Graphs - 1

Monday, November 23, 2020 12:01 PM

Prob.

Graph G.

$\deg(a) = 2$ $\qquad N(a) = \{b, f\}$

$\deg(b) = 4$ $\qquad N(b) = \{a, c, e, f\}$

$\deg(c) = 4$ $\qquad N(c) = \{b, d, e, f\}$

$\deg(d) = 1$ $\qquad N(d) = \{c\}$

$\deg(e) = 3$ $\qquad N(e) = \{b, c, f\}$

$\deg(f) = 4$ $\qquad N(f) = \{a, b, c, e\}$

$\deg(g) = 0$ $\qquad N(g) = \phi$

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Theorem: An undirected graph has even no. of vertices of odd degrees.

Proof: Let V_1 and V_2 be the sets of vertices of even degree and odd degree, respectively. Let the no. of edges in the graph be m.
Using Handshaking Theorem it follows,

$$2m = \sum_{v \in V_1} \deg(v) + \sum_{v \in V_2} \deg(v)$$

Because $\deg(v)$ is even for $v \in V_1$, the first term in the R.H.S of the equality is even.
Furthermore, the sum of two terms in the R.H.S of the equality is $2m$, which is even.

$\sum_{v \in V_2} \deg(v)$

$\sum \deg(v)$
$v \in V_2$

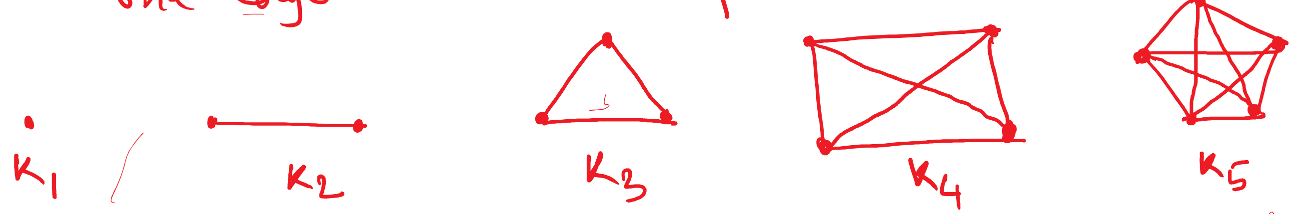
of the equality is $2m$, which is even.
Hence, the second term i.e. $\sum_{v \in V_2} \deg(v)$ is also even. Because all terms in this sum are odd, there must be even no. of such terms.
Thus, there are even no. of vertices of odd degree.

Prob.

$\deg^-(a) = 2$, $\deg^+(a) = 4$.
$\deg^-(b) = 2$, $\deg^+(b) = 1$
$\deg^-(c) = 3$, $\deg^+(c) = 2$
$\deg^-(d) = 2$, $\deg^+(d) = 2$
$\deg^-(e) = 3$, $\deg^+(e) = 3$
$\deg^-(f) = 0$, $\deg^+(f) = 0$.

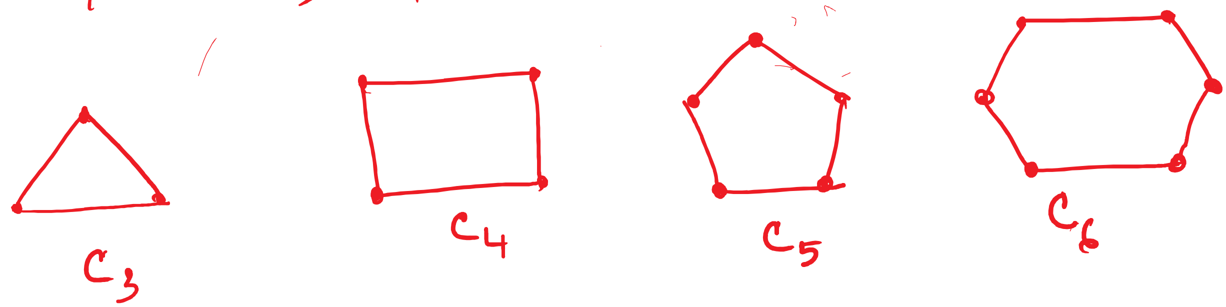
Special types of Simple Graphs.

