$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \qquad x(t)$$

① ✓
② ✓
③ ✓

Time Invariant

$$y(t) = \int_{-\infty}^{t} x(3\tau)\, d\tau \qquad x(3t)$$

① ×
② ✓
③ ✓

Time variant

$$y(t) = \int_{-\infty}^{t} \cos\tau \; x(\tau)\, d\tau \qquad \cos t \; x(t)$$

① ✓
② ×
③ ✓

Time Variant

Split system

$$y(t) = \begin{cases} x(t-1) & t<0 \\ x(t+1) & t \geq 0 \end{cases}$$

Time Variant

① ✓
② ✓
③ ✓

} Time ~~Invariant~~

$$y(t) = a(t)\, x(t-1) + b(t)\, x(t+1)$$

$$a(t) = \begin{cases} 1 & t<0 \\ 0 & t \geq 0 \end{cases} \qquad b(t) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$$

$$\xrightarrow{t_0} y(t-t_0) = a(t-t_0)\, x[(t-t_0)-1] + b(t-t_0)\, x[(t-t_0)+1]$$

$\neq$ Time Variant

$$x(t) \xrightarrow{t_0} x(t-t_0) =$$

$$Y' = a(t)\, x[(t-t_0)-1] + b(t)\, x[(t-t_0)+1]$$

02.09.2020

Linear System

The system which follows the principle of superposition is known as linear system.

Non linear System does not follows - -

Principle of superposition is the combination of two different laws.

① Law of additivity
② Law of homogeneity

Laws of additivity

$x_1(t) \longrightarrow \boxed{\text{System}} \xrightarrow{Y_1(t)}$ ⊕ $\rightarrow Y_1(t) + Y_2(t)$

$x_2(t) \longrightarrow \boxed{\text{System}} \xrightarrow{Y_2(t)}$

$x_1(t) + x_2(t) \longrightarrow \boxed{\text{System}} \rightarrow Y'(t) \begin{cases} \overset{\text{LOA}}{=} Y_1(t) + Y_2(t) \\ \neq Y_1(t) + Y_2(t) \end{cases}$

Laws of homogeneity

any constant

$x(t) \longrightarrow \boxed{\text{System}} \xrightarrow{Y(t)} \boxed{K} \rightarrow = KY(t)$

$x(t) \longrightarrow \boxed{K} \rightarrow Kx(t) \longrightarrow \boxed{\text{System}} \rightarrow Y'(t) \begin{cases} \overset{\text{LOH}}{=} KY(t) \\ \neq KY(t) \end{cases}$

ex-1: $Y(t) = x(\sin t)$ $\quad x(t) \longrightarrow \boxed{} \rightarrow Y(t) = x(\sin t)$

Linear System

① LOA $\quad Y_1(t) = x_1(\sin t)$

$\qquad Y_2(t) = x_2(\sin t)$

$Y_1(t) + Y_2(t) = x_1(\sin t) + x_2(\sin t)$

$x_1(t) + x_2(t) \longrightarrow \boxed{\text{Sys}} \rightarrow Y' = x_1(\sin t) + x_2(\sin t)$

following LOA

② LOH $\quad Y(t) = x(\sin t)$

$\qquad KY(t) = Kx(\sin t)$

$Kx(t) \longrightarrow \boxed{\text{Sys}} \rightarrow Kx(\sin t)$ following LOH

ex-2: $Y(t) = x(t^2)$ Linear System

① LOA $\quad x_1(t) \rightarrow \boxed{\text{Sys}} \rightarrow Y_1(t) = x_1(t^2)$

$\qquad x_2(t) \qquad\qquad Y_2(t) \quad x_2(t^2)$

$Y_1(t) + Y_2(t) = x_1(t^2) + x_2(t^2)$

$x_1(t) + x_2(t) \rightarrow \boxed{\text{Sys}} \rightarrow Y' = x_1(t^2) + x_2(t^2)$

