Indian Institute of Engineering Science and Technology, Shibpur

B.Tech. 3rd Semester Mid Semester Examination, December 2020

Subject: Mathematics-III (MA-2101)

Full Marks : 30 **Time : 45 minutes**

Answer any SIX questions. Only the first six questions answered by the candidate will be evaluated.

1. State and prove Bayes' theorem.

[2+3=5]

2. The chance that a doctor will diagnose a certain disease correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of the doctor, who had the disease, dies. What is the probability that the disease was diagnosed correctly?

[5]

3. Find the Laplace transforms of

$$\text{(i)}\frac{1}{t}(1-e^t) \quad \text{(ii)}\frac{1}{t}(\cos at - \cos bt).$$

[2+3=5]

4. For a periodic function $f(t)$ with period $T(>0)$, show that

$$L\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt.$$

[5]

5. Define a convex set in R^n. Show analytically that the point set:

$$X = \{(x_1, x_2) : 4x_1^2 + 9x_2^2 \leq 36\}$$

is a convex set in R^2. State its extreme points.

[1+3+1=5]

6. A fair coin is tossed n times and the tosses are assumed to be independent. Let T_n be the number of times a success (Head appears) is followed by a failure (Tail appears) in the first n trials. Calculate $E(T_n)$ and $Var(T_n)$.

[5]

7. Suppose U_1 and U_2 are independent random variables with common density $f(u) = 1$, $u \in (0, 1)$ and 0 otherwise. Calculate $P(|U_1 - U_2| > 1/6)$.

[5]

8. In a poll, n students are randomly selected and asked whether they are in favour of written examination or not in the pandemic situation caused by COVID-19. Let S denote the proportion of students in favour of written examination. Suppose $X_i = 1$ when the i-th student agrees to give written examination and $X_i = 0$ otherwise, and assume that X_i's are Bernoulli random variables with parameter p. Using Chebyshev's inequality, find the minimum value of n to ensure that the probability that the proportion of students in favour of written examination lies between $p - 0.05$ and $p + 0.05$.

[5]