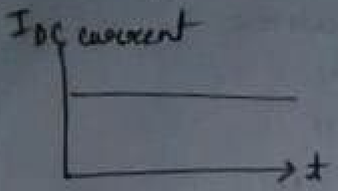
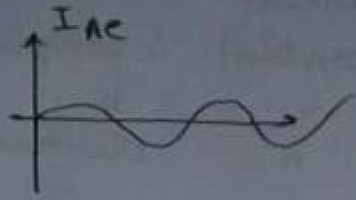
29.03.2020

# SIGNALS & SYSTEMS
## EE 2104

$i = \frac{dq}{dt}$ A ampere

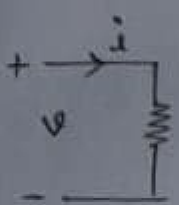



$V_{ab} = \frac{dw}{dq}$


$V_{ab} = V$
$V_{ba} = -V$

power =, $P = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v.i$




$p = v.i = +ve$


$P = v.(-i)$
$= -ve$

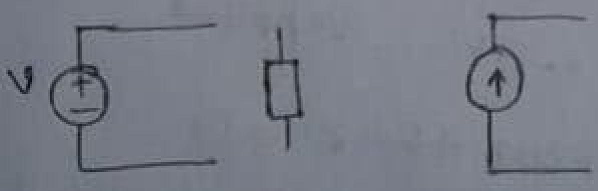
## Circuit elements

Active element → capable of generating energy
eg- battery,

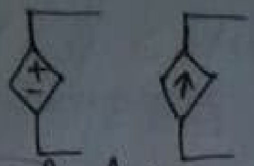
Passive element →
eg- resistor, capacitor, inductor.

## Sources

independent source →
does not depent on source.
v not changes




dependent source → (control surce)
active element in which source contity is controled by another voltage or current



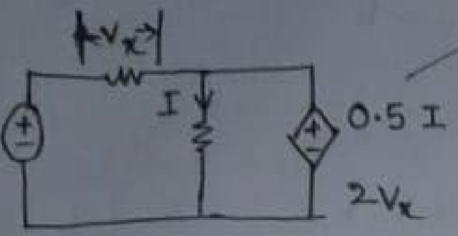
4 types of dep. sources-

VCVS → Voltage control Voltage source

CCVS → Current " " "

VCCS → Voltage " current "

CCCS → Current " " "

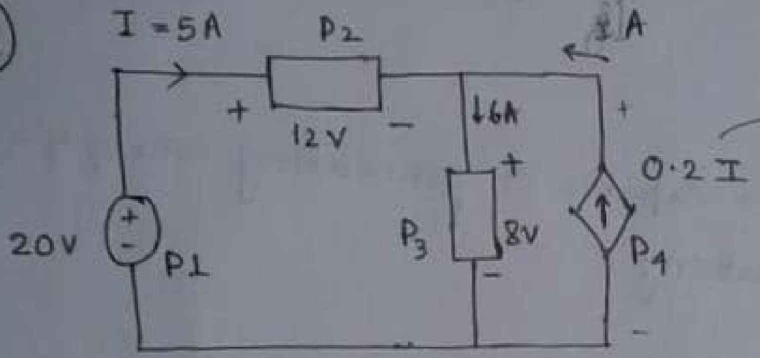



if I = 5 A, → 0.5 × 5

I = 10 A → 0.5 × 10

if $V_x$ = 20 V, $2V_x \times 20$

(P.1)




0.2 × 5
= 1 A

Calculate the power supplied or absorbed by each element as shown in fig

$P_1 = -100$ W $[20\times(-5)]$ (delivered power / supplying power)

$P_2 = 60$ W $(12\times5)$ absorbed power.

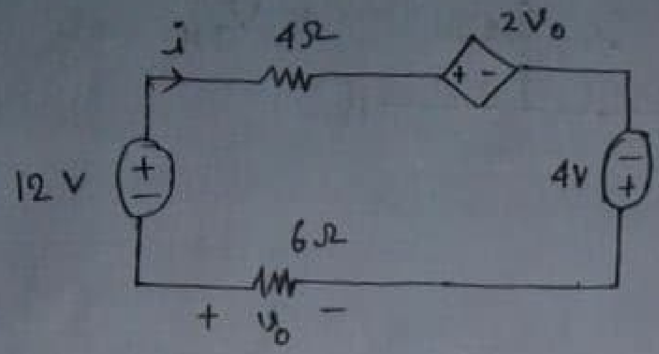
$P_3 = \cancel{5.2 \times 8}$ $\cancel{41.6}$ W  $8\times6 = 48$ W  absorbed

$P_4 = v \times i = 8 \times (-1) = -8$ W , supplied

$P_1 + P_2 + P_3 + P_4 = -100 + 60 + 48 - 8 = 0$

total power supplied = total power absorbed.

## P.2




Determine $v_0$ and $i$ in the circuit.

$$-12 + 4i + 2v_0 - 4 - V_0 = 0$$

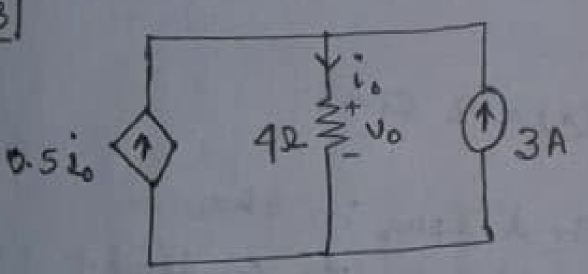
$$-12 + 4i - 12i - 4 + 6i = 0 \qquad [\because v_0 = 6\times(-i) = -6i$$

