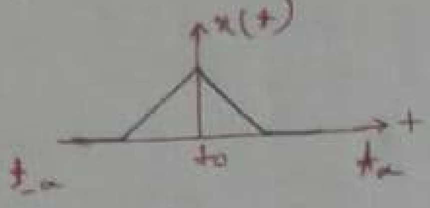
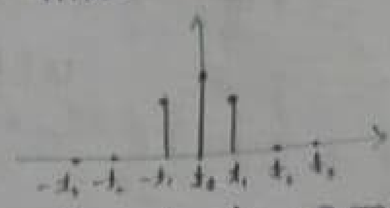
07-09-2020

Discrete Time Signals (DTS)

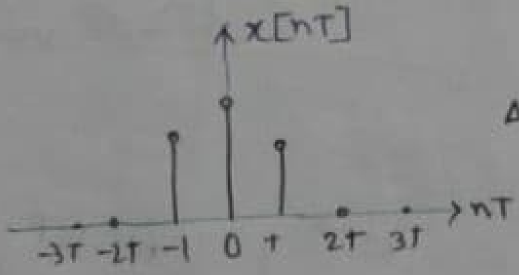
CTS	DTS
specified for every value of time (t)	specified at discrete time intervals or discrete values of time





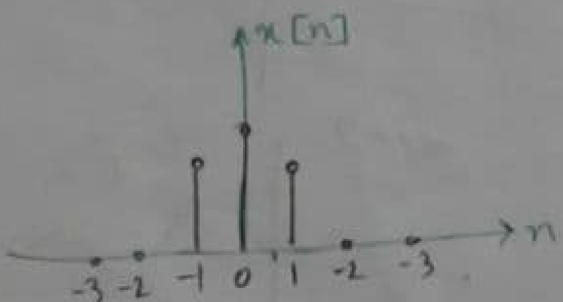
$\Delta T = t_1 - t_0 = 2\ ms$
$= t_2 - t_1 = 3\ ms$
$= t_3 - t_2 =$

* if ΔT is same, then it is uniformly sampled.
* if ΔT is not same, then it is non uniformly sampled.



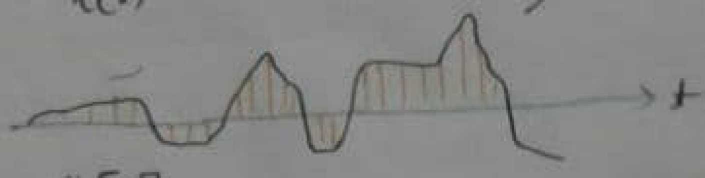
$\Delta T = t_1 - t_0 = T$
$= t_2 - t_1 = T$

n is an integer.

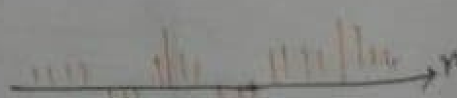


DTS representation.

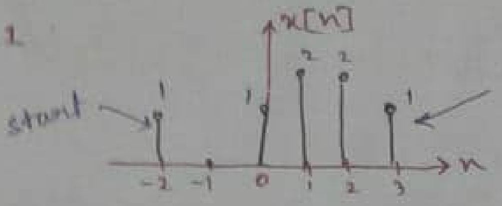
x(t)



x[n]



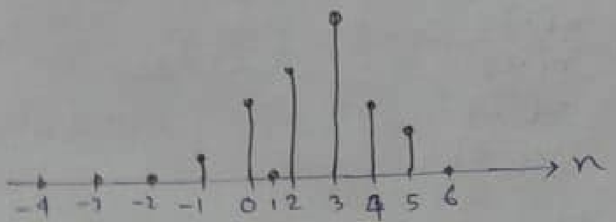
ex-1



$x[n] = \{1, 0, \underset{\uparrow}{1}, 2, 2, 1\}$

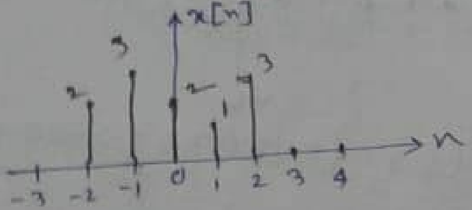
n=-2 n=-1 n=0 n=1 n=2 n=3

ex-2: $x[n] = \{0, 1, \underset{\uparrow}{3}, 0, 4, 6, 3, 2\}$



Operations on DTS

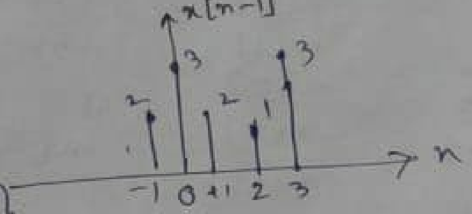
① Time shifting



$x[n] = \{0, 0, 2, 3, \underset{\uparrow}{2}, 1, 3, 0, 0\}$

$= \{2, 3, \underset{\uparrow}{2}, 1, 3\}$

① Right shifting:

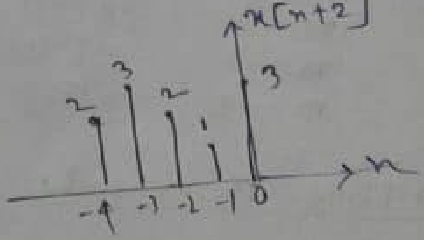


$x[n-1] = \{2, \underset{\uparrow}{3}, 2, 1, 3\}$

② left shifting:

$x[n] = \{2, 3, \underset{\uparrow}{2}, 1, 3\}$

$x[n+2] = \{2, 3, 2, 1, \underset{\uparrow}{3}\}$



② Time scaleing

$x[n] \rightarrow y[n] = x[an]$

Depending of the value of 'a' there are three types of scalling ——

① Time Compression:

$|a| > 1$

ex- $a = 2$

$x[n] \rightarrow y[n] = x[2n]$

$x[n] = \{5, 3, 7, \underset{\uparrow}{8}, -2, 4, 9\}$

$x[2n] = \{3, \underset{\uparrow}{8}, 4\}$

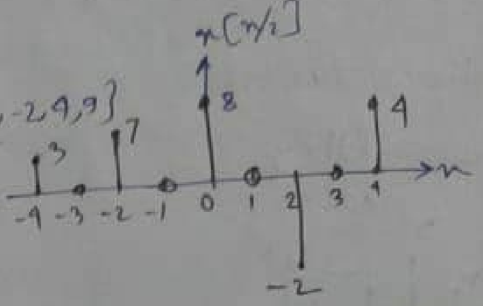
$x[n]$	$x[2n]$
$n=0$	$x[0]$
$n=1$	$x[2]$
$n=2$	$x[4]$
$n=-1$	$x[-2]$
$n=-2$	$x[-4]$

if we calculate,
$x[3n] = \{5, \underset{\uparrow}{8}, 9\}$

② Time expansion:

