SIGNAL & SYSTEM

ASSIGNMENT - 2

GROUP MEMBERS	**ENROLLMENT NO.**
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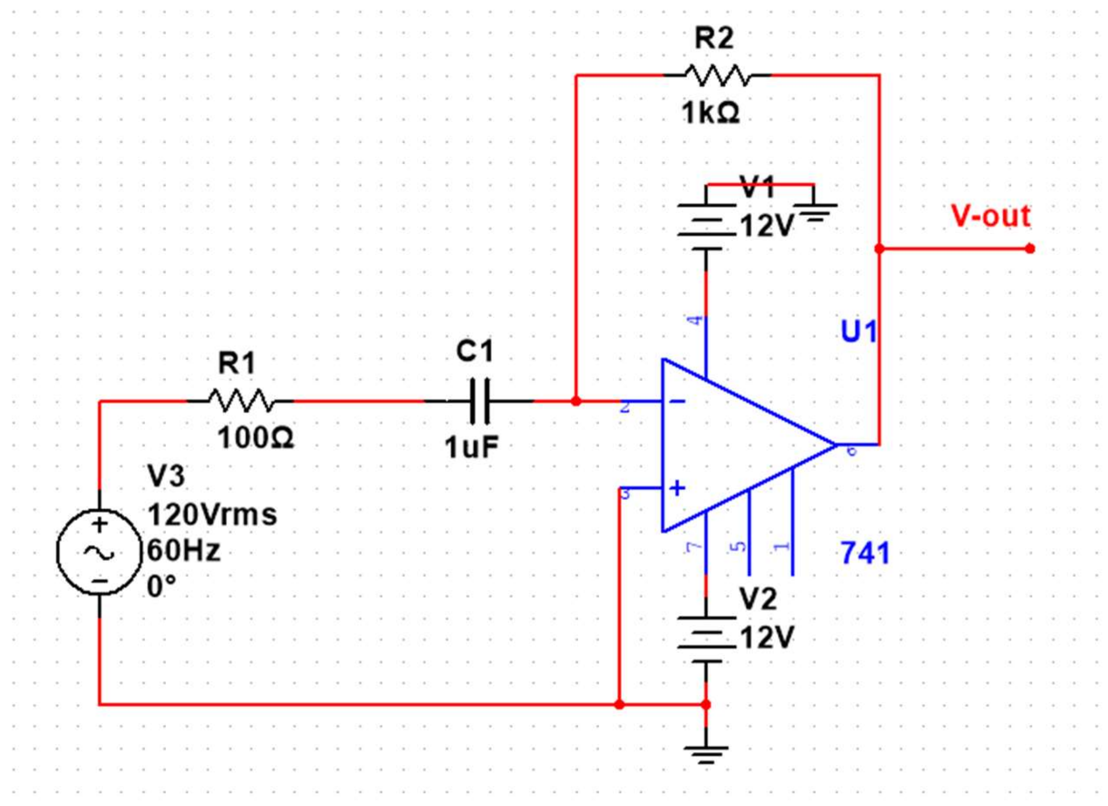
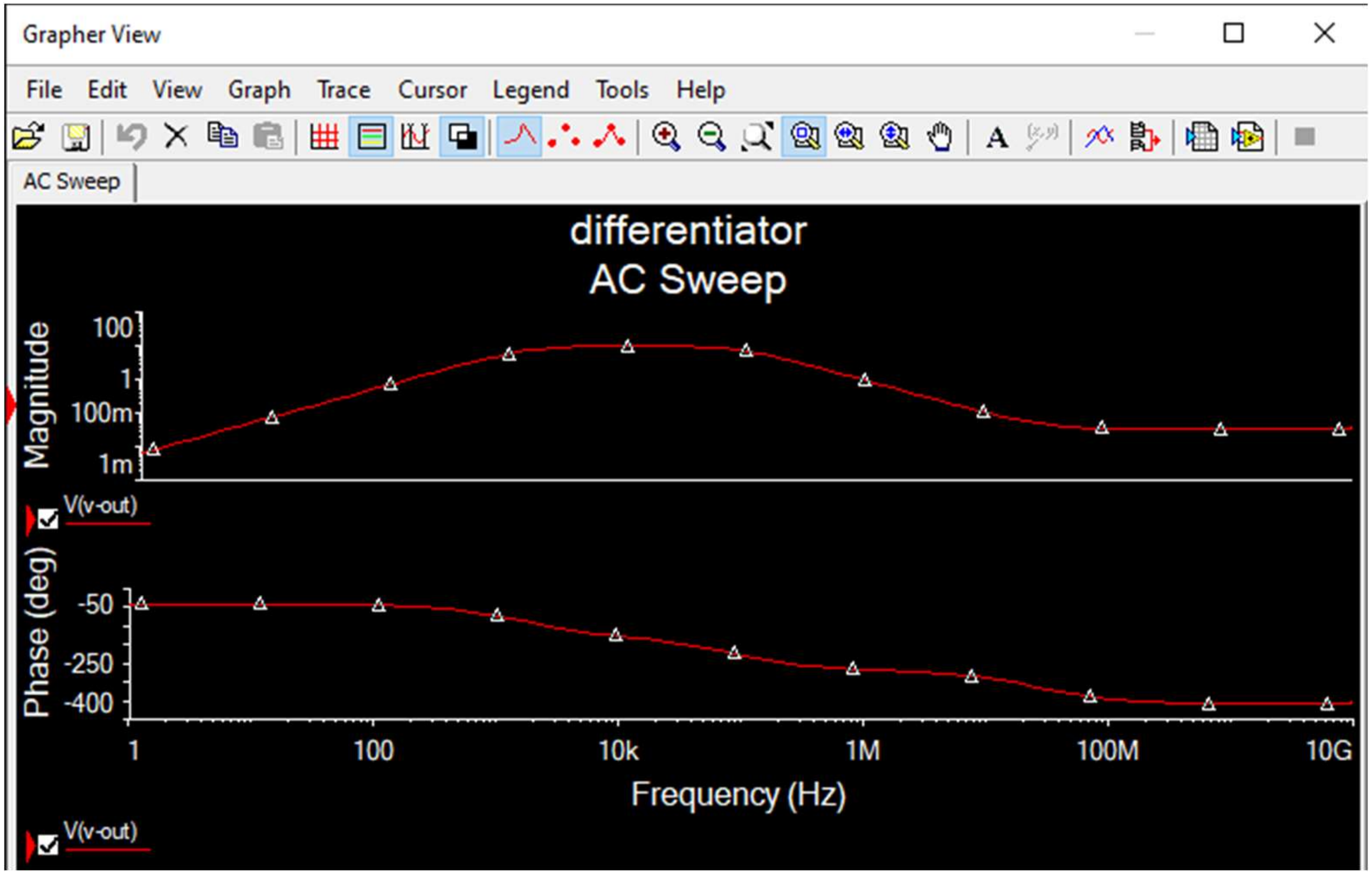
A. Perform frequency domain analysis for each of the implemented opamp circuits with R and/or C placed in both forward and feedback paths by giving a sweep from 10 Hz to 100 MHz. Choose component dimensions carefully and use spectrum analyzer, Bode plotter etc and also means to obtain Fourier coefficients. Try to explain the frequency domain behaviour from first principles.

- **DIFFERENCIATOR:**
- CIRCUIT:

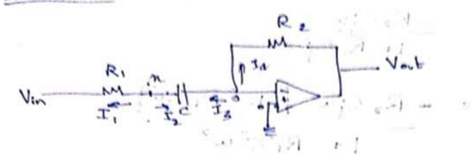


GRAPHER VIEW:



EXPLANATION:

differentiator



a) Current law in x.

$$I_1 + I_2 = 0$$

$$\frac{x - V_{in}}{R_1} + C \frac{d(x-0)}{dt} = 0$$

or $$\frac{x - V_{in}}{R_1} + C\frac{dx}{dt} = 0$$

or $$\frac{x(j\omega) - V_{in}(j\omega)}{R_1} + Cj\omega \, x(j\omega) = 0$$

or $$x(j\omega)\left[\frac{1}{R_1} + Cj\omega\right] = \frac{V_{in}(j\omega)}{R_1}$$

or $$x(j\omega) = \frac{V_{in}(j\omega)}{1 + R_1 C j\omega}$$

b) $I_4 + I_3 = 0$

$$\frac{0 - V_{out}}{R_2} + C\frac{d(0-x)}{dt} = 0$$

or $$-\frac{V_{out}}{R_2} - \frac{C\,dx}{dt} = 0$$

or $$-\frac{V_{out}(j\omega)}{R_2} - Cj\omega \, x(j\omega) = 0$$

or $$\frac{-V_{out}(j\omega)}{R_2} - \frac{Cj\omega \, V_{in}(j\omega)}{1 + R_1 C j\omega} = 0$$

or $$V_{out}(j\omega) = \frac{-R_2 C j\omega}{1 + R_1 C j\omega} \, V_{in}(j\omega)$$

$$H(j\omega) = \frac{-R_2 C j\omega}{1 + R_1 C j\omega}$$

$$= \frac{-R_2 C j\omega \left[1 - R_1 C j\omega\right]}{1 + R_1^2 C^2 \omega^2}$$

$$= \frac{-R_2 C j\omega - R_1 R_2 C^2 \omega^2}{1 + R_1^2 C^2 \omega^2}$$

$$= -\left[\frac{R_1 R_2 C^2 \omega^2 + R_2 C j\omega}{1 + R_1^2 C^2 \omega^2}\right]$$

$$|H(j\omega)| = \frac{\sqrt{R_1^2 R_2^2 C^4 \omega^4 + R_2^2 C^2 \omega^2}}{1 + R_1^2 C^2 \omega^2}$$

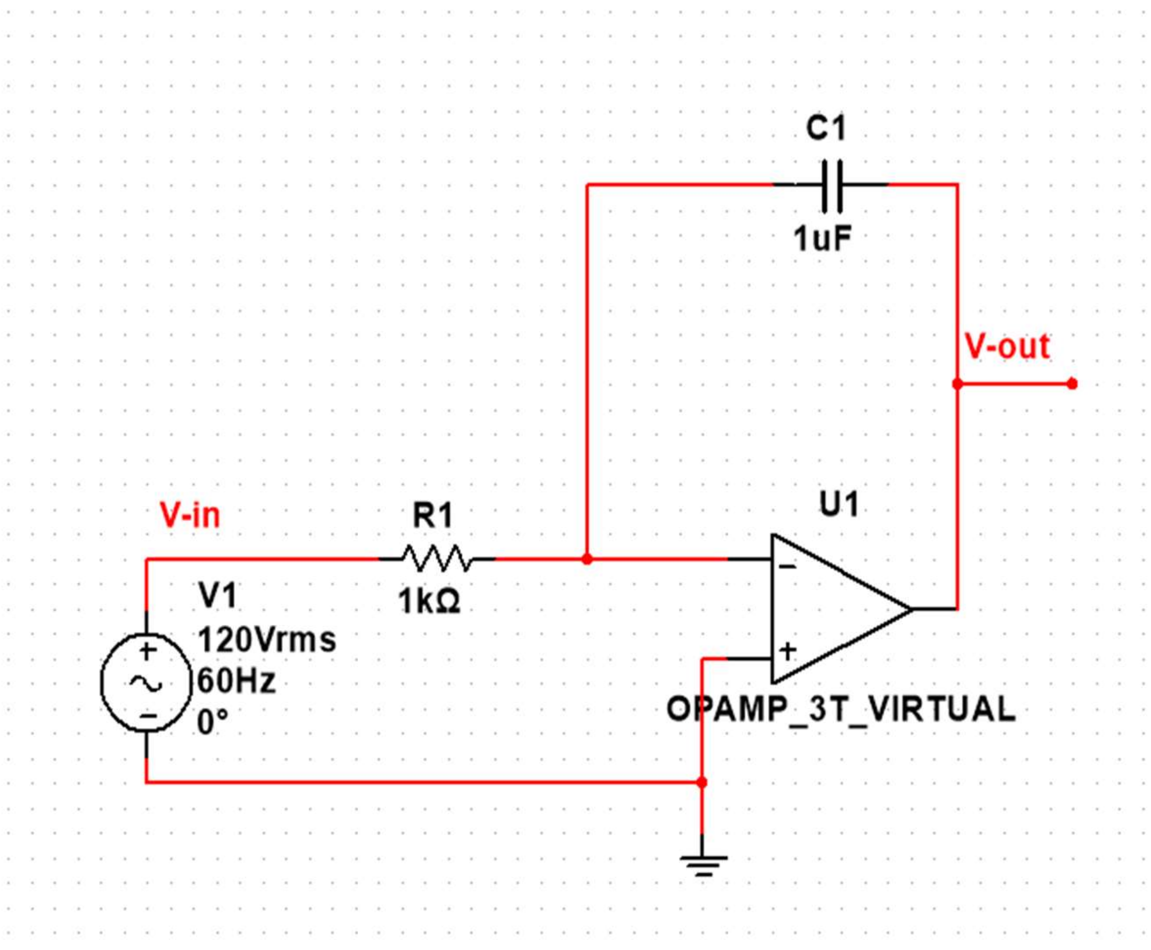
$$\angle H(j\omega) = -\tan^{-1}\left[\frac{R_2 C \omega}{R_1 R_2 C^2 \omega^2}\right]$$

