Quantifiers – 1

Monday, August 31, 2020 11:32 AM

Universal quantification $\forall x\, P(x)$ is essentially a conjunction:

$$\forall x\, P(x) \equiv P(x_1) \wedge P(x_2) \wedge \dots \wedge P(x_n)$$ → this is true iff $P(x_1), P(x_2), \dots P(x_n)$ are true.

Existential quantification $\exists x\, P(x)$ is essentially a disjunction:

$$\exists x\, Q(x) = Q(x_1) \vee Q(x_2) \vee \dots \vee Q(x_n)$$ → this is true if any one of $Q(x_1), Q(x_2), \dots Q(x_n)$ is true.

<u>Empty domain of discourse</u>.

✓ $\forall x\, P(x)$: always true (because there is no exception to prove $P(x)$ is false)

✓ $\exists x\, P(x)$: always false (because there is no element for which $P(x)$ is true)

- Every student in this class $\underbrace{\text{has studied Calculus}}_{C(x)}$.

For every student x in this class, x has studied Calculus

$\Rightarrow$ $\underline{\forall x\, C(x)}$, domain includes all students in the class.

Domain of discourse is change to <u>all people in the world</u>.

For every person x, if x is a student in this class, then has studied calculus

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$S(x) = x$ is a student in this class.

$$\forall x\,(S(x) \rightarrow C(x))$$

$$\underbrace{\forall x\,(S(x) \wedge C(x))}$$

all people are students in this class and have studied calculus. X

Let $Q(x, y) :=$ student x has studied course y.

replace $C(x)$ by $Q(x, \text{Calculus})$

(i) $\forall x\, Q(x, \text{calculus})$, '$x$' is a student in the class.

(ii) $\forall x\,(S(x) \rightarrow Q(x, \text{Calculus}))$, '$x$' is any person in the world.

✓ Some student in this class $\underbrace{\text{has visited Mexico}}_{M(x)}$

$\exists x\, M(x)$, x is a student in the class.

Expanded domain: all people in the world.

There is a person x such that x is a student in this class and x has visited Mexico $\Rightarrow \exists x\,(S(x) \wedge M(x))$

X $\exists x\,(S(x) \rightarrow M(x))$ $\Rightarrow$ true even if x is not a student in the class.

Conditionals & Conjunction

"R has propert P"

Conditionals & Conjunction

class.

"Everything that satisfies R has property P" — $\forall x\,(R(x) \rightarrow P(x))$

"Something that satisfies R has property P — $\exists x\,(R(x) \wedge P(x))$

Universal quantifier → Conditional statements
Existential quantifier → conjunctive statement

De Morgan's Laws

① $\neg(p \wedge q) \equiv \neg p \vee \neg q$

② $\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\forall x\, P(x)$ = conj. of $P(x_i)$

$\exists x\, P(x)$ = disj. of $P(x_i)$

When the domain has elements $x_1, x_2, \ldots, x_n$, then:

$$\neg \forall x\, P(x) \equiv \neg(P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n))$$
$$\equiv \neg P(x_1) \vee \neg P(x_2) \vee \ldots \vee \neg P(x_n)$$
$$\equiv \exists x\, \neg P(x)$$

$$\neg \exists x\, P(x) \equiv \neg(P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n))$$
$$\equiv \neg P(x_1) \wedge \neg P(x_2) \wedge \ldots \wedge \neg P(x_n)$$
$$\equiv \forall x\, \neg P(x)$$