

# Graph Basics

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- Graphs
- Properties of Graphs
- Graph Terminology

# ❖ Graphs

Many applications require

- a collection of items (i.e. a set)
- relationships/connections between items

Examples:

- maps: items are cities, connections are roads
- web: items are pages, connections are hyperlinks


Collection types you're familiar with

- lists ... linear sequence of items (COMP1511)
- trees ... branched hierarchy of items (Weeks 02/03)

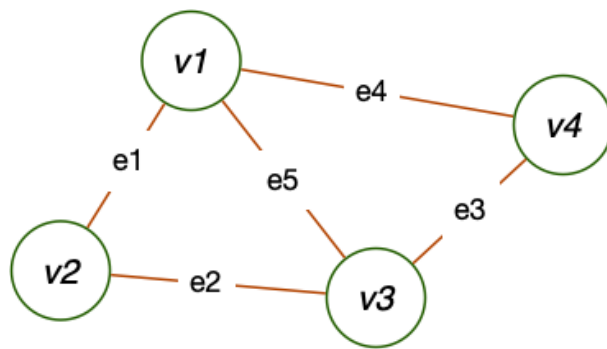
Graphs are more general ... allow arbitrary connections

## ❖ ... Graphs

A graph  $G = (V, E)$

- $V$  is a set of **vertices** 
- $E$  is a set of **edges** (subset of  $V \times V$ )

Example:



$$V = \{ v1, v2, v3, v4 \}$$

$$E = \{ e1, e2, e3, e4, e5 \}$$

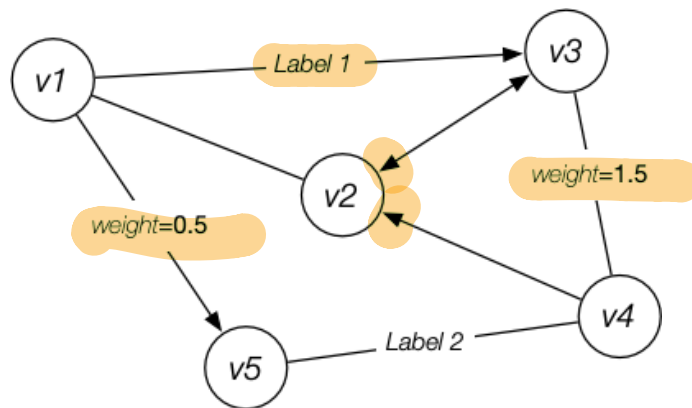
or

$$E = \{ (v1, v2), (v2, v3), (v3, v4), (v1, v4), (v1, v3) \}$$

## ❖ ... Graphs

Nodes are distinguished by a unique identifier

Edges may be (optionally) directed, labelled and/or weighted



## ❖ ... Graphs

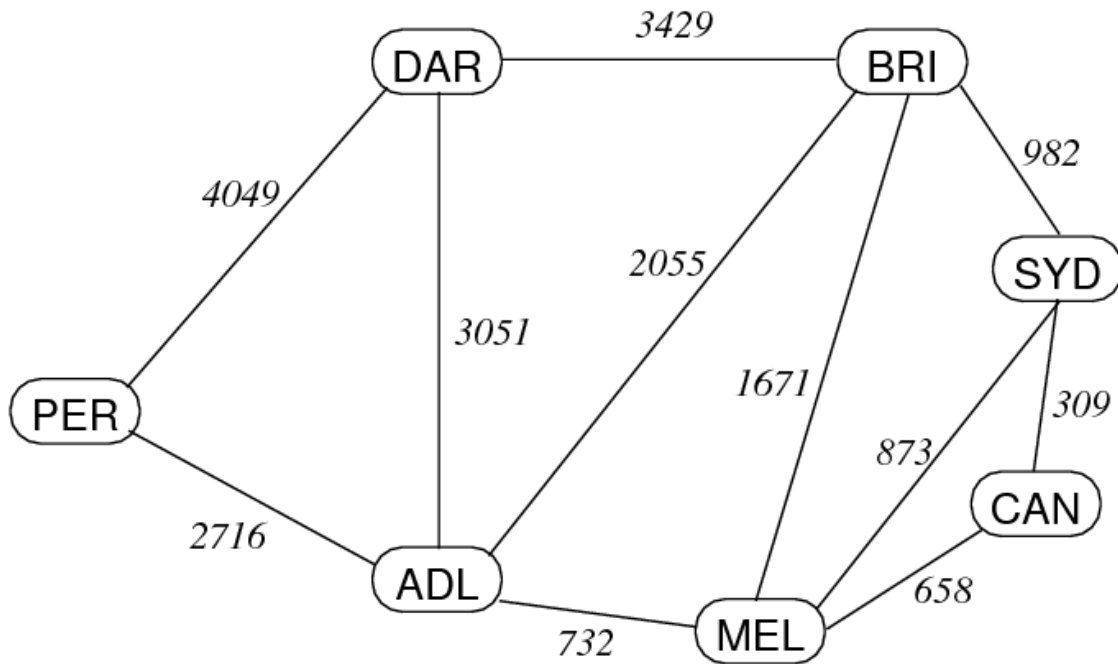
A real example: Australian road distances

