Proof Technique-3.

Wednesday, September 16, 2020 8:45 AM

Proof by cases / Exhaustive proof

Proof-15 Need to verify the inequality $(n+1)^3 \geq 3^n$ when $n = 1, 2, 3$, and 4.

Proof-16 We can prove this fact by examining all positive integers n not exceeding 100, first checking whether n is a perfect power and whether $(n+1)$ is also a perfect power.

$\mathbb{I} \in (-\infty, +\infty)$

Proof-17

We can prove that $n^2 \geq n$ for every integer by considering three cases:

(1) $n = 0$; (2) $n \geq 1$; (3) $n \leq -1$.

Case (1): When $n = 0$, because $0^2 = 0$, we see that $0^2 \geq 0$. It follows that $n^2 \geq n$.

Case (2): When $n \geq 1$, we multiply both sides of the inequality $n \geq 1$ by positive integer n. We obtain $n \cdot n \geq n \cdot 1$. This implies that $n^2 \geq n$ for $n \geq 1$.

Case (3): In this case $n \leq -1$. However, we know that $n^2 \geq 0$. It follows that $n^2 \geq n$.

that $x^2 \geq 0$. It follows that $x^2 \geq x$.

Proof-18

$$|a| = \begin{cases} a, & a \geq 0. \\ -a, & a < 0. \end{cases}$$

Four cases: (i) (p_1) x and y are both ~~positive~~ non-negative; (ii) (p_2) x is non-negative and y is negative; (iii) (p_3) x is negative and y is non-negative; (iv) (p_4) x and y are both negative.

Case (i): $p_1 \to q$ because $xy \geq 0$ when $x \geq 0$ and $y \geq 0$, so that $|xy| = |x| . |y|$.

Case (ii): To see that $p_2 \to q$, note that if $x \geq 0$ and $y < 0$, then $x \cdot y \leq 0$, so that

$$|xy| = -xy = x \cdot (-y) = |x| . |y| .$$

Case (iii): To see that $p_3 \to q$, WLOG we follow the same reasoning as case (ii).

Case (iv): To see that $p_4 \to q$, when $x < 0$ and $y < 0$, it follows that $xy > 0$. Hence,

$$|xy| = xy = (-x) \cdot (-y) = |x| . |y| .$$

in all four exhaustive

Because $|xy| = |x|\cdot|y|$, holds in all four exhaustive cases, we conclude that the given theorem is true for all real ~~to~~ numbers x & y.

Proof - ~~18~~ 19

We can quickly reduce ~~the~~ a proof to checking just a few simple cases.

$x^2 + 3y^2 = 8. \Rightarrow$ no solution exists

$x^2 > 8$, when $|x| \geq 3$ and $3y^2 > 8$ when $|y| \geq 2$. So the possible values of x are $-2, -1, 0, 1,$ or 2. and the possible values of y are $-1, 0,$ or 1. Thus, possible values of x^2 are $0, 1,$ and 4. The ~~for~~ possible values of $3y^2$ are 0 and 3. And, ~~the~~ largest ~~num~~ sum of possible values of x^2 and $3y^2$ is 7. Consequently, it is impossible for $x^2 + 3y^2 = 8$ to hold when x and y are integers.

$\neg q \to \neg p \Rightarrow$ true. $p \to$ false
$\cancel{p} \to q$ true.

Proof - ~~19~~ 20:

We will use proof by contraposition, the notion of WLOG, and proof by cases to solve this problem.

First, suppose that x and y are not both even. That is, assume that x is odd or that y is odd.

That is, assume that x is odd or that y is odd. (or both). WLOG, we assume that x is odd, so that $x = 2m+1$, for some integer m.

To complete the proof, we need to show that xy is odd or $(x+y)$ is odd. Consider two cases:

(i) y is even; $y = 2n$ for some integer n.

So, $xy = (2m+1).2n = 4mn + 2n = 2(2mn+n)$ is even. And, $(x+y) = (2m+1) + 2n = 2(m+n)+1$, is odd.

$\neg p$ is true

(ii) y is odd: $y = 2n+1$ for some integer n.

So, $x \cdot y = (2m+1)(2n+1) = 2(2mn+m+n)+1$ is odd.

$\neg p$ is true

And, $x+y = (2m+1)+(2n+1) = 2(m+n+1)$ is even.

This completes the proof by contraposition.

Proof-2:

We construct a sequence of inequalities which are as follows:

$$\frac{x+y}{2} > \sqrt{x \cdot y}$$

$$\Rightarrow \left(\frac{x+y}{2}\right)^2 > xy$$

$$\Rightarrow \left(\frac{x+y}{2}\right)^2 > xy$$

$$\Rightarrow (x+y)^2 > 4xy$$

$$\Rightarrow x^2+2xy+y^2 > 4xy$$

$$\Rightarrow x^2-2xy+y^2 > 0$$

$$\Rightarrow (x-y)^2 > 0. \Rightarrow \text{true.}$$

Because $(x-y)^2 > 0$ when $x \neq y$. It follows that the final inequality is true, Because all these inequalities are equivalent it follows that $\frac{x+y}{2} > \sqrt{x \cdot y}$.

Proof-22

- 2nd last step for the first player: Leaves 4 stones.
- 3rd last step for the first player: leaves 8 stones.
- 4th last step for the first player: leaves 12 stones.

Winning move

The first player removes three stones in his first move.