

Indian Institute of Engineering Science and Technology, Shibpur

Five year Dual Degree (B.Tech-M.Tech) 3rd Semester Final Examination, November 2018

Data Structures and Algorithms (CS-302)

FULL MARKS: 70

Answer any five (5) questions. Students are advised to go through the questions carefully before answering. Clearly mention your assumptions, if any. Credits will be given to precise answer.

TIME: 3 hrs.

1. (a) Consider an implementation of quick sort algorithm that picks the pivot which partition the array $A[n]$ in the following way
- (i) one elements in one sub-array and remaining $n - 1$ elements in the other sub-array
 - (ii) always does a balanced partitioning (i.e., $n/2$ elements in both the sub-arrays)
 - (iii) 9/10th of the elements in one sub-array and remaining 1/10th in the other sub array
 - (iv) does the partitioning of the form as in (i) and (ii) alternately
- Calculate the running time in each of the above cases.
- (b) What is merging? What is its running time?
2. (a) How many binary trees with n nodes and height n are possible? Provide justification of your answer. (Assume a binary tree with only one node is of height 1.)
- (b) Binary trees can be represented in memory by linked allocation as well as sequential allocation. In case of linked allocation left and right children of a node are accessed directly by pointers present in the node. Sequential representation of a full binary tree results from sequentially numbering the nodes, starting with nodes on level 1, then those on level 2 and so on and nodes on any level are numbered from left to right. Using these scheme any binary tree can be stored in a one dimensional array, say tree (array index starts from 1), with the node numbered i being stored in $\text{tree}[i]$, its left children at $\text{tree}[2i]$ and right children at $\text{tree}[2i+1]$. Write down a recursive routine $\text{convert}()$ to store a tree T represented using linked representation in a one dimensional array A . (take suitable parameters of your choice as parameters to $\text{convert}()$).
- (c) A k -ary tree is a tree in which a node can have at most k children. Prove that in a k -ary tree with n nodes represented using linked representation has $n(k-1) + 1$ null links and hence prove that in a 3-ary tree more than $2/3$ of the link fields are null.
3. (a) What is a binary heap? How is it represented in memory? (5)
- (b) Is it possible to use binary heap to sort, say N keys? Justify your answer. (4)
- (c) Suppose there is an array of n numbers in which only one number is duplicated (i.e., there are $n - 1$ distinct numbers). Suggest a procedure to detect the duplicate number efficiently and mention the data-structure that you have used to implement the procedure.
4. Suppose you are solving a problem that requires to maintain a few sets, say C_1, C_2, \dots, C_k . All elements of each C_i , $1 \leq i \leq k$ are somehow connected and $C_i \cap C_j = \emptyset$, for $i \neq j$. With time some C_i may get merged with some C_j , $i \neq j$ (implying all the elements of $C_i \cup C_j$ are now become connected). You have to suggest a Data Structure which should support the following three operations efficiently.
- (i) $\text{createSet}(u)$: create a set with one element u .
 - (ii) $\text{findSet}(u)$: returns the set containing u
 - (iii) $\text{Union}(u, v)$: returns set which is union of the set containing u and the set containing v .
- (a) Write down the algorithms for the above three operation.
- (b) Provide estimates of running time of each of your proposed algorithms with proper justification(s) (i.e. provide proofs, if required).
5. (a) Prove that the minimum number of nodes in an AVL tree of height h is $N_h = (\sqrt{2})^h$ and also prove that $N_h = F_{h+2} - 1$ where F_{h+2} is the $(h+2)$ th Fibonacci number. (Assume that an AVL tree with only one node is of height 1)
- (b) Prove that if a node in an AVL tree has single child then that child should be a leaf node.
- (c) If the closest leaf in an AVL tree is at level k (assuming root is at level 1) then all the levels starting from 1 to $k - 1$ has maximum possible number of nodes.

6. What would be the minimum and maximum number of nodes in a red black tree if an external node has black depth of h .

(a) What would be the minimum and maximum number of nodes in the context of a Red Black tree.

(b) Prove the following statements in the context of a Red Black tree.

(i) If a Red node has any children, it must have two children and they must be Black.

(ii) If a Black node has only one child that child must be a Red leaf.

[4+(5+5)]

7. (a) What are collisions in hashing? Mention some of the methods for resolving collisions along with their advantages and disadvantages.

(b) Consider a hash function that randomly picks any location in the hash table and throws an element there. Comment on advantages and drawbacks of the said hash function.

(c) The load factor α is defined as $\alpha = \frac{n}{m}$ where n is the number of keys and m is the length of the hash table. Prove that the average number of probe for successful search (S_n) and unsuccessful search (U_n) using uniform hash function and chaining (for collision resolution) is

$$S_n = 1 + \frac{\alpha}{2} \text{ and } U_n \approx \alpha$$

(d) In double hashing why is it required the table size to be a prime number?