Distance	Adelaide	Brisbane	Canberra	Darwin	Melbourne	Perth	Sydney
Adelaide	-	2055	1390	3051	732	2716	1605
Brisbane	2055	-	1291	3429	1671	4771	982
Canberra	1390	1291	-	4441	658	4106	309
Darwin	3051	3429	4441	-	3783	4049	4411
Melbourne	732	1671	658	3783	-	3448	873
Perth	2716	4771	4106	4049	3448	-	3972
Sydney	1605	982	309	4411	873	3972	-

Notes: vertices are cities, edges are distance between cities, symmetric

## ❖ ... Graphs

Alternative representation of above:



## ❖ ... Graphs

Questions we might ask about a graph:

- is there a way to get from item A to item B? *Yes 经过其他点*
- what is the best way to get from A to B?
- which items are directly connected (A ↔ B)?

Graph algorithms are generally more complex than tree/list ones:

- no implicit order of items
- graphs may contain cycles
- concrete representation is less obvious
- algorithm complexity depends on connection complexity

## ❖ Properties of Graphs

Terminology:  $|V|$  and  $|E|$  (cardinality) normally written just as  $V$  and  $E$ .

A graph with  $V$  vertices has at most  $V(V-1)/2$  edges.

ab ba 没区别  
so 1/2

The ratio  $E:V$  can vary considerably.

每个点除自己  
都有线

- if  $E$  is closer to  $V^2$ , the graph is **dense** 密集
- if  $E$  is closer to  $V$ , the graph is **sparse** 稀疏
  - Example: web pages and hyperlinks

Knowing whether a graph is **sparse** or **dense** is important

- may **affect** choice of data structures to **represent** graph
- may **affect** choice of **algorithms** to **process** graph



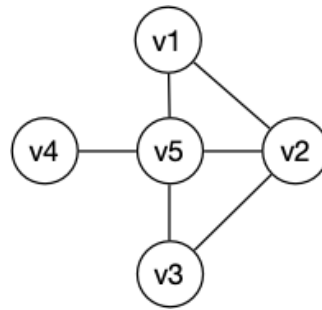
## ❖ Graph Terminology

For an edge  $e$  that connects vertices  $v$  and  $w$

- $v$  and  $w$  are **adjacent** (neighbours)
- $e$  is **incident** on both  $v$  and  $w$

**Degree** of a vertex  $v$

- number of edges incident on  $e$



$\text{degree}(v1) = 2$   
 $\text{degree}(v2) = 3$   
 $\text{degree}(v3) = 2$   
 $\text{degree}(v4) = 1$   
 $\text{degree}(v5) = 4$

Synonyms:

- vertex = **node**
- edge = **arc** = **link** (Note: some people use arc for *directed* edges)

## ❖ ... Graph Terminology

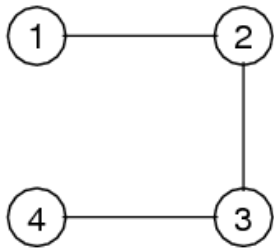
**Path:** a sequence of vertices where

- each vertex has an edge to its predecessor 前任

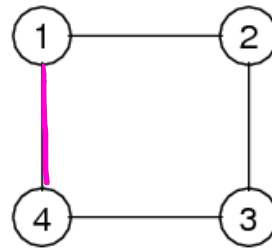
**Cycle:** a path where

- last vertex in path is same as first vertex in path

**Length** of path or cycle = #edges



*Path: 1-2, 2-3, 3-4*



*Cycle: 1-2, 2-3, 3-4, 4-1*

## ❖ ... Graph Terminology

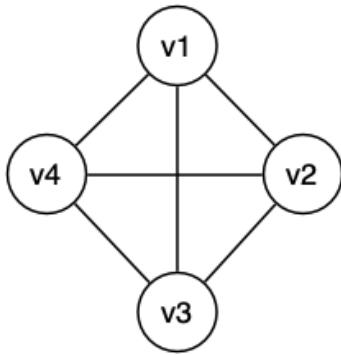
### Connected graph

- there is a path from each vertex to every other vertex
- if a graph is not connected, it has  $\geq 2$  connected components

