Graph - 2

Monday, November 30, 2020 10:44 AM

Incidence matrix

Prob-4 (i)

The incidence matrix is:

$$\begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline v_1 & 1 & 1 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 1 & 1 & 0 & 1 \\ v_3 & 0 & 0 & 0 & 0 & 1 & 1 \\ v_4 & 1 & 0 & 1 & 0 & 0 & 0 \\ v_5 & 0 & 1 & 0 & 1 & 1 & 0 \end{array}$$

Prob. 5

Graph G & H have same no. of vertices, same no. of edges, and the degrees of the vertices are the same.
The function f with $f(u_1) = v_1$, $f(u_2) = v_4$, $f(u_3) = v_3$ and $f(u_4) = v_2$ is a one-to-one correspondence b/w V and W. (where $G = (V, E)$ and $H = (W, F)$).

This correspondence also preserves adjacency as, in G the vertex u_1 is adjacent to u_2 and u_3, vertex u_2 is adjacent to u_1 and u_4, vertex u_3 is adjacent to u_1 and u_4, and vertex u_4 is adjacent to u_2 and u_3.

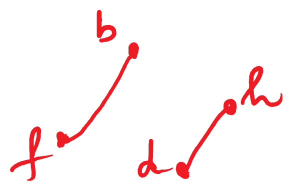
Prob-7

The graphs G and H both have eight vertices, 10 edges, and the same no. of vertices with degrees

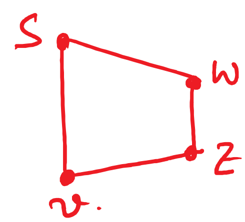
2 and 3. Because, these invariants will agree, it is still conceivable that these graphs are isomorphic.

The adjacency relationship is not preserved as in graph G, vertex a (deg(a) = 2) is adjacent to two vertices of degree 3. Whereas, all the two-degree vertices in H (t, u, x, and y) are adjacent to one three-degree vertex and one two-degree vertex

Subgraph of G



Subgraph of H.



G and H are not isomorphic as the subgraphs of G and H are made up of vertices of degree three are not isomorphic.

Prob-8

Graph invariants.

① No. of vertices : G = 6 ; H = 6.

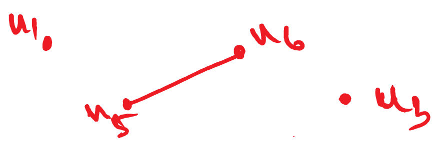
② No. of edges : G = 7 ; H = 7.

③ Degrees of the vertices : G → {deg(2) = 4 ; deg(3) = 2}
H → {deg(2) = 4 ; deg(3) = 2}

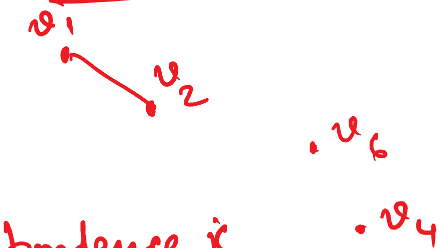
$H \rightarrow \{ \deg(2) = 4, \deg(3) = 2 \}$

Therefore, as G and H agree with respect to these invariants, it is reasonable to try and find out isomorphism f. (isomorphic function).

Subgraph of G (deg = 2)



Subgraph of H (deg = 2)

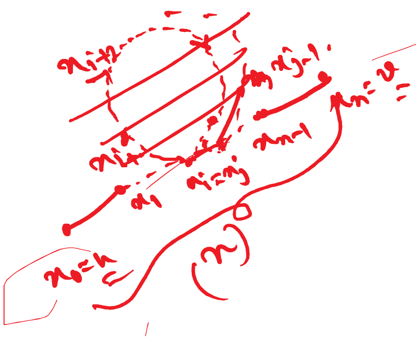


The possible one-to-one correspondence is as follows:

$f(u_3) = v_4$, $f(u_4) = v_5$, $f(u_5) = v_1$, $f(u_6) = v_2$, $f(u_1) = v_6$, and $f(u_2) = v_3$.

Theorem: There is a simple path between every pair of distinct vertices of a connected undirected graph.

Proof: Let u & v be two distinct vertices of a connected undirected graph $G = (V, E)$. Because, G is connected, there is at least one path between u and v. Let $x_0, x_1, \ldots x_n$, where $x_0 = u$ and $x_n = v$ be the vertex sequence of a path of least length. This path of least length is simple. To prove this, we suppose that it is not simple. Then $x_i = x_j$ for some i and j with $0 \le i < j$. This means there is a path from u to v of shorter length with vertex sequence $x_0, x_1, \dots, x_{i-1}, \dots x_j, \dots x_n$, obtained by deleting the edges corresponding to the vertex sequence $x_i, x_{i+1}, \dots x_{j-1}$.



Prob-9

- Cut vertices : b, c, e. (removal of one of these vertices and the adjacent edges disconnects the graph)

- Cut edges : {a, b} and {c, e} (removal of any one of them will disconnect the graph).

Vertex connectivity.

- Not all graphs have cut vertices.
- Complete graph K_n, where $n \geq 3$, has no cut vertices.
- If one of the vertices and its adjacent edges are removed the resulting subgraph is a complete graph K_{n-1}, a connected graph.
- Connected graphs with no cut vertex is called Inseparable graphs.

✓ connected graph is ...
Inseparable graphs.

* If G does not have a cut vertex, we look for the smallest set of vertices (vertex cut) that can be removed to disconnect it.

Vertex connectivity

✓ $K(K_n) = n-1$

✓ In general, $0 \leq K(G) \leq (n-1)$, if G has n vertices.

✓ The larger the value of $K(G)$, the more connected is the graph.