28.07.2020

Fourier Tranform

Fourier tranform is a mathematical tool used for frequency analysis of signals. It is a frequency domain representation of original signal.

For a continuous time periodic signal,

$$x_p(t) = \sum_{n=-\alpha}^{\alpha} a_K e^{jK\omega_0 t} \quad \text{--- (1)}$$

where $\omega_0 = \frac{2\pi}{T}$ (fundamental angular frequency)

$$a_K = \frac{1}{T}\int_T x_p(t)\, e^{-jK\omega_0 t}\, dt$$

~~Let us define $x(jk\omega_0) =$~~

$$a_K \cdot T = \int_T x_p(t) \cdot e^{-jK\omega_0 t} \cdot dt$$

$$a_K \cdot \frac{2\pi}{\omega_0} = \int_T x_p(t) \cdot e^{-jK\omega_0 t}\, dt$$

$$\text{Let } x(jK\omega_0) = \int_T x_p(t) \cdot e^{-jK\omega_0 t} \cdot dt \quad \text{--- (2)}$$

$$a_K \cdot \frac{2\pi}{\omega_0} = x(jK\omega_0)$$

$$a_K = \frac{\omega_0}{2\pi} \cdot x(jK\omega_0)$$

from eq (1)

$$x_p(t) = \frac{1}{2\pi} \sum_{K=-\alpha}^{\alpha} x(jK\omega_0)\, e^{jK\omega_0 t}\, \omega_0 \quad \text{--- (3)}$$

A non-periodic signal may be assumed as a limiting case of a periodic signal where the period of the signal approaches infinity.

A non-periodic continuous time signal $x(t)$ can be viewed as periodic signal $x_p(t)$ with time period $T \to \alpha$, freq $\omega_0 \to 0$

As $T \to \alpha$, $x_p(t) \to x(t)$ and also

$k\omega_0 \to \omega$ (continuous variable)

$\omega_0 \to d\omega$ (differential variable)

from eq ②

$$X(j\omega) = \int_{-\alpha}^{\alpha} x(t) . e^{-j\omega t} dt \quad \text{---} ④$$

from eq ③

$$x(t) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} X(j\omega) . e^{j\omega t} . d\omega \quad \text{---} ⑤$$

eq-④ is known as Fourier transform or Fourier integral of non-periodic signal.

eq-⑤ is known as inverse fourier transform of $X(j\omega)$

where $X(j\omega)$ is the frequency domain representation at time domain fuction $x(t)$

$$x(t) \xrightarrow{F.Tr} X(j\omega)$$

$X(j\omega)$ is complex function of ω and may be expressed as

$$X(j\omega) = |X(j\omega)| e^{j\theta(\omega)}$$

where $|X(j\omega)| \to$ amplitude spectrum

and $Arg[X(j\omega)] = \theta(\omega) \to$ phase spectrum

Condition for existance of Fourier transform
(Dirichlet conditions)

① Signal should have finite number of maxima and minima over any finite interval

② Signal should have finite number of discontinuities ~~at~~ over any finite interval.

③ Signal should be absolutely integrable.

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

ES is absolutely Integrable ✓
NENP is not " " ✗
impulse related NENP → absolutely integrable ✓

* this conditions are sufficient but not ~~nesse~~ necessary

properties of Fourier transform

① linearity → $x_1(t) \rightleftharpoons X_1(j\omega)$
$x_2(t) \rightleftharpoons X_2(j\omega)$

$$Ax_1(t) + Bx_2(t) = AX_1(j\omega) + BX_2(j\omega)$$

② Conjugations →

$$x(t) \rightleftharpoons X(j\omega)$$
$$x^*(t) \rightleftharpoons X^*(-j\omega)$$

③ Area under time domain $x(t)$

$$\text{Area under time domain } x(t) = \int_{-\infty}^{\infty} x(t)\, dt$$

⇓

fourier transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt$$

$\forall\ \omega = 0$

$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt = \text{Area under } x(t)$$