following LOA

② LOH $\quad y(t) = x(t^2)$

$ky(t) = kx(t^2)$

$kx(t) \to \boxed{Sys} \to kx(t^2)$ following LOH

★ System linearity is independent of time scaling

ex-3 $\quad y(t) = \sin t \cdot x(t)$ $\quad$ linear System

$x(t) \to \boxed{} \to y(t) = \sin t \cdot x(t)$

① LOA $\quad y_1(t) = \sin t \cdot x_1(t)$

$y_2(t) = \sin t \cdot x_2(t)$

$y_1(t) + y_2(t) = \sin t \cdot x_1(t) + \sin t \cdot x_2(t)$

$x_1(t) + x_2(t) \to \boxed{} \to y' = \sin t\,[x_1(t) + x_2(t)]$

$= \sin t\, x_1(t) + \sin t \cdot x_2(t)$

following LOA

② LOH $\quad y(t) = \sin t \cdot x(t)$

$ky(t) = k \sin t \cdot x(t)$

$x(t) \to k \to kx(t) \to \boxed{} \to y = k \sin t\, x(t)$

following LOH

ex-4 $\quad y(t) = e^3 x(t)$ $\quad$ linear

① LOA $\to y_1(t) \to e^3 x_1(t)$

$y_2(t) = e^3 x_2(t)$

$y_1(t) + y_2(t) = e^3 x_1(t) + e^3 x_2(t)$

$x_1(t) + x_2(t) \to \boxed{} \to y' = e^3[x_1(t) + x_2(t)]$

following LOA

② LOH $\quad y(t) = e^3 x(t)$

$ky(t) = ke^3 x(t)$

$x \to k \to kx(t) \to \boxed{} \to ke^3 x(t)$

following LOH

★ System linearity is independent of Co-efficient used in system relationship

ex-5 $y(t) = 2t + x(t)$ non linear

$x(t) \longrightarrow \square \rightarrow y(t) = 2t + x(t)$

LOA:

$x_1(t) \qquad y_1(t) = 2t + x_1(t)$

$x_2(t) \qquad y_2(t) = 2t + x_2(t)$

$y_1(t) + y_2(t) = 4t + x_1(t) + x_2(t)$

$x_1(t) + x_2(t) \longrightarrow y' = 2t + [x_1(t) + x_2(t)]$

not following LOA

ex-6: $y(t) = 2 + x(t)$ non linear

① LOA $\qquad y_1(t) = 2 + x_1(t)$

$y_2(t) = 2 + x_2(t)$

$y_1(t) + y_2(t) = 4 + x_1(t) + x_2(t)$

$x_1(t) + x_2(t) \longrightarrow y' = 2 + [x_1(t) + x_2(t)]$

not following LOA

★ If any added or substracted term other than input and output is available in the system relationship then the system will be non-linear

ex-7: $y(t) = x(t-1) + x(t+1) \qquad x(t) \rightarrow \square \rightarrow$ linear

① LOA $\qquad y_1(t) = x_1(t-1) + x_1(t+1)$

$y_2(t) = x_2(t-1) + x_2(t+1)$

$y_1(t) + y_2(t) = x_1(t-1) + x_1(t+1) + x_2(t-1) + x_2(t+1)$

$x_1(t) + x_2(t) \rightarrow \square \rightarrow y' = x_1(t-1) + x_1(t+1) + x_2(t-1) + x_2(t+1)$

following LOA

② LOH $\quad y(t) = x(t-1) + x(t+1)$

$$k\,y(t) = k\,[x(t-1) + x(t+1)]$$

$$k\,x(t) \rightarrow \square \rightarrow k\,x(t-1) + k\,x(t+1)$$

$$= k\,[x(t-1) + x(t+1)]$$ following LOH

★ If the output is summation of time shifted terms of input then the system will be linear.

ex-3 :

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$ Linear

$$x(t) \rightarrow \square \rightarrow y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

LOA: $x_1(t)$ $\qquad y_1(t) = \int_{-\infty}^{t} x_1(\tau)\,d\tau$

$x_2(t)$ $\qquad Y_2(t) = \int_{-\infty}^{t} x_2(\tau)\,d\tau)$

$$Y_1(t) + Y_2(t) = \int_{-\infty}^{t} x_1(\tau)\,d\tau + \int_{-\infty}^{t} x_2(\tau)\,d\tau$$

$$= \int_{-\infty}^{t} [x_1(\tau) + x_2(\tau)]\,d\tau.$$

$$x_1(t) + x_2(t) \longrightarrow \square \rightarrow Y' = \int_{-\infty}^{t} [x_1(\tau) + x_2(\tau)]\,d\tau$$

following LOA

LOH: $\quad k\,Y(t) = k \int_{-\infty}^{t} x(\tau)\,d\tau$

$$k\,x(t) \rightarrow \square \rightarrow Y' = \int_{-\infty}^{t} k\,x(\tau)\,d\tau$$

$$= k \int_{-\infty}^{t} x(\tau)\,d\tau$$

following LOH

★ Integral operator is a linear operator ex-9. $y(t) = \frac{d}{dt} x(t)$ Linear

LOA:

$x_1(t) \rightarrow \quad y_1(t) = \frac{d}{dt} x_1(t)$

$x_2(t) \rightarrow \quad y_2(t) = \frac{d}{dt} x_2(t)$

$y_1(t) + y_2(t) = \frac{d}{dt} x_1(t) + \frac{d}{dt} x_2(t)$

$= \frac{d}{dt} [x_1(t) + x_2(t)]$

$x_1(t) + x_2(t) \rightarrow \square \rightarrow y' = \frac{d}{dt} [x_1(t) + x_2(t)]$

following LOA

LOH: $Ky(t) = K \cdot \frac{d}{dt} x(t)$

$Kx(t) \rightarrow \square \rightarrow y' = \frac{d}{dt} Kx(t) = K \cdot \frac{d}{dt} x(t)$

following LOH

★ Both Integral & Differential operators are linear in nature ex - 9 : $Y(t) = \frac{d}{dt} x(t)$ linear

LOA

$x_1(t) \rightarrow \quad y_1(t) = \frac{d}{dt} x_1(t)$

$x_2(t) \rightarrow \quad y_2(t) = \frac{d}{dt} x_2(t)$

$$Y_1(t) + Y_2(t) = \frac{d}{dt} x_1(t) + \frac{d}{dt} x_2(t)$$

$$= \frac{d}{dt} [x_1(t) + x_2(t)]$$

$x_1(t) + x_2(t) \rightarrow \square \rightarrow Y' = \frac{d}{dt} [x_1(t) + x_2(t)]$

following LOA

LOH $\quad KY(t) = K \cdot \frac{d}{dt} x(t)$

$Kx(t) \rightarrow \square \rightarrow Y' = \frac{d}{dt} Kx(t) = K \frac{d}{dt} x(t)$

following LOH

★ Both Integral & Differential operators are linear in nature.

04.09.2020

ex-

$$y(t) = 2 \int_{-\alpha}^{t} x(\tau) \, d\tau + \frac{d}{dt} x(t^2)$$ linear

lin lin lin

ex- $y(t) = \text{Even}[x(t)] = \frac{x(t) + x(-t)}{2}$

$$= \frac{1}{2} x(t) + \frac{1}{2} x(-t)$$

(i) LOA
(ii) LOH $\Big\} \rightarrow$ linear

★ Even & Odd operators are linear operator

$y(t) = \text{Real}[x(t)]$

$a(t) + jb(t) \longrightarrow \square \longrightarrow a(t)$ o/p

$x(t) = a(t) + jb(t)$

assumed. $jx(t) = j(a) + j^2 b(t) = j(a) - b(t)$

$x(t) \longrightarrow \square \longrightarrow y(t) \xrightarrow{j} jy(t) = ja(t)$

$jx(t) \longrightarrow \square \longrightarrow Y'(t) = -b(t)$

non linear

★ Real and Imaginary operators are non linear operators

eg-
$$y(t) = \begin{cases} x(t-1) & t<0 \\ x(t+1) & t \geq 0 \end{cases}$$

$$y(t) = a(t)\,x(t-1) + b(t)\,x(t+1)$$

$$a(t) = \begin{cases} 1 & t<0 \\ 0 & t \geq 0 \end{cases} \qquad b(t) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$$

★ split systems are linear system.

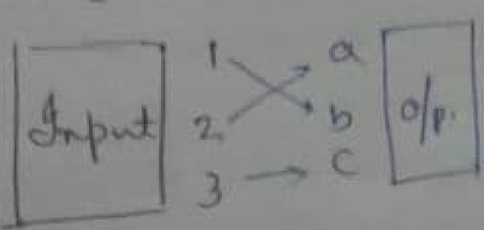
Invertible and Noninvertible system

For an invertible system there should be one to one mapping between i/p and o/p at each and every instant of time

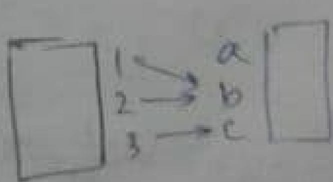
for invertible system - one to one mapping

for non invertible system - many to one mapping

one to one mapping	many to one mapping
For each and every possible unique values of input, the output will be unique	Two or more distinct possible value of i/p will produce the same output.



invertible.



non invertible

ex-1

$y(t) = x^2(t)$ non-inventable

$x(t)$	$y(t)$
-2	4
2	4

ex-2 $y(t) = x(t) + 2$ inventiable

$x(t)$	$y(t)$
0	2
1	3
-1	1

one to one mapping

ex-3 $y(t) = |x(t)|$ non inventable

$x(t)$	$y(t)$
-2	2
2	2
$2j$	2

many to one mapping

ex-4 $y(t) = x(2t)$ inventable

one to one mapping

$x(t)$	$y(t)$
$u(t)$	$u(2t)$
$-u(t)$	$-u(2t)$

ex-5 $y(t) = x(t^2)$ inventable

$x(t)$	$y(t)$
$u(t)$	$u(t^2)$
$-u(t)$	$-u(t^2$

ex-6 $y(t) = \frac{d}{dt} x(t)$

$x(t)$	$y(t)$
$u(t)$	$\delta(t)$
$r(t)$	$u(t)$
2	0
4	0

Non -Inventable

many to one mapping

Stable & Unstable system

BIBO criteria means Bounded i/p - Bounded o/p

for a stable system o/p should be bounded for bounded i/p at each and every instant of time

