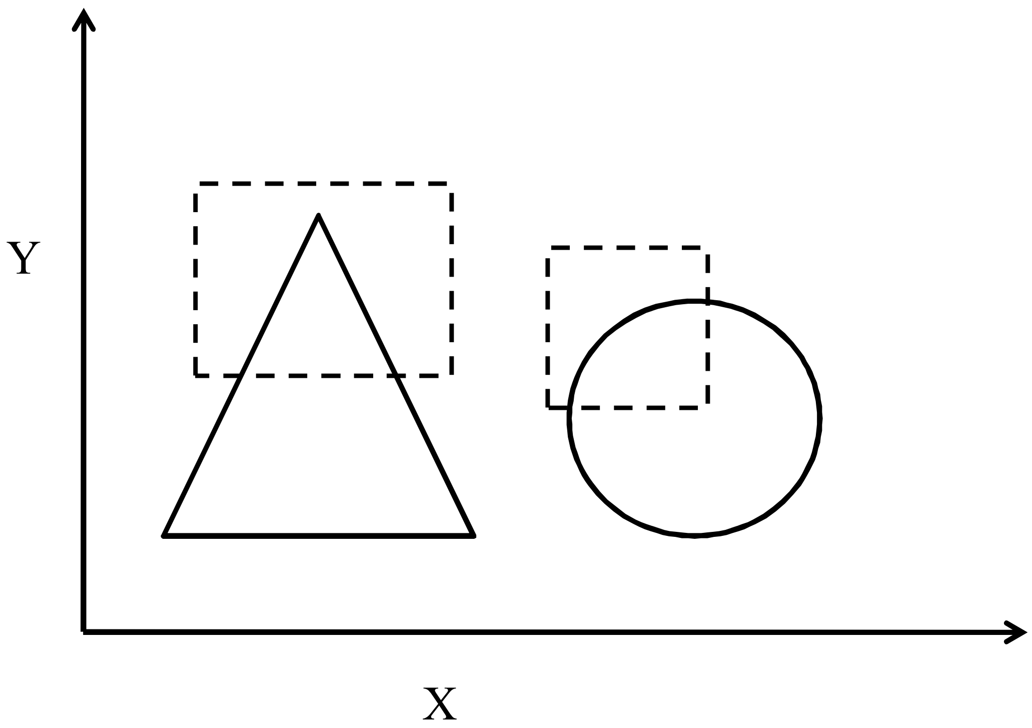
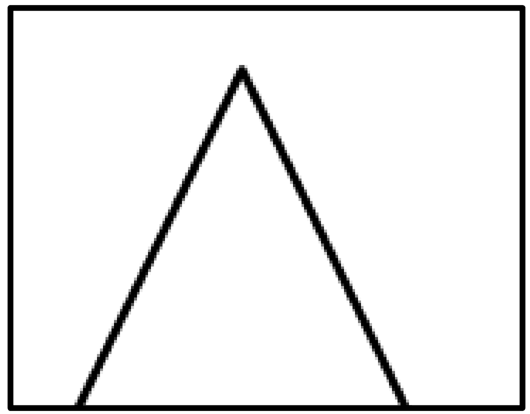
VIEWING TRANSFORMTION
&
CLIPPING

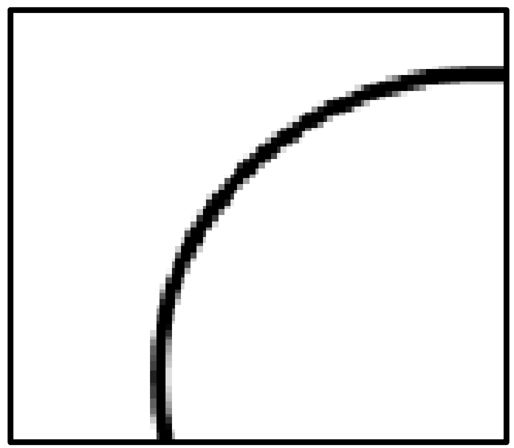
- *Window and Viewport mapping*
- *Point Clipping*
- *Line Clipping*
    - *Cohen-Sudhenland*
    - *Parametric*
- *Polygon Clipping*



World coordinate



Device screen



Device screen

- World coordinate

Cartesian coordinate w.r.t which we define diagram

- Window

An area on world coordinate selected for display.

