Computer Vision

CS308

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SUSTech CS Vision Intelligence and Perception

Week 5

南方科技大学

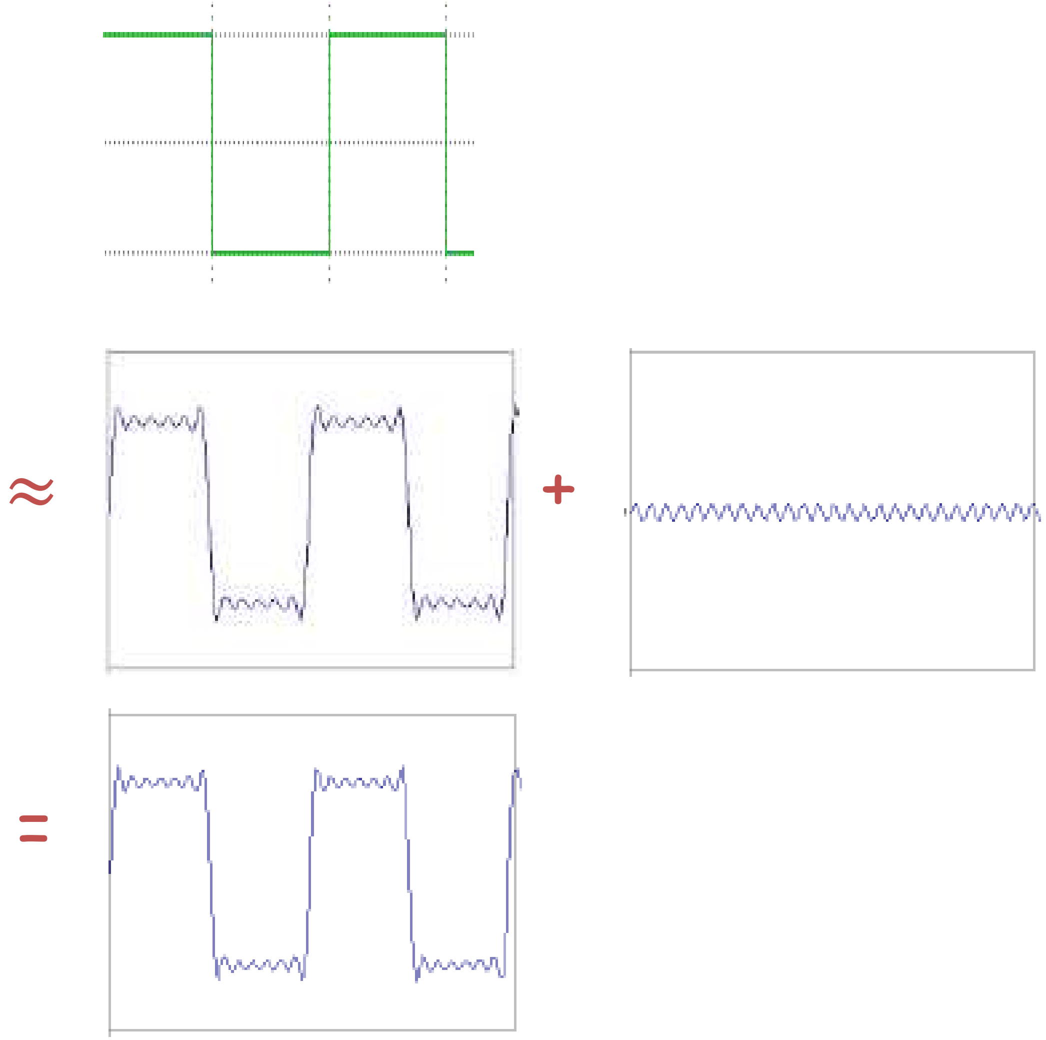
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

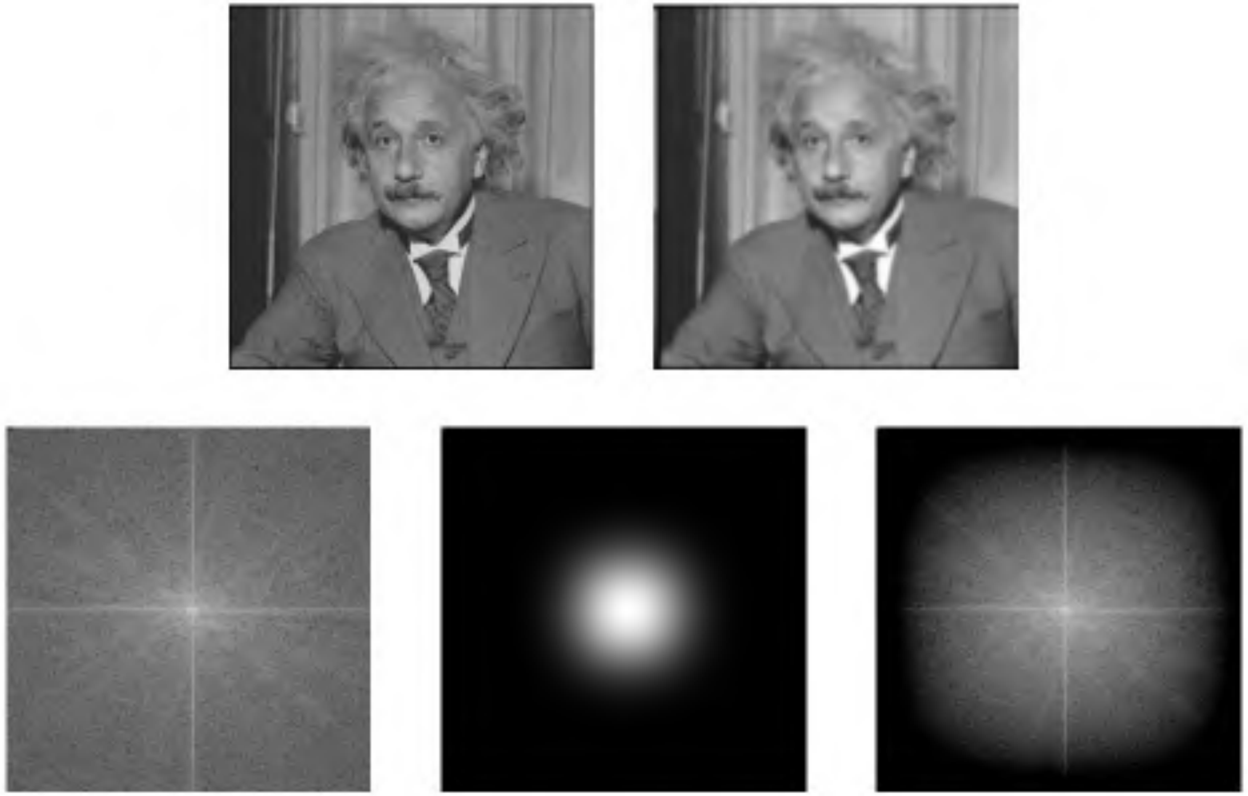
Content

- Brief Review
- Points Detection
- Points Descriptor
- Points Matching

Brief Review

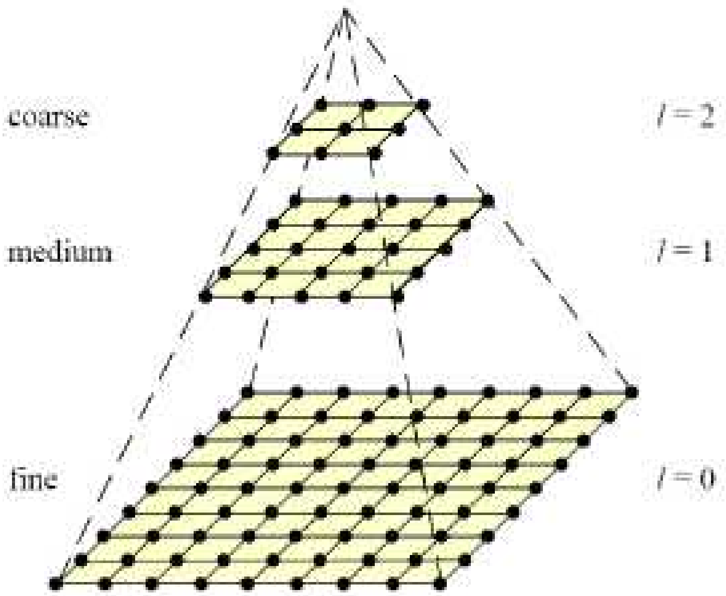
Review

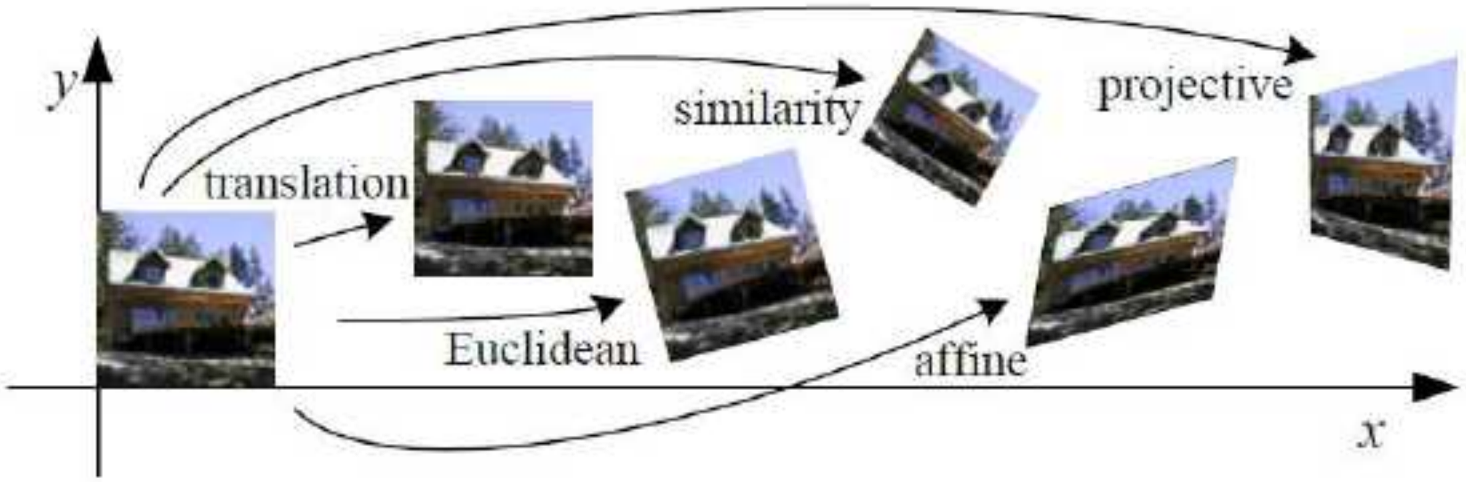




Smoothing

Review

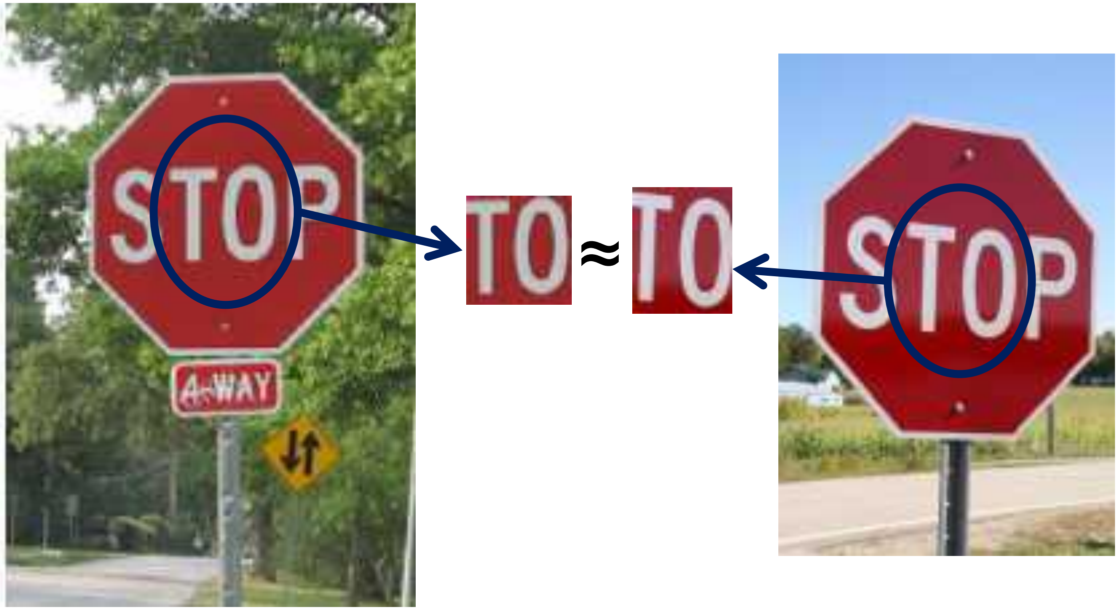

coarse
medium
fine
$l = 2$
$l = 1$
$l = 0$

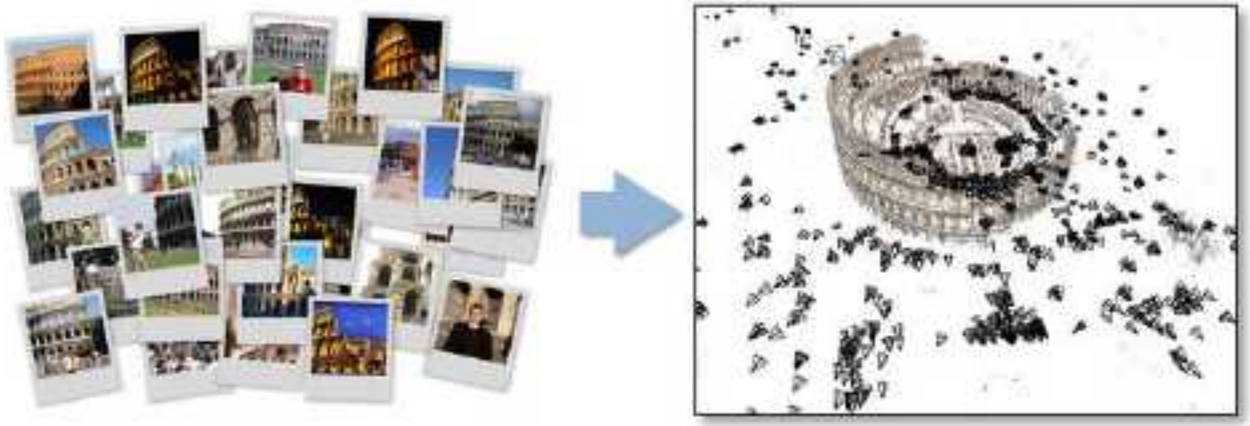

y
x
translation
similarity
projective
Euclidean
affine

Points

Correspondence Across Views

- Correspondence: matching points, patches, edges, or regions across images





Example: structure from motion

How to confirm that the two images are similar with each other?

