CS201 DISCRETE MATHEMATICS FOR COMPUTER SCIENCE

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Generating Functions

- We may use *generating functions* to characterize sequences.
  - ◇ The sequence $\{a_k\}$ with $a_k = 3$ $\qquad \displaystyle\sum_{k=0}^{\infty} 3x^k$
  - ◇ The sequence $\{a_k\}$ with $a_k = 2^k$ $\qquad \displaystyle\sum_{k=0}^{\infty} 2^k x^k$

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Definition The *generating funciton* for the sequence $a_0, a_1, \ldots, a_k, \ldots$ of real numbers is the infinite series

$$G(x) = a_0 + a_1 x + \cdots + a_k x^k$$

- **Problem 2** Find the number of solutions of

$$x_1 + x_2 + x_3 = 17,$$

where x_1, x_2, x_3 are nonnegative integers with $2 \le x_1 \le 5$, $3 \le x_2 \le 6$, $4 \le x_3 \le 7$.

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Using *generating functions*, the number is the coefficient of x^{17} in the expansion of

$$(x^2 + x^3 + x^4 + x^5)(x^3 + x^4 + x^5 + x^6)(x^4 + x^5 + x^6 + x^7)$$

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$$C(n + r - 1, r) = C(19, 17) = C(19, 2)$$

r-Combinations from a Set

- **Definition** An *r-combination* with repetition allowed, or a *multiset of size r*, chosen from a set of n elements, is an unordered selection of elements with repetition allowed.

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  Read more on pp. 537-548.

Cartesian Product

- Let $A = \{a_1, a_2, \ldots, a_m\}$ and $B = \{b_1, b_2, \ldots, b_n\}$, the *Cartesian product* $A \times B$ is the set of pairs

$$\{(a_1, b_1), (a_2, b_2), \ldots, (a_1, b_n), \ldots, (a_m, b_n)\}$$

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Cartesian product defines a set of all ordered arrangements of elements in the two sets.

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  Example: Let $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$

  ◇ Is $R = \{(a, 1), (b, 2), (c, 2)\}$ a relation from A to B?

  ◇ Is $Q = \{(1, a), (2, b)\}$ a relation from A to B?

  ◇ Is $P = \{(a, a), (b, c), (b, a)\}$ a relation from A to A?

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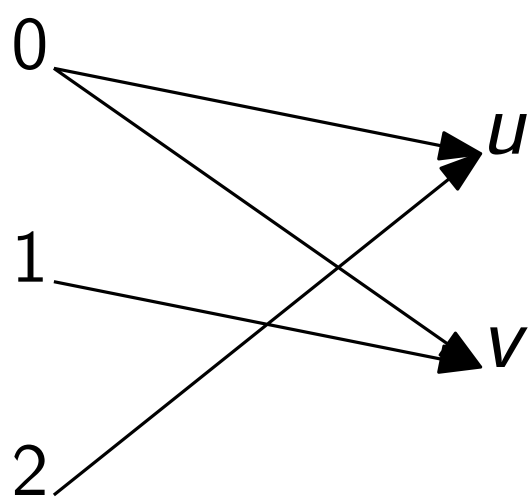

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R	u	v
0	×	×
1	×	
2		×

Relations and Functions

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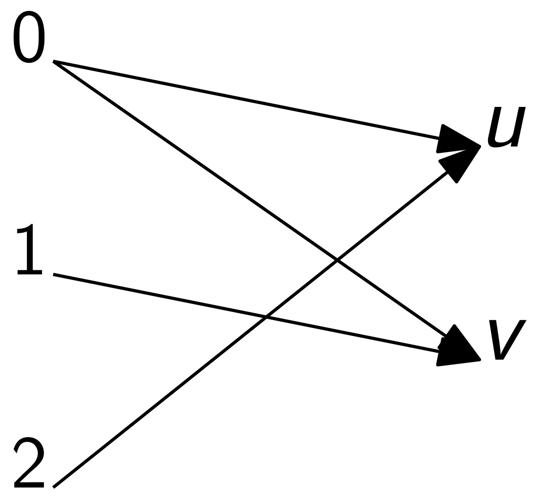