- Device Coordinate

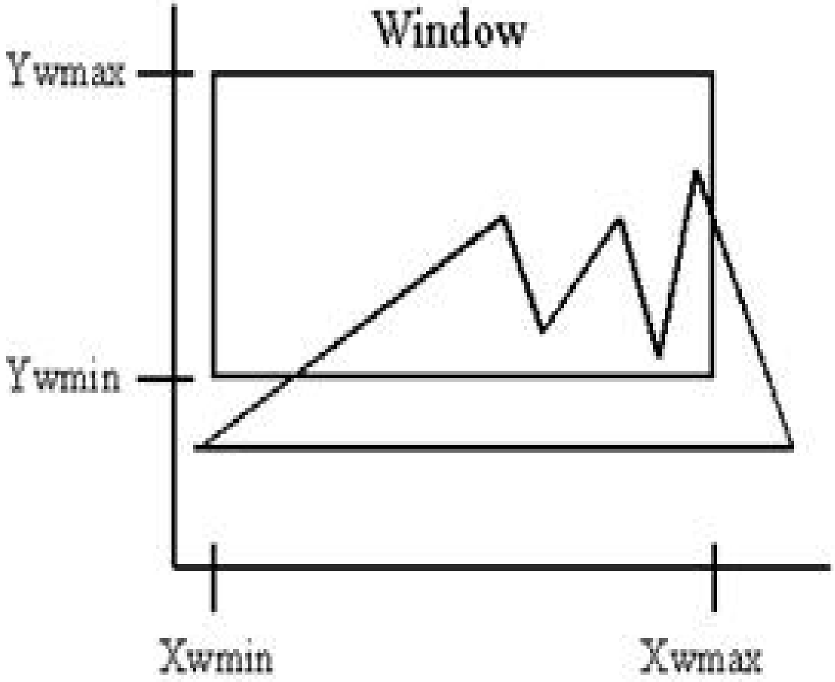
Screen Coordinate

- Viewport

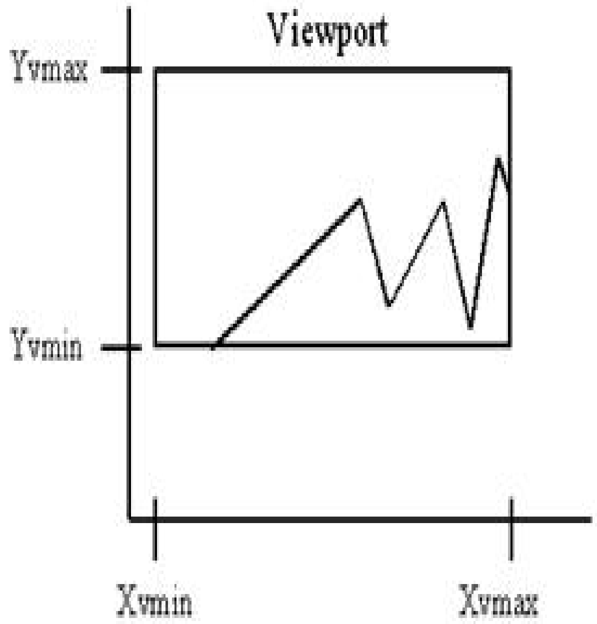
Area on device coordinate where graphics is to be displayed
The coordinate system of the frame buffer.

- Viewing transformation

Window to viewport mapping



World Coordinates



Device Coordinates

VIEWING TRANSFORMATION BY NORMALIZATION

(x_w, y_w) : A point on window
(x_v, y_v) : Corr. point on viewport

Normalized point on window $$\left(\frac{X_W - X_{W\,min}}{X_{W\,max} - X_{W\,min}}, \frac{Y_W - Y_{W\,min}}{Y_{W\,max} - Y_{W\,min}}\right)$$

Normalized point on viewport $$\left(\frac{X_v - X_{v\,min}}{X_{v\,max} - X_{v\,min}}, \frac{Y_v - Y_{v\,min}}{Y_{v\,max} - Y_{v\,min}}\right)$$

$$\frac{X_W - X_{W\,min}}{X_{W\,max} - X_{W\,min}} = \frac{X_v - X_{v\,min}}{X_{v\,max} - X_{v\,min}}$$

$$X_v - X_{v\,min} = (X_W - X_{W\,min})\frac{X_{v\,max} - X_{v\,min}}{X_{W\,max} - X_{W\,min}}$$

$$X_v - X_{v\,min} = (X_W - X_{W\,min})s_x \qquad s_x = \frac{X_{v\,max} - X_{v\,min}}{X_{W\,max} - X_{W\,min}}$$

$$X_v = X_{v\,min} + (X_W - X_{W\,min})s_x$$

$$\boxed{X_v = X_{v\,min} + (X_W - X_{W\,min})s_x} \qquad \boxed{s_x = \frac{X_{v\,max} - X_{v\,min}}{X_{W\,max} - X_{W\,min}}}$$

$$\frac{Y_W - Y_{W\,min}}{Y_{W\,max} - Y_{W\,min}} = \frac{Y_v - Y_{v\,min}}{Y_{v\,max} - Y_{v\,min}}$$

$$Y_v - Y_{v\,min} = (Y_W - Y_{W\,min})\frac{Y_{v\,max} - Y_{v\,min}}{Y_{W\,max} - Y_{W\,min}}$$

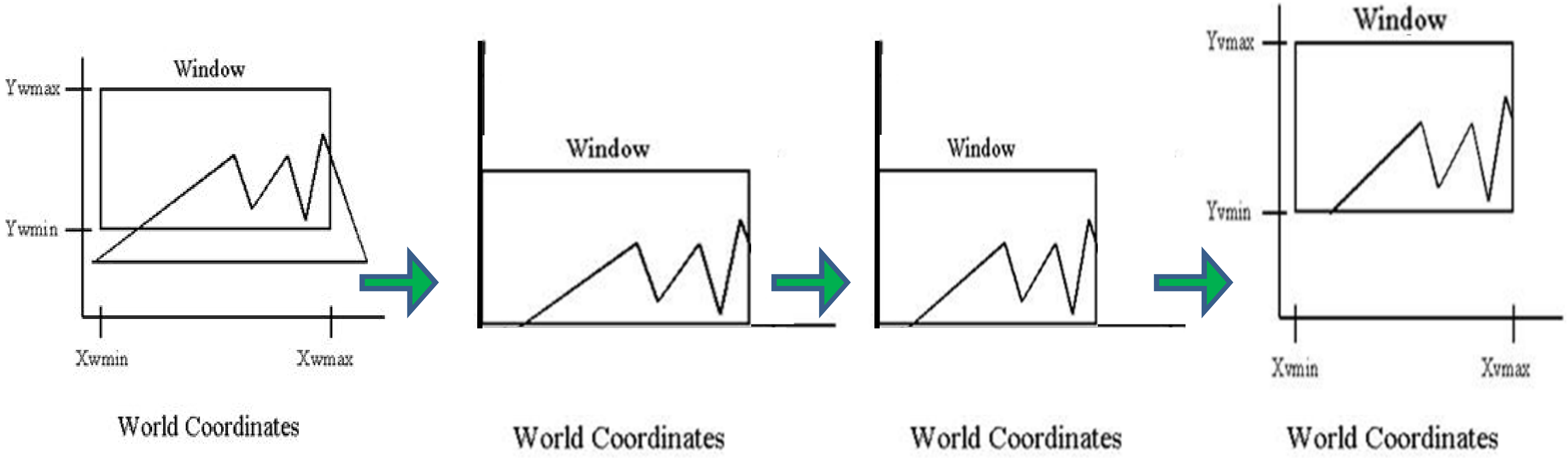
$$Y_v - Y_{v\,min} = (Y_W - Y_{W\,min})s_Y \qquad s_Y = \frac{Y_{v\,max} - Y_{v\,min}}{Y_{W\,max} - Y_{W\,min}}$$