Area under time domain function $x(t)$

$$= X(j\omega)\big|_{\omega=0}$$

4 Area under $X(j\omega)$:

$$\text{Area } X(j\omega) = \int_{-\infty}^{\infty} X(j\omega) \cdot d\omega$$

Inv F Tr, $x(t) = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} X(j\omega) \cdot e^{j\omega t} d\omega$

$$\Downarrow t = 0$$

$$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \cdot d\omega$$

$$2\pi\, x(0) = \int_{-\infty}^{\infty} X(j\omega) \cdot d\omega$$

$$= \text{Area under } X(j\omega)$$

$$\text{Area under } X(j\omega) = 2\pi\, x(t)\big|_{t=0}$$

5 Time Reversal

$$x(t) \rightleftharpoons X(j\omega)$$
$$x(-t) \rightleftharpoons X(-j\omega)$$

6 Time scaling

$$x(t) \rightleftharpoons X(j\omega)$$
$$x(at) \rightleftharpoons \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$, a is real constant

$+a$

$$x(at) = \frac{1}{a} X\left(\frac{j\omega}{a}\right)$$

$-a$

$$x(-a) = \frac{1}{a} X\left(-\frac{j\omega}{a}\right)$$

7 Time shifting:

$$x(t) \rightleftharpoons X(j\omega)$$
$$x(t+t_0) \rightleftharpoons X(j\omega) \cdot e^{j\omega t_0}$$
$$x(t-t_0) \rightleftharpoons X(j\omega) \cdot e^{-j\omega t_0}$$

⑧ Freq shifting

$$x(t) \rightleftharpoons X(j\omega)$$

$$e^{+j\omega_0 t}\, x(t) \rightleftharpoons X[j(\omega-\omega_0)]$$

$$e^{-j\omega_0 t}\, x(t) \rightleftharpoons X[j(\omega+\omega_0)]$$

⑨ Convolution in time:

$$x_1(t) \rightleftharpoons X_1(j\omega)$$

$$x_2(t) \rightleftharpoons X_2(j\omega)$$

$$x_1(t) * x_2(t) \rightleftharpoons X_1(j\omega) \cdot X_2(j\omega)$$

⑩ Multiplication in time:

$$x_1(t) \cdot x_2(t) \rightleftharpoons \frac{1}{2\pi}\{X_1(j\omega) * X_2(j\omega)\}$$

⑪ Differentiation in time:

$$x(t) \rightleftharpoons X(j\omega)$$

$$\frac{dx(t)}{dt} \rightleftharpoons (j\omega)X(j\omega)$$

$$\frac{d^2x(t)}{dt^2} = (j\omega)^2 X(j\omega)$$

$$\frac{d^n x(t)}{dt^n} = (j\omega)^n X(j\omega)$$

⑫ Integration in time.

$$x(t) \rightleftharpoons X(j\omega)$$

$$\int x(t) \rightleftharpoons \frac{X(j\omega)}{j\omega} + \pi X(0)\cdot \delta(\omega).$$

⑬ Differenciation in frequency

$$x(t) \rightleftharpoons X(j\omega)$$

$$t^n x(t) \rightleftharpoons (j)^n \frac{d^n X(j\omega)}{d\omega^n}$$

(14) Modulation:

$x(t) \rightleftharpoons X(j\omega)$ or $X(\omega)$

(i) $x(t) \cos\omega_0 t \rightleftharpoons \frac{1}{2}\left[X(\omega+\omega_0) + X(\omega-\omega_0)\right]$

(ii) $x(t) \sin\omega_0 t \rightleftharpoons \frac{j}{2}\left[X(\omega+\omega_0) - X(\omega-\omega_0)\right]$

(15) Parseval's Energy theorem:

$x(t) = X(j\omega)$

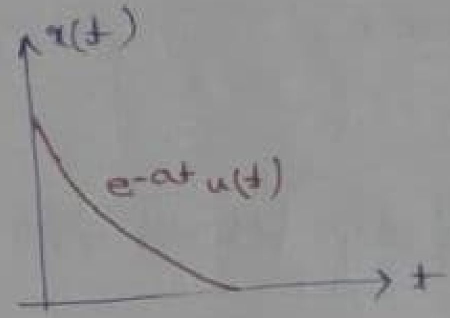
Total energy $E_{x(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

Fourier Transform pairs.

