Prove that if x is a real number, then $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$.

Prove or disprove that $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$ for all real numbers x and y.

Assume S = {0, 2, 4, 6,…..} set of even numbers. Is it countable?

Show that the set of all integers is countable.

Show that the set of positive rational numbers is countable.

Show that the $|(0, 1)| = |(0, 1]|$.