[4+3+4+3]

Indian Institute of Engineering Science & Technology, Shibpur
 B.Tech (3rd Semester CST) Mid-Semester Examination, 2018
 Digital Logic (CS 301)

F.M. 50

Time: 2 hrs

1. (a) Compare 1s and 2s complements.
 (b) Obtain the 10s complement of 90090 and 2s complement of 5.
 (c) Minimize the Boolean expression given below.

$$f(a, b, c, d) = \sum m(2, 3, 7, 9, 11, 13) + \sum d(1, 10, 15)$$

[3 + 2 + 5]

2. a) Design a circuit that compares two 4-bit numbers, A and B, to check if they are equal. The circuit will have one output Q, so that Q = 1 if A = B, and Q = 0 if A ≠ B. Draw the logic diagram of the required circuit.
 b) Design a 4-bit BCD-to-excess-3 code converter using a 4-bit full adder IC chip (containing 4 full adders). [6 + 4]
 3. Design a combinational circuit with four input lines that represent a decimal digit in BCD and four output lines that generate the 9's complement of the input digit. [10]
4. a) A combinational circuit is defined by the following three functions: [10]

$$F_1 = x'y' + xyz'$$

$$F_2 = x' + y$$

$$F_3 = xy + x'y'$$

Design the circuit with a decoder and external gates.

5. (a) Define combinational circuit and sequential circuit.
 (b) Implement the following function with a multiplexer [3 + 7]

$$F(A, B, C, D) = \Sigma(0, 1, 3, 4, 8, 9, 15)$$

Indian Institute of Engineering Science and Technology, Shibpur

Five year Dual Degree (B.Tech-M.Tech) 3rd Mid-Term Examination 2018
Data Structures and Algorithms
CS-302

FULL MARKS: 50

TIME: 2 hrs.

Answer any five (5) questions. Students are advised to go through the questions carefully before answering.
Clearly mention your assumptions, if any, while answering. Credits will be given to precise answer.

1. (a) Find out the recurrence relation for the following function and hence calculate its running time.

int hanoi(n, A, B, C)
{ // n is the number of disk, A, B, and C are the pegs

```
    if (n == 1)  
        print(move disk from A to C)  
    else  
        hanoi(n-1, A, C, B)  
        print(move top disk of A to C)  
        hanoi(n-1, B, A, C)  
}
```

1+1+1

- (b) Let $T(n)$ be defined by $T(1) = 10$ and $T(n+1) = 2n + T(n)$ for all integers $n \geq 1$. What would be the order of growth of $T(n)$?

- (c) Determine if $T(n) = n^3 + 20n$ is $\Omega(n^2)$.

[4+3+3]

2. (a) Suppose an array of n integers contains numbers ranging from 0 to $n - 2$ i.e. exactly one number is repeated. Propose an efficient method to find out the number that is repeated.

- (b) Write down a procedure to find all common numbers in given three sorted arrays.

- (c) Write down a C program to implement insertion sort algorithm that can sort any type of data. [2+4+4]

3. (a) Suggest a suitable method so that frequently searched element in a set of elements can be accessed quickly. Which data structure would you like to use and how that particular data structure would help you achieving the above requirement?

- (b) Write down an efficient procedure to delete the middle element from a linked list?

- (c) Suppose you are given a sorted linked list. Can you perform binary search on it? If so, what would be its running time?

[4+3+3]

4. (a) Suppose that you are given a binary tree of n nodes represented using linked representation, find the formula for calculating the number of null links in it.

- (b) Prove that in a binary tree with L leaves has atleast $\lceil \lg L \rceil + 1$ levels (Assume root is at level 1).

- (c) Prove that in a strictly binary tree where every node has 0 or 2 children, the number of leaf nodes is always one more than nodes with two children.

[4+3+3]

5. (a) Is it possible to implement a queue using stacks? Justify your answer.

- (b) Argue rigorously that you can't find out the minimum element in an array in lesser than $n - 1$ comparisons.

- (c) Write down an algorithm to find out the *minimum* and *second minimum* elements in an array A of integers with n elements in it. Write down the complete analysis of running time of your proposed algorithm.

[3+3+4]

6. (a) Suppose you are given a binary tree of n nodes represented using linked representation, find the formula for calculating the number of null links in it

- (b) Prove that in a binary tree with L leaves has atleast $\lceil \lg L \rceil + 1$ levels (Assume root is at level 1)

- (c) Prove that in a strictly binary tree where every node has 0 or 2 children, the number of leaf nodes is always one more than nodes with two children.

[4+3+3]

Indian Institute of Engineering Science and Technology, Shibpur

B.Tech. 3rd Semester Midterm Examination 2018

Discrete Structures (CS 303)

Full marks: 50

Time: 2 hours

(Answer all FOUR questions)

1. a) A relation R is defined on the set Z by " $a R b$ iff $a - b$ is divisible by 5" for $a, b \in Z$. Prove that R is an equivalence relation on Z . Find all the equivalence classes of Z under R .

- b) Determine the discrete numeric function corresponding to the following generating function $A(z) = 1 / (5 - 6z + z^2)$

- c) Let $A(z)$ be the generating function for a numeric function a . Find out the generating function for ∇a and $S^2 a$. [∇ and S have their usual meaning] (4+3½ +5)

- 2.a) Find the recurrence equations of the following problem.

