Sets & Functions-2

Wednesday, September 23, 2020 8:48 AM

$A_i = \{1, 2, 3, \ldots i\}$ for $i = 1, 2, 3, \ldots\ldots$

$$\bigcup_{i=1}^{\infty} A_i = \{1, 2, 3, \ldots\ldots\} = Z^+$$

set of +ve integers.

$$\bigcap_{i=1}^{\infty} A_i = \{1\}$$

$A_1 = (1)$

$A_2 = (1, 2)$

$A_3 = (1, 2, 3)$

$A_n = (1, 2, 3 \ldots\ldots n)$

$f_1(x) = x^2$; $f_2(x) = x - x^2$

$f_1, f_2 : R \to R.$

$f_1(x) + f_2(x) = (f_1 + f_2)(x)$

$= \cancel{x^2} + x - \cancel{x^2} = x$

$(f_1 f_2) x = f_1(x) . f_2(x) = x^2 . (x - x^2)$

$= x^3 - x^4.$

Prob

$f(x) = x^2 \qquad f : Z \to Z.$

$f(1) = 1 = f(-1). \Rightarrow$ The function $f(x) = x^2$ is not one-to-one.

Prob

$f(x) = x+1 \qquad f: R \to R$

the function $f(x) = x+1$ is a one-to-one as $x+1 \neq y+1$ when $x \neq y$.

Prob. $f: \{a, b, c, d\} \to \{1, 2, 3\}$

$f(a) = 3, \ f(b) = 2, \ f(c) = 1, \ f(d) = 3$

All the elements in the co-domain has at least one preimage in the domain $\{a, b, c, d\}$.

Hence, f is onto function.

Prob. $f(x) = x^2 \ ; \ f: Z \to Z$

The function f is not onto because there is no integer with $x^2 = 5$.

Prob. $f(x) = x+1 \ ; \ f: Z \to Z$.

The function f is onto because for every integer y there is an integer x such that $y = x+1$.

Inverse functions must be one-to-one correspondence $\Rightarrow$

Otherwise, some element in the codomain will be the image of more than one element in the domain.

... element 'b' in the

image of more than one element in the ...
~~We cannot assign to each element 'b' in the codomain a unique 'a' in the domain~~ s.t.
$f(a)=b$.

If f is not onto, for some element b in the codomain, no element 'a' in the domain exists for which $f(a)=b$.

Prob. $f: \{a, b, c\} \rightarrow \{1, 2, 3\}$

$f(a)=2, \ f(b)=3, \ f(c)=1$

This is one-to-one because $f(a) \neq f(b)$ for $a \neq b$.

This is onto as every image has a corresponding pre-image.

This is one-to-one correspondence and hence inverse function exists.

$f^{-1}(1)=c, \ f^{-1}(2)=a, \ f^{-1}(3)=b.$

Prob. $f: \mathbb{Z} \rightarrow \mathbb{Z}$, $f(x)=x+1$.

$\boxed{f(x)=y \quad f^{-1}(y)=x \quad f^{-1}(y)=y-1}$

This function has an inverse because it is one-to-one correspondence. Suppose y is the image of ... Then $x = y-1$

correspondence. Suppose y is the image of x such that $y = x+1$. Then, $x = y-1$. This means that $y-1$ is the unique element of $\mathbb{Z}$ that is sent to y by f. Consequently, $f^{-1}(y) = y-1$.

Prob.

$f: R \to R$ and $f(x) = x^2$.

$f(-2) = f(2) = 4$, so f is not one-to-one.

Hence, f is not invertible

f: Set of nonnegative ~~integers~~ real numbers. to set of nonnegative integers

$f(x) = x^2 \longrightarrow$ invertible. $\to$ one-to-one. $\to$ onto

Prob.

$g: \{a, b, c\} \to \{a, b, c\}$

$g(a) = b$, $g(b) = c$, and $g(c) = a$

$f: \{a, b, c\} \to \{1, 2, 3\}$

$f(a) = 3$, $f(b) = 2$, and $f(c) = 1$.

$f(a)=3$, $f(b)=2$, and $f(c)=1$.

$$f\circ g(a) = f(g(a)) = f(b) = 2$$

$$f\circ g(b) = f(g(b)) = f(c) = 1$$

$$f\circ g(c) = f(g(c)) = f(a) = 3.$$

Note $g\circ f$ is not defined, because the range of f is not a subset of the domain of g.

<u>Prob:</u>

$f: Z \to Z$, $f(x) = 2x+3$

$g: Z \to Z$, $g(x) = 3x+2$

(i) $f\circ g(x) = f(g(x)) = f(3x+2)$
$= 2\cdot(3x+2)+3$
$= 6x+7.$

(ii) $g\circ f(x) = g(f(x)) = g(2x+3)$
$= 3(2x+3)+2$
$= 6x+11.$

~~function and its inverse, in~~

Composition of a function and its inverse, in either order, generates the identity function.

→ Let $f: A \to B$ is a one-to-one correspondence from the set A to the set B. The inverse function f^{-1} exists and is given as $f^{-1}: B \to A$.

Thus, if $f(a) = b$ then $f^{-1}(b) = a$.

$$(f^{-1} \circ f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a$$

$$(f \circ f^{-1})(b) = f(f^{-1}(b)) = f(a) = b.$$

Consequently, $f^{-1} \circ f = \iota_A$ and $f \circ f^{-1} = \iota_B$, where ι_A and ι_B are identity functions on the sets A & B respectively.

Prob: We need to determine the smallest integer that is at least as large as the quotient when 100 is divided by 8.

Consequently, $\lceil 100/8 \rceil = \lceil 12.5 \rceil = 13$. bytes are reqd.