UNIVERSITY OF MISSOURI

MASTER'S PROJECT

A Survey on Character Tables for Representations of Finite Groups

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in the

Department of Mathematics

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"Thanks to my solid academic training, today I can write hundreds of words on virtually any topic without possessing a shred of information, which is how I got a good job in journalism."

Dave Barry

UNIVERSITY OF MISSOURI

Abstract

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Masters of Arts

A Survey on Character Tables for Representations of Finite Groups

by Jared Stewart

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

Acknowledgements

The acknowledgements and the people to thank go here, don't forget to include your project advisor…

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For/Dedicated to/To my…

Chapter 1

Basics of Representation Theory

1.1 Group Actions

Definition 1.1. A ***(left)* group action** of a group G on a set X is a map $\varphi : G \times X \to X$ (written as $g \cdot a$, for all $g \in G$ and $a \in A$) that satisfies the following two axoims:

$$id_G \cdot x = x \qquad \forall x \in X \tag{1.1.1}$$
$$(gh) \cdot x = g \cdot (h \cdot x) \qquad \forall g, h \in G, x \in X \tag{1.1.2}$$

Note. We could likewise define the concept of a *right* group action, where the set elements would be multiplied by group elements on the right instead of on the left. Throughout we shall use the term *group action* to mean a *left* group action.

In this section, let the group G acts on a set X. For any fixed element $g \in G$, we have an associated map $\sigma_g : X \to X$ given by $\sigma_g(x) = g \cdot x$.

Proposition 1.2. *The map σ_g is a permutation of the set X.*

Proof. We show that σ_g is a permutation of X by finding a two-sided inverse map, namely $\sigma_{g^{-1}}$. Observe that for any $x \in X$, we have

$$\begin{aligned}
(\sigma_{g^{-1}} \circ \sigma_g)(x) &= \sigma_{g^{-1}}(\sigma_g(x)) && \text{(by definition of function composition)} \\
&= g^{-1} \cdot (g \cdot x) && \text{(by definition of } \sigma_g \text{ and } \sigma_{g^{-1}}) \\
&= (g^{-1}g) \cdot x && \text{(by axiom 1.1.1 of an action)} \\
&= id_G \cdot x && \\
&= x && \text{(by axiom 1.1.2 of an action).}
\end{aligned}$$

Thus $\sigma_{g^{-1}} \circ \sigma_g$ is the identity map on X. We can reverse the roles of g and g^{-1} to see that $\sigma_g \circ \sigma_{g^{-1}}$ is also the identity map on X. Having a two-sided inverse, we conslude that σ_g is a permutation of X. □

Proposition 1.3. *The map from G to the symmetric group S_X defined by $g \mapsto \sigma_g$ is a group homomorphism.*

Proof. Let $\varphi : G \to S_X$ be defined by $\varphi(g) = \sigma_g$. We have seen from Proposition 1.2 that σ_g is indeed an element of S_X. It remains to show that $\varphi(g_1 g_2) = \varphi(g_1) \circ \varphi(g_2)$. For every $x \in X$

$$
\begin{aligned}
\varphi(g_1g_2)(x) &= \sigma_{g_1g_2}(x) && \text{(by definition of } \varphi\text{)}\\
&= (g_1g_2)\cdot x && \text{(by definition of } \sigma_{g_1g_2}\text{)}\\
&= g_1\cdot(g_2\cdot x) && \text{(by axiom 1.1.1 of an action)}\\
&= \sigma_{g_1}(\sigma g_2(x)) && \text{(by definition of } \sigma_{g_1} \text{ and } \sigma g_2\text{)}\\
&= (\varphi(g_1)\circ\varphi(g_2))(x) && \text{(by definition of } \varphi\text{).}
\end{aligned}
$$

Since their values agree on every element $x \in X$, these two permutations are equal. □

Proposition 1.4. *The actions of G on the set X are in bijective correspondence with the homomorphisms from G into the symmetric group S_X.*

Proof. We have seen already that an action of G on the set X gives rise to a homomorphism from G into S_X. Suppose conversely that we have a homomorphism $\psi =$ from G into S_X. We define a map from $G \times X$ to X by $g \cdot x = \psi(g)(x)$. This map satisfies the definition of a group action of G on X. 1.1.1: $ig_G \cdot x = \psi(id_G)(x) = x$.

