LAPLACE TRANSFORM EXAMINATION

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Q1) What is Linearity Property of Laplace Transform?

<u>Linearity Property of Laplace Transform</u>

If α and β are any constants, and there exist two functions.. $F_1(t)$ & $F_2(t)$. ~~with their~~

Then

$$L\{\alpha F_1(t) + \beta F_2(t)\} = \alpha f_1(s) + \beta f_2(s),$$

where $f_1(s)$ and $f_2(s)$ are Laplace Transform of $F_1(t)$ and $F_2(t)$ respectively

Q2) Prove that $L\{\cosh(at)\} = \frac{s}{s^2-a^2}$, if $s>|a|$

→ we know that $\cosh(x) = \frac{e^x+e^{-x}}{2}$

$\therefore \cosh(at) = \frac{e^{at}+e^{-at}}{2}$

→ also we know $L\{F(t)\} = \int_0^\infty e^{-st}F(t)\,dt$

$$\therefore L\{\cosh(at)\} = \int_0^{\infty} e^{-st}\left[\frac{e^{at}+e^{-at}}{2}\right] dt$$

$$= \int_0^{\infty} \frac{e^{-t(-a+s)}}{2}\, dt + \int_0^{\infty} \frac{e^{-t(a+s)}}{2}\, dt$$

$$= \lim_{D\to\infty}\left[-\frac{e^{-t(s-a)}}{2(s-a)}\right]_{t=0}^{t=D} + \lim_{D\to\infty}\left[-\frac{e^{-t(a+s)}}{2(a+s)}\right]_{t=0}^{t=D}$$

$$= \lim_{D\to\infty}\left[\frac{-e^{-D(s-a)}+e^{0}}{2(s-a)}\right] + \lim_{D\to\infty}\left[\frac{-e^{-D(s+a)}+e^{0}}{2(s+a)}\right]$$

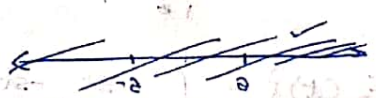
now as $D\to\infty$; $e^{-D}\to 0$ [considering $s-a>0$ ~~$s+a>0$~~]

$$\therefore L\{\cosh(at)\} = \frac{1}{2(s-a)} + \frac{1}{2(s+a)}$$

$$= \frac{1}{2}\left[\frac{s+\cancel{a}+s-\cancel{a}}{\cancel{4}(s-a)(s+a)}\right]$$

$$= \frac{s}{s^2-a^2}$$ $s-a>0$ or $s>|a|$ ~~$s+a>0$~~

∴ ~~also the intersection of $s-a>0$ and $s+a>0$~~ is



$$\therefore L\{\cosh(at)\} = \frac{s}{s^2-a^2},\quad s>|a|$$

Q3) Define Inverse Laplace Transform. Show with an example that it may not be unique.

If we know that $L\{F(t)\} = f(s)$, then we define Inverse Laplace transform $L^{-1}\{\}$, such that

$$L^{-1}\{f(s)\} = F(t)$$

Eg: $L^{-1}\left\{\frac{1}{s}\right\} = 1$

$$L^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin(at)$$

→ Consider two Functions

i) $F_1(t) = e^{-5t}$

ii) $F_2(t) = \begin{cases} 0 & t = 1 \\ e^{-5t} & \text{otherwise} \end{cases}$

~~doing~~ now

$$L\{F_1(t)\} = \frac{1}{s+5}$$

and

$$\cancel{L\{F_2(t)\} = \int_0^1 e^{-st} \times 0 \, dt + \int_1^\infty e^{-st} \, dt}$$

$$L\{F_2(t)\} = \int_0^1 e^{-5t} e^{-st} dt + \int_1^1 e^{-st} \times 0 \, dt + \int_1^\infty e^{-st} e^{-5t} dt$$

$$= \int_0^\infty e^{-st} e^{-5t} dt$$

$$= \lim_{D \to \infty} \left[\frac{-e^{-(s+5)t}}{s+5} \right]_{t=0}^{t=D}$$

$$= \lim_{D \to \infty} \frac{-e^{-D(s+5)} + 1}{s+5}$$

$$= \frac{1}{s+5}, \quad s+5 > 0$$

→ now from the Definition of Inverse Laplace Transform, we get

$$L^{-1}\left\{\frac{1}{S+5}\right\} = e^{-5t}$$

and also

$$L^{-1}\left\{\frac{1}{S+5}\right\} = \begin{cases} 0 & t=1 \\ e^{-5t} & \text{otherwise} \end{cases}$$

∴ Hence by this example. we have shown that Inverse Laplace Transform may not be unique.

Q4) Find the Inverse Laplace Transform of the following function,

$$f(s) = \frac{e^{-\frac{\pi}{3}s}}{s^2+1}$$

→ we know that $L^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin(t)$

→ Using the Second Shift property of Laplace Transform, we get

$$L^{-1}\{f(s)\} = \begin{cases} \sin\left(t-\frac{\pi}{3}\right) & t > \frac{\pi}{3} \\ 0 & \text{otherwise} \end{cases}$$