$x(t)$	$X(j\omega)$
$e^{-at} u(t)$	$\frac{1}{a+j\omega}$
$e^{at} u(-t)$	$\frac{1}{a-j\omega}$
$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$
$t\, e^{-at} u(t)$	$\frac{1}{(a+j\omega)^2}$
$\delta(t)$	1
$\delta(t \mp t_0)$	$e^{\mp j\omega t_0}$
1	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$
$\sin\omega_0 t$	$j\pi[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$

$e^{-at} \sin\omega_0 t \cdot u(t) \qquad \dfrac{\omega_0}{(a+j\omega)^2 + \omega_0^2}$

Q ① Single sided exponential function
$e^{-at} u(t) \rightarrow$ Find fourier transform.
Also draw the spectrum (where $a>0$)



$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \, dt$$

$$= \int_{-\infty}^{\infty} e^{-at} u(t) \, e^{-j\omega t} \, dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot 1 \cdot e^{-j\omega t} \, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} \, dt$$

$$= \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_0^{\infty}$$

$$= -\frac{1}{(a+j\omega)} \left[e^{-\infty} - e^{0} \right] = \frac{1}{a+j\omega}$$

$$\boxed{X(j\omega) = \frac{1}{a+j\omega}}$$

$$X(j\omega) = |X(j\omega)| \, e^{j\theta(\omega)}$$

$$X(j\omega) = \frac{1}{a+j\omega} = \frac{a-j\omega}{(a+j\omega)(a-j\omega)}$$

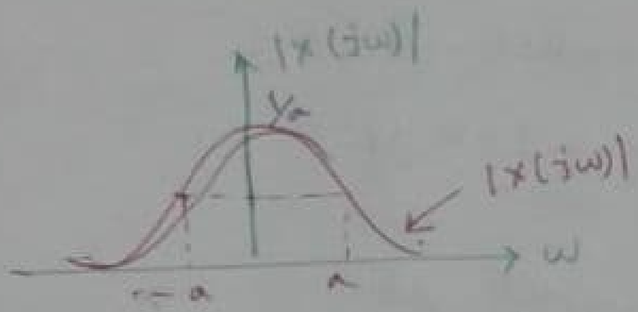
$$= \frac{a-j\omega}{a^2+\omega^2}$$

$$X(j\omega) = \frac{a}{a^2+\omega^2} - j\frac{\omega}{a^2+\omega^2}$$

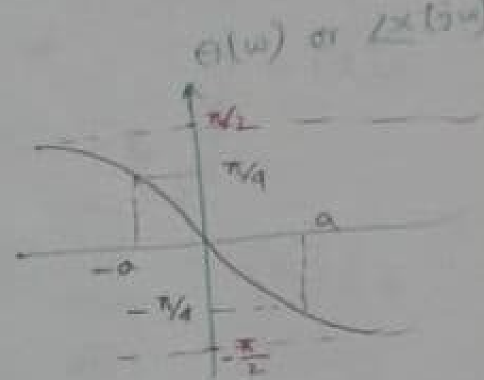
$$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$\theta(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

