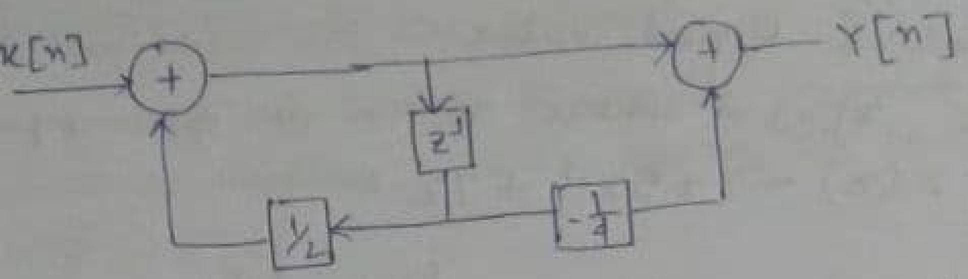
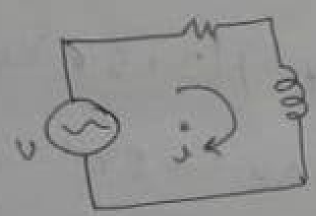


equivalent block diagram representation

12.10.2020

SYSTEM MODELLING

the process of obtaing the desired mathematical description of the system is known as modeling



v & i

$v = iR + L\cdot\frac{di}{dt}.$

plant or system → equivalent mathematical model

- Differential equation model ①
- Transfer function model ②
- State-space model ③

① Dynamics of system represented in terms of differential equations

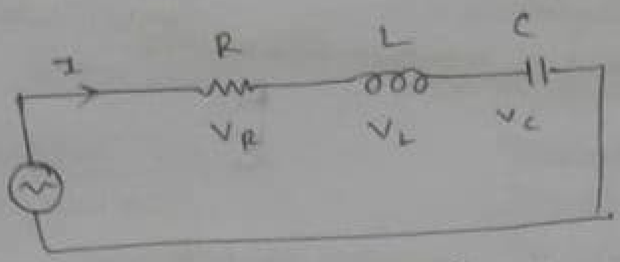
Time-domain representation of system

② Dynamics representation in terms of laplace transform expressions

Frequency domain representation of the system.

③ State is a set of variables that describes the system behaviour in conjunction with the system input.

Dinamics are represented by a set of 1st order differential equations using this state variable.



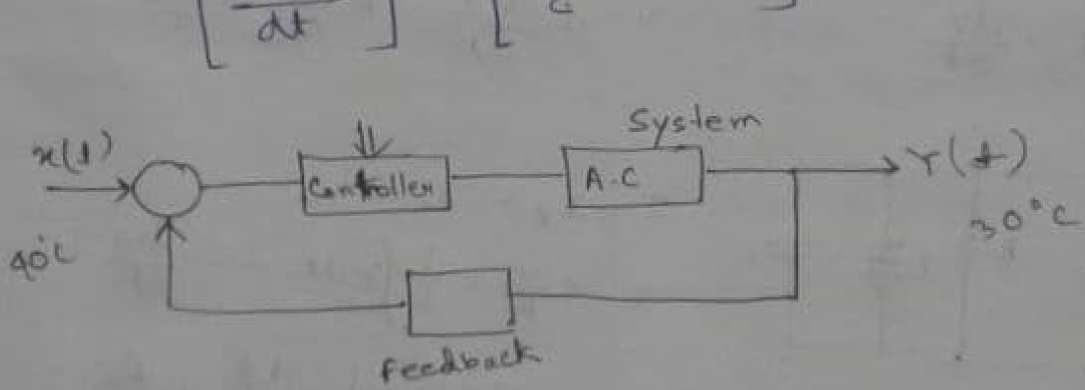
① $V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$

$$\frac{dV}{dt} = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

↓ Laplace Tr.

② $H(s) = \frac{I(s)}{V(s)} = \frac{1}{R + LS + \frac{1}{CS}}$

③ $$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_C}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \cdot V$$



Physical system

① Electrical system → R L C basic elements
② Mechanical ,,
③ Electronic ,,
④ Hydralic ,,
⑤ Thermal ,,

Electrical System

based on the type of source

- → Voltage sourced system
- → Current sourced system

Voltage sourced	Current sourced

Basic system elements

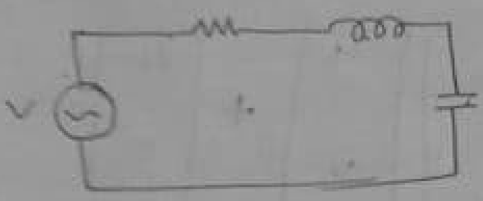
R L C	R L C

Basic system variables

V (voltage) Q (charge)	I (current) ϕ (flux)

$R \rightarrow IR$ $\quad L \rightarrow V_L = \frac{d\phi}{dt} = L\frac{dI}{dt}$

$C \rightarrow I = \frac{dv}{dt} = C\frac{dv}{dt}$

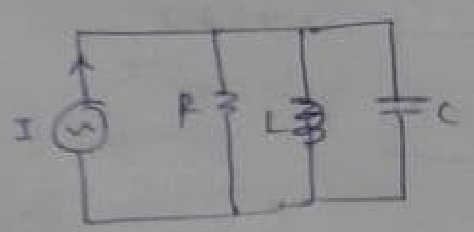


$$V = V_R + V_L + V_C$$

$$V = IR + L\cdot\frac{dI}{dt} + \frac{1}{C}\int I\,dt$$

$$V = L\frac{d^2q}{dt^2} + R\frac{dv}{dt} + \frac{q}{c}$$

$$I = \frac{dv}{dt}$$



$$I = I_R + I_L + I_C$$

$$I = \frac{V}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt}$$

$$I = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L}$$

$$v = \frac{d\phi}{dt}$$

Mechanical system

based on type of motion

- → Translation systems having Linear motion
- → Rotational systems having angular motion about a fixed axis

Translational	Rotational
Basic system elements	
Mass (M)	Inertia (J)
Damper (D)	Damper (D)
Linear spring (K)	Torsional spring (K)
Basic system variables	
Force (F)	Torque (T)
Displacement (x)	Angular Displacement (θ)

Mass

① Property of an element that stores the kinetic energy due to translational motion

② When a force is acting on a load of Mass M causing displacement x

$$F = \frac{dp}{dt} = M\frac{d^2x}{dt^2}$$

$$= M\ddot{x} = M\cdot\frac{dv}{dt}$$

Inertia

② property of an element that stores the kinetic energy due to rotational motion.

② When a torque is acting on a body of inertia (J) causing displacement θ, then

$$T = J\cdot\frac{d^2\theta}{dt^2} = J\ddot{\theta} = J\frac{d\omega}{dt}$$

DAMPER

Damper is an element that generates force which acts opposite to the direction of motion, translational or rotational.

- Damper resists motion
- Friction or dashpot are examples of damper.

Translational

$F = D \frac{dx}{dt} = D \dot{x} = DV$

D is the coefficient of viscous friction in N/m/sec

Rotational

$T = D_0 \frac{d\theta}{dt} = D_0 W$

D_0 = rotational viscous co-efficient

<u>Linear spring</u>

① Property of an element that stores the potential energy due to translational motion

② When a spring of spring constant K is applied a force F causing an elastic displacement of x then.

