

Math 170E
Summer 2022
Midterm

07/08/2022

Time Limit: 24 Hours

Name: _____

UID: _____

This exam contains 10 pages (including this cover page) and 5 questions. Total of points is 100. Make sure to write your answers in full detail, so that you may get the maximum possible partial points when applicable.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

- *You may use all the definitions and propositions stated during lectures and the textbook, unless it is otherwise stated and/or you are trying to prove the proposition itself.*
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- Please type the following statement in your handwriting, then sign and date below:

“I hereby acknowledge that I am aware I may use my textbook, lecture notes and recordings during the exam and swear on my honor as a Bruin *all* the answers I present belong solely to me, in thought and in writing.”

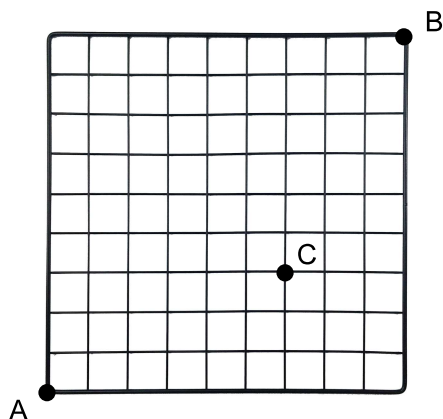
1. (20 points) (a) (10 points) Consider a 10-card poker hand. A special type of hand that has three denominations repeated three times and the last denomination repeated once is called a *chill house*. For example

King of Diamonds, King of Hearts, King of Spades, 5 of Clubs, 5 of Hearts, 5 of Spades, 2 of Clubs, 2 of Diamonds, 2 of Spades, Jack of Hearts

is a chill house.

What is the probability that in a randomly dealt hand, where all $\binom{52}{10}$ hands are equally likely, we get a chill house? (You can leave your answer in a form with binomial coefficients.)

- (b) (10 points) Consider the following 9×9 grid of cities. Assume that a travelling salesman named Tod Packer is to go from city B (top right corner) to city A (bottom left corner), while taking only either “left” (L) steps or “down” (D) steps, a total of 18 steps.



Assuming Tod chooses such a path at random (each possible path equally likely), what is the probability that he will go through city C ?

(HINT: First, find the total number of paths. How many of each of the “L” and “D” steps does this person need to go from B to A ? After that, restrict the paths to go through C . You can leave your answer in a form with binomial coefficients.)

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2. (20 points) In the town of Goslar, police records show that 20% of all the crimes are violent and 80% of all the crimes are nonviolent. 90% of violent crimes are reported, whereas only 70% of nonviolent crimes are reported.
- (a) (10 points) What is the probability that a crime goes unreported?

- (b) (10 points) Assume that a crime is reported. What is the probability that the crime is violent?

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3. (20 points) We roll two fair and independent dice, d_1 and d_2 . Let $X = \max(d_1, d_2)$, the maximum of these two dice.
- (a) (10 points) Let F be the cumulative distribution function of X . Write of F completely, as a piece-wise function, so that $F(x)$ is accounted for every $x \in \mathbb{R}$.

- (b) (10 points) Let $Y = \min(d_1, d_2)$. Are $E_a = \{X = a\}$ and $F_b = \{Y = b\}$ pairwise independent events? Either (i) find an example, a selection of a and b where these events are not independent, or (ii) show that no matter what a and b you may choose the events are independent.

4. (20 points) We are given the following probability distribution for x , the number of coffee breaks taken per day by coffee drinkers.

x	0	1	2	3	4	5
$f(x)$	0.27	0.38	0.16	0.12	0.05	0.02

- (a) (10 points) Calculate the mean, μ , and variance, σ^2 , for the number of coffee breaks per day.

- (b) (10 points) What's the probability that the number of coffee breaks falls within two standard deviations away from the mean, i.e., $(\mu - 2\sigma, \mu + 2\sigma)$?

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5. (20 points) Let X be a Negative Binomial random variable with $\mathbb{E}[X] = 50$ and $\text{Var}(X) = 200$. Calculate $F(13) - F(11)$, where F is the CDF of X . (You can leave your answer in a form with binomial coefficients.)