$$= -\tan^{-1}\left[\frac{1}{R_1 C \omega}\right]$$

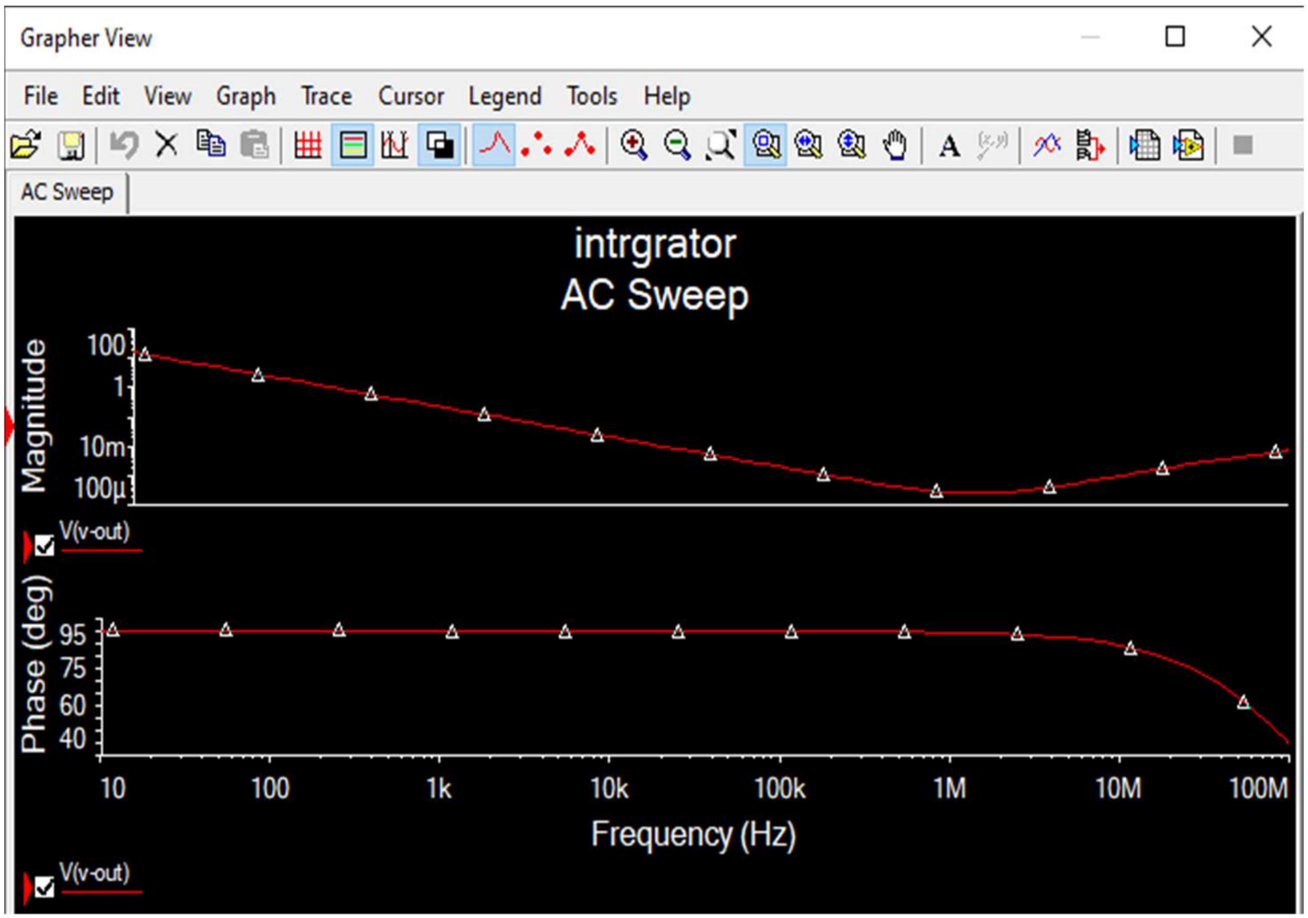
$$= -\tan^{-1}\left[\frac{1}{R_1 C \omega}\right]$$

INTEGRATOR:

- CIRCUIT:

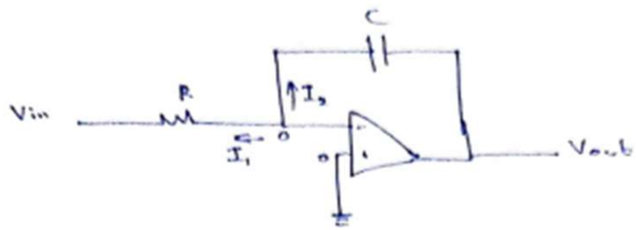


GRAPHER VIEW:



EXPLANATION:

Integrator



→ Kirchoff's

$$\frac{0 - V_{in}}{R} + C\frac{d(0 - V_{out})}{dt} = 0$$

$\cancel{-V_{in}}$

fourier

$$-\frac{V_{in}(j\omega)}{R} + -Cj\omega\, V_{out}(j\omega) = 0.$$

$$V_{out}(j\omega) = -\frac{V_{in}(j\omega)}{RCj\omega}$$

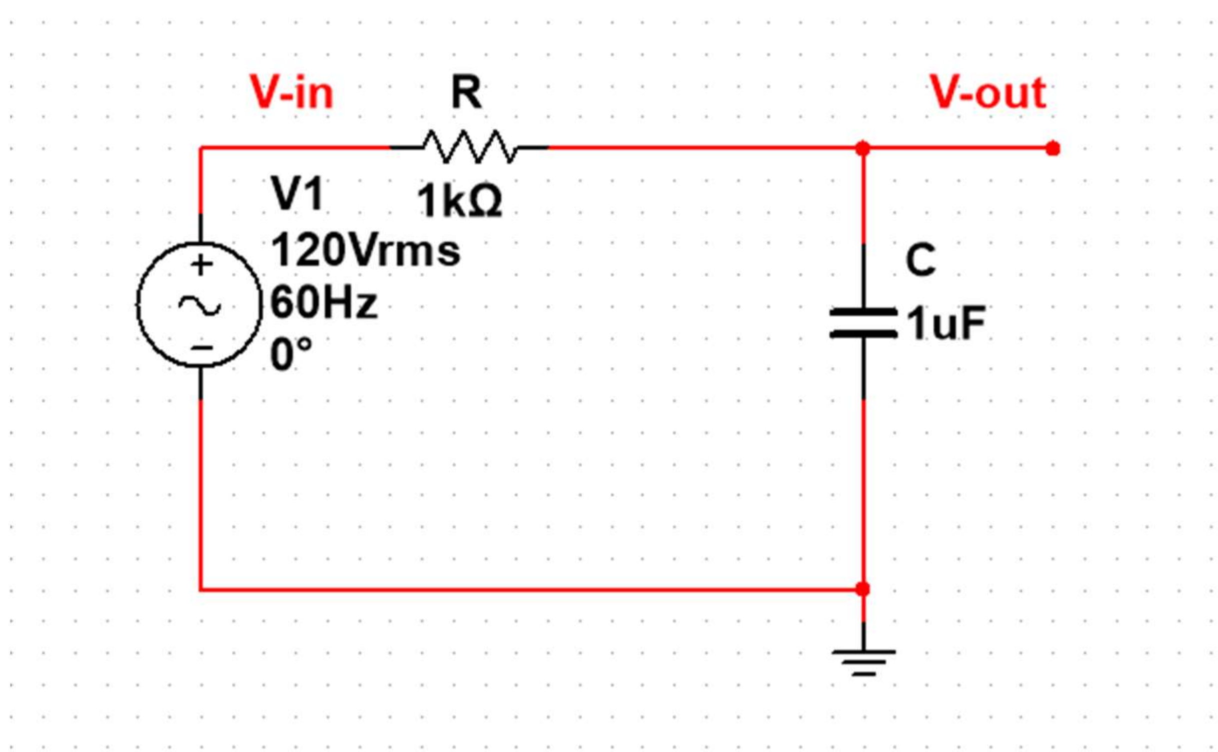
$$\therefore H(j\omega) = \frac{-1}{RCj\omega} = \frac{j}{RC\omega}$$

$$\therefore |H(j\omega)| = \frac{1}{RC\omega}$$

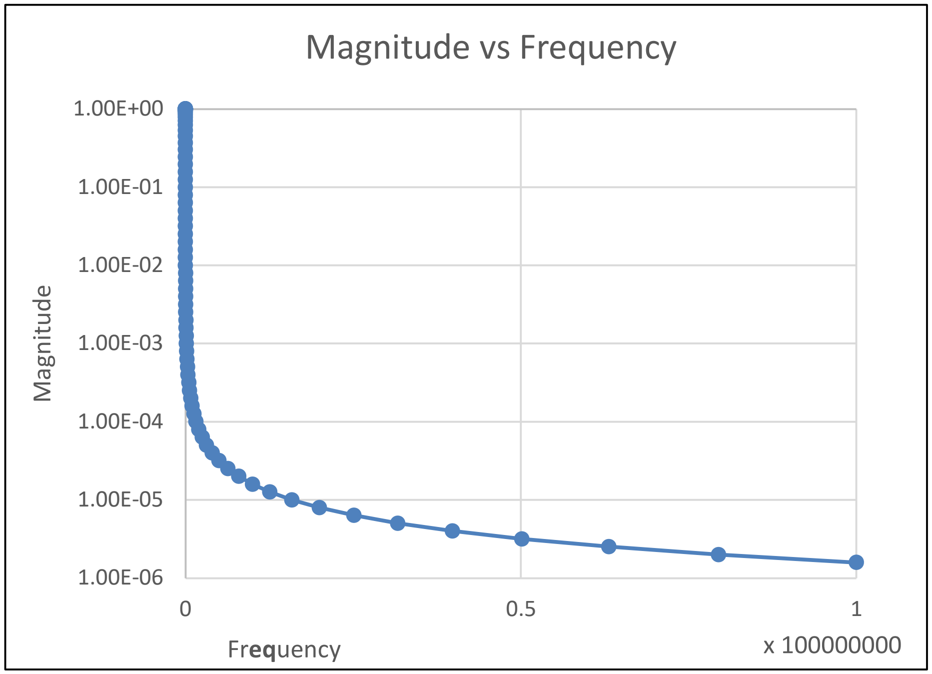
$$\angle H(j\omega) = \frac{\pi}{2}$$

B. Plot the magnitude vs frequency and phase vs frequency from theoretical circuit analysis and compare with the obtained frequency response. Implement the obtained formula in excel or write C code to suitable for Bode plot. Comment on the match and departure between the theoretically obtained response and that obtained from the simulation environment.

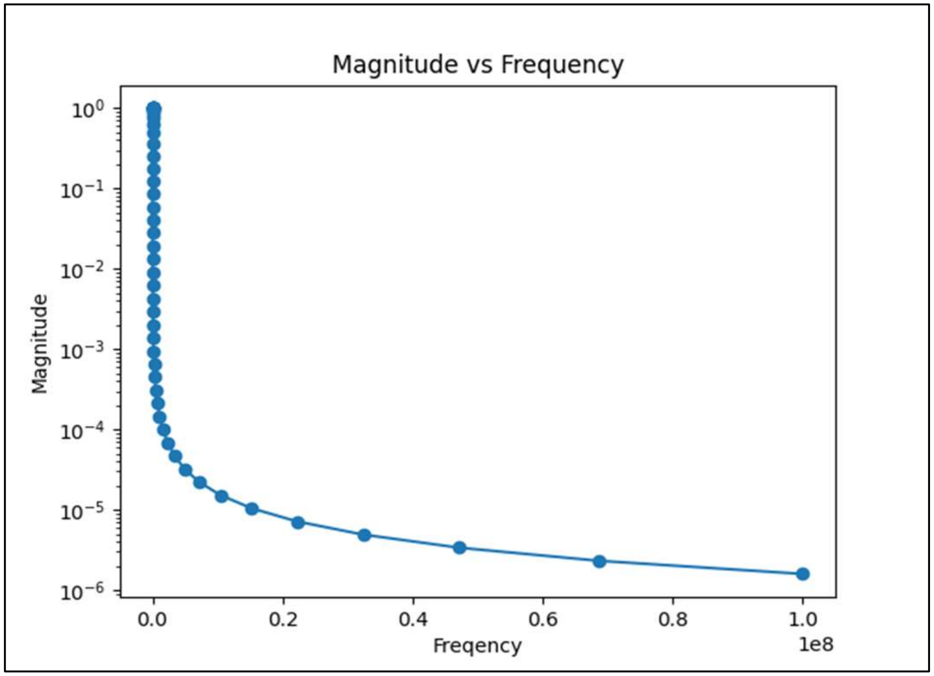
- CIRCUIT:



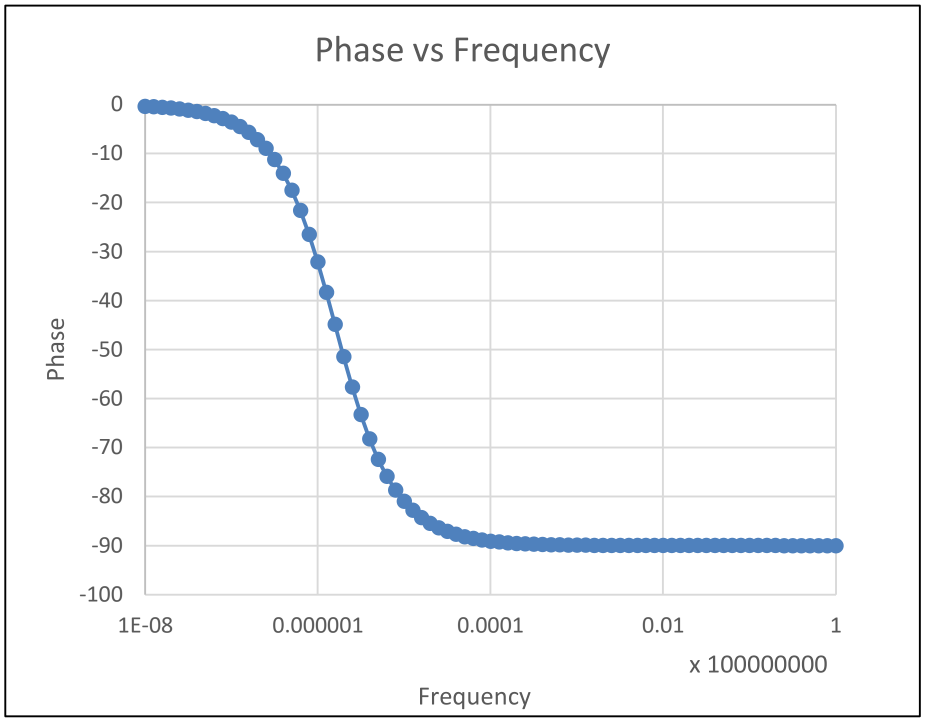
GRAPHS:



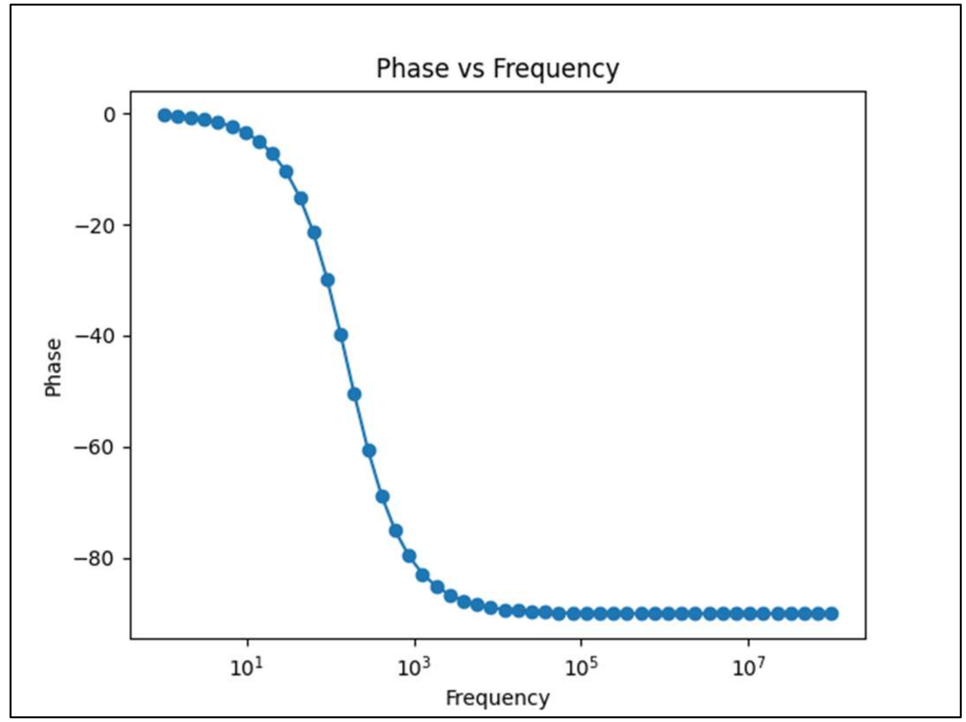
Exported from Simulator



Plotted By Code



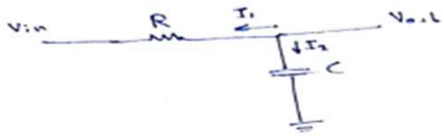
Exported from Simulator



Plotted By Code

EXPLANATION:

Low Pass Filter



using Kirchoff's Law

$$\frac{V_{out} - V_{in}}{R} + C\frac{d(V_{out}-0)}{dt} = 0$$

$$\frac{V_{out} - V_{in}}{R} + C\frac{dV_{out}}{dt} = 0$$

fourier: $$\frac{V_{out}(j\omega) - V_{in}(j\omega)}{R} + Cj\omega\, V_{out}(j\omega) = 0$$

or $$V_{out}(j\omega)\left[\frac{1}{R} + Cj\omega\right] = \frac{V_{in}(j\omega)}{R}$$

or $$V_{out}(j\omega) = \frac{V_{in}(j\omega)}{1+RCj\omega}$$

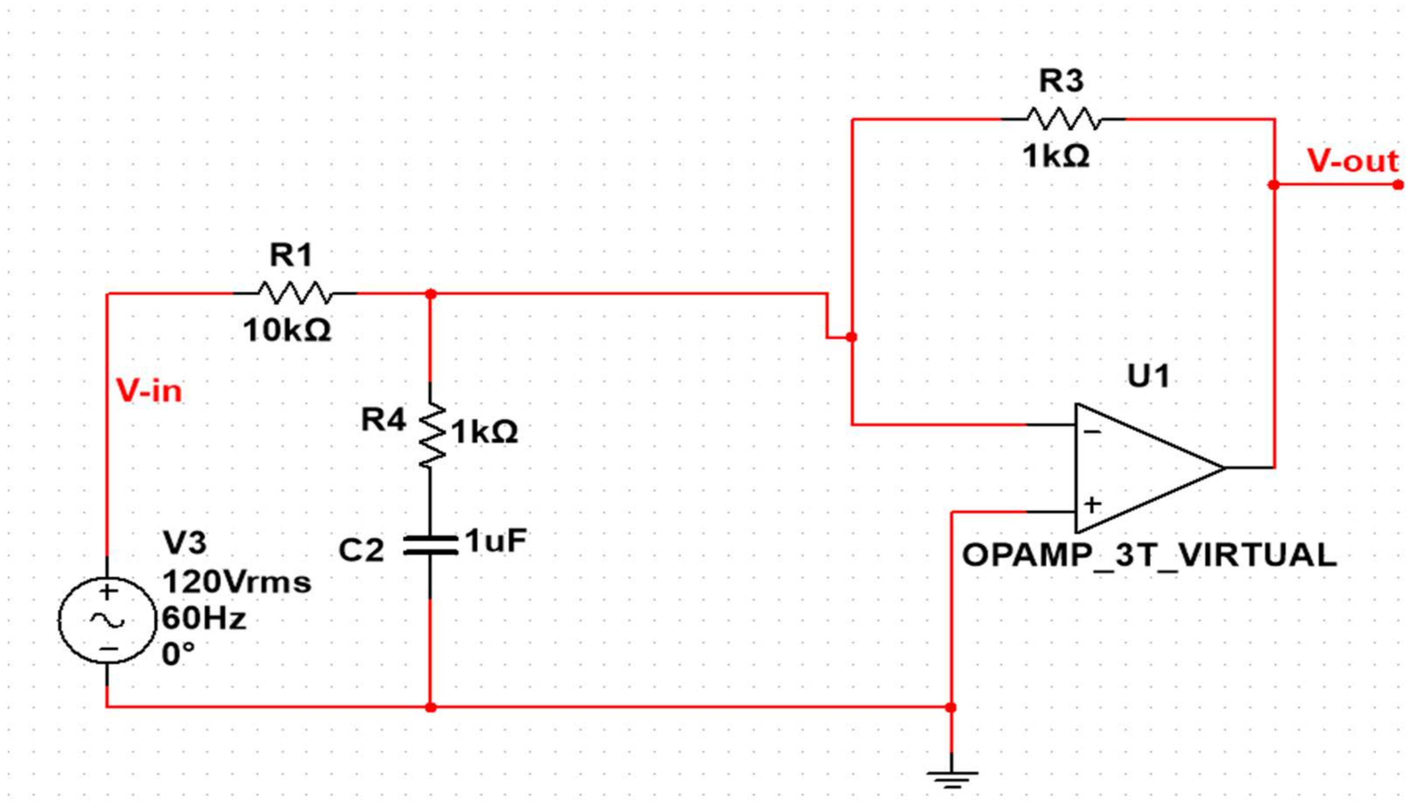
$$\therefore H(j\omega) = \frac{1}{1+RCj\omega} = \frac{1-RCj\omega}{1+R^2C^2\omega^2}$$

$$\therefore |H(j\omega)| = \frac{\sqrt{1+R^2C^2\cancel{j}\,\omega^2}}{1+R^2C^2\omega^2} = \frac{1}{\sqrt{1+R^2C^2\omega^2}}$$

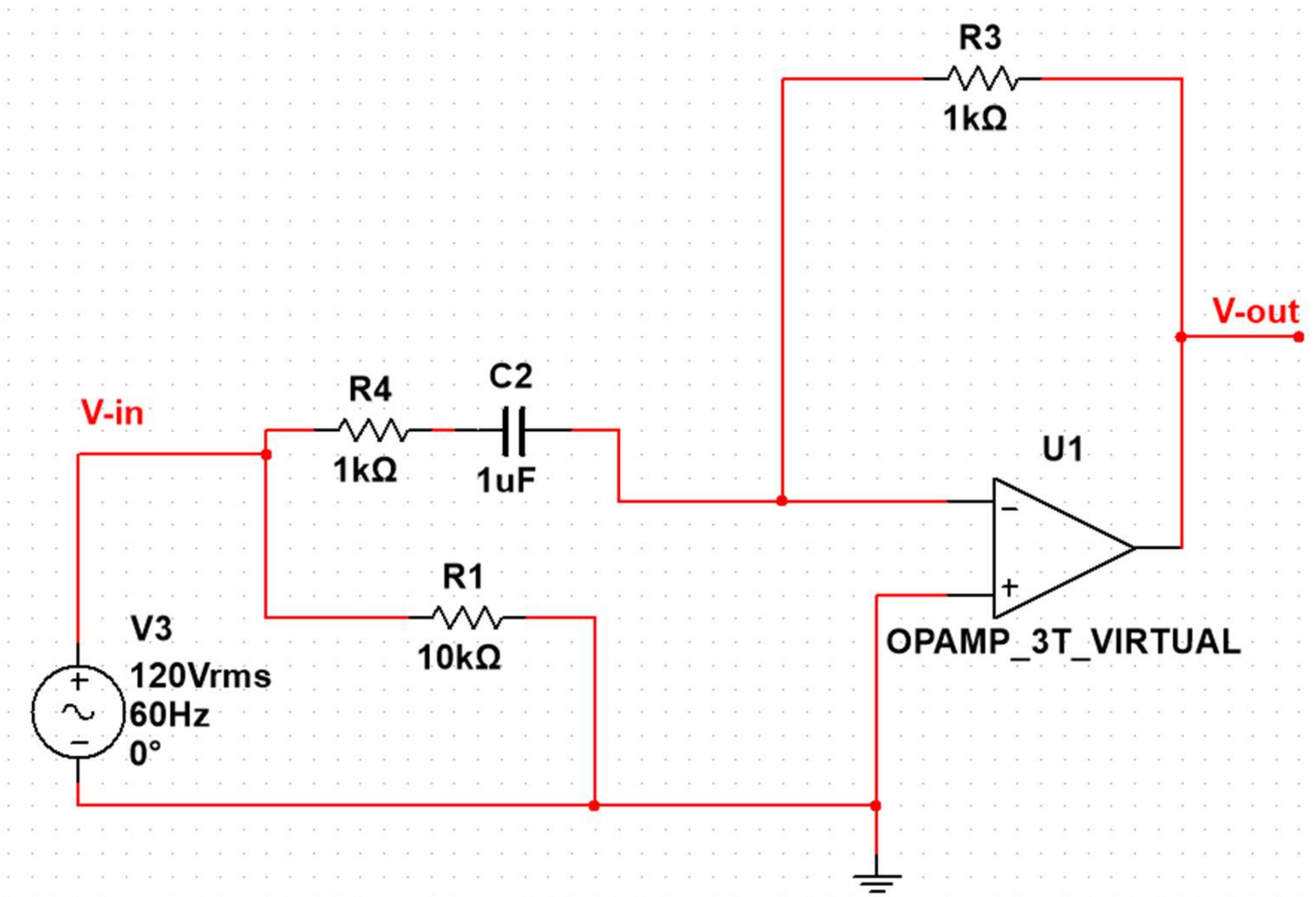
$$\angle H(j\omega) = \tan^{-1}\left[-RC\cancel{j}\,\omega\right] = -\tan^{-1}\left[RC\omega\right]$$

C. Demonstrate how the frequency response of PID circuit resembles behaviour of lowpass, highpass and bandpass filters. Note the range and nature of passband frequency zone. Note the slope with which the response rises or falls from passband to stopband. Establish the relation between circuit components and gain, phase, slope of the frequency response behaviour.

