5 (1. Halbtag) | Operationsverstärker

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September 11, 2024

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Contents

1 Introduction

This experiment deals with the opamp and its funcitonality in different circuits. The opamp can be used as a non–inverting or inverting amplifier, adder, differential amplifier, current source, monoflop, to logarithmize or exponentiate and to differentiate or integrate a signal. It is also possible to build a SCHMITT–trigger or an astable multivibrator. In this experiment the opamp will serve as a non–inverting amplifier, adder, current source, integrator and differential amplifier.

2 Theory

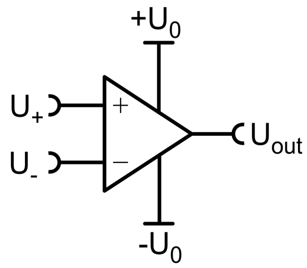

Figure 1: Schematic of an opamp; Abb. 5/6.1[?]

The most important properties of opamps are

- With stable negative feedback, the opamp regulates the output voltage such that $U_+ = U_-$.
- There is close to no input current $I_+ = I_- \approx 0$.

Important features of real opamps to keep in mind

- The maximum output voltage can't be higher than the maximum supply voltage.
- Due to internal constrains, the opamp has a finit slew rate, meaning that it can't change a signal at infinite speed.
- The open–loop gain decreases with rising frequency. The bandwidth is the cutoff frequency at which $\nu = \frac{1}{\sqrt{2}}$.
- For $U_+ = U_- = 0$ one would assume that $U_{\text{out}} = 0$. In real world opamps this is not the case. The output voltage goes to zero at a certain difference between U_+ and U_-. This difference is component specific.
- The output voltage should not change if both input voltages increae at the same rate. This is not the case in the real world. The common mode rejection ratio is the ratio between the differential gain and common mode gain.

The opamp can be utilised as a non–inverting amplifier with an ideal open–loop gain of infinity. Here $U_- = k \cdot U_{\text{out}}$ and $U_+ = U_{\text{in}}$. The gain is

$$\nu = \frac{U_{\text{out}}}{U_{\text{in}}} = \frac{1}{k} = 1 + \frac{Z_2}{Z_1}. \tag{2.1}$$

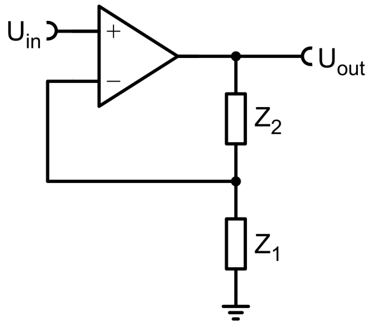

Figure 2: Non–inverting amplifier; Abb. 5/6.4[?]

The opamp can also be used as an adder. As discussed in preliminary task F the output voltage is an addition of input voltages

$$U_{\text{out}} = c_i U_i \qquad c_i = -\frac{R_0}{R_i}. \tag{2.2}$$

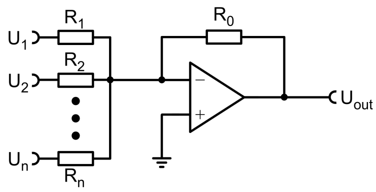

Figure 3: Adder; Abb. 5/6.6[?]

The opamp can also integrate signals by charging a capacitor to sum up the input signal

$$U_{\text{out}}(t) = -\frac{1}{R_1 C} \int_{t_0}^{t} U_{\text{in}}(t')\, \mathrm{d}t'. \tag{2.3}$$

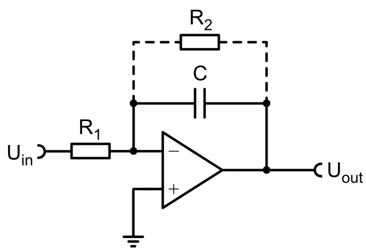

Figure 4: Integrator; Abb. 5/6.11[?]

At last the opamp is used in a circuit to function as a differential amplifier. As discussed in preliminary task G, the output voltage is an amplification of the difference of the input voltages

$$U_{\text{out}} = \frac{R_2}{R_1}\left(U_2 - U_1\right). \tag{2.4}$$

Here $\frac{R_2}{R_1}$ is the gain.

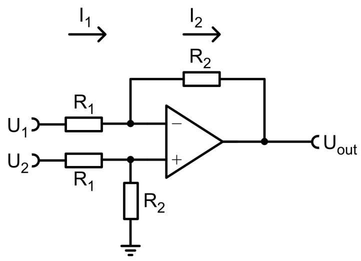

Figure 5: Differential amplifier; Abb. 5/6.7[?]

When using the opamp as an inverting amplifier, the current flowing through the negative feedback circuit does not depend on Z_2 but only on U_{in} and Z_1. Thus one can construct a current source for the resistance Z_2 which can be controlled via the input voltage.

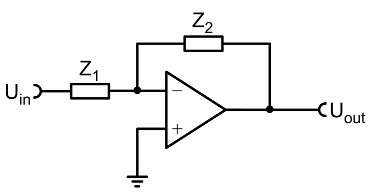

Figure 6: Inverting amplifier; Abb. 5/6.5[?]

The astable multivibrator uses the SCHMITT–trigger to construct a signal from an ideal wave form, see preliminary task K.

