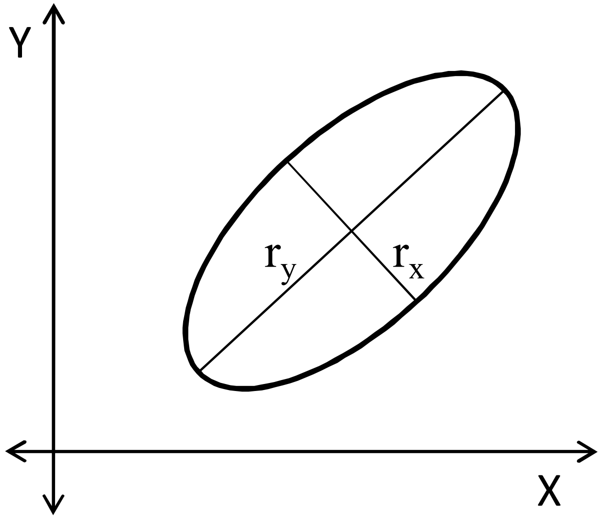
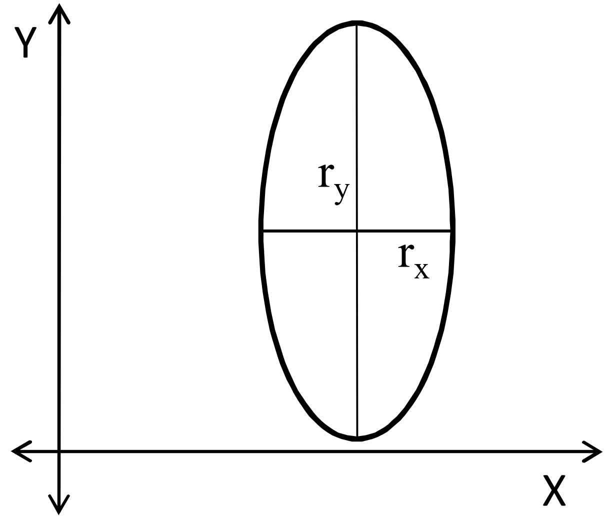
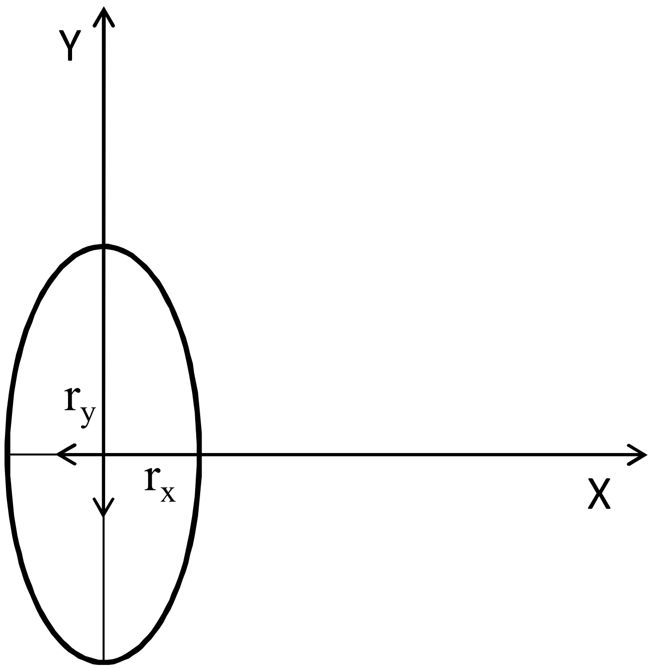
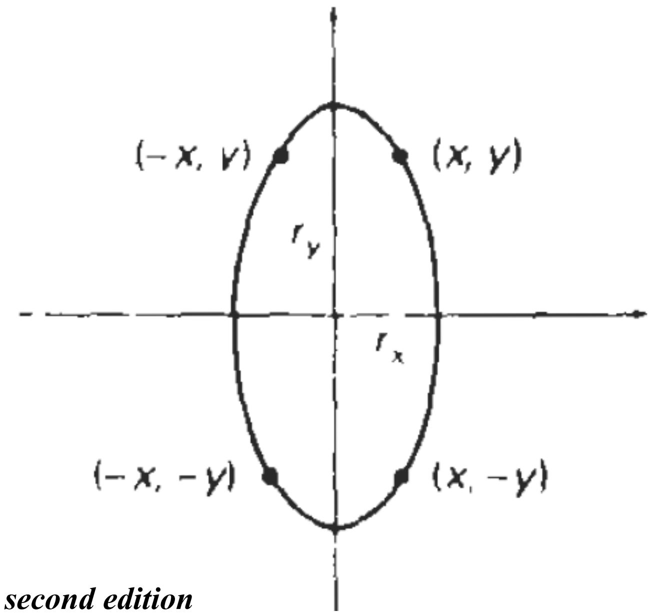
Midpoint ellipse drawing algorithm









