MATH401 Project 1

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Exercise 2.8

Suppose an economy has three sectors producing products Product 1, Product 2 and Product 3 respectively.

- It takes 0.02 units of Product 1, 0.06 units of Product 2, and 0.10 units of Product 3 to produce 1 unit of Product 1.
- It takes 0.40 units of Product 2 and 0.04 units of Product 3 to produce 1 unit of Product 2.
- It takes 0.18 units of Product 1, 0.01 units of Product 2, and 0.10 units of Product 3 to produce 1 unit of Product 3.

(a) What should the total production be set at in order to satisfy an external demand of 200 units of Product 1, 180 units of Product 2 and 175 units of Product 3?

```
>> M = [0.02 0.00 0.18;0.06 0.40 0.01;0.10 0.04 0.10];
>> d = [200;180;175];
>> inv(eye(3)-M)*d
ans =
  247.5308
  328.6957
  236.5566
```

So the production of the three product should be set at 248.7364 for Product 1, 316.5139 for Product 2, and 247.7085 for Product 3.

(b) Using the adjugate method calculate how production in sector 3 must change if the demand for sector 2 changes by +1. How about by +2? How about by −1?

$$A = (I - M) = \begin{bmatrix} 0.9800 & 0 & -0.1800 \\ -0.0600 & 0.6000 & -0.0100 \\ -0.1000 & -0.0400 & 0.9000 \end{bmatrix}$$

$$A^{-1}_{32} = (-1)^{2+3} det(A_{23})/det(A)$$

$$det(A_{23}) = det \begin{bmatrix} 0.9800 & 0 \\ -0.1000 & -0.0400 \end{bmatrix} = -0.0392$$

$$det(A) = 0.5176$$

$$A^{-1}_{32} = (-1)^{2+3}(-0.0392)/0.5176 = 0.0757$$

If the demand for sector 2 changes by +1, then the production in sector 3 must change by +0.0757;

If the demand for sector 2 changes by +2, then the production in sector 3 must change by +0.1514;

If the demand for sector 2 changes by -1, then the production in sector 3 must change by -0.0757;

(c) Find (I – M)^–1 using technology.

```
>> inv(eye(3)-M)
ans =
   1.0426    0.0139    0.2087
   0.1063    1.6693    0.0398
   0.1206    0.0757    1.1361
```

Exercise 2.10

Suppose the consumption matrix for four sectors is given by:

$$M = \begin{bmatrix} 0.02 & 0 & 0.02 & 0.04 \\ 0 & 0.01 & 0.05 & 0.04 \\ 0.03 & 0 & 0.01 & 0.02 \\ 0.02 & 0.05 & 0.01 & 0 \end{bmatrix}$$

Using the infinite series method find a quick approximation for how production in sector 1 must change if the demand for sector 1 changes by +1

```
>> M = [0.02 0 0.02 0.04;0 0.01 0.05 0.04;0.03 0 0.01 0.02;0.02 0.05 0.01 0];
>> eye(4)+M+M^2+M^3
ans =
   1.0219    0.0021    0.0212    0.0414
   0.0024    1.0122    0.0516    0.0416
   0.0314    0.0011    1.0110    0.0215
   0.0209    0.0507    0.0131    1.0031
```

If the demand for sector 1 changes by +1, then the production in sector 1 must change by +1.0219. Since the values in M are small and the matrix is very sparse, so I let the greatest powers of M be 3.

Exercise 2.15

Suppose the consumption matrix for an economy with two sectors is given by

$$M = \begin{bmatrix} 0.06 & 1.02 \\ 0.05 & 0.1 \end{bmatrix}$$

(a) This matrix is very similar to the matrix in the previous question yet this one gives reasonable results whereas the previous question does not. Explain this difference in economic terms. In other words why is it economically reasonable that $m_{12} > 1$ but not that $m_{11} > 1$?

Inorder for this economy to be more efficient, we prefer smaller internal demands. If it takes more than 1 product to make a copy of itself, then the economy is not efficient at all. In this case, it is unrealistic for matrix M have entry m_{11} greater than 1.