Bounded i/p $\rightarrow$ $-\infty$ to $+\infty$ the amplitude of the signal is finite. It should not be reach to infinite at any instant of time

Bounded o/p $\rightarrow$ "

For a unstable system, we provide bounded i/p, output of the system is un-bounded

Bounded signals: DC values

$x(t) = 6$

$x(t) = \sin t \rightarrow$ any

amp varies -1 to $+1$

$x(t) = \cos t$

$x(t) = u(t) \rightarrow 0$ or 1

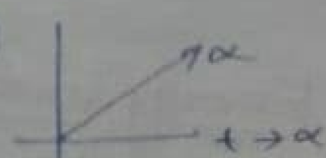
ex-1

$y(t) = t\,x(t)$ unstable

$x(t) \rightarrow \boxed{} \rightarrow y(t) = t\,x(t)$

$u(t)$

$= t\,u(t) = r(t)$



not a bounded o/p

ex-2. $y(t) = x(t) + 2$

$x(t) \rightarrow \boxed{} \rightarrow y(t) = x(t) + 2$ stable

4 $\quad = 4 + 2 = 6$

$u(t)$ $\quad = u(t) + 2$

bounded

ex-3 : $y(t) = \sin[x(t)]$ Stable

Case1 $x(t)$ = bounded

$y(t) = \sin[\text{bounded}]$

$-1 \leq \sin() \leq 1$

$-1 \leq y(t) \leq 1$

ex-4 :

$$y(t) = \int_{-\alpha}^{t} x(\tau)\, d\tau$$

un-stable

$x(t) = \cos t$

$$y(t) = \int_{-\alpha}^{t} \cos\tau \, d\tau = [\sin T]_{-\alpha}^{t}$$

$= \sin t - \sin(\alpha)$

Bounded $= \sin t + \underline{\sin(\alpha)}$

$-1 \leq \quad \leq 1$

$x(t) = u(t)$

$$y(t) = \int_{-\alpha}^{t} u(\tau)\, d\tau = \underline{r(t)}$$ - un-bounded signal

ex-5

$y(t) = \frac{d}{dt} x(t)$ un-stable

$x(t) = u(t)$ $\qquad \frac{d}{dt} u(t) = \underline{\delta(t)}$

un-bounded

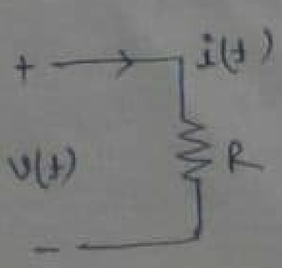
Energy and Power of continious time signals

$\text{Power} = \frac{\text{Work done}}{\text{Time}}$

Work = power × time

Energy (E) = capacity of work

= power × time



$v(t) = R \times i(t)$

$P = v(t) \cdot i(t)$

$= i^2(t) \cdot R$

$= \frac{v^2(t)}{R}$

Let assume, $R = 1$, $P = i^2(t) = v^2(t)$

Total energy, $E = \int_{-\infty}^{\infty} P(t)\,dt = \int_{-\infty}^{\infty} v^2(t)\,dt$

Average power $P = \dfrac{\text{Total energy}}{\text{Total time}}$

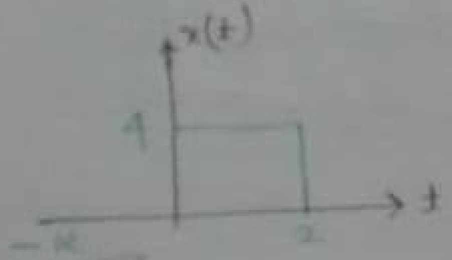
$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} P(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} v^2(t)\,dt$$

Total energy $(E) = \int_{-\infty}^{\infty} |x(t)|^2\,dt$ for periodic & nonperiodic signals

Average power, $P = \dfrac{1}{T_0} \displaystyle\int_{-T_0/2}^{T_0/2} |x(t)|^2\,dt \rightarrow$ periodic signals

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\,dt \rightarrow \text{nonperiodic signals}$$



