B-TECH 3RD SEMESTER MID TERM EXAMINATION, DECEMBER 2020

SUBJECT: SIGNAL AND SYSTEMS [CS2104]

Date of Examination: 28/12/2020

Name: Abhiroop Mukherjee

Enrolment Number: 510519109

Previous Enrolment Number: 510719007

G-Suite ID: 510519109.abhirup@students.iiests.ac.in

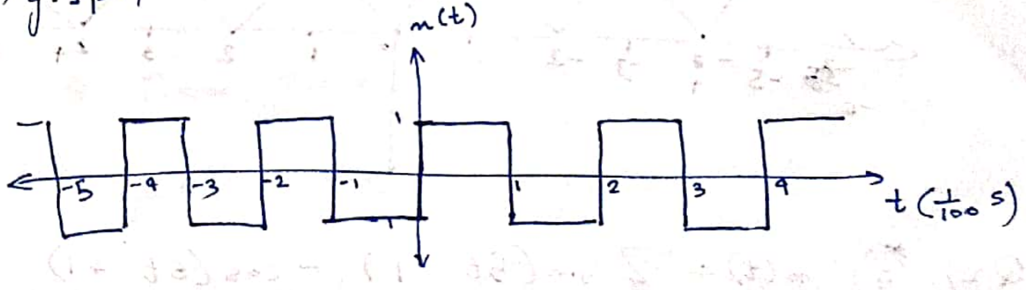
No. of sheets uploaded:

Q1) given Amplitude = 1 V, $f = 100$ hz, square wave passed through integrator system.

100 → 1s
1 → $\frac{1}{100}$ s

∴ ~~x(t)~~

∴ x(t) graph



→ passed through $y(t) = \int$ integrator.

∴ ~~for~~ → for time duration 0 to T, T being Time period,

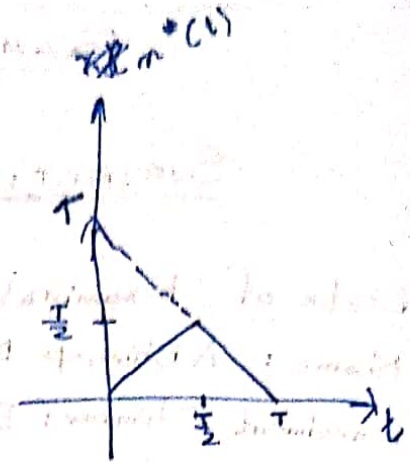
we get $x(t) = \begin{cases} 1 & 0 < t < \frac{T}{2} \\ -1 & \frac{T}{2} < t < T \end{cases}$

∴ output $y(t) = \int_{-\infty}^{T} x(t) = \int_{0}^{\frac{T}{2}} dt - \int_{\frac{T}{2}}^{T} dt = 0$

$= t - t = 0$

(hence integration of pevious cycles not required)

→ for $t < \frac{T}{2}$



$$\int_{-\infty}^{t} x(t)\,dt = \int_{0}^{t} dt = t$$

→ for $t > \frac{T}{2}$

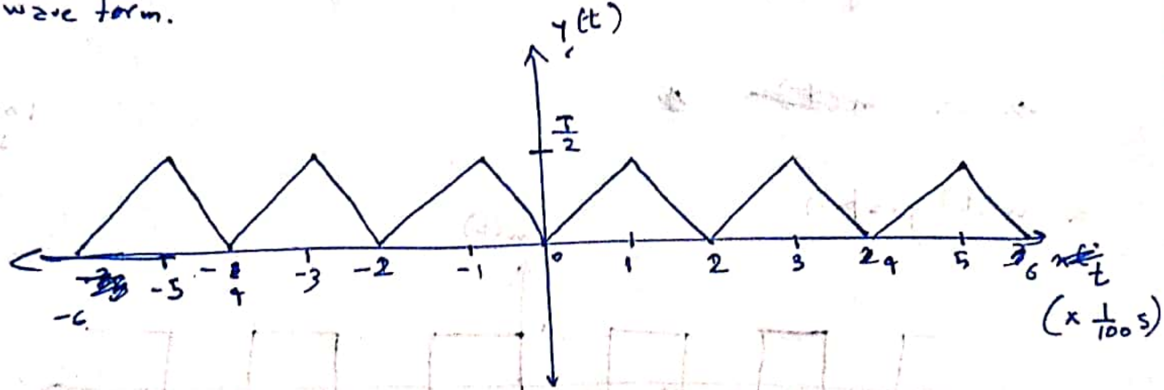
$$\int_{-\infty}^{t} x(t)\,dt = \int_{0}^{\frac{T}{2}} dt - \int_{\frac{T}{2}}^{t} dt$$

$$= \frac{T}{2} - \left(t - \frac{T}{2}\right)$$

$$= T - t$$

→ and the waveform gets repeated (due to periodicity)

∴ wave form.



