B-TECH 4TH SEMESTER MID-TERM EXAMINATION

April 2021

Subject: Theory of Computation [CS2204]

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Q1) a) Statement is True

→ A finite Language L on alphabet Σ is a finite set of strings containing symbol of Σ

→ Each string can be treated as a symbol for some other alphabet Σ', hence L can be alphabet for some other Language of L.

Q.1) b) given

$G = (V, \Sigma, R, S)$ regular iff it is context free

This is not true, consider following counter example.

Rules: $S \to aSb$

$S \to e$

$L = \{a^n b^n, n \geq 0\}$

$V = (a, S, b)$

$\Sigma = \{a, b\}$

$R = \{S \to aSb, S \to e\}$

$S = S$

→ This Language can't be made in form of xy^iz, ~~Lemma~~ ~~x~~ $y \neq e$, hence ~~&~~ it doesn't follow Pumping Lemma and hence is not Regular.

Q.1) c) conside $\Sigma = \{a, b\}$

any Language will be subset of Σ^*

$\therefore L \subseteq \Sigma^*$

now subset of Σ^* contains both ϕ and $\{e\}$

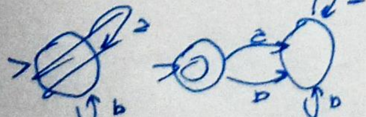
→ So ϕ and $\{e\}$ are Languages over Σ

→ Now we need to see if they are Regular or not

→ ϕ is automatically Regular (by definition of RE)

→ $\{e\}$ can be accepted by following ~~PDA~~ DFA

→ hence $\{e\}$ is also RE.



$\therefore$ Statement is True

2) a) $L = \{a^m b^n c^l \mid l, m, n \geq 0,\ m \leq l + n\}$

[CFG]

~~Rules~~

~~$S_1 \to aS_1$~~

~~$S_1 \to aS_1c$~~ ~~$S_1 \to e$~~

~~$S_1 \to aS_2$~~

~~$S_1 \to abS_2b$~~

~~$S_2 \to aS_2$~~

~~$S_2 \to aS_2b$~~ ~~$S_2 \to e$~~

Eg of strings generated: abc, abbc, abcc, acc

aabbc, aabcc, aabc

Rules

$S_1 \to S_1c$

$S_1 \to aS_1c$ $\quad S_1 \to e$

$S_1 \to S_2$

$S_2 \to S_2b$

$S_2 \to aS_2b$ $\quad S_2 \to e$.

$\therefore G = (V, \Sigma, R, S)$

$V = \{S_1, c, a, S_2, b\}$

$\Sigma = \{a, b, c\}$

$R = \{S_1 \to S_1c,\ S_1 \to aS_1c,\ S_1 \to S_2,\ S_1 \to e,\ S_2 \to S_2b,$

$S_2 \to aS_2b,\ S_2 \to e\}$

$S = S_1$

2)b) $L = \{a^m b^n \mid m,n \geq 0$ and $m \neq n\}$

[PDA]

→ first let us draw CFG, then we convert to PDA

$m \neq n \rightarrow m < n \cup m > n$

I) $L_1 = \{a^m b^n \mid m,n \geq 0, m < n\}$

1) $S_1 \to S_1 b$
2) $S_1 \to aS_1 b$
3) $S_1 \to b$

strings ~~accepted~~ generated: abb, aabbbb,

strings not generated: ab

II) $L_2 = \{a^m b^n \mid m,n \geq 0, m > n\}$

4) $S_2 \to aS_2$
5) $S_2 \to aS_2 b$
6) $S_2 \to a$

strings generated: a, aab, aaaab

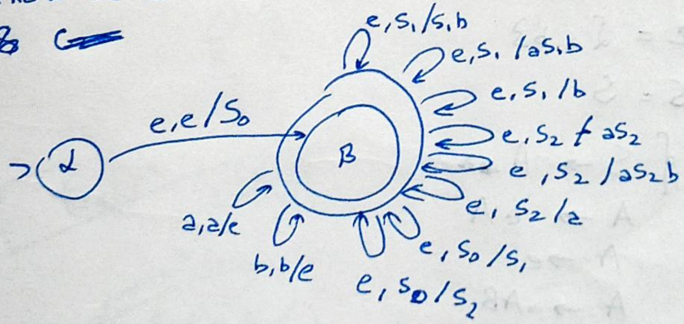
strings not generated: ab, abb

III) $L = L_1 \cup L_2$

7) $S_0 \to S_1$
8) $S_0 \to S_2$

→ now we convert to PDA

∴



$\therefore M = (K, \Sigma, \Gamma, \Delta, s, F)$

$K = \cancel{\{a, b\}}\ \{\alpha, \beta\}$

$\Sigma = \{a, b\}$

$\Gamma = \{S_0, S_1, S_2, a, b\}$

$\Delta = \{(\alpha, e, e, \beta, S_0), (\beta, e, S_1, \beta, S_1 b), (\beta, e, S_1, \beta, aS_1 b),$

