Name: Jiayu Li

UID #: 605-348-766

MIDTERM EXAM

Math 174E – Spring 2023
Srinivas Paruchuri
Due: May 4, 2023 11:59pm

Instructions: All of the following resources are allowed:

- textbook
- lecture notes and recordings
- calcultors
- Standard Ultimate Life Table (attached)

Good Luck!

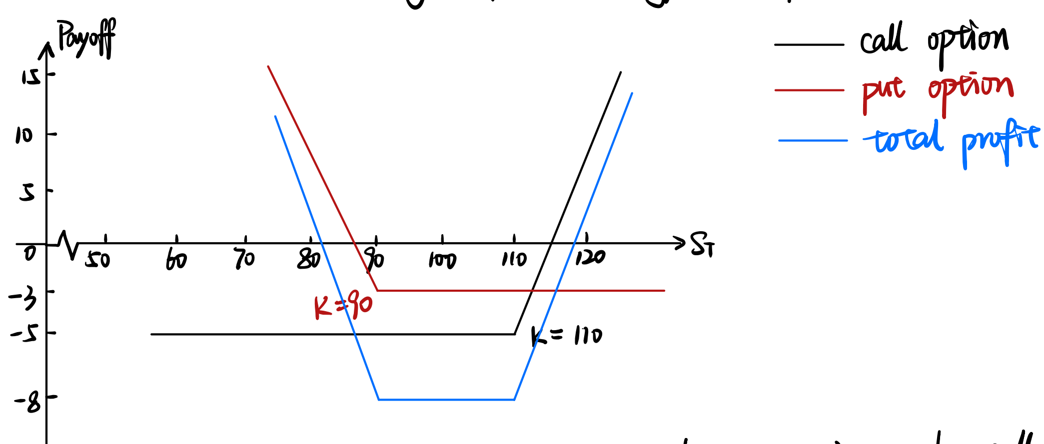
Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

1. (20 points) Suppose an investor is considering a multi option strategy on a stock with a current price of \$100. The following strategy is called a strangle. The investor purchases a call option with a strike price of K \$110 for a premium of \$5 and purchases a put option with a strike price of K \$90 for a premium of \$3.

   a) Draw a payout diagram for the strangle option strategy at expiration.
   b) Determine the breakeven points for the strangle option strategy.
   c) Suppose the stock price at expiration is \$120. What is the profit for the strangle option strategy?
   d) Suppose the stock price at expiration is \$85. What is the profit for the strangle option strategy?
   e) What is the investor speculating on with her option strategy?

a) call option : K = \$110 premium = \$5
put option : K = \$90 premium = \$3.

Profit for the strangle option strategy at expiration.



If the stock price is greater than \$110 on the expiration, the call option provides a payoff of $S_T - 110$ and the put option no payoff.
If the stock price is less than \$90 on the expiration, the put option provides a payoff of $90 - S_T$ and the call option no payoff.
If the stock price is between \$90 and \$110, neither option provides a payoff. There is a net loss of \$8.
Since the option premium cost is \$5 + \$3 = \$8, so the total profit is $S_T - 115$ for call option, $87 - S_T$ for put option.

b) by part a), we have lower breakeven = $90 - 3 - 5 = \$82$
upper breakeven = $110 + 3 + 5 = \$118$

Thus, it is profitable when stock price less than 82 or greater than 118.
The breakeven points are \$82 and \$118.

c) Suppose the stock price is \$120 at expiration, we will exercised call option.

the profit = $\$120 - \$110 - \$8 = \$2$

d) Suppose the stock price is \$85 at expiration, we will exercised put option.
the profit = $\$90 - \$85 - \$8 = -\$3$

e) The investor trade based on her educated guesses on where her believe the market is headed. When the market price is expected to fluctuate vibiration, the investor can use this strangle option strategy to hedge or reduce the risk exposure of her portfolios whether the stock price is higher than \$118 or lower than \$82.

2. (20 points) Consider a 5-year, semi-annual coupon bond with a price of \$110 and a coupon rate of 6%. The face value is \$100.

   1. What is the yield of the bond?
   2. How much does the price change if the yield is increased by 0.01%.

1. $t = 5$-year coupon rate $= 6\%$ bond price $= \$110$

Suppose that a 5-year bond with the face value of \$100 provides a coupon rate of 6% semiannually.

$$\text{coupon} = 100 \times \frac{0.06}{2} = \$3 \text{ per semiannual}$$

Let y is bond's yield, expressed with continuous compounding, it must be true that

$$3e^{-y\cdot 0.5} + 3e^{-y\cdot 1} + 3e^{-y\cdot 1.5} + 3e^{-y\cdot 2} + 3e^{-y\cdot 2.5} + 3e^{-y\cdot 3} + 3e^{-y\cdot 3.5} + 3e^{-y\cdot 4} + 3e^{-y\cdot 4.5} + 103e^{-y\cdot 5} = 110$$