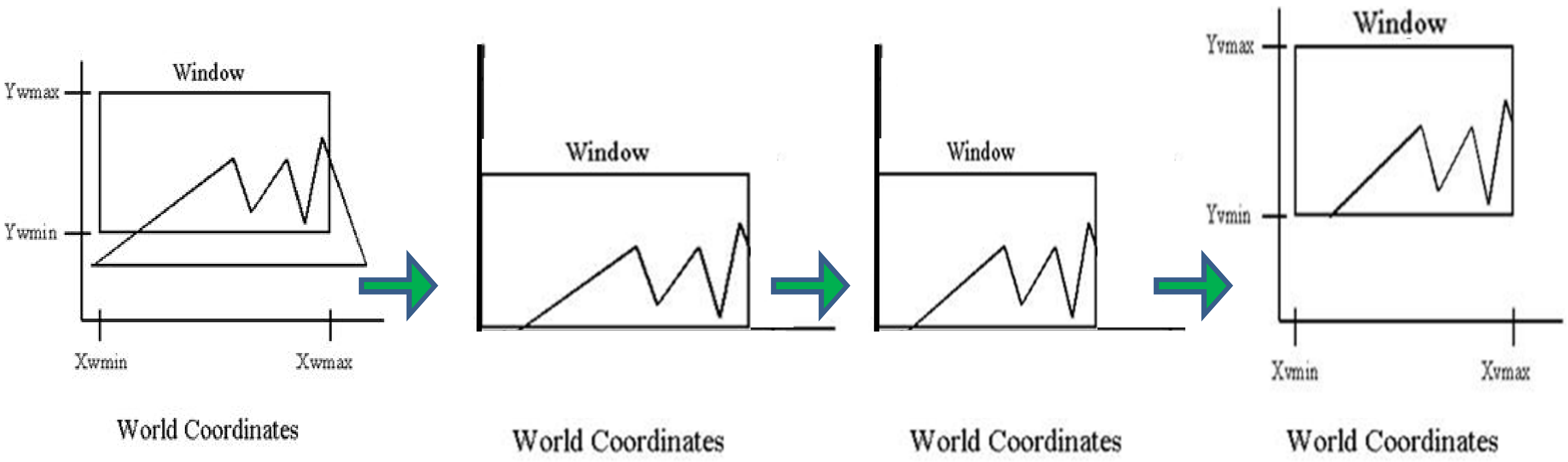
$$Y_v = Y_{v\,min} + (Y_W - Y_{W\,min})s_Y$$

$$\boxed{Y_v = Y_{v\,min} + (Y_W - Y_{W\,min})s_Y} \qquad \boxed{s_Y = \frac{Y_{v\,max} - Y_{v\,min}}{Y_{W\,max} - Y_{W\,min}}}$$

VIEWING TRANSFORMATION BY COMPOSITE TRANSFORMATION

(x_w, y_w) : A point on window
(x_v, y_v) : Corr. point on viewport





$$P_v = T_2 S T_1 (P_w)$$

$$T_1 = \begin{pmatrix} 1 & 0 & -X_{W\min} \\ 0 & 1 & -Y_{W\min} \\ 0 & 0 & 1 \end{pmatrix} \qquad T_2 = \begin{pmatrix} 1 & 0 & X_{V\min} \\ 0 & 1 & Y_{V\min} \\ 0 & 0 & 1 \end{pmatrix} \qquad S = \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_Y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} X_V \\ Y_V \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & X_{V\min} \\ 0 & 1 & Y_{V\min} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_Y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -X_{W\min} \\ 0 & 1 & -Y_{W\min} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_W \\ Y_W \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} X_V \\ Y_V \\ 1 \end{pmatrix} = \begin{pmatrix} S_x & 0 & -S_x X_{W\min} + X_{V\min} \\ 0 & S_Y & -S_Y Y_{W\min} + Y_{V\min} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_W \\ Y_W \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} X_V \\ Y_V \\ 1 \end{pmatrix} = \begin{pmatrix} S_x X_W - S_x X_{W\min} + X_{V\min} \\ S_Y Y_W - S_Y Y_{W\min} + Y_{V\min} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} X_V \\ Y_V \\ 1 \end{pmatrix} = \begin{pmatrix} X_{V\min} + S_x (X_W - X_{W\min}) \\ Y_{V\min} + S_Y (Y_W - Y_{W\min}) \\ 1 \end{pmatrix}$$

