Module 2 (Inverse Laplace Transform)

Q 1 : Compute the inverse Laplace transform of the following. (LOS APPLYING)

(1) $\dfrac{1}{s^2+9}$

(2) $\dfrac{6}{s^3}$

(3) $\dfrac{4s+5}{16s^2-25}$

(4) $\dfrac{3(s^2-1)^2}{2s^5}$ $\left\{ H\text{int} : \dfrac{(s^2-1)^2}{s^5} = \dfrac{s^4-2s^2+1}{s^5} = \dfrac{1}{s} - \dfrac{2}{s^3} + \dfrac{1}{s^5} \right\}$

Q 2 Compute the following : (HOS APPLYING)

(1) $L^{-1}\left(\dfrac{s+2}{s^2-4s+13}\right)$

(2) $L^{-1}\left(\dfrac{2s}{(2s-1)^2}\right)$

(3) $L^{-1}\left(\dfrac{s}{(s^2+2s+2)(s^2+3s+2)}\right)$ (reciprocal difference)

Q 3 Compute Inverse Laplace Transform of following : (HOS APPLY

(1) $\log\left(\dfrac{s+2}{s+1}\right)$

(2) $\log\left(\dfrac{s^2+b^2}{s^2+a^2}\right)$

(3) $\log\left(1+\frac{a^2}{s^2}\right)$

(4) $\tan^{-1}\left(\frac{2}{s}\right)$

(5) $\cot^{-1}(s+1)$

(6) $\tan h^{-1} s$

(7) $\log\sqrt{\frac{s^2+a^2}{b^2}}$

Q4 Compute the inverse Laplace transform of the following : (HOS APPLYING)

(1) $L^{-1}\left(\frac{1}{s(s^2+4)}\right)$

(2) (a) $L^{-1}\left(\frac{1}{s^3(s^2+1)}\right) = L^{-1}\left(\frac{1}{s.s^2(s^2+1)}\right)$

$$= \int_0^t L^{-1}\left(\frac{1}{s^2(s^2+1)}\right) du$$

$$= \int_0^t L^{-1}\left(\frac{1}{s^2} - \frac{1}{s^2+1^2}\right) du$$

$$= \int_0^t \left[u - \frac{1}{1}\sin u\right] du$$

$$= \left[\frac{u^2}{2} + \cos u\right]_0^t$$

(b) $L^{-1}\left(\frac{1}{s\sqrt{s+4}}\right)$ **(HOS ANALYZING)**

$$= \int_0^t L^{-1}\frac{1}{\sqrt{s+4}}\, du$$

$$= \int_0^t e^{-4u} L^{-1}\left(\frac{1}{\sqrt{s}}\right) du$$

(5) $\frac{s+2}{s^2(s+3)}$

(6) $\frac{1}{s(s+1)^3}$

(7) $\frac{1}{s\sqrt{s^2+a^2}}$

Q 5 Using convolution theorem. Find the inverse Laplace transform of the following :

(HOS ANALYZING)

(1) $\frac{s}{(s^2+a^2)^2}$

(2) $\frac{1}{s^2(s+1)^2}$

(3) $\frac{1}{(s+3)(s-1)}$

(4) $\frac{1}{(s+2)^2(s-2)}$

(5) $\frac{1}{(s+1)(s^2-1)}$

(6) $\frac{s^2}{(s^2+a^2)^2}$

(7) $\frac{s}{(s^2+4)(s^2+9)}$

(8) $\frac{2s}{(s^2+1)^2}$

(9) $\frac{s}{(s^2-a^2)^2}$

(10) $\frac{s}{(s^2+4)(s^2+1)}$

(11) $\frac{s}{s^4+8s^2+16}$

(12) $\dfrac{s^2+s}{(s^2+1)(s^2+2s+2)}$

(13) Calculate using convolution theorem

$L^{-1}\left(\dfrac{s}{s^4+13s^2+36}\right)$ & hence $L^{-1}\left(\dfrac{s^2}{s^4+13s^2+36}\right)$

(14) $\dfrac{1}{(s+1)^3}$

(15) $\dfrac{s}{(s^2+4)^3}$

Q 6 Partial Fraction :Compute the inverse Laplace transform of the following :

(HOS A

(1) $\dfrac{s^2+1}{s^3+3s^2+2s}$

(2) $\dfrac{s^2}{s^4+9s^2+20} = \dfrac{s^2}{(s^2+4)(s^2+5)} = \dfrac{x}{(x+4)(x+5)}$ put $s^2 = x$

(3) $\dfrac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} = \dfrac{x+3}{(x+2)(x+5)}$ Let $s^2+2s = x$

(4) $\dfrac{3s+7}{s^2-2s-3}$

(5) $\dfrac{2s^2-4}{(s-1)(s-2)(s-3)}$

(6) $\dfrac{2s-1}{s^3-s}$

(7) $\dfrac{11s^2-2s+5}{(s-2)(2s-1)(s+1)}$

(8) $\dfrac{5s^2-15s-11}{(s+1)(s-2)^3}$

(9) $\dfrac{s^2-2s+3}{(s-1)^2(s+1)}$

(10) $\dfrac{3s+1}{(s-1)(s^2+1)}$

(11) $$\frac{s}{(s^2-2s+2)(s^2+2s+2)}$$

(12) $$\frac{5s^2+8s-1}{(s+3)(s^2+1)}$$

(13) $$\frac{s+2}{(s^2+4s+8)(s^2+4s+13)}$$