(b) Find $(I – M)^{-1}$.

```
>> M = [0.06 1.02;0.05 0.1];
>> inv(eye(2)- M)
ans =
    1.1321    1.2830
    0.0629    1.1824
```

(c) Find $\bar{p}$ which corresponds to $\bar{d} = \begin{bmatrix} 100 \\ 200 \end{bmatrix}$.

```
>> d = [100;200];
>> p = inv(eye(2)- M)*d;
>> p
p =
  369.8113
  242.7673
```

(d) Show with some calculation that it seems that $I + M + M^2 + \ldots$ does converge to $(I – M)^{-1}$ in this case.

```
>> eye(2)+M+M^2+M^3+M^4+M^5+M^6+M^7+M^8+M^9
ans =
   1.1321    1.2830
   0.0629    1.1824
```

```
>> inv(eye(2)- M) - eye(2)+M+M^2+M^3+M^4+M^5+M^6+M^7+M^8+M^9
ans =
   1.0e-04 *
    0.0485    0.2390
    0.0117    0.0578
```

Exercise 3.9

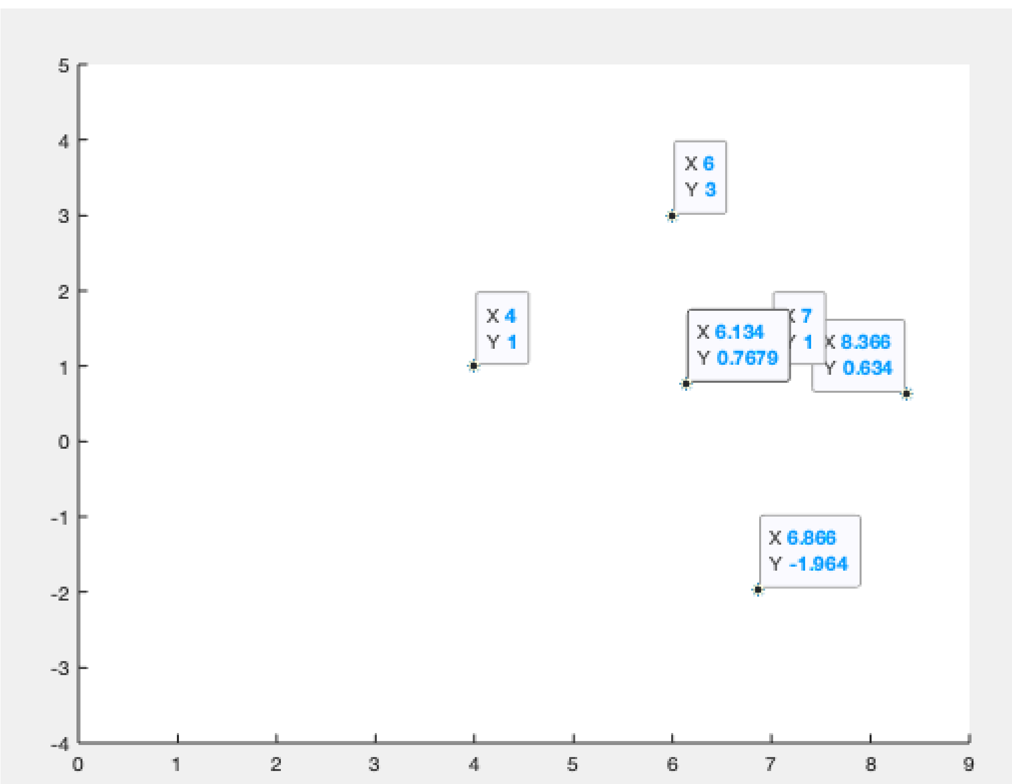
In two dimensions find the image of the three points

$$(6,3),(4,1),(7,1)$$

under rotation around the point (8, 2) by π/3 radians. Sketch the original points and the images.

