Graphs & Trees

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Formal Definition & Examples

A *graph* $G = (V, E)$ consists of V, a nonempty set of *vertices* (or *nodes*) and E, a set of *edges*. Each edge has either one or two vertices associated with it, called its *endpoints*. An edge is said to *connect* its endpoints.

- Examples of graph in real life:

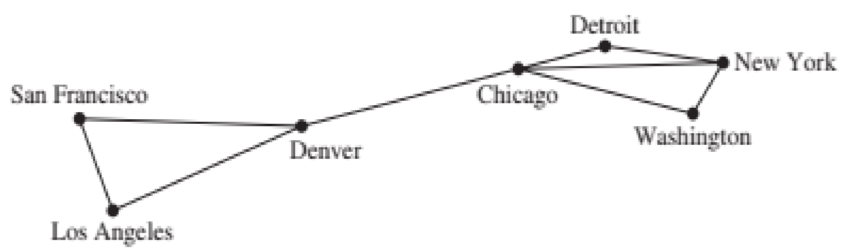

FIGURE 1 **A Computer Network.**

- Simple graph:
  - No edge connects a vertex to itself
  - Each edge connects two different vertices
  - No two edge connects same pair of vertices
  - Each edge is associated to an unordered pair of vertices

Formal Definition & Examples (*Contd…*)

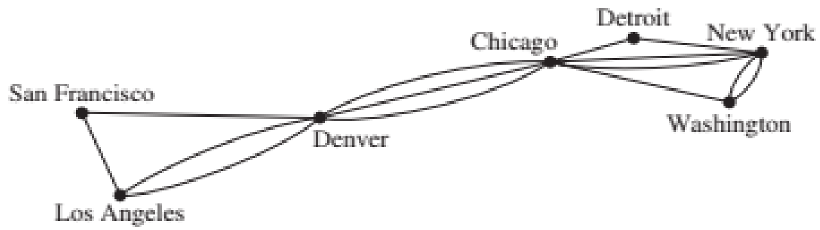

FIGURE 2 **A Computer Network with Multiple Links between Data Centers.**

- Multigraphs
  - Multiple edges connecting the same vertices

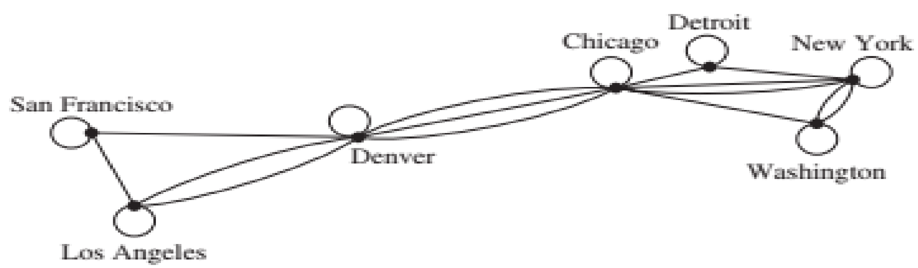

FIGURE 3 **A Computer Network with Diagnostic Links.**

- Pseudographs
  - **Loops** - edges connect a vertex to itself
  - Multiple edges may connect the same pair of vertices or vertex

Directed Graphs

A *directed graph* (or *digraph*) (V, E) consists of a nonempty set of vertices V and a set of *directed edges* (or *arcs*) E. Each directed edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair (u, v) is said to *start* at u and *end* at v.

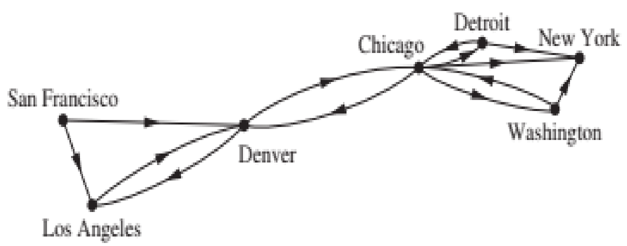

FIGURE 4 A Communications Network with One-Way Communications Links.

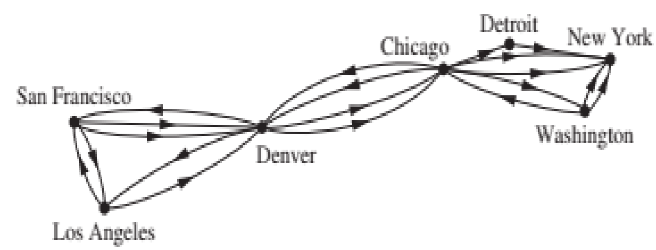

FIGURE 5 A Computer Network with Multiple One-Way Links.

TABLE 1 Graph Terminology.

Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

- Key questions
  - Are all edges of the graph undirected or directed (or both)?
  - Do multiple edges/directed edges connect the same pair of vertices?
  - Are loops present?

