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Math 174E HW3
Due April 21, 2023

7.13. Company X wishes to borrow U.S. dollars at a fixed rate of interest. Company Y wishes to borrow Japanese yen at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. The companies have been quoted the following interest rates, which have been adjusted for the impact of taxes:

	Yen	*Dollars*
Company X	5.0%	9.6%
Company Y	6.5%	10.0%

Design a swap that will net a bank, acting as intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank.

Observe: Company Y pays 1.5% more than Company X in Yen but only 0.4% more in Dollars.

Illustration:

Swap rate r equally attractive to the two companies.
Since the bank requires 50 basis point per annum.
So the total gain for both parties in the swap is $1.5\% - 0.4\% - 0.5\% = 0.6\%$
Best scenario for two companies:
net gain for company X: $\frac{0.6\%}{2} = 0.3\%$
net gain for company Y: $\frac{0.6\%}{2} = 0.3\%$
The effective borrowing rate after swap for company X: $9.6\% - 0.3\% = 9.3\%$ p.a.
effective borrowing rate after swap for company Y: $6.5\% - 0.3\% = 6.2\%$ p.a.

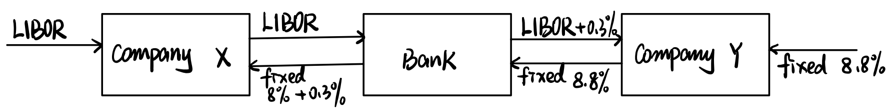
7.18. Companies X and Y have been offered the following rates per annum on a $5 million 10-year investment:

	Fixed rate 0.8% p.a.	*Floating rate* 0%
Company X	8.0%	LIBOR
Company Y	8.8%	LIBOR

Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and will appear equally attractive to X and Y.

Observe: Company Y gains 0.8% more than Company X in fixed rate investment, and 0% per annum in floating rate.

Illustration:



Swap rate equally attractive to X and Y.
Since swap brokered by a financial institution which wants to earn a spread of 0.2%.
So the total gain for both parties in the swap is $0.8\% - 0\% - 0.2\% = 0.6\%$
Best scenario for two companies:
net gain for company X: $\frac{0.6\%}{2} = 0.3\%$
net gain for company Y: $\frac{0.6\%}{2} = 0.3\%$

The fixed rate return for company X: $8\% + 0.3\% = 8.3\%$ p.a.
floating rate return for company Y: LIBOR + 0.3% p.a.

7.31. A financial institution has entered into a swap where it agreed to receive quarterly payments at a rate of 2% per annum and pay the SOFR three-month reference rate on a notional principal of $100 million. The swap now has a remaining life of 10 months. Assume the risk-free rates with continuous compounding (calculated from SOFR) for 1 month, 4 months, 7 months, and 10 months are 1.4%, 1.6%, 1.7%, and 1.8%, respectively. Assume also that the continuously compounded risk-free rate observed for the last two months is 1.1%. Estimate the value of the swap.

2% / 4 = 0.5%

notional principal : \$100 million = \$100,000,000

frequency: quarterly   fixed rate = 2%   10 months remaining.

SOFR 3-month reference rate =

Last 2 months = 1.1%

$\left.\begin{array}{l}\text{1 month} : 1.4\% \\ \text{4 months} : 1.6\% \\ \text{7 months} : 1.7\% \\ \text{10 months} : 1.8\%\end{array}\right\} \Rightarrow$

fixed rate

0.5% quarterly

The forward floating rate for payments at times 1m, 4m, 7m, 10m is calculated by quarterly compounding as follows: (risk free average monthly APR)

$$(1.1\% \times 2 + 1.4\%) \div 3 = 1.2\%$$
$$(1.6\% \times 4 - 1.4\%) \div 3 = 1.67\%$$
$$(1.7\% \times 7 - 1.6\% \times 4) \div 3 = 1.83\%$$
$$(1.8\% \times 10 - 1.7\% \times 7) \div 3 = 2.03\%$$

Estimate the value of the swap in \$100 millions :

$$B = \sum_{i=1}^{n} C_i \cdot e^{-y \cdot t_i}$$

$$= 100 \cdot (0.5\% - 1.2\% \times \tfrac{1}{4}) \cdot e^{-\frac{1.4\%}{4}} + 100 \cdot (0.5\% - 1.67\% \times \tfrac{1}{4}) \cdot e^{-\frac{1.6\%}{4}}$$
$$+ 100 (0.5\% - 1.83\% \times \tfrac{1}{4}) \cdot e^{-\frac{1.7\%}{4}} + 100 \cdot (0.5\% - 2.03\% \times \tfrac{1}{4}) \cdot e^{-\frac{1.8\%}{4}}$$

