Quantifiers - 2

Monday, August 31, 2020 12:22 PM

1. There is an honest politician.

$H(x) = x$ is honest.

$\exists x \; H(x)$, where domain of x is the politicians in the world.

$$\neg \exists x H(x) \equiv \underbrace{\forall x \neg H(x)}$$

↳ Every politician is dishonest

All politicians are not honest ✗

(not all politicians are honest)

2. ~~All~~ Americans eat Cheeseburgers.

$C(x) := x$ eats cheeseburgers.

$\forall x \; C(x)$, where x is an American

$$\neg \forall x C(x) \equiv \underbrace{\exists x \neg C(x)}$$

↳ There is an American who does not like Cheeseburger.

↳ Some Americans do not like Cheeseburgers.

ex:- If a person is female and is a parent, ~~this~~

Ex: If a person is female and is a parent, ~~this~~ then this person is someone's mother.

$F(x) := x$ is female.

$P(x) := x$ is parent

$M(x, y) := x$ is the mother of y

$$\forall x \left(\left(F(x) \wedge P(x) \right) \rightarrow \exists y \, M(x,y) \right),$$

where $x, y :=$ domain of all people.

Ex: Everyone has exactly one best friend.

$B(x,y) := y$ is the best friend of x.

$$\exists y \left(B(x,y) \wedge \forall z \left((z \neq y) \rightarrow \neg B(x,z) \right) \right)$$

Ex: There is a ~~best~~ woman who has taken a flight on every airline in the world.

$P(w, f) := w$ has taken flight f.

$Q(f, a) := f$ is a flight of airline a.

$$\exists w \forall a \exists f \left(P(w,f) \wedge Q(f,a) \right)$$

$$\exists w \forall a \exists f \, (P(w,f) \wedge Q(f,a))$$

$$\exists w \exists f \forall a \, (P(w,f) \wedge Q(f,a))$$

→ There is a flight f ~~who~~ for which Q(f,a) is true for all airline 'a' (infeasible)

Ex: Quantifiers from System Specifications

- Every mail message larger than one megabyte will be compressed.

$S(m,y) :=$ mail message m has size y MB.

$dom(m) =$ all mail messages

$dom(y) =$ positive real nos.

$C(m) :=$ mail message m will be compressed.

$$\boxed{\forall m \, (S(m,1) \rightarrow C(m))}$$

- If a user is active, at least one network link will be available.

$A(u) :=$ user u is active;

$S(n, \text{available}) :=$ Link n is available.

$S(n, \text{available}) :=$ Link n is available.

$$\boxed{\exists u\, A(u) \rightarrow \exists n\, S(n, \text{available})}$$

✓ Consider the following statements:

"All lions are fierce" $p \rightarrow q$

$\exists x$ "Some lions do not drink coffee"

"Some fierce creatures do not drink coffee"

Let $P(x)$ = "x is a lion"; $Q(x)$ = "x is fierce";

$R(x)$ = "x drinks ~~tea~~ coffee.

domain of x consists of all creatures.

1. $\forall x\, (P(x) \rightarrow Q(x))$
2. $\exists x\, (P(x) \wedge \neg R(x))$ | ✗ $\exists x\, (P(x) \rightarrow \neg R(x))$
3. $\exists x\, (Q(x) \wedge \neg R(x))$

Consider the following sentences:

1. "All humming birds are richly colored"
2. "No large birds live on honey"
3. "Birds that do not live on honey are dull in color"
4. "Humming birds are small"

$P(x) :=$ x is a humming bird.
$Q(x) :=$ x is large.
$R(x) =$ x lives on honey.
$S(x) =$ x is richly colored

domain of x consists of all birds.

① $\forall x \, (P(x) \rightarrow S(x))$
② $\neg \exists x \, (Q(x) \wedge R(x)) =$
③ $\forall x \, (\neg R(x) \rightarrow \neg S(x))$
④ $\forall x \, (P(x) \rightarrow \neg Q(x))$

The order of quantifiers.

① Let $Q(x, y, z)$ be the statement "$x + y = z$".
What are the truth values of the Statement
(i) $\forall x \forall y \exists z \, Q(x, y, z)$
(ii) $\exists z \forall x \forall y \, Q(x, y, z)$

domains of x, y, z Consist of all real numbers. $(-\infty$ to $+\infty)$

(i) $\forall x \forall y \exists z \, Q(x, y, z) \Rightarrow$ True
(ii) $\exists z \forall x \forall y \, Q(x, y, z) \Rightarrow$ False

② Let $P(x, y)$ be the statement "$x + y = y + x$"
$\forall x \forall y \, P(x, y)$ is True.

$$\left.\begin{array}{l}\forall x \forall y\, P(x,y) \\ \forall y \forall x\, P(x,y)\end{array}\right\} \text{True}$$

(commutative law for addition)

Translating the Statement

① "The sum of two positive integers is always positive"

$x, y \rightarrow$ domains consist of all integers

$\forall x \forall y\, (x+y > 0) \rightarrow$ domain consists of all positive integers.

$\forall x \forall y\, ((x>0) \wedge (y>0) \rightarrow (x+y>0))$

② For every real number x, [if $x \neq 0$, then there exists a real number y such that $xy = 1$].

(multiplicative inverse of a real number)

$\forall x\, ((x \neq 0) \rightarrow \exists y\, (xy = 1))$

③ Definition of a statement is given as following:

$$\lim_{x \to a} f(x) = L$$

For every real number $\epsilon > 0$, there is a real

For every real number $\epsilon > 0$, there is a real number $\delta > 0$, s.t. $[|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta.]$

q whenever p

$(p \to q)$

$$\forall \epsilon \exists \delta \forall x \left(0 < |x - a| < \delta \to |f(x) - L| < \epsilon\right)$$