Introduction to Probability and Statistics
Fourteenth Edition

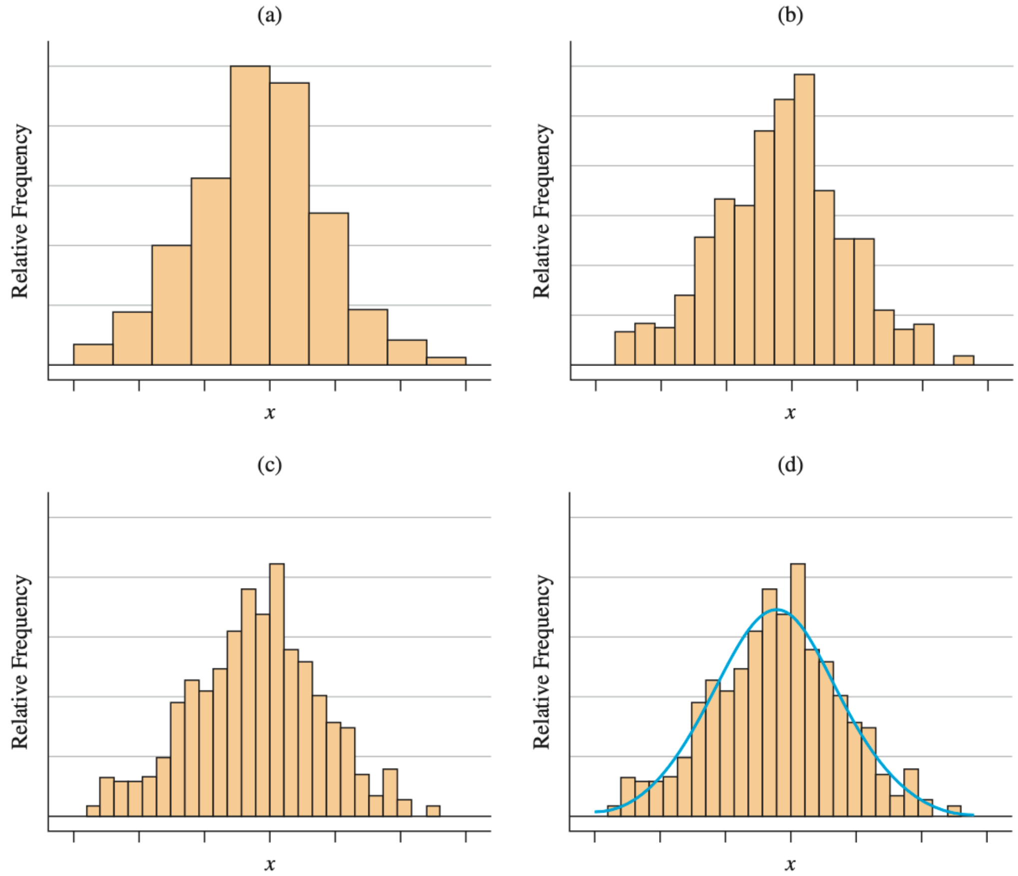
Chapter 6
The Normal Probability Distribution

6.1 Probability distributions for continuous random variables

Continuous Random Variables

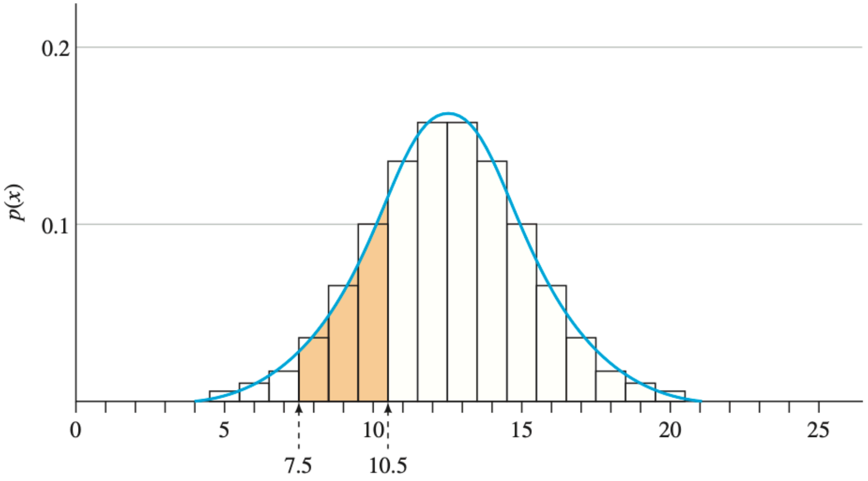
- Continuous random variables can assume the infinitely many values corresponding to points on a line interval.
- **Examples:**
  - Heights, weights
  - length of life of a particular product
  - experimental laboratory error

Figure 6.1
Relative frequency histograms for increasingly large sample sizes



Suppose $X \sim Binomial(n = 25, p = 0.5)$
Think about $P(X \le 5) = C_0^{25} 0.5^{25} + C_1^{25} 0.5^{25} + C_2^{25} 0.5^{25} + C_3^{25} 0.5^{25} + C_4^{25} 0.5^{25} + C_5^{25} 0.5^{25}$ **using a calculator!**
This is very difficult even with a caclulator.

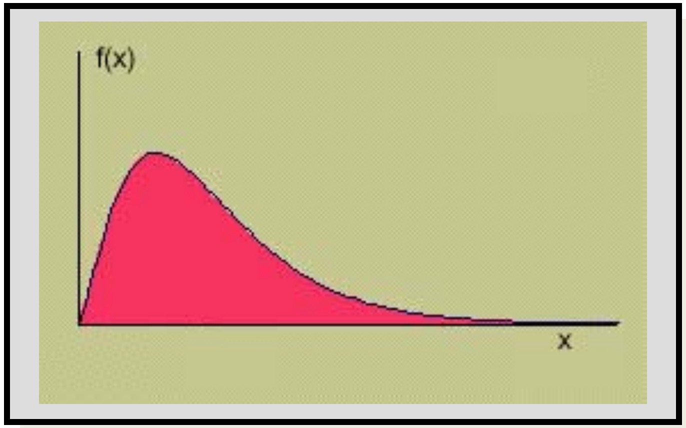
Figure 6.17
The binomial probability distribution for $n = 25$ and $p = .5$ and the approximating normal distribution with $\mu = 12.5$ and $\sigma = 2.5$



	Discrete RV	Continuous RV
Easy to understand	Yes	No
Easy to calculate its probability	No	Yes

Continuous Random Variables

- A **smooth curve** describes the probability distribution of a continuous random variable.



