Computer Vision

CS308

Feng Zheng
SUSTech CS Vision Intelligence and Perception

Week 2

南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Content

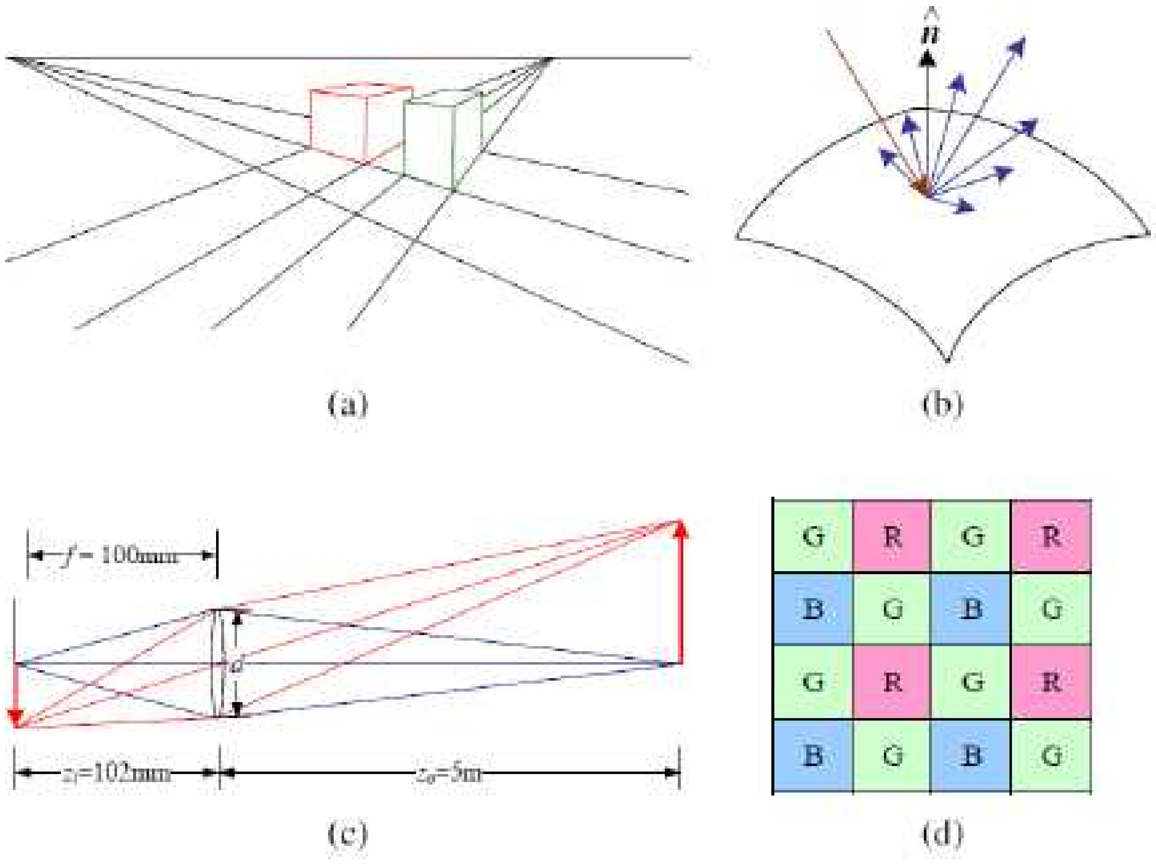
- Geometric primitives and transformations
- Projections
- Photometric image formation
- The digital camera

Image Formation

3D geometric primitives to 2D geometric primitives

Components of the Image Formation Process

- Image formation process: 3D (real-world) to 2D (matrix)
  - ➢ (a) Perspective projection
  - ➢ (b) Light scattering when hitting a surface
  - ➢ (c) Lens optics
  - ➢ (d) Bayer color filter array



Geometric primitives and transformations

Geometric Primitives

- 2D points

$$\boldsymbol{x} = (x, y) \in \mathcal{R}^2 \qquad \boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

  - Homogeneous coordinates

$$\tilde{\boldsymbol{x}} = (\tilde{x}, \tilde{y}, \tilde{w}) \in \mathcal{P}^2$$

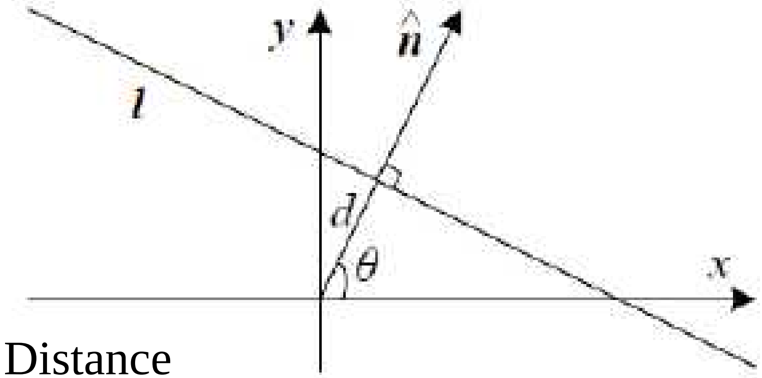
  - Augmented vector

$$\bar{\boldsymbol{x}} = (x, y, 1)$$

  - Relationship

$$\tilde{\boldsymbol{x}} = (\tilde{x}, \tilde{y}, \tilde{w}) = \tilde{w}(x, y, 1) = \tilde{w}\bar{\boldsymbol{x}},$$

Geometric Primitives



- 2D lines

$$\bar{x} \cdot \tilde{l} = \underline{ax + by + c = 0}$$

$$\tilde{l} = (a, b, c)$$

Direction Distance

  - Polar coordinates $\quad \boldsymbol{l} = (\hat{n}_x, \hat{n}_y, d) = (\hat{\boldsymbol{n}}, d)$
    - ✓The direction (normal vector) is a function of a rotation angle

- Advantageous

$$\hat{\boldsymbol{n}} = (\hat{n}_x, \hat{n}_y) = (\cos\theta, \sin\theta)$$

  - Intersection of two lines
  - Line joining two points

Cross product operation

$$\tilde{x} = \tilde{l}_1 \times \tilde{l}_2 \qquad \tilde{l} = \tilde{x}_1 \times \tilde{x}_2$$

Geometric Primitives

- 3D points

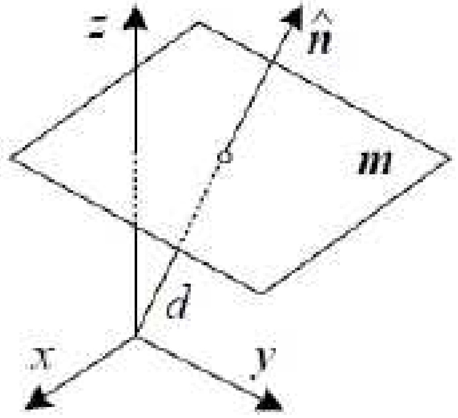
$$x = (x, y, z) \in \mathcal{R}^3 \qquad \tilde{x} = (\tilde{x}, \tilde{y}, \tilde{z}, \tilde{w}) \in \mathcal{P}^3$$

$$\bar{x} = (x, y, z, 1) \qquad \tilde{x} = \tilde{w}\bar{x}$$

- 3D planes

$$\bar{x} \cdot \tilde{m} = \underline{ax + by + cz + d = 0}$$

$$m = (\hat{n}_x, \hat{n}_y, \hat{n}_z, d) = (\hat{n}, d)$$

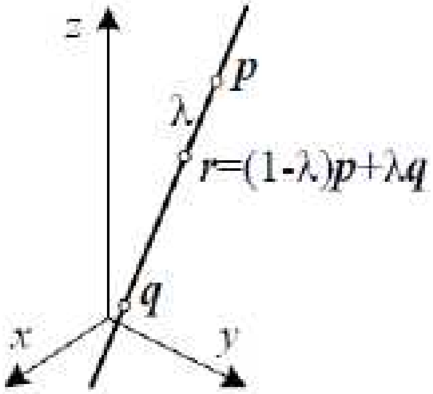


  - ➢ The direction (normal vector) is a function of two rotation angles

$$\hat{n} = (\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi)$$

- 3D lines

$$r = (1 - \lambda)p + \lambda q$$



Transformations

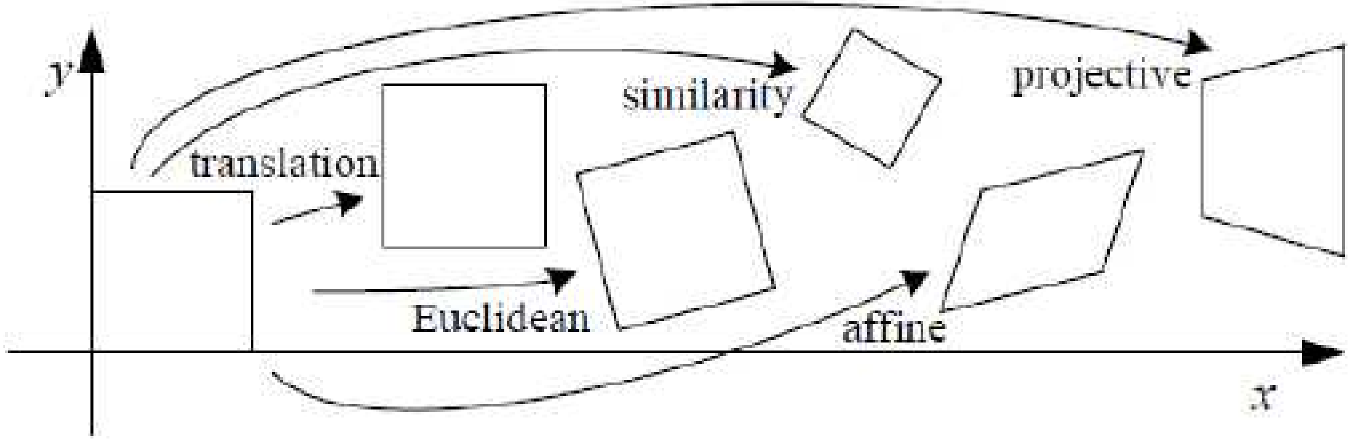
- 2D transformations
  - Translation $x' = x + t = \begin{bmatrix} I & t \end{bmatrix} \bar{x}$ $\quad \bar{x}' = \begin{bmatrix} I & t \\ 0^T & 1 \end{bmatrix} \bar{x}$

1.

2. - Rotation + translation $x' = Rx + t = \boxed{\begin{bmatrix} R & t \end{bmatrix} \bar{x}}$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

3. skewing

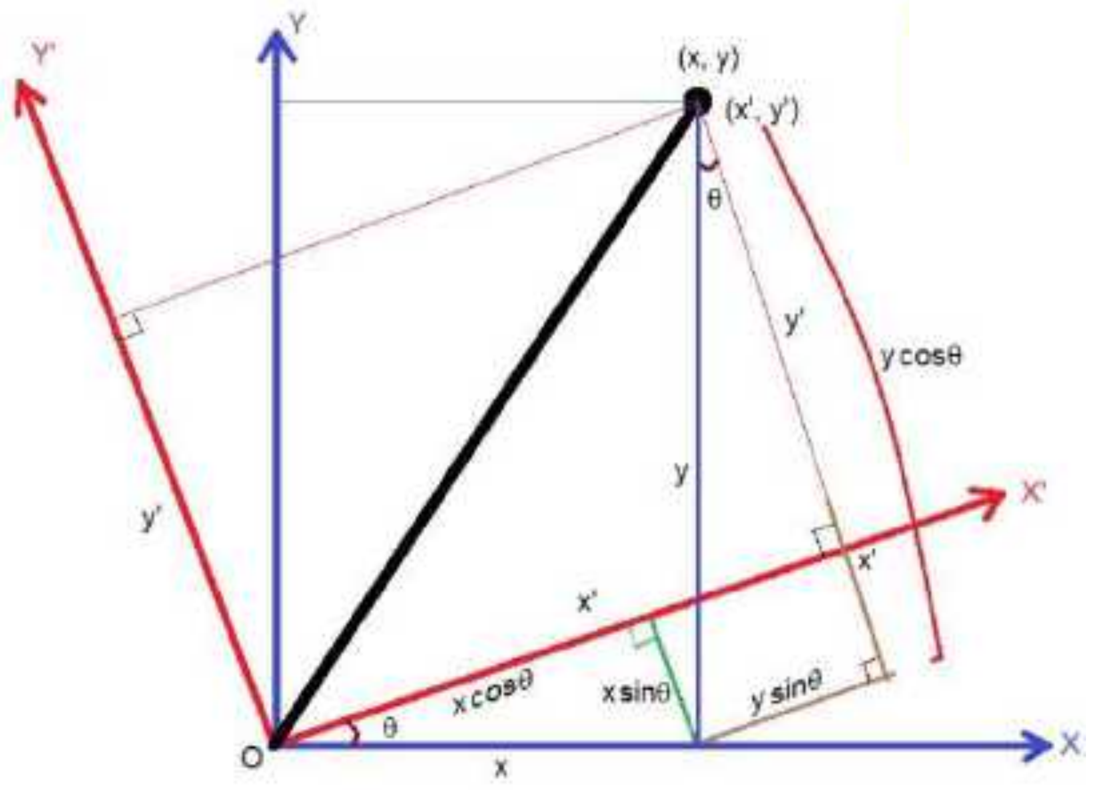


Transformations

- Rotation matrix
  - After the rectangular coordinate system is rotated by a certain angle
  - The relationship between the new and the old coordinate systems

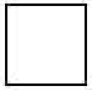
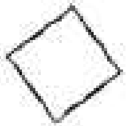
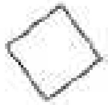
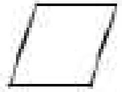
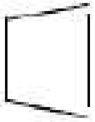
$$x' = x\cos\theta + y\sin\theta$$

$$y' = y\cos\theta - x\sin\theta$$



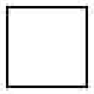
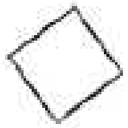
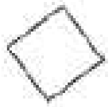
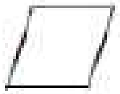
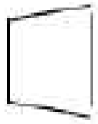
Transformations

