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Math 174E HW6
Due May 19, 2023

19.12. Calculate the delta of an at-the-money six-month European call option on a non-dividend-paying stock when the risk-free interest rate is 10% per annum and the stock price volatility is 25% per annum.

$S_0 = K$, $r = 10\%$, $\sigma = 25\%$, $T = 6 \text{ months} = 0.5 \text{ yrs}$.

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln(1) + (0.1 + 0.25^2/2) \cdot 0.5}{0.25\sqrt{0.5}}$$

$$= 0.3712$$

$$\Delta = N(d_1) = N(0.3712) = 0.6443$$

19.18. A company uses delta hedging to hedge a portfolio of long positions in put and call options on a currency. Which would give the most favorable result: (a) a virtually constant spot rate or (b) wild movements in the spot rate? Explain your answer.

A long call/put position has positive gamma. When gamma is positive, the hedger makes money when the stock price changes greatly, and the hedger loses when the stock price changes small.

For (a), a virtually constant spot rate, which can be regarded as a dividend-paying stock, the Gamma is large. So the Delta is very sensitive to the spot rate. A small change in the spot rate will cause a large change in Delta, and thus quiet risky. In this case, in order to maintain Delta neutral, the portfolio needs to be adjusted frequently, and the cost is high.

For (b), Contrary to (a) for a spot rate that wild movements. The Gamma is small, which means the Delta changes very slowly, and there is no need to frequently adjust the portfolio.

Therefore, (b) is the most favorable result.

19.29. Use the put–call parity relationship to derive, for a non-dividend-paying stock, the relationship between:

a. The delta of a European call and the delta of a European put
b. The gamma of a European call and the gamma of a European put
c. The vega of a European call and the vega of a European put
d. The theta of a European call and the theta of a European put.

Proof: For a non-dividend paying stock, the formula for call-put parity gives: (time t)

$$p + S = c + Ke^{-r(T-t)}$$

a. $\Delta = \frac{\partial c}{\partial S}$ where c is the price of the call option, S is the stock price.

Thus, the delta of a European call:

$$\frac{\partial P}{\partial S} + 1 = \frac{\partial c}{\partial S} \Rightarrow \frac{\partial P}{\partial S} = \frac{\partial c}{\partial S} - 1$$

Since $\Delta(\text{call}) = N(d_1)$

$\Delta(\text{put}) = N(d_1) - 1$

Thus, the delta of European put = the delta of European call − 1.

b. $\Gamma = \frac{\partial^2 C}{\partial S^2}$

Thus, $\frac{\partial^2 P}{\partial S^2} = \frac{\partial^2 C}{\partial S^2}$

By the put-call parity, the gamma for European call and put options with the same strike are equal.

c. $V = \frac{\partial c}{\partial \sigma}$

Thus, $\frac{\partial P}{\partial \sigma} = \frac{\partial c}{\partial \sigma}$

$V(\text{call}) = V(\text{put})$

By the put-call parity, the vega of a call option equals the vega of a put option with the same strike.

d. $\theta = -\frac{\partial c}{\partial T}$

Thus, $\frac{\partial P}{\partial t} = rKe^{-r(T-t)} + \frac{\partial c}{\partial t}$

Since for a European call option on a non-dividend-paying stock,

$$\theta(\text{call}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

for a European put option on the stock,

$$\theta(\text{put}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

Thus, $N(-d_2) = 1 - N(d_2)$, the theta of a put exceeds the theta of the corresponding call by rKe^{-rT}.

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19.30. A financial institution has the following portfolio of over-the-counter options on sterling:

Type	Position	Delta of option	Gamma of option	Vega of option
Call	−1,000	0.50	2.2	1.8
Call	−500	0.80	0.6	0.2
Put	−2,000	−0.40	1.3	0.7
Call	−500	0.70	1.8	1.4

A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.

a. What position in the traded option and in sterling would make the portfolio both gamma neutral and delta neutral?

b. What position in the traded option and in sterling would make the portfolio both vega neutral and delta neutral? Assume that all implied volatilities change by the same amount so that vegas can be aggregated.

a. The gamma of portfolio:

$-1000 \times 2.2 + (-500) \times 0.6 + (-2000) \times 1.3 + (-500) \times 1.8$

$= -2200 - 300 - 2600 - 900$

$= -6000$

The delta of portfolio:

$-1000 \times 0.5 + (-500) \times 0.8 + (-2000) \times (-0.4) + (-500) \times 0.7$

$= -500 - 400 + 800 - 350$

$= -450$

For a gamma neutral portfolio, so the gamma for long position: +6000

$6000 \div 1.5 = 4000$

So holding a long position of 4000 traded options makes the portfolio gamma neutral.

Then, the delta of the portfolio:

$4000 \times 0.6 - 450 = 1950$

Thus, in order to make the portfolio both Gamma neutral and Delta neutral, a short position of 1950 in sterling is required.

b. Similarly,

The vega of portfolio:

$-1000 \times 1.8 + (-500) \times 0.2 + (-2000) \times 0.7 + (-500) \times 1.4$

$= -1800 - 100 - 1400 - 700$

$= -4000$

For a vega neutral portfolio, so the long position of vega: +4000

$4000 \div 0.8 = 5000$

So holding a long position of 5000 traded options makes the portfolio vega neutral.

Then, the delta of the portfolio:

$5000 \times 0.6 - 450 = 2550$

Thus, in order to make the portfolio both Vega neutral and Delta neutral, a short position of 2550 in sterling is required.