There are two kinds of particles inside a nuclear reactor. In every second, one α particle will split into three β particles, and one β particle will split into one α particle and two β particles. If there is a single α particle in a reactor at time $t = 0$, how many particles are there altogether at $t = r$?

- b) In a lattice (A, \leq) , if $a \leq b$ and $c \leq d$ then prove that
(i) $a \vee c \leq b \vee d$ and (ii) $a \wedge c \leq b \wedge d$. an=3

- c) Consider the following algorithm for sorting r numbers for $r \geq 2$.
(i) Use $2r - 3$ comparisons to determine the largest and second largest of the r numbers.
(ii) Recursively, sort the remaining $r - 2$ numbers. ar=3

Let a_r denote the number of comparisons used for sorting r -numbers.
Determine a_r . (3+5+4½)

3. a) Solve the recurrence relation $a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$
b) Solve the recurrence relation $ra_r + ra_{r-1} = a_{r-1} + 2^r$ given that $a_0 = 139$.
c) Using the method of generating function find the number of ways of selecting r objects from n objects with unlimited repetitions. (4+4½+4)

4. Prove that there exists a unique finite boolean algebra of 2^n elements for any $n > 0$ (12 ½)

Answer all questions

Figures in the margin indicate full marks

1. A 10 kW, 240 V dc shunt motor draws a line current of 5.2 A while at no-load speed of 1200 rpm from a 240 V dc supply. It has an armature resistance of 0.25 Ω and a field resistance of 160 Ω . Estimate the efficiency of the motor when it delivers rated load. (10)

2. A 230 V dc series motor develops its rated output at 1500 rpm while taking 20A. Armature and series field resistance are 0.3 Ω and 0.2 Ω respectively. Neglecting saturation, determine the resistance that must be added to obtain rated torque (a) at starting (b) at 1000 rpm. (10+10)

3. A 250 V dc shunt motor has an armature resistance of 0.5 Ω and a field resistance of 250 Ω . When driving a constant torque load of 600 rpm, the motor draws 21 A. What will be the new speed of the motor if an additional 250 Ω resistance is inserted in the field circuit. (10)

4. A 250 V compound generator has a armature, series-field and shunt-field resistance of 0.4 Ω , 0.2 Ω and 125 Ω respectively. If this generator supplies 10 kW at rated voltage, find the e.m.f. generated in the armature when the machine is connected (a) long shunt (b) short shunt. Ignore armature reaction and allow 1 volt per brush for contact drop. (10+10)

5. a) Describe the clock method of angle designation for representing three-phase transformer with a suitable example. (6)

b) Explain why zigzag-connected transformer requires less copper than a star connected transformer for the same rating. Discuss the advantages and disadvantages of a delta-star and star-zigzag method of transformer connections. (4+4=8)

c) The polarities of phase 'a' winding of the low-voltage or secondary side of a Yy0 transformer are reversed keeping the high-voltage or primary side winding connections unaltered. Explain with phasor diagrams, its effect on the secondary side line voltages and phase displacements between the line voltages. (10)

6. a) Draw the phasor diagrams and the connection diagrams for following three-phase transformer connections: (i) Yz1 (ii) Dz0 (3x2=6)

b) A three-phase step-down transformer is connected to 11000 V, 50 Hz voltage source and draws a line current of 34 A from the supply. Each secondary phase has two similar windings. Neglecting losses, calculate the (i) secondary line voltage, (ii) secondary line current, (iii) output in kVA for the following transformer connections: Delta/star and Delta/zigzag (Assume per phase turns ratio to be 44)

Delta/star and Delta/zigzag (Assume $P = 1$)
 Draw a neat phasor diagram for each transformer connections and derive necessary relations. (5x2=10)

Indian Institute of Engineering Science and Technology, Shibpur,
 Dual Degree (B. Tech- M. Tech) B. Tech 3rd Semester Mid-Term Examination, 2018

Subject: Mathematics - III

Subject Code: MA-301

Time: 2 hours

Full Marks: 100

ANSWER ANY FOUR QUESTIONS

1. a) If A_1, A_2, \dots, A_n be any n events connected to a random experiment E then prove that $P(A_1 A_2 \dots A_n) \geq \sum_{i=1}^n P(A_i) - (n-1)$

b) Define conditional probability $P(A/B)$. If for any two events A and B, $P(A) = \frac{1}{4}, P(B/A) = \frac{1}{2}, P(A/B) = \frac{1}{4}$ then find $P(\bar{A}/B)$ and $P(\bar{A}/\bar{B})$.

c) A class contains 10 men and 20 women, of which half the men and half the women have brown eyes. Find the probability that a person chosen at random is a man or has brown eyes.

8+9+8=25

~~1. a) State and prove Baye's theorem.~~

b) A random variable X has the following *probability mass function*:

$$\begin{array}{cccccccc} x & : & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ P(X=x) & : & 0 & k & 2k & 2k & 3k & k^2 & 2k^2 & 7k^2 + k \end{array}$$

i) Find k.

ii) Evaluate $P(X < 6), P(X \geq 6), P(0 < X < 5)$.

