1 $y' + 2y = e^{-x}$

$$
\begin{aligned}
y' &= e^{-x} - 2y \\
y &= \left(C + \int e^{-x} e^{\int 2dx} dx\right) e^{-\int 2dx} \\
y &= \left(C + e^{C_1} \int e^{x} dx\right) e^{-2x} e^{-C_1} \\
y &= \left(C + e^{C_1} e^{x}\right) e^{-2x} e^{-C_1} \\
y &= \left(Ce^{-C_1} e^{-2x} + e^{-x}\right) \\
y &= \left(Ce^{-2x} + e^{-x}\right)
\end{aligned}
$$

2 $y' - y \operatorname{tg} x = \dfrac{1}{\cos^3 x}, y(0) = 0$

$$
\begin{aligned}
y' &= y \operatorname{tg} x + \frac{1}{\cos^3 x} \\
p(x) = \operatorname{tg} x \quad q(x) &= \frac{1}{\cos^3 x} \\
y &= \left(C + \int \frac{1}{\cos^3 x} e^{-\int \operatorname{tg} x dx} dx\right) e^{-\int \operatorname{tg} x dx} \\
y &= \left(C + \int \frac{1}{\cos^3 x} e^{-\ln\cos x - C_1} dx\right) e^{\ln\cos x + C_1} \\
y &= \left(C + \int \frac{1}{\cos^3 x} \cos^{-1} x e^{-C_1} dx\right) \cos x e^{C_1} \\
y &= \left(C + e^{-C_1} \int \frac{1}{\cos^4 x} dx\right) \cos x e^{C_1} \\
y &= \left(C + e^{-C_1} \left(\frac{1}{3} \frac{\operatorname{tg} x}{\cos^2 x} + \frac{2}{3} \int \frac{1}{\cos^2 x} dx\right) dx\right) \cos x e^{C_1} \\
y &= \left(C + e^{-C_1} \left(\frac{1}{3} \frac{\operatorname{tg} x}{\cos^2 x} + \frac{2}{3} \operatorname{tg} x\right)\right) \cos x e^{C_1} \\
y &= \left(C \cos x + e^{-C_1} \left(\frac{1}{3} \frac{\operatorname{tg} x}{\cos x} + \frac{2}{3} \sin x\right)\right) e^{C_1} \\
y &= e^{C_1} C \cos x + \left(\frac{1}{3} \frac{\operatorname{tg} x}{\cos x} + \frac{2}{3} \sin x\right) \\
0 &= e^{C_1} C + 0 \\
C &= 0
\end{aligned}
$$

$$y = \left(\frac{1}{3}\frac{\operatorname{tg} x}{\cos x} + \frac{2}{3}\sin x\right)$$

3 $y' - y\ln 2 = 2^{\sin x}(\cos x - 1)\ln 2$, y ограничено при $\to \infty$

$$y' = y\ln 2 + 2^{\sin x}(\cos x - 1)\ln 2$$
$$y \xrightarrow[x\to\infty]{} \frac{y'}{\ln 2} - 2^{\sin x}(\cos x - 1)$$

$y' - y\cos x = y^2\cos x$

$$y' = y\cos x(y+1)$$

$y \equiv 0$ подходит, $y \equiv -1$ тоже подходит.

$\lhd y \notin \{0, -1\}$

$$\frac{dy}{dx}\frac{1}{y(y+1)} = \cos x$$
$$\frac{dy}{y(y+1)} = dx\cos x$$
$$\int\frac{dy}{y(y+1)} = \int dx\cos x$$
$$\int\frac{dy}{y(y+1)} = \sin x + C$$
$$\int\frac{dy}{y(y+1)} = \sin x + C$$
$$\int\frac{dy}{y} - \int\frac{dy}{y+1} = \sin x + C$$
$$\ln|y| - \ln|y+1| = \sin x + C$$