1. Complete graphs: A complete graph on 'n' vertices, denoted by K_n, is a simple graph that contains exactly one edge between each pair of vertices.

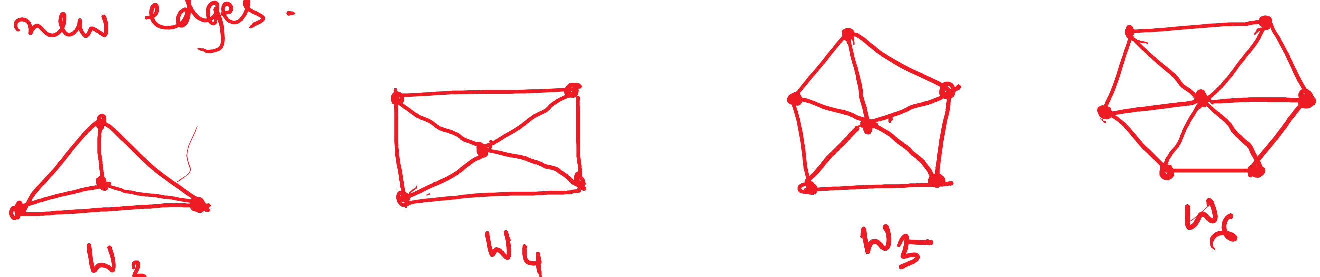


2. Cycles: A cycle C_n, $n \geq 3$, consists of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}$, $\{v_2, v_3\}$...

$v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots$
$\{v_{n-1}, v_n\}, \{v_n, v_1\}$.



(3) **Wheels:** We obtain a wheel W_n when we add an additional vertex to a cycle C_n for $n \geq 3$, and connect this new vertex to each of the n vertices of C_n, by new edges.



Theorem: A simple graph is bipartite iff it is possible to assign one of the two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color.

Proof: First, suppose that $G = (V, E)$ is a bipartite graph. Then $V = V_1 \cup V_2$ where V_1 and V_2 are disjoint sets and every edge in E connects a vertex in V_1 and a vertex in V_2. If we assign one color of each vertex in V_1 and ... then no two a second color to each vertex in V_2, then no two adjacent vertices are assigned the same color.

$p \leftrightarrow q$

Now, suppose that it is possible to assign colors to the vertices of the graph using just two colors so that no adjacent vertices are assigned the same color. Let V_1 be the set of vertices assigned one color and V_2 be the set of vertices assigned the second color.

$q \to p$

Then, V_1 and V_2 are disjoint and $V = V_1 \cup V_2$.

Furthermore, every edge connects a vertex in V_1 and a vertex in V_2 because no two adjacent vertices are either both in V_1 or both in V_2. Consequently, G is bipartite.

Complete Bipartite Graph

A complete bipartite graph $K_{m,n}$ is a graph that has its vertex set partitioned into two subsets of m and n vertices, respectively, with an edge between two vertices iff one vertex is in the first subset and the other vertex is in the second subset.

Matching: Matching M in a simple graph $G = (V, E)$ is a subset of the set E such that no two edges are incident with the same vertex.

- If $\{s, t\}$ and $\{u, v\}$ are the two edges, they have distinct vertices. Both $\{s, t\}$ and $\{u, v\}$ are included in the matching set M.

- <u>Maximum</u> matching - match with largest no. of edges.
- <u>Complete matching ~~from~~ V_1 to V_2</u>: if every vertex in V_1 is the end point of an edge in matching, or equivalently, $|M| = |V_1|$.

<u>Subgraph</u>

✓ <u>Removing or Adding edges of a Graph</u>

- Given a graph $G = (V, E)$ and an edge $e \in E$, we can produce a subgraph of G by removing the edge e. The resulting subgraph, denoted by $G - e$, has the same vertex set V and edge set $\{E - e\}$.

Instead of removing an edge, a subset of edges E' can also be removed. Then the subgraph created is $G' = \{V, E - E'\}$.

- We can also add an edge e to produce a new subgraph. induced by a set of vertices W which is a ~~subset of vertex set~~ V.

✓ <u>Removing</u> ~~the vertices of a Graph: When we~~ remove a vertex v and all edges incident to it from $G = (V, E)$, we produce a subgraph denoted by $(G - v)$.

$= (V - v, E')$, where E' is the set of edges of G not incident to v.