$\Rightarrow$ By calculator, we can get $y = 0.037498 = 3.75\%$

Thus, the bond's yield is 3.75%

2. If the yield is increased by 0.01%

Yield $y = 3.76\%$

$$B = \sum_{i=1}^{n} c_i \cdot e^{-y\cdot t_i}$$

$$= 3e^{-0.0376\cdot 0.5} + 3e^{-0.0376\cdot 1} + 3e^{-0.0376\cdot 1.5} + 3e^{-0.0376\cdot 2} + 3e^{-0.0376\cdot 2.5} + 3e^{-0.0376\cdot 3} + 3e^{-0.0376\cdot 3.5} + 3e^{-0.0376\cdot 4} + 3e^{-0.0376\cdot 4.5} + 103e^{-0.0376\cdot 5}$$

$$= 109.95$$

Same, if we use the duration to calculate.

$$D = \frac{1}{110}\left[0.5\times 3e^{-0.0375\times 0.5} + 1\times 3e^{-0.0375\times 1} + 1.5\times 3e^{-0.0375\times 1.5} + 2\times 3e^{-0.0375\times 2} + 2.5\times 3e^{-0.0375\times 2.5} + 3\times 3e^{-0.0375\times 3} + 3.5\times 3e^{-0.0375\times 3.5} + 4\times 3e^{-0.0375\times 4} + 4.5\times 3e^{-0.0375\times 4.5} + 5\times 103e^{-0.0375\times 5}\right]$$

$$= 4.43$$

$$\Delta B = -BD\Delta y = -110\times 4.43\times 0.0001 = -0.05$$

$$B = 110 - 0.05 = 109.95$$

Thus, the yield increased by 0.01%, the bond's price decrease to \$109.95.

3. (20 points) Consider two European call options with strike K and the same underlying non-dividend paying stock. The stock price is currently at S_0. Option 1 has price C_1 maturity T_1 and Option 2 has price C_2 and maturity T_2, where $T_2 > T_1$. The risk-free interest rate is r. Use a no-arbitrage argument to prove that $C_2 > C_1$.

European call option	price	maturity
Option 1	C_1	T_1
Option 2	C_2	T_2

Proof: Since same underlying non-dividend paying stock, and the risk-free interest rate is r,

so bounds for Calls on non-dividend paying stock are given by

$$\max(S_0 - Ke^{-rT}, 0) \text{ and } S_0.$$

Under the no-arbitrage assumption properties hold true.

For all $t \in [0, T]$, if $T_2 > T_1$, then

case 1: $S_T \leq K$, the amount portfolio is 0

case 2: $S_T > K$, the amount portfolio is $S_T - K$

we have: $C_1(K, T_1) \geq \max(S_0 - Ke^{-rT_1}, 0) \Rightarrow K \geq \frac{S_0 - C_1}{e^{-rT_1}}$

$C_2(K, T_2) \geq \max(S_0 - Ke^{-rT_2}, 0) \Rightarrow K \geq \frac{S_0 - C_2}{e^{-rT_2}}$

Since $T_2 > T_1$, same strike K so that

$$\frac{S_0 - C_1}{e^{-rT_1}} = \frac{S_0 - C_2}{e^{-rT_2}}$$

$\Rightarrow (S_0 - C_1)e^{-rT_2} = (S_0 - C_2)e^{-rT_1}$

Since $\frac{1}{e^{rT_2}} < \frac{1}{e^{rT_1}}$, for $r > 0$, $T_2 > T_1$

Thus, it must be $S_0 - C_1 > S_0 - C_2$

$\Rightarrow C_2 > C_1$

$\blacksquare$

S_0 T

4. (20 points) Consider a non-dividend-paying stock with a current price of \$50. A 6-month forward contract on the stock is available for purchase. The risk-free interest rate is 4% per annum, compounded continuously.

   r

   An investor believes that in 6 months, the stock price will either increase by 20% or decrease by 10%. The investor also believes that there is a 60% probability of the stock price increasing and a 40% probability of the stock price decreasing.

   The investor is considering two strategies:

   1. Strategy A: Buy the stock today and hold it for 6 months.
   2. Strategy B: Buy the 6-month forward contract on the stock.

   Part 1: Calculate the forward price of the stock.

   Part 2: Calculate the expected stock price in 6 months and the expected payoff for each strategy.

   Part 3: Calculate the minimum amount the investor would be willing to pay for an option that would allow them to buy the stock in 6 months at the forward price.

Part 1: $S_0 = \$50$ $\quad r = 4\%$ $\quad T = 6$ months $= 0.5$ yrs

Forward price:

$$F_0 = S_0 e^{rT} = 50 \cdot e^{0.04 \times 0.5} = 51.01$$

The forward price of the stock is \$51.01.

