Mathematics Linear Programming Problem

Assignment

Student:

Abhiroop Mukherjee

Enrolment Number

510519109

Q) Given $x_1 = 1$, $x_2 = 1$, $x_3 = 1$, $x_4 = 0$. is a feasible solution of the system of equations:

$$x_1 + 2x_2 + 4x_3 + x_4 = 7$$
$$2x_1 - x_2 + 3x_3 - 2x_4 = 4$$

Reduce the feasible solution to one basic feasible solution.

→ as $(1, 1, 1, 0)$ is a feasible solution, we can say that

$$1 a_1 + 1 a_2 + 1 a_3 + 0 a_4 = 0$$

where $a_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $a_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $a_3 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $a_4 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

→ But this is not a Basic Feasible solution as there are three non-zero variables. We have to reduce it to two non-zero variable.

→ as $x_4 = 0$ is already zero, ignoring a_4; we get

$$x_1 + 2x_2 + 4x_3 = 7$$
$$2x_1 - x_2 + 3x_3 = 4$$

∴ ~~we~~ Here : a_1, a_2, a_3 are linearly dependant,

that is $\lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3 = \bar{0}$

or
$$\lambda_1 + 2\lambda_2 + 4\lambda_3 = 0$$
$$2\lambda_1 - \lambda_2 + 3\lambda_3 = 0$$

or $\dfrac{\lambda_1}{6+4} = \dfrac{\lambda_2}{-(3-8)} = \dfrac{\lambda_3}{-1-4} = k \text{ (let)}$

for $k=1$, we get

$\lambda_1 = 10$
$\lambda_2 = 5$
$\lambda_3 = -5$

To reduce the solution to a basic feasible solution,

we find

$$\min_{\lambda>0} j \left\{ \frac{x_j}{\lambda_j}, \lambda_j > 0 \right\} = \min\left(\frac{1}{10}, \frac{1}{5}\right) = \frac{1}{10}$$

for $j=1$

$\therefore$ minimum occurs at $j=1$, hence we eliminate a_1

from ans to reduce a_1 to zero.

$\therefore$ ~~new ans =~~

A basic feasible solution $= \left(1-\frac{1}{10}\times 10,\ 1-\frac{1}{10}\times 5,\ 1-\frac{1}{10}(-5), 0\right)$

$$= \boxed{\left(0, \frac{1}{2}, \frac{3}{2}, 0\right)}$$

[check: $0+1+6+0=7$
$0 - \frac{1}{2} + \frac{9}{2} - 0 = 4$]

→ we can also do $\max_j \left\{ \frac{x_j}{\lambda_j}, \lambda_j < 0 \right\} = \frac{-1}{5}$ $(j=3)$

to get another basic feasible solution

$$\left(1+\frac{1}{5}(10),\ 1+\frac{1}{5}(5),\ 1+\frac{1}{5}(-5),\ 0\right)$$

$$= \boxed{(3, 2, 0, 0)}$$

[check: $3+4+0+0=7$
$6-2+0+0=4$]

$\therefore$ ~~Th~~ Two basic Feasible can be found

$\left(0, \frac{1}{2}, \frac{3}{2}, 0\right)$ and $(3, 2, 0, 0)$

Q) $x_1 = 1$, $x_2 = 2$, $x_3 = 1$, $x_4 = 0$ is a feasible solution to the system

$$11x_1 + 2x_2 - 9x_3 + 4x_4 = 6$$
$$15x_1 + 3x_2 - 12x_3 + 6x_4 = 9$$

Reduce the feasible solution to more than one Basic Solution and prove that one of them is non-degenerate and the others are degenerat

→ given (1, 2, 1, 0) solution to

$$11x_1 + 2x_2 - 9x_3 + 4x_4 = 6$$
$$15x_1 + 3x_2 - 12x_3 + 6x_4 = 9$$

→ ignoring $6x_4$ (as it is already zero), we get

$$11x_1 + 2x_2 - 9x_3 = 6$$
$$15x_1 + 3x_2 - 12x_3 = 9$$

→ as the coefficient matrices are linearly dependant, we get

$$11\lambda_1 + 2\lambda_2 - 9\lambda_3 = 0$$
$$15\lambda_1 + 3\lambda_2 - 12\lambda_3 = 0$$

or $\dfrac{\lambda_1}{-24+27} = \dfrac{\lambda_2}{-(-132+135)} = \dfrac{\lambda_3}{33-30} = k$ (let)

