

# Character Tables for Representations of Finite Groups

for Math 8190

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# Group Representations

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One may wonder more generally: Given an abstract group  $G$ , which objects  $X$  does  $G$  act on? This is the basic question of representation theory, which attempts to classify all such  $X$  up to isomorphism.

# Group Actions

## Definition

A **(left) group action** of a group  $G$  on a set  $X$  is a map  $\rho: G \times X \rightarrow X$  (written as  $g \cdot a$ , for all  $g \in G$  and  $a \in A$ ) that satisfies the following two axioms:

$$1 \cdot x = x \qquad \forall x \in X \qquad (1.1)$$

$$(gh) \cdot x = g \cdot (h \cdot x) \qquad \forall g, h \in G, x \in X \qquad (1.2)$$

we could likewise define the concept of a *right* group action, where the set elements would be multiplied by group elements on the right instead of on the left. Throughout we shall use the term *group action* to mean a *left* group action.

# The Definition of a Representation

## Definition

Let  $G$  be a group, let  $F$  be a field, and let  $V$  be a vector space over  $F$ . A **linear representation** of  $G$  is an action of  $G$  on  $V$  which preserves the linear structure of  $V$ , i.e. an action of  $G$  on  $V$  such that

$$g \cdot (v_1 + v_2) = g \cdot v_1 + g \cdot v_2 \quad \forall g \in G, v_1, v_2 \in V \quad (2.1)$$

$$g \cdot (kv) = k(g \cdot v) \quad \forall g \in G, v \in V, k \in F \quad (2.2)$$



# The Definition of a Representation

## Definition (Alternative definition)

Let  $G$  be a group, let  $F$  be a field, and let  $V$  be a vector space over  $F$ . A **linear representation** of  $G$  is any group homomorphism  $\rho: G \rightarrow GL(V)$ . If we fix a basis for  $V$ , we get a representation in the previous sense.

# Metropolis titleformats

themesupports 4 different titleformats:

- Regular
- SMALLCAPS
- ALLSMALLCAPS
- ALLCAPS

They can either be set at once for every title type or individually.

# Small caps

This frame uses the `smallcaps` titleformat.

## Potential Problems

Be aware, that not every font supports small caps. If for example you typeset your presentation with pdfTeX and the Computer Modern Sans Serif font, every text in `smallcaps` will be typeset with the Computer Modern Serif font instead.

# All small caps

This frame uses the `allsmallcaps` titleformat.

## Potential problems

As this titleformat also uses smallcaps you face the same problems as with the `smallcaps` titleformat. Additionally this format can cause some other problems. Please refer to the documentation if you consider using it.

As a rule of thumb: Just use it for plaintext-only titles.

# All caps

This frame uses the `allcaps` titleformat.

## Potential Problems

This titleformat is not as problematic as the `allsmallcaps` format, but basically suffers from the same deficiencies. So please have a look at the documentation if you want to use it.

# Typography

The theme provides sensible defaults to  
`\emph{emphasize}` text, `\alert{accent}` parts  
or show `\textbf{bold}` results.

becomes

The theme provides sensible defaults to *emphasize* text, **accent** parts or  
show **bold** results.

# Font feature test

- Regular
- *Italic*
- SMALLCAPS
- **Bold**
- **Bold Italic**
- **Bold SmallCaps**
- Monospace
- *Monospace Italic*
- Monospace Bold
- *Monospace Bold Italic*

# Lists

## Items

- Milk
- Eggs
- Potatos

## Enumerations

- ① First,
- ② Second and
- ③ Last.

## Descriptions

PowerPoint Meeh.  
Beamer Yeeeha.



# Animation

- This is important

# Animation

- This is important
- Now this

# Animation

- This is important
- Now this
- And now this

# Animation

- This is really important
- Now this
- And now this

# Figures

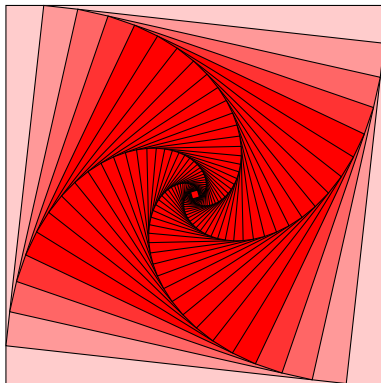


Figure: Rotated square from [texample.net](http://texample.net).

# Tables

**Table:** Largest cities in the world (source: Wikipedia)

City	Population
Mexico City	20,116,842
Shanghai	19,210,000
Peking	15,796,450
Istanbul	14,160,467

# Blocks

Three different block environments are pre-defined and may be styled with an optional background color.

Default

Block content.

Default

Block content.

Alert

Block content.

Alert

Block content.

Example

Block content.

Example

Block content.

# Math

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$



# Quotes

*Veni, Vidi, Vici*

# Frame footer

themedefines a custom beamer template to add a text to the footer. It can be set via

```
\setbeamertemplate{frame footer}{My custom footer}
```

# References

Some references to showcase [allowframebreaks] [?, ?, ?, ?, ?]

# Summary

Get the source of this theme and the demo presentation from

`github.com/matze/mtheme`

The theme *itself* is licensed under a Creative Commons Attribution-ShareAlike 4.0 International License.



Questions?

# Backup slides

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.

The best way to do this is to include the `appendixnumberbeamer` package in your preamble and call `\appendix` before your backup slides. This will automatically turn off slide numbering and progress bars for slides in the appendix.

# References I