B-TECH 3RD SEMESTER, FINAL EXAMINATIONS
JANUARY ~~2021~~ 2021 [CST]

SUBJECT: DISCRETE STRUCTURES [CS2101]

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Q1) i) c) Trivial Proof

ii) no. of sets in $A \times B = 3 \times 4 = 12$

No. of subsets of $A \times B = 2^{12} = 4096$

$\therefore$ d) 4096

iii) A lattice is a partially ordered set in which every pair of elements has both ~~greatest upper bound and~~ least upper bound and greatest lower bound

iv) $R = \{(1,1), (3,1), (2,3), (4,2)\}$

$R^2 = R \circ R = \{(1,1), (3,1), (2,1), (4,3)\}$

$\therefore$ a) $\{(1,1), (2,1), (4,3), (3,1)\}$

v) given $f(x) = x^3 + 2$

let $y = x^3 + 2$

$\therefore y - 2 = x^3$

or $x = [y-2]^{\frac{1}{3}}$

$\therefore f^{-1}(x) = [x-2]^{\frac{1}{3}}$

vi) 12 indishtinguishable books
4 distinguishable shelves

$\therefore$ 12 books, ~~the~~ 3 divider

$$Ans = {}^{12+3}C_3 = {}^{15}C_3 = 455$$

$\therefore$ 455 ways

vii) given 10 vertices, each of degree six

$\therefore$ By handshaking theorem, (m is no. of edges)

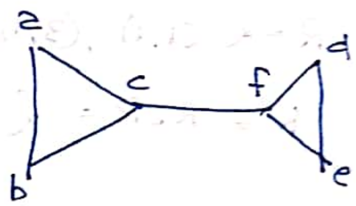
$$2m = 6 \times 10$$
$$m = 30$$

$\therefore$ 30 edges.

viii) Euler Circuit: Edges

~~→ No euler path exist.~~

→ No euler path exist as the ~~as~~ edge (c,f) is being repeated.

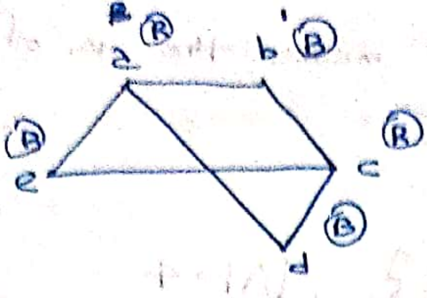


Hamilton Circuit: Vertices

→ Following Hamilton Circuit was found.

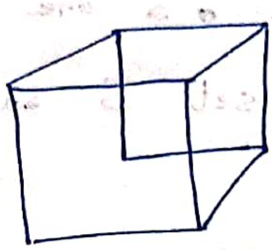
i) a, b, c, f, e, d
ii) b, a, c, f, d, e
iii) a, b, c, f, d, e.
iv) b, a, c, f, e, d.

ix)

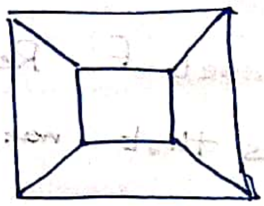


chromatic no. = 2 as we can ~~clour~~ colour the graph by ~~2~~ two colors only

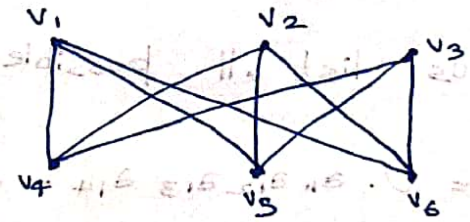
x) I) given G



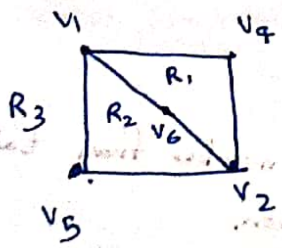
~~The~~ G is planar
it's planar representation



II) given H



Let us try to make planar repre-sentation to H



→ now if we place V_3 in any region R_1, R_2, R_3, the edges are getting intersected.

→ ~~we~~ ∴ we can't draw planar ~~representation~~ representation of H

→ H is not planar.

Q2) a) Cardinality of a set means the no. of elements in the set.

Eg: $A = \{1, 2, 3, 4\}$, $|A| = 4$

b) To determine if an infinite Set S is countable, we try to find a one to one correspondence between set S and set of positive integers Z^+

c) We have to prove that set of Real numbers is uncountable.

→ Let us assume set of Real no. is countable, so it also means that numbers in range $(0,1)$ is also countable.

