Math 170E - Summer 2022 - 07/01/2022
Quiz 2
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Please remember that your work is graded on the quality of your writing and explanation as well as the validity of the calculations.

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(1) (6 points) There are 6 bakeries on campus. Each bakery is open with probability 30% on Independence Day, mutually independent of other bakeries. 4 bakeries are located in the east campus, and 2 bakeries are located in the west campus. Suppose a person twice more likely to go to east campus than west campus on Independence Day to purchase bread, without any information. If there was *exactly* 1 bakery open on the side of campus the individual went to, what is the probability that this person went to the west campus?

West campus No.1 bakery No.2 bakery

case 1: P(open) = 30% P(close) = 70%

case 2: P(close) = 70% P(open) = 30%

P(case 1) = 30% × 70% = 21%

P(case 2) = 70% × 30% = 21%

P(go to west campus) = $\frac{2}{6} = \frac{1}{3}$

P(exactly 1 bakery open on the west campus the individual went to)

$= \frac{1}{3} \times (21\% + 21\%)$

$= 14\%$.

(2) (4 points) Suppose you're playing a game of prize wheel with nine arcs of equal size. In six of these arcs, you win 5 dollars. In one of them, you win 100 dollars. In the remaining two, you have to spin the wheel again. You must play till you win something. What is the expected value of your winnings in this game?

the expected value of X to be

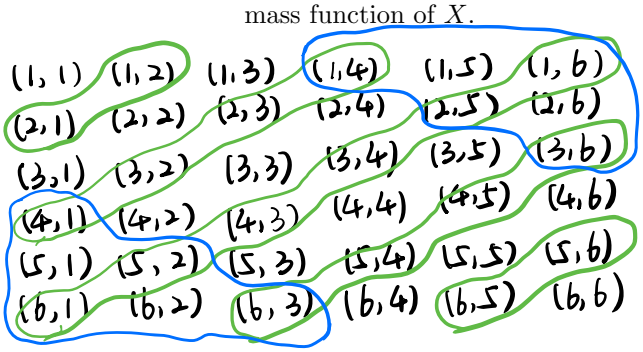
$$\mathbb{E}[X] = \sum_{x \in S} x \cdot f(x)$$

$$f(\$5) = \frac{6}{9}, \quad f(\$100) = \frac{1}{9}, \quad f(\text{zero}) = \frac{2}{9}$$

$$\mathbb{E}[X] = 5 \times \frac{6}{9} + 100 \times \frac{1}{9} + 0 \times \frac{2}{9} = \frac{130}{9}$$

(3) (10 points) Suppose you roll two 6-sided fair dice, d_1 and d_2. Define event A as their absolute difference, $|d_1 - d_2|$, is odd. And define event B as their absolute difference is ≥ 3.

(a) (4 pts) Let $X = |d_1 - d_2|$ be the absolute difference of the rolls of two dice. Find the probability mass function of X.



$$f(x) = \begin{cases} P(X=0) = \frac{6}{36} = \frac{1}{6} \\ P(X=1) = \frac{10}{36} = \frac{5}{18} \\ P(X=2) = \frac{8}{36} = \frac{1}{4} \\ P(X=3) = \frac{6}{36} = \frac{1}{6} \\ P(X=4) = \frac{4}{36} = \frac{1}{9} \\ P(X=5) = \frac{2}{36} = \frac{1}{18} \end{cases}$$

(b) (2 pts) Calculate the probabilities of event A and event B.

$$P(\text{event A}) = \frac{18}{36} = \frac{1}{2}$$

$$P(\text{event B}) = \frac{12}{36} = \frac{1}{3}$$

(c) (4 pts) Are event A and event B independent? Why?

Event A and event B are not independent.

Because $P(A \cap B) = \frac{8}{36} \neq 0$

so they are not independent.