$F = Kx = K \int_{-\infty}^{t} v(t)\, dt$

$= K \left[\int_{0}^{t} v(t)\, dt + x(0) \right]$

<u>Torsional spring</u>

① Property of an element that stores the potential energy due to rational motion

② When a torsional spring of constant K_θ is applied a torque T causing an angular displacement θ then.

$T = K_\theta \theta = K_\theta \int_{-\infty}^{t} \omega(t)\, dt$

$= K_\theta \left[\int_{0}^{t} \omega(t)\, dt + \theta(0) \right]$

ANALOGOUS SYSTEM.

Mechanical systems can be represented using electrical element by the following analogies.

① Force (Torque) - Voltage analogy
(F-V) analogy

② Force (Torque) - Current analogy
(F-I) analogy

Mechanical system		Electrical system	
Translation	Rotation	F-V	F-I
Force (F)	Torque (T)	Voltage (V)	Current (I)
Mass (M)	Inertia (J)	Inductor (L)	capacitor (C)
Friction (D)	Friction (D_θ)	Resistor (R)	Conductance ($\frac{1}{R}$)
Linear spring (K)	Torsional spring (K_θ)	Capacitor ($\frac{1}{C}$)	Inductor ($\frac{1}{L}$)
Displacement (x)	Displacement (θ)	charge (Q)	Flux (ϕ)
velocity (v)	Angular velocity (ω)	current (I)	voltage (V)

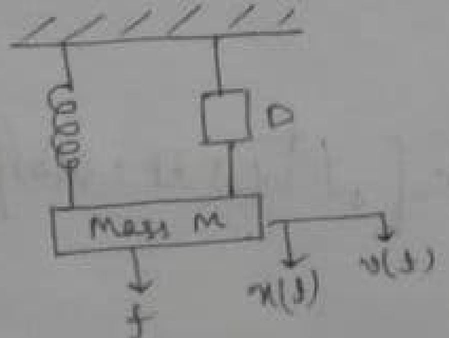
$$f = M\cdot\frac{d^2x(t)}{dt^2} + D\cdot\frac{dx}{dt} \rightarrow \text{mech} \qquad f_1 = M\frac{dv_1}{dt} + Dv_1$$

F-V
$$V = L\cdot\frac{d^2Q}{dt^2} + R\cdot\frac{dQ}{dt} \qquad V_1 = L_1\frac{dI_1}{dt} + RI_1$$

F-I
$$I = C\cdot\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} \qquad I_1 = C_1\frac{dV_1}{dt} + \frac{1}{R}V_1$$

D' ALEMBERT'S PRINCIPLE

"For any body, the algebraic sum of externally applied forces and the forces resisting motion in any given direction is zero.

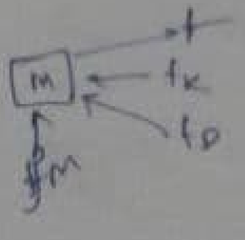


External force = f

Resisting force

(A) Inertial force

$$f_M = -M\cdot\frac{dv(t)}{dt}$$



(B) Damping force

$$f_D = -D\,v(t)$$

(C) Spring force

$$f_K = -K\left[\int_0^t v(t)\,dt + x(0)\right]$$

According to D' Alembert's principle

$$f + f_m + f_D + f_K = 0$$

$$f = M\frac{dv(t)}{dt} + Dv(t) + K\left[\int_0^t v(t)\,dt + x(0)\right]$$

$\Rightarrow$ dinamic equation for the translation mechanical system.

For any body, the algebric sum of externally applied torques and the torques resisting motion about any axis is zero.

External torque $= T$

Resisting torque :-

Ⓐ Inertial torque.

$$T_I = -J\frac{d\omega(t)}{dt}$$

Ⓑ Damping torque.

$$T_D = -D_0\,\omega(t)$$

Ⓒ spring Torque.

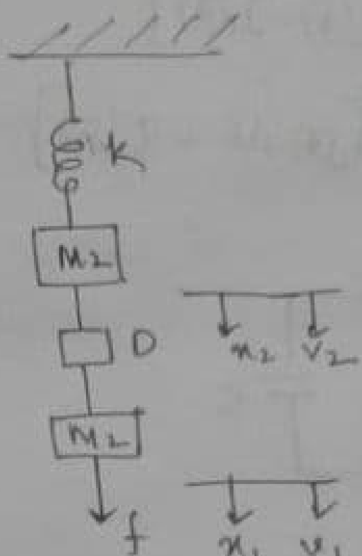
$$T_K = -K_0\left[\int_0^t \omega(t)\,.dt + \theta(0)\right]$$

By D' Alembert's Principal

$$T + T_I + T_D + T_K = 0$$

$$T = J\frac{d\omega(t)}{dt} + D_0\,\omega(t) + K_0\left[\int_0^t \omega(t)\,dt + \theta(0)\right]$$

ex-1: Write the system dynamic equations and draw the electrical analogous circuit of the mechanical system show in fig using force-voltage analogy.



The forces on mass M_1,

External force → f

Resisting forces

(A) Inertial force

$$f_{M_1} = -M_1 \frac{dv_1(t)}{dt}$$

(B) Damping force

$$f_{D_1} = -D[v_1(t) - v_2(t)]$$

$$f = M_1 \frac{dv_1(t)}{dt} + D[v_1(t) - v_2(t)] \quad \text{— (1)}$$

mass – M_2

External force = 0

Resisting forces

(A) Inertial force

$$f_{M_2} = -M_2 \frac{dv_2(t)}{dt}$$

(B) Damping force

$$f_{D_2} = -D[v_2(t) - v_1(t)]$$

(C) Spring force

$$f_K = -K\left[\int_0^t v_2(t)\,dt - x_2(0)\right]$$

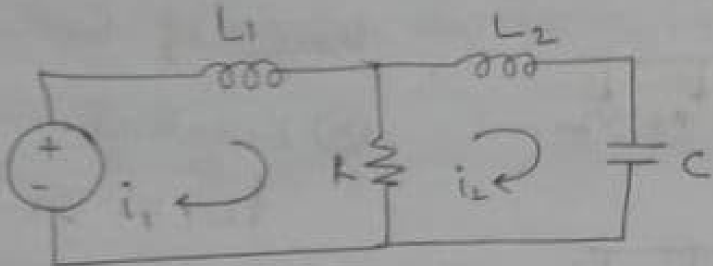
$$0 = M_2 \frac{dv_2(t)}{dt} + D[v_2(t) - v_1(t)] + K\left[\int_0^t v_2(t)\,dt + x_2(0)\right]$$

→ (2)

F-V $$f = M_1 \frac{dv_1(t)}{dt} + D\left[v_1(t) - v_2(t)\right]$$

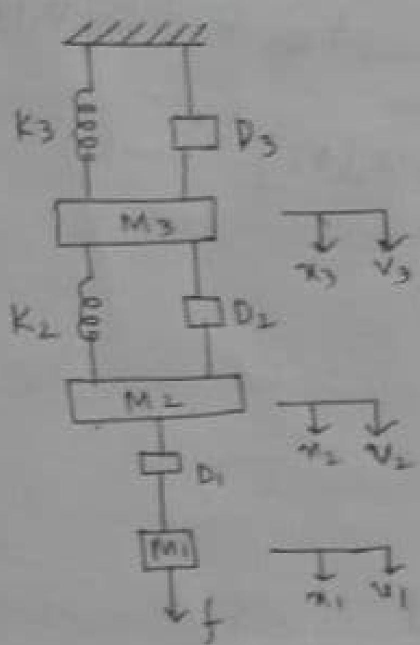
$$V = L_1 \frac{di_1(t)}{dt} + R\left[i_1(t) - i_2(t)\right] \quad \text{— (3)}$$

$$0 = L_2 \frac{di_2(t)}{dt} + R\left[i_2(t) - i_1(t)\right] + \frac{1}{C}\left[\int_0^t i_2(t)\,dt + q_2(0)\right]$$