$|a| < 1$

$x[n] = \{5, 3, 7, \underset{\uparrow}{8}, -2, 4, 9\}$



ex- $a = \frac{1}{2}$

$x[n]$	$\longrightarrow x[\frac{n}{2}]$
$n=0$	$x[0]$
$n=1$	$x[\frac{1}{2}]$
$n=2$	$x[1]$
$n=3$	$x[3/2]$
$n=4$	$x[2]$
$n=-1$	$x[-\frac{1}{2}]$
$n=-2$	$x[-1]$
$n=-3$	$x[-3/2]$
$n=-4$	$x[-2]$

$x[\frac{n}{2}] = \{5, 0, 3, 0, 7, 0, \underset{\uparrow}{8}, 0, -2, 0, 4, 0, 9\}$

if -
$x[n/3] = \{5, 0, 0, 3, 0, 0, 7, 0, 0, \underset{\uparrow}{8}, 0, 0, -2, 0, 0, 4, 0, 0, 9\}$

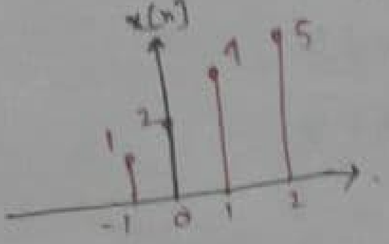
③ Time reversal:

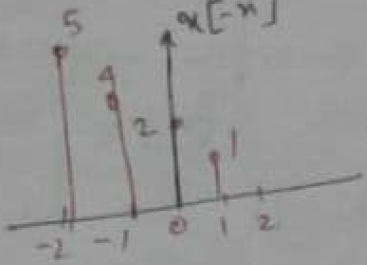
$a = -1$

$x[n] \longrightarrow x[-n]$

the whole webform will flip about the Y-axis ex- $x[n] = \{1, \underset{\uparrow}{2}, 4, 5\}$

$x[-n] = \{5, 4, \underset{\uparrow}{2}, 1\}$





ex-1: $x_1[n] = \{2, 4, 6, 8, 10\}$

Find $y_1[n] = x_1\left[\frac{2n}{3}\right]$

$x_1[2n] = \{2, \underset{\uparrow}{6}, 10\}$

$x_1\left[\frac{2n}{3}\right] = \{2, 0, 0, 6, 0, 0, 10\}$

ex-2: $x_2[n] = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$y_2[n] = x_2[-2n]$

$= \{5, \underset{\uparrow}{3}, 1\}$

ex-3: $x_3[n] = \{7, 1, \underset{\uparrow}{2}, 6, 3\}$

$y_3[n] = x_3[-n-1]$ ~~$= \{3, 6, 2, 1, 7\}$~~

$x_3[n-1] = \{7, \underset{\uparrow}{1}, 2, 6, 3\}$

$x_3[-n-1] = \{3, 6, 2, \underset{\uparrow}{1}, 7\}$

① Time shifting
② Time scaling
③ Time reversal

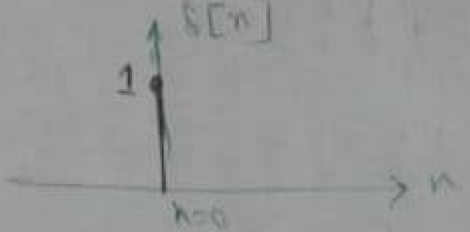
or

$x_3[-n] = \{3, 6, \underset{\uparrow}{2}, 1, 7\}$

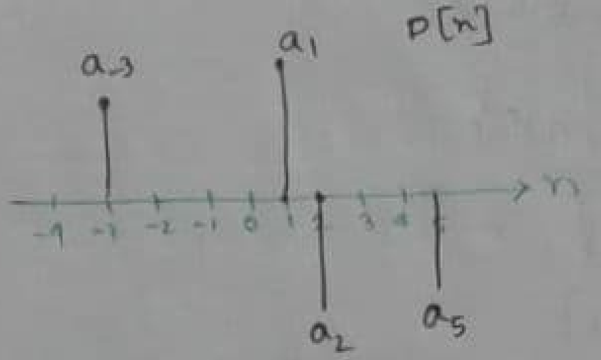
$x_3[-(n+1)] = \{3, 6, 2, \underset{\uparrow}{1}, 7\}$

Unit sample sequence / Discrete time impulse / Impulse in DTS

The unit sample sequence is defined as the sequence $\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$



any arbitorary sequence can be represented as a sum of sealed, delayed impulse.



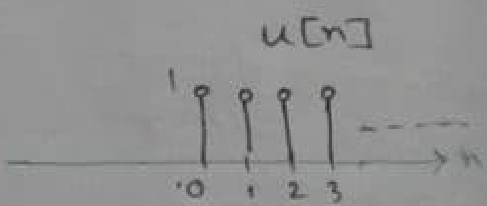
$$p[n] = a_{-3}\,\delta[n+3] + a_1\,\delta[n-1] + a_2\,\delta[n-2] + a_5\,\delta[n-5]$$

more generally

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,.\,\delta[n-k]$$

Unit Step sequence

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



$$u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \cdots$$

$$= \sum_{k=0}^{\infty} \delta[n-k]$$

$$\delta[n] = u[n] - u[n-1]$$

Exponential

$x[n] = A\alpha^n$, where A and α are any number

$\alpha = a + jb$

if A and α are real number then the $x[n]$ will be real.

if $0<\alpha<1$, A is positive then the sequence values are positive and decrease with increasing n

$|\alpha| > 1$

Let $\alpha = |\alpha| . e^{jw_0}$, $A = |A| . e^{j\phi}$

$x[n] = A\alpha^n = |A| e^{j\phi} . |\alpha|^n . e^{jw_0 n}$

$= |A| . |\alpha|^n . e^{j(w_0 n + \phi)}$

$= |A| . |n|^n \cos(w_0 n + \phi) + j |A| . |\alpha|^n \sin(w_0 n + \phi)$

if $|\alpha| > 1 \longrightarrow$ exponentially growing envelope.

$|\alpha| < 1 \longrightarrow$ " decaying "

$|\alpha| = 1$ $\quad x[n] = |A| . e^{j(w_0 n + \phi)}$

$= |A| . \cos(w_0 n + \phi) + j |A| . \sin(w_0 n + \phi)$

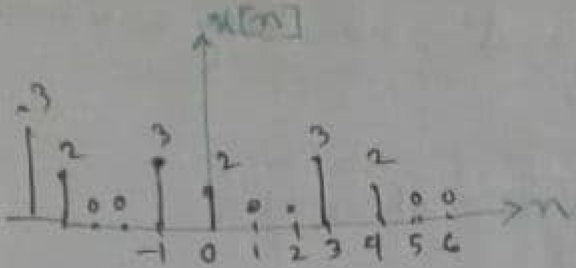
The real and imaginary parts of $e^{jw_0 n}$ vary sinusoidally with n.

periodic in DTS

$x(t) = x(t+T) \rightarrow$ CTS

$x[n] = x[n \pm N]$, N = fundamental time period.

