COMP2521: Recursion (Linked List)

Term: 20T1

[Credits: Lecture slides from COMP1511 (18s2)]

Recursion

递归

- **Recursion** is a programming pattern where a function calls ***itself***

- For example, we define *factorial* as below,

  n! = 1*2*3* … *(n-1)*n

  n×(n-1)×(n-2)…1

- We can ***recursively*** define *factorial* function as below,

  f(n) = 1 , if (n=0) 0! = 1

  f(n) = n * f(n-1) , for others

Pattern for a Recursive function

- Base case(s)
  - Situations when we **do not** call the same function (no recursive call), because the problem can be solved easily without a recursion.
  - All recursive calls eventually lead to one of the base cases.
- Recursive Case
  - We **call** the **same function** for a problem with ***smaller size***.
  - Decrease in a problem size eventually leads to one of the base cases.

```
// return sum of list data fields: using recursive call

int sum(struct node *head) {
    if (head == NULL) {
        return 0;
    }
    return head->data + sum(head->next);
}
```

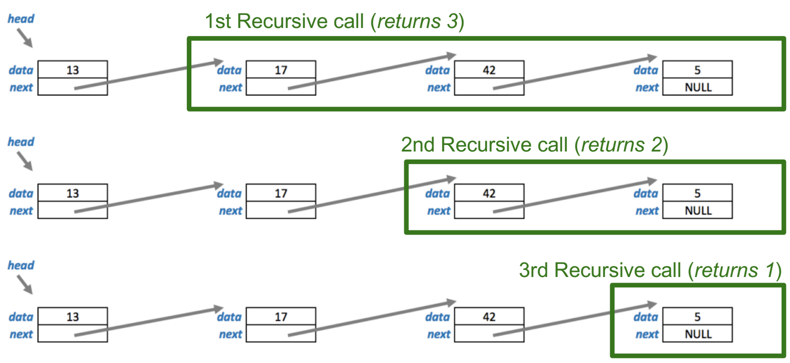
Base case

Recursive case,
Recursive call for a
smaller problem
(size-1)

Linked List with Recursion

```
// return count of nodes in list

int length(struct node *head) {
    if (head == NULL) {
        return 0;
    }
    return 1 + length(head->next);
}
```

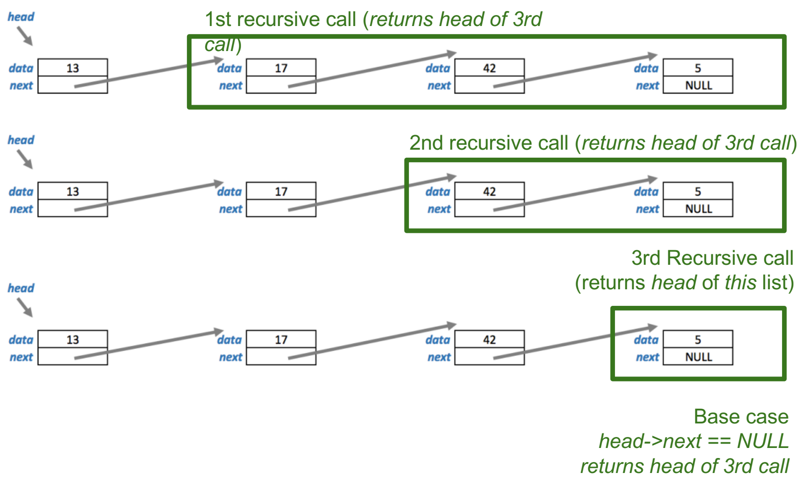
Recursive call



Base case
Head == NULL
returns 0

Last Node using Recursion

```c
struct node *last(struct node *head) {
    // list is empty
    if(head == NULL) {
        return NULL;
    }
    // found the last node! return it.
    else if (head->next == NULL) {
        return head;
    }
    // return last node from the rest of the list
    // using a recursion
    else {
        return last(head->next);
    }

}
```



Find Node using Recursion

```c
// return pointer to first node with specified data value
// return NULL if no such node

struct node *find_node(struct node *head, int data) {
    // empty list, so return NULL
    if (head == NULL) {
        return NULL;
    }
    // Data at "head" is same as the "data" we are searching,
    // Found the node! so return head.
    else if (head->data == data) {
        return head;
    }
    // Find "data" in the rest of the list, using recursion,
    // return whatever answer we get from the recursion
    else {
        return find_node(head->next, data);
    }

}
```

Recursive call

head = head -> next

Delete From List using Recursion

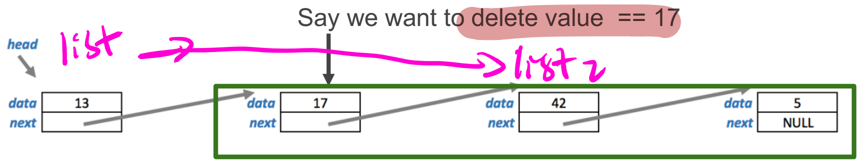
```
// Delete a Node from a List: Recursive
struct node *deleteR(struct node *list, int value) {
   if (list == NULL) {
      fprintf(stderr, "warning: value %d is not in list\n", value);

   } else if (list->data == value) {
      struct node *tmp = list;
      list = list->next;          // remove first item
      free(tmp);

   } else {
      list->next = deleteR(list->next, value);
   }
   return list;
}
```

empty list

head

Recursive call

Say we want to delete value == 17



1st recursive call (node to delete is same as "head" of this call, *returns updated list, pointing to node with 42)*