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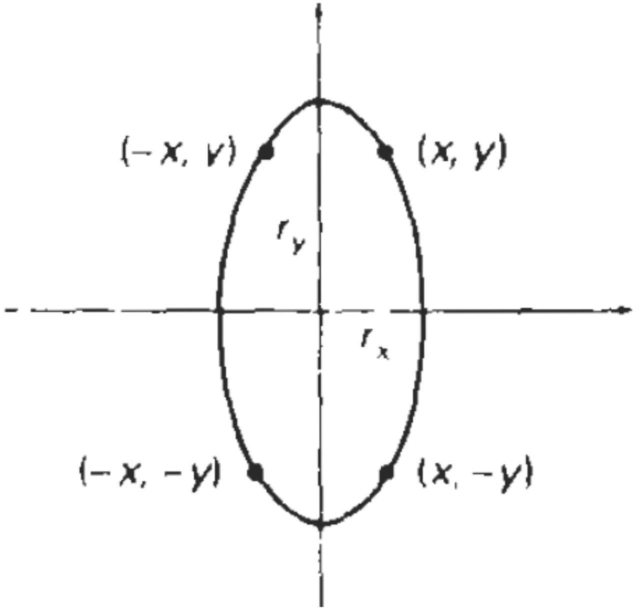

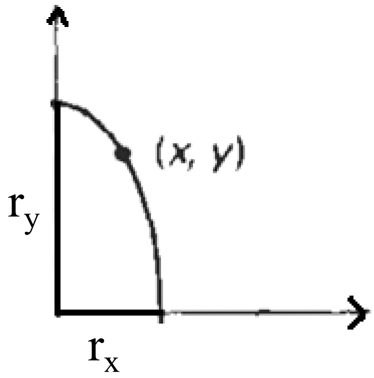

$$\left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2 = 1$$

$$x^2 r_y^2 + y^2 r_x^2 - r_x^2 r_y^2 = 0$$

$$f_{\text{Ellipse}}(x, y) = x^2 r_y^2 + y^2 r_x^2 - r_x^2 r_y^2$$

$f_{\text{Ellipse}}(x, y) = 0$ If (x,y) is on the boundary

$f_{\text{Ellipse}}(x, y) < 0$ If (x,y) is inside the boundary

$f_{\text{Ellipse}}(x, y) > 0$ If (x,y) is outside the boundary

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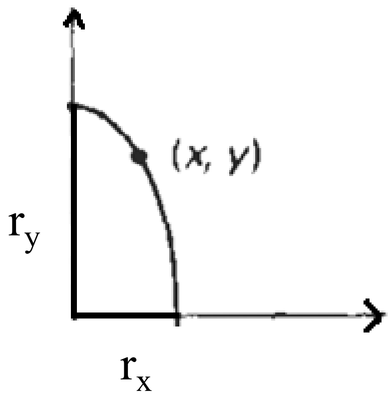