•The depth or density of the probability, which varies with *x*, may be described by a mathematical formula *f* (*x*), called the **probability distribution** or **probability density function (pdf)** for the random variable *x*.

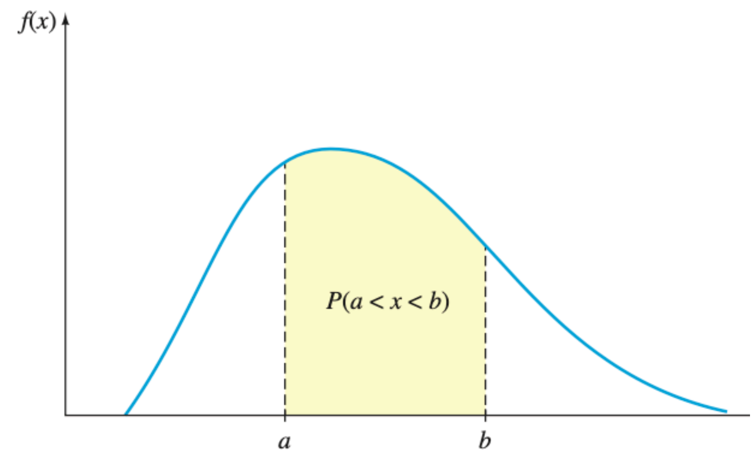
Mean and Variance of a continuous random variable

- The mean is $\mu = E[X] = \int x f(x) dx$
- The variance is
  $\sigma^2 = E[(X - mu)^2] = \int (x - \mu)^2 f(x) dx$

Properties of Continuous Probability Distributions

- The area under the curve is equal to **1.**
- P(a < x < b) = **area under the curve** between a and b.

Figure 6.2
The probability distribution $f(x)$; $P(a<x<b)$ is equal to the shaded area under the curve



- There is no probability attached to any single value of x. That is, **P(x = a) = 0.**

THE CONTINUOUS UNIFORM PROBABILITY DISTRIBUTION

The Continuous Uniform Probability Distribution

The **continuous uniform random variable** is used to model the behavior of a random variable whose values are uniformly or evenly distributed over a given interval. If we describe this interval in general as an interval from a to b, the formula or probability density function (pdf) that describes this random variable x is given by

$$f(x) = \frac{1}{b-a} \text{ for } a \le x \le b.$$

A graph of this probability distribution is shown in Figure 6.3.

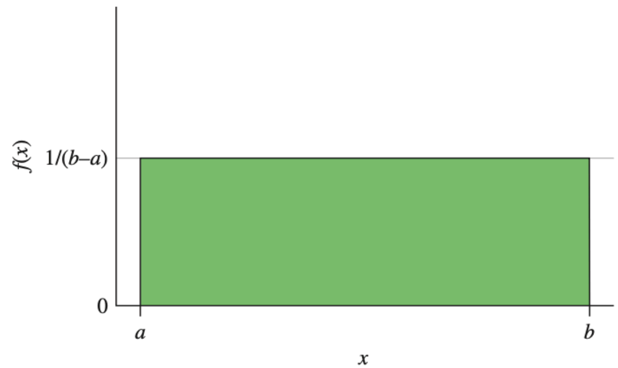


Figure 6.3 The continuous uniform probability distribution

$$\mu = \frac{b-a}{2} \text{ and } \sigma^2 = \frac{(b-a)^2}{12}.$$

Example 6.1

EXAMPLE 6.1 The error introduced by rounding an observation to the nearest centimeter has a uniform distribution over the interval from $-.5$ to $.5$. What is the probability that the rounding error is less than $.2$ in absolute value?

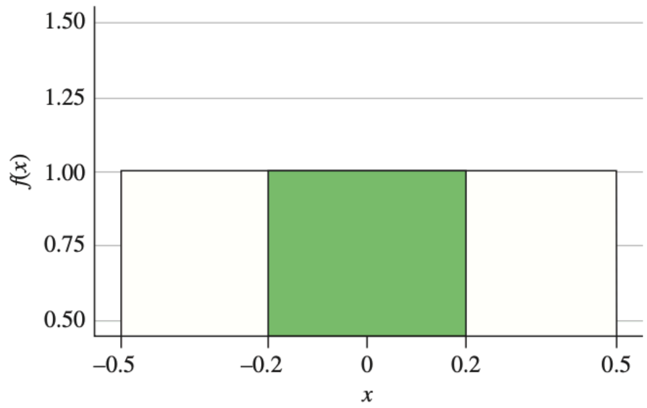


Figure 6.4 Uniform probability distribution for Example 6.1

Solution This probability corresponds to the area under the distribution between $x = -.2$ and $x = .2$, as shown in Figure 6.4. Since the height of the rectangle is 1,

$$P(-.2 < x < .2) = [.2 - (-.2)] \times 1 = .4$$

THE EXPONENTIAL PROBABILITY DISTRIBUTION (SKIPPED)

The Exponential Probability Distribution

The **exponential random variable** is used to model positive continuous random variables such as waiting times or lifetimes associated with electronic components. The probability density function is given by

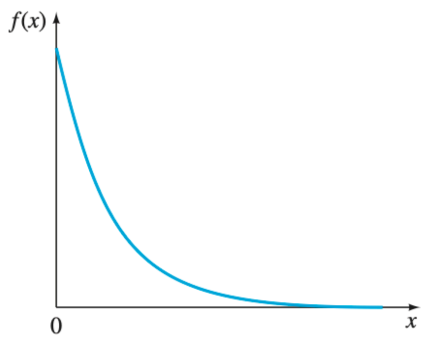
$$f(x) = \lambda e^{-\lambda x} \text{ for } x \geq 0 \text{ and } \lambda > 0$$

and is 0 otherwise. The parameter λ (the Greek letter "lambda") is often referred to as the *intensity* and is related to the mean and variance as

$$\boxed{\mu = 1/\lambda \text{ and } \sigma^2 = 1/\lambda^2}$$

so that $\mu = \sigma$. A graph of an exponential distribution is shown in Figure 6.5.