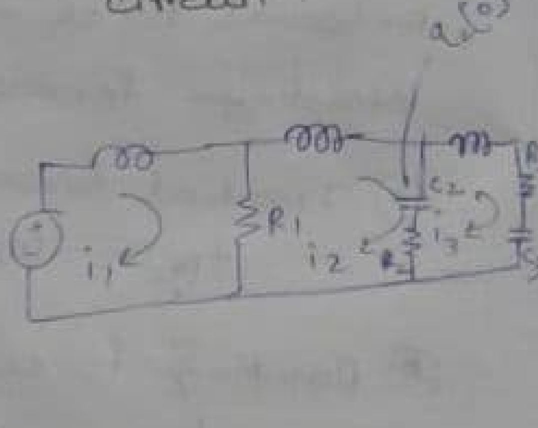


14.10.2020

ex-2



f-v electrical analogous circuit



$$f_1 = M_1 \frac{dv_1}{dt} + D_1\left[v_1(t) - v_2(t)\right]$$

$$V_1 = L_1 \frac{di_1(t)}{dt} + R_1\left[i_1(t) - i_2(t)\right]$$

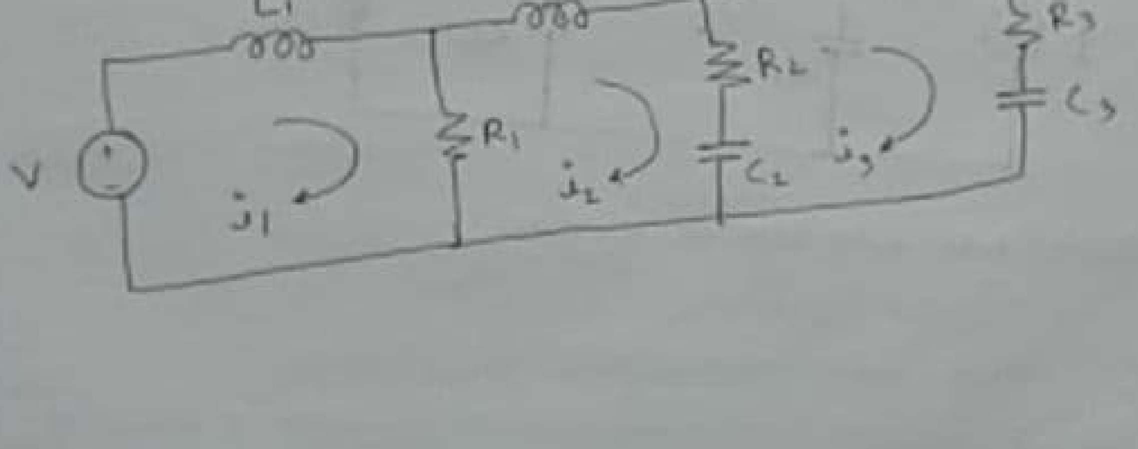
$$0 = M_2 \frac{dv_2(t)}{dt} + D_2[v_2(t) - v_3(t)] + D_1[v_2(t) - v_1(t)] + K_1\left[\int_0^t [v_2(t) - v_3(t)]\,dt + x_2(0)\right]$$

$$0 = L_2 \frac{di_2(t)}{dt} + R_2(i_2(t) - i_3(t)) + R_1(i_2(t) - i_1(t)) + \frac{1}{C_2}\left[\int_0^t (i_2(t) - i_3(t))\,dt + q_2(0)\right]$$

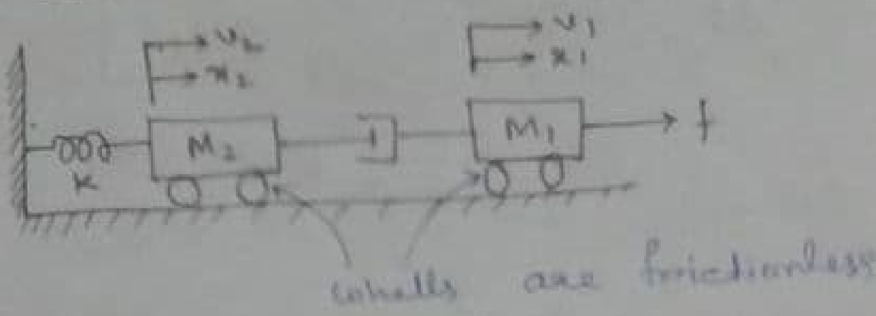
for M_3

$$0 = M_3 \frac{dv_3(t)}{dt} + D_3 v_3(t) + D_2[v_3(t) - v_2(t)] + K_3\left[\int_0^t v_3(t)\,dt + x_3(0)\right] + K_2\left[\int_0^t (v_3(t) - v_2(t))\,dt + x_2(0)\right]$$

$$0 = L_3 \frac{di_3(t)}{dt} + R_3 i_3(t) + R_2[i_3(t) - i_2(t)] + \frac{1}{C_3}\left[\int_0^t i_3(t)\,dt + q_1(0)\right] + \frac{1}{C_2}\left[\int_0^t [i_3(t) - i_2(t)]\,dt + q_2(0)\right]$$



Force - current



mass M_1

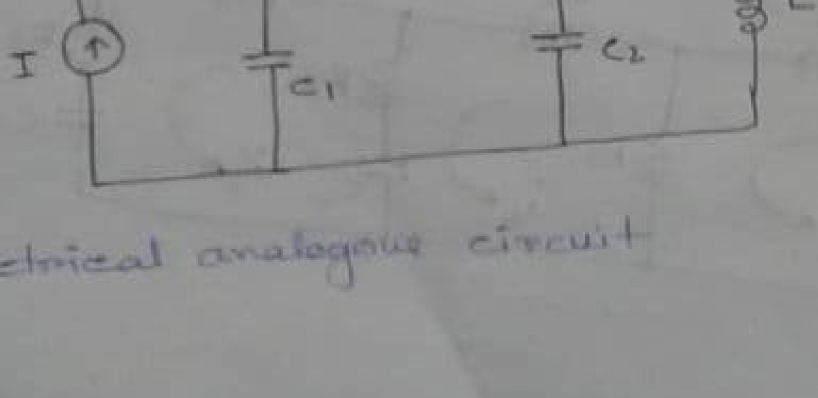
$$f = M_1 \frac{dv_1(t)}{dt} + D\left[v_1(t) - v_2(t)\right]$$

$$I = C_1 \frac{dv_1(t)}{dt} + \frac{1}{R}\left[v_1(t) - v_2(t)\right]$$

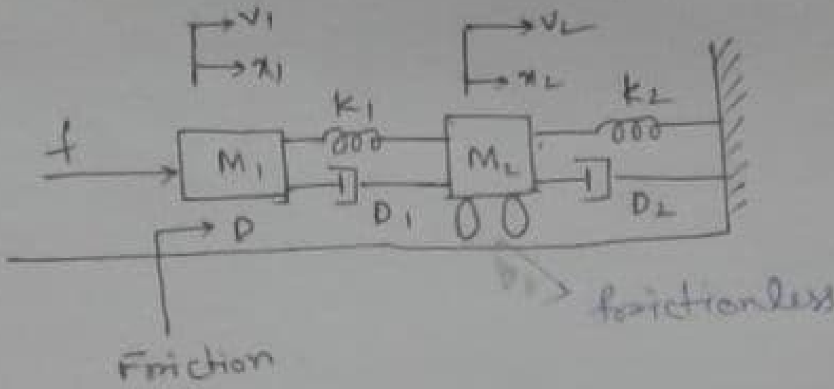
for M_2

$$0 = M_2 \frac{dv_2(t)}{dt} + D\left[v_2(t) - v_1(t)\right] + K\left[\int_0^t v_2(t)\,dt + x_2(0)\right]$$

