PROPOSITION is

Tautology ⟶ Always True

Contradiction ⟶ Always false

Contingency ⟶ Sometimes True Sometimes False

✓ **Logical equivalence**

Each statements in ~~both~~ Truth Table has same truth value for two different expressions.

✓ **ATOMIC PROPOSITION**

✓ **COMPOUND PROPOSITION**

↓ Well Formed Formulas (wffs)

✓ **inverse**

$p \rightarrow q \xrightarrow[\text{inverse}]{} \sim p \rightarrow \sim q$

✓ Representation using BOOLEAN ALGEBRA LOGIC GATE

✓ **converse**

$p \rightarrow q \xrightarrow[\text{converse}]{} q \rightarrow p$

✓ **contra-positive**

$p \rightarrow q \xrightarrow[\text{contra-positive}]{} \sim q \rightarrow \sim p$

Duality Principle

Dual of $(A \cap B) \cup C$ is $(A \cup B) \cap C$

- interchange union into intersection & vice versa
- interchange Null set with universial set & vice versa

Deductions

P

If Vijay eats his vegetable then he can have cookie. (Q)

Vijay ate his vegetable.

Therefore, he gets a cookie.

$$\frac{P \rightarrow Q \qquad P}{\therefore Q}$$

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

$P \rightarrow Q$ $\sim P \vee Q$

$\overline{P} + Q$

$(P \rightarrow Q)$ and P

$(P \rightarrow Q) : P = T$

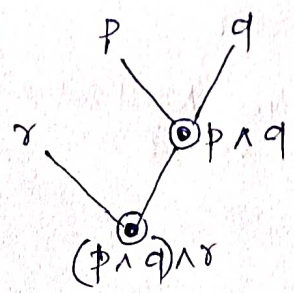
T	T	T
F	T	F
T	F	T
T	T	T

When $P \rightarrow Q = T$, $P = T$ then $Q = T$

(1st row of the truth table)

This is a valid deduction rule.

Expression tree



Normal Forms

- Disjunctive Normal Form (DNF)

$$(P \wedge q) \vee (\neg P \wedge \neg q)$$

- Conjunctive Normal Form (CNF)

$$(P \vee q) \wedge (\neg P \vee \neg q)$$

$P.q + \bar{P}\bar{q}$ DNF

$(P+q).(\bar{P}+\bar{q})$ CNF

Conjunction $P \wedge q$

Disjunction $P \vee q$

Arguments

$P_1, P_2, P_3 \ldots P_n \vdash Q$ is valid if

Q is true whenever $P_1, P_2, P_3 \ldots P_n$ are true.

If an argument is not valid then it is called fallacy.

$$\frac{\begin{array}{c} P \rightarrow Q \\ P \end{array}}{\therefore Q} \longrightarrow P \rightarrow Q, P \vdash Q.$$

Valuation

p	q	F
T	T	T
T	F	F
F	T	T
F	F	F

Each row of truth table corresponds to valuations

$V(x)$

Valuation is a function that takes proposition x & produces truth value $\{T, F\}$

Interpretation

An interpretation function takes a proposition formula Φ and a valuation V, and returns truth value of the formula.

$\Phi = p \rightarrow (p \vee q)$

Valuation $V(p) = T$ and $V(q) = F$

Then interpretation function

$$\Phi^V = V(p) \rightarrow (V(p) \vee V(q))$$
$$= T \rightarrow (T \vee F)$$
$$= T \rightarrow T$$
$$= T$$

Satisfiability & Validity

A proposition that has at least one true interpretation is called satisfiable.

A proposition that is false in all interpretations is called unsatisfiable.

True in all interpretation — valid
False in at least one interpretation — invalid