$$X(j\omega) = \frac{1}{\sqrt{a^2+\omega^2}}\, e^{-j\tan^{-1}\frac{\omega}{a}}$$

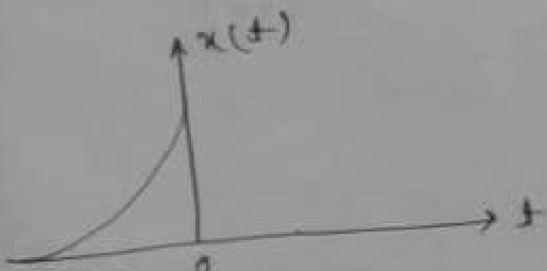


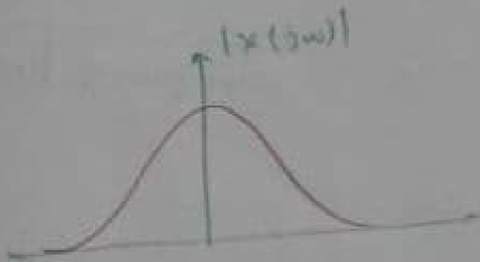
Amplitude spectrum

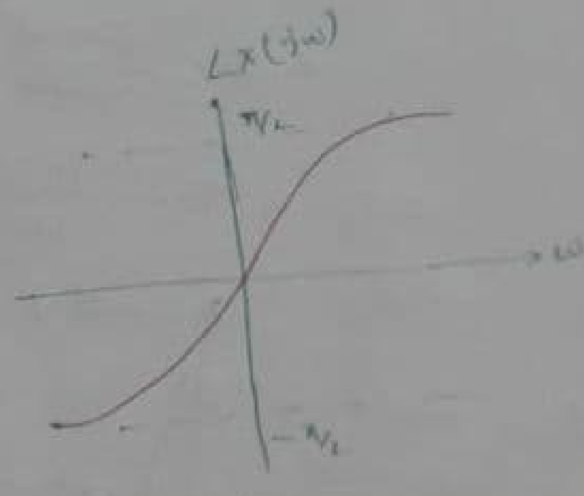


phase spectrum

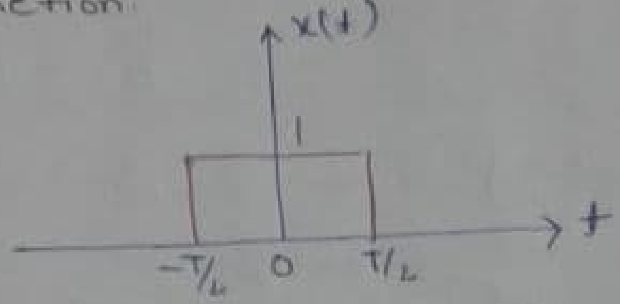
② $e^{at}u(-t) \quad a>0$







③ Find the Fourier Transform of gate function.



$$x(t) = \begin{cases} 1 & \text{for } -T/2 < t < T/2 \\ 0 & \text{otherwise} \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \, dt$$

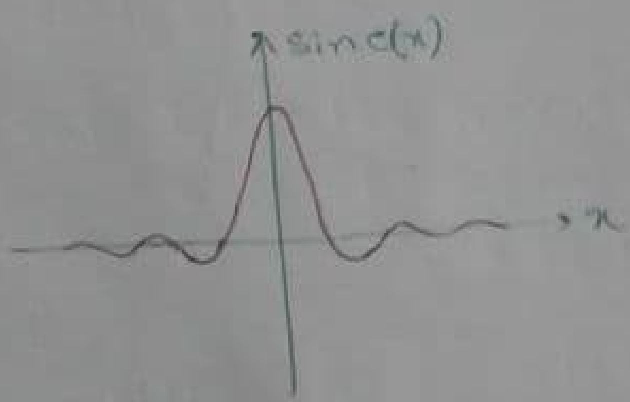
$$= \int_{-T/2}^{T/2} 1 \cdot e^{-j\omega t} \, dt$$

