UNIVERSITY OF MISSOURI

MASTER'S PROJECT

A Survey on Character Tables for Representations of Finite Groups

Author:
Jared Stewart

Supervisor:
Dr. Calin Chindris

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in the

Department of Mathematics

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"Thanks to my solid academic training, today I can write hundreds of words on virtually any topic without possessing a shred of information, which is how I got a good job in journalism."

Dave Barry

UNIVERSITY OF MISSOURI

Abstract

Calin Chindris
Department of Mathematics

Masters of Arts

A Survey on Character Tables for Representations of Finite Groups

by Jared Stewart

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too…

Acknowledgements

The acknowledgements and the people to thank go here, don’t forget to include your project advisor…

Contents

For/Dedicated to/To my…

Chapter 1

Basics of Representation Theory

1.1 Definition of a Representation

Definition 1.1. A ***(left)* group action** of a group G on a set X is a map $\varphi : G \times X \to X$ (written as $g \cdot a$, for all $g \in G$ and $a \in A$) that satisfies the following two axoims:

$$id_G \cdot x = x \qquad \forall x \in X \tag{1.1.1}$$
$$(gh) \cdot x = g \cdot (h \cdot x) \qquad \forall g, h \in G, x \in X \tag{1.1.2}$$

Note. We could define the concept of a *right* group action, where we multiply the set elements by group elements on the right instead of on the left. We shall use the term *group action* throughout to mean a *left* group action.

Definition 1.2. A **linear representation** of a group G on a vector space V is a group homomorphism from G to $GL(V)$, the general linear group on V.

Definition 1.3. A **linear representation** ρ of a group G on a vector space V over a field F is a group action of G on V which preserves the linear structure of V, that is,

$$\rho(g)(v_1 + v_2) = \rho(g)(v_1) + \rho(g)(v_2) \qquad \forall g \in G, v_1, v_2 \in V \tag{1.3.1}$$
$$\rho(g)(kv) = k \cdot \rho(g)v \qquad \forall g \in G, v \in V, k \in F \tag{1.3.2}$$

Proposition 1.4. *The definitions of a linear representation given in 1.2 and 1.3 are equivalent.*

Proof. $(\to)$ Suppose that we have a homomorphism $\varphi : G \to GL(V)$. We can define an action of G on V by taking:

$$g \cdot v = \varphi(g)(v) \quad \forall g \in G, v \in V.$$

We verify that indeed we have obtained group action.
1.1.1 For any $v \in V$, we have: $id_G \cdot v = \varphi(id_G)(v) = id_V(v) = v$.
1.1.2 For any $v \in eV$ and g, h in G we have: $(gh) \cdot v = \varphi(gh)(v) = (\varphi(g)\varphi(h))(v) = \varphi(g)(\varphi(h)(v)) = g \cdot (h \cdot v)$.
Next, we check that this action preserves the linear structure of V.
1.3.1 For any $g \in G$, $v_1, v_2 \in V$ we have: $g \cdot (v_1 + v_2) = \varphi(g)(v_1 + v_2) = \varphi(g)(v_1) + \varphi(g)(v_2) = g \cdot v_1 + g \cdot v_2$

□

1.1.1 Subsection 1

Definition 1.5. Here is a new definition.

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1.1.2 Subsection 2

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Definition 1.6. A **linear representation** ρ of a group G on a vector space V over a field K is a group action of G on V which preserves the linear structure of V. That is,

$$\begin{aligned} \rho(g)(v_1 + v_2) &= \rho(g)(v_1) + \rho(g)(v_2) \quad \forall g \in G, \forall v_1, v_2 \in V \\ \rho(g)(kv) &= k \cdot \rho(g)v \quad \forall g \in G, v \in V, k \in K \end{aligned} \tag{1.6.1}$$

1.2 Main Section 2

Definition 1.7. Here is a new definition.

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Chapter 2

Spaghetti

2.1 Definition of a Representation AGAIN

Definition 2.1. A **linear representation** of a group G on a vector space V is a group homomorphism from G to $GL(V)$, the general linear group on V.

More explicitly, a representation is a map $\rho : G \to GL(V)$ such that

$$\rho(g_1 g_2) = \rho(g_1)\rho(g_2) \quad \forall g_1, g_2 \in G.$$

Definition 2.2. A **linear representation** ρ of a group G on a vector space V over a field K is a group action of G on V which preserves the linear structure of V. That is,

1. $\rho(g)(v_1 + v_2) = \rho(g)(v_1) + \rho(g)(v_2) \quad \forall g \in G, v_1, v_2 \in V$
2. $\rho(g)(kv) = k \cdot \rho(g)v \quad \forall g \in G, v \in V, k \in K$

2.1.1 Subsection 1

Definition 2.3. Here is a new definition.

$$E = mc^2 \tag{2.3.1}$$

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2.1.2 Subsection 2

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Definition 2.4. A **linear representation** ρ of a group G on a vector space V over a field K is a group action of G on V which preserves the linear structure of V. That is,

$$\rho(g)(v_1 + v_2) = \rho(g)(v_1) + \rho(g)(v_2) \quad \forall g \in G, \forall v_1, v_2 \in V \tag{2.4.1}$$

$$\rho(g)(kv) = k \cdot \rho(g)v \quad \forall g \in G, v \in V, k \in K$$

2.2 Main Section 2

Definition 2.5. Here is a new definition.

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Appendix A

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