DTS



$N = 2-(-1) + 1$ for $n=0$
$= 4$

Fundamental time period $= 4$

for $n=0$
$N = 6-3+1 = 4$

Composite DTS

$x[n] = x_1[n] + x_2[n]$

$\downarrow$ $N_1 =$ $\quad$ $\downarrow$ $N_2 =$

$\frac{N_1}{N_2}$ = Rational → $x[n]$ is periodic
↓
Fundamental time period, of the Composite signal, $N = LCM(N_1, N_2)$

$\frac{N_1}{N_2}$ = Irrational → aperiodic

$x[n] = A_0 \cdot e^{j\omega_0 n}$

$x[n] = x[n+N]$

$A_0 \cdot e^{j\omega_0 n} = A_0 e^{j\omega_0 (n+N)}$

$A_0 \cdot e^{j\omega_0 n} = A_0 \cdot e^{j\omega_0 n} \cdot e^{j\omega_0 N}$

$e^{j\omega_0 N} = 1$
$= e^{j2\pi k}$, k is an integer.

$\omega_0 N = 2\pi K$

$\frac{N}{K} = \frac{2\pi}{\omega_0}$ = Rational number → Complex exponential signal ~~periodic~~ / Sinusoidal signal is periodic

ex-1

$x[n] = e^{j2n}$

$x[n] = e^{j\omega_0 n}$

Aperiodic signal
no time period

$\frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi$ = Irrational number

ex-2:

$x[n] = \cos\left[\frac{3\pi}{4} n\right]$

$\omega_0 = \frac{3\pi}{4}$

Periodic

$\frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi/4} = \frac{8}{3}$ = Rational number.

$\frac{2\pi}{\omega_0} = \frac{N}{K}$

$N = \frac{2\pi}{\omega_0} \cdot K = \frac{8}{3} \cdot K$

$N = \frac{8}{3} \times 3 = 8$

K = 3 is the minimum value for making N to integer

Fundamental time period, N = 8

ex-3: $x[n] = \sin\left[\frac{3\pi}{4} \cdot n\right] + \sin\left[\frac{5\pi}{7} \cdot n\right]$

$\downarrow$ $x_1[n]$ $\downarrow$ $N_1 = 8$

$\downarrow$ $x_2[n]$ $\downarrow$ $N_2 = 14$

$N_1 = \frac{2\pi}{\omega_0} \times K$

$= \frac{2\pi}{3\pi/4} \times K = \frac{8}{3} \times 3$

$= 8$

$N_2 = \frac{2\pi}{\omega_0} \times K$

$= \frac{2\pi}{5\pi/7} \times K$

$= \frac{14}{5} \times 5$, $K_{min} = 5$

$= 14$

$\frac{N_1}{N_2} = \frac{8}{14}$

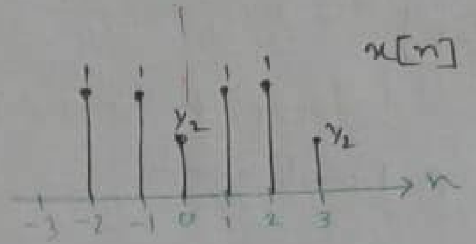
$N = LCM(8, 14) = 56$ ← fundamental time of $x[n]$

8.

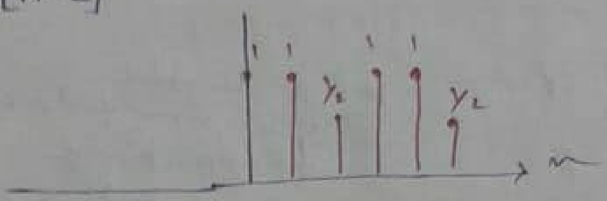
For the discrete time signal shown in fig draw the following signals.

$x[n-2]$, $x[2-n]$, $x[3n+2]$, $x[n/2]$, $x[$

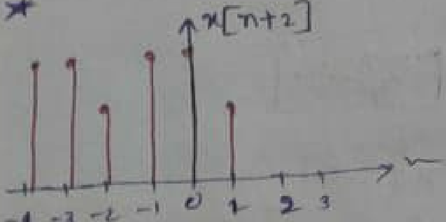
$x[n]+x[-n]$

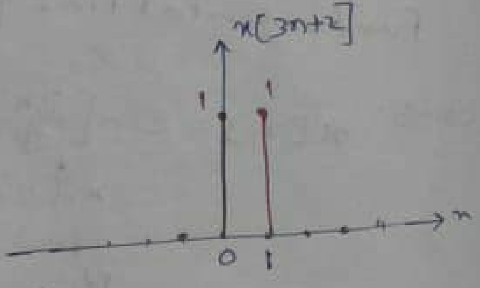


★ $x[n-2]$



★





$x[n+2] \longrightarrow x[3n+2]$

$n+2=0 \rightarrow n=-2$ $\quad x[3(-2)+2] = x[-4] = 1$

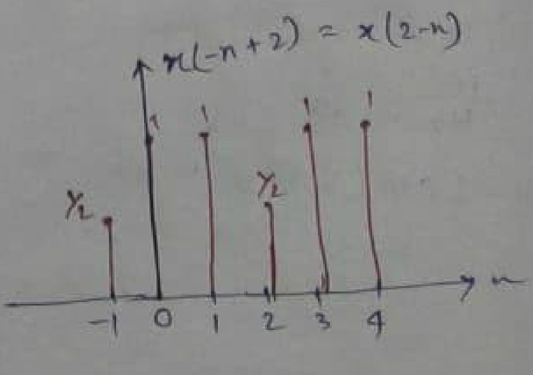
$n+2=1 \rightarrow n=-1$ $\quad x[3\times(-1)+2] = x(-1) = 1$

$n+2=2 \rightarrow n=0$ $\quad x[3\times 0+2] = x(2) = 0$

$n+2=-1 \rightarrow n=-3$ $\quad x[3(-3)+2] = x(-7) = 0$

★ $x(2-n) = x[-n+2]$

$x[n+2] \xrightarrow[\text{rev}]{\text{Time}} x[-n+2]$

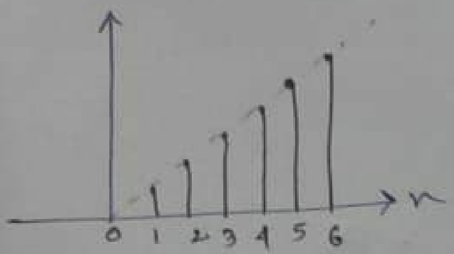


09.09.2020

Energy & Power in DTS

CTS	DTS
$E = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2\, dt$	$E = \sum_{n=-\infty}^{\infty} \lvert x(n)\rvert^2$
$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \lvert x(t)\rvert^2\, dt$	$P_{av} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} \lvert x(n)\rvert^2$

ex-1: $x[n] = \text{ramp } n = \begin{cases} 0 & \\ & n \geq 0 \end{cases}$



$$P_{av} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} \lvert x(n)\rvert^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[\sum_{n=-N}^{0} 0 + \sum_{n=0}^{N} n^2 \right]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} n^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \frac{N(N+1)(2N+1)}{6}$$

$$= \lim_{N\to\infty} \frac{N(N+1)}{6} = \infty$$

NENP

$$E = \sum_{n=-\infty}^{\infty} \lvert x(n)\rvert^2$$

$$= \sum_{n=-\infty}^{0} 0 + \sum_{n=0}^{N} n^2$$

$$= 0 + 1 + 4 + \cdots = \infty$$

ex-2

$Y[n] = A$

$$P_{avg} = \lim_{N\to\alpha} \frac{1}{2N+1} \sum_{n=-N}^{N} |Y(n)|^2$$

$$= \lim_{N\to\alpha} \frac{1}{2N+1} \cdot \sum_{n=-N}^{N} A^2$$

$$= \lim_{N\to\alpha} \frac{A^2}{2N+1} \cdot \sum_{n=-N}^{N} 1$$

$$= \lim_{N\to\alpha} \frac{A^2}{2N+1} (2N+1)$$

$= A^2$ power.

