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## 1 Basic

### 1.1 vimrc

```
se nu rnu ai hls et ru ic is sc
se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a
syntax on
se bg=dark
ino {<CR> {<CR>}<Esc>ko<tab>
```

### 1.2 readchar [a419b9]

```
inline char readchar() {
    static const size_t bufsize = 65536;
    static char buf[bufsize];
    static char *p = buf, *end = buf;
    if (p == end) end = buf +
        fread_unlocked(buf, 1, bufsize, stdin), p = buf;
    return *p++;
}
```

### 1.3 Black Magic [afb343]

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> // rb_tree
#include <ext/rope> // rope
using namespace __gnu_pbds;
using namespace __gnu_cxx; // rope
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2; // max heap
    h1.push(1), h1.push(3), h2.push(2), h2.push(4);
    h1.join(h2); // h1 = {1, 2, 3, 4}, h2 = {};
    tree<ll, null_type, less<ll>, rb_tree_tag
        , tree_order_statistics_node_update> st;
    tree<ll, ll, less<ll>, rb_tree_tag
        , tree_order_statistics_node_update> mp;
```

```
for (int x : {0, 3, 20, 50}) st.insert(x);
assert(st.
    order_of_key(3) == 1 && st.order_of_key(4) == 2);
assert(*st.find_by_order
    (2) == 20 && *st.lower_bound(4) == 20);
rope<char> *root[10]; // nsqrt(n)
root[0] = new rope<char>();
root[1] = new rope<char>(*root[0]);
// root[1]->insert(pos, 'a');
// root[1]->at(pos); 0-base
// root[1]->erase(pos, size);
}
// __int128_t, __float128_t
// for (int i = bs._Find_first
    ()); i < bs.size(); i = bs._Find_next(i));
```

### 1.4 Pragma Optimization [0ca2dd]

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8040)
```

### 1.5 Default Code [fdcdff]

```
#include <bits/stdc++.h>
#define int long long
#define fi first
#define se second
#define all(a) a.begin(),a.end()
#define SZ(a) (int)(a.size())
#define pb push_back
#define eb emplace_back
#define rep(i,a,b) for(int i=a,Z=b;i<Z;++i)
#define rep1(i,a,b) for(int i=a,Z=b;i<=Z;++i)
using namespace std;
using ll = long long;
using pii = pair<int,int>;
using vi = vector<int>;
using vvi = vector<vi>;
using vp = vector<pii>;
using vvp = vector<vvp>;

void foo(){
}

signed main(){
    ios::sync_with_stdio(0);cin.tie(0);
    int t = 1;
    // cin >> t;
    while(t--) foo();
}
```

### 1.6 cmp [57e6a1]

```
struct cmp {
    bool operator()(T a, T b) const {
        return 0;
    }
};
priority_queue<T, vector<T>, cmp> pq;
set<T, cmp> st;
```

### 1.7 Builtin Functions [4cc12f]

```
int __builtin_popcount(int a); // count number of 1s
int __lg(int a); // floor(log_2(a))
int __gcd(int a,int b); // gcd
iota(a.begin(), a.end(), 1); // fill a, start from 1
bool next_permutation(all(a)); // return 0 if end
```

## 2 Graph

### 2.1 BCC Vertex\* [6763ed]

```
struct BCC { // 0-base
    int n, dft, nbcc;
    vi low, dfn, bln, stk, is_ap, cir;
    vvi G, bcc, nG;
    void make_bcc(int u) {
        bcc.eb(1, u);
        for (; stk.back() != u; stk.pop_back())
            bln[stk.back()] = nbcc, bcc[nbcc].pb(stk.back());
        stk.pop_back(), bln[u] = nbcc++;
    }
    void dfs(int u, int p) {
```

```

int child = 0;
low[u] = dfn[u] = ++dft, stk.pb(u);
for (int v : G[u])
    if (!dfn[v]) {
        dfs(v, u), ++child;
        low[u] = min(low[u], low[v]);
        if (dfn[u] <= low[v]) {
            is_ap[u] = 1, bln[u] = nbcc;
            make_bcc(v), bcc.back().pb(u);
        }
    } else if (dfn[v] < dfn[u] && v != p)
        low[u] = min(low[u], dfn[v]);
if (p == -1 && child < 2) is_ap[u] = 0;
if (p == -1 && child == 0) make_bcc(u);
}
BCC(int _n): n(_n), dft(),
    nbcc(), low(n), dfn(n), bln(n), is_ap(n), G(n) {}
void add_edge(int u, int v) {
    G[u].pb(v), G[v].pb(u);
}
void solve() {
    for (int i = 0; i < n; ++i)
        if (!dfn[i]) dfs(i, -1);
}
void block_cut_tree() {
    cir.resize(nbcc);
    for (int i = 0; i < n; ++i)
        if (is_ap[i])
            bln[i] = nbcc++;
    cir.resize(nbcc, 1), nG.resize(nbcc);
    for (int i = 0; i < nbcc && !cir[i]; ++i)
        for (int j : bcc[i])
            if (is_ap[j])
                nG[i].pb(bln[j]), nG[bln[j]].pb(i);
} // up to 2 * n - 2 nodes!! bln[i] for id
};

```

## 2.2 Bridge\* [4da29a]

```

struct ECC { // 0-base
    int n, dft, ecnt, necc;
    vector<int> low, dfn, bln, is_bridge, stk;
    vector<vector<pii>> G;
    void dfs(int u, int f) {
        dfn[u] = low[u] = ++dft, stk.pb(u);
        for (auto [v, e] : G[u])
            if (!dfn[v])
                dfs(v, e), low[u] = min(low[u], low[v]);
            else if (e != f)
                low[u] = min(low[u], dfn[v]);
        if (low[u] == dfn[u]) {
            if (f != -1) is_bridge[f] = 1;
            for (; stk.back() != u; stk.pop_back())
                bln[stk.back()] = necc;
            bln[u] = necc++, stk.pop_back();
        }
    }
    ECC(int _n): n(_n), dft(),
        , ecnt(), necc(), low(n), dfn(n), bln(n), G(n) {}
    void add_edge(int u, int v) {
        G[u].pb(pii(v, ecnt)), G[v].pb(pii(u, ecnt++));
    }
    void solve() {
        is_bridge.resize(ecnt);
        for (int i = 0; i < n; ++i)
            if (!dfn[i]) dfs(i, -1);
    }
}; // ecc_id(i): bln[i]

```

## 2.3 SCC\* [7ae459]

```

struct SCC { // 0-base
    int n, dft, nsc;
    vi low, dfn, bln, instack, stk;
    vvi G;
    void dfs(int u) {
        low[u] = dfn[u] = ++dft;
        instack[u] = 1, stk.pb(u);
        for (int v : G[u])
            if (!dfn[v])
                dfs(v), low[u] = min(low[u], low[v]);
            else if (instack[v] && dfn[v] < dfn[u])
                low[u] = min(low[u], dfn[v]);
        if (low[u] == dfn[u]) {
            for (; stk.back() != u; stk.pop_back())
                bln[stk.back()] = nsc, instack[stk.back()] = 0;
        }
    }

```

```

        instack[u] = 0, bln[u] = nsc++, stk.pop_back();
    }
}
SCC(int _n): n(_n), dft(), nsc
    (), low(n), dfn(n), bln(n), instack(n), G(n) {}
void add_edge(int u, int v) {
    G[u].pb(v);
}
void solve() {
    for (int i = 0; i < n; ++i)
        if (!dfn[i]) dfs(i);
}
}; // scc_id(i): bln[i]

```

## 2.4 2SAT\* [f5630a]

```

struct SAT { // 0-base
    int n;
    vector<bool> istrue;
    SCC scc;
    SAT(int _n): n(_n), istrue(n + n), scc(n + n) {}
    int rv(int a) {
        return a >= n ? a - n : a + n;
    }
    void add_clause(int a, int b) {
        scc.add_edge(rv(a), b), scc.add_edge(rv(b), a);
    }
    bool solve() {
        scc.solve();
        for (int i = 0; i < n; ++i) {
            if (scc.bln[i] == scc.bln[i + n]) return false;
            istrue[i] = scc.bln[i] < scc.bln[i + n];
            istrue[i + n] = !istrue[i];
        }
        return true;
    }
};

```

## 2.5 MinimumMeanCycle\* [3e5d2b]

```

ll road[N][N]; // input here
struct MinimumMeanCycle {
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for (int i = 2; i <= L; ++i)
            for (int k = 0; k < n; ++k)
                for (int j = 0; j < n; ++j)
                    dp[i][j] =
                        min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for (int i = 0; i < n; ++i) {
            if (dp[L][i] >= INF) continue;
            ll ta = 0, tb = 1;
            for (int j = 1; j < n; ++j)
                if (dp[j][i] < INF &&
                    ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if (ta == 0) continue;
            if (a == -1 || a * tb > ta * b) a = ta, b = tb;
        }
        if (a != -1) {
            ll g = __gcd(a, b);
            return pll(a / g, b / g);
        }
        return pll(-1LL, -1LL);
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
    }
};

```

## 2.6 Virtual Tree\* [1b641b]

```

vector<int> vG[N];
int top, st[N];

void insert(int u) {
    if (top == -1) return st[++top] = u, void();
    int p = LCA(st[top], u);
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if (st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

```

```

}

void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
    // do something
    reset(v[0]);
}

```

## 2.7 Maximum Clique Dyn\* [d50aa9]

```

struct MaxClique { // fast when N <= 100
    bitset<N> G[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void add_edge(int u, int v) {
        G[u][v] = G[v][u] = 1;
    }
    void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
        if (l < 4) {
            for (int i : r) d[i] = (G[i] & mask).count();
            sort(ALL(r),
                [&](int x, int y) { return d[x] > d[y]; });
        }
        vector<int> c(SZ(r));
        int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
        cs[1].reset(), cs[2].reset();
        for (int p : r) {
            int k = 1;
            while ((cs[k] & G[p]).any()) ++k;
            if (k > rgt) cs[++rgt].reset();
            cs[k][p] = 1;
            if (k < lft) r[tp++] = p;
        }
        for (int k = lft; k <= rgt; ++k)
            for (int p = cs[k]._Find_first(); p < N; p = cs[k]._Find_next(p))
                r[tp] = p, c[tp] = k, ++tp;
        dfs(r, c, l + 1, mask);
    }
    void dfs(vector<
        int> &r, vector<int> &c, int l, bitset<N> mask) {
        while (!r.empty()) {
            int p = r.back();
            r.pop_back(), mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr;
            for (int i : r) if (G[p][i]) nr.pb(i);
            if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
            else if (q > ans) ans = q, copy_n(cur, q, sol);
            c.pop_back(), --q;
        }
    }
    int solve() {
        vector<int> r(n);
        ans = q = 0, iota(ALL(r), 0);
        pre_dfs(r, 0, bitset<N>(string(n, '1')));
        return ans;
    }
};