$$\boxed{X_v = X_{v\min} + (X_W - X_{W\min}) s_x}$$

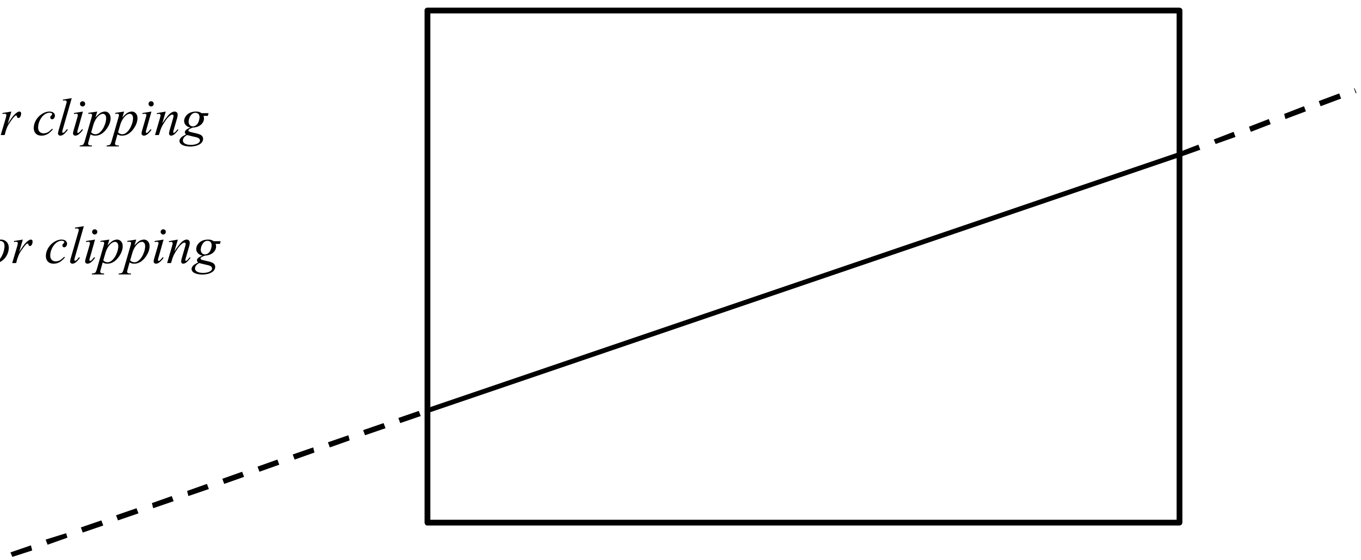
$$\boxed{Y_v = Y_{v\min} + (Y_W - Y_{W\min}) s_Y}$$