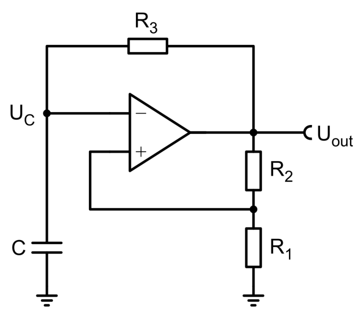

Figure 7: Astable multivibrator; Abb. 5/6.15[?]

3 Preliminary Tasks

3.1 A

The equation hold

$$\frac{1}{\nu} = \frac{1}{\nu_0} + k \qquad \nu = \frac{1}{\frac{1}{\nu_0} + k}. \tag{3.1}$$

For $k = 0.1$, $\nu_0 = 10^4$ and $\nu_0 = 10^5$

$$\nu_1 \approx 9.990 \qquad \nu_2 \approx 9.999. \tag{3.2}$$

The approximation $\nu = \frac{1}{k}$ results in

$$\nu_{\text{Näh}} = 10. \tag{3.3}$$

The deviation of ν_1 and ν_2 from $\nu_{\text{Näh}}$ lie at 0.001% and 0.0001% respectively.

3.2 B

It hold

$$U_x = U_{\text{in}} - kU_{\text{out}} \tag{3.4}$$
$$\Leftrightarrow \quad = U_{\text{in}} - kv_0 U_x$$
$$\Leftrightarrow \quad = \frac{U_{\text{in}}}{1 + v_0 k}. \tag{3.5}$$

For $k = 0.1$, $v_0 = 10^5$ and $U_{\text{in}} = 1\,\text{V}$

$$U_x \approx 0.0001\,\text{V}. \tag{3.6}$$

3.3 C

Let there be a common mode signal with $\Delta U_+ = \Delta U_- = +\Delta U_{\text{in}}$. then

$$\Delta U_+ = \Delta U_E + \Delta U_1 \quad \Delta U_- = \Delta U_E + \Delta U_1. \tag{3.7}$$

from this follows $\Delta U_{\text{in}} = \Delta U_E + \Delta U_1$. The output voltage is

$$\Delta U_{\text{out}} = R_C \cdot \Delta I_C. \tag{3.8}$$

At point 1,

$$I_1 = 2I_E. \tag{3.9}$$

Therefore

$$\begin{aligned}\Delta U_{\text{in}} &= R_E \cdot \Delta I_E + R_1 \cdot 2\Delta I_E \\ &= \Delta I_E\left(R_E + 2R_1\right) \approx \Delta I_E \cdot 2R_1. \end{aligned} \tag{3.10}$$

At the node U_{out} applies

$$\Delta I_E = \Delta I_C \Rightarrow \Delta U_{\text{out}} = R_C \cdot \Delta I_E. \tag{3.11}$$

The amplification results in

$$v_{CM} = \frac{\Delta U_{\text{out}}}{\Delta U_{\text{in}}} = \frac{R_C}{2R_1}. \tag{3.12}$$

The common mode suppression is

$$10\log\left(\frac{R_E}{R_1}\right) = 10\log\left(\frac{1\,\text{k}\Omega}{100\,\text{k}\Omega}\right) = -20\,\text{dB}. \tag{3.13}$$

3.4 D

The frequency dependence of the impedance of a capacitor is

$$Z_1 = \frac{1}{\mathrm{i}\omega C} = \frac{1}{\mathrm{i}2\pi f C} \tag{3.14}$$

$$|Z_1| = \left|\frac{1}{\mathrm{i}\omega C}\right| = \frac{1}{2\pi f C}. \tag{3.15}$$

The gain as a function of frequency is

$$v(f) = 1 + \frac{Z_2}{|Z_1|} = 1 + R2\pi f C. \tag{3.16}$$

The limits are

$$\lim_{f\to 0}\left[1 + R2\pi f C\right] = 1 \quad \lim_{f\to\infty}\left[1 + R2\pi f C\right] = \infty. \tag{3.17}$$

For $|Z_1| = R$ it has to hold that

$$\frac{1}{2\pi f C} = R \Leftrightarrow f = \frac{1}{2\pi R C}. \tag{3.18}$$

With concrete values $Z_1 = R = 100\,\text{k}\Omega$ and $Z_1 = C = 100\,\text{nF}$, the frequency is

$$f = \frac{1}{2\pi R C} \approx 15.92\,\text{Hz} \Rightarrow v(f) \approx 2. \tag{3.19}$$

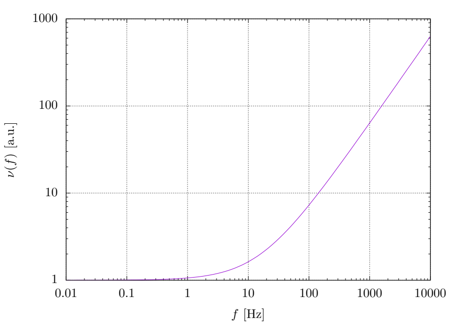

Figure 8: Time dependend amplification of a non–invertible amplifier as a BODE–plot

3.5 E

Let

$$v = \frac{U_{\text{out}}}{U_{\text{in}}} = -\frac{Z_2}{Z_1}. \tag{3.20}$$

The minus sign results from the negative feedback. Because of the golden rule $U_- = U_+ = 0\,\text{V}$, the negative feedback has a different sign compared to the input signal.

The input impedance is very high and the output impedance very low.