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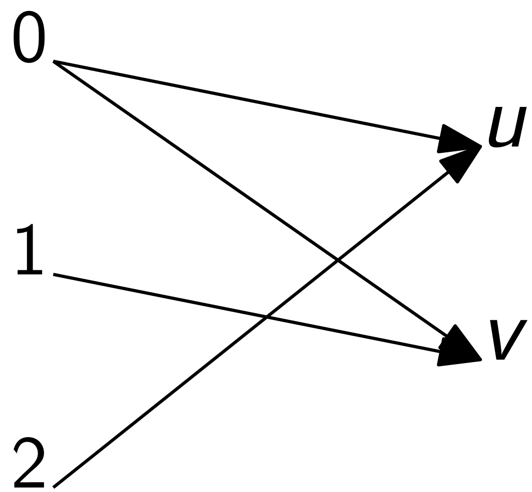

What is the difference between a relation and a function from A to B?

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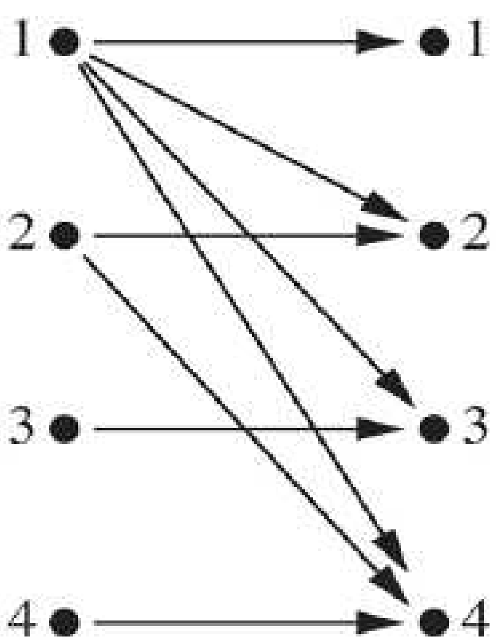

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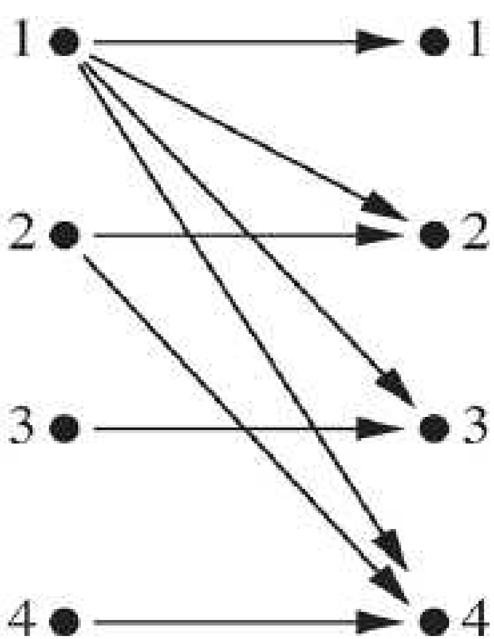

R	1	2	3	4
1	×	×	×	×
2		×		×
3			×	
4				×

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  The number of subsets of a set with k elements is 2^k

Properties of Relations

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$$\mathrm{MR_{div}} = \begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}$$

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A relation R is reflexive if and only if MR has 1 in every position on its main diagonal.

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  No. $(1,1) \notin R$

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A relation R is symmetric if and only if MR is symmetric.

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$$MR_{div} = \begin{matrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

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  No. $(1,2),(2,1) \in R_{\neq}$ but $(1,1) \notin R_{\neq}$.

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  Set operations: union, intersection, difference, etc.

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  We may also combine relations by matrix operations.

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  How many subsets on $n(n-1)$ elements are there?

