

Signals and Systems

Tutorial problems with solutions

Abhik

CST Dept, IEST Shibpur

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Find the fundamental period of continuous signal
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- Thus the derivative can be expressed as sum of two infinite impulse trains $g(t) = \sum \delta(t - 2k)$

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