CS208 · Algorithm Design and Analysis

# Assignment 1

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## Ch.1 - Ex.1

**False** Consider such counter-example:

| man\pref | #1 | #2 |
|---|---|---|
| $m_1$ | $w_1$ | $w_2$ |
| $m_2$ | $w_2$ | $w_1$ |

(a) Men's preference list

| woman\pref | #1 | #2 |
|---|---|---|
| $w_1$ | $m_2$ | $m_1$ |
| $w_2$ | $m_1$ | $m_2$ |

(b) Women's preference list

Run the G-S algorithm to get a stable matching $\{m_1 - w_1, m_2 - w_2\}$. It's trivial that there's no such pair (m, w) that m is ranked first on the preference list of w and w is ranked first on the preference list of m.

## Ch.1 - Ex.2

**True** Suppose that, there exists a "stable matching" S for this instance, in which $m$ is not pairing with $w$. By the fact that *$m$ is ranked first for $w$ and $w$ is ranked first for $m$*, we know that $m$ is pairing with someone he prefers less than $w$, similar for $w$. By the definition of *unstable pair*, the pair $m - w$ is unstable and thus let S cannot be a stable matching.

By the above contradiction, there does not exist any "stable matching" for this instance s.t. $m$ is not pairing with $w$. A.k.a. in every stable matching S for this instance, the pair (m, w) belongs to S.

## Ch.1 - Ex.3

**(b) No stable pair of schedules** Let the $n$ in this example be 2. Consider that Network $\mathcal{A}$ has two shows: $A_1$ and $A_2$, while Network $\mathcal{B}$ has two shows as $B_1$ and $B_2$.

Assume that the ratings are $A_1 > B_1 > A_2 > B_2$. Let's check all the 4 possible arrangements.

| Slot | $\mathcal{A}$ | $\mathcal{B}$ |
|---|---|---|
| 1 | $A_1$ | $B_1$ |
| 2 | $A_2$ | $B_2$ |

(a)

| Slot | $\mathcal{A}$ | $\mathcal{B}$ |
|---|---|---|
| 1 | $A_1$ | $B_2$ |
| 2 | $A_2$ | $B_1$ |

(b)

| Slot | $\mathcal{A}$ | $\mathcal{B}$ |
|---|---|---|
| 1 | $A_2$ | $B_1$ |
| 2 | $A_1$ | $B_2$ |

(c)

| Slot | $\mathcal{A}$ | $\mathcal{B}$ |
|---|---|---|
| 1 | $A_2$ | $B_2$ |
| 2 | $A_1$ | $B_1$ |

(d)

For (a), Network $\mathcal{B}$ can reorder its schedule to (b) s.t. it wins $0 \rightarrow 1$ slot.

For (b), Network $\mathcal{A}$ can reorder its schedule to (d) s.t. it wins $1 \rightarrow 2$ slots.

For (c), Network $\mathcal{A}$ can reorder its schedule to (a) s.t. it wins $1 \rightarrow 2$ slots.

For (d), Network $\mathcal{B}$ can reorder its schedule to (c) s.t. it wins $0 \rightarrow 1$ slot.

All the possible arrangements for this example are therefore not stable. Q.E.D.

## Ch.1 - Ex.8

**(b) Could improve the partner of a woman** Let $w_1$ be the lying woman, assume that her actual preference sequence is $m_1 - m_2 - m_3$ but her lies that $m_1 - m_3 - m_2$. We then construct others preference list as below:

| man\pref | #1 | #2 | #3 |
|---|---|---|---|
| $m_1$ | $w_3$ | $w_1$ | $w_2$ |
| $m_2$ | $w_1$ | $w_3$ | $w_2$ |
| $m_3$ | $w_1$ | $w_3$ | $w_2$ |

(a) Men's preference list

| woman\pref | #1 | #2 | #3 |
|---|---|---|---|
| $w_1$ (fake) | $m_1$ | $m_3$ | $m_2$ |
| $w_2$ | $m_2$ | $m_1$ | $m_3$ |
| $w_3$ | $m_2$ | $m_1$ | $m_3$ |

(b) Women's preference list

Base on the fake preference list, we run the G-S algorithm and will get a stable matching $\{m_1 - w_1, m_2 - w_3, m_3 - w_2\}$. But when we run the G-S algorithm on the real preference list we will get $\{m_1 - w_3, m_2 - w_1, m_3 - w_2\}$. By lying, $w_1$ successfully got married with the man $m_1$, who is actually more preferred by $w_1$ than the other two men.