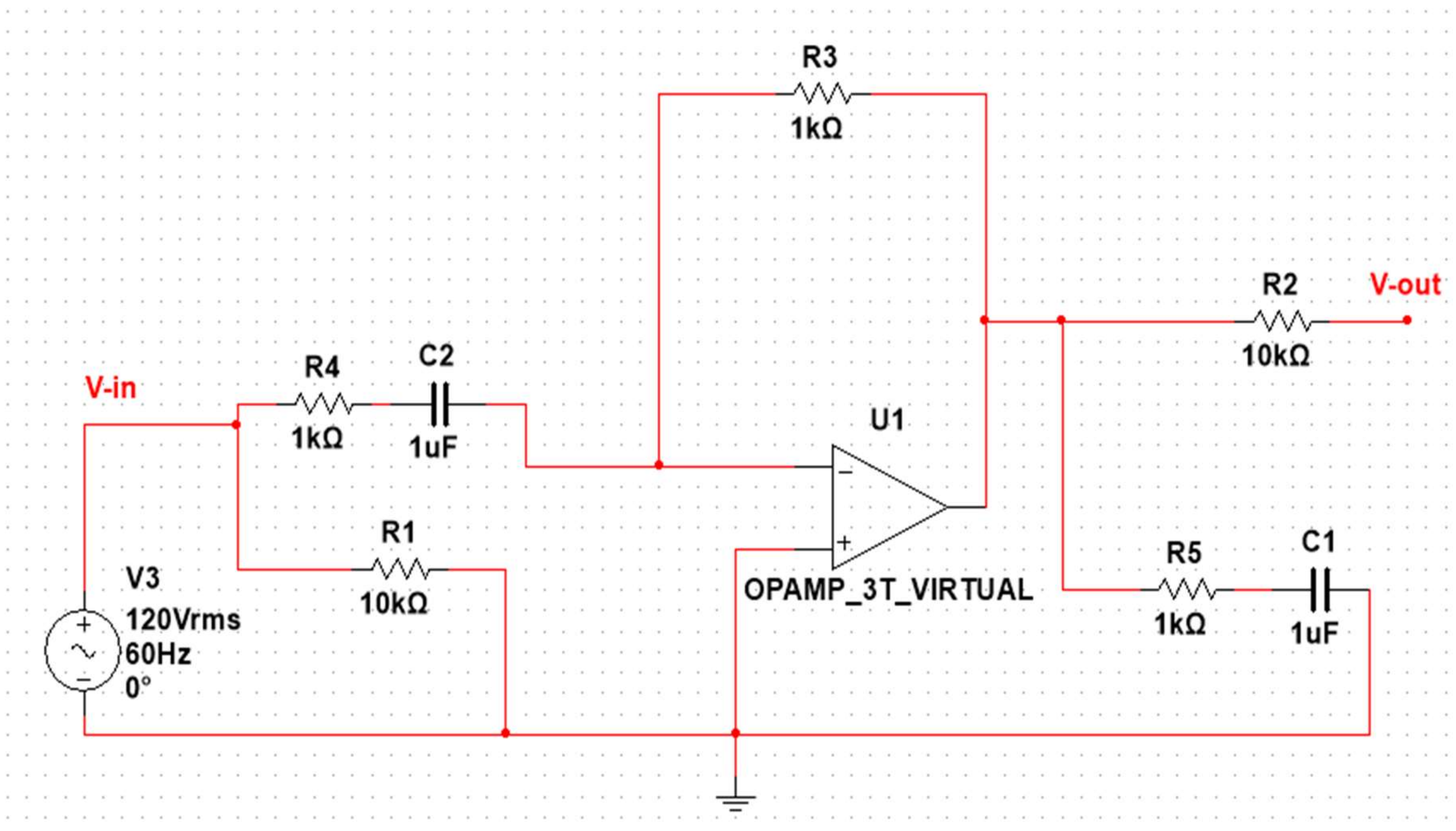
- CIRCUIT:(LOW PASS)



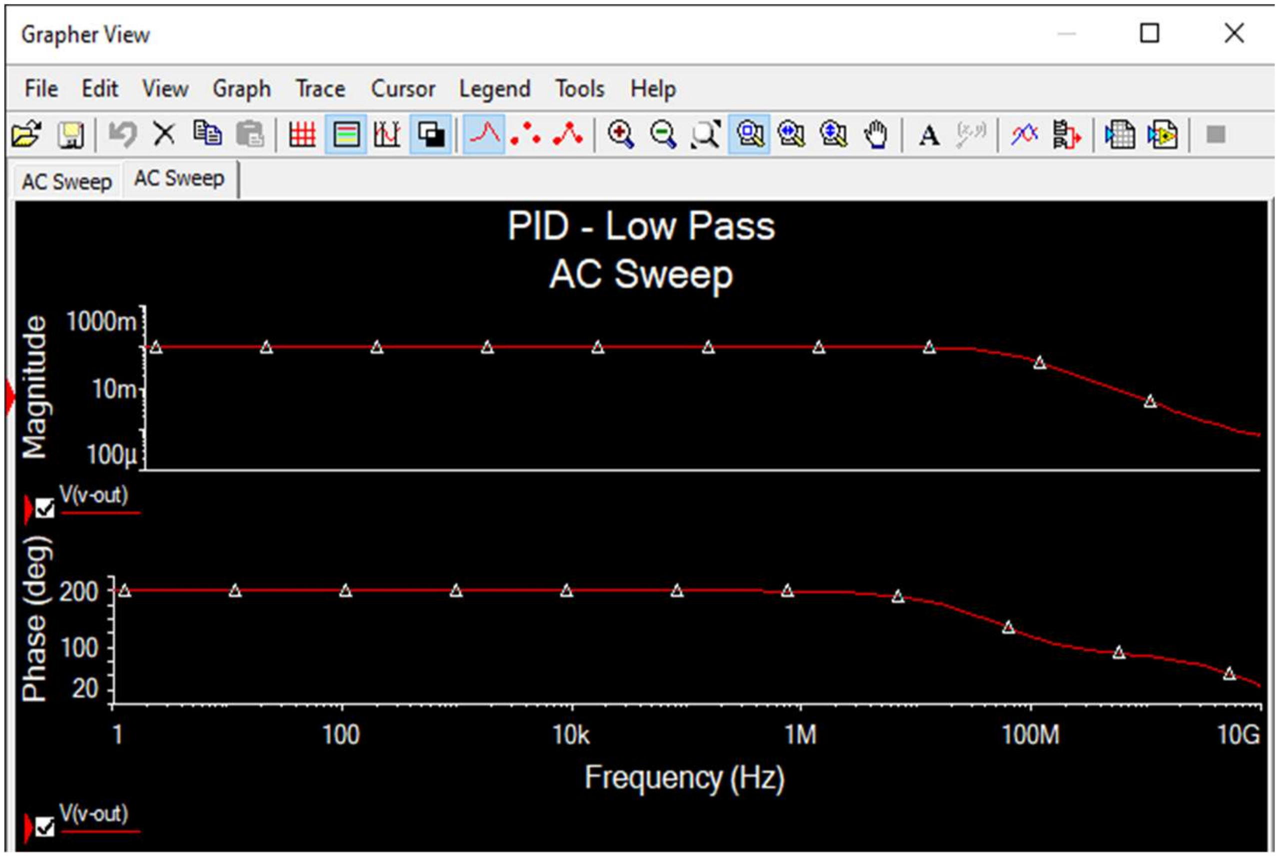
CIRCUIT:(HIGH PASS)



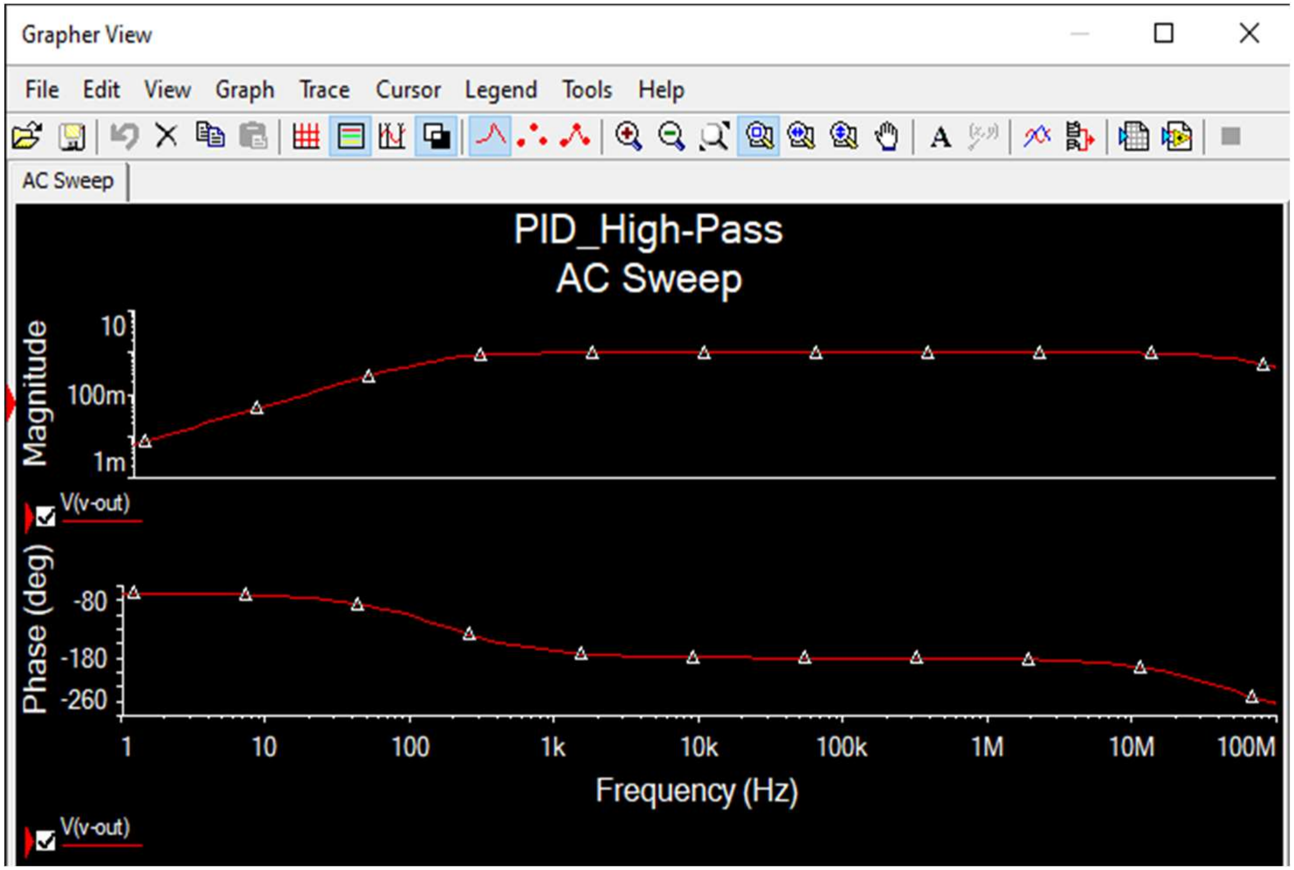
CIRCUIT:(BAND PASS)



