1, 2, 3, 4
✓ 1c doubt
✓ 1d again
✓ PDA/CFG not

B-Tech 4TH Semester
End-Term Examination
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Subject: Theory of Computation [CS 2204]

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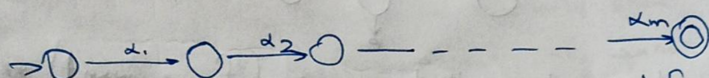
1) a) given L is a finite language.

→ let ~~of~~ ~~L⊆Σ~~ $L \subset \Sigma^*$ contains m symbols at max.

→ that is ~~the~~ any string w in language L will have ~~at max~~ it's length m

→ so we can create a DFA ~~with~~ as follows



→ i.e at each transition we consume one symbol from the string

→ so we can ~~use~~ represent given language L using an DFA

→ so we can say that L is a Regular Language

→ as we know that all Regular Languages are ~~are~~ also context-free language, we can say that L is also a Context-Free Language.

→ Hence we see that if L is a finite ~~a~~ language, we can make ~~an~~ a FA which accepts that language, which make ~~A~~ L Regular, and hence also Context-Free

∴ Given Statement is **True**

b) A Parse Tree is a pictorial way to display all type of derivation that could happen for a Context - Free - Grammar

Eg: consider CFG $G = (V, \Sigma, R, S)$ with

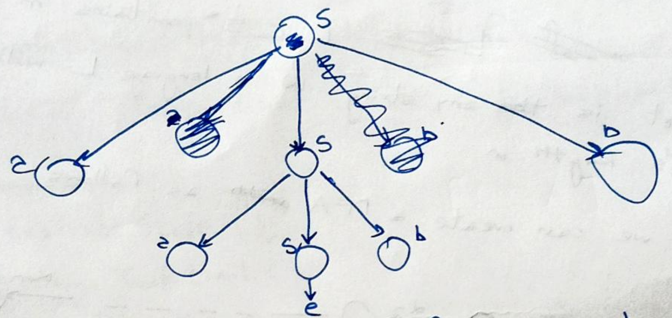
$R = \{ S \to aSb, S \to e \}$

$V = \{ S, a, b \}$

$\Sigma = \{ a, b \}$

$S = S$

it's Parse tree will be as follows:



→ we observe here that we can (if we want), use the rule $S \to aSb$ indefinatly, so we see that there can't be an upper limit on the length of yield W of a parse tree T under a CFG $G = (V, \Sigma, R, S)$

∴ given statement is False

now intersection of L_1 and L_2 is as follows

$$L_3 = L_1 \cap L_2 = \{ w \in \{a,b,c\}^* \mid a^n b^n c^n , n \geq 0 \}$$

→ we can show by using Context Free Language Pumping Theorem, that L_3 is not context free.

∴ Intersection of two Context-Free Language can never be a Context-Free Language.

∴ Statement is True

Q2) a) $L = \{ w_1 w_2 : w_1 \in \{a,b\}^*, w_2 \in \{c,d\}^+ \mid |w_1| = 2|w_2| \}$ [PDA]

Let's first make an Context Free Grammar Rules for L

$S \rightarrow aaSc$
$S \rightarrow aaSd$
$S \rightarrow abSc$
$S \rightarrow abSd$
$S \rightarrow baSc$
$S \rightarrow baSd$
$S \rightarrow bbSc$
$S \rightarrow bbSd$