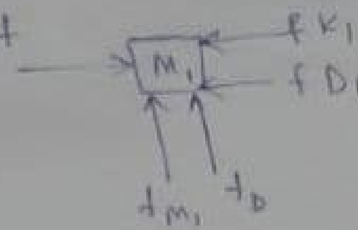
$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{R}\left[v_2(t) - v_1(t)\right] + \frac{1}{L}\left[\int_0^t v_2(t)\,dt + \phi(0)\right]$$

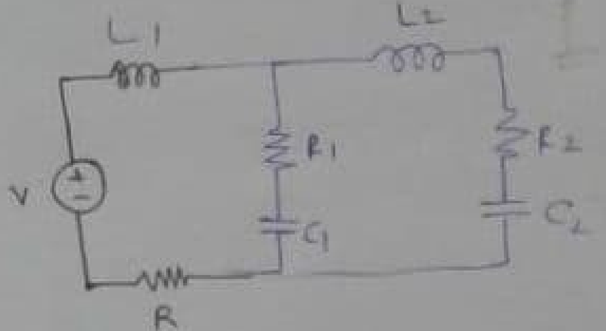


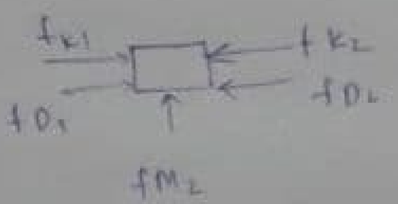
Electrical analogous circuit


v1
x1
v2
x2
K1
K2
f
M1
M2
D1
D2
D
frictionless
Friction

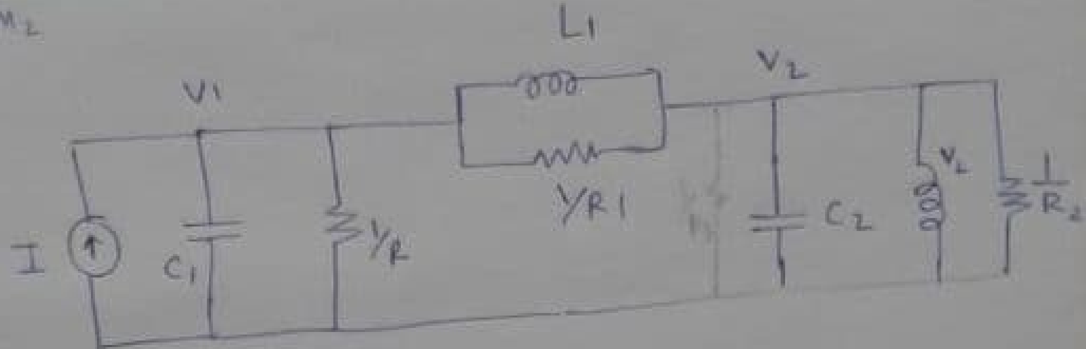
F-V


f
fK1
fD1
M1
fm1
fD


L1
L2
V
R1
R2
C1
C2
R

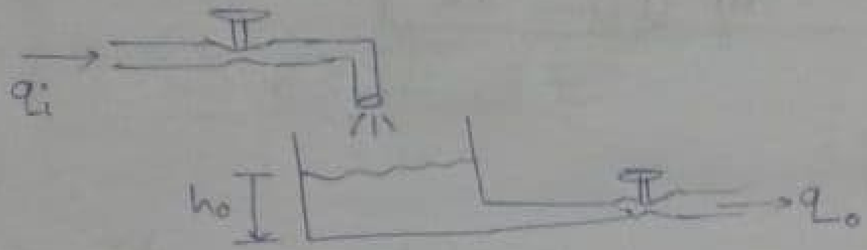

fK1
fK2
fD2
fD1
fM2

F-I


L1
V1
V2
1/R1
I
C1
1/R
C2
1/R2

16.10.2020

LIQUID LEVEL SYSTEM



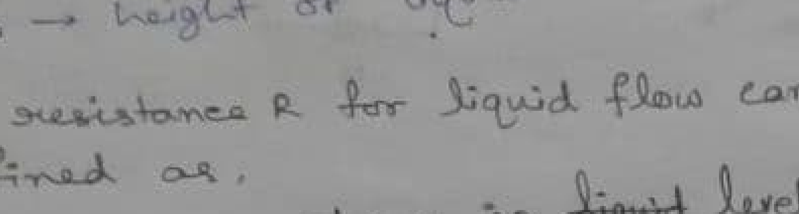
$q_i \rightarrow$ Inflow rate of the liq in m^3/sec
$q_o \rightarrow$ outflow rate of the liq in m^3/sec
$h_o \rightarrow$ height of liquid in tank in m

The resistance R for liquid flow can be ~~def~~ defined as,

$$R = \frac{\text{change in liquid level difference}}{\text{change in flow rate.}}$$

For laminar flow, the flow resistance,

$R = \frac{\Delta h}{\Delta q}$ is constant and analogous to electrical resistance



laminar flow   Turbulence flow

under steady-state condition we have $q_i = q_o$ and h_o is the steady-liquid level in the tank

Let small increase in liquid inflow rate by

$\Delta q_i \longrightarrow \Delta h_o \uparrow$, $\Delta q_o \uparrow$ $\Delta q_o = \frac{\Delta h}{R}$

liquid flow rate balanced equation

Rate of liquid inflow - rate of liquid outflow = rate of liquid storage in the tank.

$$\Delta q_i - \Delta q_o = C \frac{d(\Delta h)}{dt}$$, C is cross sectional area of the tank and analogous to electrical capacitance

$$RC \frac{d(\Delta h)}{dt} + \Delta h = R(\Delta q_i) \quad \text{--- ①}$$

eq ① is the mathematical model of given figure

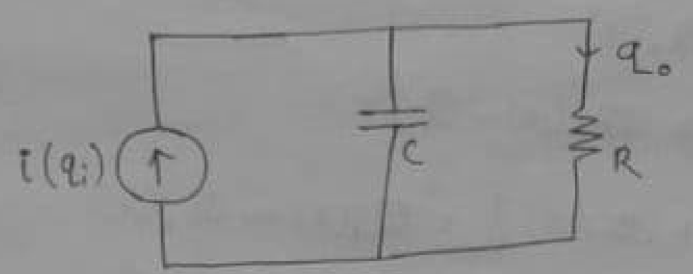
Taking the Laplace transform of eq ①

$$RC\, sH(s) + H(s) = RQ_i(s)$$

$$\frac{H(s)}{Q_i(s)} = \frac{R}{RCs+1}$$

wher $H(s) \rightarrow$ Laplace Tr. of Δh

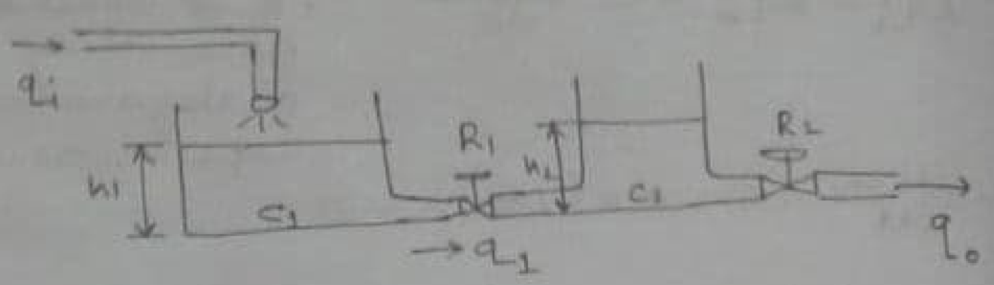
$Q_i(s) \rightarrow$ " " " Δq_i



Electrical analogous of liquid level system.