10+15=25

iii) Determine the distribution function of X.

iv) Find mean and variance of the distribution.

~~3. a) Prove that under certain conditions (to be stated by you), Binomial distribution approximates to the Poisson distribution.~~

b) Let X follows binomial distribution, whose mean and variance are 4 and $\frac{4}{3}$ respectively then find $P(X \geq 1)$.

c) A bank receives on an average 2.5 customers per hour. Find the probability that in a certain hour the bank receives (i) no customer, (ii) exactly four customers (iii) more than five customers. Assume that the number of customers received in an hour is poissonly distributed.

$X(MB)^{\frac{1}{2}}$

8+9+8=25

~~4.~~ a) Define Laplace transform of a function $f(t)$. If $L\{f(t)\} = F(s)$ then show that

$$L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right), \text{ where } a > 0.$$

b) Evaluate the Laplace transform of the following functions:

$$\checkmark i) f(t) = e^{-4t} \frac{\sin 3t}{t} \quad ii) f(t) = e^{-2t} \cos^2 5t \quad iii) f(t) = \sinh(t) \sin\left(\frac{\sqrt{3}}{2} t\right) \quad \underline{7+18=25}$$

~~5.~~ a) Define convex set. Prove that the set of all feasible solutions of a LPP is a convex set.

b) Let $x_1 = 2, x_2 = 3, x_3 = 1$ be a feasible solution (F.S.) of the system

$$2x_1 + x_2 + 4x_3 = 11$$

$$3x_1 + x_2 + 5x_3 = 14$$

$$x_1, x_2, x_3 \geq 0.$$

Reduce the above F.S. into two different B.F.S.

c) Find the extreme points of the convex set of feasible solution of the LPP:

$$\text{Minimize } Z = 2x_1 + 3x_2 + 4x_3 + 5x_4$$

$$\text{Subject to } 2x_1 + 3x_2 + 5x_3 + 6x_4 = 16$$

$$x_1 + 2x_2 + 2x_3 + 3x_4 = 9$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

$$\underline{6+10+9=25}$$

Hence find the minimum value of Z .

Indian Institute of Engineering Science & Technology, Shibpur
 B.Tech 3rd Semester (CST) Examination, 2018
 Digital Logic (CS 301)

F.M. 70

Answer any 5 questions.

Time: 3 hrs

1. (a) Simplify the following Boolean function to minimum number of literals

$$y(w\bar{z} + wz) + xy.$$

- (b) Express the Boolean function $F = xy + \bar{x}z$ as a product of maxterms.

- (c) Draw the voltage transfer characteristic of an inverter and define the following parameters: V_{OH} , V_{OL} , V_{IL} , and V_{IH} . Obtain the expression for noise margins from the said parameters.

[3 + 4 + 7]

2. (a) Why wired-logic connection is not allowed with totem-pole (TTL) output circuit?

- (b) Which logic level limits the fan-out of DTL gates and why?

- (c) Draw the circuit diagram of a two inputs DTL NAND gate. Connect the output of the DTL gate to N inputs of other similar gates. Assume that the output transistor is saturated and its base current is 0.44mA. Let h_{FE} of the transistor be 20. Find the value of N that will keep the transistor in saturation and what is the fan-out of the gate?

[4 + 3 + 7]

3. (a) Using four XOR gates and minimum number of full-adders, construct a 4-bit parallel adder/subtractor circuit. Use an input select variable S so that when $S = 0$, the circuit adds and when $S = 1$, the circuit subtracts.

- (b) Design a combinational circuit that will act as 4-bit binary to gray code converter.

- (c) Draw the circuit diagram of a tri-state TTL inverter.

[5 + 5 + 4]

4. (a) Design a four (three message bits and one parity bit) bit even parity-generator-cum-checker circuit and also draw the logic diagram of the circuit.

- (b) An 8×1 multiplexer has inputs A, B, and C connected to the selection inputs S_2 , S_1 and S_0 respectively. The data inputs I_0 through I_7 are as follows:

- $I_1 = I_2 = I_7 = 0$; $I_3 = I_5 = 1$; $I_0 = I_4 = D$ and $I_6 = \bar{D}$. Determine the Boolean function that the multiplexer implements.

[7 + 7]

5. (a) Draw the circuit diagram of a mod-16 ripple counter and also draw the waveforms (timing diagram) of the said counter.
- b) Define state equations of sequential circuits? Design a sequential circuit with JK flip-flops to satisfy the following state equations:

$$\begin{aligned}A(t+1) &= xAB + y\bar{A}C + xy \\B(t+1) &= xAC + \bar{y}BC \\C(t+1) &= \bar{x}B + yA\bar{B}\end{aligned}$$

(6 + 8)

6. (a) Draw the block diagram of a 4-bit bi-directional shift register and explain its operation.
- (b) Design a counter with the following binary sequence: 0, 1, 3, 2, 6, 4, 5, 7 and repeat. Use RS flip-flops in the design process. (7 + 7)
7. Define lockout of counter. Design a mod-5 counter using JK flip-flop so that if the unused states 111, 110 and 101 occur, the next clock rise will reset the counter to 000. [3 + 11]

Indian Institute of Engineering Science and Technology, Shibpur
B.Tech. 3rd Semester Final Examination, November, 2018
Discrete Structures (CS 303)

Full marks: 70

Time: 3 hours

Attempt any five from question number 1 to question number 7, and any two from question number 8 to question number 10.