→ Let us list all possible numbers between $(0,1)$

$a_1 = 0.\, a_{11}\, a_{12}\, a_{13}\, a_{14} \ldots$

$a_2 = 0.\, a_{21}\, a_{22}\, a_{23}\, a_{24} \ldots$

$a_3 = 0.\, a_{31}\, a_{32}\, a_{33}\, a_{34} \ldots$

$a_4 = 0.\, a_{41}\, a_{42}\, a_{43}\, a_{44} \ldots$

→ Now Let us construct a new number from taking diagonal digits, in the following fashion.

[new no. is s]

$$s_i = \begin{cases} 1 & a_{ii} = 0 \\ 0 & \text{otherwise} \end{cases}$$

$s = 0.\, s_1 s_2 s_3 \ldots$

→ as components ~~on~~ of s_i are taken from all the possible $a_1, a_2, \ldots\ldots$, s_i is different ~~than~~ no. than $a_1, a_2, \ldots$

→ but also, s_i ~~is~~ also exists in $(0,1)$, so there ~~m~~ is a number $t \in (0,1)$, which is equal to s

$$0.t_1 t_2 t_3 \ldots = 0.s_1 s_2 s_3 \ldots$$

→ This contradicts the fact that s_i is different number.

→ So, what we first assumed was wrong, Therfore. Set of Real number is not countable.

3) a) <u>Zero one matrix method</u>

→ to find. M_R, ~~R~~ $M_R[2]$, --- $M_R[n]$, we need $(n-1)$ boolean multiplication.

→ To generate a single element in $M_R[i]$, n products, $(n-1)$ joins. So for a single element, $2n-1$ operation

→ This has to repeat n^2 times for all element

→ and $(n-1)$ multiplication

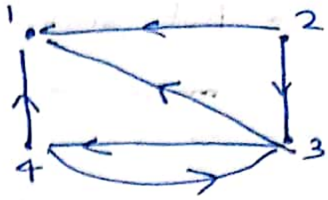
$$\therefore O[n^2(2n-1)(n-1)] = O[n^4]$$

~~For~~ <u>Warshall's Algorithm</u> $O[n^3]$

3) b) given Relation

$$R = \{(2,1), (2,3), (3,1), (3,4), (4,1), (4,3)\}$$

→ W_0 will be ~~th~~ equal to M_R



$$\therefore W_0 = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

→ W_1 will ~~be~~ have 1 in those places, where we can find a path, which have "1" as it's internal vertices

→ There is no extra path found.

$$W_1 = W_0$$

→ W_2 will have 1 in those place, through which we can have a path with "1" and/or "2" as it's internal vertices.

→ There is no such path found.

$$\therefore W_2 = W_1 = W_0$$

→ W_3 will have 1 in those places, through which we can have a path with "1", "2" and/or "3" as it's internal vertex

→ following paths are found.

→ (2, 4)

→ (4, 4)

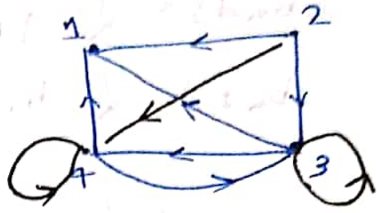
$$\therefore W_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

→ W_4 will have 1 in those places, through which we can make a path with "1", "2", "3" and /or 4 as it's internal vertex

→ following paths are found.

→ (3, 3)

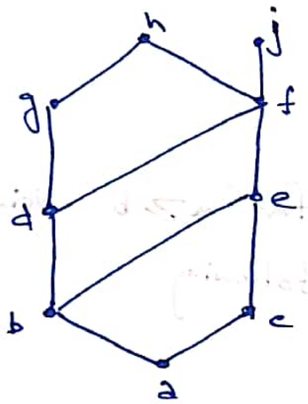
$$\therefore W_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$



→ as all the vertices are ~~not~~ considered,

$\therefore W_4$ is the matrix of The transitive closure of R

4)



I) $\{a, b, c\}$

$ub\{a\} = \{a, b, c, d, e, f, g, h, j\}$

$ub\{b\} = \{b, d, e, g, f, h, j\}$

$ub\{c\} = \{c, e, f, h, j\}$

$\therefore ub\{a, b, c\} = \{e, f, h, j\}$

$lb\{a\} = \{a\}$

$lb\{b\} = \{b, a\}$

$lb\{c\} = \{c, a\}$

$\therefore lb\{a, b, c\} = \{a\}$

II) $\{j, h\}$

$ub\{j\} = \{j\}$

$ub\{h\} = \{h\}$

$\therefore ub\{j, h\} = \phi$

$lb\{j\} = \{f, d, e, b, c, a\}$

$lb\{h\} = \{g, f, d, e, b, c, a\}$

$\therefore lb\{j, h\} = \{f, d, e, b, c, a\}$

III) $\{a, c, d, f\}$

$ub\{a\} = \{a, b, c, d, e, g, f, h, j\}$
$ub\{c\} = \{e, f, h, j\}$
$ub\{d\} = \{g, f, h, j\}$
$ub\{f\} = \{h, j\}$