$$= 0.1993 + 0.0822 + 0.0423 - 0.0075$$

$$= 0.316 \text{ millions}$$

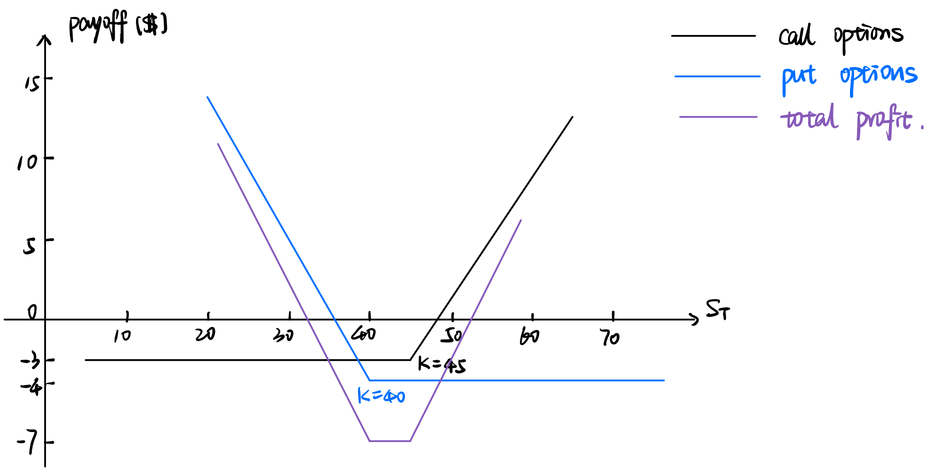
Therefore, the value of swap is 0.316 millions.

10.19. A trader buys a call option with a strike price of $45 and a put option with a strike price of $40. Both options have the same maturity. The call costs $3 and the put costs $4. Draw a diagram showing the variation of the trader's profit with the asset price.

a call option: strike price $K = \$45$, option price $= \$3$

a put option: strike price $K = \$40$. option price $= \$4$ same T.

Profit from buying a European call/put option on one share of a stock



If the stock price is above $45 on the expiration date, the call option will be exercise. And the payoff of $S_T - 45$, using " ——— " expressed.

If the stock price is below $40 on the expiration date, the put option will be exercise. And the payoff of $40 - S_T$, using " ——— " expressed.

Since the trader option cost is $\$3 + \$4 = \$7$, so the total profit is $S_T - 52$ for call option; $33 - S_T$ for put option. using " ——— " expressed.

10.29. Calculate the intrinsic value and time value from the mid market (average of bid and ask) prices for the September call options in **Table 1.2**. Do the same for the September put options in **Table 1.3**. Assume in each case that the current mid market stock price is $316.00.

Table 1.2 Prices of call options on Apple, May 21, 2020; stock price: bid $316.23, ask $316.50 (*Source*: CBOE).

Strike price	June 2020		September 2020		December 2020	
($)	Bid	Ask	Bid	Ask	Bid	Ask
290	29.80	30.85	39.35	40.40	46.20	47.60
300	21.55	22.40	32.50	33.90	40.00	41.15
310	14.35	15.30	26.35	27.25	34.25	35.65
320	8.65	9.00	20.45	21.70	28.65	29.75
330	4.20	5.00	15.85	16.25	23.90	24.75
340	1.90	2.12	11.35	12.00	19.50	20.30

Table 1.3 Prices of put options on Apple, May 21, 2020; stock price: bid $316.23, ask $316.50 (*Source*: CBOE).

Strike price	June 2020		September 2020		December 2020	
($)	Bid	Ask	Bid	Ask	Bid	Ask
290	3.00	3.30	12.70	13.65	20.05	21.30
300	4.80	5.20	15.85	16.85	23.60	24.90
310	7.15	7.85	19.75	20.50	28.00	28.95
320	11.25	12.05	24.05	24.80	32.45	33.35
330	17.10	17.85	28.75	29.85	37.45	38.40
340	24.40	25.45	34.45	35.65	42.95	44.05

The current mid market stock price is $S_T = \$316$.

By definition: Time value = option price − intrinsic value

(Prices of call options)

Strike price (K)	Sept. 2020 Bid	Ask	Mid market values of options	Instrinsic value $(S_T - K)^+$	Time value
290	39.35	40.40	39.875	26	13.875
300	32.50	33.90	33.20	16	17.20
310	26.35	27.25	26.80	6	20.80
320	20.45	21.70	21.075	0	21.075
330	15.85	16.25	16.05	0	16.05
340	11.35	12.00	11.675	0	11.675

(Prices of put options)

Strike price (K)	Sept. 2020 Bid	Ask	Mid market values of options	Instrinsic value $(K - S_T)^+$	Time value
290	12.70	13.65	13.175	0	13.175
300	15.85	16.85	16.35	0	16.35
310	19.75	20.50	20.125	0	20.125
320	24.05	24.80	24.425	4	20.425
330	28.75	29.85	29.30	14	15.30
340	34.45	35.65	35.05	24	11.05