

# Presentation Title

## Presentation Subtitle

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Date / Occasion

# Outline

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# Group Representations

Groups arise naturally as sets of symmetries of some object which are closed under composition and taking inverses. For example,

- 1 The **symmetric group** of degree  $n$ ,  $S_n$ , is the group of all symmetries of the set  $\{1, \dots, n\}$ .
- 2 The **dihedral group** of order  $2n$ ,  $D_n$ , is the group of all symmetries of the regular  $n$ -gon in the plane.

One may wonder more generally: Given an abstract group  $G$ , which objects  $X$  does  $G$  act on? This is the basic question of representation theory, which attempts to classify all such  $X$  up to isomorphism.

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# Group Actions

## Definition

A **(left) group action** of a group  $G$  on a set  $X$  is a map  $\rho: G \times X \rightarrow X$  (written as  $g \cdot a$ , for all  $g \in G$  and  $a \in A$ ) that satisfies the following two axioms:

$$1 \cdot x = x \quad \forall x \in X \quad (1.1)$$

$$(gh) \cdot x = g \cdot (h \cdot x) \quad \forall g, h \in G, x \in X \quad (1.2)$$

One could likewise define the concept of a *right* group action, where the set elements would be multiplied by group elements on the right instead of on the left. Throughout we shall use the term *group action* to mean a *left* group action.



# The Definition of a Representation

## Definition

Let  $G$  be a group, let  $F$  be a field, and let  $V$  be a vector space over  $F$ . A **linear representation** of  $G$  is an action of  $G$  on  $V$  which preserves the linear structure of  $V$ , i.e. an action of  $G$  on  $V$  such that

$$g \cdot (v_1 + v_2) = g \cdot v_1 + g \cdot v_2 \quad \forall g \in G, v_1, v_2 \in V \quad (2.1)$$

$$g \cdot (kv) = k(g \cdot v) \quad \forall g \in G, v \in V, k \in F \quad (2.2)$$

# The Definition of a Representation

## Definition (Alternative definition)

Let  $G$  be a group, let  $F$  be a field, and let  $V$  be a vector space over  $F$ . A **linear representation** of  $G$  is any group homomorphism  $\rho: G \rightarrow GL(V)$ . If we fix a basis for  $V$ , we get a representation in the previous sense.

# Make Titles Informative.

You can create overlays. . .

- using the `pause` command:
  - First item.
  - Second item.
- using overlay specifications:
  - First item.
  - Second item.
- using the general `uncover` command:
  - First item.
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# Summary

- The **first main message** of your talk in one or two lines.
- The **second main message** of your talk in one or two lines.
- Perhaps a **third message**, but not more than that.
- Outlook
  - Something you haven't solved.
  - Something else you haven't solved.