LAB 2: Finding angle between two polar curves, curvature and radius of curvature.

2.1 Objectives:

Use python

1. To find angle between two polar curves.
2. To find radius of curvature.

Syntax for the commands used:

1. `diff()`

```
diff(function,variable)
```

2. `Derivative()`

```
Derivative(expression, reference variable)
```

- `expression` – A SymPy expression whose unevaluated derivative is found.
- `reference variable` – Variable with respect to which derivative is found.
- **Returns:** Returns an unevaluated derivative of the given expression.

3. `doit()`

```
doit(x)
```

4. **Return** : evaluated object

5. `simplify()`

```
simplify(expression)
```

6. `expression` – It is the mathematical expression which needs to be simplified.

7. **Returns:** Returns a simplified mathematical expression corresponding to the input expression.

8. `display()`

```
display(expression)
```

9. `expression` – It is the mathematical expression which needs to be simplified.

10. **Returns:** Displays the expression.

11. syntax of Substitute : `subs()`

$r = 2a\cos\theta$ [illegible]

```
math_expression.subs(variable, substitute)
```

12. `variable` – It is the variable or expression which will be substituted.
13. `substitute` – It is the variable or expression or value which comes as substitute.
14. **Returns:** Returns the expression after the substitution.

2.2 1. Angle between two polar curves

Angle between radius vector and tangent is given by $\tan\phi = r\frac{d\theta}{dr}$.

If $\tan\phi_1$ and $\tan\phi_2$ are angle between radius vector and tangent of two curves then $|\phi_1 - \phi_2|$ is the angle between two curves at the point of intersection.

1. Find the angle between the curves $r = 4(1 + \cos t)$ and $r = 5(1 - \cos t)$. 1.57

```
from sympy import *

r,t =symbols('r,t') # Define the variables required as symbols

r1=4*(1+cos(t)); #Input first polar curve
r2=5*(1-cos(t)); #Input first polar curve
dr1=diff(r1,t)   # find the derivative of first function
dr2=diff(r2,t)   # find the derivative of secodn function
t1=r1/dr1
t2=r2/dr2
q=solve(r1-r2,t)   # solve r1==r2, to find the point of intersection
                              between curves
w1=t1.subs({t:float(q[1])}) # substitute the value of "t" in t1
w2=t2.subs({t:float(q[1])})  # substitute the value of "t" in t2
y1=atan(w1)    # to find the inverse tan of w1
y2=atan(w2)   # to find the inverse tan of w2
w=abs(y1-y2)   # angle between two curves is abs(w1-w2)
print('Angle between curves in radians is %0.3f'%(w))
```

2. Find the angle between the curves $r = 4\cos t$ and $r = 5\sin t$. 1.5708

```
from sympy import *
r,t =symbols('r,t')

r1=4*(cos(t));
r2=5*(sin(t));


dr1=diff(r1,t)
dr2=diff(r2,t)
t1=r1/dr1
t2=r2/dr2

q=solve(r1-r2,t)
```

```
w1=t1.subs({t:float(q[0])})
w2=t2.subs({t:float(q[0])})

y1=atan(w1)
y2=atan(w2)
w=abs(y1-y2)
print('Angle between curves in radians is %0.4f'%float(w))
```

2.3 2. Radius of curvature

Formula to calculate Radius of curvature in polar form is $\rho = \dfrac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$

1. Find the radius of curvature, $r = 4(1 + \cos t)$ at t=$\pi/2$. 2.8284

```
from sympy import *
t=Symbol('t')   # define t as symbol
r=Symbol('r')
r=4*(1+cos(t))
r1=Derivative(r,t).doit() #find the first derivative of r w.r.t "t"
r2=Derivative(r1,t).doit() #find the second derivative of r w.r.t "t"
rho=(r**2+r1**2)**(1.5)/(r**2+2*r1**2-r*r2);   # Substitute r1 and r2 in
                                                  formula
rho1=rho.subs(t,pi/2) # substitute t in  rho
print('The radius of curvature is %3.4f units'%rho1)
```

