10 AM 25/4/17

Indian Institute of Engineering Science and Technology, Shibpur

Dual Degree (B.Tech & M. Tech.) 4^{th} Semester End-Term Examination, April 2017

Theory of Computation (CS 404)

Full Marks: 70 **Time: 3 hours**

- **Question 1. is compulsory. Attempt Question 1. and any four from the rest.**
- Two marks are reserved for clarity of presentation
- Answers should be **brief, to the point** and **in your own words** as far as practicable.
- Make your own assumptions as and when necessary and state them at proper places.

1. State and justify the truth or falsity of **any three** of the following statements.

   (a) There are uncountably infinite number of languages over some alphabet Σ.

   (b) Given a Deterministic Finite Automaton M_{dfa}, one can always mechanically construct one Push Down Automaton M_{pda}, such that, $L(M_{dfa}) = L(M_{pda})$.

   (c) Grammars cannot compute functions from natural numbers to natural numbers.

   (d) Given a Deterministic Turing Machine $M = (K, \Sigma, \delta, s)$, one can always construct one Non-Deterministic Turing Machine $M' = (K', \Sigma', \Delta', s')$ doing what M does. [4 × 3]

2. Construct Finite Automaton (Deterministic or Non-Deterministic) or Pushdown Automaton for the following languages.

   (a) $\{\omega_1\omega_2 \; : \; \omega_1 \in \{a, b\}^*, \; \omega_2 \in \{c, d\}^* \text{ and } |\omega_1| = |\omega_2|\}$

   (b) $\{\omega \in \{a, b\}^* \; : \; \omega$ does not have $abaab$ as a substring$\}$

   (c) $\{0^i 1^j 2^k \; : \; i, j, k \geq 0 \text{ and } i \neq j\}$

   (d) $\{a^i b^j c^{2i-1} d^k \; : \; i, j, k > 0\}$ [3 + 3 + 4 + 4]

3. Construct Grammar (Regular or Context-Free or Unrestricted) for the following languages.

   (a) $\{\omega c^m d^n \; : \; \omega \in \{a, b\}^*, m =$ the number of a's in ω and $n =$ the number of b's in $\omega\}$

   (b) $\{\omega \in \{a, b\}^* \; : \; |\omega|$ is odd and the middle symbol of ω is 'b'$\}$

   (c) $\{\omega\omega'^R \; : \; \omega, \omega' \in \{a, b\}^*, \; |\omega| = |\omega'|, \; \omega(i) = a \text{ iff } \omega' = b, \text{ and } \omega'^R \text{ is reverse of } \omega'\}$

   (d) $\{a^i b^j c^j d^{2i} \; : \; i, j \geq 0\}$ [5 + 3 + 3 + 3]

4. (a) Formally define the concepts of a **2-Tape Turing machine**, **configuration** of such a machine, its **yields in one step relation** and the idea that such a machine computes a function from natural numbers to natural numbers.

   (b) Construct a Turing machine (standard Turing machine or any extension) M that conputes the function $inc : \mathcal{N} \rightarrow \mathcal{N}, \; inc(n_1) = n_1 + 1$. M accepts a natural number in **binary representation** (as input) and produces the next natural number in **binary representation** (as the output). [6+8]

5. (a) Propose a scheme for encoding any arbitrary Turing machine $M = (K, \Sigma, \delta, s)$ as a string over some fixed alphabet so that it can be fed as an input to the Universal Turing Machine.

   (b) Given a Context-Free Grammar $G = (V, \Sigma, R, S)$, formally define the idea of "*Derives in one step*" relation between $u, v \in V^*$ under G ($u \overset{G}{\Rightarrow} v$).

   (c) Formally prove that, given a Context-Free Grammar $G = (V, \Sigma, R, S)$, for every derivation $S \overset{G}{\Rightarrow}^* \omega$ under G, there exists a **leftmost derivation** of ω under G. [7+2+5]

(Contd.)

6. Write short notes on the following.

   (a) Nondeterminism in Automata

   (b) Pumping Lemma for the class of Context-Free Languages

   (c) Turing acceptable languages [5+5+4]