INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR

B.TECH. 4th SEMESTER (CST) EXAMINATION, 2018

Analysis and Design of Algorithms (CS 401)

TIME: 3 Hrs

FULL MARKS: 70

Answer Q.1 and any three from the rest.

One mark is reserved for neatness.

1. **Answer any four** from the following. Without proper explanation of the algorithm/method used, there will be no credit. **6x4=24**

(a) Find the connected components of a given graph with ten vertices [A ... J] with edges (A,B); (C,D); (G,F); (E,B); (D,G); (A,G); (H,J); (I,J); (A,E) using appropriate disjoint set operations and data structures. What is the count of such operations?

(b) Consider a scheduling problem where all tasks have unit execution time and the table below provides their deadlines and penalty imposed for missing of deadline. Find the schedule that minimizes the penalty.

Task	A	B	C	D	E	F
Deadline	2	2	1	3	6	3
Penalty	10	20	30	40	50	60

(c) Derive an expression for the count of all possible paths of a robot to follow the 45° diagonal using $2n$ unit step moves only along left-to-right and back-to-front orthogonal directions and explain why this count is exponential.

(d) A polygon consists of five vertices with coordinates *0,0; 6,0; 0,6; 6,8; 8,6*. Triangulate the polygon using diagonals that minimize the total perimeter of the triangles.

(e) Perform modular exponentiation for 7^13 (mod 37). Find the multiplicative inverse of the result (mod 37). You have to show your calculation steps using number theoretic algorithm for fast exponentiation and GCD finding and report on the number of operations needed.

2. Describe the coefficient-based and value-based representation of polynomials. Discuss the time complexity of the basic operations – evaluate, add and multiply - involved in working with polynomials using the above representations. Describe the FFT algorithm that performs fast multiplication of two polynomials and explain its complexity. 3+4+8

3. What are the properties of the complexity class P, NP and NPC? Prove from set theoretic viewpoint that if any of the problems in NPC is solvable in polynomial time, P=NP. Show that Travelling Salesman Problem is NP Complete with Hamiltonian Cycle of graph as starting point. 5+5+5

4. Point out the differences in terms of data structure, algorithm and complexity:
   (a) Kruskal's algorithm and Prim's algorithm for finding Minimum Spanning Tree.
   (b) Chaining and Open addressing schemes for dynamic set operations.

   2 X 7 ½

5. *(a) A problem of size* n *is divided into* a subproblems of size n/b each and n^c amount of additional work is done for problem size n. Show that $T(n) = \Theta(n^c \log n)$ when $\log_b a = c$ using the concept of recursion tree.
   (b) Derive the average case complexity of randomized quick sort algorithm.

   7+8

6. Provide argument/proof for the following statements: 2 X 7 ½
   (a) Finding the Clique of graph is NP complete.
   (b) It is hard to decrypt messages of Public key cryptosystem.