Application of Graphs

- Social media
  - Acquaintanceship/Friendship graph
  - Influence graph

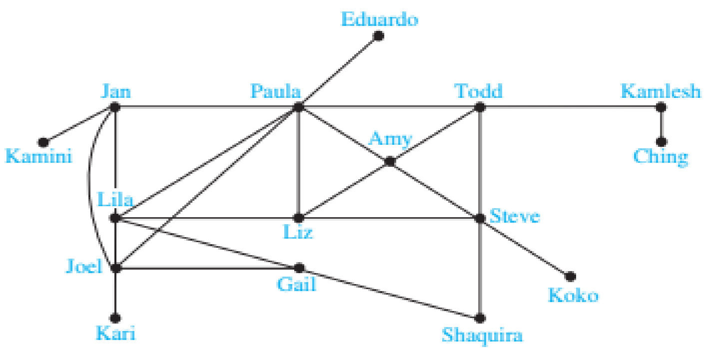

FIGURE 6 **An Acquaintanceship Graph.**

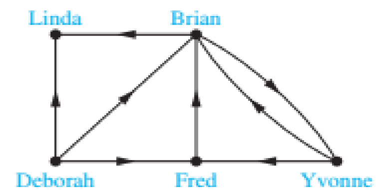

FIGURE 7 **An Influence Graph.**

- Communication networks
  - Call graphs

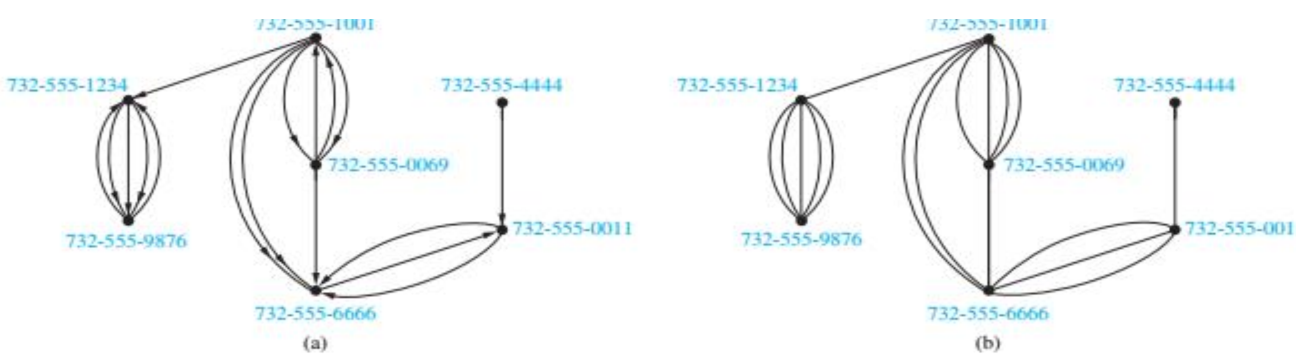

FIGURE 8 **A Call Graph.**

Application of Graphs (*Contd...*)

- Information networks
  - The Web Graph
  - Citation graphs
- Transportation networks
  - Airline routes
  - Road networks
- Software design applications
  - Module dependency graphs
  - Precedence graphs and concurrent processing

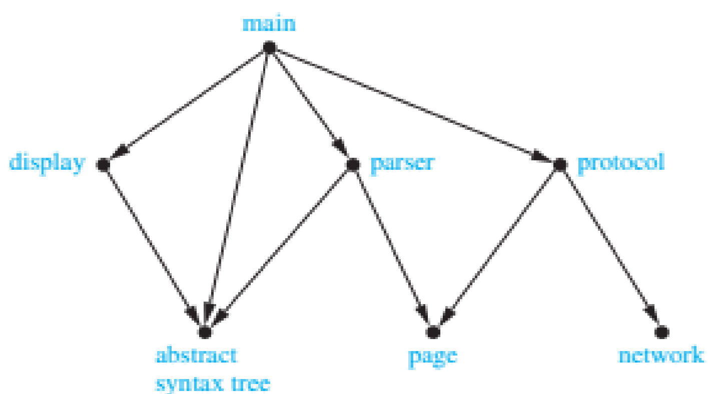

FIGURE 9 **A Module Dependency Graph.**

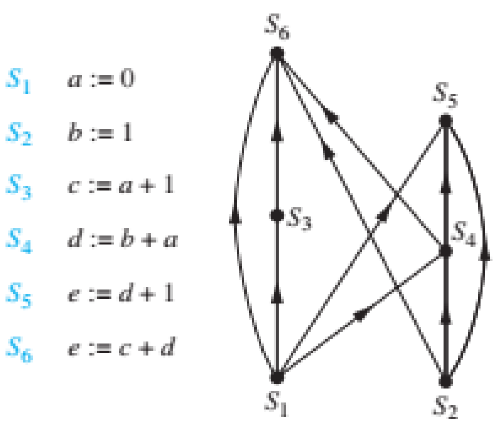

FIGURE 10 **A Precedence Graph.**

Application of Graphs (*Contd…*)