→ for $k=1$, we get

$\lambda_1 = 3$
$\lambda_2 = -3$
$\lambda_3 = 3$

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→ ignoring $6x_4$ (as it is already zero), we get

$$\boxed{\begin{aligned} 11x_1 + 2x_2 - 9x_3 &= 6 \\ 15x_1 + 3x_2 - 12x_3 &= 0 \end{aligned}}$$

→ as the coefficient matrices are linearly dependant, we get

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or $\dfrac{\lambda_1}{-24+27} = \dfrac{\lambda_2}{-(-132+135)} = \dfrac{\lambda_3}{33-30} = k$ (let)

∴ for $k = 1$, we get

$\lambda_1 = 3$
$\lambda_2 = -3$
$\lambda_3 = 3$

→ ~~for~~ we get a basic feasible solution by

$$\min_j \left(\frac{x_j}{\lambda_j}, \lambda_j > 0 \right) = \min_j \left(\frac{1}{3}, \frac{1}{3} \right) = \frac{1}{3}$$

for $j = 1$ or 3

$$\therefore bfs = \left(1 - \frac{1}{3}\times 3, \; 2 + \frac{1}{3}\times(+3), \; 1 - \frac{1}{3}\times 3, \; 0 \right)$$

$$= \boxed{(0, 3, 0, 0)}$$

as one of the basic variable Vanished, this solution is ~~non~~ degenerate

→ we get another basic feasible solution by

$$\max_j \left\{ \frac{x_j}{\lambda_j}, \lambda_j < 0 \right\} = \frac{-2}{3} \text{ for } j = 2$$

$$\therefore \text{another } bfs = \left(1 + \frac{2}{3}(3), \; 2 + \frac{2}{3}(-3), \; 1 + \frac{2}{3}(3), \; 0 \right)$$

$$= \boxed{(3, 0, 3, 0)}$$

this solution is non-degenerate

Q) Show that $x_1 = 5, x_2 = 0, x_3 = -1$ is a basic solution of the system of equation

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

Find other basic solution if there are any

→ ~~A=F~~

→ converting system of equations to matrix form, we get;

$$\underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 4 \\ 5 \end{bmatrix}}_{b}$$

here Rank (A) = 2 = m

$\therefore n - m = 3 - 2 = 1$

∴ 1 zero must be present in solution for it to be basic

→ also checking (5, 0, −1) in the system

$$5 + 0 - 1 = 4$$
$$10 + 0 - 5 = 5$$

∴ (5, 0, −1) is a basic solution for the system

→ To get more basic solutions, we set $x_1 = 0$ & $x_3 = 0$

I) $x_1 = 0$

system becomes: $2x_2 + x_3 = 4$
$x_2 + 5x_3 = 5$

or $x_2 = \frac{5}{3}$ $\quad x_3 = \frac{2}{3}$

$\therefore \left(0, \frac{5}{3}, \frac{2}{3}\right)$ is another basic solⁿ.

II) $x_3 = 0$

system becomes

$$x_1 + 2x_2 = 4$$
$$2x_1 + x_2 = 5$$

or $x_1 = 2$

$x_2 = 1$

$\therefore (2,1,0)$ is another basic solution.

Q) The two linearly independent equations with three variables are given by:

$$2x_1 - 3x_2 + 5x_3 = 10$$
$$4x_1 + x_2 + 10x_3 = 20$$

Find, if possible, a basic solution with x_2 as a non-basic variable.

To set x_2 as non-basic variable, we do

$x_2 = 0$

~~The~~ so, the system becomes

$$2x_1 + 5x_3 = 10$$
$$4x_1 + 10x_3 = 20$$

as for these two lines:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

→ the ~~lines~~ system have no solution

∴ The solution doesn't exist for x_2 as a non-basic variable.

