Assignment 1

1) a) Simplify the boolean function to minimise the number of literals using the rules of linear algebra

$$y(w\bar{z} + wz) + xy$$

LHS $= wy(\bar{z}+z) + xy$

$= wy + xy$

$= y(x+w)$

b) Express the boolean function $F = xy + \bar{x}z$ as a product of maxterms.

$F = xy + \bar{x}z$

$= (xy+\bar{x})(xy+z)$

$= (x+\bar{x})(y+\bar{x})(x+z)(y+z)$

$= (\bar{x}+y)(x+z)(y+z)$

$= (\bar{x}+y+z\bar{z})(x+y\bar{y}+z)(x\bar{x}+y+z)$

$= (\bar{x}+y+z)(\bar{x}+y+\bar{z})(x+y+z)(x+\bar{y}+z)$
$(x+y+z)(\bar{x}+y+z)$

$= (\bar{x}+y+z)(\bar{x}+y+\bar{z})(x+y+z)(x+\bar{y}+z)$

c) Simplify (using Map Method) the boolean function F together with don't care condition d in (i) sum of products and ii) product of sums

$$F(A,B,C,D) = \Sigma(0,1,2,3,7,8,10) + \Sigma d(5,6,11,15)$$

i)

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	1 (1)	1 (3)	1 (2)
$\bar{A}B$	(4)	X (5)	1 (7)	X (6)
AB	(12)	(13)	X (15)	(14)
$A\bar{B}$	1 (8)	(9)	X (11)	1 (10)

→ combining the squares with 1's gives the simplified function in Sum of Products

$$F(A,B,C,D) = \bar{B}\bar{D} + \bar{A}D$$

ii)

	$C+D$	$C+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
$A+B$	1 (0)	1 (1)	1 (3)	1 (2)
$A+\bar{B}$	0 (4)	X (5)	1 (7)	X (6)
$\bar{A}+\bar{B}$	0 (12)	0 (13)	X (15)	0 (14)
$\bar{A}+B$	1 (8)	0 (9)	X (11)	1 (10)

→ if squares marked 0's are combined, we get the function in the product of Sum form.

$$\therefore F = (\bar{B}+D)(\bar{A}+\bar{D})$$

or $F(A,B,C,D) = (D+\bar{B})(\bar{A}+\bar{D})$

d) Simplify the following Boolean Function using Tabulation method.

$$F(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

I) writing every number in their binary format and sorting them in the basis of number of 1's.

number	w	x	y	z	
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
5	0	1	0	1	✓
9	1	0	0	1	✓
10	1	0	1	0	✓
7	0	1	1	1	✓
13	1	1	0	1	✓
15	1	1	1	1	

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II) grouping two number of different group having only 1 bit different

number	w	x	y	z
(0,1)	0	0	0	-
(0,2)	0	0	-	0
(0,8)	-	0	0	0
(1,5)	0	-	0	1
(1,9)	-	0	0	1
(2,10)	-	0	1	0
(8,9)	1	0	0	-
(8,10)	1	0	-	0
(5,7)	0	1	-	1
(5,13)	-	1	0	1
(9,13)	1	-	0	1
(7,15)	-	1	1	1
(13,15)	1	1	-	1

III) grouping again.

numbers	w	x	y	z
(0,1,8,9)	-	0	0	-
(0,2,8,10)	-	0	-	0
(0,8,1,9)	-	0	0	-
(0,8,2,10)	-	0	-	0
(1,5,9,13)	-	-	0	1
(1,9,5,13)	-	-	0	1
(5,7,13,15)	-	1	-	1
(5,13,7,15)	-	1	-	1

→ from this group we see that no two group have only 1 bit difference, hence we have found the prime implicants.