return list 2

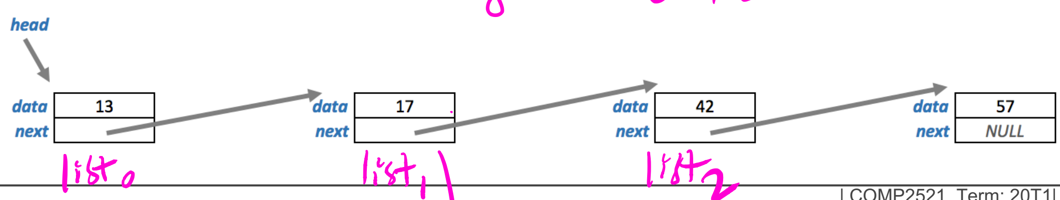
Linked List with Recursion

```
// Insert a Node into an Ordered List: recursive
struct node *insertR(struct node *list, int value) {
   if (list == NULL || list->data >= value) {

      struct node *newHead = create_node(value, NULL);
      newHead->next = list;
      return newHead;

      // Alternatively, in one line
      // return create_node(value, list) ;
   }

   list->next = insertR(list->next, value);

   return list;
}
```

or NULL

Recursive call

eg. value 42



call 2

42

new head

Print Python List using Recursion

```c
// print contents of list in Python syntax

void print_list(struct node *head) {
    printf("[");
    if (head != NULL) {
        print_list_items(head);
    }
    printf("]");
}

void print_list_items(struct node *head) {
    printf("%d", head->data);
    if (head->next != NULL) {
        printf(", ");
        print_list_items(head->next);
    }
}
```

Recursive function

Recursive call

head head

2 → 4 → Null

Compilation and Makefiles

- Compilers
- Make/Makefiles

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❖ Compilers

Compilers are programs that

可执行

- convert program source code to executable form
- "**executable**" might be machine code or bytecode 字节码

The Gnu C compiler (`gcc`)

预处理

- applies source-to-source transformation (pre-processor)
- compiles source code to produce object files
- links object files and libraries to produce executables

`clang` is an alternative C compiler (also available in CSE)

包装器

Note that `dcc` and `3c` are wrappers around `gcc`/`clang`

- providing more checking and more detailed/understandable error messages
- better run-time support (e.g. array bounds, use of dynamic memory)

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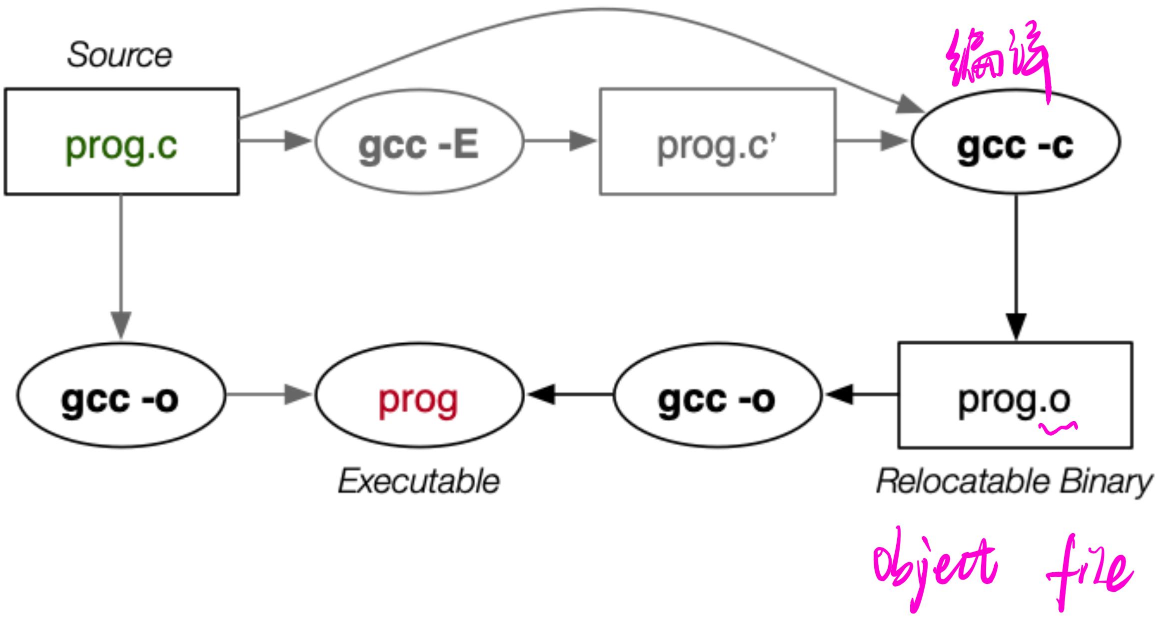
❖ ... Compilers

