Counting - 1

Wednesday, November 11, 2020 10:09 AM

Pigeonhole principle

Theorem: If k is a positive integer and $(k+1)$ or more objects are to be placed in k boxes, there there is at least one box which Contains two or more objects.

Proof: We will proof by contradiction. S'pose that one of the k boxes contain more than one object.
Then, ~~the total no. of objects~~ would be at most k.
This is a contradiction, because there are $(k+1)$ objects.

Generalized PH

Proof: We will be using the proof by contradiction.
Suppose none of the boxes contains more that $\lceil N/k \rceil - 1$ objects. Then the total no. of objects is at most

$$k \cdot \left(\lceil N/k \rceil - 1 \right)$$

$$< k\left(\left(\frac{N}{k} + 1\right) - 1\right)$$

$$< N$$

$$\lceil N/k \rceil \equiv \left(\frac{N}{k} + 1\right)$$

$\lceil N \rceil < \left(N + 1\right)$ has

$< N$

Here, the inequality $\lceil \frac{N}{k} \rceil < \left(\frac{N}{k}+1\right)$ has been used. This is a contradiction because there are a total of N objects.

$\lceil N/k \rceil \geq r.$

The smallest integer N with $\frac{N}{k}+1 > r$, namely,

$\boxed{N = k.(r-1)+1}$ satisfies the inequality

$\lceil N/k \rceil \geq r.$

Is $\boxed{N = k(r-1)+1}$ the smallest integer?

→ Value smaller than N is not suffice, because if we had $k.(r-1)$ objects, then we could put $(r-1)$ objects in each of k boxes and no box would have r objects.

A, B, C, D, F

Prob: Possible grades, $k = 5$

Minimum No. of students getting the same grade $\Rightarrow r = 6.$

$\therefore$ Minimum no. of students in the class, $N = k(r-1)+1$

$= 5.(6-1)+1$

$= \underline{26}.$

Prob.

No. of suits, $K = 4$.

Min. no. of cards to be chosen from same suit, $r = 3$.

$\therefore$ Min. No. of cards to be chosen, $N = K(r-1)+1$

$= 4.(3-1)+1 = \underline{9}$

In the worst case, we can select all the clubs, diamonds, and spades, 39 in all, before we select a single heart. the next three cards will be definitely all hearts, So we may need $(39+3)$ i.e. 42 cards to get three hearts.

Prob.

Possible remainder, $K = 4$ $(0, 1, 2, 3)$

No. of integers, $N = 5$.

$\therefore$ If r is the least no. of integers which produces the same remainder then,

$$N = K(r-1)+1$$

or, $$5 = 4(r-1)+1$$

$$\Rightarrow \boxed{r = 2.}$$