## Dual Degree (B.Tech & M.Tech) 3rd Semester (CST), Final Examination, 2016 Indian Institute of Engineering Science and Technology, Shibpur, Howrah

## Discrete Structures (CS 303)

Full Marks: 70

Time: 3 hrs

Use ONE Answer Script.

Answer any FIVE questions (attempt at least two questions from each group)

## Group A

- 1. a) Define the following terms:
  - (i) Poset

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- (ii) Lattice
- (iii) Atom

(iv) Universal lower bound

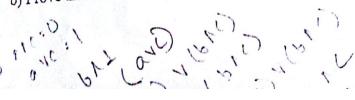
- (v) Universal upper bound
- > vi b) Prove that in a distributive Lattice, if an element has a complement then this complement is
  - c) Let the number of ways selecting r objects from n objects is denoted by the numeric function  $a_r$ . With the help of generating function prove that  $a_r = C(n+r-1, r)$ .
  - 2. a) Transform the Propositional Logic formula  $(P \land (Q \rightarrow R)) \rightarrow S$  into a Conjunctive Normal
    - b) Given:  $F_1, F_2, ..., F_n$  and G are formulas in a Propositional Logic. Prove that G is a logical
      - i) if and only if the formula  $((F_1 \wedge F_2 \wedge .... \wedge F_n) \rightarrow G)$  is valid consequence of  $F_1, F_2, \ldots, F_n$ :
        - ii) if and only if the formula  $(F_1 \wedge F_2 \wedge ... \wedge F_n \wedge \sim G)$  is inconsistent
    - e) Evaluate the truth value of the formula  $(\forall x)$   $(P(x) \rightarrow Q(f(x), a))$  under the following interpretation.

Domain:  $D = \{1, 2\}$ 

Assignment for a: a = 1

Assignment for f: f(1) = 2 and f(2) = 1Assignment for Q: Q(1,1) = T, Q(1,2) = T, Q(2,1) = F, Q(2,2) = T(3+6+5)Assignment for P: P(1) = F, P(2) = T

- 3. a) Solve the following recurrence relation using generating function: b) Let  $a_r$  be the number of comparisons required in Bubble sort algorithm for sorting a list of r
  - (i) Derive a recurrence relation for  $a_r$  in terms of  $a_{r-1}$ . elements.
  - c) Let a be an arbitrary numeric function and b is the accumulated sum of a. Find the generating function of b in terms of the generating function of a.
  - 4. a) Find the particular solution of the recurrence equation  $a_r 2$   $a_{r-1} = 3$ .  $2^r$ .
    - b) Prove that there exists a unique finite Boolean algebra of  $2^n$  elements for any n > 0. (4+10)



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## Group B

- 5. a) Translate the following statements into formula:
  - (i) "Every man is mortal. Confucious is a man. Therefore, Confucious is mortal".
  - (ii) "For every number other than zero, there is one and only one immediate predecessor".
  - (ii) "Nobody can be a good student unless he is smart and his father supports him".
  - b) Define Prenex Normal Form. Write an algorithm to convert a First Order Predicate Logic (FOPL) into a Prenex Normal Form.
  - c) Transform the following formula into Prenex Normal Form:  $(\exists x) \; (\sim ((\exists y) \; P(x,y)) \to ((\exists z) \; Q(z) \to R(x)))$

(5+5+4)

- 6. a) Define Kuratowski's two graphs. List any four properties common to both the graphs.
  - b) In any simple, connected planar graph with n vertices and e edges (e > 2), prove that  $e \le 3n 6$
  - (5+4+5)c) Write an algorithm to conclude if a graph is a planar graph or not.
- 7. a) Define Chromatic number and Independence number of a graph with n vertices and establish a relationship between them.
  - b) Prove that every tree with two or more vertices is 2-chromatic.
  - c) Define Minimal Spanning Tree (MST). Write an algorithm for finding an MST of a graph and demonstrate your algorithm considering a suitable graph.

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