$$-2i = 16$$

$$i = -8\ \text{A}$$

$$v_0 = -6i = 48\ \text{V}$$

## P.3




Find $i_0$ & $V_0$

$\cancel{0.5 i_0 \times V_0 + 3V_0 + 4i_0 = 0}$

$$V_0 = 4i_0$$

$\cancel{3V_0 + 4i_0} = 0$

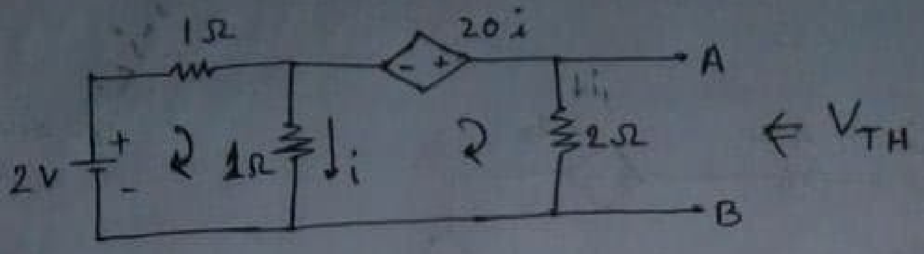
$\cancel{12 i_0}$

$$3 + 0.5 i_0 = i_0$$

$$i_0 = 6\ \text{A}$$

$$\therefore V_0 = 24\ \text{V}$$

P.4




Find $V_{TH}$

$V_{TH} = 2 \times i_1$

KVL

$2 = 1(i+i_1) + i$

$2i + i_1 = 2$

$1i = -20i + 2i_1$

$21i = 2i_1$

$\rightarrow i = 0.16$

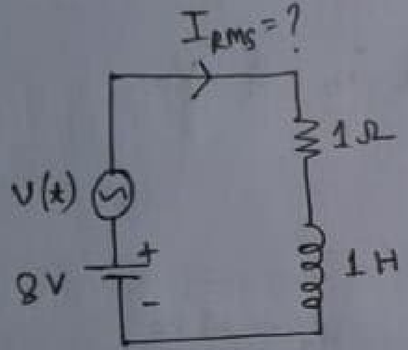
$i_1 = \frac{21}{2} \times 0.16$

$= 1.68$ A

$V_{TH} = 2 \times 1.68$

$= 3.36$ V

P.5




$v(t) = 12 \sin t$

Find $I_{RMS}$ in the stedy state condition.

only 8V,

$I_{dc} = \frac{8}{1} = 8$ A

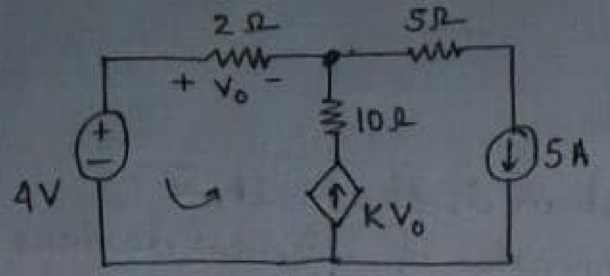
$X_L = \omega L$
$= 1 \times 1 = 1$

$I_{AC} = \frac{12}{\sqrt{1^2+1^2}} = 8.485$ A

$I_{RMS} = \sqrt{I_{dc}^2 + \left(\frac{I_{AC}}{\sqrt{2}}\right)^2} = \sqrt{8^2 + \left(\frac{8.485}{\sqrt{2}}\right)^2}$

$= 9.99$ A

P.6




K = ?

Power dissipated in the 2Ω resistance is 12.5 W

$12.5 = i^2 \times 2$

$i^2 = \frac{12.5}{2}$

$i = 2.5$

$12.5 = V_o \cdot 2.5$

$V_o = 5$

$4 + 2K + 10 = 0$

$-2 \times 2.5 + 4 + 2K + 10 \times (2.5-5) =$

$-5 + 4 + 2K ~~5~~ - 25 = 0$

$\therefore 2K = 2$

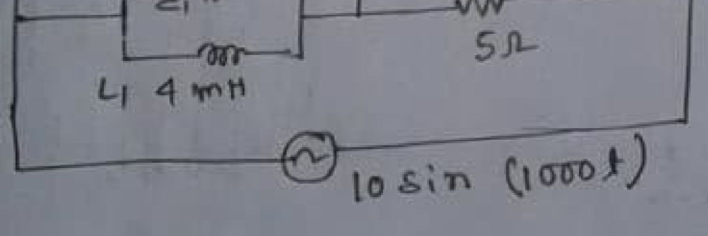
KCL

$2.5 + KV_o = 5$

$K = \frac{2.5}{5} = 0.5$

$R + jX_L$
$R - jX_C$

P.7




Peak value of current through 1Ω

$\omega = 1000$

$X_{C1} = \frac{1}{1000 \times 250 \times 10^{-6}} = 4\,\Omega$

$X_{C2} = \cancel{1000 \times 4 \times 10^{-3}} = 500\,\Omega$

$X_{L1} = 4\,\Omega$

$X_{L2} = 500\,\Omega$

$Z_1 = (-j4) \parallel (j4) = \infty$, open circuit

$Z_2 = (-j500) \parallel (j500) = \infty$ ,,

$10 = i \times 10$
$i = 1$ A

$i_{1\Omega} = \frac{10 \sin(1000t)}{10}$

$= \sin(1000t)$

$(i_{peak})_{1\Omega} = 1$ A

26.08.2020

* What are signals & systems

Defination of a system is that it is meaningfull interconnection of phisical devices of components.

exitation (response)

Input signal $\longrightarrow$ $\boxed{\text{System}}$ $\longrightarrow$ output

$f(x_1, x_2, x_3 \ldots x_n)$ $\quad$ $y(x_1, x_2, x_3 \ldots x_n)$

$x(t)$ $\quad$ $y(t)$

Electrical $\longrightarrow$ $\boxed{\text{Motor}}$ $\longrightarrow$ mechanical.

system

Input signal $\longrightarrow$ $\boxed{\text{system}}$ $\longrightarrow$ output

given $\quad$ given $\quad$ ??

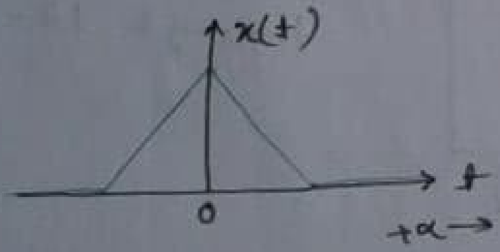
Analisis
solution always exist.

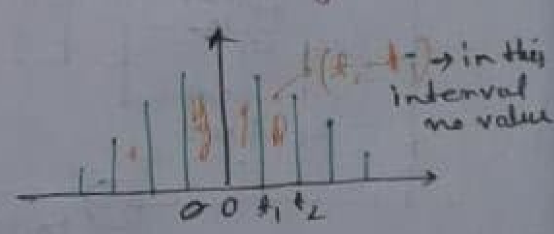
Input $\longrightarrow$ $\boxed{\text{System}}$ $\longrightarrow$ output

given $\quad$ ?? $\quad$ given

Synthesis
Solution may or may not exist.

