' = z^2$$
$$\frac{z'}{z^2} = 1$$

$$-\frac{1}{z} = x + C$$
$$-\frac{1}{y - e^x} = x + C$$

В ответе решение $-\dfrac{1}{y - e^x} = x + C$ и $y = e^x$