$$E = \sum_{n=\alpha}^{\alpha} A^2 = A^2 \sum_{n=\alpha}^{\alpha} 1 = \alpha$$

① The discrete time input $x(n)$ and output $Y(n)$ relationship as,

$$Y(n) = \sqrt{n\,x(n)}$$

Then the properties of the system are

① Linear, time invarient, causal, stable
② Non linear, time invarient, causal, stable.
③ Non, linear, time varient, causal, unstable.
④ Linear, time invarient, causal, unstable.

LOA:
$x_1(n) \rightarrow Y_1(n) = \sqrt{n\,x_1(n)}$
$x_2(n) \rightarrow Y_2(n) = \sqrt{n\,x_2(n)}$
$Y_1(n) + Y_2(n) = \sqrt{n x_1(n)} + \sqrt{n\,x_2(n)}$
$x_1(n) + x_2(n) \rightarrow \square \rightarrow Y' = \sqrt{n \cdot [x_1(n) + x_2(n)]}$

not following LOA.

Non-linear.

$y(n) = \sqrt{n\,x(n)}$

$y(n-n_0) = \sqrt{(n-n_0)\,x(n-n_0)}$

$x(n) \rightarrow x(n-n_0) \rightarrow \square \rightarrow y' = \sqrt{n\,x(n-n_0)}$

<u>time variant</u>

$x(n) \rightarrow y(n) = \sqrt{n\,x(n)}$

$n = 0 \quad y(0) = \sqrt{0\,x(0)} = 0$ <u>Causal</u>

$n = n_0 \quad y(n_0) = \sqrt{n\,x(n_0)}$

$x(n_0)^x$

$x(n) = 5$

$y(n) = \sqrt{n\,(5)}$

$n \rightarrow \infty, \; y(n) \rightarrow \infty$. unstabe.

Q. which of the following system is time invariant.

(a) $y(n) = x[n] - x[n-1]$

(b) $y[n] = n\,x[n]$

(c) $y[n] = x[-n]$

(d) $y[n] = n\,x^2[n]$

11.09.202

Linear time Invarient system
L T I

$x(t) \longrightarrow$ [LTI sys] $\longrightarrow Y(t)$

i/p — o/p

LOA — LDH

Impulse response.

$\delta(t) \longrightarrow \boxed{LTI} \longrightarrow Y(t) = h(t)$

i/p

Impulse response is used to define LTI system

Transfer function is also used to define LTI "

Impulse response is in time domain.

$h(t) \longrightarrow$ i/p $\longrightarrow$ o/p

Laplace transform / Inv. Laplace transform

$$H(s) = \frac{Y(s)}{X(s)} = ?!$$

The transfer function of a fixed linear system as the ratio of laplace transform of the system output to the laplace transform of the system input when all initial condition are zero.

$$H(s) = \frac{Y(s)}{X(s)} \Bigg|_{\text{all initial cond. are zero}}$$

ex-1 $Y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

$$\Big\downarrow \text{L.T}$$

$Y(s) = \frac{X(s)}{s} \Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s}$

$$\Big\downarrow \text{Inv LT}$$

$h(t) = u(t)$

So, the impulse response of this system is $u(t)$. (unit step)

ex-2 $Y(t) = ?$ $h(t) =$ given.

Convolution is a mathematical tool which is used to calculate the output of the LTI system, when the impulse response input is available.

$h(t)$.

$$\Big\downarrow \text{L.T}$$

$H(s) = \frac{Y(s)}{X(s)}$

$Y(s) = H(s)\,X(s)$

$$\Big\downarrow \text{Inv LT}$$

Convolution operator

$Y(t) = h(t) * x(t)$

$= \int_{-\infty}^{\infty} x(\tau).h(t-\tau)\,d\tau$ ← in case of CTS system

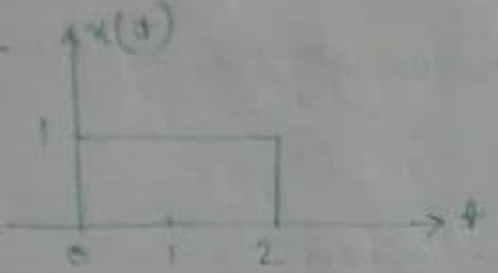
$= \int_{-\infty}^{\infty} h(\tau).x(t-\tau)\,d\tau$

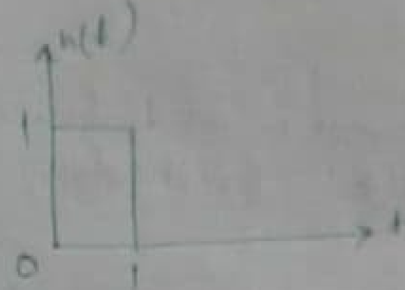
A convolution is an integral that express amount of overlap of one function when it is shifted over another function

Convolution in DTS

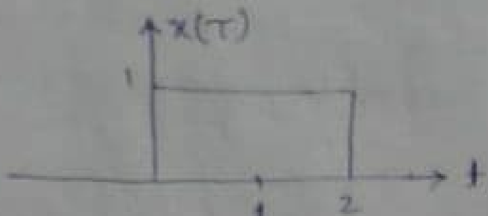
$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= h[n] * x[n] \\ &= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \\ &= \sum_{k=-\infty}^{\infty} h[k]\cdot x[n-k] \end{aligned}$$

Ex-3

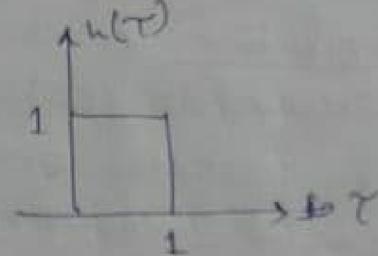




$t = \tau$



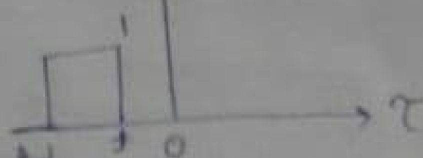
$t = \tau$



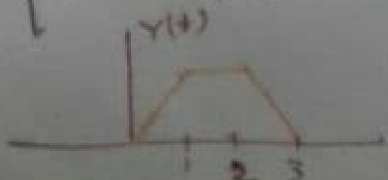
$$\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)$$