Applications

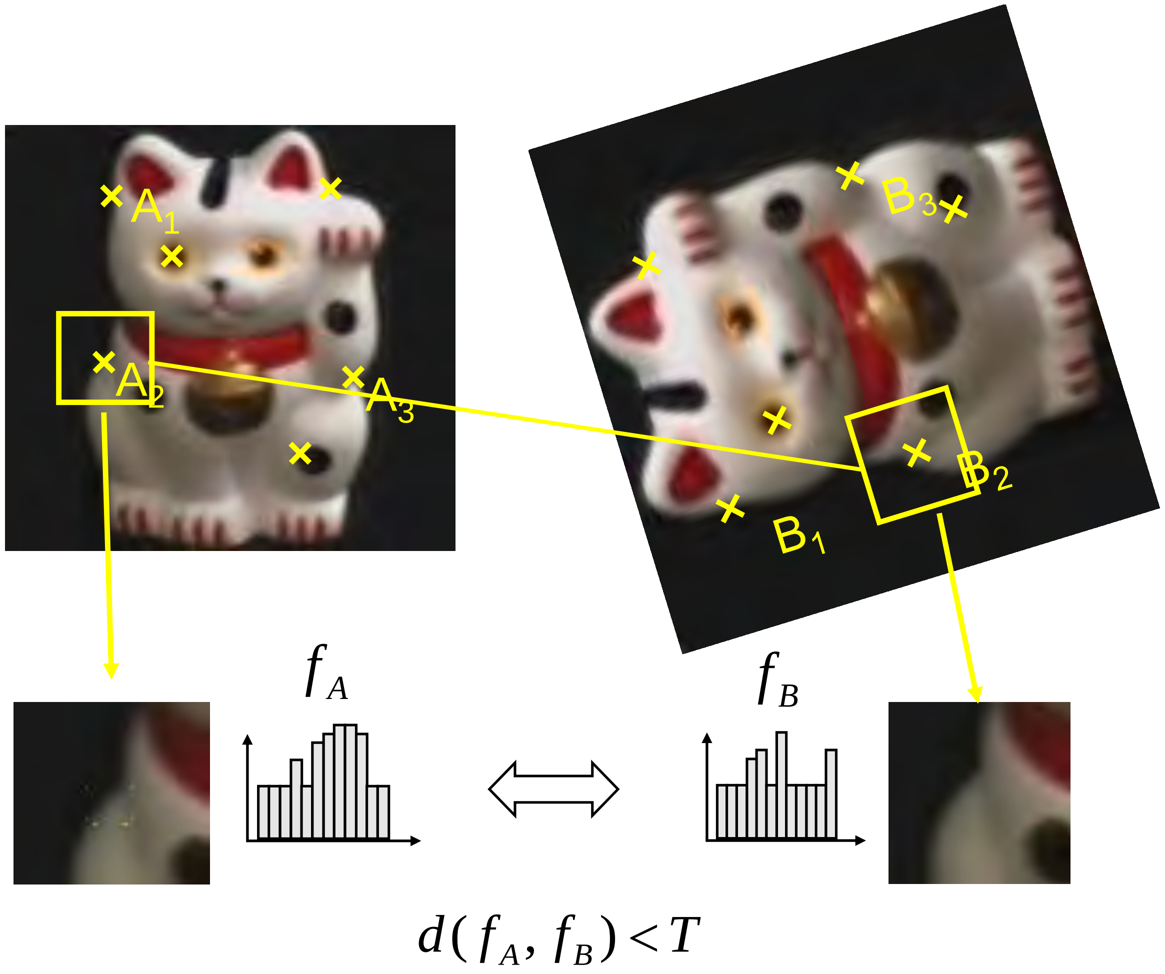
- Feature points are used for:
  - Image alignment
  - 3D reconstruction
  - Motion tracking
  - Indexing and database retrieval
  - Object recognition

An Example

- Motivation: panorama stitching
  - ➢ We have two images – how do we combine them?

Overview of Keypoint Matching

- Steps
  - Find a set of distinctive keypoints
  - Define a region around each keypoint
  - Compute a local descriptor from the region
  - Match local descriptors
- Goals
  - Detect points that are repeatable and distinctive



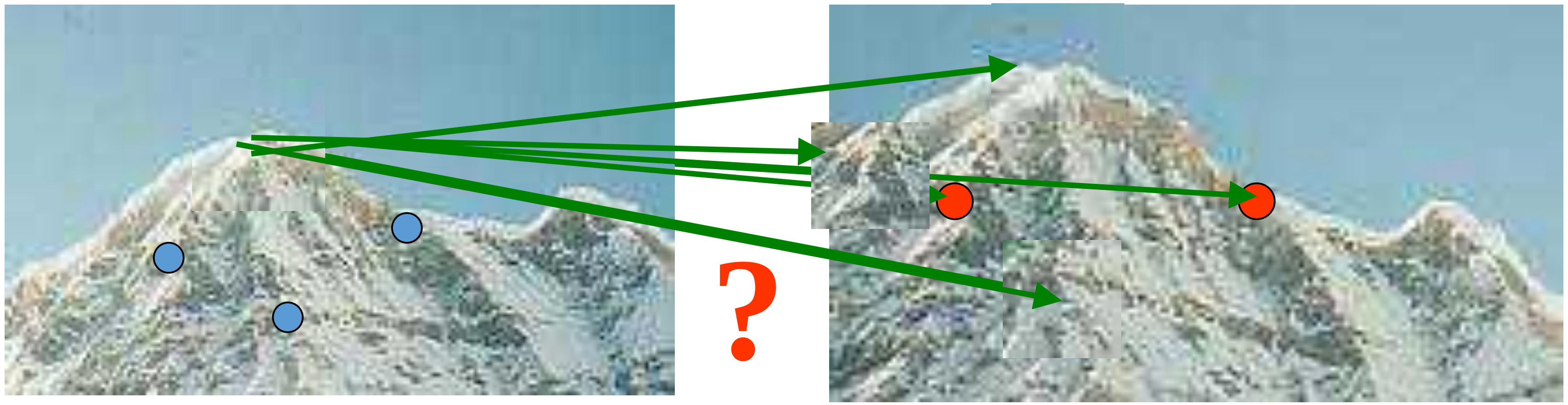
Interesting Points

- Goal: interest operator **repeatability**
  - Detect (at least some of) the same points in both images.
  - Run the detection procedure independently per image

No chance to find true matches!

Interesting Points

- Goal: descriptor **distinctiveness**
  - ➢ Reliably determine which point goes with which
  - ➢ Must provide some invariance to geometric and photometric differences between the two views



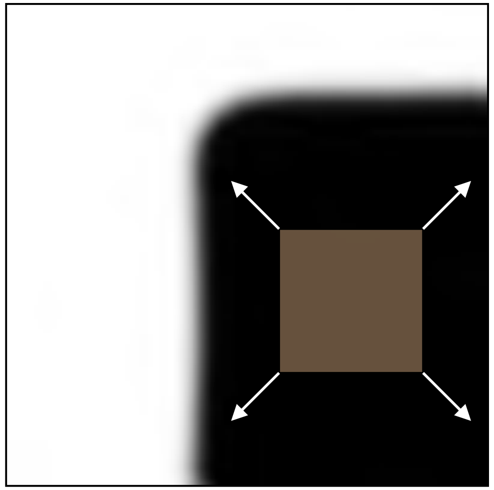
Interesting Points

- What are the characteristics of good features?
  - ➢ Repeatability
    - ✓ The same feature can be found in several images despite geometric and photometric transformations
  - ➢ Saliency
    - ✓ Each feature is distinctive
  - ➢ Compactness and efficiency
    - ✓ Many fewer features than image pixels
  - ➢ Locality
    - ✓ Relatively small area of the image
    - ✓ Robust to clutter and occlusion

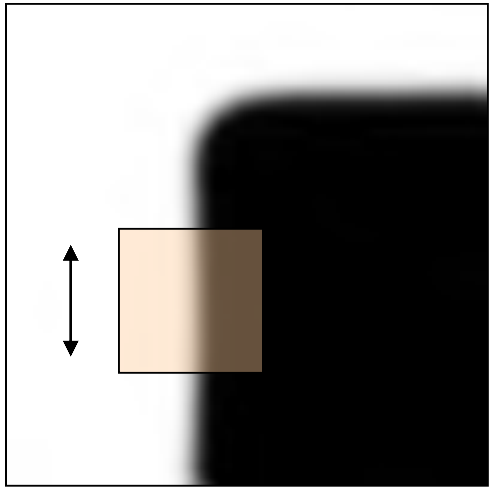

Points Detection

Corner Detection: Basic Idea

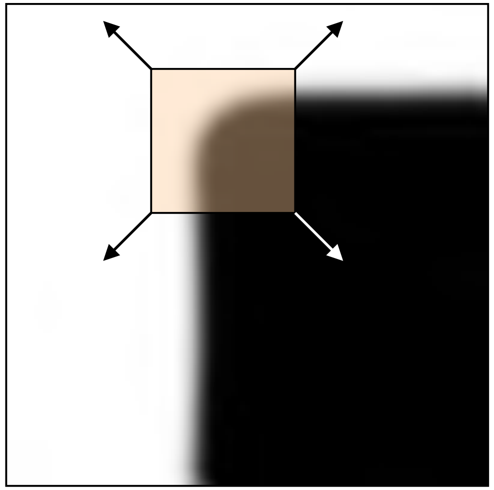
- We should easily recognize the point by looking through a small window
- **Shifting a window in any direction should give a large change in intensity**



"flat" region: no change in all directions



"edge": no change along the edge direction



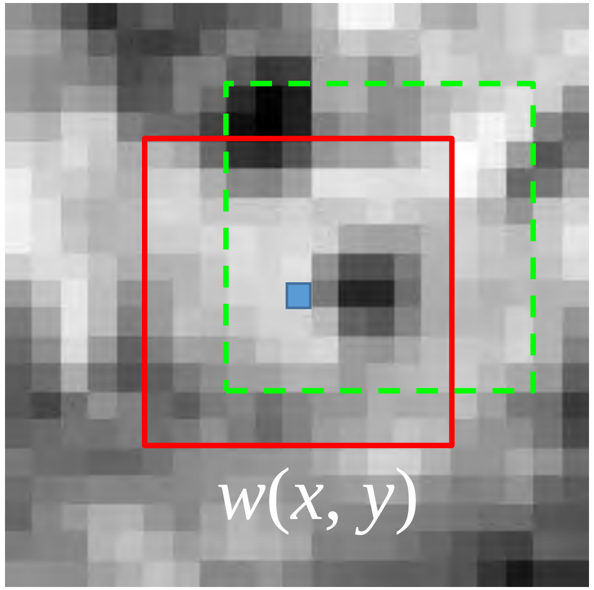
"corner": significant change in all directions

Corner Detection: Mathematics

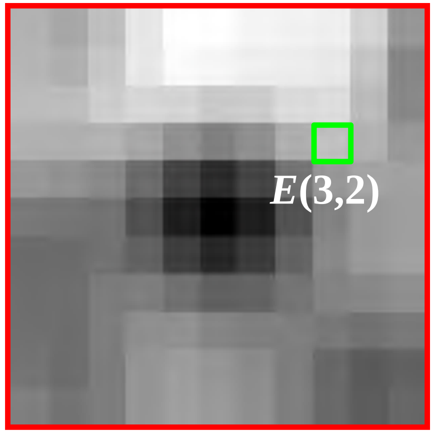
- Change in appearance of window w(x, y) for the shift [u, v]:

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

$I(x, y)$



$E(u, v)$



Corner Detection: Mathematics

- Change in appearance of window w(x, y) for the shift [u, v]:

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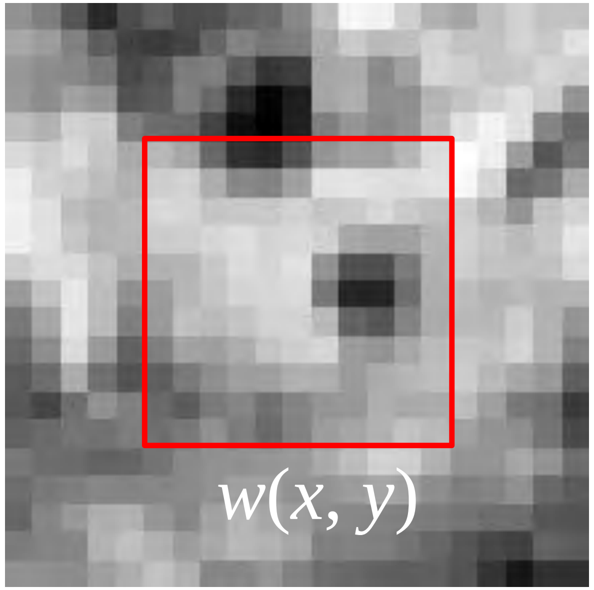
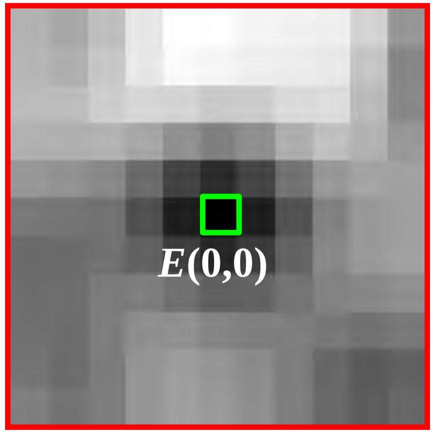


$E(u, v)$



Corner Detection: Mathematics

- Change in appearance of window w(x, y) for the shift [u, v]:

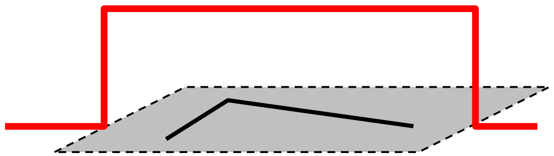
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

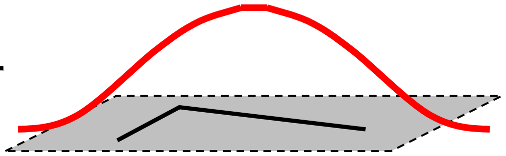
Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Corner Detection: Mathematics

- Change in appearance of window w(x, y) for the shift [u, v]:

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

- We want to find out how this function behaves for small shifts

$E(u, v)$

Corner Detection: Mathematics

- Change in appearance of window w(x, y) for the shift [u, v]:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

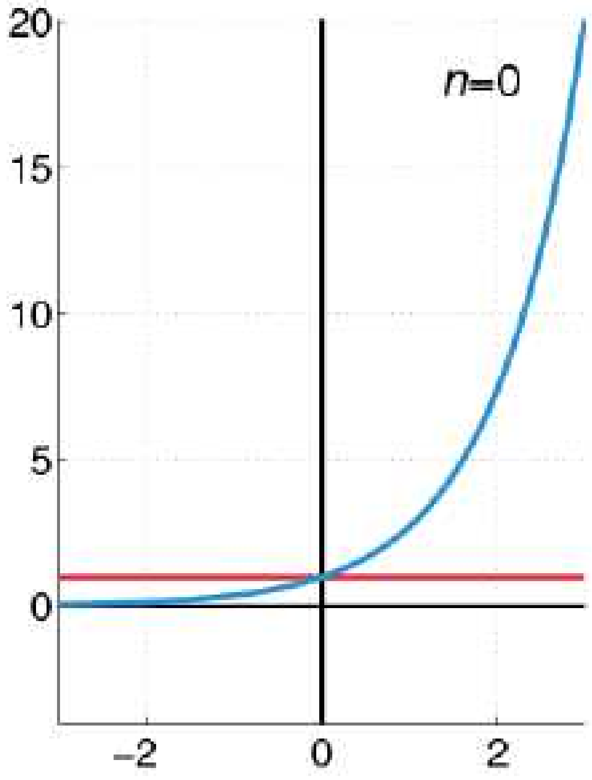
- We want to find out how this function behaves for small shifts
- But this is very slow to compute naively
  - ➢ O(window_width^2 * shift_range^2 * image_width^2)
  - ➢ O(11^2 * 11^2 * 600^2) = 5.2 billion of these
  - ➢ 14.6 thousand per pixel in your image