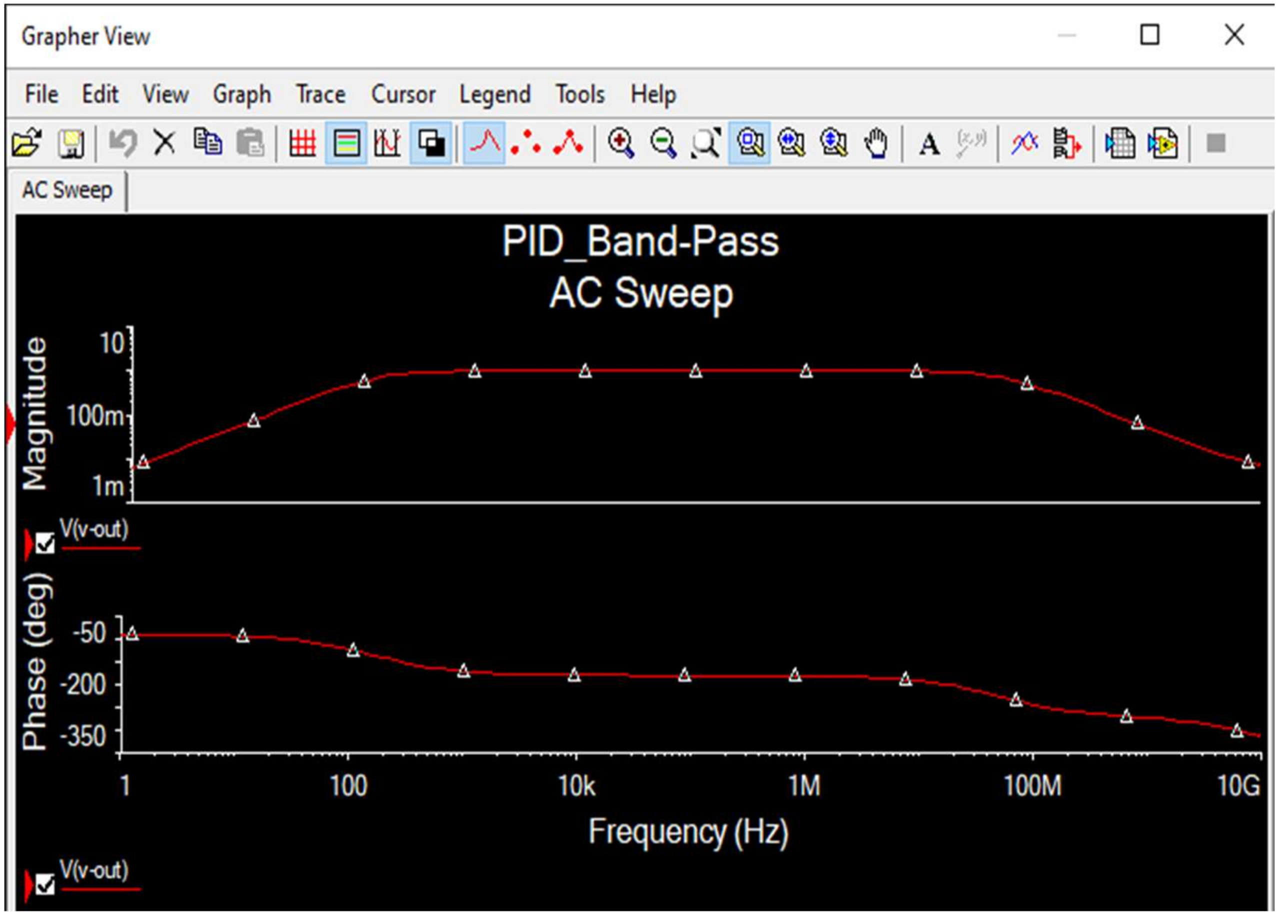
GRAPHER VIEW:(LOW PASS)



GRAPHER VIEW:(HIGH PASS)

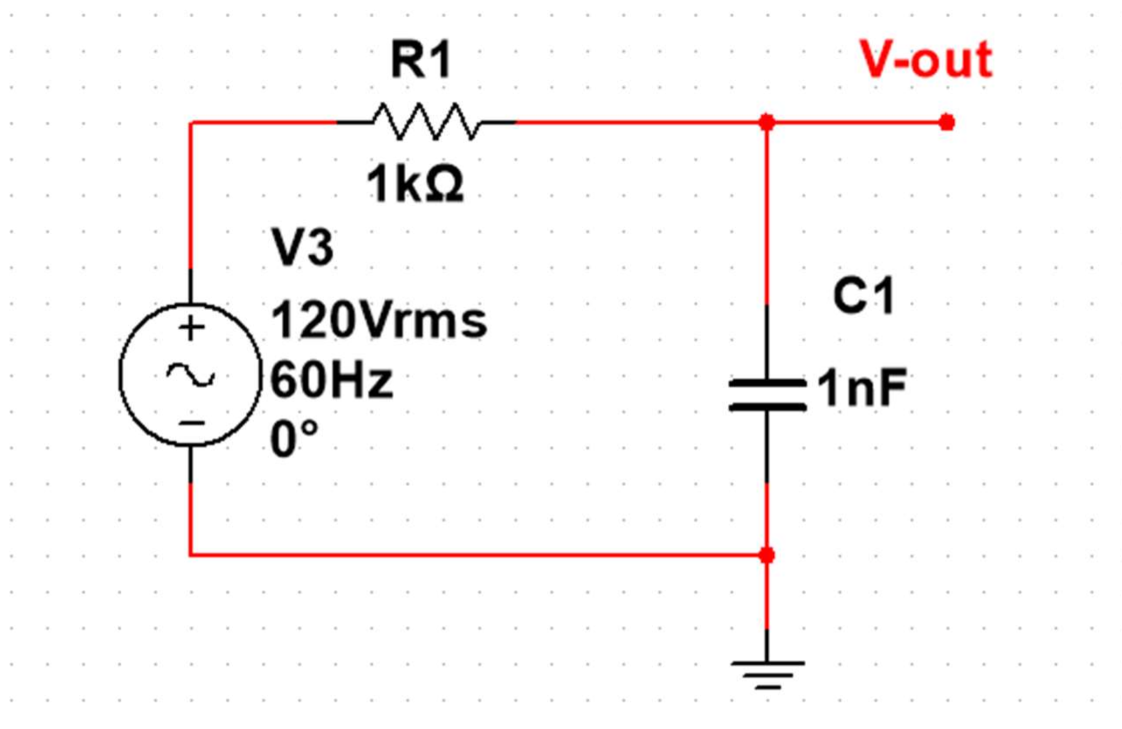


GRAPHER VIEW:(BAND PASS)

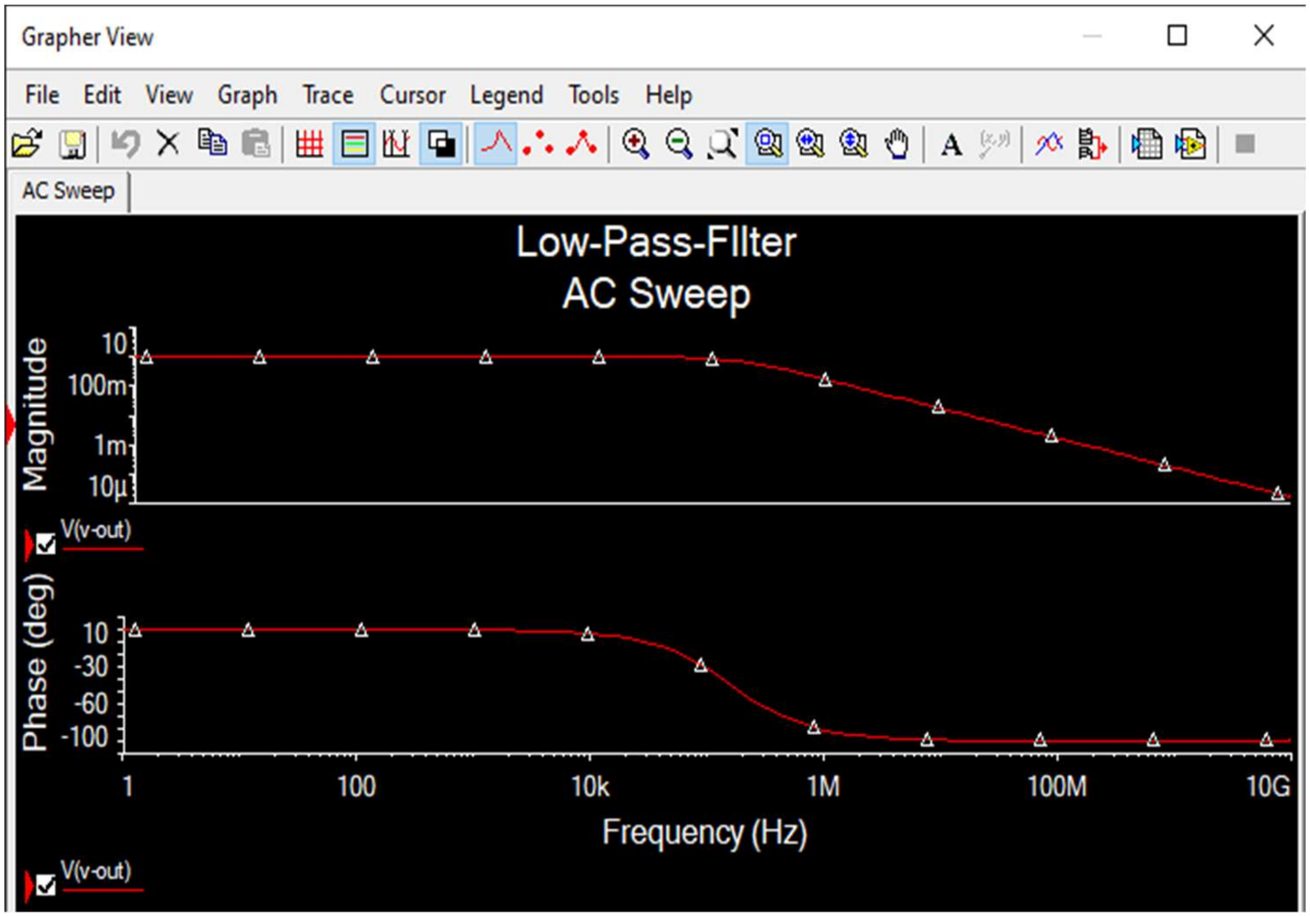


D. Show how the generic frequency shaping filters can be designed for desired cutoff frequency and report the corresponding frequency domain characteristics. Study and explore first the theoretical design and then implement by choosing the circuit components. You can try for lowpass, highpass and bandpass filters.

- CIRCUIT:

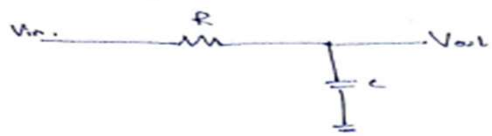


GRAPHER VIEW:



EXPLANATION:

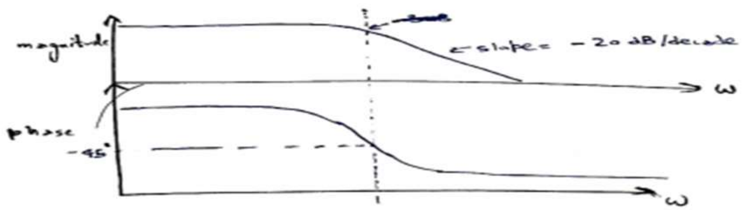
Low Pass Filter



$$H(j\omega) = \frac{1}{1 + RCj\omega}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + R^2C^2\omega^2}}$$