Time reversal

Time shifting



$$y(t) = \begin{cases} 0 & t<0 \\ t & 0<t<1 \\ 1 & 1<t<2 \\ 3-t & 2<t<3 \\ 0 & t>3 \end{cases}$$



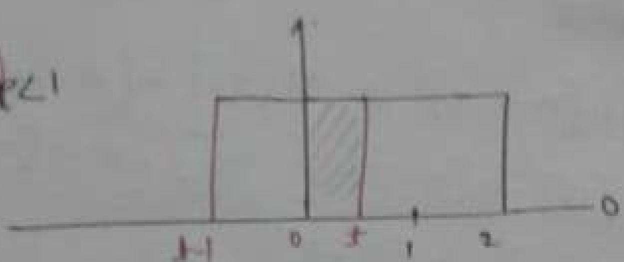
$x(\tau)\ h(t-\tau)$

Case-1 $\quad t < 0$

No overlap, multiplication is zero

Case-2

$0 < t < 1$

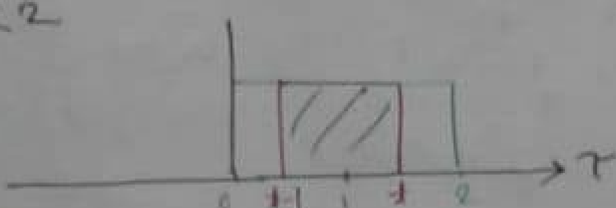


$$y(t) = \int_0^t x(\tau)\ h(t-\tau)\, d\tau$$

$$= \int_0^t 1\, d\tau = \tau \Big|_0^t = t$$

Case-3.

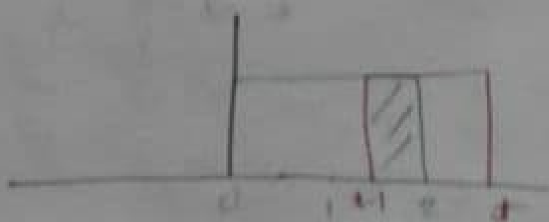
$1 < t < 2$



$$y(t) = \int_{t-1}^{t} 1\, d\tau = 1$$

Case-4

$2 < t < 3$



$$y(t) = \int_{t-1}^{2} 1 \cdot d\tau = 2 - t + 1$$

$$= 3 - t$$

Case-5

$t > 3$

no overlap

$y(t) = 0$

ex- DTS

$$x[n] = \{1, 2, \underset{\uparrow}{-3}, 2, 1\}$$

$$h[n] = \{4, \underset{\uparrow}{-2}, 1\}$$

$$x[n] * h[n] = \sum_{k-\alpha}^{\alpha} x[k] \cdot [n-k]$$

$$= \sum_{k=-2}^{2} x[k] \cdot [h-k]$$

$$= x[-2] \cdot h[n-(-2) + x[-1] \cdot h[n-(-1)]$$

$$+ x[0] \cdot h[n-0] + x(1] + h[n-1]$$

$$+ x[2] \cdot h[n-2]$$

$$= 1 \cdot h[n+2] + 2 \cdot h[n+1] - 3 \cdot h[n] + 2\, h[n-1]$$

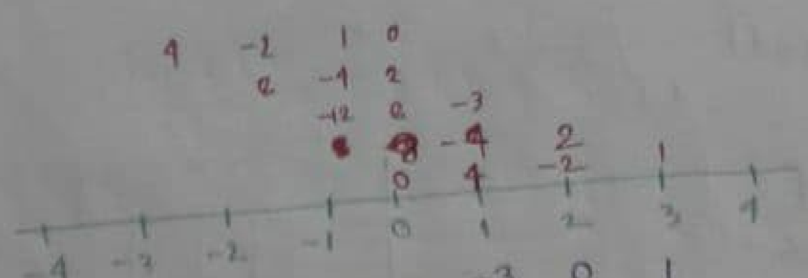
$$+ 1\, h[n-2]$$

$h[n] = \{4, \underset{\uparrow}{-2}, 1\}$ $\quad$ $h[n+1] = 4, -2, \underset{\uparrow}{1}$

$h[n-1] = \underset{\uparrow}{4}, -2, 1$ $\quad$ $h[n+2] = 4-2, 1, \underset{\uparrow}{0}$

$h[n-2] = \underset{\uparrow}{0}, 4, -2, 1$

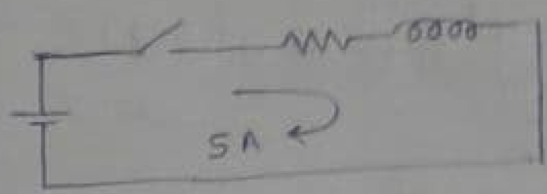
~~=1~~



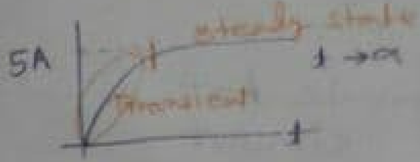
4 6 -15 16 -3 0 1

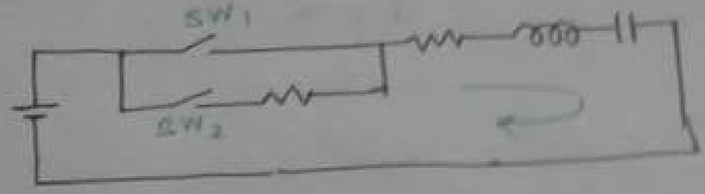
$$x[n] * h[n] = \{4, 6, -15, \underset{\uparrow}{16}, -3, 0, 1\}$$

Transient Response

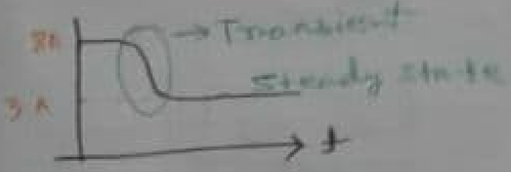


SW → open
$i = 0$
SW - close
i = some value





$i = 8A \to SW_1$ close, SW_2 open
$i = 3A \to SW_2$ close, SW_1 open



Linear differential eq = Complementary function (Transient part) + Particular solution (steady state part)

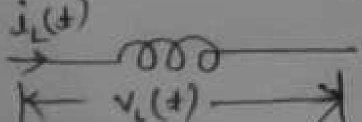
$t = 0^-$ instant immedeatly before swiching
$t = 0^+$ instant immedeaty after swiching

