Properties of Boolean Function

Sekhar Mandal

August 23, 2020

Minterm and Maxterm

- In a Boolean function, a literal is defined as variable in uncomplemented or complemented form. For example, $x,\ \overline{x},\ y,\ \overline{y}$ etc.
- Consider an n-variable Boolean function $f(x_1, x_2, \ldots, x_n)$.
  Minterm: A product term (AND operation) of all the n literals is called a *minterm*.
  Maxterm: A sum term (OR operation) of all the n literals is called *maxterm*.
- Consider a function $f(x, y, z)$, the minters are $\overline{x}\ \overline{y}\ \overline{z}$, $\overline{x}\ \overline{y}\ z$, $\overline{x}\ y\ \overline{z}$, $\overline{x}\ y\ z$, $x\ \overline{y}\ \overline{z}$, $x\ \overline{y}\ z$, $x\ y\ \overline{z}$, $x\ y\ z$
  Maxterms are: $(x + y + z)$, $(x + y + \overline{z})$, $(x + \overline{y} + z)$, $(x + \overline{y} + \overline{z})$, $(\overline{x} + y + z)$, $(\overline{x} + y + \overline{z})$, $(\overline{x} + \overline{y} + z)$, $(\overline{x} + \overline{y} + \overline{z})$

Properties of Minterms and Maxterms

- A particular minterm assumes value 1 for exactly one combination of variables.
  Consider a function $f(x, y, z) = x\overline{y}z + xy\overline{z} + xyz$
  The first minterm will be 1 when $x = 1, y = 0$ and $z = 1$, the second minter will be 1 when $x = 1, y = 1$ and $z = 0$ and the third minterm will be 1 when $x = y = z = 1$
- A particular maxterm assumes value 0 for exactly one combination of variables.
  For example, the first maxterm of the following function will be 0 when $x = 0, y = 1$ and $z = 0$
  $f(x, y, z) = (x + \overline{y} + z).(\overline{x} + \overline{y} + \overline{z}).(x + y + z)$

Properties of Minterms and Maxterms

For a given Boolean function, and for a given values of input variables:

- All the minterms that have the value 1 are called the true minterms.
- All the minterms that have the value 0 are called the false minterms.
- $f(x, y, z) = \overline{x}y + xyz$, for this function the true minterms are $\overline{x}yz$, $\overline{x}y\overline{z}$ and xyz. As $\overline{x}y = \overline{x}y(z + \overline{z})$
- All the maxterms that have the value 1 are called the true maxterms.
- All the maxterms that have the value 0 are called the false maxterms.

Canonical Form of Representing Function

- A canonical form is unique representation of a function.
- We can obtain two canonical representation directly from the truth table.
  (i) Canonical sum-of-product (disjunction normal form).
  (ii) Canonical product-of-sum (conjunctive normal form.

Canonical Sum-of-Product

- From the truth table, identify all the true miterms - corresponding to rows for which the output of the function is 1.
- Take the sum of all the true minterms.
- Example: Consider following truth table, the canonical sum-of-product form is $s = \overline{x}\ \overline{y}z + \overline{x}y\overline{z} + x\overline{y}\ \overline{z} + xyz$

x	y	z	s
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Canonical Sum-of-Product

We can write the canonical sum-of-product expression in a compact way by noting the decimal equivalent values of the true minterms.
$s = \sum(1, 2, 4, 7)$

Canonical Product-of-Sum

- From the truth table, identify the false minterms. - corresponding to the rows for which the output of the function is 0.
- For each false minterm, form a sum term where a variable will appear in uncomplemented (complemented) form if it has value 0 (1) in the row.
- For example, s can be written as
  $s = (x + y + z)(x + \overline{y} + \overline{z})(\overline{x} + y + \overline{z})(\overline{x} + \overline{y} + z)$
- We can write the canonical product-of-sum expression in compact way by noting the decimal equivalent value of false minterm.
  $s = \prod(0, 3, 5, 6)$