1. a) Define the following terms:

(i) Partial Order Relation
 (iii) Complemented Lattice

(ii) Chain and Antichain
 (iv) Boolean Lattice

- b) Let (A, \leq) be a Partially Order set. Suppose the length of the longest chains in A is m . Then prove that the elements in A can be partitioned into m disjoint antichains. (4+6)

2. a) In a distributive lattice, if $x \wedge \bar{y} = 0$, then prove that $x \leq y$. The symbols have their usual meanings.

- b) For any a, b, c, d in a lattice (A, \leq) , if $b \leq a$ and $d \leq c$ then prove that
 (i) $b \vee d \leq a \vee c$ and (ii) $b \wedge d \leq a \wedge c$

(5+5)

3. a) Solve the difference equation, $u_r = \sqrt{u_{r-1} + \sqrt{u_{r-2} + \sqrt{u_{r-3} + \dots}}}$, given that $u_0 = 4$.

- b) Let e_n be the number of edges in a complete graph of n vertices.

- (i) Derive a recurrence relation for e_n in terms of e_{n-1} .
 (ii) Solve the recurrence relation. (5+5)

4. a) Find out the particular solution of the difference equation $a_r + a_{r-1} = 3r2^r$
 b) Define the convolution of two numeric functions a and b . Compute it when $a = 3^r; r \geq 0$ and $b = 2^r; r \geq 0$.
 c) Use generating function to evaluate the sum $1^2 + 2^2 + 3^2 + \dots + n^2$ (3+3+4)

5. a) Using numeric function, determine the number of ways in which r objects can be selected from n objects with unlimited repetitions.
 b) Derive only the recurrence relations for solving the following problems:

- (i) Suppose a coin is tossed r times. You want to determine the number of sequences of outcomes in which heads never appear on successive tosses.

- (ii) Consider a certain nuclear reaction inside of a reactor containing nuclei and high and low energy free particles. There are two kinds of events:

- A high-energy particle strikes a nucleus and is absorbed, causing it to emit two high energy particles and three low-energy particles; and
- A low-energy particle strikes a nucleus and is absorbed, causing it to emit two high energy particles and one low-energy particle.

You want to determine the number of high-energy and low-energy particles in the system at r -th μs .

Assumptions: (I) Every free particle causes an event 1 μs after it is emitted. (II) A single high energy particle is injected at time 0 into a system containing only nuclei. $(5+(2+3))$

6. a) Define: term, atom and well formed formula (wff) in First Order Predicate Logic (FOPL) and give suitable example of each of the above.
- b) Translate the following basic axioms of whole numbers into formulas:
- For every number, there is one and only one immediate successor
 - There is no number for which 0 is the immediate successor
 - For every number, other than 0, there is one and only one immediate predecessor
- c) Obtain Prenex Normal Form (PNF) of the following formula: $\forall x \forall y (\exists z P(x, y, z)) \wedge \exists x \exists u (Q(x, u) \rightarrow \exists y \exists w Q(y, u))$ $(3+3+4)$

7. a) Categorize the following well formed formulas (wffs) into valid, invalid, satisfiable, or unsatisfiable.

(i) $\neg((p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (q \rightarrow r))$

(ii) $(p \rightarrow (\neg q \rightarrow r)) \wedge (p \rightarrow \neg q) \rightarrow (p \rightarrow r)$

- b) Solve the following problem using the concept of logical consequence of a formula to the other formulas, without using truth Table.

Problem: Given that if the congress refuses to enact new laws, then the strike will not be over unless it lasts more than one year and the President of the firm resigns. Will the strike not be over if the congress refuses to act and the strike just starts?

- c) Justify whether the following arguments are valid.

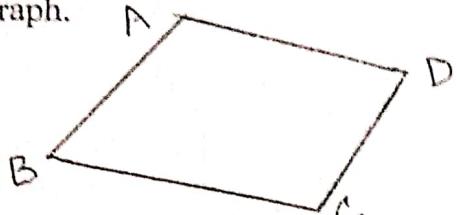
- (i) All men are mortal. Confucius is a man. Prove that Confucius is mortal
- (ii) If Sita was at the fair, then her father was negligent or her mother was not at home. If her mother was not at home, then her father was not negligent. Her mother was at home. Therefore, Sita was at the fair. $(3+4+3)$

8. Prove that in a binary tree

- (i) number of vertices is always odd; and
- (ii) number of pendant vertices is one more than the number of internal vertices.