Corner Detection: Mathematics

- Recall Taylor series expansion. A function f can be approximated around point a as

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots.$$

Approximation of
$f(x) = e^x$
centered at f(0)



Corner Detection: Mathematics

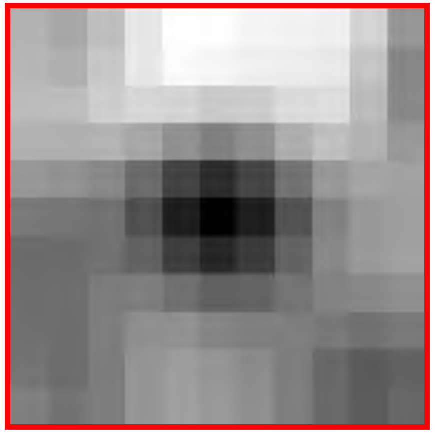
- Local quadratic approximation of E(u,v) in the neighborhood of (0,0) is given by the second-order Taylor expansion:

$$E(u,v) \approx E(0,0) + [u \;\; v]\begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2}[u \;\; v]\begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

Always 0

First derivative is 0

$E(u, v)$



Corner Detection: Mathematics

- Second-order Taylor expansion of E(u, v) about (0, 0):

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

$$\begin{aligned}
E(u,v) &\approx E(0,0) \\
&+ \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} \\
&+ \frac{1}{2} \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}
\end{aligned}$$

$$E_u(u,v)=\sum_{x,y} 2w(x,y)\left[I(x+u,y+v)-I(x,y)\right]I_x(x+u,y+v)$$

$$\begin{aligned}
E_{uu}(u,v) &= \sum_{x,y} 2w(x,y) I_x(x+u,y+v) I_x(x+u,y+v) \\
&+ \sum_{x,y} 2w(x,y)\left[I(x+u,y+v)-I(x,y)\right]I_{xx}(x+u,y+v)
\end{aligned}$$

$$\begin{aligned}
E_{uv}(u,v) &= \sum_{x,y} 2w(x,y) I_y(x+u,y+v) I_x(x+u,y+v) \\
&+ \sum_{x,y} 2w(x,y)\left[I(x+u,y+v)-I(x,y)\right]I_{xy}(x+u,y+v)
\end{aligned}$$

Corner Detection: Mathematics

- Second-order Taylor expansion of E(u, v) about (0,0):

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

$$E(u,v) \approx E(0,0) + \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(0,0)=0$$

$$E_u(0,0)=0$$

$$E_v(0,0)=0$$

$$E_{uu}(0,0)=\sum_{x,y} 2w(x,y)I_x(x,y)I_x(x,y)$$

$$E_{vv}(0,0)=\sum_{x,y} 2w(x,y)I_y(x,y)I_y(x,y)$$

$$E_{uv}(0,0)=\sum_{x,y} 2w(x,y)I_x(x,y)I_y(x,y)$$

Corner Detection: Mathematics

- The quadratic approximation simplifies to

$$E(u,v) \approx [u \ \ v] \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2(x,y) & \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y) \\ \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y) & \sum_{x,y} w(x,y) I_y^2(x,y) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(u,v) \approx [u \ \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix}$$

- M is a second moment matrix computed from image derivatives:

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y] = \sum \nabla I (\nabla I)^T$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Corner Detection: Mathematics

- **Corners** as distinctive interest points

$$M = \sum w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

- 2 x 2 matrix of image derivatives (averaged in neighborhood of a point)

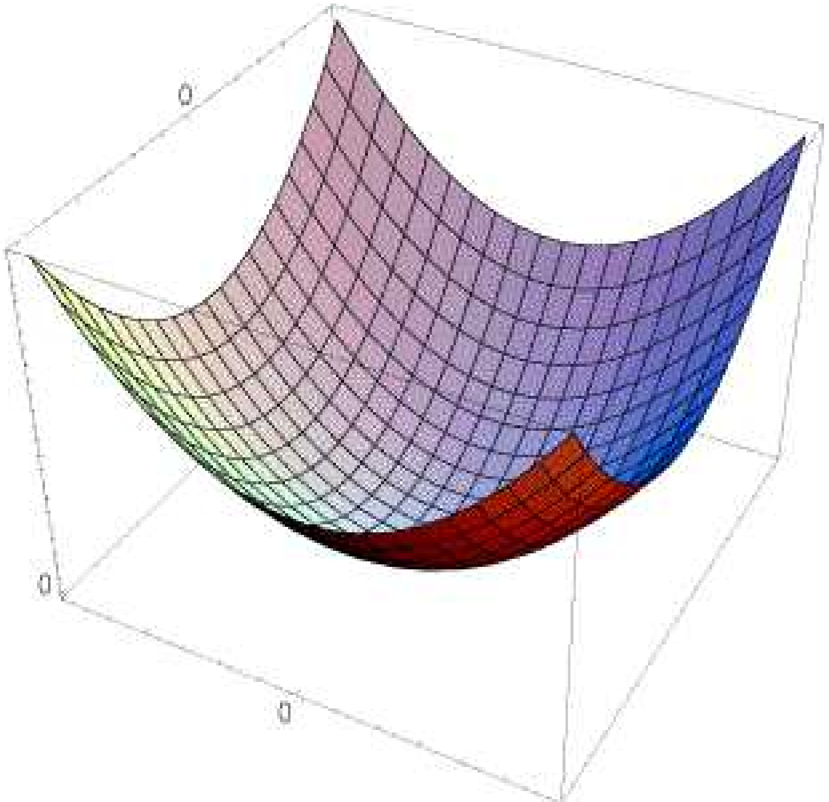
Notation: $I_x \Leftrightarrow \frac{\partial I}{\partial x}$ $I_y \Leftrightarrow \frac{\partial I}{\partial y}$ $I_x I_y \Leftrightarrow \frac{\partial I}{\partial x}\frac{\partial I}{\partial y}$

Corner Detection: Mathematics

- Interpreting the second moment matrix
  - The surface E(u,v) is locally approximated by a quadratic form
  - Let's try to understand its shape

$$E(u,v) \approx [u \ \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix}$$

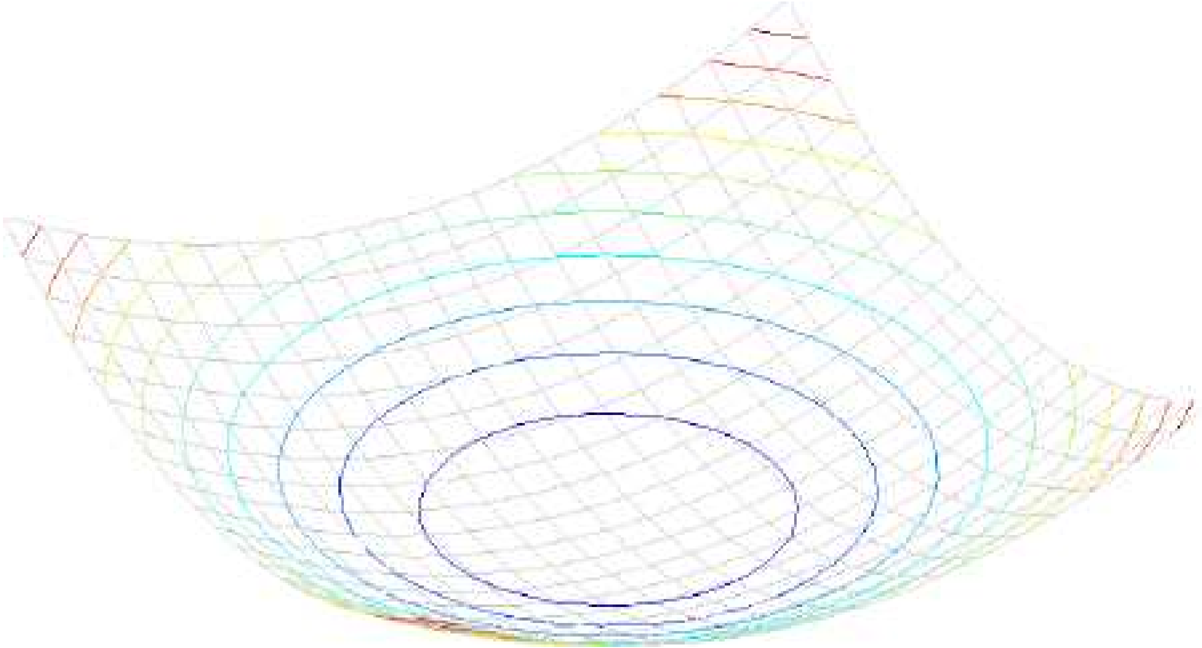
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Corner Detection: Mathematics

- Interpreting the second moment matrix
  - Consider a horizontal "slice" of E(u, v):
  - This is the equation of an ellipse

$$[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



Corner Detection: Mathematics

- Interpreting the second moment matrix
  - First, consider the axis-aligned case (gradients are either horizontal or vertical)

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \quad M = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

  - If either λ is close to 0, then this is not a corner, so look for locations where both are large

Corner Detection: Mathematics

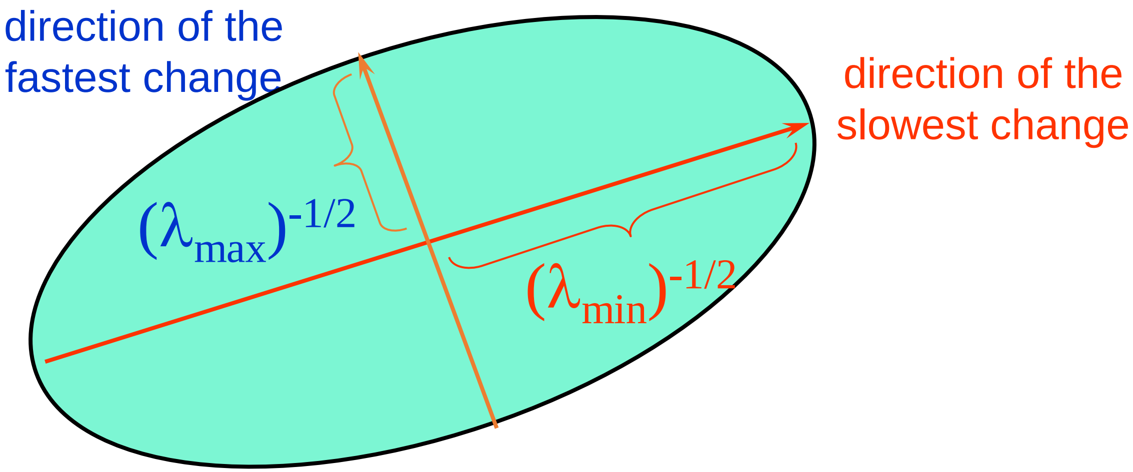
- Interpreting the second moment matrix
  - Consider a horizontal "slice" of E(u, v):
  - This is the equation of an ellipse

$$[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

- Diagonalization of M:

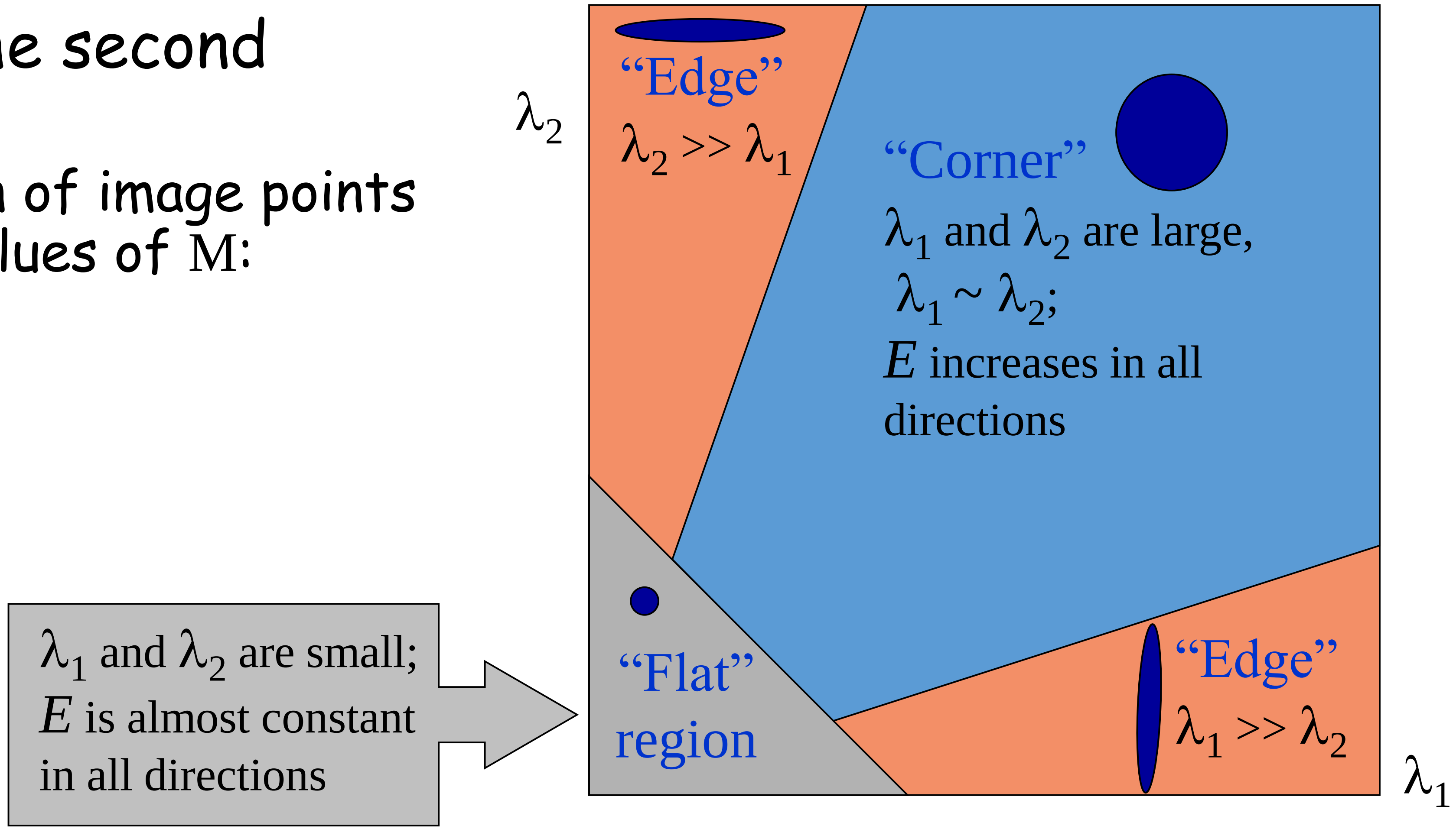
$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

