Assignment4

ACDE 1. [2 pts] (Multiple choice) Assume there is a decider M, and a language L(M) recognized by M. Which of the following statements is/are **TRUE**?

A. M must be a Turing Machine

B. L(M) must be a finite set

C. L(M) must be Turing-decidable

D. L(M) must be Turing-recognizable

E. M will halt on all inputs

ABCD 2. [2 pts] (Multiple choice) If a language A is Turing-decidable, then the subset of A can be ______.

A. Turing-decidable

B. Turing-undecidable

C. Turing-recognizable

D. Turing-unrecognizable

2. [2 pts] ABCD （对于 BCD，考虑 Σ^*、A_{TM}、${A_{TM}}^C$，Σ^* 是图灵可判定的，A_{TM}是图灵可识别的、图灵不可判定的，${A_{TM}}^C$是图灵不可识别的)

false 3. [2 pts] (True or False) A language L is Turing-recognizable if and only if both L and its complement L^C are Turing-decidable.

true 4. [1 pts] (True or False) If $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $f(n) = O(h(n))$.

false 5. [1 pts] (True or False) For an arbitrary size input, an algorithm with O(1) time complexity will definitely solve the problem faster than an algorithm with O(n) time complexity.

6. [2 pts] Give the Big-O estimates for the following functions:

1) $f(n) = 2n(n^2 + 1) + 10nlogn$

2) $f(n) = 1^4 + 2^4 + 3^4 + \cdots + n^4$

Q. 6-1 $f(n) = 2n^3 + 2n + 10n\log n = O(n^3)$

Q. 6-2 By telescoping:

$$\Sigma[(n+1)^5 - n^5] = 5\Sigma n^4 + 10\Sigma n^3 + 10\Sigma n^2 + 5\Sigma n + n$$

$$(\text{w.r.t } \Sigma n^3 = \frac{n^2(n+1)^2}{4}; \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}; \Sigma n = \frac{n(n+1)}{2})$$

$$= 5\Sigma n^4 + O(n^4)$$

$$= (n+1)^5 - 1^5 = O(n^5)$$

$$\Sigma n^4 = O(n^5)$$