Quiz 5 王和 12012929

(1) $\begin{cases} P_{bull} = 0.8\ P_{bull} + 0.1\ P_{bear} \\ P_{bull} + P_{bear} = 1 \end{cases} \Rightarrow \begin{cases} P_{bull} = \frac{1}{3} \\ P_{bear} = \frac{2}{3} \end{cases}$

(2) $\alpha(z_1) = P(x_1|z_1)\,P(z_1) = \begin{bmatrix} 0.4 \\ 0.7 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{2}{15} \\ \frac{7}{15} \end{bmatrix}$

$\alpha(z_2) = P(x_2|z_2) \cdot \sum_{z_1} P(z_2|z_1)\,\alpha(z_1) = \begin{bmatrix} 0.4 \\ 0.7 \end{bmatrix} \cdot \begin{bmatrix} 0.8 \cdot \frac{2}{15} + 0.1 \times \frac{7}{15} \\ 0.2 \times \frac{2}{15} + 0.9 \times \frac{7}{15} \end{bmatrix} = \begin{bmatrix} \frac{23}{375} \\ \frac{469}{1600} \end{bmatrix}$

$\alpha(z_3) = P(x_3|z_3) \cdot \sum_{z_2} P(z_3|z_2)\,\alpha(z_2) = \begin{bmatrix} 0.0482 \\ 0.0841 \end{bmatrix}$

$P(x_1 = \downarrow,\ x_2 = \downarrow,\ x_3 = \uparrow) = 0.1363$

(3) $\delta(z_1) = \begin{bmatrix} \frac{1}{15} \\ \frac{2}{15} \end{bmatrix}$

$\delta(z_2) = \begin{bmatrix} 0.4 \\ 0.7 \end{bmatrix} \begin{bmatrix} \frac{8}{75} \\ \frac{21}{80} \end{bmatrix} = \begin{bmatrix} \frac{16}{375} \\ \frac{147}{500} \end{bmatrix}$

$\delta(z_3) = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix} \cdot \begin{bmatrix} \frac{64}{1875} \\ \frac{1323}{5000} \end{bmatrix} \begin{matrix} (bull) \\ (bear) \end{matrix} = \begin{bmatrix} 0.02048 \\ 0.07938 \end{bmatrix}$

so sequence is bear bear bear