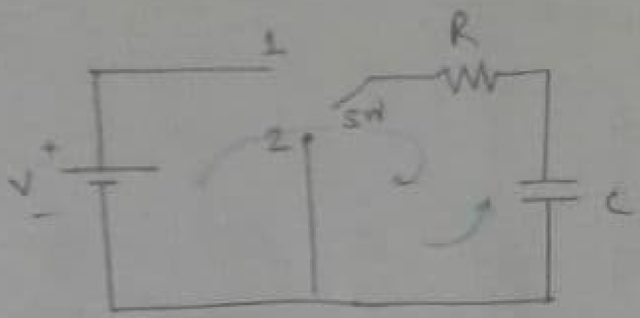
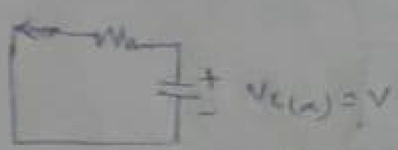
19.03.2020




The switch is closed at position 1 for a long time. At $t=0$ it moves to position 2. Find $i(t)$.

vte $V_{c(\infty)} = V$ = supply voltage




KVL at $t=0^+$

$$R\,i(t) + \frac{1}{C}\int_0^t i(t)\,dt + V_c(0^+) = 0$$

$$R\,i(t) + \frac{1}{C}\int_0^t i(t)\,dt = -V_c(0^+)$$

$$\frac{R\,d\,i(t)}{dt} + \frac{1}{C}\,i(t) = 0$$

$$\frac{d\,i(t)}{dt} + \frac{1}{RC}\,i(t) = 0$$

$$i(t) = K\,e^{-\frac{1}{RC}t}$$

at $t=0^+$ $v_c(0^+) = -V$

$$i(0^+) = -\frac{V}{R}$$

$$-\frac{V}{R} = K\,e^{-\frac{1}{RC}\times 0}$$

$$K = -\frac{V}{R}$$

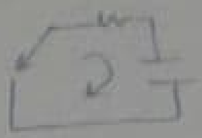
$$i(t) = -\frac{V}{R}\,e^{-\frac{1}{RC}t}$$

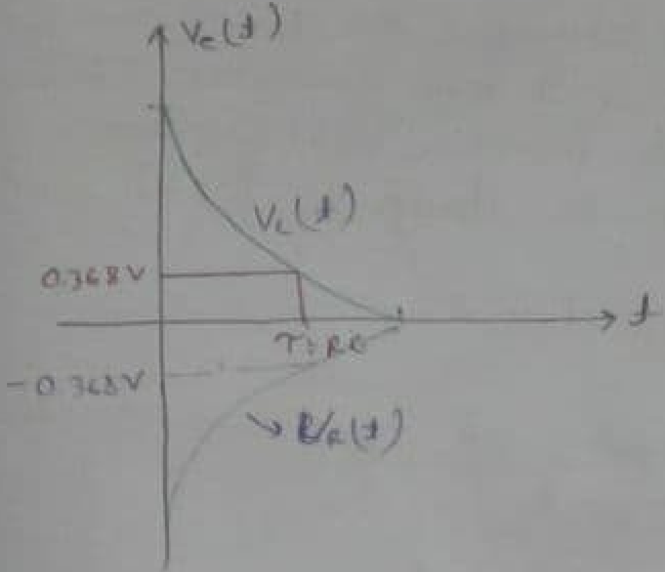
$$V_R(t) = R\,.\,i(t) = -V\,.\,e^{-\frac{1}{RC}t}$$

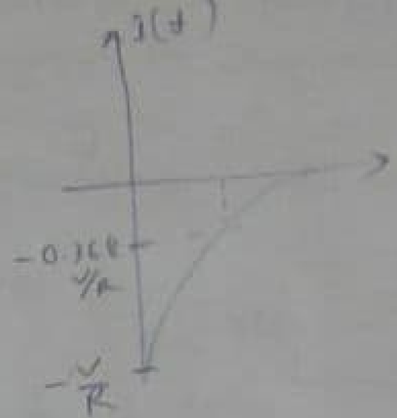
$V_R(t) + V_c(t) = 0$

$V_c(t) = -V_R(t)$

$V_c(t) = V e^{-\frac{1}{RC} \cdot t}$

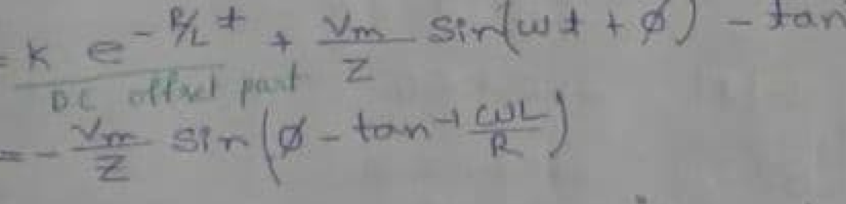





(P.1) Gate - 2019

In the circuit shown below, the switch is closed at $t=0$, the value of $\theta$ in degrees which will give the maximum value of DC offset of the current at the time of switching is ________ ?




$v(t) = 150 \sin(377t + \theta)$

$$i(t) = \underbrace{K e^{-\frac{R}{L}t}}_{\text{DC offset part}} + \frac{V_m}{Z} \sin(\omega t + \phi) - \tan^{-1}\frac{\omega L}{R}$$

$$K = -\frac{V_m}{Z} \sin\left(\phi - \tan^{-1}\frac{\omega L}{R}\right)$$

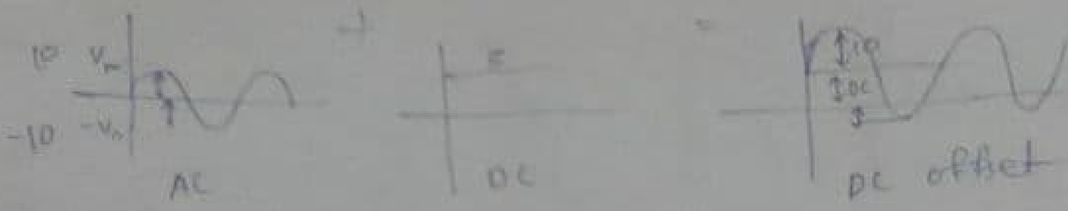
at $t=0$ DC offset value is maximum.

so $-\left(\theta - \tan^{-1}\frac{\omega L}{R}\right) = 90^\circ$

$\theta - \tan^{-1} \frac{377 \times 10 \times 10^{-3}}{3.77} = -90^\circ$

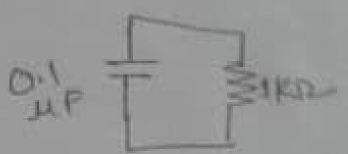
$\theta - 45^\circ = -90^\circ$

$\boxed{\theta = -45^\circ}$




**(P.2)**

A 0.1 μF capacitor charged to 100 V is discharged through a 1 kΩ resistor. The time in ms required for the voltage across the capacitor to drop to 1 V is ___?




$v_c(0^-) = 100\ V$

discharging of capacitor equation

$$V_c(t) = V_0\, e^{-\frac{t}{RC}}$$

$$\tau = RC = 10^3 \times 0.1 \times 10^{-6}$$
$$= 10^{-4}\ sec$$

$$V_c(t) = 100\, e^{-10^4 t}$$

$$1 = 100\, e^{-10^4 t}$$

$$t = 0.46\ m\,sec$$

**(P3)** Gate-2017

Consider the two CTS defined bellow

$$x_1(t) = \begin{cases} |t| & -1 \le t \le 0 \\ 0 & \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} 1-|t| & -1 \le t \le 0 \\ 0 & \text{otherwise} \end{cases}$$

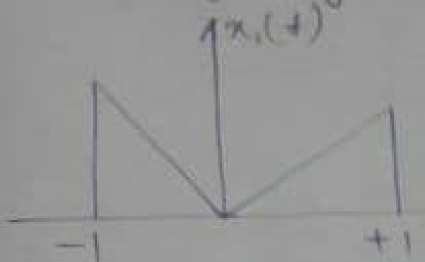
These signals are sampled with a sampling period of $T = 0.25$ sec to obtain discrete time signals $x_1[n]$ and $x_2[n]$ respectively which one of the following statements are ~~true~~ true?