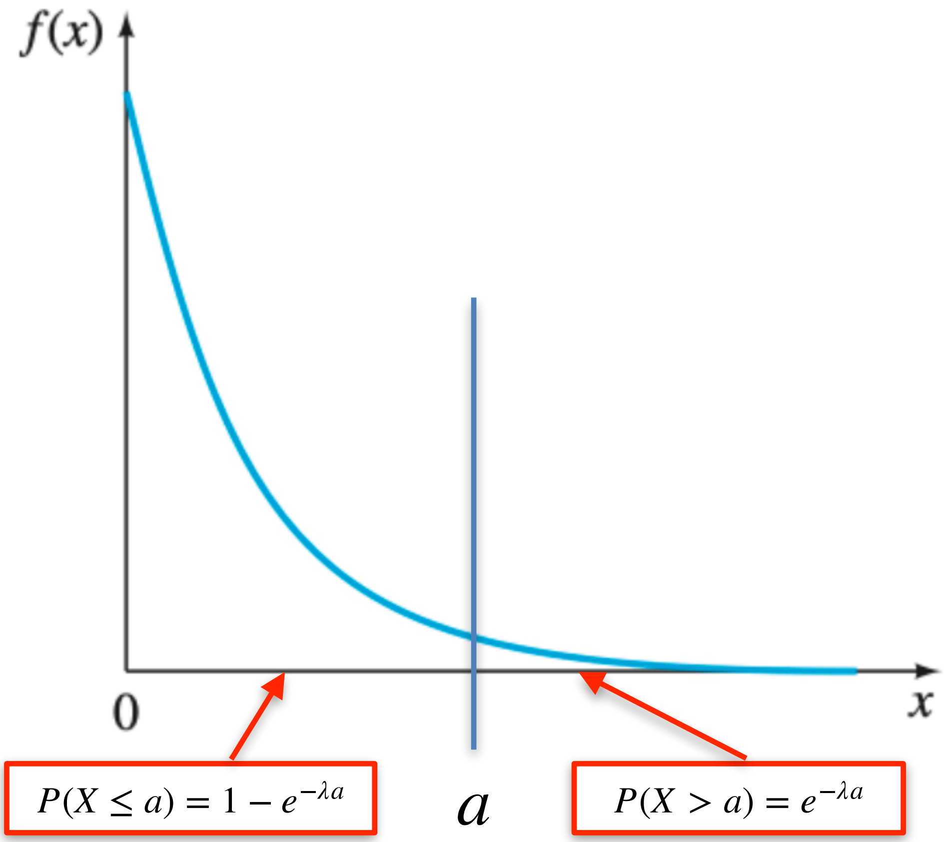
Figure 6.5
An exponential probability distribution



To find areas under this curve, you can use the fact that $P(x > a) = e^{-\lambda a}$ for $a > 0$ to calculate right-tailed probabilities. The left-tailed probabilities can be calculated using the complement rule as $\boxed{P(x \leq a) = 1 - e^{-\lambda a} \text{ for } a > 0.}$

$$\boxed{P(X > a) = 1 - P(X \leq a) = e^{-\lambda a} \textbf{ for } a > 0}$$

PROBABILITY OF THE EXPONENTIAL DISTRIBUTION (SKIPPED)



$P(X \leq a) = 1 - e^{-\lambda a}$

$P(X > a) = e^{-\lambda a}$

EXAMPLE 6.2 (SKIPPED)

EXAMPLE 6.2 The waiting time at a supermarket checkout counter has an exponential distribution with an average waiting time of 5 minutes. What is the probability that you will have to wait more than 10 minutes at the checkout counter?

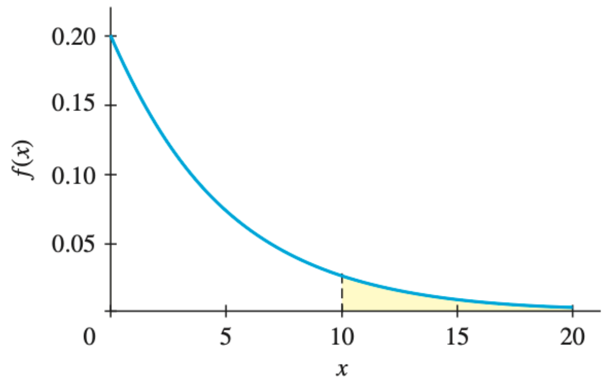


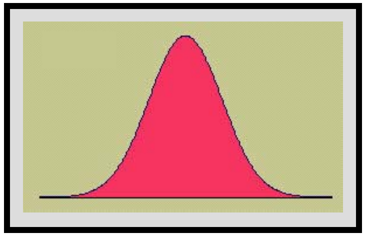
Figure 6.6
Exponential probability distribution for Example 6.2

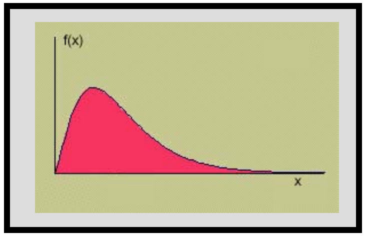
Solution Since the average waiting time at the checkout counter is $\mu = 5$ minutes and because $\mu = 1/\lambda$, we can find $5 = 1/\lambda$ or $\lambda = .2$. The probability density function then is given by $f(x) = .2e^{-.2x}$ for $x > 0$ (shown in Figure 6.6) and the probability to be calculated is the shaded area in the figure. Use the general formula for $P(x > a)$ to find

$$P(x > 10) = e^{-.2(10)} = .135$$

6.2 The normal probability distribution

Continuous Probability Distributions

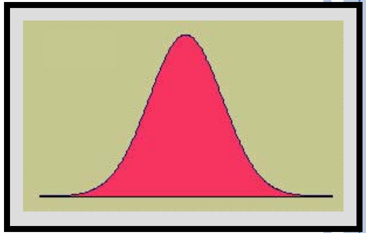




- There are many different types of continuous random variables
- We try to pick a model that
  - Fits the data well
  - Allows us to make the best possible inferences using the data.
- One important continuous random variable is the **normal random variable**.


DEUTSCHE BUNDESBANK
Banknote
1777–1855 Carl Friedr. Gauß

THE NORMAL DISTRIBUTION



- The formula that generates the normal probability distribution is:

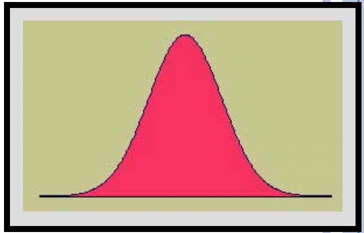
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for } -\propto < x < \propto$$

$$e = 2.7183 \qquad \pi = 3.1416$$

μ and σ are the population mean and standard deviation.