$$\angle H(j\omega) = -\tan^{-1}(RC\omega)$$



$\therefore$ cutoff frequency $\Rightarrow$ $-45^\circ = -\tan^{-1}(RC\omega)$

$$\therefore\ 1 = RC\omega_c$$

$$\text{or } 1 = RC\,2\pi f_c$$

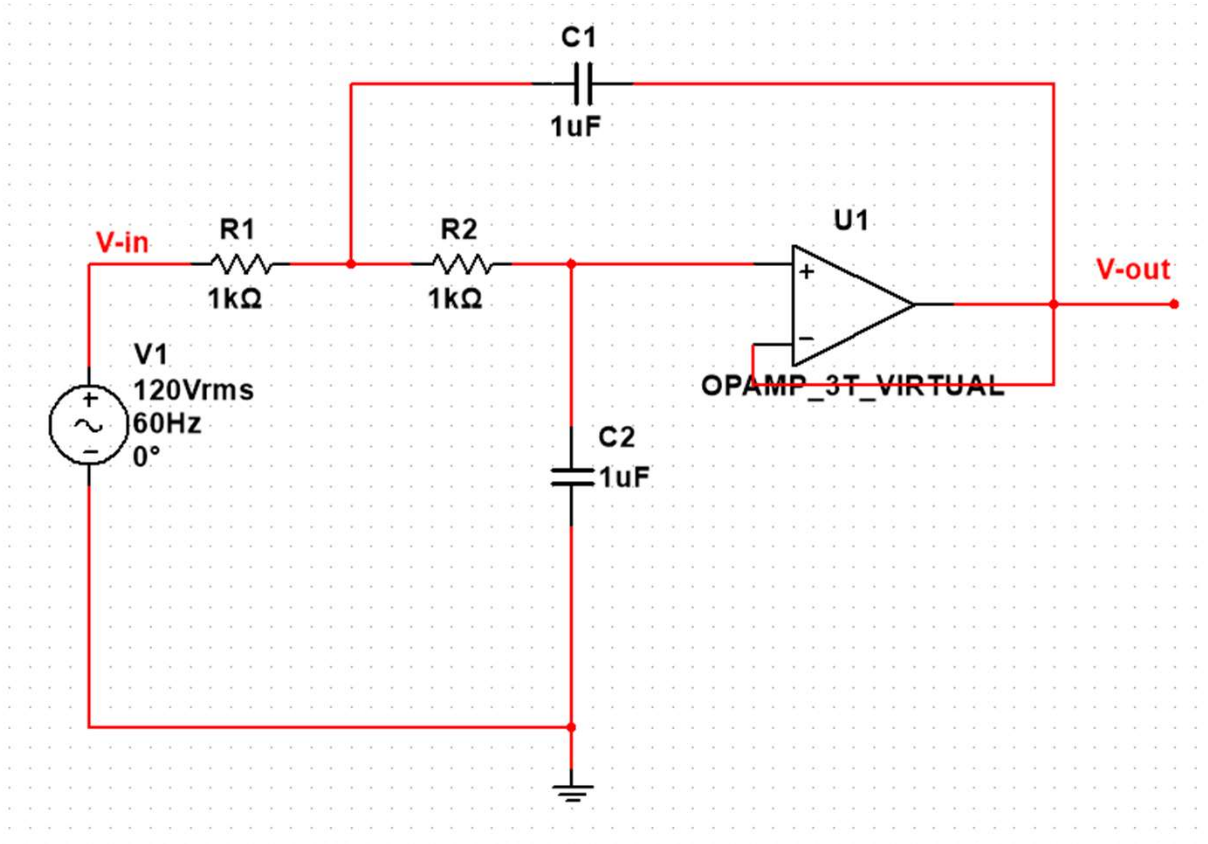
$$\therefore f_c = \frac{1}{2\pi RC}$$

$\therefore$ If we are given a cutoff frequency, we can find R & C

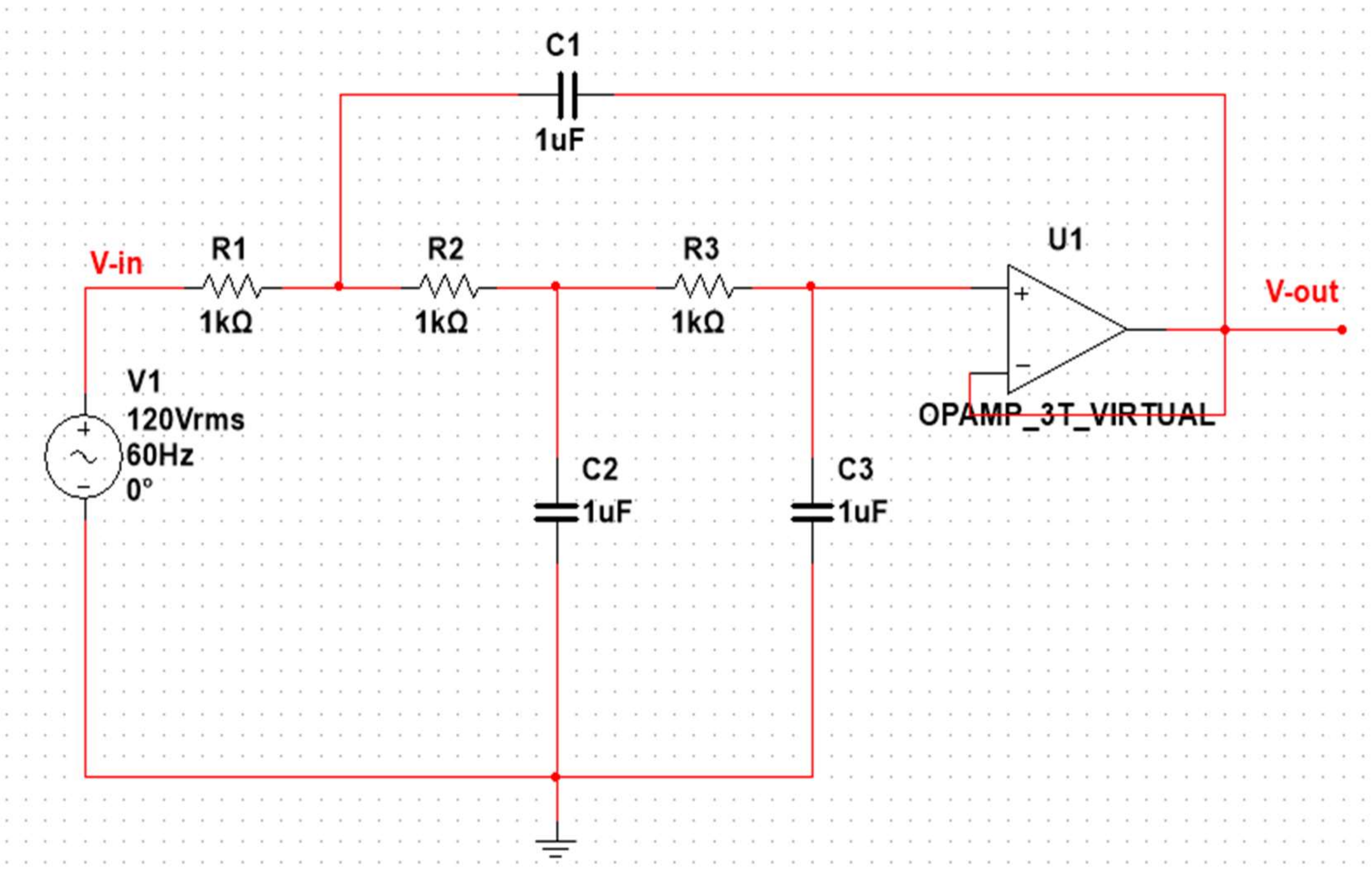
~~not~~ required for getting f_c

E. Increase the filter order for lowpass filter and demonstrate that the slope of cutoff zone indeed gets sharper with increase in order of the filters. Use Butterworth filter design approach.

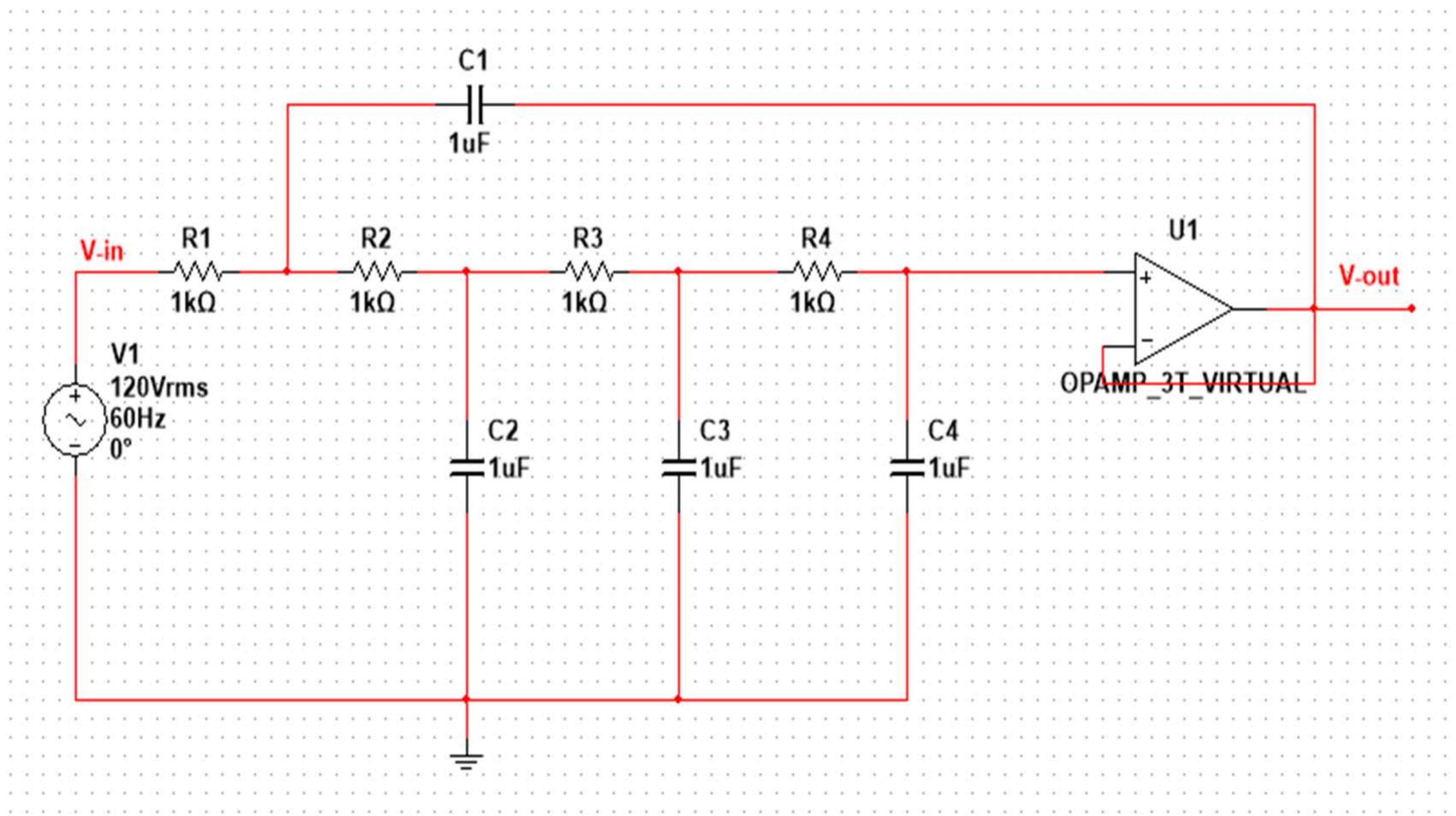
- CIRCUIT:(2ND ORDER)



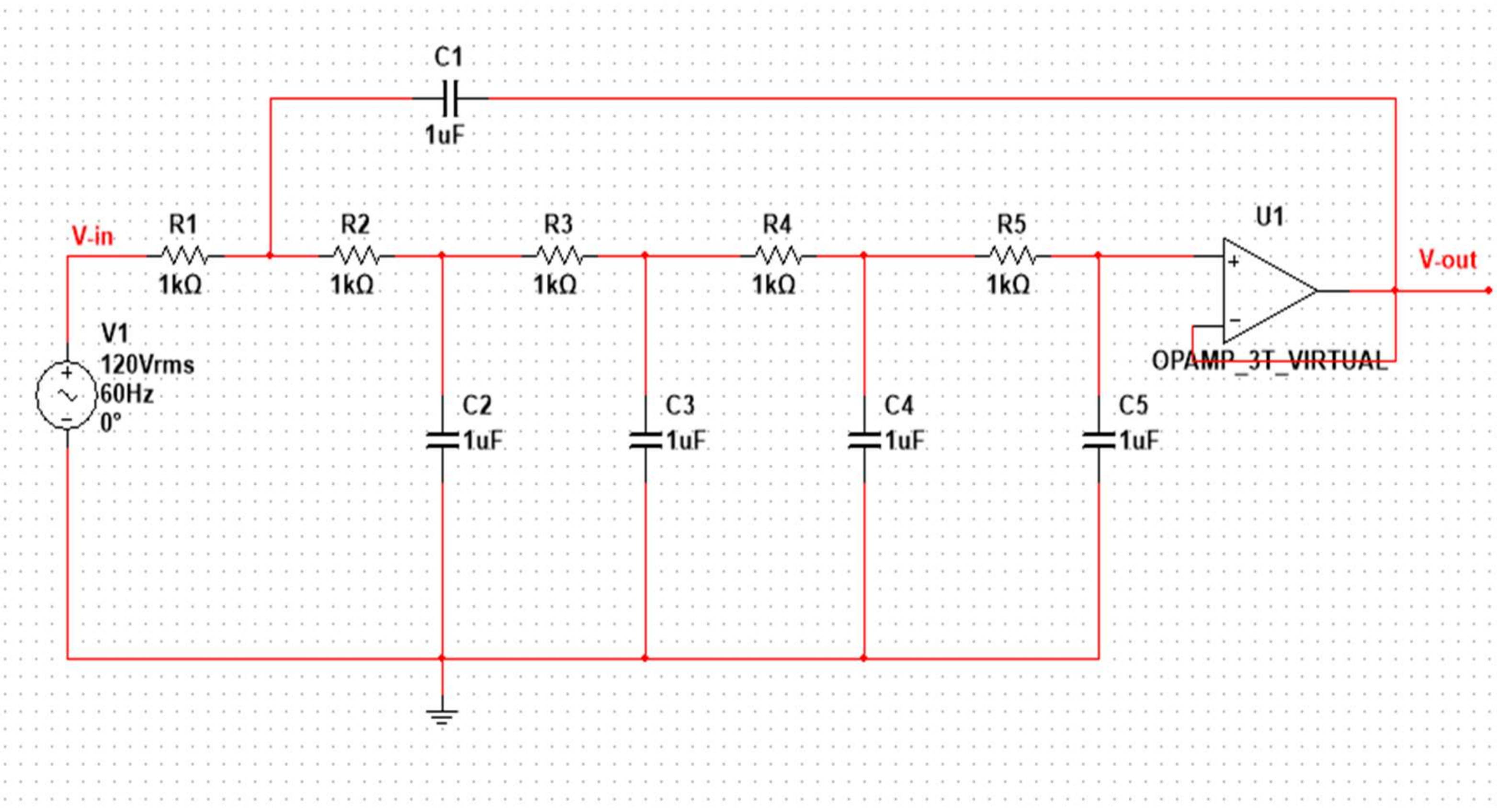
CIRCUIT:(3RD ORDER)



