1 $y'' - 3y' + 2 = 1/(1+e^x)$

Найдём решение частного уравнения $y'' - 3y' = 0$

$$y'' - 3y' = 0$$
$$\lambda^2 - 3\lambda = 0$$
$$\left[\begin{array}{l}\lambda = 0\\ \lambda = 3\end{array}\right.$$
$$y = C_1 + C_2 e^{3x}$$

Пусть $C_1 = Z_1(x), C_2 = Z_2(x)$

$$\begin{cases} Z_1' + Z_2' e^{3x} = 0 \\ 3Z_2' e^{3x} = \frac{1}{1+e^x} - 2 \end{cases}$$
$$\begin{cases} Z_1' + \frac{1}{3(1+e^x)} - \frac{2}{3} = 0 \\ Z_2' e^{3x} = \frac{1}{3(1+e^x)} - \frac{2}{3} \end{cases}$$
$$\begin{cases} Z_1' = \frac{2}{3} - \frac{1}{3(1+e^x)} \\ Z_2' = \frac{1}{3e^{3x}(1+e^x)} - \frac{2}{3e^{3x}} \end{cases}$$
$$\begin{cases} Z_1 = \int \left(\frac{2}{3} - \frac{1}{3(1+e^x)}\right) dx \\ Z_2 = \int \left(\frac{1}{3e^{3x}(1+e^x)} - \frac{2}{3e^{3x}}\right) dx \end{cases}$$
$$\begin{cases} Z_1 = \frac{2x}{3} - \frac{1}{3}\int \frac{1}{1+e^x} dx \\ Z_2 = \frac{2}{9e^{3x}} + \frac{1}{3}\int \frac{1}{e^{3x}(1+e^x)} dx \end{cases}$$
$$\begin{cases} Z_1 = \frac{2x}{3} - \frac{1}{3}(x - \ln(e^x+1)) + C_3 \\ Z_2 = \frac{2}{9e^{3x}} + \frac{1}{3}\left(-x - \frac{1}{3e^{-3x}} + \frac{1}{2e^{-2x}} - \frac{1}{e^x} + \ln(e^x+1)\right) + C_4 \end{cases}$$
$$\begin{cases} Z_1 = \frac{x}{3} + \frac{\ln(e^x+1)}{3} + C_3 \\ Z_2 = \frac{1}{9e^{3x}} - \frac{x}{3} + \frac{1}{6e^{2x}} - \frac{1}{3e^x} + \frac{\ln(e^x+1)}{3} + C_4 \end{cases}$$

$$y = \frac{x}{3} + \frac{\ln(e^x+1)}{3} + C_3 + \frac{1}{9} - \frac{xe^{3x}}{3} + \frac{e^x}{6} - \frac{e^{2x}}{3} + \frac{e^{3x}\ln(e^x+1)}{3} + e^{3x}C_4$$

2 $y'' - y' = -(x+1)/x^2$

Найдём решение частного уравнения $y'' - y' = 0$

$$\lambda^2 - \lambda = 0$$
$$\left[\begin{array}{l}\lambda = 0\\ \lambda = 1\end{array}\right.$$
$$y = C_1 + C_2 e^x$$

Пусть $C_1 = Z_1(x), C_2 = Z_2(x)$

$$\begin{cases} Z_1' + Z_2' e^x = 0 \\ Z_2' e^x = -\frac{x+1}{x^2} \end{cases}$$
$$\begin{cases} Z_1' = \frac{x+1}{x^2} \\ Z_2' = -\frac{x+1}{x^2 e^x} \end{cases}$$
$$\begin{cases} Z_1 = \int \frac{x+1}{x^2} dx \\ Z_2 = -\int \frac{x+1}{x^2 e^x} dx \end{cases}$$
$$\begin{cases} Z_1 = \ln|x| - \frac{1}{x} + C_3 \\ Z_2 = \frac{1}{xe^x} + C_4 \end{cases}$$

$$y = \ln|x| - \frac{1}{x} + C_3 + C_4 e^x + \frac{1}{x}$$
$$y = \ln|x| + C_3 + C_4 e^x$$

3 $x^2 y'' + 2xy' - 12y = 0$

$$x^2 y'' + 2xy' - 12y = 0$$
$$x := e^t \quad y_t' = y_x' x_t' = y_x' e^t \quad y_{tt}'' = y_{xx}'' e^{2t} + y_x' e^t$$
$$e^{2t} y_{xx}'' + 2e^t y_x' - 12y = 0$$
$$y_t'' + y_t' - 12y = 0$$
$$\lambda^2 + \lambda - 12 = 0$$

$$\left[\begin{array}{l}\lambda = -4 \\ \lambda = 3\end{array}\right.$$

$$y = C_1 e^{-4t} + C_2 e^{3t}$$
$$y = \frac{C_1}{x^4} + C_2 x^3$$

4 $x^2 y'' - 2y = 4/x^2$

$$x^2 y'' - 2y = 4/x^2$$
$$x := e^t \quad y'_t = y'_x x'_t = y'_x e^t \quad y''_{tt} = y''_{xx} e^{2t} + y'_x e^t$$
$$e^{2t} y''_{xx} - 2y = \frac{4}{e^{2t}}$$
$$y''_{tt} - y'_t - 2y = \frac{4}{e^{2t}}$$

Найдём решение частного уравнения $y''_{tt} - y'_t - 2y = 0$

$$\lambda^2 - \lambda - 2 = 0$$
$$\left[\begin{array}{l}\lambda = -1 \\ \lambda = 2\end{array}\right.$$
$$y = C_1 e^{-t} + C_2 e^{2t}$$

$$\begin{cases} Z'_1 e^{-t} + Z'_2 e^{2t} = 0 \\ -Z'_1 e^{-t} + 2 Z'_2 e^{2t} = \frac{4}{e^{2t}} \end{cases}$$
$$\begin{cases} Z'_1 = -Z'_2 e^{3t} \\ 3 Z'_2 e^{2t} = \frac{4}{e^{2t}} \end{cases}$$
$$\begin{cases} Z'_1 = -\frac{4}{3e^t} \\ Z'_2 = \frac{4}{3e^{4t}} \end{cases}$$
$$\begin{cases} Z_1 = \frac{4t}{3e^t} + C_3 \\ Z_2 = -\frac{1}{3e^{4t}} + C_4 \end{cases}$$

$$y = \frac{4}{3e^{2t}} + \frac{C_3}{e^t} + C_4e^{2t} - \frac{1}{3e^{2t}}$$
$$y = \frac{1}{e^{2t}} + \frac{C_3}{e^t} + C_4e^{2t}$$

5 $x' = 4x - 8y + \operatorname{tg} 4t, y' = 4x - 4y$