Dual Degree (B.Tech & M.Tech) 3rd Semester (CST), Final Examination, 2016
Indian Institute of Engineering Science and Technology, Shibpur, Howrah

Discrete Structures (CS 303)

Full Marks: 70 **Time: 3 hrs**

Use ONE Answer Script.

Answer any FIVE questions (attempt at least two questions from each group)

Group A

1. a) Define the following terms:
(i) Poset (ii) Lattice (iii) Atom
(iv) Universal lower bound (v) Universal upper bound

b) Prove that in a distributive Lattice, if an element has a complement then this complement is unique.

c) Let the number of ways selecting r objects from n objects is denoted by the numeric function a_r. With the help of generating function prove that $a_r = C(n + r - 1, r)$. (5+5+4)

2. a) Transform the Propositional Logic formula $(P \wedge (Q \rightarrow R)) \rightarrow S$ into a Conjunctive Normal Form.

b) Given: $F_1, F_2, \ldots, F_n$ and G are formulas in a Propositional Logic. Prove that G is a logical consequence of $F_1, F_2, \ldots, F_n$:
i) if and only if the formula $((F_1 \wedge F_2 \wedge \ldots \wedge F_n) \rightarrow G)$ is valid
ii) if and only if the formula $(F_1 \wedge F_2 \wedge \ldots \wedge F_n \wedge \neg G)$ is inconsistent

c) Evaluate the truth value of the formula $(\forall x)\,(P(x) \rightarrow Q(f(x), a))$ under the following interpretation.
Domain: $D = \{1, 2\}$
Assignment for a: $a = 1$
Assignment for f: $f(1) = 2$ and $f(2) = 1$
Assignment for P: $P(1) = F$, $P(2) = T$
Assignment for Q: $Q(1,1) = T$, $Q(1, 2) = T$, $Q(2, 1) = F$, $Q(2, 2) = T$ (3+6+5)

3. a) Solve the following recurrence relation using generating function:
$a_r - 3a_{r-1} = 2$ where $r \geq 1$, given that $a_0 = 1$.

b) Let a_r be the number of comparisons required in Bubble sort algorithm for sorting a list of r elements.
(i) Derive a recurrence relation for a_r in terms of a_{r-1}.
(ii) Solve the recurrence relation.

c) Let a be an arbitrary numeric function and b is the accumulated sum of a. Find the generating function of b in terms of the generating function of a. (5+5+4)

4. a) Find the particular solution of the recurrence equation $a_r - 2\,a_{r-1} = 3.\,2^r$.
b) Prove that there exists a unique finite Boolean algebra of 2^n elements for any $n > 0$. (4+10)

P.T.O.

Group B

5. a) Translate the following statements into formula:
   (i) "Every man is mortal. Confucious is a man. Therefore, Confucious is mortal".
   (ii) "For every number other than zero, there is one and only one immediate predecessor".
   (ii) "Nobody can be a good student unless he is smart and his father supports him".

   b) Define Prenex Normal Form. Write an algorithm to convert a First Order Predicate Logic (FOPL) into a Prenex Normal Form.

   c) Transform the following formula into Prenex Normal Form: (5+5+4)
   $(\exists x)\ (\sim ((\exists y)\ P(x, y)) \rightarrow ((\exists z)\ Q(z) \rightarrow R(x)))$

6. a) Define Kuratowski's two graphs. List any four properties common to both the graphs.

   b) In any simple, connected planar graph with n vertices and e edges ($e > 2$), prove that $e \leq 3n - 6$

   c) Write an algorithm to conclude if a graph is a planar graph or not. (5+4+5)

7. a) Define Chromatic number and Independence number of a graph with n vertices and establish a relationship between them.

   b) Prove that every tree with two or more vertices is 2-chromatic.

   c) Define Minimal Spanning Tree (MST). Write an algorithm for finding an MST of a graph and demonstrate your algorithm considering a suitable graph. (5+3+6)