```

## 2.8 Dominator Tree\* [2da9bb]

```

struct Dominator {
    int n;
    vector<vector<int>> g, r, rdom; int tk;
    vector<int> dfn, rev, fa, sd, dom, val, rp;
    Dominator
        (int _n) : n(_n), g(n), r(n), rdom(n), tk(0) {
        dfn = rev = fa = sd = dom =
        val = rp = vector<int>(n, -1);
    }
    void add_edge(int x, int y) { g[x].push_back(y); }
    void dfs(int x) {
        rev[dfn[x]] = tk = x;
        fa[tk] = sd[tk] = val[tk] = tk; tk++;
    }
};

```

```

for (int u : g[x]) {
    if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
    r[dfn[u]].push_back(dfn[x]);
}
}

void merge(int x, int y) { fa[x] = y; }
int find(int x, int c = 0) {
    if (fa[x] == x) return c ? -1 : x;
    if (int p = find(fa[x], 1); p != -1) {
        if (sd[fa[x]] > sd[p])
            val[x] = val[p];
        fa[x] = p;
        return c ? p : val[x];
    } else return c ? fa[x] : val[x];
}

vector<int> build(int s) {
    // return the father of each node in dominator tree
    dfs(s); // p[i] = -2 if i is unreachable, p[s] = -1
    for (int i = tk - 1; i >= 0; --i) {
        for (int u : r[i])
            sd[u] = min(sd[u], sd[find(u)]);
        if (i) rdom[sd[i]].push_back(i);
        for (int u : rdom[i]) {
            int p = find(u);
            dom[u] = (sd[p] == i ? i : p);
        }
        if (i) merge(i, rp[i]);
    }
    vector<int> p(n, -2); p[s] = -1;
    for (int i = 1; i < tk; ++i)
        if (sd[i] != dom[i]) dom[i] = dom[dom[i]];
    for (int i = 1; i < tk; ++i)
        p[rev[i]] = rev[dom[i]];
    return p;
}
};

```

## 2.9 Vizing's theorem\* [2b5b01]

```

namespace vizing { // returns
    edge coloring in adjacent matrix G. 1 - based
const int N = 105;
int C[N][N], G[N][N], X[N], vst[N], n;
void init(int _n) { n = _n;
    for (int i = 0; i <= n; ++i)
        for (int j = 0; j <= n; ++j)
            C[i][j] = G[i][j] = 0;
}

void solve(vector<pii> &E) {
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; ++X[u]);
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };

    auto flip = [&](int u, int c1, int c2) {
        int p = C[u][c1];
        swap(C[u][c1], C[u][c2]);
        if (p) G[u][p] = G[p][u] = c2;
        if (!C[u][c1]) X[u] = c1;
        if (!C[u][c2]) X[u] = c2;
        return p;
    };

    fill_n(X + 1, n, 1);
    for (int t = 0; t < SZ(E); ++t) {
        int u = E[t].X, v0 = E[t].Y, v = v0, c0 = X[u], c = c0, d;
        vector<pii> L;
        fill_n(vst + 1, n, 0);
        while (!G[u][v0]) {
            L.emplace_back(v, d = X[v]);
            if (!C[v][c]) for (int a = SZ(L) - 1; a >= 0; --a) c = color(u, L[a].X, c);
            else if (!C[u][d]) for (int a = SZ(L) - 1; a >= 0; --a) color(u, L[a].X, L[a].Y);
            else if (vst[d]) break;
            else vst[d] = 1, v = C[u][d];
        }
        if (!G[u][v0]) {
            for (; v; v = flip(v, c, d), swap(c, d));
            if (int a; C[u][c0]) {

```

```

    for (
        a = SZ(L) - 2; a >= 0 && L[a].Y != c; --a);
    for (; a >= 0; --a) color(u, L[a].X, L[a].Y);
    }
    else --t;
    }
}
}
// namespace vizing

```

## 2.10 Minimum Clique Cover\* [879472]

```

struct Clique_Cover { // 0-base, 0(n2^n)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
    void add_edge(int u, int v) {
        E[u] |= 1 << v, E[v] |= 1 << u;
    }
    int solve() {
        for (int i = 0; i < n; ++i)
            co[1 << i] = E[i] | (1 << i);
        co[0] = (1 << n) - 1;
        dp[0] = (n & 1) * 2 - 1;
        for (int i = 1; i < (1 << n); ++i) {
            int t = i & -i;
            dp[i] = -dp[i ^ t];
            co[i] = co[i ^ t] & co[t];
        }
        for (int i = 0; i < (1 << n); ++i)
            co[i] = (co[i] & i) == i;
        fwt(co, 1 << n, 1);
        for (int ans = 1; ans < n; ++ans) {
            int sum = 0; // probabilistic
            for (int i = 0; i < (1 << n); ++i)
                sum += (dp[i] * co[i]);
            if (sum) return ans;
        }
        return n;
    }
};

```

## 2.11 NumberofMaximalClique\* [11fa26]

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsn = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsn++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsn, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
};

```

# 3 Data Structure

## 3.1 Discrete Trick

```

vector<int> val;
// build
sort(ALL
    (val)), val.resize(unique(ALL(val)) - val.begin());
// index of x
upper_bound(ALL(val), x) - val.begin();
// max idx <= x
upper_bound(ALL(val), x) - val.begin();
// max idx < x
lower_bound(ALL(val), x) - val.begin();

```

## 3.2 zkwseg [aae74d]

```

template<class T, T (*op)(T, T), T (*e)()>
struct segtree {
    int n, sz;
    vector<T> d;
    explicit
        segtree(int n): segtree(vector<T>(n, e())) {}
    explicit segtree(const vector<T> &v) {
        n = v.size(), sz = 1;
        while (sz < n) sz <= 1;
        d.assign(sz * 2, e());
        for (int i = 0; i < n; ++i) d[sz + i] = v[i];
        for (int i = sz - 1; i >= 1; --i) d[i] = op(d[i << 1], d[i << 1 | 1]);
    }
    void set(int p, const T &x) {
        p += sz, d[p] = x;
        while (p >= 1) d[p] = op(d[p << 1], d[p << 1 | 1]);
    }
    T get(int p) const { return d[sz + p]; }
    T prod(int l, int r) const {
        T sl = e(), sr = e();
        l += sz, r += sz + 1;
        while (l < r) {
            if (l & 1) sl = op(sl, d[l++]);
            if (r & 1) sr = op(d[--r], sr);
            l >>= 1, r >>= 1;
        }
        return op(sl, sr);
    }
};

```

## 3.3 BIT kth\* [e39485]

```

int bit[N + 1]; // N = 2 ^ k
int query_kth(int k) {
    int res = 0;
    for (int i = N >> 1; i >= 1; i >>= 1)
        if (bit[res + i] < k)
            k -= bit[res + i];
    return res + 1;
}

```

## 3.4 Centroid Decomposition\* [5a24da]

```

struct Cent_Dec { // 1-base
    vector<pll> G[N];
    pll info[N]; // store info. of itself
    pll upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    ll dis[__lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
        for (int i = 1; i <= n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pll(b, w)), G[b].pb(pll(a, w));
    }
    void get_cent(
        int u, int f, int &mx, int &c, int num) {
        int mxsz = 0;
        sz[u] = 1;
        for (pll e : G[u])
            if (!done[e.X] && e.X != f) {
                get_cent(e.X, u, mx, c, num);
                sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
            }
        if (mx > max(mxsz, num - sz[u]))
            mx = max(mxsz, num - sz[u]), c = u;
    }
    void dfs(int u, int f, ll d, int org) {
        // if required, add self info or climbing info
        dis[layer[org]][u] = d;
        for (pll e : G[u])

```

```

    if (!done[e.X] && e.X != f)
        dfs(e.X, u, d + e.Y, org);
}
int cut(int u, int f, int num) {
    int mx = 1e9, c = 0, lc;
    get_cent(u, f, mx, c, num);
    done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
    for (pll e : G[c])
        if (!done[e.X]) {
            if (sz[e.X] > sz[c])
                lc = cut(e.X, c, num - sz[c]);
            else lc = cut(e.X, c, sz[e.X]);
            upinfo[lc] = pll(), dfs(e.X, c, e.Y, c);
        }
    return done[c] = 0, c;
}
void build() { cut(1, 0, n); }
void modify(int u) {
    for (int a = u, ly = layer[a]; a;
         a = pa[a], --ly) {
        info[a].X += dis[ly][u], ++info[a].Y;
        if (pa[a])
            upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
}
ll query(int u) {
    ll rt = 0;
    for (int a = u, ly = layer[a]; a;
         a = pa[a], --ly) {
        rt += info[a].X + info[a].Y * dis[ly][u];
        if (pa[a])
            rt += upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
}
};

```

### 3.5 LiChaoST\* [817716]

```

ll INF = 3e18;
struct LiChaoTree { // max
    using L = pair<ll, ll>; // l.first * x + l.second
    int sz;
    vector<L> data;
    vi xs;
    static ll eval
        (L l, ll x) { return l.first * x + l.second; }
    LiChaoTree(vi _xs) : xs(_xs) {
        int n = (int)(xs.size());
        int lg = 1;
        while ((1 << lg) < n) lg++;
        sz = 1 << lg;
        while
            ((int)(xs.size()) < sz) xs.push_back(1e9);
        data = vector<L>(2 * sz, L(0, INF));
    }
    void add(L line, int l, int r) {
        l = lower_bound
            (xs.begin(), xs.end(), l) - xs.begin();
        r = lower_bound
            (xs.begin(), xs.end(), r) - xs.begin();
        add(line, l, r, 0, sz, 1);
    }
    ll query(ll x) {
        int k = (int)(lower_bound
            (xs.begin(), xs.end(), x) - xs.begin());
        assert(0 <=
            k && k < (int)(xs.size()) && xs[k] == x);
        k += sz;
        ll ans = INF;
        while (k >= 1) {
            ans = min(ans, eval(data[k], x));
            k >>= 1;
        }
        return ans;
    }
private:
    void add
        (L line, int ql, int qr, int l, int r, int k) {
        if (qr <= l || r <= ql) {
            return;
        } else if (ql <= l && r <= qr) {
            int mid = (l + r) / 2;
            ll mx = xs[mid];
            if (eval(line, mx) < eval(data[k], mx)) {

```

```

                swap(line, data[k]);
            }
            if (l + 1 == r) return;
            if (line.first > data[k].first) {
                add(line, ql, qr, l, mid, 2 * k);
            } else if (line.first < data[k].first) {
                add(line, ql, qr, mid, r, 2 * k + 1);
            }
        } else {
            int mid = (l + r) / 2;
            add(line, ql, qr, l, mid, 2 * k);
            add(line, ql, qr, mid, r, 2 * k + 1);
        }
    }
};

```

## 4 Flow/Matching

### 4.1 Bipartite Matching\* [784535]