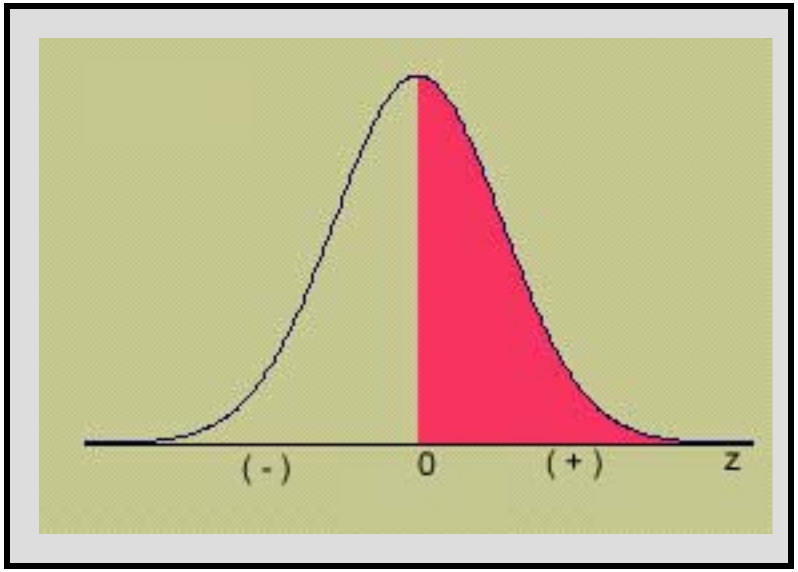
- The shape and location of the normal curve changes as the mean and standard deviation change.

The Standard Normal Distribution



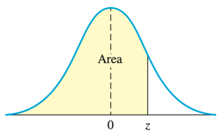
- To find $P(a < x < b)$, we need to find the area under the appropriate normal curve.
- To simplify the tabulation of these areas, we **standardize** each value of x by expressing it as a z-score, the number of standard deviations σ it lies from the mean μ.

$$z = \frac{x - \mu}{\sigma}$$



The Standard Normal (z) Distribution

- Mean = 0; Standard deviation = 1
- Symmetric about $z = 0$
- Values of z to the left of center are negative
- Values of z to the right of center are positive
- Total area under the curve is 1.



■ Table 3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

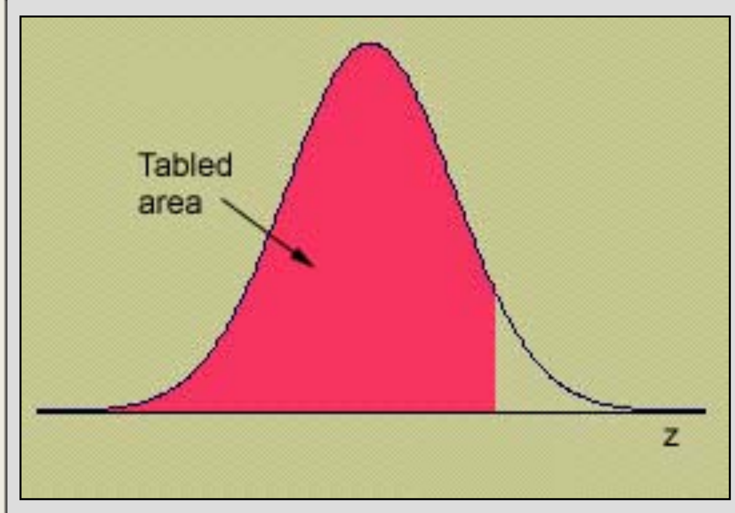
■ Table 3 (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

USING TABLE 3

The four digit probability in a particular row and column of Table 3 gives the area under the *z* curve to the left that particular value of *z*.

z	.00	.01	.02	.03	.04	.05	.06
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051
0.9	0.8159	0.8186	0.8212	0.8328	0.8264	0.8289	0.8315
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278



Area for $z = 1.36$

Example

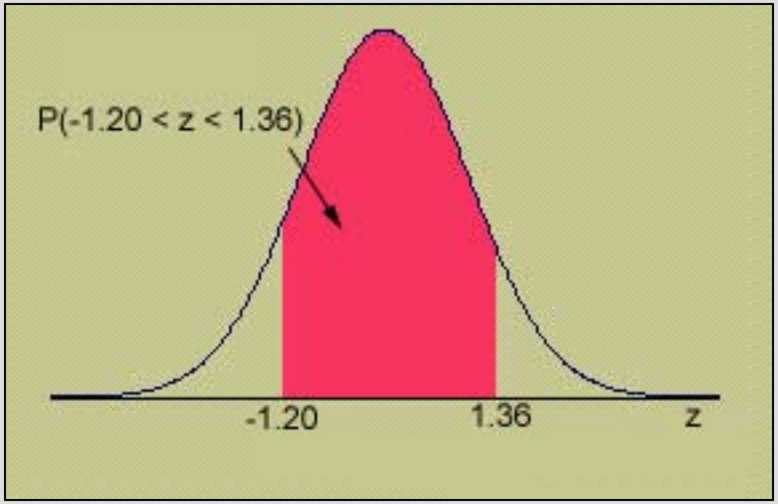
Use Table 3 to calculate these probabilities:

$P(z \le 1.36) = .9131$

$P(z > 1.36)$

$= 1 - .9131 = .0869$

$P(-1.20 \le z \le 1.36) =$
$.9131 - .1151 = .7980$



Using Table 3

✓To find an area to the left of a *z*-value, find the area directly from the table:

$$P(z < a)$$

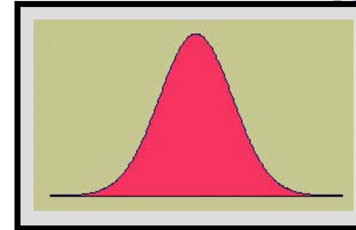
✓To find an area to the right of a *z*-value, find the area in Table 3 and subtract from 1:

$$P(z > a) = 1 - P(z < a)$$

✓To find the area between two values of *z*, find the two areas in Table 3, and subtract one from the other:

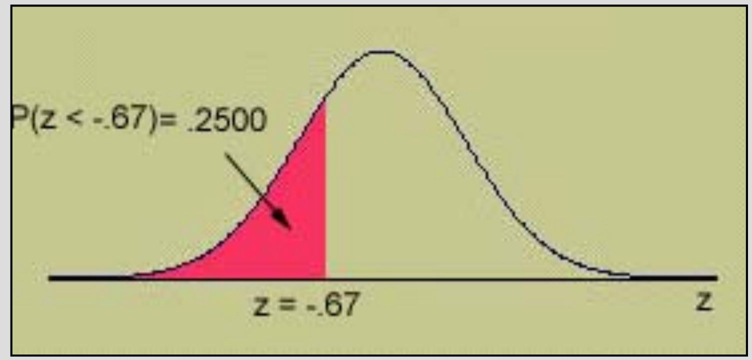
$$P(a < z < b) = P(z < b) - P(z < a)$$

Working Backwards



Find the value of z that has area .25 to its left.

