

CARDINAL UTILITY ANALYSIS
OR
MARGINAL UTILITY ANALYSIS

Assumptions

1. **Rationality:** The consumer is rational. He aims at the maximisation of his utility subject to the constraint imposed by his given income.
2. **Cardinal utility:** The utility of each commodity is measurable. Utility is a cardinal concept. The most convenient measure is money: the utility is measured by the monetary units that the consumer is prepared to pay for another unit of the commodity.
3. **Constant marginal utility of money:** This assumption is necessary if the monetary unit is used as the measure of utility. The essential feature of a standard unit of measurement is that it be constant. If the marginal utility of money changes as income increases (or decreases) the measuring-rod for utility becomes like an elastic ruler, inappropriate for measurement.
4. **Diminishing marginal utility:** The utility gained from successive units of a commodity diminishes. In other words, the marginal utility of a commodity diminishes as the consumer acquires larger quantities of it. This is the axiom of diminishing marginal utility.
5. **Total utility is additive:** $U = U_1(x_1) + U_2(x_2) + \dots + U_n(x_n)$

Price of Apple is Rs 10/piece.

Units of Apple	Total Utility	Marginal Utility
1	30	30
2	45	15
3	50	5
4	50	0
5	45	-5
6	30	-15

Law of Equimarginal Utility

Suppose there are only two goods X and Y on which a consumer has to spend a given income.

The consumer's behavior will be governed by two factors: first, the marginal utilities of the goods and secondly, the prices of two goods.

Suppose the prices of the goods are given for the consumer.

The law of equimarginal utility states that the consumer will distribute his money income between the goods in such a way that the utility derived from the last rupee spend on each good is equal. In other words, consumer is in equilibrium position when marginal utility of money expenditure on each goods is the same.

Law of Equi-marginal Utility with the aid of an Arithmetical Table

Table-Marginal Utility of Goods X and Y:

Units	MU_x (units)	MU_y (Units)
1	20	24
2	18	21
3	16	18
4	14	15
5	12	9
6	10	2

Let the prices of goods X and Y be Rs. 2 and Rs. 5 respectively.

Reconstructing the above table by dividing marginal utilities of X (MU_x) by Rs. 2 and marginal utilities of Y (MU_y) by Rs. 5 we get the table of Marginal Utility of Money Expenditure in the next slide.

Table-Marginal Utility of Money Expenditure

Units	$\frac{MU_x}{P_x}$	$\frac{MU_y}{P_y}$
1	10	8
2	9	7
3	8	6
4	7	5
5	6	3
6	5	1

Suppose, the consumer has Rupees 19 with him to spend on the two goods X and Y. By looking at the table it is clear that $\frac{MU_x}{P_x}$ is equal to 6 units when the consumer purchases 5 units of goods X; and $\frac{MU_y}{P_y}$ is equal to 6 units when he buys 3 units of goods y. Therefore, consumer will be in equilibrium when he is buying 5 units of good X and 3 units of goods Y and will be spending (Rs. 2×5+ Rs. 3×3) = Rs.19 on them.

- The equimarginal condition for the equilibrium of the consumer can be stated in three ways.

(1) A consumer is in equilibrium when he equalizes weighted marginal utilities of all goods, that is, when the marginal utility of each good weighted by its price is equal.

- In other words, when $\frac{MU_x}{P_x} = \frac{MU_y}{P_y} = \frac{MU_n}{P_n}$

(2) A consumer is in equilibrium when he equalizes the ratios of marginal utilities of goods with the ratio of corresponding prices for each pair of goods consumed, that is, when $\frac{MU_x}{MU_y} = P_x/P_y$ and $\frac{MU_y}{MU_z} = P_y/P_z$ and so forth

(3) Since $\frac{MU_x}{P_x}$ measures the marginal utility of a rupee's worth of each good consumed at the given prices, consumer can be said to be in equilibrium when the marginal utility of a rupee spent on each good purchased is equal. Marginal utility of a rupee spent on a good means the marginal utility of a rupee's worth of the good.