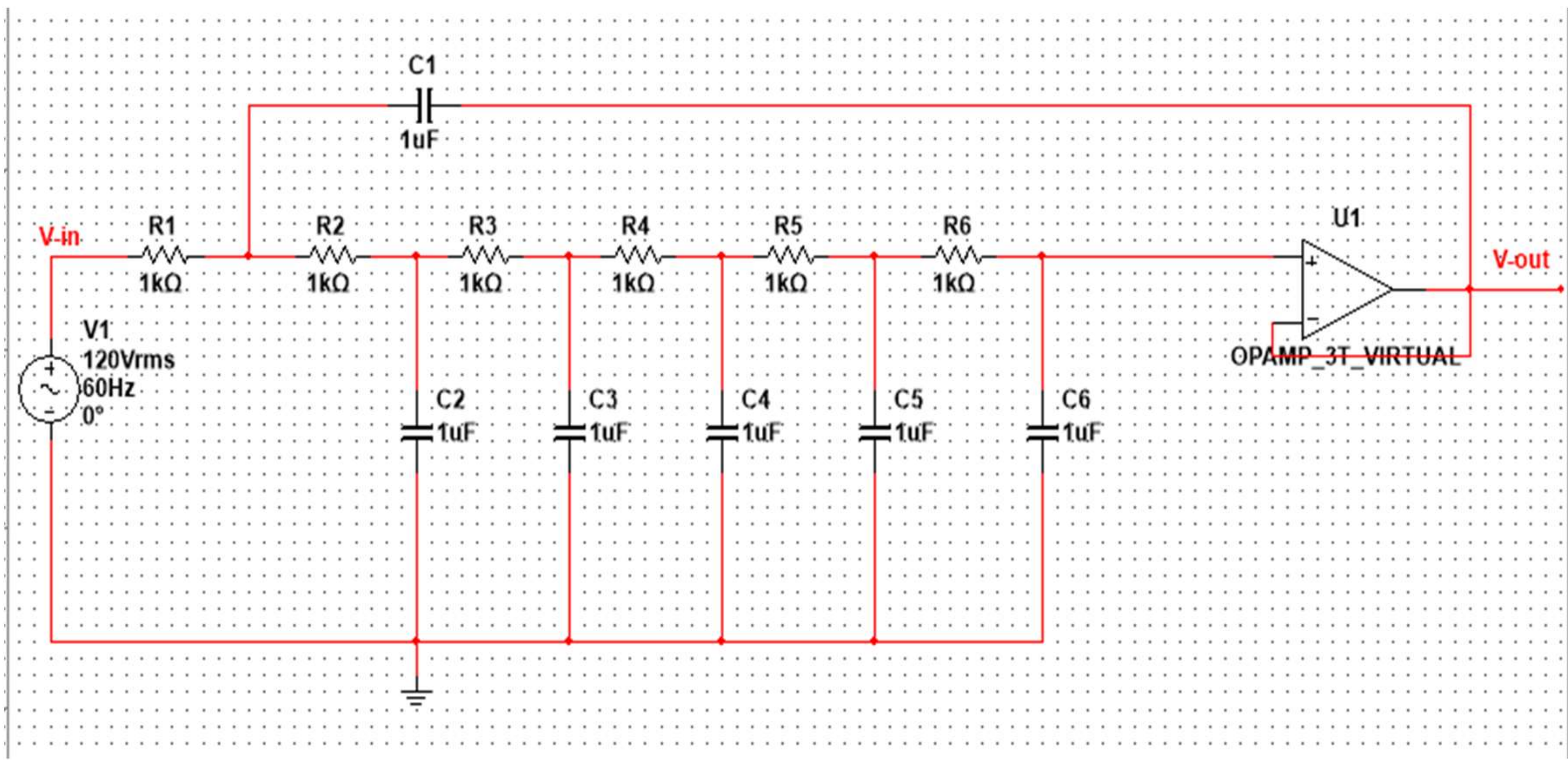
CIRCUIT:(4TH ORDER)



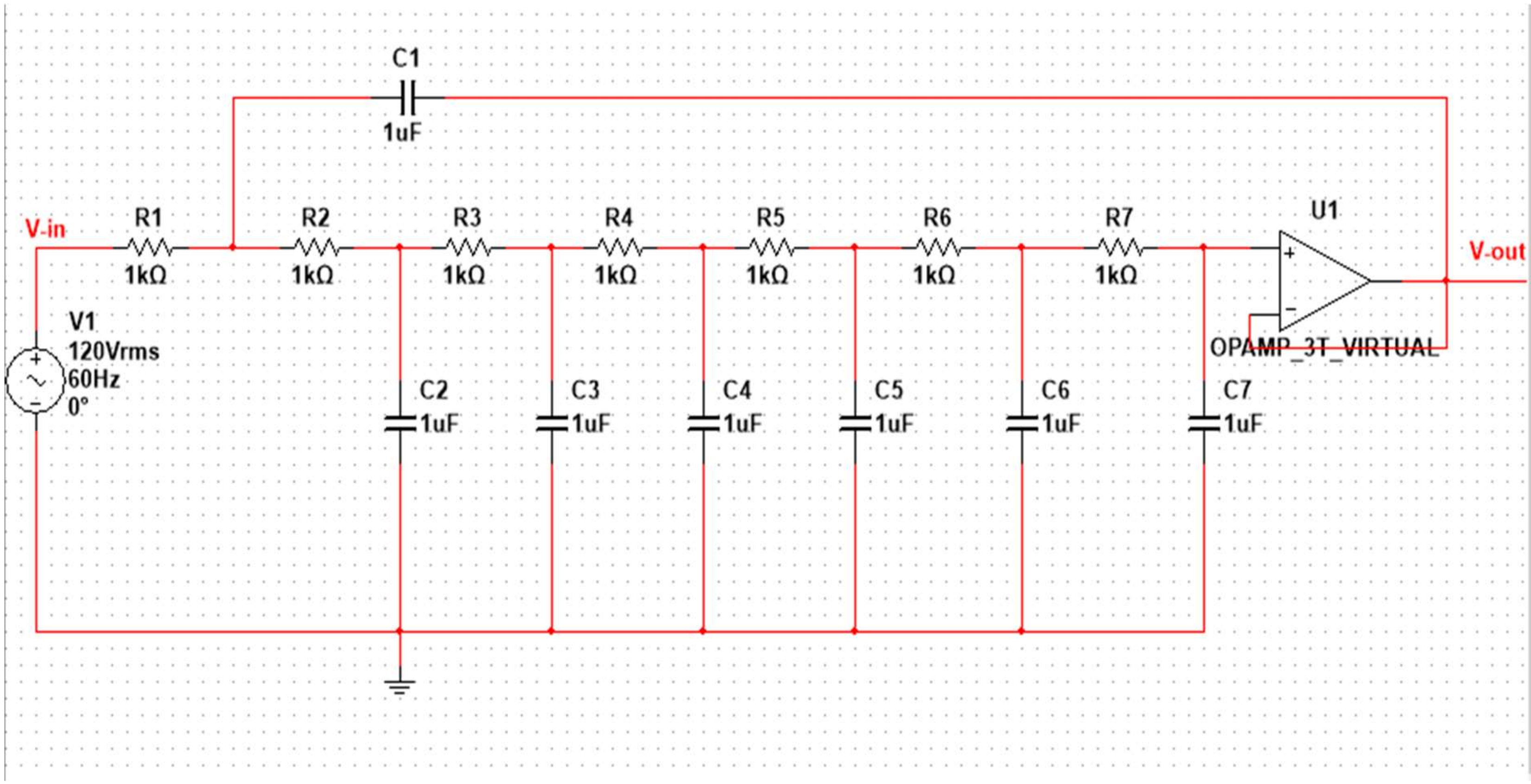
CIRCUIT:(5TH ORDER)



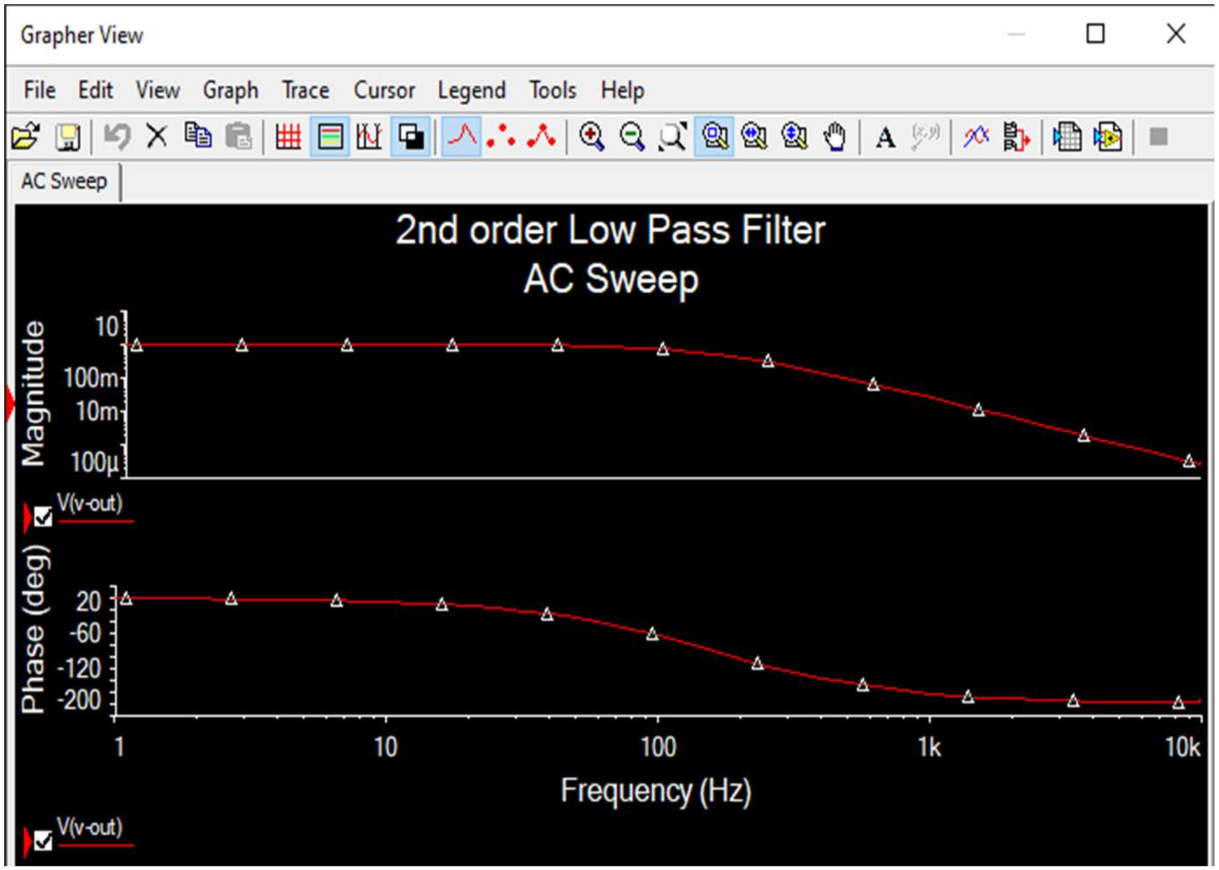
CIRCUIT:(6TH ORDER)