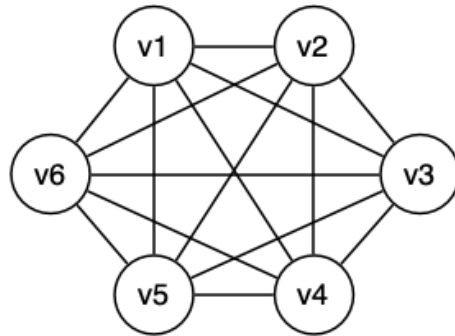
一个连通图

### Complete graph $K_V$

- there is an edge from each vertex to every other vertex
- in a complete graph,  $E = V(V-1)/2$



Complete  
Graphs



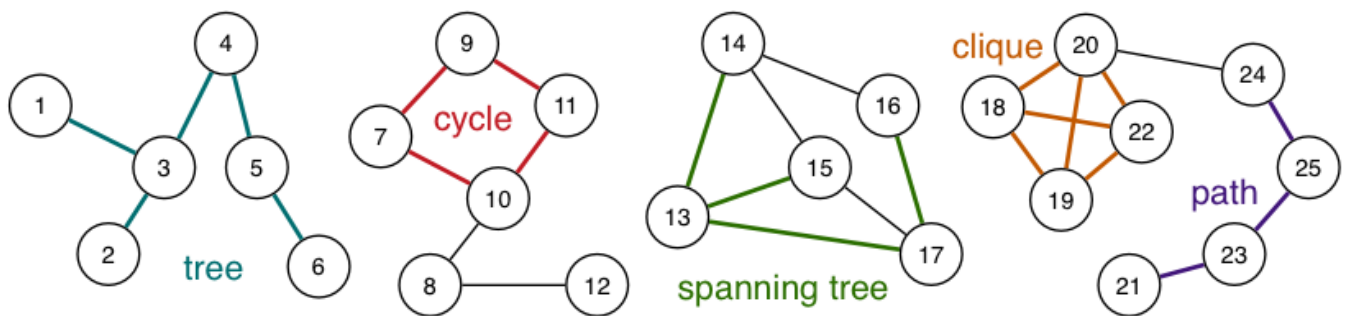
## ❖ ... Graph Terminology

**Tree:** connected (sub)graph with no cycles

**Spanning tree:** tree containing all vertices

**Clique:** complete subgraph

Consider the following *single graph*:



This graph has 25 vertices, 32 edges, and 4 connected components

Note: The entire graph has no spanning tree; what is shown in green is a spanning tree of the third connected component

## ❖ ... Graph Terminology

A **spanning tree** of **connected graph**  $G = (V, E)$

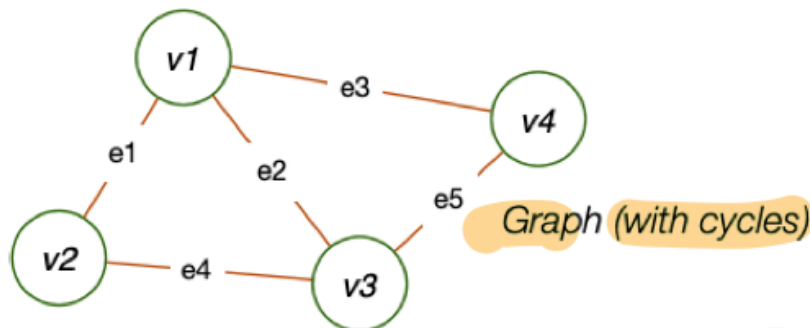
- is a **subgraph of  $G$  containing all of  $V$**
- and is a **single tree (connected, no cycles)** ✱

