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INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR

B. TECH 5^{th} SEMESTER MID-TERM EXAMINATION, OCTOBER 2021

GRAPH ALGORITHMS [CS3104]

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1) a) Simple graph is a graph with no loop and only one edge between two vertices.

→ ~~for a Simple graph with n elements, ... edge possibles~~

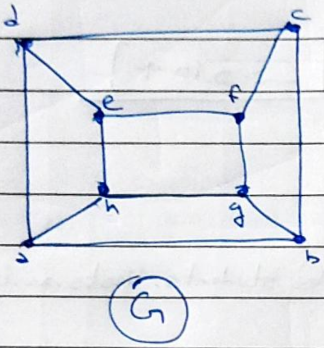
→ for a Simple graph with n element, we can get max edges if we have an edge between every pair of vertices in the graph

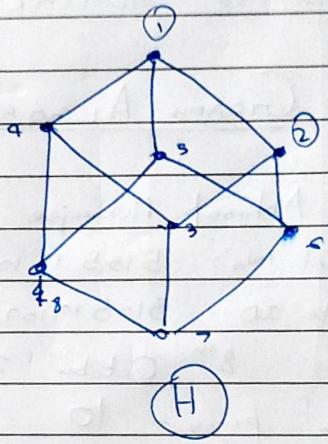
→ Maximum no. of edges = Total ways of selecting two vertices from graph & providing them edge

$$= {}^nC_2 = \frac{n!}{2!\,(n-2)!} = \frac{n(n-1)}{2}$$

∴ Hence Proved.

1) b) given graph





Let's check graph invariants

(i) No. of vertices → 8 both
(ii) No. of edges = 12 both
(iii) 2 degree ~~ee~~ vertices → 0 in both
(iii) 3 degree vertices → 8 both

Let's do mapping

I) map inside square of G to upper square of H

$f(e) = 1$
$f(f) = 2$
$f(g) = 3$
$f(h) = 4$

II) Now try to map outside square of G to lower square of H maintaining neighbouring relation.

$f(d) = 5$ $\quad f(c) = 6$ $\quad f(b) = 7$ $\quad f(a) = 8$

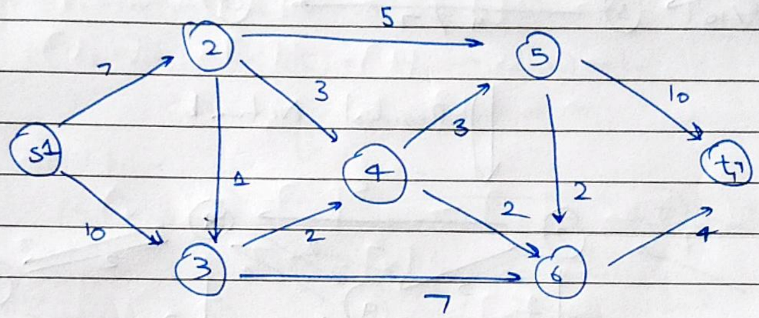
∴ ~~We found~~ a ~~one~~-to-one ~~onto relation between~~

∴ We found one-to-one ~~and~~ correspondence between vertices of G & H, while maintaining adjacency relationship

∴ G & H are isomorphic.

Q) 3) a) Initial graph [residual network same as initial graph

(I)



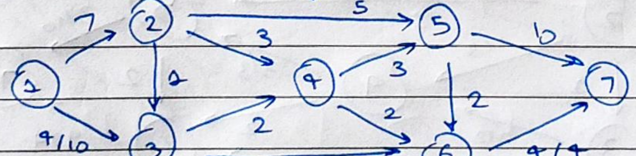
→ path found

$1 \xrightarrow{10} 3 \xrightarrow{7} 6 \xrightarrow{4} 7$

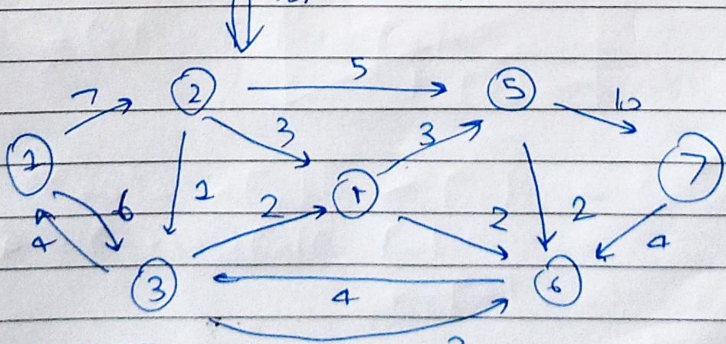
min. $c_f(p) = 4$ → add to path flow

⇓

(II)



