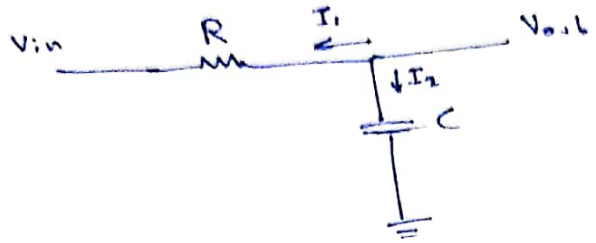
Low Pass Filter



Using Kirchoff's Law

$$\frac{V_{out} - V_{in}}{R} + C\frac{d(V_{out} - 0)}{dt} = 0$$

$$\frac{V_{out} - V_{in}}{R} + C\frac{d\,V_{out}}{dt} = 0$$

fourier: $$\frac{V_{out}(j\omega) - V_{in}(j\omega)}{R} + Cj\omega\, V_{out}(j\omega) = 0$$

or $$V_{out}(j\omega)\left[\frac{1}{R} + Cj\omega\right] = \frac{V_{in}(j\omega)}{R}$$

or $$V_{out}(j\omega) = \frac{V_{in}(j\omega)}{1 + RCj\omega}$$

$$\therefore H(j\omega) = \frac{1}{1 + RCj\omega} = \frac{1 - RCj\omega}{1 + R^2C^2\omega^2}$$

$$\therefore |H(j\omega)| = \frac{\sqrt{1 + R^2C^2\omega^2}}{1 + R^2C^2\omega^2} = \frac{1}{\sqrt{1 + R^2C^2\omega^2}}$$

$$\angle H(j\omega) = \tan^{-1}\left[-RC\omega\right] = -\tan^{-1}\left[RC\omega\right]$$