Stages in C compilation: pre-processing, compilation, linking



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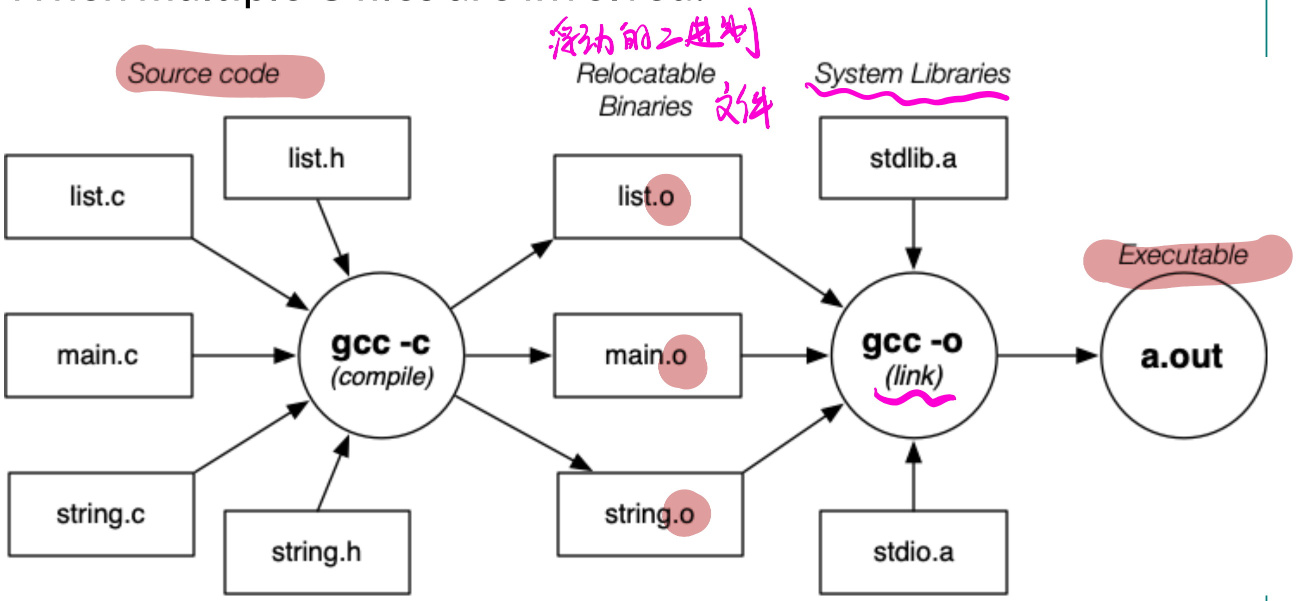
❖ ... Compilers

When multiple C files are involved:



❖ ... Compilers

Compilation/linking with **`gcc`**

```
gcc -c Stack.c
  produces Stack.o, from Stack.c and Stack.h
gcc -c bracket.c
  produces bracket.o, from bracket.c and Stack.h
gcc -o rbt bracket.o Stack.o
  links bracket.o, Stack.o and libraries
  producing executable program called rbt
```

Note that **`stdio,assert`** included implicitly.

`gcc` is a multi-purpose tool

- compiles (**`-c`**), links, makes executables (**`-o`**)

❖ Make/Makefiles

Compilation process is complex for large systems.

How much to compile?

- ideally, what's changed since last compile
- practically, recompile everything, to be sure

The `make` command assists by allowing

- programmers to document dependencies in code
- minimal re-compilation, based on dependencies

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❖ ... Make/Makefiles

Example multi-module program ...

main.c

```
#include <stdio.h>
#include "world.h"
#include "graphics.h"

int main(void)
{
   ...
   drawPlayer(p);

   spin(...);
}
```

world.h

```
typedef ... Ob;
typedef ... Pl;

extern addObject(Ob);
extern remObject(Ob);
extern movePlayer(Pl);
```

world.c

```
#include <stdlib.h>

addObject(...)
{ ... }

remObject(...)
{ ... }

movePlayer(...)
{ ... }
```

graphics.h

```
extern drawObject(Ob);
extern drawPlayer(Pl);
extern spin(...);
```

graphics.c

```
#include <stdio.h>
#include "world.h"

drawObject(Ob o);
{ ... }

drawPlayer(Pl p)
{ ... }

spin(...)
{ ... }
```

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❖ ... Make/Makefiles

make is driven by *dependencies* given in a **Makefile**

A dependency specifies

```
target : source1 source2 ...
        commands to build target from sources
```

e.g.

```
game : main.o graphics.o world.o
        gcc -o game main.o graphics.o world.o
```

from

Rule: *target* is rebuilt if older than any $source_i$ (applied recursively)

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❖ ... Make/Makefiles

depends on

target

```
game : main.o graphics.o world.o
	gcc -o game main.o graphics.o world.o

main.o : main.c graphics.h world.h
	gcc -Wall -Werror -c main.c

graphics.o : graphics.c world.h
	gcc -Wall -Werror -c graphics.c

world.o : world.c
	gcc -Wall -Werror -c world.c
```

action

Things to note:

- A target (**game**, **main.o**, ...) is on a newline
  - followed by a **:**
  - then followed by the files that the target is dependent on
- The action (**gcc** ...) is always on a newline
  - and must be indented with a TAB

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❖ ... Make/Makefiles

If **`make`** arguments are targets, build just those targets:

```
prompt$ make world.o
gcc -Wall -Werror -c world.c
```

make + target name

If no args, build first target in the **`Makefile`**.

```
prompt$ make
gcc -Wall -Werror -c main.c
gcc -Wall -Werror -c graphics.c
gcc -Wall -Werror -c world.c
gcc -o game main.o graphics.o world.o
```

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❖ ... Make/Makefiles

`Makefiles` can contain "variables"

- e.g. **`CC`**, **`CFLAGS`**, **`LDFLAGS`**
- can easily change which C compiler used, etc

`make` has rules, which allow it to interpret e.g.

```
Stack.o : Stack.c Stack.h
```

as

```
Stack.o : Stack.c Stack.h
          $(CC) $(CFLAGS) -c Stack.c
```

|| gcc

|| -Wall -Werror

- gcc q3.c List.c -o q3
- ./q3

= - gcc q3.c List.c
- ./a.out

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Abstract Data Types

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❖ Abstract Data Types

A data type is ...

