# B-TECH 3RD SEMESTER FINAL EXAMINATION

## JANUARY 2021

## SUBJECT: SIGNAL AND SYSTEMS [CS2104]

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Name: Abhiroop Mukherjee
Enrolment Number: 510519109
Previous Enrolment Number: 510719007
G-Suite ID: 510519109.abhirup@students.iiests.ac.in
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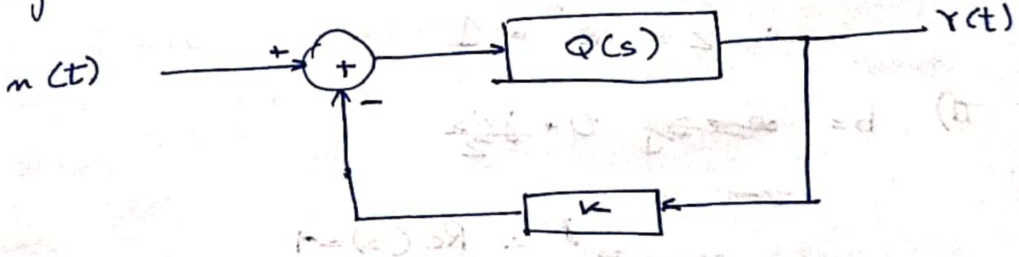
Q.1) → An unstable system can be stabilised using feedback control

→ considere the following situation.

$$Q(s) = \frac{b}{s-a}$$ → unstable at $s=a$. $Q(s) \to \infty$

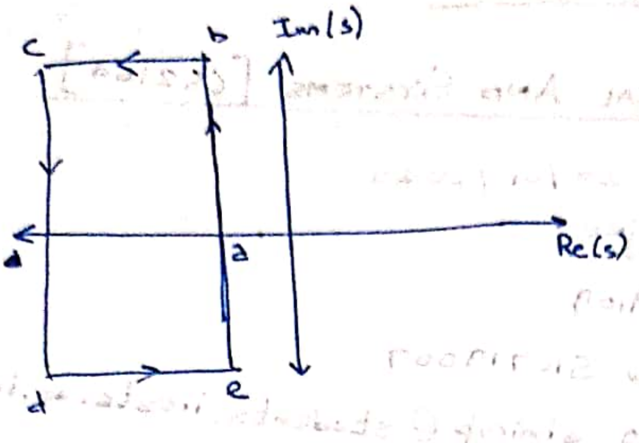
but using feed back



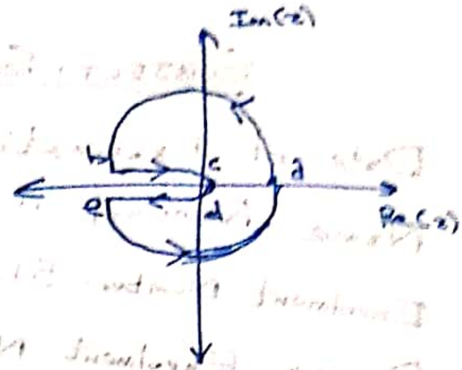

$$H(s) = \frac{Q(s)}{1+kQ(s)} = \frac{b}{s-a+kb}$$

→ hence by using k, we can change the pole ~~operation~~ of system to shift it outside our operating input, to make system stable for operation.

## Map of s-plane to z-plane




s plane




z plane.

→ we know that the relationship between S-plane and z-plane is as follows

$$\cancel{z = e^{st}} \quad z = e^{sT}$$

→ consider ~~p~~ a path ~~from a~~ a, b, c, d, e, a in s-plane

I) $a = 0 + 0j$

$\therefore z = e^{0} = 1$

II) $b = \cancel{\infty + 0j}\ 0 + \frac{jW_s}{2}$

$\cancel{z =}$

$z = \cancel{e}$ $\therefore Re(z) = -1$

$Im(z) = 0^{+}$

III) $c = \infty + \frac{jW_s}{2}$

$\therefore Re(z) = \infty$

$Im(z) \approx 0^{+}$

IV) $d = \infty - \frac{jw_s}{2}$

$Re(z) > 0$

$Im(z) = 0^{-}$

V) $e = 0 - \frac{jw_s}{2}$

$Re(z) = 1$

$Im(z) = 0^{-}$

→ now ~~the~~ form the points in z plane, we can determine the following

I + II) path from a to b will be a circular arc from z = 1 to z = -1 in anticlockwise direction

