Practice problems

1) Express $f(x) = |x|$, $-\pi \le x \le \pi$ as Fourier series.

2) Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \le x \le \pi$. Deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots = \frac{\pi - 2}{4}$.

3) Find the fourier series expansion of the following periodic function with period 2π,

$$f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ 0, & 0 \le x < \pi \end{cases}$$

$f(x + 2\pi) = f(x)$. Hence deduce that,

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}.$$

4) Show that for $-\pi < x < \pi$,

$$\sin ax = \frac{2 \sin a\pi}{\pi} \left[\frac{\sin x}{1^2 - a^2} - \frac{2 \sin 2x}{2^2 - a^2} + \frac{3 \sin 3x}{3^2 - a^2} - \cdots \right]$$

5) Obtain the fourier series for the function

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$$

Hence deduce that,

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}.$$