(2) 4/12/17

INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR

B.Tech. 1st Semester Examination, 2017

Subject: Mathematics-I (MA-101)

Full Marks: 70 Time :3 hrs.

Use separate answer scripts for each half

Answer **SIX** questions, taking **THREE** from each half

Two marks are reserved for general proficiency in each half.

Symbols have their usual meanings.

First Half

1. a) Find the value of y_n for $x = 0$ where $y = e^{a\sin^{-1}x}$.

b) Using Lagrange's Mean Value theorem prove that

$$\frac{x}{1+x^2} < \tan^{-1}x < x \ , \ 0 < x < \frac{\pi}{2} .$$

c) Verify Cauchy's Mean Value theorem for

$f(x) = \sqrt{x}$, $g(x) = \frac{1}{\sqrt{x}}$ in (a,b) where $a > 0$.

5+3+3

2. a) Prove that the asymptotes of the cubic equation

$(x^2 - y^2)y - 2ay^2 + 5x - 7 = 0$ form a triangle of area a^2.

b) If $f(x,y) = \frac{xy}{x^2+y^2}$ when $(x,y) \neq (0,0)$,

$= 0$ when $(x,y) = (0,0)$,

show that the partial derivatives exist at (0,0) but f(x,y) is not continuous thereat.

c) Find the Taylor series expansion of the function $f(x,y) = e^x \cos y$ about the origin up to second degree term.

3+5+3

3. a) State Euler's theorem for homogeneous functions. If $u = tan^{-1}\frac{x^3+y^3}{x-y}$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$.

b) Find the radius of curvature at the point (r, θ) on the curve

$r = a(1 - \cos\theta)$ and show that it varies as $\sqrt{r}$.

(1+6)+4

4. a) Find the maxima of the following function given by
$f(x,y) = x^2 + y^2 + (x+y+1)^2$.

b) Show that the series $\sum \frac{1}{n}$ does not converge.

c) Test for convergence:

i) $\sum \frac{n^2-1}{n^2+1} x^n$, where $x > 0$

ii) $\frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \cdots\cdots$

4+1+(3+3)

5. a) Using the technique of Lagrange's multipliers, find the minimum value of

$x^2 + y^2 + z^2$ subject to the condition $2x + 3y + 5z = 30$.

b) Show that the functions $u(x, y, z)$, $v(x, y, z)$, $w(x, y, z)$ given by

$$u = x + y - z$$
$$v = x - y + z$$
$$w = x^2 + y^2 + z^2 - 2yz$$

are not independent and find the relation among them . 6+5

Second Half

6. a) Evaluate $\int_{0}^{\pi/2}\int_{\pi/2}^{\pi}\cos(x+y)dxdy$.

b) Change the order of integration and hence evaluate the integral

$$\int_{0}^{a}\int_{y}^{a}\frac{xdxdy}{x^2+y^2}.$$

c) Compute the value of $\iint_R ydxdy$, where R is the region in the first quadrant

bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 3+4+4

7. a) Examine the convergence of the improper integral $\int_{\infty}^{-\infty}\frac{dx}{1+x^2}$; if possible,

evaluate the integral.

b) Find the value of $\int_{0}^{\infty}e^{-x^2}dx$.

c) Evaluate $\int_{0}^{1}x^3(1-x^2)^{\frac{5}{2}}dx$. 4+3+4

8. a) Show that $\int_{-1}^{1}\frac{1}{x^3}dx$ does not exist but it exists in Cauchy's principal value sense.

b) Solve the following differential equations:

(i) $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{3x}$ (ii) $(D^2+1)y=\cos x$, $D\equiv\frac{d}{dx}$ 5+3+3

9. a) Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10(x+\frac{1}{x})$.

b) Using the method of variation of parameters, solve

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x\log x.$$ 6+5

10. a) Find the series solution of the differential equation

$\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+3y=0$ near the ordinary point $x=0$.

b) Write down the standard form of

i) Legendre's differential equation

ii) Bessel's differential equation

iii) Legendre's polynomial of degree n

iv) Bessel's function of first kind of order n. 7+4