II + III) path from b to c will be a straight line

III + IV) path from c to d will be a circular arc [clockwise]

IV + V) path from d to e will be straight line

~~V + III~~

V + I) path from e to a will be anticlockwise circular arc

Q3) Limitation of Amplitude Modulation

⇒ In ~~AM,~~ Amplitude modulation, the message signal is encoded in Amplitude, which creates a huge variations in amplitude of output signal, hence it will not always operate in peak power

~~→ Also~~

→ Noise can also be added in signal, which is ~~harded~~ harder to remove in Amplitude Modulation.

# Frequency Modulation : Narrowband vs Wideband

→ consider $m(t) = A\cos[\omega_m t]$

So $\omega_i(t) = \omega_c + k_f A\cos(\omega_m t)$

$$y(t) = \cos\left[\omega_c t + \int m(t)\,dt\right]$$

$$= \cos\left[\omega_c t + \frac{k_f A}{\omega_m}\sin\omega_m t\right]$$

now $\frac{k_f A}{\omega_m}$ is called modulating index

→ also $\omega_i(t)$ varies between $\omega_c \pm k_f A$

$$\therefore \Delta\omega = k_f A$$

$$\therefore y(t) = \cos\left[\omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t\right]$$

$$\text{modulation index} = \frac{\Delta\omega}{\omega_m} = m$$

## Narrowband

$\frac{\Delta\omega}{\omega_m} << \frac{\pi}{2}$ → $\cos[m\sin\omega_m t] \Rightarrow 1$

$\sin[m\sin\omega_m t] \rightarrow m\sin\omega_m t$

$$\therefore y(t) = \cos[\omega_c t]\cos[m\sin\omega_m t] - \sin[\omega_c t]\sin[m\sin\omega_m t]$$

$$= \cos[\omega_c t] - m\sin[\omega_c t]\sin[\omega_m t]$$

## Wideband

$\frac{\Delta\omega}{\omega_m} \rightarrow \infty$, Fourier Coefficients become Bessel Function of First Kind.

## Demodulation of Frequency Modulated Signals

→ Demodulation of FM signals can be done in following way

- i) convert ~~it~~ ~~to~~ ~~Fre~~ FM to AM via ~~differentiat~~ diffrentiation
- ii) do AM demodulation

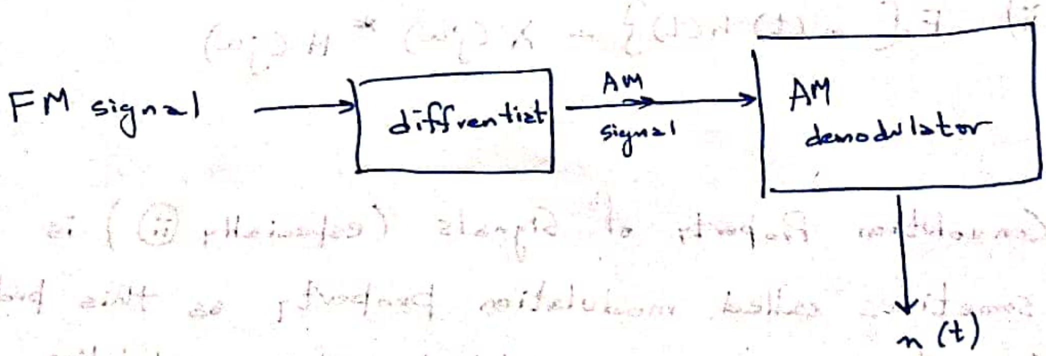
→ ~~Other ways~~

→ ~~div~~

Eg: $y(t) = \cos\left[\omega_c t + \cancel{\frac{\Delta\omega}{\omega_m}\sin\omega_m t} + \int m(t)\,dt\right]$

$$\frac{dy(t)}{dt} = \left[\omega_c + m(t)\right] \cancel{\cos} - \sin\left[\omega_c t + \int m(t)\,dt\right]$$

