THEORY OF PRODUCTION

Will learn throughout:

- Production Function
- Short Run Production
- Law of Variable Proportions
- Long Run Production
- Returns to Scale

Production functions involve (and can provide measurements of) concepts which are useful tools in all fields of economics.

The production function is a purely technical relation which connects factor inputs

and outputs.

It describes the laws of proportion, that is, the transformation of factor inputs into products (outputs) at any particular time period.

The production function represents the technology of a firm of an industry, or of the economy as a whole.

The production function includes all the technically efficient methods or production.

The main concepts are: 1. The marginal productivity of the factors of production. 2. The marginal rate of substitution and the elasticity of substitution. 3. Factor intensity. 4. The efficiency of production. 5. The returns to scale.

A method of production (process, activity) is a combination of factor inputs required for the production of one unit of output. Usually a commodity may be produced by various methods of production. The basic theory of production concentrates only on efficient methods. Inefficient methods will not be used by rational entrepreneurs.

Short Run and Long Run

A period of time so brief that at least one factor of production cannot be varied practically. A factor that cannot be varied practically in the short run is called a fixed input. In contrast, a variable input is a factor of production whose quantity can be changed readily by the firm during the relevant time period. The long run is lengthy enough period of time that all inputs can be varied. In the long run all factors of production are variable inputs.

Suppose that a painting company's customers all want the paint job on their homes to be finished by the end of the day. The firm could complete these projects on time if it had fewer job. To complete all the jobs, it needs to use more inputs. The firm does not have time to buy or rent an extra truck and buy another compressor to run a power sprayer, these inputs are fixed in the short run. To get the work done that afternoon, the firm uses the company's one truck to pick up and drop off temporary workers, each equipped with only a brush and paint, at the last job. However, in the long run, the firm can adjust all its inputs. If the firm wants to paint more houses every day, it hires more full-time workers, gets a second truck, purchases more compressors to run the power sprayers, and uses a computer to keep a track of all its projects.

How long it takes for all inputs to be variable depends on the factors a firm uses. For a janitorial service whose only major input is workers, the short run is a very brief period of time. In contrast, an automobile manufacturer may need many years to build a new manufacturing plant or to design and construct a new type of machine. A pistachio farmer needs the better part of a decade before newly planted trees yield a substantial crop of nuts.

Short Run Production

- It is a period in which at least one input is fixed.
- Consider a production process with only two inputs in which capital is fixed input and labour is a variable input. The firm can increase output by increasing the amount of labour it uses. In the short run, the firm's production is
- Q= f (L, $\overline{K}$)

Total Product, Marginal Product, and Average Product of Labor with Fixed Capital

The exact relationship between the output or total product and labor can be illustrated by using a particular function. This table shows the relationship between output and labor when capital is fixed for a firm. The first column lists the fixed amount of capital: eight fully equipped workbenches. As the number of workers (the amount of labor, second column) increases, total output (the number of computers assembled in a day, third column) first increases and then decreases.

Capital ($\overline{K}$)	Labor (L)	Output/ Total Product (q)	Marginal Product of Labor, $MP_L = \frac{\Delta q}{\Delta L}$	Average Product of Labor, $AP_L = \frac{q}{L}$
8	0	0	-	-
8	1	5	5	5
8	2	18	13	9
8	3	36	18	12
8	4	56	20	14
8	5	75	19	15
8	6	90	15	15
8	7	98	8	14
8	8	104	6	13
8	9	108	4	12
8	10	110	2	11
8	11	110	0	10
8	12	108	-2	9
8	13	104	-4	8

