Shruti Bhatnagar

Indian Institute of Engineering Science and Technology, Shibpur

Dual Degree (B. Tech.- M. Tech.) 3rd Semester Midterm Examination, September 2016

Subject: Mathematics III (MA-301)

Time: 2 hours Full Marks: 100

Answer any FOUR questions.

1. (a) State and prove Bayes' theorem.

   (b) A and B are two independent witnesses in a case. The probability that A will speak the truth is x and the probability that B will speak the truth is y. A and B agree in a certain statement. Show that the probability that this statement is true is

   $$\frac{xy}{1-x-y+xy}.$$

   (c) Suppose their are 10 pairs of shoes in a closet. 4 shoes are taken out at random. Find the probability that among the 4 that are taken out, there is at least one complete pair?

   $8+9+8=25$

2. (a) 7 Mathematics and 3 Physics books are placed at random on a book shelf. Find the probability that none of the Physics books are placed consecutively.

   (b) A problem in statistics is given to 3 students A, B and C whose chances of solving it are 1/3, 2/5 and 3/7 respectively. What is the probability that the problem will be solved?

   (c) Consider events A and B such that $P(A)=1/4$, $P(B|A)=1/2$, $P(A|B)=1/4$. Find $P(A^c|B^c)$ and $P(A|B)+P(A|B^c)$.

   $8+8+9=25$

3. (a) Define pairwise independence and mutual independence of events. Give an example to show that pairwise independence need not imply mutual independence.

   (b) A coin, whose probability of turning up heads is 3/5, is tossed till the 7th head appears. Suppose X is the number of tosses required. Find the probability mass function(p.m.f.) of X.

   (c) Determine the value of the constant C such that $f(x)$ defined by $f(x)=Cx^{5/2}(1-x)^{7/2}$ for $0<x<1$ and 0 elsewhere, is a probability density function.

   $8+8+9=25$

4. (a) Find the Laplace transforms of the following functions: (i) $f(t)=e^t\sin t\cos t$, (ii) $f(t)=\sin^3 t$, (iii) $f(t)=\int_0^t \sin 2u\,du$

   (b) Let $f(t)$ be a periodic function with period T. Show that

   $$L\{f(t)\}=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt$$

   $15+10=25$

5. (a) Find (i) $L^{-1}\left\{\frac{1}{(s^2+a^2)(s^2+b^2)}\right\}$

   (ii) $L^{-1}\left\{\frac{s+1}{(s^2+s+1)}\right\}$

6, 7

(v) Define a convex set. Show that the feasible region of the LPP

$$\text{Max } c'x \text{ subject to}$$

$$Ax \leq b, \quad x \geq 0$$

(where c is a $1 \times n$ vector, x is a $n \times 1$ vector, A is a $m \times n$ matrix and b is a $m \times 1$ vector), is a convex set.

$12 + 13 = 25$

6. (a) Find, if possible, a basic solution with x_2 as a non-basic variable, of the following system

$$\begin{aligned} 2x_1 - 3x_2 + 5x_3 &= 10 \\ 4x_1 + x_2 + 10x_3 &= 20 \end{aligned}$$

(b) Determine which of the the following sets are convex:

(i) $X = \{(x, y) | x^2 + y^2 \leq 4\}$

(ii) $X = \{(x, y) | \frac{x^2}{9} + \frac{y^2}{16} = 1\}$

(iii) $X = \{(x, y) | (x + y - 1)(2x + y - 2) = 0\}$

$10 + 15 = 25$

Indian Institute of Engineering Science and Technology, Shibpur

Dual Degree (B.Tech-M.Tech) 3rd Semester Final Examination

November 2016

Subject: Mathematics-III (MA-301)

Time : 3 hours | Full Marks : 70

(Use separate answer script for each half)

First Half (Full Marks-40)

Answer any eight questions

1. State axiomatic definition of probability. Show that for any two events A and B:

$$P(\overline{A}B) = P(B) - P(AB).$$

[5]

2. State and prove Baye's theorem.

[5]

3. The chance that a doctor will diagnose a certain disease correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of the doctor, who had the disease, dies. What is the probability that the disease was diagnosed correctly?

[5]

4. Find the mean and variance of the Binomial distribution.

[5]

5. Suppose that an airplane engine will fail, when in flight, with probability $(1-p)$ independently from engine to engine. Suppose that the airplane will make a successful

$P(X \leq 50)$ flight if at least 50% of its engines remain operative. For what values of p is a four-engine plane preferable to a two-engine plane?

[5]

6. Prove that Poisson distribution is a limiting case of Binomial distribution. $n \to \infty$ $p \to 0$ $np =$ finite constant

[5]

7. State and prove Tchebycheff's inequality for a continuous random variable.

[5]

$(X \geq 50)$

8. Using Tchebycheff's inequality, find a lower bound for the probability of getting 64 to 184 driving licences issued by Road Transport Authority in a specific month. It is given that the number of driving licences issued per month be a random variable having mean $m = 124$ and standard deviation $\sigma = 7.5$.

