Math 170E - Summer 2022 - 07/15/2022
Quiz 3
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Please remember that your work is graded on the quality of your writing and explanation as well as the validity of the calculations.

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(1) (6 points) You buy a lottery ticket in 50 local lotteries, in each of which your chance of winning a prize is $\frac{1}{100}$. Using an appropriate approximate Poisson distribution to Binomial, calculate the probability that you will win the prize

(a) (2 points) at least once?

Poisson distribution's pmf form:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad \lambda > 0 \qquad n \cdot p = 50 \cdot \frac{1}{100} = 0.5$$

$$P(x=0) = e^{-\frac{1}{2}} = 60.65\% \qquad P(x=1) = \frac{e^{-\frac{1}{2}}(\frac{1}{2})^1}{1} = \frac{1}{2}e^{-\frac{1}{2}} = 30.325\%$$

$$P(x=2) = \frac{e^{-\frac{1}{2}} \cdot \frac{1}{4}}{2!} = \frac{1}{8}e^{-\frac{1}{2}} = 7.58125\%$$

$$P(\text{at least one}) = P(x \geq 1) = 1 - P(x=0) = 1 - \frac{e^{-\frac{1}{2}}(\frac{1}{2})^0}{0!} = 1 - 60.65\% = \boxed{39.35\%}$$

(b) (2 points) exactly once?

$$P(\text{exactly one}) = P(x=1)$$
$$= \frac{e^{-\frac{1}{2}}(\frac{1}{2})^1}{1!} = \frac{1}{2}e^{-\frac{1}{2}} = \boxed{30.325\%}$$

(c) (2 points) at least twice?

$$P(\text{at least twice}) = P(x \geq 2) = 1 - P(x=0) - P(x=1$$
$$= 1 - e^{-\frac{1}{2}} - \frac{1}{2}e^{-\frac{1}{2}}$$
$$= 1 - \frac{3}{2}e^{-\frac{1}{2}}$$
$$= \boxed{9.025\%}$$

(2) (6 points) The probability density function of the random variable X is given by

$$f(x) = \begin{cases} a + bx^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If $E[X] = \frac{3}{5}$, what are the values of a and b?

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx = \frac{3}{5}$$

$$\Rightarrow \int_0^1 (a+bx^2)x\, dx = \frac{3}{5}$$

$$\Rightarrow \frac{a}{2} + \frac{b}{4} = \frac{3}{5} \Rightarrow a + \frac{b}{2} = \frac{6}{5} \quad ①$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^1 a+bx^2\, dx \Rightarrow a + \frac{b}{3} = 1 \quad ②$$

compute ① and ②: $b = \frac{6}{5}$ $a = \frac{3}{5}$

(3) (8 points) Ron Graham is a prolific author of mathematical papers. His friend, Don Knuth, reads all of Ron's papers and realizes that, on average, there are 4 typos for every 100 page of writing.

(a) (4 pts) Ron writes a new paper, that is 20 pages long. Don, before reading the actual paper, would like to anticipate the probability that the first half of the paper has no typos, using an exponential random variable. Which exponential r.v. would Don use? What is the probability Don calculates?

mean typos for every 100 page $= \frac{4}{100}$

mean typos for first half the page $= \frac{4}{100} \times 0.5 \times 20 = 0.4$

$P(T > 10) = \int_{10}^{\infty} 0.04 e^{-0.04t} dt = e^{-0.4} = 67.03\%$

(b) (4 pts) What is the expected page number for the second typo?

$$P(T>0) = \int_1^\infty 0.04 e^{-0.04t}\,dt = e^{-0.04}$$

$$P = 1 - e^{-0.04}$$

$$\frac{2}{P} = \frac{2}{1-e^{-0.04}} = 51$$

the expected page number is 51.