- Hierarchy of 2D coordinate transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} I \mid t \end{bmatrix}_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} R \mid t \end{bmatrix}_{2\times 3}$	3	lengths	
similarity	$\begin{bmatrix} sR \mid t \end{bmatrix}_{2\times 3}$	4	angles	
affine	$\begin{bmatrix} A \end{bmatrix}_{2\times 3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3\times 3}$	8	straight lines	

Transformations

- Hierarchy of 3D coordinate transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{3\times 4}$	3	orientation	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{3\times 4}$	6	lengths	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{3\times 4}$	7	angles	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{3\times 4}$	12	parallelism	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{4\times 4}$	15	straight lines	

Transformations

- 3D to 2D projections (what information you want to preserved)
  - Specify how 3D primitives are projected onto the image plane
  - Use a linear 3D to 2D projection matrix
- Orthography
  - Orthographic projection

$$x = [I_{2\times 2}|0]\, p$$

2D

$$\tilde{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tilde{p}$$

3D

  - Scaled orthography
    - ✓ First project the world points onto a local fronto-parallel image plane
    - ✓ Then scale this image using regular perspective projection

z

$$x = [sI_{2\times 2}|0]\, p$$

Transformations

透视变换

- Perspective
  - The most commonly used projection
  - Points projected onto the image plane by dividing them by their z component

inhomogeneous $\bar{x} = \mathcal{P}_z(p) = \begin{bmatrix} x/z \\ y/z \\ 1 \end{bmatrix}$ homogeneous $\tilde{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tilde{p}$

  - A two-step projection
    - ✓First project 3D points into normalized device coordinates in the range
    - ✓Then rescale these coordinates to integer pixel coordinates

the near and far z *clipping planes*

$$z_{\text{range}} = z_{\text{far}} - z_{\text{near}}$$

$$\tilde{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -z_{\text{far}}/z_{\text{range}} & z_{\text{near}} z_{\text{far}}/z_{\text{range}} \\ 0 & 0 & 1 & 0 \end{bmatrix} \tilde{p}$$

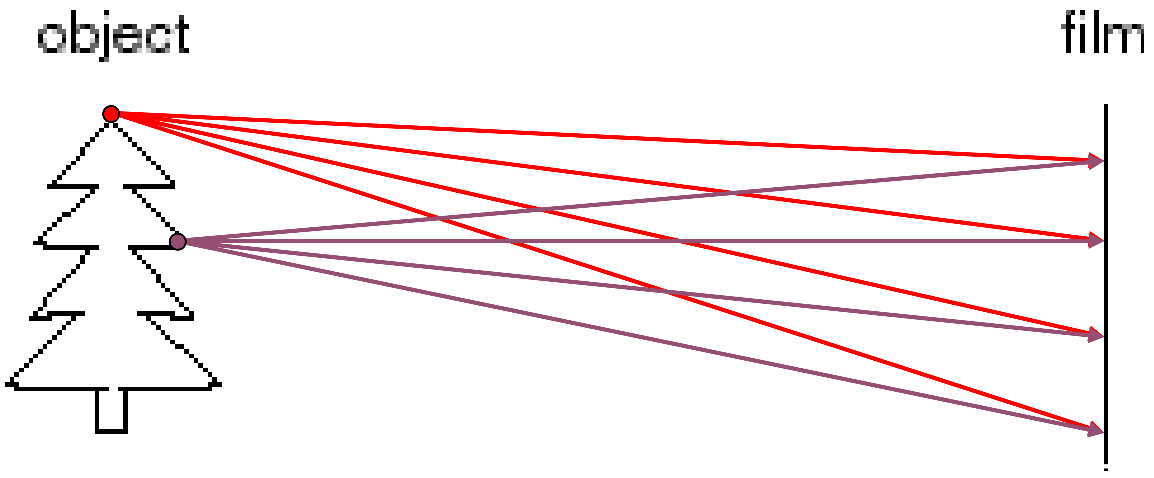
Projections

The Geometry of Image Formation

- Mapping between image and world coordinates
  - ➢ Pinhole camera model
  - ➢ Projective geometry
    - ✓ Vanishing points and lines
  - ➢ Projection matrix

Image Formation

- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

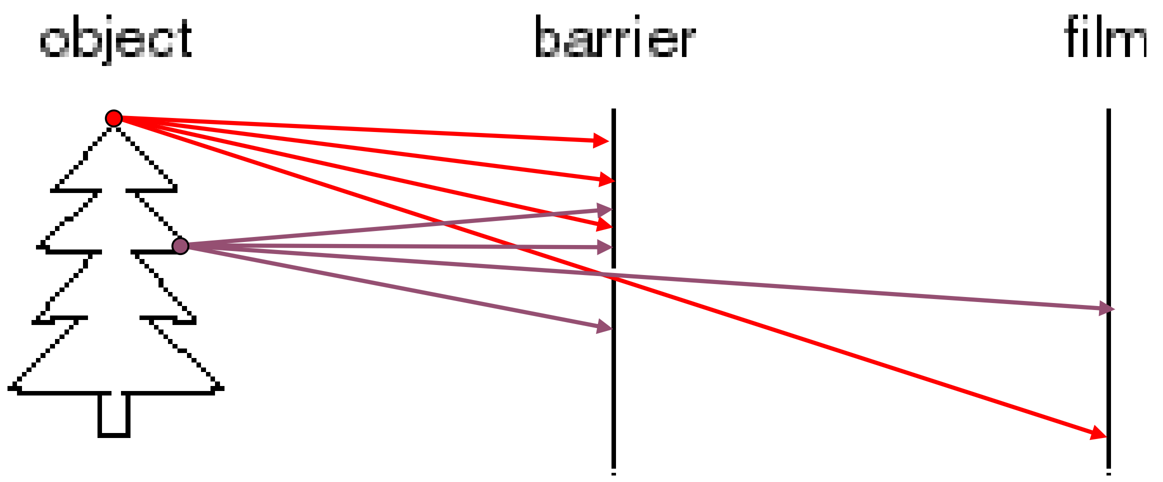


blured → seperate ?

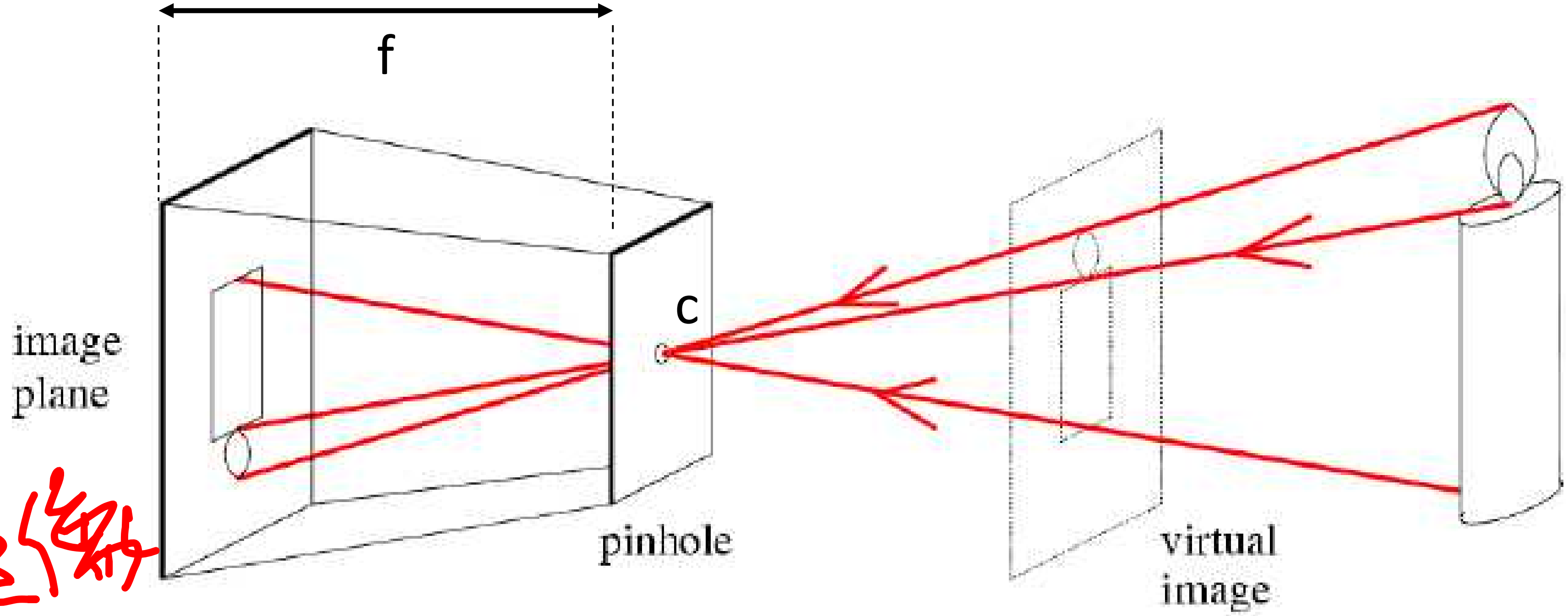
Slide source: Seitz

Pinhole Camera

- Idea 2: add a barrier to block off most of the rays
  - ➢ This reduces blurring
  - ➢ The opening known as the aperture



Pinhole Camera



成像

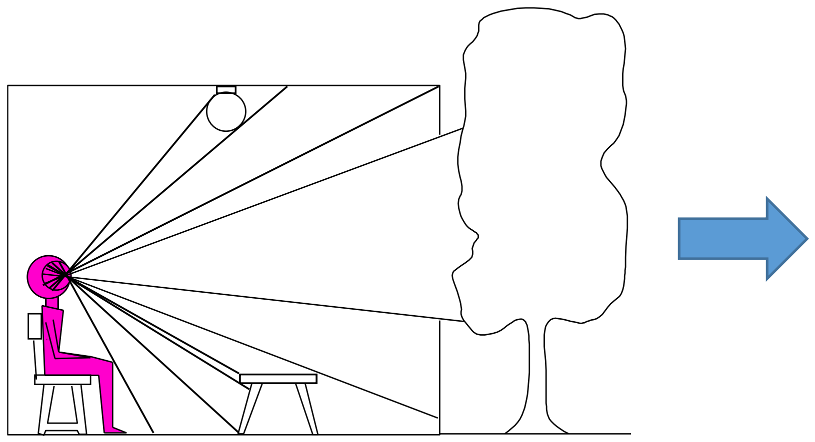
f = focal length
c = center of the camera

Camera and World Geometry

- Questions:
  - How tall is this woman?
  - How high is the camera?
  - What is the camera rotation?
  - What is the focal length of the camera?
  - Which ball is closer?

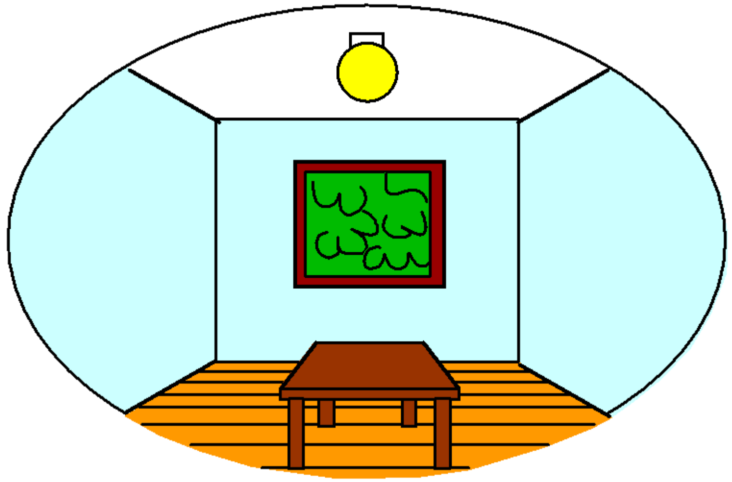
Dimensionality Reduction Machine (3D to 2D)