- a set of values (atomic or structured values)
- a collection of operations on those values

An abstract data type is ...

- an approach to implementing data types
- separates interface 接口 from implementation 实现
- users of the ADT see only the interface
- builders of the ADT provide an implementation

E.g. do you know what a (`FILE *`) looks like? do you want/need to know?

large data structure

❖ DTs, ADTs, GADTs

We want to distinguish ...

DT = (non-abstract) data type (e.g. C strings)

- internals of data structures are visible (e.g. `char s[10];`)

ADT = abstract data type (e.g. C files)

- can have multiple instances (e.g. `Set a, b, c;`)

GADT = generic (polymorphic) abstract data type

- can have multiple instances (e.g. `Set<int> a, b, c;`)
- can have multiple types (e.g. `Set<int> a; Set<char> b;`)
- not available natively in the C language

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❖ Interface/Implementation

接口

ADT interface provides

原型

- a user-view of the data structure (e.g. **`FILE*`**)
- function signatures (prototypes) for all operations
- semantics of operations (via documentation)
- a contract between ADT and its clients 用户

ADT implementation gives

- concrete definition of the data structures
- definition of functions for all operations

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❖ Collections

Many of the ADTs we deal with ...

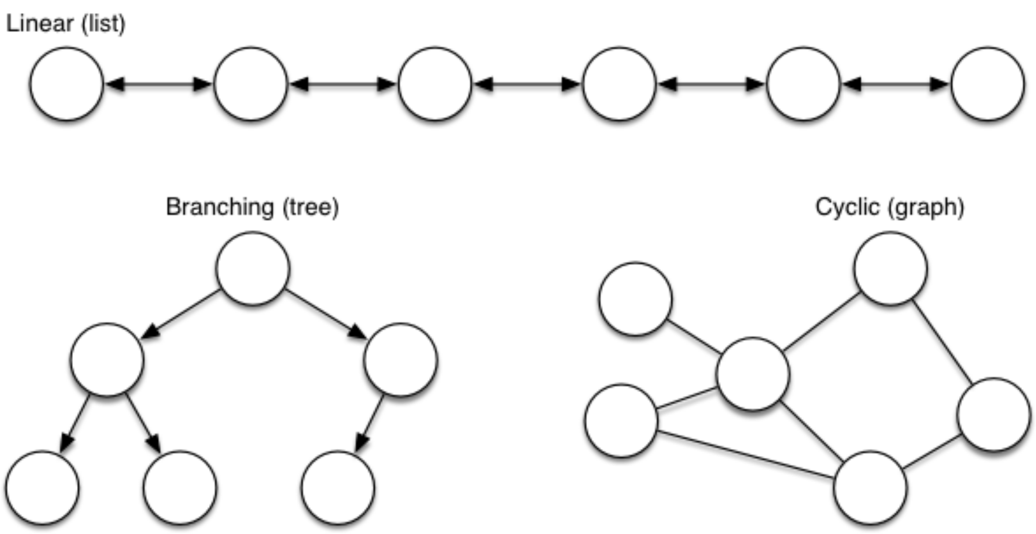
- consist of a collection of items eg. char, int
- where each item may be a simple type or an ADT
- and items often have a key (to identify them)

Collections may be categorised by ...

- structure: linear (list), branching (tree), cyclic (graph)
- usage: set, matrix, stack, queue, search-tree, dictionary, ...

❖ ... Collections

Collection structures:



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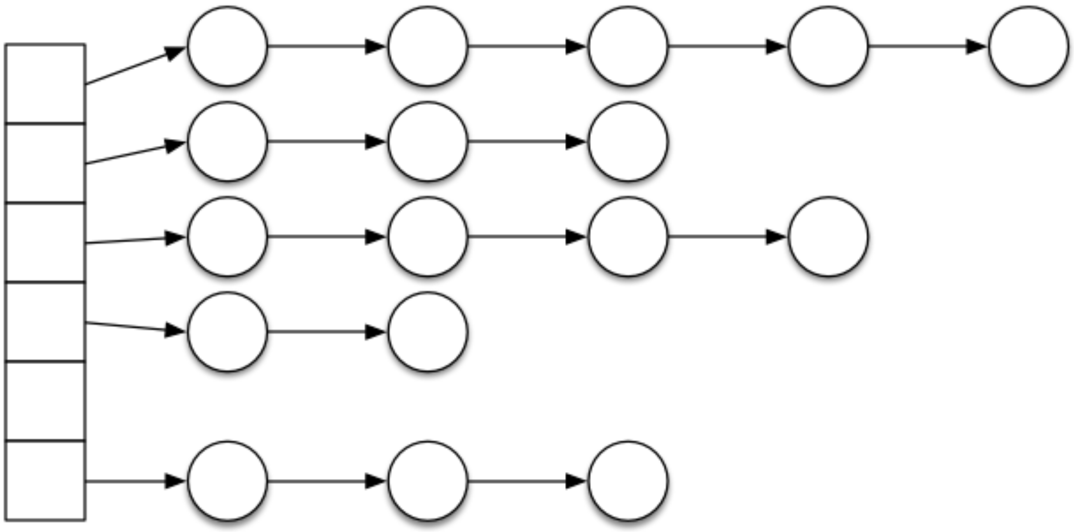
❖ ... Collections