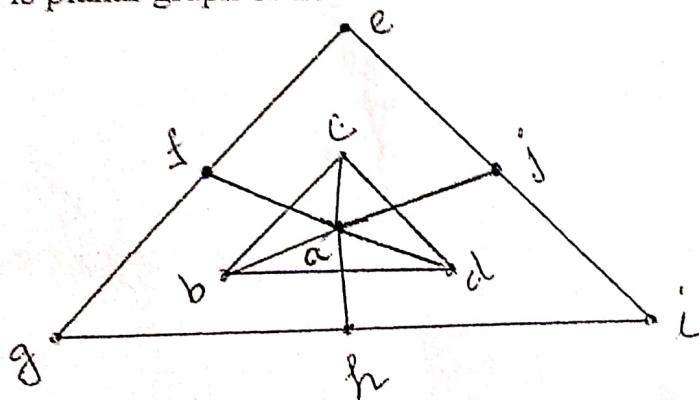
b) Define chromatic number and chromatic polynomial of a graph. State the decomposition theorem and apply it to find the chromatic polynomial and chromatic number of the following graph. (2+2)

(2+(1+2+1))



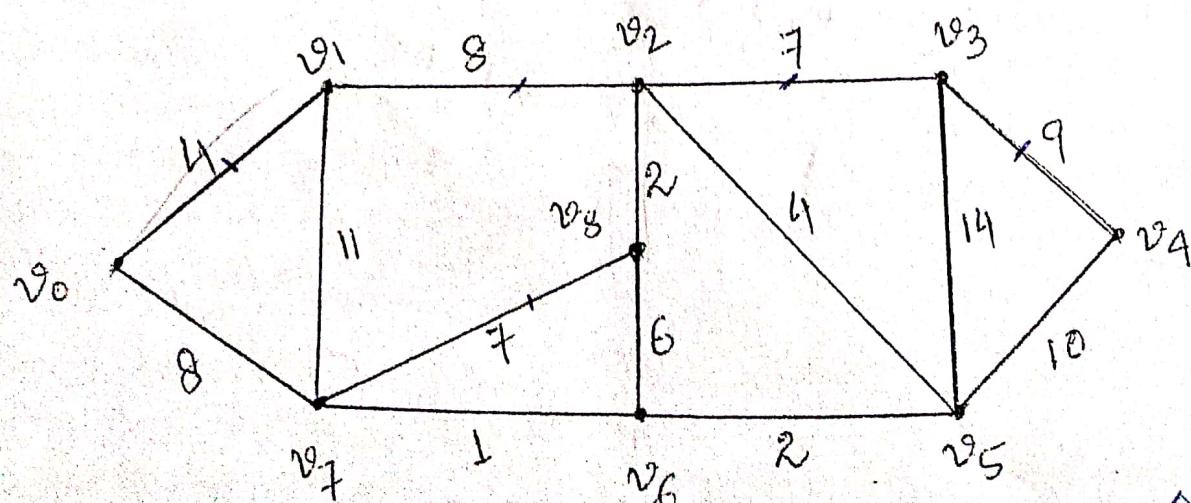
9. a) Prove the Euler's Formula that a connected graph with n vertices and e edges has $e - n + 2$ regions.

b) Write an algorithm to detect the planarity of a graph and apply it to determine if the following graph is planar graph or not. (4+6)



10. a) Define fundamental circuit and spanning tree of a connected graph. How many fundamental circuits does a graph have? Explain. (2+2)

b) Write the PRIM's Algorithm and apply it on the following graph to determine a minimal spanning tree. (6) 2



INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR
Dual Degree (B. Tech. - M.Tech.) 3rd Semester (CST) Part-I Examination, November-2018
Subject:- Electrical Machines (EE-304)

Full Marks: 70

Time: 3 hours

- (i) Answer any **three** questions from **first half** and in **second half**, answer Q. No. 6 or Q. No. 7, and the rest **two** questions from among Q. No. 8 to Q. No. 11.
(ii) All questions carry equal marks
(iii) Two marks are reserved in each half for neatness and organized answer-script.

FIRST HALF

- 1) a) Explain the working principle of 'permanent magnet' type stepper motor.
b) A 20 kW, 250 V DC shunt motor has a full-load armature current of 85 A at 1100 rpm. The armature resistance is 0.18Ω . Determine (i) the internal torque developed, (ii) the internal torque, if the field current is suddenly reduced to 80% of its original value. Also find (iii) the steady motor speed in part(ii) assuming the load to be remaining constant.

[3+(4+4)]

- 2) a) Explain the working principle of 3-phase induction motor.
b) A 3-phase, 4-pole, 50-Hz induction motor at standstill has 120 V induced across its star-connected terminals. The rotor resistance and standstill reactance per phase are 0.2Ω and 1.0Ω respectively. Calculate the speed when the rotor is drawing a current of 16 A at a particular load. Also calculate the speed at which the torque is maximum and the corresponding value of rotor input.

[4+ (3+4)]

- 3) a) Discuss any two methods of speed control of 3-phase squirrel cage induction motors.
b) A 440 V, 50 Hz, 6-pole, 3-phase induction motor draws an input power of 76 kW from the mains. The rotor emf makes 120 complete cycles per minute. Its stator losses are 1 kW and rotor current per phase is 62 A. Calculate (a) the rotor copper losses per phase, (b) the rotor resistance per phase and (c) the torque developed.