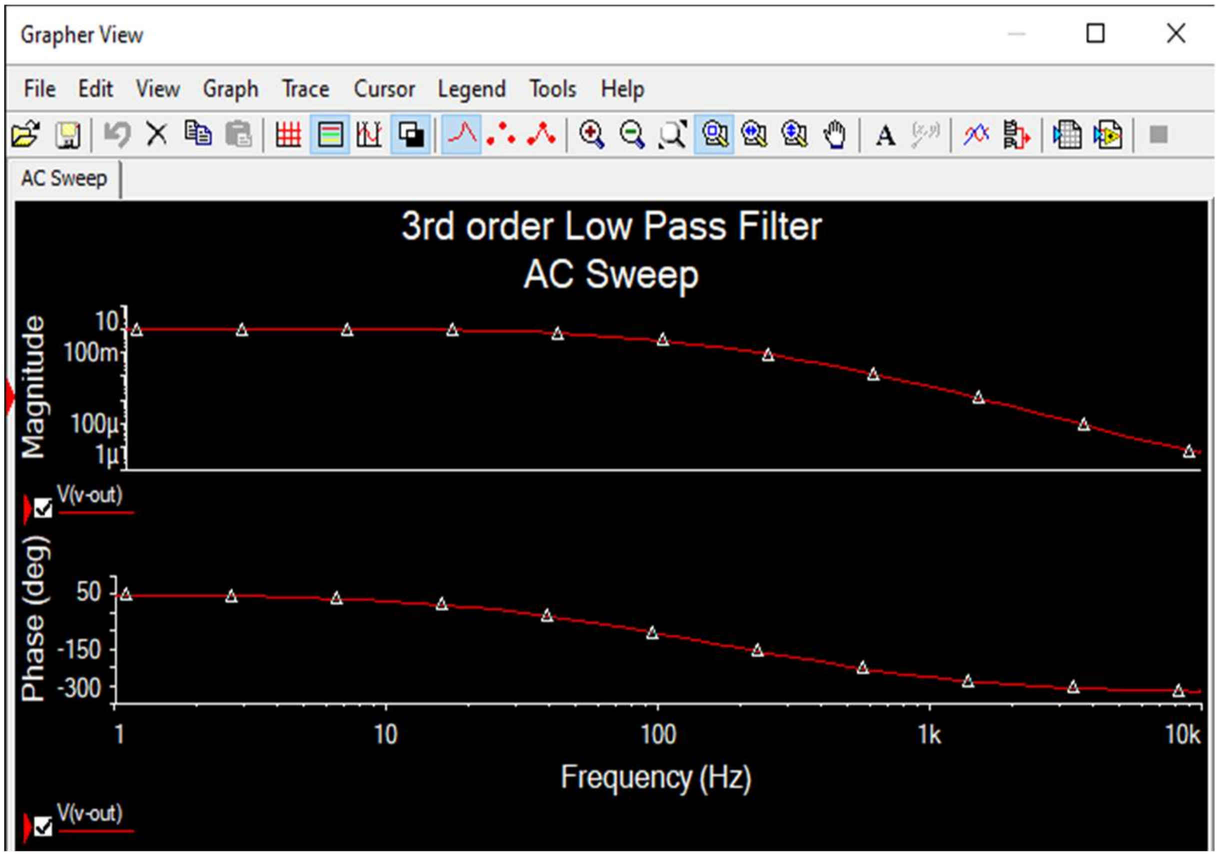
CIRCUIT:(7TH ORDER)



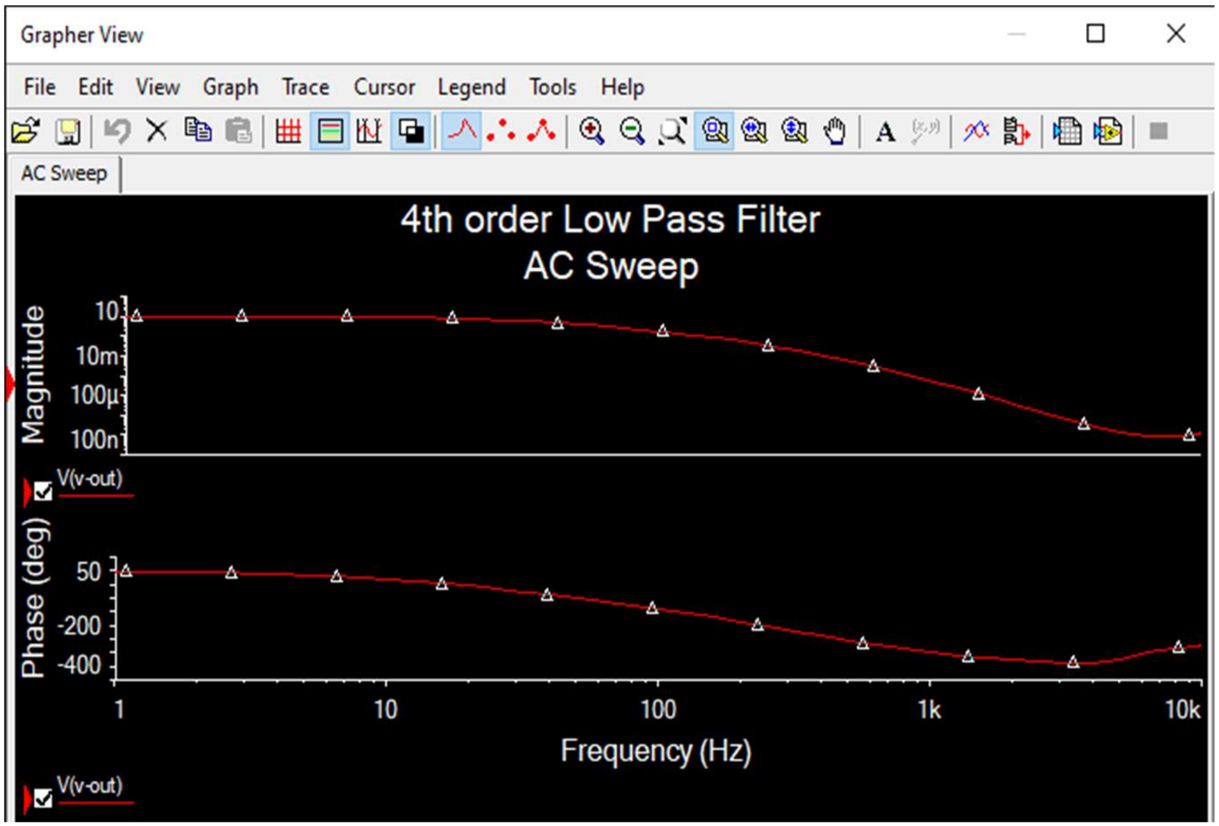
GRAPHER VIEW:(2ND ORDER)



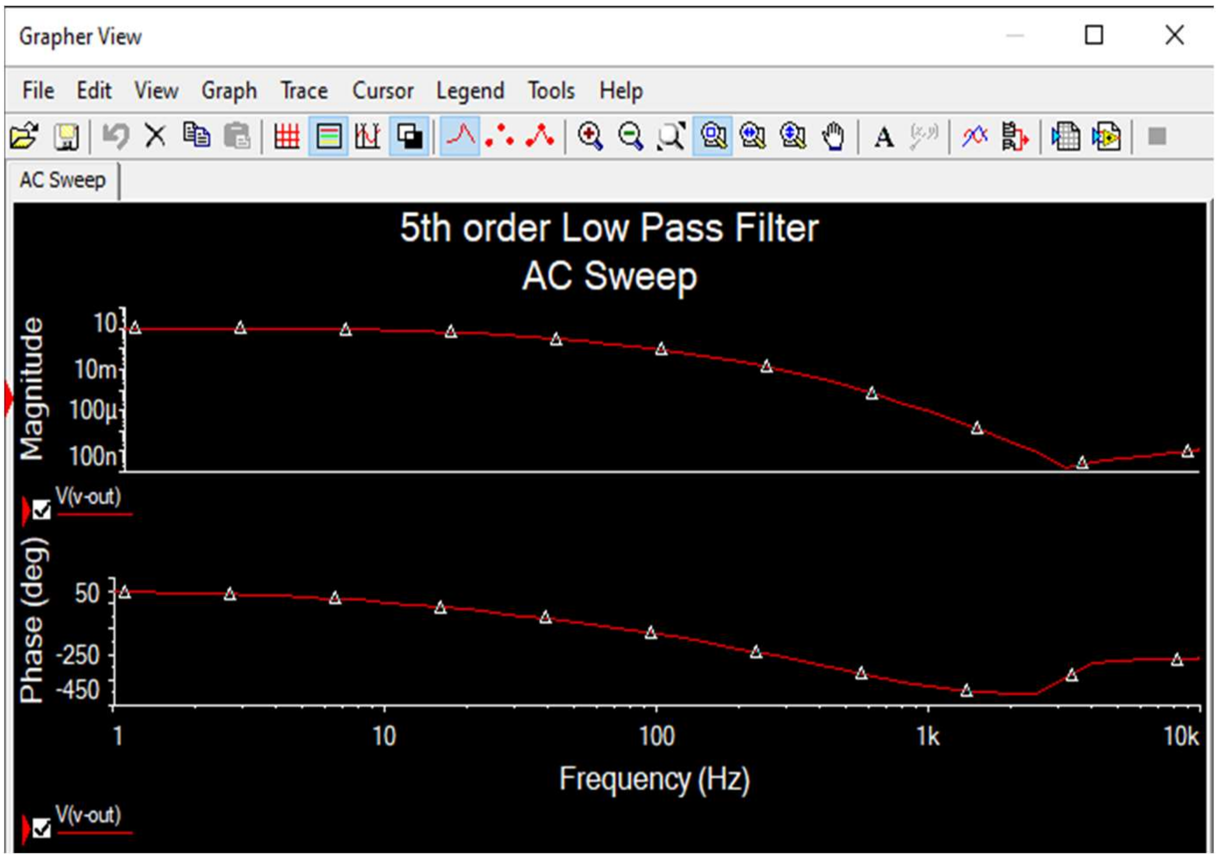
GRAPHER VIEW:(3RD ORDER)



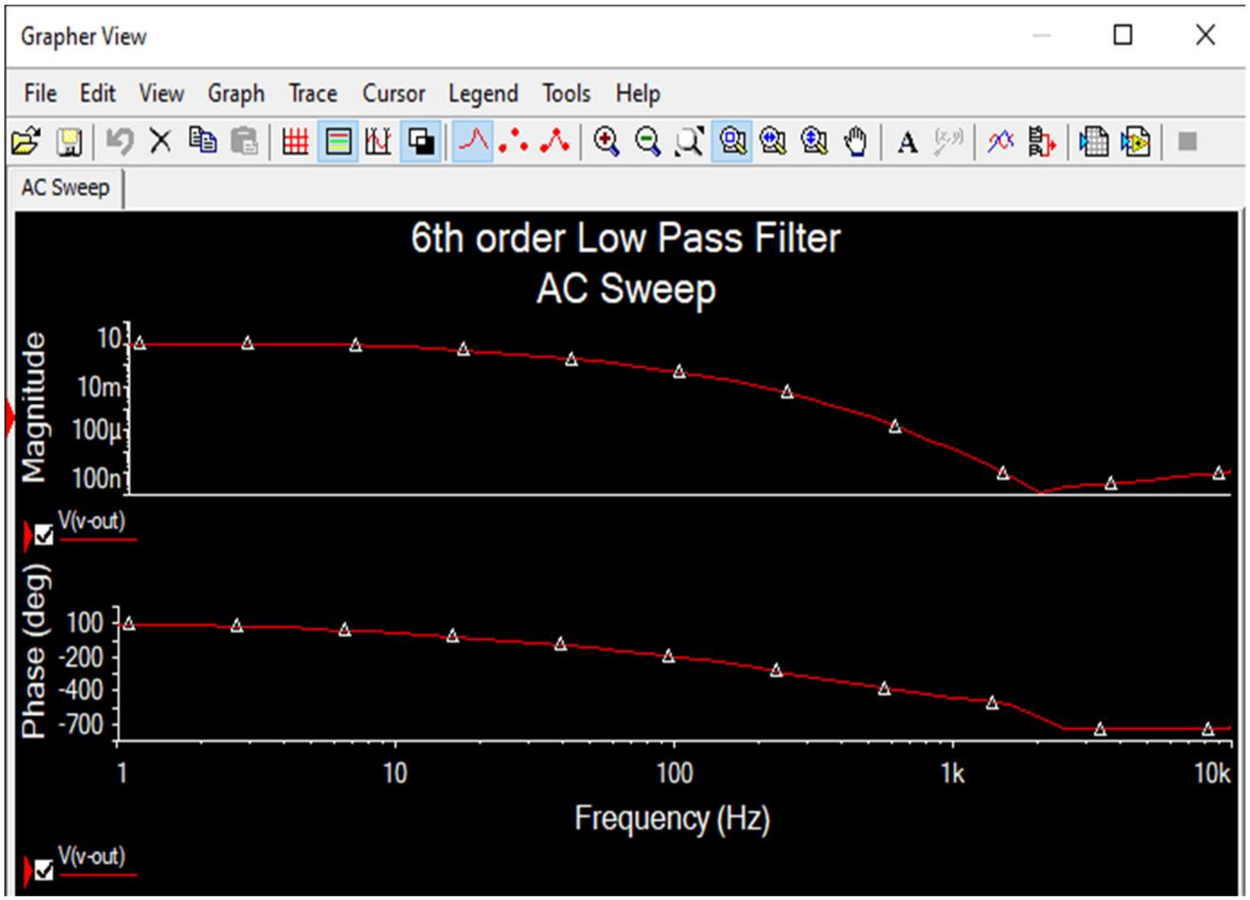
GRAPHER VIEW:(4TH ORDER)



GRAPHER VIEW:(5TH ORDER)



GRAPHER VIEW:(6TH ORDER)



GRAPHER VIEW:(7TH ORDER)