Convolution

Example-1: The impulse response of LTI system is- **h[n] = {1, 2, 1, -1}** and the input sequence is **x[n] = {1, 2, 3, 1}**. Determine y[n].

$\boxed{\begin{array}{l} k \Rightarrow x_{low} = 0,\ x_{high} = 3 \quad | \\ \qquad\qquad \mathbf{0 \leq k \leq 3} \\ n \Rightarrow h_{low} + x_{low} = -1 + 0 = -1 \\ h_{high} + x_{high} = 2+3 = 5 \quad | \\ \qquad\qquad \mathbf{-1 \leq n \leq 5} \end{array}}$

Sol^n: We have -

h[n] = {1, 2, 1, -1}

x[n] = {1, 2, 3, 1}

We know

$$y[n] = \sum_{k=-\alpha}^{\alpha} x[k].h[n-k]$$

$$y[-1] = \sum_{k=0}^{3} x[k].h[-1-k]$$

= x [0].h [-1] + x [1].h [-2] + x [2].h [-3] + x [3].h [-4]

= 1.1 + 2.0 + 3.0 + 1.0

= 1 + 0 + 0 + 0

= 1

$$y[0] = \sum_{k=0}^{3} x[k].h[0-k]$$

= x [0].h [0] + x [1].h [-1] + x [2].h [-2] + x [3].h [-3]

= 1.2 + 2.1 + 3.0 + 1.0

= 2 + 2 + 0 + 0

= 4

$$y[1] = \sum_{k=0}^{3} x[k].h[1-k]$$

= x [0].h [1] + x [1].h [0] + x [2].h [-1] + x [3].h [-2]

= 1.1 + 2.2 + 3.1 + 1.0

= 1 + 4 + 3 + 0

= 8

$$y[2] = \sum_{k=0}^{3} x[k].h[2-k]$$

= x [0].h [2] + x [1].h [1] + x [2].h [0] + x [3].h [-1]

= 1.-1 + 2.1 + 3.2 + 1.1

= -1 + 2 + 6 + 1

= 8

y [3] = $\sum_{k=0}^{3} x[k].h[3-k]$

= x [0].h [3] + x [1].h [2] + x [2].h [1] + x [3].h [0]

= 1.0 + 2.-1 + 3.1 + 1.2

= 0 – 2 + 3 + 2

= 3

y [4] = $\sum_{k=0}^{3} x[k].h[4-k]$

= x [0].h [4] + x [1].h [3] + x [2].h [2] + x [3].h [1]

= 1.0 + 2.0 + 3.-1 + 1.1

= 0 + 0 – 3 + 1

= -2

y [5] = $\sum_{k=0}^{3} x[k].h[5-k]$

= x [0].h [5] + x [1].h [4] + x [2].h [3] + x [3].h [2]

= 1.0 + 2.0 + 3.0 + 1.-1

= 0 + 0 + 0 – 1

= -1

So, y[n] = {1, 4, 8, 8, 3, -2, -1}

………………. O ……………………

<u>**Example-2:**</u> The impulse response of LTI system is- **h[n] = {1, 2, -1}** and the input sequence is **x[n] = {4, 1, 2, 5}**. Determine y[n].

x[n] \ h[n]	1	2	-1
4	4	8	-4
1	1	2	-1
2	2	4	-2
5	5	10	-5

Using the formula-

$$y[n] = \sum_{k=-\alpha}^{\alpha} x[k].h[n-k]$$

We get,

y[n] = {4, 9, 0, 8, 8, -5}

Correlation

Example-1: Determine the Cross-Correlation sequence $r_{xy}(l)$ of the following sequences: -

$x[n] = [1, 2, 3, 4]$ and $y[n] = [4, 3, 2, 1]$.

Sol^n: We have,

$x[n] = [1, 2, 3, 4]$ and

$y[n] = [4, 3, 2, 1]$.

