I. Calculate the Laplace transform of the following :

1. $f(t)=t \qquad 0\le t\le \frac{1}{2}$
$=t-1 \qquad \frac{1}{2}\le t\le 1$
$=0 \qquad t\ge 1$

2. $f(t)=\cos t \qquad 0<t<2\pi$
$=0 \qquad t\ge 2\pi$

3. $f(t)=0 \qquad 0<t<\pi$
$=\sin^2(t-\pi) \qquad t\ge \pi$

4. $f(t)=t+1 \qquad 0<t<2$
$=3 \qquad t>2$

5. $f(t)=\sin 2t \qquad 0<t<\pi$
$=0 \qquad t\ge \pi$

6. $f(t)=\frac{t}{a} \qquad 0<t<a$
$=\frac{2a-t}{a} \qquad a<t<2a$

7. $\cosh^4 t$

8. $\left(\sqrt{t}-1\right)^2$

9. $\cos(\omega t-b)$

10. $4t^2+\sin t+e^{-2t}$

11. $(\cos 2t-\sin 2t)^2$

12. $\sin^4 t$

13. $\frac{1}{\sqrt{\pi t}}+6^{2t}$

14. $\sin\sqrt{t}$

15. $\frac{\cos\sqrt{t}}{\sqrt{t}}$

16. Prove that $L(\sin^5 t)=\frac{5!}{(s^2+1)(s^2+9)(s^2+25)}$

II. Find the Laplace transform of the following :

17. $\frac{\cos 2t\ \sin t}{e^t}$

18. $(t\sinh 2t)^2$

19. $\frac{\cos t-\cos 3t}{t}$

20. $\frac{\sin t}{t\,e^t}$

21. $\frac{\sin^2 t}{t^2}$

22. $\frac{\sinh 2t}{t}$

23. $t\sqrt{1+\sin 2t}$

24. $\sinh at\ \sin at$

25. $\sinh\frac{t}{2}\sin\frac{\sqrt{3}t}{2}$

26. $e^{-3t}\,t^4$

27. $(t+2)^2 e^t$

28. $\left(1+te^{-t}\right)^3$

29. $\cosh t\int_0^t e^u \sinh u\,du$

30. $t^{-1}\int_0^t e^{-3u}\sin 3u\,du$

31. $t\int_0^t e^{-2u}\sin 4u\,du$

32. $e^{-t}\int_0^t \frac{\sin t}{t}dt$

33. $\int_0^t \frac{\sin t}{e^t\,t}dt$

34. $\int_0^t \frac{1-e^{-at}}{t}dt$

35. $\frac{\sin t\ \sin 5t}{t}$

36. $\int_0^t u\,e^u \sin u\,du$

37. If $L[f(t)]=\frac{1}{\sqrt{s^2+1}}$ then Determine $L[\sinh t\, f(t)]$

38. If $L[f(t)]=\frac{8(3-s)}{s^2-6s+25}$, find $L[f(2t)]$, $L\left[f\left(\frac{t}{2}\right)\right]$

39. If $L[f(t)]=\frac{2}{s^3}e^{-s}$ then find $L\left[f\left(\frac{t}{3}\right)\right]$, $L[f(3t)]$

40. $L[f(t)]=\frac{s+2}{s^2+2}$, then Calculate $L[f(t)]$

III. Solve the following :

41. If $\int_0^\infty e^{-2t}\sin(t+\alpha)\cos(t-\alpha)dt=\frac{3}{8}$ then find α.

42. If $L[f(t)]=\frac{1}{\sqrt{s^2+1}}$ Then Determine $\int_0^\infty t\,e^{-3t}\,f(4t)dt$

IV. Evaluate the following Integrals :

43. $\int_0^\infty \frac{\cos 6t-\cos 4t}{t}dt$

44. $\int_0^\infty \frac{e^{-t}\sin t}{t}\,dt$

45. $\int_0^\infty e^{-t}\left(t^2-3t+5\right)dt$

46. $\int_0^\infty e^{-t}\left[t\int_0^t e^{-4u}\cos 4u\,du\right]dt$

47. $\int_0^\infty e^{-t}\left[\int_0^t u\cos^2 u\,du\right]dt$

48. $\int_0^\infty \frac{\cos 7t - \cos 3t}{t}\,dt$

49. $\int_0^\infty \frac{e^{-t}\sin^2 t}{t}\,dt$

50. $\int_0^\infty \frac{t^2 \sin 3t}{e^{2t}}\,dt$