Graph - 4.

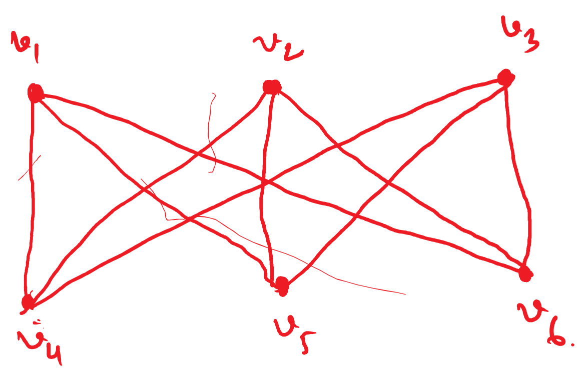
Monday, December 7, 2020 10:53 AM

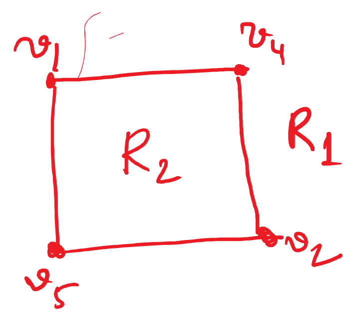
Best case
Avg. case
Worst case. → Lower bound.

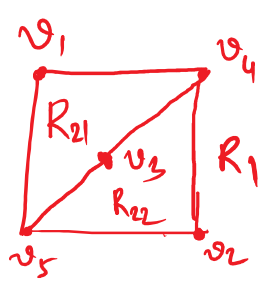
Travelling Salesman Problem (TSP)

- No algorithm with polynomial worst-case time complexity exists.
- Alternate approach — approximation algorithms.
- They do not necessarily produce the exact solution to the problem, but instead guarantees to produce a solution that is close to the exact solution.
- They may produce a HAM-CIRCUIT with a total length W' such that $W \leq W' \leq c.W$, where W is the total length of an exact solution and 'c' is a constant.

$K_{3,3}$.







There is no way to place the final vertex v_6 without forcing a crossing,

Hence, $K_{3,3}$ is not a planar graph.

Prob. No. of vertices = 20.

Degree of each vertex = 3.

$\therefore$ Sum of the degrees of the vertices $= (20\times 3) = \underline{60}$

We know, $$2e = \sum_{v\in V} \deg(v)$$

$$\therefore \quad e = 60/2 = \underline{30}.$$

$\therefore$ No of regions in the planar representation $=$ $e - v + 2 = 30 - 20 + 2 = \underline{12}.$

Corollary-1

Proof: Let a connected planar simple graph divides a plane into r regions.

The degree of each region is at least three as the no. of vertices, $v \geq 3$.

As each edge occurs on the boundary of regions exactly twice, the sum of the degrees of the regions is exactly twice the no. of edges.

Because the degree of each region is at least three, it follows,

$$2e = \sum_{\text{all region } R} \deg(R) \geq 3r.$$

Hence, $(2/3)e \geq r$.

Hence, $\left(\frac{2}{3}\right)e \geq r$.

$r = e - v + 2$

Using Euler's formula we get,

$$e - v + 2 \leq \left(\frac{2}{3}\right)e$$

$$\text{or, } \left(\frac{1}{3}\right)e \leq v - 2$$

$$\therefore \boxed{e \leq 3v - 6}$$

Corollary-2

Proof: If G has one or two vertices, the result is true. If G has at least three vertices, by Corollary-1, we have $e \leq 3v - 6$ so that

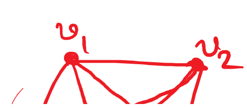
$$\boxed{2e \leq 6v - 12}$$

If the degree of every vertex were at least six then because $2e = \sum_{v \in V} \deg(v)$, we could have $\boxed{2e \geq 6v.}$

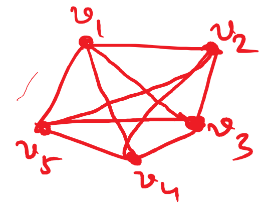
But this is a contradiction to the inequality $2e \leq 6v - 12$. It follows that there must be a vertex with degree no greater than five.

Prob. Show that K_5 is nonplanar using Corollary-1.

→ In graph K_5, there are 5 vertices and



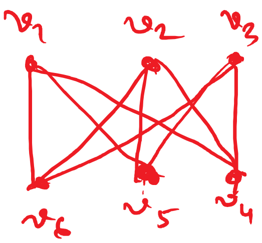
→ In graph K_5, there are 5 vertices and 10 edges.



However, the inequality $e \leq 3v - 6$. is not satisfied. Therefore, K_5 is not planar.

Prob: Show that $K_{3,3}$ is planar or non-planar using Corollary-1.

→ The graph $K_{3,3}$ has 6 vertices and 9 edges. the inequality $e \leq 3v - 6$



The inequality satisfies. So according to Corollary-1 $K_{3,3}$ is planar. However, it can be shown that $K_{3,3}$ is nonplanar. Therefore, even if the inequality in Corollary-1 is satisfied, it does not imply that the graph is planar.