Simplification of Boolean Functions using Karnaugh Map

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Karnaugh Map

- What is Karnaugh map?
  A Karnaugh map may be considered as a pictorial representation of a truth table.
  It provides straightforward procedure for minimizing Boolean functions.
- For an n-variable function, there are 2^n cells in the map (one for each minterm).
- Adjacent 2 cells differ in only one variable.
- Adjacent $2^2 = 4$ cells differ in 2 variables.
- Adjacent 2^m cells differ in m variables.

Karnaugh Map

- We try to group 2^m cells corresponding true minterms. Try to make cubs (groups) bigger, ensure all the minterms are covered.
- How does you label the Karnaugh map such that its property is ensured?
  (i) such that two adjacent differ in the value of one variable.
  (ii) help in combing cells into cubs.
  $ABC + A\,\overline{B}C = AC$
  $A\overline{B}\;\overline{C} + A\,\overline{B}C + AB\,\overline{C} + ABC = A$
  <u>Drawback</u>

- Since it is pictorial approach, it is difficult to visualize function with more than 5 or 6 variables.
- We shall discuss examples with upto 4 variables.

Basic Approach

1. Fill up the cells of K-map with true minterms of the function.
2. Group the true minterms of the function into cubs such that
   - The size of the cubes are maximized.
   - Every true minterms is covered by at least one cube.
3. Write down the minimized sum-of-product expression for the function by creating one product term out of every cube that has been selected.

Three Variable Karnaugh Map

Examples:

Four Variable Karnaugh Map

The basic concept of labelling the variables remains the same:

- Adjacent cells differ in value of a single variable.
- Top-bottom and left-right cells are also adjacent.
- The four corner cells are considered adjacent to each other.

Examples:

Handling Don't Care Inputs

- There exists functions for which some of the inputs are treated as don't cares, and the corresponding output values do not matter.
  - Such inputs never appear. For example, in case of BCD number miterms 1010 to 1111 never come in the inputs.
- The don't care minterms are labelled as'X' in the K-map.
- When creating the cubes, we can include cells marked 'X' along with those marked as '1' to make larger cube.
- But it is not necessary to cover all the cells marked by 'X'.

Examples:

Some Definitions

- **Implicant:** Given a function f of n variables, a minterm p is an implicant of f if and only if, for all combinations of the n variables for which $p = 1$, f is also 1.
  Consider the the function $f = A\overline{B}C + A\overline{C} + \overline{B}\ \overline{C}$
  The term $A\overline{B}C$ is an implicant because when A = 1, B = 0 and C = 1, the term is 1 as well as the function is also 1.
- **Prime implicant:** An implicant is said to be prime implicant if after removing any literal from it, the resulting product is no longer an implicant..
- With respect to K-map, it is a cube that is not completely covered by another implicant represent a larger cube.

Consider the function$f = \overline{A}B + AC + \overline{B}\ \overline{C}$
$\overline{A}B$ is an prime implicant. A B C (010 and 0 1 1), for these combinations of the inputs, $\overline{A}B = 1$. If we remove B from $\overline{A}B$, the resulting term $\overline{A}$ is not an implicant. Because 0 0 0 and 0 0 1 for this two combinations of the inputs the function is not equal to 1.

Essential Prime Implicant:

A prime implicant of a function is said to be essential prime implicant if it covers at least one minterm of the function that is not cover by any other prime implicant.

Some Results:

- Every irredundant sum-of-product expression equivalent to a function F is a union of prime implicant of F.
- The set of all essential prime implicants must be present in any irredundant sum-of-product expression.
- Any prime implicant covered by the some of the essential prime implicants must not be present in an irredundant sum-of-product expression.