(A) Energy of $x_1[n]$ > Energy of $x_2[n]$

(B) Energy of $x_2[n]$ > Energy of $x_1[n]$

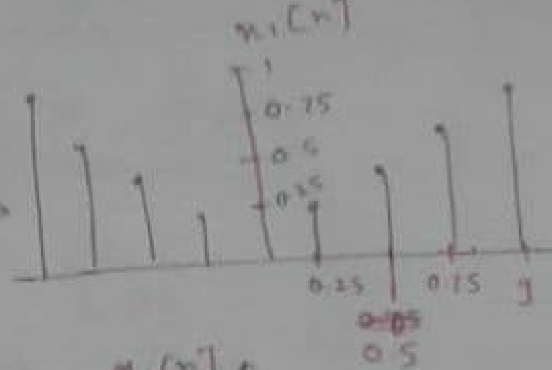
(C) $x_1[n]$ & $x_2[n]$ have eq energy

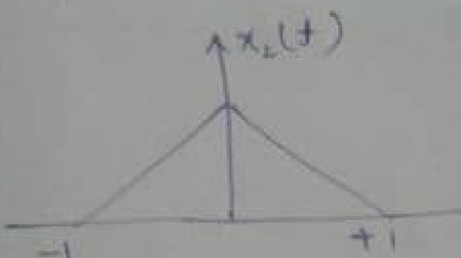
(D) Neither $x_1[n]$ nor $x_2[n]$ is a finite energy signals.



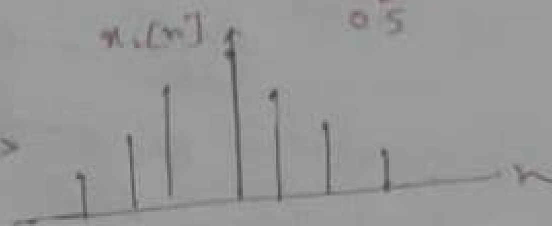

$T_S = 0.25\,s$

Sampling




$T_S = 0.25$

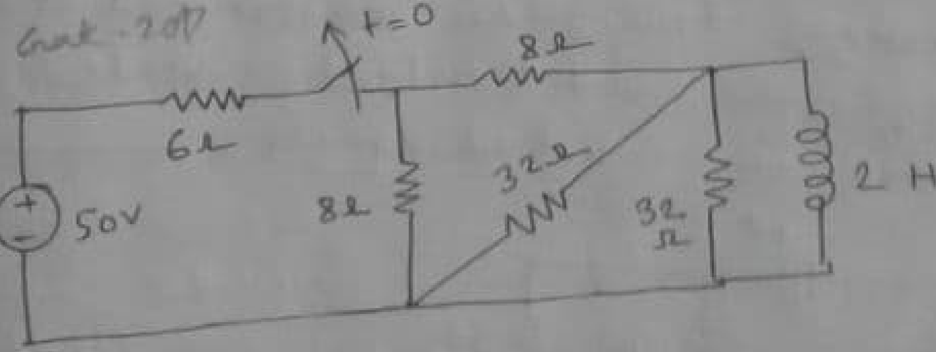



$$E_1 = \sum_{n=-\infty}^{\infty} |x_1(n)^2| = 0^2 + 2\left[1^2 + 0.75^2 + 0.5^2 + 0.25^2\right]$$

$$E_2 = \sum_{n=-\infty}^{\infty} |x_2[n]^2| = 1^2 + 2\left[0.75^2 + 0.5^2 + 0.25^2 + 0^2\right]$$

$E_1 > E_2$

Q.9 Gate 20[illegible]

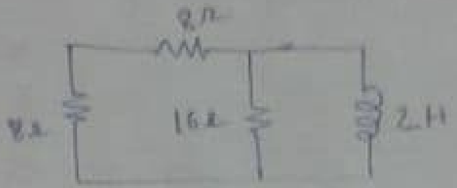



the switch in the fig was closed for a long time, It is opened at $t=0$, the current in the inductor of 2H for $t \geq 0$ is ______ at $t = 0^-$

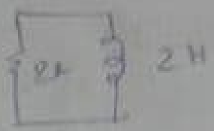

$I = 5A$

$I_L = \frac{5}{2} = 2.5\ A$

at $t = 0^+$




$\Rightarrow$




$$2\frac{d\,i(t)}{dt} + 8\,i(t) = 0$$

$$\frac{di(t)}{dt} + 4\,i(t) = 0$$

$$i(t) = K\,e^{-4t}$$

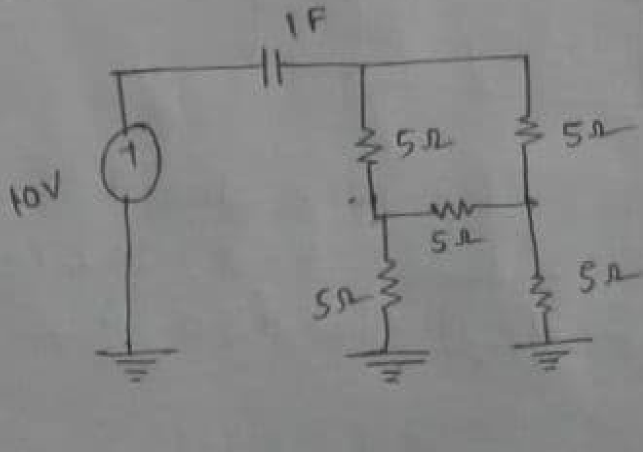
at $t = 0^+$ $\quad i_L(0^-) = i_L(0^+) = 2.5$

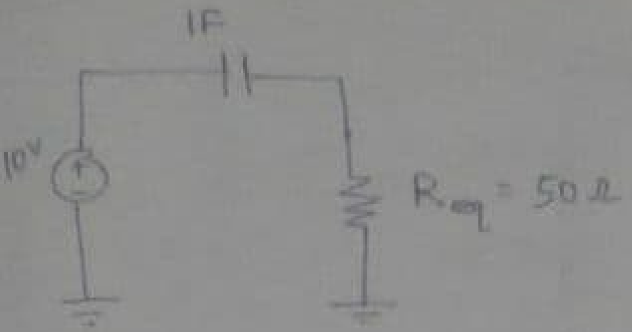
$$K = 2.5$$

$$\therefore\ i(t) = 2.5\,e^{-4t}$$

(P5) gate-2017

The initial charge in the 1F capacitor present int the circuit show is zero. The energy in judu trasformed from the DC source until steady state condition is reached equals ________






at $t = 0^-$, $v_c(0^-) = 0$
final voltage $v_c(t) = 0$

$$R_{eq}\, i(t) + \frac{1}{C}\int_0^t i(t)\,dt = 10$$

$$\frac{di(t)}{dt} + \frac{1}{R_{eq}C}\, i(t) = 0$$

$$\frac{di(t)}{dt} + \frac{1}{5}\, i(t) = 0$$

$$i(t) = K \cdot e^{-\frac{1}{5} t}$$

at $t = 0^+$ $\quad i(0^+) = \frac{10}{5} = 2\,A$

$K = 2$

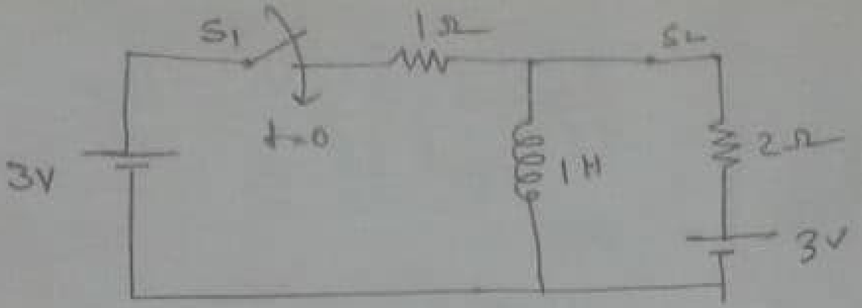
$i(t) = 2\, e^{-t/5}$

instantaneous power, $p = v \times i_n$
$= 10 \times 2 \cdot e^{-t/5}$
$= 20 \cdot e^{-t/5}$

Energy transferred $= \int_0^{\infty} p\,dt = \int_0^{\infty} 20 \cdot e^{-t/5}\,dt$
$= 100\ J$

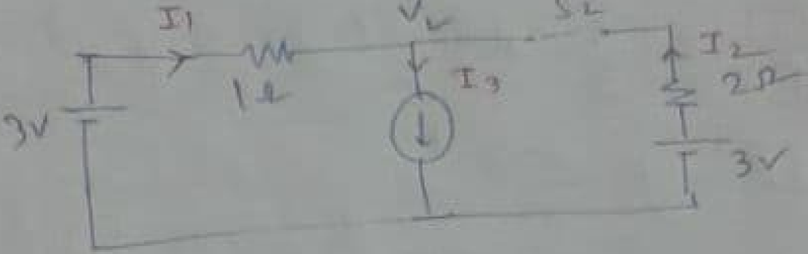
(P 6) gate-2016

In the ckt ~~phr~~ shown, switch $S_2$ has been closed for a long time, at time $t=0$, switch $S_1$ is closed.
At $t=0^+$, the rate of change of current through the inductor in ampere/sec is _?




