# Character Tables for Representations of Finite Groups

## for Math 8190

Jared Stewart

Advisor: Dr. Calin Chindris

University of Missouri

April 2, 2016

# Table of contents

# Group Representations

Groups arise naturally as sets of symmetries of some object which are closed under composition and taking inverses.

# Group Representations

Groups arise naturally as sets of symmetries of some object which are closed under composition and taking inverses. For example,

1. The **symmetric group** of degree $n$, $S_n$, is the group of all symmetries of the set $\{1, \ldots, n\}$.

# Group Representations

Groups arise naturally as sets of symmetries of some object which are closed under composition and taking inverses. For example,

1. The **symmetric group** of degree $n$, $S_n$, is the group of all symmetries of the set $\{1, \ldots, n\}$.

2. The **dihedral group** of order $2n$, $D_n$, is the group of all symmetries of the regular $n$-gon in the plane.

# Group Representations

Groups arise naturally as sets of symmetries of some object which are closed under composition and taking inverses. For example,

1. The **symmetric group** of degree $n$, $S_n$, is the group of all symmetries of the set $\{1, \ldots, n\}$.

2. The **dihedral group** of order $2n$, $D_n$, is the group of all symmetries of the regular $n$-gon in the plane.

One may wonder more generally: Given an abstract group $G$, which objects $X$ does $G$ act on? This is the basic question of representation theory, which attempts to classify all such $X$ up to isomorphism.

# Group Actions

### Definition

A **(left) group action** of a group $G$ on a set $X$ is a map $\rho\colon G \times X \to X$ (written as $g \cdot a$, for all $g \in G$ and $a \in A$) that satisfies the following two axoims:

$$1 \cdot x = x \qquad\qquad \forall x \in X \qquad\qquad (1.1)$$

$$(gh) \cdot x = g \cdot (h \cdot x) \qquad \forall g, h \in G, x \in X \qquad (1.2)$$

e could likewise define the concept of a *right* group action, where the set elements would be multiplied by group elements on the right instead of on the left. Throughout we shall use the term *group action* to mean a *left* group action.

# The Definition of a Representation

### Definition

Let $G$ be a group, let $F$ be a field, and let $V$ be a vector space over $F$. A **linear representation** of $G$ is an action of $G$ on $V$ which preserves the linear structure of $V$, i.e. an action of $G$ on $V$ such that

$$g \cdot (v_1 + v_2) = g \cdot v_1 + g \cdot v_2 \qquad \forall g \in G, v_1, v_2 \in V \qquad (2.1)$$

$$g \cdot (kv) = k(g \cdot v) \qquad \forall g \in G, v \in V, k \in F \qquad (2.2)$$

# The Definition of a Representation

### Definition (Alternative definition)

Let $G$ be a group, let $F$ be a field, and let $V$ be a vector space over $F$. A **linear representation** of G is any group homomorphism $\rho\colon G \to GL(V)$. If we fix a basis for $V$, we get a representation in the previous sense.

# Metropolis titleformats

themesupports 4 different titleformats:

- Regular
- SMALLCAPS
- ALLSMALLCAPS
- ALLCAPS

They can either be set at once for every title type or individually.

# Small caps

This frame uses the `smallcaps` titleformat.

## Potential Problems

Be aware, that not every font supports small caps. If for example you typeset your presentation with pdfTeX and the Computer Modern Sans Serif font, every text in smallcaps will be typeset with the Computer Modern Serif font instead.

# All small caps

This frame uses the `allsmallcaps` titleformat.

### Potential problems

As this titleformat also uses smallcaps you face the same problems as with the `smallcaps` titleformat. Additionally this format can cause some other problems. Please refer to the documentation if you consider using it.
As a rule of thumb: Just use it for plaintext-only titles.

# All caps

This frame uses the `allcaps` titleformat.

**Potential Problems**

This titleformat is not as problematic as the `allsmallcaps` format, but basically suffers from the same deficiencies. So please have a look at the documentation if you want to use it.

## Typography

```
The theme provides sensible defaults to
\emph{emphasize} text, \alert{accent} parts
or show \textbf{bold} results.
```

becomes

The theme provides sensible defaults to *emphasize* text, accent parts or show **bold** results.

## Font feature test

- Regular
- *Italic*
- SMALLCAPS
- **Bold**
- **Bold Italic**
- **Bold SmallCaps**
- `Monospace`
- `Monospace Italic`
- `Monospace Bold`
- `Monospace Bold Italic`

# Lists

Items

- Milk

- Eggs

- Potatos

Enumerations

1. First,

2. Second and

3. Last.

Descriptions

PowerPoint Meeh.

Beamer Yeeeha.

# Animation

- This is important

# Animation

- This is important
- Now this

# Animation

- This is important
- Now this
- And now this

# Animation

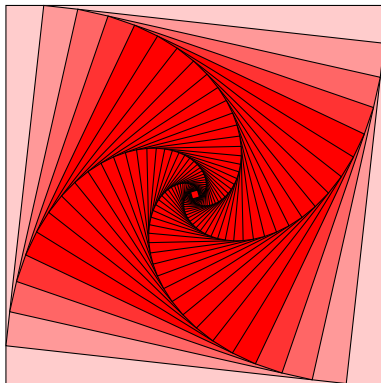- This is really important
- Now this
- And now this

# Figures



Figure: Rotated square from texample.net.

# Tables

Table: Largest cities in the world (source: Wikipedia)

| City | Population |
| --- | --- |
| Mexico City | 20,116,842 |
| Shanghai | 19,210,000 |
| Peking | 15,796,450 |
| Istanbul | 14,160,467 |

# Blocks

Three different block environments are pre-defined and may be styled with an optional background color.

| Default | Default |
|---|---|
| Block content. | Block content. |

| Alert | Alert |
|---|---|
| Block content. | Block content. |

| Example | Example |
|---|---|
| Block content. | Block content. |

# Math

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

# Quotes

*Veni, Vidi, Vici*

## Frame footer

themedefines a custom beamer template to add a text to the footer. It can be set via

`\setbeamertemplate{frame footer}{My custom footer}`

# References

Some references to showcase [allowframebreaks] [**?**, **?**, **?**, **?**, **?**]

# Summary

Get the source of this theme and the demo presentation from

github.com/matze/mtheme

The theme *itself* is licensed under a Creative Commons Attribution-ShareAlike 4.0 International License.

Questions?

# Backup slides

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.

The best way to do this is to include the `appendixnumberbeamer` package in your preamble and call `\appendix` before your backup slides.

themewill automatically turn off slide numbering and progress bars for slides in the appendix.