Block Diagram




We can also

→ Directly Track Phase / Frequency of FM signal and find $m(t)$

## Q7) Parseval's Relation for Signal Energy

### Continuous

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega$$

### Discrete

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} \cancel{X(j\omega)} |X(e^{j\omega})|^2 \, d\omega$$

### Convolution Property of Signals

i) $F\{x(t) * h(t)\} = X(j\omega) \, H(j\omega)$

ii) $F\{x(t) \, h(t)\} = X(j\omega) * H(j\omega)$

→ Convolution Property of Signals (especially ii) is sometimes called modulation property as this property is extensively used ~~modulated~~ Amplitude modulation

→ in Amplitude modulation output signal $y(t)$ is in the form

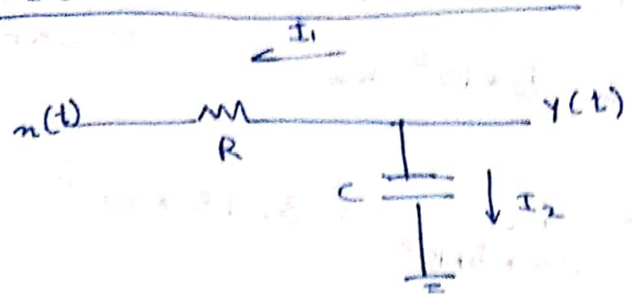
$$y(t) = c(t) \, m(t).$$

where $c(t)$ is carrier signal and $m(t)$ is message signal

→ Here we use property ii to simplify the mathematics

$$Y(j\omega) = C(j\omega) * M(j\omega)$$

Q5) First Order low pass filter




$\rightarrow I_1 + I_2 = 0 \quad (V = IR) \quad [q = CV]$

$$\frac{y(t) - x(t)}{R} + C\frac{dy(t)}{dt} = 0$$

or $$y(t) + RC\frac{dy(t)}{dt} = x(t)$$

Fourier $$Y(j\omega) + RCj\omega\, Y(j\omega) = X(j\omega)$$

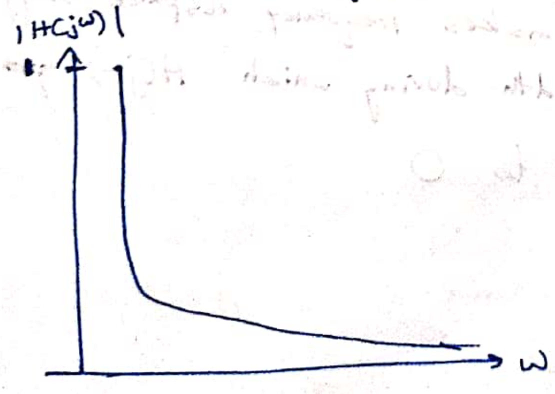
$$\cancel{Y(j\omega)} \Rightarrow$$

$$Y(j\omega) = \left[\frac{1}{1+RCj\omega}\right] X(j\omega)$$

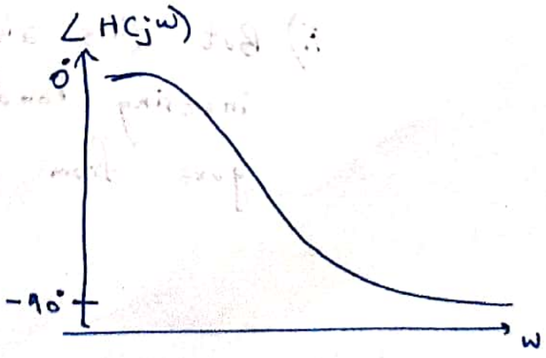
$$\therefore H(j\omega) = \frac{1}{1+RCj\omega} = \cancel{\frac{1-RCj\omega}{\sqrt{1+R^2C^2\omega^2}}}$$

$$= \frac{1-RCj\omega}{1+R^2C^2\omega^2}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+R^2C^2\omega^2}} \qquad \angle H(j\omega) = \tan^{-1}[-RC\omega]$$