Continuous time Signal (CTS) $\quad$ Discrete time signal (DTS)




$t_2 - t_1 \rightarrow 2\,ms$ $\quad$ 2 ms

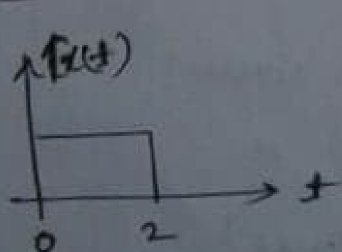
$t_3 - t_2 \rightarrow \cancel{5\,ms}$ $\quad$ 2 ms

$t_4 - t_3 \rightarrow 6\,ms$ $\quad$ 2 ms

Time shifting

$x(t) \xrightarrow[K=\text{Const}]{TS} y(t) = x(t+K)$

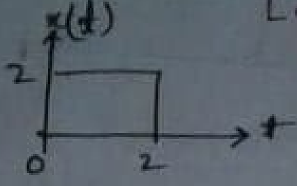
$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$

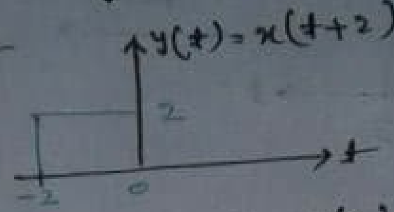


Case I : $k > 0$ $\underline{k \text{ is } +ve}$ $k = +2$

$y(t) = x(t+2)$

Left shifting

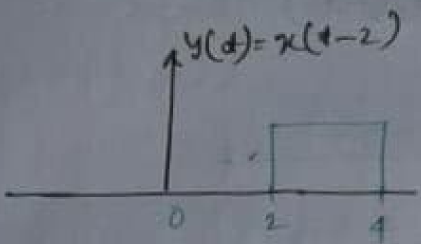



$x(0) = 2$ → $t = -2$, $y(t) = x(0)$

$x(2) = 2$ → $t = 0$, $y(t) = x(2)$

Case II : $k < 0$ $\underline{k \text{ is } -ve}$ $y = x(t-2)$




Right shifting / Time delay

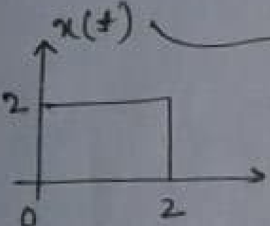
$t = 0$, $y(t) = x(-2) = 0$

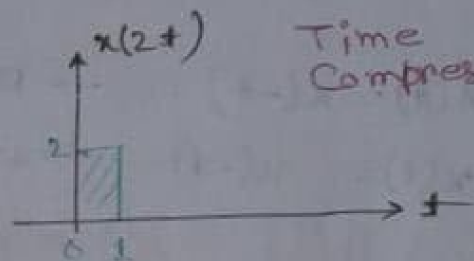
$t = 2$, $y(t) = x(0) = 2$

$t = 4$, $y(t) = x(2) = 2$

<u>Time scaling</u> compression or expansion of a signal in time

$x(t) \longrightarrow \xrightarrow{2 \times t}$




Time Compression

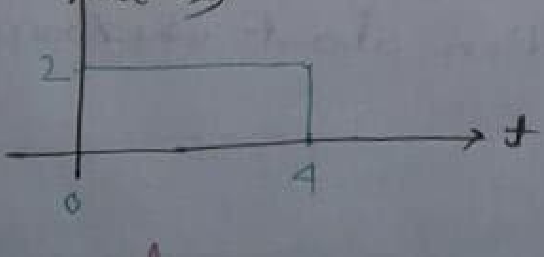
$x(0) = 2$

$x(2) = 2$

$t = 0$ $x(2 \cdot 0) = x(0) = 2$

$t = 1$ $x(2 \cdot 1) = x(2) = 2$

$x(t) \xrightarrow{0.5} x(0.5t)$ Time expansion




$t = 0$ $x(0) = 2$

$t = 4$ $x(0.5 \times 4)$

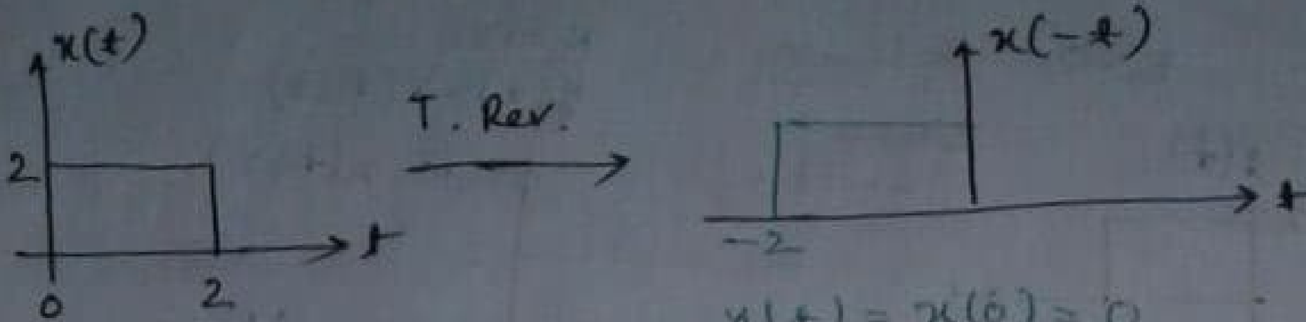
$= x(2)$

<u>Time Reversal</u>

$x(t) \longrightarrow x(-t)$

$x(t) \xrightarrow{T.S} y(t) = (x t)$

$= x((-1)t)$

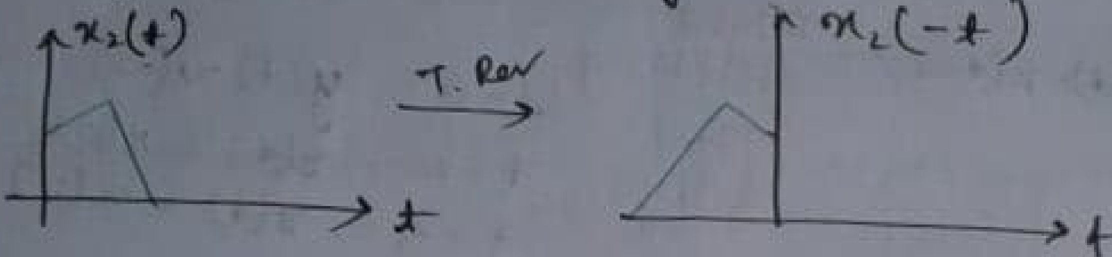



$x(2) = 2$

$y(t) = x(0) = 0$

$t = -2 \quad y(t) = x(-1 \times -2)$
$= x(2) = 2$

It is a reflection of the original signal about the y axis.

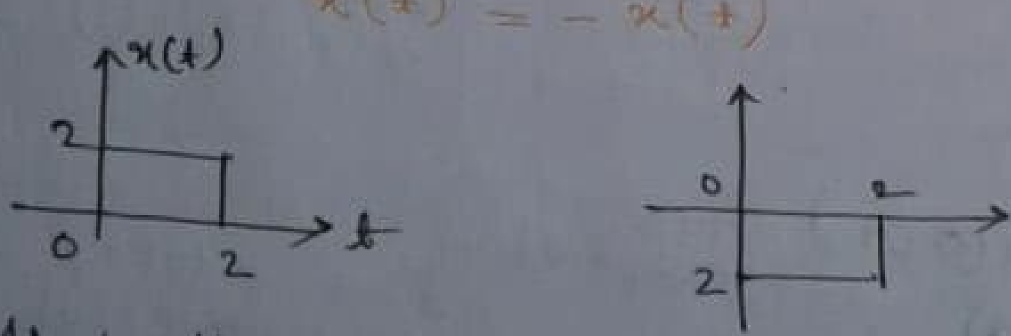




If $x(t) = x(-t)$ → Even signal
$x(t) = -x(-t)$ → odd signal.

