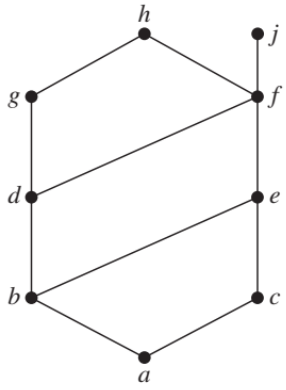
Find the lower and upper bounds of the subsets *{a, b, c}*, *{j, h}*, and *{a, c, d, f}* in the poset with the following Hasse diagram. Also find the greatest lower bounds (*glb*) and least upper bounds (*lub*) of these subsets.



Find the greatest lower bound and least upper bound of *{b, d, g}*, if they exist, in the poset shown above.

Find the greatest lower bound and the least upper bound of the sets {3, 9, 12} and {1, 2, 4, 5, 10}, if they exist, in the poset $(\mathbf{Z}^+, |)$.

Is the poset $(\mathbf{Z}^+, |)$ a lattice?

Determine whether the posets $(\{1, 2, 3, 4, 5\}, |)$ and $(\{1, 2, 4, 8, 16\}, |)$ are lattices.

Determine whether $(P(S), \subseteq)$ is a lattice where S is a set.

Determine whether the posets with the following Hasse diagrams are lattices.

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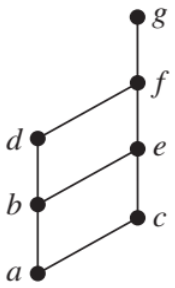
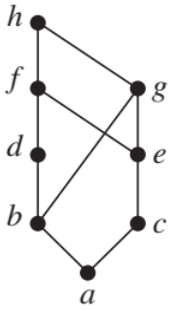
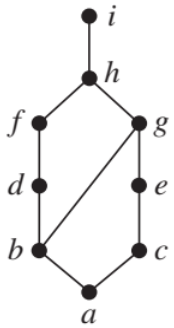
a)



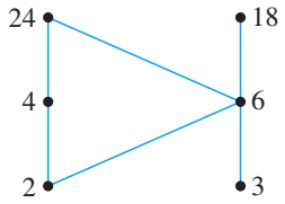
b)



c)



Consider the set $A = \{2, 3, 4, 6, 18, 24\}$ ordered by the "divides" relation $|$. The Hasse diagram of this relation is the following:



The ordinary "less than or equal to" relation $\leq$ on this set is a topological sorting for it since for positive integers a and b, if $a \mid b$ then $a \leq b$. Find another topological sorting for this set.

Find a compatible total ordering for the poset $(\{1, 2, 4, 5, 12, 20\}, |)$.