Proof technique - 2

Wednesday, September 9, 2020 10:26 AM

Proof-6 Let us assume that proposition p is "$\sqrt{2}$ is irrational." To start a proof by contradiction, we have to consider $\neg p$ is true where $\neg p$ is the statement "$\sqrt{2}$ is rational".

$\neg p \to$ r
$\neg p \to$ ir

By the definition of rational numbers, there ~~there~~ exists two integers a and b with $\sqrt{2} = a/b$, where $b \neq 0$ and a & b do not have a common factor. (So the fraction a/b is in lowest term).

Because, $\sqrt{2} = \frac{a}{b}$, squaring both sides,

$$2 = a^2/b^2 \Rightarrow 2b^2 = a^2$$

By the defn. of even integers, it follows that a^2 is even. There exists an integer c such that $a = 2c$. Thus, $2b^2 = 4c^2 \Rightarrow$ $b^2 = 2c^2 \Rightarrow b^2$ is also even by defn. of even numbers.

We have now shown that $\neg p$ leads to the equation $\sqrt{2} = a/b$, where a and b are even and 2 divides both a & b.

Because our assumption of $\neg p$ leads to a contradiction, it follows that $\neg p$ must

Because our assum[illegible] ... a contradiction, it follows that $\neg p$ must be false. That is the statement p, "$\sqrt{2}$ is irrational" is true.

Proof-7 "If $3n+2$ is odd, then n is odd" =

To construct proof by contradiction, we assume both p & $\neg q$ are true. That is, $3n+2$ is odd and n is not odd i.e. n is even. If n is even, there exists an integer K such that $n = 2K$. This implies, $3n+2 = 3.(2K)+2 = 6K+2 = 2.(3K+1)$. is also even. Thus we have both p and $\neg p$ to be true, which is a contradiction. This implies that our initial assumption of $\neg q$ is false i.e. q is true. Henceforth, if $3n+2$ is odd, then n is also odd.

Proof-8 We can take the negation of the statement and suppose it to be true. i.e there exists a greatest integer N. Then $N \geq m$ for every integer m. Let $M = N+1$. Now, M is another integer since it is the sum of two integers. Also, $M > N$ since $M = N+1$. Thus, M is an integer which is greater than N. So, N is the greatest integer and N is not the greatest integer, which is a contradiction. This shows that our supposition is false and hence the statement is true.

$r \wedge \neg r$

contradiction. This shows that our supposition is false, and hence the statement is true.

Proof-9

We take the negation of the hypothesis and suppose it is to be true i.e. there is at least one integer which both odd and even. By defn. of even, $n = 2K$, for some integer K. And, by defn of odd, $n = 2l+1$ for some integer l. Consequently,

$$2K = 2l+1$$

$$\text{or, } K - l = 1/2.$$

Since, K & l are both integers, their difference must also be an integer. But $K - l = \frac{1}{2}$, which is not an integer. Thus, $K-l$ is an integer and $K-l$ is not an integer, which is a contradiction. This contradiction shows that our supposition was false, and hence the hypothesis is true.

Proof-10

$r = p/q,\ q \neq 0$

We take the negation of the ~~theorem~~ hypothesis and suppose it to be true i.e. the sum of rational and an irrational number is rational. Let r be a rational and s be a irrational no. By defn. of rational numbers, rational number $r = a/b$ and the sum of r and $s = c/d$ for some integers a, b, c, and d, with $b \neq 0$ and $d \neq 0$. By and d, with $b \neq 0$ and $d \neq 0$. By Substitution,

$$\frac{a}{b} + s = \frac{c}{d}.$$

$$\text{or, } s = \left(\frac{c}{d} - \frac{a}{b}\right) = \left(\frac{bc - ad}{bd}\right)$$

Now, $(bc - ad)$ and bd are both integers and as $b \neq 0$, $d \neq 0$, it follows $bd \neq 0$. Therefore, by the definition of rational nos., s is also rational. This is a contradiction to our supposition that s is irrational. Therefore, the given statement is true.