Liquid level	Electrical System
Liquid flow, m^3	charge, coulombs
Liquid flow rate, m^3/sec	current, amps
head/height, m	Voltage, volts
Liquid resistance	Resistance, Ω
Liquid Capacitance (cross-sectional area of the tank)	Capacitance

ex-1:



$$R_1 = \frac{h_1 - h_2}{q_1} \qquad C_1 \frac{dh_1}{dt} = q_i - q_1$$

$$R_2 = \frac{h_2}{q_0} \qquad C_2 \frac{dh_2}{dt} = q_1 - q_0$$

Taking laplace Tr.

$$\begin{cases} H_1(s) - H_2(s) = R_1 Q_1(s) \\ C_1 s H_1(s) = Q_i(s) - Q_1(s) \end{cases}$$

$$\begin{cases} H_2(s) = R_2 Q_0(s) \\ C_2 s H_2(s) = Q_1(s) - Q_0(s) \end{cases}$$

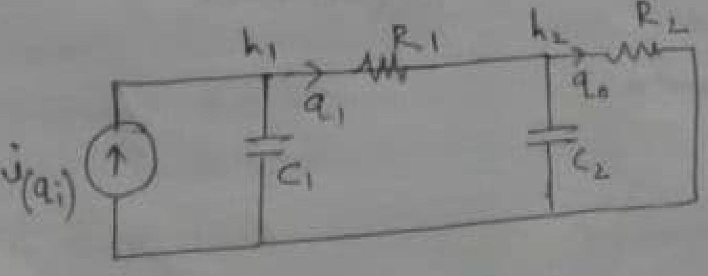
$\to C_1 s \left[H_2(s) + R_1 Q_1(s) \right] = Q_i(s) - Q_1(s)$

$\to C_2 s \left[R_2 Q_0(s) \right] = Q_1(s) - Q_0(s)$

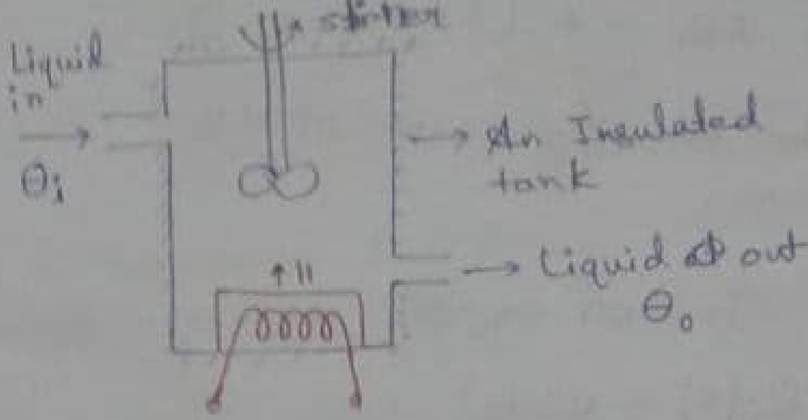
$$C_1 s \left[R_2 Q_0(s) + R_1 \{ 1 + R_2 C_2 s \} Q_0(s) \right] = Q_i(s) - (1 + R_2 C_2 s) Q_0(s)$$

$$\frac{Q_0(s)}{Q_i(s)} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_1 + R_2 C_2) s + 1}$$

Transfer function of the given liquid level system



THERMAL SYSTEM



H → steady state heat input rate from the heater.

Let small increase in ΔH (heat input rate) from its steady state value.

$$\Delta H = \Delta H_1 + \Delta H_2$$

ΔH_1 → heat outflow rate

ΔH_2 → heat storage rate of the liquid.

Temp of the liquid in tank is also increase.

The increase in outflow rate

$$\Delta H_1 = QS.\Delta\theta_0$$

where Q = steady liquid flow rate.

S = specific heat of the liquid.

$$\Delta H_1 = \frac{\Delta\theta_0}{R} \quad \text{—— (1)}$$

where $R = \frac{1}{QS}$ is called the thermal resistance.

The rate of heat storage in the tank, $\Delta H_2 = M.S\frac{d(\Delta\theta_0)}{dt}$

where M → Mass of liquid in tank.

$$\Delta H_2 = C\frac{d(\Delta\theta_0)}{dt} \quad \text{—— (2)}$$

where $C = M.S.$ is called the thermal capacitance

$$\Delta H = \Delta H_1 + \Delta H_2$$

$$\Delta H = \frac{\Delta\theta_0}{R} + C \cdot \frac{d(\Delta\theta_0)}{dt}$$

$$RC \frac{d}{dt}(\Delta\theta_0) + \Delta\theta_0 = R(\Delta H) \quad \text{—— (3)}$$

eq (3) describes the dynamics of the thermal system.

taking Laplace Tr of eq (3)

$$RCs\,\theta_0(s) + \theta_0(s) = RH(s)$$

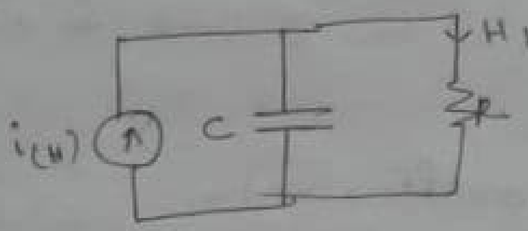
$$\frac{\theta_0(s)}{H(s)} = \frac{R}{RCs+1}$$

where $\theta_0(s)$ is L Tr of $\Delta\theta_0$

$H(s)$ is L Tr of ΔH

θ_0 is the Temp of outflow liquid

H is the heat input rate from the heater.



Electrical analogous of the thermal system of given fig

Thermal	Electrical
Heat flow	Charge
Heat flow rate	current
Temp	Voltage.
Thermal Resistance	R.
Thermal Capacitance	C.