$$= -\frac{1}{j\omega} e^{-j\omega t} \Big|_{-T/2}^{T/2}$$

$$= \frac{1}{j\omega} \left[e^{j\omega T/2} - e^{-j\omega T/2} \right]$$

$$= 2 \cdot \frac{\sin \omega T/2}{\omega}$$

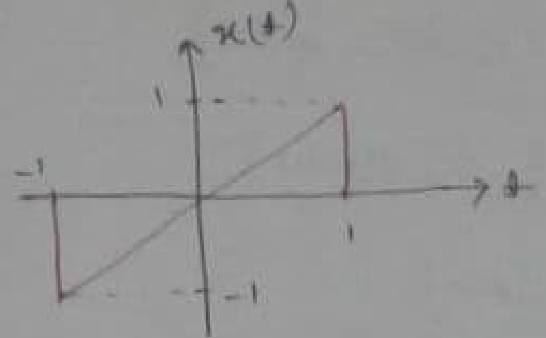
$$= T \cdot \frac{\sin \omega T/2}{\omega T/2} = \underbrace{T \cdot \operatorname{sinc}\left(\frac{\omega T}{2}\right)}_{\text{sampling function}}$$



Sampling function or Interpolating function

$$\operatorname{sinc} x = \frac{\sin x}{x}$$

(9)



Find Fourier transform of the time signals shown in figure

$$x(t) = \begin{cases} t & -1 < t < 1 \\ 0 & \cancel{t > -1}\ \ t < -1 \text{ and } t > 1 \end{cases}$$

$$X(j\omega) = \int_{-\alpha}^{\alpha} x(t) \cdot e^{-j\omega t}\, dt$$

$$= \int_{-1}^{0} (-1)\, e^{-j\omega t}\, dt + \int_{0}^{1} 1\, e^{-j\omega t}\, dt$$

$$= -1 \left[\cancel{e^{-0}} \cdot \frac{e^{-j\omega t}}{-j\omega}\right|_{-1}^{0} + \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{0}^{1}$$

$$= \frac{1}{j\omega}\left[e^{-0} - e^{j\omega}\right] + \frac{1}{-j\omega}\left[e^{-j\omega} - e^{0}\right]$$

$$= -\frac{e^{j\omega}}{j\omega} - \frac{e^{-j\omega}}{j\omega}$$

$$= -\frac{1}{j\omega}\left[e^{j\omega} + e^{-j\omega}\right]$$

$$= -2 \frac{\sin\omega}{\omega}$$

$$= \int_{-1}^{1} t\; e^{-j\omega t}\, dt$$

$$= t \cdot \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-1}^{1} - \int_{-1}^{1} 1 \cdot \frac{e^{-j\omega t}}{-j\omega}\, dt$$

$$= \left[\frac{1 \cdot e^{-j\omega} + 1 \cdot e^{j\omega}}{-j\omega}\right] + \frac{1}{j\omega}\int_{-1}^{1} e^{-j\omega t}\, dt$$

$$= j \cdot \frac{2}{\omega} \cos\omega + \frac{1}{j\omega} \cdot \frac{1}{-j\omega} \cdot \left. e^{-j\omega t}\right|_{-1}^{1}$$

$= j\,\frac{2}{\omega}\cos\omega + \frac{1}{\omega^2}\left(e^{-j\omega} - e^{j\omega}\right)$

$= j\,\frac{2}{\omega}\cos\omega + \frac{1}{\omega^2}\left(-2j\sin\omega\right)$

$= j\,\frac{2}{\omega}\left[\cos\omega - \frac{1}{\omega}\sin\omega\right]$

ex: Find the inverse Fourier Transform of $x(-j\omega) = \dfrac{j\omega+1}{(j\omega)^2+5j\omega+6}$

$$x(j\omega) = \frac{K_1}{j\omega+2} + \frac{K_2}{j\omega+3}$$

$$K_1 = \left.\frac{j\omega+1}{j\omega+3}\right|_{j\omega=-2} = -1$$

$$K_2 = \left.\frac{j\omega+1}{j\omega+2}\right|_{j\omega=-3} = 2$$

$$x(j\omega) = -\frac{1}{j\omega+2} + \frac{2}{j\omega+3}$$

$\Downarrow$ Iinverse fourier transform

$$x(t) = -e^{-2t}u(t) + 2e^{-3t}u(t)$$

$$x(t) = \left(2e^{-3t} - e^{-2t}\right)u(t)$$

ex: Calculate the inverse fourier transform of $x(j\omega) = 2\pi\,\delta(\omega) + \pi\,\delta(\omega-4\pi) + \pi\,\delta(\omega+4\pi)$

