INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

B. TECH (CST) 6^{TH} SEMESTER, MID TERM EXAMINATION, 2022

GRAPH THEORY [CS3224]

Date: 12/03/2022
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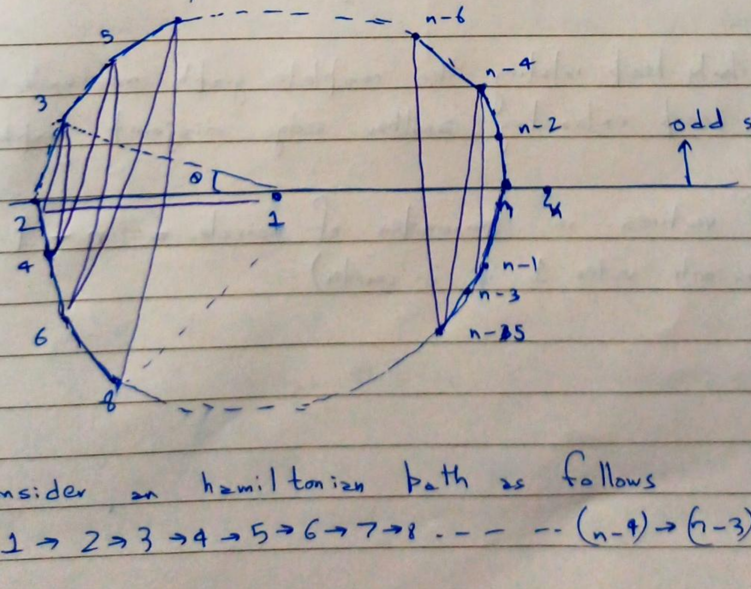
Q3) a) To prove that in a complete graph with n vertices there are $\frac{(n-1)}{2}$ ~~edge disj~~ edge-disjoint Hamiltonian Cycles, in n is odd & $n \geq 3$

<u>Proof</u> consider we arranged n vertices complete graph as follows (every vertex connected with every other vertex

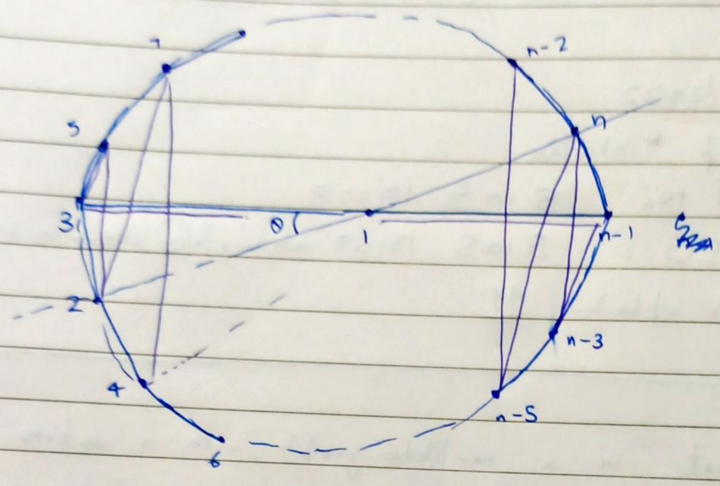


→ consider an hamiltonian path as follows

$1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 7 \to 8 \cdots (n-4) \to (n-3) \to (n-2) \to (n-1) \to n \to 1$

→ This is an hamiltonian circuit as well

→ now consider another hamiltonian circuit created by rotating entire structure by θ



→ now we get another hamiltonian path by same structure rotated by θ

$1 \to 3 \to 5 \to 2 \to 7 \to 4 \to - - - - - (n-2) \to (n-5) \to n \to (n-3) \to (n-1) \to 1$

→ this hamiltonian circuit is edge disjoint with previous one

→ we similarly keep rotating the complete graph one node at a time & keep extracting another edge disjoint graph

→ No. of vertices in perimeter of circle = $n-1$
(as only vertex 1 is in center)

→ noting the first edge of the $n-1$ hamiltonian circuits

$1 \leftrightarrow 2$
$1 \leftrightarrow 3$
$1 \leftrightarrow 4$
$1 \leftrightarrow 5$
|
|
|
|
$1 \leftrightarrow (n-1)$
$1 \leftrightarrow n$

Same hamiltonian circuit

same hamiltonian circuits

∴ There is a symmetry between $1 \leftrightarrow i$ & $1 \leftrightarrow (n-i+2)$ starting hamiltonian circuit

∴ Total No. of edge disjoint hamiltonian circuit $= \frac{n-1}{2}$

Q3) b) ~~R~~ Ring Sum

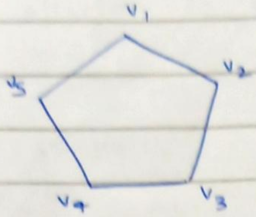
→ Ring Sum of two graphs G_1 & G_2 is ~~defined as~~ defined as