  - The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by R



Corner Detection: Mathematics

- Interpreting the second moment matrix
  - Classification of image points using eigenvalues of M:

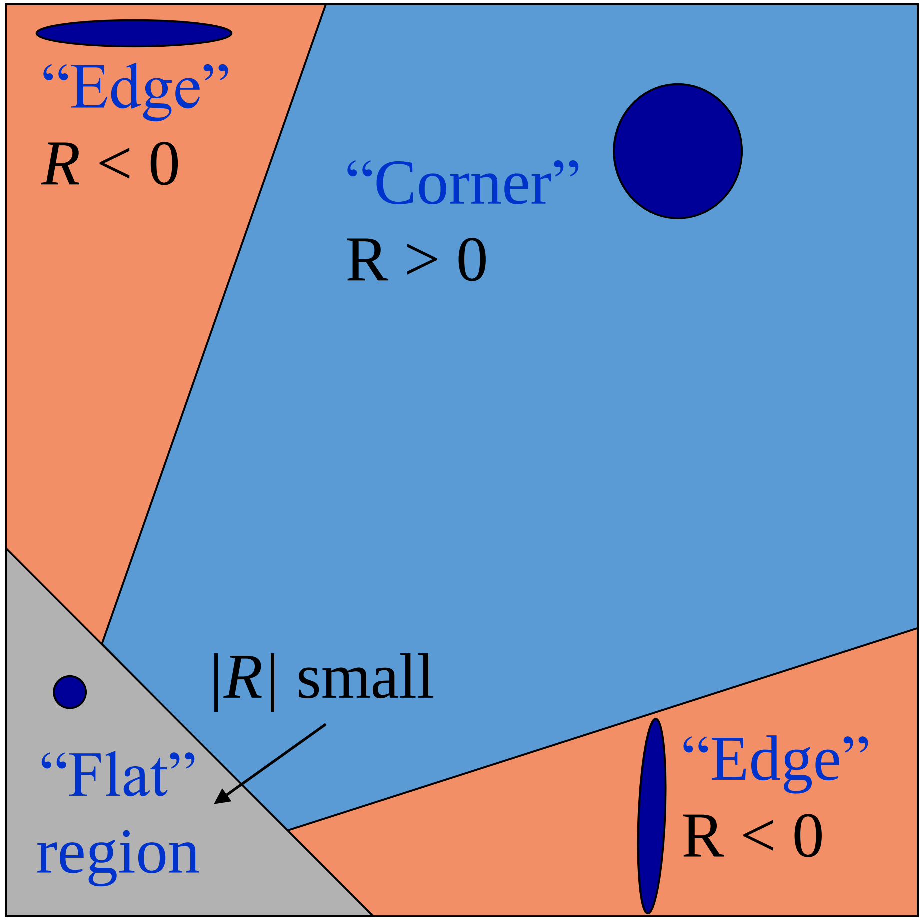


Corner Detection: Mathematics

- Corner response function

α: constant (0.04 to 0.06)

$$R = \det(M) - \alpha \operatorname{trace}(M)^2$$
$$= \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

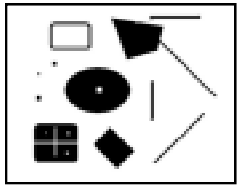


Harris Corner Detector

- Steps
  - Compute M matrix for each image window to get their cornerness scores
  - Find points whose surrounding window gave large corner response ($R >$ threshold)
  - Take the points of local maxima, i.e., perform non-maximum suppression

C.Harris and M.Stephens. [“A Combined Corner and Edge Detector.”]
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

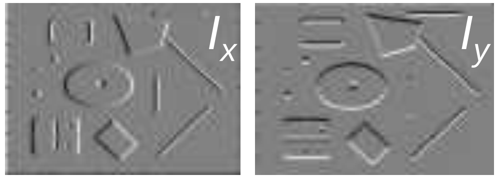
Harris Corner Detector: Steps



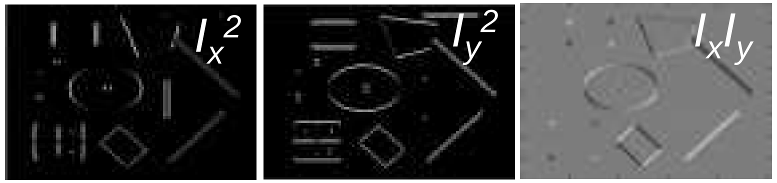
- Second moment matrix

$$\mu(\sigma_I,\sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

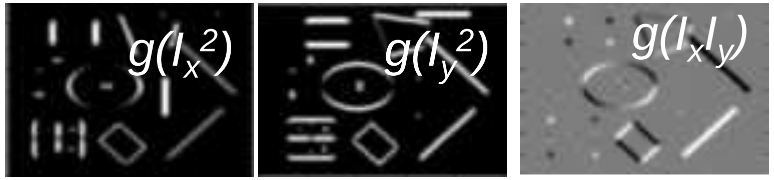
1. Image derivatives (optionally, blur first)



2. Square of derivatives



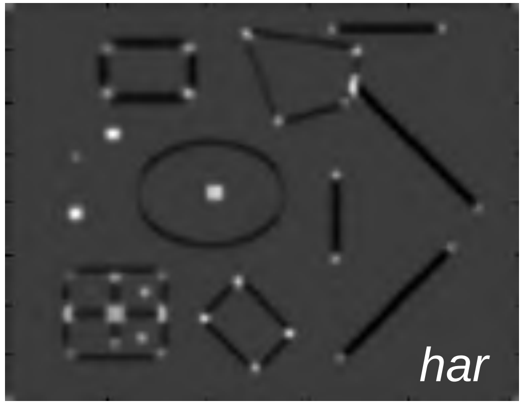
3. Gaussian filter $g(\sigma_I)$



4. Cornerness function – both eigenvalues are strong

$$har = \det[\mu(\sigma_I,\sigma_D)] - \alpha[\mathrm{trace}(\mu(\sigma_I,\sigma_D))^2] =$$

$$g(I_x^2)g(I_y^2) - [g(I_xI_y)]^2 - \alpha[g(I_x^2)+g(I_y^2)]^2$$

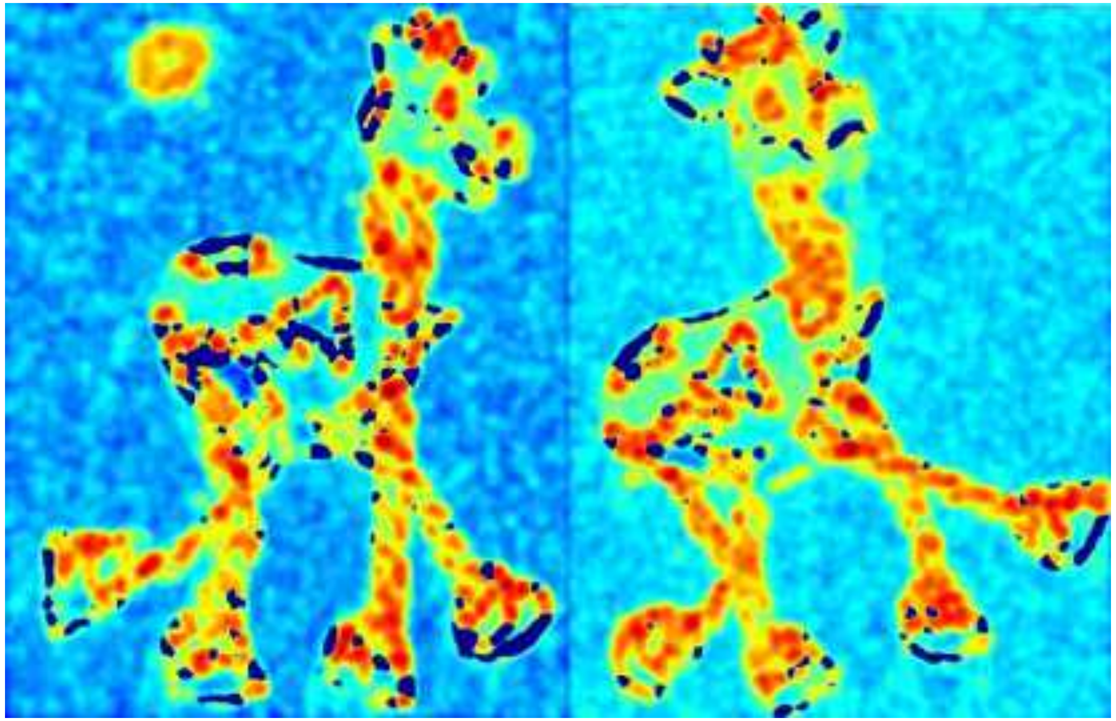


5. Non-maxima suppression

Harris Detector: Example

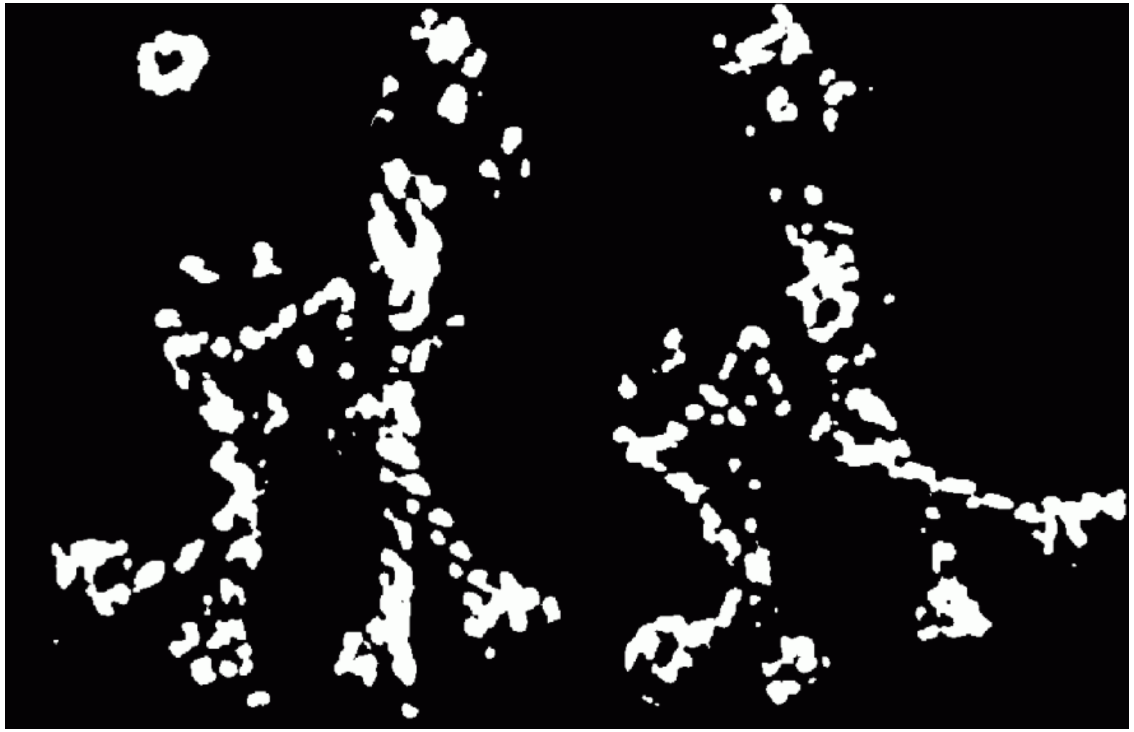
- Compute corner response R
  - ➢ R_harris(x,y) = det(M) – a*trace(M)

Repeatability



Harris Detector: Example

- Find points with large corner response: $R > \text{threshold}$

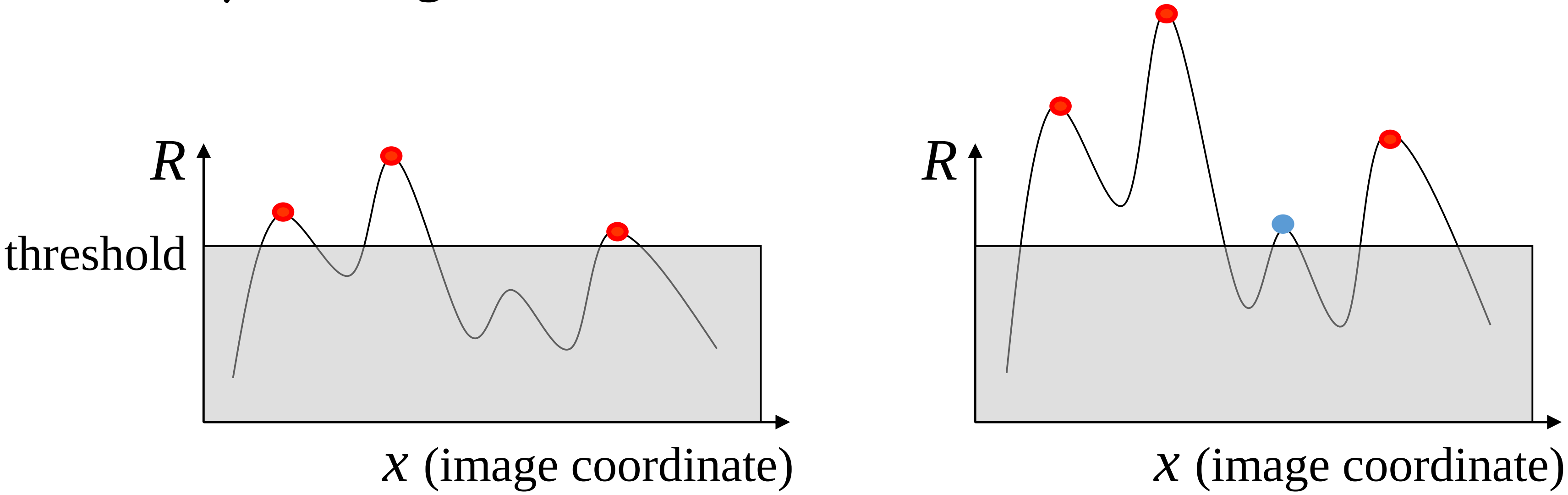


Invariance and Covariance

- We want corner locations to be invariant to photometric transformations and covariant to geometric transformations
  - Invariance: image is transformed and corner locations do not change
  - Covariance: if we have two transformed versions of the same image, features should be detected in corresponding locations

