PROBLEMS ON LAPLACE TRANSFORM

1. Find the Laplace transform of the function f(x), if

$$f(x) = \begin{Bmatrix} x^n, & when \ \ x > 0 \\ 0, & when \ \ x < 0 \end{Bmatrix}.$$

2. Find the Laplace transform of a periodic function f(x) of period τ i.e. f(x+ τ) = f(x).

3. Given the function

$$\text{f(t)} = \begin{Bmatrix} sint, when \ 0 < t < \pi \\ 0, \ \ when \ \pi < t < 2\pi \end{Bmatrix}$$

extended periodically with period 2π. Find L { f(t) }.

4. If $L\{f(t); t \rightarrow \ s\} = \frac{e^{-\frac{1}{s}}}{s}$, then find $L\{e^{-\ t} f(3t)\}$.

5. Find $L\left\{\sin\sqrt{t}\right\}$.

6. Prove that $L\ [H(x-a)] = \frac{e^{-ap}}{p}$, where H(x - a) is Heaviside's unit step function

7. Find $L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\}$.

8. Evaluate $L^{-1}\left\{\frac{1}{p(p^2+4)^2}\right\}$, by using convolution theorem.

9. Find $L^{-1}\left\{\frac{2s^3+10s^2+8s+40}{s^2(s^2+9)}\right\}$.

10. Evaluate $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$, by using convolution theorem.

11. Find $L^{-1}\left\{\frac{8e^{-\ 3s}}{s^2+4}\right\}$.

12. Find $L^{-\ 1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$.

13. Find $L^{-1}\left\{log\left(\frac{s+1}{s-1}\right)\right\}$.

14. Find $L^{-1}\left\{\frac{s}{s^4+4a^4}\right\}$.

15.Find $L^{-1}\left\{\frac{e^{-p}(1-e^{-p})}{(p^2+1)p}\right\}$.

16.Prove that $\beta\,(m\,,n) = \int_0^1 x^{m-1}\,(1-x)^{n-1}dx = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)},\ where\ m > 0,$ $n > 0$, by using convolution theorem.

17.Prove that $L\left\{\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}\,du\right\} = \frac{1}{s\sqrt{s+1}}$.

18.Use Laplace transform technique to solve the initial value problem

$$\frac{d^2y}{dt^2} - 3\,\frac{dy}{dt} + 2y = 4\,e^{2t}, y(0) = -3, \frac{dy}{dt} = 5, when\ t = 0.$$

19.Use Laplace transform technique to solve the initial value problem

$$\frac{d^2x}{dt^2} + 3\,\frac{dx}{dt} + 2x = t\,e^{-t}, x(0) = 1,\ \frac{dx}{dt} = 0, when\ t = 0.$$

20.Use Laplace transform technique to solve the initial value problem

$$Y''' - 3Y'' + 3Y' - Y = t^2e^t, Y(0) = 1, Y'(0) = 0, Y''(0) = -2.$$