1. Look for the four digit area closest to .2500 in Table 3.
2. What row and column does this value correspond to?
3. $z = -.67$

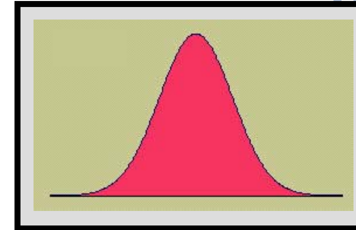


4. What percentile does this value represent?

z	.00	.01	.02	.03	.04	.05	.06	.07	.08
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810

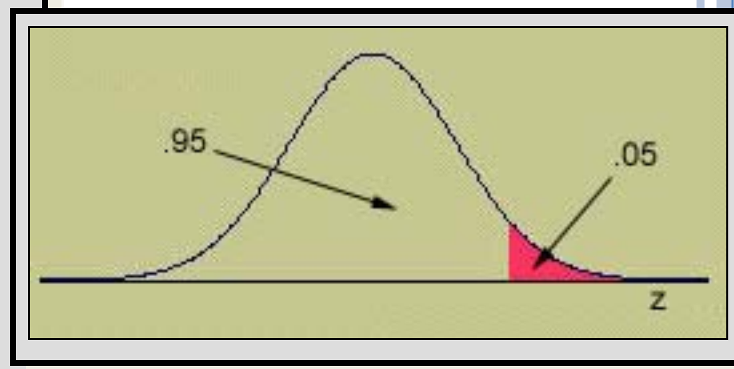
25th percentile, or 1st quartile (Q_1)

Working Backwards



Find the value of z that has area .05 to its right.

1. The area to its left will be 1 - .05 = .95
2. Look for the four digit area closest to .9500 in Table 3.
3. Since the value .9500 is halfway between .9495 and .9505, we choose z halfway between 1.64 and 1.65.
4. $z = 1.645$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

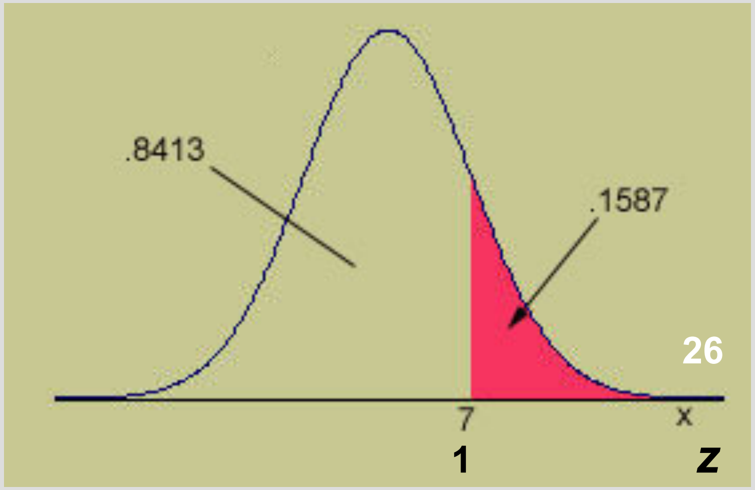
Finding Probabilities for the General Normal Random Variable

✓To find an area for a normal random variable *x* with mean μ and standard deviation σ, *standardize or rescale* the interval in terms of *z.*
✓Find the appropriate area using Table 3.

Example: *x* has a normal distribution with μ = 5 and σ = 2. Find P(*x* > 7).

$$P(x > 7) = P(z > \frac{7-5}{2})$$

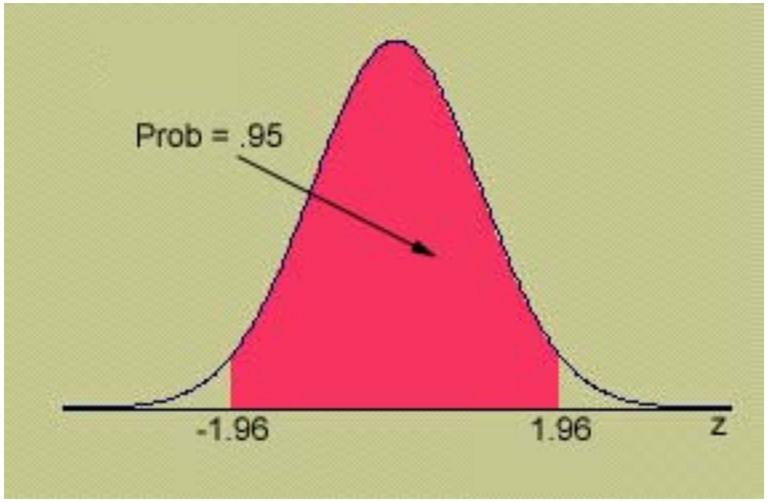
$$= P(z > 1) = 1 - .8413 = .1587$$



Empirical Rule

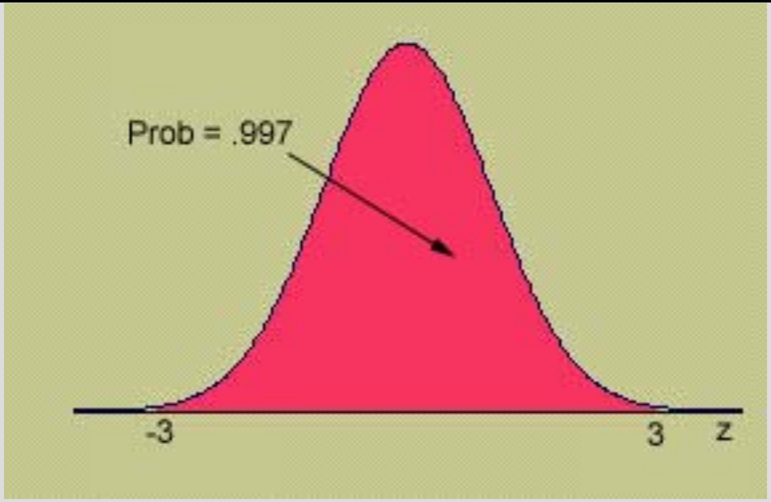
Approximately 95% of the measurements lie within 2 (1.96) standard deviations of the mean.