$S \rightarrow e$

→ now it's PDA will be

PDA formal definition

$M = (K, \Sigma, \Gamma, \Delta, s, F)$

$K = \{\alpha, \beta\}$

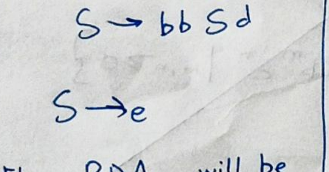
$\Sigma = \{a, b, c, d\}$

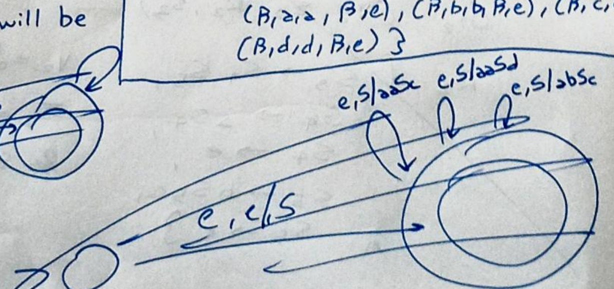
$\Gamma = \{a, b, c, d, S\}$

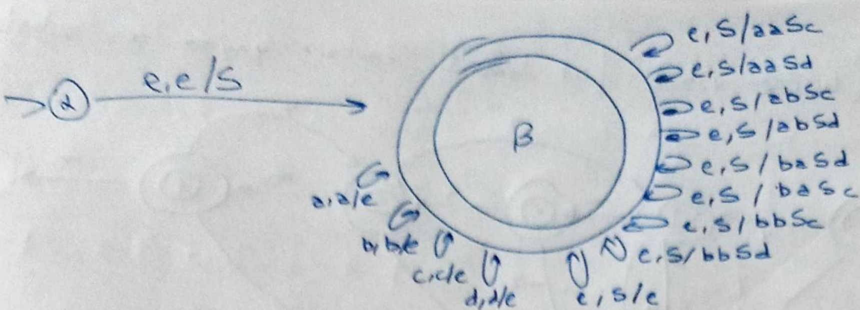
$s = \alpha$

$F = \{\beta\}$

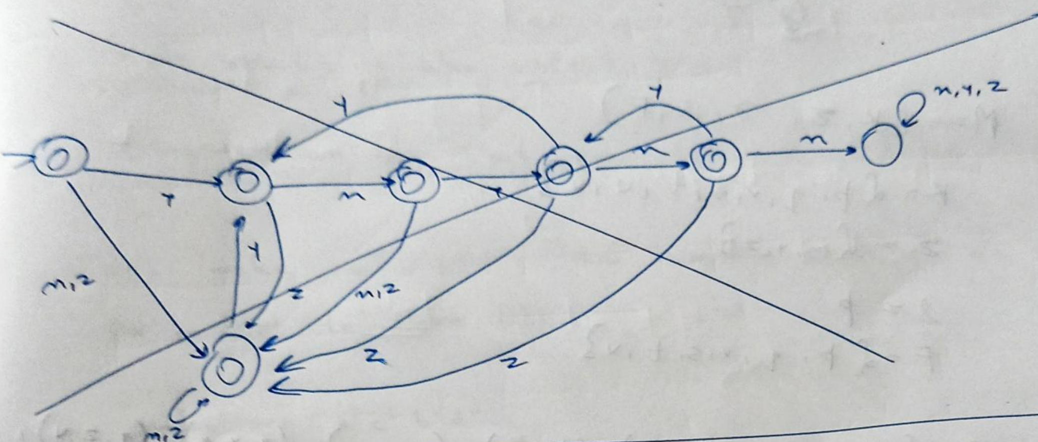
$\Delta = \{ (\alpha, e, e, \beta, S), (\beta, e, S, \beta, aaSc), (\beta, e, S, \beta, aaSd), (\beta, e, S, \beta, abSc), (\beta, e, S, \beta, abSd), (\beta, e, S, \beta, baSc), (\beta, e, S, \beta, baSd), (\beta, e, S, \beta, bbSc), (\beta, e, S, \beta, bbSd), (\beta, e, S, \beta, e), (\beta, a, a, \beta, e), (\beta, b, b, \beta, e), (\beta, c, c, \beta, e), (\beta, d, d, \beta, e) \}$







b) $L = \{w \in \{x,y,z\}^* \mid w$ doesn't contain yxyxx as substring$\}$



c) $L = \{w \in \{a,b,c\}^* \mid a^i b^j c^i : i,j \geq 0\}$

let us first make it's CFG Rules

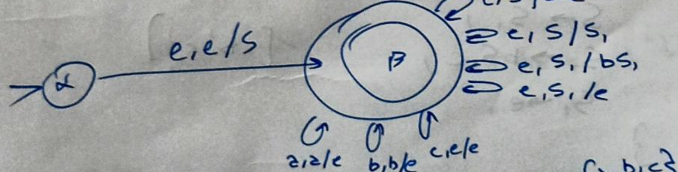
$S \rightarrow aSc$ [generate $a^i c^i$]

$S \rightarrow S_1$

$S_1 \rightarrow bS_1$ [generate b^j]

$S_1 \rightarrow e$

so it's equivalent ~~DF~~ a PDA will be



$\Sigma = \{a,b,c\}$; $\Gamma = \{a,b,S,c\}$

so PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$; $K = \{\alpha, \beta\}$;

$s = \alpha$; $F = \{\beta\}$;

$\Delta = \{(\alpha,e,e,\beta,S), (\beta,e,S,\beta,aSc), (\beta,e,S,\beta,S_1), (\beta,e,S_1,\beta,bS_1),$
$(\beta,e,S_1,\beta,e), (\beta,a,a,\beta,e), (\beta,b,b,\beta,e), (\beta,c,c,\beta,e)\}$