Law of Diminishing Marginal Utility

- According to the Law of Diminishing Marginal Utility, marginal utility of a good diminishes as an individual consumes more units of a good. In other words, as a consumer takes more units of a good, the extra utility or satisfaction that he derives from an extra unit of the good goes on falling. The law of diminishing marginal utility means that the total utility increases but at a decreasing rate.
- This law is based upon two important facts. Firstly, while the total wants of a man are virtually unlimited, each single want is satiable. Therefore, as an individual consumes more and more units of goods, intensity of his want for the goods goes on falling and a point is reached where the individual no longer wants any more units of the goods. That is, when saturation point is reached, marginal utility of goods becomes zero. Zero marginal utility of goods implies that the individual has all that he wants of the goods in question.
- The second fact on which the law of diminishing marginal utility is based is that the different goods are not perfect substitutes for each other in the satisfaction of various particular wants. When an individual consumes more and more units of a goods, the intensity of particular want for the goods diminishes but if the units of that goods could be devoted to the satisfaction of other wants and yield as much satisfaction as they did initially in the satisfaction of the first want, marginal utility of the good would not have diminished.

Table: Diminishing Marginal Utility

Cups of Tea Consumed per day	Total Utility (units)	Marginal utility (units)	Consider this table in which we have presented the total and marginal utilities derived by a person from cups of tea consumed per day. When one cup of tea is taken per day, the total utility derived by the person is 12 units. And because this is the first cup its marginal utility is also 12.
1	12	12	With the consumption of 2nd cup per day, the total utility rises to 22 but marginal utility falls to 10. It will be seen from the table that as the consumption of tea increases to six cups per day, marginal utility from the additional cups goes on diminishing (i.e., the total utility goes on increasing at a diminishing rate).
2	22	10	
3	30	8	
4	36	6	
5	40	4	
6	41	1	
7	39	- 2	However, when the cups of tea consumed per day increase to seven, then instead of giving positive marginal utility, the seventh cup gives negative marginal utility equal to -2. This is because too many cups of tea consumed per day (say more than six for a particular individual) may cause him acidity or other health issues. Thus, the extra cups of tea beyond six to the individual in question give him disutility rather than positive satisfaction.
8	34	- 5	

Critique of the cardinal approach

- The assumption of cardinal utility is extremely doubtful. The satisfaction derived from various commodities cannot be measured objectively.
- The assumption of constant utility of money is also unrealistic. As income increases the marginal utility of money changes. Thus money cannot be used as a measuring-rod since its own utility changes.
- Additive utility is also not possible in the real world.

ORDINAL UTILITY ANALYSIS
OR
INDIFFERENCE CURVE ANALYSIS

Assumptions

1. **Rationality:** The consumer is assumed to be rational- he aims at the maximization of his utility, given his income and market prices. It is assumed he has full knowledge (certainty) of all relevant information.
2. **Utility is ordinal:** It is taken as axiomatically true that the consumer can rank his preferences (order the various 'baskets of goods') according to the satisfaction of each basket. He need not know precisely the amount of satisfaction.
3. **Diminishing marginal rate of substitution:** Preferences are ranked in terms of indifference curves, which are assumed to be convex to the origin. This implies that the slope of the indifference curves increases. The slope of the indifference curve is called the marginal rate of substitution of the commodities.
4. The total utility of the consumer depends on the quantities of the commodities consumed. $U = f(q_1, q_2, \dots, q_n)$.
5. **Consistency and transitivity of choice:** It is assumed that the consumer is consistent in his choice, that is, if in one period he chooses bundle A over B, he will not choose B over A in another period if both bundles are available to him. The consistency assumption may be symbolically written as follows: If $A > B$, then $B > A$.
Similarly, it is assumed that consumer's choices are characterized by transitivity: if bundle A is preferred to B, and B is preferred to C, then bundle A, is preferred to C. Symbolically we may write the transitivity assumption as follows: If $A > B$, and $B > C$, then $A > C$.

