

# Practicing Feedback Control

In this unit we are going to explore basic feedback control on a highly simplified, 1-dimensional model of the quadrotor-camera system.

## Simple Slide Camera

Imagine a camera attached to a cart that is allowed to slide along a 1-dimensional track. The track is elevated some height  $h$  above the ground and the camera is pointing downward. There is some target on the ground that the camera can observe; more specifically the camera can measure the angle  $-\pi/2 < \gamma < \pi/2$  from the centerline of the camera to the target. The objective is to issue velocity commands,  $v_{cmd}$ , in order to move the cart to a position such that  $\gamma \rightarrow \gamma_{des}$  (e.g. if  $\gamma_{des} = 0$ , then the objective is simply to move the cart directly over the target).

This system is depicted in the below diagram:



This somewhat contrived system can be thought of as a highly simplified model of the quadrotor and its downward-facing camera. If the quadrotor is constrained to move in one dimension, can be controlled via velocity commands, and we ignore the pitch that is induced when changing velocity (valid assumption for low-acceleration maneuvers), then we can roughly model the quadrotor as this simple slide camera

Note that the position of the cart,  $x$ , and position of the target  $x_{des}$  are not directly measured, only  $\gamma$  is measured.

Now we will provide the code necessary to simulate this system

```
In [1]: from __future__ import division, print_function
import numpy as np
import matplotlib.pyplot as plt

_HEIGHT = 1.0
_VEL_CONST = 1.0
_TIME_STEP = 0.1
```

## Plant Dynamics, Sensors, and Actuators

the following object contains functions for the plant dynamics, sensing of the target angle  $\gamma$ , and actuator for  $v_{cmd}$

```
In [6]: class SimpleSlideCamera():
        '''Object that defines the dynamics of the simple slide-camera'''

        def __init__(self, x_0, v_0, x_d, gamma_d=0.0, h=_HEIGHT):
```

```

    # state variables (hidden)
    self.__x = x_0
    self.__v = v_0

    # reference position (hidden)
    self.__x_d = x_d

    # reference angle (observed)
    self.gamma_d = gamma_d

    # parameters
    self.__h = h

    def sense_gamma(self):
        # calculate angle from camera center line to target
        return np.arctan2(self.__x - self.__x_d, self.__h)

    def _get_hidden_position(self):

        return self.__x

    def _get_hidden_position_desired(self):
        return self.__x_d

    def _get_hidden_velocity(self):
        return self.__v

    def actuate_velocity_command(self, vel_cmd, dt=_TIME_STEP):
        self.__v = vel_cmd
        self.__x += self.__v*dt

    def actuate_disturbed_velocity_command(self, vel_cmd, dt=_TIME_STEP,
        self.__v = vel_cmd + np.random.normal(vel_bias, vel_std)
        self.__x += self.__v*dt

    def actuate_acceleration_command(self, acc_cmd, dt=_TIME_STEP):
        self.__v += acc_cmd*dt
        self.__x += self.__v*dt

```

## Controller

```

In [7]: def p_control(y_err, kp):
    ''' compute the actuator command based on proportional error between
    Args:
        y_err: y_des - y where y is the output variable of the plant
    Returns:
        cmd: actuator command to correct error (e.g. velocity command)
    '''

    # TODO: write a proportional control law (hint: it is a single line,
    # YOUR CODE HERE

    cmd = y_err * kp

    return cmd

```

```

In [8]: # p_control tests

```

```

assert np.isclose(p_control(0.0, 1.0), 0.0)
assert np.isclose(p_control(1.0, 1.0), 1.0)
assert np.isclose(p_control(-10.0, 0.1), -1.0)

```

## Simulation Script

below is a script for testing various controllers for the SimpleSlideCamera plant as well as plotting the results.

```

In [23]: # Proportional gain
# TODO: assign an appropriate value to kp
# YOUR CODE HERE

kp = 100000
kd = 0

# Control inputs
dt = _TIME_STEP
t_final = 10.0

# initial conditions (position, velocity and target position)
x_0 = 0.0
v_0 = 0.0
x_des = 1.0

# create SimpleSlideCamera with initial conditions
sscam = SimpleSlideCamera(x_0, v_0, x_des)

# initialize data storage
data = dict()
data['t'] = []
data['acc_cmd'] = []
data['vel_cmd'] = []
data['err_gamma'] = []
data['x_hidden'] = []
data['v_hidden'] = []
t = 0.0
err_prev = 0.0
while t < t_final:
    t += dt

    # SENSOR: sense output variable gamma (angle from camera centerline to target)
    err_gamma = sscam.gamma_d - sscam.sense_gamma()

    # CONTROLLER: call velocity control algorithm
    vel_cmd = p_control(err_gamma, kp)

    # ACTUATOR: send velocity command to plant
    sscam.actuate_acceleration_command(vel_cmd)

    # store data
    err_gamma_prev = err_gamma
    data['t'].append(t)
    data['vel_cmd'].append(vel_cmd)
    data['err_gamma'].append(err_gamma)
    data['x_hidden'].append(sscam._get_hidden_position())
    data['v_hidden'].append(sscam._get_hidden_velocity())

```

```
# Plot Data
handle_position, = plt.plot(data['t'], data['x_hidden'], label='position')
handle_velocity, = plt.plot(data['t'], data['v_hidden'], label='velocity')
handle_err_gamma, = plt.plot(data['t'], data['err_gamma'], label='gamma error')
plt.legend(handles=[handle_position, handle_velocity, handle_err_gamma])
plt.show()
```



## Questions

**Q1.** Does your gamma error (i.e. output converge to 0.0? - If so how quickly (i.e. how long does it take for the error to be 5% of the original error) - If not, what is happening? Is the error never changing or is it oscillating back and forth across zero? Can you change the proportional gain  $k_p$  to change this behavior?

Yes, it will probably take a very very long time.

**Q2.** What values should your system's position and velocity be converging upon? Since this is a practice problem and we want to gain understanding of how this system behaves, we've cheated and let you observe the position and velocity of the system which were meant to be unobservable in the real world. Are the position and velocity converging to values you expected?

No, my system's position should be 1, and velocity should be 0.

**Q3.** As you increase the proportional gain, what happens to the rate at which the error converges to zero?

Can you find a proportional gain that converges within 1 second (i.e error reaches 5% of it's original value)? Note: if you start to see an oscillatory, saw-tooth pattern, you're

gain is too high.

The position values and the gamma error decreases.

**Q4.** What happens if you cannot perfectly control the velocity, for example there is some form of velocity disturbance that you cannot control (e.g. wind, actuator noise)? Let's try it out:

In the #ACTUATOR portion of the simulation script, replace `actuate_velocity_command` with `actuate_disturbed_velocity_command`, restart the kernel, and run the script. Does output error (gamma error) still converge to zero? If not, what does the gamma error converge to? How might you change your controller to fix this "steady state error"

Yes, it's probably because I put some crazy value for  $k_p$ .

**Q5.** Often times we control a system by applying a force. In this way we are not directly controlling the velocity of a system, but instead the acceleration. Let's try it out:

In the ACTUATOR portion of the simulation script replace `actuate_velocity_command` with `actuate_acceleration_command`. In the CONTROLLER portion of the script, have the controller output a `acc_cmd` instead of `vel_cmd` (just change the variable name, but this way you know it represents an acceleration instead of a velocity). Restart the kernel and rerun the script

Now what is happening to the gamma error, is it converging to zero? If not, is there anything you could think of to change in you controller in order to converge the error to zero?

Yes, it is still converging to 0, the velocity graph is crazily zigzagging up and down, and the position is also constantly changing.