Q2) a) $x(t) = 2\sin(5t-1) - \cos(6t+1)$

$$x(t+T) = 2\sin(5t-1+5T) - \cos(6t+1+6T)$$

$$= 2\sin(5t-1)\cos(5T) + 2\sin(5T)\cos(5t-1)$$

$$- \cos(6t+1)\cos(6T) + \sin(6T)\sin(6t+1)$$

for $x(t) = x(t+T)$ to be come true.

i) $\cos(5T) = 1$, ~~Sin 5T~~

ii) $\sin(5T) = 0$

iii) $\cos(6T) = 1$

iv) $\sin(6T) = 0$

now we know that $\cos(2\alpha\pi) = 1$ (α is integer)

$\sin(\beta\pi) = 0$ (β is integer)

~~but for sin~~

$\therefore$ minimum value of T, for this to be true is: $T = 2\pi$.

$\therefore$ Fundamental Period $= 2\pi$.

b) $x[n] = 2 + e^{j\frac{2\pi}{3}n} - e^{j\frac{6\pi n}{7}}$

$x[n+N] = 2 + e^{j\frac{2\pi N}{3}} e^{j\frac{2\pi n}{3}} - e^{j\frac{6\pi N}{7}} e^{\frac{j6\pi n}{7}}$

for $x[n] = x[n+N]$

i) $e^{j\frac{2\pi}{3}N} = 1$

ii) $e^{j\frac{6\pi N}{7}} = 1$

as $e^{j\theta} = \cancel{\sin\theta + j}\ \cos\theta + j\sin\theta$

for $e^{j\theta} = 1$

$\theta = \cancel{2\pi}.\ 2\alpha\pi.$

$\therefore \frac{2\pi}{3}N = 2\beta\pi.$ $\quad \frac{6\pi N}{7} = 2\gamma\pi$

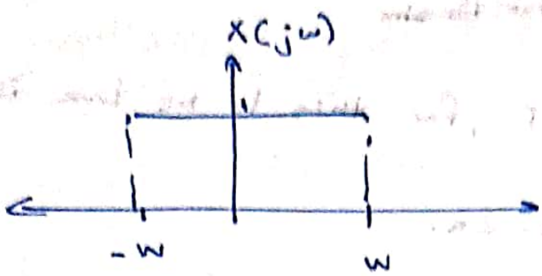
(α, β, γ are integer)

$\rightarrow$ smallest value of N for which this is true:

$N = 3 \times \frac{7}{\cancel{3}} = 35$

$\frac{3}{\cancel{6}\frac{\pi}{7}}N = 2\pi$

Q3) given $X(j\omega) = \begin{cases} 1 & |\omega| < w \\ 0 & |\omega| > w \end{cases}$



now using inverse fourier formula, i.e

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \cdot e^{j\omega t} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-w}^{w} e^{j\omega t} \, d\omega$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega t}}{jt} \right]_{-w}^{w}$$

$$= \frac{1}{2\pi jt} \left[e^{jwt} - e^{-jwt} \right]$$

$$= \frac{1}{2\pi jt} \times \left[\cancel{\cos(wt)} + j\sin(wt) - \cancel{\cos(wt)} + j\sin(w\ldots \right.$$

$$= \frac{1}{\cancel{2}\pi \cancel{j} t} \times \cancel{2}\cancel{j} \sin(wt)$$

$$= \frac{\sin(wt)}{\pi t}$$

→ The significance of w here is that, w acts as a cut off frequency, i.e ~~th~~ X(jω) acts as a low pass filter, with w as it's cutoff frequency.

Q4) given $h[n] = \left(\frac{1}{3}\right)^n u[n]$.

→ now as $F\{\alpha^n u[n]\} = \frac{1}{1-\alpha e^{-j\omega}}$, $|\alpha| < 1$

$$\therefore H(j\omega) = \frac{1}{1 - \frac{1}{3}e^{-j\omega}}$$

also we know that $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$

$$\therefore \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{1 - \frac{1}{3}e^{-j\omega}}$$

or $Y(j\omega) - \frac{1}{3}e^{-j\omega}Y(j\omega) = X(j\omega)$

now as $F\{y[n-1]\} = e^{-j\omega}Y(j\omega)$, we get

$$y[n] - \frac{1}{3}y[n-1] = x[n]$$

which is the required difference equation.