```

struct Bipartite_Matching { // 0-base
    int mp[N], mq[N], dis[N + 1], cur[N], l, r;
    vector<int> G[N + 1];
    bool dfs(int u) {
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            int e = G[u][i];
            if (mq[e] == l
                || (dis[mq[e]] == dis[u] + 1 && dfs(mq[e])))
                return mp[mq[e] = u] = e, 1;
        }
        return dis[u] = -1, 0;
    }
    bool bfs() {
        queue<int> q;
        fill_n(dis, l + 1, -1);
        for (int i = 0; i < l; ++i)
            if (!mp[i])
                q.push(i), dis[i] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int e : G[u])
                if (!dis[mq[e]])
                    q.push(mq[e]), dis[mq[e]] = dis[u] + 1;
        }
        return dis[l] != -1;
    }
    int matching() {
        int res = 0;
        fill_n(mp, l, -1), fill_n(mq, r, l);
        while (bfs()) {
            fill_n(cur, l, 0);
            for (int i = 0; i < l; ++i)
                res += (!mp[i] && dfs(i));
        }
        return res; // (i, mp[i] != -1)
    }
    void add_edge(int s, int t) { G[s].pb(t); }
    void init(int _l, int _r) {
        l = _l, r = _r;
        for (int i = 0; i <= l; ++i)
            G[i].clear();
    }
};

```

### 4.2 MincostMaxflow\* [1c78db]

```

struct MinCostMaxFlow { // 0-base
    struct Edge {
        ll from, to, cap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    int inq[N], n, s, t;
    ll dis[N], up[N], pot[N];
    bool BellmanFord() {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, ll d, ll cap, Edge *e) {
            if (cap > 0 && dis[u] > d) {
                dis[u] = d, up[u] = cap, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, INF, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;

```

```

    for (auto &e : G[u]) {
        ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
        relax
            (e.to, d2, min(up[u], e.cap - e.flow), &e);
    }
    return dis[t] != INF;
}
void solve(int _s
    , int _t, ll &flow, ll &cost, bool neg = true) {
    s = _s, t = _t, flow = 0, cost = 0;
    if (neg) BellmanFord(), copy_n(dis, n, pot);
    for (; BellmanFord(); copy_n(dis, n, pot)) {
        for (int
            i = 0; i < n; ++i) dis[i] += pot[i] - pot[s];
        flow += up[t], cost += up[t] * dis[t];
        for (int i = t; past[i]; i = past[i]->from) {
            auto &e = *past[i];
            e.flow += up[t], G[e.to][e.rev].flow -= up[t];
        }
    }
}
void init(int _n) {
    n = _n, fill_n(pot, n, 0);
    for (int i = 0; i < n; ++i) G[i].clear();
}
void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].pb(Edge{a, b, cap, 0, cost, SZ(G[b]))});
    G[b].pb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
}
};

```

### 4.3 Maximum Simple Graph Matching\* [0fe1c3]

```

struct Matching { // 0-base
    queue<int> q; int n;
    vector<int> fa, s, vis, pre, match;
    vector<vector<int>> G;
    int Find(int u)
    { return u == fa[u] ? u : fa[u] = Find(fa[u]); }
    int LCA(int x, int y) {
        static int tk = 0; tk++; x = Find(x); y = Find(y);
        for (; swap(x, y)) if (x != n) {
            if (vis[x] == tk) return x;
            vis[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
    void Blossom(int x, int y, int l) {
        for (; Find(x) != l; x = pre[y]) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            for (int z : {x, y}) if (fa[z] == z) fa[z] = l;
        }
    }
    bool Bfs(int r) {
        iota(ALL(fa), 0); fill(ALL(s), -1);
        q = queue<int>(); q.push(r); s[r] = 0;
        for (; !q.empty(); q.pop()) {
            for (int x = q.front(); int u : G[x])
                if (s[u] == -1) {
                    if (pre[u] = x, s[u] = 1, match[u] == n) {
                        for (int a = u, b = x, last;
                            b != n; a = last, b = pre[a])
                            last = match[b], match[b] = a, match[a] = b;
                        return true;
                    }
                    q.push(match[u]); s[match[u]] = 0;
                } else if (!s[u] && Find(u) != Find(x)) {
                    int l = LCA(u, x);
                    Blossom(x, u, l); Blossom(u, x, l);
                }
            }
        return false;
    }
    Matching(int _n : n(_n), fa(n + 1), s(n + 1), vis
        (n + 1), pre(n + 1, n), match(n + 1, n), G(n) {})
    void add_edge(int u, int v)
    { G[u].pb(v), G[v].pb(u); }
    int solve() {
        int ans = 0;
        for (int x = 0; x < n; ++x)
            if (match[x] == n) ans += Bfs(x);
        return ans;
    }
    // match[x] == n means not matched
};

```

### 4.4 Maximum Weight Matching\* [9ffb94]

```

#define REP(i, l, r) for (int i=(l); i<=(r); ++i)
struct WeightGraph { // 1-based
    struct edge { int u, v, w; }; int n, nx;
    vector<int> lab; vector<vector<edge>> g;
    vector<int> slk, match, st, pa, S, vis;
    vector<vector<int>> flo, flo_from; queue<int> q;
    WeightGraph(int n_) : n(n_), nx(n * 2), lab(nx + 1),
        g(nx + 1, vector<edge>(nx + 1)), slk(nx + 1),
        flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
        match = st = pa = S = vis = slk;
        REP(u, 1, n) REP(v, 1, n) g[u][v] = {u, v, 0};
    }
    int E(edge e)
    { return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2; }
    void update_slk(int u, int x, int &s)
    { if (!s || E(g[u][x]) < E(g[s][x])) s = u; }
    void set_slk(int x) {
        slk[x] = 0;
        REP(u, 1, n)
            if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
                update_slk(u, x, slk[x]);
    }
    void q_push(int x) {
        if (x <= n) q.push(x);
        else for (int y : flo[x]) q.push(y);
    }
    void set_st(int x, int b) {
        st[x] = b;
        if (x > n) for (int y : flo[x]) set_st(y, b);
    }
    vector<int> split_flo(auto &f, int xr) {
        auto it = find(ALL(f), xr);
        if (auto pr = it - f.begin(); pr % 2 == 1)
            reverse(1 + ALL(f), it = f.end() - pr);
        auto res = vector(f.begin(), it);
        return f.erase(f.begin(), it), res;
    }
    void set_match(int u, int v) {
        match[u] = g[u][v].v;
        if (u <= n) return;
        int xr = flo_from[u][g[u][v].u];
        auto &f = flo[u], z = split_flo(f, xr);
        REP(i, 0, SZ(z) - 1) set_match(z[i], z[i ^ 1]);
        set_match(xr, v); f.insert(f.end(), ALL(z));
    }
    void augment(int u, int v) {
        for (;;) {
            int xnv = st[match[u]]; set_match(u, v);
            if (!xnv) return;
            set_match(v = xnv, u = st[pa[xnv]]);
        }
    }
    int lca(int u, int v) {
        static int t = 0; ++t;
        for (++t; u || v; swap(u, v)) if (u) {
            if (vis[u] == t) return u;
            vis[u] = t, u = st[match[u]];
            if (u) u = st[pa[u]];
        }
        return 0;
    }
    void add_blossom(int u, int o, int v) {
        int b = find(n + 1 + ALL(st), 0) - begin(st);
        lab[b] = 0, S[b] = 0, match[b] = match[o];
        vector<int> f = {o};
        for (int t : {u, v}) {
            reverse(1 + ALL(f));
            for (int x = t, y; x != o; x = st[pa[y]])
                f.pb(x), f.pb(y = st[match[x]]), q_push(y);
        }
        flo[b] = f; set_st(b, b);
        REP(x, 1, nx) g[b][x].w = g[x][b].w = 0;
        fill(ALL(flo_from[b]), 0);
        for (int xs : flo[b]) {
            REP(x, 1, nx)
                if (g[b][x].w == 0 || E(g[xs][x]) < E(g[b][x]))
                    g[b][x] = g[xs][x], g[x][b] = g[x][xs];
            REP(x, 1, n)
                if (flo_from[xs][x] flo_from[b][x] = xs;
        }
        set_slk(b);
    }
    void expand_blossom(int b) {
        for (int x : flo[b]) set_st(x, x);
    }
};

```



```

int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
for (int x : split_flo(flo[b], xr)) {
    if (xs == -1) { xs = x; continue; }
    pa[xs] = g[x][xs].u, S[xs] = 1, S[x] = 0;
    slk[xs] = 0, set_slk(x), q_push(x), xs = -1;
}
for (int x : flo[b])
    if (x == xr) S[x] = 1, pa[x] = pa[b];
    else S[x] = -1, set_slk(x);
st[b] = 0;
}
bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
        int nu = st[match[v]]; pa[v] = e.u; S[v] = 1;
        slk[v] = slk[nu] = S[nu] = 0; q_push(nu);
    } else if (S[v] == 0) {
        if (int o = lca(u, v)) add_blossom(u, o, v);
        else return augment(u, v), augment(v, u), true;
    }
    return false;
}
bool matching() {
    fill(ALL(S), -1), fill(ALL(slk), 0);
    q = queue<int>();
    REP(x, 1, nx) if (st[x] == x && !match[x])
        pa[x] = S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
        while (SZ(q)) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            REP(v, 1, n)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (E(g[u][v]) != 0)
                        update_slk(u, st[v], slk[st[v]]);
                    else if
                        (on_found_edge(g[u][v])) return true;
                }
        }
        int d = INF;
        REP(b, n + 1, nx) if (st[b] == b && S[b] == 1)
            d = min(d, lab[b] / 2);
        REP(x, 1, nx)
            if (int
                s = slk[x]; st[x] == x && s && S[x] <= 0)
                d = min(d, E(g[s][x]) / (S[x] + 2));
        REP(u, 1, n)
            if (S[st[u]] == 1) lab[u] += d;
            else if (S[st[u]] == 0) {
                if (lab[u] <= d) return false;
                lab[u] -= d;
            }
        REP(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
            lab[b] += d * (2 - 4 * S[b]);
        REP(x, 1, nx)
            if (int s = slk[x]; st[x] == x &&
                s && st[s] != x && E(g[s][x]) == 0)
                if (on_found_edge(g[s][x])) return true;
        REP(b, n + 1, nx)
            if (st[b] == b && S[b] == 1 && lab[b] == 0)
                expand_blossom(b);
    }
    return false;
}
pair<ll, int> solve() {
    fill(ALL(match), 0);
    REP(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    REP(u, 1, n) REP(v, 1, n) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
    }
    fill(ALL(lab), w_max);
    int n_matches = 0; ll tot_weight = 0;
    while (matching()) ++n_matches;
    REP(u, 1, n) if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void add_edge(int u, int v, int w)
{ g[u][v].w = g[v][u].w = w; }
};

```

## 4.5 BoundedFlow\*(Dinic\*) [4a793f]