$G = G_1 \oplus G_2$
$= (V, E)$

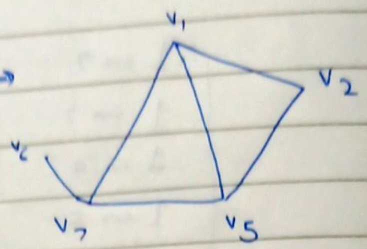
where $V = V_1 \cup V_2$
E = set of edges that are in G_1 & G_2 (E_1 & E_2) but not in both
$= (E_1 \cup E_2) - (E_1 \cap E_2)$

given two graphs

$G_1 \rightarrow$

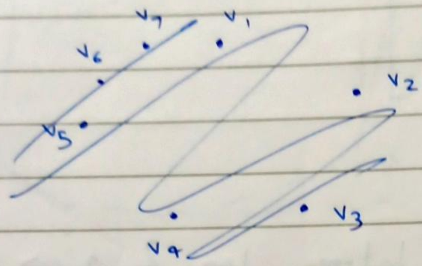


$G_2 \rightarrow$

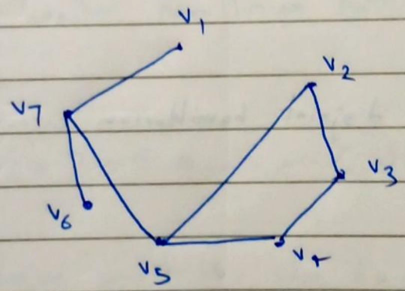


$G_1 \oplus G_2$

$\rightarrow$



$\rightarrow$



Q1) c) Complement of a graph

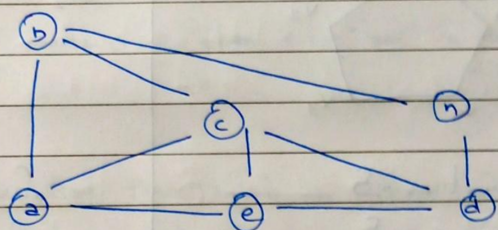
→ complement of a simple graph G is a simple graph G' having all the vertices of G and an edge between two vertices u & v iff there exists no edge between u & v in original graph G

$\therefore G' = (V', E')$ is complement of G

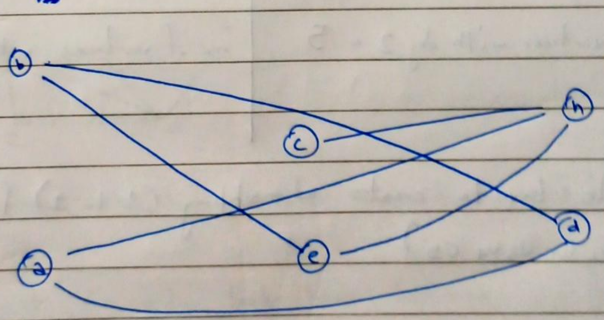
where $V' = V$

$E' = V \times V - E$

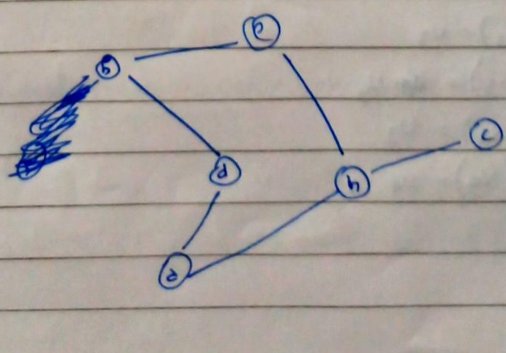
given graph $G \Rightarrow$



so $G' =$



or



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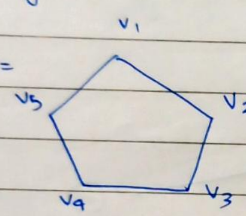
Q1) b) Isomorphic Graphs

→ Two graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are called isomorphic to each other if there exist one-to-one correspondence between the vertex set & edge set such that incidence relationship of G_1 & G_2 is preserved.

Given graphs

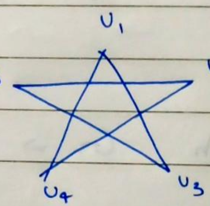
$G_1 =$



no. of vertices = 5
no. of edges = 5

no. of vertices with deg 2 = 5

$G_2 =$



no. of vertices = 5
no. of edges = 5

no. of vertices with deg 2 = 5

→ let's try to create a mapping (1 to 1) from $\{v_1\ v_2\ v_3\ v_4\ v_5\}$ & $\{u_1\ u_2\ u_3\ u_4\ u_5\}$

$f(v_1) = u_1$
$f(v_2) = u_3$
$f(v_3) = u_5$
$f(v_4) = u_2$
$f(v_5) = u_4$

→ checking for incidence relationship

$$g(\{v_1, v_2\}) = \{u_1, u_3\}$$
$$g(\{v_2\ v_3\}) = \{u_3\ u_5\}$$
$$g(\{v_4, v_5\}) = \{u_2, u_4\}$$
$$g(\{v_3\ v_4\}) = \{u_5, u_2\}$$
$$g(\{v_5\ v_6\}) = \{u_4, u_1\}$$

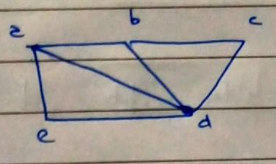
→ Incidence relationship is also preserved, ~~tha~~ hence graphs are isomorphic.