NETWORK

① Transform impedance

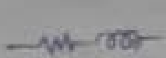
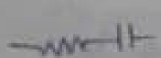
$$z(s) = \frac{V(s)}{I(s)}$$

$$S = \sigma + j\omega$$

Transform admittance

$$Y(s) = \frac{1}{Z(s)} = \frac{I(s)}{V(s)}$$

<u>Impedance functions</u>

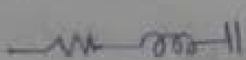
	Impedance	Admittance
R	R	$1/R$
L	SL	$1/SL$
C	$1/SC$	SC
	$R + SL$	$\frac{1}{R+SL}$
	$R + \frac{1}{SC}$	$\frac{1}{R} + SC$

for series circuit

$$z(s) = Z_1(s) + Z_2(s) + Z_3(s)$$

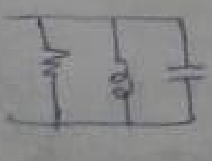
for parallel circuit

$$Y(s) = Y_1(s) + Y_2(s) + Y_3(s)$$



$$z(s) = R + SL + \frac{1}{SC}$$

$$= \frac{RCS + LCS^2 + 1}{SC}$$



$$Y(s) = \frac{1}{R} + \frac{1}{SL} + SC$$

$$= \frac{SL + R + S^2RLC}{SL\,R}$$

Transfer Impedance function, $Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$

Transfer Admittance function, $Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$

Voltage transfer function, $G_{21}(s) = \frac{V_2(s)}{V_1(s)}$

Current transfer function $L_{21}(s) = \frac{I_2(s)}{I_1(s)}$

POLES and ZEROS of Network functions

All network functions $T(s)$ are the rational functions of s and may be expressed as the ratio of two polynomials $N(s)$ & $D(s)$

$N(s) \rightarrow$ Numerator polynomial

$D(s) \rightarrow$ Denominator "

$$T(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} \ldots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} \ldots + b_1 s + b_0}$$

$$= K \frac{s^n + c_{n-1} s^{n-1} + \ldots + c_1 s + c_0}{s^m + d_{m-1} s^{m-1} \ldots + d_1 s + d_0}$$

$K = \frac{a_n}{b_m}$ is a positive constant and know as scalar function

The polynomial $N(s) = 0$ has n roots

They are called as 'zeros' of the network function $T(s)$

The polynomial $D(s) = 0$ has m roots

They as called as poles of the network functions $T(s)$

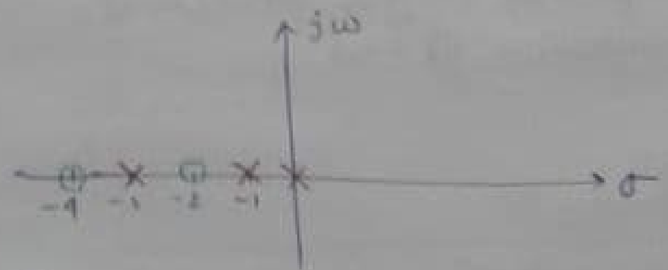
$$T(s) = K\frac{(s-z_1)(s-z_2)\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots(s-p_m)}$$

$z_i \rightarrow$ The zeros of $T(s)$
$p_i \rightarrow$ The poles of $T(s)$

For any rational network function
The total no of poles = the total number of zeroes

$$T(s) = \frac{(s+2)(s+4)}{s(s+1)(s+3)}$$

x poles at $s=0$, $s=-1$, $s=-3$
0 zeros at $s=-2$, $s=-4$.



Time response

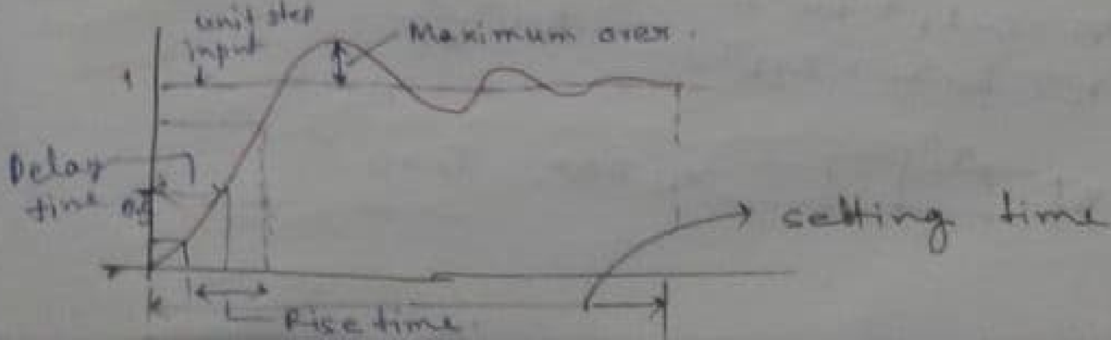
For a unit step input, $r(t) = u(t)$ or $R(s) = \frac{1}{s}$

the output response is obtained by taking the I.L.Tr of the $C(s)$

$$C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n+\omega_n^2)}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left(\omega_n\sqrt{1-\xi^2}\,t + \cos^{-1}\xi\right) \quad t \geq 0$$

The typical unit-step response of second order control system



Frequency Response

ω

$H(s) \rightarrow H(j\omega)$

$\sigma + j\omega$

$H(j\omega) \rightarrow$ frequency domain transfer function

$H(j\omega) = |H(j\omega)| \angle H(j\omega)$

$= M_r \angle \phi_r$

$M_r \rightarrow$ Resultant magnitude which is function of ω

$\phi_r \rightarrow$ Resultant phase angle which is also a function of ω.

$$H(s) = \frac{20}{(s+1)(s+3)}$$

$$H(j\omega) = \frac{20}{(j\omega+1)(j\omega+3)}$$

$$= M_r \angle \phi_r$$

$$M_r = \frac{20}{\sqrt{\omega^2+1}\,\sqrt{\omega^2+3^2}}$$

$$\phi_r = 0 - \tan^{-1}\frac{\omega}{1} - \tan^{-1}\frac{\omega}{3}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

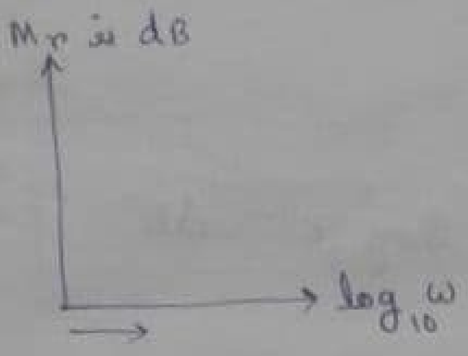
$V_1 = Z_{11} I_1 + Z_{12} I_2$ $\qquad Z_{21} = \frac{V_2}{I_1}$