$$T(8,2)R(\pi/3)T(-8,-2) = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\pi/3) & -sin(\pi/3) & 0 \\ sin(\pi/3) & cos(\pi/3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0.5000 & -0.8660 & 5.7321 \\ 0.8660 & 0.5000 & -5.9282 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.5000 & -0.8660 & 5.7321 \\ 0.8660 & 0.5000 & -5.9282 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 4 & 7 \\ 3 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 6.1340 & 6.8660 & 8.3660 \\ 0.7679 & -1.9641 & 0.6340 \\ 1 & 1 & 1 \end{bmatrix}$$



Exercise 3.16

In three dimensions find the image of the three points

$$(0, 3, 1), (-6, 3, 1), (3, 1, 10)$$

under shifting by (+2, –3, –6) followed by rotation around the z-axis by 7π/6.

To rotate around the z-axis by 7π/6: $R_Z(7\pi/6) = \begin{bmatrix} cos(7\pi/6) & -sin(7\pi/6) & 0 & 0 \\ sin(7\pi/6) & cos(7\pi/6) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$R_Z(7\pi/6)\begin{bmatrix}0\\3\\1\\1\end{bmatrix} + \begin{bmatrix}2\\-3\\-6\\1\end{bmatrix} = \begin{bmatrix}1.5\\-2.5981\\1\\1\end{bmatrix} + \begin{bmatrix}2\\-3\\-6\\1\end{bmatrix} = \begin{bmatrix}3.5\\-5.5981\\-5\\2\end{bmatrix}$$

so (0,3,1) become (3.5 , -2.5981, -5)

$$R_Z(7\pi/6)\begin{bmatrix}-6\\3\\1\\1\end{bmatrix} + \begin{bmatrix}2\\-3\\-6\\1\end{bmatrix} = \begin{bmatrix}6.6962\\0.4019\\1\\1\end{bmatrix} + \begin{bmatrix}2\\-3\\-6\\1\end{bmatrix} = \begin{bmatrix}8.6962\\-2.5981\\-5\\2\end{bmatrix}$$

so (-6,3,1) become (8.6962 , -2.5981, -5)

$$R_Z(7\pi/6)\begin{bmatrix}3\\1\\10\\1\end{bmatrix} + \begin{bmatrix}2\\-3\\-6\\1\end{bmatrix} = \begin{bmatrix}-2.0981\\-2.3660\\10\\1\end{bmatrix} + \begin{bmatrix}2\\-3\\-6\\1\end{bmatrix} = \begin{bmatrix}-0.0981\\-5.3660\\4\\2\end{bmatrix}$$

so (-6,3,1) become (-0.0981, -5.3660, 4)

Exercise 3.17

In three dimensions find the image of the three points

$$(1, 2, 3), (-2, 3, 1), (3, 2, 2)$$

under the perspective projection with center of perspective at z = 10.

$$P(10)A = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & -1/10 & 1\end{bmatrix}\begin{bmatrix}1\\2\\3\\1\end{bmatrix} = \begin{bmatrix}1\\2\\0\\0.7\end{bmatrix} = \begin{bmatrix}1.4286\\2.8571\\0\\1\end{bmatrix}$$

The point will be (1.4286, 2.8571, 0)

$$P(10)A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/10 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \\ 0.9 \end{bmatrix} = \begin{bmatrix} -2.2222 \\ 3.3333 \\ 0 \\ 1 \end{bmatrix}$$

The point will be (-2.2222, 3.3333, 0)

$$P(10)A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/10 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 3.75 \\ 2.5 \\ 0 \\ 1 \end{bmatrix}$$

The point will be (3.75, 2.5, 0)

Exercise 3.27

Design/construct a three-dimensional object using points. Store those points in a 4 × k matrix and then apply a series of interesting transformations. Find the resulting points and plot. In your submission plot the points before the transformations, give the series of transformations both by description and by matrix and give the overall resulting matrix. Finally plot the points after the transformations.