$$
\begin{aligned}
&P(\mu - 1.96\sigma < x < \mu + 1.96\sigma)\\
&= P\left(\frac{\mu - 1.96\sigma - \mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{\mu + 1.96\sigma - \mu}{\sigma}\right)\\
&= P(-1.96 < z < 1.96)\\
&= 0.975 - 0.0250\\
&= 0.9500
\end{aligned}
$$



Approximately 99.7% of the measurements lie within 3 standard deviations of the mean.

$$
\begin{aligned}
&P(\mu - 3\sigma < x < \mu + 3\sigma)\\
&= P\left(\frac{\mu - 3\sigma - \mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{\mu + 3\sigma - \mu}{\sigma}\right)\\
&= P(-3 < z < 3)\\
&= 0.9987 - 0.0013\\
&= 0.9974
\end{aligned}
$$



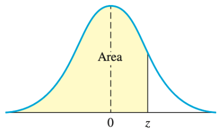


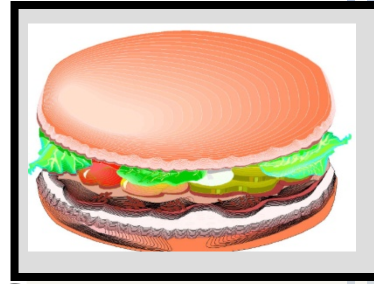
Table 3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table 3 (continued)

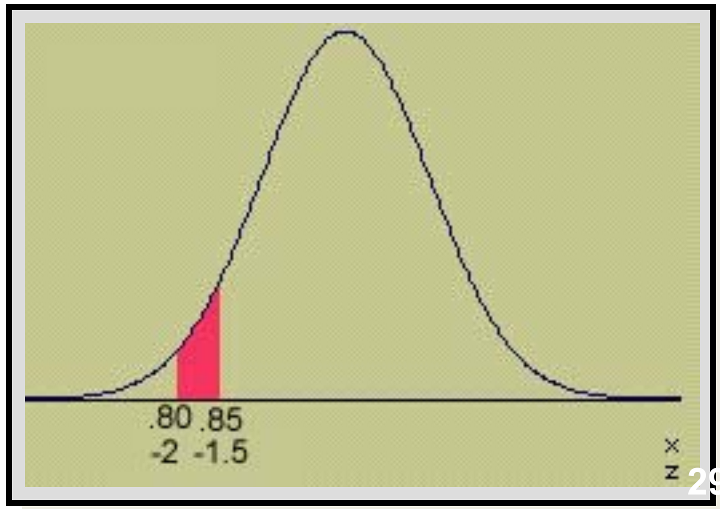
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Example

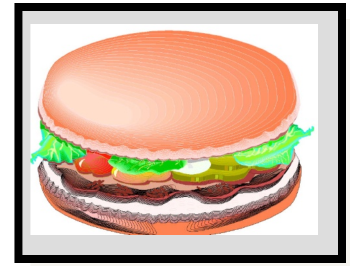


The weights of packages of ground beef are normally distributed with mean 1 pound and standard deviation .10. What is the probability that a randomly selected package weighs between 0.80 and 0.85 pounds?

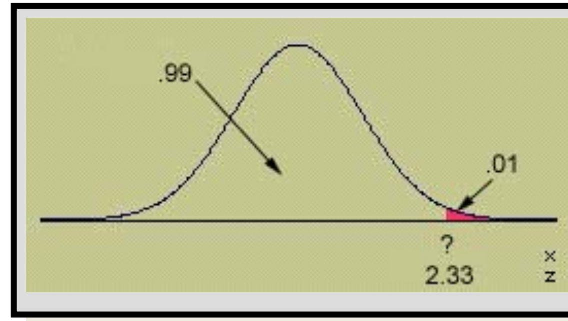
$$P(.80 < x < .85) =$$
$$P(-2 < z < -1.5) =$$
$$.0668 - .0228 = .0440$$



Example



What is the weight of a package such that only 1% of all packages exceed this weight?



$$P(x > ?) = .01$$

$$P\left(z > \frac{? - 1}{.1}\right) = .01$$

$$\text{From Table 3, } \frac{? - 1}{.1} = 2.33$$

$$? = 2.33(.1) + 1 = 1.233$$

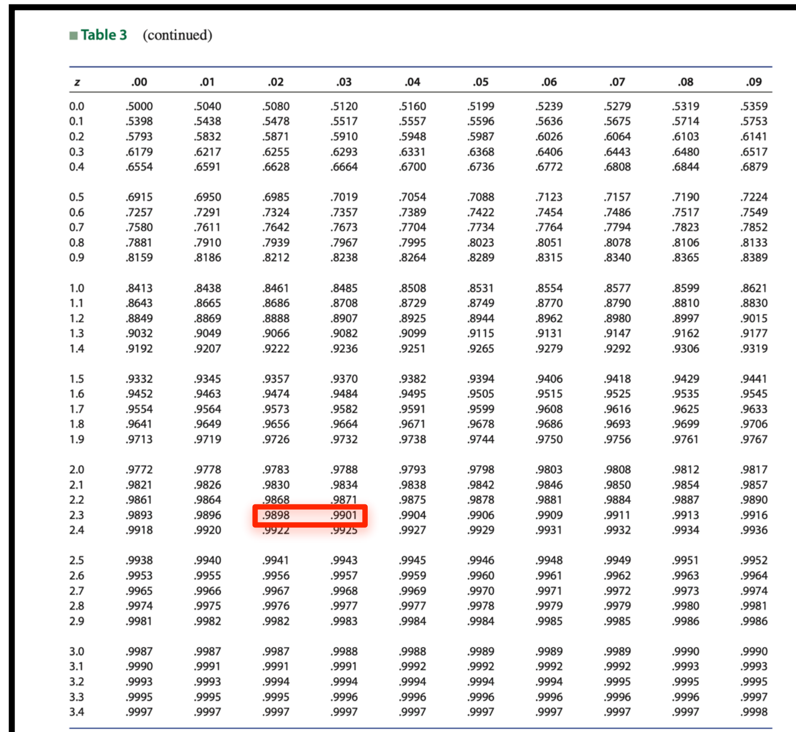


Table 3 (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

6.3 The normal Approximation to the Binomial probability distribution

The Normal Approximation to the Binomial

- We can calculate binomial probabilities using
  - The binomial formula: $C_k^n p^k (1-p)^{n-k}$ for $k = 0,1,\ldots,n$
  - The cumulative binomial tables
- When n is large, and p is not too close to zero or one, areas under the normal curve with mean np and variance npq can be used to approximate binomial probabilities:

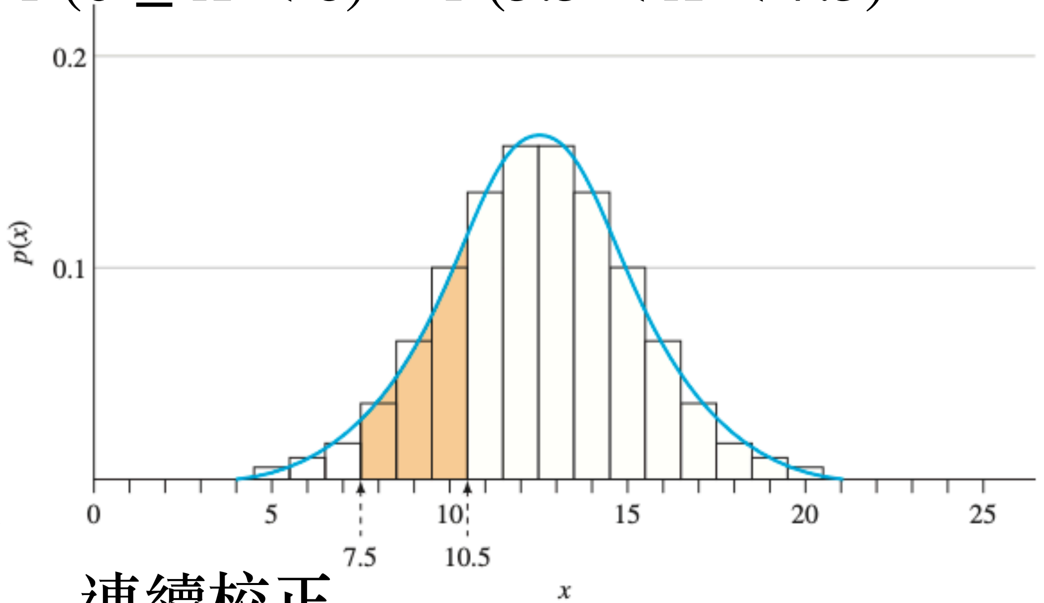
$$Binomial(n,p) \approx N(\mu = np, \sigma^2 = npq)$$

MOTIVATIONS

$$P(6 \le X \le 8) \approx P(5.5 < X < 8.5)$$

$$P(6 \le X < 8) \approx P(5.5 < X < 7.5)$$

Figure 6.17
The binomial probability distribution for $n = 25$ and $p = .5$ and the approximating normal distribution with $\mu = 12.5$ and $\sigma = 2.5$



連續校正例子

連續校正

$$P(X < 5) \approx P(X < 4.5) = P(\frac{X - 12.5}{2.5} < \frac{4.5 - 12.5}{2.5}) = P(Z < hello)$$

$$P(X < 5) \approx P(X < 5.5) = P(\frac{X - 12.5}{2.5} < \frac{5.5 - 12.5}{2.5}) = P(Z < h2)$$

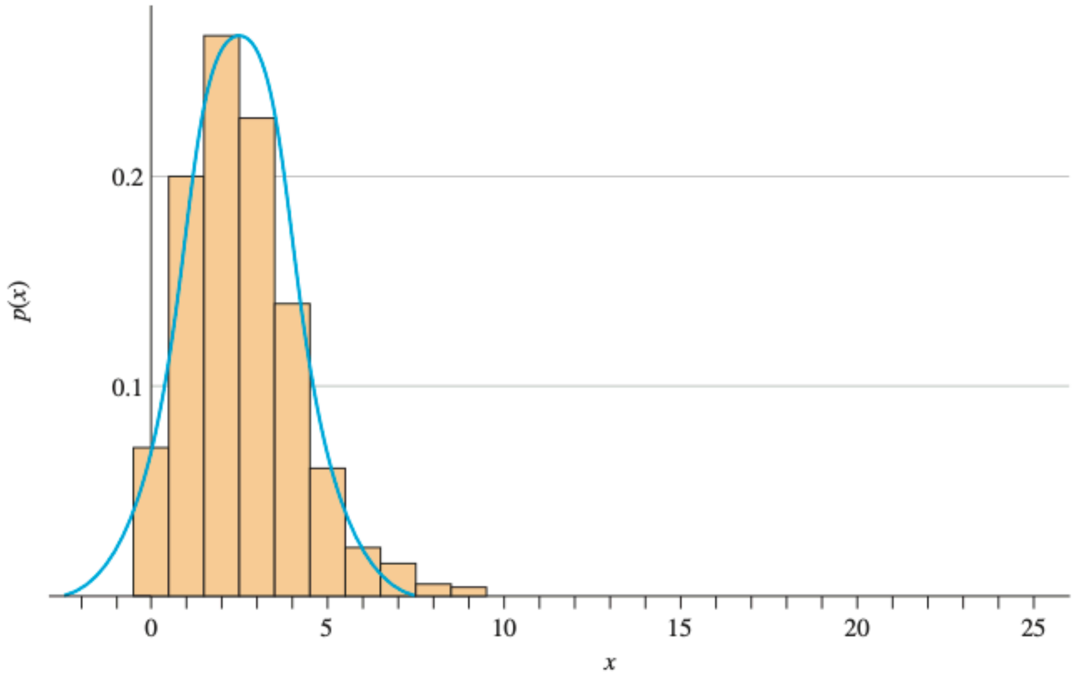
Rule of Thumb

Rule of Thumb

The normal approximation to the binomial probabilities will be adequate if both

$np > 5$ and $nq > 5$

Figure 6.18
The binomial probability distribution and the approximating normal distribution for $n = 25$ and $p = .1$



How to calculate Binomail Probabilities using the Normal Approximation

1. Find the necessary values of n and p.
2. Make sure $np > 5$ and $nq > 5$
3. Calculate $\mu = np$ and $\sigma^2 = npq$
4. Write the probability you need in x and locate the approximate area on the curve
5. Continuity correct for x by ± 0.5
6. Convert the necessary x-values to z-values using

$$z = \frac{x \pm 0.5 - np}{\sqrt{np}}$$

7. Use Table 3 in Appendix I to calcualte the approximate probabiltiy.

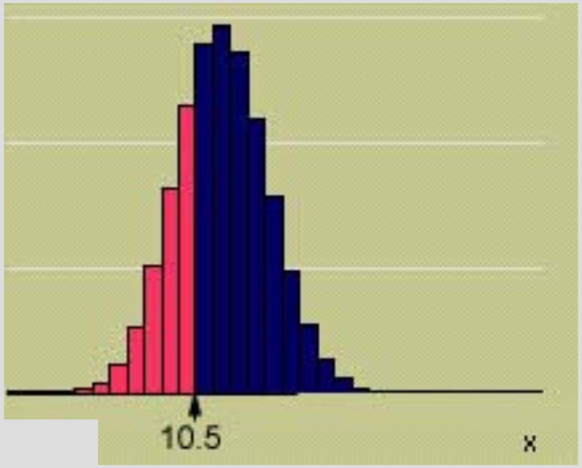
Example

Suppose *x* is a binomial random variable with *n* = 30 and *p* = .4. Using the normal approximation to find P(*x* ≤ 10).