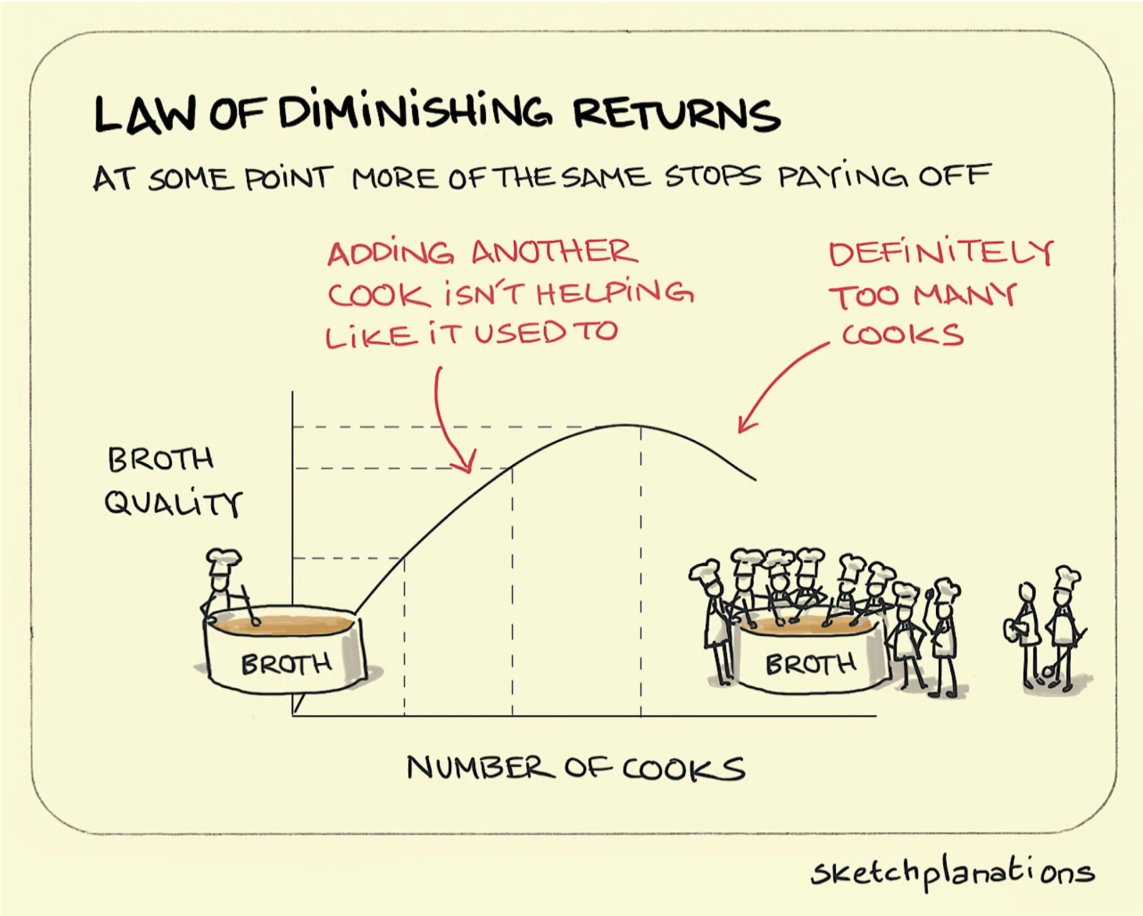
Marginal Product of Labor (MP_L)

- The change in total output, Δq, resulting from using an extra unit of labor, ΔL, holding other factors constant. $MP_L = \frac{\Delta q}{\Delta L}$.
- As the table shows, if the number of workers increases from 1 to 2, , ΔL=1, output rises by Δq = 13=18-5, so the marginal product of labor is 13.

Average Product of Labor (AP_L)

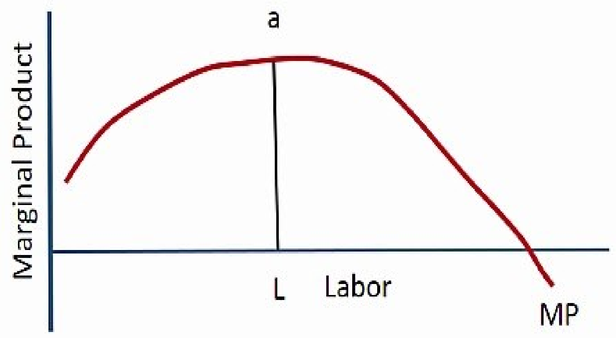
- Before hiring extra workers, a manager may also want to know whether output will rise in proportion to this extra labor. To answer this question, the firm determines how extra workers affect the average product of labor (AP_L) i.e. the ratio of output to the number of workers used to produce that output. $AP_L = \frac{q}{L}$

Table shows that 9 workers can assemble 108 computers a day, so the average product of labor for 9 workers is 12 (=108/9). Ten workers can assemble 110 computers in a day, so the average product of labor for 10 workers is 11 (=110/10) computers. Thus, increasing the labor force from 9 to 10 workers lowers the average product per worker.