$\therefore ub\{a, c, d, f\} = \{h, j\}$

$lb\{a\} = \{a\}$
$lb\{c\} = \{c, a\}$
$lb\{d\} = \{b, a\}$
$lb\{f\} = \{f, e, b, c, a\}$

$\therefore lb\{a, c, d, f\} = \{a\}$

5) a) To prove a theorem $P(n)$ for $n \geq b$ using strong Induction, we do the following

i) Basis Step

→ ~~check if~~ show that $P(b)$ is true ~~or not~~

ii) Inductive Step

→ assum $P(j)$ is true for ~~$b \geq j$~~ $b \leq j \leq k$

iii) Inductive Hypothesis

→ using the above assumption, Prove $P(k+1)$ is true

b) given $H_j = 1 + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{j}$

T.P. $\rightarrow H_{2^n} \geq 1 + \frac{n}{2}$

(n is non negative)

Basis Step

$\rightarrow H_{2^0} = 1 \geq 1 + \frac{0}{2}$

which is true.

Inductive Step

$\rightarrow$ Let us assume that the statement is true for an arbitrary non negative number k

$\therefore$ $H_{2^k} \geq 1 + \frac{k}{2}$ (i)

and $H_{2^k} = 1 + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{2^k}$ (ii)

Inductive Hypothes

$\rightarrow$ Now using above assumption, we have to prove that

$H_{2^{k+1}} \geq 1 + \frac{k+1}{2}$

$\rightarrow$ By definition.

$H_{2^{k+1}} = 1 + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{2^k} + \frac{1}{2^k+1} + \frac{1}{2^k+2} + \dots \frac{1}{2^{k+1}}$

$\rightarrow$ using (ii)

$H_{2^{k+1}} = H_{2^k} + \frac{1}{2^k+1} + \frac{1}{2^k+2} + \dots \frac{1}{2^{k+1}}$

$\rightarrow$ using (i)

$$H_{2^{k+1}} \geq 1 + \frac{k}{2} + \underbrace{\frac{1}{2^k+1} + \frac{1}{2^k+2} + \dots \frac{1}{2^{k+1}}}_{2^k \text{ terms}}$$

for now we can say that

$$\frac{1}{2^k+1} \geq \frac{1}{2^{k+1}}$$

$$\frac{1}{2^k+2} \geq \frac{1}{2^{k+1}}$$

$$\vdots$$

$$\frac{1}{2^{k+1}} \geq \frac{1}{2^{k+1}}$$

$$\therefore H_{2^{k+1}} \geq 1 + \frac{k}{2} + 2^k \times \frac{1}{2^{k+1}}$$

$$\geq 1 + \frac{k}{2} + \frac{1}{2}$$

$$\geq 1 + \frac{k+1}{2}$$

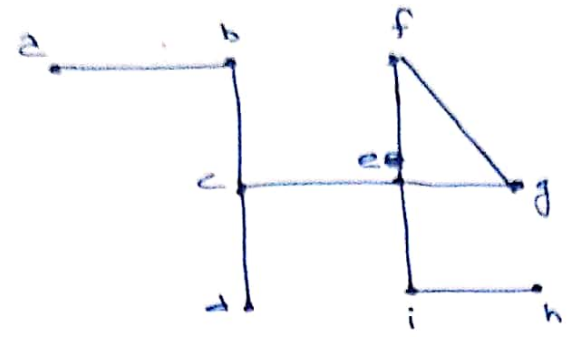
⇒ Conclusion

→ we saw that this statement is true for $k=0$.

→ we also saw that if we assume statement to be true for an arbitrary k, the statement is also true for $k+1$

$\therefore$ We can say that the statement is true for all non-negative integers.

6) given



cut vertex = ~~e~~ b, c, e, i

vertex cuts = {b, c}, {c, d}, {c, e}, {e, i}, {e, g}, etc.

Vertex connectivity = 1 (as min. no. of vertex to remove for graph to get disconnected is 1)

cut edge = (a, b), (b, c), (c, e), (e, i),

~~ver edge cut = {~~

→ removing any of these edge disconnects the graph

∴ Edge connectivity = 1

→ Edge cut can have a lots of sets of edges, some of them are

{(a, b)}, {(b, c)}, {(c, e)}, {(e, i)}; {(a, b), (b, c)},
{(b, c), (c, e)}, etc.