CLIPPING OPERATION

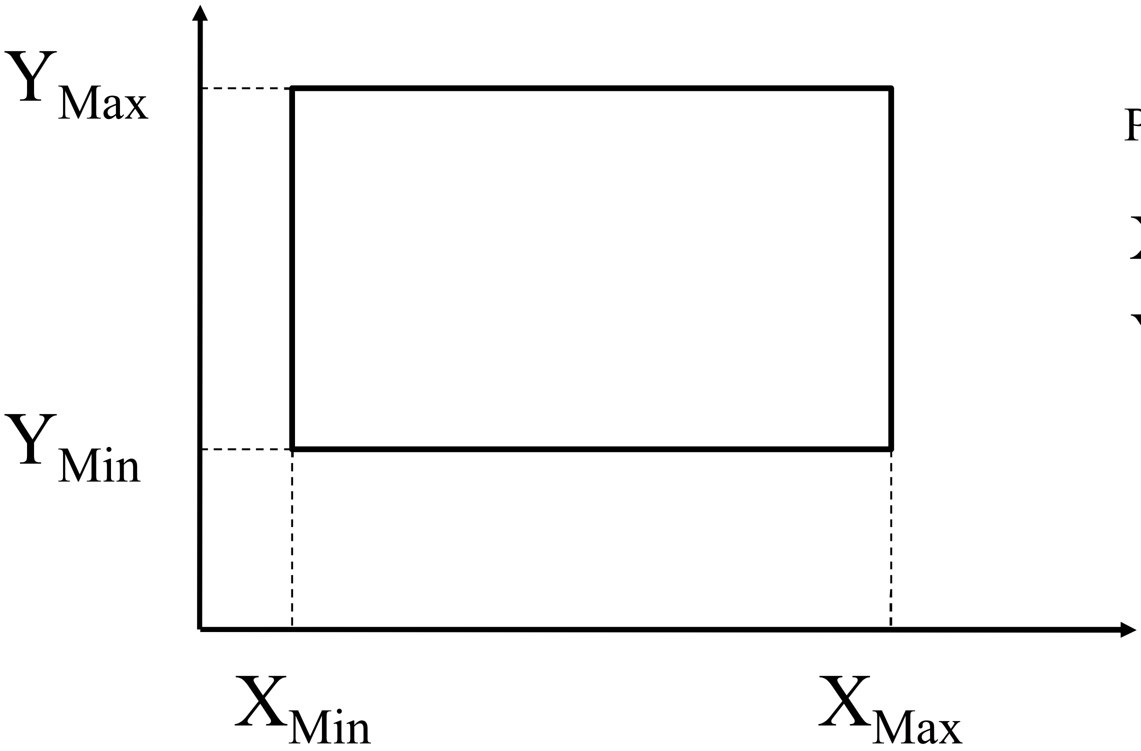
Clip window

Interior clipping

Exterior clipping



POINT CLIPPING



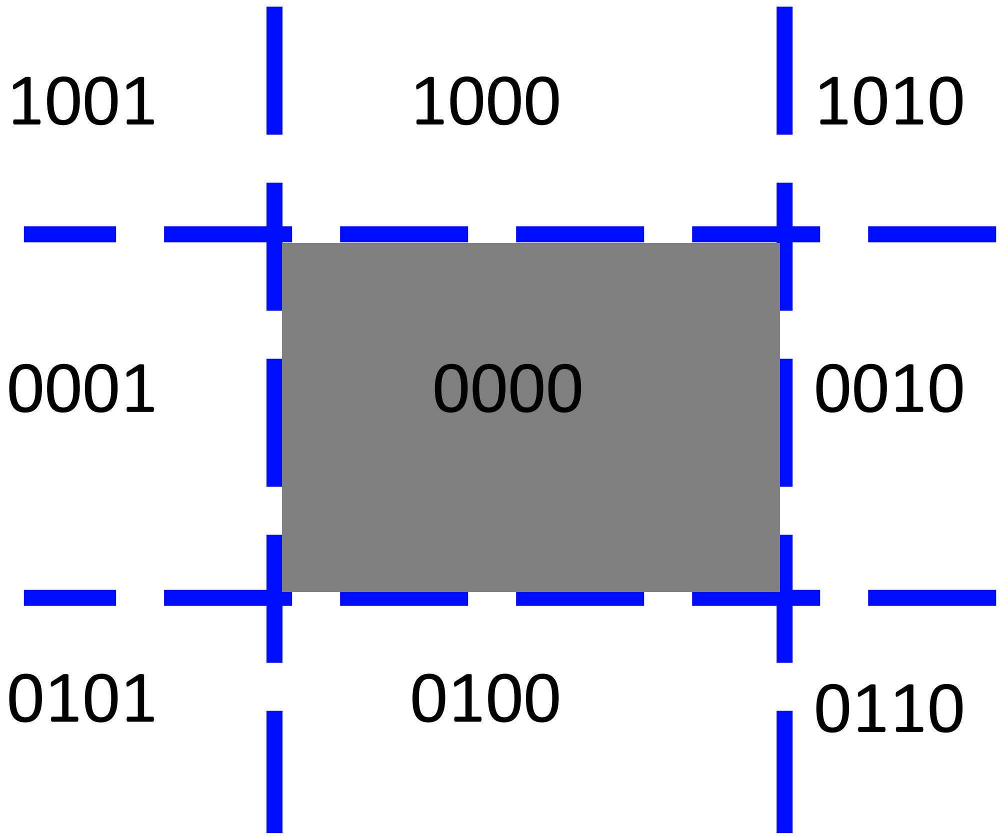
Plot a point (x, y)