Equilibrium of the Consumer

- To define the equilibrium of the consumer (that is, his choice of the bundle that maximizes his utility) we must introduce the concept of indifference curves and of their slope (the marginal rate of substitution), and the concept of the budget line. These are the basic tools of the indifference curves approach.

Indifference Curve

- An indifference curve is the locus of points- particular combinations or bundles of goods-which yield the same utility (level of satisfaction) to the consumer, so that he is indifferent as to the particular combination he consumes. An indifference map shows all the indifference curves which rank the preferences of the consumer. Combinations of goods situated on an indifference curve yield the same utility. Combinations of goods lying on a higher indifference curve yield higher level of satisfaction and are preferred. Combinations of goods on a lower indifference curve yield a lower utility.

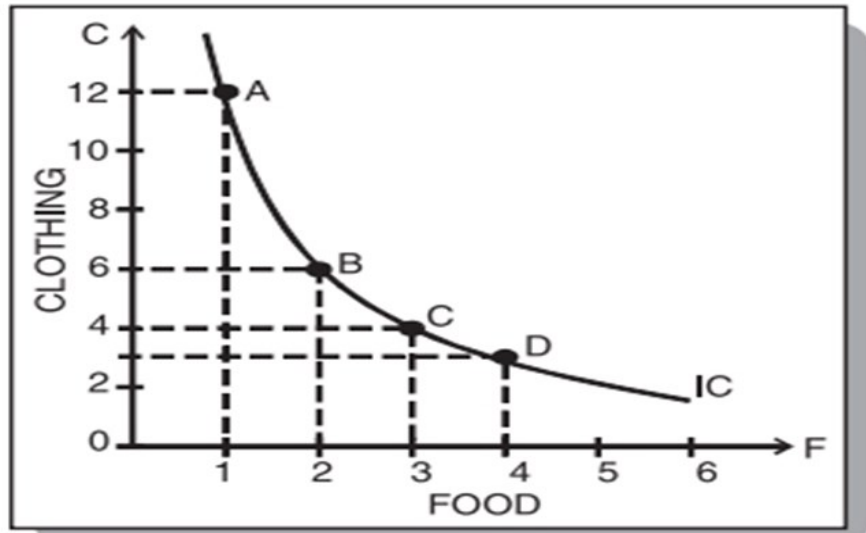
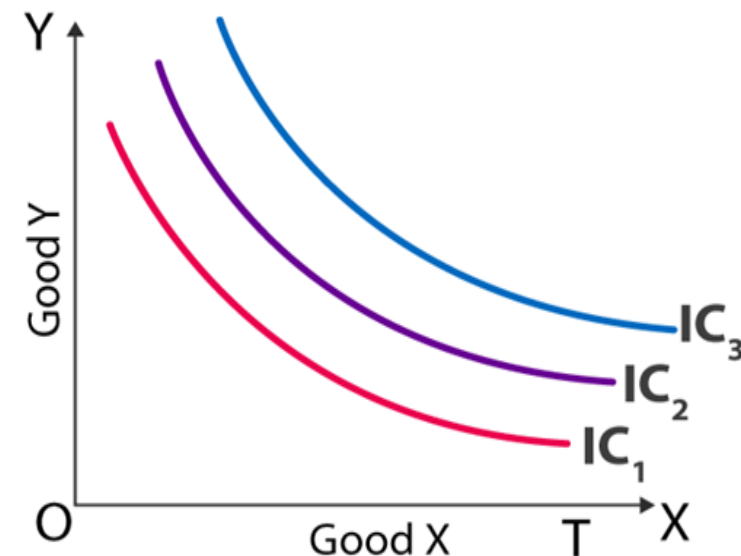
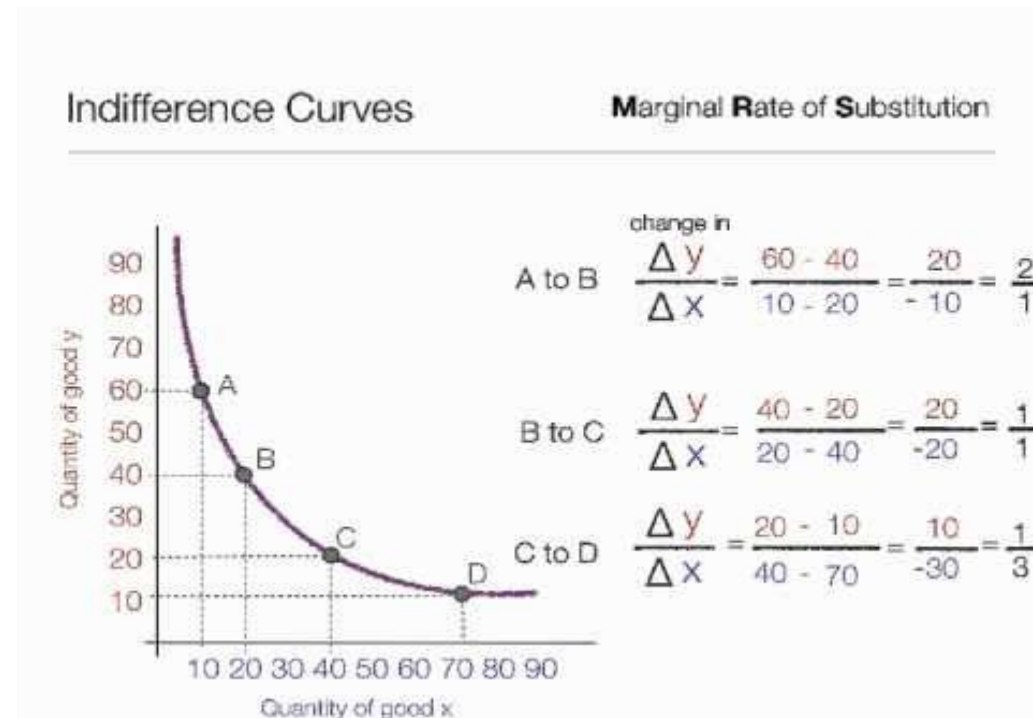
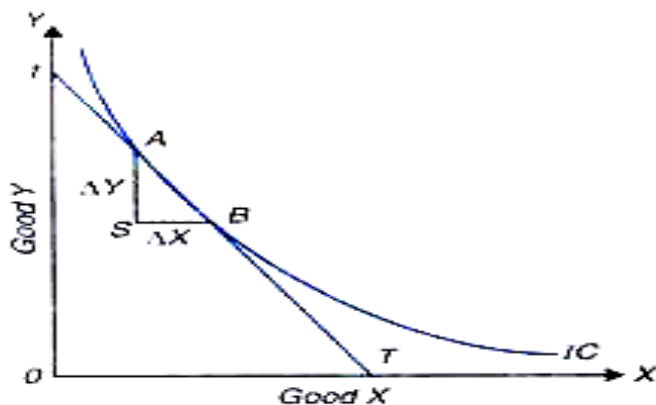