★ A signal is energy signal if and only if the total energy is finite

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

$$= \int_{-\infty}^{0} 0\,dt + \int_{0}^{2} 4^2\,dt + \int_{2}^{\infty} 0\,dt$$

$$= [16t]_0^2 = 32 \text{ Joule}$$

This signal is energy signal

Average power $= \dfrac{\text{Total power}}{\text{Total time}} = \dfrac{\int_{-\infty}^{\infty} P(t)\,dt}{\text{of } -\infty \text{ to } +\infty}$

$$= \frac{32}{\infty} = 0$$

$x_1(t) \rightarrow$ Total energy = finite value $\quad P_{avg} = 0$

Energy signal

Properties –

① ES are absolutely integrable signals

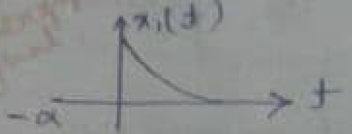
$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \text{finite} < \infty$$

② Total energy of a signal = Area under $|x(t)|^2$ graph

③ Avarage power, $P = \lim_{T \to \infty} \frac{E}{T}$

ex-1 $x_1(t) = e^{-at} u(t)$, $a > 0$
Calculate total energy of this signal.

energy signal



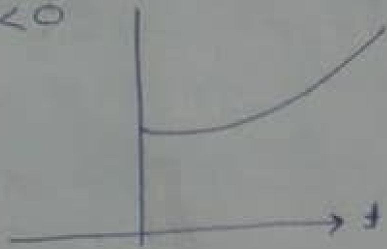
$$E = \int_{-\infty}^{\infty} |x(t)^2| \, dt = \int_{-\infty} (e^{-at} u(t))^2 \, dt = \int_0^{\infty} (e^{-at} \cdot 1)^2 \, dt = \int_0^{\infty} e^{-2at} \, dt$$

$$= -\frac{1}{2a} \left[e^{-2at} \right]_0^{\infty}$$

$$= -\frac{1}{2a} \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{1}{2a} \quad \text{finite}$$

in same equation,
let $a < 0$

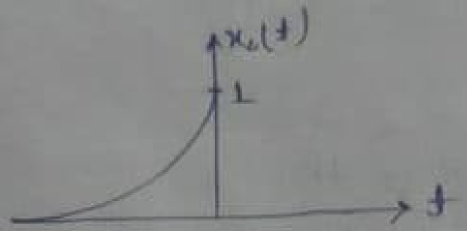


$E = \infty$

Not energy signals

ex-2

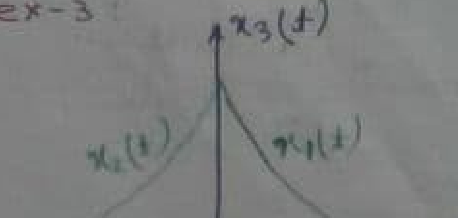
$x_2(t) = x_1(-t)$



$E = \frac{1}{2a}$

★ No effect of time reversal to total energy

ex-3



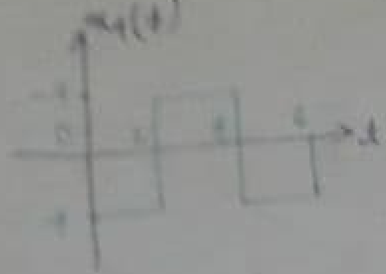
$$E_{x_3(t)} = E_{x_1(t)} + E_{x_2(t)}$$

$$= \frac{1}{2a} + \frac{1}{2a}$$

$$= \frac{1}{a}$$

thus it is also an energy signal

ex-4 :



find total energy

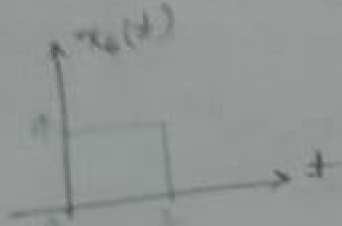
$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{0} 0 \, dt + \int_{0}^{2} 16 \, dt + \int_{2}^{4} 16 \, dt + \int_{4}^{6} 16 \, dt + \int_{6}^{\infty} 0 \, dt$$

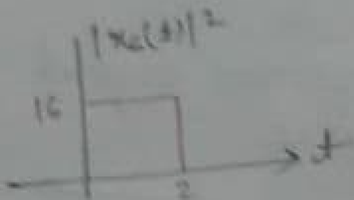
$$= 32 + 16(4-2) + 16(6-4)$$

$$= 96 \text{ Joule}$$

ex-6



$E = 32$ Joule



Area $= 16 \times 2 = 32$ Joule.

