Time: $1^1/_2$ hours   **Full Marks: 50**


Answer *all* Questions
(Write all parts of the same question together; marks will be deducted if intermediate steps are skipped)

1. Give short answers to the following: [2x10 = 20]

   i.   A proof that $p \rightarrow q$ is true based on the fact that q is true, such proofs are known as
        a) Direct proof   b) Vacuous proof   c) Trivial proof   d) Proof by cases

   ii. If set A and B have 3 and 4 elements respectively then the number of subsets of set $(A \: X \: B)$ is
        a) 1024   b) 2048   c) 512   d) 4096

   iii. A lattice is a partially ordered set in which every pair of elements has both _____ and
        _____.

   iv. If R = *{(1, 1), (3, 1), (2, 3), (4, 2)}*, then which of the following represents R^2, where R^2 is R
        composite R?
        a) {(1, 1), (2, 1), (4, 3), (3, 1)}   b) {(1, 1), (3, 1), (2, 3), (4, 2)}
        c) {(1, 3), (3, 3), (3, 4), (3, 2)}   d) {(1, 1), (9, 1), (4, 9), (16, 4)}

   v.   Find the inverse of function $f(x) = x^3 + 2$.

   vi. In how many ways can a dozen indistinguishable books be placed on four distinguishable
        shelves?

   vii. How many edges are there in a graph with 10 vertices each of degree six?

   viii. Determine the Euler path and Hamilton path in the graph below.
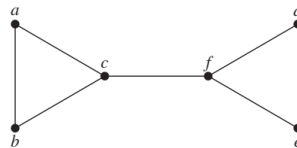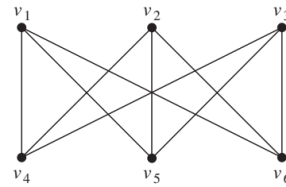

   ix. Find the chromatic number of the following graph?
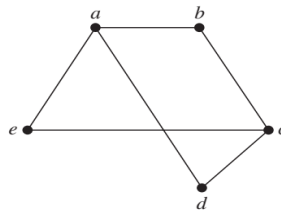
   x.   Are the graphs G & H planar? If so, draw their planar representations.
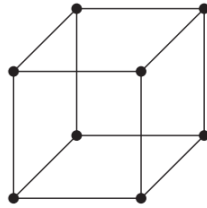
G   **H**

2. a) What do you mean by the "cardinality" of a set?
   b) How to determine if an infinite set S is countable?
   c) Use *Cantor's diagonalization argument* to prove that the set of real numbers is uncountable.
   $$[1 + 1 + 4 = 6]$$

3. a) How many bit operations are required to find the transitive closure of a set with n elements using
   (i) *zero-one matrix method* (ii) *Warshall's algorithm*?
   b) Use Warshall's algorithm to find the transitive closure of the relation $R = \{(2, 1), (2,3), (3,1),$
   $(3,4), (4,1), (4, 3)\}$ on the set $S = \{1, 2, 3, 4\}$ $[(1 + 1) + 4 = 6]$

4. Find the *upper bound(s)* and *lower bound(s)* of the subsets $\{a, b, c\}$, $\{j, h\}$, and $\{a, c, d, f\}$ in the poset
   with the Hasse diagram shown below (show intermediate steps) $[2 \times 3 = 6]$
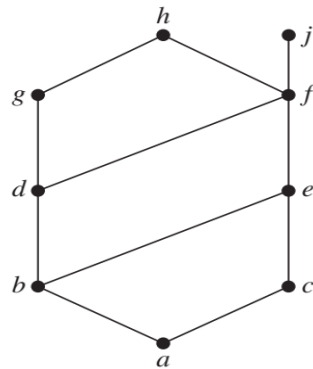
5. a) Write the basis and inductive steps to be followed in proofs by strong induction method.
   b) The harmonic numbers H_j, $j = 1, 2, 3, \dots$, are defined by $H_j = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{j}$. For

   instance, $H_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$. Use mathematical induction to show that $H_{2^n} \geq 1 +$
   $\frac{n}{2}$, whenever n is a nonnegative integer. $[2 + 4 = 6]$

6. Find the *cut vertex, cut edge, vertex cut(s), edge cut(s), vertex connectivity*, and *edge connectivity* of
   the graph shown below (give brief justifications). $[1 \times 6 = 6]$