Indian Institute of Engineering Science and Technology, Shibpur

B. Tech. (CST) 4^{th} Semester End-Term Examination, May 2021

Theory of Computation (CS2204)

Full Marks: 50 Time: 90 Minutes

- **Attempt Question No. 1 and any 2 from the remaining 3.**
- **The number within square brackets ([ ]) at the end of each question indicates its marks.**
- **Answers should be precise and to the point.**
- **Make your own assumptions, if necessary, and state them at proper places.**

1. State whether the statement is true or false (1 mark)! **Formally** justify your answer (remaining marks)!
   (a) If L is a finite language then it is context-free. [1 + 2]
   (b) There exists an **upper bound** on the length of the yield ω of a parse-tree T under a Context-Free Grammar $G = (V, \Sigma, R, S)$. [1 + 3]
   (c) Pumping theorem for the class of **Regular Languages** can be used to show that a language is **Context-Free**. [1 + 3]
   (d) Intersection of two Context-Free Languages can never be a Context-Free Language. [1 + 2]

2. As stated at its side, for **each** of the following languages construct a Deterministic or Non-Deterministic Finite Automaton (**FA**) **or** a Pushdown Automaton (**PDA**) that accepts the language.
   (a) $\{\omega_1\omega_2 \ : \ \omega_1 \in \{a, b\}^*, \ \omega_2 \in \{c, d\}^*$ and $|\omega_1| = 2|\omega_2|$, that is, length of ω_1 is twice that of $\omega_2\}$ [**PDA**] [6]
   (b) $\{\omega \in \{x, y, z\}^* \ : \ \omega$ does not contain $yxyxx$ as a substring$\}$ [**FA**] [6]
   (c) $\{a^i b^j c^i \ : \ i, j \geq 0\}$ [**PDA**] [6]

3. As stated at its side, for **each** of the following languages construct a Context-Free Grammar (**CFG**) or an Unrestricted Grammar (**UG**) that generates the language.
   (a) $\{a^i b^j c^j d^i \ : \ i, j \geq 0\}$ [**CFG**] [8]
   (b) $\{a^i b^j c^i d^j \ : \ i, j \geq 0\}$ [**UG**] [10]

4. As stated at its side, show that each of the following functions is Grammatically Computable (or Turing Computable) by constructing a Grammar (or a Turing machine) that computes the function.
   (a) $f : \mathcal{N} \to \mathcal{N}, \ f(n) = 2^n$ [**Grammatically Computable**] [10]
   (b) $f : \{a, b\}^* \to \mathcal{N}, \ f(\omega) = |\omega|$, where $|\omega|$ denotes the length of the string ω. [**Turing Computable**] [8]