Amplitude scaling / reversal

$x(t) = -x(t)$



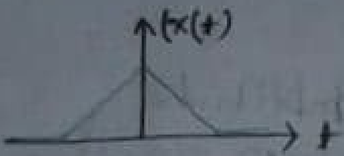
28.08.2020

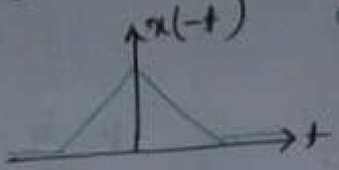
## Even Signal

$\boxed{x(t) = x(-t)}$ for all $t$

$x(t) \xrightarrow{\text{Time Rev}} x(-t)$

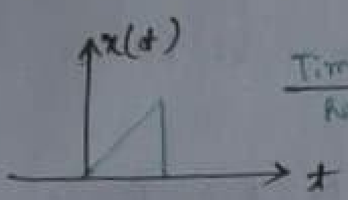
Remains identical under folding operation



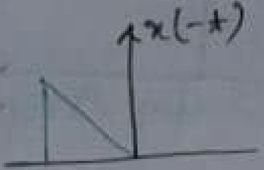



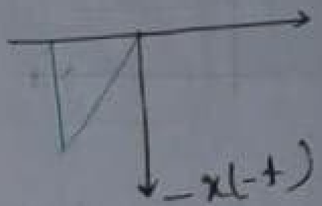

## Odd Signal

$x(t) = -x(-t)$ folded it and $-$ve.




Time Reversal




① the odd signals must be zero at $t=0$

② Avg or mean of any odd signals is equals to zero

General $x(t)$ can be rewritten as even and odd parts

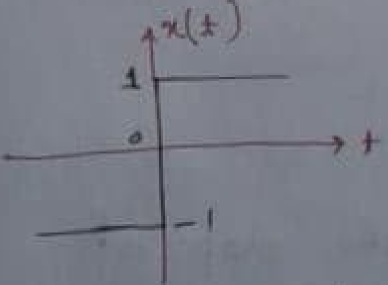
$$x(t) = Ev[x(t)] + od[x(t)] \quad \text{——— ①}$$

$$x(-t) = Ev[x(-t)] + od[x(-t)]$$

$$= Ev[x(t)] - od[x(t)] \quad \text{——— ②}$$

$$\left.\begin{aligned} Ev[x(t)] &= \frac{x(t)+x(-t)}{2} \\ od[x(t)] &= \frac{x(t)-x(-t)}{2} \end{aligned}\right\} \text{true for any arbitrary signals}$$

(P.1)




Even and odd parts of a unit step function $u(t)$ are respectively

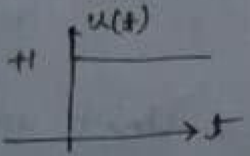
A) $\frac{1}{2}, \frac{1}{2}x(t)$  C) $\frac{1}{2}, -\frac{1}{2}x(t)$

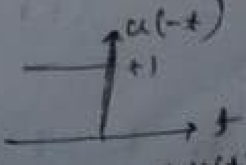
B) $-\frac{1}{2}, \frac{1}{2}x(t)$  D) $-\frac{1}{2}, -\frac{1}{2}x(t)$

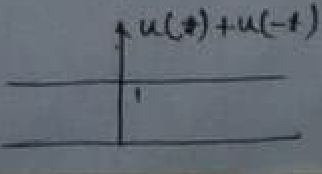
$$x_e(t) = \frac{x(t)+x(-t)}{2}$$

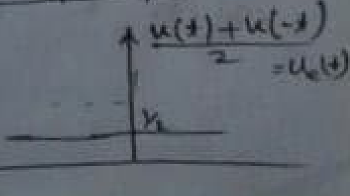
$$u_e(t) = \frac{u(t)+u(-t)}{2}$$

$$u_e(t) = \frac{1}{2}$$



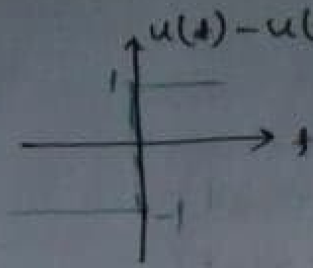



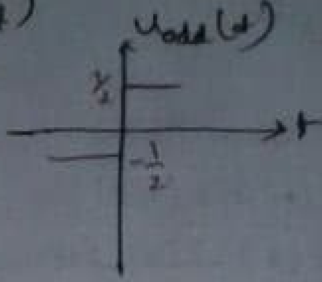



$$U_{odd}(t) = \frac{u(t) - u(-t)}{2}$$

$$U_{odd}(t) = \frac{1}{2} x(t)$$

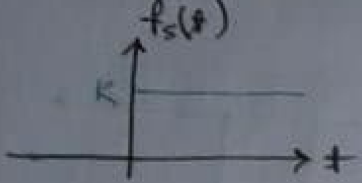


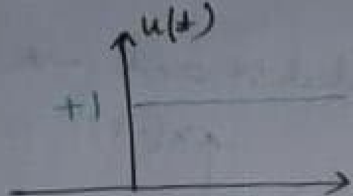




## Step Signal

$$f_s(t) = \begin{cases} 0 & t<0 \\ k & t>0 \end{cases}$$

k is the amplitude of the step signal.



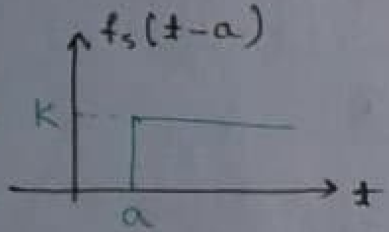




unit step signal.

$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$$

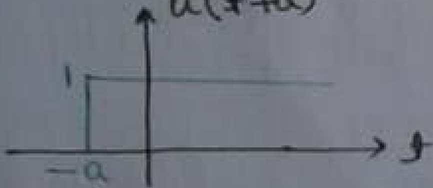
at $t=0$ the function is undified and discontinioue.