3D world



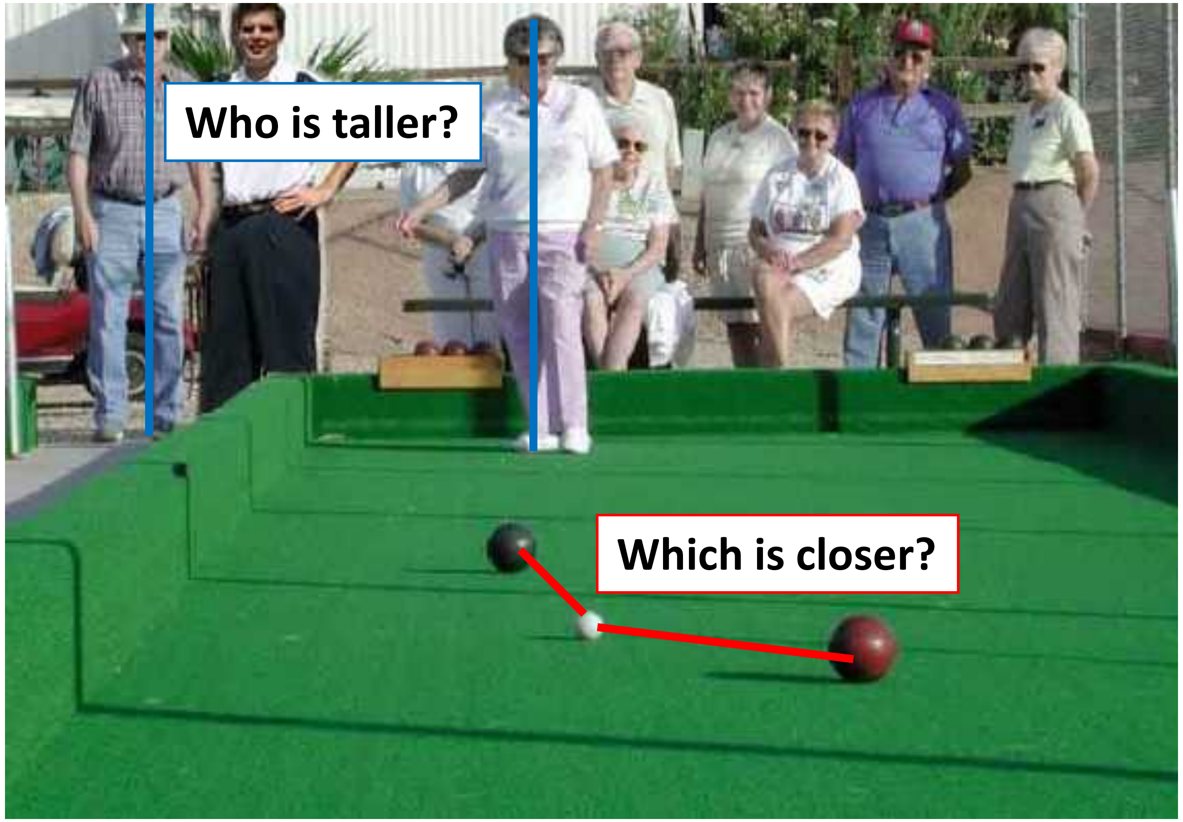
Point of observation

2D image



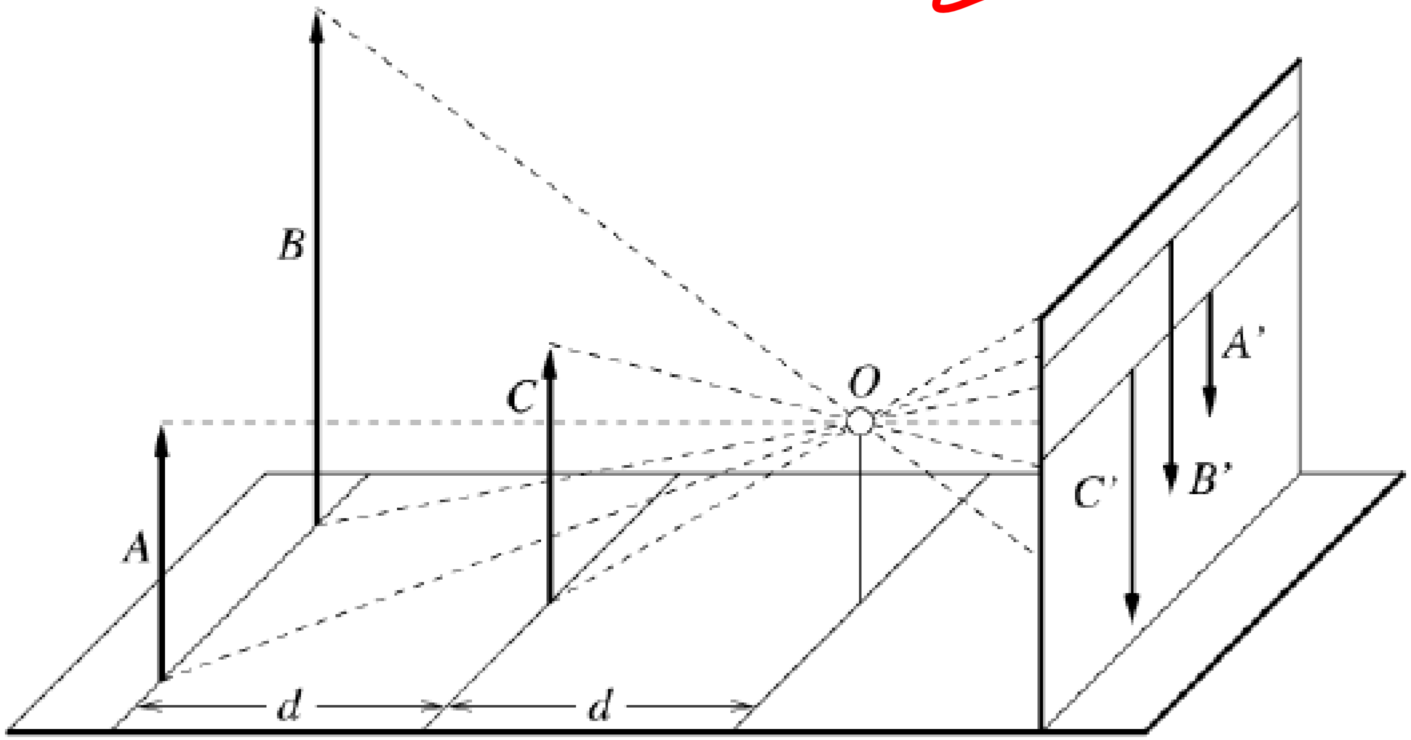
Projective Geometry

- What is lost?
  - ➢ Length



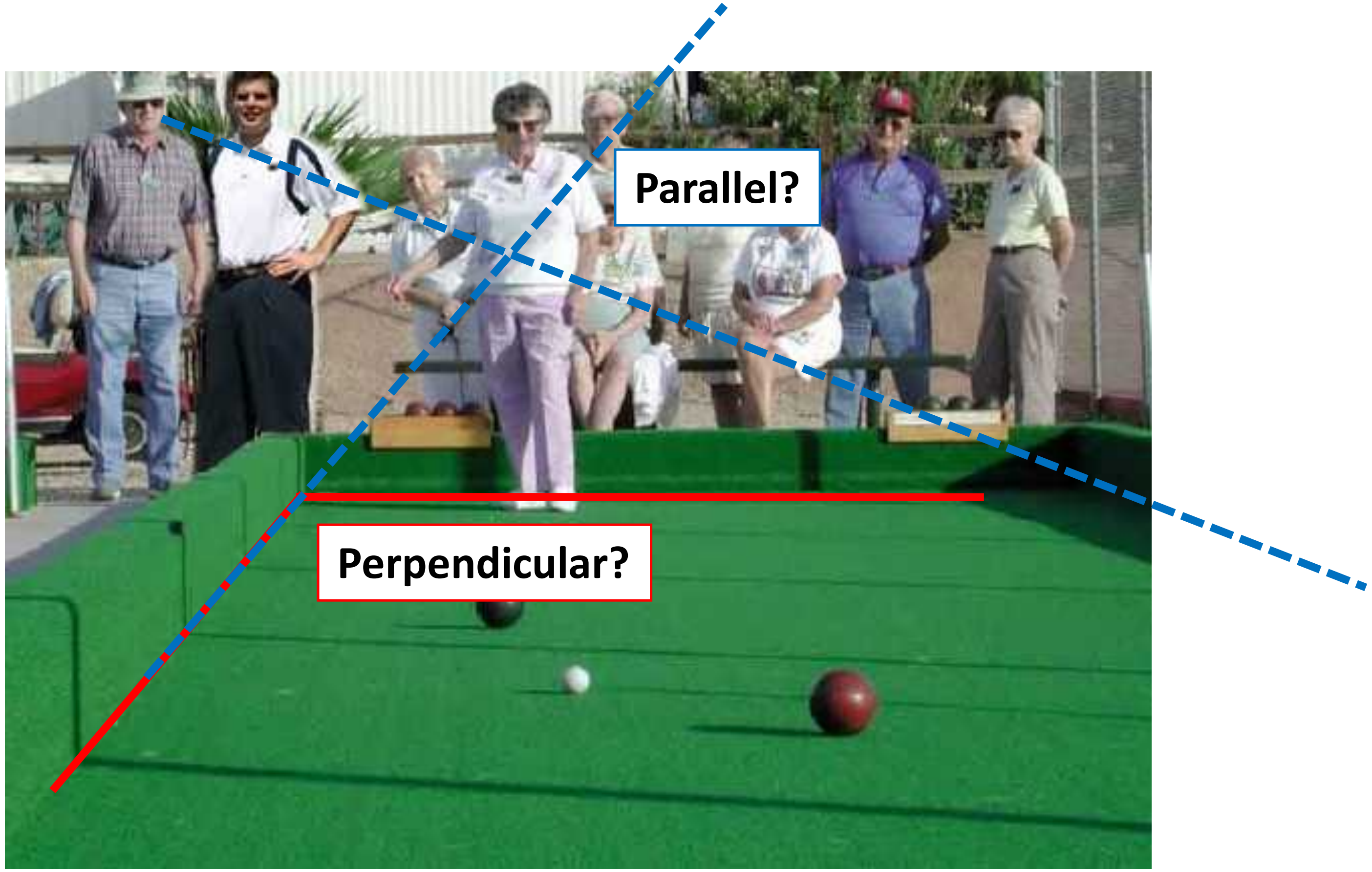
Projective Geometry

- What is lost?
  - Length and area are not preserved



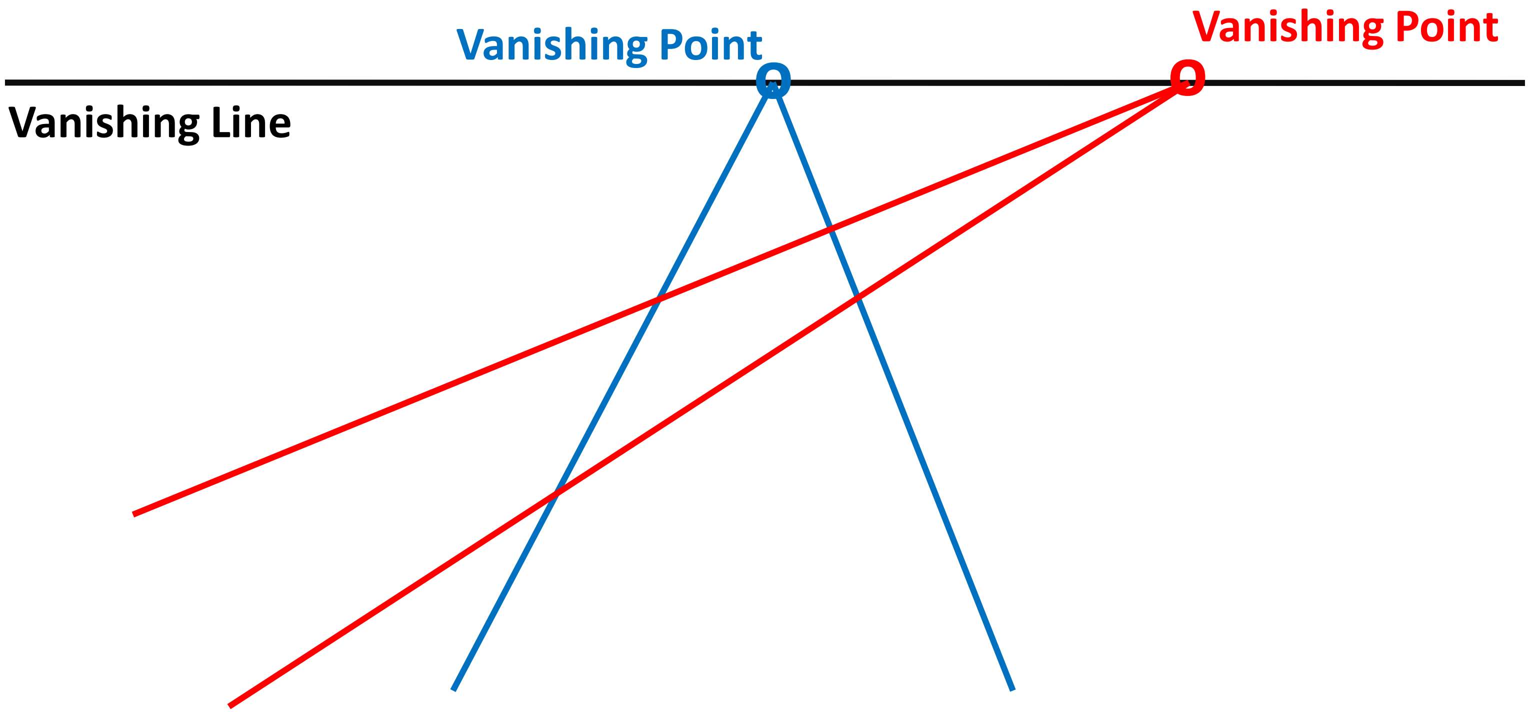
Projective Geometry

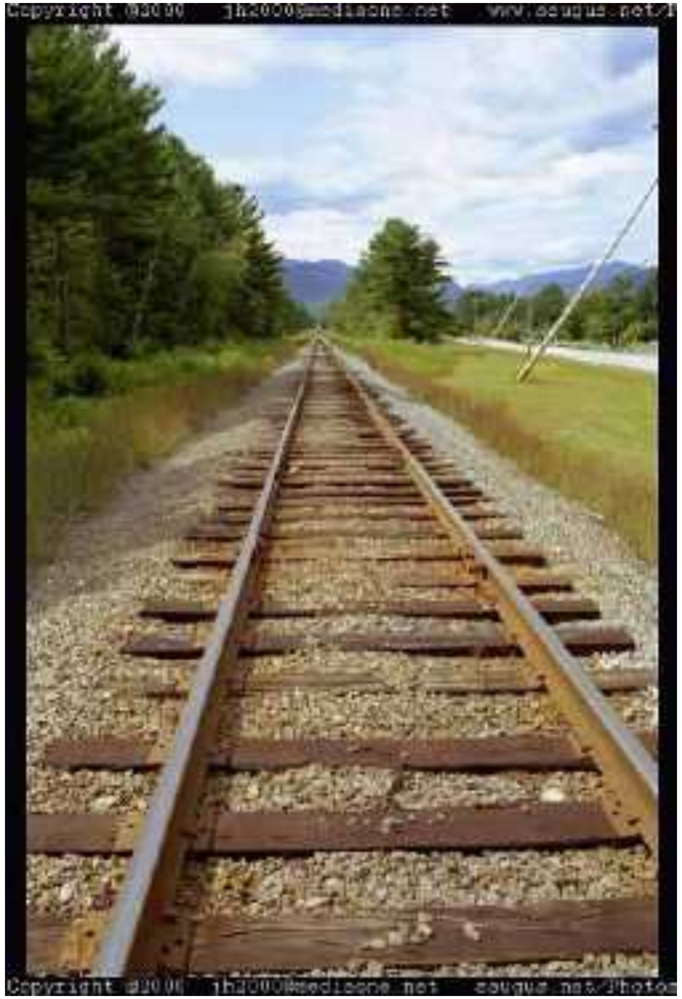
- What is lost?
  - Length
  - Angles
- What is preserved?
  - Straight lines are still straight