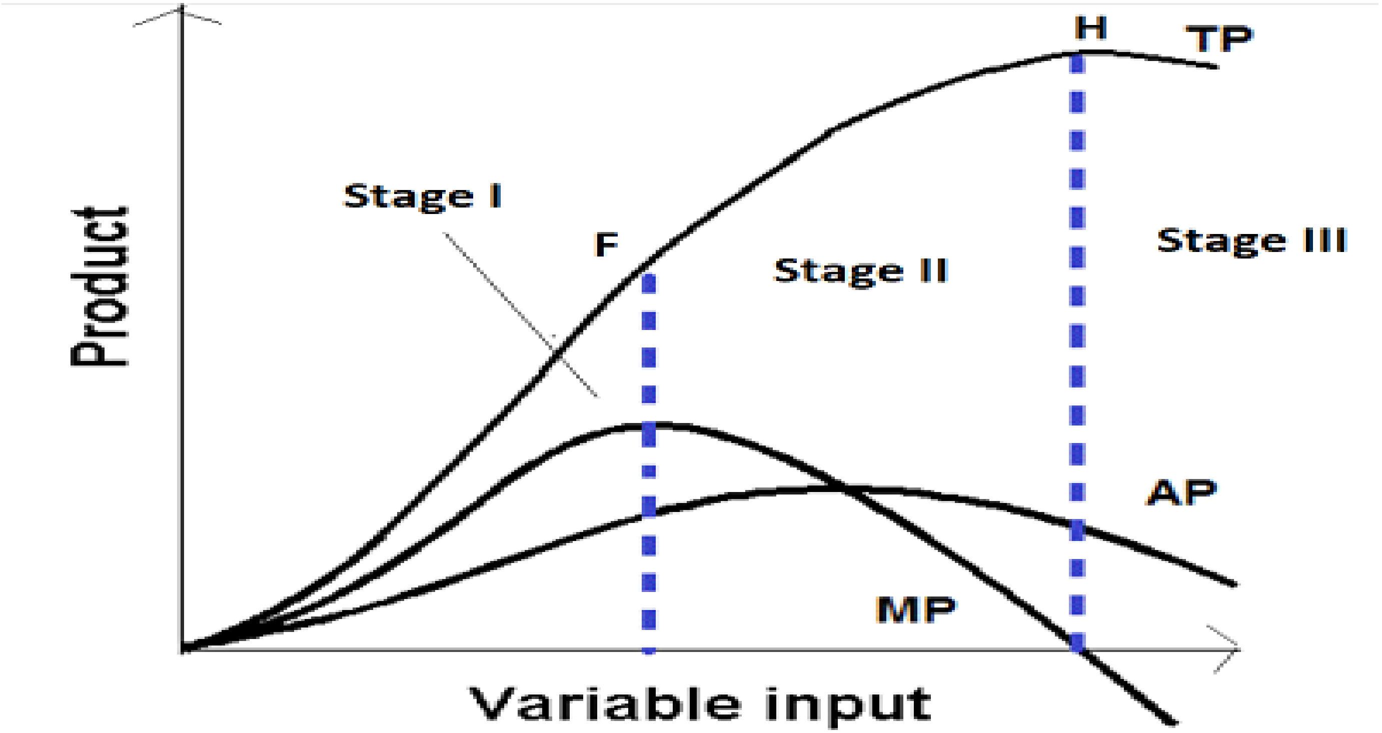

LAW OF DIMINISHING RETURNS
AT SOME POINT MORE OF THE SAME STOPS PAYING OFF
ADDING ANOTHER COOK ISN'T HELPING LIKE IT USED TO
DEFINITELY TOO MANY COOKS
BROTH QUALITY
BROTH
BROTH
NUMBER OF COOKS
sketchplanations

Law of Diminishing Marginal Product

Land (acre)	Labour (Units)	Total Product (qty)	Marginal Product (Qty)
5	1	50	50
5	2	110	60
5	3	180	70
5	4	260	80
5	5	340	80
5	6	410	70
5	7	470	60
5	8	520	50
5	9	550	30
5	10	560	10
5	11	560	0
5	12	550	-10
5	13	530	-20



Law of Variable Proportions/Law of diminishing Returns



Assumptions of The law of variable proportions

Constant State of Technology: First, the state of technology is assumed to be given and unchanged. If there is improvement in the technology, then the marginal product may rise instead of diminishing.

Fixed Amount of Other Factors: Secondly, there must be some inputs whose quantity is kept fixed. It is only in this way that we can alter the factor proportions and know its effects on output. The law does not apply if all factors are proportionately varied.

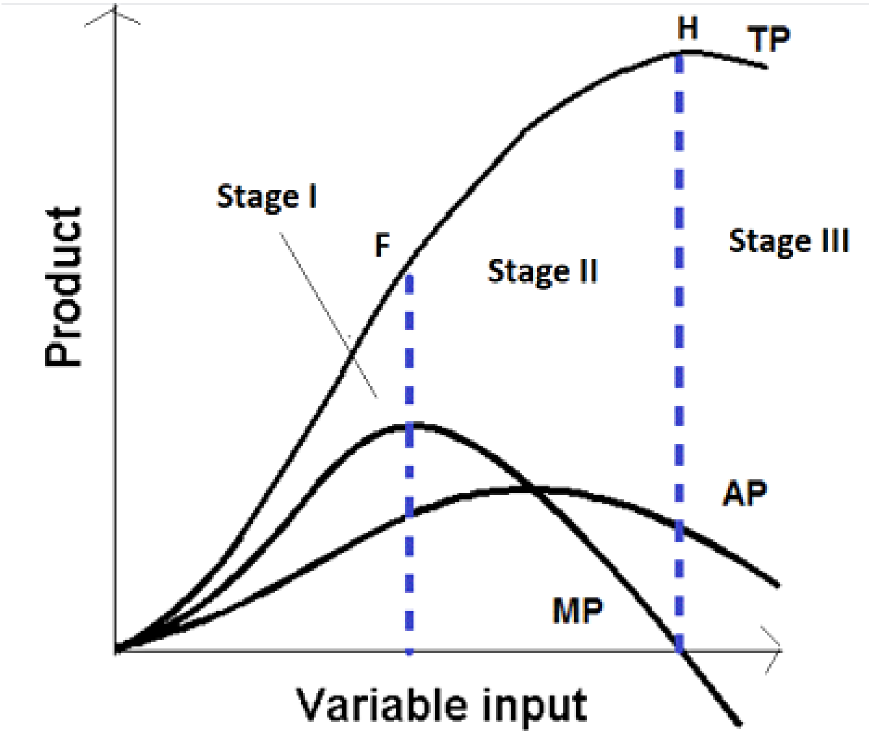
Possibility of Varying the Factor proportions: Thirdly, the law is based upon the possibility of varying the proportions in which the various factors can be combined to produce a product. The law does not apply if the factors must be used in fixed proportions to yield a product.