at $t = 0^-$ $\quad i_L(0^-) = \frac{3}{2} = 1.5$ A

at $t = 0^+$

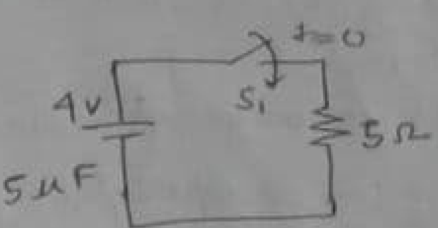

$$\frac{3-V_L}{1} + \frac{3-V_L}{2} = I_3 = 1.5$$

$$V_L = 2 = L \cdot \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{2}{1} = 2 \text{ A/sec}$$

(P.7) Gate-2016

In the ckt shown bellow, the initial capacitor voltage is 4V. Switch $S_1$ is closed at $t=0$. The charge in μC lost by the capacitor from $t = 25$ μs to $t = 100$ μs is ______ ?




$$v_c(0^-) = 4\text{V}$$

$$R\,i(t) + \frac{1}{C}\int_0^t i(t)\,dt + 4 = 0$$

$$R\frac{di(t)}{dt} + \frac{1}{C}\,i(t) = 0$$

$$\frac{di(t)}{dt} + \frac{1}{5\times 5\times 10^{-6}}\,i(t) = 0$$

$$i(t) = Ke^{-40000t}$$

at $t = 0^+$ $\quad i(0^+) = -\frac{4}{5} = -0.8$

$$K = -0.8$$

$$i(t) = -0.8 \cdot e^{-40000t}$$

$$v_R = R\, i(t) = -4 \cdot e^{-40000t}$$

$$v_C = 4 \cdot e^{-40000t}$$

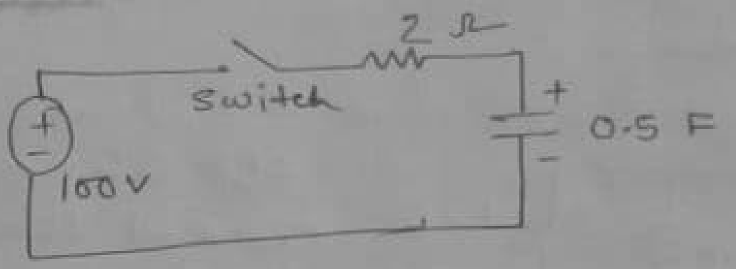
at, $t = 25\ \mu s$ , $v_c = 1.47$

$t = 100\ \mu s$ $\quad v_c = 0.073$

$$\therefore \Delta Q = C \cdot \Delta V = 5 \times 10^{-8} \times (1.47 - 0.73)$$

$$\boxed{\Delta Q = 6.99\ \mu C}$$

(P8)




In fig the capacitor initially has a charge of 10 coulomb. The current in the circuit 1 s after the switch S is closed will be ______ ?

$Q = 10\ C$

initial voltage $V_0 = \frac{Q}{C} = \frac{10}{0.5} = 20\ V$

voltage across the capacitor $= 100 + (20 - 100)e^{-t}$

$= 100 - 80e^{-t}$

$\tau = RC = 1$

The current through the capacitor

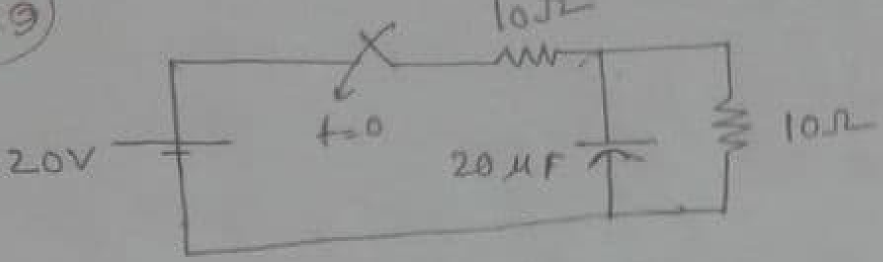
$$I = C\frac{dv(t)}{dt} = 0.5 \frac{d}{dt}(100 - 80e^{-t})$$

$$= 40\ e^{-t}$$

The current after one sec

$i_c = 40 \cdot e^{-1} = 14.71$ A

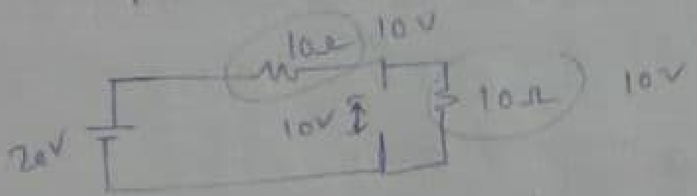
(P 9)



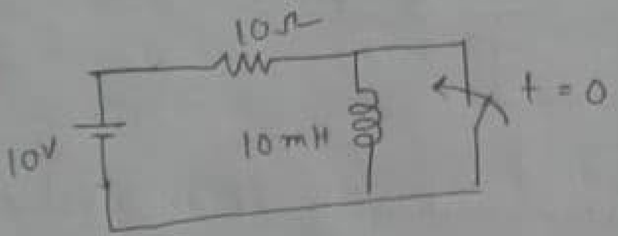

Initial capacitor voltage is zero.

Final steady state voltage across the capacitor is — ?

Ans - 10 V




(P.10)

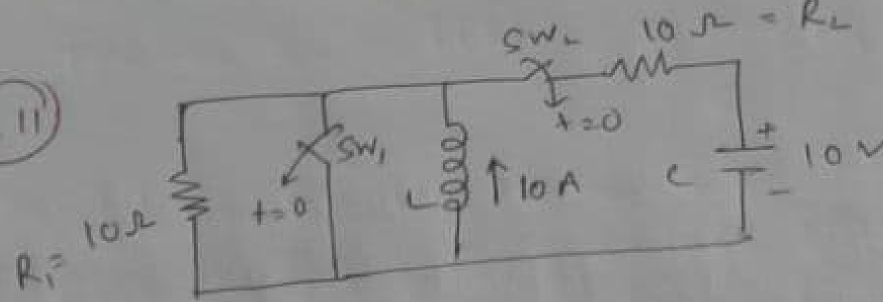



at, t = 0+ current through inductor — ?

for inductor, $i_L(0^-) = i_L(0^+)$

1 A = 1 A

(P.11)



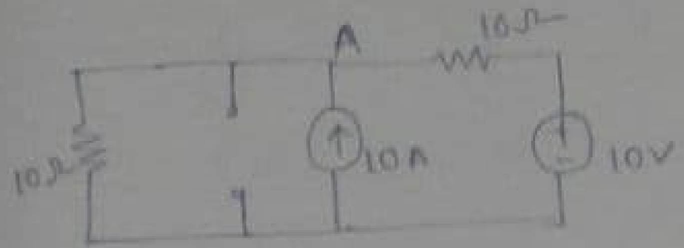

$SW_1$ is initialy closed. & $SW_2$ is open

$I_{inductor} = 10$ A

capacitor is charged 10 V with polarity

at t = 0 . $SW_2$ closed, $SW_1$ open

find current through C and Voltage across L at $t = 0^+$ ?




$$\frac{V_A}{10} + \frac{V_A - 10}{10} - 10 = 0$$

$V_A = 55$ V

$$I_c = \frac{V_A - 10}{10} = 4.5\text{ A}$$

4.5A , 55V
Ans

16.09.2020

# LAPLACE TRANSFORM

Laplace transform is an integral transform

A transform having

$$g(\alpha) = \int_a^b f(t) \cdot K(\alpha t)\, dt$$

is called integral transform.

