

JANUARY 2021 [CST]

SUBJECT: DATA STRUCTURES [CG 2103]

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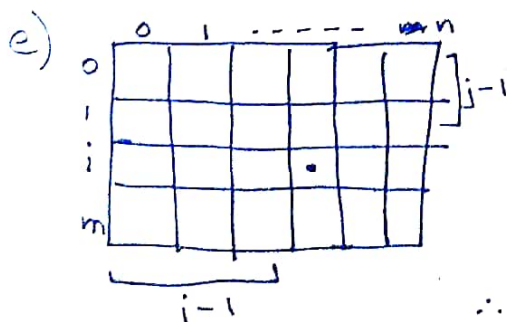
Q.1) a) ~~Abstract~~ Abstract Data Type [ADT] is functional definition of the data structure, it specifies, what the data structure should do, but is ~~indep~~ independent of its implementation

Eg: Stack is an ADT with push() and pop() functions.

d) Advantages of Linked List:

i) Deletion of element is fast [$O(1)$]

ii) We can ~~make~~ do dynamic allocation with Linked List, saving space in memory



Given $A[0,0] = \alpha$
to find $A[i,j]$

→ assuming row major.

$$\therefore A[i,j] = \alpha + W[(j-1)n + (i-1)]$$

where W = size of datatype of array

Pg 1

f) Although Binary Search is faster than Linear Search [BS: $O(\log n)$, LS: $O(n)$], Binary Search can't work if we don't have following things

i) ~~Sorted & Array~~ Sorted Data.

ii) Easy to Access Data.

→ if we have unsorted data, we should prefer Linear Search ~~over~~

→ if we are using Linked List [it is harder to access middle element], we should prefer Linear Search.

[although Binary Search for Linked List is ~~possible~~ possible].

g) Component Sum hash code map is method where all the components of a data type [string for example] are summed to create a integer input for compression map,

→ The problem arise in the following example

a b c d
d a c b
b a c d
c b a d
⋮

} → all have same component sum and will have lot of collision in a single key, hampering performance.

2) a) I) Link List Based Queue

Advantages

- all the advantage of Linked List
- Dynamic Allocation of memory
- ~~Easier deletion~~
- Easier Deletion of incorrectly fed ~~value~~ data

Disadvantage

- Takes up more space than similarly sized array queue due to the fact that ~~the~~ linked list also have to store the addresses
- Due to extra steps of link arrangements, this is slower than Array based Implementation

II) Array Based Queue.

Advantage

- Faster as there is no link ~~arr~~ rearrangements
-

Disadvantage

- Fixed Size
- Harder deletion of incorrectly fed data.

2) b) structure

```
struct stack_max  
{  
    int max;  
    int stack [100], stack data[100];  
    int top;  
}
```

```
typedef struct stack_max Stack;
```

```
void print_max (Stack s)
```

```
{  
    printf ("%d \n", s.max);  
}
```

```
bool push (Stack s, int value)
```

```
{  
    if (s.top == 100)  
    {  
        printf ("Stack Full");  
        return false;  
    }  
    if (value > s.max) if (value > s.max)  
        s.max = value;  
    s.top = s.top + 1; s.top = s.top + 1;  
    s.stack s.data[s.top] = value;  
    return true;  
}
```

```
int pop (stack s)
```

```
{ if (s.top == 0)
```

```
{ printf ("Already Empty");
```

```
return NULL -1; //error
```

```
}
```

```
int value = s.data[s.top];
```

```
s.top --;
```

```
return value;
```

```
}
```

```
int main ()
```

```
{
```

```
stack s; s.max = 0; s.top = 0;
```

```
s test
```

```
bool test = push (s, 5);
```

```
test = push (s, 10);
```

```
print print_max (s);
```

```
int value = pop (s);
```

```
return 0;
```

```
}
```


3) a) ^{node*}
~~void~~ rearrange (node* L, int n)