Stage 1. Stage of Increasing Returns: In this stage, total product increases at an increasing rate up to a point. This is because the efficiency of the fixed factors increases as additional units of the variable factors are added to it. In the figure, from the origin to the point F, slope of the total product curve TP is increasing i.e. the curve TP is concave upwards up to the point F, which means that the marginal product MP of labour rises. The point F where the total product stops increasing at an increasing rate and starts increasing at a diminishing rate is called the point of inflection. Corresponding vertically to this point of inflection marginal product of labour is maximum, after which it diminishes. This stage is called the stage of increasing returns because the average product of the variable factor increases throughout this stage. This stage ends at the point where the average product curve reaches its highest point.

Stage 2. Stage of Diminishing Returns: In this stage, total product continues to increase but at a diminishing rate until it reaches its maximum point H where the second stage ends. In this stage both the marginal product and average product of labour are diminishing but are positive. This is because the fixed factor becomes inadequate relative to the quantity of the variable factor. At the end of the second stage, i.e., at point M marginal product of labour is zero which corresponds to the maximum point H of the total product curve TP. This stage is important because the firm will seek to produce in this range.

Stage 3. Stage of Negative Returns: In stage 3, total product declines and therefore the TP curve slopes downward. As a result, marginal product of labour is negative and the MP curve falls below the X-axis. In this stage the variable factor (labour) is too much relative to the fixed factor.

Variable inputs (units)	TP (units)	MP (units)	Stages of law of variable proportion	
1	4	4	Stage I	TP is increasing at an increasing rate and MP is also increasing.
2	9	5		
3	13	4	Stage II	TP is increasing at a diminishing rate and MP starts declining.
4	15	2		
5	15	0		
6	12	−3	Stage III	MP becomes negative and TP falls.



An isoquant is a firm's counterpart of the consumer's indifference curve. An isoquant is a curve that shows all the combinations of inputs that yield the same level of output. 'Iso' means equal and 'quant' means quantity. Therefore, an isoquant represents a constant quantity of output. The isoquant curve is also known as an "Equal Product Curve" or "Production Indifference Curve" or Iso-Product Curve."

The concept of isoquants can be easily explained with the help of the table given below:

Combinations of Labour and Capital	Units of Labour	Units of capital	Output of cloth (meters)
A	5	9	100
B	10	6	100
C	15	4	100
D	20	3	100

The above table is based on the assumption that only two factors of production, namely, Labor and Capital are used for producing 100 meters of cloth.

Combination A = 5L + 9K = 100 meters of cloth

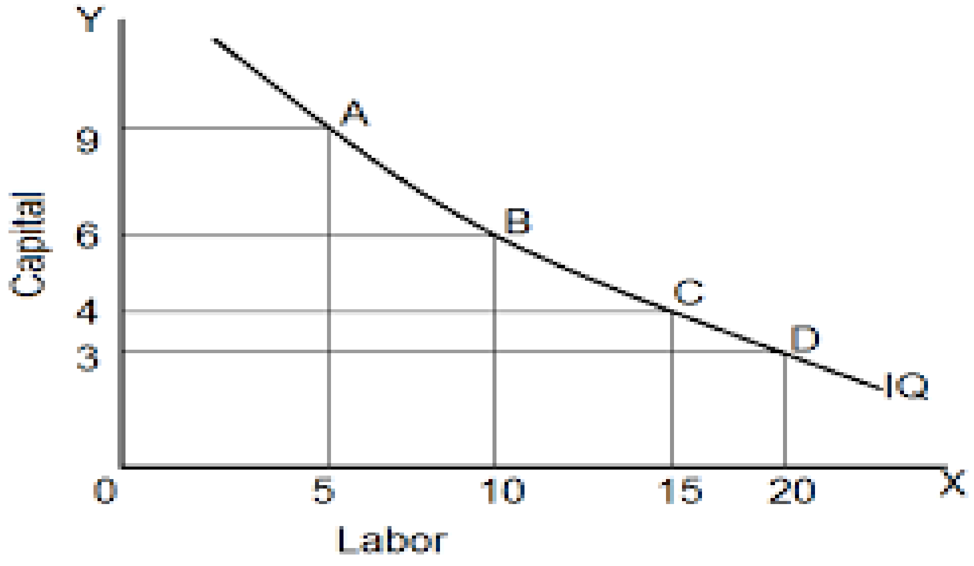
Combination B = 10L + 6K = 100 meters of cloth