→ ~~To test for Time invariance, we go the following.~~

→ We know that difference equation based system ~~are~~ is a class of LTI system, so this is Time invariace

→ To Test Time in variace, we do the following.

i) $y[n-n_0] - \frac{1}{3}y[n-n_0-1] = x[n-n_0]$ [time delay followed by system]

ii) system followed by time delay.

$$y[n-n_0] - \frac{1}{3}y[n-n_0-1] = x[n-n_0]$$

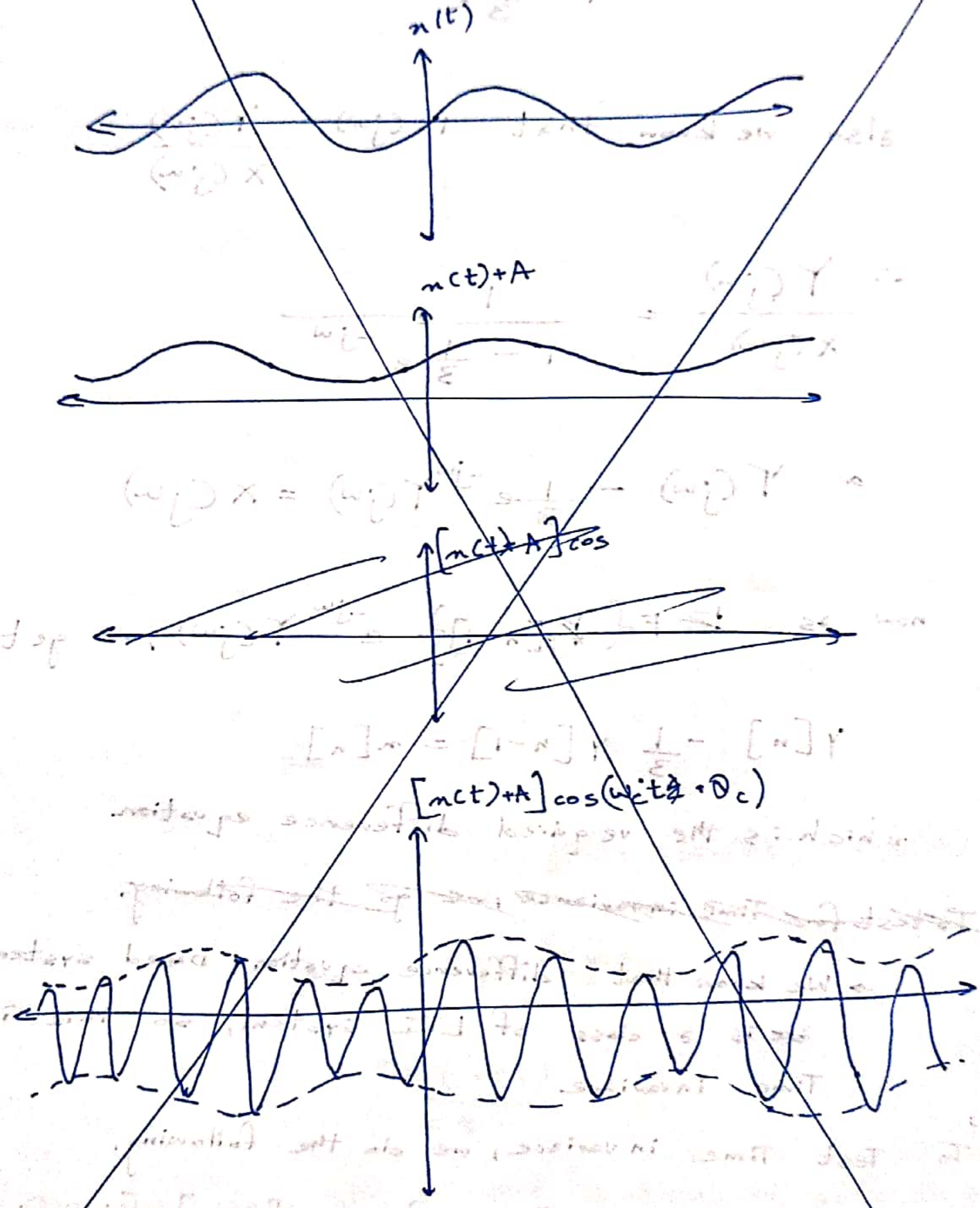
as both are same, this is Time Invariant.