[5]

9. State Law of Large Numbers. Suppose X_i takes the values $-\sqrt{2i-1}$ and $\sqrt{2i-1}$ with equal probabilities. Show that the law of large numbers can not be applied to the independent random variables $X_1, X_2, \cdots$.

[5]

P.T.O

10. The probability density function of a two-dimensional random variable (X, Y) is given by

$$f(x,y) = \begin{cases} k(x+y), & 0 < x+y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find k and evaluate $P(X < \frac{1}{2}, Y > \frac{1}{4})$.

[5]

11. Prove that the sample mean $\overline{X}$ is an unbiased estimator of the population mean. A random sample $\{X_1, X_2, X_3, X_4\}$ of size 4 is drawn from an infinite population having mean μ and variance σ^2. Define the two estimators of μ as follows:

$$T_1 = \frac{1}{5}(X_1 + X_2 + X_3 + 2X_4) \text{ and } T_2 = \frac{1}{8}(X_1 + 2X_2 + 3X_3 + 2X_4).$$

Which one is better? Why? Which one is best among $T_1, T_2, \overline{X}$ as an estimator of μ?

[5]

12. Let $x_1, x_2, \cdots, x_n$ be a particular sample of size n drawn from a population with probability density function:

$$f(x,\theta) = 1; \theta - \frac{1}{2} \leq x \leq \theta + \frac{1}{2}, -\infty < \theta < \infty.$$

Obtain maximum likelihood estimate of θ.

[5]

13. Fit a second degree parabola to the following data:

x_i	y_i
0	1
1	1.8
2	1.3
3	2.5
4	6.3

[5]

14. In a partially destroyed Laboratory record of an analysis of correlation data, the following results only are legible:

Var(X)=9.

Regression equations: $8X - 10Y + 66 = 0$, $40X - 18Y = 214$.

Find (i) the mean values of X and Y,

(ii) the correlation coefficient between X and Y, and

(iii) the standard deviation of Y.

[5]

P.T.O

2^{nd} Half -MA-301

Answer any <u>THREE</u> questions

15. a) Define Laplace transform of a function $F(t)$. Find the Laplace transform of t^n, when n is a positive but not necessarily an integer.

b) Evaluate (i) $L^{-1}\left\{\frac{8s+29}{s^2-12s+3^2}\right\}$

(ii) $L^{-1}\left\{\frac{4}{s^2}+\frac{(\sqrt{s}-1)^2}{s^2}-\frac{5}{3s+4}\right\}$

(1+3) + (3+3)

16. a) If $L\{F(t)\}=f(s)$, then prove that $L\{e^{at}F(t)\}=f(s-a)$.
Hence evaluate $L\{e^{-t}(3\sin 2t-5\cos 2t)\}$.

b) Use convolution theorem to find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

(3+2)+5

17. a) Use Laplace transform to solve the differential equation $\frac{d^2y}{dt^2}+y=6\cos 2t$, given that $y=3, \frac{dy}{dt}=1$ when $t=0$.

b) Obtain the Laplace inversion of $f(s)=\frac{1}{\sqrt{1+s^2}}$ as a series of powers of t.

6+4

18. a) Explain basic solution of a system of linear equations.
Find all the basic solutions of the set of equations

$$4x_1+2x_2+3x_3-8x_4=6$$
$$3x_1+5x_2+4x_3-6x_4=8$$

b) Let $x_1=1, x_2=3, x_3=2$ be a feasible solution of the set of equations

$$2x_1+4x_2-2x_3=10$$
$$10x_1+3x_2+7x_3=33.$$

Reduce the feasible solution to a basic feasible solution.

(2+3)+5

19. a) Define hyperplane and convex set.

Prove that a hyperplane is a convex set.

b) Use simplex method to maximize $5x_1 + 2x_2 + 2x_3$

subject to $2x_1 + 2x_2 - 2x_3 \le 30$

$x_1 + 3x_2 + x_3 \le 36$

$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$ (2+2)+6

20. a) Solve the following L. P. P. by big M method

$Max Z = -4x_1 - 3x_2$

subject to

$x_1 + 2x_2 \ge 8$

$3x_1 + 2x_2 \ge 12$

$x_1 \ge 0, x_2 \ge 0$

b) Solve the following L. P. P. by using two-phase simplex method:

$Min\ Z = x_1 + x_2$

subject to

$2x_1 + 4x_2 \ge 4$

$x_1 + 7x_2 \ge 7$

$x_1 \ge 0, x_2 \ge 0$

5+5