Simplification of Boolean Functions using Tabulation Method

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Motivation

- The K-map method of minimization is convenient when the number of variables less than six.
- The map method is a trial-and-error procedure that depends on the human ability to recognize certain pattern.
- It is also difficult to automate the method.
- For large functions, a more systematic procedure is required and for that Quine-MeCluskey method can be used.
- The Quine-MeCluskey method is easy to automate.

Basic Concept

- The basic concept remain the same.
- Repeated application of the theorem $A\overline{X} + AX = A$ to all adjacent pair term. This will produce the set of all prime implicants at the end.
- This is a two steps process. First generate all the prime implicants then, select the prime implicants that will cover the all miniterms of the function.
- Example:
  $F(A, B, C, D) = \overline{A}\ \overline{B}\ \overline{C}D + \overline{A}\ \overline{B}\ \overline{C}\ \overline{D} + A\ \overline{B}\ \overline{C}\ \overline{D} + A\ \overline{B}\ \overline{C}D$
  $F(A, B, C, D) = \overline{A}\ \overline{B}\ \overline{C} + A\ \overline{B}\ \overline{C}$
  $F(A, B, C, D) = \overline{B}\ \overline{C} \Rightarrow$ Prime implicant.
- We can go combing any pair of product terms that differ in the value of single literal.

Basic Concept

1. Two k-variable terms can be combined into single (k -1) variable term, if and only if they differ in only one literal.
2. We use binary representation of the minterms for convenience.
3. Two minterms can be combined if their binary representation differ in only one position.
4. We use the symbol '-' to indicate the absence of a literal.

Identification of all Prime Implicant (step -1)

- Arrange all the minterms in groups based on the number of 1's. The number of 1's in a term is called in index.
- Compare every term of a group (index i) with the each term of another group having index (i +1).
  - Merge the two term where possible using the rule $AX + A\overline{X} = A$.
  - Place a check mark to the each term that has been combined with at least one term.
- Now compare the terms that are generated as the output of the previous iteration in the same fashion, generate a new term by combining two terms that differ by only one position.
- The process continues until no further combination are possible.
- The remaining unchecked terms are the prime implicants of the function.

Example

Consider the following Boolean function
$F(w, x, y, z) = \sum(0, 1, 2, 8, 10, 11, 14, 15)$

	w	x	y	z
0	0	0	0	0 $\surd$
1	0	0	0	1 $\surd$
2	0	0	1	0 $\surd$
8	1	0	0	0 $\surd$
10	1	0	1	0 $\surd$
11	1	0	1	1 $\surd$
14	1	1	1	0 $\surd$
15	1	1	1	1 $\surd$

	w	x	y	z
0,1	0	0	0	-
0,2	0	0	-	0 $\surd$
0,8	-	0	0	0 $\surd$
2,10	-	0	1	0 $\surd$
8,10	1	0	-	0 $\surd$
10,11	1	0	1	- $\surd$
10,14	1	-	1	0 $\surd$
11,15	1	-	1	1 $\surd$
14,15	1	1	1	- $\surd$

Example Cont.

	w	x	y	z
0,2,8,10	-	0	-	0
0,8,2,10	-	0	-	0
10,11,14,15	1	-	1	-
10,14,11,15	1	-	1	-

The set of prime implicants are $\{\overline{w}\ \overline{x}\ \overline{y},\ \overline{x}\ \overline{z},\ w\ y\}$

Prime Implicant Chart

How to select the smallest set of prime implicants that cover all the minters of the function.
For this purpose, We shall use prime implicant chart.

- It is a tabular data structure, which pictorially depicts the covering relationship between prime implicants and minterms.
- Useful to select the minimum set of prime implicants.
- Minterms are listed along columns, while prime implicants listed along rows.
- A 'X' is entered in the table if corresponding prime implicant covers corresponding minterm.
- If a column has a single 'X', the prime implicant corresponding to the row in which the 'X' appears is an essential prime implicant.

Selection of essential Prim implicants

- A check mark is placed in the chart to the essential prime implicants to indicate that they have selected.
- Next check each column whose minterm is covered by the essential prime implicants.
- If the essential prime implicants do not cover all the minterms of the function, select the prime implicants that will cover the uncovered minterms.

Example of Prime Implicant Chart

Consider the following Boolean function:
$F(w, x, y, z) = \sum(1, 4, 6, 7, 8, 9, 10, 11, 15)$ and the prime implicates are
$\overline{x}\ \overline{y}\ z$, $\overline{w}\ x\ \overline{z}$, $\overline{w}\ x\ y$, $x\ y\ z$, $w\ y\ z$, and $w\ \overline{x}$.
We have to create a prime implicant chart (table)

		1	4	6	7	8	9	10	11	15
$\surd\ \overline{x}\ \overline{y}\ z$	1,9	X					X			
$\surd\ \overline{w}\ x\ \overline{z}$	4,6		X	X						
$\overline{w}\ x\ y$	6,7			X	X					
$x\ y\ z$	7,15				X					X
$w\ y\ z$	11,15								X	X
$\surd\ w\ \overline{x}$	8,9,10,11					X	X	X	X	
		$\surd$	$\surd$	$\surd$		$\surd$	$\surd$	$\surd$	$\surd$	