Q5) given $\omega_m = 10$ kHz

$\omega_c = 1$ MHz = ~~10 × 10 kHz~~

→ Assuming we are doing Asynchronous ~~D~~ Modulation

$\therefore y(t) = (x(t) + A)\cos(\omega_c t + \theta_c)$



Q5) given $\omega_m = 10\ kHz$
$\omega_c = 1\ MHz$

→ assuming we are modulating with a sinusoidal carrier

∴ $y(t) = m(t)\, c(t)$

where $c(t) = \cos \omega_c t = \dfrac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$

$$\therefore C(j\omega) = \frac{2\pi}{2}\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

now $Y(j\omega) = X(j\omega) * C(j\omega)$

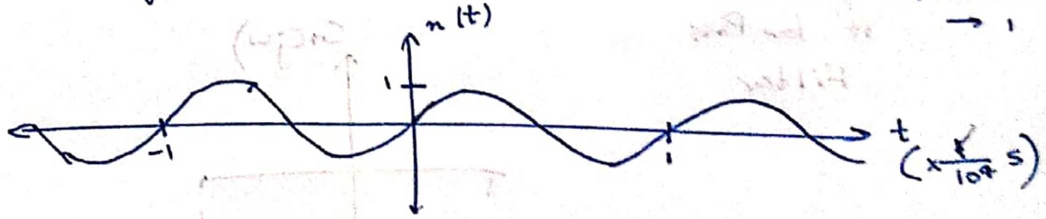
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\, C[j(\omega-\theta)]\, d\theta$$

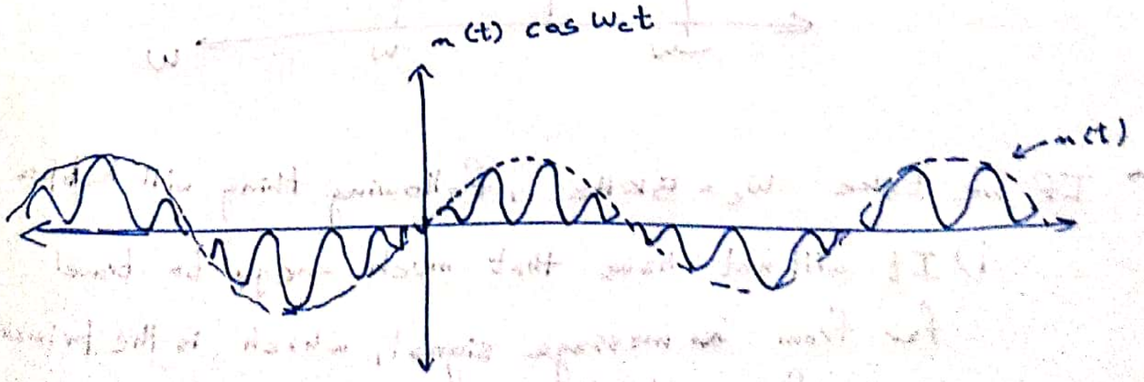
$$= \frac{1}{2\pi}\left[X(j\omega - j\omega_c) + X(j\omega + j\omega_c)\right]$$

Sketches

→ assuming message is pure sinusoid, with $\omega_m = 10\ kHz$

$1 \to 10\times10^3$
$\to 1$





→ drawn approximately

Scanned with CamScanner

→ To demoulate, we again modulate it and apply low pass filter.

$\therefore \quad y(t) = m(t) \cos \omega_c t$

$w(t) = y(t) \cancel{\cos\omega_c(t)} \cos \omega_c t$

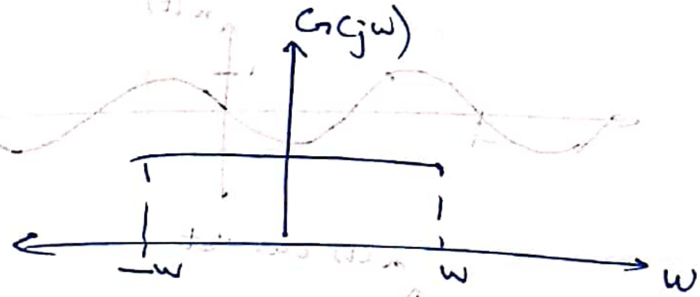
$= m(t) \cos^2(\omega_c t)$

$= \dfrac{m(t)}{2} + \dfrac{m(t) \cos(2\omega_c t)}{2}$

now low pass filter, can have ~~freq~~ it's cutoff frequency ~~fro~~ anywhere from $\omega_m + \underbrace{1000}_{\text{protection}}$ to $2\omega_c - \underbrace{1000}_{\text{protection}}$ hz

(let it be w)

$$\therefore \underbrace{G(j\omega)}_{\substack{\text{Impulse Response}\\ \text{of low Pass Filter}}} = \begin{cases} 1 & |\omega| < W \\ 0 & \text{otherwise} \end{cases}$$



→ If we take $\omega_c = 5\text{KHz}$, following thing will happen

i) It will not have that much enegy to travel far from ~~as~~ message signal, which is the primary reason for modulation.

ii) as $\omega_c < 2\omega_m$, it ~~d~~ will not follow sampling Theorem, hence Aliasing will occur. ~~which~~ ~~as~~ ~~which~~

and due to aliasing, data loss will be experienced.