$x_6(-t) \xrightarrow{\text{Time reversal}} E = 32 \text{ J}$

$x_6(t-2) \xrightarrow{\text{Time shifting}} E = 32 \text{ J}$

$x_6(2t) \xrightarrow{\text{Time scaling}} E \neq 32$

$$\boxed{x(kt) \longrightarrow \frac{E}{|k|} \quad k \neq 0}$$

$$x_6(2t) \longrightarrow \frac{E}{|2|} = \frac{32}{2} = 16 \text{ J}$$

Amplitude scaling

$$2x_6(t) \longrightarrow |2^2| \cdot 32 = 128$$

$$\boxed{k\,x(t) \longrightarrow |k|^2 E}$$

ex-7

$x_7(t) \longrightarrow E = 1\text{ J}$

$Y(t) \longrightarrow 2j\, x_7(2t-1) \rightarrow E = ?$

$$E = \frac{1}{|2|} |2j|^2 = 8$$

$x_7(t) \longrightarrow E = 1$

$x_7(t-1) \rightarrow E = 1$

$x_7(2t-1) \rightarrow E = \frac{1}{|2|} = 2$

$2j\, x_7(2t-1) \rightarrow E = |2j|^2\, 2 = 8\text{ J}$

Power Signal: If and only if average power is finite and total energy is ∞

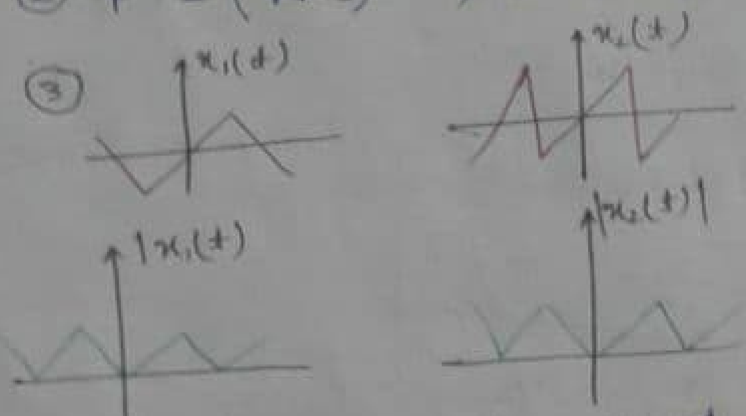
Range of finite power $0 < P < \infty$
cannot be -ve

Properties -

① periodic signals are the power signal but vise-versa is not true.

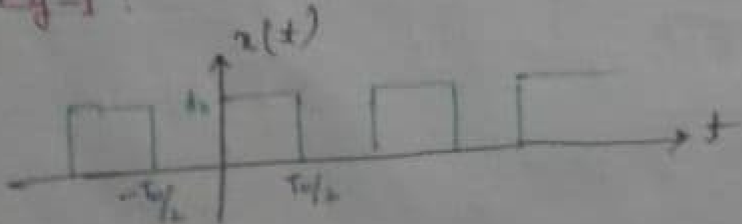
② $P = (RMS)^2 \Rightarrow RMS = \sqrt{P}$

③



Avg power of $|x_1(t)|$ = Avg power of $|x_2(t)|$

eg-1 :



$$P = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2\, dt$$

$$= \frac{1}{T_0}\left[\int_{-T_0/2}^{0} 0 \, dt + \int_{0}^{T_0/2} A_0^2 \, dt\right] = \frac{A_0^2}{2} \rightarrow \text{finite}$$

This $x(t)$ is a power signal

$$RMS = \sqrt{P} = \sqrt{\frac{A_0^2}{2}} = \frac{A_0}{\sqrt{2}}$$

eg-2

$$x_1(t) = A_0 \sin \omega_0 t$$

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (A_0^2 \sin^2 \omega_0 t) \, dt = \frac{A_0^2}{2}$$

$$RMS = \sqrt{P} = \sqrt{\frac{A_0^2}{2}} = \frac{A_0}{\sqrt{2}}$$

perform time scaling

$$x_2(t) = x_1(2t)$$
$$= A_0 \sin(2\omega_0 t) \rightarrow P_{av} = \frac{A_0^2}{2}$$

phase shift

$$x_3(t) = A_0 \sin(\omega_0 t + \phi) \longrightarrow P_{av} = \frac{A_0^2}{2}$$

time reversal

$$x_4(t) = x_1(-t)$$
$$= A_0 \sin(-\omega_0 t)$$
$$= -A_0 \sin \omega_0 t \longrightarrow P_{av} = \frac{A_0^2}{2}$$

time shifting

$$x_5(t) = x_1(t+2)$$
$$= A_0 \sin[\omega_0(t+2)]$$
$$= A_0 \sin[\omega_0 t + 2\omega_0] \longrightarrow P_{av} = \frac{A_0^2}{2}$$

Amplitude shifting

$$x_6(t) = -x_1(t)$$
$$= -A_0 \sin \omega_0 t \longrightarrow P_{av} = \frac{A_0^2}{2}$$