Magnitude vs Frequency




Phase vs Frequency

## Cutoff Frequency

$\rightarrow |H(j\omega)|$ cutoff when $\omega = RC$ or $2\pi f = RC$

$$f_{cutoff} = \frac{1}{2\pi RC} = 5 \times 10^3 \text{ hz}$$

$$\therefore RC = \frac{1}{2\pi \times 5 \times 10^3} = 3.18 \times 10^{-5}$$

$\therefore$ for any R, C with $RC = 3.18 \times 10^{-5}\ \Omega F$
5kHz cutoff can be ~~for~~ made.

## Metrics of Performance of Filter Design

→ For a filter, following metrics are considered

i) Frequency Response, i.e. how precise/small ~~ii) Fm.~~ is the frequency cutoff region., i.e the part where $H(j\omega)$ goes from 1 to 0

ii) ~~s~~ time Responce, i.e how fast does Filter act in responce to change in frequency.

## Improvements in Butterworth Filter

→ Using Butterworth Filter Design, we can increse the order of filter., ~~but~~ which has following effects

i) Higer Order allows filter to have fast time responses, i.e filters are fast

ii) But this also makes frequency responce sluggish, incresing bandwidth during which $H(j\omega)$ ~~ges~~ goes from 1 to 0

Q6) b) Properties of discrete Fourier Transform

i) Periodicity

$$X(e^{j(w+2\pi)}) = X(e^{jw})$$

i.e, $X(e^{jw})$ ~~has~~ is periodic with $T = 2\pi$

ii) Time Shift

$$F\{x[n-n_0]\} = e^{-jw_0} X(e^{jw})$$

$$= e^{-jw_0} F\{x[n]\}$$

iii) ~~F~~ Conjugation

$$F\{x^*[n]\} = X^*(e^{-jw})$$

iv) Frequency Shift

→ Duality of Time Shift

$$F\{x[n]e^{jw_0 n}\} = X(e^{j(w-w_0)})$$

v) Linearity

$$F\{\alpha x[n] + \beta y[n]\} = \alpha X(e^{jw}) + \beta Y(e^{jw})$$

vi) ~~Division~~

$$\cancel{F\{x[\tfrac{n}{k}]\} = \tfrac{1}{k}}$$

vi) Diffrentiation of Frequency

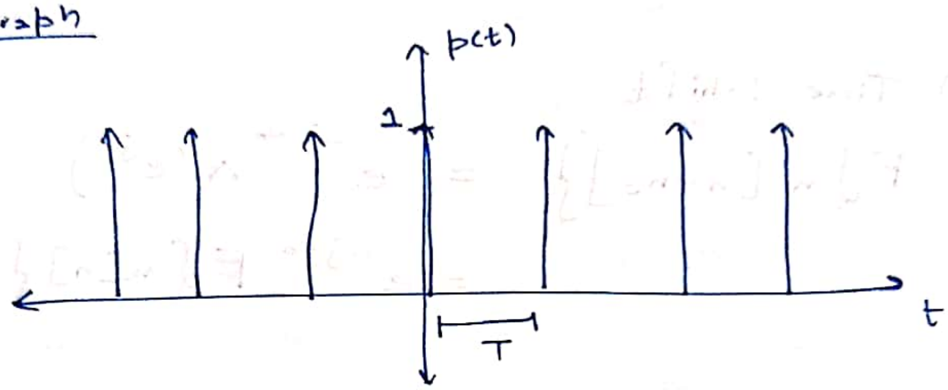
$$F\{n x[n]\} = j \frac{dX(e^{jw})}{dw}$$