{

// node* have value & next

node* less = NULL; node* temp-less = NULL;

node* more = NULL; node* temp-more = NULL;

node* temp = L;

node* next = L;

L = NULL; // avoid dangling pointer

while (^{temp}~~next~~ != NULL)

{

next = next → next;

if (temp → value ≥ n)

{

if (more == NULL)

{

~~temp~~ more = temp;

temp-more = more;

~~more~~ → next

temp-more → next = NULL;

}

else

{

temp → more → next = temp;

temp-more = temp;

temp-more → next = NULL;

}

} // end of if

else {

↱ (next pg.)

```

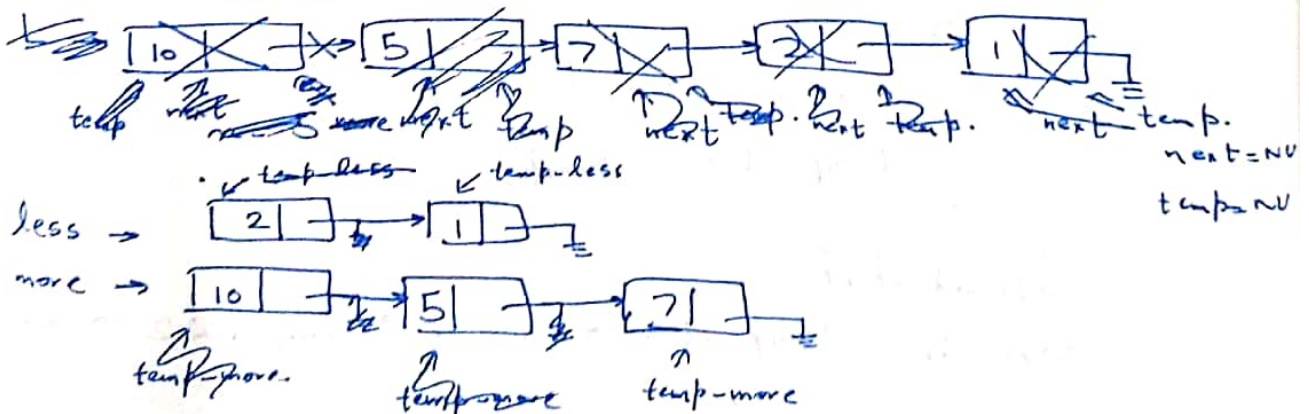
if (less == NULL)
{
    less = temp;
    temp->less = less;
    temp->less->next = NULL;
}
else
{
    temp->less->next = temp;
    temp->less = temp;
    temp->less->next = NULL;
}
} //end of else
temp = next;
} //end of while loop.

```

temp
temp->less->next = more;

return less;
} //end of function.

Dry Run $n=5$



3) b) node^{*} intersection (node^{*} L₁, node^{*} L₂)

{ // L₁ and L₂ sorted in increasing order.

// node have value & next

~~int~~ node^{*} temp-L1 = L1;

node^{*} temp-L2 = L2;

node^{*} ~~temp~~ L3 = NULL;

~~while~~ node^{*} temp-L3 = NULL;

while (temp-L1 != NULL && temp-L2 != NULL)

{

if (temp-L1->value == temp-L2->value)

{

~~if (L3 == NULL)~~

node^{*} temp = (node^{*}) malloc (size of (node));

temp->value = temp-L1->value;

temp->next = NULL;

temp-L1 = temp-L1->next;

temp-L2 = temp-L2->next;

if (L3 == NULL)

{ L3 = temp;

temp-L3 = L3;

}

else {

temp-L3->next = temp;

temp-L3 = temp;

}

} //end of if

else if (temp-L1->value < temp-L2->value)

{

temp-L1 = temp-L1->next;

}

Pg 3

else

{

temp - L2 = temp - L2 → next;

