B-TECH 3RD SEMESTER MID TERM EXAMINATION
DECEMBER 2020

SUBJECT: DISCRETE STRUCTURES [CS 2101]

Date of Examination: 22/12/2020
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Q1) If howard can swim across the lake (p), then Howard can swim to the island. (q)

If p, then $q \Rightarrow p \rightarrow q$

- Converse: $q \rightarrow p$

  $\therefore$ If Howard can swim to the island, then Howard can swim across the lake.

  Inverse: $\neg p \rightarrow \neg q$

  $\therefore$ If howard cannot swim across the lake, then Howard cannot swim to the island

2) A compound proposition that is always false is called a contradiction, and the one which is neither a tautology nor a contradiction is called a Contingency

3) d) $\forall x \, [C(x) \rightarrow F(x)]$

4) i) Every Computer Science Student needs a course in discrete mathematics.

let $C(x) \Rightarrow x$ is a computer Science Student
$D(x) \Rightarrow x$ needs a course is Discrete Mathematics

$\therefore \forall x \, [C(x) \rightarrow D(x)]$

domain of x: All person in the world.

4) ii) " There is a student in this class who has taken at least one course in Computer Science

let $C(x) \rightarrow x$ is a student in this class

~~$CS(y) \rightarrow$~~

$CS(x,y) \rightarrow x$ has taken course y in Computer Science.

$\therefore \exists x \; \exists y \; [\, C(x) \wedge CS(x,y) \,]$

[domain of x: ~~Pee~~ Set of all people, domain of y: Subjects of Computer Science]

5) let

$ah \rightarrow$ andy works hard

$adb \rightarrow$ andy is a dull boy

$aj \rightarrow$ andy will not get job

Premise	Conclusion
i) ah	aj
ii) $ah \rightarrow adb$	
iii) $adb \rightarrow aj$	

Steps	Reason
1) ah	Premise (i)
2) $ah \rightarrow adb$	Premise (ii)
3) adb	Modus Ponens on 1 and 2
4) $adb \rightarrow aj$	Premise (iii)
5) aj	Modus Ponens on 3 and 4

6) given $f: A \to B$

→ A contains m element

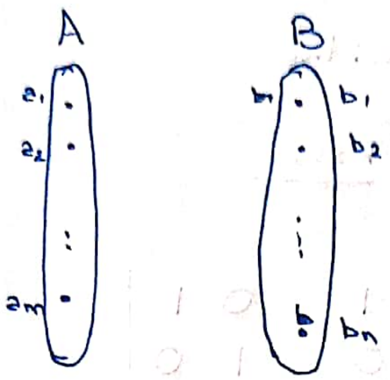
→ B contains n element

→ $m \leq n$

→ To find no. of one-to-one functions

→ Let A have elements $a_1, a_2, \dots a_m$

→ Let B have elements $b_1, b_2, \dots b_n$



→ To find out number of one-to-one function from A to B, we do the following

→ for $f(a_1)$, we have n elements from B to chose from

→ for $f(a_2)$, we have n-1 element from B to chose from, as 1 element has been assigned to $f(a_1)$

→ similarly $f(a_3)$ will have $n-2$ options to choose from

→ following this trend, $f(a_m)$ will have $(n-m+1)$ ~~choise~~ choices

∴ No. of one-to-one functions:

$= n(n-1)(n-2) \dots (n-m+1)$

7) a) We know that an equivalence Relation is reflexive, anti-symmetric and transitive

∴ Statement is False

b) We know that a Partially ordered Relation is, Reflexive Anti-Symmetric and Transitive.

∴ Statement is False

8) given $\cancel{R}\ M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

→ This ~~matrix~~ Relation is not reflexive as all ~~of~~ the elements of the main diagonal ~~e~~ are not 1.