Affine Intensity Change

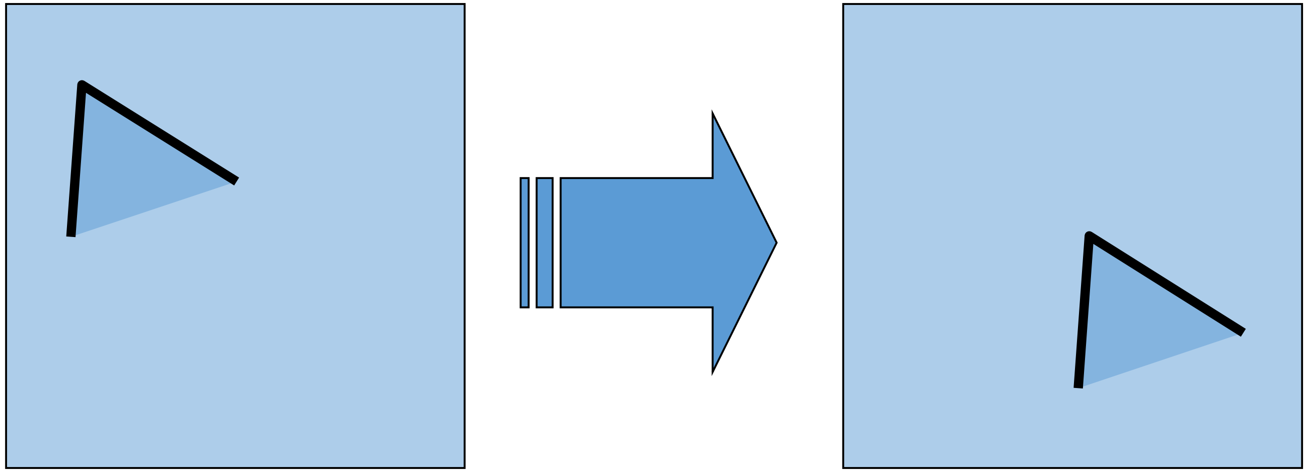
- Intensity change $I \rightarrow a\,I + b$
  - Only derivatives are used => invariance to intensity shift: $I \rightarrow I + b$
  - Intensity scaling: $I \rightarrow a\,I$



Partially invariant to affine intensity change

Image Translation

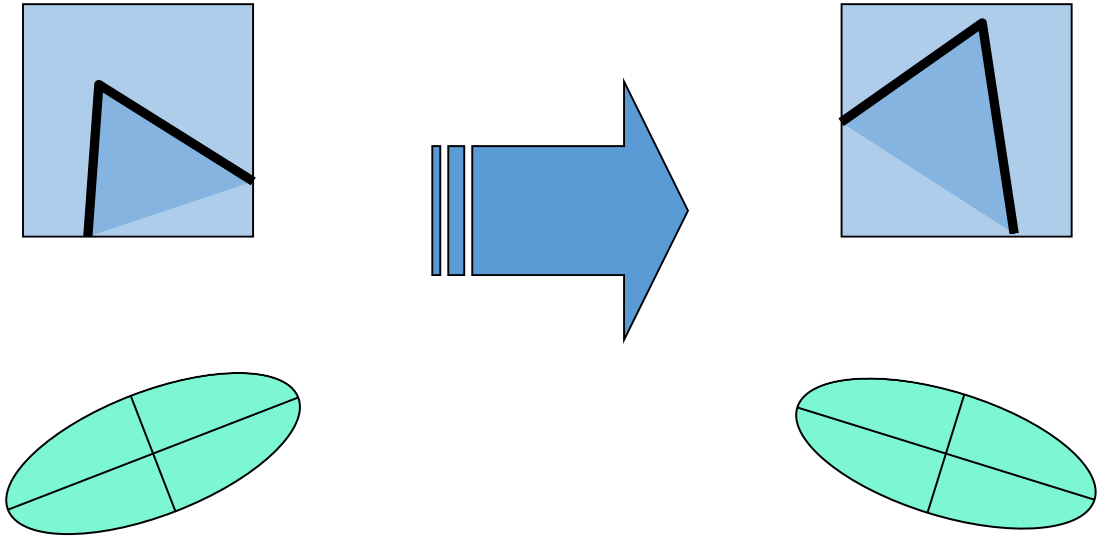
- Derivatives and window function are shift-invariant



Corner location is covariant w.r.t. translation

Image Rotation

- Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

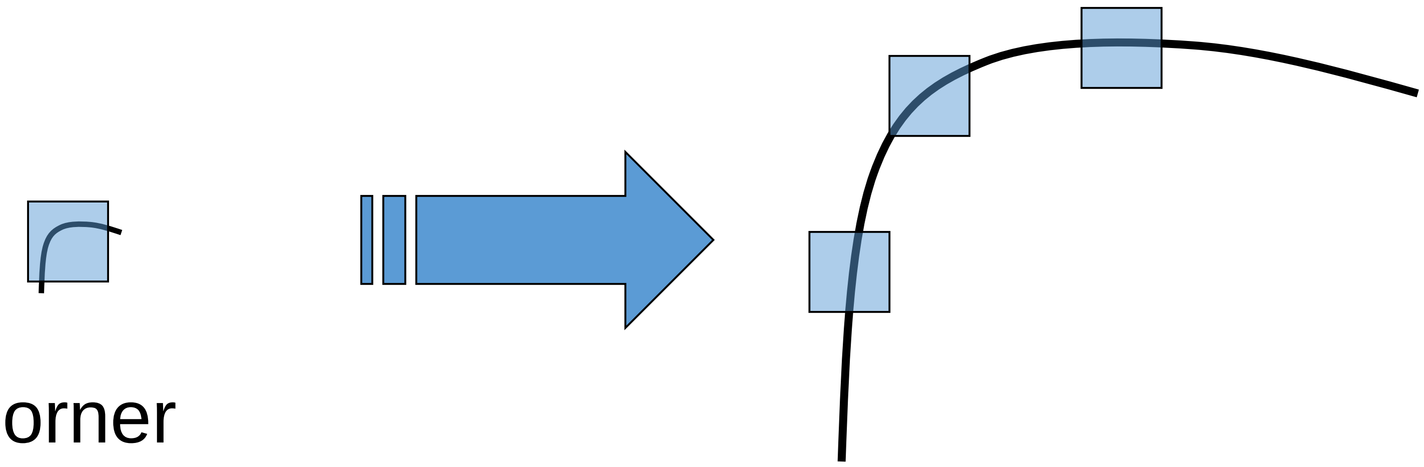


Corner location is covariant w.r.t. rotation

Scaling

- All points will be classified as edges

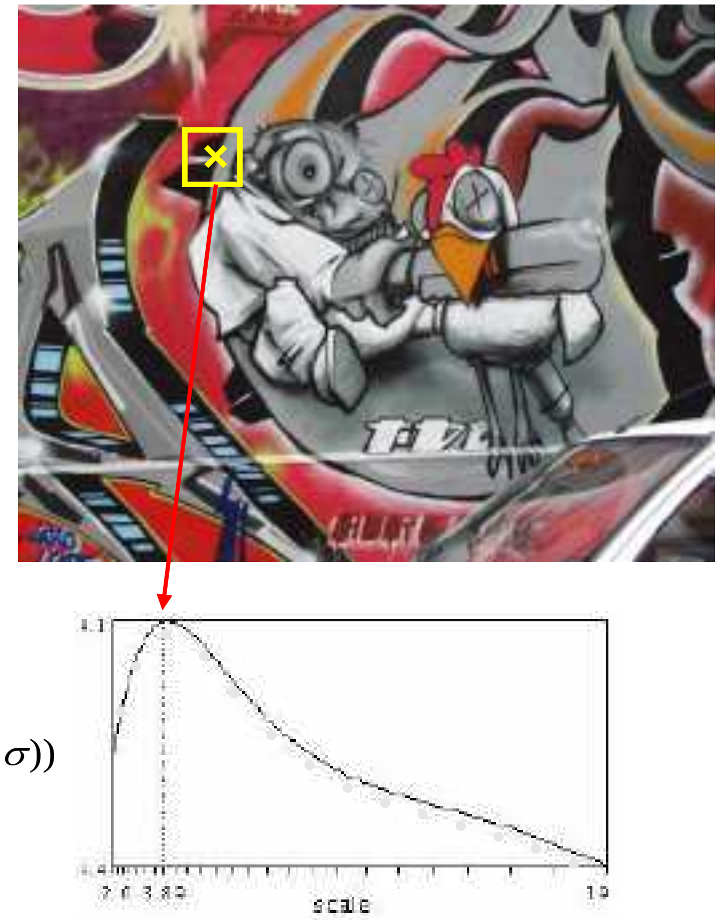
A Problem!



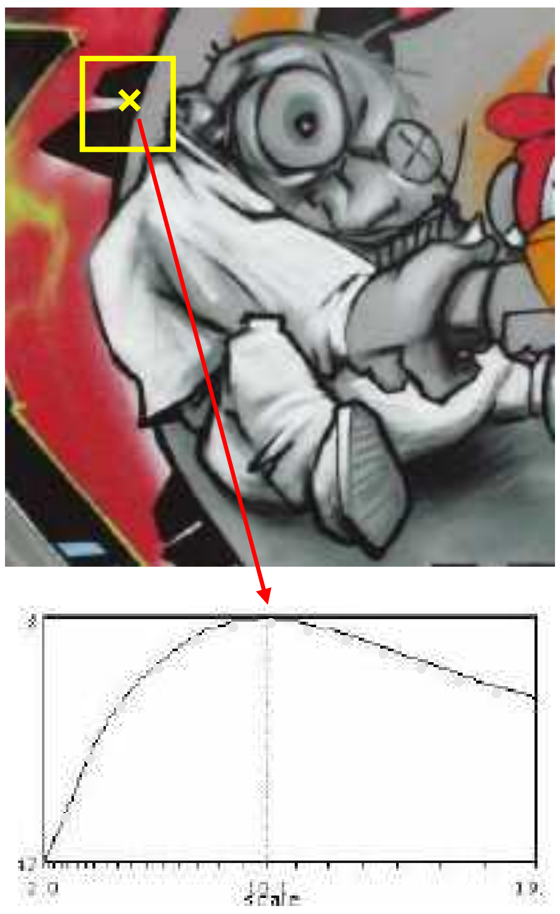
Corner location is not covariant to scaling!

Automatic Scale Selection

- Function responses for increasing scale (scale signature)



$f(I_{i_1 \ldots i_m}(x,\sigma))$



$f(I_{i_1 \ldots i_m}(x',\sigma'))$

Difference-of-Gaussian (DoG)

- **Laplace Operator**

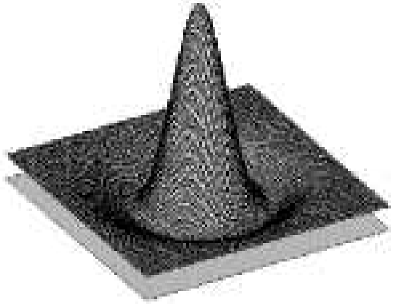
$$\triangle = \nabla \cdot \nabla = \nabla^2 = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

- **Laplacian of Gaussian (LoG)**

$$\triangle[G_\sigma(x,y) * f(x,y)] = [\triangle G_\sigma(x,y)] * f(x,y) = LoG * f(x,y)$$

$$LoG \triangleq \triangle G_\sigma(x,y) = \frac{\partial^2}{\partial x^2} G_\sigma(x,y) + \frac{\partial^2}{\partial y^2} G_\sigma(x,y) = \frac{x^2+y^2-2\sigma^2}{\sigma^4} e^{-(x^2+y^2)/2\sigma^2}$$

- **Difference of Gaussian (DoG)**



$$g_1(x,y) - g_2(x,y) = G_{\sigma_1} * f(x,y) - G_{\sigma_2} * f(x,y) = \underline{(G_{\sigma_1} - G_{\sigma_2})} * f(x,y) = DoG * f(x,y)$$

$$DoG \triangleq G_{\sigma_1} - G_{\sigma_2} = \frac{1}{\sqrt{2\pi}} \left(\frac{1}{\sigma_1} e^{-(x^2+y^2)/2\sigma_1^2} - \frac{1}{\sigma_2} e^{-(x^2+y^2)/2\sigma_2^2} \right)$$

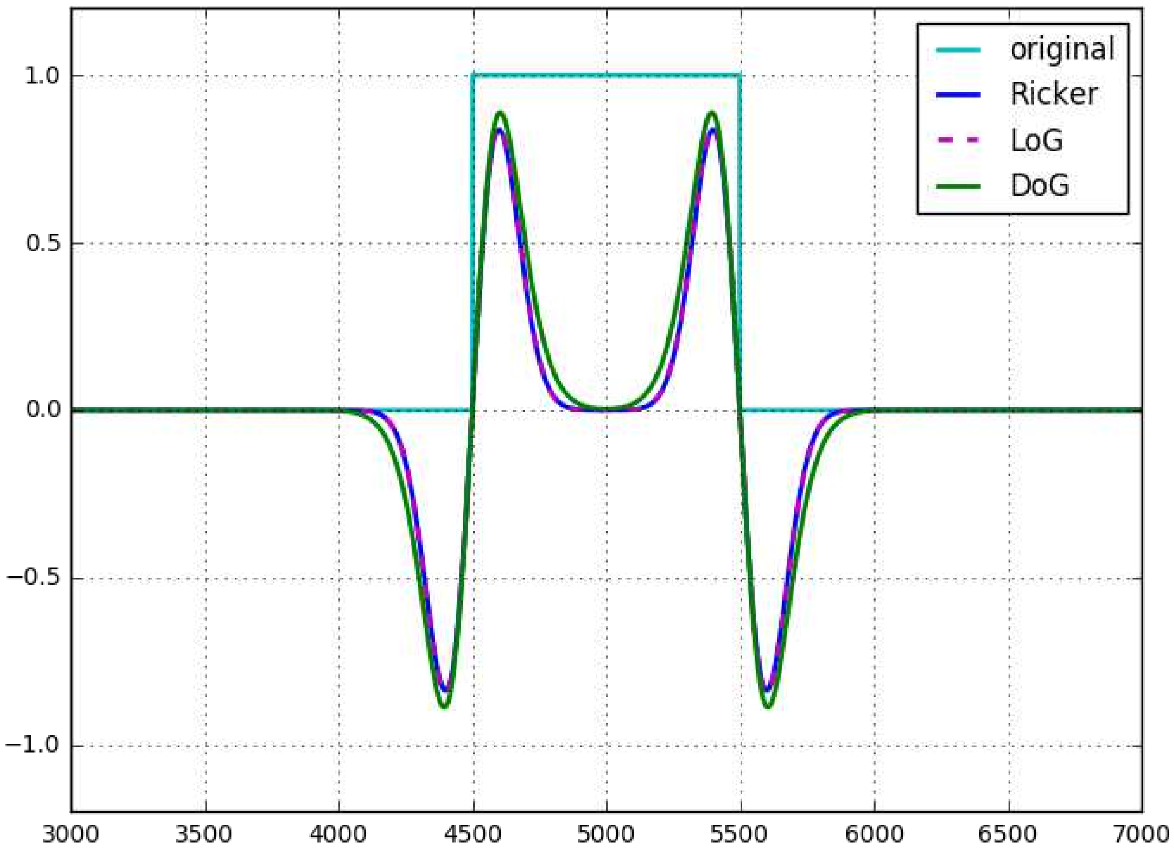
$$g_1(x,y) = \underline{G_{\sigma_1}}(x,y) * f(x,y)$$

$$g_2(x,y) = \underline{G_{\sigma_2}}(x,y) * f(x,y)$$

-

=

Difference-of-Gaussian (DoG)