Region I

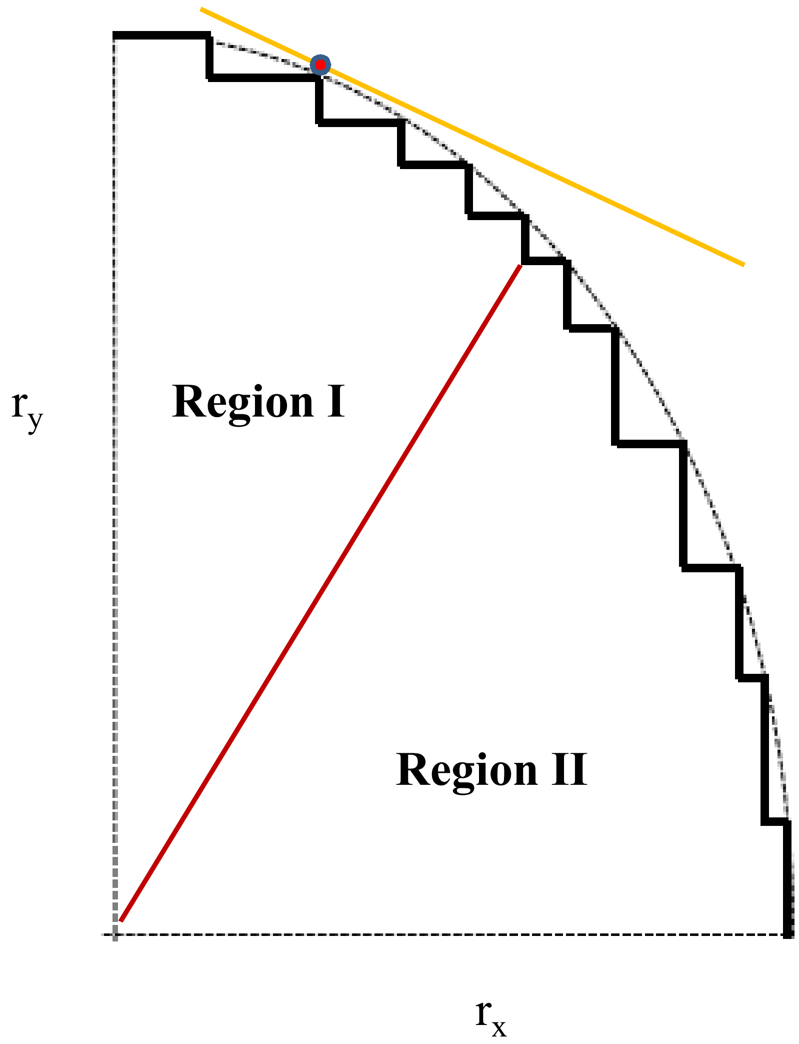
$$| dy | \le | dx |$$

$$\frac{| dy |}{| dx |} \le 1$$

Region II

$$| dy | > | dx |$$

$$\frac{| dy |}{| dx |} > 1$$



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$$\left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2 = 1$$

$$\left(\frac{y}{r_y}\right)^2 = 1 - \left(\frac{x}{r_x}\right)^2$$

$$\frac{|dy|}{|dx|} = \frac{x r_y^2}{y r_x^2}$$

Region I

$$\frac{|dy|}{|dx|} \leq 1$$

$$\frac{x r_y^2}{y r_x^2} \leq 1$$

$$\boxed{x r_y^2 \leq y r_x^2}$$

Region II

$$\frac{|dy|}{|dx|} > 1$$

$$\frac{x r_y^2}{y r_x^2} > 1$$

$$\boxed{x r_y^2 > y r_x^2}$$

Region I

We have plotted (x_k,y_k) and next pixel is (x_{k+1},y_{k+1}) where $x_{k+1} = x_k+1$

$y_{k+1} = y_k$ or y_k-1

$$pl_k = f_{Ellipse}(x_k+1, y_k-\frac{1}{2})$$

$$= (x_k+1)^2 r_y^2 + (y_k-\frac{1}{2})^2 r_x^2 - r_x^2 r_y^2$$

$$pl_{k+1} = f_{Ellipse}(x_{k+1}+1, y_{k+1}-\frac{1}{2})$$

$$= f_{Ellipse}((x_k+1)+1, y_{k+1}-\frac{1}{2})$$

$$= (x_k+2)^2 r_y^2 + (y_{k+1}-\frac{1}{2})^2 r_x^2 - r_x^2 r_y^2$$

$$pl_{k+1} - pl_k = ((x_k+2)^2 r_y^2 + (y_{k+1}-\frac{1}{2})^2 r_x^2 - r_x^2 r_y^2) - ((x_k+1)^2 r_y^2 + (y_k-\frac{1}{2})^2 r_x^2 - r_x^2 r_y^2)$$

$$pl_{k+1} = pl_k + ((x_k+2)^2 r_y^2 + (y_{k+1}-\frac{1}{2})^2 r_x^2 - r_x^2 r_y^2) - ((x_k+1)^2 r_y^2 + (y_k-\frac{1}{2})^2 r_x^2 - r_x^2 r_y^2)$$

If $pl_k < 0$ $\quad y_{k+1} = y_k$ $\quad pl_{k+1} = pl_k + 2r_y^2 x_{k+1} + r_y^2$

$pl_k > 0$ $\quad y_{k+1} = y_k - 1$ $\quad pl_{k+1} = pl_k + 2r_y^2 x_{k+1} + r_y^2 - 2r_x^2 y_{k+1}$

