Graph-3

Wednesday, December 2, 2020 8:48 AM

Graph Connectivity

Vertex connectivity, $K(G)$

$G = K_n$, a complete graph of n vertices.

$K(K_n) = n-1$ → no. of vertices to be removed to produce a graph with single vertex.

In general, $0 \leq K(G) \leq (n-1)$, if G has n vertices.

Special cases.

- $K(G) = 0$, iff G is disconnected, $G = K_1$.
- Larger the value of $\underline{K(G)}$, more connected the graph.

Prob.

G_1: cut vertex: b, c, e.

Vertex cut = {b} or {c} or {e}

$K(G_1) = 1$.

G_2: cut vertex: c.

Vertex cut = {c}

$K(G_2) = 1.$

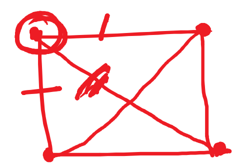
G_5 : cut vertex $= \phi$

vertex cut $= \{b, f, g\}$

$K(G_5) = 3.$

Edge connectivity

$\lambda(G) = 0$, if G is not connected and consists of one vertex.

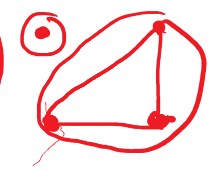


K_4.

For a graph with n vertices,

$$0 \leq \lambda(G) \leq (n-1)$$

For complete graphs (K_n), $\lambda(G) = (n-1)$



${}^nC_2$

Prob.

G_3:

cut edge $= \phi$

edge cut $= \{(a, b), (a, g)\}, \{(b, c), (g, f)\} \ldots$

Edge connectivity, $\lambda(G_3) = 2.$

G_4: cut edge $= \phi$

edge cut = {(b, c), (a, f), (f, g)}.

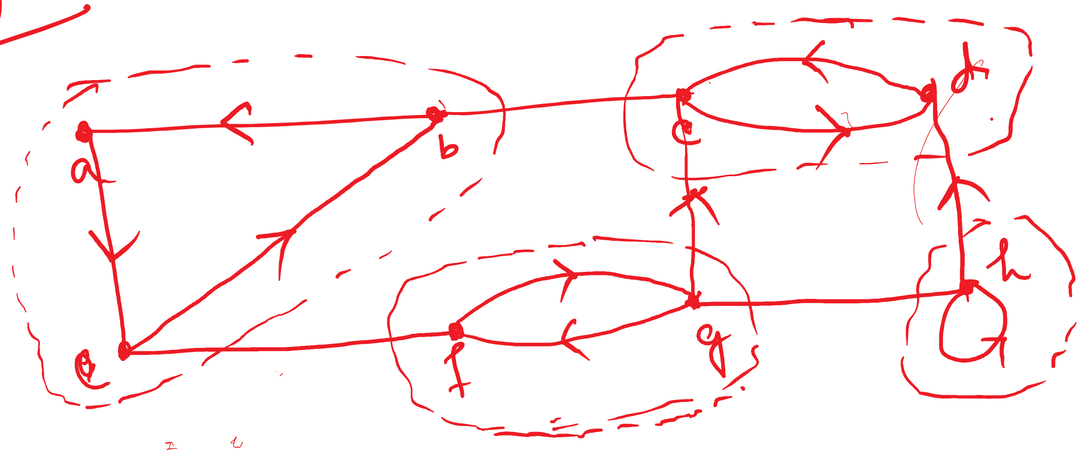
$\lambda(G_3) = \underline{3}.$

* For $G = (V, E)$ to be a non-complete connected graph with $|V| \geq 3$, the minimum degree of a vertex of G is the upper bound for both the vertex connectivity and edge connectivity of G.

i.e. $\kappa(G) \leq \min_{v \in V} \deg(v)$ and

$$\lambda(G) \leq \min_{v \in V} \deg(v)$$

Strongly Connected Components (SCCs)



- 4 SCCs.
- For a graph to have SSCs, it must have cycles.

Prob.

Both G & H have simple ~~circuits~~ of length three, four, and five.

All four isomorphic invariants agree.

G & H are potentially isomorphic.