```

struct BoundedFlow { // 0-base
    struct edge {

```

```

        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n + 2; ++i)
            G[i].clear(), cnt[i] = 0;
    }
    void add_edge(int u, int v, int lcap, int rcap) {
        cnt[u] -= lcap, cnt[v] += lcap;
        G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df, G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n + 3, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (edge &e : G[u])
                if (!dis[e.to] && e.flow != e.cap)
                    q.push(e.to), dis[e.to] = dis[u] + 1;
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(cur, n + 3, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    bool solve() {
        int sum = 0;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0)
                add_edge(n + 1, i, cnt[i]), sum += cnt[i];
            else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
        if (sum != maxflow(n + 1, n + 2)) sum = -1;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0)
                G[n + 1].pop_back(), G[i].pop_back();
            else if (cnt[i] < 0)
                G[i].pop_back(), G[n + 2].pop_back();
        return sum != -1;
    }
    int solve(int _s, int _t) {
        add_edge(_t, _s, INF);
        if (!solve()) return -1; // invalid flow
        int x = G[_t].back().flow;
        return G[_t].pop_back(), G[_s].pop_back(), x;
    }
};

```

## 4.6 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem

1. Construct super source  $S$  and sink  $T$ .
2. For each edge  $(x, y, l, u)$ , connect  $x \rightarrow y$  with capacity  $u - l$ .
3. For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
4. If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .

- To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
- To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
- 5. The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.
- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X, Y)$ 
  1. Redirect every edge:  $y \rightarrow x$  if  $(x, y) \in M$ ,  $x \rightarrow y$  otherwise.
  2. DFS from unmatched vertices in  $X$ .
  3.  $x \in X$  is chosen iff  $x$  is unvisited.
  4.  $y \in Y$  is chosen iff  $y$  is visited.
- Minimum cost cyclic flow
  1. Construct super source  $S$  and sink  $T$
  2. For each edge  $(x, y, c)$ , connect  $x \rightarrow y$  with  $(cost, cap) = (c, 1)$  if  $c > 0$ , otherwise connect  $y \rightarrow x$  with  $(cost, cap) = (-c, 1)$
  3. For each edge with  $c < 0$ , sum these cost as  $K$ , then increase  $d(y)$  by 1, decrease  $d(x)$  by 1
  4. For each vertex  $v$  with  $d(v) > 0$ , connect  $S \rightarrow v$  with  $(cost, cap) = (0, d(v))$
  5. For each vertex  $v$  with  $d(v) < 0$ , connect  $v \rightarrow T$  with  $(cost, cap) = (0, -d(v))$
  6. Flow from  $S$  to  $T$ , the answer is the cost of the flow  $C + K$
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer  $T$
  2. Construct a max flow model, let  $K$  be the sum of all weights
  3. Connect source  $s \rightarrow v, v \in G$  with capacity  $K$
  4. For each edge  $(u, v, w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$
  5. For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  6.  $T$  is a valid answer if the maximum flow  $f < K|V|$
- Minimum weight edge cover
  1. For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u, v)$ .
  2. Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
  3. Find the minimum weight perfect matching on  $G'$ .
- Project selection problem
  1. If  $p_v > 0$ , create edge  $(s, v)$  with capacity  $p_v$ ; otherwise, create edge  $(v, t)$  with capacity  $-p_v$ .
  2. Create edge  $(u, v)$  with capacity  $w$  with  $w$  being the cost of choosing  $u$  without choosing  $v$ .
  3. The mincut is equivalent to the maximum profit of a subset of projects.
- Dual of minimum cost maximum flow
  1. Capacity  $c_{uv}$ , Flow  $f_{uv}$ , Cost  $w_{uv}$ , Required Flow difference for vertex  $b_u$ .
  2. If all  $w_{uv}$  are integers, then optimal solution can happen when all  $p_u$  are integers.

$$\min \sum_{uv} w_{uv} f_{uv} \quad \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv})$$

$$-f_{uv} \geq -c_{uv} \Leftrightarrow \sum_v f_{vu} - \sum_v f_{uv} = -b_u \quad p_u \geq 0$$

## 5 String

### 5.1 KMP [db6286]

```
vector<int> fail_func(const string &s) {
    int n = s.size();
    vector<int> f(n);
    for(int i=1; i<n; ++i) {
        int j = f[i-1];
        while(j > 0 && s[i] != s[j]) j = f[j-1];
        f[i] = j + (s[i] == s[j]);
    }
    return f;
}

int kmp_count(const string &s, const string &p) {
    int n = s.size(), m = p.size();
    vector<int> f = fail_func(p + "$");
    int cnt = 0, j = 0;
    for(int i=0; i<n; ++i) {
        while(j > 0 && s[i] != p[j]) j = f[j-1];
        j += (s[i] == p[j]);
        if(j == m) cnt++;
    }
    return cnt;
}
```

### 5.2 Z-value\* [e4e6b3]

```
vi z_value(string s) {
    int n = s.size(), b = 0;
    vi z(n);
    for(int i=1; i<n; ++i) {
        if(z[b]+b < i) z[i] = 0;
        else z[i] = min(z[b]+b-i, z[i-b]);
        while(s[z[i]] == s[z[i]+i]) z[i]++;
        if(z[i]+i > z[b]+b) b = i;
    }
    return z;
}
```

### 5.3 Manacher\* [ae9cc4]

```
int z[N]; // 0-base
/* center i: radius z[i * 2 + 1] / 2
   center i, i + 1: radius z[i * 2 + 2] / 2
   both aba, abba have radius 2 */
void Manacher(string tmp) {
    string s = "%";
    int l = 0, r = 0;
    for(char c : tmp) s.push_back(c), s.push_back('%');
    for(int i = 0; i < s.size(); ++i) {
        z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
        while(i - z[i] >= 0 && i + z[i] < s.size()
            && s[i - z[i]] == s[i + z[i]]) ++z[i];
        if(z[i] + i > r) r = z[i] + i, l = i;
    }
}
```

### 5.4 SAIS\* [6f26bc]

```
auto sais(const auto &s) {
    const int n = SZ(s), z = ranges::max(s) + 1;
    if(n == 1) return vector{0};
    vector<int> c(z); for(int x : s) ++c[x];
    partial_sum(ALL(c), begin(c));
    vector<int> sa(n); auto I = views::iota(0, n);
    vector<bool> t(n, true);
    for(int i = n - 2; i >= 0; --i)
        t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    auto is_lms = views::filter([&t](int x) {
        return x && t[x] && !t[x - 1];
    });
    auto induce = [&] {
        for(auto x = c; int y : sa)
            if(y-- && (!t[y])) sa[x[s[y] - 1]++] = y;
        for(auto x = c; int y : sa | views::reverse)
            if(y-- && (t[y])) sa[--x[s[y]]] = y;
    };
    vector<int> lms, q(n); lms.reserve(n);
    for(auto x = c; int i : I | is_lms)
        q[i] = SZ(lms), lms.pb(sa[--x[s[i]]] = i);
    induce(); vector<int> ns(SZ(lms));
    for(int j = -1, nz = 0; int i : sa | is_lms) {
        if(j >= 0) {
            int len = min({n - i, n - j, lms[q[i] + 1] - i});
            ns[q[i]] = nz += lexicographical_compare(
                begin(s) + j, begin(s) + j + len,
                begin(s) + i, begin(s) + i + len);
        }
        j = i;
    }
    fill(ALL(sa), 0); auto nsa = sais(ns);
    for(auto x = c; int y : nsa | views::reverse)
        y = lms[y], sa[--x[s[y]]] = y;
    return induce(), sa;
}

// sa[i]: sa[i]-th suffix
// is the i-th lexicographically smallest suffix.
// hi[i]: LCP of suffix sa[i] and suffix sa[i - 1].
struct Suffix {
    int n; vector<int> sa, hi, ra;
    Suffix(const auto &s, int _n) : n(_n), hi(n), ra(n) {
        vector<int> s(n + 1); // s[n] = 0;
        copy_n(_s, n, begin(s)); // _s shouldn't contain 0
        sa = sais(s); sa.erase(sa.begin());
        for(int i = 0; i < n; ++i) ra[sa[i]] = i;
        for(int i = 0, h = 0; i < n; ++i) {
            if(!ra[i]) { h = 0; continue; }
            for(int j = sa[ra[i] - 1]; max(
                (i, j) + h < n && s[i + h] == s[j + h]) ++h;
            hi[ra[i]] = h ? h - 1 : 0;
        }
    }
}
```



```

    }
}
};

```

## 5.5 Aho-Corasick Automatan\* [794a77]

```

struct AC_Automatan {
    int nx[LEN][SIGMA], fl[LEN], cnt[LEN], ord[LEN], top;
    int rnx[LEN][SIGMA]; // node actually be reached
    int newnode() {
        fill_n(nx[top], sigma, -1);
        return top++;
    }
    void init() { top = 1; newnode(); }
    int input(string &s) {
        int X = 1;
        for (char c : s) {
            if (!nx[X][c - 'A']) nx[X][c - 'A'] = newnode();
            X = nx[X][c - 'A'];
        }
        return X; // return the end node of string
    }
    void make_fl() {
        queue<int> q;
        q.push(1), fl[1] = 0;
        for (int t = 0; !q.empty(); ) {
            int R = q.front();
            q.pop(), ord[t++] = R;
            for (int i = 0; i < sigma; ++i)
                if (~nx[R][i]) {
                    int X = rnx[R][i] = nx[R][i], Z = fl[R];
                    for (; Z && !~nx[Z][i]; ) Z = fl[Z];
                    fl[X] = Z ? nx[Z][i] : 1, q.push(X);
                }
            else rnx[R][i] = R > 1 ? rnx[fl[R]][i] : 1;
        }
    }
    void solve() {
        for (int i = top - 2; i > 0; --i)
            cnt[fl[ord[i]]] += cnt[ord[i]];
    }
} ac;

```

## 5.6 Extended SAM\* [64c3b7]

```

struct exSAM {
    int len[N * 2], link[N * 2]; // maxlength, suflink
    int next[N * 2][CNUM], tot; // [0, tot), root = 0
    int lenSorted[N * 2]; // topo. order
    int cnt[N * 2]; // occurence
    int newnode() {
        fill_n(next[tot], CNUM, 0);
        len[tot] = cnt[tot] = link[tot] = 0;
        return tot++;
    }
    void init() { tot = 0; newnode(), link[0] = -1; }
    int insertSAM(int last, int c) {
        int cur = next[last][c];
        len[cur] = len[last] + 1;
        int p = link[last];
        while (p != -1 && !next[p][c])
            next[p][c] = cur, p = link[p];
        if (p == -1) return link[cur] = 0, cur;
        int q = next[p][c];
        if (len[p] + 1 == len[q]) return link[cur] = q, cur;
        int clone = newnode();
        for (int i = 0; i < CNUM; ++i)
            next[clone][i] = len[next[q][i]] ? next[q][i] : 0;
        len[clone] = len[p] + 1;
        while (p != -1 && next[p][c] == q)
            next[p][c] = clone, p = link[p];
        link[link[cur]] = clone;
        link[q] = clone;
        return cur;
    }
    void insert(const string &s) {
        int cur = 0;
        for (auto ch : s) {
            int &nxt = next[cur][ch - 'a'];
            if (!nxt) nxt = newnode();
            cnt[cur = nxt] += 1;
        }
    }
    void build() {
        queue<int> q;

```