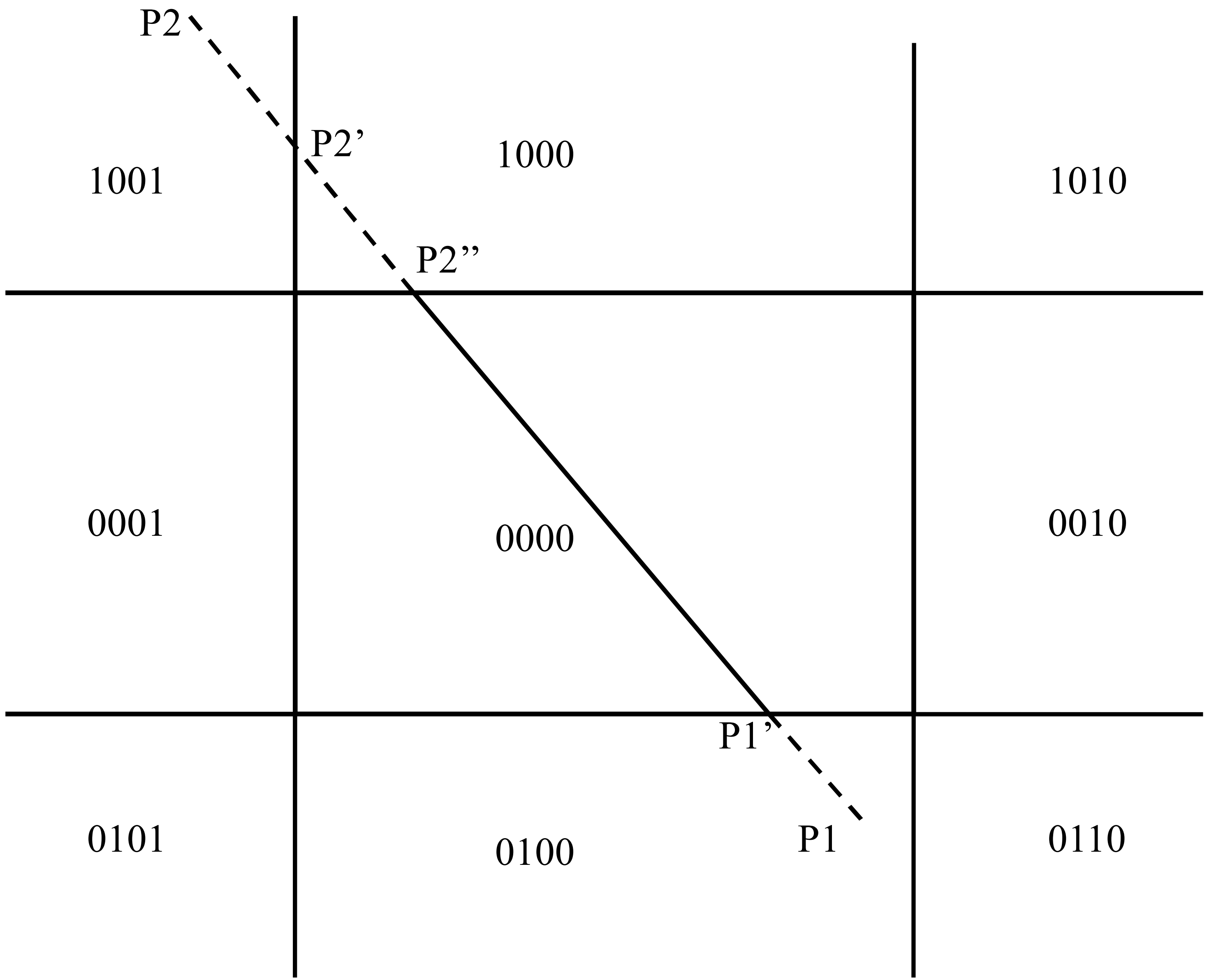
$$X_{Min} \le x \le X_{Max}$$

$$Y_{Min} \le y \le Y_{Max}$$

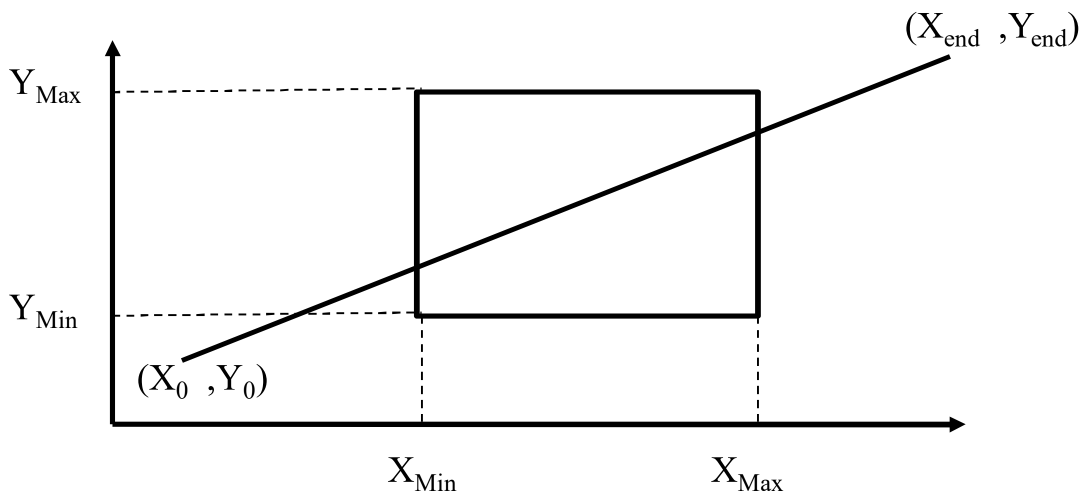
Cohen-Sutherland LINE CLIPPING algorithm

bit 1: left
bit 2: right
bit 3: below
bit 4: above




P2
P2'
1000
1001
1010
P2''
0001
0000
0010
P1'
0101
0100
P1
0110

Liang-Barsky Line Clipping



Parametric definition of a line:

$x = x_0 + u\Delta x$

$y = y_0 + u\Delta y$

$\Delta x = (x_{end} - x_0)$, $\Delta y = (y_{end} - y_0)$, $0 \leq u \leq 1$

From point clipping strategy

$$x_{min} \le x_0 + u\Delta x \le x_{max}$$
$$y_{min} \le y_0 + u\Delta y \le y_{max}$$

- $-u\Delta x \le (x_0 - x_{min})$
- $u\Delta x \le (x_{max} - x_0)$
- $-u\Delta y \le (y_0 - y_{min})$
- $u\Delta y \le (y_{max} - y_0)$

$$u\, p_k \le q_k$$

$$p_1 = -\Delta x \qquad q_1 = x_0 - x_{Min} \qquad \text{Left Boundary}$$

$$p_2 = \Delta x \qquad q_2 = x_{Max} - x_0 \qquad \text{Right Boundary}$$

$$p_3 = -\Delta y \qquad q_3 = y_0 - y_{Min} \qquad \text{Bottom Boundary}$$

$$p_4 = \Delta y \qquad q_4 = y_{Max} - y_0 \qquad \text{Top Boundary}$$

$$u_k = \frac{q_k}{p_k}$$

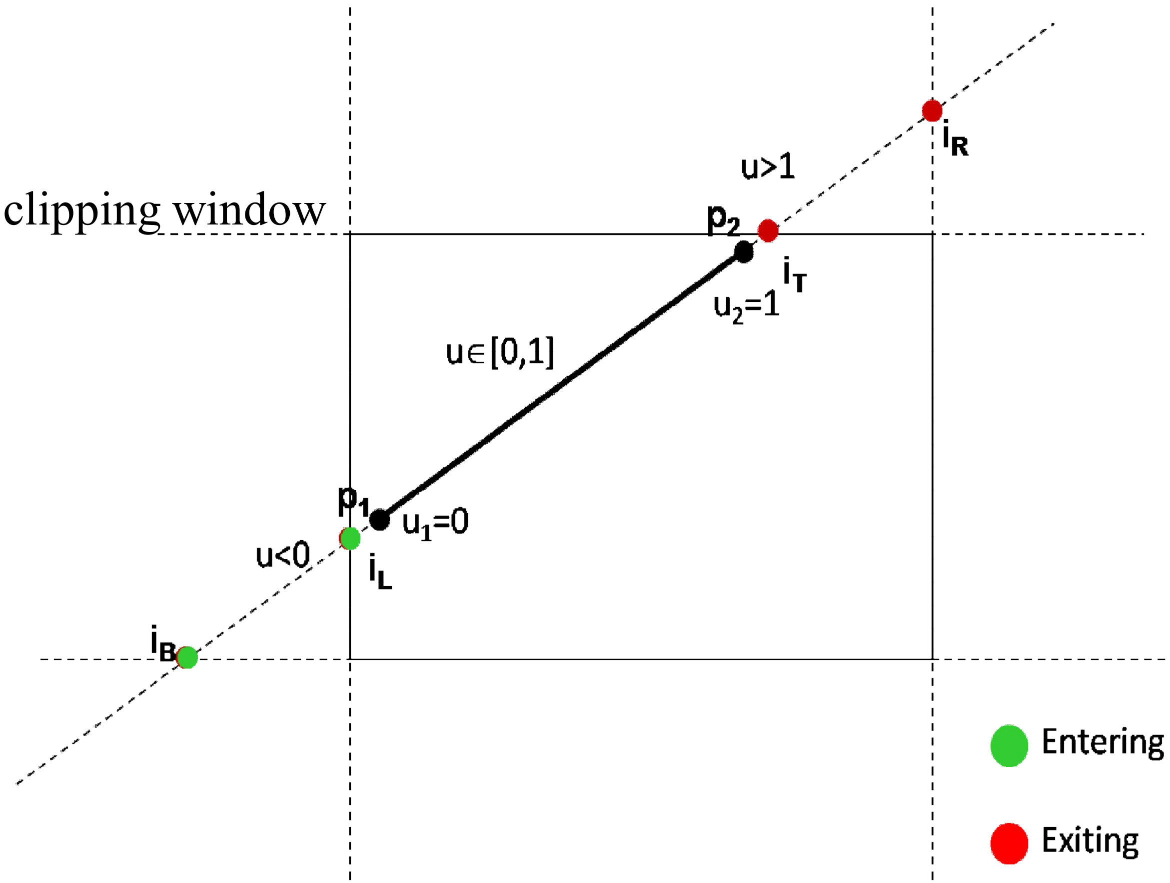
$P_k = 0$ Line is parallel to clipping window