Amplitude scaling

$$x_7(t) = 2x_1(t)$$
$$= 2A_0 \sin \omega_0 t$$

$$\boxed{\begin{array}{l} x(t) \rightarrow P \\ K x(t) \rightarrow |K|^2 P \end{array}}$$

$$\hookrightarrow P_{av} = 4 \cdot \frac{A_0^2}{2} = 2A_0^2$$

$$RMS = \sqrt{P} = \sqrt{2} A_0$$

ex-2 :

$x_8(t) \longrightarrow P = 4$

$y(t) \longrightarrow 4j\, x_8(2t+4)$

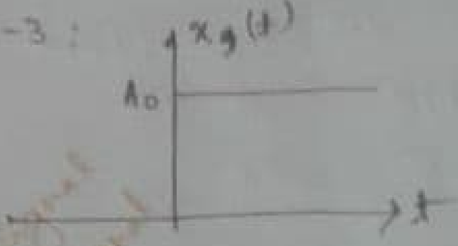
$x_8(t) \longrightarrow P = 4$

$x_8(t+4) \longrightarrow P = 4$

$x_8(2t+4) \rightarrow P = 4$

$4j\, x_8(2t+4) \rightarrow P = |4j|^2 \times 4 = 64$

ex-3 :



Step signal is a power signal

non-periodic signal

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x_9(t)|^2\, dt$$

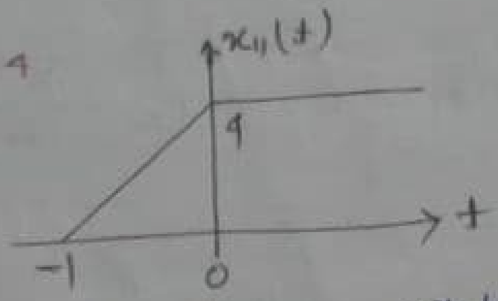
$$= \lim_{T\to\infty} \frac{1}{T}\left[\int_{-T/2}^{0} 0 \cdot dt + \int_{0}^{T/2} A_0^2\, dt\right]$$

$$= \lim_{T\to\infty} \frac{1}{T} A_0^2 \cdot [t]_0^{T/2}$$

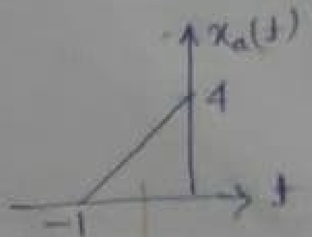
$$= \lim_{T\to\infty} \frac{A_0^2}{T} \cdot T/2 = \frac{A_0^2}{2}$$

$$\text{RMS} = \sqrt{P} = \frac{A_0}{\sqrt{2}}$$

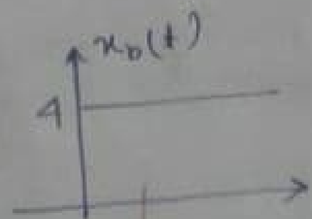
ex-4



total avg. power = ?



+



$P_{avg} = 0 = P_a$

$P_{avg} = \frac{4^2}{2} = 8 = P_b$

Total av. power $= P_a + P_b = 0 + 8 = 8$

$\text{RMS} = \sqrt{8} = 2\sqrt{2}$

$$P_x = \lim_{T\to\alpha} \frac{1}{T} \int_{-T/2}^{T/2} |x_a(t)|^2 \, dt$$

$$= \lim_{T\to\alpha} \frac{1}{T} \int_{-1}^{0} (4.t)^2 \, dt$$

$$= \lim_{T\to\alpha} \frac{1}{T} \int_{-1}^{0} 16.t^2 \, dt$$

$$= \lim_{T\to\alpha} \frac{1}{T} \, 16 \cdot \frac{t^3}{3} \Big|_{-1}^{0} \qquad = \frac{\text{finite value}}{\alpha}$$

$$= 0 = P_a$$

$=$

ex-5

$$x_{12}(t) = \underbrace{5\cos(10t+\phi)}_{x_a(t)} + \underbrace{10\sin(5t+\phi)}_{x_b(t)}$$

find the total avg. power of the signal.

$$A_0 \sin \omega_0 t \to P = \frac{A_0^2}{2}$$

$$x_b(t) = 10\sin(5t+\phi)$$

$$P_b = \frac{A_0^2}{2} = \frac{(10)^2}{2} = 50$$

$x_a(t)$

$$A_0 \sin(\omega_0 t + \pi/2 - \phi') \to P = \frac{A_0^2}{2}$$

$$A_0 (\omega t - \phi) = P = \frac{A_0^2}{2}$$

$$\phi' = -\phi$$

$$A_0 \cos(\omega_0 t + \phi) \to \frac{A_0^2}{2}$$

$$x_a(t) = 5\cos(10t+\phi) \to \frac{A_0^2}{2} = \frac{5^2}{2} = 12.5$$

Total power $= 12.5 + 50 = 62.5$

Neither energy nor power signal (NENP)

