Jiayu Li
Math 174E HW1
Due April 7, 2023

1.19. Suppose that a March call option to buy a share for $50 costs $2.50 and is held until March. a. Under what circumstances will the holder of the option make a profit? b. Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity of the option.

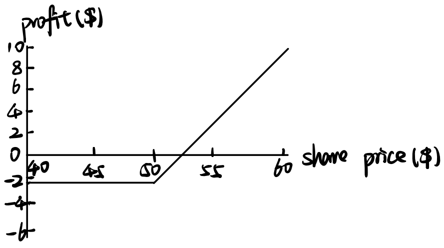
a. The buyer of a call option seeks to make profit if and when the price of the underlying asset increases to a price higher than the option strike price.
For this circumstances, if the March share price is more than \$50 + \$2.5 = \$52.5, the option holder will profit.

b. Call options are usually only exercised if the price of the underlying is trading above the strike price.
For example, in this circumstances, if holder own a call option with a strike price of \$50, and the stock closes at \$50.01 (above \$50) in March on the expiration date of the call option, the option will be exercised.

The diagram of profit as a function of the stock price will be:

write call option

$P = 2.5$
$K = 52.5$



1.29. In the 1980s, Bankers Trust developed *index currency option notes* (ICONs). These were bonds in which the amount received by the holder at maturity varied with a foreign exchange rate. One example was its trade with the Long Term Credit Bank of Japan. The ICON specified that if the yen/USD exchange rate, $S_T$, is greater than 169 yen per dollar at maturity (in 1995), the holder of the bond receives $1,000. If it is less than 169 yen per dollar, the amount received by the holder of the bond is

$$1{,}000 - \max\left[0, 1{,}000\left(\frac{169}{S_T} - 1\right)\right]$$

When the exchange rate is below 84.5, nothing is received by the holder at maturity. Show that this ICON is a combination of a regular bond and two options.

$$1000 - \max\left[0,\ 1000\left(\frac{169}{S_T} - 1\right)\right] = \begin{cases} 1000 & S_T > 169 & \text{short call} \\ 1000\left(2 - \frac{169}{S_T}\right) & 169 > S_T > 84.5 & \text{regular bond} \\ 0 & S_T < 84.5 & \text{long call} \end{cases}$$

payoff at $84.5 \le S_T \le 169$

$$= 2000 - \frac{169000}{S_T}$$

| | bond | short call | long call | total |
|---|---|---|---|---|
| $S_T > 169$ | 1000 | 0 | 0 | 1000 |
| $84.5 \le S_T \le 169$ | 1000 | $-169000\left(\frac{1}{S_T} - \frac{1}{169}\right)$ | 0 | $2000 - \frac{16900}{S_T}$ |
| $S_T < 84.5$ | 1000 | $-169000\left(\frac{1}{S_T} - \frac{1}{169}\right)$ | $169000\left(\frac{1}{S_T} - \frac{1}{169}\right)$ | 0 |

A short call for 169000 yen at $\frac{1}{169}$,

long call for 169000 yen at $\frac{1}{84.5}$.

1.37. Trader A enters into a forward contract to buy an asset for $1,000 in one year. Trader B buys a call option to buy the asset for $1,000 in one year. The cost of the option is $100. What is the difference between the positions of the traders? Show the profit as a function of the price of the asset in one year for the two traders.

Forward contract A must buy, so its position is $1000.

Call option B pays $100 premium to get the right to buy or not to buy.

So its position is $1000 + $100 = $1100.

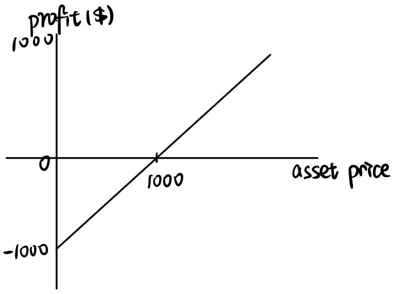
Let $y$ is profit, $x$ is price of the asset in one year.