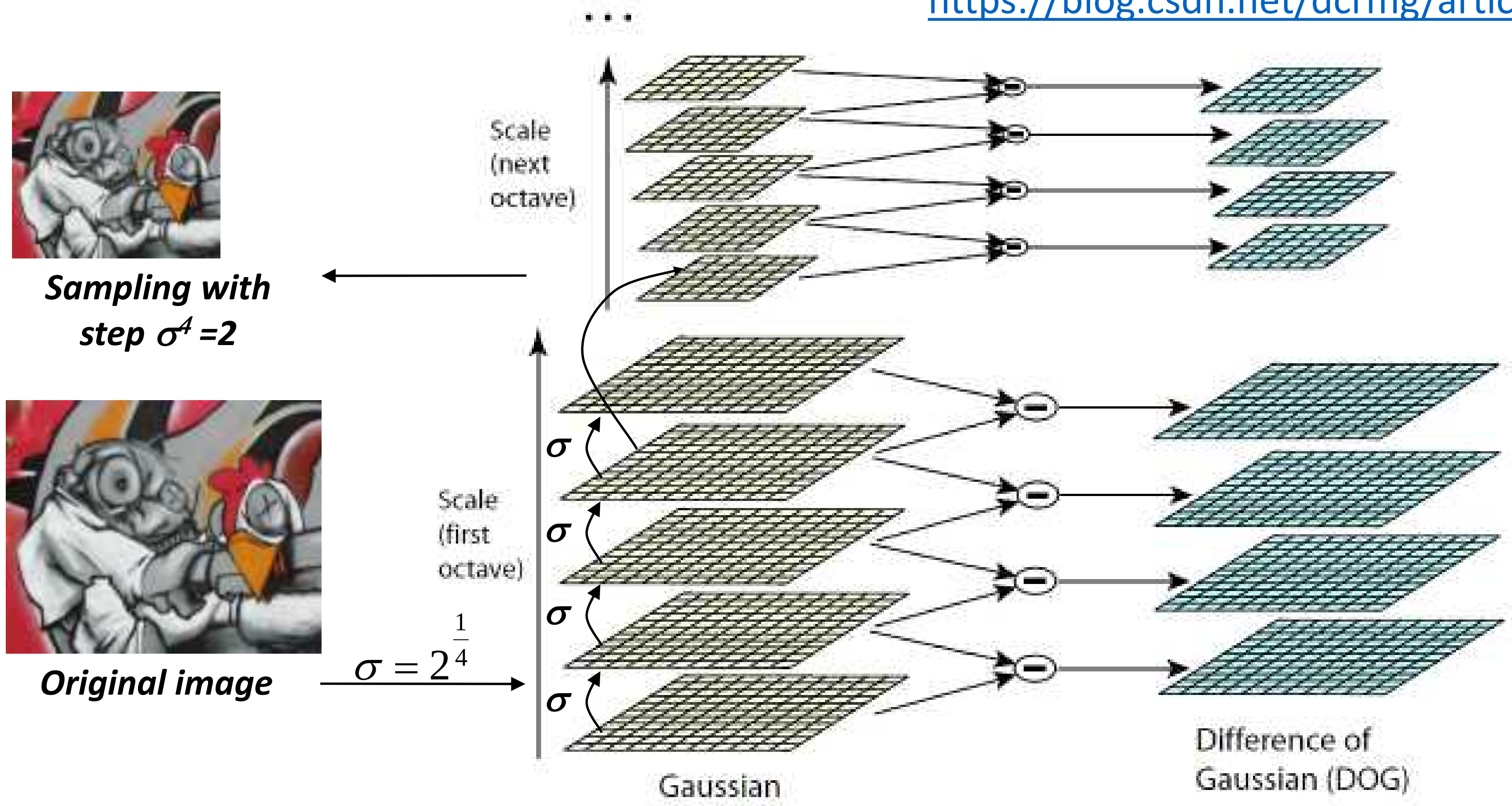


https://dsp.stackexchange.com/questions/37673/what-is-the-difference-between-difference-of-gaussian-laplace-of-gaussian-and

DoG – Efficient Computation

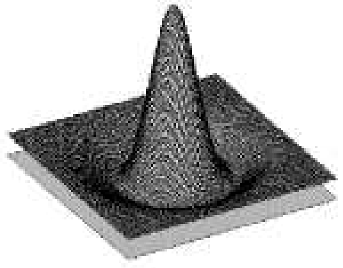
- Computation in Gaussian scale pyramid

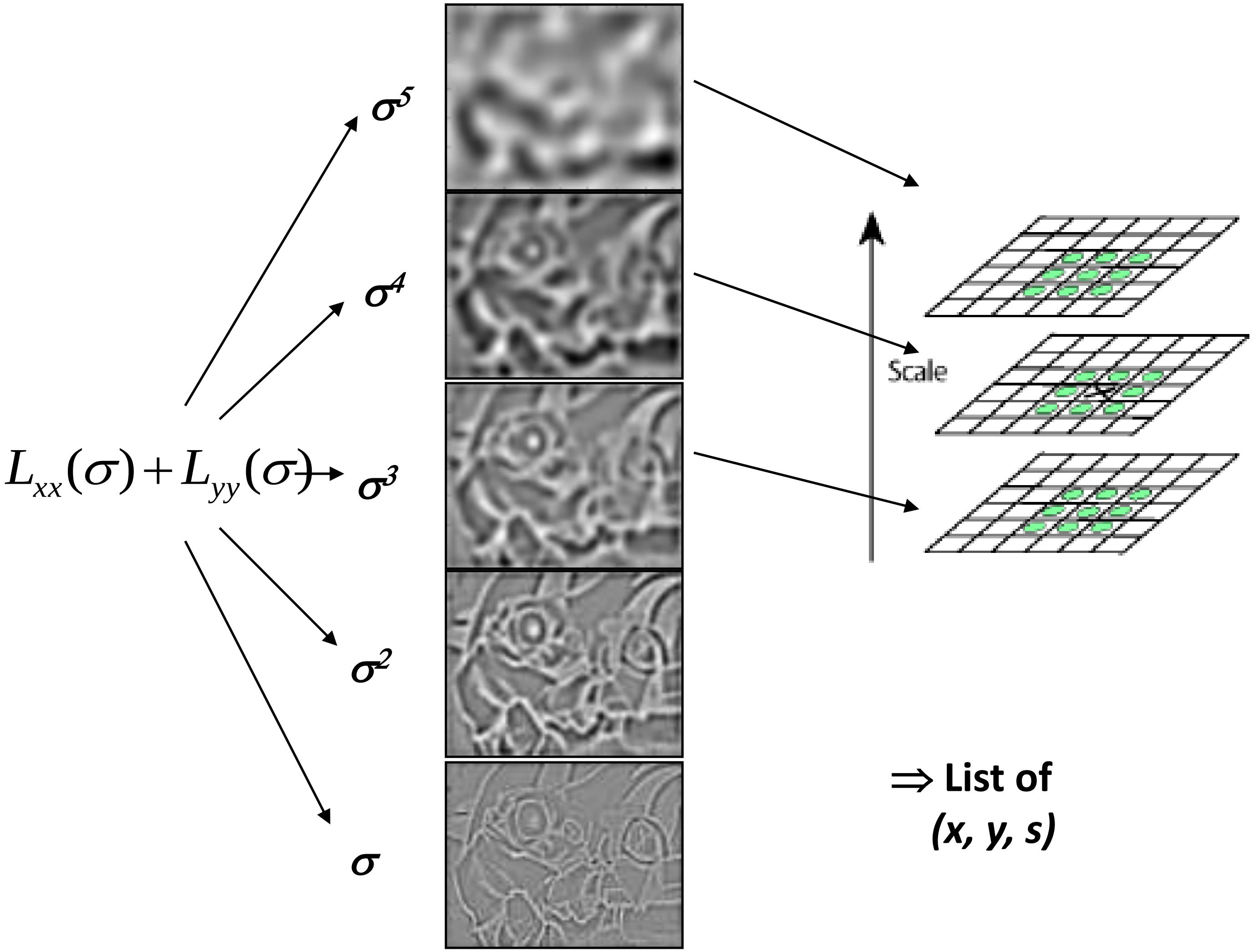
https://blog.csdn.net/dcrmg/article/details/52561656



DoG – Efficient Computation

- Find **local maxima** in **position-scale** space of Difference-of-Gaussian





DoG – Efficient Computation

- Keypoint Localization
  - The approach is similar to the one used in the Harris Corner Detector for removing edge features.

- Reject flats:
  - $|D(\hat{\mathbf{x}})| < 0.03$
- Reject edges:

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

Let α be the eigenvalue with larger magnitude and β the smaller.

$$\mathrm{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta,$$

$$\mathrm{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta.$$

Let $r = \alpha/\beta$.
So $\alpha = r\beta$

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r},$$

$(r+1)^2/r$ is at a min when the 2 eigenvalues are equal.

- $r < 10$

Points Descriptor

Image Representations

- Templates
  - ➢ Intensity, gradients, etc.

- Histograms
  - ➢ Color, texture, SIFT descriptors, etc.

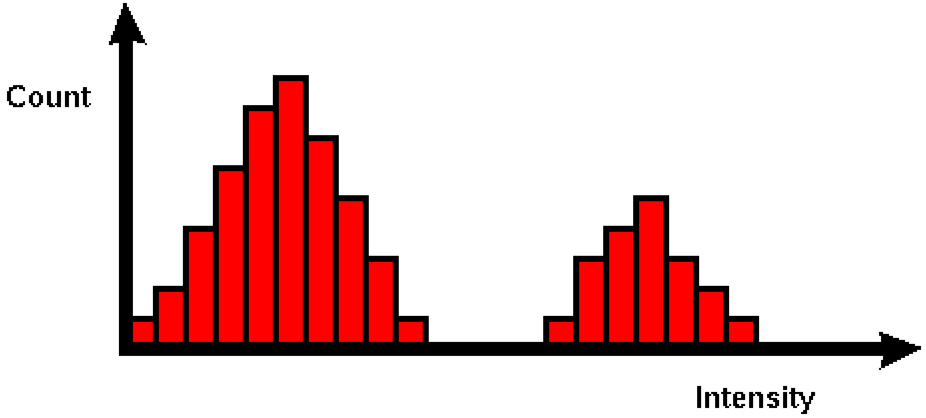


Image Representations: Histograms

- Global histogram
  - Represent distribution of features
  - Color, texture, depth, …

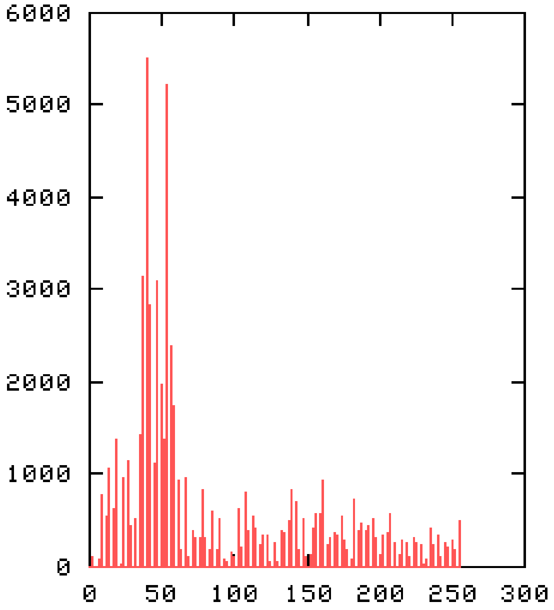
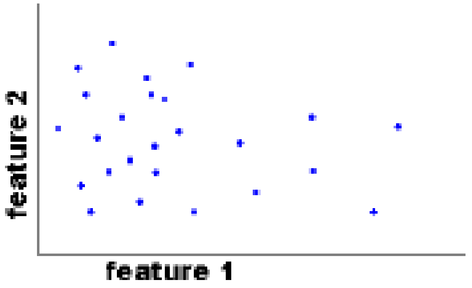
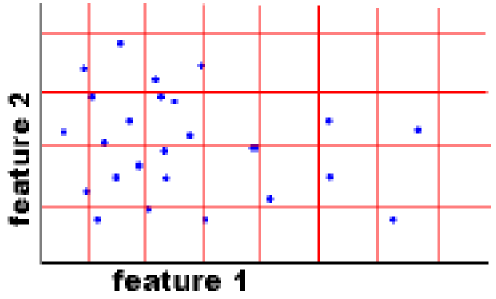


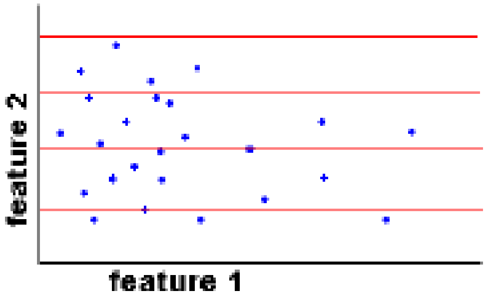
Image Representations: Histograms

- Histogram: probability or count of data in each bin
- Joint histogram
  - ➢ Requires lots of data
  - ➢ Loss of resolution to avoid empty bins





- Marginal histogram
  - ➢ Require independent features
  - ➢ More data/bin than joint histogram



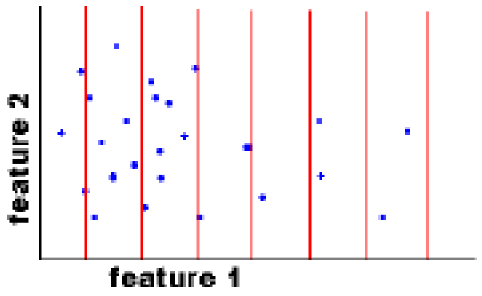
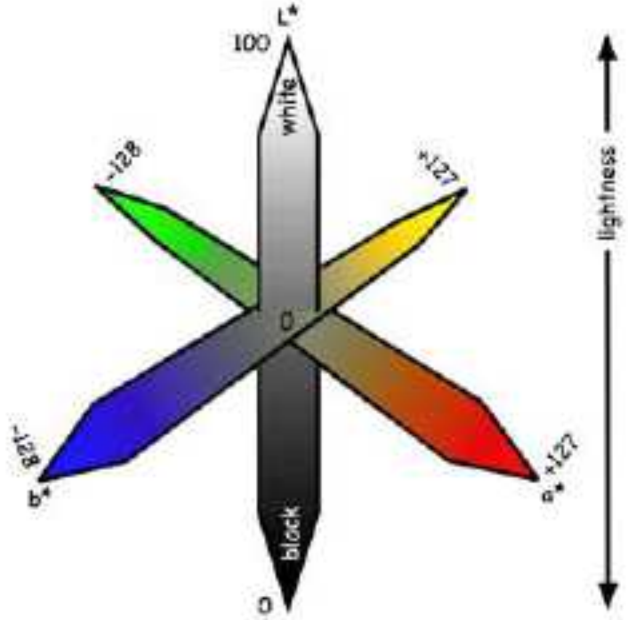