## c) Sampling of Continous Signals to Discrete Signals

→ For sampling, we use a signal called pulse train

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

graph



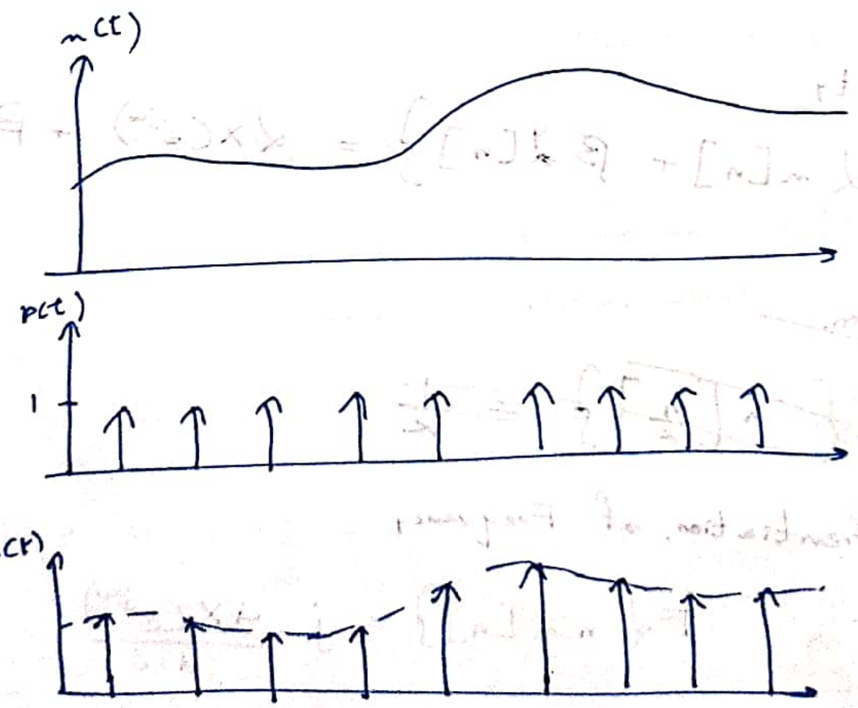

$$\omega_s = \frac{2\pi}{T}$$

→ For sampling of a continous signal $x(t)$, we do the following

$$x_p(t) = \cancel{p(t) * x(t)}$$

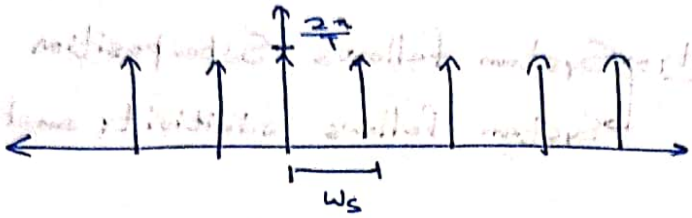
$$\left[\text{sampled } x(t)\right] = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$

Eg:



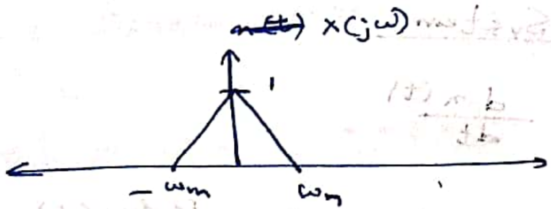
## Sample Recovery

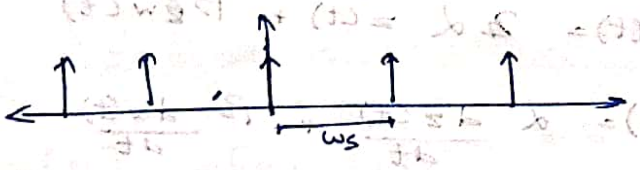
$$P(j\omega) = \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

