

Quiz 1

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Please remember that your work is graded on the quality of your writing and explanation as well as the validity of the calculations.

- (1) (6 points) Suppose we are choosing one random integer between 1 and 720, inclusive. Assume that every integer between 1 and 720 is equally likely to be chosen.

a) Give the sample space. (2p)

$$S = \{1, 2, 3, 4, \dots, 719, 720\}.$$

b) Find the probability that the chosen number is even. (1p)

$$P(\text{chosen number is even}) = \frac{360}{720} = \frac{1}{2}$$

c) Find the probability that the chosen number is a multiple of 3. (1p)

$$P(\text{multiple of 3}) = \frac{240}{720} = \frac{1}{3}$$

d) Find the probability that the chosen number is even or a multiple of 3. (2p)

$$\text{Let } A = \{\text{the number is even}\} = \{2, 4, 6, 8, \dots, 720\}$$

$$B = \{\text{the number is a multiple of 3}\} = \{3, 6, 9, 12, \dots, 720\}.$$

$$A \cap B = \{6, 12, 18, \dots, 720\}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$$

$$= \frac{2}{3}$$

- (2) (6 points) Randomly choose a permutation of the letters MISSISSIPPI such that every permutation is equally likely. (You may leave your answers in factorial form.)

a) How many such permutations are there? (3p)

Number of times the M occurs : 1
Number of times the I occurs : 4
Number of times the S occurs : 4
Number of times the P occurs : 2

$$\frac{11!}{1! 2! 4! 4!} = 34650$$

b) What is the probability that a permutation begins with M? (3p)

M _ I _ S _ S _ I _ S _ S _ I _ P _ P _ I

$$P(\text{permutation begins with M}) = \frac{1}{11}$$

(3) (8 points) A fair coin is flipped three times.

a) Give the sample space. (2p)

The sample space is

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

8 outcomes.

b) Given that one of the flips lands heads, find the probability that all three flips land heads. (2p)

$$S = \{HH, HT, TH, TT\}$$

$$\text{one of the flips lands heads and all three flips land heads} = P(\#H=3) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

c) Given that two of the flips land heads, find the probability that all three flips land heads. (2p)

$$\text{two of the flips land heads and all three flips land heads} = P(\#H=3) = \frac{1}{2}.$$

d) Given that the first two flips land heads, find the probability that all three flips land heads. (2p)

$$\text{the first two flips land heads and all three flips land heads} = P(\#H=3) = \frac{1}{2}$$