```

        q.push(0);
        while (!q.empty()) {
            int cur = q.front();
            q.pop();
            for (int i = 0; i < CNUM; ++i)
                if (next[cur][i])
                    q.push(insertSAM(cur, i));
        }
        vector<int> lc(tot);
        for (int i = 1; i < tot; ++i) ++lc[len[i]];
        partial_sum(ALL(lc), lc.begin());
        for (int i = 1; i < tot; ++i) lenSorted[--lc[len[i]]] = i;
    }
    void solve() {
        for (int i = tot - 2; i >= 0; --i)
            cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
    }
};

```

## 5.7 Main Lorentz [e8313e]

```

vector<pair<int,int>> rep[300005]; // [l,r]
void main_lorentz(const string &s, int sft = 0) {
    const int n = s.size();
    if (n == 1) return;
    const int nu = n / 2, nv = n - nu;
    const string u = s.substr(0, nu), v = s.substr(nu, ru(u)
        .rbegin(), u.rend(), rv(v.rbegin(), v.rend()));
    main_lorentz(u, sft), main_lorentz(v, sft + nu);
    const auto z1 = Zalgo(ru), z2 = Zalgo(v + '#' + u);
    const auto z3 = Zalgo(ru + '#' + rv), z4 = Zalgo(v);
    auto get_z = [&](const vector<int> &z, int i) {
        return (0 <= i && i < (int)z.size()) ? z[i] : 0;
    };
    auto add_rep =
        [=](bool left, int c, int l, int k1, int k2) {
            const int L = max(1LL, l - k2), R = min(l - left, k1);
            if (L > R) return;
            if (left) rep[l].eb(sft + c - R, sft + c - L);
            else rep[l].eb(sft + c - R - l + 1, sft + c - L - l + 1);
        };
    for (int cntr = 0; cntr < n; cntr++) {
        int l, k1, k2;
        if (cntr < nu) {
            l = nu - cntr;
            k1 = get_z(z1, nu - cntr);
            k2 = get_z(z2, nv + 1 + cntr);
        }
        else {
            l = cntr - nu + 1;
            k1 = get_z(z3, nu + 1 + nv - 1 - (cntr - nu));
            k2 = get_z(z4, (cntr - nu) + 1);
        }
        if (k1 + k2 >= l) add_rep(cntr < nu, cntr, l, k1, k2);
    }
}

```

## 6 Math

### 6.1 ax+by=gcd(only exgcd\*) [7b833d]

```

pll exgcd(ll a, ll b) {
    if (b == 0) return pll(1, 0);
    ll p = a / b;
    pll q = exgcd(b, a % b);
    return pll(q.Y, q.X - q.Y * p);
}
/* ax+by=res, let x be minimum non-negative
g, p = gcd(a, b), exgcd(a, b) * res / g
if p.X < 0: t = (abs(p.X) + b / g - 1) / (b / g)
else: t = -(p.X / (b / g))
p += (b / g, -a / g) * t */

```

### 6.2 Floor and Ceil [692c04]

```

int floor(int a, int b)
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
int ceil(int a, int b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }

```

### 6.3 Mod Min [9118e1]

```

// min{k | l <= ((ak) mod m) <= r}, no solution -> -1
ll mod_min(ll a, ll m, ll l, ll r) {
    if (a == 0) return l ? -1 : 0;
    if (ll k = (l + a - 1) / a; k * a <= r)
        return k;
}

```

```

ll b = m / a, c = m % a;
if (ll y = mod_min(c, a, a - r % a, a - l % a))
    return (l + y * c + a - 1) / a + y * b;
return -1;
}

```

#### 6.4 Miller Rabin\* [da9a82]

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633 4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383 6 : primes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a, ll n) {
    if ((a = a % n) == 0) return 1;
    if (n % 2 == 0) return n == 2;
    ll tmp = (n - 1) / ((n - 1) & (1 - n));
    ll t = __lg((n - 1) & (1 - n)), x = 1;
    for (; tmp; tmp >>= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}
bool prime(ll n) { // SCOPE HASH
    vector<ll> tmp = {2,
        325, 9375, 28178, 450775, 9780504, 1795265022};
    for (ll i : tmp)
        if (!Miller_Rabin(i, n)) return false;
    return true;
}

```

#### 6.5 Pollard Rho\* [cfe72f]

```

map<ll, int> cnt;
void PollardRho(ll n) {
    if (n == 1) return;
    if (prime(n)) return ++cnt[n], void();
    if (n % 2
        == 0) return PollardRho(n / 2), ++cnt[2], void();
    ll x = 2, y = 2, d = 1, p = 1;
    #define f(x, n, p) ((mul(x, x, n) + p) % n)
    while (true) {
        if (d != n && d != 1) {
            PollardRho(n / d);
            PollardRho(d);
            return;
        }
        if (d == n) ++p;
        x = f(x, n, p), y = f(f(y, n, p), n, p);
        d = gcd(abs(x - y), n);
    }
}

```

#### 6.6 chineseRemainder [a53b6d]

```

ll solve(ll x1, ll m1, ll x2, ll m2) {
    ll g = gcd(m1, m2);
    if ((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pll p = exgcd(m1, m2);
    ll lcm = m1 * m2 * g;
    ll res = p.first * (x2 - x1) * m1 + x1;
    // be careful with overflow
    return (res % lcm + lcm) % lcm;
}

```

#### 6.7 Factorial without prime factor\* [c324f3]

```

// O(p^k + log^2 n), pk = p^k
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
    prod[0] = 1;
    for (int i = 1; i <= pk; ++i)
        if (i % p) prod[i] = prod[i - 1] * i % pk;
        else prod[i] = prod[i - 1];
    ll rt = 1;
    for (; n; n /= p) {
        rt = rt * mpow(prod[pk], n / pk, pk) % pk;
        rt = rt * prod[n % pk] % pk;
    }
    return rt;
}
// (n! without factor p) % p^k

```

#### 6.8 QuadraticResidue\* [e0bf30]

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL *
                g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL
            * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

#### 6.9 PiCount\* [cad6d4]

```

ll PrimeCount(ll n) { // n ~ 10^13 => < 2s
    if (n <= 1) return 0;
    int v = sqrt(n), s = (v + 1) / 2, pc = 0;
    vector<int> smalls(v + 1), skip(v + 1), roughs(s);
    vector<ll> larges(s);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    for (int i = 0; i < s; ++i) {
        roughs[i] = 2 * i + 1;
        larges[i] = (n / (2 * i + 1) + 1) / 2;
    }
    for (int p = 3; p <= v; ++p) {
        if (smalls[p] > smalls[p - 1]) {
            int q = p * p;
            ++pc;
            if (1LL * q * q > n) break;
            skip[p] = 1;
            for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
            int ns = 0;
            for (int k = 0; k < s; ++k) {
                int i = roughs[k];
                if (skip[i]) continue;
                ll d = 1LL * i * p;
                larges[ns] = larges[k] - (d <= v ? larges
                    [smalls[d] - pc] : smalls[n / d]) + pc;
                roughs[ns++] = i;
            }
            s = ns;
            for (int j = v / p; j >= p; --j) {
                int c =
                    smalls[j] - pc, e = min(j * p + p, v + 1);
                for (int i = j * p; i < e; ++i) smalls[i] -= c;
            }
        }
    }
    for (int k = 1; k < s; ++k) {
        const ll m = n / roughs[k];
        ll t = larges[k] - (pc + k - 1);
        for (int l = 1; l < k; ++l) {
            int p = roughs[l];
            if (1LL * p * p > m) break;
            t -= smalls[m / p] - (pc + l - 1);
        }
    }
}

```

```

    larges[0] -= t;
}
return larges[0];
}

```

## 6.10 Discrete Log\* [da27bf]

```

int DiscreteLog(int s, int x, int y, int m) {
    constexpr int kStep = 32000;
    unordered_map<int, int> p;
    int b = 1;
    for (int i = 0; i < kStep; ++i) {
        p[y] = i;
        y = 1LL * y * x % m;
        b = 1LL * b * x % m;
    }
    for (int i = 0; i < m + 10; i += kStep) {
        s = 1LL * s * b % m;
        if (p.find(s) != p.end()) return i + kStep - p[s];
    }
    return -1;
}

int DiscreteLog(int x, int y, int m) {
    if (m == 1) return 0;
    int s = 1;
    for (int i = 0; i < 100; ++i) {
        if (s == y) return i;
        s = 1LL * s * x % m;
    }
    if (s == y) return 100;
    int p = 100 + DiscreteLog(s, x, y, m);
    if (fpow(x, p, m) != y) return -1;
    return p;
}

```

## 6.11 Primes

```

/* 12721 13331 14341 75577 123457 222557
   556679 999983 1097774749 1076767633 100102021
   999997771 1001010013 1000512343 987654361 999991231
   999888733 98789101 987777733 999991921 1010101333
   1010102101 1000000000039 100000000000037
   2305843009213693951 4611686018427387847
   9223372036854775783 18446744073709551557 */

```

## 6.12 Theorem

$$\sum k^1 = -\frac{1}{12}$$

$$\sum k^4 = \frac{1}{30}[n(n+1)(2n+1)(3n^2+3n-1)]$$

$$\sum k^5 = \frac{1}{12}[n^2(n+1)^2(2n^2+2n-1)]$$

$$\sum k^6 = \frac{1}{42}[n(n+1)(2n+1)(3n^4+6n^3-3n+1)]$$

### • Cramer's rule

$$\begin{aligned} ax+by &= e & x &= \frac{ed-bf}{ad-bc} \\ cx+dy &= f & y &= \frac{af-ec}{ad-bc} \end{aligned}$$

### • Vandermonde's Identity

$$C(n+m, k) = \sum_{i=0}^k C(n, i) C(m, k-i)$$

### • Kirchhoff's Theorem

Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where  $c$  is the number of edge  $(i, j)$  in  $G$ .

- The number of undirected spanning in  $G$  is  $|\det(\tilde{L}_{11})|$ .
- The number of directed spanning tree rooted at  $r$  in  $G$  is  $|\det(\tilde{L}_{rr})|$ .

### • Tutte's Matrix

Let  $D$  be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if  $i < j$  and  $(i, j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching on  $G$ .

### • Cayley's Formula

- Given a degree sequence  $d_1, d_2, \dots, d_n$  for each *labeled* vertices, there are  $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$  spanning trees.
- Let  $T_{n,k}$  be the number of *labeled* forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .

### • Erdős–Gallai theorem

A sequence of nonnegative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if

$$d_1 + \dots + d_n \text{ is even and } \sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k) \text{ holds for every } 1 \leq k \leq n.$$

### • Gale–Ryser theorem

A pair of sequences of nonnegative integers  $a_1 \geq \dots \geq a_n$  and  $b_1, \dots, b_n$

is bigraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

### • Fulkerson–Chen–Anstee theorem

A sequence  $(a_1, b_1), \dots, (a_n, b_n)$  of nonnegative integer pairs with  $a_1 \geq \dots \geq a_n$  is digraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and

$$\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k) \text{ holds for every } 1 \leq k \leq n.$$

### • Pick's theorem

For simple polygon, when points are all integer, we have

$$A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1.$$

### • Möbius inversion formula

$$\begin{aligned} f(n) &= \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) \\ f(n) &= \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d) \end{aligned}$$

### • Spherical cap

- A portion of a sphere cut off by a plane.
- $r$ : sphere radius,  $a$ : radius of the base of the cap,  $h$ : height of the cap,  $\theta$ :  $\arcsin(a/r)$ .
- Volume  $= \pi h^2(3r - h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2 + \cos\theta)(1 - \cos\theta)^2/3$ .
- Area  $= 2\pi r h = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos\theta)$ .

### • Lagrange multiplier

- Optimize  $f(x_1, \dots, x_n)$  when  $k$  constraints  $g_i(x_1, \dots, x_n) = 0$ .
- Lagrangian function  $\mathcal{L}(x_1, \dots, x_n, \lambda_1, \dots, \lambda_k) = f(x_1, \dots, x_n) - \sum_{i=1}^k \lambda_i g_i(x_1, \dots, x_n)$ .
- The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.

### • Nearest points of two skew lines

- Line 1:  $v_1 = p_1 + t_1 d_1$
- Line 2:  $v_2 = p_2 + t_2 d_2$
- $n = d_1 \times d_2$
- $n_1 = d_1 \times n$
- $n_2 = d_2 \times n$
- $c_1 = p_1 + \frac{(p_2 - p_1) \cdot n_2}{d_1 \cdot n_2} d_1$
- $c_2 = p_2 + \frac{(p_1 - p_2) \cdot n_1}{d_2 \cdot n_1} d_2$

### • Derivatives/Integrals

Integration by parts:  $\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$

$$\begin{aligned} \left| \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \right| & \quad \left| \frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}} \right| & \quad \left| \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2} \right| \\ \frac{d}{dx} \tan x = 1 + \tan^2 x & \quad \int \tan ax = -\frac{\ln|\cos ax|}{a} \\ \int e^{-x^2} = \frac{\sqrt{\pi}}{2} \text{erf}(x) & \quad \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1) \\ \int \sqrt{a^2 + x^2} = \frac{1}{2} \left( x \sqrt{a^2 + x^2} + a^2 \text{asin}(x/a) \right) \end{aligned}$$

### • Spherical Coordinate

$$(x, y, z) = (r \sin\theta \cos\phi, r \sin\theta \sin\phi, r \cos\theta)$$

$$(r, \theta, \phi) = (\sqrt{x^2 + y^2 + z^2}, \arccos(z/\sqrt{x^2 + y^2 + z^2}), \arctan2(y, x))$$

### • Rotation Matrix

$$M(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix}$$

## 6.13 Estimation

$n$	2	3	4	5	6	7	8	9	20	30	40	50	100		
$p(n)$	2	3	5	7	11	15	22	30	627	5604	4e4	2e5	2e8		
$n$	100	1e3	1e6	1e9	1e12	1e15	1e18								
$d(i)$	12	32	240	1344	6720	26880	103680								
$n$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\binom{2n}{n}$	2	6	20	70	252	924	3432	12870	48620	184756	7e5	2e6	1e7	4e7	1.5e8
$n$	2	3	4	5	6	7	8	9	10	11	12	13			
$B_n$	2	5	15	52	203	877	4140	21147	115975	7e5	4e6	3e7			

## 6.14 General Purpose Numbers

- Bernoulli numbers

$$B_0=1, B_1^{\pm}=\pm\frac{1}{2}, B_2=\frac{1}{6}, B_3=0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1-x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left( x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$  j's s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$  j's s.t.  $\pi(j) \geq j$ ,  $k$  j's s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

## 6.15 Tips for Generating Functions

- Ordinary Generating Function  $A(x) = \sum_{i \geq 0} a_i x^i$

- $A(rx) \Rightarrow r^n a_n$
- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$
- $x A(x)' \Rightarrow n a_n$
- $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^n a_i$

- Exponential Generating Function  $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x^i$

- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A^{(k)}(x) \Rightarrow a_{n+k}$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n \binom{n}{i} a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$
- $x A(x) \Rightarrow n a_n$

- Special Generating Function

- $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
- $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{n-1}{i} x^i$

## 7 Polynomial

### 7.1 Fast Fourier Transform [56bdd7]

```
template<int MAXN>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n); // see NTT
    void trans
        (val_t *a, int n, bool inv = false); // see NTT;
        // remember to replace LL with val_t
};
```

### 7.2 Number Theory Transform\* [f68103]

```
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int MAXN, ll P, ll RT> //MAXN must be 2^k
struct NTT {
    ll w[MAXN];
    ll mpow(ll a, ll n);
    ll minv(ll a) { return mpow(a, P - 2); }
    NTT() {
        ll dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int
            i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw % P;
```

```

    }
    void bitrev(ll *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^= k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(
        ll *a, int n, bool inv = false) { //0 <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int
                    j = i, x = 0; j < i + dl; ++j, x += dx) {
                    ll tmp = a[j + dl] * w[x] % P;
                    if ((a[j
                        + dl] = a[j] - tmp) < 0) a[j + dl] += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a + 1, a + n);
            ll invn = minv(n);
            for (int
                i = 0; i < n; ++i) a[i] = a[i] * invn % P;
        }
    }
};
```

### 7.3 Polynomial Operation [105808]

```
#define
    fi(s, n) for (int i = (int)(s); i < (int)(n); ++i)
template<int MAXN, ll P, ll RT> // MAXN = 2^k
struct Poly : vector<ll> { // coefficients in [0, P)
    using vector<ll>::vector;
    static NTT<MAXN, P, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int m) : vector<ll>(m) {
        copy_n(p.data(), min(p.n(), m), data());
    }
    Poly& irev()
        { return reverse(data(), data() + n()), *this; }
    Poly& isz(int m) { return resize(m), *this; }
    Poly& iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if
            (((*this)[i] += rhs[i]) >= P) (*this)[i] -= P;
        return *this;
    }
    Poly& imul(ll k) {
        fi(0, n()) (*this)[i] = (*this)[i] * k % P;
        return *this;
    }
    Poly Mul(const Poly &rhs) const {
        int m = 1;
        while (m < n() + rhs.n() - 1) m <= 1;
        Poly X(*this, m), Y(rhs, m);
        ntt(X.data(), m), ntt(Y.data(), m);
        fi(0, m) X[i] = X[i] * Y[i] % P;
        ntt(X.data(), m, true);
        return X.isz(n() + rhs.n() - 1);
    }
    Poly Inv() const { // (*this)[0] != 0, 1e5/95ms
        if (n() == 1) return {ntt.minv((*this)[0])};
        int m = 1;
        while (m < n() * 2) m <= 1;
        Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(m);
        Poly Y(*this, m);
        ntt(Xi.data(), m), ntt(Y.data(), m);
        fi(0, m) {
            Xi[i] *= (2 - Xi[i] * Y[i]) % P;
            if ((Xi[i] %= P) < 0) Xi[i] += P;
        }
        ntt(Xi.data(), m, true);
        return Xi.isz(n());
    }
    Poly Sqrt()
        const { // Jacobi((*this)[0], P) = 1, 1e5/235ms
        if (n()
            == 1) return {QuadraticResidue((*this)[0], P)};
        Poly
            X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
        return
            X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 + 1);
```

```

}
pair<Poly, Poly> DivMod
( const Poly &rhs) const { // (rhs.)back() != 0
if (n() < rhs.n()) return {{0}, *this};
const int m = n() - rhs.n() + 1;
Poly X(rhs); X.irev().isz(m);
Poly Y(*this); Y.irev().isz(m);
Poly Q = Y.Mul(X.Inv()).isz(m).irev();
X = rhs.Mul(Q), Y = *this;
fi(0, n()) if ((Y[i] - X[i]) < 0) Y[i] += P;
return {Q, Y.isz(max(1, rhs.n() - 1))};
}
Poly Dx() const {
Poly ret(n() - 1);
fi(0,
ret.n()) ret[i] = (i + 1) * (*this)[i + 1] % P;
return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
Poly ret(n() + 1);
fi(0, n())
ret[i + 1] = ntt.minv(i + 1) * (*this)[i] % P;
return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
Poly Y = Mul(rhs).isz(n() + nn - 1);
return Poly(Y.data() + n() - 1, Y.data() + Y.n());
}
vector<ll> _eval(const
vector<ll> &x, const vector<Poly> &up) const {
const int m = (int)x.size();
if (!m) return {};
vector<Poly> down(m * 2);
// down[1] = DivMod(up[1]).second;
// fi(2, m *
2) down[i] = down[i / 2].DivMod(up[i]).second;
down[1] = Poly(up[1])
.irev().isz(n()).Inv().irev()._tmul(m, *this);
fi(2, m * 2) down[i]
= up[i ^ 1]._tmul(up[i].n() - 1, down[i / 2]);
vector<ll> y(m);
fi(0, m) y[i] = down[m + i][0];
return y;
}
static vector<Poly> _tree1(const vector<ll> &x) {
const int m = (int)x.size();
vector<Poly> up(m * 2);
fi(0, m) up[m + i] = {(x[i] ? P - x[i] : 0), 1};
for (int i = m - 1; i
> 0; --i) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
return up;
}
vector
<ll> Eval(const vector<ll> &x) const { // 1e5, 1s
auto up = _tree1(x); return _eval(x, up);
}
static Poly Interpolate(const vector
<ll> &x, const vector<ll> &y) { // 1e5, 1.4s
const int m = (int)x.size();
vector<Poly> up = _tree1(x), down(m * 2);
vector<ll> z = up[1].Dx()._eval(x, up);
fi(0, m) z[i] = y[i] * ntt.minv(z[i]) % P;
fi(0, m) down[m + i] = {z[i]};
for (int i = m -
1; i > 0; --i) down[i] = down[i * 2].Mul(up[i
* 2 + 1]).iadd(down[i * 2 + 1].Mul(up[i * 2]));
return down[1];
}
Poly Ln() const { // (*this)[0] == 1, 1e5/170ms
return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // (*this)[0] == 0, 1e5/360ms
if (n() == 1) return {1};
Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
Poly Y = X.Ln(); Y[0] = P - 1;
fi(0, n())
if ((Y[i] = (*this)[i] - Y[i]) < 0) Y[i] += P;
return X.Mul(Y).isz(n());
}
// M := P(P - 1). If k >= M, k := k % M + M.
Poly Pow(ll k) const {
int nz = 0;
while (nz < n() && !(*this)[nz]) ++nz;
if (nz * min(k, (ll)n()) >= n()) return Poly(n());
if (!k) return Poly(Poly{1}, n());
Poly X(data() + nz, data() + nz + n() - nz * k);

