Let R be the relation on the set of real numbers such that aRb if and only if $a - b$ is an integer. Is R an equivalence relation?

Congruence Modulo m Let m be an integer with $m > 1$. Show that the relation

$$R = \{(a, b) \mid a \equiv b \pmod{m}\}$$

is an equivalence relation on the set of integers.

Suppose that R is the relation on the set of strings of English letters such that aRb if and only if $l(a) = l(b)$, where $l(x)$ is the length of the string x. Is R an equivalence relation?

Let R be the relation on the set of real numbers such that xRy if and only if x and y are real numbers that differ by less than 1, that is $|x - y| < 1$. Show that R is not an equivalence relation.

What are the equivalence classes of 0 and 1 for congruence modulo 4?

List the ordered pairs in the equivalence relation R produced by the partition $A_1 = \{1, 2, 3\}$, $A_2 = \{4, 5\}$, and $A_3 = \{6\}$ of $S = \{1, 2, 3, 4, 5, 6\}$.

What are the sets in the partition of the integers arising from congruence modulo 4?

Show that the "divisibility" relation (|) is a partial ordering on the set of positive integers (Z^+).

Show that the inclusion relation $\subseteq$ is a partial ordering on the power set of a set S.

Let R be the relation on the set of people such that xRy if x and y are people and x is older than y. Show that R is not a partial ordering.

In the poset $(\mathbf{Z}^+, |)$, are the integers 3 and 9 comparable? Are 5 and 7 comparable?

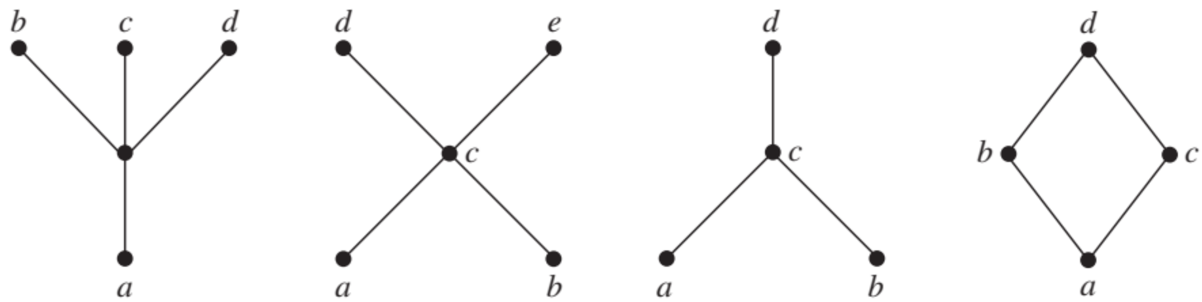
Determine whether $(3, 5) \prec (4, 8)$, whether $(3, 8) \prec (4, 5)$, and whether $(4, 9) \prec (4, 11)$ in the poset $(\mathbf{Z} \times \mathbf{Z}, \preccurlyeq)$, where $\preccurlyeq$ is the lexicographic ordering constructed from the usual $\leq$ relation on $\mathbf{Z}$.

Draw the Hasse diagram representing the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 9, 18\}$.

Draw the Hasse diagram for the partial ordering $\{(A, B) \mid A \subseteq B\}$ on the power set $P(S)$ where $S = \{a, b, c\}$.

Which elements of the poset $(\{2, 4, 5, 10, 12, 20, 25\}, |)$ are maximal, and which are minimal?

Determine whether the posets represented by each of the Hasse diagram have a greatest element and a least element.



(a) (b) (c) (d)

Let S be a set. Determine whether there is a greatest element and a least element in the poset $(P(S), \subseteq)$.

Is there a greatest element and a least element in the poset $(\mathbf{Z}^+, |)$?