Quantifiers - 2

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(1) There is an honest politician.

$H(x) \equiv x$ is honest.

$\exists_x H(x)$, where domain of x is the politicians in the world.

$\neg \exists_x H(x) \equiv \forall_x \neg H(x)$

↳ Every politician is dishonest
All politicians are not honest ✗
(not all politicians are honest)

(2) All Americans eat cheeseburgers.

$C(x) :=$ x eats cheeseburgers.

$\forall_x C(x)$, where x is an American

$\neg \forall_x C(x) \equiv \exists_x \neg C(x)$

↳ There is an American who does not like Cheeseburger.
↳ Some Americans dogsnot like Cheeseburgers.

Ex: If a person is female and is a parent, then this person is someone's mother.

$F(x) :=$ x is female.

$P(x) :=$ x is parent

$M(x,y) :=$ x is the mother of y

$$\forall x \left((F(x) \wedge P(x)) \rightarrow \exists y \, M(x,y) \right),$$

where $x, y :=$ domain of all people.

Ex: Everyone has exactly one best friend.

$B(x,y) :=$ y is the best friend of x.

$$\exists y \left(B(x,y) \wedge \forall z \left((z \neq y) \rightarrow \neg B(x,z) \right) \right)$$

Ex: There is a woman who has taken a flight on every airline in the world.

$P(w,f) :=$ w has taken flight f.

$Q(f,a) :=$ f is a flight of airline a.

$$\exists w \forall a \exists f \left(P(w,f) \wedge Q(f,a) \right)$$

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↳ There is a flight f for which $Q(f, a)$ is true for all airline 'a' (infeasible)

Ex: Quantifiers from System Specifications

- Every mail message larger than one megabyte will be compressed.

$S(m, y) :=$ mail message m has size y MB.
dom(m) = all mail messages
dom(y) = positive real nos.

$C(m) :=$ mail message m will be compressed.

$$\forall m \left(S(m, 1) \rightarrow C(m) \right)$$

- If a user is active, at least one network link will be available.

$A(u) :=$ user u is active;
$S(n, \text{available}) :=$ Link n is available.

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$S(n, \text{available}) :=$ Link n is available.

$$\exists_h A(h) \rightarrow \exists_n S(n, \text{available})$$