Proof-II

We take the negation of the statement and suppose it to be true. i.e. $1 + 3\sqrt{2}$ is rational. Then, by the definition of rational numbers,

$$1 + 3\sqrt{2} = \frac{a}{b},$$ for some integers a, b with $b \neq 0$.

$$\Rightarrow 3\sqrt{2} = \frac{a}{b} - 1 = \frac{a-b}{b}$$

$$\Rightarrow \sqrt{2} = \left(\frac{a-b}{3b}\right)$$

$(a-b)$ and $3b$ are integers. and $3b \neq 0$. Hence, $\sqrt{2}$ is a quotient of two integers $(a-b)$ and $3b$ with $3b \neq 0$. It follows

Hence, $\sqrt{2}$ is a quotient of two integers $(a-b)$ and $3b$ with $3b \neq 0$. It follows that $\sqrt{2}$ is rational. But this contradicts our previous proof of $\sqrt{2}$ being irrational. This is a contradiction and hence our initial supposition was false. Hence, $1+3\sqrt{2}$ is irrational.

if and only if (iff) $\Rightarrow$ bi-conditional Statement

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

If n is odd, then n^2 is odd $(p \rightarrow q) =$

If n^2 is odd, then n is odd $(q \rightarrow p) =$

→ direct proof

→ proof by Contraposition

$p_1, p_2, \ldots p_n$ ~~po~~ propositions.

All these propositions are equivalent.

$$p_1 \leftrightarrow p_2 \leftrightarrow p_3 \leftrightarrow \ldots \leftrightarrow p_n$$

All n propositions have the same truth table and consequently for all i and j with $1 \leq i \leq n$, and $1 \leq j \leq n$, p_i and p_j are equivalent.

Tautology

$$(p_1 \leftrightarrow p_2 \leftrightarrow \ldots \rightarrow p_n) \equiv$$

Tanto. $(p_1 \leftrightarrow p_2 \leftrightarrow \ldots \leftrightarrow p_n) \equiv$

$(p_1 \to p_2) \wedge (p_2 \to p_3) \wedge \ldots \wedge (p_n \to p_1)$ ✓

Show that n conditional statements are true then the propositions $p_1, p_2, \ldots, p_n$ are all equivalent.

p_1, p_2, p_3

$(p_1 \to p_2), (p_2 \to p_3)$

$(p_3 \to p_1)$

This is much efficient than proving $p_i \to p_j$ for all $i \neq j$ with $1 \leq i \leq n$ and $1 \leq j \leq n$.

(No. of conditional statements to be proved true $= n^2 - n$)

$p_1 \to p_2,\ p_2 \to p_1$

$p_1 \to p_3,\ p_3 \to p_1$

$p_2 \to p_3,\ p_3 \to p_2$

Proof-3

$$p_1 \leftrightarrow p_2 \leftrightarrow p_3 \equiv (p_1 \to p_2) \wedge (p_2 \to p_3) \wedge (p_3 \to p_1)$$

p_1: n is even.

p_2: $(n-1)$ is odd.

p_3: n^2 is even.

We will show that these statements are equivalent by showing that conditional statements $p_1 \to p_2$, $p_2 \to p_3$ and $p_3 \to p_1$ are true.

(i) To show $p_1 \to p_2$ we can use direct proof.

(i) To show $p_1 \to p_2$ we can use direct proof.
Suppose n is even. Then $n = 2k$ for some integer k. Consequently, $(n-1) = 2k-1 = 2(k-1)+1$. This means $(n-1)$ is odd because it is of the form $2t+1$, where $t = k-1$ is another integer.

(ii) We ~~again~~ again use direct proof to show $p_2 \to p_3$. We suppose $(n-1)$ is odd. Then $n-1 = 2k+1$ for some integer k. Hence, $n = 2k+2$ which follows $n^2 = (2k+2)^2 = 4k^2+8k+4 = 2(2k^2+4k+2)$.
This can be expressed c. $n^2 = 2t$, where $t = 2k^2+4k+2$ is another integer. $(\neg p_1 \to \neg p_3)$

(iii) We use the proof by contraposition to show $p_3 \to p_1$. That is, we prove that if n is not even then n^2 is not even i.e. if n is odd, then n^2 is odd.