For Inductor $i_L(0^-) = i_L(0^+)$ at $t = 0$
5A 5A

For capacitor $V_C(0^-) = V_C(0^+)$
10 V 10 V

Voltage across resistance, $V_R(t) = R\, i_R(t)$

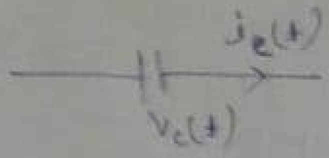
$$i_R(t) = \frac{v_R(t)}{R}$$



$$v_L(t) = L \cdot \frac{di_L(t)}{dt}$$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\, dt$$

$$= \frac{1}{L}\int_{0}^{t} v_L(t) \quad ??$$



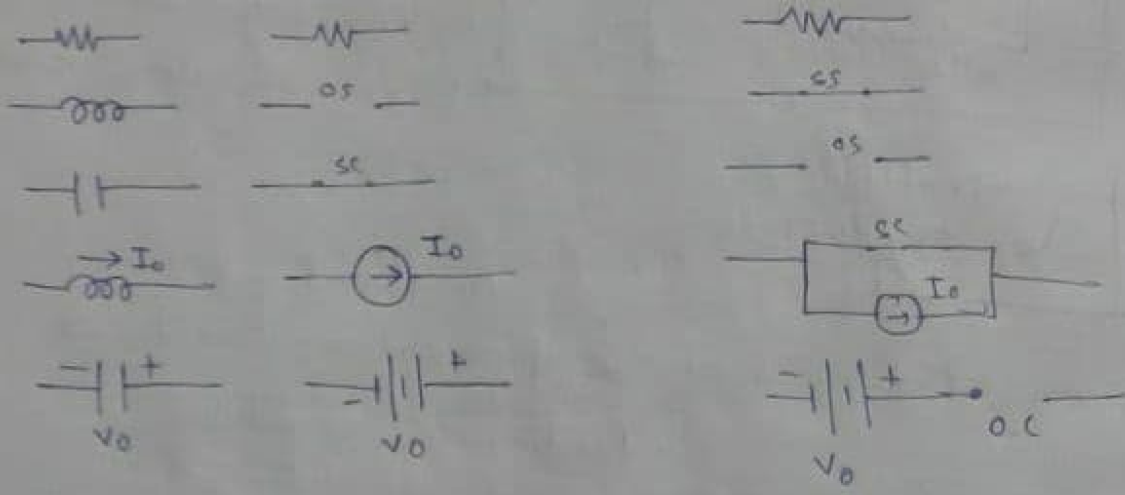
$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)\, dt$$

$$= \frac{1}{C}\int_{0}^{t} i_c(t)\, dt + V_c(0^+)$$

$$i_c = C \cdot \frac{dV_c(t)}{dt}$$

equivalent circuit $t = 0^+$

equivalent circuit $t = \infty$



Type-1:

$$\frac{dy(t)}{dt} + P\,y(t) = 0 \quad , P = \text{Const}$$

$$y(t) = K\, e^{-Pt} \quad , K = \text{Const}$$

Type-2:

$$\frac{dy(t)}{dt} + P y(t) = Q$$

$$y(t) = \underbrace{e^{-Pt}\int Q\, e^{Pt}\, dt}_{\text{Particular Integral (P.I)}} + \underbrace{K\, e^{-Pt}}_{\text{Complementary function}}$$

$$y(t) = \frac{Q}{P} + K \cdot e^{-Pt}$$

Type-3:

$$A\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + c\,y(t) = 0$$

$$y(t) = K_1 e^{P_1 t} + K_2 e^{P_2 t}$$

where P_1, P_2 are the roots of the quadratic equation

$$i(t) = \frac{V}{R} + K\, e^{-\frac{R}{L} t}$$

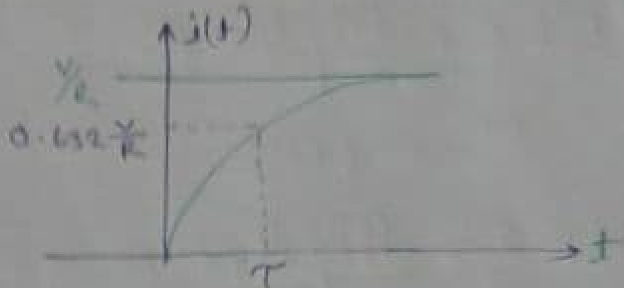
at $t = 0^+$ inductor behaves as a open circuit

$$i(0^+) = i(0^-) = 0$$

$$0 = \frac{V}{R} + K\, e^{-\frac{R}{L} \cdot 0} \Rightarrow K = -\frac{V}{R}$$

$$i(t) = \frac{V}{R} - \frac{V}{R}\, e^{-\frac{R}{L} t}$$

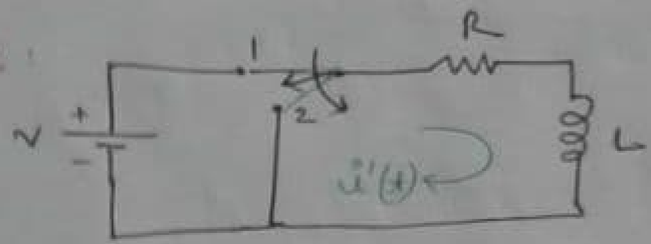
$$= \frac{V}{R}\left(1 - e^{-\frac{R}{L} t}\right)$$



$\tau = \frac{L}{R}$

after 5τ Steady state

ex-2 :



at $t = 0$, sw changes ~~position~~ 1 to position 2

$$L \cdot \frac{d i'(t)}{dt} + R\, i'(t) = 0$$

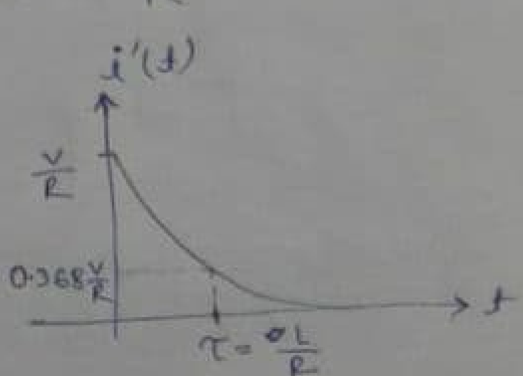
$$i'(t) = K' e^{-\frac{R}{L} t}$$

at $t = 0^+$ $\quad i_L(0^-) = i_L(0^+) = \frac{V}{R}$

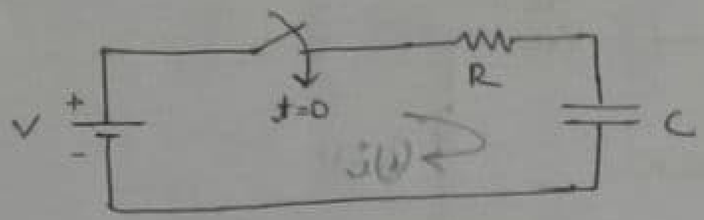
$$\frac{V}{R} = K' e^{-\frac{R}{L} \cdot 0}$$

$$K' = \frac{V}{R}$$

$$i'(t) = \frac{V}{R} \cdot e^{-\frac{R}{L} t}$$



ex-3:



KVL

$$R\,i(t) + \frac{1}{C}\int_{-\infty}^{t} i(t).dt = V$$

$$R\,i(t) + \frac{1}{C}\cdot\int_{0}^{t} i(t)\,dt + V_c(0^+) = V$$

Assume capacitor is uncharged

$$V_c(0^+) = 0$$

$$R\,i(t) + \frac{1}{C}\cdot\int_{0}^{t} i(t).\,dt = V$$

differentiating,

$$R\,\frac{di(t)}{dt} + \frac{1}{C}\,i(t) = 0$$

$$\frac{d\,i(t)}{dt} + \frac{1}{RC}\,i(t) = 0$$

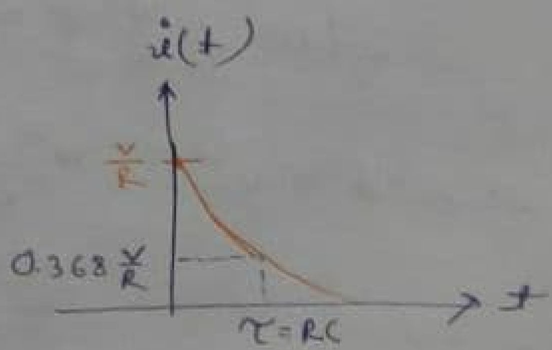
$$i(t) = K.e^{-\frac{1}{RC}t}$$

at $t = 0^+$ $\quad i(0^+) = \frac{V}{R}$

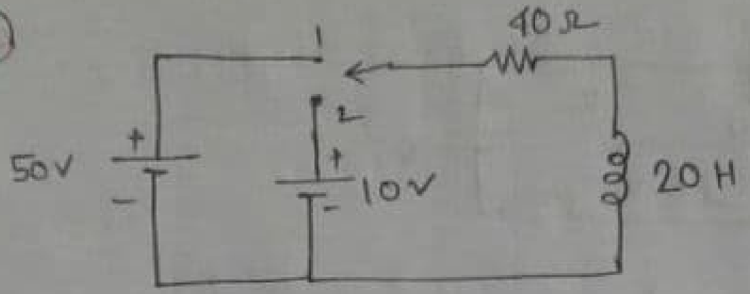
$$\frac{V}{R} = K.e^{-\frac{1}{RC}\times 0}$$

$$\therefore K = \frac{V}{R}$$

$$i(t) = \frac{V}{R}\cdot e^{-\frac{1}{RC}t}$$



(P.1)



The switch in fig has ~~between in~~ been in position 1 for a long time. It is moved to position 2 at $t=0$. Obtain the expression for current.

$$L \frac{di'(t)}{dt} + R i'(t) = \cancel{50}\ 10$$

$$\cancel{i'(t) = 10}$$

$$\frac{di'(t)}{dt} + \frac{R}{L} i'(t) = \frac{10}{L}$$

$$i'(t) = \frac{\frac{10}{L}}{\frac{R}{L}} + K e^{-\frac{R}{L}t}$$

$$i'(t) = \frac{10}{R} + K e^{-\frac{R}{L}t}$$

at, $t = 0^+$ $\quad \frac{50}{R} = \frac{10}{R} + K e^{-\frac{R}{L} \cdot 0}$

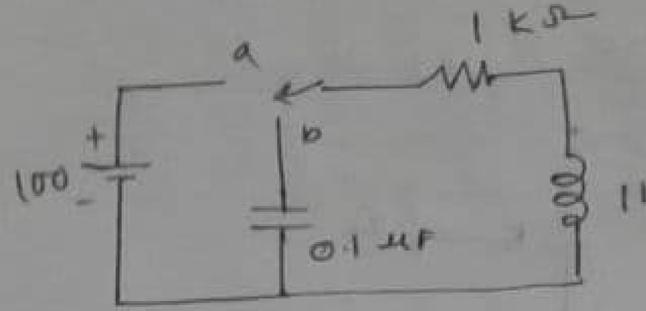
$$\frac{40}{R} = K$$

$$i'(t) = \frac{10}{R} + \frac{40}{R} e^{-\frac{R}{L}t}$$

$$= \frac{10}{40} + \frac{40}{40} e^{-\frac{40}{20}t}$$

$$= \frac{1}{4} + e^{-2t}$$

ex-2



Find the value of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t = 0^+$

sw is 'a' $\rightarrow$ 'b' at $t = 0$

at position 'a'

steady state value of current $i(0^-)$

$$= \frac{100}{1000} = 0.1\,A$$

at position 'b',

$$i(0^+) = i(0^-) = 0.1\,A$$

?

KVL

$$R\,i(t) + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = 0$$

$$R\,i(t) + L\cdot\frac{di}{dt} + \frac{1}{C}\int_{0}^{t} i(t)\cdot dt = 0$$

diff.

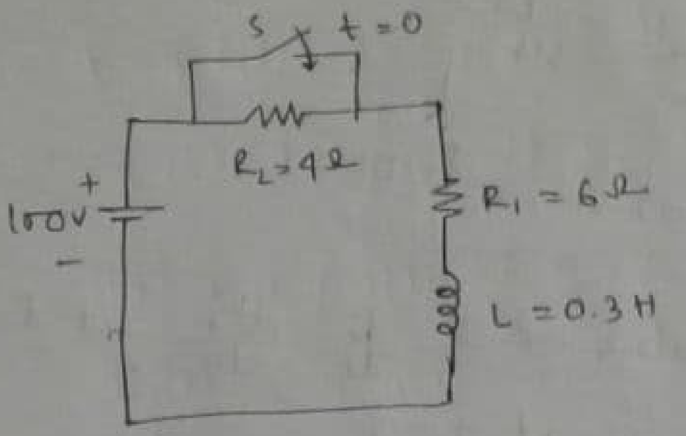
$$R\,\frac{di(t)}{dt} + L\cdot\frac{d^2i(t)}{dt^2} + \frac{1}{C}\cdot i(t) = 0$$

$$\frac{d^2i(t)}{dt^2} = -R\frac{d\,i(t)}{dt} - \frac{1}{C}\cdot i(t)$$

$$= -1000\times(-100) - \frac{1}{0.1\times10^{-6}}\times 0.1$$

$$= -9\times10^5\ A/sec^2$$

P-3



Solve for the current as a function of time.

Steady state current

$$i(0^-) = \frac{V}{R_1+R_2} = \frac{100}{4+6} = 10\ A$$

KVL → when sw is closed

R_2 → short circuit

$$L \cdot \frac{di(t)}{dt} + R_1 i(t) = V$$

$$\frac{d\,i(t)}{dt} + \frac{R_1}{L} \cdot i(t) = \frac{V}{L}$$

$$i(t) = \frac{V}{R_1} + K \cdot e^{-\frac{R_1}{L} t}$$

at $t = 0^+$ $\quad i(0^+) = 10\ A$

$$10 = \frac{100}{6} + K \cdot e^{-\frac{R_1}{L} \cdot 0}$$

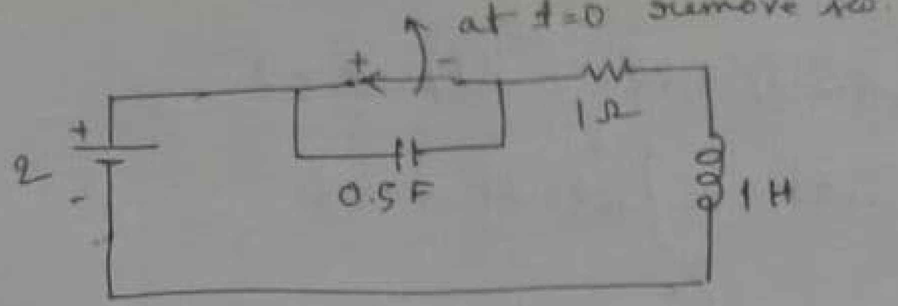
$$K = 10 - 16.66$$

$$= -6.66$$

$$i(t) = \frac{V}{R_1}\left(1 - \frac{R_2}{R_1+R_2} \cdot e^{-\frac{R_1}{L} t}\right)$$

