10 AM

21.4.17

INDIAN INSTITUTE OF ENGINEERING SCIENCE AND TECHNOLOGY, SHIBPUR

B.TECH. 4th SEMESTER (CST) EXAMINATION, 2017

Analysis and Design of Algorithms (CS 401)

FULL MARKS: 70 **TIME: 3 Hrs**

Answer Q.1 and any three from the rest.

1. **Answer any five** from the following with proper explanation of the algorithm/method:

5x5=25

(a) A matrix chain consists of five matrices with dimensions *3X4; 4Xn; nX5; 5X6; 6X3*. Suggest value(s) of *n* for which the optimal order of parenthesization would be *(A1*((A2*A3)*(A4*A5)))*.

(b) A Boolean formula in 3CNF form is $(x \vee \sim y \vee \sim z) \wedge (x \vee y \vee z) \wedge (\sim x \vee y \vee z)$. In order to prove that if 3CNF is NP complete, then Clique is also NP complete - Map this 3CNF instance to a graph and count the size 3 cliques in polynomial time.

(c) Show through calculations how Miller Rabin test for primality is able to detect $p=561$ as a pseudo-prime when Fermat's theorem for primality is applied for $a=7$ and $a=13$. You are permitted to use calculator but you have to show your calculation steps.

(d) Consider a graph consisting of 5 vertices (A,B,C,D,E) and the 10 edge weights for edges (A,B;A,C;A,D;A,E;B,C;B,D;B,E;C,D;C,E;D,E) are respectively (3;8;2;6;1;7;4;9;10;5). Find the minimum spanning tree following Kruskal's algorithm.

(e) Consider a set of six tasks for which the deadlines and penalty incurred for missing the deadlines are as follows: (2,50; 3,30; 2,20; 3,10; 5,60; 4,40). All the tasks need unit time to execute. Schedule the tasks in such a way that the penalty is minimized.

(f) For a divide-and-conquer problem where a problem of size n is broken into a sub-problems each with size (n/b) it is found that $T(n) = a\ T(n/b) + n^c$ if $n>1$ and $T(n)=d$ if $n=1$. Find an expression for $T(n)$ in terms of $\Theta(n)$ when $\log_b a \geq c$.

2. Prove a lower bound for sorting algorithm based on comparisons. Describe an algorithm that performs sorting of floating point numbers in linear time under certain assumptions. 7+8

3. Consider the following two algorithms for selection of middlemost element of an array of n elements. The first one keeps finding the minimum element and eliminates it from the array until the middlemost element appears. The second one uses a randomized pivotal element to partition the array and seeks the middlemost positioned element recursively. Compare their time complexity. Now describe an algorithm that betters these methods in worst case. 8+7

4. Define the basic operations for working with disjoint sets. Suggest data structures for fast execution of the amortized operations. 6+9

5. Derive the search time complexity for hashing based on open addressing scheme. Describe storage scheme for polynomials and discuss an algorithm that runs in $O(n \log n)$ for multiplication of two polynomials of same degree n. 7+8

6. Write short notes on: **2 X 7 ½**

   (a) Circuit satisfiability problem (b) Public key cryptosystem