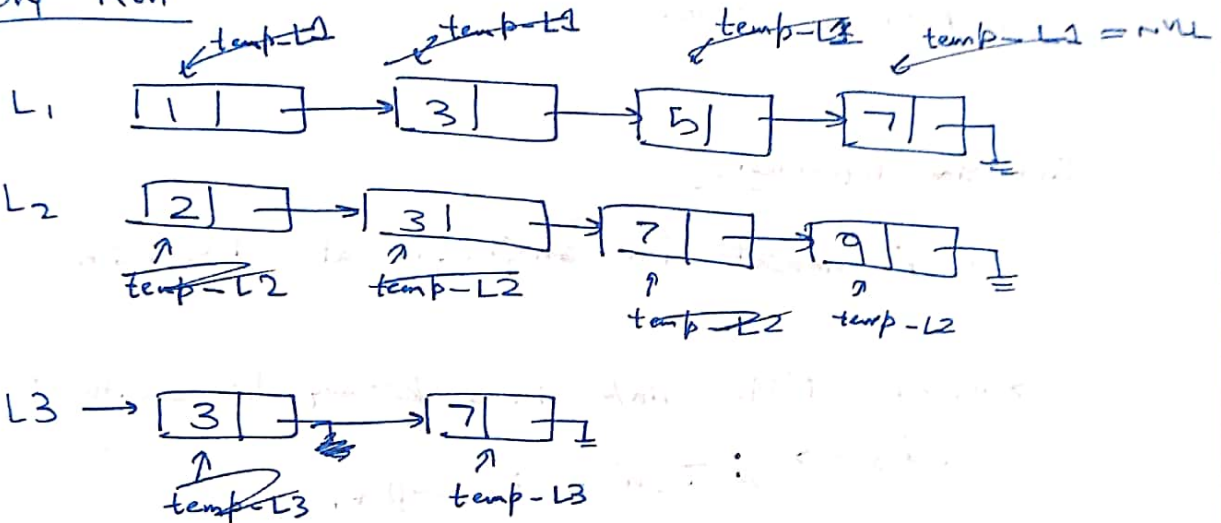
}

} // end of while

return L3;

} // end of function.

Dry Run



4) a) \forall T.P in a k -ary tree, ^{with n nodes} no. of NULL links $\Rightarrow n(k-1) + 1$

Base Case

$$n=1$$



$$\text{NULL links} = k = 1(k-1) + 1$$

$\therefore n=1$ is true.

Induction Hypothesis

let i ~~not~~ for ~~all $i \leq n$~~ all $1 \leq i \leq n$

\rightarrow no. of NULL links in a k -ary tree with i nodes $\rightarrow \forall T(n) = i(k-1) + 1$

\rightarrow we know that a tree with i nodes have $i-1$ ~~total~~ non-NULL links ~~and i ~~total~~ links~~

\therefore for i nodes $\rightarrow i(k-1) + 1$ null links
 $\rightarrow i-1$ non-null link
 $\rightarrow ik$ total links

Induction Step

→ Let a node be added in the i -node tree
so following things happen.

$$\rightarrow \text{Total links} = (i+1)k$$

$$\rightarrow \text{non-null links} = \frac{i+1+1}{2} = i+1$$

\therefore ~~Non~~

$$\text{Null links} = \text{Total} - (\text{non-null}) \\ = (i+1)k - i$$

$$\rightarrow \text{Null links} \Rightarrow T(n+1)$$

→ now addition of a node will remove a null link;
but introduce more k null links.

$$\therefore T(n+1) = T(n) - 1 + k$$

$$= n(k-1) + 1 - 1 + k$$

$$= n(k-1) + 1 + (k-1)$$

$$= (n+1)(k-1) + 1$$

\therefore ~~Prove~~

→ If we assume $T(n)$, we can prove $T(n+1)$

→ $T(1)$ is true, so $T(2)$ is true, so $T(3)$ is true
and so on and so forth

$\therefore T(n)$ is proved.

Pg 11

4)b) Let us prove this using contradiction.