$$= 16.66\left(1 - 0.4\, e^{-20t}\right)$$

P-4



Determine the voltage across the switch V_s and $\frac{dV_s}{dt}$ at $t=0^+$

$V_c(0^-) = 0$

$i(0^-) = \frac{V}{R} = \frac{2}{1} = 2A$

When sw is open at $t=0$, capacitor behaved as a short circit

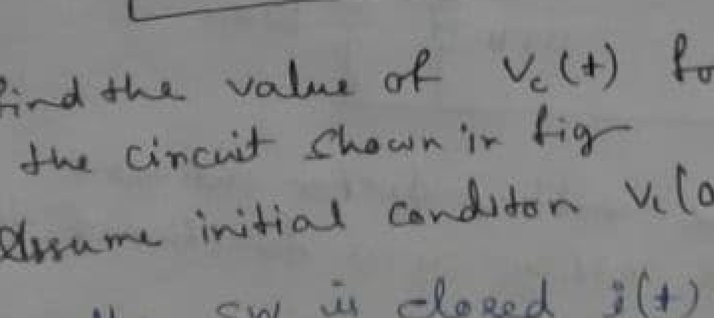
V_{sw} at $t=0^+$ $\underline{V_{sw}=0}$

$\cancel{V_c =}$ $i(t) = C\frac{dV_s}{dt}$

$\frac{dV_c}{dt} = \frac{i(t)}{C}$

at $t=0^+$ $\frac{dV_s(0^+)}{dt} = \frac{i(0^+)}{C} = \frac{2}{0.5} = \underline{4V}$

P-5



find the value of $V_c(t)$ for $t>0$ in the circuit shown in fig. Assume initial conditon $V_c(0^-) = 9V$

when the sw is closed $i(t)$ is flowing through the circuit

KVL $1 = 4\,i(t) + V_c(t)$

$V_c(t) = 9 + \frac{1}{C}\int_0^t i(t)\,dt$

$$1 = 4 \cdot i(t) + 16 \cdot \int_0^t i(t) \cdot dt + 9$$

$$4\, i(t) + 16 \int_0^t i(t) \cdot dt = -8$$

$$i(t) + 4 \int_0^t i(t) \cdot dt = -2$$

$$\frac{d\, i(t)}{dt} + 4\, i(t) = 0$$

$$i(t) = k \cdot e^{-4t}$$

at $t = 0^+$ $\quad i(0^+) = \frac{1-9}{4} = -2$ $\quad -2 = k\, e^{-4 \cdot 0}$

$$i(t) = -2 \cdot e^{-4t}$$ $\quad k = -2$

$$v_c(t) = 9 + \frac{1}{c} \int_0^t i(t)\, dt$$

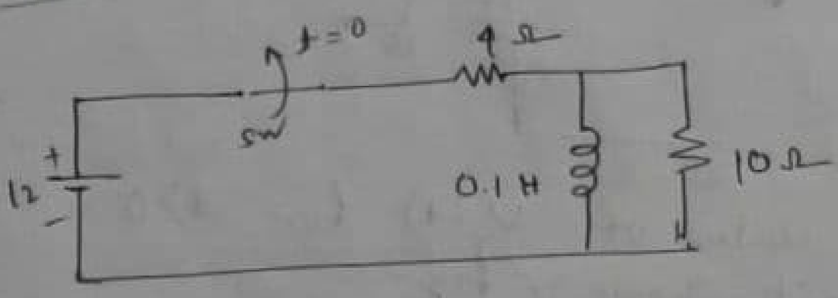
$$= 9 + 16 \cdot \int_0^t (-2) \cdot e^{-4t}$$

$$= 9 - 32 \cdot \left. \frac{e^{-4t}}{-4} \right|_0^t$$

$$= 9 + 8 \cdot e^{-4t} - 8$$

$$v_c(t) = 1 + 8 \cdot e^{-4t}$$

P-6



Find the ~~volt~~ inductor current and voltage as a function of time

Before $t = 0$, inductor short circuit

$$v_L(0^-) = 0, \quad i_L(0^-) = \frac{12}{4} = 3\ A$$

Apply KVL. $\quad 0.1 \frac{d i_2(t)}{dt} + 10\, i_2(t) = 0$

$$\frac{d i_2(t)}{dt} = 100\, i_2(t) = 0$$

$$i_2(t) = K \cdot e^{-100t}$$

$$i_L(0-) = i_L(0+) = 3$$

$$3 = K\, e^{-100\times 0}$$

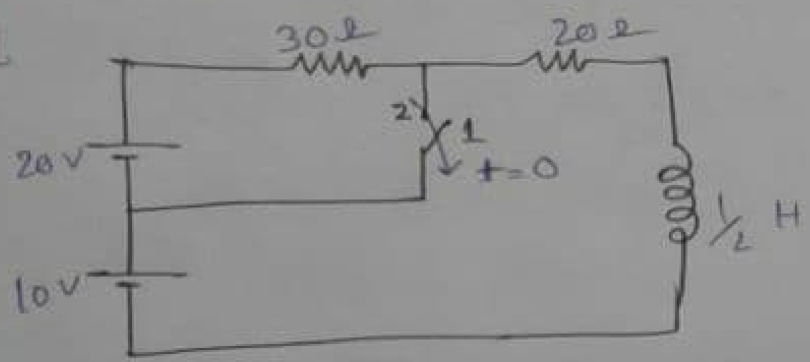
$$\therefore K = 3$$

$$i_L(t) = 3\, e^{-100t}$$

$$V_L(t) = L \cdot \frac{d i_2(t)}{dt} = 0.1 \times 3 \times (-100) \times e^{-100t}$$

$$= \left(-30 \cdot e^{-100t}\right) V$$

P-7



steady state in position 2 at t=0 the SW is moved to position 1, find i(t)

Stead state current

$$i(0-) = \frac{10}{20} = 0.5\ A$$

now SW is open

KVL $$30 = 30\, i(t) + 20\, i(t) + \frac{1}{2}\frac{di(t)}{dt}$$

$$\frac{di(t)}{dt} + 100\, i(t) = 60$$

$$i(t) = \frac{60}{100} + K\, e^{-100t}$$

t = 0: $$i(0-) = i(0+) = 0.5\ A$$

$$0.5 = \frac{60}{100} + K \cdot e^{-100\times 0}$$

$$K = -0.1$$

$$\therefore i(t) = 0.6 - 0.1\, e^{-100t}$$