Proof technique - 2

Wednesday, September 9, 2020 10:26 AM

Proof-6

Let us assume that proposition p is "$\sqrt{2}$ is irrational." To start a proof by contradiction, we have to consider $\neg p$ is true where $\neg p$ is the statement "$\sqrt{2}$ is rational".

$\neg p \to$

$\neg p \to$

By the definition of rational numbers, there ~~there~~ exists two integers a and b with $\sqrt{2} = a/b$, where $b \neq 0$ and a & b do not have a common factor. (So the fraction a/b is in lowest term).

Because $\sqrt{2} = \frac{a}{b}$, squaring both sides,

$$2 = a^2/b^2 \Rightarrow 2b^2 = a^2$$

By the defn. of even integers, it follows that a^2 is even. There exists an integer c such that $a = 2c$. Thus, $2b^2 = 4c^2 \Rightarrow b^2 = 2c^2 \Rightarrow b^2$ is also even by defn. of even numbers.

We have now shown that $\neg p$ leads to the equation $\sqrt{2} = a/b$, where a and b are even and 2 divides both a & b.

$\therefore$ P $\neg p$ leads to

are even and 2 divides both ...

Because our assumption of $\neg p$ leads to a contradiction, it follows that $\neg p$ must be false. That is the statement p, "$\sqrt{2}$ is irrational" is true.

Proof-7 "If $3n+2$ is odd, then n is odd":

To construct proof by contradiction, we assume both p & $\neg q$ are true. That is, $3n+2$ is odd and n is not odd i.e. n is even. If n is even, there exists an integer k such that $n = 2k$. This implies, $3n+2 = 3.(2k)+2 = 6k+2 = 2.(3k+1)$. is also even. Thus we have both p and $\neg p$ to be true, which is a contradiction. This implies that our initial assumption of $\neg q$ is false. i.e. q is true. Henceforth, if $3n+2$ is odd, then n is also odd.