混合

Or even a hybrid structure like ...



array to link list

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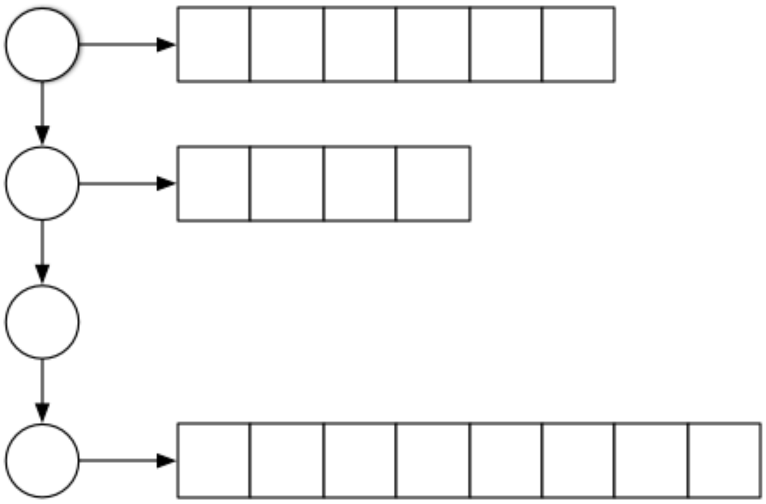
❖ ... Collections

Or this ... link list to array



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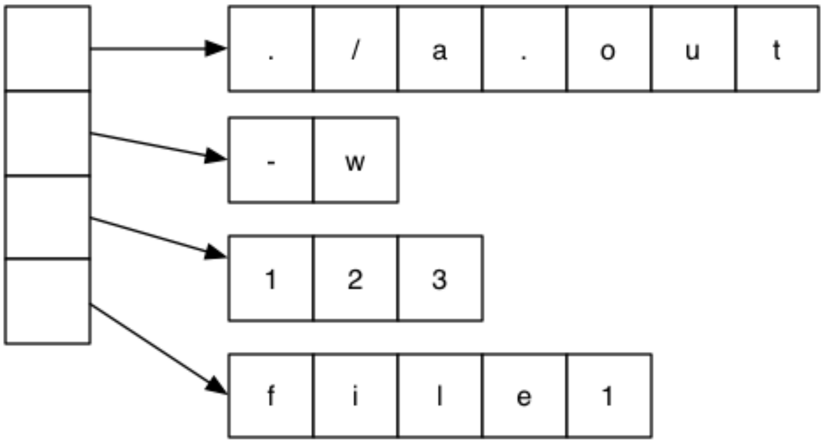
❖ ... Collections

Or this ...



array of array

❖ ... Collections

Typical operations on collections

- create an empty collection
- insert one item into the collection
- remove one item from the collection
- find an item in the collection
- check properties of the collection (size,empty?)
- drop the entire collection
- display the collection

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❖ Example: Set ADT

Set data type: collection of unique integer values.

"Book-keeping" operations:

- `Set newSet()` ... create new empty set
- `void dropSet(Set)` ... free memory used by set
- `void showSet(Set)` ... display as `{1,2,3...}`

Assignment operations:

- `void readSet(FILE*,Set)` ... read+insert set values
- `Set SetCopy(Set)` ... make a copy of a set

❖ ... Example: Set ADT

Data-type operations:

- **`void SetInsert(Set,int)`** ... add number into set
- **`void SetDelete(Set,int)`** ... remove number from set
- **`int SetMember(Set,int)`** ... set membership test
- **`Set SetUnion(Set,Set)`** ... union 交集
- **`Set SetIntersect(Set,Set)`** ... intersection
- **`int SetCard(Set)`** ... cardinality (#elements)

基数

Note: union and intersection return a newly-created **`Set`**

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❖ Set ADT Interface

```
// Set.h ... interface to Set ADT

#ifndef SET_H
#define SET_H

#include <stdio.h>
#include <stdbool.h>

typedef struct SetRep *Set;

Set newSet();                 // create new empty set
void dropSet(Set);            // free memory used by set
Set SetCopy(Set);             // make a copy of a set
void SetInsert(Set,int);      // add value into set
void SetDelete(Set,int);      // remove value from set
bool SetMember(Set,int);      // set membership
Set SetUnion(Set,Set);        // union
Set SetIntersect(Set,Set);    // intersection
int SetCard(Set);             // cardinality
void showSet(Set);            // display set on stdout
void readSet(FILE *, Set);    // read+insert set values

#endif
```

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❖ ... Set ADT Interface

Example set client: set of small odd numbers

```
#include "Set.h"
...
Set s = newSet();
for (int i = 1; i < 26; i += 2)
    SetInsert(s,i);
showSet(s); putchar('\n');
```

= printf ("\n");

Outputs:

```
{1,3,5,7,9,11,13,15,17,19,21,23,25}
```

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❖ Set Applications

Example: eliminating duplicates

```
#include "Set.h"
...
// scan a list of items in a file
int item;
Set seenItems = newSet();
FILE *in = fopen(FileName,"r");
while (fscanf(in, "%d", &item) == 1) {
    if (!SetMember(seenItems, item)) {
        SetInsert(seenItems, item);
        process item;
    }
}
fclose(in);
```

into

is it item in the set seenItems?