ans: $x(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} x(j\omega)\,e^{j\omega t}\,d\omega$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[2\pi\,\delta(\omega) + \pi\,\delta(\omega-4\pi) + \pi\,\delta(\omega+4\pi)\right] e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi}\left[2\pi \cdot e^{j\omega t}\Big|_{t=0} + \pi \cdot e^{j\omega t}\Big|_{t=4\pi} + \pi \cdot e^{j\omega t}\Big|_{t=-4\pi} \right]$$

$$\boxed{x(t_1) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)}$$

$$= \frac{1}{2\pi}\left[2\pi + \pi \cdot e^{j4\pi t} + \pi \cdot e^{-j4\pi t} \right]$$

$$= 1 + \cos 4\pi t$$

$$x(\omega_1) \cdot \delta(\omega - \omega_0) = x(\omega_0) \cdot \delta(\omega - \omega_0)$$

$$= \frac{1}{2\pi}\left[2\pi \cdot e^{j\omega t}\Big|_{t=0} + \pi \cdot e^{j\omega t}\Big|_{t=4\pi} + \pi \cdot e^{j\omega t}\Big|_{t=-4\pi} \right]$$

$$\boxed{x(t_1) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)}$$

$$= \frac{1}{2\pi}\left[2\pi + \pi \cdot e^{j4\pi t} + \pi \cdot e^{-j4\pi t} \right]$$

$$= 1 + \cos 4\pi t$$

$$x(\omega_1) \cdot \delta(\omega - \omega_0) = x(\omega_0) \cdot \delta(\omega - \omega_0)$$

30.09.2020

Unit Impulse Signal

$$\delta(t) = 0 \quad , t \neq 0$$

$$\int_{-\alpha}^{\alpha} \delta(t)\, dt = 1 \quad \text{or} \quad \delta(t) = 1 , \; t = 0$$

$$\int_{-\alpha}^{\alpha} x(t) \cdot \delta(t) \longrightarrow x(t)\Big|_{t=0} = x(0)$$

$$\int_{-\alpha}^{\alpha} x(t) \cdot \delta(t-0) = x(t)\Big|_{t=0}$$

$$\int_{-\alpha}^{\alpha} x(t) \cdot \delta(t - t_0) = x(t)\Big|_{t=t_0} = x(t_0)$$

ex-1

$$\int_{-\alpha}^{\alpha} (t^2+1) \cdot \delta(t) \cdot dt$$

$$= \int_{-\alpha}^{\alpha} t^2 \cdot \delta(t) \cdot dt + \int_{-\alpha}^{\alpha} \delta(t) \cdot dt$$

$$= t^2\Big|_{t=0} + 1 = 0 + 1 = 1$$

ex-2 :

$$\int_{-1}^{2} (t^4+1)\,\delta(t-1)\,dt$$

$$= \int_{-1}^{2} t^4\,\delta(t-1)\,dt + \int_{-1}^{2} \delta(t-1)\,dt$$

$$= t^4\Big|_{t=1} + \delta(1) = 1+1 = 2$$

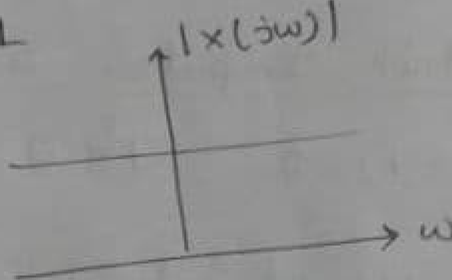
Fourier transform of an impulse function

$$x(t) = \delta(t)$$

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt$$

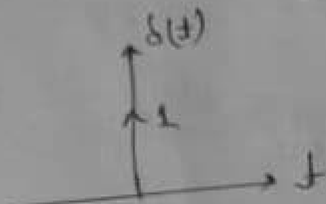
$$= \int_{-\infty}^{\infty} \delta(t)\,e^{-j\omega t}\,dt$$