⇓ residual network



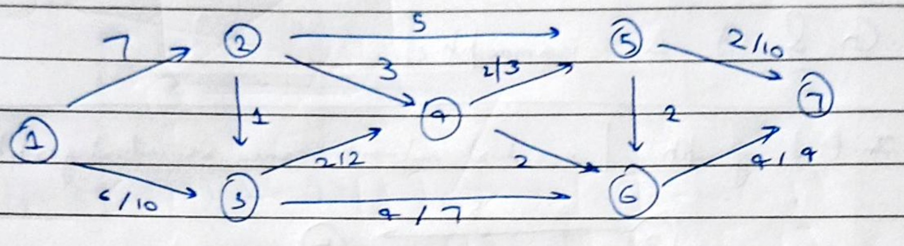
Path: ~~1 → 3 → 4 → 5 →~~

(3)

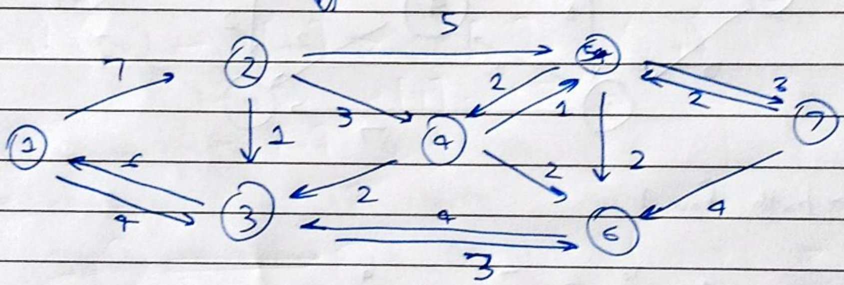
path found → $1 \xrightarrow{6} 3 \xrightarrow{2} 4 \xrightarrow{3} 5 \xrightarrow{10} 7$

min $c_f(p) = 2$ → add to p

III



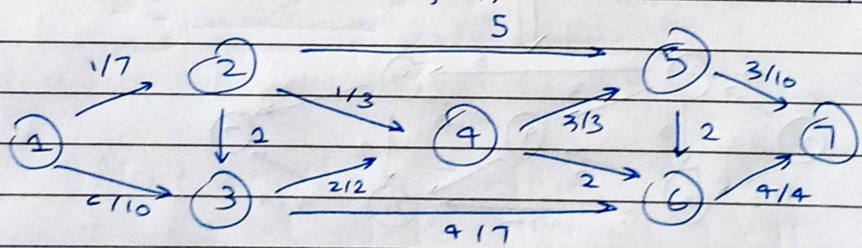
Residual Network



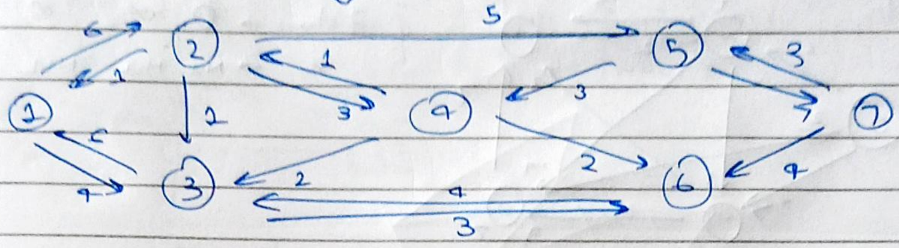
path found → $1 \xrightarrow{7} 2 \xrightarrow{3} 4 \xrightarrow{1} 5 \xrightarrow{8} 7$