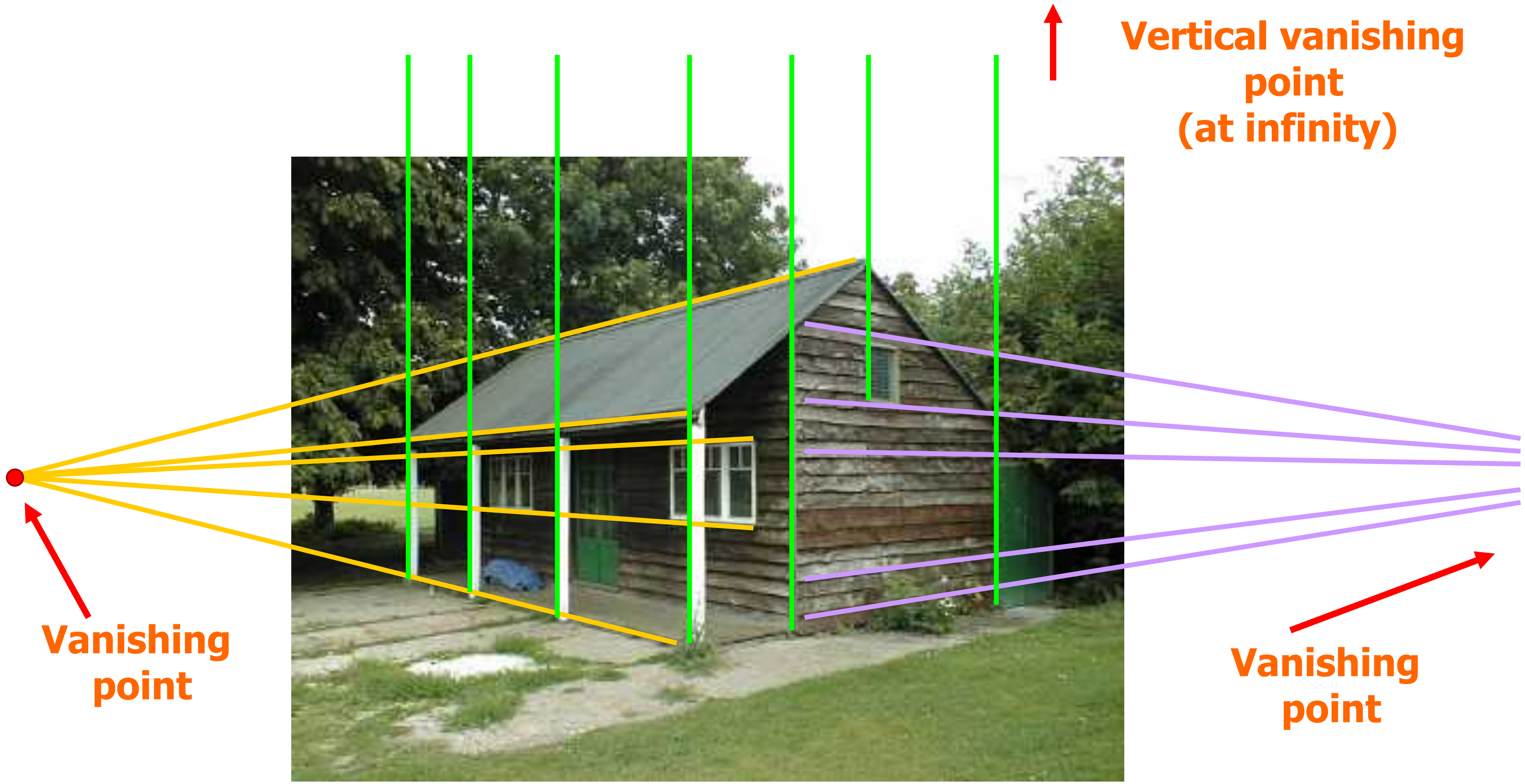
Projective Geometry

- Vanishing points and lines
  - ➢ Parallel lines in the world intersect in the image at a "vanishing point"

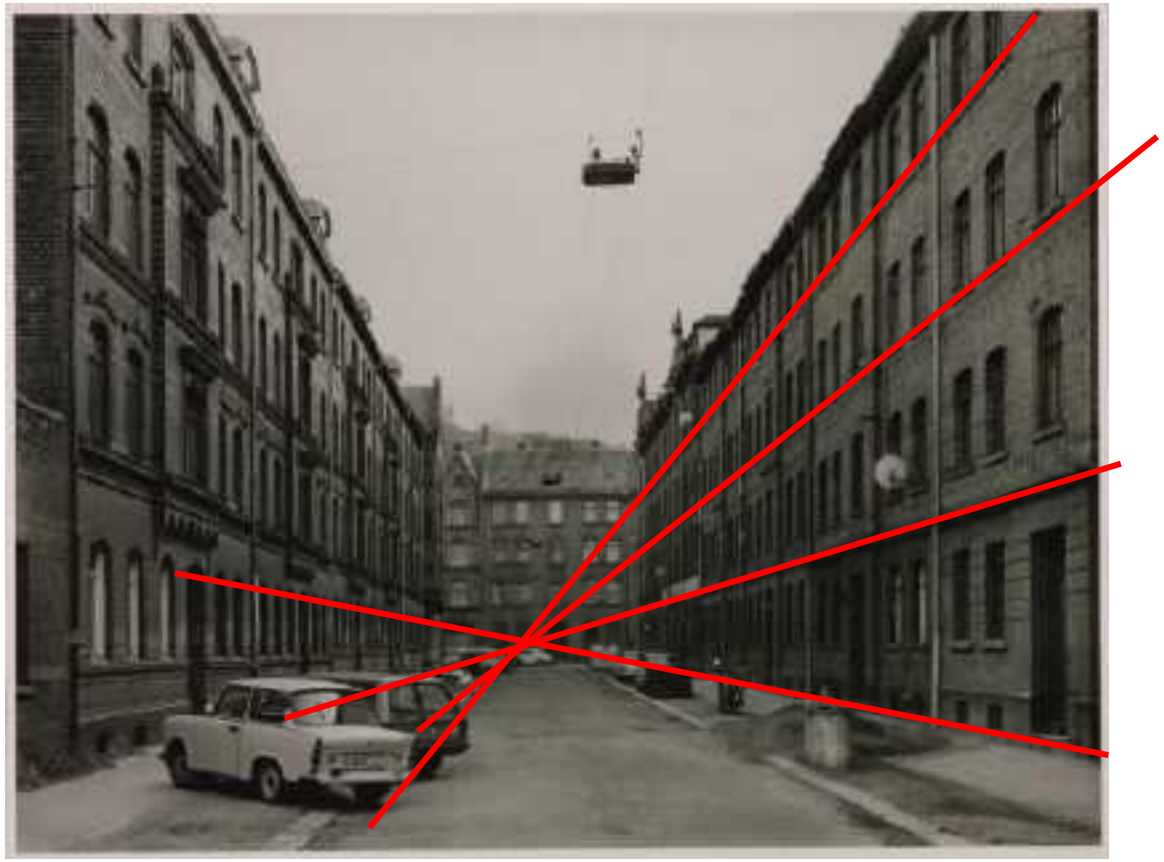




Projective Geometry



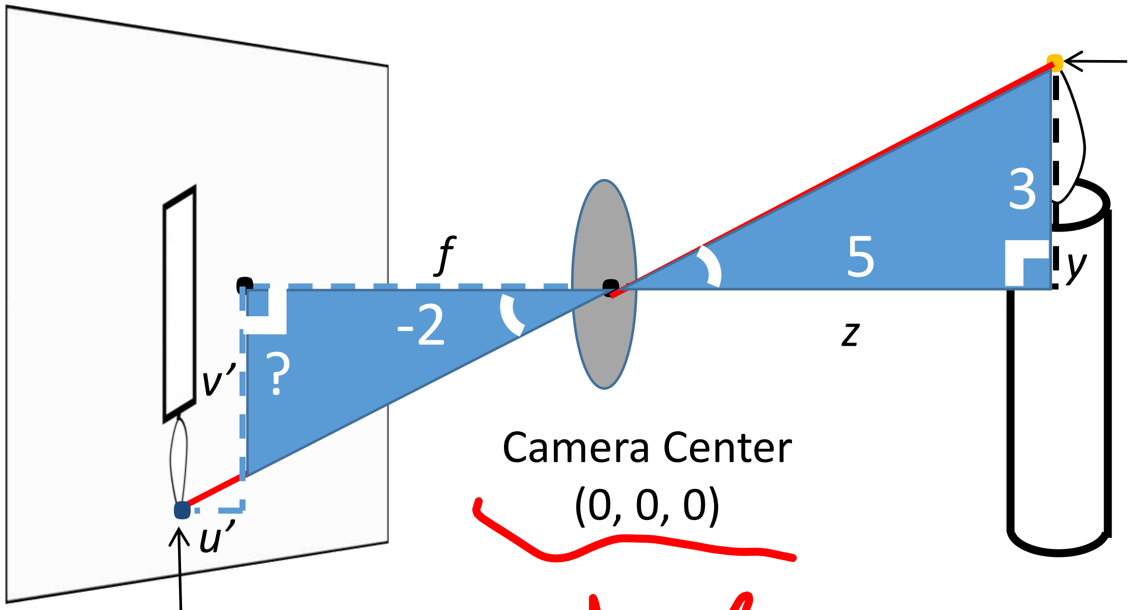
Projective Geometry



Questions: Why vertical parallel lines haven't have a finite vanishing point?

Projection

- World coordinates → image coordinates



$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$x = \begin{bmatrix} u' \\ v' \end{bmatrix}$$

lens

sensor

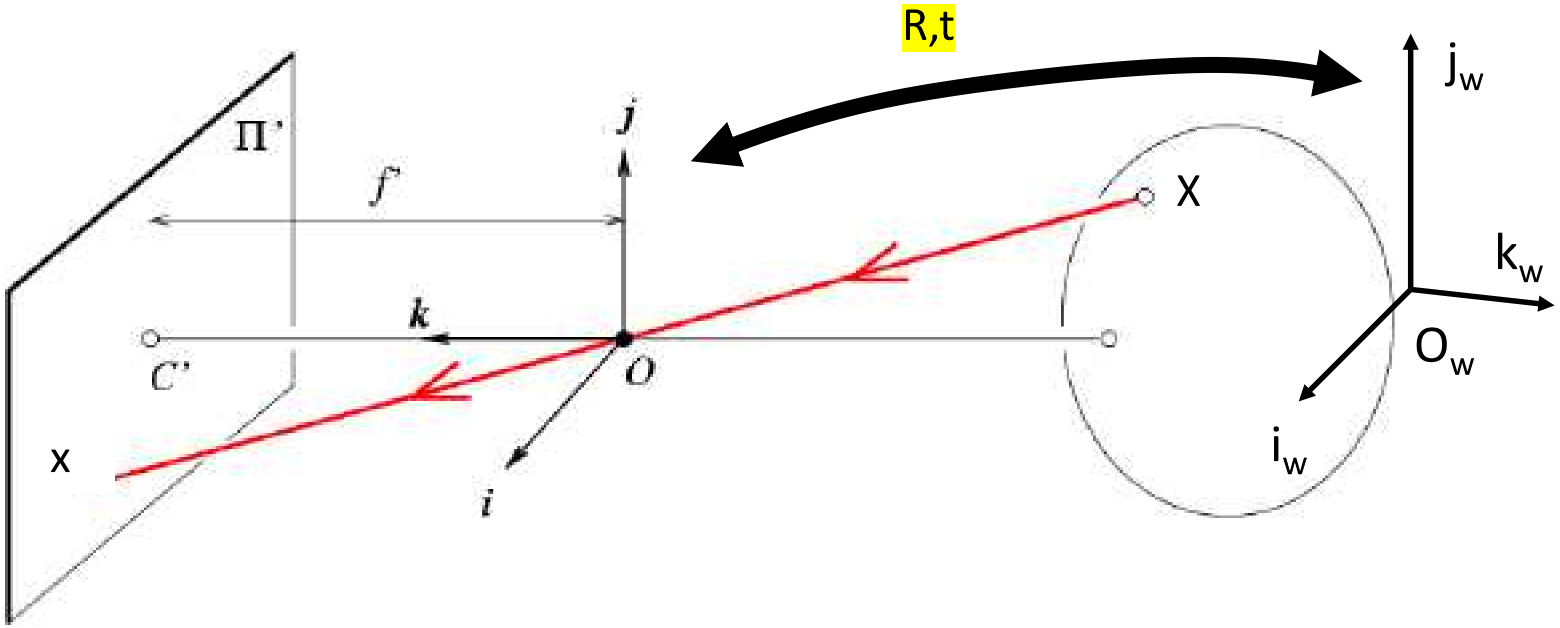
If X = 2, Y = 3,
Z = 5, and f = 2
What are U and V?