[5)+(2+2+2)]

- 4) a) Explain the AC and DC dynamic braking of a 3-phase induction motor.
b) A 30 kW rated output, 400 V, 3-phase delta-connected, 4-pole, 50 Hz induction motor has full-load slip of 5%. If the ratio of standstill reactance to resistance per rotor phase is 4, estimate the plugging torque at full speed. Ignore stator leakage impedance and magnetizing reactance.

[6+(5)]

- 5) a) State various methods of starting of a 3-phase induction motor. Explain, with the help of a diagram, the working principle of star-delta starter.
b) A squirrel cage type induction motor, when started by means of a star-delta starter takes 200% of full-load current (line) and develops 44% of full-load torque at starting. Calculate the starting torque and current, if an auto-transformer with 75% tapping were employed.

[(2+5)+4]

Full Marks: 70

Time: 3 hours

SECOND HALF

- 6) a) Why starters are required to start DC motors?
b) A 500 V, 45 kW, 600 rpm DC shunt motor has a full load efficiency of 90%. The field resistance is 200Ω and the armature resistance is 0.2Ω . Find the speed, under each of the following conditions, at which the machine will develop an electro-magnetic torque equal to rated value:- (i) regenerative braking: no limiting resistance, (ii) plugging: external limiting resistance of 5.5Ω inserted, (iii) dynamic braking: External limiting resistance of 2.6Ω inserted. The field current is maintained constant. The armature reaction effects and the brush drop may be neglected.

[2+(3+3+3)]

OR

- 7) a) What are the various losses occurring in rotating machines? What are the constant losses in a DC machine?
b) A long-shunt generator running at 1000 rpm supplies 22 kW at a terminal voltage 220 V. The resistance of armature, shunt field and series field are 0.05Ω , 110Ω and 0.06Ω respectively. The overall efficiency at the above load is 88%. Find (a) the copper losses and (b) the iron and friction losses.

[(3+2)+(3+3)]

- 8) a) What are the different methods of conversion of three phase to six phase? Describe any two methods of conversion from three to six phases using transformers with suitable circuits and phasor diagrams.
b) With the help of a neat diagram show that if the load is balanced on the two-phase side of Scott-connected transformers then it will give a balanced system on the three phase side.

[(1+4) + (1+5)]

- 9) a) Discuss the advantages and disadvantages of the various configurations of three-phase to six-phase transformer connections.
b) Two Scott-connected transformers supply two single-phase loads at 400 V. The load across the teaser transformer secondary is 300 kVA at unity power factor and that across the main transformer secondary is 200 kVA at unity power factor. For a three-phase input voltage of 6600 V, calculate the primary line currents. (*The magnetizing currents and leakage impedance drops are neglected*). Mark the respective currents and voltages on a schematic connection diagram.

[5 + (5+1)]

INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR
Dual Degree (B. Tech. – M.Tech.) 3rd Semester (CST) Part-I Examination, November-2018
Subject:- Electrical Machine (EE-304)

Full Marks: 70

Time: 3 hours

- 10) a) Classify single-phase induction motors in accordance with the methods of starting. Discuss the capacitor-split phase type of motor with circuit diagram, phasor diagram at starting and a typical torque-speed characteristic.
b) Explain the construction and principle of working of a universal motor and mention some of its applications. What additional design features are incorporated in an AC series motor compared to a DC series motor? [(1+4) + (3+3)]
- 11) a) Explain the working principle of a capacitor start capacitor run single phase induction motor with the help of connection and phasor diagram. Draw its speed-torque characteristics and mention some of its applications.
b) In the light of double revolving field theory explain why a single-phase AC supply fed to a single winding induction motor cannot produce rotation. [5 + 6]

Indian Institute of Engineering Science and Technology, Shibpur

Dual Degree (B.Tech-M.Tech) 3rd Semester Examination, 2018

Subject: Mathematics-III(MA-301)

(All Engineering Branches)

Use separate script for each half

Time : 3 hours

Full Marks : 70

First Half

Answer any eight questions

1. State axiomatic definition of probability. Using this definition show that for any two events A and B :

$$P(\overline{AB}) = P(B) - P(AB)$$

[5]

2. The chance that a doctor will diagnose a certain disease correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of the doctor, who had the disease, dies. What is the probability that the disease was diagnosed correctly? [5]

3. A man seeks advice regarding one of two possible courses of action from three advisers, who arrive at their recommendations independently. He follows the recommendations of the majority. The probabilities that the individual advisers are wrong are 0.1, 0.05, and 0.05 respectively. What is the probability that the man takes incorrect advice? [5]

4. Find the constant k so that the function

$$f(x) = \begin{cases} kx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

- is a probability density function. Find (i) distribution function and (ii) $P(1 < X < 2)$. [5]

5. Find the mean and variance of the Binomial distribution. [5]

6. The length of bolts produced by a machine is normally distributed with mean 4 and standard deviation 0.5. A bolt is defective if its length does not lie in the interval (3.8, 4.3). Find the percentage of defective bolts produced by the machine. Given:

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.6} e^{-t^2/2} dt = 0.7257, \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.4} e^{-t^2/2} dt = 0.6554.$$

[5]

7. (i) State and prove Tchebycheff's inequality for a continuous random variable.