min $c_f(p) = 1$ → add to p

IV



⇓ Residual

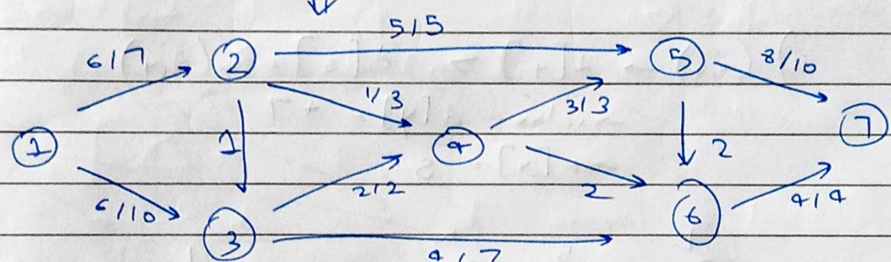


path found $\Rightarrow 1 \xrightarrow{6} 2 \xrightarrow{5} 5 \xrightarrow{7} 7$

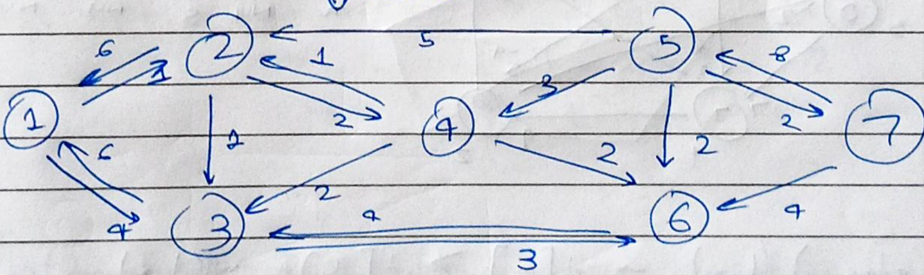
min $c_f(p) = 5 \rightarrow$ add to p

⇓

V



⇓ Residual



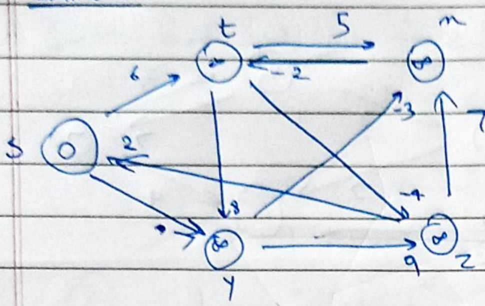
No other path found.

∴ V is the flow graph.

maximum flow = 12

Q4) Initial

I



	s	t	x	z	y
distance →	0	∞	∞	∞	∞
π →	NUL	NUL	NUL	NUL	NUL

edges i) $(s,t) \to d[t] > d[s] + w(s,t)$

$\to$ relax $d[t] = d[s] + w(s,t) = 6$

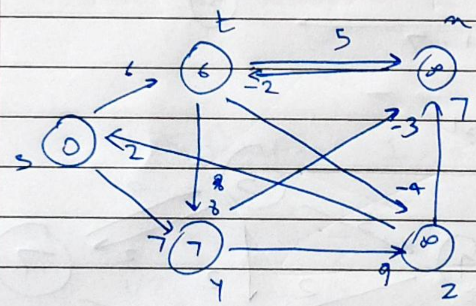
$\to \pi[t] = s$

ii) $(s,y) \to d[y] > d[s] + w(s,y)$

$\to$ relax $d[y] = +7$

$\to \pi[y] = s$

II



	s	t	x	z	y
d	0	6	∞	∞	7
π	NUL	s	NUL	NUL	s

edges i) $(t,x) \to d[x] > d[t] + w(t,x)$

$\to$ set $d[x] = d[t] + w(t,x) = 11$

$\to \pi[x] = t$

ii) $(t,z) \to d[z] > d[t] + w(t,z)$

$\to$ relax $d[z] = d[t] + w(t,z) = 2$

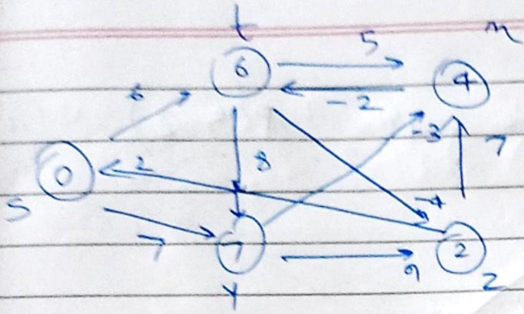
$\to \pi[z] = t$

iii) $(y,x) \to d[x] > d[y] + w(y,x)$

$\to$ relax $d[x] = d[y] + w(y,x) = 4$

$\to \pi[x] = y$

III)



	s	t	x	z	y
d	0	6	4	2	7
π	NUL	s	y	t	s

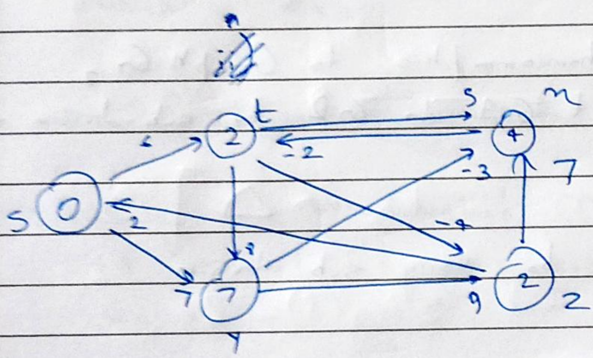
edges: i) $(x,t) \rightarrow d[t] > d[x] + w(x,t)$

relax $d[t] = 4 - 2 = 2$

$\pi[t] = x$

IV)

ii)



