CS201: Discrete Math for Computer Science
2021 Fall Semester Written Assignment # 5
Due: Dec. 15th, 2021, please submit at the beginning of class

Q.1 Let S be the set of all strings of English letters. Determine whether these relations are *reflexive, irreflexive, symmetric, antisymmetric*, and/or *transitive*.

(1) $R_1 = \{(a, b)|a$ and b have no letters in common$\}$

(2) $R_2 = \{(a, b)|a$ and b are not the same length$\}$

(3) $R_3 = \{(a, b)|a$ is longer than $b\}$

Q.2 How many relations are there on a set with n elements that are

(a) symmetric?

(b) antisymmetric?

(c) irreflexive?

(d) both reflexive and symmetric?

(e) neither reflexive nor irreflexive?

(f) both reflexive and antisymmetric?

(g) symmetric, antisymmetric and transitive?

Q.3 Suppose that the relation R is irreflexive. Is the relation R^2 necessarily irreflexive?

Q.4 Give an example of a relation R such that its transitive closure R^* satisfies $R^* = R \cup R^2 \cup R^3$, but $R^* \neq R \cup R^2$.

Q.5 Suppose that R_1 and R_2 are both *reflexive* relations on a set A.

(1) Show that $R_1 \oplus R_2$ is *irreflexive*.

(2) Is $R_1 \cap R_2$ also *reflexive*? Explain your answer.

(3) Is $R_1 \cup R_2$ also *reflexive*? Explain your answer.

Q.6 Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad = bc$.

(a) Show that R is an equivalence relation.

(b) What is the equivalence class of $(1, 2)$ with respect to the equivalence relation R?

(c) Give an interpretation of the equivalence classes for the equivalence relation R.

Q.7 Show that the relation R on $\mathbb{Z} \times \mathbb{Z}$ defined on $(a, b)\mathbb{R}(c, d)$ if and only if $a + d = b + c$ is an *equivalence* relation.

Q.8 How many different equivalence relations with exactly three different equivalence classes are there on a set with five elements?

Q.9 Let A be a set, let R and S be relations on the set A. Let T be another relation on the set A defined by $(x, y) \in T$ if and only if $(x, y) \in R$ and $(x, y) \in S$. Prove or disprove: If R and S are both *equivalence relations*, then T is also an equivalence relation.

Q.10 Let $\sim$ be a relation defined on $\mathbb{N}$ by the rule that $x \sim y$ if $x = 2^k y$ or $y = 2^k x$ for some $k \in \mathbb{N}$. Show that $\sim$ is an equivalence relation.

Q.11 Given functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$, f is **dominated** by g if $f(x) \leq g(x)$ for all $x \in \mathbb{R}$. Write $f \preceq g$ if f is dominated by g.

(a) Prove that $\preceq$ is a partial ordering.

(b) Prove or disprove: $\preceq$ is a total ordering.

Q.12 Which of these are posets?

(a) $(\mathbf{R}, =)$

(b) $(\mathbf{R}, <)$

(c) $(\mathbf{R}, \leq)$

(d) $(\mathbf{R}, \neq)$

Q.13 Consider a relation $\propto$ on the set of functions from $\mathbb{N}^+$ to $\mathbb{R}$, such that $f \propto g$ if and only if $f = O(g)$.

(a) Is $\propto$ an equivalence relation?

(b) Is $\propto$ a partial ordering?

(c) Is $\propto$ a total ordering?

Q.14 Answer these questions for the partial order represented by this Hasse diagram.

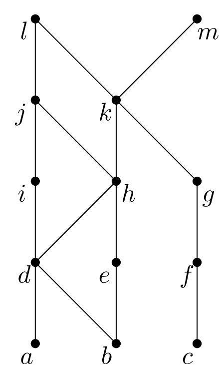

Figure 1: Q.14

(a) Find the maximal elements.

(b) Find the minimal elements.

(c) Is there a greatest element?

(d) Is there a least element?

(e) Find all upper bounds of $\{a, b, c\}$.

(f) Find the least upper bound of $\{a, b, c\}$, if it exists.

(g) Find all lower bounds of $\{f, g, h\}$.

(h) Find the greatest lower bound of $\{f, g, h\}$, if it exists.

$\square$