A **spanning forest** of **non-connected graph**  $G = (V, E)$

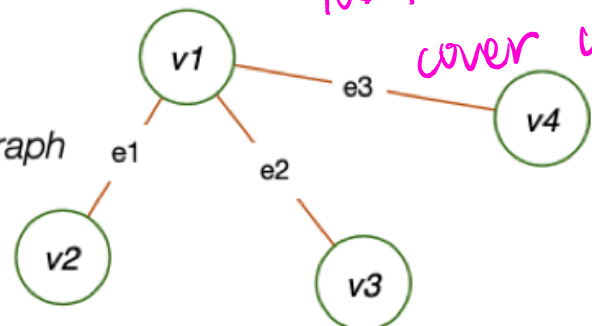
- is a subgraph of  $G$  containing all of  $V$
- and is a **set of trees (not connected, no cycles)**,
  - with **one tree for each connected component**

COMP2521 20T2 ^ Graph Basics [12/45]

Can form **spanning tree** from graph by **removing edges**



A **spanning tree of graph**  
(no cycles)



沒有 cycle 并  
cover all  $V$ .

Many possible spanning trees can be formed. Which is "best"?

## ❖ ... Graph Terminology

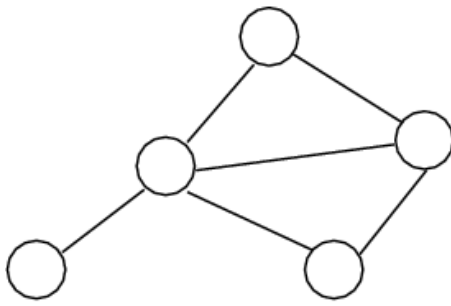
### Undirected graph

- $\text{edge}(u,v) = \text{edge}(v,u)$ , no self-loops (i.e. no  $\text{edge}(v,v)$ )

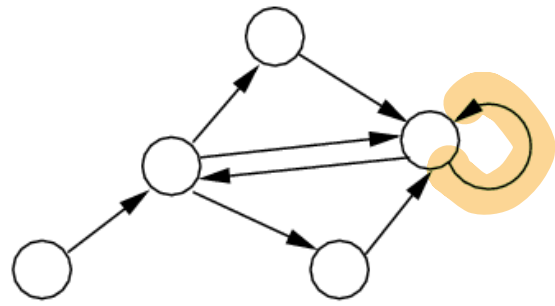
### Directed graph

- $\text{edge}(u,v) \neq \text{edge}(v,u)$ , can have self-loops (i.e.  $\text{edge}(v,v)$ )

Examples:



*Undirected graph*



*Directed graph*

## ❖ ... Graph Terminology

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Other types of graphs ...

### Weighted graph

- each edge has an associated value (weight)
- e.g. road map (weights on edges are distances between cities)

### Multi-graph

- allow multiple edges between two vertices
- e.g. function call graph ( $f()$  calls  $g()$  in several places)

### Labelled graph

- edges have associated labels
- can be used to add semantic information

# Graph Representations

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- Graph Representations
- Array-of-edges Representation
- Array-of-edges Cost Analysis
- Adjacency Matrix Representation
- Adjacency Matrix Cost Analysis
- Adjacency List Representation
- Adjacency List Cost Analysis
- Comparison of Graph Representations

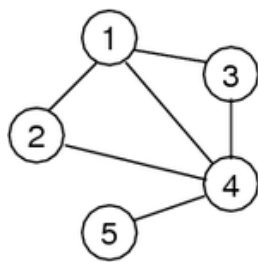


## ❖ Graph Representations

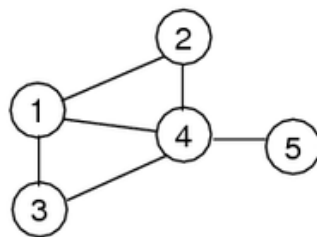
Describing graphs:

- could describe via a diagram showing edges and vertices
- could describe by giving a list of edges
- assume we identify vertices by distinct integers

E.g. four representations of the same graph:



(a)



(b)

1-2 1-3 1-4  
2-4  
3-4  
4-5

(c)

1-3  
2-1 2-4  
4-1 4-3  
5-4

(d)

## ❖ ... Graph Representations

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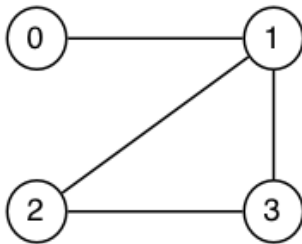
We discuss three different graph data structures:

1. Array of edges
  - explicit representation of edges as  $(v,w)$  pairs
2. Adjacency matrix
  - edges defined by presence value in  $V \times V$  matrix
3. Adjacency list
  - edges defined by entries in array of  $V$  lists

