

# Linear algebra

1. 
$$\left[ \begin{array}{ccc|c} a_{11} & \dots & & x_1 \\ a_{21} & \dots & & x_2 \\ \vdots & & & \vdots \\ a_{n1} & \dots & & x_n \end{array} \right] \xrightarrow{R_n + R_x} \left[ \begin{array}{ccc|c} R_1 & & & x_1 \\ R_2 & & & x_2 \\ \vdots & & & \vdots \\ R_n & & & x_n \end{array} \right]$$

$a_{11}x_1 + \dots + a_{1n}x_n = b_1$   
 $a_{21}x_1 + \dots + a_{2n}x_n = b_2$   
 $\vdots$   
 $a_{n1}x_1 + \dots + a_{nn}x_n = b_n$

$\left[ \begin{array}{ccc|c} R_1 & & & x_1 \\ R_2 & & & x_2 \\ \vdots & & & \vdots \\ R_n + R_x & & & x_n \end{array} \right] \xrightarrow{i?} \underbrace{a_{11}x_1 + \dots + a_{1n}x_n}_{\text{original } b_x} + \underbrace{a_{n1}x_1 + \dots + a_{nn}x_n}_{b_i} = \boxed{bx + bi}$

2. 
$$\left[ \begin{array}{ccc|c} 1 & -2 & 4 & 12 \\ 2 & 1 & -1 & 4 \end{array} \right]$$

(1)  $4x + 2y - 2z = 8$   
 (2)  $2x + y - z = 4$

(4)  $x + 2z = 4$   
 $x = 4$   
 $y = -4$   
 $z = 0$

$$\left[ \begin{array}{ccc|c} 4 & -2 & 4 & 8 \\ 2 & 1 & -1 & 4 \\ 1 & 0 & 2 & 4 \end{array} \right]$$

3. 
$$\left[ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right]$$

if  $x_1 \begin{bmatrix} a \\ b \\ c \end{bmatrix} + x_2 \begin{bmatrix} d \\ e \\ f \end{bmatrix} + \dots = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$   
 determinant = 0, not linear independent  
 ↓ if they all get squished onto a plane lower dimension.  
 or 4 equations  
 sometimes? 6 unknowns  
 never

- ② Sometimes?
- ③ never?
- ④ sometimes
- ⑤ sometimes
- ⑥ sometimes



7. 
$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = x \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix} + y \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + z \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x = -0.5$   
 $y = 1.5$   
 $z = 0.5$

8. A:  $\begin{pmatrix} 10 \text{ m/s} & 5 \text{ kg} \\ 15 \text{ m/s} & 3 \text{ kg} \end{pmatrix}$   
 B

100 m  
 25 kg

$15x + 10y = 100$

~~$5x + 3y = 25$~~   
 $3x + 5y = 25$

~~$15x + 9y = 75$~~   
 $15x + 25y = 125$

$5y = 25$   
 $y = 5$

$\begin{bmatrix} 10 & 15 & 100 \\ 15 & 5 & 25 \end{bmatrix}$

5 3

~~2A + B~~  
 $A = 5, B = 5$

$2 \cdot 5 + 5 \cdot 3 = 25$

$(10A + 15B)T = 100$

~~$15x + 3y = 100$~~

$20 \text{ m/s} \cdot 2 \cdot t$   
 $+ 15 \text{ m/s} \cdot 3t = 100$

$B = 5$   
 5 dtimes

$65T = 100$



$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -\frac{5}{9} \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -\frac{5}{9} \end{array} \right]$$

$$-5R_3$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & \frac{4}{3} \\ 0 & 1 & 0 & \frac{7}{9} \\ 0 & 0 & 1 & -\frac{5}{9} \end{array} \right]$$

$$3 \rightarrow -2 + \frac{5}{9} \cdot 5 = -\frac{18 + 25}{9} = -\frac{7}{9}$$

$$R_1 + R_2$$

$$\rightarrow U$$