Rules of Inference-2

Monday, September 7, 2020 10:37 AM

p: "You send me an email message"

q: "I will finish writing the program"

r: "I will go to sleep early"

s: "I will wake up feeling refreshed"

Premises

① $p \to q$ ③ $r \to s$

② $\neg p \to r$

Conclusion : $\boxed{\neg q \to s}$

Steps	Reason
1. $p \to q$	Premise
2. $\neg q \to \neg p$	Contrapositive of (1).
3. $\neg p \to r$	Premise
4. $\neg q \to r$	Hypothetical Syllogism on (2)&(3)
5. $r \to s$	Premise
6. $\boxed{\neg q \to s}$	Hypothetical Syllogism on (4)&(5)

Prob-4

RK := I was reading newspaper in the kitchen.
GK := My glasses are on the kitchen table.
SB := I saw them at breakfast.
RL := I was reading the newspaper in the living room.

GC := My glasses are on the coffee table.

Steps	Reasons
1. $RK \to GK$	Premise (a)
2. $GK \to SB$	Premise (b)
3. $RK \to SB$	Hypothetical syllogism on (1) and (2)
4. $\neg SB$	Premise (c)
5. $\neg RK$	Modus tollens on (3) & (4)
6. $RL \vee RK$	Premise (d)
7. RL	Disjunctive syllogism on (5) & (6)
8. $RL \to GC$	Premise (e)
9. GC	Modus ponens on (7) & (8)

→ My glasses are on the coffee table.

Prob-5 p := "It rained"

q := "It is foggy"

from $r :=$ "It rained"

$f :=$ "It is foggy"

$s :=$ "The sailing race will be held"

$d :=$ "The life saving demo will go on"

$t :=$ "The trophy will be awarded".

Premises:

(1) $(\neg r \vee \neg f) \to (s \wedge d)$

(2) $s \to t$

(3) $\neg t$

Conclusion: r

Steps	Reasons
(1) $(\neg r \vee \neg f) \to (s \wedge d) \equiv$	Premise
(2) $s \to t$	Premise
(3) $\neg t$	Premise
(4) $\neg s$	Modus tollens on (2) & (3)
(5) $\neg(s \wedge d) \to \neg(\neg r \vee \neg f)$	Equivalency $(p \to q \equiv \neg q \to \neg p)$ on (1)
(6) $\neg s \vee \neg d \to r \wedge f$	Applying De Morgan's Laws on (5)
(7)	Addition on (4)

$p \to q \equiv \neg q \to \neg p$

7	$\neg s \vee \neg d$	Addition on 4
8	$r \wedge f$	Modus ponens on (6) & (7)
9	r	Simplification on 8

Prob 6

$D(x) :=$ x is in this discrete mathematics class.

$C(x) :=$ x has taken a course in computer Sc.

Premises:

1. $\forall x \, (D(x) \rightarrow C(x))$
2. $D(\text{Marla})$

Conclusion:

$C(\text{Marla})$

Steps		Reason
1	$\forall x \, (D(x) \rightarrow C(x))$	Premise
2	$D(\text{Marla}) \rightarrow C(\text{Marla})$	Universal instantiation on (1)
3	$D(\text{Marla})$	Premise
4	$C(\text{Marla})$	Modus ponens on (2) & (3).

(4) $C(\text{Marla})$ Modus ponens on (2) & (3).

Prob-7 $C(x)$ = "x is in this class"

$B(x)$ = "x has read the book"

$P(x)$ = "x has passed the first exam"

Premises.

(1) $\exists x\,(C(x) \wedge \neg B(x))$

(2) $\forall x\,(C(x) \rightarrow P(x))$

Conclusion: $\exists x\,(P(x) \wedge \neg B(x))$ $\models$ $P(a) \wedge \neg B(a)$

Steps	Reason
1. $\exists x\,(C(x) \wedge \neg B(x))$	Premise
2. $C(a) \wedge \neg B(a)$	Existential instantiation on (1)
3. $C(a)$	Simplification on (2)
4. $\forall x\,(C(x) \rightarrow P(x))$	Premise
5. $C(a) \rightarrow P(a)$	Universal instantiation on (4)
6. $P(a)$	Modus ponens on (3) & (5)

6. $P(a)$ — Modus ponens on (3) & (5)

7. $\neg B(a)$ — Simplification on (2)

8. $P(a) \wedge \neg B(a)$ — Conjunction on (6) & (7)

9. $\exists x\,(P(x) \wedge \neg B(x))$ — Existential generalization on (8)

~~Combining Rules of Inference for Propositions and Quantified statements.~~

① Universal Modus tollens: Combines modus tollens and universal instantiation.

$$\forall x\,(P(x) \rightarrow Q(x))$$

$\neg Q(a)$, where a is a particular element in the domain

$\therefore \neg P(a)$

② Universal Modus ponens: Combines modus ponens and universal instantiation.

$$\forall x\,(P(x) \rightarrow Q(x))$$

$P(a)$, where a is a particular element in the domain

$\therefore Q(a)$