3.6 F

The first KIRCHHOFF's law states (EINSTEIN notation)

$$\sum_i I_i = -I_{\text{out}} \tag{3.21}$$

$$\Leftrightarrow \quad \frac{U_i}{R_i} = -\frac{U_{\text{out}}}{R_0} \tag{3.22}$$

$$\Leftrightarrow \quad -\frac{R_0}{R_i}U_i = U_{\text{out}} \tag{3.23}$$

$$\Leftrightarrow \quad c_i U_i = U_{\text{out}}. \tag{3.24}$$

3.7 G

$$U_+ = U_2\frac{R_2}{R_1 + R_2} \qquad |\text{voltage divider} \tag{3.25}$$

$$U_- = U_+ \qquad |\text{golden rule} \tag{3.26}$$

$$I_1 = \frac{U_1 - U_-}{R_1} \qquad |\text{OHM's law} \tag{3.27}$$

$$I_2 = I_1 \qquad |\text{golden rule} \tag{3.28}$$

$$I_2 = \frac{U_- U_{\text{out}}}{R_2} \qquad |\text{OHM's law.} \tag{3.29}$$

The final result $U_{\text{out}} = \frac{R_2}{R_1}(U_2 - U_1)$ can be calculated using the above relations.

3.8 H

A constant negative input voltage provides a continuous charge to the capacitor, resulting in a steadily rising output voltage. One could also argue that for a constant input voltage the integral will increase linearly.

3.9 I

For an inverting amplifier $\nu = -\frac{Z_2}{Z_1}$. With $Z_1 = R$ and $Z_2 = \frac{1}{\mathrm{i}\omega C}$

$$\nu = -\frac{1}{\mathrm{i}\omega C R}. \tag{3.30}$$

The phase relation between the output– and inputsignal is then

$$\tan\Phi = \frac{\Im(\nu)}{\Re(\nu)} = \lim_{\alpha\to 0}\frac{-\frac{1}{\omega CR}}{\alpha} = -\infty. \tag{3.31}$$

Thus $\Phi = -\frac{\pi}{2}$.

3.10 J

The AD711 opamp has a slew rate of $s = 20\,\mathrm{V\,\mu s^{-1}}$. The peak to peak voltage lies at $U_{pp} = 28\,\mathrm{V}$. This means that the AD711 needs

$$t = \frac{U_{pp}}{s} = 1.4\,\mathrm{\mu s} \tag{3.32}$$

to get from $-14\,\mathrm{V}$ to $14\,\mathrm{V}$. The switching frequency is therefore

$$\tfrac{1}{t} = 0.714\,\mathrm{\mu s^{-1}}. \tag{3.33}$$

3.11 K

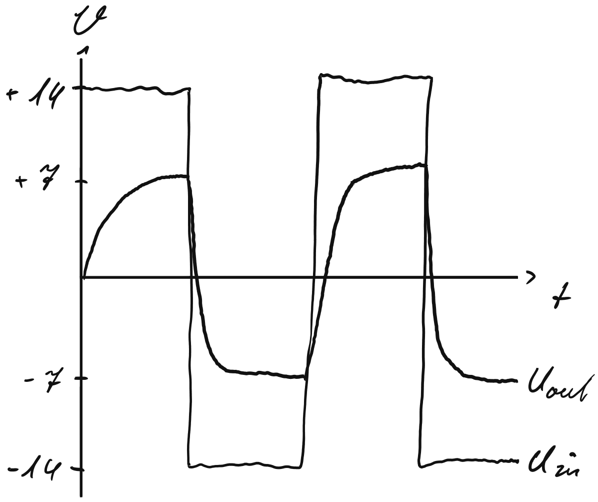

Figure 9: Astable multivibrator, voltage curve

The ideal signal is a square–wave with $U_{\mathrm{max,min}} \pm 14\,\mathrm{V}$. Due to the slew rate of the opamp, the signal need time to rise to $\pm 7V$. The output voltage can be modeled with an exponential function.

4 Analysis

4.1 Non–inverting amplifier

The non–inverting amplifier is built with a gain of $\nu = 11$ (i.e. $R_1 = 1\,\mathrm{k}\Omega$ and $R_2 = 10\,\mathrm{k}\Omega$). A signal of $U_{pp} = 1\,\mathrm{V}$ is applied to the circuit and the gain is observed for different frequencies.

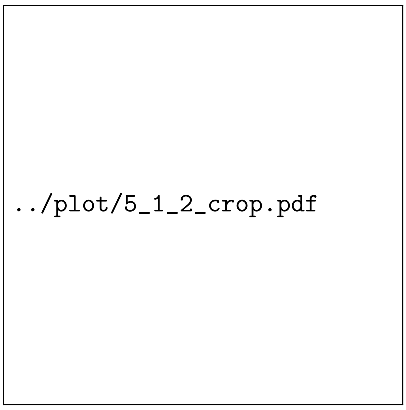

Figure 10: BODE–plot of a non–inverting amplifier with $\nu = 11$

4.2 Adder

With the help of Tri-Sinewave-Generator we generate three Sinewaves with 50 Hz, 100 Hz and 150 Hz. The goal is to produce a sawtooth-signal. For that we look at the taylor expansion and find the amplitudes for our frequencies.

$$F(t) = \frac{2}{\pi} A \cdot \left(\sin \omega t + \frac{1}{2} \sin 2\omega t + \frac{1}{3} \sin 3\omega t + \dots \right) \tag{4.1}$$

So we choose for our 50 Hz Signal full amplitude, for 100 Hz halve amplitude and for 150 Hz one third amplitude. With this configuration we achieve a signal thats displayed in fig. ??.