$$= e^{-j\omega t}\Big|_{t=0} = 1$$



$$f[\delta(t)] = 1$$

$$x(j\omega) = 1$$



Inverse Fourier transform of $\delta(\omega)$

$$F^{-1}[X(j\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega)\,e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi}\,e^{j\omega t}\Big|_{\omega=0}$$

$$F^{-1}[\delta(\omega)] = \frac{1}{2\pi}$$

$$F\left[\frac{1}{2\pi}\right] = \delta(\omega)$$

$$F[1] = 2\pi\,\delta(\omega)$$

Q. Find the inverse Fourier transform of $\delta(\omega-\omega_0)$

$$F^{-1}[\delta(\omega-\omega_0)] = \frac{1}{2\pi}\int_{-\alpha}^{\alpha} \delta(\omega-\omega_0)e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi} e^{j\omega t}\Big|_{\omega=\omega_0}$$

$$F^{-1}[\delta(\omega-\omega_0)] = \frac{1}{2\pi} e^{j\omega_0 t}$$

$$F[e^{j\omega_0 t}] = 2\pi\,\delta(\omega-\omega_0)$$

Similarly

$$F[e^{-j\omega_0 t}] = 2\pi\,\delta(\omega+\omega_0)$$

Find the fourier transform of $\cos\omega_0 t$

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$\cos\omega_0 t = \frac{1}{2}\left[e^{j\omega_0 t}+e^{-j\omega_0 t}\right]$$

$$F[\cos\omega_0 t] = \frac{1}{2}\,F\left[e^{j\omega_0 t}+e^{-j\omega_0 t}\right]$$

$$= \frac{1}{2}\left[2\pi\delta(\omega-\omega_0)+2\pi\delta(\omega+\omega_0)\right]$$

$$F[\cos\omega_0 t] = \pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$$

$$F[\sin\omega_0 t] = \frac{1}{j}\,\pi\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$$

~~Fourier Transfer of periodic Signal~~

$$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$

$$\sin\omega_0 t = \frac{e^{j\omega_0 t}-e^{-j\omega_0 t}}{2j}$$

$$= \frac{1}{2j}\left[2\pi\delta(\omega-\omega_0)-2\pi\delta(\omega+\omega_0)\right]$$

$$F(\sin\omega_0 t) = \frac{\pi}{j}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$$

$$= j\pi\left[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\right]$$

Fourier transform of the periodic Signal.

$x(t) \longrightarrow$ periodic signal

$\downarrow X(\omega)$

$$x(t) = \sum_{n=-\alpha}^{\alpha} C_n e^{jn\omega_0 t}$$

$$F[1] = 2\pi \delta(\omega)$$

$$[C_n \cdot 1] = 2\pi C_n \delta(\omega)$$

$$[C_n \cdot 1 \cdot e^{jn\omega_0 t}] = 2\pi C_n \delta(\omega - n\omega_0)$$

$\rightarrow$ Freq shifting

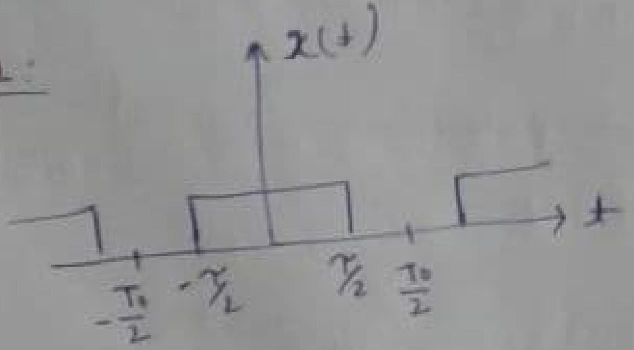
$$x(t) = \sum_{n=-\alpha}^{\alpha} C_n \cdot 1 \cdot e^{jn\omega_0 t}$$

$$= \sum_{n=-\alpha}^{\alpha} 2\pi C_n \delta(\omega - n\omega_0)$$

$$\boxed{X(\omega) = \sum_{n-\alpha}^{\alpha} 2\pi C_n \delta(\omega - n\omega_0)}$$

Fourier transform of periodic signals.