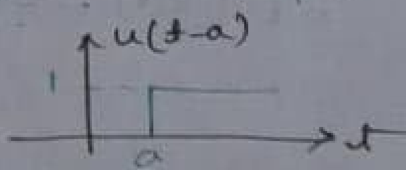
## Shifted Step Signal




$$f_s(t-a) = \begin{cases} 0 & t<a \\ k & t>a \end{cases}$$

if $k=1$, then it is a shifted unit step signal



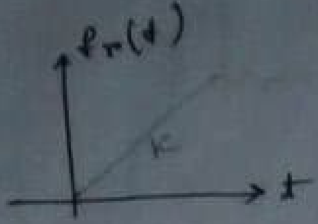



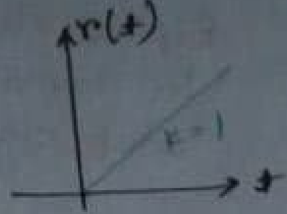

## Ramp Signal

$$f_r(t) = \begin{cases} 0 & t<0 \\ kt & t\geq 0 \end{cases}$$

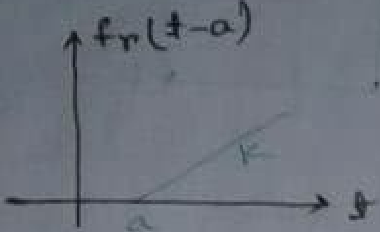
k is the slope of ramp signal.

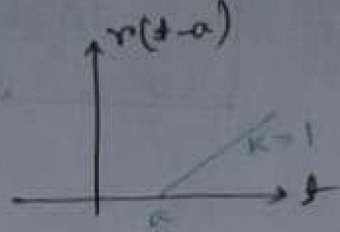
when $k=1 \rightarrow$ Unit ramp signal.




## Shifted Ramp Signal




$$f_r(t-a) = \begin{cases} 0 & t < a \\ k(t-a) & t \geq a \end{cases}$$

$$r(t-a) = \begin{cases} 0 & t < a \\ t-a & t \geq a \end{cases}$$

ramp fr

$$r(t) = t\,u(t)$$
$$r(t-a) = (t-a)\,u(t-a)$$

step fn

$$f_r(t) = k\,r(t) = kt\,u(t)$$
$$f_r(t-a) = k\,r(t-a) = k(t-a)\cdot u(t-a)$$

(I) $f(t) = \sin\omega t$

(II) $f(t)\cdot u(t) = \sin\omega t \cdot u(t)$

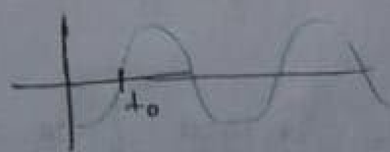
(III) $f(t-t_0) = \sin\omega(t-t_0)$

(IV) $f(t-t_0)\cdot u(t) = \sin\omega(t-t_0)\cdot u(t)$

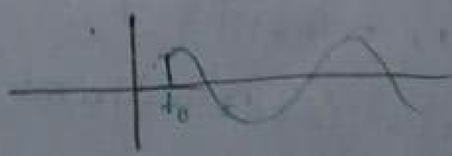
(V) $f(t)\cdot u(t-t_0) = \sin\omega t\cdot u(t-t_0)$

(VI) $f(t-t_0)\cdot u(t-t_0) = \sin\omega(t-t_0)\cdot u(t-t_0)$

(I)


(IV)


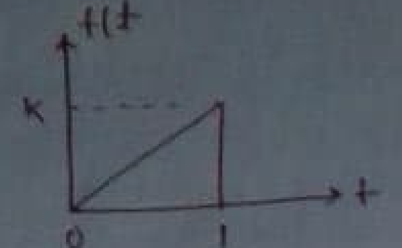

(II)


(V)


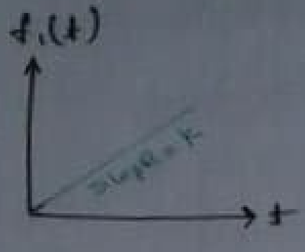

(III)

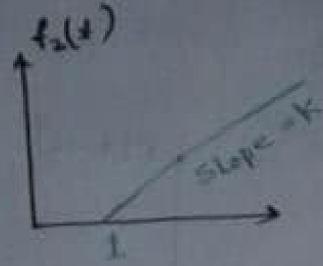


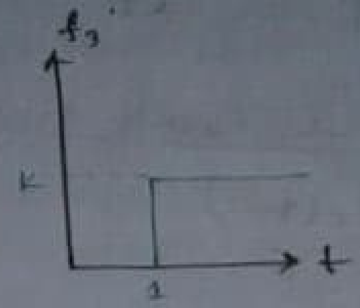
(VI)

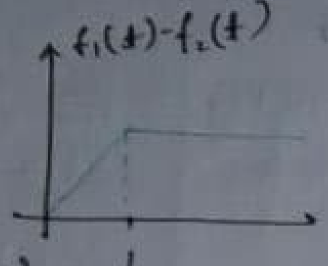

(P.1) 


Express the given fig in terms of ramp & step signals.

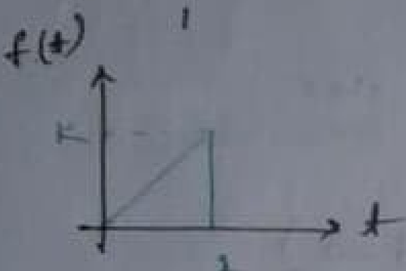



$-f_3(t)$




$$\begin{aligned} f(t) &= f_1(t) - f_2(t) - f_3(t) \\ &= kr(t) - kr(t-1) - kU(t-1) \\ &= kt\,u(t) - k(t-1)\,u(t-1) - kU(t-1) \\ &= k\left[t\,u(t) - (t-1+1)\,u(t-1)\right] \\ &= k\left[t\,u(t) - t\,u(t-1)\right] \\ &= kt\left[u(t) - u(t-1)\right] \end{aligned}$$

## Periodic & Aperiodic Signal.

$x(t) = x(t+T)$

The smallest value of T which satisfies the above condition is called fundamental time period. $f = \frac{1}{T}$. $\omega = 2\pi f = \frac{2\pi}{T}$

$$\left.\begin{aligned} x_1(t) &= \sin 100t \\ x_2(t) &= \cos 100t \end{aligned}\right\} x = x_1(t) + x_2(t)$$

$x(t) = \cos 4t + \sin \pi t$

Method 1 Consider two signal $x_1(t)$ & $x_2(t)$ with periods $T_1$ & $T_2$ respectively. When they are summed the resultant signal is said to be periodic when $\frac{T_1}{T_2}$ = Rational number.

Method 2 : GCD of frequency is possible then we can say they are periodic signals

Ex-1

$x(t) = \cos 4t + \sin \pi t$

$\omega_1 = 4$ $\quad \omega_2 = \pi$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4} = \frac{\pi}{2}$

$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{2} = 2$

$\frac{T_1}{T_2} = \frac{\pi}{4}$ (not a rational number)

So it is a aperiodic signal

Ex-2. $x(t) = \cos 2\pi t + \cos 4\pi t$

$\omega_1 = 2\pi$ $\quad \omega_2 = 4\pi$

$T_1 = \frac{2\pi}{2\pi} = 1$

$T_2 = \frac{2\pi}{4\pi} = \frac{1}{2}$

$\frac{T_1}{T_2} = 2$ (rational)

$\therefore$ periodic

$f_1 = 10$
$f_2 = 45$

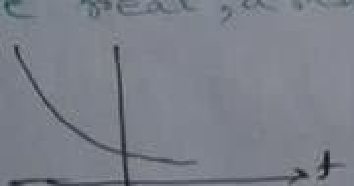
GCD(10, 45) periodic | GCD(2, 3.33) not posible, aperiodic

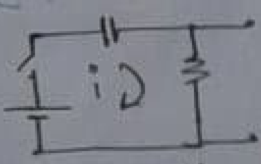
## Exponential Signal (CTS)

$Ce^{at}$ where C & a can be real, +ve, −ve and complex also.