If no, insert item in the set.

close file

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❖ Set ADT Pre/Post-conditions

Each **Set** operation has well-defined semantics. function

Express these semantics in detail via statements of:

- what conditions need to hold at start of function
- what will hold at end of function (assuming successful)

Could implement condition-checking via **assert()**s

But only during the development/testing phase

- **assert()** does not provide useful error-handling

At the very least, implement as comments at start of functions.

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❖ ... Set ADT Pre/Post-conditions

is a set

If **`x`** is a variable of type **`T`**, where **`T`** is an ADT

- $ptr(x)$ is the pointer stored in **`x`**
- $val(x)$ is the abstract value represented by **`*x`**
- $valid(T, x)$ indicates that
    - the collection of values in **`*x`**
      satisfies all constraints on "correct" values of type **`T`**
- x' is an updated version of x (note: $ptr(x') == ptr(x)$)
- res is the value returned by a function

Can also use math/logic notation as used in pseudocode.

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❖ ... Set ADT Pre/Post-conditions

Examples of defining pre-/post-conditions:

```
// pre:  true
// post: valid(Set,res) and res = {}
Set newSet() { ... }

// pre:  valid(Set,s) and valid(int n)
// post: n ∈ s'
void SetInsert(Set s, int n) { ... }

// pre:  valid(Set,s1) and valid(Set,s2)
// post: ∀ n ∈ res, n ∈ s1 or n ∈ s2
Set SetUnion(Set s1, Set s2) { ... }

// pre:  valid(Set,s)
// post: res = |s|
int SetCard(Set s)  { ... }
```

should empty

合并 s_1 s_2

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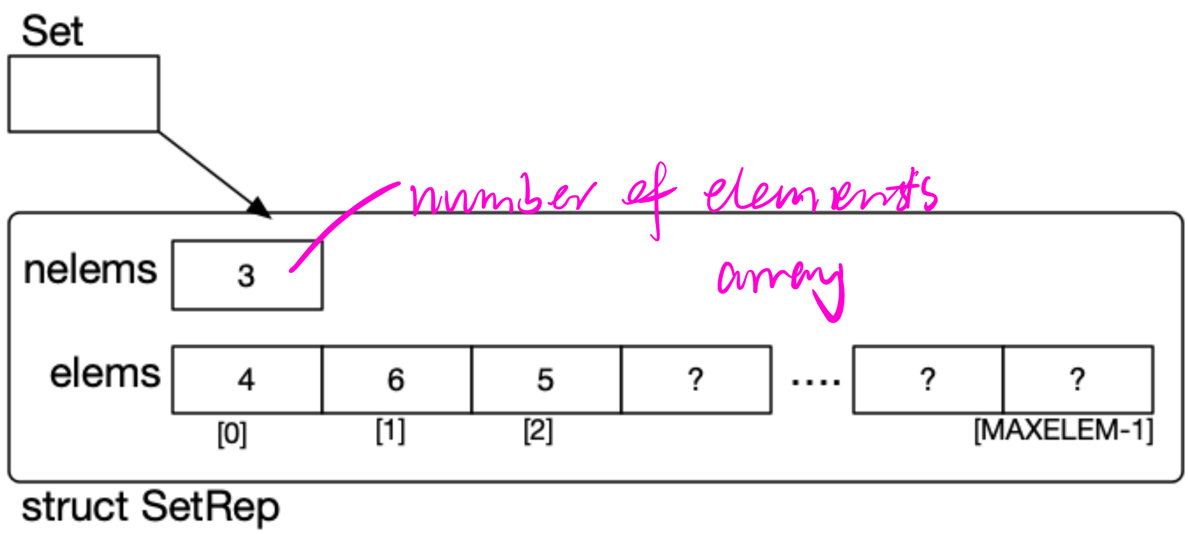
❖ Sets as Unsorted Arrays

Concrete data structure:



❖ ... Sets as Unsorted Arrays

Concrete data structure (in C):

```
#define MAXELEMS 1000

// concrete data structure
struct SetRep {
     int elems[MAXELEMS]; array
     int nelems;
};
```

Need to set upper bound on number of elements

Could do statically (as above) or dynamically

```
Set newSet(int maxElems) { ... }
                1000
```

❖ ... Sets as Unsorted Arrays

Set creation:

```
// create new empty set
Set newSet()
{
   Set s = malloc(sizeof(struct SetRep));
   if (s == NULL) {
      fprintf(stderr, "Insufficient memory\n");
      exit(EXIT_FAILURE);
   }
   s->nelems = 0;
   // assert(isValid(s));
   return s;
}
```

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❖ ... Sets as Unsorted Arrays

Checking membership:

```
// set membership test
int SetMember(Set s, int n)
{
    // assert(isValid(s));
    int i;
    for (i = 0; i < s->nelems; i++)
        if (s->elems[i] == n) return TRUE;
    return FALSE;
}
```

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❖ ... Sets as Unsorted Arrays

Costs for set operations on unsorted array:

- card: read from struct; constant cost $O(1)$
- member: scan list from start; linear cost $O(n)$
- insert: duplicate check, add at end; linear cost $O(n)$
- delete: find, copy last into gap; linear cost $O(n)$
- union: copy s1, insert each item from s2; quadratic cost $O(nm)$
- intersect: scan for each item in s1; quadratic cost $O(nm)$

Assuming: s1 has n items, s2 has m items

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❖ Sets as Sorted Arrays

Same data structure as for unsorted array.