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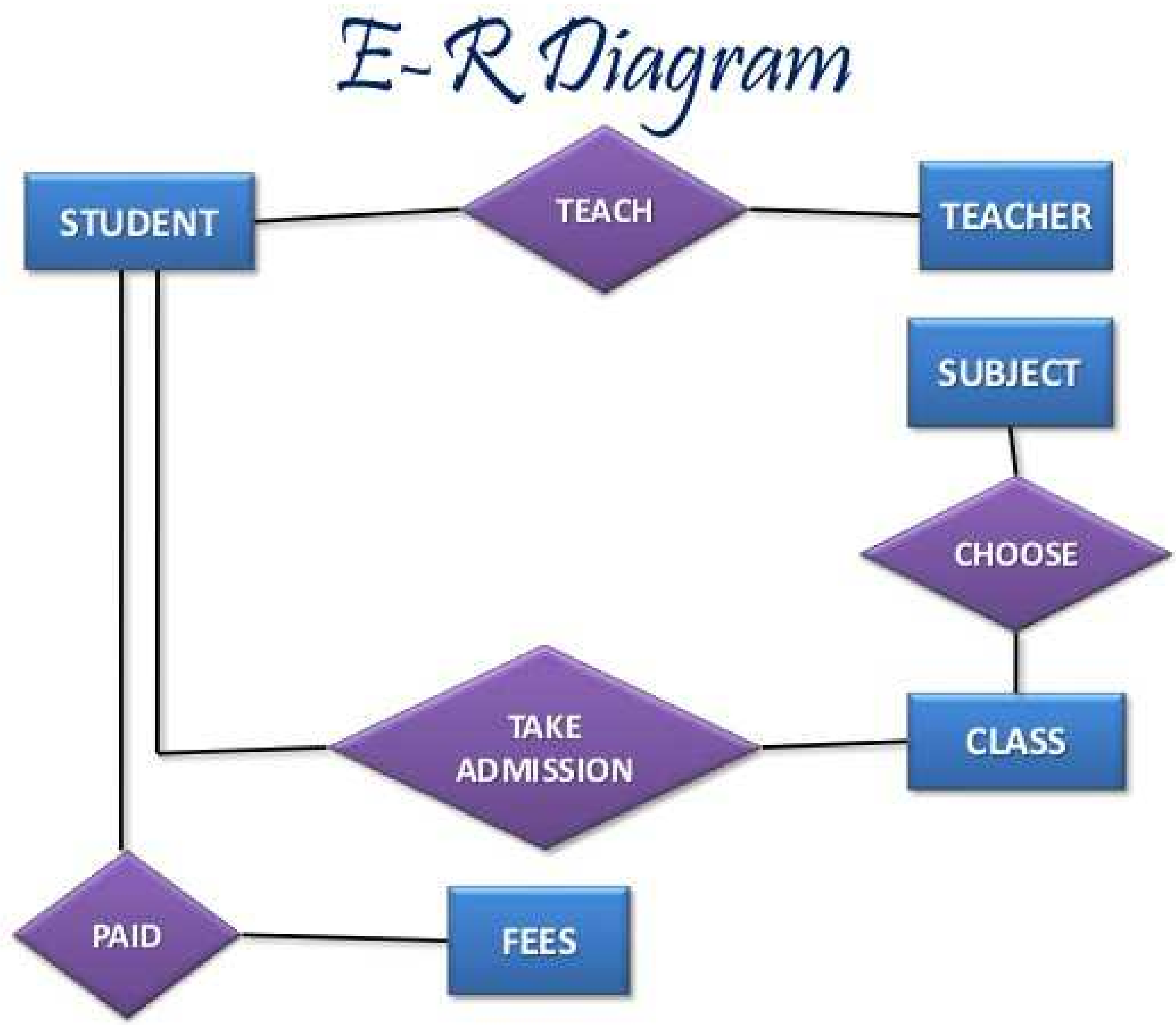
Relational Databases



Selection Operators

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  – $\forall R \subseteq A$,

$$\begin{aligned} s_C(R) &= R \cap \{a \in A \mid s_C(a) = T\} \\ &= \{a \in R \mid s_C(a) = T\}. \end{aligned}$$

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- Then, $s_{UpperLevel}$ is the selection operator that takes any relation R on A (database of students) and produces a relation consisting of just the upper-level classes (juniors and seniors).

Projection Operators

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- Then the *projection operator* on n-tuples

  $$P_{\{i_k\}} : A \rightarrow A_{i_1} \times \cdots \times A_{i_m}$$

  is defined by

  $$P_{\{i_k\}}(a_1, \cdots, a_n) = (a_{i_1}, \cdots, a_{i_m})$$

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  $Cars = Model \times Year \times Color$ $(n = 3)$

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- This operator can be usefully applied to a whole relation $R \subseteq Cars$ (database of cars) to obtain a list of *model*/*color* combinations available.

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- A, B, C can also be sequences of elements rather that single elements.

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- Suppose that R_2 is a room assignment table relating *Courses* to *Rooms* and *Times*.

- Then $J(R_1, R_2)$ is like your class schedule, listing $(professor, course, room, time)$.

Next Lecture

- relation II ...