C real, a real and +ve

C real, a real and −ve





$i = \frac{E}{R} e^{-t/\tau}$ $\quad (\tau = RC)$

C real, a is purely imaginary

$x(t) = e^{j\omega_0 t}$

$x(t) = x(t+T)$

$e^{j\omega_0 t} = e^{j\omega_0 (t+T)} = e^{j\omega_0 t} \cdot e^{j\omega_0 T}$

for periodicity $e^{j\omega_0 T} = 1$

$\omega_0 T = 2\pi$ $\quad \therefore T = \frac{2\pi}{\omega_0}$

→ is the fundamental

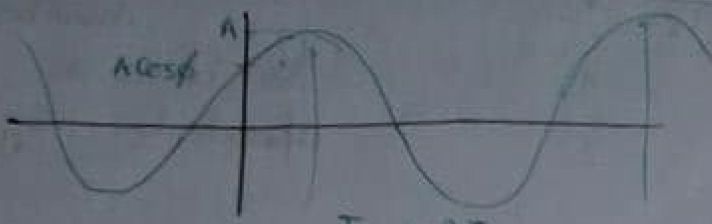
$e^{-j\omega_0 T}$

$e^{j\omega_0 T} = \cos\omega_0 t + j\sin\omega_0 t$

$x(t) = A\cos(\omega_0 t + \phi) = \frac{A}{2} e^{j\phi} e^{j\omega_0 t} + \frac{A}{2} e^{-j\phi} e^{-j\omega_0 t}$

Real part of

$= A\,\mathrm{Re}\left\{ e^{j(\omega_0 t + \phi)} \right\}$




$T_0 = \frac{2\pi}{\omega_0}$

$\cos x = \frac{e^{jx} + e^{-jx}}{2}$

$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$

$x_1(t) = e^{j\omega_0 t}$

$x_2(t) = e^{jm\omega_0 t}$, where $m$ is an integer.

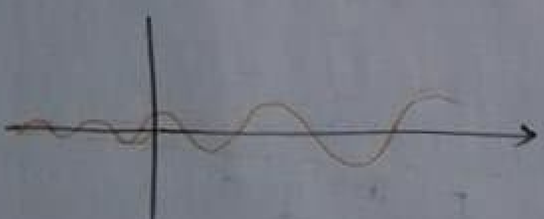
$f_1 = \frac{\omega_0}{2\pi}$ $\qquad f_2 = \frac{m\omega_0}{2\pi} = mf_1$

$x_2$ is the mth harmonic of $f_1$

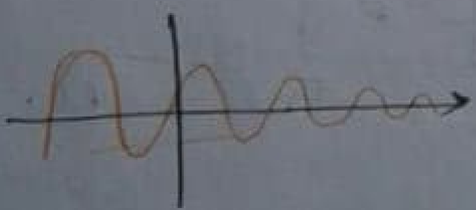
$$\left.\begin{aligned} c\cdot e^{at} &= |c|\cdot e^{j\theta}\cdot e^{(r+j\omega_0)t} \\ &= |c| \cdot e^{rt}\cdot e^{j(\omega_0 t + \theta)} \end{aligned}\right\}$$

where
$a = r + j\omega_0$
$c = |c|\cdot e^{j\theta}$

$= |c|\, e^{r(t)} \cos(\omega_0 t + \theta) + j\,|c|\cdot e^{rt}\cdot \sin(\omega_0 t + \theta)$



for $r > 0$

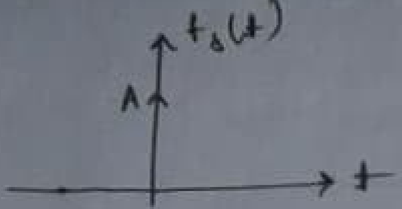


for $r < 0$

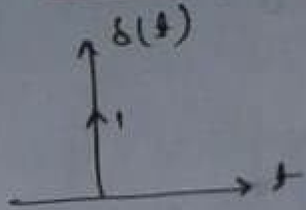
## Impulse signal

$$f_\delta(t) = \begin{cases} 0 & t \neq 0 \\ A & t = 0 \end{cases}$$

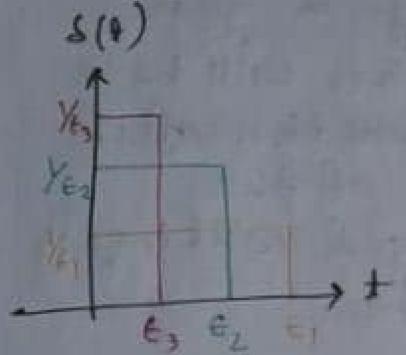
where A is the area of the impulse signal and it is also called strength of impulse.




A = 1




$$\text{Area} = \int_{-\alpha}^{\alpha} \delta(t) \cdot dt = 1$$




where $\epsilon_1 > \epsilon_2 > \epsilon_3$

## Shifted impulse signal

$$f_\delta(t-a) = \begin{cases} 0 & t \neq a \\ A & t = a \end{cases}$$

shifted unit impulse

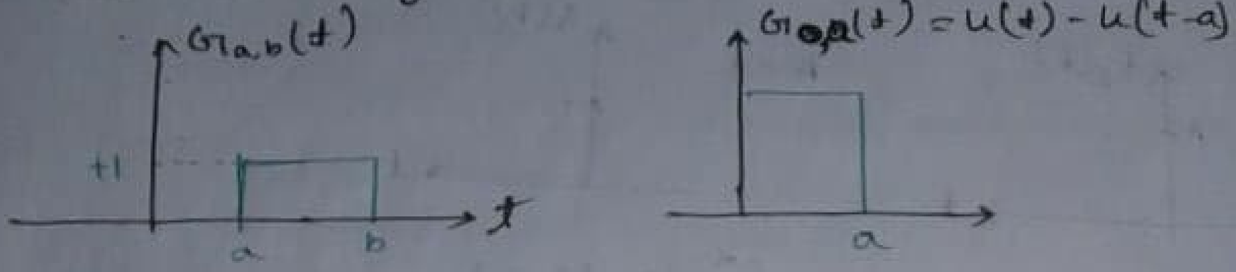
$$\delta(t-a) = \begin{cases} 0 & t \neq a \\ 1 & t = a \end{cases}$$

Relationship between step, ramp, impulse
Derivative of step signal = Impulse signal
" " ramp signal = step signal

$$\frac{d}{dt}[f_s(t)] = f_\delta(t) \quad \text{or} \quad \int f_\delta(t)\, dt = f_s(t)$$

$$\frac{d}{dt}[f_r(t)] = f_s(t) \quad \text{or} \quad \int f_s(t)\, dt = f_r(t)$$

Gate Signal : A rectangular pulse at unit height starting at $t = a$ and ending at $t = b$ as shown in fig and represented as $G_{a,b}(t) = u(t-a) - u(t-b)$ is called a gate function.