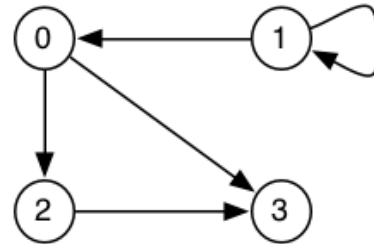
## ❖ Array-of-edges Representation

Edges are represented as an array of **Edge** values (= pairs of vertices)

- space efficient representation
- adding and deleting edges is slightly complex
- undirected: order of vertices in an **Edge** doesn't matter
- directed: order of vertices in an **Edge** encodes direction



[ (0,1), (1,2), (1,3), (2,3) ]



[ (1,0), (1,1), (0,2), (0,3), (2,3) ]

For simplicity, we always assume vertices to be numbered  $0 \dots V-1$

## ❖ ... Array-of-edges Representation

Graph initialisation

```
newGraph(V):  
    Input  number of nodes V  
    Output new empty graph (no edges)  
    f → nV = V  
    g.nV = V    // #vertices (numbered 0..V-1)  
    g.nE = 0    // #edges  
    allocate enough memory for g.edges[]  
    return g
```

Assumes  $\equiv$  `struct Graph { int nV; int nE; Edge edges[]; }`

## ❖ ... Array-of-edges Representation

Edge insertion

```
insertEdge(g, (v,w)) :
  Input  graph g, edge (v,w)
  Output graph g containing (v,w)

  i=0
  while i < g.nE ^ g.edges[i] ≠ (v,w) do
    i=i+1
  end while
  if i=g.nE then // (v,w) not found
    g.edges[i]=(v,w)
    g.nE=g.nE+1
  end if
```

*i = g.nE 时*  
*边数*

We "normalise" edges so that e.g.  $(v < w)$  in all  $(v,w)$

## ❖ ... Array-of-edges Representation

Edge removal

```
removeEdge(g, (v,w)) :
  Input  graph g, edge (v,w)
  Output graph g without (v,w)

  i=0
  while i < g.nE ^ g.edges[i] ≠ (v,w) do
    i=i+1
  end while
  if i < g.nE then // (v,w) found
    g.edges[i]=g.edges[g.nE-1]
    // replace by last edge in array
    g.nE=g.nE-1
  end if
```

## ❖ ... Array-of-edges Representation

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Print a list of edges

```
showEdges(g):  
  Input graph g  
  
  for all i=0 to g.nE-1 do  
    (v,w)=g.edges[i]  
    print v-"-w"  
  end for
```

## ❖ Array-of-edges Cost Analysis

Storage cost:  $O(E)$

Cost of operations:

- initialisation:  $O(1)$
- insert edge:  $O(E)$  (need to check for edge in array) *add*
- delete edge:  $O(E)$  (need to find edge in edge array)

If array is full on insert

- allocate space for a bigger array, copy edges across  $\Rightarrow$  still  $O(E)$  *realloc 复制*

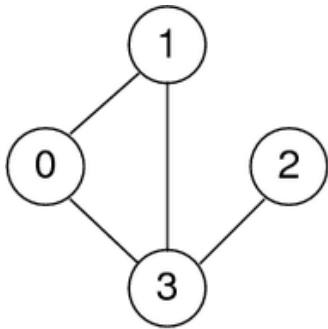
If we maintain edges in order

- use binary search to find edge  $\Rightarrow O(\log E)$



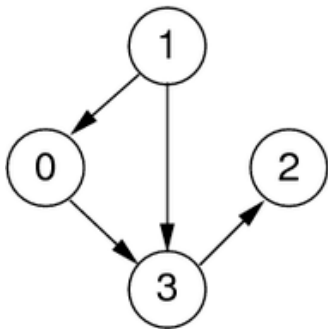
## ❖ Adjacency Matrix Representation

Edges represented by a  $V \times V$  matrix



*Undirected graph*

A	0	1	2	3
0	0	1	0	1
1	1	0	0	1
2	0	0	0	1
3	1	1	1	0



*Directed graph*

A	→ 0	1	2	3
0	0	0	0	1
1	→ 1	0	0	1
2	0	0	0	0
3	0	0	1	0

## ❖ ... Adjacency Matrix Representation

### Advantages

- easily implemented as 2-dimensional array
- can represent graphs, digraphs and weighted graphs
  - graphs: symmetric boolean matrix
  - digraphs: non-symmetric boolean matrix
  - weighted: non-symmetric matrix of weight values

### Disadvantages:

- if few edges (sparse)  $\Rightarrow$  memory-inefficient ( $O(V^2)$  space)