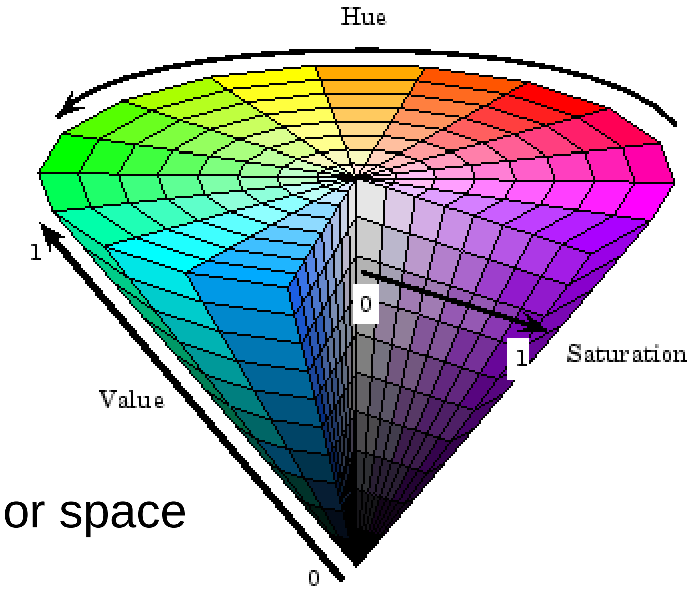
Image Representations

- What kind of things do we compute histograms of?
  - Color

L*a*b* color space



HSV color space



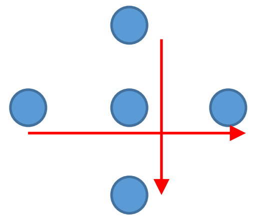
  - Texture (filter banks or HOG over regions)

Histogram of Oriented Gradients

- Gaussian-smoothed image at the **keypoint's scale** $L(x, y, \sigma)$
  - ➢ Every pixel in a neighboring region around the keypoint
    - ✓ Magnitude
    - ✓ Direction

$$m(x,y)=\sqrt{(L(x+1,y)-L(x-1,y))^2+(L(x,y+1)-L(x,y-1))^2}$$

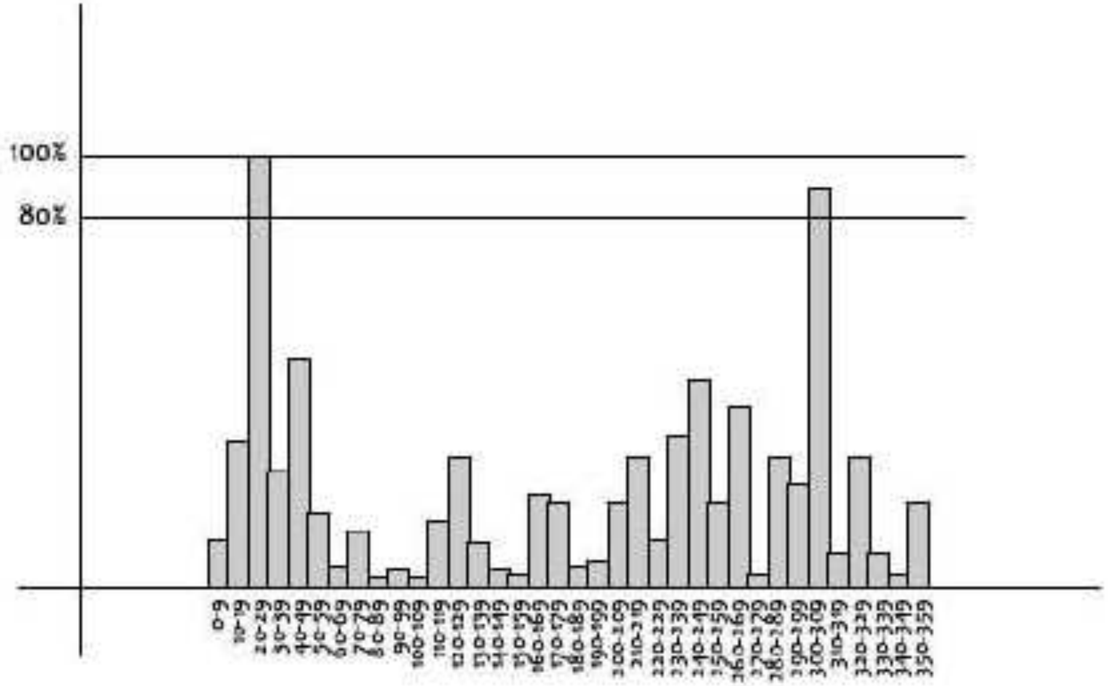
$$\theta(x,y)=\operatorname{atan2}(L(x,y+1)-L(x,y-1),L(x+1,y)-L(x-1,y))$$



- An **orientation** histogram is formed
  - ➢ Each sample in the neighboring window added to a histogram bin is weighted by its gradient magnitude and by a Gaussian-weighted circular window
  - ➢ The peaks in this histogram correspond to **dominant** orientations

Histogram of Oriented Gradients

- An orientation histogram with 36 bins covering 360 degrees is created
  - A histogram bin is weighted by its gradient magnitude



Orientation Normalization

- Compute orientation histogram
- Select dominant orientation
- Normalize: rotate to fixed orientation

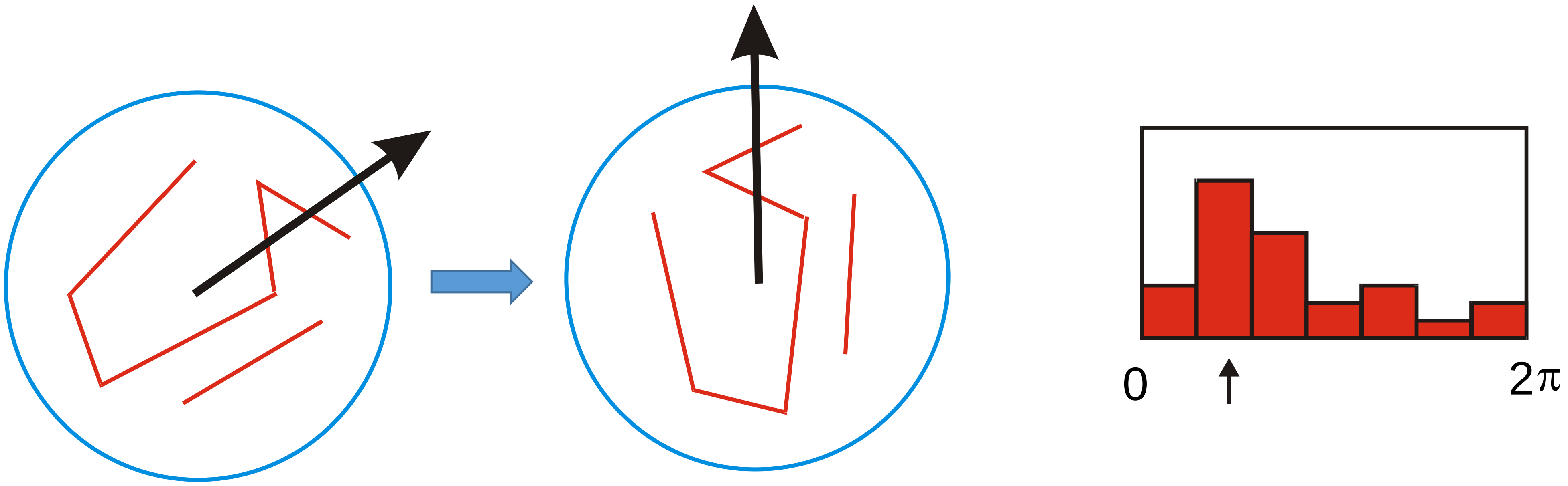
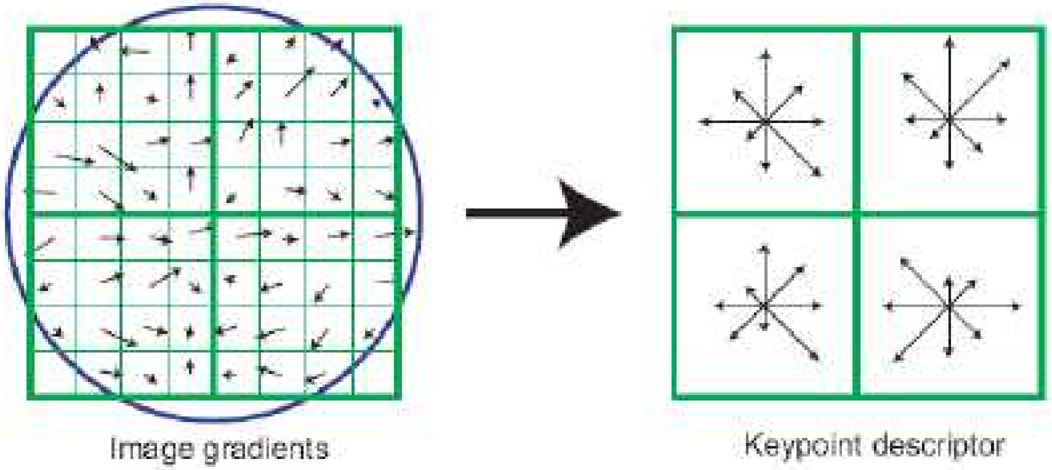


Image Representations

- At this point, each keypoint has a location, scale, orientation
- What kind of things do we compute histograms of?



SIFT – Lowe IJCV 2004

SIFT Vector Formation

- Computed on rotated and scaled version of window according to computed orientation & scale
  - Resample the window
- Based on gradients weighted by a Gaussian of variance half the window (for smooth falloff)

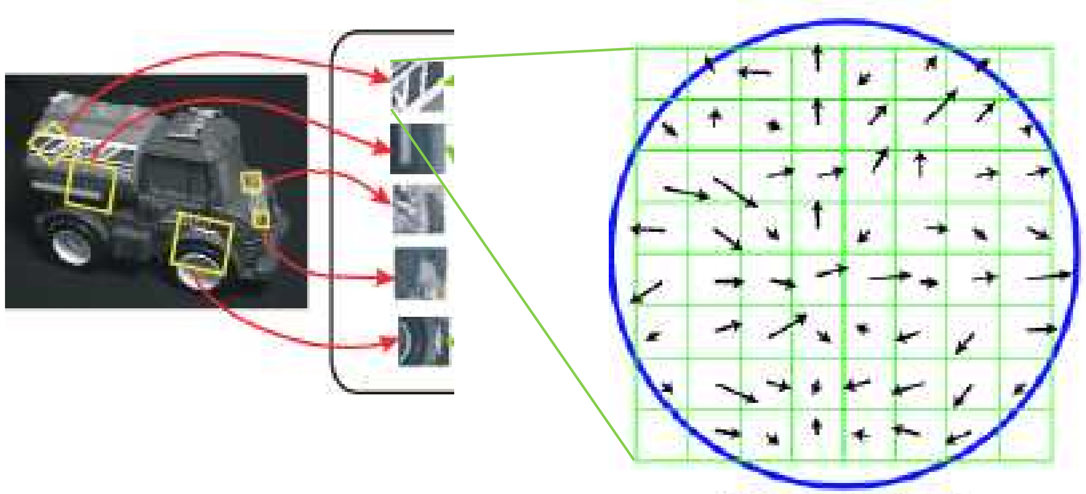
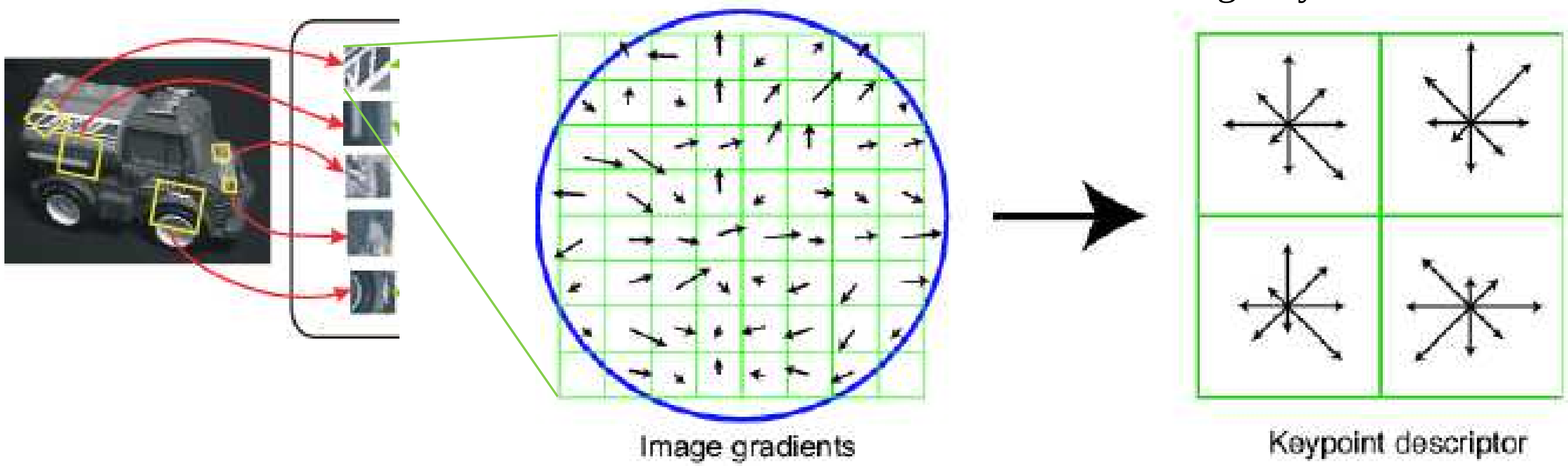


Image gradients

SIFT Vector Formation

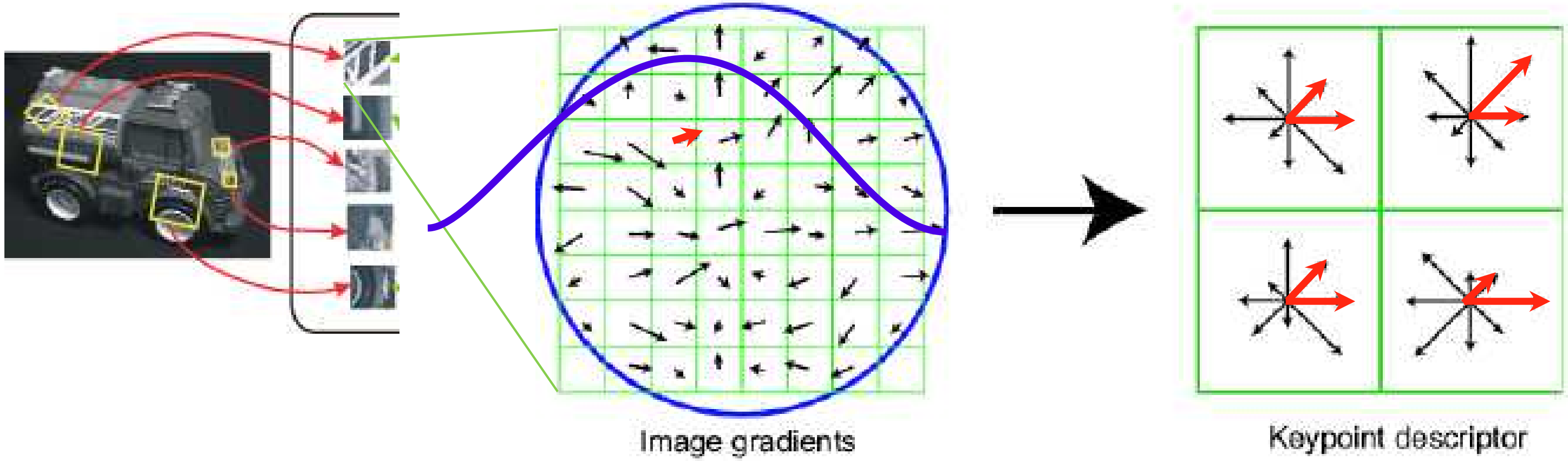
- 4x4 array of gradient orientation histogram weighted by magnitude
- 8 orientations x 4x4 array = 128 dimensions
- Motivation: some sensitivity to spatial layout, but not too much.

