Cheat Sheet – Regression Analysis

What is Regression Analysis?

Fitting a function f(.) to datapoints y_i=f(x_i) under some error function. Based on the estimated function and error, we have the following types of regression

1. Linear Regression:

Fits a line minimizing the sum of mean-squared error for each datapoint.

$$min_\beta \sum_i \|y_i - f_\beta^{linear}(x_i)\|^2$$

$$f_\beta^{linear}(x_i) = \beta_0 + \beta_1 x_i$$

2. Polynomial Regression:

Fits a polynomial of order k (k+1 unknowns) minimizing the sum of mean-squared error for each datapoint.

$$min_\beta \sum_{i=0}^{m} \|y_i - f_\beta^{poly}(x_i)\|^2$$

$$f_\beta^{poly}(x_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \ldots + \beta_k x_i^k$$

3. Bayesian Regression:

For each datapoint, fits a gaussian distribution by minimizing the mean-squared error. As the number of data points x_i increases, it converges to point estimates i.e. $n \to \infty, \sigma^2 \to 0$

$$min_\beta \sum_i \|y_i - \mathcal{N}\left(f_\beta(x_i), \sigma^2\right)\|^2$$

$$f_\beta(x_i) = f_\beta^{poly}(x_i) \ or \ f_\beta^{linear}(x_i)$$

$\mathcal{N}(\mu, \sigma^2) \to$ Gaussian with mean μ and variance σ^2

4. Ridge Regression:

Can fit either a line, or polynomial minimizing the sum of mean-squared error for each datapoint and the weighted L2 norm of the function parameters beta.

$$min_\beta \sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2 + \sum_{j=0}^{k} \beta_j^2$$

$$f_\beta(x_i) = f_\beta^{poly}(x_i) \ or \ f_\beta^{linear}(x_i)$$

5. LASSO Regression:

Can fit either a line, or polynomial minimizing the the sum of mean-squared error for each datapoint and the weighted L1 norm of the function parameters beta.

$$min_\beta \sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2 + \sum_{j=0}^{k} |\beta_j|$$

$$f_\beta(x_i) = f_\beta^{poly}(x_i) \ or \ f_\beta^{linear}(x_i)$$

6. Logistic Regression (NOT regression, but classification):

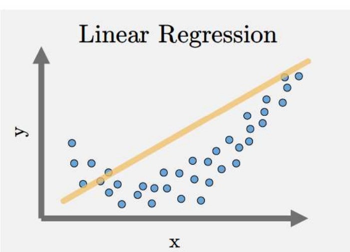
Can fit either a line, or polynomial with sigmoid activation minimizing the sum of mean-squared error for each datapoint. The labels y are binary class labels.

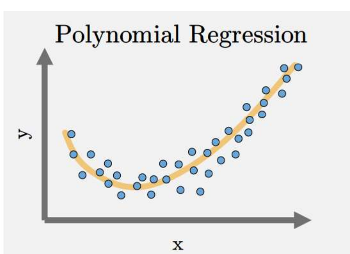
$$min_\beta \sum_i \|y_i - \sigma(f_\beta(x_i))\|^2$$

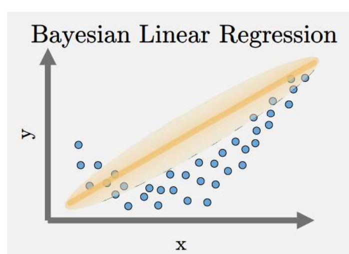
$$f_\beta(x_i) = f_\beta^{poly}(x_i) \ or \ f_\beta^{linear}(x_i)$$

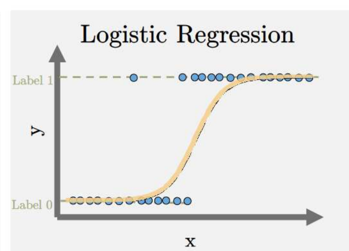
$$\sigma(t) = \frac{1}{1+e^{-t}}$$

Visual Representation:









Summary:

	What does it fit?	Estimated function	Error Function
Linear	A line in n dimensions	$f_\beta^{linear}(x_i) = \beta_0 + \beta_1 x_i$	$\sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2$
Polynomial	A polynomial of order k	$f_\beta^{poly}(x_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \ldots$	$\sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2$
Bayesian Linear	Gaussian distribution for each point	$\mathcal{N}\left(f_\beta(x_i), \sigma^2\right)$	$\sum_i \|y_i - \mathcal{N}\left(f_\beta(x_i), \sigma^2\right)\|^2$
Ridge	Linear/polynomial	$f_\beta^{poly}(x_i) \ or \ f_\beta^{linear}(x_i)$	$\sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2 + \sum_{j=0}^{n} \beta_j^2$
LASSO	Linear/polynomial	$f_\beta^{poly}(x_i) \ or \ f_\beta^{linear}(x_i)$	$\sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2 + \sum_{j=0}^{n} \|\beta_j\|$
Logistic	Linear/polynomial with sigmoid	$\sigma(f_\beta(x_i))$	$\sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2$

Source: https://www.cheatsheets.aqeel-anwar.com