
LAB 1, CSE/Mech/Civil:

Programme to compute area, volume and center of gravity.

Objectives:

Use python

1. to evaluate double integration.
 2. to compute area and volume.
 3. to calculate center of gravity of 2D object.
-

Syntax for the commands used

1. `pprint()` ### "pretty-print" the command prints in a well-formatted and more readable way!
2. `integrate(function,(variable, min_limit, max_limit))` ### Returns integration of a mathematical expression w.r.t a variable between the given limits.

I Double and triple integration

1. Evaluate the integral $\int_0^1 \int_0^x (x^2 + y^2) dy dx$

```
In [ ]: from sympy import *
x,y,z=symbols('x y z')
w1=integrate((x**2+y**2),(y,0,x),(x,0,1))
print(w1)
```

1/3

1. Evaluate the integral $\int_0^3 \int_0^{3-x} \int_0^{3-x-y} (xyz) dz dy dx$

```
In [ ]: from sympy import *
x,y,z=symbols('x y z')
w2=integrate((x*y*z),(z,0,3-x-y),(y,0,3-x),(x,0,3))
print(w2)
```

1. Prove that $\int \int (x^2 + y^2) dy dx = \int \int (x^2 + y^2) dx dy$

```
In [ ]: from sympy import *
x,y,z=symbols('x y z')
w3=integrate(x**2+y**2,y,x)
pprint(w3)
w4=integrate(x**2+y**2,x,y)
pprint(w4)
#display(w4)
```

$$\frac{x^3 \cdot y}{3} + \frac{x \cdot y^3}{3}$$

$$\frac{x^3 \cdot y}{3} + \frac{x \cdot y^3}{3}$$

II Area and Volume

Area of the region R in the cartesian form is $\int_R \int dx dy$.

1. Find the area of an ellipse by double integration. $A=4 \int_0^a \int_0^{(b/a)\sqrt{a^2-x^2}} dy dx$.

```
In [ ]: from sympy import *
x=Symbol('x')
y=Symbol('y')
#a=Symbol('a')
#b=Symbol('b')
a=4
b=6
w3=4*integrate(1,(y,0,(b/a)*sqrt(a**2-x**2)),(x,0,a))
print(w3)
```

24.0*pi

Area of the region R in the polar form is $\int_R \int r dr d\theta$.

1. Find the area of the cardioid $r = a(1 + \cos\theta)$ by double intgration.

```
In [ ]: from sympy import *
r=Symbol('r')
t=Symbol('t')
a=Symbol('a')
#a=4

w3=2*integrate(r,(r,0,a*(1+cos(t))),(t,0,pi))
pprint(w3)
```

$$\frac{3 \cdot \pi \cdot a^2}{2}$$

Volume of a solid is given by $\int_V \int \int dx dy dz$.

1. Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0$ and $z = 0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

```
In [ ]: from sympy import *
x,y,z,a,b,c=symbols('x y z a b c')
w2=integrate(1,(z,0,c*(1-x/a-y/b)),(y,0,b*(1-x/a)),(x,0,a))
print(w2)

a*b*c/6
```

Center of Gravity

1. Find the center of gravity of cardioid . Plot the graph of cardioid and mark the center of gravity.

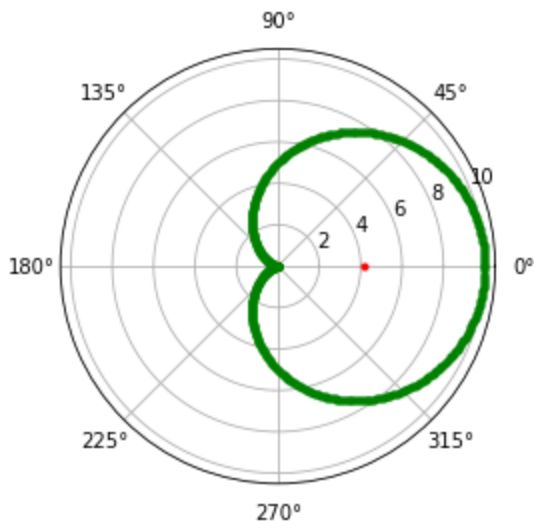
```
In [ ]: import numpy as np
import matplotlib.pyplot as plt
import math
from sympy import *
r=Symbol('r')
t=Symbol('t')
a=Symbol('a')
I1=integrate(cos(t)*r**2,(r,0,a*(1+cos(t))),(t,-pi,pi))
I2=integrate(r,(r,0,a*(1+cos(t))),(t,-pi,pi))
I=I1/I2
print(I)
I=I.subs(a,5)
plt.axes(projection = 'polar')
a=5

rad = np.arange(0, (2 * np.pi), 0.01)

# plotting the cardioid
for i in rad:
    r = a + (a*np.cos(i))
    plt.polar(i,r,'g.')

plt.polar(0,I,'r.')
plt.show()

5*a/6
```



Exercise:

1. Evaluate $\int_0^1 \int_0^x (x + y) dy dx$.

Ans: 0.5

1. Find the $\int_0^{\log(2)} \int_0^x \int_0^{x+\log(y)} (e^{x+y+z}) dz dy dx$.

Ans:-0.2627

1. Find the area of positive quadrant of the circle $x^2 + y^2 = 16$.

Ans: 4π

1. Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0$ and $z = 0, \frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$.

Ans: 4