$g(\alpha) \rightarrow$ is the function of $\alpha$ and it is o/p

$f(t) \rightarrow$ is the " of $t$ " " i/p

$K(\alpha, t) \rightarrow$ " " " of both ($\alpha$ & $t$) and it is known as integral Kernel

Let $f(t) \longrightarrow \quad \mathcal{L}\{f(t)\} = F(s) = \int_{-\infty}^{\infty} f(t) \cdot e^{-st}\, dt$

o/p  i/p  integral kernel (I.K)

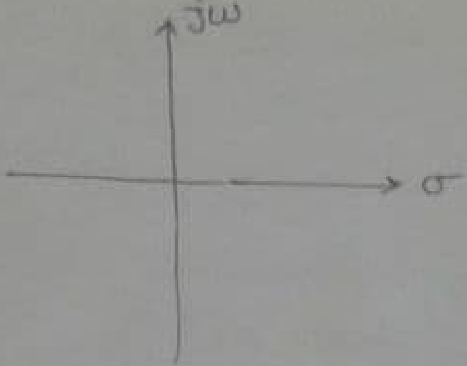
$s$ is a complex variable

$s = \sigma + j\omega$

$\sigma$ is the damping factor

It tells about the stability

$\omega \rightarrow$ is the angular frequency rad/sec




$\mathcal{L}$ is a complex function

Bilateral $\mathcal{L}$

$$F(s) = \int_{-\infty}^{\infty} f(t).e^{-st}.dt$$

If $f(t)$ such that $t \geq 0$

$$F(s) = \int_{0}^{\infty} f(t).e^{-st}.dt$$

$= \text{unilateral } \mathcal{L}$

When deal with Bilateral $\mathcal{L}$ it becomes important to maintain region of convergence (ROC along with $\mathcal{L}$

ROC is the region of the s plane, where $\mathcal{L}$ is finite in that region.

$$f(t) \rightarrow F(s)$$

$$\text{Inverse } \mathcal{L} = f(t) = \frac{1}{2\pi j} \int_{\sigma - jw}^{\sigma + jw} F(s).e^{st}\,ds$$

Condition for existance of $\mathcal{L}$

$$F(s) = \int_{-\infty}^{\infty} f(t).e^{-st}\,dt$$

$$= \int_{-\infty}^{\infty} f(t).e^{-(\sigma + jw)t}\,dt$$

$$= \int_{-\infty}^{\infty} \underbrace{f(t).e^{-\sigma t}}_{f_1(t)}.e^{-jwt}.dt$$

$$= \int_{-\infty}^{\infty} f_1(t).e^{-jwt}.dt$$

where $f_1(t)$ is the absolutely integrable signal.

Condition for signal to be absolutely integrable

$$\int_{-\infty}^{\infty} |f_1(t)|\, dt < \infty$$

$$\boxed{\int_{-\infty}^{\infty} |f(t).e^{-\sigma t}|\, dt < \infty}$$ This is the condition for the existance of $\mathcal{L}$

↓

range of $\sigma \rightarrow$ ROC

ex-1 Find the ROC of signal, $f(t) = e^{2t}.u(t)$

$$\int_{-\infty}^{\infty} |f(t).e^{-\sigma t}|\, dt = \int_{-\infty}^{\infty} |e^{2t}\, u(t).\, e^{-\sigma t}|\, dt$$

$$= \int_{0}^{\infty} |e^{(2-\sigma)t}|\, dt$$

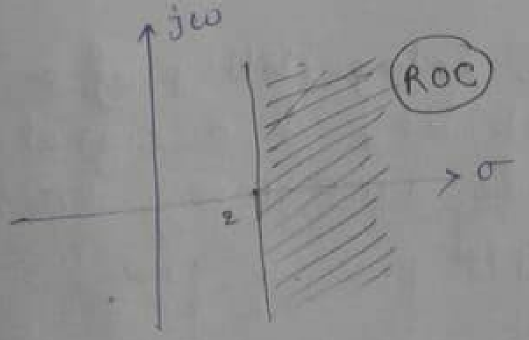
$$\int_{-\infty}^{\infty} |f(t).e^{-\sigma t}|\, dt < \infty$$

$$= \int_{0}^{\infty} |e^{(2-\sigma)t}|\, dt < \infty$$

$2 - \sigma < 0$

$\therefore -\sigma < -2$

$\therefore \sigma > 2 \longrightarrow$ range of $\sigma$



$f_1(t) \xrightarrow{\mathcal{L}} F_1(s)$ $\qquad F_1(s) \downarrow \sigma > 2$

$f_2(t) \xrightarrow{\mathcal{L}} F_2(s)$ $\qquad F_2(s) \downarrow \sigma < -1$


Properties

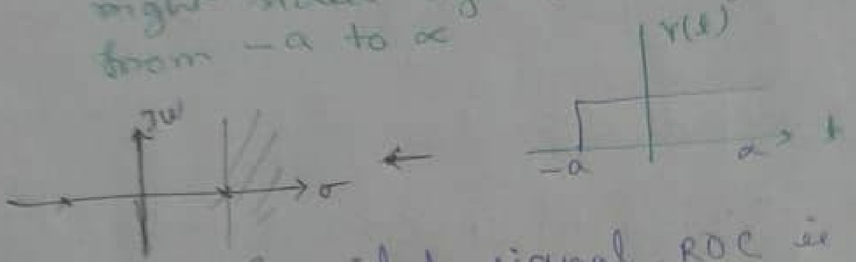
① ROC doesnot include any poles

$$F(s) = \frac{1}{s+2} \qquad s+2=0 \Rightarrow s=-2$$

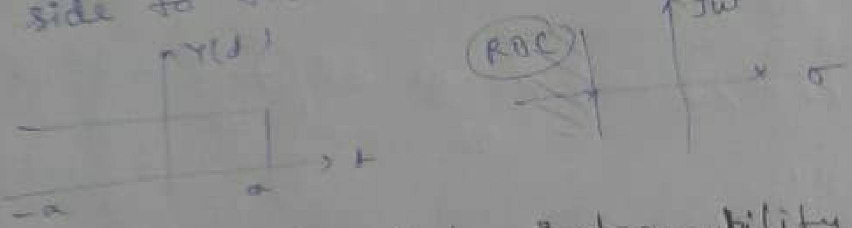
$$F(-2) = \frac{1}{-2+2} = \frac{1}{0} = \infty$$

② For right sided signal, ROC is right side to the right most pole.

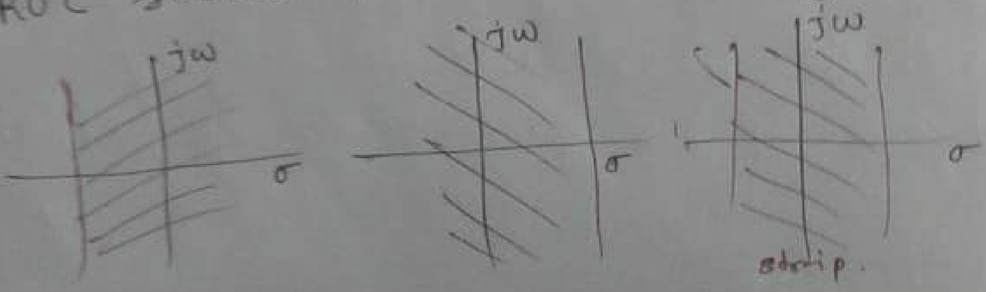
right sided signals having non-zero value from $-a$ to $\infty$



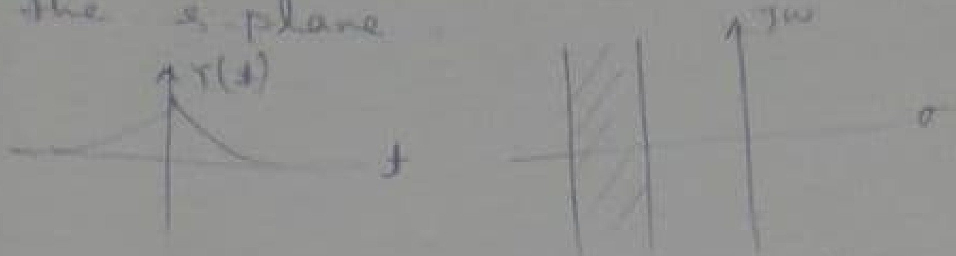

③ For left sided signal ROC is left side to the left most pole



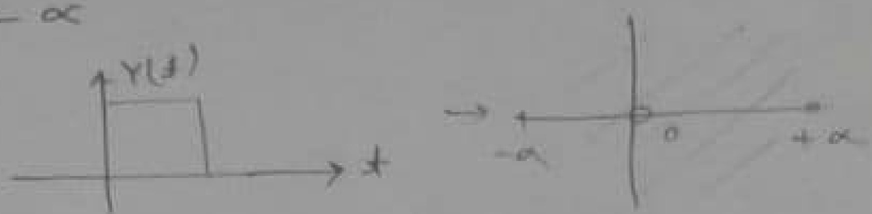

④ For the absolute integrability of a signal or a stability of a signal, ROC should include imaginary axis.



