

## INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR

B.TECH. 4<sup>th</sup> SEMESTER (CST) EXAMINATION, 2016

## Analysis and Design of Algorithms (CS 401)

FULL MARKS: 70

TIME: 3 Hrs

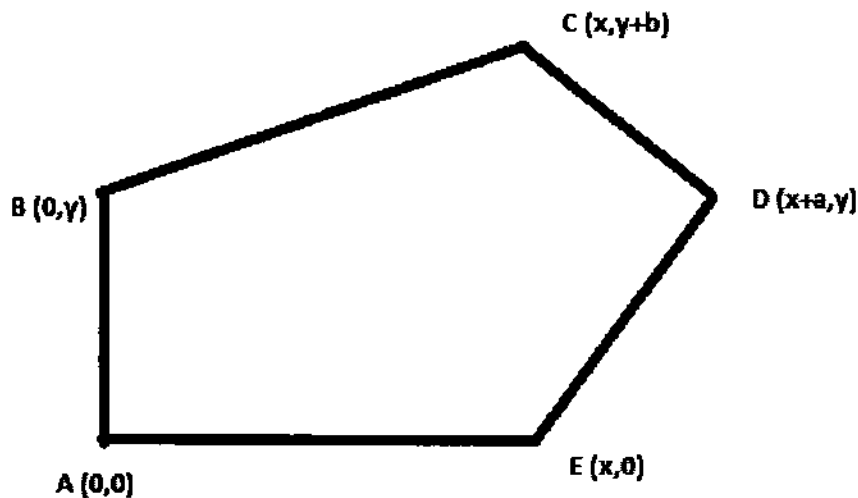
**Answer any five questions.**

1. What is the time complexity involved in multiplication of two  $n$ -degree polynomials represented using their coefficients? Which properties involving the roots of unity can be used for making the multiplication faster? For  $n=4$ , show the steps involved in the multiplication. 3+6+5
2. Describe an algorithm that performs modular exponentiation and analyze its complexity – illustrate for  $7^{11} \pmod{41}$ . Find the multiplicative inverse of 19  $\pmod{73}$  using Euclid's GCD finding algorithm and analyze its complexity. Show the importance of these two algorithms in public key RSA cryptosystem. 4+4+6
3. Describe two schemes for collision resolution, along with their data structures, involved in dynamic set operations. An element with key 'x' was inserted as the  $k$ -th element and another element with key 'y' was inserted as the  $(n-k)$ -th element – both getting mapped to same value (out of  $m$  slots) upon hashing. For the two schemes, what would be the expected count of operations needed when the key  $x$  and  $y$  are respectively searched after  $n$  elements got inserted? Suggest how the searching time can be improved. 6+8
4. Consider a scheduling problem where all tasks have unit execution time and are provided with their deadlines and penalty imposed for missing of deadline. Show how this problem can be mapped to a greedy algorithm for finding the maximal weight independent subset for weighted matroids. Find the optimal solution for the numerical example tabulated below: 8+6

Task	A	B	C	D	E	F
Deadline	2	1	2	3	6	3
Penalty	10	20	30	40	50	60

5. Given a polygon with  $n$  vertices, its triangulation needs to split the polygon into non-overlapping triangles. The best possible triangulation is the instance where the perimeter of the constituent triangles is minimum. Show that count of all possible triangulations is exponential in  $n$ . Explain how dynamic programming can be used to solve the problem without having to check all possible triangles. In particular, consider a pentagon having the five vertices located at  $A(0,0)$ ;  $B(0,y)$ ;  $C(x,y+b)$ ;  $D(x+a,y)$ ;  $E(x,0)$  as shown in the figure. Find out which of the five vertices stands the chance of becoming the common vertex (vertex from which both the diagonals involved in the triangulation emerge) in the best triangulation for the condition  $a > 0$ ,  $b > 0$  and along with any additional inequalities involving  $a$  and  $b$ .

3+5+6



6. Explain the term NP completeness. Show schematically from first principles that CLIQUE of a graph is NP complete. 4+10
7. What are the basic operations involving disjoint sets? Illustrate how these operations are performed in finding the connected components of a given graph with seven vertices  $[A..G]$  with edges  $(A,B)$ ;  $(C,D)$ ;  $(G,F)$ ;  $(E,B)$ ;  $(D,G)$ ;  $(A,G)$ . How can the performance improve by using some data structure with path compression heuristics? 2+4+8
8. Write short notes on:  $3\frac{1}{2} \times 4$
- (a) Comparison of Kruskal and Prim algorithm; (b) Primality testing algorithm;
- (c) Travelling salesman problem; (d) Lower bound of sorting problems.