→ This Relation is not irreflexive as all the elements of the main diagonal are not 0

→ $M_R^T = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} = \cancel{R}\ M_R$

→ as $\cancel{R}\ M_R^T = M_R$, the Relation is Symmetric

→ This relation is not antisymmetric as

$M_R[2][3] = \cancel{R}.\ M_R[3][2] = 1$

$\rightarrow M_{RoR} = M_R \odot M_R$

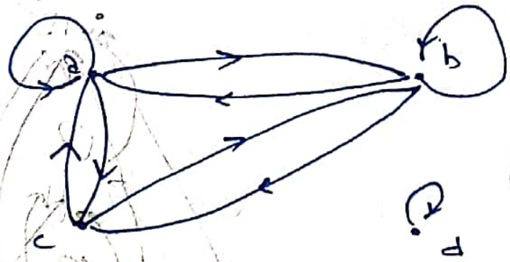
$$= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore \cancel{R}\ M_{RoR} \neq M_R$

$\therefore$ Relation is not Transitive

9) given ~~the~~ Relation in form of Digraph



→ This relation is not reflexive as c doesn't have a loop

→ This relation is symmetric as ~~if any element (k,j) exist~~, it there exist an ~~to~~ "arrow" from an element to ~~th~~ other element, it's reverse "arrow" also exists

→ This relation is not anti symmetric, as ~~to~~ (a, b) and (b, a) exists, but a ≠ b

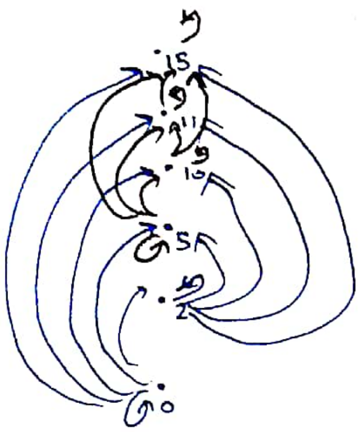
→ This relation is not Transitive as (c, b) and (b, c) exists, but not (c, c).

10) ~~To draw~~ To draw Hasse diagram of "less than or equal to" relation on $\{0, 2, 5, 10, 11, 15\}$

let $S = \{0, 2, 5, 10, 11, 15\}$

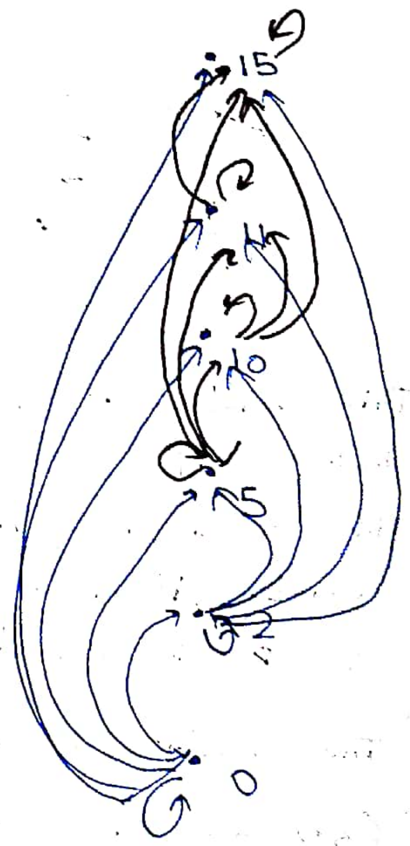
$\therefore R : S \to S = \{(0,0), (0,2), (0,5), (0,10), (0,11), (0,15), (2,2), (2,5), (2,10), (2,11), (2,15), (5,5), (5,10), (5,11), (5,15), (10,10), (10,11), (10,15), (11,11), (11,15), (15,15)\}$

drawing digraph of R

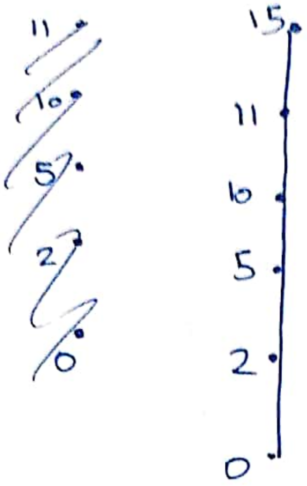


(drawn with different color, so as to make cleanliness)

(drawing Bigger Graph)



→ Removing all loops, ~~rem~~ removing directions, and removing all Transitive Arrows, we get



→ maximal element ~~&~~ = {15}

→ greatest element = {15}

→ minimal element = {0}

→ least element = {0}