$P_k > 0$ Line is exiting

$P_k < 0$ Line is entering

u_{max} Min(Exiting u,1)

u_{min} Max(Entering u,0)



ALGORITHM

1. Initialize $U_{min}=0$ and $U_{max}=1$
2. Calculate 'u' values (eg. u_{left} ,u_{right} ,u_{top} ,u_{bottom})
3. If $u<U_{min}$ or $u>U_{max}$ ignore it.

   Otherwise update U_{min} and U_{max}
4. If $U_{min} < U_{max}$

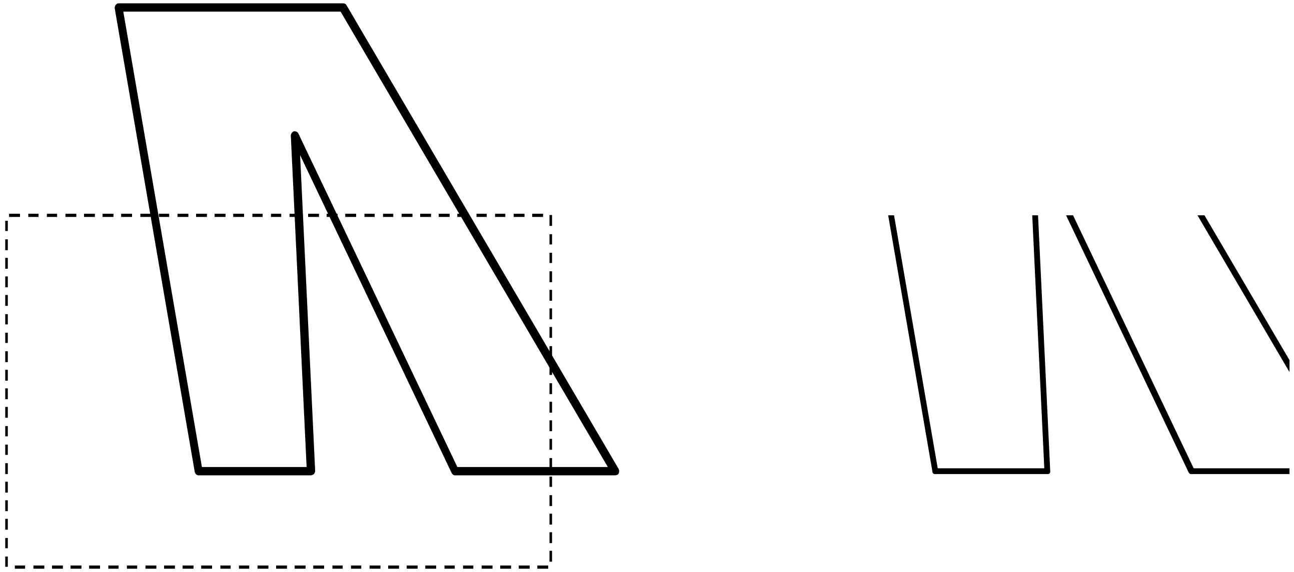
   Draw a line between the following points

   1. $(x_0+ U_{min}\,\Delta x,\ y_0+ U_{min}\,\Delta y)$
   2. $(x_0+ U_{max}\,\Delta x,\ y_0+ U_{max}\,\Delta y)$

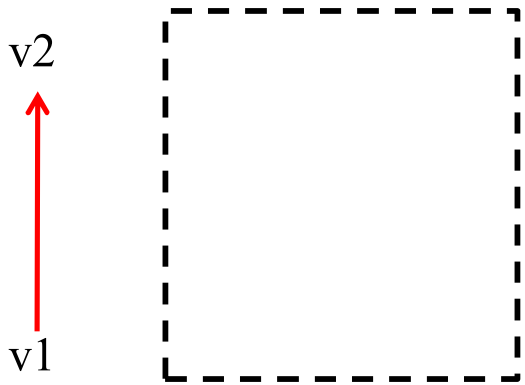
Otherwise if $U_{min} > U_{max}$

No line segment to draw.