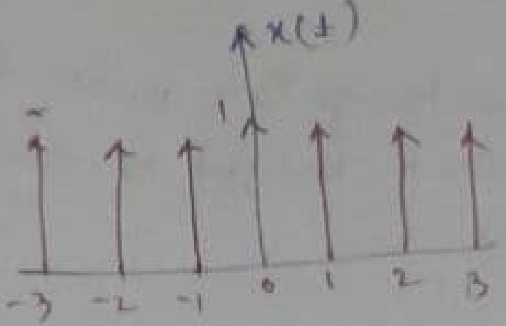
ex-1:



$$C_n = \frac{A_0 \tau}{T_0} \text{ samp}\left(\frac{n\omega_0 \tau}{2}\right)$$

$$X(\omega) = \sum_{n=-\alpha}^{\alpha} 2\pi \cdot \frac{A_0 \cdot \tau}{T_0} \cdot \text{samp}\left(\frac{n\omega_0 \tau}{2}\right) \cdot \delta(\omega - n\omega_0)$$

ex-2



$$x(\omega) = \sum_{n=-\infty}^{\infty} 2\pi \cdot C_n \; \delta(\omega - n\omega_0)$$

$$C_n = \frac{1}{T_0}\int_{T_0} x(t) \cdot e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t) \cdot e^{-jn\omega_0 t}\, dt = \frac{1}{T_0}$$

$$x(\omega) = \sum_{n=-\infty}^{\infty} \cancel{\frac{1}{2\pi} \cdot \frac{1}{T}}\; 2\pi \cdot \frac{1}{T_0}\; \delta(\omega - n\omega_0)$$

$$= \frac{2\pi}{T_0} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

$$= \omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

ex-3 $\quad x(t) = \sin\omega_0 t$

The Fourier series co-efficients for the signal are

$$a_1 = \frac{1}{2j} \qquad a_{-1} = -\frac{1}{2j}$$

$$a_k = 0 \; ; \; k \neq 1 \text{ or } -1$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi \cdot a_k \cdot \delta(\omega - k\omega_0)$$

$$= 2\pi \cdot \left[-\frac{1}{2j}\delta(\omega + \omega_0) + \frac{1}{2j}\delta(\omega - \omega_0)\right]$$

$$= \frac{\pi}{j}\left[-\delta(\omega + \omega_0) + \delta(\omega - \omega_0)\right]$$

$$= \frac{\pi}{j}\left[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right]$$

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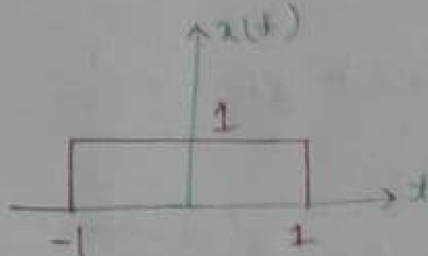
$x(t)$ is a positive rectangular pulse from $t=1$ to $t=1$ with unit height as shown in figure the value of

$$\int_{-\alpha}^{\alpha} |x(\omega)|^2 \, d\omega \quad \text{is}$$

(A) 2 (B) 2π (C) 4 (D) 4π



$$I = \int_{-\alpha}^{\alpha} |X(\omega)|^2 \, d\omega$$

$$= \frac{2\pi}{2\pi} \int_{-\alpha}^{\alpha} |X(\omega)|^2 \, d\omega$$

$$E_{x(t)} = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} |X(\omega)|^2 \, d\omega \qquad \text{parcivals energy theorem}$$

$$I = 2\pi \, E_{x(t)}$$

$E_{x(t)}$ = Total energy

= Area under $|x(t)|^2$

$I = 2\pi \times$ Area under $|x(t)|^2$

$= 2\pi \times 2 = 4\pi$