Let us ~~assume~~ ^{suppose} that there is a non-leaf node at level $k-2$ with children < 2 , i.e. 1 children in an AVL Tree



→ node B is a leaf node, it has height 1

→ node d has height 2, and is height balanced.

→ u is not height balanced as its lchild have height 2 and rchild have height 0

∴ The tree is not AVL, which contradicts the fact that we assumed the tree is AVL

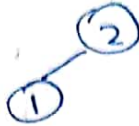
∴ What we ~~ass~~ assumed was wrong

∴ If the closest leaf in an AVL tree is at level k , then all the levels from 1 to $k-1$ has maximum possible no. of nodes.

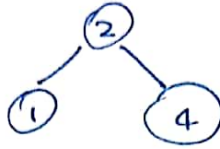
4c) I) 2



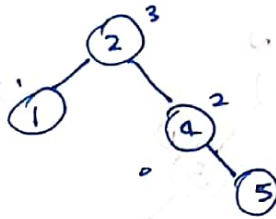
II) 1



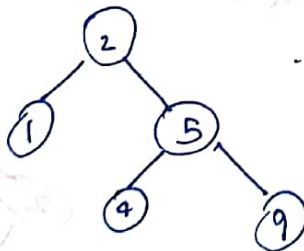
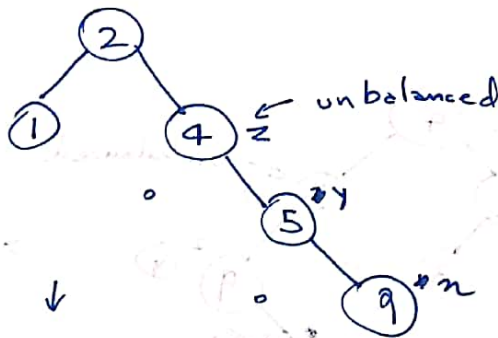
III) 4



