Q) Find the basic solution of the following equations identifying the basic vectors and basic variables in each case:

$$x_1 + x_2 + x_3 = 4$$
$$2x_1 + 5x_2 \mp 2x_3 = 3$$

Find also basic feasible solutions.

~~→ here n = 3, m = 2~~

→ here n = no. of variables = 3
m = no. of equations = 2

So to ~~make~~ find basic solution, we have to keep m (2) ~~bas~~ variables non-zero and $n-m = 1$ variable = 0

→ No. of ways to do this = ${}^3C_2 = 3$

I) keep $x_1 = 0$

∴ the ~~equation~~ system becomes

$$x_2 + x_3 = 4$$
$$5x_2 - 2x_3 = 3$$

which gives, $x_2 = \frac{11}{7}$, $x_3 = \frac{17}{7}$

→ hence the solution $(0, \frac{11}{7}, \frac{17}{7})$ is also ~~basic feasi~~ feasible.

II) keep $x_2 = 0$

∴ The system becomes

$$x_1 + x_3 = 4$$
$$2x_1 - 2x_3 = 3$$

which gives, $x_1 = 2.75$, $x_3 = 1.25$

→ hence the solution $(2.75, 0, 1.25)$ is also feasible

III) keep $x_3 = 0$

$\therefore$ The system becomes

$$x_1 + x_2 = 4$$
$$2x_1 + 5x_2 = 3$$

which gives, $x_1 = \frac{17}{3}$, $x_2 = -\frac{5}{3}$

$\rightarrow$ which can't be feasible as $x_2 < 0$

$\therefore$ For the system,

Basic Feasible Solutions: $\left(0, \frac{11}{7}, \frac{17}{7}\right)$, $(2.75, 0, 1.25)$

Basic Non-Feasible Solution: $\left(\frac{17}{3}, -\frac{5}{3}, 0\right)$