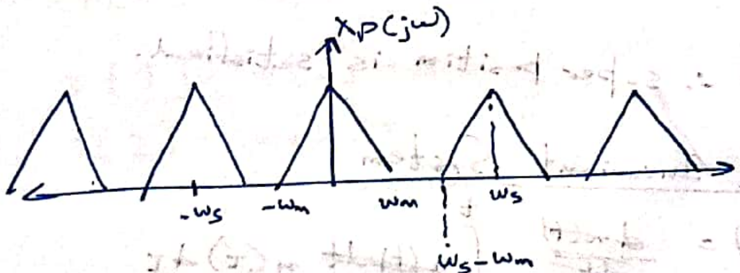



→ also sing $X(j\omega) * \delta(\omega - \omega_0) = X[j(\omega - \omega_0)]$

$$X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X[j(\omega - k\omega_s)]$$




→ all is well and good, when $\omega_s - \omega_m < \omega_m$ or $\omega_s < 2\omega_m$

→ but when $\omega_s > 2\omega_m$, $X_p(j\omega)$ experiences ailasing and sample recovery is not possible

→ when $\omega_s < 2\omega_m$, we can use low pass filter to get $X(j\omega)$, and hence we get $x(t)$.

## Q7) Linear Time Invariant System

→ LTI systems are the system which follows Linearity as well as are Time Invariant

Linearity → System follows Superposition
↳ System follows additivity and ~~two~~ homogeneity

Time Invariance : System followed by time delay is equivalent to Time delay followed by system.

### Eg. of Linear System

$$y(t) = \frac{d\,x(t)}{dt}$$

for $x(t) = \alpha\, z(t) + \beta\, w(t)$

$$y(t) = \alpha \frac{d z(t)}{dt} + \beta \frac{d w(t)}{dt}$$

∴ super position is satisfied.

### Eg. of Time Invariant System

$$y(t) = \cancel{\frac{d x(t)}{dt}} \int_{-\infty}^{t} \cancel{x(t)\,dt}\; x(\tau)\, d\tau$$

Time delay followed by system.

$$y(t-t_0) = \int_{-\infty}^{t-t_0} \cancel{x(t)}\; x(\tau)\, d\tau$$

System followed by Time delay

$$y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau$$

→ as they are equivalent y(t) is time invariant

## Finite Impulse Response. System [FIR]
[discrete]

→ Non-recursive system, which doesn't use previous outputs

→ Only convolution of inputs are present

Eg: $$y[n] = \sum_{k=0}^{M} \frac{b_k}{a} x[n-k]$$

where $a$, and $b_i$, $i=0,1,\dots M$ are constants.

## Infinite Impulse Response System [IIR]

→ Recursive discrete system, uses previous outputs

Eg: $y[n] - 2y[n-1] = x[n]$

let $y[-1] = 0$, $x[n] = k\,\delta[n]$

$\therefore y[0] = x[0] + 2y[-1] = k$

$y[1] = x[1] + 2y[0] = 2k$

$y[2] = x[2] + 2y[1] = 4k$

⋮

# Impulse Signal Diffrentiation and Integration

I) consider followingying system

$$x(t) \longrightarrow \boxed{\frac{d}{dt}} \xrightarrow{w(t)} \boxed{\int_{-\infty}^{t} dt} \longrightarrow y(t)$$

for $x(t) = \delta(t)$

$$w(t) = \cancel{f_r}\ \cancel{d}\ \frac{d\,\delta(t)}{dt}$$

$$y(t) = \int_{-\infty}^{t} \cancel{\frac{d}{dt}}\, w(t)\, dt$$

$$= \int_{-\infty}^{t} \frac{d\,\delta(t)}{dt}\, dt = \delta(t) = x(t)$$

II) consider following system

$$x(t) \longrightarrow \boxed{\int_{-\infty}^{t} dt} \xrightarrow{w(t)} \boxed{\frac{d}{dt}} \longrightarrow y(t)$$

for $x(t) = \delta(t)$

$$w(t) = \int_{-\infty}^{t} \delta(t)\, dt$$

$$y(t) = \frac{d}{dt}\left[\int_{-\infty}^{t} \delta(\tau)\, d\tau\right]$$

$$= \delta(t) = x(t)$$

$\therefore$ integration and diffentiation are inverse operation of one another.