Differences in

- membership test ... can use binary search
- insertion ... binary search and then shift up and insert
- deletion ... binary search and then shift down

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❖ ... Sets as Sorted Arrays

Costs for set operations on sorted array:

- card: read from struct; *O(1)*
- member: binary search; *O(log n)*
- insert: find, shift up, insert; *O(n)*
- delete: find, shift down; *O(n)*
- union: merge = scan s1, scan s2; *O(n)* (technically *O(n+m)*)
- intersect: merge = scan s1, scan s2; *O(n)* (technically *O(n+m)*)

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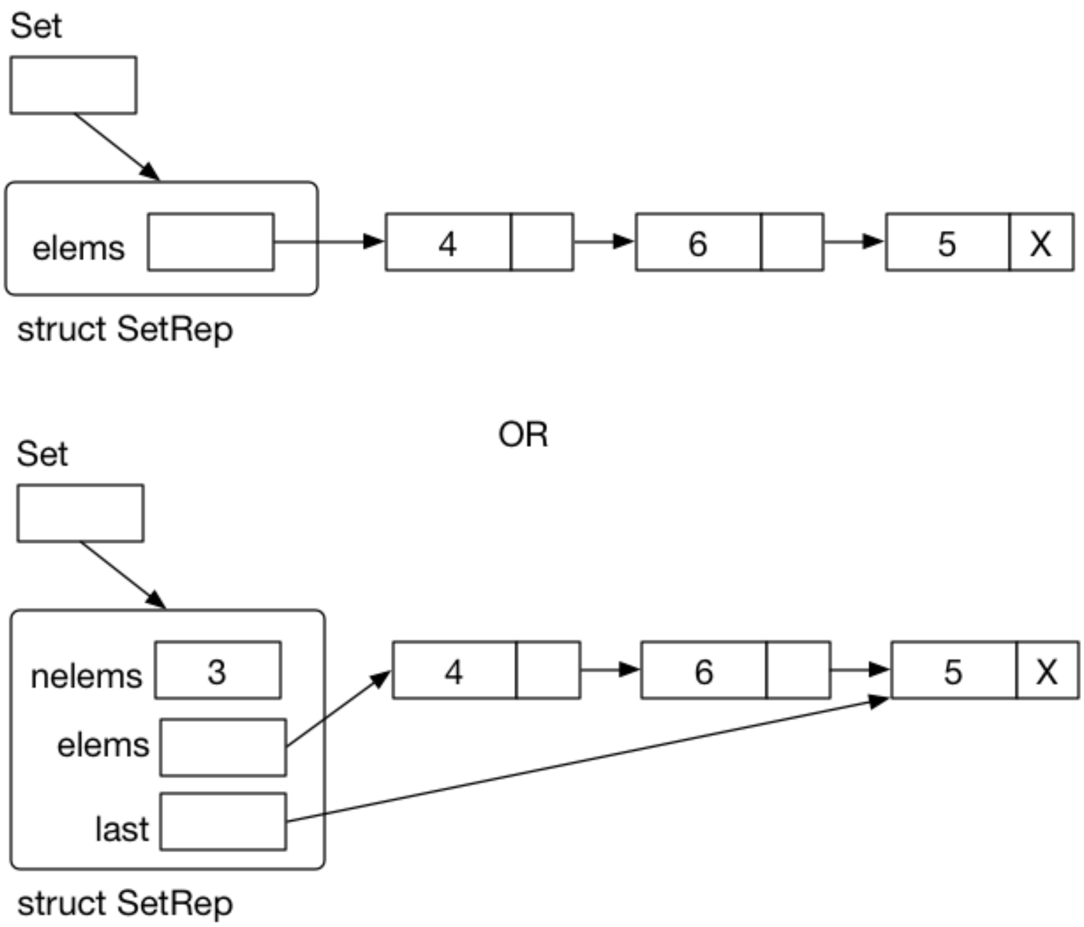
❖ Sets as Linked Lists

Concrete data structure:



❖ ... Sets as Linked Lists

Concrete data structure (in C):

```
typedef struct Node {
   int  value;
   struct Node *next;
} Node;

struct SetRep {
   Node *elems;   // pointer to first node
   Node *last;    // pointer to last node
   int  nelems;   // number of nodes
};
```

❖ ... Sets as Linked Lists

Set creation:

```
// create new empty set
Set newSet()
{
    Set s = malloc(sizeof(struct SetRep));
    if (s == NULL) {...}
    s->nelems = 0;
    s->elems = s->last = NULL;
    return s;
}
```

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❖ ... Sets as Linked Lists

Checking membership:

```
// set membership test
int SetMember(Set s, int n)
{
    // assert(isValid(s));
    Node *cur = s->elems;
    while (cur != NULL) {
        if (cur->value == n) return true;
        cur = cur->next;
    }
    return false;
}
```

first

done find

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❖ ... Sets as Linked Lists

Costs for set operations on linked list:

- insert: duplicate check, insert at head; $O(n)$
- delete: find, unlink; $O(n)$
- member: linear search; $O(n)$
- card: lookup; $O(1)$
- union: copy s1, insert each item from s2; $O(nm)$
- intersect: scan for each item in s1; $O(nm)$

Assume n = size of $s1$, m = size of $s2$

If we don't have **`nelems`**, card becomes $O(n)$

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❖ Sets as Bit-strings

Set is a very long bit-string, typically an array of **`words`**.

