BCD and Gray Code

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Binary Coded Decimal (BCD) Number Representation

- It is sometimes desirable to manipulate numbers in decimal instead of converting them to binary.
- Decimal to binary and binary to decimal conversion process is complex.
- One popular code to represent decimal is BCD. Each decimal digit is represented by 4-bit binary equivalent. conversion is much easier.

Examples:
Decimal 391 in BCD 0011 1001 0001
Decimal 13.34 in BCD 0001 0011.0011 0100
There are six combinations 1010 1011 1100 1101 1110 1111

Addition of BCD Numbers

When two BCD numbers are added, there may be needed for correction step where 6(0110) will be added to one of the nibble. This correction is required when a nibble is one of the six unused combinations or there is a carry in from the previous nibble.

Examples:

$23 + 46 = 69$

0 0 1 0 0 0 1 1

$\underline{0\ 1\ 0\ 0\ 0\ 1\ 1\ 0}$

0 1 1 0 1 0 0 1

Addition of BCD Numbers

Example:

$23 + 48 = 71$

0 0 1 0 0 0 1 1

$\underline{0\ 1\ 0\ 0\ 1\ 0\ 0\ 0}$

0 1 1 0 1 0 1 1 $\Rightarrow$ Here the right most nibble is one of the unused combination. Hence, we have to add [0 1 1 0] with this nibble.

0 1 1 0 1 0 1 1

$\underline{0\ 0\ 0\ 0\ 0\ 1\ 1\ 0}$

0 1 1 1 0 0 0 1

Addition of BCD Numbers

Example:

$28 + 39 = 67$

0 0 1 0 1 0 0 0

$\underline{0\ 0\ 1\ 1\ 1\ 0\ 0\ 1}$ 0 1 1 0 0 0 0 1

Here, carry is flowing from one nibble to the other nibble. So, correction is required.

0 1 1 0 0 0 0 1

$\underline{0\ 0\ 0\ 0\ 0\ 1\ 1\ 0}$

0 1 1 0 0 1 1 1

Gray Code

⇒ There are some applications, where if multiple bits are changing between two consecutive number, that may create problem. To elevate the problem, a new code was introduced which is called Gray code.
⇒ Gray code is a type of non-weighted binary code where successive code words differ in only one bit. Any code with this property is called cyclic code.
⇒ Gray code is used in many practical applications that require analog to digital conversion.

- To reduce error in conversion.
- Also, binary to gray and gray to binary conversions are easy.
- Example: to measure the angle of rotation of a wheel.

Gray Code

- Gray code also called self-reflecting code. Suppose we have the Gray code representation for m-bit.
- To obtain the Gray code representation for $(m+1)$-bit, we write down two m-bit representation one below the other with second one being the mirror image of the first one.
- We then add 0 at the beginning of every code in the first group and 1 at the beginning of every code in the second group.

Binary to Gray code conversion

- Let $g_{n-1}g_{n-2}\ldots g_2g_1g_0$ and $b_{n-1}b_{n-2}\ldots b_2b_1b_0$ denote n-bit Gray code and its equivalent binary representation respectively.
  $g_i = b_i \oplus b_{i+1}$ for $0 \le i \le n-2$ and
  $g_{n-1} = b_{n-1}$
- $0 \oplus 0 = 0 \quad 0 \oplus 1 = 1 \quad 1 \oplus 0 = 1 \quad 1 \oplus 1 = 0$

Decimal	binary representation	Gray code
0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

Gray code to binary code conversion

Start with the MSB and proceed to the LSB and set

- $b_i = g_i$, if number of 1's preceding g_i is even.
- $b_i = \overline{g_i}$, if number of 1's preceding g_i is odd.

$g_2 g_1 g_0$	$b_2 b_1 b_0$
000	000
001	001
011	010
010	011
110	100
111	101
101	110
100	111