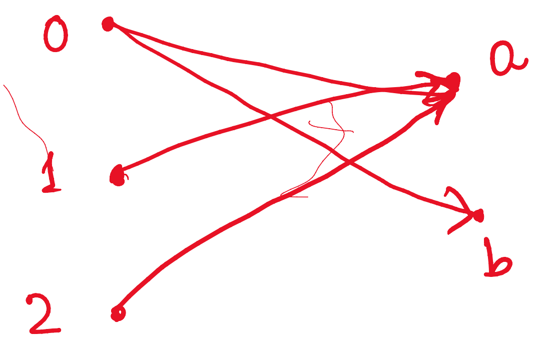
Relations - I

Wednesday, September 30, 2020 9:53 AM

$A = \{0, 1, 2\}$, $B = \{a, b\}$

$R = \{(0,a), (0,b), (1,a), (2,b)\}$

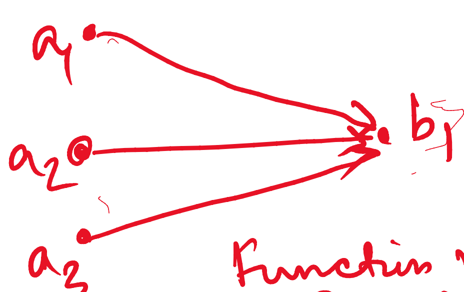
Graphically:



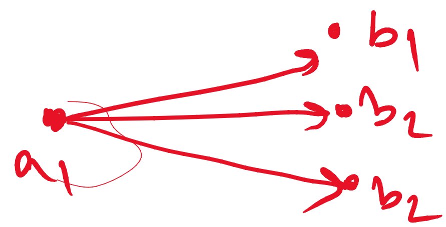
Tables.

R	a	b
0	X	X
1	X	
2		X

Function: It represents a relation where exactly one element of B is related to each element of A.



Function ✓
Relation ✓



Function X
Relation ✓

A relation can be used to express a

A relation can be used to express a one-to-many relationship b/w the elements of the sets A & B.

$$\boxed{R \subseteq A \times A}$$

Ex) $A = \{1, 2, 3, 4\}$

$$R = \{(a, b) \mid a \text{ divides } b\}$$

$$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$$

P-2

$(1,1) \in R_1, R_3, R_4, R_6.$

$\cancel{(1,2) \in R_1, R_6}$

$(2,1) \in R_2, R_5, R_6$

$(1,-1) \in R_2, R_3, R_6.$

$(2,2) \in R_1, R_3, R_4.$

Prob-3

A relation on a set $A \subseteq A \times A$. Therefore, $A \times A$ has n^2 elements. We know that if a set has n elements, then it has n^2 no. of if a set has n elements, then it has 2^n subsets. Thus, there are 2^{n^2} no. of subsets in $A \times A$. This implies that there are 2^{n^2} no. of possible relations on a set with n elements.

For example, there are $2^{3^2} = 2^9 = 512$ relations on a set $\{a, b, c\}$.

S: R_3 & R_5 — reflexive.

R_1, R_2, R_4, R_6 — not reflexive.

P: R_1, R_3, R_4 — reflexive.

R_2, R_5, R_6 — not reflexive.

Prob-4.

x divides y

$x \mid y$; $x \nmid y$

Because $a|a$ whenever a is a positive integer, the "divides" relation is reflexive. (If we replace the set of +ve integers with the set of all integers, the relation is not reflexive because by ~~definion~~ definition 0 divides 0 is undefined.)

S: R_2 & R_3 — Symmetric

R_4, R_5, R_6 — Antisymmetric.

R_1 — Neither symmetric nor anti-symmetric.

p: R_3, R_4, R_6 — Symmetric.

R_1, R_2, R_4, R_5 — Antisymmetric.

'=' relation is both symmetric & anti-symmetric.

Prob-5

The "divides" relation is not symmetric because $1 | 2$, but $2 \nmid 1$.

It is antisymmetric, for if a and b are positive integers with $a|b$ and $b|a$, then it implies $a = b$.

S: R_3, R_5, R_6 — transitive.

R_1, R_2, R_4 — Not transitive.

p: R_1, R_2, R_3, R_4 — transitive

R_5, R_6 — Not transitive.

Prob-6

Suppose a divides b and b divides c. Then there are positive integers k & l such that $b = ak$ and $c = bl$. Hence, $c = (ak).l = a.(kl)$.

$b = ak$ and $c = bl$. Hence, $c = (ak).l = a.(kl)$.
This implies $a|c$. Therefore, the "divides" relation is transitive.

$$R_1 = \{(1,1), (2,2), (3,3)\}$$

$$R_2 = \{(1,1), (1,2), (1,3), (1,4)\}$$

$$R_1 \cup R_2 = \{(1,1), (2,2), (3,3), (1,2), (1,3), (1,4)\}$$

$$R_1 \cap R_2 = \{(1,1)\}$$

$$R_1 - R_2 = \{(2,2), (3,3)\}$$

$$R_1 \oplus R_2 = (R_1 \cup R_2) - (R_1 \cap R_2)$$
$$= \{(1,2), (1,3), (1,4), (2,2), (3,3)\}$$

Prob-7

(i) $R_1 \cup R_2$: all ordered pairs (a, b), where a ~~is~~ a student who ~~has~~ eithere has taken course b or needs course b to graduate.

(ii) $R_1 \cap R_2$: Student a has taken course b and needs this course to graduate.

(iii) $R_1 \oplus R_2$: student 'a' has taken course 'b' but does not need it to graduate or needs course b to graduate but has not taken it.

(iv) $R_1 - R_2$: student a has taken course b, but does not need it to graduate.

(v) $R_2 - R_1$: Student a has taken course b, which is required to graduate.

Prob-8

$$R_1 = \{(x, y) \mid x < y\}$$
$$R_2 = \{(x, y) \mid x > y\}.$$

(i) $R_1 \cup R_2 = \{(x, y) \mid x \neq y\}$

(ii) $R_1 \cap R_2 = \emptyset$ (it is impossible to have $x < y$ and $x > y$)

(iii) $R_1 - R_2 = \{(x, y) \mid x < y\}$

(iv) $R_2 - R_1 = \{(x, y) \mid x > y\}$

(v) $R_1 \oplus R_2 = (R_1 \cup R_2) - (R_1 \cap R_2)$
$$= \{(x, y) \mid x \neq y\}.$$

Prob-8

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$

$S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$

R composite S $\Rightarrow$ $\underline{\underline{S \circ R}} = \{(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)\}$.

Let R be the relation on the set of all people such that $(a,b) \in R$ if is a person 'a' is a parent of person 'b'.

$(a,c) \in R \circ R$ iff there is a person b such that $(a,b) \in R$ and $(b,c) \in R$.

Then, $(a,c) \in R \circ R$ iff a is a ~~grand~~ grandparent of c.

Prob-10

$R = \{(1,1), (2,1), (3,2), (4,3)\}$.

$R = \{(1,1), (2,1), (3,2), (4,3)\}$

$R^2 = R \circ R = \{(1,1), (2,1), (3,1), (4,2)\}$.

$R^3 = R^2 \circ R = \{(1,1), (2,1), (3,1), (4,1)\}$ =

$\{(1,1), (2,1), (3,1), (4,1)\}$ =

$R = R \circ R = \{$

$R^4 = R^3 \circ R = \{(1,1),(2,1),(3,1),(4,1)\} =$

Prob-11

$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4),$

$(a_3, b_1), (a_3, b_3), (a_3, b_5)\}$