If any function multiplied by a gate fuction, then that function will have zero value outside the duration of the gate and the value of the function will be unchanged withing the duration of the gate.

Direct formula:

If a function is a combination of various gate function, then we can developed a formula to represent the function directly in terms of step function. This formula is called the direct formula.

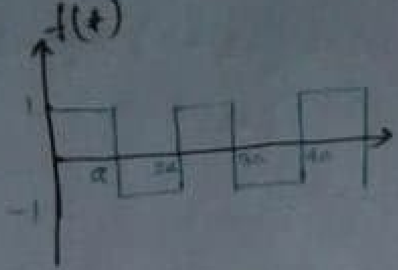
$$f(t) = \sum_{T=0}^{\infty} (A_f - A_i)\, u(t-T)$$

where $A_f$ is the final value at the corresponding time instant and

$A_i$ is the inital value at ...

$T$ is the time instant at which function $f(t)$ changes its values.

(P.1)




Synthesize the webform using standard signal

using Gate function

$$f(t) = G_{0,a}(t) + (-1)\,G_{a,2a}(t) + G_{2a,3a}(t) + (-1)\,G_{3a,4a}(t) + \dots$$

$$= 1[u(t)-u(t-a)] + (-1)[u(t-a)-u(t-2a)] + 1[u(t-2a)-u(t-3a)] + (-1)[u(t-3a)-u(t-4a)]$$

$$= u(t) - 2u(t-a) + 2u(t-2a) - 2u(t-3a) + \dots$$

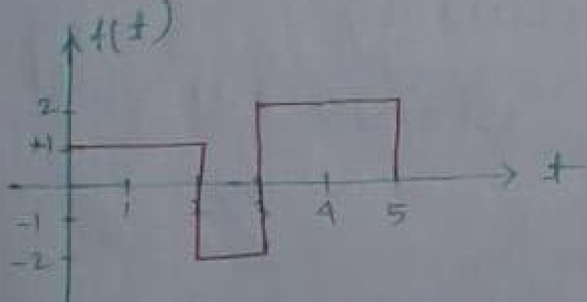
using Direct formula,

$$f(t) = \sum_{T=0}^{\infty} (A_f - A_i)\,u(t-T)$$

$$= (1-0)\,u(t-0) + (-1-1)\,u(t-a) + [1-(-1)]\,u(t-2a) + (-1-1)\,u(t-3a) + \dots$$

$$= u(t) - 2u(t-a) + 2u(t-2a) - 2u(t-3a) + \dots$$

(P.2)




Synthesize the webform using step function

Gate function,

$$f(t) = G_{0,2}(t) + (-2)\,G_{2,3}(t) + (+2)\cdot G_{3,5}(t)$$

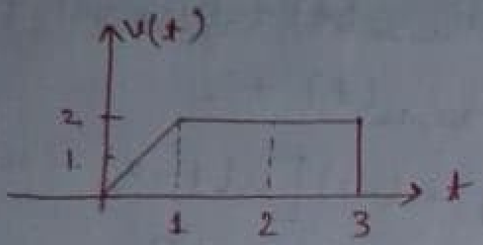
$$= 1[u(t)-u(t-2)] + (-2)[u(t-2)-u(t-3)] + 2[u(t-3)-u(t-5)]$$

$$= u(t) - 3u(t-2) + 4u(t-3) - 2u(t-5)$$

Direct, $f(t) = (1-0)u(t-0) + (-2-1)u(t-2)$
$+ (2-(-2)]u(t-3)$
$+ (0-2)u(t-5)$

$= u(t) - 3u(t-2) + 4u(t-3) - 2u(t-5)$

P.3




Express the given waveform using standard signal.

1) Gate function,

$v(t) = 2t\, G_{0,1}(t) + 2\, G_{1,3}(t)$

$= 2t[u(t) - u(t-1)] + 2[u(t-1) - u(t-3)]$

$= 2t\,u(t) - 2t\,u(t-1) + 2u(t-1)$
$- 2u(t-3)$

$= 2t\,u(t) - 2(t-1)\,u(t-1) - 2u(t-3)$

2) ramp, step

$v(t) = 2[r(t) - r(t-1)] - 2u(t-3)$

$\Rightarrow r(t) = t\,u(t)$

$r(t-1) = (t-1)\,u(t-1)$

$v(t) = 2t\,u(t) - 2(t-1)u(t-1) - 2u(t-3)$

31.08.2020

## Basic System Properties.

1) Static and dynamic system
2) Causal and non-causal system
3) Time Invariant and time variant system
4) Linear and non-linear system
5) Invertible and non invertible system
6) Stable and unstable system.

i/p $\xrightarrow{x(t)}$ $\boxed{\text{System}}$ $\longrightarrow$ o/p $y(t)$

must have finite amplitude from $-\infty$ to $+\infty$

must have finite amp. from $-\infty$ to $+\infty$

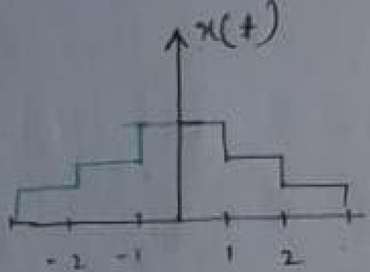
(i) $y(t) = x(t) \longrightarrow t=0,\ x(0) = y(0)$

(ii) $y(t) = x(t-1) \longrightarrow t=0\ \ x(-1) = y(0)$ past i/p

$x(1) = y(2)$

(iii) $y(t) = x(t+1) \longrightarrow t=0,\ x(+1) = y(0)$ future i/p

$y(2) = x(3)$




Static system: o/p of system depends only in present value at i/p.

Dynamic System: o/p of system depends on past or future value of i/p at any instant of time.

Ex - 1.

