Relation - 6

Wednesday, October 21, 2020 8:56 AM

Partial Ordering

Prob. $x \underline{\underline{R}} y \Rightarrow x$ and y are people and x is older than y.

(x, y)

(y, x)

Note that R is anti-symmetric because if a person x is older than y, then y is not older than x i.e. if $x \underline{\underline{R}} y$, then $y \underline{\underline{\not R}} x$

The relation R is transitive because if x is older than y, y is older than z, then x is older than z i.e. xRy and yRz implies xRz.

The relation R is not reflexive, because no person is older than himself i.e. $x \not R x$. for all x. It follows that R is not a partial ordering.

Prob. (i) $(3,5) \prec (4,8)$; (ii) $(3,8) \prec (4,5)$

(iii) $(4,9) \prec (4,11)$

Because $3 < 4$, it follows that $(3,5) \prec (4,8)$ and $(3,8) \prec (4,5)$.

We have $(4,9) \preceq (4,11)$ because the first entries are the same but $9 < 11$.

$(1, 2, 3, 5) \preceq (1, 2, 4, 3)$

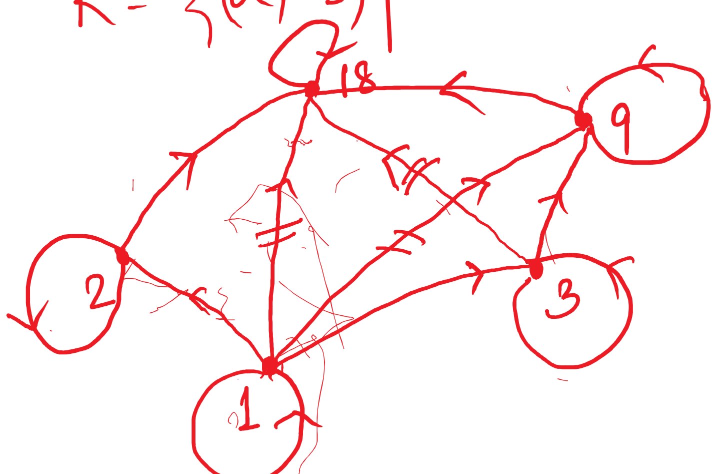
discreet $\preceq$ discrete as $e \preceq t$ in the 7^{th} position.

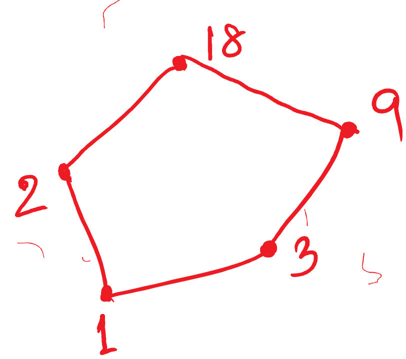
discreet $\preceq$ discreetness, because the first either letters they agree and the second string is longer.

Prob.

Let $A = \{1, 2, 3, 9, 18\}$

$R = \{(a, b) \mid a \text{ divides } b\}$.





← Hasse diagram

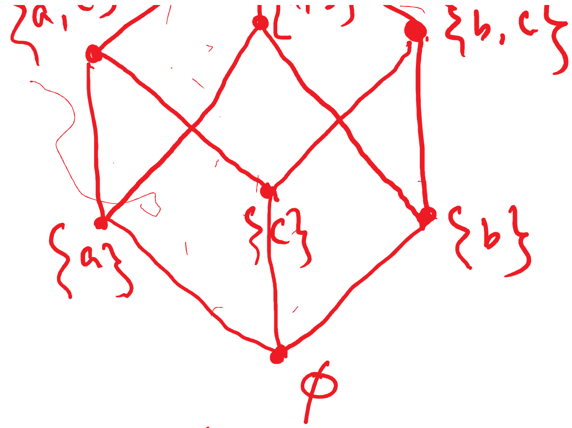
$S = \{a, b, c\}$

$P(S) = \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}.$

(i) Remove all the loops present at every vertex

(ii) Remove all edges that occur from transitivity, namely, $(\phi, \{a, b\})$, $(\phi, \{a, c\})$, $(\phi, \{b, c\})$, $(\phi, \{a, b, c\})$, $(\{a\}, \{a, b, c\})$, $(\{b\}, \{a, b, c\})$, and $(\{c\}, \{a, b, c\})$.

(iii) Point all the edges upward and delete the arrows.

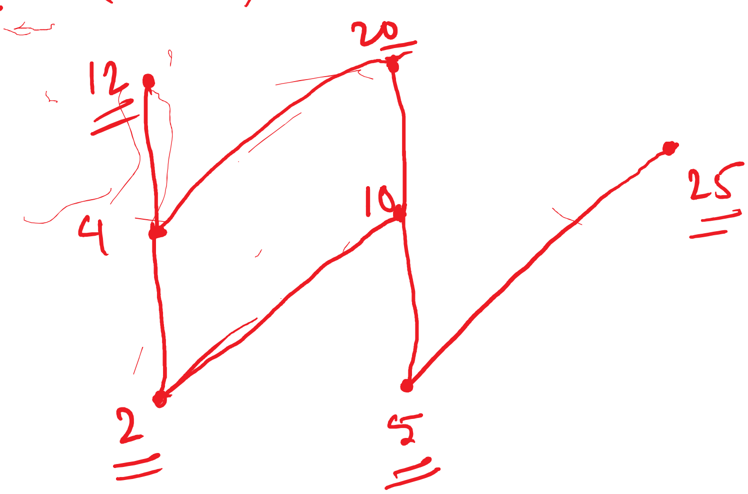


Hasse diagram.