Combination C = 15L + 4K = 100 meters of cloth

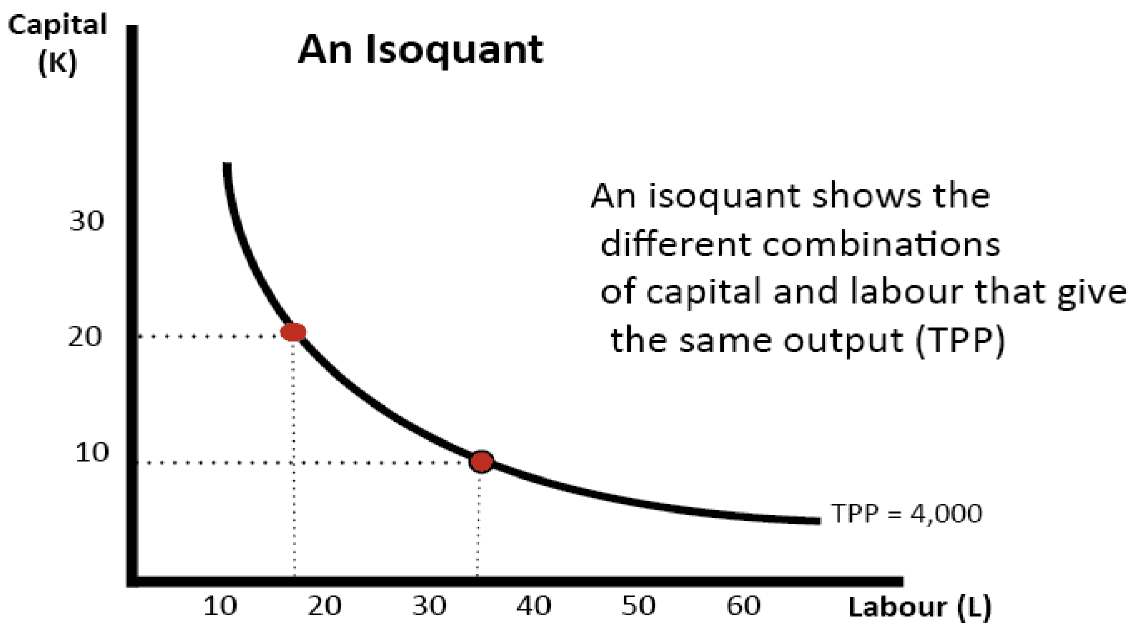
Combination D = 20L + 3K = 100 meters of cloth

The combinations A, B, C, and D show the possibility of producing 100 meters of cloth by applying various combinations of labor and capital. Thus, an isoquant schedule is a schedule of different combinations of factors of production yielding the same quantity of output.

Figure 1



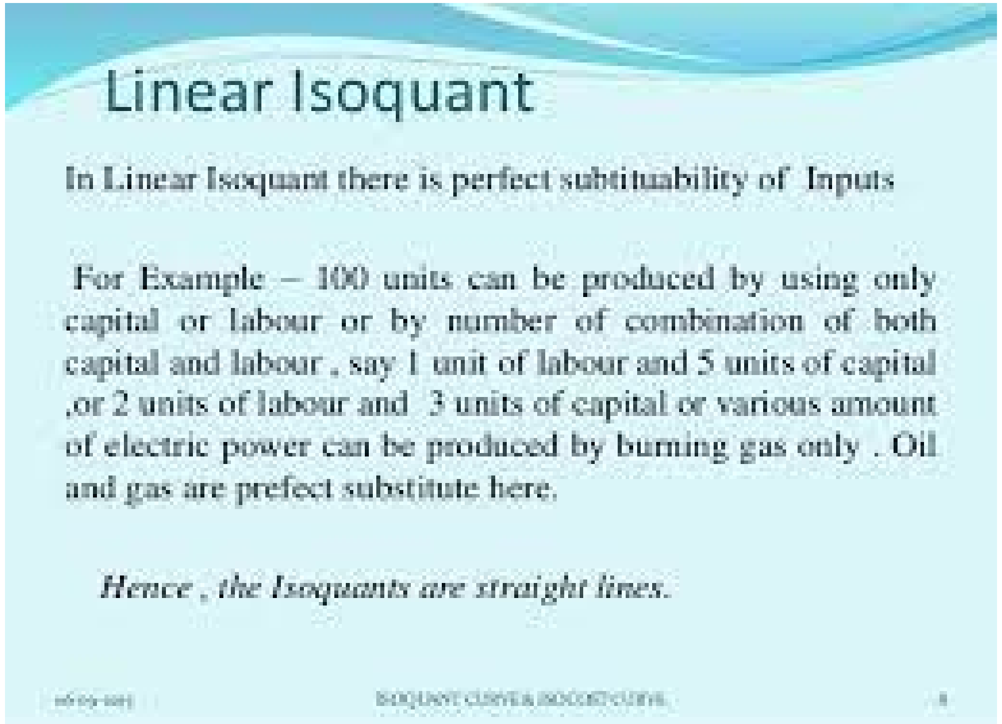
- An isoquant includes (is the locus of) all the technically efficient methods (or all the combinations of factors of production) for producing a given level of output. The production isoquant may assume various shapes depending on the degree of substitutability of factors.