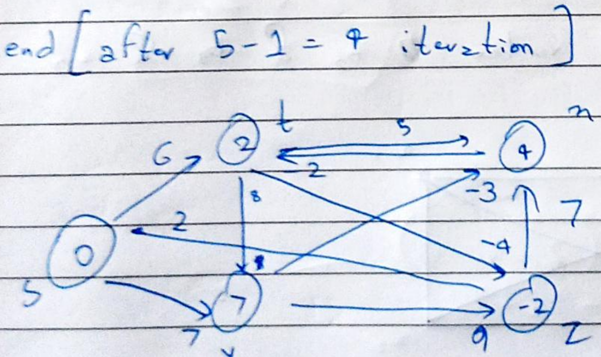
	s	t	x	z	y
d	0	2	4	2	7
π	NUL	x	y	t	s

edge. i) $(t,z) \rightarrow d[z] > d[t] + w(t,z)$

$\rightarrow$ relax $d[z] = 2 - 4 = -2$

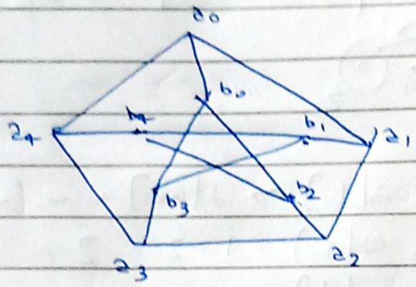
$\rightarrow \pi[z] = t$

end [after 5 − 1 = 4 iteration]



	s	t	x	z	y
d	0	2	4	−2	7
π	NUL	x	y	t	s

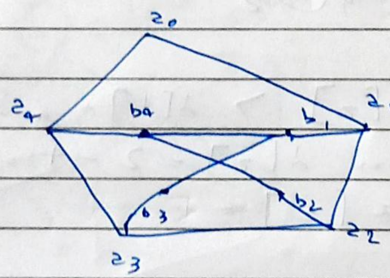
Q3) a) consider following peterson graph P(5, 2)



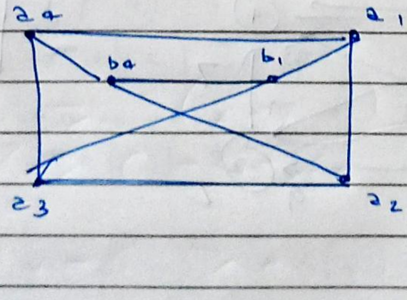
~~we will try~~

→ Graph G1 is homeomorphic to Graph G2 if we can convert G1 to G2 by sub-divison or smoothing

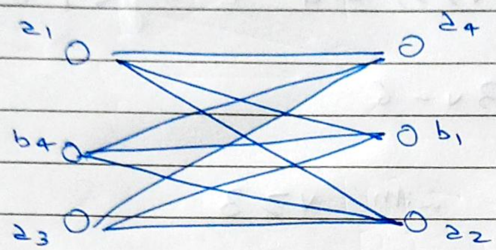
(i) First we remove b_0 using sub division



(ii) next we smooth vertex ~~a~~ b_2 & b_3 & a_0



iii) Rearranging, we get



which is $K_{3,3}$

→ ~~we know $K_{3,3}$ is non-planar~~

→ According to Kuratowski's Theorem, a graph is non-planar iff one of it's ~~gr~~ subgraph is homeomorphic to $K_{3,3}$ or K_5

→ given any peterson graph → we can use sub-division & smoothing to reduce it to K_{33} & K_5

→ so using Kuratowsk's Theorem we proved Pertersen's graph non-planar.

Q3) b) Every planar graph, which has no loops or multiple edge, and v vertices $(v \geq 3)$ and e edges, then

$$e \leq 3v - 6$$

Proof planar graph with $v \geq 3$

$\therefore$ ~~face de~~ minimum face degree of a face $= 3$

$\therefore$ we can say that

$$\sum \deg(f) \geq 3f \quad (i)$$

[equality holds in $k_3 \Rightarrow \triangle$]

→ we also know that $\sum \deg(f) = 2e$ (ii)

using (i) & (ii)

$$2e \geq 3f \quad (iii)$$

→ now using euler formula → $f - e + v = 2$

or $\boxed{f = 2 - v + e}$ (iv)

using (iii) & (iv)

$$2e \geq 3(2 - v + e)$$

$$\cancel{2e} \geq 6 - 3v + \cancel{3}e$$

$$3v - 6 \geq e$$

$\boxed{e \leq 3v - 6}$ $\therefore$ Hence Proved

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