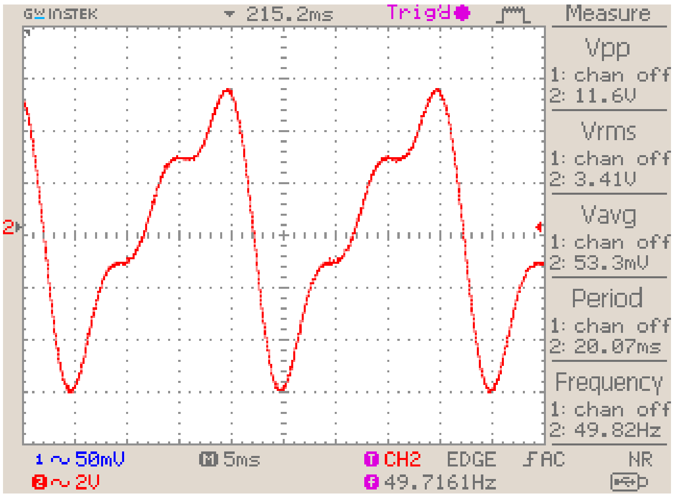

Figure 11: Sawtooth-Signal with calculated amplitudes

We see that it allready approximates a Sawtooth-Signal pretty well. But we think we can make it even better. So we adjust our amplitudes and find an even better approximation. It is displayed in fig. ??.

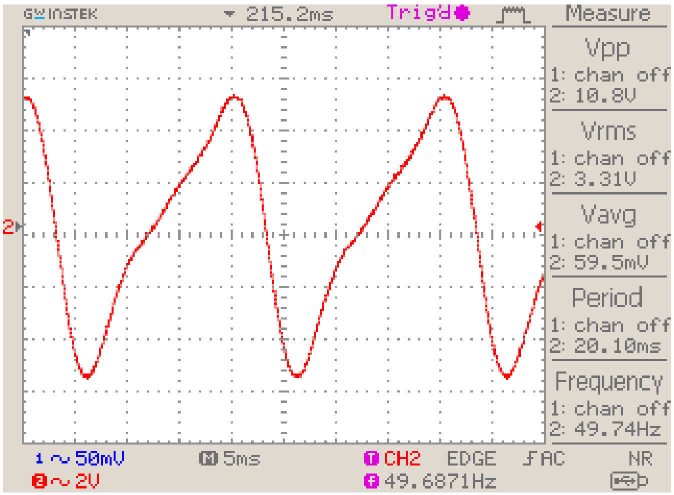

Figure 12: Sawtooth-Signal with found amplitudes

For this Outputsignal we used

$$\begin{aligned} U(f = 50\,\mathrm{Hz}) &= 1 \cdot U_0, \\ U(f = 100\,\mathrm{Hz}) &= \frac{1}{2} \cdot U_0 \text{ and} \\ U(f = 150\,\mathrm{Hz}) &= \frac{1}{6} \cdot U_0. \end{aligned}$$

This difference from calculated to experimental optimum, could be a result of frequency dependent volatge, voltage loss from overlooked resistance or simply bad equipment.

4.3 Constant current source

In this subsection we build a constant current source circut, thats shown in the schematic in fig. ??. We expect a current of $I_{R_2} = \frac{U}{R_1} = \frac{9.4\,\text{V}}{47\,\text{k}\Omega} = 0.2\,\text{mA}$. For R_2 we choose $10\,\text{k}\Omega$.

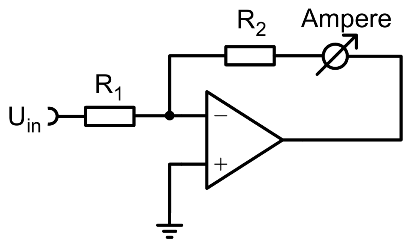

Figure 13: Schematic of a constant current source [?]

Before resistor 2 we meassure a current of 0.171 mA. Thats diviation of 16.96 % from the calculated current. Causes could be again and as always resitance that we have overlooked or bad equipment. Especially with souch low current (micro to milli ampere), we can expect some diviations alone from the resistance in the cables and bad connections.

Afterwards we replace the second resistor with a potentiometer and variate the resistance. We meassure no difference in the current. Thats as expected, since the feedback loop is supposed to do exactly that; regulate itself, since the output voltage will grow proportional to the input voltage, wich is less regulated by the higher resistor. With this idea we see that the outputvoltage will change with the change of the potentiometer. So we have to ways of reducing the current. Either we increase the first resistor or we reduce the input voltage.

4.4 Integrator

Now we substitute the second resistor / potentiometer and amperemeter with an $1\,\text{M}\Omega$ resistor and place a $100\,\mu\text{F}$ condesator parallel to it. We also set a square wave signal with 100 Hz and $1\,\text{V}_{\text{PP}}$. The resulting oscillogramm is displayed in fig. **??** with CH1 beeing the input signal and CH2 the output signal.

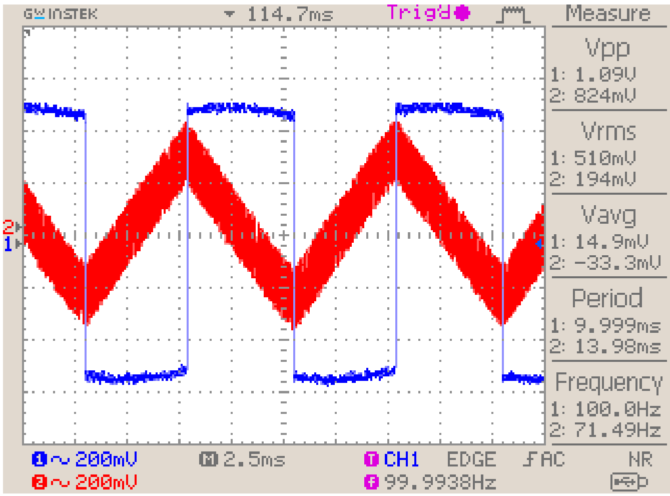

Figure 14: Integrator with a square wave signal with 100 Hz and $1\,\text{V}_{\text{PP}}$.