Q2) a) <u>Unicarsal Graph</u>

→ A graph G which contains a open euler line is called an unicarsal graph

→ open euler lines means that there exist a path in G which covers all edges of G but is not closed

<u>Eg</u>



~~euler~~ open euler path exists → a b d a e d c b

→ We just need an edge ~~betw~~ ~~have~~ between a & b to make this graph Euler Graph (ie have an euler circuit)

→ We can say that unicarsal graph is an Euler graph with one edge removed.

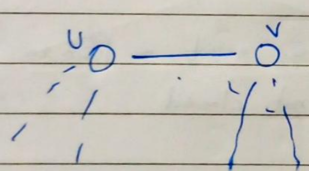
Prove that a connected graph is unicursal iff it has exactly two vertices of odd degree

(I) Consider we have a ~~unicor~~ connected unicursal graph G & we need to prove that it has exactly two vertices of odd degree

→ We can consider that G is an Euler graph with one arbitrary edge removed.

→ In an ~~edge~~ Euler graph we know that every ~~a~~ vertices has even degree

→ ~~If we remove an arbitrary edge from an Euler Graph to create G, then two vertices of G will have od~~



consider an edge (u,v) of an euler graph, let u have α degree & v have β degree

→ we know that α & β is even

in

→ let's remove edge (u,v) from the euler graph to create a unicursal graph G

→ ~~to~~ in G, u & v will have α-1 (odd) & β-1 (odd) degree resp, & all other ~~nodes~~ vertices will have even degree.

→ hence part (I) is proved, only two vertices with odd degree.

(II) consider we have a connected graph G with exactly two vertices ~~of~~ of odd degree, we need to prove that G is unicursal

→ consider the two vertices with odd degree in G be u & v

let $\deg(u) = \alpha$ (odd)
$\deg(v) = \beta$ (odd
& all other vertices has even degree.

→ let us add an edge (u,v) to G to create $G' = (V, E \cup (u,v))$
(it doesn't matter if (u,v) already existed in G, we create $\parallel^{el}$ edges then)

→ now $\deg(u) = \alpha+1$ (even)
$\deg(v) = \beta+1$ (even)
and all other vertices has even degree

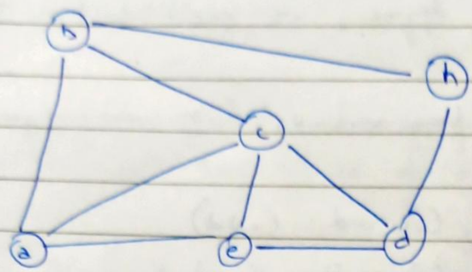
→ hence all vertices of G' ~~are~~ have even degree, therefore G' is an Euler Graph

→ ~~now $G' - G$~~ G' is G

→ now we know that if we add (u,v) to G, we get an euler graph

→ hence by definition G is unicursal graph.

Q2) b) given graph



& starting vertex c

I) BFS (queue)

front ↓ rear ↓

(i) Q: | c |

(ii) c Q | b | a | e | d |

(iii) cb Q | ~~b~~ | a | e | d | h

(iv) cba Q | ~~a~~ | e | d | h

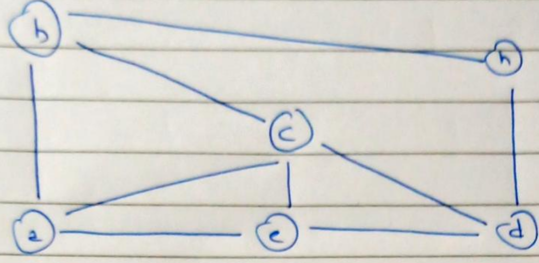
(v) cbae Q | d | h

(vi) cbaed Q | h |

(vii) cbaedh Q [ ] → Q empty end algo

∴ Traversal → $c \rightarrow \underbrace{b\ a\ e\ d}_{\text{arbitrary order}} \rightarrow h$

II dfs (stack)



(i) stk | c | ← top

(ii) c stk | b | a | e | d

(iii) cd stk | b | a | e | h

(iv) cdh stk | b | a | e

(v) cdhe stk | b | a

(vi) cdhea stk | b |

(vii) cdheab stk empty → end algo.