LAB 2: Evaluation of improper integrals, Beta and Gamma functions

2.1 Objectives:

Use python

1. to evaluate improper integrals using Beta function.

2. to evaluate improper integrals using Gamma function.

Syntax for the commands used:

1. gamma

```
math.gamma(x)
```

Parameters :

x : The number whose gamma value needs to be computed.

2. beta

```
math.beta(x,y)
```

Parameters :

x ,y: The numbers whose beta value needs to be computed.

3. **Note:** We can evaluate improper integral involving infinity by using inf.

Example 1:

Evaluate $\int\limits_0^\infty e^{-x}\,dx$.

```
from sympy import *
x=symbols('x')
w1=integrate(exp(-x),(x,0,float('inf')))
print(simplify(w1))
```

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Gamma function is $x(n) = \int_0^\infty e^{-x} x^{n-1}\,dx$

Example 2:

Evaluate $\Gamma(5)$ by using definition

```
from sympy import *
x=symbols('x')
w1=integrate(exp(-x)*x**4,(x,0,float('inf')))
print(simplify(w1))
```

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Example 3:

Evaluate $\int_0^\infty e^{-st} \cos(4t)dt$. That is Laplace transform of $\cos(4t)$

```
from sympy import *
t,s=symbols('t,s')
# for infinity in sympy we use oo
w1=integrate(exp(-s*t)*cos(4*t),(t,0,oo))
display(simplify(w1))
```

$$\begin{cases} \frac{s}{s^2+16} & \text{for } 2\,|arg\,(s)| < \pi \\ \int_0^\infty e^{-st} \cos(4t)\, dt & \text{otherwise} \end{cases}$$

Example 4:

Find Beta(3,5), Gamma(5)

```
#beta and gamma functions
from sympy import beta, gamma
m=input('m : ');
n=input('n : ');
m=float(m);
n=float(n);
s=beta(m,n);
t=gamma(n)
print('gamma (',n,') is %3.3f '%t)
print('Beta (',m,n,') is  %3.3f '%s)
```

m :3

n :5

gamma (5.0) is 24.000
Beta (3.0 5.0) is 0.010

Example 5:

Calculate Beta(5/2,7/2) and Gamma(5/2).

```
#beta and gamma functions
# If the number is a fraction give it in decimals. Eg 5/2=2.5
from sympy import beta, gamma
m=float(input('m : '));
n=float(input('n :'));

s=beta(m,n);
t=gamma(n)
print('gamma (',n,') is %3.3f '%t)
print('Beta (',m,n,') is %3.3f '%s)
```

```
m : 2.5
n :3.5
gamma ( 3.5 ) is 3.323
Beta ( 2.5 3.5 ) is 0.037
```

Example 6:

Verify that $Beta(m, n) = Gamma(m)Gamma(n)/Gamma(m + n)$ for m=5 and n=7

```
from sympy import beta, gamma
m=5;
n=7;
m=float(m);
n=float(n);
s=beta(m,n);
t=(gamma(m)*gamma(n))/gamma(m+n);
print(s,t)
if (abs(s-t)<=0.00001):
    print('beta and gamma are related')
else:
    print('given values are wrong')
```

```
0.000432900432900433 0.000432900432900433
beta and gamma are related
```

2.2   Exercise:

1. Evaluate $\int_0^\infty e^{-t}\cos(2t)dt$

   Ans: 1/5

2. Find the value of Beta(5/2,9/2)

   Ans: 0.0214

3. Find the value of Gamma(13)

   Ans: 479001600

4. Verify that $Beta(m,n) = Gamma(m)Gamma(n)/Gamma(m+n)$ for m=7/2 and n=11/2

   Ans: True