$(\beta, e, S_1, \beta, b), (\beta, e, S_2, \beta, aS_2), (\beta, e, S_2, \beta, aS_2 b),$

$(\beta, e, S_2, \beta, a), (\beta, e, S_0, \beta, S_1), (\beta, e, S_0, \beta, S_2),$

$(\beta, b, b, \beta, e), (\beta, a, a, \beta, e)\}$

$s = \cancel{s}\ \alpha$

$F = \{\beta\}$

Q2) c) $L = \{w \in \{a, b\}^* \mid w \text{ has more } a\text{'s than } b\text{'s}\}$

strings to be generated: aab, aba, baa, a

strings not to be generated: ab, ba

$G = (V, \Sigma, R, S)$

$V = \{S, A, B, a, b\}$

$\Sigma = \{a, b\}$

$S = S$

$R = \{S \rightarrow Aa$

$A \rightarrow aA$

$A \rightarrow e$

$A \rightarrow aAB$

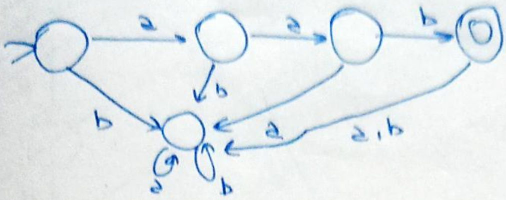
$A \rightarrow BaA$

$B \rightarrow b$

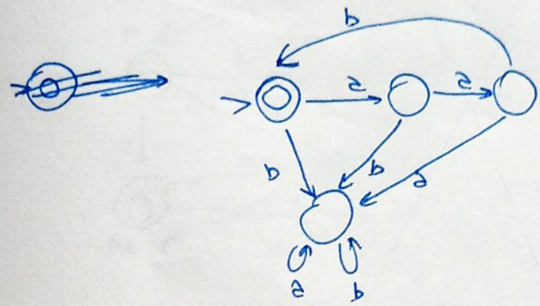
$B \rightarrow e\}$

Q4) a) RE ⇒ $(aab)^* \cdot (bba)^*$

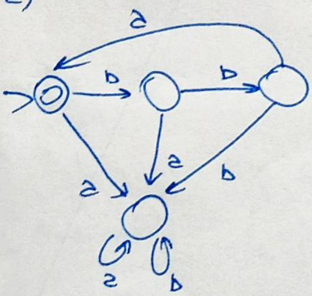
I) for aab



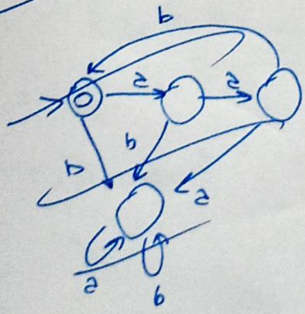
II) ~~aa~~ for $(aab)^*$



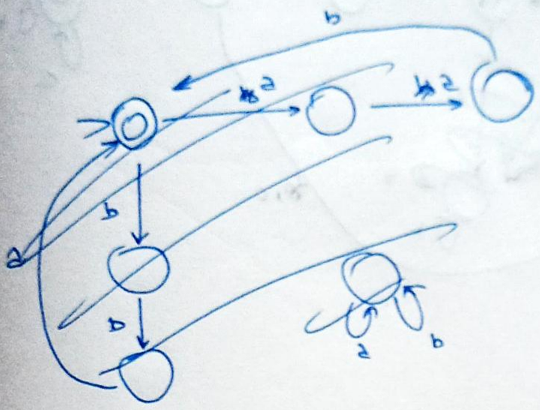
III) for $(bba)^*$

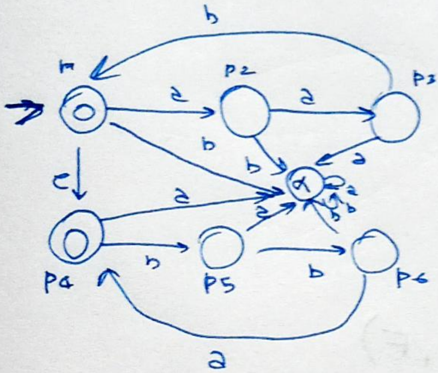


~~IV) for~~ $(aab)^* \cdot (bba)^*$



II) for $(aab)^* . (bba)^*$





$\therefore M = (K, \Sigma, \delta, s, F)$

$K = \{p_1, p_2, p_3, p_4, p_5, p_6, \alpha\}$