□

1.2 Definition of a Representation

Definition 1.5. A **linear representation** of a group G on a vector space V is a group homomorphism from G to $GL(V)$, the general linear group on V.

Definition 1.6. A **linear representation** ρ of a group G on a vector space V over a field F is a group action of G on V which preserves the linear structure of V, that is,

$$\rho(g)(v_1+v_2) = \rho(g)(v_1) + \rho(g)(v_2) \qquad \forall g \in G, v_1, v_2 \in V \tag{1.6.1}$$

$$\rho(g)(kv) = k\cdot\rho(g)v \qquad \forall g \in G, v \in V, k \in F \tag{1.6.2}$$

Proposition 1.7. *The definitions of a linear representation given in 1.5 and 1.6 are equivalent.*

Proof. ($\rightarrow$) Suppose that we have a homomorphism $\varphi : G \rightarrow GL(V)$. We can define an action of G on V by taking:

$$g\cdot v = \varphi(g)(v) \quad \forall g \in G, v \in V.$$

We verify that indeed we have obtained group action.

1.1.1 For any $v \in V$, we have: $id_G \cdot v = \varphi(id_G)(v) = id_V(v) = v$.

1.1.2 For any $v \in eV$ and g, h in G we have: $(gh)\cdot v = \varphi(gh)(v) = (\varphi(g)\varphi(h))(v) = \varphi(g)(\varphi(h)(v)) = g\cdot(h\cdot v)$.

Next, we check that this action preserves the linear structure of V.

1.6.1 For any $g \in G$, $v_1, v_2 \in V$ we have: $g\cdot(v_1+v_2) = \varphi(g)(v_1+v_2) = \varphi(g)(v_1) + \varphi(g)(v_2) = g\cdot v_1 + g\cdot v_2$

□

1.2.1 Subsection 1

Definition 1.8. Here is a new definition.

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1.2.2 Subsection 2

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Definition 1.9. A **linear representation** ρ of a group G on a vector space V over a field K is a group action of G on V which preserves the linear structure of V. That is,

$$\begin{gathered}\rho(g)(v_1 + v_2) = \rho(g)(v_1) + \rho(g)(v_2) \quad \forall g \in G, \forall v_1, v_2 \in V \\ \rho(g)(kv) = k \cdot \rho(g)v \quad \forall g \in G, v \in V, k \in K\end{gathered} \tag{1.9.1}$$

1.3 Main Section 2

Definition 1.10. Here is a new definition.

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Chapter 2

Spaghetti

2.1 Definition of a Representation AGAIN

Definition 2.1. A **linear representation** of a group G on a vector space V is a group homomorphism from G to $GL(V)$, the general linear group on V.

More explicitly, a representation is a map $\rho : G \to GL(V)$ such that

$$\rho(g_1 g_2) = \rho(g_1)\rho(g_2) \quad \forall g_1, g_2 \in G.$$

Definition 2.2. A **linear representation** ρ of a group G on a vector space V over a field K is a group action of G on V which preserves the linear structure of V. That is,

1. $\rho(g)(v_1 + v_2) = \rho(g)(v_1) + \rho(g)(v_2) \quad \forall g \in G, v_1, v_2 \in V$
2. $\rho(g)(kv) = k \cdot \rho(g)v \quad \forall g \in G, v \in V, k \in K$

2.1.1 Subsection 1

Definition 2.3. Here is a new definition.

$$E = mc^2 \tag{2.3.1}$$

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2.1.2 Subsection 2

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Definition 2.4. A **linear representation** ρ of a group G on a vector space V over a field K is a group action of G on V which preserves the linear structure of V. That is,

$$\rho(g)(v_1 + v_2) = \rho(g)(v_1) + \rho(g)(v_2) \quad \forall g \in G, \forall v_1, v_2 \in V \tag{2.4.1}$$

$$\rho(g)(kv) = k \cdot \rho(g)v \quad \forall g \in G, v \in V, k \in K$$

2.2 Main Section 2

Definition 2.5. Here is a new definition.

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Appendix A

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