(5) For both sided signals ROC is a strip in the s-plane




(6) For finite duration signal ROC is the entire s-plane excluding $s = 0$ or $+\alpha$ or $-\alpha$




Step-1 - Compare $\sigma$ with real part at the co-efficient of $t$ in power of e

Step-2 - check if the signal is left sided or right sided and decide $<$ or $>$

ex-2

$$f(t) = e^{-at}\, u(-t)$$

$\sigma$ = real part at s

setep-1 . coeff at $t = -a$

$$\sigma < -a$$

ex-3 $f(t) = e^{(5+j3)t}\, u(-t-5)$ → left sided

$$\sigma < 5$$

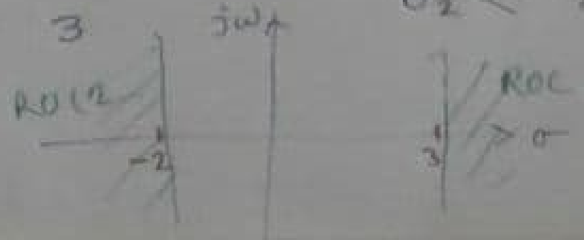
ex-4 $f(t) = e^{3t}\, u(t) + e^{-2t}\, u(-t)$

ROC 1 ROC 2

$u(-t)$ left sided

ROC $\sigma_1 > 3$ $\sigma_2 < -2$

u(t) Right sided.




no common ROC
↓
no $\mathcal{L}$

18.09.2020

# Properties of laplace transform

① $\mathcal{L}[Kf(t)] = KF(s)$

② $\mathcal{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$

③ $\mathcal{L}\left[\frac{d}{dt} f(t)\right] = sF(s) - \lim_{t \to 0} f(t)$

$$= sF(s) - f(0^+)$$

$$\mathcal{L}\left[\frac{d^2}{dt^2} f(t)\right] = s^2 F(s) - s f(0^+) - f'(0^+)$$

$$= s^2 F(s) - s f(0^+) - \frac{d}{dt} f(0^+)$$

④ $\mathcal{L}\left[\int_0^t f(t)\, dt\right] = \frac{F(s)}{s}$

$$\mathcal{L}\left[\int f(t) \cdot dt\right] = \mathcal{L}\left[\int_0^t f(t) \cdot dt + f^{-1}(0^+)\right]$$

$$= \frac{F(s)}{s} + \frac{f^{-1}(0^+)}{s}$$

where $f^{-1}(0^+)$ is the value of the integral $f(t)$ as $t$ approaches zero.

$$\mathcal{L}\left[\int_0^{t_1} \int_0^{t_2} \int_0^{t_3} \cdots \int_0^{t_n} f(t) \cdot dt_1\, dt_2 \cdots dt_n\right] = \frac{F(s)}{s^n}$$

⑤ $\mathcal{L}[t\, f(t)] = -\frac{d}{ds} F(s)$

⑥ $\mathcal{L}\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(s)\, ds$

⑦ $\mathcal{L}[f(t-a)\, u(t-a)] = e^{-as} F(s)$

⑧ $\mathcal{L}\left[e^{-at} \cdot f(t)\right] = F(s+a)$

$\mathcal{L}\left[e^{at} f(t)\right] = F(s-a)$

⑨ Initial value theorem

$$f(0^+) = \lim_{t \to 0} f(t) = \lim_{s \to \infty} \left[s \cdot F(s)\right]$$

⑩ Final value theorem

$$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0} \left[s \cdot F(s)\right]$$

⑪ Convolution theorem

given two function $f_1(t)$ and $f_2(t)$ which are zero for $t<0$

if $\mathcal{L}\left[f_1(t)\right] = F_1(s)$

$\mathcal{L}\left[f_2(t)\right] = F_2(s)$

then $\mathcal{L}^{-1}\left[F_1(s) \cdot F_2(s)\right] = f_1(t) * f_2(t)$

$$= \int_0^t f_1(t-\tau) \cdot f_2(\tau)\, d\tau$$

$$= \int_0^t f_1(\tau) \cdot f_2(t-\tau)\, d\tau$$

⑫ $\mathcal{L}\left[f(at)\right] = \frac{1}{|a|} F\left(\frac{s}{a}\right)$

Laplace transform pairs

| $f(t)$ | $F(s)$ |
|---|---|
| $1$ or $u(t)$ | $1/s$ |
| $K$ | $K/s$ |
| $t$ | $1/s^2$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ |
| $\delta(t)$ | $1$ |
| $e^{\pm at}$ | $\frac{1}{s \mp a}$ |
| $t\, e^{\pm at}$ | $\frac{1}{(s \mp a)^2}$ |

| | |
|---|---|
| $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ |
| $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ |
| $e^{-at} \sin \omega t$ | $\frac{\omega}{(s+a)^2+\omega^2}$ |
| $e^{+at} \sin \omega t$ | $\frac{\omega}{(s-a)^2+\omega^2}$ |
| $e^{-at} \cos \omega t$ | $\frac{s+a}{(s+a)^2+\omega^2}$ |
| ~~e-at co~~ | |
| $\sinh at$ | $\frac{a}{s^2-\omega^2}$ |
| $\cosh at$ | $\frac{s}{s^2-\omega^2}$ |

ex-1 $f(t) = u(t)$

$$F(s) = \int_{-\alpha}^{\alpha} f(t) \cdot e^{-st} \, dt$$

$$= \int_{-\alpha}^{\alpha} u(t) \cdot e^{-st} \, dt$$

$$= \int_{0}^{\alpha} e^{-st} \, dt = \frac{e^{-st}}{-s}\Big|_0^{\alpha} = \frac{1}{s}$$

ex-2 $f(t) = K \, t \, u(t)$

$$F(s) = \int_{-\alpha}^{\alpha} f(t) \cdot e^{-st} \, dt$$

$$= \int_{-\alpha}^{\alpha} K \, t \cdot u(t) \cdot e^{-st} \, dt$$

$$= K \int_{0}^{\alpha} t \cdot e^{-st} \, dt$$

$250 \times 12 + 12^2 \times 12500$

$250 \times 4 + 4^2 \times 0.5$

$= \frac{1088 - 1000}{1000} \times 100\%$

$8.8\%$

9231654811 - PBC mam

$$= k\left[t\,\frac{e^{-st}}{-s}\Big|_0^{\infty}\right] - k\int_0^{\infty}\frac{1\,e^{-st}}{-s}\,dt$$

$$= 0 + \frac{k}{s}\int_0^{\infty} e^{-st}\,dt$$

$$= \frac{k}{s^2}$$

$$a_0\,\frac{d^n i}{dt^n} + a_1\,\frac{d^{n-1} i}{dt^{n-1}} + \dots\, a_{n-1}\frac{di}{dt} + a_n i = v(t)$$

$$I(s) = \frac{\mathcal{L}\,v(t) + \text{Initial condition}}{a_0 s^n + a_1 s^{n-1} + \dots\, a_{n-1} s + a_n}$$

$$= \frac{P(s)}{Q(s)}$$

where $Q(s) = a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n$

$= a_0 (s + s_1) \cdots (s + s_n)$

① When all the roots of $Q(s)$ are simple

$Q(s) = 0$ then

$$I(s) = \frac{P(s)}{(s+s_1)\cdot(s+s_2)\cdots(s+s_n)} = \frac{k_1}{s+s_1} + \frac{k_2}{s+s_2} + \dots \frac{k_n}{s+s_n}$$

$$k_j = (s+s_j)\,I_s\Big|_{s=-s_j}$$

ex-1 $$F(s) = \frac{2}{(s-1)(s-2)}$$

$$= \frac{c_1}{s-1} + \frac{s_2}{s-2}$$

$$c_1 = (s-1)\cdot F(s)\big|_{s=1} = (s-1)\frac{2}{(s-1)(s-2)}\Big|_{s=1}$$

$$= \frac{2}{1-2} = -2$$

$$c_2 = (s-2)\cdot F(s)\Big|_{s=2} = \cancel{s-2}\ 2$$

$$F(s) = \frac{-2}{s-1} + \frac{2}{s-2}$$

$f(t) = -2e^{t} + 2e^{2t}$ [ILT]

(II) When some roots of $Q(s)$ are of multiple order

if a root of $Q(s) = 0$, is of multiplicity $r$ then

$$I(s) = \frac{P(s)}{(s+s_1)^r \, Q_1(s)} = \frac{K_{11}}{s+s_1} + \frac{K_{12}}{(s+s_1)^2} + \cdots \frac{K_{1r}}{(s+s_1)^r}$$

$$K_{1r} = (s+s_1)^r \, I(s) \Big|_{s=-s_1}$$

$$K_{1(r-1)} = \frac{d}{ds}\left[(s+s_1)^r \, I(s)\right] \Big|_{s=-s_1}$$

$$K_{1(r-2)} = \frac{1}{2!} \frac{d^2}{ds^2}\left[(s+s_1)^r \, I(s)\right] s=-s_1$$

$$K_{11} = \frac{1}{(r-1)!} \frac{d^{r-1}}{ds^{r-1}}\left[(s+s_1)^r \, I(s)\right] \Big|_{s=-s_1}$$

example, $$F(s) = \frac{2s+1}{(s+2)^3}$$

$$= \frac{C_{11}}{s+2} + \frac{C_{21}}{(s+2)^2} + \frac{C_{31}}{(s+2)^3}$$

$$C_{31} = (s+2)^3 \, F(s) \Big|_{s=-2}$$

$$= (s+2)^3 \frac{(2s+1)}{(s+2)^3} \Big|_{s=-2} = -2 \cdot 2 + 1 = -3$$

$$C_{21} = \frac{d}{ds}\left[(s+2)^3 \, F(s)\right] \Big|_{s=-2}$$

$$= \frac{d}{ds}\left[(s+2)^3 \cdot \frac{2s+1}{(s+2)^3}\right] \Big|_{s=-2}$$

$$= \frac{d}{ds}\left[2s+1\right] \Big|_{s=-2}$$

$$= 2$$

$$C_{11} = \frac{1}{(3-1)!} \frac{d^2}{ds^2} \left[ (s+2)^3 \; F(s) \right] \Big|_{s=-2}$$

$$= \frac{1}{2!} \left[ \frac{d^2}{ds^2} (2s+1) \right] \Big|_{s=-2} = 0$$

$$F(s) = \frac{2}{(s+2)^2} - \frac{3}{(s+2)^3}$$

(iii) Partial expansion when two roots of $Q(s)$ are of complex conjugate pair.