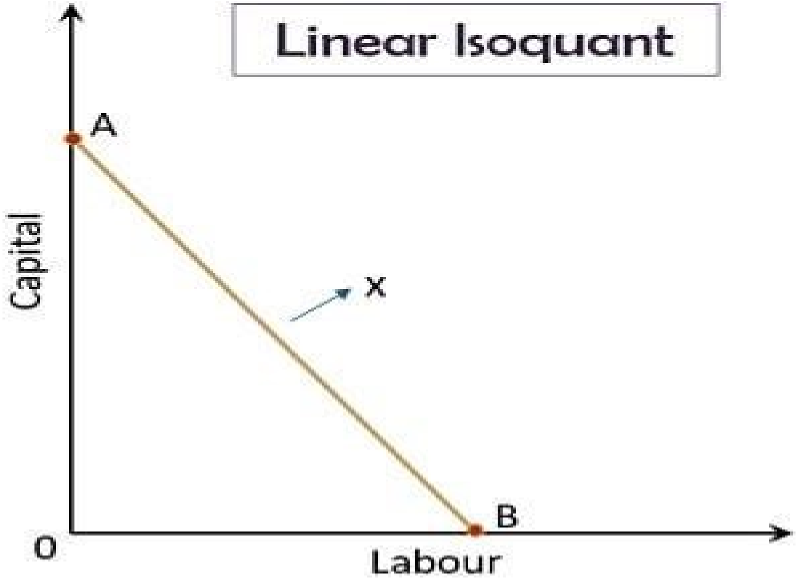


Properties of Isoquants

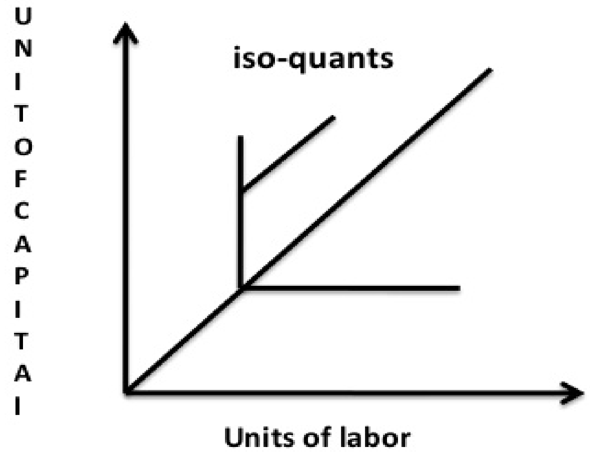
- Two isoquants can never cross. Since each isoquant refers to a specific level of output, no two isoquants intersect, for such an intersection would indicate that the same combination of resources could, with equal efficiency, produce two different amounts of output.
- Every possible combination of inputs is on an isoquant.
- Isoquants further from the origin represent greater output levels
- Isoquants slope down to the right. Consider the capital vs. labour isoquant. In any practical situation, the quantity of labour employed is inversely related to the quantity of capital employed, so isoquants have negative slopes.
- Isoquants are usually convex to the origin, meaning that the slope of the isoquant gets flatter down along the curve.

Linear Isoquant





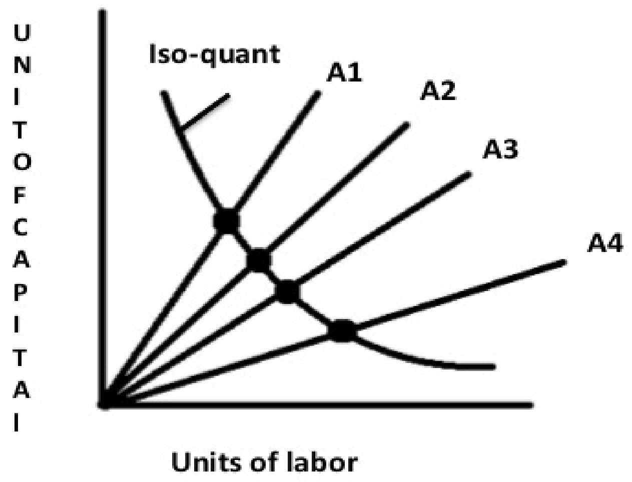
Input output iso-quants



Zero Substitutability

- ❖This type of Iso-Quant assumes perfect complementary or zero substitutability between the inputs.
- ❖When there is only one method of production for any product, its Iso-quant is of right-angled shape. This type of iso-quant is also known as Leontief iso-quant.