Q) Reduce the feasible solution

$x_1 = 2, x_2 = 1, x_3 = 1$ of the given system

$$x_1 + 4x_2 - x_3 = 5$$
$$2x_1 + 3x_2 + x_3 = 8$$

to a basic feasible solution.

as the coefficient matrix of the given system are dependent,

$$\lambda_1 + 4\lambda_2 - \lambda_3 = 0$$
$$2\lambda_1 + 3\lambda_2 + \lambda_3 = 0$$

or $$\frac{\lambda_1}{4+3} = \frac{\lambda_2}{-(1+2)} = \frac{\lambda_3}{3-8} = k \text{ (let)}$$

for $k=1$, we get

$\lambda_1 = 7$
$\lambda_2 = -3$
$\lambda_3 = -5$

→ for a basic feasible solution, we find

$$\min_j \left\{ \frac{x_j}{\lambda_j}, \lambda_j > 0 \right\} = \frac{2}{7} \text{ for } j=1$$

$$\therefore bfs = \left(2 - \frac{2}{7}(7),\ 1 - \frac{2}{7}(-3),\ 1 - \frac{2}{7}(-5)\right)$$

$$= \boxed{\left(0, \frac{13}{7}, \frac{17}{7}\right)}$$

→ for anothe basic feasible solution, we find

$$\max_j \left\{ \frac{x_j}{\lambda_j}, \lambda_j < 0 \right\} = \max\left\{ \frac{-1}{3}, \frac{-1}{5} \right\} = \frac{-1}{5} \text{ for } j=3$$

$$\therefore bfs = \left(2 + \frac{1}{5}(7),\ 1 + \frac{1}{5}(-3),\ 1 + \frac{1}{5}(-5)\right)$$

$$= \boxed{\left(\frac{17}{5}, \frac{2}{5}, 0\right)}$$

Q) Show by Simplex Method that the following LPP has an unbounded solution

$$\max \quad z = 3x_1 + 4x_2$$

st

$$x_1 - x_2 \geq 0$$

$$-x_1 + 3x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

→ converting the given LPP to it's standard form, we get

$$\max \quad z = 3x_1 + 4x_2 + 0x_3 + 0x_4$$

st

$$x_1 - x_2 - x_3 + 0x_4 = 0$$

$$-x_1 + 3x_2 + 0x_3 + x_4 = 3$$

$$x_1, x_2, x_3, x_4 \geq 0$$

In matrix for

$$z = \begin{bmatrix} 3 & 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & -1 & -1 & 0 \\ -1 & 3 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 3 \end{bmatrix}}_{b}$$

I) let $B = [a_1 \; a_2] = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$

$|B| = 3 - 1 = 2$

$B^{-1} = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$

$x_B = B^{-1}b = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$c_B = [3 \quad 4]$

$Y_1 = B^{-1}a_1 = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$z_1 - c_1 = c_B Y_1 - c_1 = [3 \; 4]\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 3 = 0$

$Y_2 = B^{-1}a_2 = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$z_2 - c_2 = c_B Y_2 - c_2 = [3 \; 4]\begin{bmatrix} 0 \\ 1 \end{bmatrix} - 4 = 0$

$Y_3 = B^{-1}a_3 = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -3 \\ -1 \end{bmatrix}$

$z_3 - c_3 = c_B Y_3 - c_3 = \frac{1}{2}[3 \; 4]\begin{bmatrix} -3 \\ -1 \end{bmatrix} - 0 = -\frac{13}{2}$

$Y_4 = B^{-1}a_4 = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$z_4 - c_4 = c_B Y_4 - c_4 = \frac{1}{2}[3 \; 4]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{7}{2}$

→ as $z_3 - c_3$ is the minimum -ve ans, ∴ a_3 will be ~~inclu~~ the entering vector

→ as both the values of Y_3 are less than zero, there is no exiting vector, suggesting that the LPP has unbounded solution.

Q) Solve

$$\max z = 5x_1 + 2x_2 + 2x_3$$

$$\text{st} \quad x_1 + 2x_2 - 2x_3 \le 30$$

$$x_1 + 3x_2 + x_3 \le 36$$

$$x_1, x_2, x_3 \ge 0$$

converting LPP to standard form

$$\max z = 5x_1 + 2x_2 + 2x_3 + 0x_4 + 0x_5$$

$$\text{st} \quad x_1 + 2x_2 - 2x_3 + x_4 + 0x_5 = 30$$

$$x_1 + 3x_2 + x_3 + 0x_4 + x_5 = 36$$

writing it in matrix form

$$\max z = \begin{bmatrix} 5 & 2 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$\text{st} \quad \underbrace{\begin{bmatrix} 1 & 2 & -2 & 1 & 0 \\ 1 & 3 & 1 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 30 \\ 36 \end{bmatrix}}_{b}$$