If two roots of $Q(s) = 0$, which form a complex conjugate pair, then

$$I(s) = \frac{P(s)}{(s+\alpha+j\omega)(s+\alpha-j\omega)\,Q(s)} = \frac{K_1}{s+\alpha+j\omega} + \frac{K_1^*}{s+\alpha-j\omega} + \cdots$$

$$K_1 = (s+\alpha+j\omega)\, I(s) \Big|_{s=-(\alpha+j\omega)}$$

$K_1^*$ is a complex conjugate of $K_1$

example,

$$s^2+2s+2 = 0$$

$$s = \frac{-2 \pm \sqrt{-4}}{2}$$

$$= -1 \pm j1$$

$$F(s) = \frac{s}{s^2+2s+2}$$

$$= \frac{c_1}{s+1+j1} + \frac{c_2}{s+1-j1}$$

$$c_1 = (s+1+j1) \cdot F(s) \Big|_{s=-1-j1}$$

$$= (s+1+j1) \frac{s}{(s+1+j1)(s+1-j1)} \Big|_{s=-1-j1}$$

$$= \frac{-1-j1}{-j2} = 0.5 - j0.5$$

$$c_2 = (s+1-j1) \frac{s}{(s+1+j1)(s+1-j1)} \Big|_{s=-1+j1}$$

$$= \frac{-1+j1}{j2} = 0.5 + j0.5$$

$$c_2 = c_1^*$$

example. $F(s) = \frac{2s+3}{(s+2)\,(s^2+4s+8)}$

roots of denominator $-2, -2 \pm j2$

$F(s) = \frac{A}{s+2} + \frac{B}{s+2+j2} + \frac{B^*}{s+2-j2}$

$= \frac{A}{s+2} + \frac{Bs+C}{s^2+4s+8}$ (alternative method)

$= \frac{A(s^2+4s+8) + (s+2)(Bs+C)}{(s+2)(s^2+4s+8)}$

from which. $2s+3 = A(s^2+4s+8) + (s+2)(Bs+C)$
$= (A+B)s^2 + (4A+2B+C)s + (8A+2C)$

$A+B=0$, $4A+2B+C=2$, $8A+2C=3$

$\therefore A = -\frac{1}{4}$, $B = \frac{1}{4}$, $C = 5/2$

$F(s) = \frac{-1}{4(s+2)} + \frac{\frac{1}{4}s + \frac{5}{2}}{s^2+4s+8}$

Q solve the differential equation.

$x'' + 3x' + 2x = 0$, $x(0^+) = 2$, $x'(0^+) = -3$

taking Laplace transform

$s^2x(s) - s\,x(0^+) - x'(0^+) + 3.s\,x(s) - 3x(0^+) + 2x(s) = 0$

$(s^2+3s+2)\,x(s) = s\,x(0^+) + x'(0^+) + 3x(0^+)$

$2s - 3 + 6$

$(s^2+2s+2)\,x(s) = 2s+3$

$x(s) = \frac{2s+3}{s^2+2s+2} = \frac{2s+3}{(s+1)(s+2)} = \frac{k_1}{s+1} + \frac{k_2}{s+2}$

$k_1 = (s+1)\,x(s)\,\big|_{s=-1} = \frac{(s+1)(2s+3)}{(s+1)(s+2)}\Big|_{s=-1} = \frac{-2+3}{-1+2} = 1$

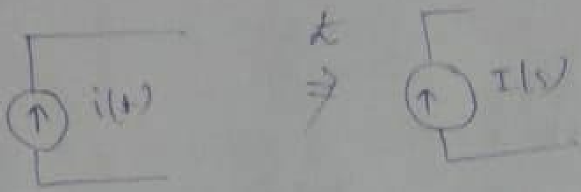
$k_2 = (s+2).x(s)\,\big|_{s=-2} = \frac{(s+2)(2s+3)}{(s+2)(s+1)}\Big|_{s=-2} = \frac{-4+3}{-2+1} = 1$

$x(s) = \frac{1}{s+1} + \frac{1}{s+2}$

$x(t) = \mathcal{L}^{-1}[x(s)] = \mathcal{L}^{-1}\left[\frac{1}{s+1} + \frac{1}{s+2}\right] = e^{-t} + e^{-2t}$

# Transformed circuit component Representations

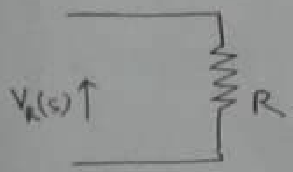
① Independent sources:


② Resistance:

$$v_R(t) = R \cdot i_R(t)$$

$$V_R(s) = R \cdot I_R(s)$$

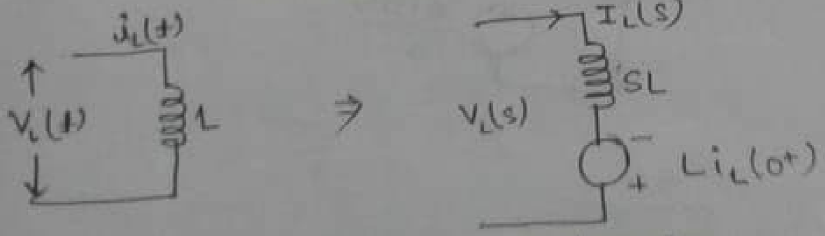



③ Inductance

$$v_L(t) = L \frac{di_L(t)}{dt}$$

$$v_L(s) = L\left[ sI_L(s) - i_L(0^+) \right]$$

$$= LSI_L(s) - L\, i_L(0^+)$$

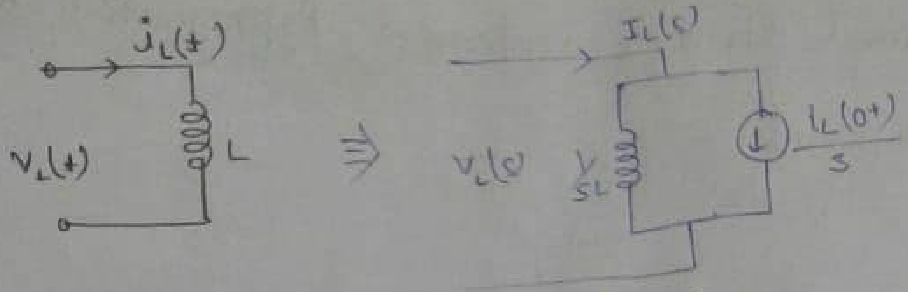



time domain

s-domain.

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)\,dt + i_L(0^+)$$

$$I_L(s) = \frac{V_L(s)}{SL} + \frac{i_L(0^+)}{S}$$

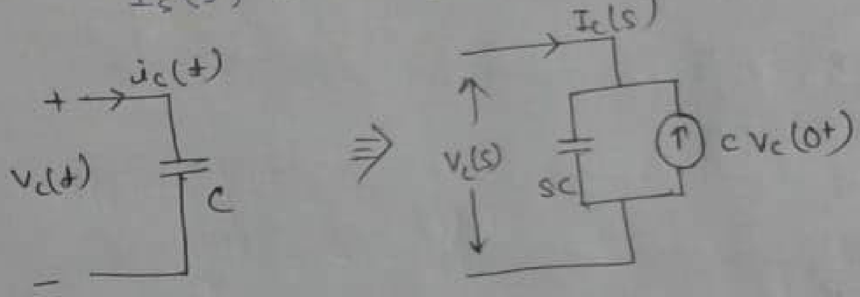



time domain　　　　　　　s-domain

④ Capacitor

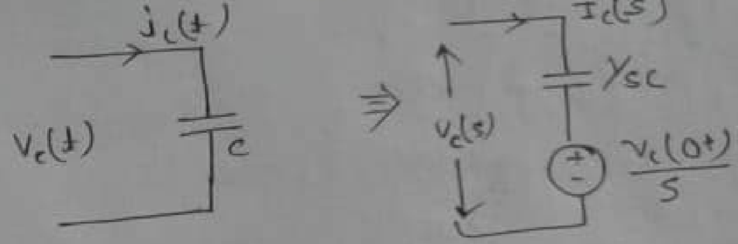
$$i_c(t) = C\,\frac{dv_c(t)}{dt}$$

$$I_c(s) = sCV_c(s) - CV_c(0^+)$$

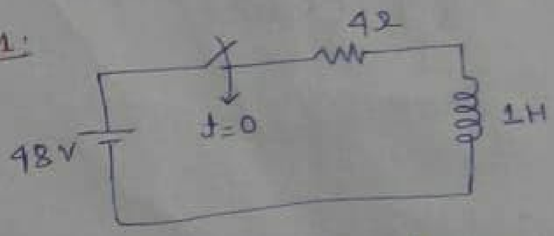



$$v_c(t) = \frac{1}{C}\int_0^t i_c(t)\,dt + v_c(0^+)$$

$$V_c(s) = \frac{I_c(s)}{Cs} + \frac{v_c(0^+)}{s}$$




ex-1:




Assume the initial current through the inductor is 3A. Find $i(t)$ $t \geq 0$

Also draw the s-domain representation of the circuit

Apply KVL

$$R\, i(t) + L\frac{di(t)}{dt} = 48$$

$$R\, I(s) + L\left[s I(s) - i_L(0^+)\right] = \frac{48}{s}$$

$$4 I(s) + 1\left[s I(s) - 3\right] = \frac{48}{s}$$

$$I(s) = \frac{3s + 48}{s(s+4)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+4}$$

$$K_1 = s.\, I(s)\Big|_{s=0} = 12$$

$$K_2 = (s+4)\, I_s\Big|_{s=-4} = -9$$

$$I(s) = \frac{12}{s} - \frac{9}{s+4}$$

$$i(t) = (12 - 9.\, e^{-4t})\ A$$

or.

$$4\, i(t) + 1\,\frac{di(t)}{dt} = 48$$

$$\frac{di(t)}{dt} + 4\, i(t) = 48$$

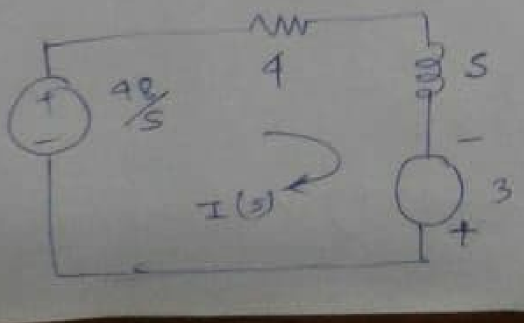
$$i(t) = \frac{48}{4} + K.\, e^{-4t}$$

$$i_L(0^-) = 3A$$

$$3 = 12 + K$$

$$K = -9$$

$$i(t) = 12 - 9.\, e^{-4t}$$



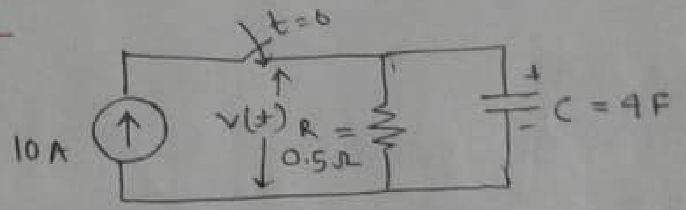

$V = \frac{48}{s} + 3$

$Z = 4 + s$

$I = \left(\frac{48}{s} + 3\right)/s+4$

s-domain

ex-2




assume initial voltage across the capacitor is 2 v

draw the s- domain capacitor.

apply KCL

$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} = 0$$

$$\frac{V(s)}{R} + C\left[s\,V(s) - v(0^+)\right] = 10/s$$

$$2\,V(s) + 4\left[s.V(s) - 2\right] = 10/s$$

$$V(s) = \frac{8s+10}{s(4s+2)}$$

$$= \frac{2s+2.5}{s(s+0.5)}$$

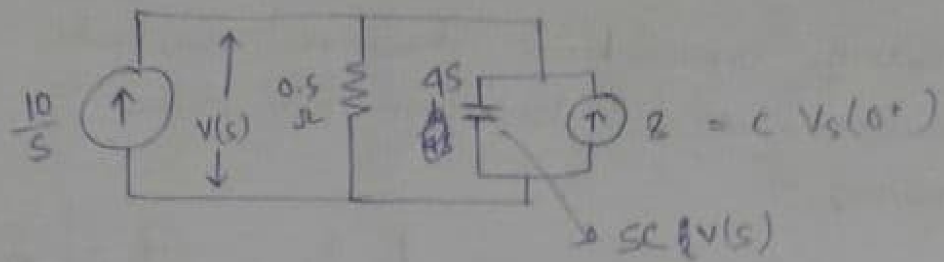
$$= \frac{K_1}{s} + \frac{K_2}{s+0.5}$$

$$K_1 = s.V(s)\Big|_{s=0} = 5$$

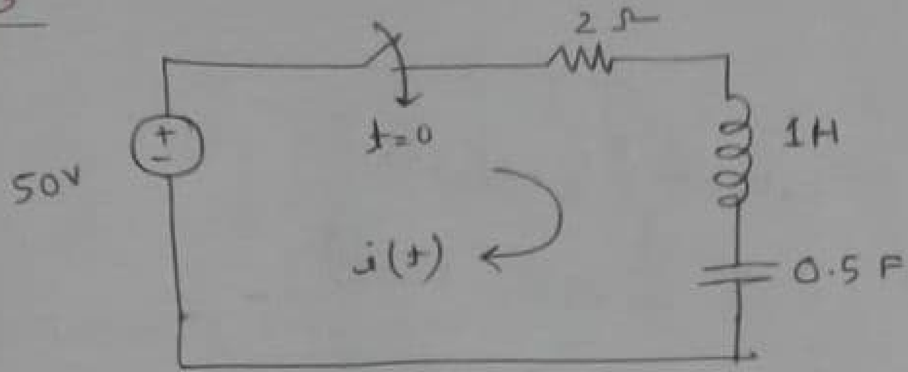
$$K_2 = (s+0.5)\,V(s)\Big|_{s=-0.5} = -3$$

$$V(s) = \frac{5}{s} - \frac{3}{s+0.5}$$

$$v(t) = \left(5 - 3\,e^{-0.5t}\right) v$$




ex-3




no initial charge in capacitor. Find i(t), $t \geq 0$, all initial Condition are zero.

KVL.
$$2\,i(t) + 1\,\frac{di(t)}{dt} + \frac{1}{0.5}\int i(t)\,dt = 50$$

$$2\,i(t) + \frac{di(t)}{dt} + 2.\int i(t)\,dt = 50$$

$$2\,I(s) + s\,I(s) - i(0^+) + 2\,\frac{I(s)}{s} = 50/s$$

$$I(s) = \frac{50}{s^2+2s+2} = \frac{50}{(s+1)^2+1}$$

$$i(t) = 50\,e^{-t}.\sin t \quad A$$

Ex. Unsing convolution theorem evalute the inverse laplace tranfom for the following

(i) $\frac{1}{(s+a)^2}$ (ii) $\frac{1}{s(s+a)}$ (iii) $\frac{1}{(s+a)(s+b)}$ (iv) $\frac{1}{(s^2+1)^2}$

(i) $F(s) = \frac{1}{(s+a)^2} = \frac{1}{(s+a)(s+a)}$

$= F_1(s) \cdot F_2(s)$

Where $F_1(s) = \frac{1}{s+a}$ $F_2(s) = \frac{1}{s+a}$

$f_1(t) = e^{-at}$ $f_2(t) = e^{-at}$

$$\mathcal{L}^{-1}\left[F_1(s) \cdot F_2(s)\right] = f_1(t) * f_2(t)$$

$$= \int_0^t f_1(\tau) \cdot f_2(t-\tau) \cdot d\tau$$

$$= \int_0^t e^{-a\tau} e^{-a(t-\tau)} d\tau$$

$$= e^{-at} \int_0^t e^{-a\tau} e^{a\tau} d\tau$$

$$= e^{-at} \int_0^t d\tau = t e^{-at}$$

$$f(t) = e^{-at} \int_0^t d\tau = t e^{-at}$$

$f(t) \rightarrow t e^{-at}$

(11) $F(s) = \frac{1}{s(s+a)}$ $\quad F_1(s) = \frac{1}{s}$ $\quad f_1(t) = u(t)$