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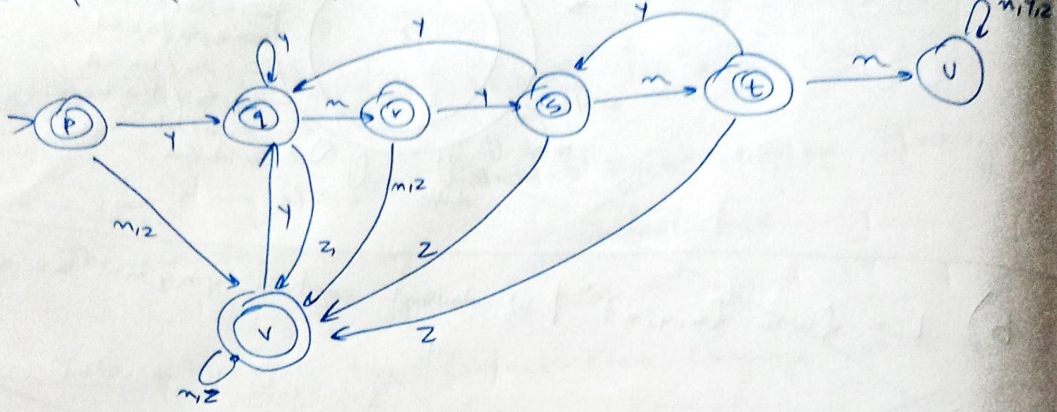
b) $L = \{ w \in \{x,y,z\}^* \mid w$ doesn't contain $yxyxx$ as substring $\}$



$M = (K, \Sigma, \delta, s, F)$

$K = \{p, q, r, s, t, u, v\}$

$\Sigma = \{x, y, z\}$

$s = p$

$F = \{p, q, r, s, t, v\}$

$\delta = \{ (p,x,v), (p,y,q), (p,z,v), (q,x,r), (q,y,q), (q,z,v),$
$(r,x,v), (r,y,s), (r,z,v), (s,x,t), (s,y,q), (s,z,v),$
$(t,x,u), (t,y,s), (t,z,v), (u,x,u), (u,y,u), (u,z,u),$
$(v,x,v), (v,y,q), (v,z,v) \}$

Q3) a) $L = \{ a^i b^j c^j d^i \; ; \; i,j \geq 0 \}$

~~Rules~~ ~~R~~ $G = (V, \Sigma, R, S)$

$R = \{ S \to aSd,$ $[a^i d^i]$
$S \to S_1,$
$S_1 \to bS_1c,$ $[b^j c^j]$
$S_1 \to e,$
$\}$

$V = \{a, b, c, d, S, S_1\}$

$\Sigma = \{a, b, c, d\}$

$s = S$

1) d) → We know that context free Languages are not closed under intersection

But consider this example

~~a~~ i) $L_1 = \{w \in \{a,b\}^* \mid w$ contains at most one $b\}$

ii) $L_2 = \{w \in \{a,b\}^* \mid w$ contains at most one $a\}$

~~&~~ so $L_3 = L_1 \cap L_2 = \{w \in \{a,b\}^* \mid w$ contains at most one a and one $b\}$

→ we see L_3 is also context free.

→ so we can say that given statement is <u>false</u>

3 b) Idea: make $a^i c^i b^j d^j$, then ~~swap~~ shift all b's to left to ~~take~~ make $a^i b^j c^i d^j$

so, generationg $a^i c^i b^j d^j$

1) $S \to S_1 S_2$
2) $S_1 \to a S_1 C$
3) $S_2 \to B S_2 d$
4) $S_1 \to [$
5) $S_2 \to]$

[generate a^i [C^i B^j] d^j]

now shifting all B's to left of [

6) $CB \to BC$
7) $[B \to b[$

[will have $a^i b^j [C^i] d^j$]

now shifting all C's to right of]

8) $C] \to]c$

[will have $a^i b^j [] c^i d^j$]

ending conversion

9) $[] \to e$

[will have $a^i b^j c^i d^j$]

Q1) c) → Pumping Theorem for the class of regular language can be used to show that if a language is regular or not

→ Regular Language is a subset of Context Free Language, ~~thus~~ Using this theorem, we ~~might~~ can say a Langage L is not regular, but we can't say that if L is Context free or not

→ So we can't use Pumping Theorem for class of Regular Language to show if a language L is Context - Free or not

→ So given statement is <u>False</u>