=> so the profit as functions:

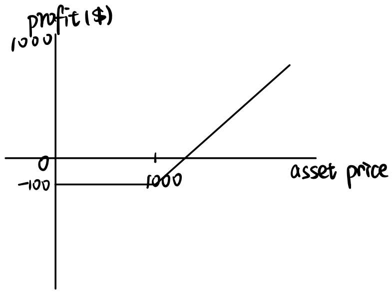
$$A: \quad y = x - 1000$$

$$B: \quad y = \begin{cases} x - 1100 & x > 1000 \quad \text{buy} \\ -100 & x < 1000 \quad \text{do not buy} \end{cases}$$

we also can write: $B: \; y = \max\{x - 1000, 0\} - 100$




Trader A




Trader B

2.32. Trader A enters into futures contracts to buy 1 million euros for 1.1 million dollars in three months. Trader B enters in a forward contract to do the same thing. The exchange rate (dollars per euro) declines sharply ↘ during the first two months and then increases ↗ for the third month to close at 1.1300. Ignoring daily settlement, what is the total profit of each trader? When the impact of daily settlement is taken into account, which trader has done better?

A: Buy 1m EUR / 1.1m USD futures

B: 1m EUR / 1.1m USD forward

Let's ignore the daily settlement, the total profit of A and B is the same.

$$(1.13 - 1.10) \times \$1,000,000 = \$30,000$$

the total profit of each trader is $30,000.

A: profit day-by-day during 3-months

B: profit realized at the end of the 3 months.

Due to the sharp decline in the first two months, trader A may face insufficient margin and need to make up the margin. If the margin is not paid in time, the position may be closed. And the forward contract does not need daily settlement. thus, trader B has done better.

2.37. A bank's derivatives transactions with a counterparty are worth +$10 million to the bank and are cleared bilaterally. The counterparty has posted $10 million of cash collateral. What credit exposure does the bank have?

Asymmetrics of expected gains and losses may occur.

The bank faces the risk that the counterparty defaults and does not return the cash provided by the bank as collateral.

3.14. A company has a $20 million portfolio with a beta of 1.2. It would like to use futures contracts on a stock index to hedge its risk. The index futures price is currently standing at 1080, and each contract is for delivery of $250 times the index. What is the hedge that minimizes risk? What should the company do if it wants to reduce the beta of the portfolio to 0.6?

Based equation (3.5) from textbook,

$$N^* = \beta \frac{V_A}{V_F}$$

where $N^*$: the number of futures contracts that should be shorted to hedge the portfolio

$V_A$: Current value of the portfolio

$V_F$: Current value of one futures contract

$\beta = 1.2$ $V_A = \$ 20$ million $V_F = 1080 \times \$250$

$$\Rightarrow N^* = 1.2 \times \frac{\$20,000,000}{1080 \times \$250} = 88.9 \approx 89 \text{ contracts}$$

Thus, the hedge that minimizes risk is to buy 89 short contracts.

If the company wants to reduce the $\beta = 0.6$,

$$\frac{88.9}{2} = 44.45 = 44 \text{ contracts}$$

it should short position of 44 contracts at least.

3.25. On July 1, an investor holds 50,000 shares of a certain stock. The market price is $30 per share. The investor is interested in hedging against movements in the market over the next month and decides to use an index futures contract. The index futures price is currently 1,500 and one contract is for delivery of $50 times the index. The beta of the stock is 1.3. What strategy should the investor follow? Under what circumstances will it be profitable?

holder: 50000 shares $30 per stock

$$50,000 \times \$30 = \$1,500,000$$

total value of a contract $= 1500 \times \$50 = \$75000$

So we calculated: $\frac{1,500,000}{75000} = 20$ hedging contracts

Then suppose the market volatility rate is $x$,

$$1,500,000 \cdot 1.3x - 20 \cdot (1500 \cdot 50 \cdot x) > 0$$

$$450000\, x > 0$$ ↳ profit balance point

$$\Rightarrow x > 0$$

Thus, we got $x > 0$, the stock index increase $x\%$. The return on the stock is higher than predicted market pricing return.

So the investor should $1.3 \times \frac{1,500,000}{75000} = 26$ future contracts that should be shorted to hedge, then it will be profitable.

3.34. A portfolio manager has maintained an actively managed portfolio with a beta of 0.2. During the last year, the risk-free rate was 5% and equities performed very badly providing a return of −30%. The portfolio manager produced a return of −10% and claims that in the circumstances it was a good performance. Discuss this claim.

$\beta = 0.2$ risk-free rate $= 5\%$ expected return on the market: $-30\%$

$$5\% + 0.2(-30\% - 5\%) = -2\% > -10\%$$

$-10\%$ actual return is worse than expected return by $-8\%$.

So portfolio manager produced a return of $-10\%$ is not a good performance.