If magnitude of signal is infinite at any instant of time then signal will be NENP

ex-1 : $x(t) = t\,u(t)$

∞ NENP

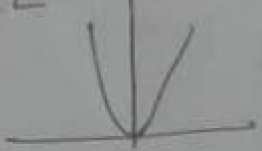
ex-2

$x(t) = \tan(t)$

at $\pi/2$ $x(t) = \infty$ NENP

$3\pi/2$ $x(t) = \infty$

ex-3 $x(t) = t^2$



NENP

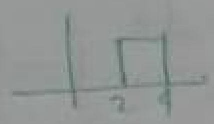
① find P_{avg}

② then $P_{avg} = 0 \rightarrow$ E.S

$= \text{finite} \rightarrow$ P.S

$= \infty \rightarrow$ NENP

Energy E = finite $\rightarrow P = 0$



① finite duration signal

② infinite extention signal (amplitude decreasing in nature)

NENP

① magnitude = ∞

② infinte extention signal (amplitude increasing in nature)

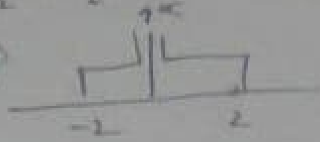
★ Periodic signals
- Power signal $\rightarrow \sin(t), \cos(t)$
- NENP $\rightarrow \tan(t)$

★ nonperiodic signals
- ES $e^{-2t}u(t)$
- PS $1 \cdot u(t)$
- NENP $r(t) \cdot \frac{1}{t}$

★ finite duration signal

ES

NENP



★ infinite duration signal

ES $e^{-t}u(t)$

PS $u(t)$

NENP $e^{t}u(t)$, $r(t)$, $\frac{1}{t}$

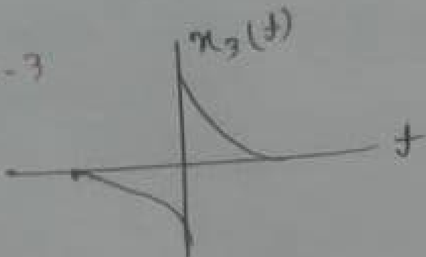
ex-1

ES

ex-2

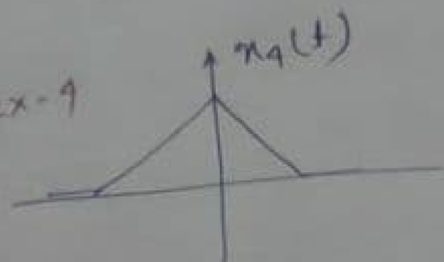
NENP

ex-3



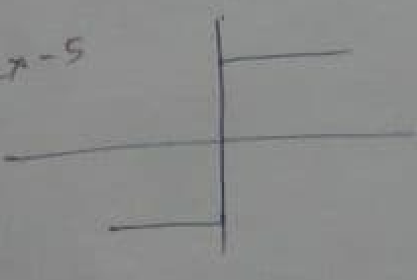
ES

ex-4



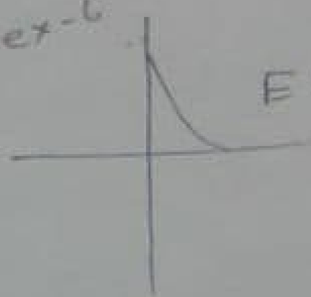
ES

ex-5



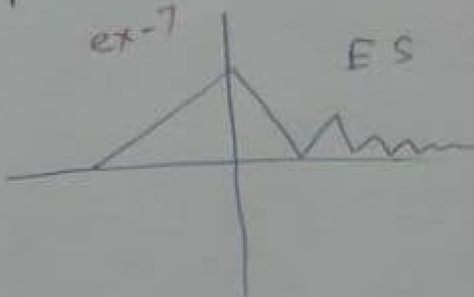
PS

ex-6



ES

ex-7

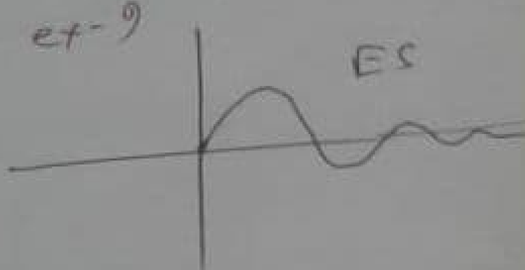


ES

ex-8

NENP

ex-9



ES