As expected the result is the integration of the input signal.

Now we change the frequency. The result is displayed in fig. **??** and **??**.

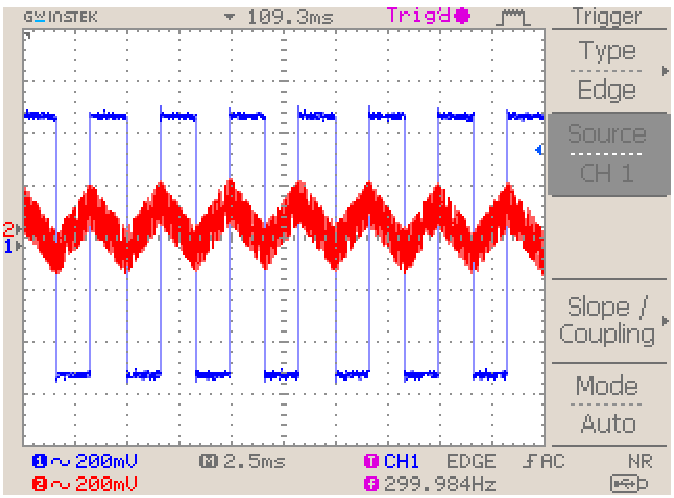

Figure 15: Integrator with a square wave signal with 300 Hz and $1\,\text{V}_{\text{PP}}$.

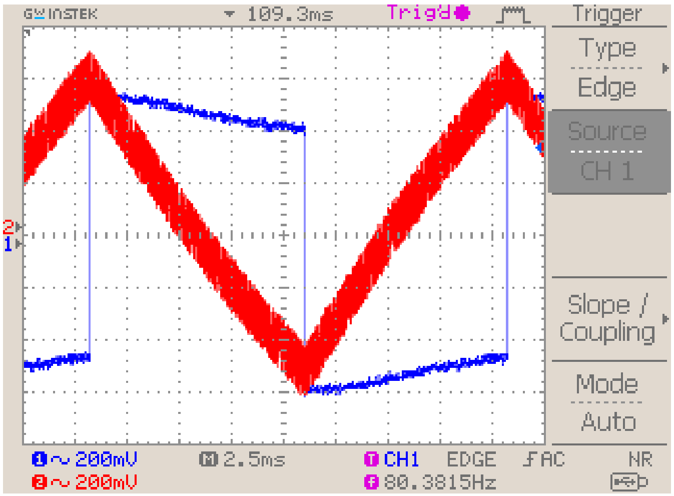

Figure 16: Integrator with a square wave signal with 80 Hz and 1 V_{PP}.

We observe that with higher frequencies, the integrated signal looses peak-to-peak voltage. Thats logical since, firstly the condensator increases resistence with higher frequencies, and secondly if we think about it in a mathmatical sense, integrating a square wave signal with shorter pulses, gives the integration less time to increase, so the voltage does not increase as much.

Now we change input voltage. The result is displayed in fig. ??, ?? and ??.

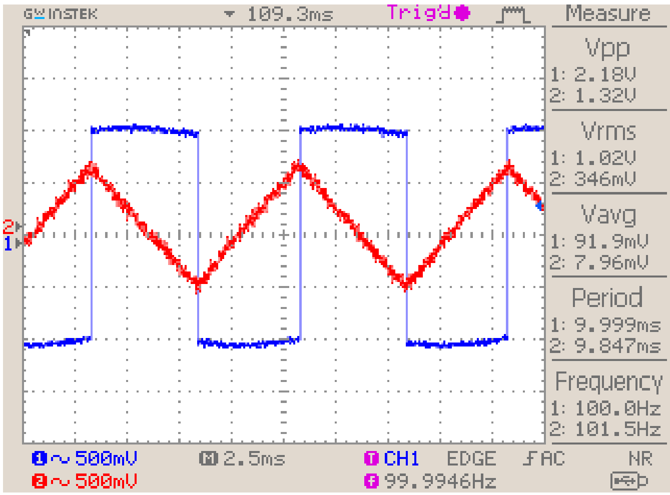

Figure 17: Integrator with a square wave signal with 100 Hz and 2 V_{PP}.

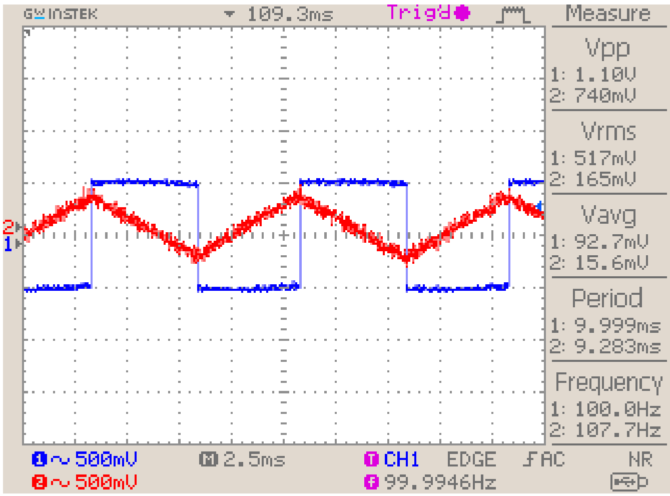

Figure 18: Integrator with a square wave signal with 100 Hz and 1 V_{PP}.