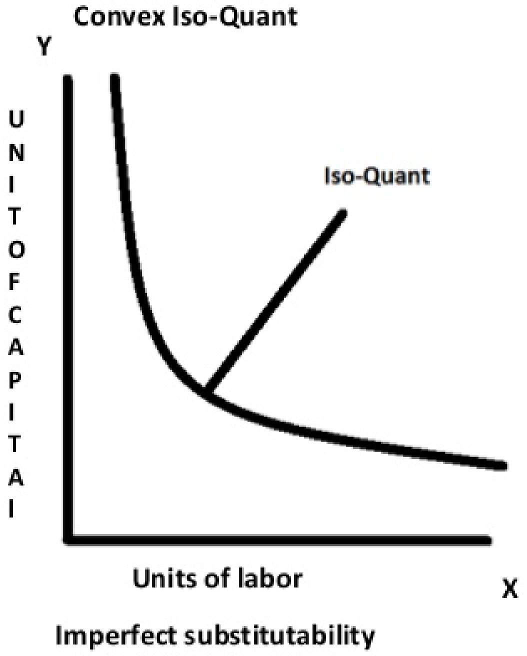
Kinked Iso-Quant



Linear Programming Iso-Quant

- This is also known as linear programming iso-quant.
- It assumes limited substitutability of labour and capital.
- As there are only a few processes which are available for producing any commodity (say A1, A2, A3, A4) substitutability of factors is possible only at kinks. This can be illustrated as on the left side.

Smooth Convex Iso-Quant



- It assumes continuous substitutability of labour and capital only over a certain range, beyond which factors cannot be substituted for each other.
- This iso-quant appears as a smooth curve convex to origin.

Marginal Rate of Technical Substitution

- The marginal rate of technical substitution (MRTS) can be defined as, keeping constant the total output, how much input 1 have to decrease if input 2 increases by one extra unit. In other words, it shows the relation between inputs, and the trade-offs amongst them, without changing the level of total output. When using common inputs such as capital (K) and labour (L), the MRTS can be obtained using the following formula:

$$\mathrm{MRTS}\ (\mathrm{L}, \mathrm{K}) = -\frac{\Delta\ \mathrm{K}}{\Delta\ \mathrm{L}} = \frac{\mathrm{MP\ L}}{\mathrm{MP\ K}}$$

- The MRTS is equal to the slope of isoquants.

Elasticity of Substitution

The marginal rate of substitution as a measure of the degree of substitutability of factors has a serious defect: it depends on the units of measurement of the factors. A better measure of the ease of factor substitution is provided by the elasticity of substitution. The elasticity of substitution is defined as the percentage change in the capital labour ratio, divided by the percentage change in the rate of technical substitution. The elasticity of substitution is a pure number independent of the units of measurement of K and L, since both the numerator and the denominator are measured in the same units.

$$\sigma = d(K/L)/(K/L)/d(MRS)/(MRS)$$

Isoquants and Returns to Scale

- Let us now examine the responses in output when all inputs are varied in equal proportions.
- Returns to scale refer to output responses to an equi-proportionate, change in all inputs. Suppose labor and capital are doubled, and then if output doubles, we have constant returns to scale. If output is less than double, we have decreasing returns to scale, and if output is more than double, we have increasing returns to scale.

Increasing returns to scale

When output increases by a proportion that exceeds the proportion by which inputs increase, increasing returns to scale prevail. The line OP is the scale line because a movement along this line shows only a change in the scale of production. The proportion of labor to capital along this line remains the same because it has the same sloe throughout. The operation of increasing returns to scale is shown by the gradual decrease in the distance between the isoquant. For example OA > AB > BC.

Causes of increasing returns to scale

Several technical and/or managerial factors contribute to the operation of increasing returns to scale.

1. Increasing specialization of labor Increasing returns to scale can be the result of increase in the productivity of inputs caused by increased specialization and division of labor as the scale of operations increases.