$y(t) = 2x(t) = f\,x(t)$

$t = 0 \quad y(0) = 2 \cdot x(0)$

$t = 1 \quad y(1) = 2 \cdot x(1)$

$t = -1 \quad y(-1) = 2\ x(-1)$

static system

ex-2 :

$$y(t) = x(t) + x(t-1)$$

$t = 0$ $\quad \underbrace{y(0)}_{\text{Present}} = \underbrace{x(0)}_{\text{Present}} + \underbrace{x(-1)}_{\text{Past}}$ $\quad$ Dynamic System

depends on

* Static systems are also known memoryless system. Dynamic systems are known as with memory System.

ex-3: $y(t) = x(2t) \rightarrow$ dynamic system

$t = 0, \; y(0) = x(0)$ not a static system

$t = 1, \; y(1) = x(2)$

$t = -1, \; y(-1) = x(-2)$

ex-4: $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$ Dynamic (all integration based system)



ex-5: $y(t) = \frac{d}{dt} x(t)$

e-6:

$y(t)$

Causal system: o/p of system is ~~true~~ independent of future values of i/p

Noncausal system: o/p of system depends on the future values of i/p at any instant of time.

ex-1

$y(t) = x(t)$ → o/p depends on i/p only → causal.

Causal system.

future value of i/p should not be there.

→ o/p = i/p
→ o/p = Present + past
→ o/p = past value of i/p

ex-2:

$y(t) = x(t) + x(t-1)$

$t=0 \quad y(0) = x(0) + x(-1)$ → causal.

ex-3:

$y(t) = x(t) + x(t+1)$

$t=0, \quad y(0) = x(0) + \underbrace{x(+1)}_{\text{future}}$ → Non causal.

Anti-causal system. o/p depends only on future values at i/p. no past, no present value.

* all anticausal systems are non causal but all noncausal are not anticausal.

$y(t) = x(t+2)$ → anticausal system.

ex-4:

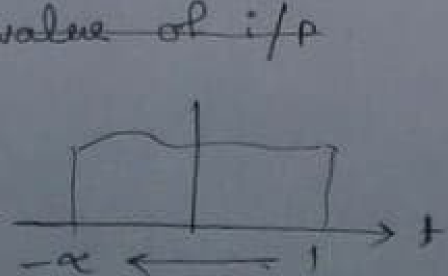
$y(t) = x(3t)$ → non-causal (not anticausal) because

$t=0 \quad y(0) = x(0)$ present

$t=1 \quad y(1) = \underline{x(3)}$ future.

ex-5: ~~o/p = i/p + past value of i/p~~

$y(t) = \int_{-\alpha}^{t} x(\tau)\,d\tau$



* $y(t) = \int_{-\infty}^{t+1} x(\tau)\, d\tau$ $\quad x(t+1) \rightarrow t=0 \quad x(1)$

future value of i/P

Non causal.

* $y(t) = \int_{-\infty}^{t-1} x(\tau)\, d\tau$ . $\quad x(t-1) \rightarrow t=0 \quad x(-1)$

→ Past

Past value of input

Causal.

* $y(t) = \int_{-\infty}^{t} x(3\tau)\, d\tau$ $\quad x(3t) \rightarrow t=0 \quad x(0)$

$t=1 \quad x(3)$

future value of i/P

Non Causal.

## Split system:

$$y(t) = \begin{cases} x(3t) & t<0 \\ x(t-1) & t \geq 0 \end{cases}$$

Causal

When $t<0$, $y(t) = x(3t)$

$t=-1 \quad y(-1) = x(-3)$ Causal

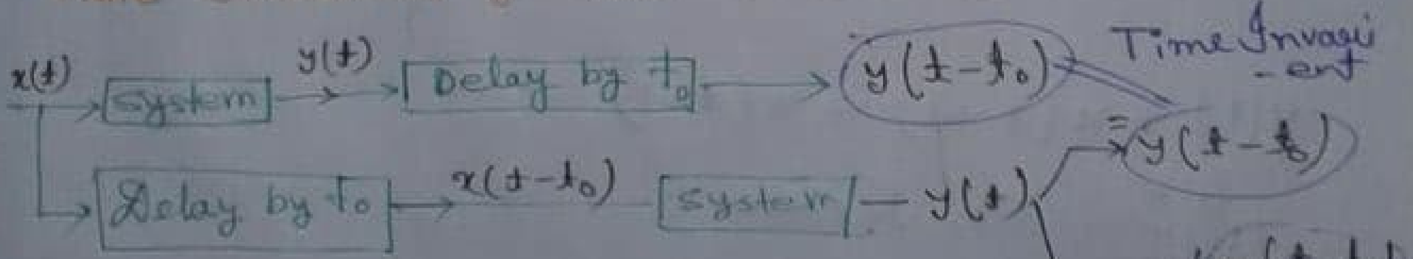
Past value of i/P

$t \geq 0 \quad y(t) = x(t-1)$

$t=0,\ y(0) = x(-1)$ Causal.

Past

## Time Invariant & Time variant system




any delay provided in input must be reflected in the output

ex-1: $y(t) = x(2t)$

$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t) = x(2t)$

PTO

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \qquad x(t)$$

$x(t) \to \square$

① ✓
② ✓
③ ✓
Time Invariant

$$y(t) = \int_{-\infty}^{t} x(3\tau)\, d\tau \qquad x(3t)$$

① ×
② ✓
③ ✓
Time variant

$$y(t) = \int_{-\infty}^{t} \cos\tau \cdot x(\tau)\, d\tau \qquad \cos t\, x(t)$$

① ✓
② ×
③ ✓
Time Variant

Split system

$$y(t) = \begin{cases} x(t-1) & t<0 \\ x(t+1) & t \geq 0 \end{cases}$$

Time Variant

① ✓
② ✓
③ ✓
Time ~~Invariant~~ variant

$$y(t) = a(t)\, x(t-1) + b(t)\, x(t+1)$$

$$a(t) = \begin{cases} 1 & t<0 \\ 0 & t \geq 0 \end{cases} \qquad b(t) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$$

Step 1: $y(t) \xrightarrow{t_0} y(t-t_0) = x[2(t-t_0)]$

time varient system $= x[2t-2t_0] \neq$

step-2: $x(t) \xrightarrow{t_0} x(t-t_0) \longrightarrow \text{system} \rightarrow x(2t-t_0)$

ex-2: $y(t) = 2 + x(t) \rightarrow$ time invarient

Step-1: $y(t) \xrightarrow{t_0} 2 + x(t-t_0)$ =

Step-2: $x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow \boxed{\text{Sys}} \rightarrow 2 + x(t-t_0)$

Conditions for time-invariant system:

1) No time scaling (i/p or o/p)

2) Co-efficient should be constant
Should not be function of time.

3) any added or substracted term in the system relationship (except i/p and o/p must be constant or zero)

ex-3:

$y(t) = \cos t_0 \, x(t)$ time varient

ex-4: $y(t) = e^{-2k} x(t)$ time invarient

$y(t) = 8\,x(t)$ time Invarient

$y(t) = x(t+1) + x(t-1)$

1 ✓
2 ✓
3 ✓

Time Invarient

$y(t) = x(2t+1) + x(2t-1)$

Time varient