(2) Find a basis of E^3 containing the vectors $(1,1,2)$ and $(3,5,2)$

~~let~~ we take us consider a standard basis

$$\{(1,0,0),(0,1,0),(0,0,1)\} = \{e_1, e_2, e_3\}$$

now $(1,1,2) = 1\times(1,0,0) + 1\times(0,1,0) + 2\times(0,0,1)$

$\therefore$ as $(1,1,2)$ ~~is dependent~~ can be expressed as a linear combination of $\{e_1, e_2, e_3\}$, we can replace any vector in the basic by $(1,1,2)$ and call it basis

$\therefore$ $\{(1,1,2), e_2, e_3\}$ is a basis.

now ~~let~~ expressing $(3,5,2)$ as linear combination of basis vectors,

$$(3,5,2) = \alpha(1,1,2) + \beta(0,1,0) + \gamma(0,0,1)$$

where α, β, γ are some constant.

$$\therefore \left.\begin{aligned} 3 &= \alpha \\ 5 &= \alpha + \beta \\ 2 &= 2\alpha + \gamma \end{aligned}\right\} \rightarrow \begin{aligned} \alpha &= 3 \\ \beta &= 2 \\ \gamma &= -4 \end{aligned}$$

$\therefore$ $(3,5,2) = 3(1,1,2) + 2(0,1,0) - 4(0,0,1)$

$\therefore$ as we can express $(3,5,2)$ as a linear combination of basis $\{(1,1,2), e_2, e_3\}$, we can use replacement Theorem to say that:

$\rightarrow$ $\boxed{\{(1,1,2), (3,5,2), (0,0,1)\}}$ forms a basis in $E3$

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