$$\frac{v'}{f} = \frac{y}{z}$$

$$u' = -x * \frac{f}{z} \qquad u' = -2 * \frac{2}{5}$$

$$v' = -y * \frac{f}{z} \qquad v' = -3 * \frac{2}{5}$$

Projection Matrix



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Projection Matrix

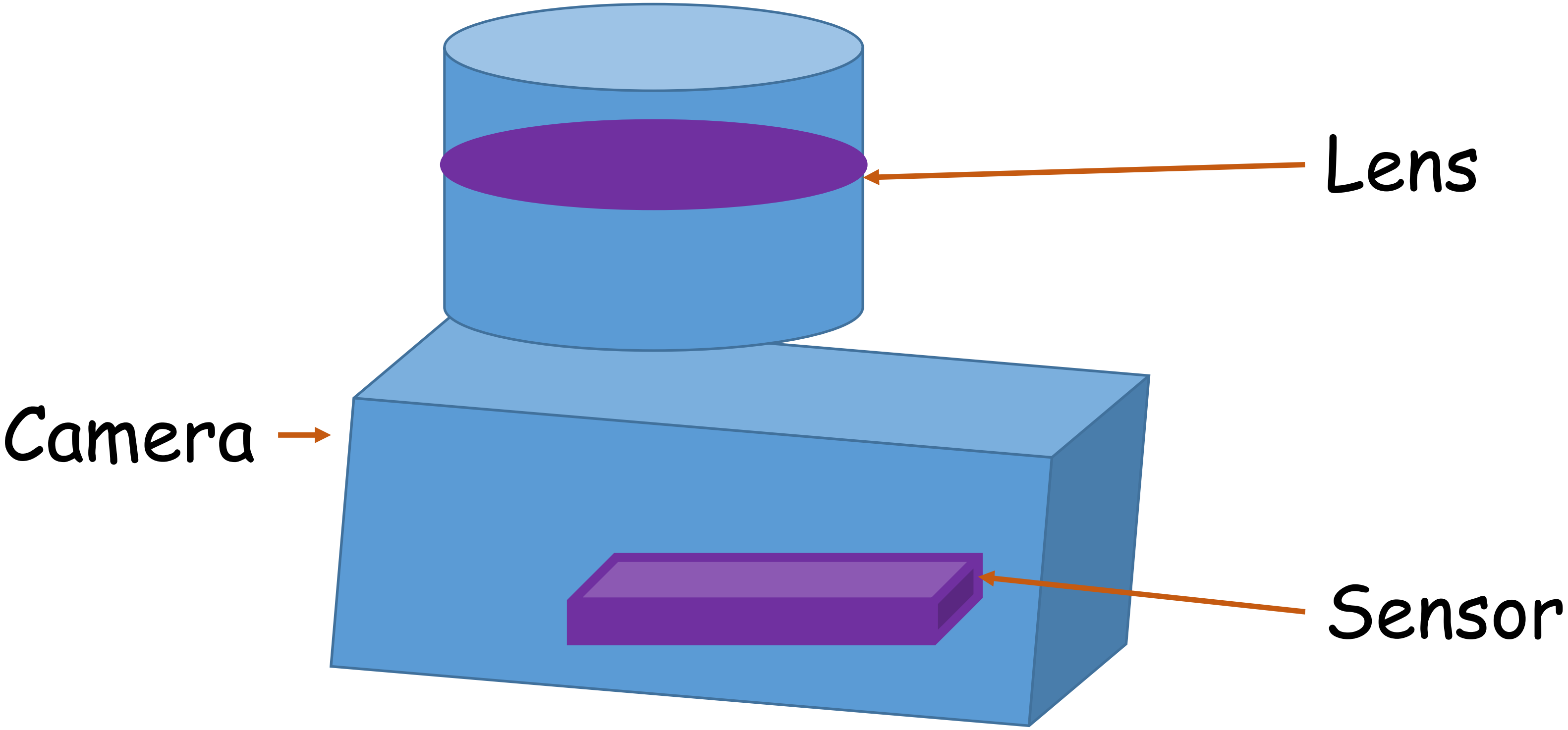
- Inserting photographed objects into images (SIGGRAPH 2007)

Original

Created

Camera Intrinsic

- Potential problems caused by the production process



Camera Intrinsic

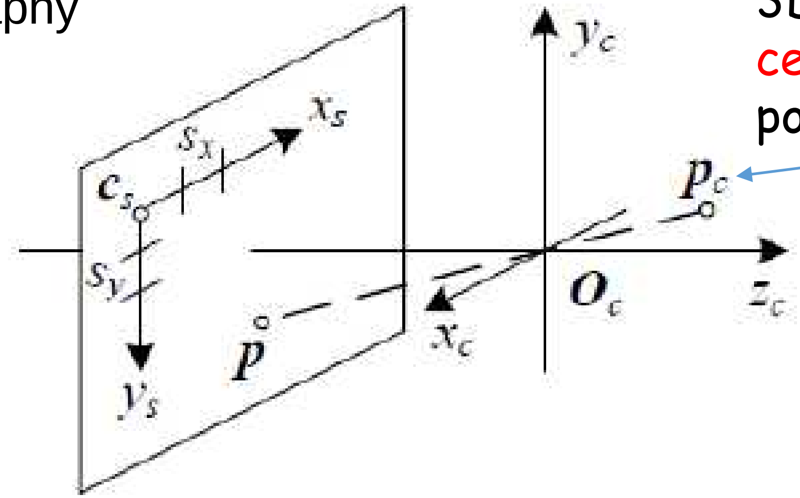
- Pixel values indexed by integer pixel coordinates
- Starting at the upper-left corner of the image
  - ✓ First scale the pixel values by the pixel spacing
  - ✓ Then describe the orientation of the sensor array relative to the camera projection center

the sensor planes at location

a sensor homography

$$p = \begin{bmatrix} R_s & | & c_s \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ 1 \end{bmatrix} = M_s \bar{x}_s$$

3D rotation | origin | scale | integer pixel coordinates



3D camera-centered points

Camera Intrinsic

- The relationship between the 3D pixel center and the 3D camera-centered point is given by an unknown scaling s
  - ➢ The calibration matrix describes the camera intrinsics

$$p = s p_c$$

the sensor planes at location

3D camera-centered points

$$\tilde{x}_s = s M_s^{-1} p_c = K p_c$$

scale

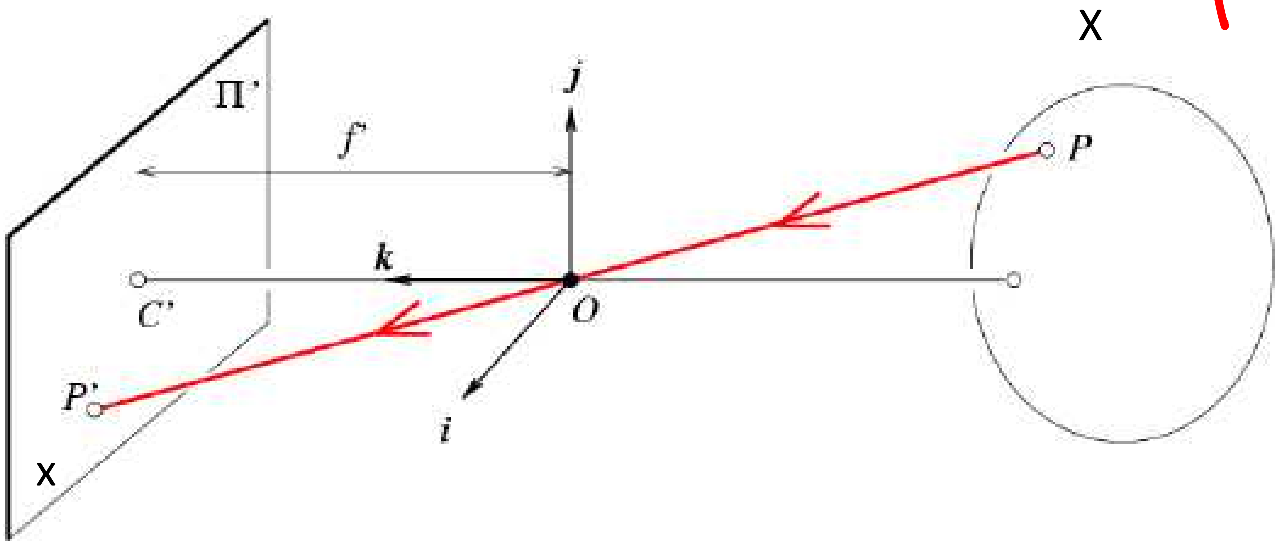
pixel address

calibration matrix

Projection (Camera) matrix

$S_x = S_y$

- Intrinsic Assumptions
  - Unit aspect ratio
  - Optical center at (0,0)
  - No skew
- Extrinsic Assumptions
  - No rotation
  - Camera at (0,0,0)

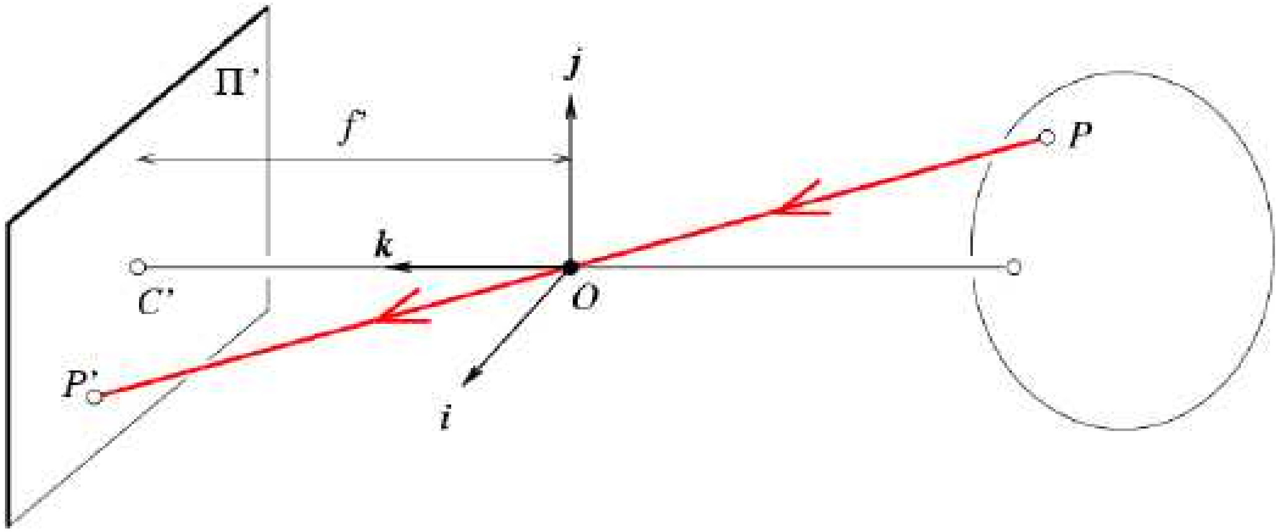


K

$$\mathbf{x} = \mathbf{K}\underbrace{[\mathbf{I} \quad \mathbf{0}]}_{\text{Perspective}}\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Projection (Camera) matrix

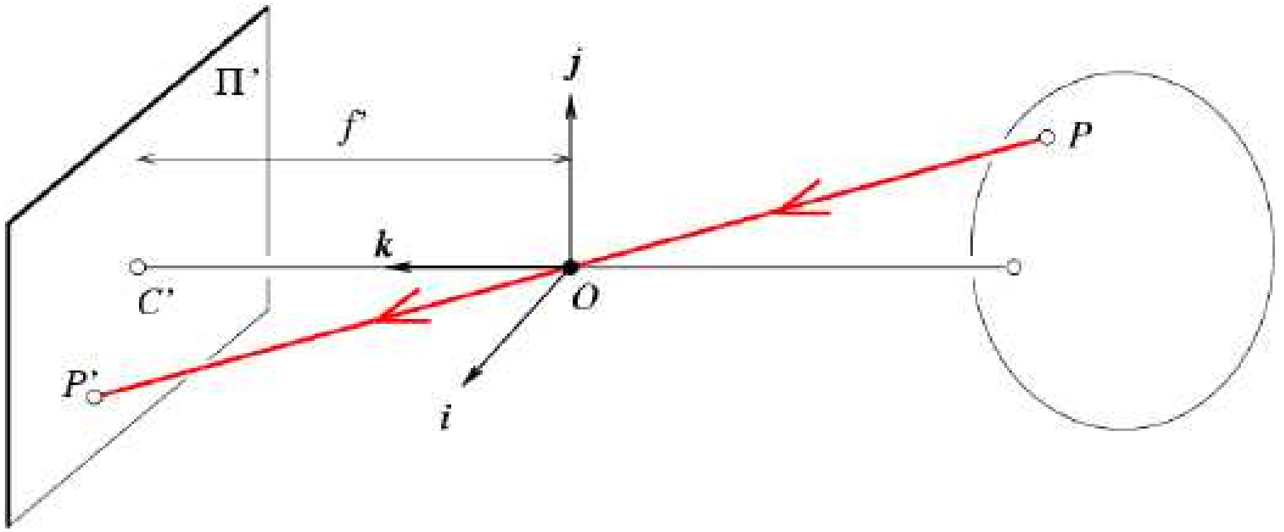
- Intrinsic Assumptions
  - Unit aspect ratio
  -
  - No skew
- Extrinsic Assumptions
  - No rotation
  - Camera at (0,0,0)



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \quad \Rightarrow \quad w\begin{bmatrix}u\\ v\\ 1\end{bmatrix} = \begin{bmatrix}f & 0 & u_0 & 0\\ 0 & f & v_0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix}$$