Fig. 1 : A Consumer's Indifference Curve



Indifference Map

- The marginal rate of substitution of x for y is defined as the number of units of commodity y that must be given up in exchange for an extra unit of commodity x so that the consumer maintains the same level of satisfaction.
- The slope of an indifference curve at any one point is called the marginal rate of substitution of the two commodities, x and y, and is given by the slope of the tangent at that point: [slope of indifference curve = $-\frac{dy}{dx} = MRS_{x,y}$]



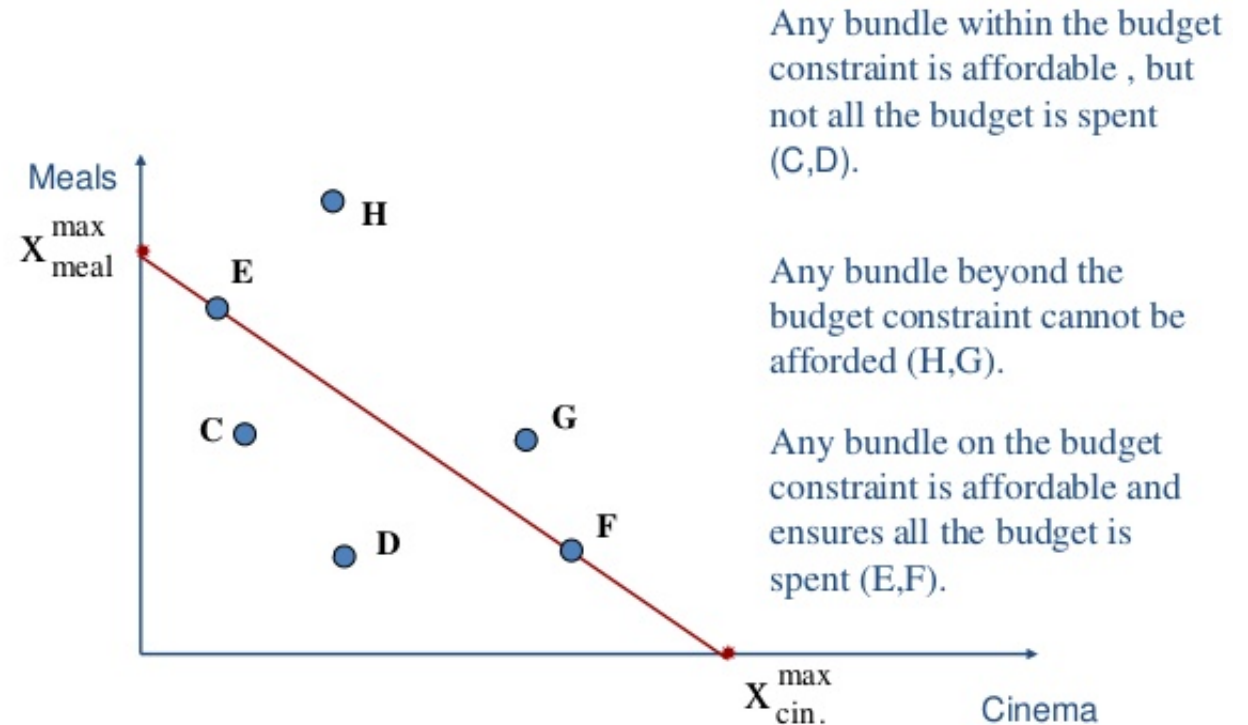
Properties of Indifference Curve

- An indifference curve has a negative slope, which denotes that if the quantity of one commodity (y) decreases, the quantity of the other (x) must increase, if the consumer is to stay on the same level of satisfaction.
- The further away from the origin an indifference curve lies, the higher the level of utility it denotes: bundles of goods on a higher indifference curve are preferred by the rational consumer.
- Indifference curves do not intersect. If they did, the point of their intersection would imply two different levels of satisfaction, which is impossible.
- The indifference curves are convex to the origin. This implies that the slope of an indifference curve decreases (in absolute terms) as we move along the curve from the left downwards to the right: the marginal rate of substitution of the commodities is diminishing.

