3-12-15

INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR
B.TECH+M.TECH DUAL DEGREE 3rd SEMESTER CS EXAMINATION, 2015

DISCRETE STRUCTURE (CS 303)

FULL MARKS: 70 **TIMES: 3 Hrs**

Use ONE Answer Script.

All answers of a particular group must appear consecutively, failing which marks will be deducted.

Group A
Answer any THREE questions.

1. a) Prove that the power set of a set is of larger cardinality.
   b) Prove that a non-empty subset of an enumerable set is either finite or enumerable.
   [3+4]

2. a) Prove that the set of all ordered pairs $\{(m, n) \mid m \in N, n \in N\}$ is enumerable.
   b) Prove that the set of all integral multiples of 5 is enumerable.
   [4+3]

3. a) Give an example of two uncountable sets A and B such that $A \cap B$ is
   i) finite ii) countably infinite iii) uncountable.
   b) Show that the set of all finite bit strings is countable. [3+4]

4. a) Define inverse and constant mapping.
   b) Prove that a mapping f : A→ B is invertible if and only if f is a bijection.
   [3+4]

5. a) Let $S = \{1,2,3,4,5,6,7,8,9,10\}$
   i) Let $x \leq y$ mean x is a divisor of y, then find the maximal element/elements in the poset $(S, \leq)$.
   ii) Let $x \leq y$ have its usual meaning as in the set R. Then find the maximal element/elements in the poset $(S, \leq)$.
   b) Suppose that (S, r_1) and (T, r_2) are posets. Show that $(S \times T, r)$ is a poset where $(s, t)\ r\ (u, v)$ if and only if $s\ r_1\ u$ and $t\ r_2\ v$.
   [4+3]

Group B
Answer any FOUR questions.

6. a) Solve the following recurrence relation by the method of generating function:
   $a_r - 3ra_{r-1} = 2r!$ where $r \geq 1$, given that $a_0 = 1$.
   b) Using the concept of Numeric function, determine the number of sequences of length 4 made up of 6 English letters and 5 Greek letters where first portion of each sequence contains English letters and the second portion Greek letters.
   [5+2]

7. a) Let a_r denotes the number of edges in a complete graph on r vertices.
   i) Derive a recurrence relation for a_r in terms of a_{r-1}.
   ii) Solve the recurrence relation by Iterative method.

   b) Using generating function determine the number of ways in which eleven books can be distributed among three students so that no one will get more than five books.
   [4+3]

8. a) Using the concept of generating function prove that,
$1^2 + 2^2 + 3^2 + \dots \dots + r^2 = \frac{r(r+1)(2r+1)}{6}$.
b) Given a generating function, $A(z) = \frac{2}{1-4z^2}$, find its corresponding Numeric function.

[5+2]

9. a) Prove that a tree with n vertices has (n – 1) edges.
b) What do you mean by Center of a graph? Prove that every tree has either one or two centers.

[3+4]

10. a) Prove that number of pendant vertices in a binary tree with n vertices is $\frac{n+1}{2}$.
b) Define the terms ***Rank*** and ***Nullity*** of a disconnected graph with k components.
c) Prove that every connected graph has at least one spanning tree.

[2+2+3]

11. a) Define ***Complete Graph***, ***Perfect Graph*** and ***Arbitrarily Traceable Graph***.
b) Prove that in a complete graph with n vertices there are $\frac{n-1}{2}$ edge-disjoint Hamiltonian circuits, if n is an odd number ≥ 3.

[3+4]

Group C
Answer any THREE questions.

12. a) Categorize the following wffs into valid, invalid, satisfiable, or unsatisfiable.
i) $\neg((p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (q \rightarrow r))$
ii) $(p \rightarrow (\neg q \rightarrow r)) \wedge (p \rightarrow \neg q) \rightarrow (p \rightarrow r)$
b) Simplify the following proposition
$\neg p \wedge q \wedge (p \rightarrow (q \vee \neg r))$

[5+2]

13. Justify whether the following arguments are valid.
a) If Sam was at the fair, then his father was negligent or his mother was not at home. If his mother was not at home, then his father was not negligent. His mother was at home. Therefore, Sam was at the fair.
b) Either the program does not terminate or m is eventually 0. If m is eventually 0, then n also becomes eventually 0. The program is known not to terminate. Hence n is eventually 0.

[3.5x2]

14. a) Define: function, term, predicate, atom in First Order Predicate Logic (FOPL) and give suitable example of each of the above.
b) Why is it necessary to define a domain in First Order Predicate Logic for interpretation of a formula? How would you interpret a formula in FOPL? How would you evaluate the interpretations of a Universal Quantifier and an Existential Quantifier in a formula in FOPL?

[4+3]

15. a) Write an algorithm to obtain Prenex Normal Form (PNF) from a FOPL formula.
b) Obtain PNF of the following:
$\exists x\, (P(x) \rightarrow \neg \exists y\, (P(y) \rightarrow (Q(x) \rightarrow Q(y)))) \wedge \forall x\, (P(x) \rightarrow \forall y\, Q(z))$

[4+3]

16. a) Some people like all doctors. No people like any quack. Prove that no doctor is a quack.
b) Discuss limitations of First Order Predicate Logic with appropriate examples.

[5+2]