$n = 30 \quad p = .4 \quad q = .6$

$np = 12 \quad nq = 18$

The normal approximation is ok!

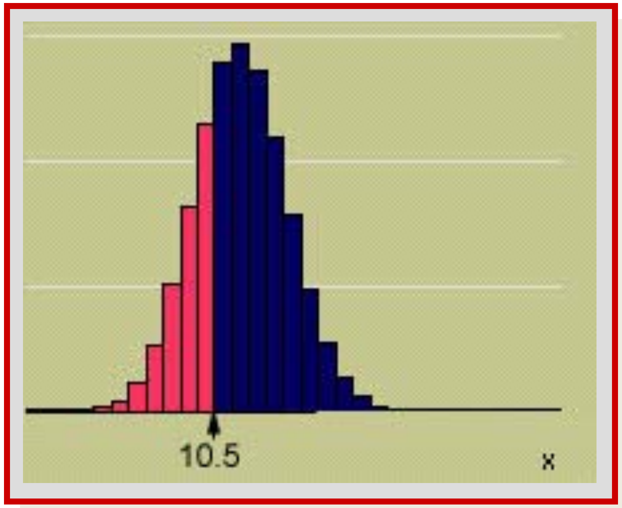


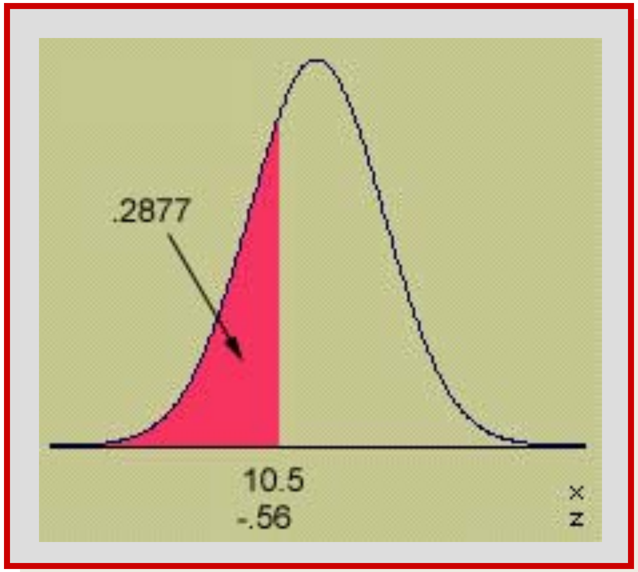
Calculate

$$\mu = np = 30(.4) = 12$$

$$\sigma = \sqrt{npq} = \sqrt{30(.4)(.6)} = 2.683$$

Example





$$
\begin{aligned}
P(X \le 10) &\approx P(X < 10.5) \\
&= P\left(\frac{X-12}{2.683} < \frac{10.5-12}{2.683}\right) \\
&= P(Z < -0.56) \\
&= 0.2877
\end{aligned}
$$

$$
\begin{aligned}
P(X > 10) &\approx P(X > 10.5) \\
&= P\left(\frac{X-12}{2.683} > \frac{10.5-12}{2.683}\right) \\
&= P(Z > -0.56) \\
&= 1 - 0.2877
\end{aligned}
$$

Example

A production line produces AA batteries with a reliability rate of 95%. A sample of n = 200 batteries is selected. Find the probability that at least 195 of the batteries work.

Success = working battery

Binomial(n=200, p=0.95)

n **= 200**

We need to find $P(X \geq 195)$

p = .95 np = 190
nq = 10

The normal approximation is ok!

$$
\begin{aligned}
P(X \geq 195) &\approx P(X > 194.5) \\
&= P\left(\frac{X - 190}{\sqrt{200 \times 0.95 \times 0.05}} > \frac{194.5 - 190}{\sqrt{200 \times 0.95 \times 0.05}}\right) \\
&= P(Z > 1.46) \\
&= 1 - 0.9278 = 0.0722
\end{aligned}
$$

EXAMPLE (MOVE FROM CH 7)

The soda bottler in the previous example claims that only 5% of the soda cans are underfilled. A quality control technician randomly samples 200 cans of soda. What is the probability that more than 10% of the cans are underfilled?

P(X>20 \approx P(X>20.5)

X(n=200, p=0.05)

***n* = 200**

S: underfilled can

***p* = P(S) = .05**

***q* = .95**

***np* = 10 *nq* = 190**

OK to use the normal approximation

$$P(\hat{p} > .10)$$

$$= P\left(z > \frac{.10 - .05}{\sqrt{\frac{.05(.95)}{200}}}\right) = P(z > 3.24)$$

$$= 1 - .9994 = .0006$$

This would be very unusual, if indeed p = .05!

Key Concepts

I. Continuous Probability Distributions

1. Continuous random variables
2. Probability distributions or probability density functions
   a. Curves are smooth.
   b. The area under the curve between a and b represents the probability that x falls between a and b.
   c. $P(x = a) = 0$ for continuous random variables.

II. The Normal Probability Distribution

1. Symmetric about its mean μ.
2. Shape determined by its standard deviation σ.

Key Concepts

III. The Standard Normal Distribution

1. The normal random variable z has mean 0 and standard deviation 1.
2. Any normal random variable x can be transformed to a standard normal random variable using

$$z = \frac{x - \mu}{\sigma}$$

3. Convert necessary values of x to z.
4. Use Table 3 in Appendix I to compute standard normal probabilities.
5. Several important z-values have tail areas as follows:

Tail Area:	.005	.01	.025	.05	.10
z-Value:	2.58	2.33	1.96	1.645	1.28