I) let $B = [a_1 \;\; a_2] = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$$|B| = 3 - 2 = 1$$

$$B^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$x_B = B^{-1} b = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 36 \end{bmatrix} = \begin{bmatrix} 18 \\ 6 \end{bmatrix}$$

$$c_B = \begin{bmatrix} 5 & 2 \end{bmatrix}$$

$$Y_1 = B^{-1} a_1 = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$z_1 - c_1 = c_B Y_1 - c_1 = [5 \;\; 2] \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 5 = 0$$

$$Y_2 = B^{-1} a_2 = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$z_2 - c_2 = c_B Y_2 - c_2 = [5 \;\; 2] \begin{bmatrix} 0 \\ 1 \end{bmatrix} - 2 = 0$$

$$Y_3 = B^{-1} a_3 = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \end{bmatrix}$$

$$z_3 - c_3 = c_B Y_3 - c_3 = [5 \;\; 2] \begin{bmatrix} -8 \\ 3 \end{bmatrix} - 2 = \boxed{-36}$$

$$Y_4 = B^{-1} a_4 = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$z_4 - c_4 = c_B Y_4 - c_4 = [5 \;\; 2] \begin{bmatrix} 3 \\ -1 \end{bmatrix} - 0 = 13$$

$$Y_5 = B^{-1} a_5 = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$z_5 - c_5 = c_B Y_5 - c_5 = [5 \;\; 2] \begin{bmatrix} -2 \\ 1 \end{bmatrix} - 0 = \boxed{-8}$$

→ as $z_3 - c_3$ is the least -ve value of all $z_j - c_j$
a_3 will be the entering vector

→ min $\left(\frac{6}{3} \right) = 2$ *, i.e a_2 will be exiting vector.

II) $B = [a_1 \;\; a_3] = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$

$|B| = 1 + 2 = 3$

~~det~~ $B^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \times \frac{1}{3}$

~~$C_B =$~~ $x_B = B^{-1} b = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 36 \end{bmatrix}$

$= \frac{1}{3} \begin{bmatrix} 102 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} 34 \\ 2 \end{bmatrix}$

$C_B = [5 \;\; 2]$

$Y_1 = B^{-1} a_1 = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$z_1 - c_1 = C_B Y_1 - c_1 = [5 \;\; 2] \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 5 = 0$

$Y_2 = B^{-1} a_2 = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 8 \\ 1 \end{bmatrix}$

$z_2 - c_2 = C_B Y_2 - c_2 = \frac{1}{3} [5 \;\; 2] \begin{bmatrix} 8 \\ 1 \end{bmatrix} - 2 = 12$

$Y_3 = B^{-1} a_3 = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$z_3 - c_3 = C_B Y_3 - c_2 = \cancel{\frac{1}{3}} [5 \;\; 2] \begin{bmatrix} 0 \\ 1 \end{bmatrix} - 2 = 0$

$Y_4 = B^{-1} a_4 = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$z_4 - c_4 = C_B Y_4 - c_4 = \frac{1}{3} [5 \;\; 2] \begin{bmatrix} 1 \\ -1 \end{bmatrix} - 0 = 1$

$Y_5 = B^{-1} a_5 = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$z_5 - c_5 = C_B Y_5 - c_5 = \frac{1}{3} [5 \;\; 2] \begin{bmatrix} 2 \\ 1 \end{bmatrix} - 0 = 4$

→ as none of the $z_j - c_j < 0$, $j = 1, 2, 3, 4, 5$

→ the optimal solution for the LPP is

$x_1 = 34$

$x_2 = 0$, i.e $(34, 0, 2)$

$x_3 = 2$

~~$x_4 = 12$~~

with $z_{max} = 174$

Q) solve LPP by big M method and prove that the problem has finite optimal solution