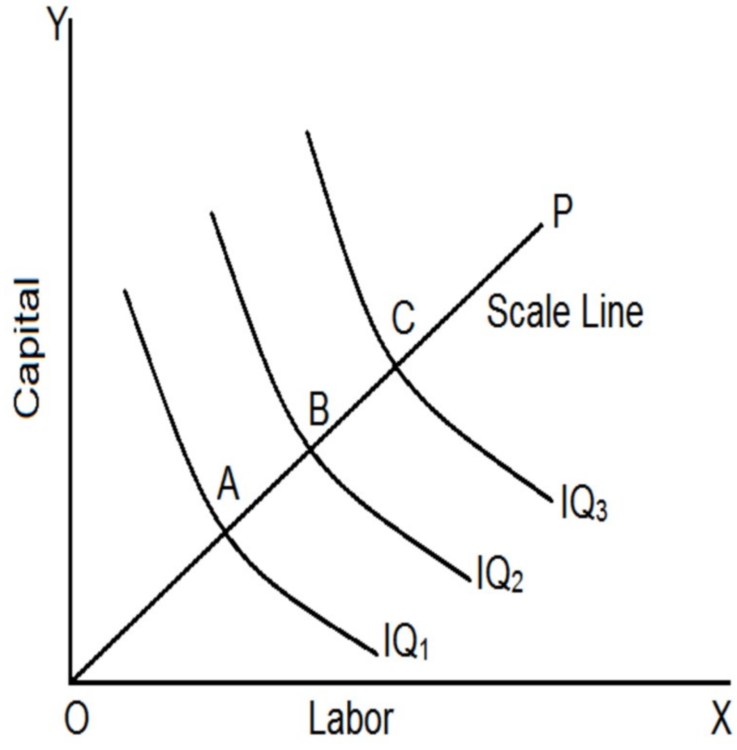
2. Indivisibilities

In general, indivisibility implies that equipment is available only in minimum sizes or in definite ranges of size. Specialized machines are generally far more productive than less specialized machines. In large-scale operations, the possibility of using specialized machines is higher, so productivity will also be higher.

3. Geometric necessity

For some production processes, it is a matter of geometric necessity. A larger scale of operation makes it more efficient. For example, to double the grazing area, a farmer need not have to double the length of fencing. Similarly, doubling the cylindrical equipment (like pipes and smoke stacks) and spherical equipment (like storage tanks) requires less than twice the quantity of metal.

Figure 2

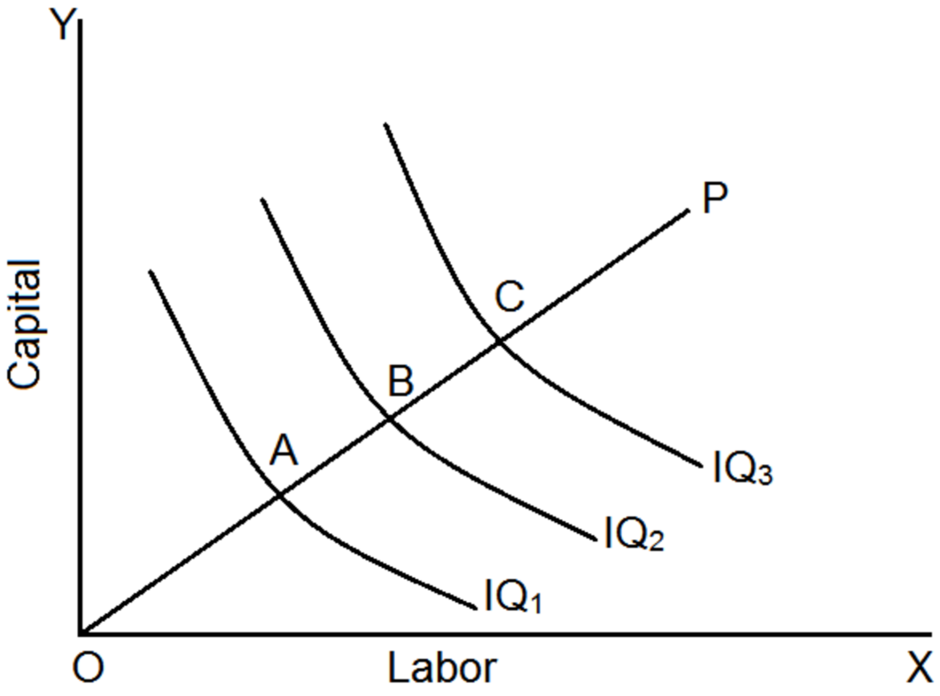


Decreasing returns to scale

Decreasing returns to scale prevail when the distance between consecutive isoquants increases. For example, OA < AB < BC.

Decreasing returns arise when diseconomies are greater than economies. Difficulties in coordinating the operations of many factories and communication problems with employees may contribute to decreasing returns to scale. More than proportionate increases in managerial inputs may be required to expand output when an organization becomes very large.

Figure 3



Constant returns to scale

Constant returns to scale prevail when output also increases by the same proportion in which input increases. In the case of constant returns to scale, the distance between successive isoquants remains constant. For example OA = AB = BC

Constant returns arise when economies exactly balance with diseconomies. As economies of scale are exhausted, a phase of constant returns to scale may set in operation.

Figure 4