Proof Techniques - 1

Wednesday, September 9, 2020 8:40 AM

Proof-1: We assume that the hypothesis of the given conditional statement is true i.e. n is an odd integer. By the definition of odd integers, there exists an integer K, such that $n = 2K+1$. Squaring both sides, $n^2 = (2K+1)^2 = 4K^2+4K+1 = 2(\underset{=}{2K^2+2K})+1$. As $(2K^2+2K)$ is an integer, say t, we have $n^2 = 2t+1$. By definition of odd integers, n^2 is ~~even~~ odd. Consequently, it has been proved that if n is an odd integer, n^2 is also odd.

Proof-2: We assume that our hypothesis is true. i.e. m and n are perfect squares. By using the definition of perfect squares, it follows that there exists integers s and t, such that $m = s^2$ and $n = t^2$. This gives us $m \cdot n = s^2 \cdot t^2 = (st)^2$, using the commutativity and associativity of multiplication ⊛. By the definition of perfect square it follows that $m \cdot n$ is also a perfect square.

⊛ Since s and t are integers $s^2 t^2$ is also an integer.

Proof-3

We first assume that the hypothesis is true i.e. $3n+2$ is odd. By the definition of odd integers, there exists an integer k such that $3n+2 = 2k+1$. Simplifying, we get, $\cancel{3n = 2k}$ $3n+1 = 2k$. There is exists no direct way to conclude that n is odd.

$n = 2t+1$

Proof by contraposition

We assume that the conclusion of the given conditional statement "If $3n+2$ is odd, then n is odd" is false i.e. We assume n is even. Then by the defn. of even integers there exists an integer k such that $n = 2k$. Substituting $2k$ for n, we get $3n+2 = 3.(2k)+2$ $= 6k+2 = 2.(3k+1)$. As k is an integer, $3k+1$ is also an integer and it follows that $(3n+2)$ is even. This is a negation of the hypothesis of the given statement. As negation of the conclusion of the contrapositive statement implies that the hypothesis of the original statement is false, it follows that the original statement is true.

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true.

Proof-4

Direct proof: We assume that the hypothesis $n = a \cdot b$ is true. There is no obvious way to show that $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$ from the given equation $n = ab$, where a & b are positive integers

Proof by contraposition: $p \to q \equiv \neg q \to \neg p$

Demorgan's law

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

The first step is to assume that the conclusion of the given conditional statement is false i.e. $\neg((a \leq \sqrt{n}) \vee (b \leq \sqrt{n}))$. This means $a > \sqrt{n}$ and $b > \sqrt{n}$. We multiply the inequalities together to obtain $a \cdot b > \sqrt{n} \cdot \sqrt{n} > n$. This shows $ab \neq n$ which contradicts the hypothesis $n = ab$ of the original conditional statement.

As the hypothesis of the original conditional statement is false, it follows that the statement is true.

$\forall \ (P(\ldots) \ldots P(\ldots))$

$\forall x\,(P(x) \rightarrow Q(x))$

- Evaluate whether direct proof looks promising
- Begin with hypothesis, expand it using definitions/axioms and start reasoning.
- If the direct proof doesn't work and it seems to go nowhere, then try with the proof by contraposition.

Proof-5

$r = p/q$ s.t $q \neq 0$

Direct Proof:

Suppose r and s are rational numbers. From the definition, there are integers p & q with $q \neq 0$, such that $r = p/q$, and there are integers t and u with $u \neq 0$, such that $s = t/u$.

Adding r and s we have,

$$r + s = p/q + t/u = \frac{pu + tq}{uq}$$

As $q \neq 0$, $u \neq 0$ it follows that $uq \neq 0$. Similarly, $(pu + tq)$ and uq are all integers. Consequently, we have proved that $r+s$ is

$(p \rightarrow q) \not\equiv (q \rightarrow p)$

Consequently, we have proved that ... also rational.

$p \to q \not\equiv q \to p$

Proof-6 Show "If n^2 is odd, then n is odd".

Direct proof: Since n is an integer and n^2 is odd, then there exists an integer k such that $n^2 = 2k+1$. Which gives $n = \pm\sqrt{2k+1}$. There is no obvious approach to show that n is odd because the equation $n = \pm\sqrt{2k+1}$ is not useful.

Proof by Contraposition: We take the negation of the conclusion of the given statement as the hypothesis i.e. n is even. There exists an integer k such that $n = 2k$. By squaring on both sides we have $n^2 = 4k^2 = 2.(2k^2)$. This we can write as $n^2 = 2.t$ where $t = 2k^2$ is an integer which follows that n^2 is even. Consequently, we have shown ~~the~~ the ~~Conclu~~ hypotheses of the original statement is false which implies that the given statement is true.