showing only 2x2 here but is 4x4



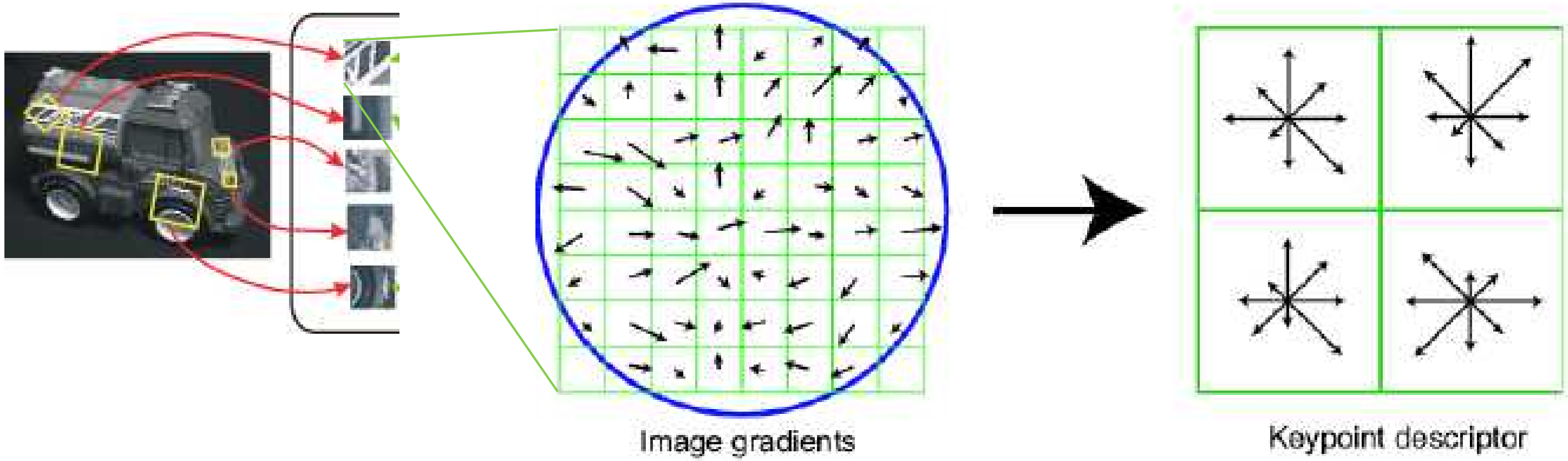
Ensure Smoothness

- Gaussian weight
- Interpolation
  - ➢ A given gradient contributes to 8 bins: 4 in space times 2 in orientation



Reduce Effect of Illumination

- 128-dim vector normalized to 1
- Threshold gradient magnitudes to avoid excessive influence of high gradients
  - ➢ After normalization, clamp gradients > 0.2
  - ➢ Renormalize



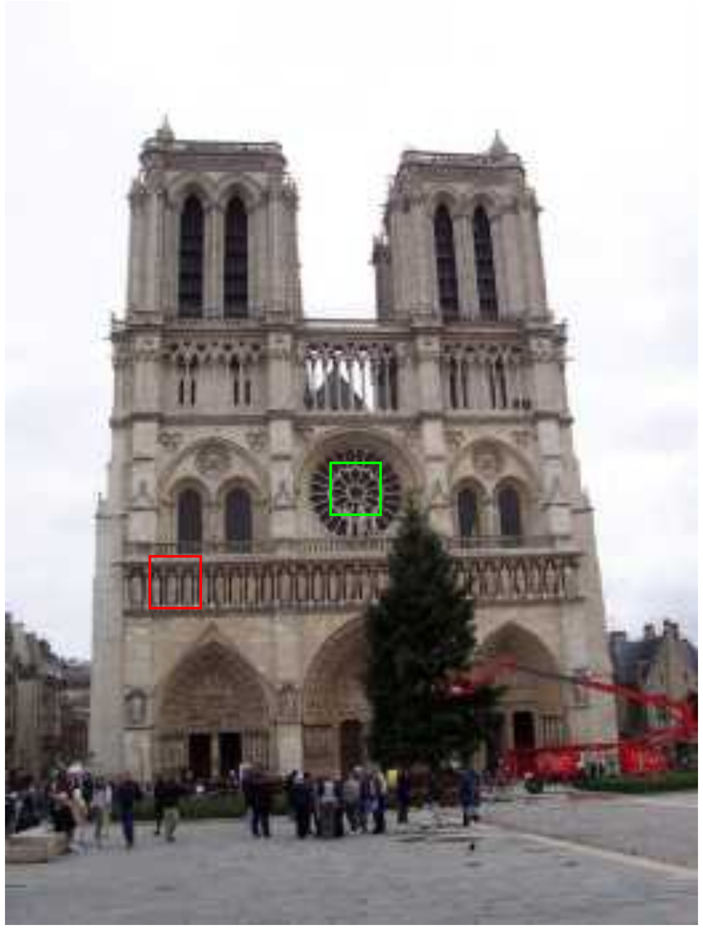
Local Descriptors

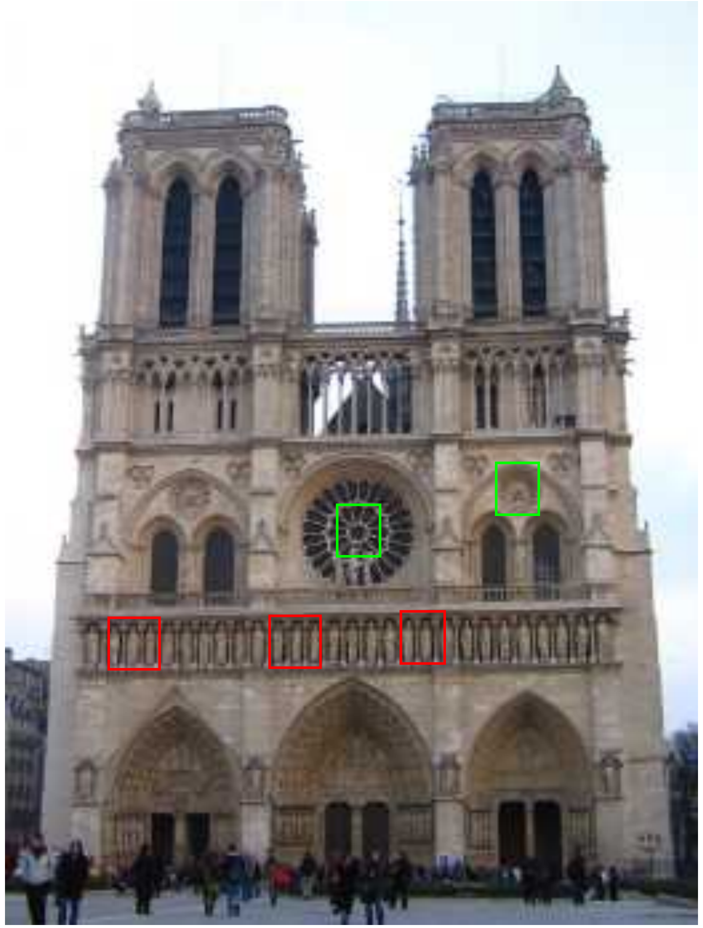
- Most features can be thought of as templates, histograms (counts), or combinations
- The ideal descriptor should be
  - Robust
  - Distinctive
  - Compact
  - Efficient
- Most available descriptors focus on edge/gradient information
  - Capture texture information
  - Color rarely used

Points Matching

Matching

- Simplest approach: Pick the nearest neighbor
- Threshold on absolute distance
- Problem: Lots of self similarity in many photos



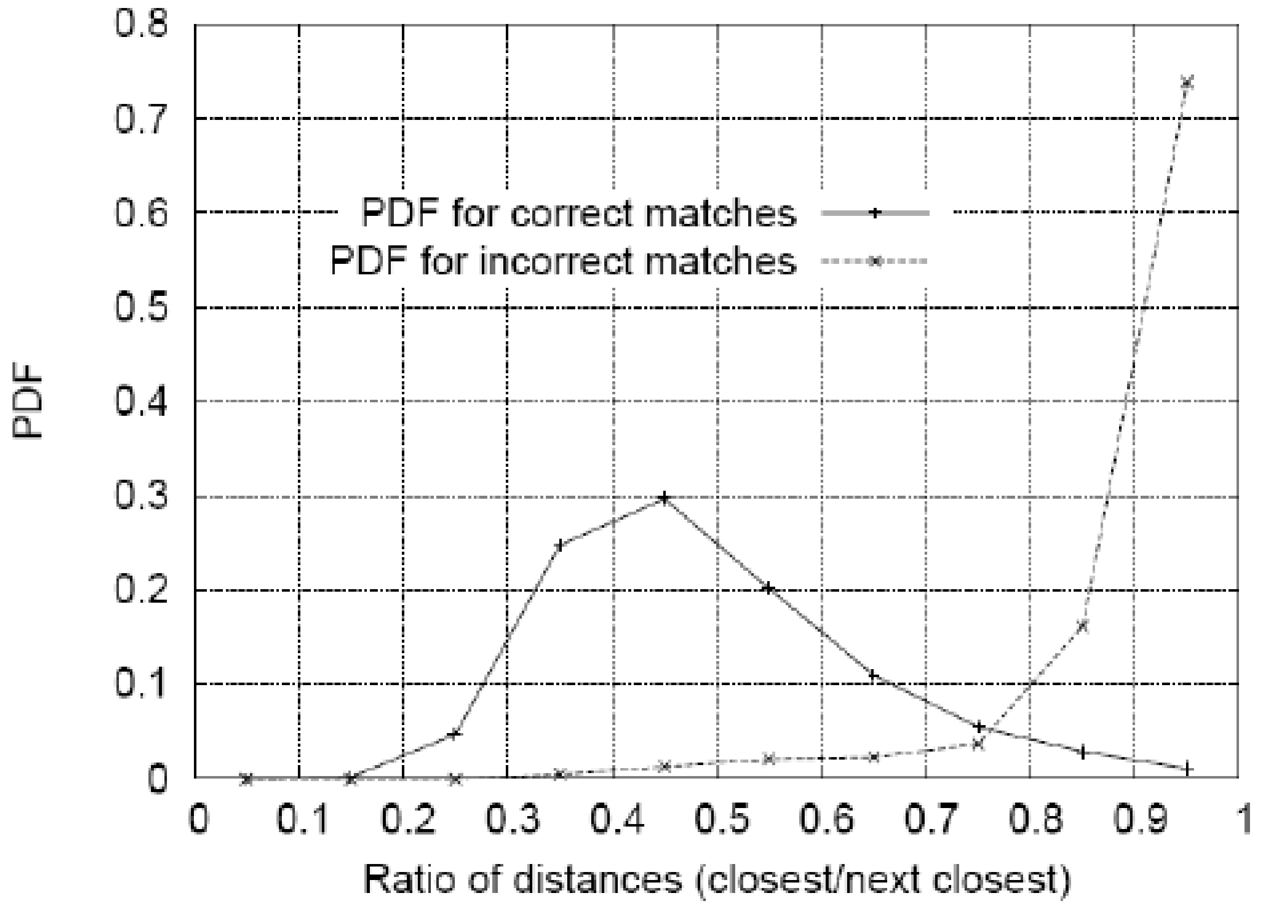


Distance: 0.34, 0.30, 0.40

Distance: 0.61, 1.22

Nearest Neighbor Distance Ratio

- $\frac{NN1}{NN2}$ where NN1 is the distance to the first nearest neighbor and NN2 is the distance to the second nearest neighbor
- Sorting by this ratio puts matches in order of confidence



Conclusions

Choosing a Detector

- What do you want it for?
  - Precise localization in x-y: Harris
  - Good localization in scale: Difference of Gaussian
- Best choice often application dependent
  - Harris-/Hessian-Laplace/DoG work well for many natural categories
- There have been extensive evaluations/comparisons
  - [Mikolajczyk et al., IJCV'05, PAMI'05]
  - All detectors/descriptors shown here work well

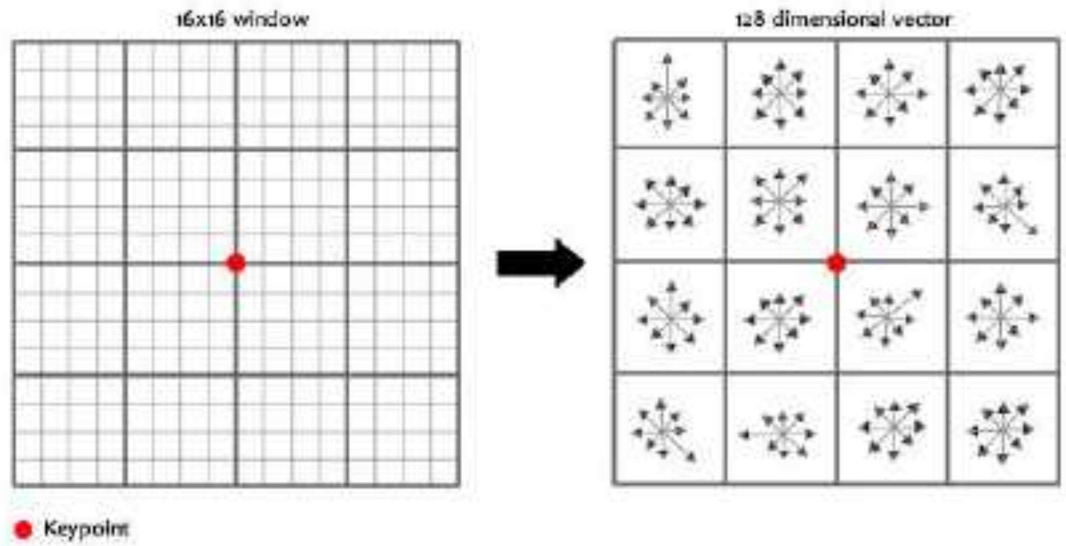
Comparison of Keypoint Detectors

Table 7.1 Overview of feature detectors.

Feature Detector	Corner	Blob	Region	Rotation invariant	Scale invariant	Affine invariant	Repeatability	Localization accuracy	Robustness	Efficiency
Harris	√			√			+++	+++	+++	++
Hessian		√		√			++	++	++	+
SUSAN	√			√			++	++	++	+++
Harris-Laplace	√	(√)		√	√		+++	+++	++	+
Hessian-Laplace	(√)	√		√	√		+++	+++	+++	+
DoG	(√)	√		√	√		++	++	++	++
SURF	(√)	√		√	√		++	++	++	+++
Harris-Affine	√	(√)		√	√	√	+++	+++	++	++
Hessian-Affine	(√)	√		√	√	√	+++	+++	+++	++
Salient Regions	(√)	√		√	√	(√)	+	+	++	+
Edge-based	√			√	√	√	+++	+++	+	+
MSER			√	√	√	√	+++	+++	++	+++
Intensity-based			√	√	√	√	++	++	++	++
Superpixels			√	√	(√)	(√)	+	+	+	+

Things to Remember

- Keypoint detection: repeatable and distinctive
  - Corners, blobs, stable regions
  - Harris, DoG
- Descriptors: robust and selective
  - Spatial histograms of orientation
  - SIFT



Thanks

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