2. Find the radius of curvature for $r = asin(nt)$ at $t = pi/2$ and $n = 1$. $\dfrac{(a^2)^{1.5}}{2a^2}$

```
from sympy import *
t,r,a,n=symbols('t r a n')
r=a*sin(n*t)
r1=Derivative(r,t).doit()
r2=Derivative(r1,t).doit()
rho=(r**2+r1**2)**1.5/(r**2+2*r1**2-r*r2);
rho1=rho.subs(t,pi/2)
rho1=rho1.subs(n,1)
print("The radius of curvature is")
display(simplify(rho1))
```

2.4 Parametric curves

The formula to calculate Radius of curvature is $\rho = \frac{(x'^2+y'^2)^{\frac{3}{2}}}{y''x'-x''y'}$.

$x' = \frac{dx}{dt}, x'' = \frac{d^2x}{dt^2}, y' = \frac{dy}{dt}, y'' = \frac{d^2y}{dt^2}$

1. Find radius of curvature of $x = acos(t)$, $y = asin(t)$.

```
from sympy import *
from sympy.abc import rho, x,y,r,K,t,a,b,c,alpha    # define all symbols
                                                 required
y=(sqrt(x)-4)**2
y=a*sin(t) #input the parametric equation
x=a*cos(t)
dydx=simplify(Derivative(y,t).doit())/simplify(Derivative(x,t).doit())
                                                 # find the derivative of parametric
                                                  equation
rho=simplify((1+dydx**2)**1.5/(Derivative(dydx,t).doit()/(Derivative(x,
                                                 t).doit()))) #substitute the
                                                 derivative in radius of curvature
                                                 formula
print('Radius of curvature is')
display(ratsimp(rho))
t1=pi/2
r1=5
rho1=rho.subs(t,t1);
rho2=rho1.subs(a,r1);
print('\n\nRadius of curvature at r=5 and t= pi/2 is', simplify(rho2));
curvature=1/rho2;
print('\n\n Curvature at (5,pi/2) is',float(curvature))
```

2. Find the radius of curvature of $y = (asin(t))^{3/2}$; $x = (acos(t))^{3/2}$.

```
from sympy import *
from sympy.abc import rho, x,y,r,K,t,a,b,c,alpha
y=(a*sin(t))**(3/2)
x=(a*cos(t))**(3/2)
dydx=simplify(Derivative(y,t).doit())/simplify(Derivative(x,t).doit())
rho=simplify((1+dydx**2)**1.5/(Derivative(dydx,t).doit()/(Derivative(x,
                                                 t).doit())))
print('Radius of curvature is')
display(ratsimp(rho))
t1=pi/4
r1=1;
rho1=rho.subs(t,t1);
rho2=rho1.subs(a,r1);
display('Radius of curvature at r=1 and t=pi/4 is',simplify(rho2));
curvature=1/rho2;
print('\n\n Curvature at (1,pi/4) is',float(curvature))
```

2.5 Exercise:

Plot the following:

1. Find the angle between radius vector and tangent to the folloing polar curves
   a) $r = a\theta$ and $r = \frac{a}{\theta}$
   Ans: Angle between curves in radians is 90.000
   b) $r = 2sin(\theta)$ and $r = 2cos(\theta)$
   Ans: Angle between curves in radians is 90.000

2. Find the radius of curvature of $r = a(1 - cos(t))$ at $t = \frac{\pi}{2}$.
Ans: $\frac{0.942809041582063(a^2)^{1.5}}{a^2}$

3. Find radius of curvature of $x = acos^3(t)$, $y = asin^3(t)$ at $t = 0$.
Ans: $\rho = 0.75\sqrt{3}$ and $\kappa = 0.769800$

4. Find the radius of curvature of $r = acos(t)$ at $t = \frac{\pi}{4}$.
Ans: $\frac{(a^2)^{1.5}}{2a^2}$

5. Find the radius of curvature of $x = a(t - sin(t))$ and$y = a(1 - cos(t))$ at $t = \pi/2$.
Ans: $\rho = 2.82842712$ and $\kappa = 0.353553$