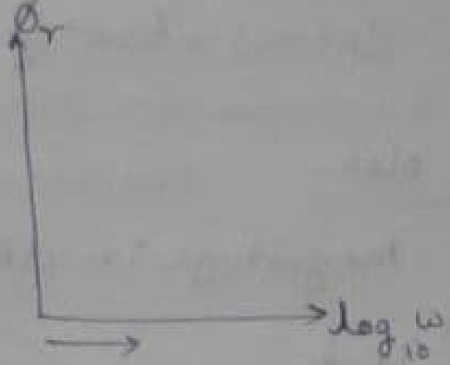
$V_2 = Z_{21} I_1 + Z_{22} I_2$

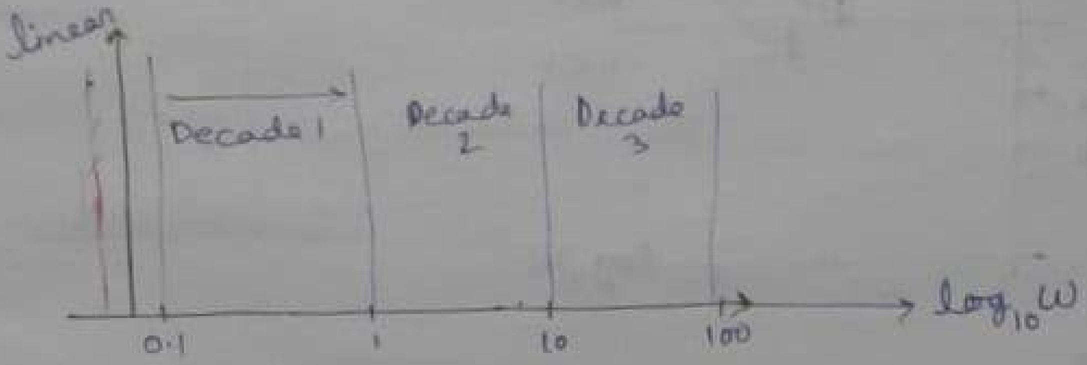
V_2 open $\quad I_2 = 0$

BODE PLOT

$$H(j\omega) = |H(j\omega)| \angle H(j\omega)$$
$$= M_r \angle \phi_r$$







$\longrightarrow$ Non-linear

$$|H(j\omega)| \text{ in dB} = M_r \text{ in dB}$$
$$= 20 \log_{10} |M_r|$$

$$\omega \to 0 \text{ to } \alpha$$

$$\angle H(j\omega) = 0 - \tan^{-1}\left(\frac{\omega}{1}\right)$$

$$\underline{\underline{H(j\omega)}} = \frac{\boxed{K}(1 + T_1 j\omega)(1 + T_2 j\omega)}{(j\omega)^{\pm P}(1 + T_a j\omega)(1 + T_b j\omega)}$$

freq domain transfer function

$$\left[1 + \frac{2\xi}{\omega_n} j\omega + \frac{1}{\omega_n^2}(j\omega)^2\right]^{\pm 1}$$

System Gain K

Let $H(s) = K$

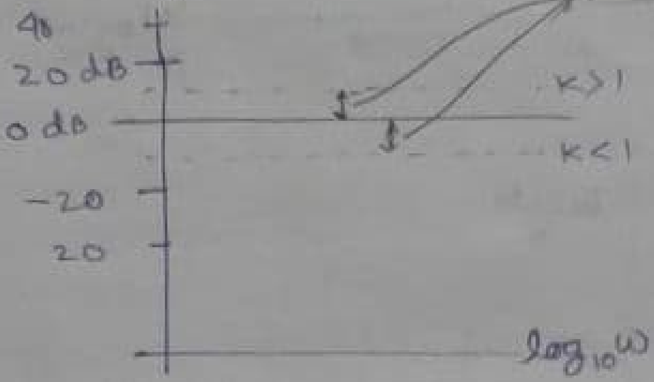
$H(j\omega) = K + j0$

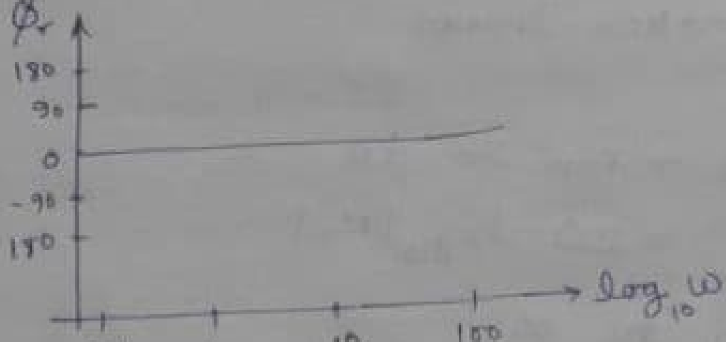
$H(j\omega) = \sqrt{K^2 + 0} = K$

$\angle H(j\omega) = \tan^{-1}\frac{0}{K} = 0$

<u>Mr plot.</u>

Magnitude in dB $= \boxed{20 \log_{10} K}$ dB





$H(s) = \frac{1}{s}$

$H(j\omega) = \frac{1}{j\omega} = \frac{1}{0 + j\omega}$

$|H(j\omega)| = \frac{1}{\sqrt{0^2 + \omega^2}} = \frac{1}{\omega}$

$\angle H(j\omega) = \angle\frac{1}{j\omega} \quad -90^\circ$

magnitude in dB $= 20 \log_{10} |M_r|$

$= 20 \log_{10} \frac{1}{\omega}$

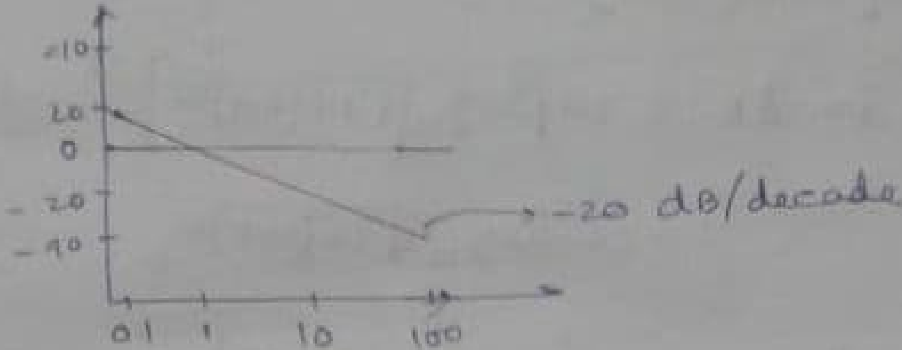
$= -20 \log_{10} \omega$

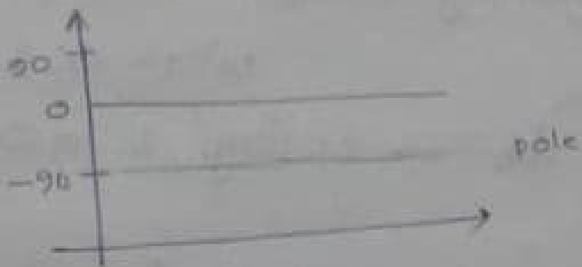
$= 0 \text{ dB} \longrightarrow \omega = 1$

$-20 \text{ dB} \longrightarrow \omega = 10$

$-40 \text{ dB} \longrightarrow \omega = 100$

$+20 \text{ dB} \longrightarrow \omega = 0.1$

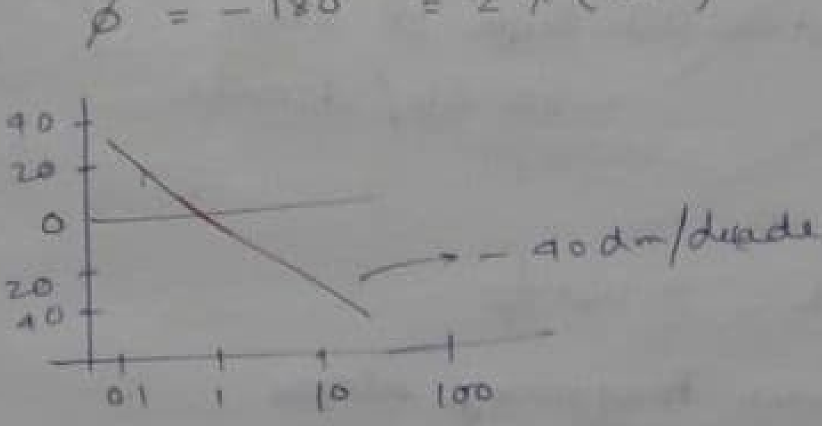




$H(s) = \frac{1}{s^2}$

$M_r = -40 \text{ dB/decade}$

$\phi = -180 = 2 \times (-90^\circ)$



$$H(s) = \frac{1}{1+Ts}$$

$$H(j\omega) = \frac{1}{1+Tj\omega}$$

$$|H(j\omega)| = \left[\sqrt{1+(\omega T)^2}\right]^{-1}$$

$$\text{Magnitude in dB} = 20\left\{\log_{10}\left[\sqrt{1+(\omega T)^2}\right]^{-1}\right.$$