Projection (Camera) matrix

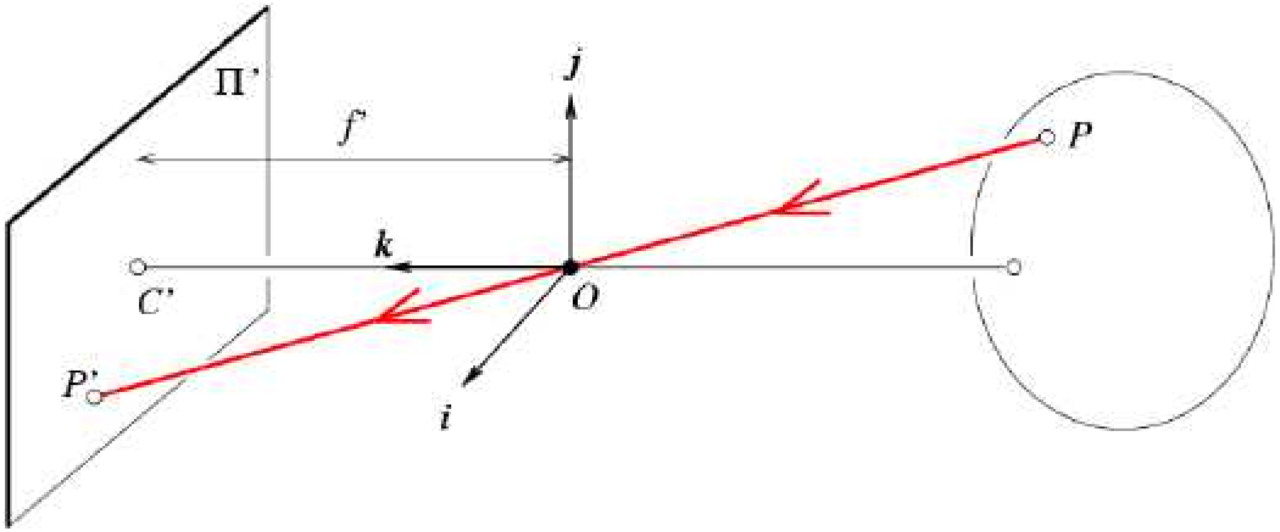
- Intrinsic Assumptions
  -
  -
  - No skew
- Extrinsic Assumptions
  - No rotation
  - Camera at (0,0,0)



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Projection (Camera) matrix

- Intrinsic Assumptions
  -
  -
  -
- Extrinsic Assumptions
  - No rotation
  - Camera at (0,0,0)

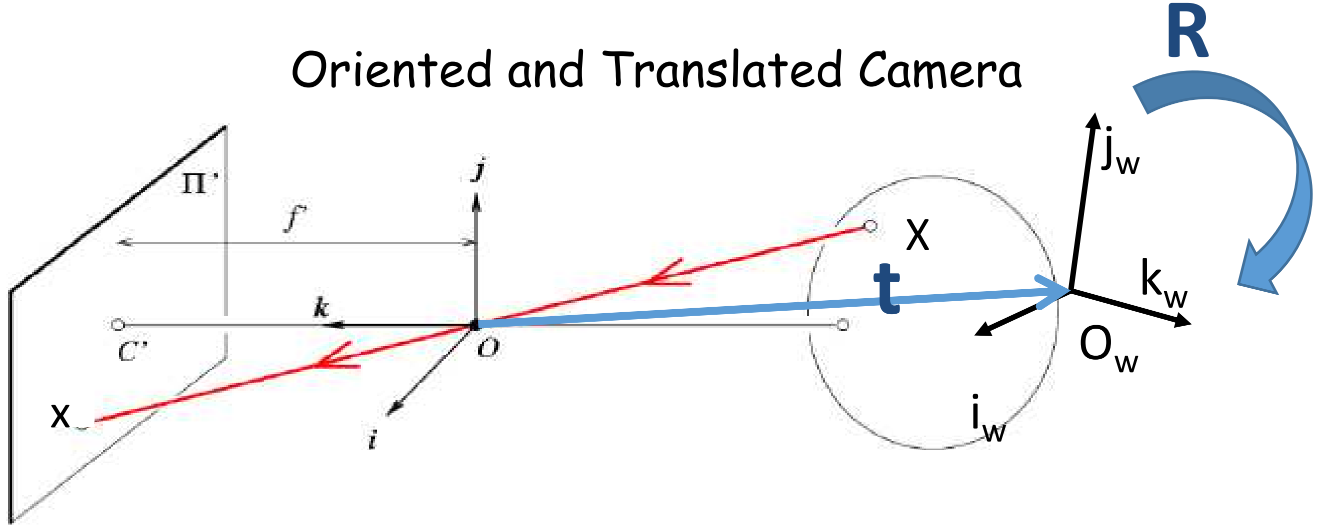


$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0 & 0\\0 & \beta & v_0 & 0\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

S encodes any possible skew between the sensor axes due to the sensor not being mounted perpendicular to the optical axis

Projection (Camera) matrix

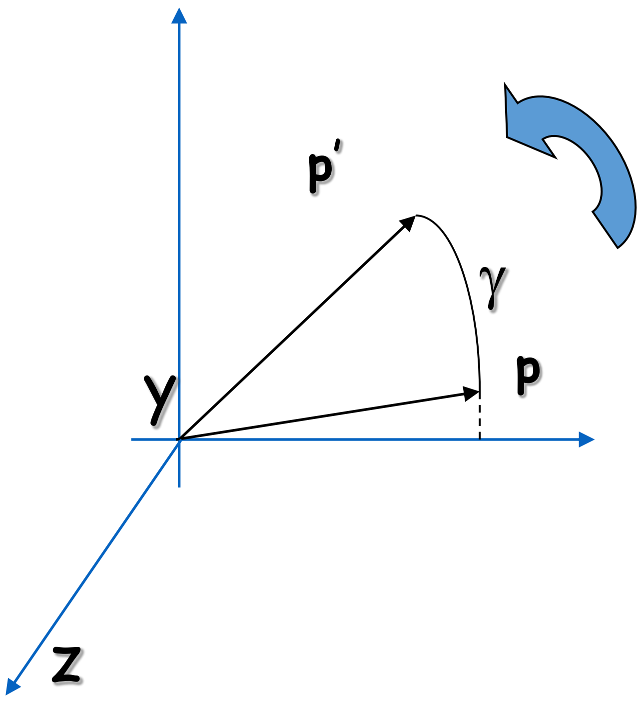
- Intrinsic Assumptions
  -
  -
  -
- Extrinsic Assumptions
  - No rotation
  -



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{t}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z\end{bmatrix}\begin{bmatrix}x \\ y \\ z \\ 1\end{bmatrix}$$

Projection (Camera) matrix

- 3D Rotation of Points
  - ➢ Rotation around the coordinate axes, counter-clockwise:



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Projection (Camera) matrix

- Allow camera rotation

$$\mathbf{x} = \mathbf{K}\left[\mathbf{R} \quad \mathbf{t}\right]\mathbf{X}$$

$$\Downarrow$$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

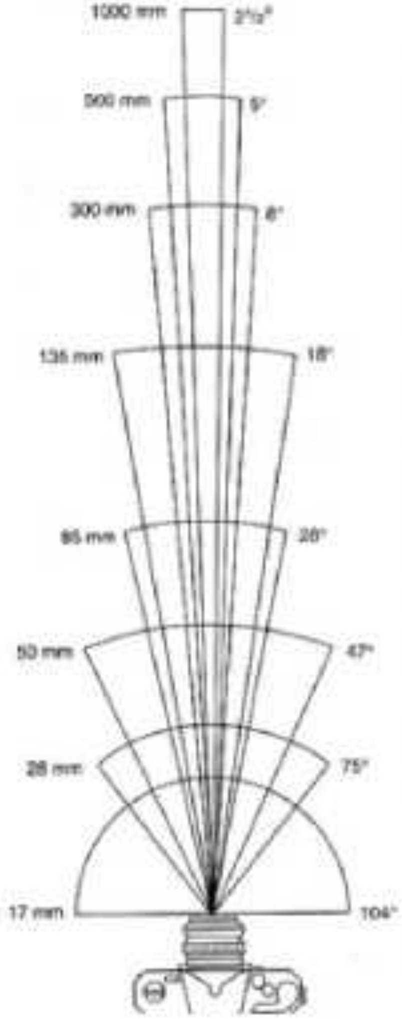
Projection (Camera) matrix

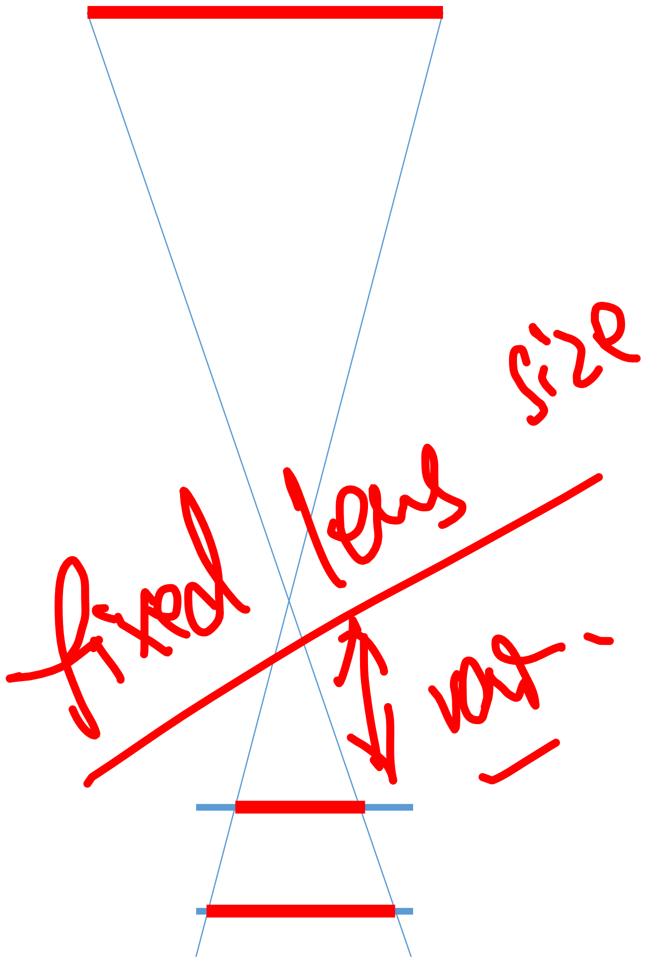
- Vanishing point = Projection from infinity

$$\mathbf{p} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} \Rightarrow \mathbf{p} = \mathbf{KR}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \mathbf{p} = \mathbf{K}\begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix}$$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix} \Rightarrow \quad u = \frac{f x_R}{z_R} + u_0 \qquad v = \frac{f y_R}{z_R} + v_0$$

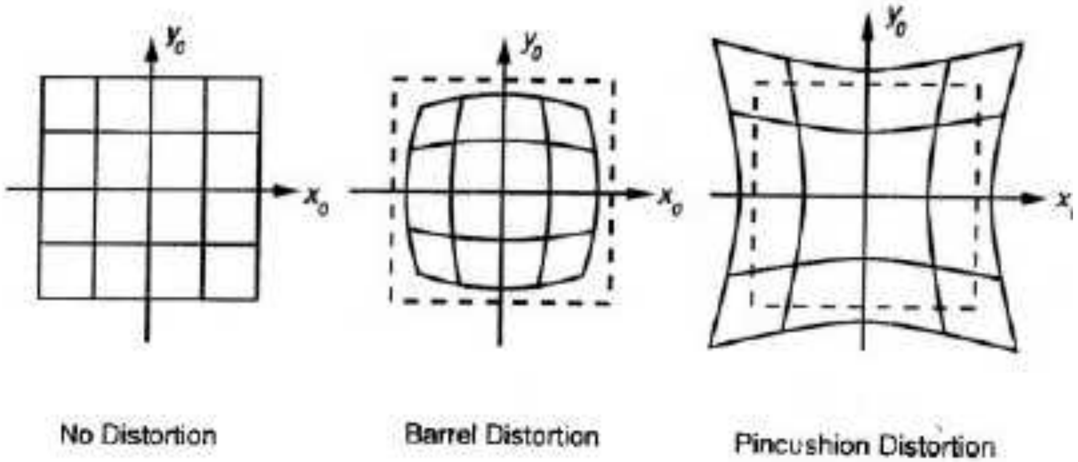
Field of View (Zoom, Focal Length)