Restrict possible values that can be stored in the Set

- typically restricted to 0..N-1, (where N%32 == 0) 32 bits
- represent each value by position in large array of bits
- insertion means set a bit to 1 (**`bit|1`**)
- deletion means set a bit to 0 (**`bit&0`**)
- bit position for value **`i`** is easy to compute

0 2 3 1

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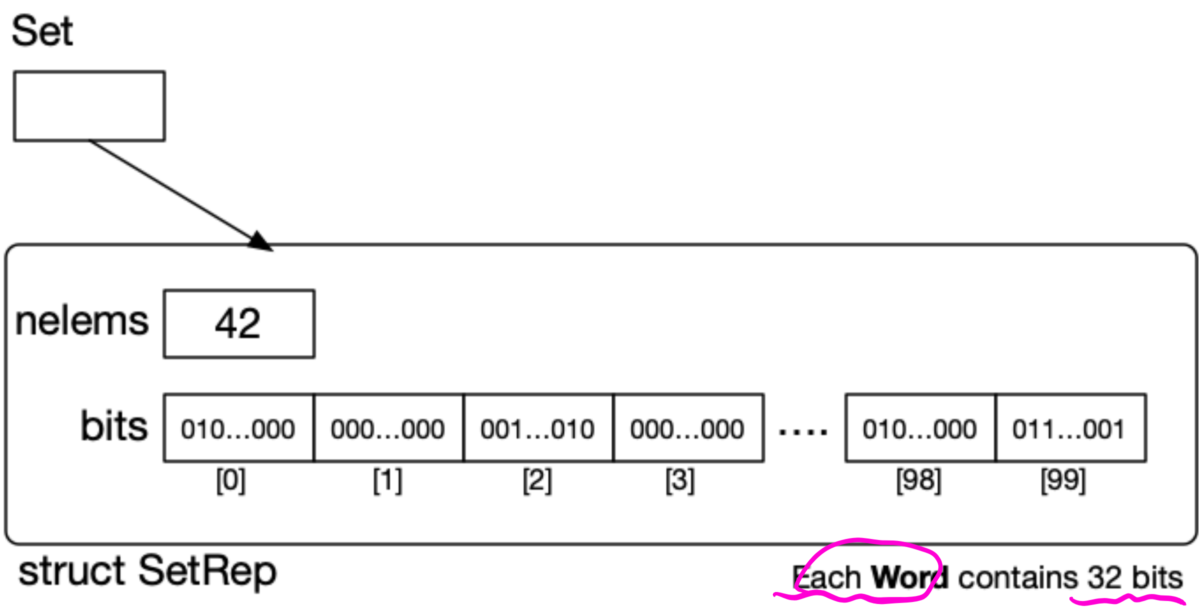
❖ ... Sets as Bit-strings

Concrete data structure:



❖ ... Sets as Bit-strings

Concrete data structure (in C):

```
#define NBITS 1024
#define NWORDS (NBITS/32)
typedef unsigned int Word;
typedef Word Bits[NWORDS];

struct SetRep {
   int nelems;
   Bits bits;    // Word bits[NWORDS]
};
```

$= 1024/32 = 32$

Sets defined like this can hold values in range 0..1023

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❖ ... Sets as Bit-strings

Implementation as bit-strings requires extra functions:

- **`getBit(Bits b, int i)`** ... get value of i'th bit, 0 or 1
- **`setBit(Bits b, int i)`** ... ensure i'th bit is set to 1
- **`unsetBit(Bits b, int i)`** ... ensure i'th bit is set to 0

Can be implemented efficiently, e.g.

array

```
getBit(Bits b, int i) {
   int whichWord = i / 32;
   int whichBit  = i % 32;
   Word mask = (1 << whichBit)
   return (b[whichWord] & mask) >> whichBit;
}
```

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❖ Setting and unsetting bits

Setting and unsetting bits by **&** and **|**

`0xFF;` all set to 1

```
unsigned char x, y, z;
```

x 00000111	y 10000001	`z = x & y;`	交集	z 00000001
x 00000111	y 10000001	`z = x | y;`	并集	z 10000111
x 00000011		`z = x & 0xFF;`	11111111	z 00000011
x 00000001		`z = x | 0xFF;`		z 11111111
x 00000000		`z = x | (1 << 2);`	set 1 左移 2位	z 00000100
x 11111111		`z = x & ~(1 << 2);`	000100	z 11111011

The last two switch on/off bit 2

~(1<<2) : 11111011

❖ ... Setting and unsetting bits

Powers of two by bit-shifting - don't use **`pow(...)`** from **`math.h`**!

x		x'
00000001	左移1位 `x = x << 1`	00000010
00000001	`x = x << 2`	00000100
00000001	`x = x << 99`	00000000
00000010	`x = x >> 1`	00000001
00011000	`x = x >> 2`	00000110
11111111	`x = x >> 99`	00000000

`x = x << n`

is

`x = x *` 2^n

```
assume:
unsigned char x;
```

`x = x >> n`

is

`x = x /` 2^n

<< ^

❖ Performance of Set Implementations

Performance comparison:

Data Structure	insert	delete	member	∪, ∩	storage
unsorted array	$O(n)$	$O(n)$	$O(n)$	$O(n.m)$	$O(N)$
sorted array	$O(n)$	$O(n)$	$O(\log_2 n)$	$O(n+m)$	$O(N)$
unsorted linked list	$O(n)$	$O(n)$	$O(n)$	$O(n.m)$	$O(n)$
sorted linked list	$O(n)$	$O(n)$	$O(n)$	$O(n+m)$	$O(n)$
bit-maps	$O(1)$	$O(1)$	$O(1)$	$O(N)$	$O(N)$

n,m = #elems, N = max #elems,