

# Application of satisfiability

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Four propositions we need to assert:

- ① Every row contains every number.
  - ② Every column contains every number.
  - ③ Every  $3 \times 3$  block contains every number.
  - ④ Each cell contains no more than one number.
- Compound proposition  $\leftarrow \textcircled{1} \wedge \textcircled{2} \wedge \textcircled{3} \wedge \textcircled{4}$

For ①: Asserting that row 'i' contains number  $n$ , we have:

$$\bigvee_{j=1}^9 p(i, j, n)$$

Asserting that row 'i' contains all  $n$  numbers, we have:

$$\bigwedge_{n=1}^9 \bigvee_{j=1}^9 p(i, j, n)$$

Asserting that all rows contains all  $n$  numbers, we have:

$$\bigwedge_{i=1}^9 \bigwedge_{n=1}^9 \bigvee_{j=1}^9 p(i, j, n)$$

For ②

Asserting that all columns contain all  $n$  numbers, we have:

$$\bigwedge_{j=1}^9 \bigwedge_{n=1}^9 \bigvee_{i=1}^9 p(i, j, n)$$

$$\boxed{\begin{matrix} 3 \times 3 \\ \text{block} \end{matrix} \left\{ \begin{matrix} i \rightarrow 1 \text{ to } 3 \\ j \rightarrow 1 \text{ to } 3 \end{matrix} \right.}$$

For ③

Asserting that row  $i$  of a given  $3 \times 3$  block contains

for ①

Asserting that row  $i$  of a given  $3 \times 3$  block contains number  $n$ , we have:  $\bigvee_{j=1}^3 p(3r+i, 3s+j, n)$

Asserting that column  $j$  of a given  $3 \times 3$  block contains number  $n$ , we have:  $\bigvee_{i=1}^3 \bigvee_{j=1}^3 p(3r+i, 3s+j, n)$

Asserting that a given  $3 \times 3$  block contains all numbers we have:  $\bigwedge_{n=1}^9 \bigvee_{i=1}^3 \bigvee_{j=1}^3 p(3r+i, 3s+j, n)$

To assert every  $3 \times 3$  block contains all numbers we have:  $\bigwedge_{r=0}^2 \bigwedge_{s=0}^2 \bigvee_{n=1}^9 \bigvee_{i=1}^3 \bigvee_{j=1}^3 p(3r+i, 3s+j, n)$

for ④

For all  $n \neq m$ , if  $p(i, j, n) \rightarrow \neg p(i, j, m)$