Indian Institute of Engineering Science and Technology, Shibpur

B.Tech.(All Branches) 3rd Semester Final Examination, January 2021

Mathematics III

(MA-2101)

Full Marks: 50 Time: 01 hour 30 minutes

Answer any **FIVE** (05) questions.

[**Only the first five answers will be considered for evaluation**]

1. a) Define Normal distribution and prove that $\int_{-\infty}^{\infty} f(x)dx = 1$, where $f(x)$ represents probability density function of this distribution.
   b) Let $F(x)$ be a distribution function. Prove that for any fixed $h > 0$, the function $G(x) = \frac{1}{2h}\int_{x-h}^{x+h} F(u)du$ is also a distribution function. [(1+3)+6]

2. a) Find the mean and variance for the Binomial distribution.
   b) Evaluate the mean and variance of the continuous distribution with probability density function given by

$$f(x) = 1 - |1 - x| \,, 0 < x < 2$$
$$= 0 \,, elsewhere.$$

[2x(2+3)]

3. a) Two random variables X and Y are connected by the relation 3X+4Y+5 = 0. A sample (x_i, y_i) , i = 1,2, … n is taken from the bivariate population of (X , Y). Obtain the correlation coefficient of the sample.
   b) Find the most likely price in Chennai corresponding to the price of Rs. 70 in Calcutta from the following data obtained from 25 observations on the price of a commodity in each of two States.
   Average prices in Calcutta and Chennai are respectively Rs. 65 and Rs. 67.
   Standard deviations of prices in Calcutta and Chennai are respectively Rs. 2.5 and Rs. 3.5.
   Coefficient of correlation between the prices of the commodity in two States is 0.8. [6+4]

4. Solve the following differential equation by Laplace transform method

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t}, \quad \text{given that } y(0) = y'(0) = 1. \qquad [10]$$

5. a) Evaluate $\mathcal{L}^{-1}\left[\log\left(1 + \frac{1}{s^2}\right);\ t\right]$.
   b) Define extreme point of a convex set.

Find the extreme points, if any, of the following sets

i) $X_1 = \{(x_1, x_2): |x_1| \le 1, |x_2| \le 1\}$

ii) $X_2 = \{(x_1, x_2): x_1 + 2x_2 = 9\}$

iii) $X_3 = \{(x_1, x_2): x_1^2 + x_2^2 \le 1, x_1 \ge 0\}$. [5+(2+1+1+1)]

6. Using Charnes Big-M method solve the following LPP

$$\begin{aligned} \textit{Maximize} \quad & Z = 2x_1 - 3x_2 \\ \textit{Subject to} \quad & -x_1 + x_2 \ge -2 \\ & 5x_1 + 4x_2 \le 46 \\ & 7x_1 + 2x_2 \ge 32 \\ & x_1,\ x_2 \ge 0. \end{aligned}$$

[10]