Part 2: Expected stock price:

$$50 \cdot (1 + 20\%) \cdot 60\% + 50 \cdot (1 - 10\%) \cdot 40\%$$
$$= 36 + 18$$
$$= \$54$$

Thus, expected payoff for strategy A: $54 - 50 = \$4$

expected payoff for strategy B: $54 - 51.01 = \$2.99$

Therefore, the expected stock price is \$54 in 6 months.

The expected payoff of strategy A is \$4, strategy B is \$2.99.

Part 3: By part 1 and 2, we have

$S_0 = \$54$ $K = \$51.01$ $r = 4\%$ $T = 0.5\,yr$

When there are no dividends, it can be shown

lower bound: $S_0 - Ke^{-rT} = 54 - 51.01 \cdot e^{-0.04 \times 0.5} = \$4.00$

Thus, the minimum amount the investor would be willing to pay for this option is $4.

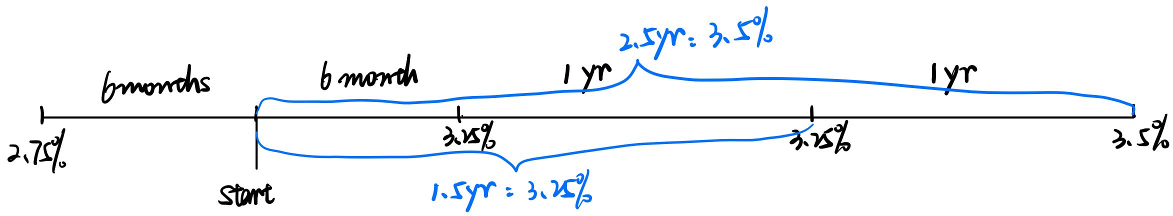
5. (20 points) Suppose you have a 2.5-year remaining on an interest rate swap with a notional principal of $\$10,000,000$ between Company A and Company B. Company A pays fixed rate and Company B pays the float rate. Fixed and float payments are exchanged every year and the last payment was exchanged 6 months ago. The fixed rate is 3.5% per annum, and the floating rate is tied to the annual LIBOR. The previous 1-year LIBOR rate, set 6 months ago, is 2.75%, 6 month LIBOR is 3.25%. the 1.5-year LIBOR is 3.25%, and the 2.5-year LIBOR is 3.50%.

Calculate the present value of the fixed and floating legs of the swap, and determine the swap's net present value from Company A's perspective. Assume annual compounding for discounting.

notional principal = \$10,000,000

frequency = annually   2.5yrs remaining.   fixed rate = 3.5% p.a.

There will be 3 transactions, in 0.5, 1.5, 2.5 years.



The floating rate for the first transaction is found using the 2.75% rate for the past 6 months. and the 3.25% forward rate for the upcoming 6-month. The continuously compounded rate for the annual is

$$2.75\% \times \frac{6}{12} + 3.25\% \times \frac{6}{12} = 3\% .$$

Then we have to exercised swap for the second transaction. The floating rate is 3.25% for 1 yr,

$$3.25\% \times 1.5 - 3.25\% \times 0.5 = 3.25\% .$$

Then the third transaction. The floating rate is 3.5%.

$$3.5\% \times 2.5 - 3.25\% \times 1.5 = 3.875\% .$$

Then let's compute the profit/loss from company A's perspective

discount	Floating legs (millions)	Fixed legs (millions)
$e^{3\% \times \frac{1}{2}} - 1 = 0.0151$	$0.0151 \times 10 = 0.151$	$\frac{1}{2} \times 3.5\% \times 10 = -0.175$
$e^{3.25\%} - 1 = 0.033$	$0.33 \times 10 = -0.33$ (swap)	$3.5\% \times 10 = 0.35$
$e^{3.875\%} - 1 = 0.0395$	$0.0395 \times 10 = 0.395$	$3.5\% \times 10 = -0.35$

The present value of floating legs of swap:

$$0.151 + (-0.33) + 0.395 = 0.216 \text{ millions} = \$216000$$

The present value of fixed legs of swap:

$$-0.175 + 0.35 + (-0.35) = -0.175 \text{ millions} = -\$175000$$

Thus, the swap's net present value:

$$B = \sum_{i=1}^{n} c_i \cdot e^{-y \cdot t_i}$$

$$= (0.151 - 0.175) e^{-0.03 \times \frac{6}{12}} + (0.35 - 0.33) \cdot e^{-0.0325 \times 1.5} + (0.395 - 0.35) e^{-0.03875 \times 2.5}$$

$$= 0.03625082 \text{ millions}$$

$$= \$36250.82$$