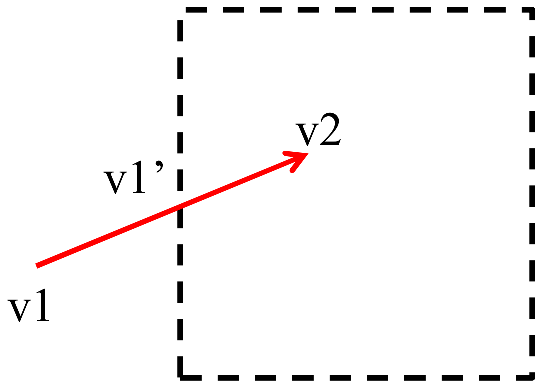
POLYGON CLIPPING



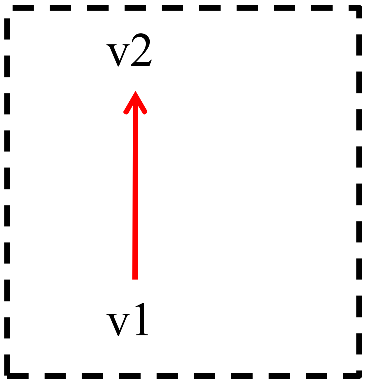
Sutherland-Hodgeman POLYGON CLIPPING algorithm



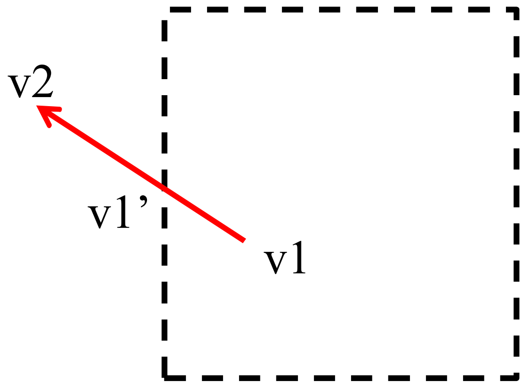
SAVE NOTHING



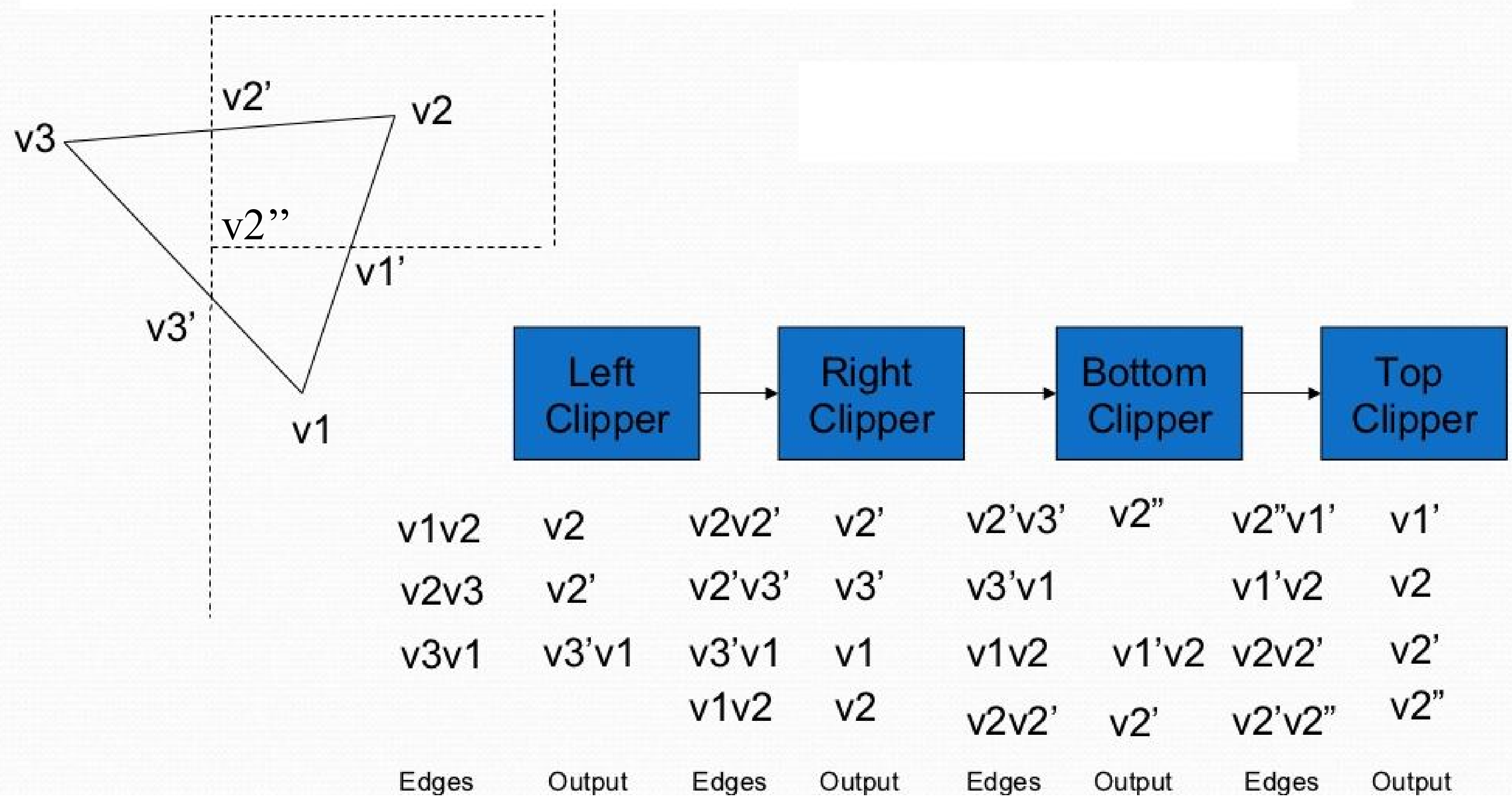
SAVE v1’and v2



SAVE v2



SAVE v1’


v3
v2'
v2
v2''
v1'
v3'
v1
Left Clipper
Right Clipper
Bottom Clipper
Top Clipper
v1v2
v2v3
v3v1
Edges
v2
v2'
v3'v1
Output
v2v2'
v2'v3'
v3'v1
v1v2
Edges
v2'
v3'
v1
v2
Output
v2'v3'
v3'v1
v1v2
v2v2'
Edges
v2''
v1'v2
v2'
Output
v2''v1'
v1'v2
v2v2'
v2'v2''
Edges
v1'
v2
v2'
v2''
Output

END OF CHAPTER