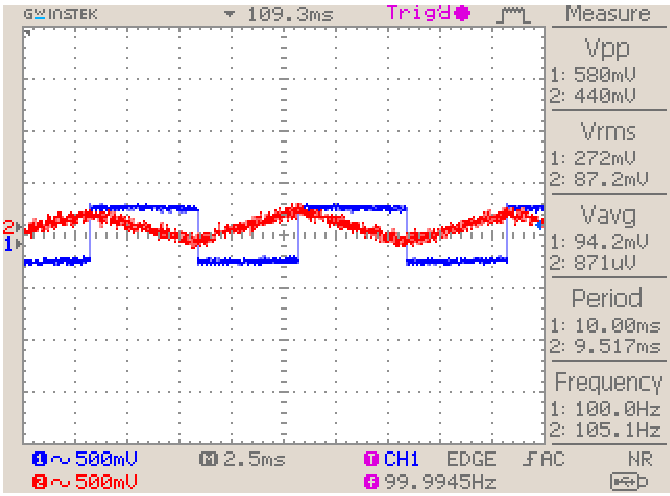

Figure 19: Integrator with a square wave signal with 100 Hz and 0.5 V_{PP}.

As we see, the input and output voltages are proportionaly lowerd as we descend from 2 V_{PP} to 0.5 V_{PP}. Thats not so surprising since every component scales the voltage linealy through its resistance.

If we remove the 1 MΩ resistor we can occasionlly observe fracmentations, wich are displayed in fig. ??.

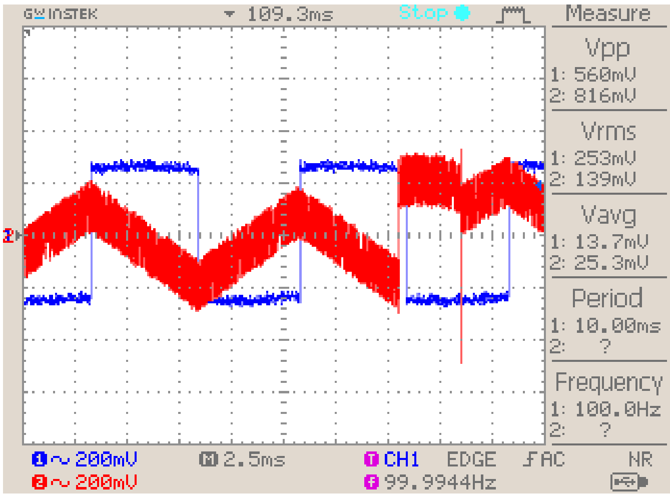

Figure 20: Integrator wihtout 1 MΩ resistor with a square wave signal with 100 Hz and 0.5 V_{PP}.

We suspect that the base-line voltage is the cause of the phenomenon, since there is no resistance over wich it can decrease and every fluctuation, noise and base-voltage is captured by the capacitor/integrator.

Switching the input signal to a sine wave, gives us the expected cosine wave displayd in fig. **??**. The drop in voltage is explainable through overlooked resistances or frequency dependent resistance.

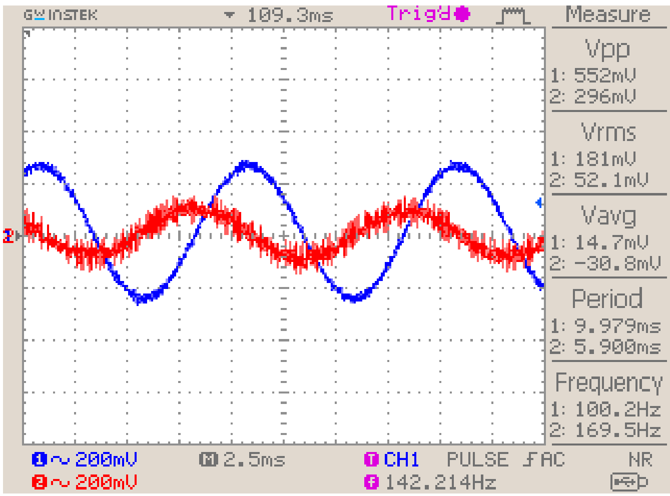

Figure 21: Integration of a sine wave.

Displaying the phases through XY configuration we can observe the Lissajour figure in fig. **??**

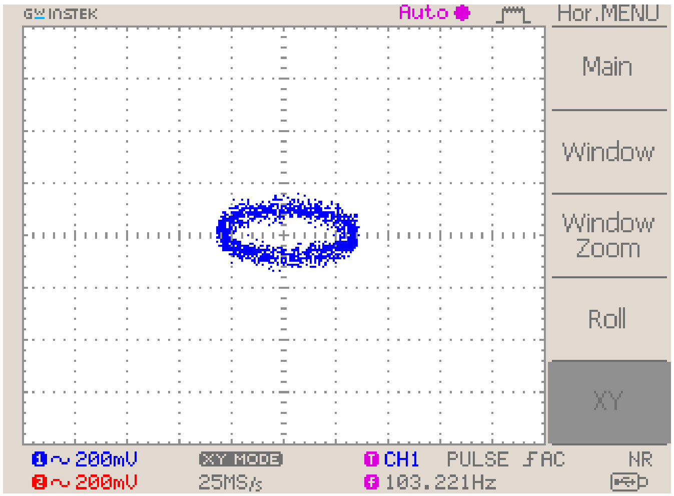

Figure 22: Lissajour figure of the sine wave and its integrated cosine output.