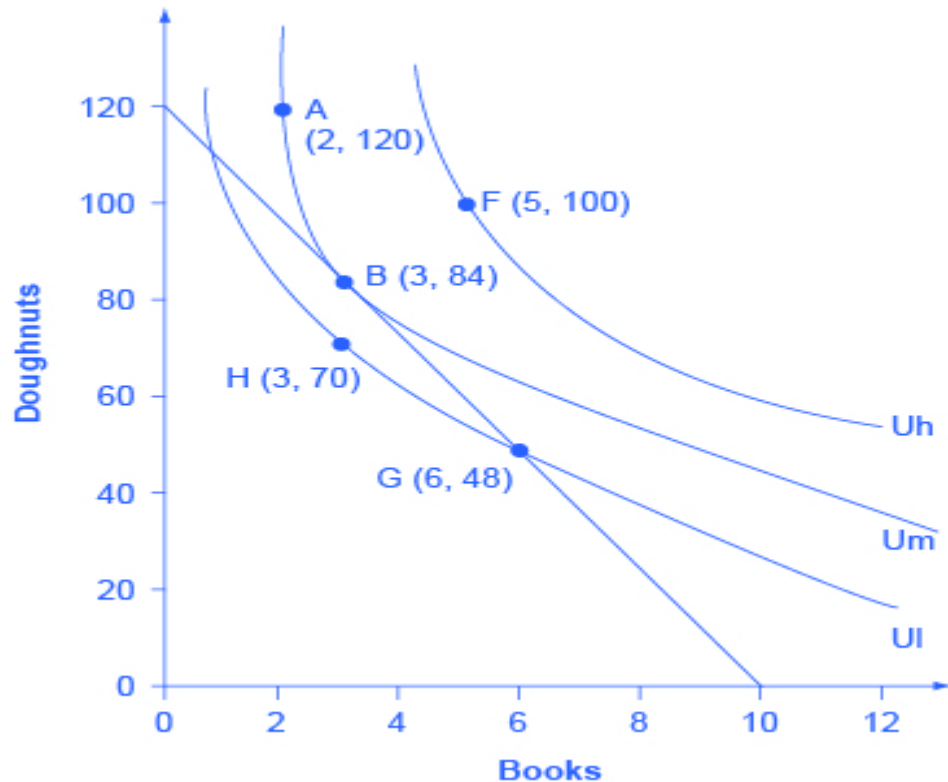
The budget constraint of the consumer

The consumer has a given income which sets limits to his maximizing behavior. Income acts as a constraint in the attempt for maximizing utility. The income constraint, in the case of two commodities, may be written as $Y = p_x q_x + p_y q_y$ where Y is the income, p is the price and q is the quantity consumed.

The budget constraint



Consumer's equilibrium



Indifference Curves and a Budget Constraint. Lilly's preferences are shown by the indifference curves. Lilly's budget constraint, given the prices of books and doughnuts and her income, is shown by the straight line. Lilly's optimal choice will be point B, where the budget line is tangent to the indifference curve U_m . Lilly would have more utility at a point like F on the higher indifference curve U_h , but the budget line does not touch the higher indifference curve U_h at any point, so she cannot afford this choice. A choice like G is affordable to Lilly, but it lies on indifference curve U_l and thus provides less utility than choice B, which is on indifference curve U_m .

- Lilly's choice between paperback books and doughnuts: Books cost \$6, doughnuts are 50 cents each, and that Lilly has \$60 to spend. This information provides the basis for the budget line shown in Figure 1. Along with the budget line are shown the three indifference curves from the figure. What is Lilly's utility-maximization equilibrium?
- The choice of F with five books and 100 doughnuts is highly desirable, since it is on the highest indifference curve U_h of those shown in the diagram. However, it is not affordable given Lilly's budget constraint. The choice of H with three books and 70 doughnuts on indifference curve U_l is a wasteful choice, since it is inside Lilly's budget set, and as a utility-maximizer, Lilly will always prefer a choice on the budget constraint itself. Choices B and G are both on the opportunity set. However, choice G of six books and 48 doughnuts is on lower indifference curve U_l than choice B of three books and 84 doughnuts, which is on the indifference curve U_m . If Lilly were to start at choice G, and then thought about whether the marginal utility she was deriving from doughnuts and books, she would decide that some additional doughnuts and fewer books would make her happier—which would cause her to move toward her preferred choice B. Given the combination of Lilly's personal preferences, as identified by her indifference curves, and Lilly's opportunity set, which is determined by prices and income, B will be her utility-maximizing choice.

Equilibrium Condition

Derivation of the equilibrium of the consumer: The consumer is in equilibrium when he maximizes his utility, given his income and the market prices. Two conditions must be fulfilled for the consumer to be in equilibrium.

a) Marginal rate of substitution is equal to the ratio of commodity prices

$MRS_{x,y} = MU_x / MU_y = P_x / P_y$ This is a necessary but not sufficient condition for equilibrium.

b) The indifference curves be convex to the origin. This condition is fulfilled by the axiom of diminishing $MRS_{x,y}$ which states that the slope of the indifference curve decreases (in absolute terms) as we move along the curve from the left downwards to the right.