$$\min z = 3x_1 + 5x_2$$

st $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60$

$x_1, x_2 \geq 0$

→ introducing slack, surplus, and artificial variables

$$\min z = 3x_1 + 5x_2 + 0x_3 + 0x_4 + 0x_5 + Mx_6 + Mx_7$$

st $x_1 + 2x_2 - x_3 + 0x_4 + 0x_5 + x_6 + 0x_7 = 8$

$3x_1 + 2x_2 - x_4 + x_7 = 12$

$5x_1 + 6x_2 + x_5 = 60$

Simplex Table

		c_j	3	5	0	0	0	M	M	Remark
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	a_7	
M	a_6	x_6	8	1	2	-1	0	0	1	0	max $z_j - c_j = 4M-3$
M	a_7	x_7	12	$\boxed{3}$	2	0	-1	0	0	1	∴ a_1 entering
0	a_5	x_5	60	5	6	0	0	1	0	0	min $(\frac{8}{1}, \frac{12}{3}, \frac{60}{5})$ = 4
		$z_j - c_j$	$4M-3$ ↑	$4M-5$	$-M$	$-M$	0	0	0	∴ a_7 leaving, key 3
M	a_6	x_6	4	0	$\boxed{\frac{4}{3}}$	-1	$\frac{1}{3}$	0	1	–	max $z_j - c_j = \frac{4}{3}M-3$
3	a_3	x_3	4	1	$\frac{2}{3}$	0	$-\frac{1}{3}$	0	0	–	∴ a_2 enter
0	a_5	x_5	40	0	$\frac{8}{3}$	0	$\frac{5}{3}$	1	0	–	min (3, 6, 15) = 3
		$z_j - c_j$	0	$\frac{4}{3}M-3$ ↑	$-M$	$\frac{M}{3}-1$	0	0	–	∴ a_6 exit, key = $\frac{4}{3}$
5	a_2	x_2	3	0	1	$-\frac{3}{4}$	$\frac{1}{4}$	0	–	–	
3	a_1	x_1	2	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	–	–	
0	a_5	x_5	32	0	0	2	1	1	–	–	
		$z_j - c_j$	0	0	$-\frac{9}{4}$	$-\frac{1}{4}$	0	–	–	

as all $z_j - c_j \leq 0$, optimal solution exist

$x_1 = 2 \quad x_2 = 3$

$\min z = 21$

Q) solve the following LPP by Big M method & prove that the problem has no feasible sol^n

$$\max z = 5x_1 + 11x_2$$

$$\text{st} \quad 2x_1 + x_2 \le 4$$
$$3x_1 + 4x_2 \ge 24$$
$$2x_1 - 3x_2 \ge 6$$
$$x_1, x_2 \ge 0$$

→ introducing slack / surplus, and artificial variables

$$\max z = 5x_1 + 11x_2 + 0x_3 + 0x_4 + 0x_5 - Mx_6 - Mx_7$$

st

$$2x_1 + x_2 + x_3 = 4$$
$$3x_1 + 4x_2 - x_4 + x_6 = 24$$
$$2x_1 - 3x_2 - x_5 + x_7 = 6$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \ge 0$$

C_B	B	x_B	c_j / b	5 / a_1	11 / a_2	0 / a_3	0 / a_4	0 / a_5	$-M$ / a_6	$-M$ / a_7	Remark
0	a_3	x_3	4	$\boxed{2}$	1	1	0	0	0	0	min $-$ve $z_j-c_j = -5M-5$
$-M$	a_6	x_6	24	3	4	0	-1	0	1	0	∴ a_1 enter.
$-M$	a_7	x_7	6	2	-3	0	0	-1	0	1	$\min\left(\frac{4}{2}, \frac{24}{3}, \frac{6}{2}\right) = 2$
			z_j-c_j	$-5M-5$ ↑	$-M-11$	0	M	M	0	0	∴ a_3 leaving 2 key
5	a_1	x_1	2	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	all $z_j - c_j \ge 0$
$-M$	a_6	x_6	18	0	$\frac{5}{2}$	$-\frac{3}{2}$	-1	0	1	0	but artificial vectors
$-M$	a_7	x_7	2	0	-4	-1	0	-1	0	1	a_6, a_7 appears in
			z_j-c_j	0	$\frac{3M}{2}-\frac{17}{2}$	$\frac{5M}{2}+\frac{5}{2}$	M	M	0	0	the basis at +ve level