Now we increase the voltage and observe a proportional increase in the output signal (fig. **??**), where as the ellips (fig. **??**) just increases in size in x and y direction.

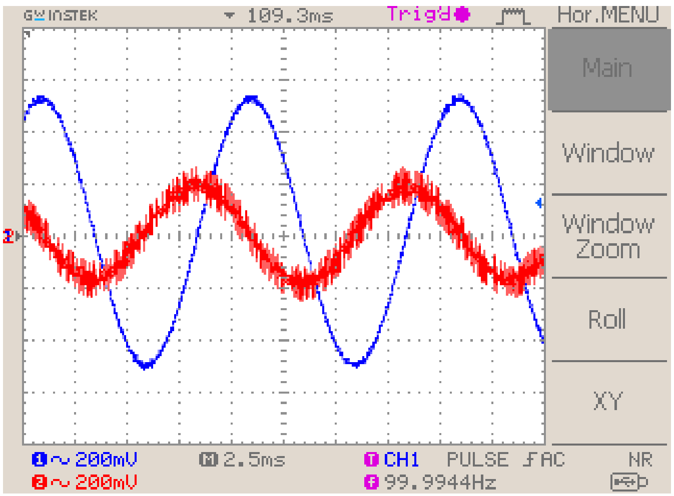

Figure 23: Integration of a sine wave with 100 Hz and 1 V_{PP}

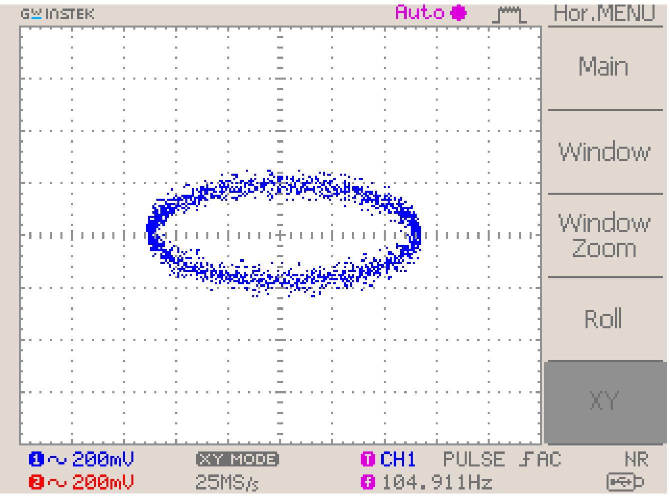

Figure 24: Integration of a sine wave with 100 Hz and 1 V_{PP} in XY channel configuration as Lissajour figure.

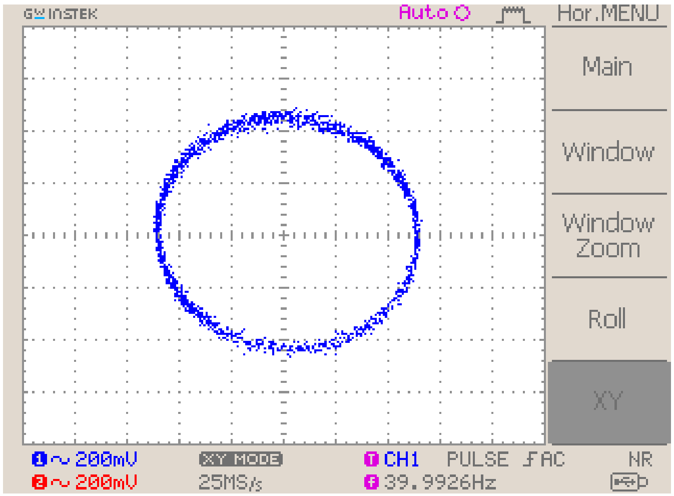

Figure 26: Integration of a sine wave with 40 Hz and 1 V_{PP} in XY channel configuration as Lissajour figure.

Now we can also decrease the frequency from 100 Hz to 40 Hz. We observe that the ouput signal is increased to nearly the level of the input signal. This behaviour suggests that indeed the voltage is frequeny dependent, wich makes sense, since the impedance of the capacitor is also frequency dependent. We also observe a nearly perfect circel instead of a ellipse.

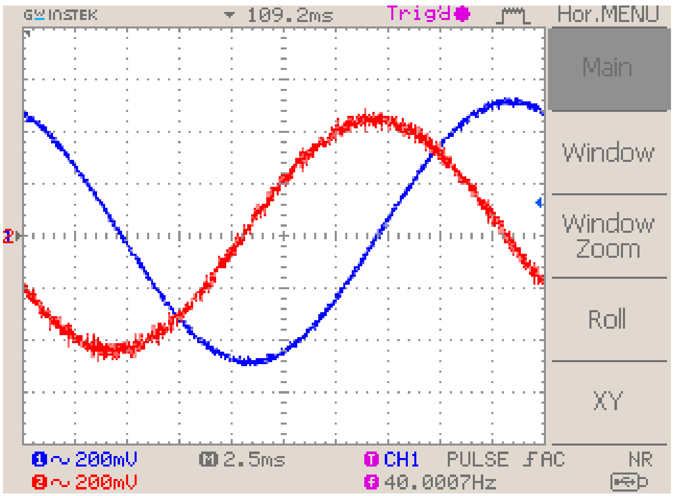

Figure 25: Integration of a sine wave with 40 Hz and 1 V_{PP}

4.5 Differential amplifier

At last we look back again at the differential amplifier and analyse the effect of applying a positiv base voltage at the the positiv input, wich is parallel connected to ground. As we can see in fig. **??**, **??** and **??**, that the increasing base line voltage increases the shift in its output.