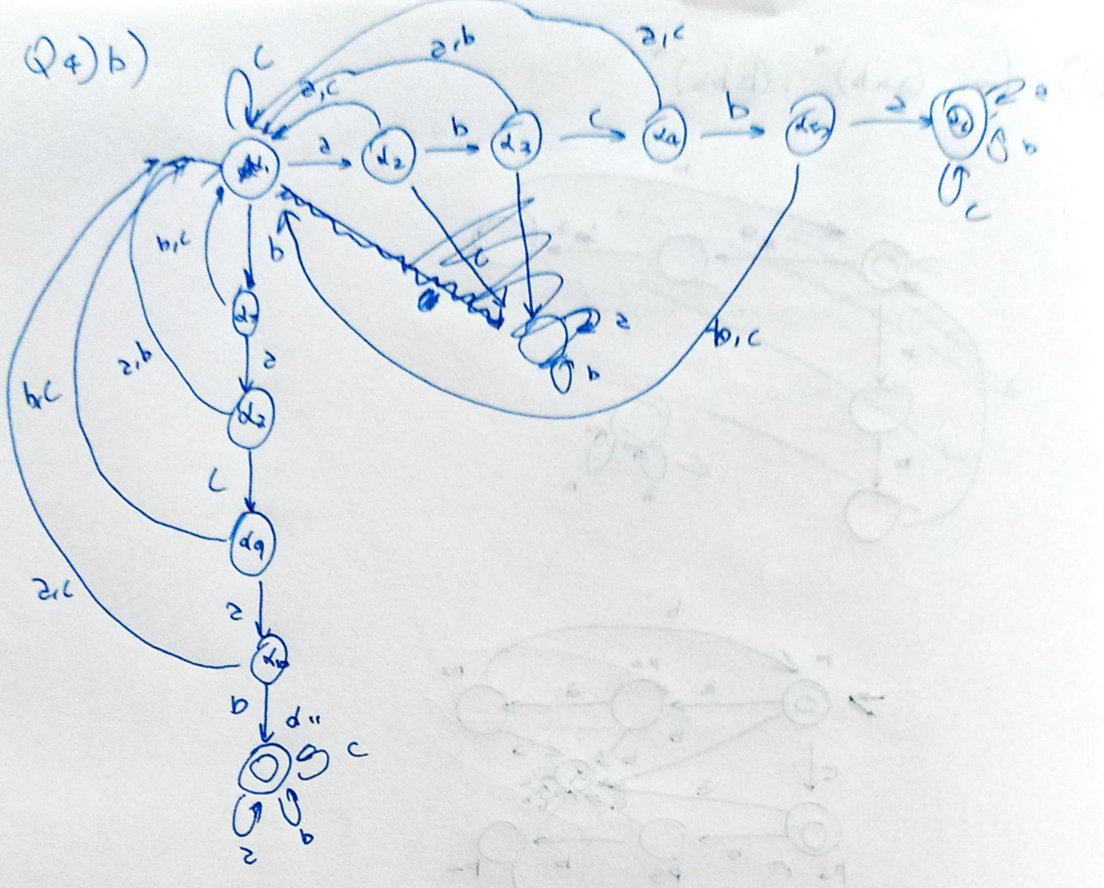
$\Sigma = \{a, b\}$

$\delta = \{(p_1, a, p_2), (p_2, a, p_3), (p_3, b, p_1), (p_1, b, \alpha),$
$(p_2, b, \alpha), (p_3, a, \alpha), (\alpha, a, \alpha), (\alpha, b, \alpha), (p_1, e, p_4)$
$(p_4, b, p_5), (p_5, b, p_6), (p_6, a, p_4), (p_4, a, \alpha),$
$(p_5, a, \alpha), (p_6, b, \alpha)\}$

$s = p_1$

$F = \{p_1, p_4\}$

(Q4) b)



$M = (K, \Sigma, \partial, s, F)$

$K = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}\}$

$\partial = \{(\alpha_1, c, \alpha_1), (\alpha_1, a, \alpha_2), (\alpha_2, b, \alpha_3), (\alpha_3, c, \alpha_4), (\alpha_4, b, \alpha_5), (\alpha_5, a, \alpha_6), (\alpha_6, a, \alpha_6), (\alpha_6, b, \alpha_6), (\alpha_6, c, \alpha_6), (\alpha_2, a, \alpha_1), (\alpha_2, c, \alpha_1), (\alpha_3, a, \alpha_1), (\alpha_3, b, \alpha_1), (\alpha_4, a, \alpha_1), (\alpha_4, c, \alpha_1), (\alpha_5, b, \alpha_1), (\alpha_5, c, \alpha_1), (\alpha_1, b, \alpha_7), (\alpha_7, a, \alpha_8), (\alpha_8, c, \alpha_9), (\alpha_9, a, \alpha_{10}), (\alpha_{10}, b, \alpha_{11})\}$

~~$s = \alpha_1$~~

~~$F = \{\alpha_6, \alpha_{11}\}$~~

$s = \alpha_1$

$F = \{\alpha_6, \alpha_{11}\}$