(ii) Using Chebycheff's inequality, find a lower bound for the probability of getting 0 to 184 driving licences issued by Road Transport Authority in a specific month. It is given that the number of driving licences issued per month be a random variable having mean $m = 124$ and standard deviation $\sigma = 7.5$.

[3+2]

Q8) The probability density function a two-dimensional random variable (X, Y) is given by

$$f(x) = \begin{cases} k(x+y), & 0 \leq x+y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find k and evaluate $P(X < \frac{1}{2}, Y > \frac{1}{4})$.

[5]

9. Fit a straight line to the following data:

x_i	y_i
1	2.4
2	3
3	3.6
4	4
6	5
8	6

[5]

10. Define correlation coefficient between two random variables and show that it lies between -1 and +1.

[5]

11. In a partially destroyed Laboratory record of an analysis of correlation data, the following results only are legible: $\text{Var}(X)=9$.

Regression equations: $8X - 10Y + 66 = 0$, $40X - 18Y = 214$.

Find (i) the mean values of X and Y , (ii) $\rho(X, Y)$, (iii) σ_Y .

[5]

12) If S^2 be the sample variance of a random sample $\{X_1, X_2, \dots, X_n\}$ of size n drawn from a population with mean μ and standard deviation σ , then prove that the statistic $s^2 = \frac{n}{n-1}S^2$ is an unbiased estimator of σ^2 .

[5]

13) (i) Define consistent estimator for a population parameter.

(ii). Let $\{X_1, X_2, \dots, X_n\}$ be a random sample drawn from a population having a parameter θ and $T_n = T(X_1, X_2, \dots, X_n)$. If $E(T_n) \rightarrow \theta$ and $\text{Var}(T_n) \rightarrow 0$ as $n \rightarrow \infty$, then show that T_n is a consistent estimator of θ .

[2+3]

14) Find the maximum likelihood estimate of p for a Binomial (N, p) population.

[5]

SECOND HALF

Answer any THREE (03) questions

15. a) If $f(t)$ is a periodic function with period T , then show that

$$\mathcal{L}[f(t); s] = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt.$$

Hence obtain the Laplace transform of the periodic function given by $f(t) = \frac{t}{T}$ of period T , $0 < t < T$.

- b) State convolution theorem.

Apply convolution theorem to evaluate the inverse Laplace transform of

$$\frac{s}{(s^2 + a^2)^2}$$

$[(3+2)+(1+4)]$

16. a) Solve the following ordinary differential equation using Laplace transformation technique

$$\frac{d^2x}{dt^2} + 3 \frac{dx}{dt} + 2x = te^{-t}$$

It is given that at $t = 0$, $x = 0$ and $\frac{dx}{dt} = 0$.

- b) Evaluate the followings :

$$i) \mathcal{L}[t(3\sin 2t - 2\cos 2t); s], \quad ii) \mathcal{L}^{-1}\left[\frac{2s - 3}{s^2 - s - \frac{3}{4}}; t\right].$$

$[5 + (2\frac{1}{2} + 2\frac{1}{2})]$

17. a) Reduce the feasible solution $x_1 = 2, x_2 = 1, x_3 = 1$ of the system of equations

$$x_1 + 4x_2 - x_3 = 5$$

$$2x_1 + 3x_2 + x_3 = 8$$

to two basic feasible solutions.

- b) Find the extreme points, if any, of the following sets :

i) $X = \{(x_1, x_2) : |x_1| \leq 1, |x_2| \leq 1\}$

ii) $X = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 1; x_1 \geq 0, x_2 \geq 0\}$.

- c) Evaluate the value of

$$\mathcal{L}\left[\frac{1 - \cos t}{t}; s\right].$$

18. a) Solve the following LPP by simplex method :

$$\text{Maximize } Z = 10x_1 + x_2 + 2x_3$$

$$\text{Subject to } x_1 + x_2 - 2x_3 \leq 10$$

$$4x_1 + x_2 + x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0.$$

b) Show that the set $X = \{\bar{x} : |\bar{x}| \leq 2\}$ is a convex set.

c) Check whether the following sets are convex or not :

i) $X = \{(x_1, x_2) : x_2^2 \geq 4x_1\}$

ii) $X = \{(x_1, x_2) : x_1x_2 \leq 4\}$

[5 + 3 + 2]

19. Solve the following LPP using Charne's Big M method :

$$\text{Maximize } Z = 2x_1 + 3x_2$$

$$\text{Subject to } x_1 + x_2 \leq 8$$

$$x_1 + 2x_2 = 5$$

$$2x_1 + x_2 \leq 8$$

$$x_1, x_2 \geq 0.$$

OR

Use Two Phase method to solve the following LPP :

$$\text{Maximize } Z = 3x_1 - x_2$$

$$\text{Subject to } 2x_1 + x_2 \geq 2$$

$$x_1 + 3x_2 \leq 2$$

$$x_1 \leq 4$$

$$x_1, x_2 \geq 0.$$

[10]

$$\int_0^\infty e^{-st} t^n dt$$