```

```

const ll c = ntt.mpow(X[0], k % (P - 1));
return X.Ln().imul
(k % P).Exp().imul(c).irev().isz(n()).irev();
}
static ll
LinearRecursion(const vector<ll> &a, const vector
<ll> &coef, ll n) { // a_n = |sum c_j a_{n-j}
const int k = (int)a.size();
assert((int)coef.size() == k + 1);
Poly C(k + 1), W(Poly{1}, k), M = {0, 1};
fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
C[k] = 1;
while (n) {
if (n % 2) W = W.Mul(M).DivMod(C).second;
n /= 2, M = M.Mul(M).DivMod(C).second;
}
ll ret = 0;
fi(0, k) ret = (ret + W[i] * a[i]) % P;
return ret;
}
};
#undef fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

## 7.4 Newton's Method

Given  $F(x)$  where

$$F(x) = \sum_{i=0}^{\infty} \alpha_i (x - \beta)^i$$

for  $\beta$  being some constant. Polynomial  $P$  such that  $F(P) = 0$  can be found iteratively. Denote by  $Q_k$  the polynomial such that  $F(Q_k) = 0 \pmod{x^{2^k}}$ , then

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

## 8 Geometry

### 8.1 Default Code [2e3f42]

```

using ld = ll;
using pdd = pair<ld, ld>;
using Line = pair<pdd, pdd>;
struct Cir{ pdd O; ld R; };
#define X first
#define Y second
// const ld eps = 1e-7;

pdd operator+(pdd a, pdd b)
{ return {a.X + b.X, a.Y + b.Y}; }
pdd operator-(pdd a, pdd b)
{ return {a.X - b.X, a.Y - b.Y}; }
pdd operator*(ld i, pdd v)
{ return {i * v.X, i * v.Y}; }
pdd operator*(pdd v, ld i)
{ return {i * v.X, i * v.Y}; }
pdd operator/(pdd v, ld i)
{ return {v.X / i, v.Y / i}; }
ld dot(pdd a, pdd b)
{ return a.X * b.X + a.Y * b.Y; }
ld cross(pdd a, pdd b)
{ return a.X * b.Y - a.Y * b.X; }
ld abs2(pdd v)
{ return dot(v, v); }
ld abs(pdd v)
{ return sqrt(dot(v, v)); }
int sgn(ld v)
{ return v > 0 ? 1 : (v < 0 ? -1 : 0); }
// int sgn(
ld v){ return v > eps ? 1 : (v < -eps ? -1 : 0); }
int ori(pdd a, pdd b, pdd c)
{ return sgn(cross(b - a, c - a)); }
bool collinearity(pdd a, pdd b, pdd c)
{ return ori(a, b, c) == 0; }
bool btw(pdd p, pdd a, pdd b)
{ return collinearity
(p, a, b) && sgn(dot(a - p, b - p)) <= 0; }

bool seg_intersect(pdd p1, pdd p2, pdd p3, pdd p4){
if(btw(p1, p3, p4) || btw(p2
, p3, p4) || btw(p3, p1, p2) || btw(p4, p1, p2))
return true;
return ori(p1, p2, p3) * ori(p1, p2, p4) < 0 &&
ori(p3, p4, p1) * ori(p3, p4, p2) < 0;
}

```



```

pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4){
    ld a123 = cross(p2 - p1, p3 - p1);
    ld a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}
pdd perp(pdd p1)
{ return pdd(-p1.Y, p1.X); }
pdd projection(pdd p1, pdd p2, pdd p3)
{ return p1 + (
    p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1); }
pdd reflection(pdd p1, pdd p2, pdd p3)
{ return p3 + perp(p2 - p1
    ) * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2; }
pdd linearTransformation
(pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
    pdd dp = p1 - p0
    , dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
    return q0 + pdd(
        cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r

```

## 8.2 PointSegDist\* [57b6de]

```

double PointSegDist(pdd q0, pdd q1, pdd p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign(dot(q1 - q0,
        p - q0)) >= 0 && sign(dot(q0 - q1, p - q1)) >= 0)
        return fabs(cross(q1 - q0, p - q0) / abs(q0 - q1));
    return min(abs(p - q0), abs(p - q1));
}

```

## 8.3 Heart [468aeb]

```

pair<pdd
    , ld> circenter(pdd a, pdd b, pdd c){ // SCOPE HASH
    pdd m1 = (a+b)/2, m2 = (b+c)/2;
    pdd cent = intersect
        (m1, m1 + perp(b-a), m2, m2 + perp(c-b));
    return {cent, abs(a-cent)};
}
pair<pdd, ld> incenter
(pdd p1, pdd p2, pdd p3) { // radius = area / s * 2
    ld a =
        abs(p2 - p3), b = abs(p1 - p3), c = abs(p1 - p2);
    pdd cent = (a * p1 + b * p2 + c * p3) / (a + b + c);
    return {cent, abs(a-cent)};
}
pdd masscenter(pdd p1, pdd p2, pdd p3)
{ return (p1 + p2 + p3) / 3; }
pdd orthcenter(pdd p1, pdd p2, pdd p3)
{ return masscenter
    (p1, p2, p3) * 3 - circenter(p1, p2, p3) * 2; }

```

## 8.4 point in circle [02a7ca]

```

// return q'
// s relation with circumcircle of tri(p[0],p[1],p[2])
bool inCC(const array<pdd, 3> &p, pdd q) {
    __int128 det = 0;
    for (int i = 0; i < 3; ++i)
        det += __int128(abs2(p[i]) - abs2(q)) *
            cross(p[(i + 1) % 3] - q, p[(i + 2) % 3] - q);
    return det > 0; // in: >0, on: =0, out: <0
}

```

## 8.5 Convex hull\* [9b329e]

```

vector<int> getConvexHull(vector<pdd>& pts){
    vector<int> id(SZ(pts));
    iota(all(id), 0);
    sort(all(id
        ), [&](int x, int y){ return pts[x] < pts[y]; });
    vector<int> hull;
    for(int tt = 0; tt < 2; tt++){
        int sz = SZ(hull);
        for(int j : id){
            pdd p = pts[j];
            while(SZ(hull) - sz >= 2 &&
                cross(pts
                    [hull.back()] - pts[hull[SZ(hull) - 2]],
                    p - pts[hull[SZ(hull) - 2]]) <= 0)
                hull.pop_back();
            hull.pb(j);
        }
        hull.pop_back();
        reverse(all(id));
    }
    return hull;
}

```

## 8.6 PointInConvex\* [82b81e]

```

bool PointInConvex
(const vector<pdd> &C, pdd p, bool strict = true) {
    int a = 1, b = SZ(C) - 1, r = !strict;
    if (SZ(C) == 0) return false;
    if (SZ(C) < 3) return r && btw(p, C[0], C.back());
    if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if (ori
        (C[0], C[a], p) >= r || ori(C[0], C[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(C[0], C[c], p) > 0 ? b : a) = c;
    }
    return ori(C[a], C[b], p) < r;
}

```

## 8.7 TangentPointToHull\* [523bc1]

```

/* The point should be strictly out of hull
return arbitrary point on the tangent line */
pii get_tangent(vector<pdd> &C, pdd p) {
    auto gao = [&](int s) {
        return cyc_tsearch(SZ(C), [&](int x, int y)
            { return ori(p, C[x], C[y]) == s; });
    };
    return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0

```

## 8.8 Intersection of line and convex [157258]

```

int TangentDir(vector<pdd> &C, pdd dir) {
    return cyc_tsearch(SZ(C), [&](int a, int b) {
        return cross(dir, C[a]) > cross(dir, C[b]);
    });
}
#define cml(i) sign(cross(C[i] - a, b - a))
pii lineHull(pdd a, pdd b, vector<pdd> &C) {
    int A = TangentDir(C, a - b);
    int B = TangentDir(C, b - a);
    int n = SZ(C);
    if (cml(A) < 0 || cml(B) > 0)
        return pii(-1, -1); // no collision
    auto gao = [&](int l, int r) {
        for (int t = l; (l + 1) % n != r; ) {
            int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
            (cml(m) == cml(t) ? l : r) = m;
        }
        return (l + !cml(r)) % n;
    };
    pii res = pii(gao(B, A), gao(A, B)); // (i, j)
    if (res.X == res.Y) // touching the corner i
        return pii(res.X, -1);
    if (!
        cml(res.X) && !cml(res.Y)) // along side i, i+1
        switch ((res.X - res.Y + n + 1) % n) {
            case 0: return pii(res.X, res.X);
            case 2: return pii(res.Y, res.Y);
        }
    /* crossing sides (i, i+1) and (j, j+1)
    crossing corner i is treated as side (i, i+1)
    returned
        in the same order as the line hits the convex */
    return res;
} // convex cut: (r, l]

```

## 8.9 minMaxEnclosingRectangle\* [180fb8]

```

const double INF = 1e18, qi = acos(-1) / 2 * 3;
pdd solve(vector<pdd> &dots) {
    #define diff(u, v) (dots[u] - dots[v])
    #define vec(v) (dots[v] - dots[i])
    hull(dots);
    double Max = 0, Min = INF, deg;
    int n = SZ(dots);
    dots.pb(dots[0]);
    for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
        pdd nw = vec(i + 1);
        while (cross(nw, vec(u + 1)) > cross(nw, vec(u)))
            u = (u + 1) % n;
        while (dot(nw, vec(r + 1)) > dot(nw, vec(r)))
            r = (r + 1) % n;
        if (!l) l = (r + 1) % n;
        while (dot(nw, vec(l + 1)) < dot(nw, vec(l)))
            l = (l + 1) % n;
        Min = min(Min, (double)(dot(nw, vec(r)) - dot
            (nw, vec(l)) * cross(nw, vec(u)) / abs2(nw)));
    }
}

```



```

deg = acos(dot(diff(r
, l), vec(u)) / abs(diff(r, l)) / abs(vec(u)));
deg = (q1 - deg) / 2;
Max = max(Max, abs(diff
(r, l)) * abs(vec(u)) * sin(deg) * sin(deg));
}
return pdd(Min, Max);
}

```

## 8.10 VectorInPoly\* [c6d0fa]

