Math 170E - Summer 2022 - 07/22/2022
Quiz 4
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Please remember that your work is graded on the quality of your writing and explanation as well as the validity of the calculations.

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(1) (10 points) Normal distribution.

(a) (6 points) Assume the lifetime X (in hours) of a certain brand of light bulb follows normal distribution with mean $\mu = 160$ and variance $\sigma^2 = 20^2$. Four bulbs are chosen randomly and independently. Compute the probability that none of them has a lifetime lower than 180 hours. Use $\Phi(1) = 0.8413$.

$X \sim N(160, 20^2)$

$P(180 \le X) = P\left(\frac{180-160}{20} \le \frac{X-160}{20}\right)$

$= 1 - \Phi(1)$

$= 1 - 0.8413$

$= 0.1587$

(b) (4 points) Let $X \sim N(0, \sigma^2)$. Compute the fourth moment, $\mathrm{E}(X^4)$. (Hint: use the moment generating function)

$X \sim N(0, \sigma^2)$

$M(t) = e^{\mu t + \frac{\sigma^2}{2} t^2}$

$M'(t) = e^{\mu t + \frac{\sigma^2}{2} t^2} (\mu + \sigma^2 t)\Big|_{t=0} = \mu$

$\vdots$

$M^{(4)}(t) = \mu^4 + 6\sigma^2\mu^2 + 3\sigma^4$

$E(X^4) = M^{(4)}(t) = \mu^4 + 6\sigma^2\mu^2 + 3\sigma^4$

$= 3\sigma^4 = 3 \cdot (20^2)^2$

$= 480000$

(2) (10 points) X and Y are two discrete random variables following the joint distribution below.

Y \ X	1	2	3
-1	0.2	0.1	0
0	0.1	0	0.3
1	0.1	0.1	0.1

(a) (3 points) Compute the marginal pmf for X and Y, respectively.

$$P_X(X)=\begin{cases}0.4 & X=1\\ 0.2 & X=2\\ 0.4 & X=3\end{cases}$$

$$P_Y(Y)=\begin{cases}0.3 & Y=-1\\ 0.4 & Y=0\\ 0.3 & Y=1\end{cases}$$

(b) (4 points) Find $\mathrm{E}(X), \mathrm{E}(Y), \mathrm{Var}(X), \mathrm{Var}(Y)$.

$$E(X)=1\times 0.4+2\times 0.2+3\times 0.4=2.0$$

$$E(Y)=-1\times 0.3+0\times 0.4+1\times 0.3=0$$

$$\mathrm{Var}(X)=(1-2)^2\times 0.4+(2-2)^2\times 0.2+(3-2)^2\times 0.4$$

$$=0.8$$

$$\mathrm{Var}(Y)=(-1-0)^2\times 0.3+(0-0)^2\times 0.4+(1-0)^2\times 0.3$$

$$=0.6$$

(c) (3 points) Find $\mathrm{Cov}(X, Y)$ and correlation coefficient ρ. (Keep the square root in your expression. You need not compute it.)

$f(x,y) = f_x(x) \cdot f_{y|x}(y)$

$f(x,y)$:

Y \ X	1	2	3
-1	0.12	0.06	0.12
0	0.16	0.08	0.16
1	0.12	0.06	0.12

$Cov(X,Y) = E(XY) - \mu_x \cdot \mu_y$

$= (-1 \times 1 \times 0.12) + (-1 \times 2 \times 0.06) + (-1 \times 3 \times 0.12) + 0$

$+ (1 \times 1 \times 0.12) + (1 \times 2 \times 0.06) + (1 \times 3 \times 0.12) - (2 \times 0)$

$= -0.12 - 0.12 - 0.36 + 0.12 + 0.12 + 0.36 - 0$

$= 0$

correlation coefficient

$\rho = \frac{Cov(X,Y)}{\sigma_x \cdot \sigma_y} = \frac{0}{\sqrt{0.8} \cdot \sqrt{0.6}} = 0$