$$= -20\log_{10}\sqrt{1+(\omega T)^2}$$

(i) For low frequency range $\omega \ll \frac{1}{T}$

$$\omega^2 T^2 \ll 1$$

$\therefore$ magnitude in dB $= -20\log_{10} 1 = 0$

(ii) For High frequency $\omega \gg \frac{1}{T}$

$$\omega^2 T^2 \gg 1$$

$\therefore$ magnitude in dB $= -20\log_{10}\omega T$

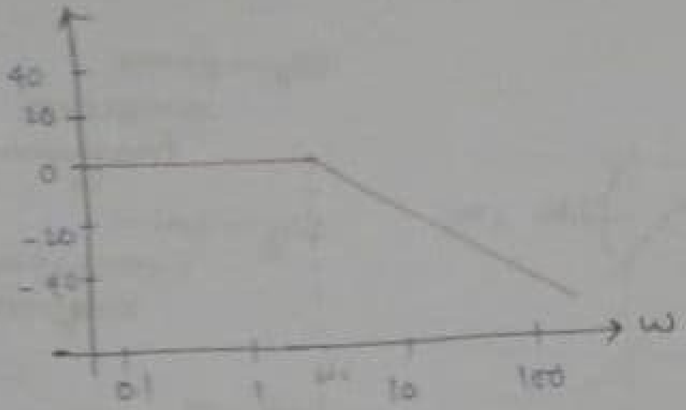
$\therefore$ It is straight ~~line~~ line of slope -20 dB/decade

$$-\log_{10}\omega T = 0$$

$\Rightarrow \omega_c T = 1 \quad \Rightarrow \omega_c = \frac{1}{T}$

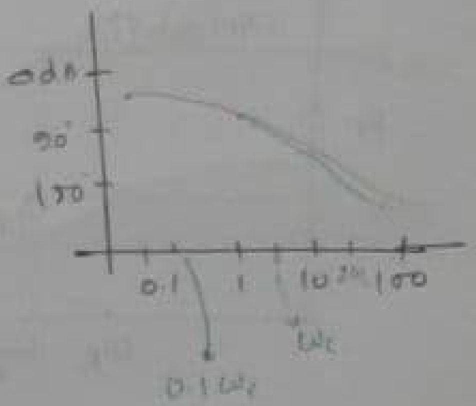
ω_c = corner frequency ~~slope~~

In this freq slope change from 0 dB to -20 dB/decade



$\angle \phi_T = 0 - \tan^{-1} \frac{\omega T}{1} = -\tan^{-1} \omega T$

ω	$\tan^{-1} \omega T$
$0.1\,\omega_c = \frac{1}{10T}$	$-5.71°$
$\omega_c = \frac{1}{T}$	$-45°$
$2\omega_c = \frac{2}{T}$	$-63.4°$
$10\,\omega_c = \frac{10}{T}$	$-84.3°$

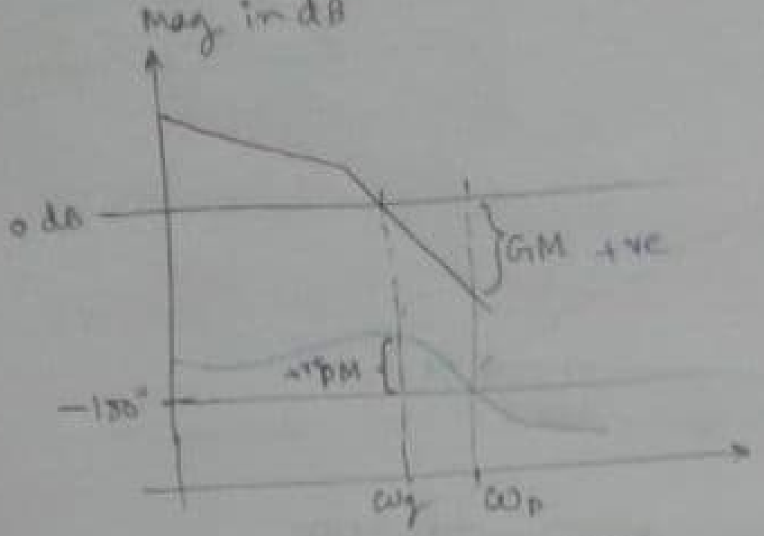


① Phase crossover freq → ω_p
② Gain crossover freq → ω_g
③ Gain margin (GM)
④ Phase margin (PM)

$$GM = \frac{1}{|G(j\omega_p)|}$$

$$GM \text{ in dB} = 20 \log_{10} \frac{1}{|G(j\omega_p)|}$$
$$= -20 \log_{10} |G(j\omega_p)|$$

$$PM = 180° + \phi$$
$$\phi = \angle G(j\omega_g)$$

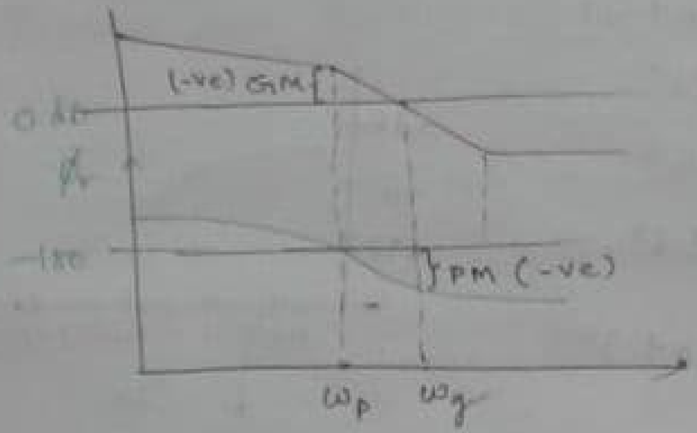


$\omega_p \rightarrow$ phase crossover frequency

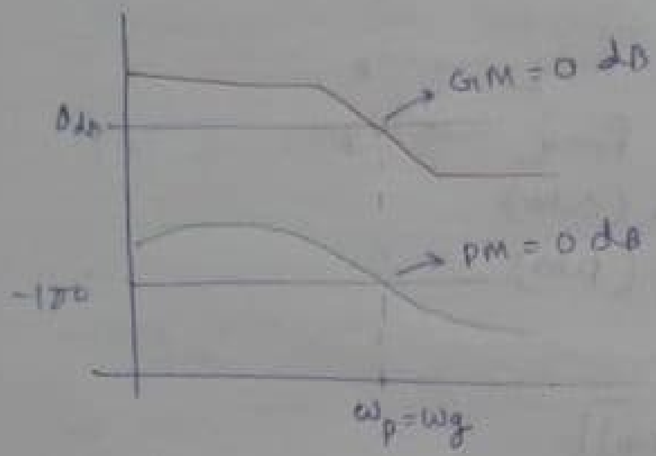
$\omega_g \rightarrow$ gain crossover frequency

stable

$GM = -20 \log_{10} |G(j\omega_p)|$



Unstable



marginal stable system