We know the equation for Cross-Correlation –

$$r_{xy}(l) = \sum_{n=-\alpha}^{\alpha} x[n].y[n-l]$$

xlen = 4 and ylen = 4

$k \Rightarrow x_{low} = 0, x_{high} = 3$ | **$0 \leq n \leq 3$**

tlen = xlen+ylen-1 = 4+4-1 = 7

l_{min} = -(tlen – xlen) = -(7 – 4) = -3

l_{max} = tlen – ylen = 7 – 4 = 3

$-3 \leq l \leq 3$

$$r_{xy}(-3) = \sum_{n=0}^{3} x[n].y[n+3]$$

$= x[0].y[3] + x[1].y[4] + x[2].y[5] + x[3].y[6]$

$= 1.1 + 2.0 + 3.0 + 4.0$

$= 1 + 0 + 0 + 0$

$= 1$

$$r_{xy}(-2) = \sum_{n=0}^{3} x[n].y[n+2]$$

$= x[0].y[2] + x[1].y[3] + x[2].y[4] + x[3].y[5]$

$= 1.2 + 2.1 + 3.0 + 4.0$

$= 2 + 2 + 0 + 0$

$= 4$

$$r_{xy}(-1) = \sum_{n=0}^{3} x[n].y[n+1]$$

$= x[0].y[1] + x[1].y[2] + x[2].y[3] + x[3].y[4]$

$= 1.3 + 2.2 + 3.1 + 4.0$

$= 3 + 4 + 3 + 0$

$= 10$

$$r_{xy}(0) = \sum_{n=0}^{3} x[n].y[n+0]$$

$= x[0].y[0] + x[1].y[1] + x[2].y[2] + x[3].y[3]$

$= 1.4 + 2.3 + 3.2 + 4.1$

$= 4 + 6 + 6 + 4$

$= 20$

$r_{xy}(1) = \sum_{n=0}^{3} x[n].\, y[n-1]$

$= x[0].\, y[-1] + x[1].\, y[0] + x[2].\, y[1] + x[3].\, y[2]$

$= 1.0 + 2.4 + 3.\,3 + 4.2$

$= 0 + 8 + 9 + 8$

$= 25$

$r_{xy}(2) = \sum_{n=0}^{3} x[n].\, y[n-2]$

$= x[0].\, y[-2] + x[1].\, y[-3] + x[2].\, y[-4] + x[3].\, y[-5]$

$= 1.0 + 2.0 + 3.\,4 + 4.3$

$= 0 + 0 + 12 + 12$

$= 124$

$r_{xy}(3) = \sum_{n=0}^{3} x[n].\, y[n-3]$

$= x[0].\, y[-3] + x[1].\, y[-2] + x[2].\, y[-1] + x[3].\, y[0]$

$= 1.0 + 2.0 + 3.\,0 + 4.4$

$= 0 + 0 + 0 + 16$

$= 16$

So, we have-

$r_{xy}(l) = \{1, 4, 10, 20, 25, 24, 16\}$

y[n] / x[n]	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

DFT

Example-1: Find the DFT of the sequence x[n] = {1, 1, 0, 0}. Sketch the magnitude and phase spectrum, real part and imaginary part. Also find the power spectrum: -

We know the equation for Cross-Correlation –

$$X(k) = \sum_{n=-\alpha}^{\alpha} x[n].e^{-j.2.\pi.k.n/N}$$

Here, k and n = 0, 1, 3……... N-1

$$X(0) = \sum_{n=0}^{3} x[n].e^{-j.2.\pi.0.n/4}$$

$$= \sum_{n=0}^{3} x[n].e^{0}$$

$$= x[0] + x[1] + x[2] + x[3]$$

$$= 1 + 1 + 0 + 0$$

$$= 2$$

$$X(1) = \sum_{n=0}^{3} x[n].e^{-j.2.\pi.1.n/4}$$

$$= \sum_{n=0}^{3} x[n].e^{-j.\pi.n/2}$$

$$= 1 - j$$

$$X(2) = 0$$

$$X(3) = 1 + j$$

DFT: X (k) = {2, 1 – j, 0, 1 + j}

Real Part: Real(X(k)) = {2, 1, 0, 1}

Imaginary part: Imag(X(k)) = {0, -1, 0, 1}

Magnitude Spectrum: abs(X(k)) = {2, $\sqrt{2}$, 0, $\sqrt{2}$ }

Phase Spectrum: phase(X(k)) = {0, -45, -90, 45} # in degree

Power Spectrum: pow(abs(X(k)), 2) = {4, 2, 0, 2}