Region II

We have plotted (x_k,y_k) and next pixel is (x_{k+1},y_{k+1}) where $x_{k+1} = x_k$ or x_k+1
$y_{k+1} = y_k - 1$

$$p2_k = f_{Ellipse}(x_k + \frac{1}{2}, y_k - 1)$$
$$= (x_k + \frac{1}{2})^2 r_y^2 + (y_k - 1)^2 r_x^2 - r_x^2 r_y^2$$

$$p2_{k+1} = f_{Ellipse}(x_{k+1} + \frac{1}{2}, y_{k+1} - 1)$$
$$= f_{Ellipse}((x_{k+1} + \frac{1}{2}) + 1, (y_k - 1) - 1)$$
$$= (x_{k+1} + \frac{1}{2})^2 r_y^2 + (y_k - 2)^2 r_x^2 - r_x^2 r_y^2$$
$$p2_{k+1} - p2_k = ((x_k + \frac{1}{2})^2 r_y^2 + (y_k - 1)^2 r_x^2 - r_x^2 r_y^2) - ((x_{k+1} + \frac{1}{2})^2 r_y^2 + (y_k - 2)^2 r_x^2 - r_x^2 r_y^2)$$
$$p2_{k+1} = p2_k + ((x_k + \frac{1}{2})^2 r_y^2 + (y_k - 1)^2 r_x^2 - r_x^2 r_y^2) - ((x_{k+1} + \frac{1}{2})^2 r_y^2 + (y_k - 2)^2 r_x^2 - r_x^2 r_y^2)$$

If $p2_k < 0$ $\quad x_{k+1} = x_k$ $\quad p2_{k+1} = p2_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$

$p2_k > 0$ $\quad x_{k+1} = x_k + 1$ $\quad p2_{k+1} = p2_k - 2r_x^2 y_{k+1} + r_x^2$

$$p1_0 = f_{Ellipse}(0+1, r_y - \frac{1}{2})$$

$$= r_y^2 + (r_y - \frac{1}{2})^2 r_x^2 - r_x^2 r_y^2$$

$$\boxed{p1_0 = r_y^2 + \frac{1}{4} r_x^2 - r_x^2 r_y}$$

$$p2_0 = f_{Ellipse}(x_0 + \frac{1}{2}, y_0 - 1)$$

$$\boxed{p2_0 = r_y^2 (x_0 + \frac{1}{2})^2 + (y_0 - 1)^2 r_x^2 - r_x^2 r_y^2}$$

(x_0,y_0) is the last point of region I

Midpoint Ellipse Algorithm

1. Input r_x, r_y, and ellipse center (x_c, y_c), and obtain the first point on an ellipse centered on the origin as

$$(x_0, y_0) = (0, r_y)$$

2. Calculate the initial value of the decision parameter in region 1 as

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$$

3. At each x_k position in region 1, starting at $k = 0$, perform the following test: If $p1_k < 0$, the next point along the ellipse centered on (0, 0) is (x_{k+1}, y_k) and

$$p1_{k+1} = p1_k + 2r_y^2 x_{k+1} + r_y^2$$

Otherwise, the next point along the circle is $(x_k + 1, y_k - 1)$ and

$$p1_{k+1} = p1_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$$

with

$$2r_y^2 x_{k+1} = 2r_y^2 x_k + 2r_y^2, \qquad 2r_x^2 y_{k+1} = 2r_x^2 y_k - 2r_x^2$$

and continue until $2r_y^2 x \geq 2r_x^2 y$.

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4. Calculate the initial value of the decision parameter in region 2 using the last point (x_0, y_0) calculated in region 1 as

$$p2_0 = r_y^2\left(x_0 + \frac{1}{2}\right)^2 + r_x^2(y_0 - 1)^2 - r_x^2 r_y^2$$

5. At each y_k position in region 2, starting at $k = 0$, perform the following test: If $p2_k > 0$, the next point along the ellipse centered on $(0, 0)$ is $(x_k, y_k - 1)$ and

$$p2_{k+1} = p2_k - 2r_x^2 y_{k+1} + r_x^2$$

Otherwise, the next point along the circle is $(x_k + 1, y_k - 1)$ and

$$p2_{k+1} = p2_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$$

using the same incremental calculations for x and y as in region 1.

6. Determine symmetry points in the other three quadrants.
7. Move each calculated pixel position (x, y) onto the elliptical path centered on (x_c, y_c) and plot the coordinate values:

$$x = x + x_c, \qquad y = y + y_c$$

8. Repeat the steps for region 1 until $2r_y^2 x \geq 2r_x^2 y$.

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