```

// ori(a
, b, c) >= 0, valid: "strict" angle from a-b to a-c
bool btwangle(pll a, pll b, pll c, pll p, int strict) {
return
ori(a, b, p) >= strict && ori(a, p, c) >= strict;
}
// whether vector
{cur, p} in counter-clockwise order prv, cur, nxt
bool inside
(pll prv, pll cur, pll nxt, pll p, int strict) {
if (ori(cur, nxt, prv) >= 0)
return btwangle(cur, nxt, prv, p, strict);
return !btwangle(cur, prv, nxt, p, !strict);
}

```

## 8.11 PolyUnion\* [434d93]

```

ld rat(pll a, pll b) {
return sgn(b.X) ? (ld)a.X / b.X : (ld)a.Y / b.Y;
} // all poly. should be ccw
ld polyUnion(vector<vector<pll>> &poly) {
ld res = 0;
for (auto &p : poly)
for (int a = 0; a < SZ(p); ++a) {
pll A = p[a], B = p[(a + 1) % SZ(p)];
vector<pair<ld, int>> segs = {{0, 0}, {1, 0}};
for (auto &q : poly) {
if (&p == &q) continue;
for (int b = 0; b < SZ(q); ++b) {
pll C = q[b], D = q[(b + 1) % SZ(q)];
int sc = ori(A, B, C), sd = ori(A, B, D);
if (sc != sd && min(sc, sd) < 0) {
ld sa = cross(D
- C, A - C), sb = cross(D - C, B - C);
segs.eb(sa / (sa - sb), sgn(sc - sd));
}
if (!sc && !sd &&
&q < &p && sgn(dot(B - A, D - C)) > 0) {
segs.eb(rat(C - A, B - A), 1);
segs.eb(rat(D - A, B - A), -1);
}
}
}
sort(all(segs));
for (auto &s : segs) s.X = clamp(s.X, 0.0, 1.0);
ld sum = 0;
int cnt = segs[0].second;
for (int j = 1; j < SZ(segs); ++j) {
if (!cnt) sum += segs[j].X - segs[j - 1].X;
cnt += segs[j].Y;
}
res += cross(A, B) * sum;
}
return res / 2;
}

```

## 8.12 Polar Angle Sort\* [b20533]

```

// -1: a // b (if same), 0/1: a < b
int cmp(pll a, pll b, bool same = true) {
#define is_neg(k) (
sign(k.Y) < 0 || (sign(k.Y) == 0 && sign(k.X) < 0))
int A = is_neg(a), B = is_neg(b);
if (A != B) return A < B;
if (sign(cross(a, b)) == 0)
return same ? abs2(a) < abs2(b) : -1;
return sign(cross(a, b)) > 0;
}

```

## 8.13 Half plane intersection\* [e13bb6]

```

pll area_pair(Line a, Line b)
{ return pll(cross(a.Y
- a.X, b.X - a.X), cross(a.Y - a.X, b.Y - a.X)); }
bool isin(Line l0, Line l1, Line l2) {
// Check inter(l1, l2) strictly in l0
auto [a02X, a02Y] = area_pair(l0, l2);

```

```

auto [a12X, a12Y] = area_pair(l1, l2);
if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
return (__int128)
a02Y * a12X - (__int128) a02X * a12Y > 0; // C^4
}
/* Having solution, check size > 2 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> arr) {
sort(all(arr), [&](Line a, Line b) -> int {
if (cmp(a.Y - a.X, b.Y - b.X, 0) != -1)
return cmp(a.Y - a.X, b.Y - b.X, 0);
return ori(a.X, a.Y, b.Y) < 0;
});
deque<Line> dq(1, arr[0]);
for (auto p : arr) {
if (cmp(
dq.back().Y - dq.back().X, p.Y - p.X, 0) == -1)
continue;
while (SZ(dq)
>= 2 && !isin(p, dq[SZ(dq) - 2], dq.back()))
dq.pop_back();
while (SZ(dq) >= 2 && !isin(p, dq[0], dq[1]))
dq.pop_front();
dq.pb(p);
}
while (SZ(dq)
>= 3 && !isin(dq[0], dq[SZ(dq) - 2], dq.back()))
dq.pop_back();
while (SZ(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
dq.pop_front();
return vector<Line>(all(dq));
}

```

## 8.14 RotatingSweepLine [37eaad]

```

struct Event {
pll d; int u, v;
bool operator<(const Event &b) const {
int ret = cmp(d, b.d, false);
return ret == -1 ? false : ret; } // no tie-break
};
void rotatingSweepLine(const vector<pll> &p) {
const int n = SZ(p);
vector<Event> e; e.reserve(n * (n - 1));
for (int i = 0; i < n; i++)
for (int j = 0; j < n
; j++) // pos[i] < pos[j] when the event occurs
if (i != j) e.pb(p[j] - p[i], i, j);
sort(all(e));
vector<int> ord(n), pos(n);
iota(all(ord), 0);
sort(all(ord), [&](int i, int j) { // initial order
return p[i].Y != p[
j].Y ? p[i].Y < p[j].Y : p[i].X < p[j].X; });
for (int i = 0; i < n; i++) pos[ord[i]] = i;
// initialize
for (int i = 0, j = 0; i < SZ(e); i = j) {
// do something
vector<pii> tmp;
for (; j < SZ(e) && !(e[i] < e[j]); j++)
tmp.pb(pii(e[j].u, e[j].v));
sort(all(tmp), [&](pii x, pii y){
return pii(pos[x.fi],
pos[x.se]) < pii(pos[y.fi], pos[y.se]); });
for (auto [x, y] : tmp) // pos[x] + 1 == pos[y]
tie(ord[pos[x]], ord[pos[y]], pos[x], pos[y]) =
make_tuple
(ord[pos[y]], ord[pos[x]], pos[y], pos[x]);
}
}

```

## 8.15 Minimum Enclosing Circle\* [2c7ccd]

```

using ld = long double;
mt19937 rng(950223); // O(n) expected time
pair<pdd, ld> minimumEnclosingCircle(vector<pdd> &pts){
shuffle(all(pts), rng);
pdd c = pts[0];
ld r = 0;
for(int i = 1; i < SZ(pts); i++){
if(abs(pts[i] - c) <= r) continue;
c = pts[i]; r = 0;
for(int j = 0; j < i; j++){
if(abs(pts[j] - c) <= r) continue;
c = (pts[i] + pts[j]) / 2;
r = abs(pts[i] - c);
for(int k = 0; k < j; k++){

```

```

        if(abs(pts[k] - c) > r)
            auto [c, r] = circenter(pts[i], pts[j], pts[k]);
    }
}
return {c, r};
}

```

## 8.16 Intersection of two circles\* [f7a2fe]

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.O, o2 = b.O;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2), d = sqrt(d2);
    if(d < max(r1, r2) - min(r1, r2) || d > r1 + r2) return 0;
    pdd u = (o1 + o2) * 0.5 + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
    double A = sqrt((r1 + r2 + d) * (r1 - r2 + d) * (r1 + r2 - d) * (-r1 + r2 + d));
    pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2);
    p1 = u + v, p2 = u - v;
    return 1;
}

```

## 8.17 Intersection of polygon and circle\* [d4d295]

```

// Divides into multiple triangle, and sum up
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb), b=abs(pa), c=abs(pb-pa);
    double cosB = dot(pb, pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa, pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}
double area_poly_circle(const vector<pdd> poly, const pdd &O, const double r){
    double S=0;
    for(int i=0; i<SZ(poly); ++i)
        S += _area(poly[i]-O, poly[(i+1)%SZ(poly)]-O, r)*ori(O, poly[i], poly[(i+1)%SZ(poly)]);
    return fabs(S);
}

```

## 8.18 Intersection of line and circle\* [76e533]

```

vector<pdd> circleLine(pdd c, double r, pdd a, pdd b) {
    pdd p = a + (b - a) * dot(c - a, b - a) / abs2(b - a);
    double s = cross(b - a, c - a), h2 = r * r - s * s / abs2(b - a);
    if (sgn(h2) < 0) return {};
    if (sgn(h2) == 0) return {p};
    pdd h = (b - a) / abs(b - a) * sqrt(h2);
    return {p - h, p + h};
}

```

## 8.19 Tangent line of two circles [5ac5a5]

```

vector<Line>
> CCtan(const Cir& c1, const Cir& c2, int sign1){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = abs2(c1.O - c2.O);
    if (sgn(d_sq) == 0) return ret;
    double d = sqrt(d_sq);
    pdd v = (c2.O - c1.O) / d;
    double c = (c1.R - sign1 * c2.R) / d;
    if (c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        pdd n = pdd(v.X * c - sign2 * h * v.Y,

```

```

        v.Y * c + sign2 * h * v.X);
        pdd p1 = c1.O + n * c1.R;
        pdd p2 = c2.O + n * (c2.R * sign1);
        if (!sgn(p1.X - p2.X) && !sgn(p1.Y - p2.Y))
            p2 = p1 + perp(c2.O - c1.O);
        ret.pb(Line(p1, p2));
    }
    return ret;
}

```

## 8.20 Minkowski Sum\* [399d43]

```

void reorder_poly(vector<pdd>& pnts){
    int mn = 0;
    for(int i = 1; i < SZ(pnts); i++){
        if(pnts[i].Y < pnts[mn].Y || (pnts[i].Y == pnts[mn].Y && pnts[i].X < pnts[mn].X)) mn = i;
        rotate(pnts.begin(), pnts.begin() + mn, pnts.end());
    }
    vector<pdd> minkowski(vector<pdd> P, vector<pdd> Q){
        reorder_poly(P); reorder_poly(Q);
        int psz = SZ(P), qsz = SZ(Q);
        P.pb(P[0]); P.pb(P[1]); Q.pb(Q[0]); Q.pb(Q[1]);
        vector<pdd> ans; int i = 0, j = 0;
        while(i < psz || j < qsz){
            ans.pb(P[i] + Q[j]);
            int t = sgn(cross(P[i+1]-P[i], Q[j+1]-Q[j]));
            if(t >= 0) i++; if(t <= 0) j++;
        }
        return ans;
    }
}

```

## 9 Else

### 9.1 Cyclic Ternary Search\* [9017cc]

```

/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);
    while (r - l > 1) {
        int m = (l + r) / 2;
        if (pred(0, m) ? rv : pred(m, (m + 1) % n)) r = m;
        else l = m;
    }
    return pred(l, r % n) ? l : r % n;
}

```

### 9.2 Tree Hash\* [34aae5]

```

ull seed;
ull shift(ull x) {
    x ^= x << 13;
    x ^= x >> 7;
    x ^= x << 17;
    return x;
}
ull dfs(int u, int f) {
    ull sum = seed;
    for (int i : G[u])
        if (i != f)
            sum += shift(dfs(i, u));
    return sum;
}

```

## 10 Python

### 10.1 Misc

```

from decimal import *
setcontext(Context(prec=MAX_PREC, Emax=MAX_EMAX, rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction('3.14159').limit_denominator(10).numerator # 22
sys.set_int_max_str_digits(int(1e6))
a, b = map(int, input().split())
a = map(int, input().split()) # array

```