From London and Upton

Beyond Pinholes: Radial Distortion

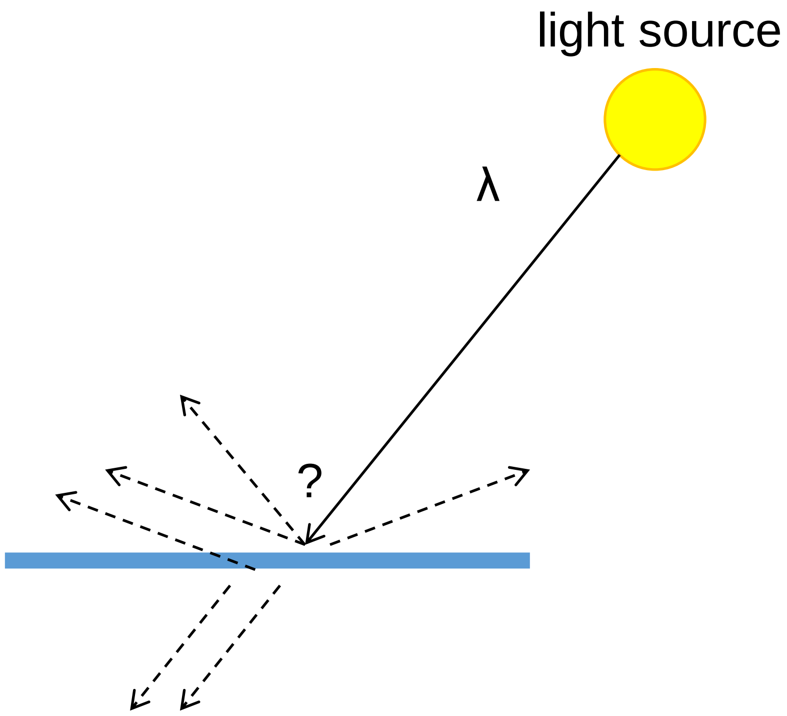


Corrected Barrel Distortion

Photometric image formation

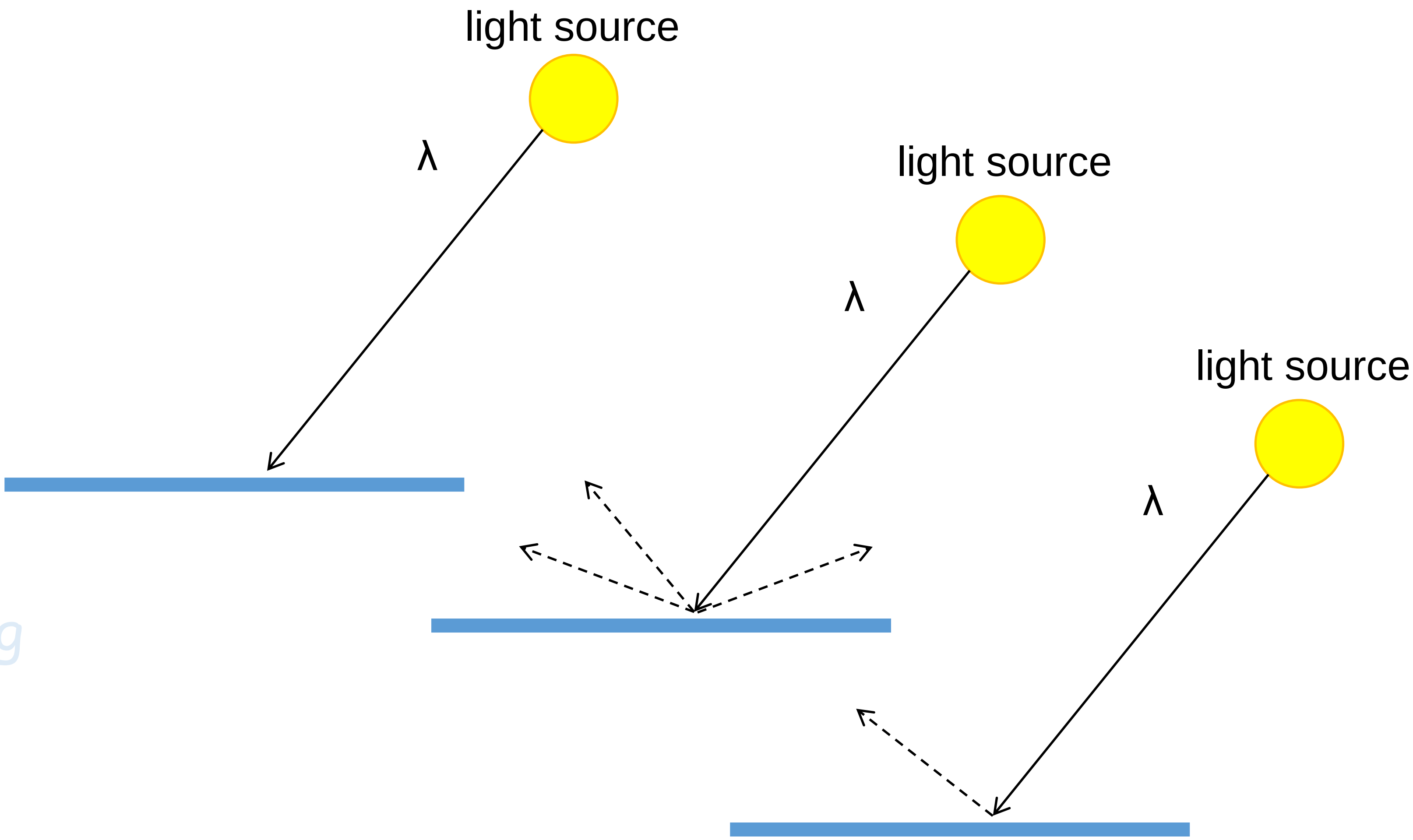
A photon's life choices

- Absorption 吸收
- Diffusion 漫射
- Reflection反射
- Transparency 透射
- Refraction 折射
- Fluorescence 荧光反应
- Subsurface scattering 次表面散射
- Phosphorescence 磷光
- Interreflection 相互反射



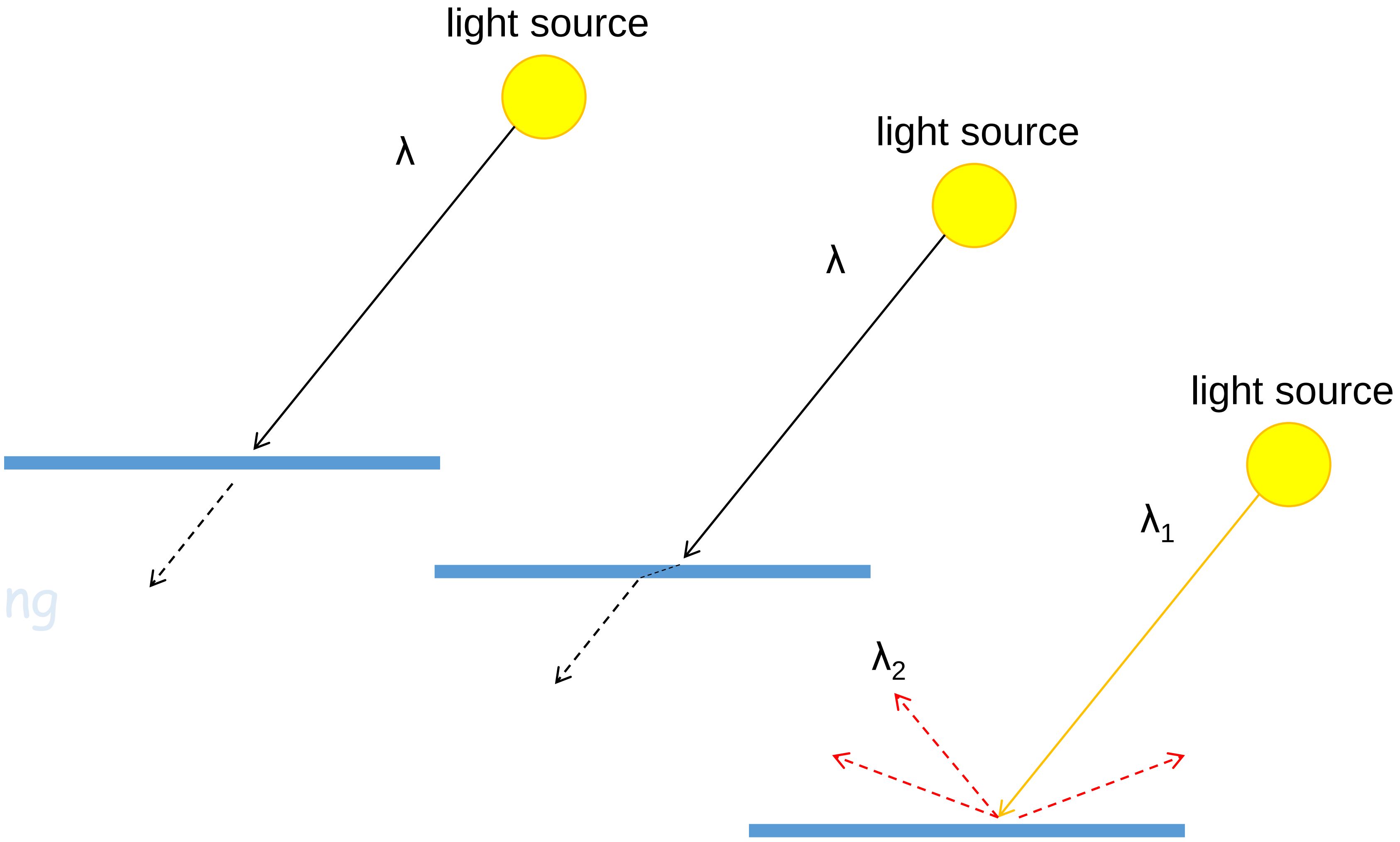
A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



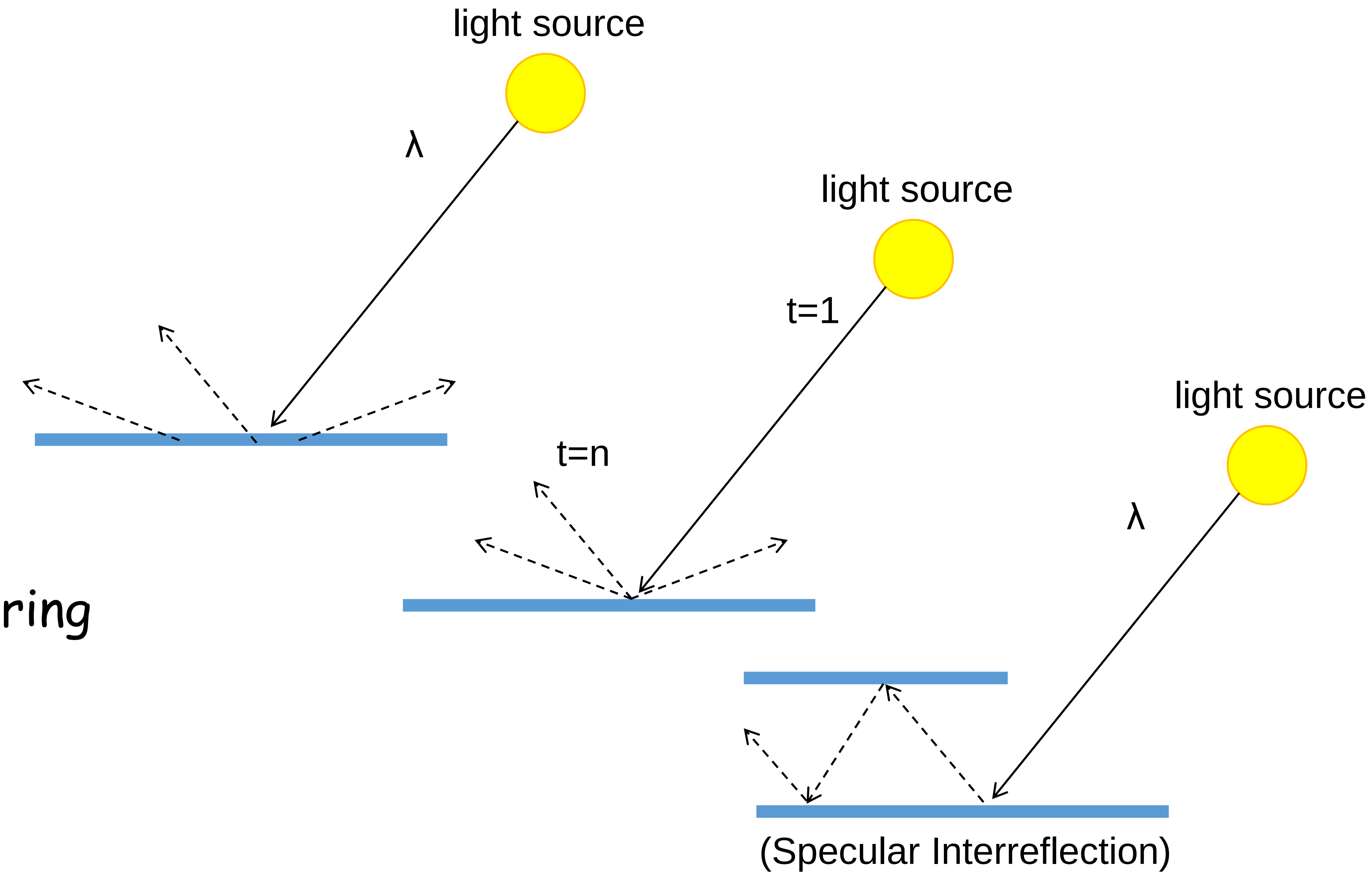
A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



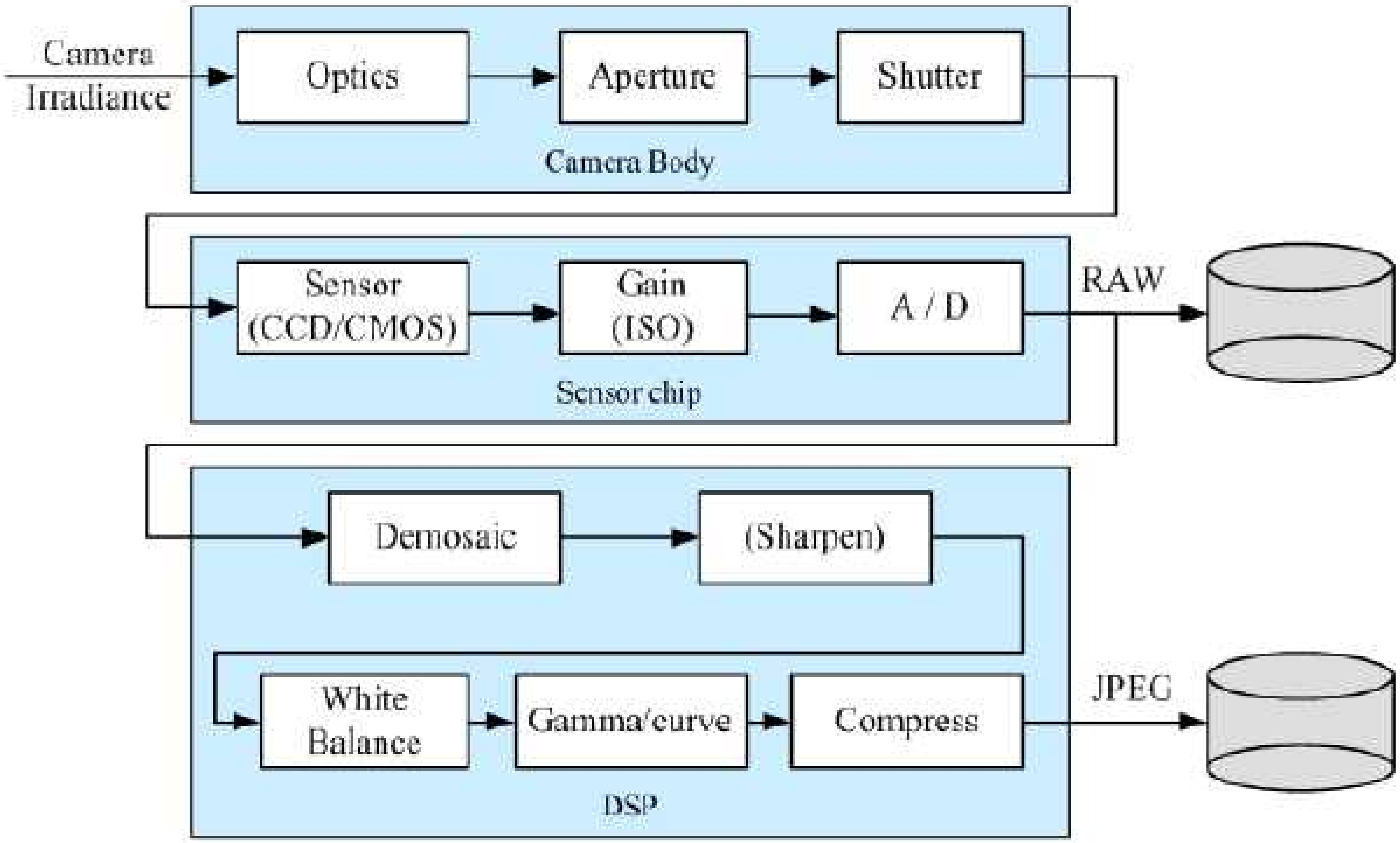
A photon's life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection



The digital camera

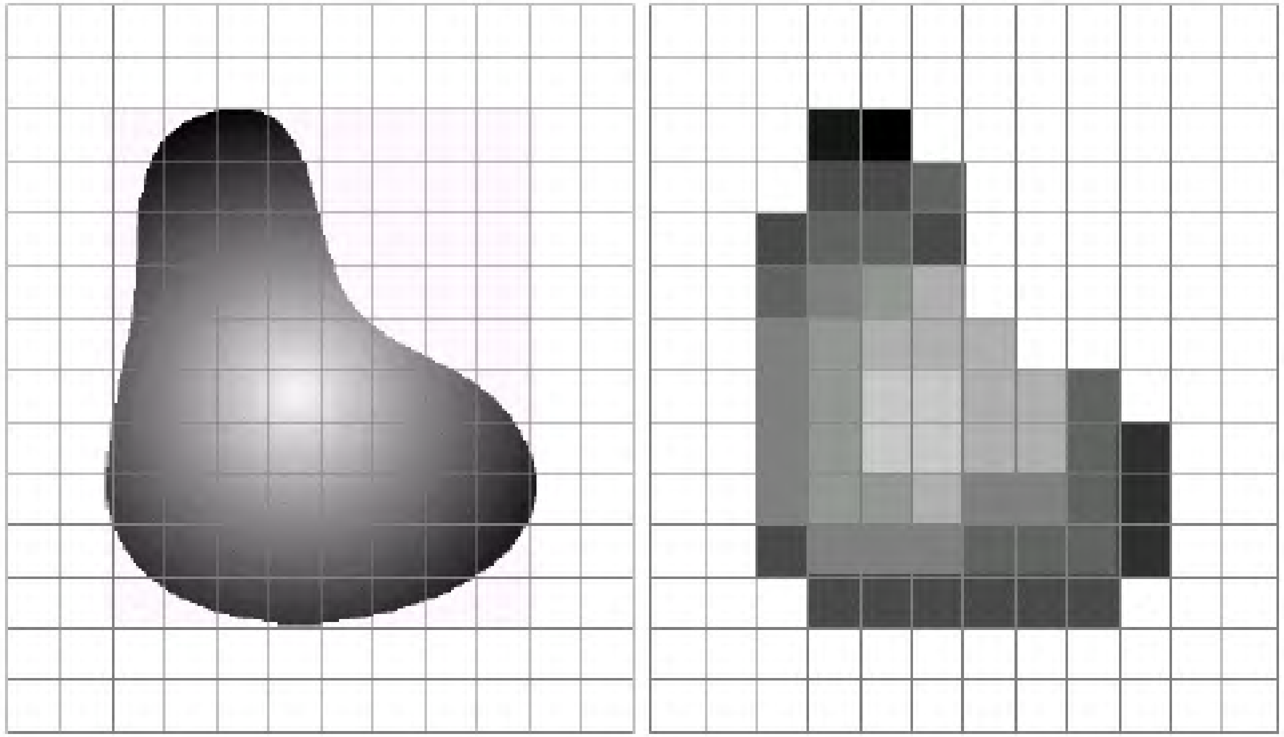
Image sensing pipeline



Digital Camera

- A digital camera replaces film with a sensor array
  - Each cell in the array is light-sensitive diode （光敏二极管） that converts photons to electrons
  - Two common types
    - ✓ Charge Coupled Device (CCD)
    - ✓ CMOS

Sensor Array



a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

CMOS sensor

Sampling and Quantization

- Shannon's Sampling Theorem

$$f_s \geq 2f_{\max}$$

0-255

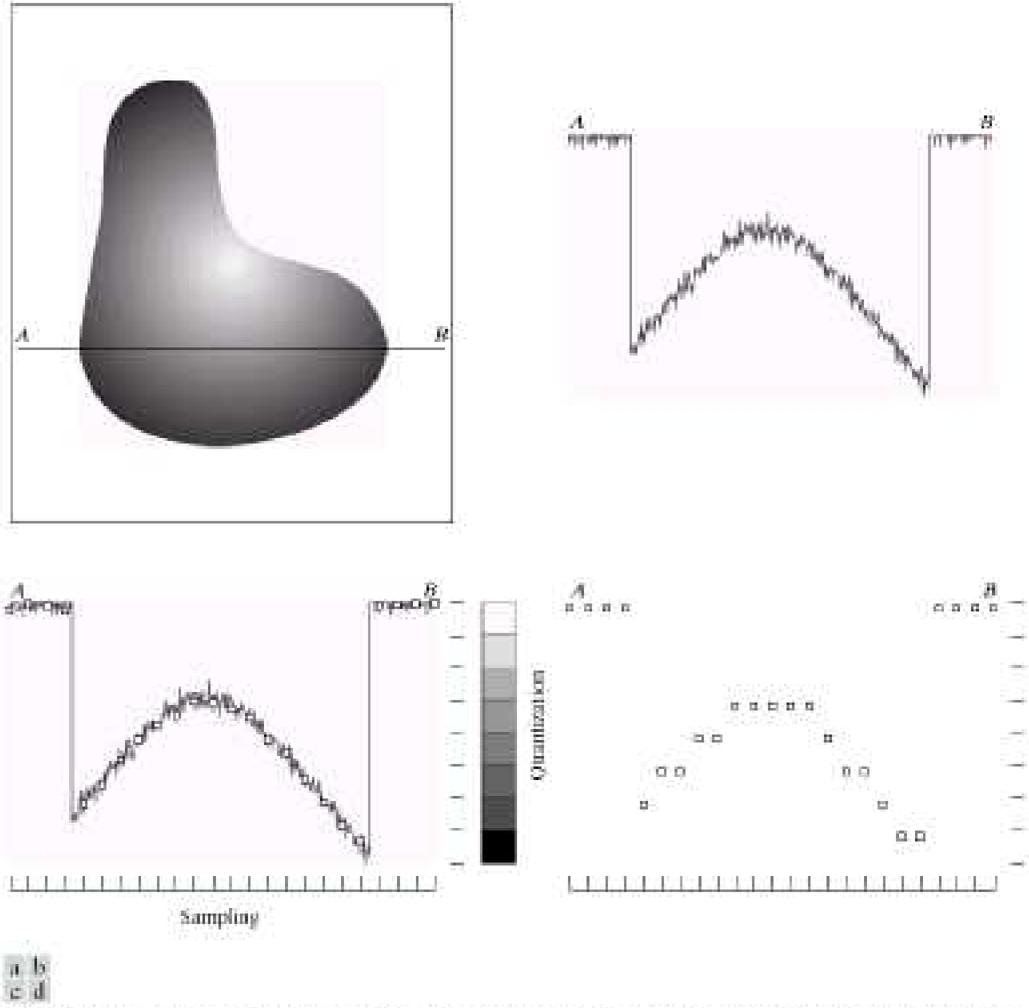
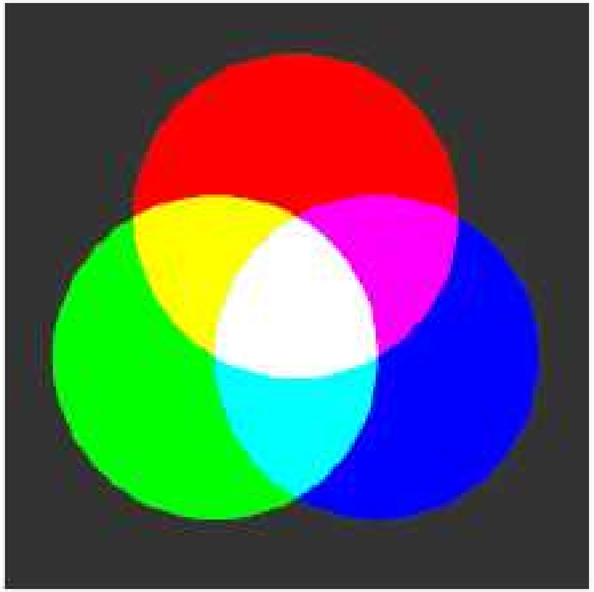


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Color

- Primary and secondary colors



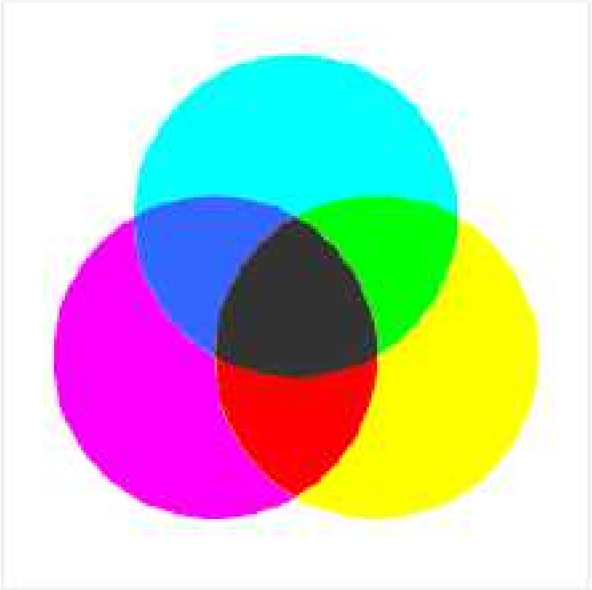
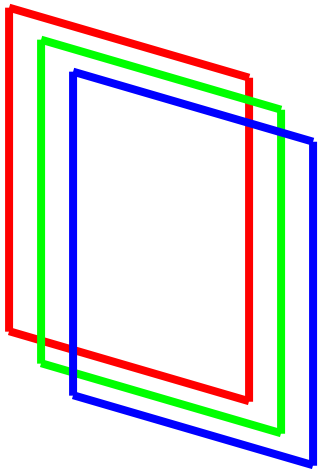
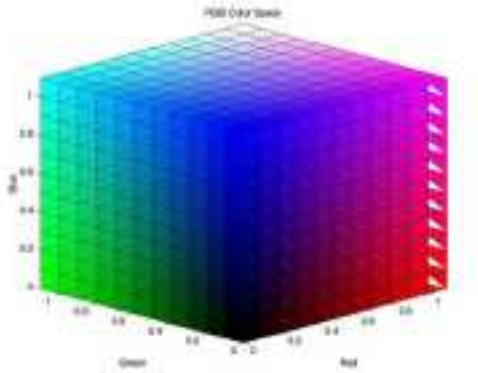


Image: three matrices



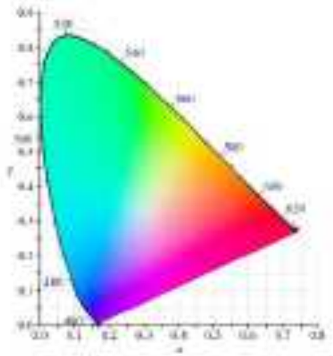
Color Spaces

- RGB



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

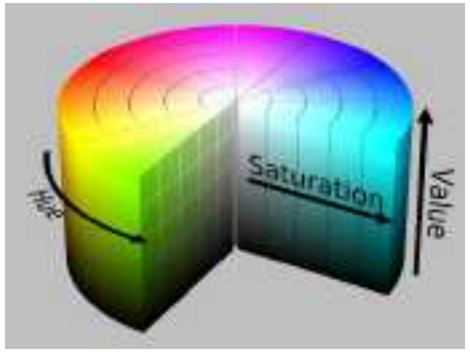
- CIE XYZ



Luminance

$$x = \frac{X}{X+Y+Z}, \quad y = \frac{Y}{X+Y+Z}, \quad z = \frac{Z}{X+Y+Z}$$

- HSV
  - ➢ Hue
  - ➢ Saturation
  - ➢ Value



$$C = V \times S_{HSV}$$

$$H' = \frac{H}{60^\circ}$$

$$X = C \times (1 - |H' \bmod 2 - 1|)$$

$$m = V - C$$

$$(R_1, G_1, B_1) = \begin{cases} (0,0,0) & \text{if } H \text{ is undefined} \\ (C, X, 0) & \text{if } 0 \le H' \le 1 \\ (X, C, 0) & \text{if } 1 < H' \le 2 \\ (0, C, X) & \text{if } 2 < H' \le 3 \\ (0, X, C) & \text{if } 3 < H' \le 4 \\ (X, 0, C) & \text{if } 4 < H' \le 5 \\ (C, 0, X) & \text{if } 5 < H' \le 6 \end{cases}$$

$$(R, G, B) = (R_1 + m, G_1 + m, B_1 + m)$$

Color Filter Arrays

- Color filter array layout
- Interpolated pixel values
  - ➢ The luminance signal is mostly determined by green values
  - ➢ The visual system is much more sensitive to high frequency detail in luminance than in chrominance

G	R	G	R
B	G	B	G
G	R	G	R
B	G	B	G

rGb	Rgb	rGb	Rgb
rgB	rGb	rgB	rGb
rGb	Rgb	rGb	Rgb
rgB	rGb	rgB	rGb

Conclusions

Conclusions

- Vanishing points and vanishing lines
- Pinhole camera model and camera projection matrix
- Homogeneous coordinates
- Digital camera

Thanks

zhengf@sustc.edu.cn