4/12/17

Indian Institute of Engineering Science and Technology, Shibpur

Five year Dual Degree (B.Tech-M.Tech) 3rd Semester Examination 2017

Discrete Structures (CS-303)

Full marks: 70 Time: 3 hours

Group A

(Answer any FOUR questions)

1. a) What is the recurrence relation for solving the following problem:

The lifespan of a rabbit is exactly 10 years. Suppose that there are two newborn rabbits at the beginning, and the number of newborn rabbits in each year is two times that in the previous year. Determine the number of rabbits there are in the r-th year.

b) Solve the recurrence relation: $a_r^2 - 2a_{r-1} = 0$, given that $a_0 = 4$. (2+5)

2. a) Determine the discrete numeric function corresponding to the following generating function:

$$A(z) = \frac{z^5}{5-6z+z^2}$$

b) Using the method of generating function find the number of ways in which 3 balls can be selected from 10 balls among which a particular ball B will be selected at most twice and the others will be selected at most once. (3+4)

3. Find the recurrence equations of the following problem and solve it.

There are two kinds of particles inside a nuclear reactor. In every second, one α particle will split into three β particles, and one β particle will split into one α particle and two β particles. If there is a single α particle in a reactor at time t = 0, how many particles are there altogether at t = 100? (7)

4. a) Define Boolean Lattice and Boolean Algebra.

b) Let (A, ∨, ∧, ~) be a finite Boolean algebra. Let b be any non-zero element in A, and $a_1, a_2, \ldots, a_r$ be all the atoms of A such that $a_i \leq b$. Prove that b = $a_1 \vee a_2 \vee \ldots \vee a_r$. Here, '∨' and '∧' are the two binary operations on A and '~' is the unary operation on A with their usual meaning. (2+5)

5. a) Determine the particular solution of the following recurrence relations:

i) $a_r - 2a_{r-1} = 3.2^r$ ii) $a_r = a_{r-1} + 7$

b) Find the generating functions for the following numeric functions:

i) Accumulated sum of a ii) Forward difference of a

where, 'a' is a given numeric function ((2+2) + (1½ + 1½))

Group B
(Answer any THREE questions)

6. a) State Konigsberg Bridge Problem. What is the solution of the problem? Explain

b) Consider following two graphs:

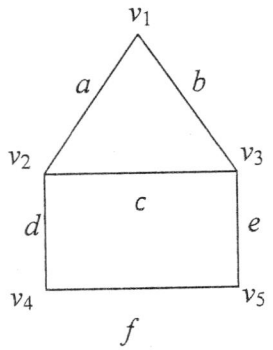

Fig. 1: Graph G_1

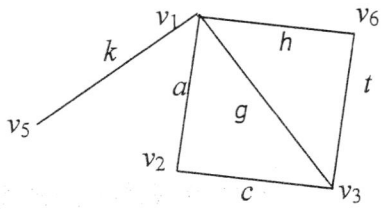

Fig. 2: Graph G_2

Construct the resultant graph after applying the following operations:

(i) Ring sum of G_1 and G_2 (ii) Fusion of v_1 and v_3 in G_1

(iii) Delete the vertex v_1 from G_2 (iv) Delete edges a, c, and d from G_1 (3+4)

7. a) Draw a graph in which an Euler line is also a Hamiltonian circuit.

b) Prove that a graph G with n vertices always has a Hamiltonian path if the sum of the degrees of every pair of vertices v_i, v_j in G satisfies the following condition

$d(v_i) + d(v_j) \geq n - 1$ (2+5)

8. a) (i) How many centers a graph may have? Explain

(ii) Prove that every tree has either one or two centers.

b) Define a spanning tree. Prove that every connected graph has at least one spanning tree.

((1½ + 2½)+3)

9. a) Define the terms ***Rank*** and ***Nullity*** of a disconnected graph with k components.

b) Prove that in a complete graph with n vertices there are $\frac{n-1}{2}$ edge-disjoint Hamiltonian circuits, if n is an odd number ≥ 3. (3+4)

Group C
(Answer any THREE questions)

10. a) Categorize the following wffs into valid, invalid, satisfiable, or unsatisfiable.

i) $\sim ((p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (q \rightarrow r))$

ii) $(p \rightarrow (\sim q \rightarrow r)) \wedge (p \rightarrow \sim q) \rightarrow (p \rightarrow r)$

b) Justify whether the following arguments are valid.

If Ram went to see a movie, then his father was negligent or his mother was not at home. If his mother was not at home, then his father was not negligent. His mother was at home. Therefore, Ram went to see a movie. (3+4)

11. a) Consider the formula $\forall x\ (P(x) \rightarrow R(Q(x), a))$ and the following interpretation:

Domain D = {1, 2}
Assignment for a: $a = 1$
Assignment for P, Q, and R:

P(1)	P(2)	Q(1)	Q(2)	R(1, 1)	R(1, 2)	R(2, 1)	R(2, 2)
False	True	2	1	True	True	False	True

Mention which are functions and which are predicates in the above formula. Evaluate the formula under above interpretation.

b) Convert the formula $(P \wedge (Q \rightarrow R)) \rightarrow S$ into Conjunctive Normal Form (CNF). (4+3)

12. a) Write an algorithm to obtain Prenex Normal Form (PNF) of a FOPL formula.
b) Obtain PNF of the following formula:

$$\exists x\ (\sim \exists y\ P(x, y)) \rightarrow \exists z\ (Q(z) \rightarrow R(x))$$ (3+4)

13. a) Some people like all doctors. No people like any quack. Prove that no doctor is a quack.
b) All men are mortal. Confucius is a man. Prove that Confucius is mortal. (3½ + 3½)