Prob.

$S = \{2, 4, 5, 10, 12, 20, 25\}$

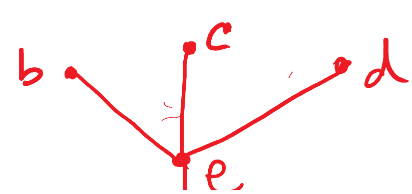
Poset: $(S, |)$



Maximal elements = 12, 20, 25

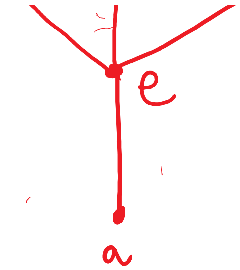
Minimal elements = 2, 5 → which are not preceded by another element.

Prob.



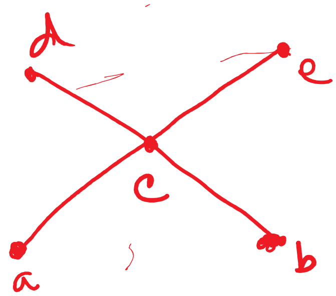
least element – e.

Exe:



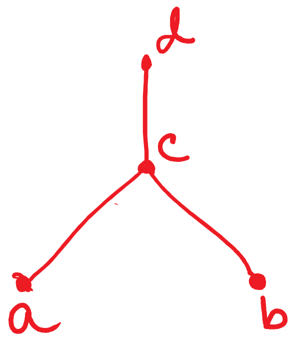
least element - a.

greatest element - not present



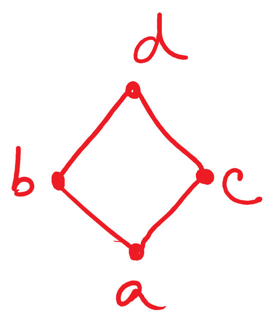
least element - not present

greatest element - not present.



least element - no present

greatest element - d.

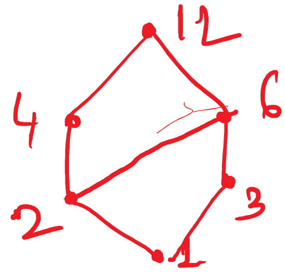


least element - a

greatest element - d.

Ex:
$S = \{1, 2, 3, 4, 6, 12\}$

Draw the Hasse diagram of $(S, |)$



Maximal element - 12

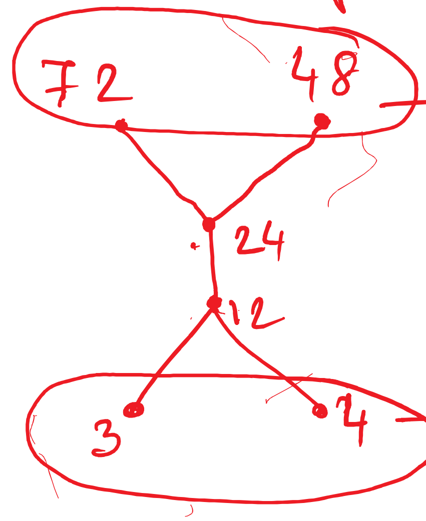
greatest element - 12

Minimal element - 1

least element - 1.

Ex: $S = \{3, 4, 12, 24, 48, 72\}$

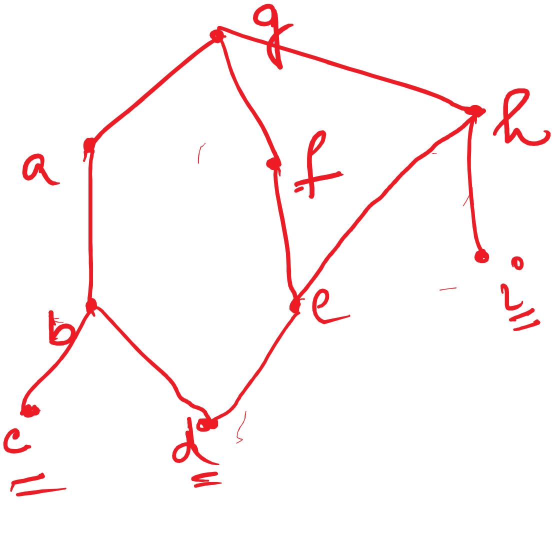
Draw the Hasse diagram of $(S, |)$.



maximal elements. (no greatest element)

minimal elements. (no least element)

Ex: $A = \{a, b, c, d, e, f, g, h, i\}$ has the partial ording $\leq$ defined by the following Hasse diagram. Find all maximal, minimal, greatest and least element of A.



$i \leq h$

$c \leq b$

$d \leq b$

$d \leq e$.

Maximal element — g
Greatest element — g
Minimal elements — c, d, i
Least element — not present.

Prob

Poset : $(P(S), \subseteq)$

✓ Least element — empty set, because
(minimal) $\phi \subseteq T$ for any subset T of S.

✓ Greatest element — Set S because $T \subseteq S$
(maximal) whenever T is a subset of S.

Prob

Poset: $(Z^+, |)$.

✓ Least element :— 1 ; because $1 \mid n$ whenever $n \in Z^+$.

✓ Greatest element — no greatest element as there is no integer that is divisible by all positive integers.