- Tournaments
  - Round-robin tournaments
  - Single-elimination tournaments

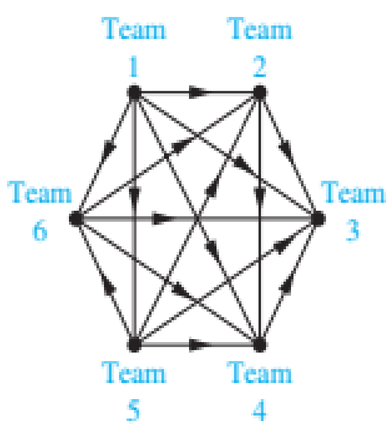

FIGURE 13 A Graph Model of a Round-Robin Tournament.

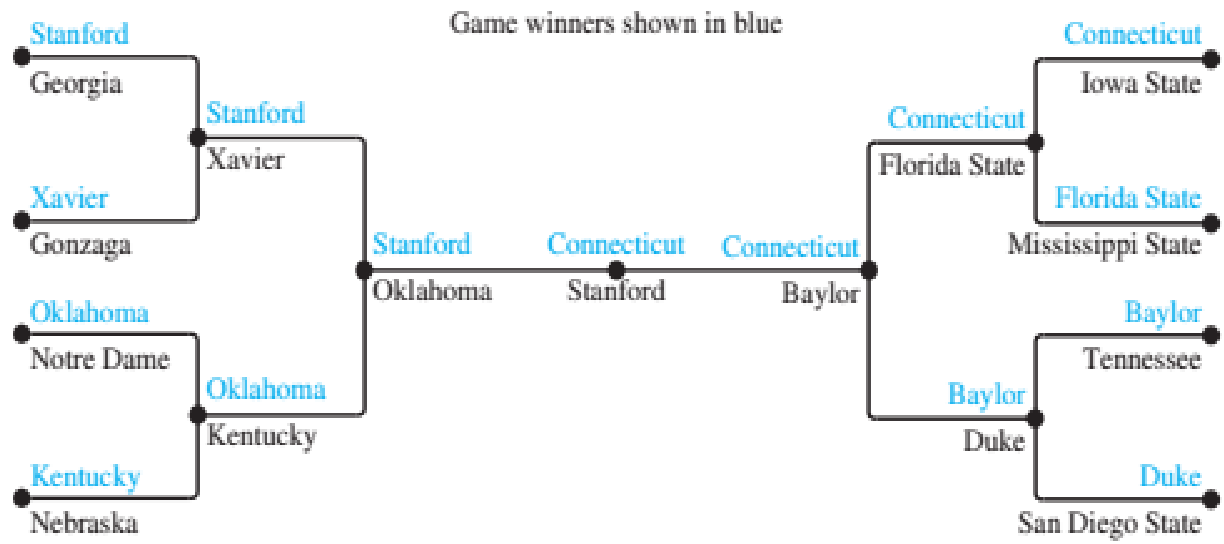

FIGURE 14 A Single-Elimination Tournament.

Graph Terminologies

- Terminologies to describe vertices and edges of undirected graph

Two vertices u and v in an undirected graph G are called *adjacent* (or *neighbors*) in G if u and v are endpoints of an edge e of G. Such an edge e is called *incident with* the vertices u and v and e is said to *connect* u and v.

The set of all neighbors of a vertex v of $G = (V, E)$, denoted by $N(v)$, is called the *neighborhood* of v. If A is a subset of V, we denote by $N(A)$ the set of all vertices in G that are adjacent to at least one vertex in A. So, $N(A) = \bigcup_{v \in A} N(v)$.

The *degree of a vertex in an undirected graph* is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex. The degree of the vertex v is denoted by $\deg(v)$.

- Isolated vertex: a vertex of degree zero (not adjacent to any of the other vertices)
- Pendant vertex: a vertex is pendant if and only if it has degree one (adjacent to exactly one vertex)

Graph Terminologies (*Contd...*)

Theorem 1:

THE HANDSHAKING THEOREM Let $G = (V, E)$ be an undirected graph with m edges. Then

$$2m = \sum_{v \in V} \deg(v).$$

(Note that this applies even if multiple edges and loops are present.)

Theorem 2:

An undirected graph has an even number of vertices of odd degree.

Definition:

When (u, v) is an edge of the graph G with directed edges, u is said to be *adjacent to* v and v is said to be *adjacent from* u. The vertex u is called the *initial vertex* of (u, v), and v is called the *terminal* or *end vertex* of (u, v). The initial vertex and terminal vertex of a loop are the same.