Graphs & Trees (Lecture – 6)

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Planar Graphs (*Contd…*)

EULER'S FORMULA Let G be a connected planar simple graph with e edges and v vertices. Let r be the number of regions in a planar representation of G. Then $r = e - v + 2$.

- Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane?
- Corollary-1:

If G is a connected planar simple graph with e edges and v vertices, where $v \geq 3$, then $e \leq 3v - 6$.

- Corollary-2:

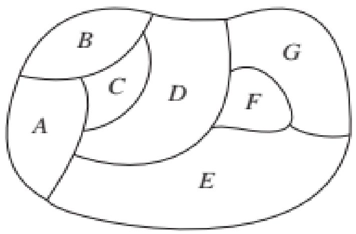
If G is a connected planar simple graph, then G has a vertex of degree not exceeding five.

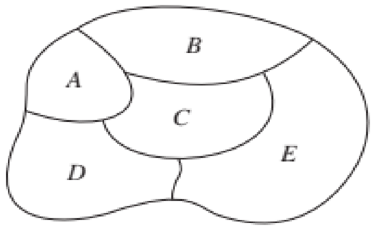
- Corollary-3:

If a connected planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length three, then $e \leq 2v - 4$.

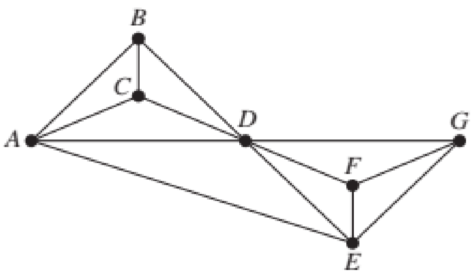
Graph Coloring

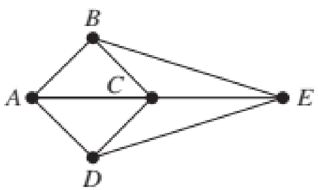
- Derived from the problem of map coloring
  - *Determining the least number of colors that can be used to color a map so that adjacent regions never have the same color.*
- Find the minimum number of colors required to color the following maps





- Each map in the plane can be represented by a graph
  - Each region of the map is represented by a vertex.
  - Edges connect two vertices if the regions represented by these vertices have a common border.
  - Resulting graph is known as **dual graph**
- The problem of coloring the regions of a map is equivalent to the problem of coloring the vertices of the dual graph so that no two adjacent vertices in this graph have the same color.
- The dual graphs generated from the maps are planar.





Graph Coloring (*Contd…*)

- Definitions:

A *coloring* of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

The *chromatic number* of a graph is the least number of colors needed for a coloring of this graph. The chromatic number of a graph G is denoted by $\chi(G)$. (Here χ is the Greek letter *chi*.)

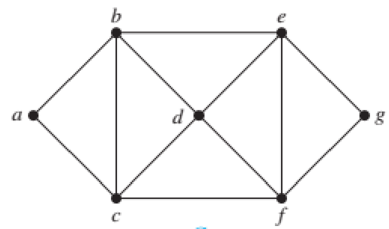
- Finding the chromatic number of a planar graph is equivalent to find the minimum number of colors required to color a planar map so that no two adjacent regions are assigned the same color.
- This question has been studied for more than 100 years.
- The answer is provided by one of the most famous theorems in mathematics.

THE FOUR COLOR THEOREM The chromatic number of a planar graph is no greater than four.

- Nonplanar graphs can have arbitrarily large chromatic number
- Two things are required to show that the chromatic number of a graph is k.
  - Show that the graph can be colored with k colors.
  - Show that the graph cannot be colored using fewer than k colors

Graph Coloring (*Contd…*)

- What are the chromatic numbers of the following graphs?



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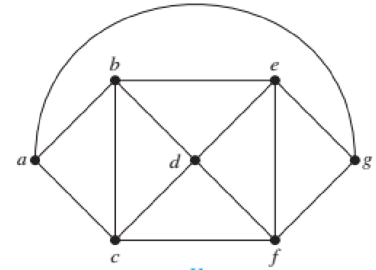

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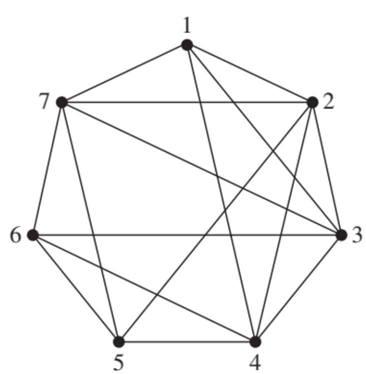
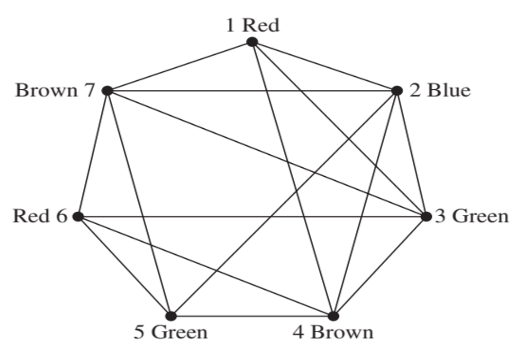
- What is the chromatic number of K_n?
- What is the chromatic number of the complete bipartite graph $K_{m,n}$, where m and n are positive integers?
- What is the chromatic number of the graph C_n, where $n \geq 3$? (Recall that C_n is the cycle with n vertices.)
- Applications of graph colorings
  - Graph coloring has a variety of applications to problems involving *scheduling* and *assignments*.





Time Period	Courses
I	1, 6
II	2
III	3, 5
IV	4, 7