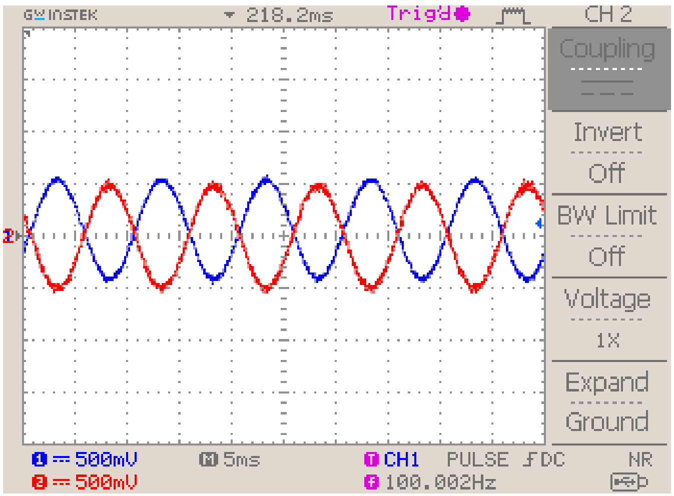

Figure 27: Amplification of a sine wave with no base voltage.

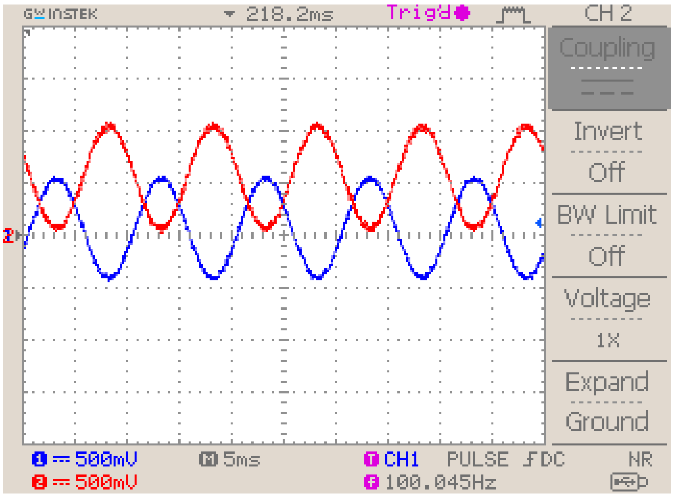

Figure 28: Amplification of a sine wave with a base voltage of 0.5 V.

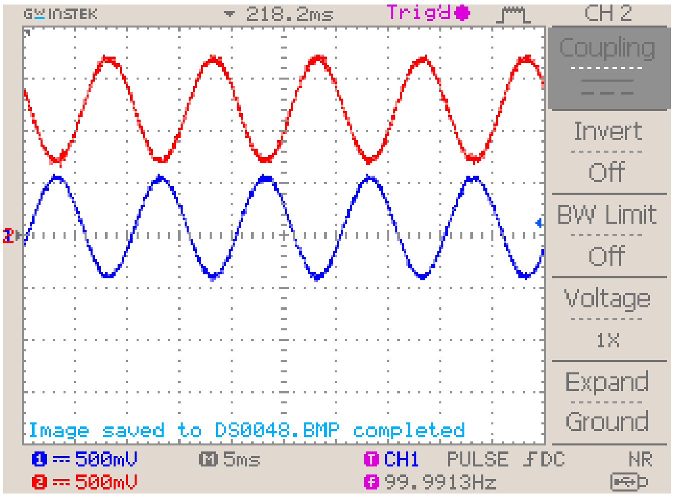

Figure 29: Amplification of a sine wave with a base voltage of 1.5 V.

At this point we were tasked to create a beat with two very slightly different frequencies. Since at this point we ran out of time, we were not able to do this. We are expecting to see the typical image of a beat, where two sine wave interfere with each other and produce a seemingly standing wave wich does not propergate. We would have used the normal frequency generator where we would set the variable frequency to the fixed frequency of another generator.

5 Conclusion

Afterwards we used an Adder to produce a sawtooth signal where our experimental optimum was slightly diffrent to the calculated one. We calculated for the 150 Hz sine wave the best amplitude to be $\frac{1}{3}U_0$. We found that $\frac{1}{6}U_0$ was better in approximating the sawtooth signal.

With the constant current source we showed that any resistor in the feedback loop had no influence in the current and the only way to modify it would be to change the voltage or resistance of the source.

Proceeding, we build an intergrator wich integrated a square wave signal and found an frequency dependncy. Removing a resistor that should be parallel to the condesator showed fragmentations in the output. Switching to sine waves we oberseved the integration to a cosine and could produce, through varying the frequency and input voltage, a nearly perfect circel of a Lissajour figure.

Lastly we build a differential amplifier, where we observed the shifting of the output through increasing the grounded positiv voltage. Because we had no time left, we could not create a beat, but the theory and outcome is fully understood.

Overall the lab course was very successfull.

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