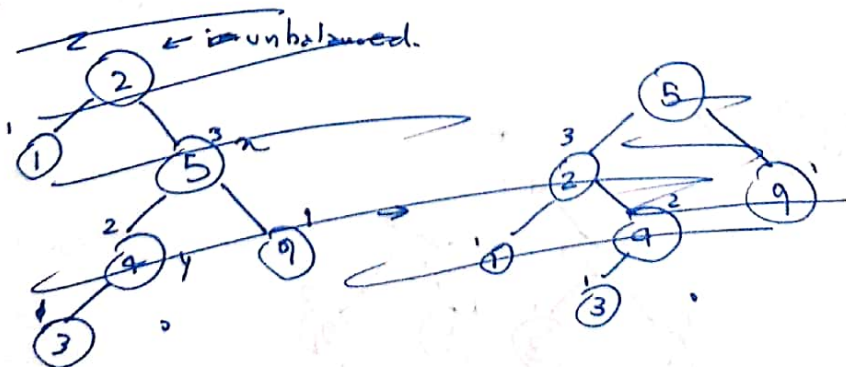
IV) 5



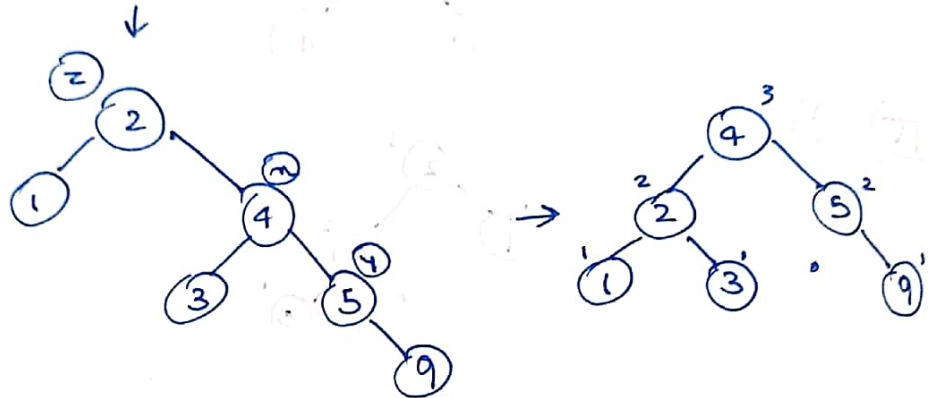
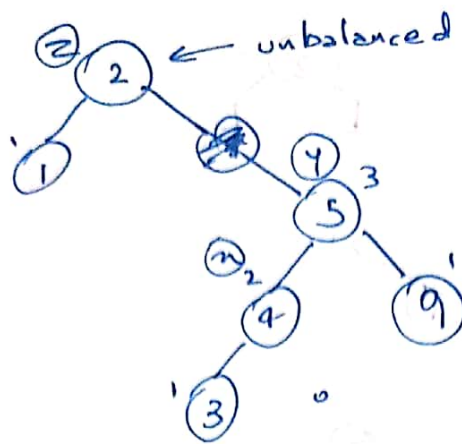
V) 9



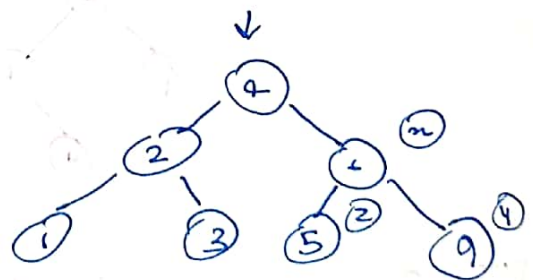
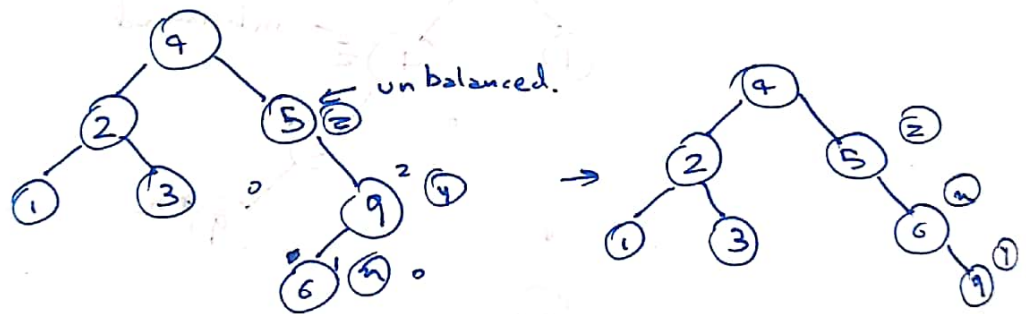
~~VI) 3~~