$F_2(s) = \frac{1}{s+a}$ $\quad f_2(t) = e^{-at}$

$f(t)$

$$f(t) = \int_0^t f_1(\tau)\, f_2(t-\tau)\, d\tau$$

$$= \int_0^t e^{-a(t-\tau)}\, d\tau$$

$$= \cancel{\int_0^t} e^{-at} \int_0^t e^{a\tau}\, d\tau$$

$$= \frac{1 - e^{-at}}{a}$$

gate 2013

P.① The output response of a system is denoted by $y(t)$, as its Laplace transform is given by $Y(s) = \frac{10}{s(s^2+s+100\sqrt{2})}$

The steady state value of $Y(t)$ is ___?

$$Y(s) = \frac{10}{s(s^2+s+100\sqrt{2})}$$

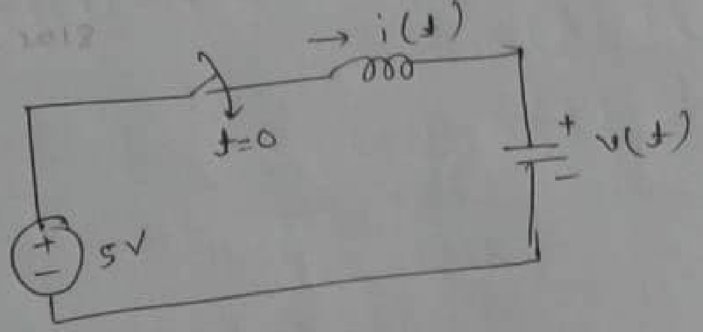
$$Y(t) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} Y(s)$$

$$= \lim_{s\to 0} s\,\frac{10}{s(s^2+s+100\sqrt{2})}$$

$$= \frac{1}{10\sqrt{2}}$$

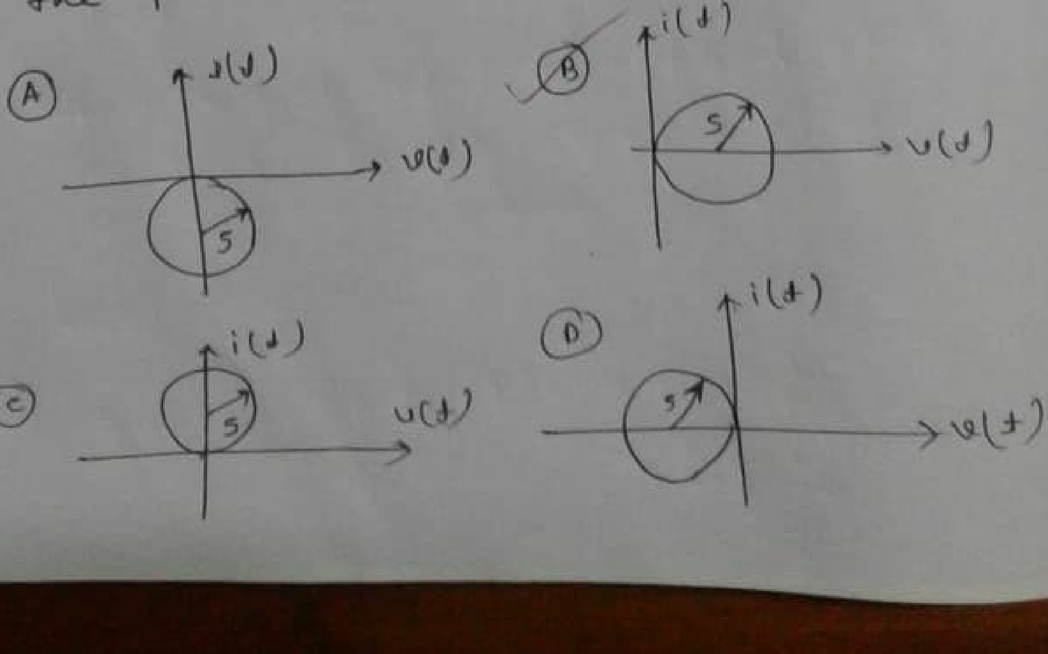
gate 2012

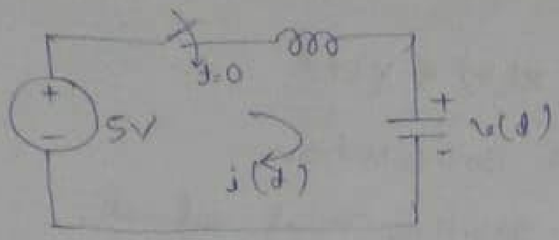
P.②

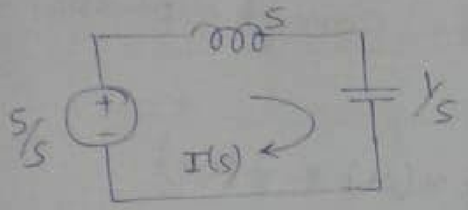



$i_L(0) = 0$, $v_c(0) = 0$

which one of the following loci presents the plot of $i(t)$ vs $v(t)$





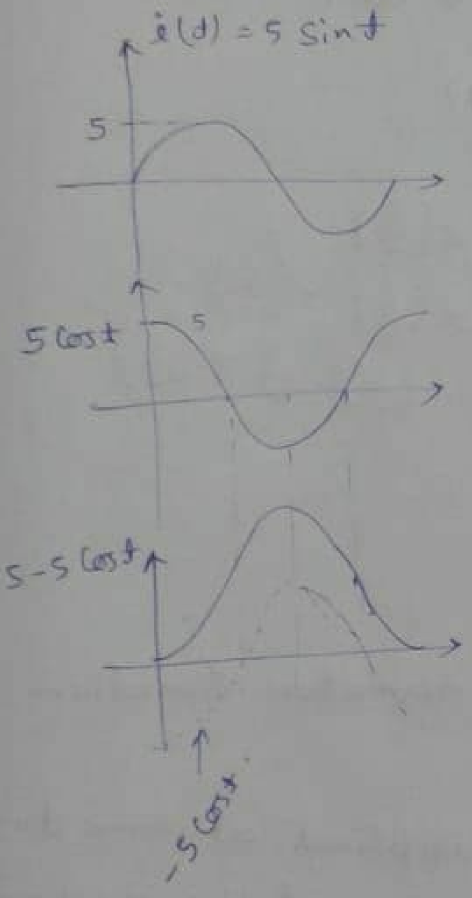

$$I(s) = \frac{5/s}{s + 1/s} = \frac{5}{s^2+1}$$

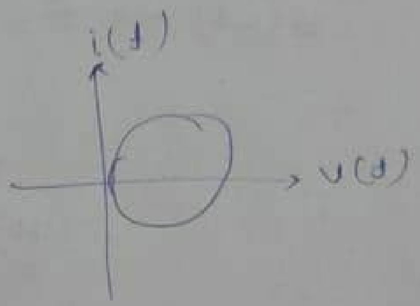
$$i(t) = 5\sin t$$

$$v(t) = \frac{1}{C}\int_0^t i(t)\,dt$$

$$= 1 \cdot \int_0^t 5\sin t \; dt$$

$$= 5 \cdot (1 - \cos t)$$



$$r(t) = u(t) - 2.e^{-t}u(t)$$

$t = 1.5$ sec. $\quad r(1.5) = 1 - 2e^{-1.5}$

$$= 1 - 0.446 = \underline{0.554}$$

P (5) The laplace transform of $f(t) =$
$e^{2t} . \sin 5t . u(t)$ is

$$F(s) = \frac{5}{(s-2)^2 + 25} = \frac{5}{s^2 - 4s + 29}$$

P. 3. Let $z(t) = x(t) * y(t)$

$*$ denotes convolution

Let $c$ be a positive real-valued constant choose the correct expression for $z(ct)$

(A) $c \cdot x(ct) * y(ct)$ (B) $x(ct) * y(ct)$

(C) $c \cdot x(t) * y(t)$ (D) $c \cdot x(ct) * y(t)$

Ans.

$$z(t) = x(t) * y(t)$$
$$z(s) = X(s) \cdot Y(s)$$
$$z(ct) \xrightarrow{\mathcal{L}} \frac{1}{c} \cdot z\left(\frac{s}{c}\right)$$
$$= \frac{1}{c} \cdot X\left(\frac{s}{c}\right) \cdot Y\left(\frac{s}{c}\right)$$
$$= c \cdot \frac{1}{c} \cdot X\left(\frac{s}{c}\right) \cdot \frac{1}{c} \cdot Y\left(\frac{s}{c}\right)$$

ILT

$$c \cdot x(ct) * y(ct)$$

P. 4. For a system having transfer function

$$G(s) = \frac{-s+1}{s+1}$$

A unit step input is applied at time $t=0$. The value of the response of the system at $t = 1.5\,s$ ____?____.

$$G(s) = \frac{-s+1}{s+1}$$

System output $Y(s) = G(s) \frac{1}{s}$

$$= \frac{-s+1}{s+1} \cdot \frac{1}{s}$$

$$Y(s) = \frac{1}{s} - \frac{2}{s+1}$$