The object I designed is a straight line in the three-dimensional space, the line starts at (0,0,0) and ends at (20,30,40).I want to rotate it by 0.7 radians about the axis y = x, z = 0.

In ordet to do this, I rotate by $\pi/4$ about the z-axis to move this axis to the y-axis, then rotate, then rotate back. In other words we use the matrix: $R_Z(-\pi/4)R_Y(0.7)R_Z(\pi/4)$

$$R_Z(-\pi/4)R_Y(0.7)R_Z(\pi/4)$$

$$= \begin{bmatrix} cos(-\pi/4) & -sin(-\pi/4) & 0 & 0 \\ sin(-\pi/4) & cos(-\pi/4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(0.7) & 0 & sin(0.7) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(0.7) & 0 & cos(0.7) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\pi/4) & -sin(\pi/4) & 0 & 0 \\ sin(\pi/4) & cos(\pi/4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8824 & 0.1176 & 0.4555 & 0 \\ 0.1176 & 0.8824 & -0.4555 & 0 \\ -0.4555 & 0.4555 & 0.7648 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The point (20,30,40) become:

$$\begin{bmatrix} 0.8824 & 0.1176 & 0.4555 & 0 \\ 0.1176 & 0.8824 & -0.4555 & 0 \\ -0.4555 & 0.4555 & 0.7648 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 30 \\ 40 \\ 1 \end{bmatrix} = \begin{bmatrix} 39.3970 \\ 10.6030 \\ 35.1490 \\ 1 \end{bmatrix}$$

Thus my line after a series of interesting transformations become a new line that has a starts point (0,0,0) and end point (39.3970, 10.6030, 35.1490)

Exercise 6.5

Suppose four teams play a number of games with the following results:

- Team 1 beats Team 2 by 5 points. Team 1 at home.
- Team 1 beats Team 3 by 10 points. Team 3 at home.
- Team 2 beats Team 3 by -10 points. Team 2 at home.
- Team 2 beats Team 3 by 7 points. Team 3 at home.
- Team 2 beats Team 4 by -14 points. Team 2 at home.
- Team 3 beats Team 4 by 21 points. Team 3 at home.

(a) Find the team rankings, ignore the "at home" aspect.

$$Ar = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} r1 \\ r2 \\ r3 \\ r4 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ -10 \\ 7 \\ -14 \\ 21 \end{bmatrix} = b$$

$$A^T A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 4 & -2 & -1 \\ -1 & -2 & 4 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 15 \\ -22 \\ 14 \\ -7 \end{bmatrix}$$

New matrix

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 4 & -2 & -1 \\ -1 & -2 & 4 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} r = \begin{bmatrix} 15 \\ -22 \\ 14 \\ 0 \end{bmatrix}$$

$$r = \begin{bmatrix} 6.5 \\ -4 \\ 2 \\ -4.5 \end{bmatrix}$$

Team 1 has ranking r1 = 6.5
Team 2 has ranking r2 = -4
Team 3 has ranking r3 = 2
Team 4 has ranking r4 = -4.5

(b) Suppose when a team plays at home it has a point advantage. Suppose history suggests that to factor this into the scores if a team wins at home the score should be lowered by 25% before calculating the ranking. Find the new team rankings; do they change?

$$Ar = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} r1 \\ r2 \\ r3 \\ r4 \end{bmatrix} = \begin{bmatrix} 3.75 \\ 10 \\ -10 \\ 7 \\ -14 \\ 15.75 \end{bmatrix} = b$$

New matrix

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 4 & -2 & -1 \\ -1 & -2 & 4 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} r = \begin{bmatrix} 13.75 \\ -20.75 \\ 8.75 \\ 0 \end{bmatrix}$$

$$r = \begin{bmatrix} 5.3750 \\ -3.9583 \\ 0.9583 \\ -2.3750 \end{bmatrix}$$

Team 1 has ranking r1 = 5.3750
Team 2 has ranking r2 = -3.9583
Team 3 has ranking r3 = 0.9583
Team 4 has ranking r4 = -2.3750