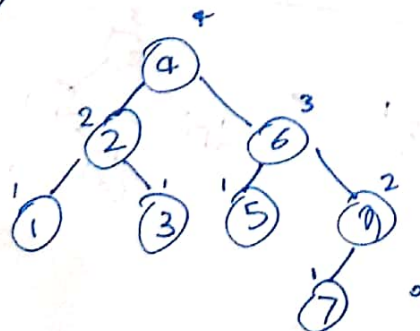
VII) 3



VII) 6



VIII) 7

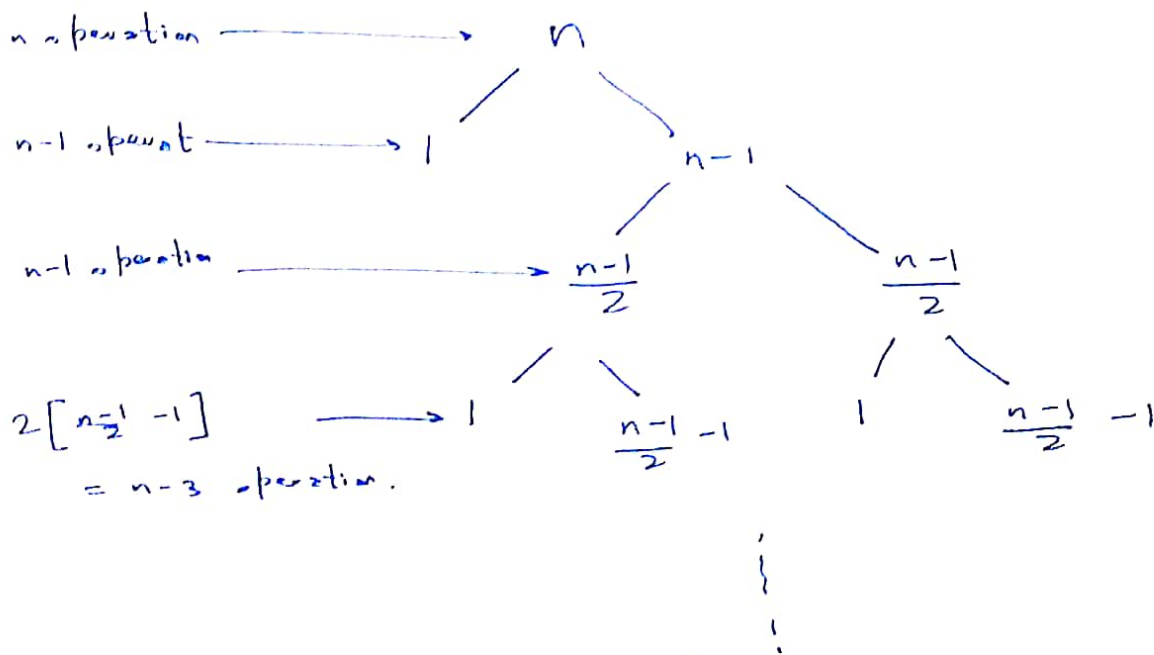


P. 4

5) 2) i) as data is almost sorted, quicksort will perform poorly, Here we can use bubble ~~sort~~
~~ii)~~ sort, as it works fast on almost ~~order~~ sorted data.

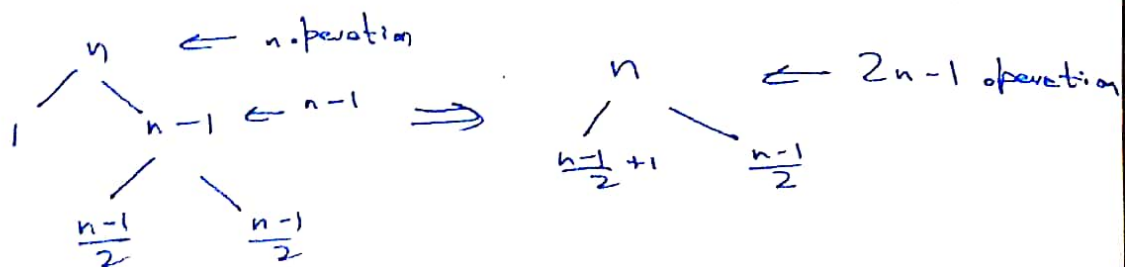
ii) as all the data are random, quicksort will work in $O(n \log n)$, so quicksort will be applicable.

b) Alternate Lucky and unlucky



→ This will be the order of partitions.

→ also we can do this.



\therefore It's Recursion will be

$$L(n) = 2U\left(\frac{n}{2}\right) + O(n)$$

$$U(n) = L(n-1) + O(n)$$

where $U \rightarrow$ unlucky case

$L \rightarrow$ Lucky case

$$\therefore L(n) = 2\left[L\left(\frac{n}{2}-1\right) + O\left(\frac{n}{2}\right)\right] + O(n)$$

$$= 2L\left(\frac{n}{2}-1\right) + 2O\left(\frac{n}{2}\right) + O(n)$$

which simplifies to

$$L(n) = O(n \log n)$$

\therefore ~~hence~~ \therefore Hence Proved.