∴ There is no feasible solution

Q) Use simplex method to solve

$$\text{Max } z = 2x_2 + x_3$$

st $\quad x_1 + x_2 - 2x_3 \le 7$

$\quad -3x_1 + x_2 + 2x_3 \le 3$

$\quad x_1, x_2, x_3 \ge 0$

introducing slack variables

$\max Z = 0x_1 + 2x_2 + x_3 + 0x_4 + 0x_5$

st $\quad x_1 + x_2 - 2x_3 + x_4 = 7$

$\quad -3x_1 + x_2 + 2x_6 + x_5 = 3$

$\quad x_1, x_2, x_3, x_4, x_5 \ge 0$

		c_j		0	2	1	0	0	
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	Remark
0	a_4	x_4	7	1	1	−2	1	0	min −ve $z_j - c_j = -2$ ∴ a_2 enter
0	a_5	x_5	3	−3	$\boxed{1}$	2	0	1	$\min\left(\frac{7}{1}, \frac{3}{1}\right) = 3$ ∴ a_5 exit, 1 key
			$z_j - c_j$	0	$-2 \uparrow$	−1	0	0	
0	a_4	x_4	4	$\boxed{4}$	0	−4	1	−1	min −ve $z_j - c_j = -6$ ∴ a_1 enter
2	a_2	x_2	3	−3	1	2	0	1	$\min\left(\frac{4}{4}\right) = 1$ ∴ a_4 exit, key = 4
			$z_j - c_j$	$-6 \uparrow$	0	3	0	2	
0	a_1	x_1	1	1	0	−1	$\frac{1}{4}$	$-\frac{1}{4}$	min −ve $z_j - c_j = -3$ ∴ a_3 exit
2	a_2	x_2	6	0	1	−1	$-\frac{3}{4}$	$\frac{1}{4}$	but all $y_{ij} < 0$
			$z_j - c_j$	0	0	$-3 \uparrow$	$\frac{3}{2}$	$\frac{1}{2}$	

∴ Solution to give LPP is unbounded.

Q) Solve LPP and prove that alternative optimal solution exist

$$\max\ z = 2x_1 - x_2 + 3x_3 + x_4$$

st
$$2x_1 + x_2 + 3x_3 + 5x_4 \le 12$$
$$3x_1 + 2x_2 + x_3 + 4x_4 \le 15$$
$$x_1, x_2, x_3, x_4 \ge 0$$

introducing slack variables:

$$\max\ Z = 2x_1 - x_2 + 3x_3 + x_4 + 0x_5 + 0x_6$$

st
$$2x_1 + x_2 + 3x_3 + 5x_4 + x_5 = 12$$
$$3x_1 + 2x_2 + x_3 + 4x_4 + x_6 = 15$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

			c_j	2	-1	3	1	0	0	
CB	B	x_B	b	a_1	a_2	a_3	a_4	a_5	a_6	Remarks
0	a_5	x_5	12	2	1	$\boxed{3}$	5	1	0	min -ve $z_j - c_j = -3$
0	a_6	x_6	15	3	2	1	4	0	1	∴ a_3 enter
			$z_j - c_j$	-2	1	-3 ↑	-1	0	0	$\min\left(\frac{12}{3}, \frac{15}{1}\right) = 4$ ∴ a_5 exit, 3 key
3	a_3	x_3	4	$\cancel{\frac{2}{3}}$ $\frac{2}{3}$	$\frac{1}{3}$	1	$\frac{5}{3}$	$\frac{1}{3}$	0	all $z_j - c_j \ge 0$
0	a_6	x_6	11	$\boxed{\frac{7}{3}}$	$\frac{5}{3}$	0	$\frac{7}{3}$	$-\frac{1}{3}$	1	∴ $\boxed{x_1 = 0, x_2 = 0, x_3 = 4, x_4 = 0, z_{max} = 12}$
			$z_j - c_j$	0 ↑	2	0	4	1	0	here $z_1 - c_1 = 0$ but a_1 not in basis ∴ ~~exit a_5 and enter a_1 key $\frac{5}{3}$~~
~~3~~	~~a_3~~	~~x_3~~								$\min\left(\frac{4}{\frac{2}{3}}, \frac{11}{\frac{7}{3}}\right) = \frac{33}{7}$
~~2~~	~~a_1~~	~~x_1~~								∴ a_1 enter a_6 exit
3	a_3	x_3	$\frac{6}{7}$	0	$-\frac{1}{7}$	1	1	$\frac{3}{7}$	$-\frac{2}{7}$	all $z_j - c_j \ge 0$
2	a_1	x_1	$\frac{33}{7}$	1	$\frac{5}{7}$	0	1	$-\frac{1}{7}$	$\frac{3}{7}$	∴ $\boxed{x_1 = \frac{33}{7}, x_2 = 0, x_3 = \frac{6}{7}, x_4 = 0, z_{max} = 12}$
			$z_j - c_j$	0	2	0	4	1	0	

∴ Two optimal solution: $(0, 0, 4, 0)$ & $\left(\frac{33}{7}, 0, \frac{6}{7}, 0\right)$

with $z_{max} = 12$