∴ Prime implicants = $\bar{x}\bar{y}$ (0,1,8,9), $\bar{x}\bar{z}$ (0,2,8,10), $\bar{y}z$ (1,5,9,13), xz (5,7,13,15)

IV) → Now finding essential prime implicants from these Prime implicants:

		0	1	2	5	7	8	9	10	13	15
$\bar{x}\bar{y}$	0, 1, 8, 9	×	×				×	×			
✓ $\bar{x}z$	0, 2, 8, 10	×		×			×		×		
$\bar{y}z$	1, 5, 9, 13		×		×			×		×	
✓ xz	5, 7, 13, 15				×	×				×	×
		✓		✓	✓	✓	✓		✓	✓	✓

→ from the table, we can see that $\bar{x}z$ and xz are essential prime implicant, but we need another prime implicant to cover 1 and 9, hence we can pick either of $\bar{x}\bar{y}$ or $\bar{y}z$

$$\therefore F(w,x,y,z) = \bar{x}z + xz + \bar{x}\bar{y}$$

or

$$F(w,x,y,z) = \bar{x}z + xz + \bar{y}z$$

e) Determine the value of base x if

$$(211)_x = (152)_8$$

→ The above statement implies that

$$1\times x^0 + 1\times x + 2\times x^2 = 2\times 8^0 + 5\times 8^1 + 1\times 8^2$$

or $2x^2 + x + 1 = 106$

or $2x^2 + x - 105 = 0$

using quadratic formula : $x = \dfrac{-1 \pm \sqrt{1^2 - 4\times 2\times(-105)}}{2\times 2}$

or $x = \dfrac{-1 \pm \sqrt{841}}{4}$

or $x = \dfrac{-1 \pm 29}{4}$

or $x = 7$ or -7.5

→ base can not be negative

$\therefore x = 7$

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f) Find 10's complement of 132900

10's complement = 9's complement + 1

9's complement of 132900

→

$$\begin{array}{r} 999999 \\ -132900 \\ \hline 867099 \end{array}$$

10's complement of 132900

→

$$\begin{array}{r} 867099 \\ +000001 \\ \hline 867100 \end{array}$$

∴ 10's complement of 132900 = 867100

g) Perform the arithematic operations (+42) + (−13) and (−42) − (−13) in binary using signed 2's complement representation for negative numbers.

→ Before doing any arithmetic operation, let's find out binary representation of 42, −42, 13, −13

a) $(42)_{10} = (101010)_2$

$$\begin{array}{r|l l} 2 & 42 & 0 \\ 2 & 21 & 1 \\ 2 & 10 & 0 \\ 2 & 5 & 1 \\ 2 & 2 & 0 \\ 2 & 1 & 1 \\ & 0 & \end{array}$$

b) $(-42)_{10}$ = 010101 + 000001
= $(010110)_2$, 2's complement

c) $(13)_{10} = (1101)_2 = (001101)_2$

$$\begin{array}{r|l l} 2 & 13 & 1 \\ 2 & 6 & 0 \\ 2 & 3 & 1 \\ 2 & 1 & 1 \\ & 0 & \end{array}$$

d) $(-13)_{10}$ = 110010 + 000001
= $(110011)_2$, 2's complement

II)

I) finding +42 + (-13)

~~42~~ →

$$\begin{array}{r} 10\ 10\ 10 \\ +\ 11\ 00\ 11 \\ \hline 1\quad 01\ 11\ 01 \end{array}$$

→ carry is present → discard it
→ number is positive

$\therefore\ 42 + (-13) = (011101)_2 = 1\times2^0 + 1\times2^2 + 1\times2^3 + 1\times2^4$
$= 1 + 4 + 8 + 16$
$= 29$

II) finding (-42) - (-13)

→ $(-42) - (-13) = (-42) + 13$

→

$$\begin{array}{r} 010\ 110 \\ +\ 001\ 101 \\ \hline 100\ 011 \end{array}$$

→ no carry occured → number is -ve and is in it's 2's complement form

$\therefore$ 1's complement $= 100011 - 000001$
$= 100010$

number's magnitude $= (011101)_2 = 1+4+8+16$
$= 29$

$\therefore\ (-42) - (-13) = -29$