内存占用太多

## ❖ ... Adjacency Matrix Representation

Graph initialisation

```
newGraph(V):  
    Input   number of nodes V  
    Output new empty graph  
  
    g.nV = V    // #vertices (numbered 0..V-1)  
    g.nE = 0    // #edges  
    allocate memory for g.edges[][]  
    for all i,j=0..V-1 do  
        g.edges[i][j]=0    // false  
    end for  
    return g
```

## ❖ ... Adjacency Matrix Representation

Edge insertion

```
insertEdge(g, (v,w)):  
    Input   graph g, edge (v,w)  
    Output graph g containing (v,w)  
  
    if g.edges[v][w] = 0 then    // (v,w) not in graph  
        g.edges[v][w]=1          // set to true  
        g.edges[w][v]=1  
        g.nE=g.nE+1  
    end if
```

## ❖ ... Adjacency Matrix Representation

Edge removal

```
removeEdge(g, (v,w)) :  
    Input   graph g, edge (v,w)  
    Output graph g without (v,w)  
  
    if g.edges[v][w] ≠ 0 then    // (v,w) in graph  
        g.edges[v][w] = 0        // set to false  
        g.edges[w][v] = 0  
        g.nE = g.nE - 1  
    end if
```

## ❖ ... Adjacency Matrix Representation

Print a list of edges

```
showEdges(g):  
    Input graph g  
  
    for all i=0 to g.nV-1 do  
        for all j=i+1 to g.nV-1 do  
            if g.edges[i][j] ≠ 0 then  
                print i-"-j  
            end if  
        end for  
    end for
```

## ❖ Adjacency Matrix Cost Analysis

Storage cost:  $O(V^2)$



If the graph is sparse, most storage is wasted.

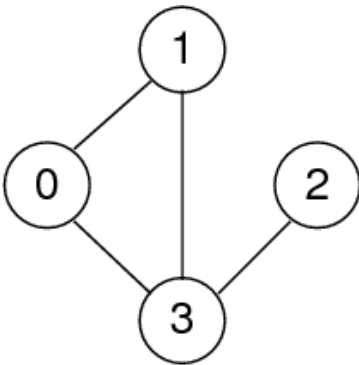
稀疏

Cost of operations:

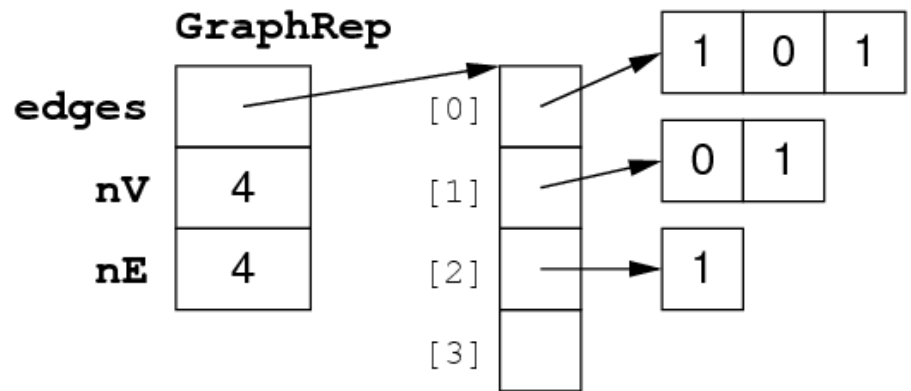
- initialisation:  $O(V^2)$  (initialise  $V \times V$  matrix)
- insert edge:  $O(1)$  (set two cells in matrix)
- delete edge:  $O(1)$  (unset two cells in matrix)

## ❖ ... Adjacency Matrix Cost Analysis

A storage optimisation: store **only top-right part** of matrix.



*Undirected graph*



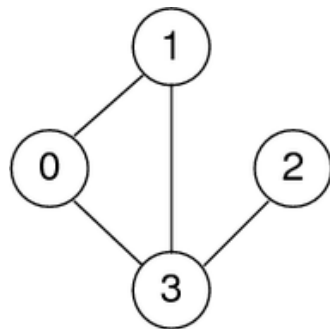
New storage cost:  $V-1$  int ptrs +  $V(V+1)/2$  ints (but still  $O(V^2)$ )

Requires us to always use **edges  $(v,w)$**  such that  $v < w$ .



## ❖ Adjacency List Representation

For each vertex, store linked list of adjacent vertices:



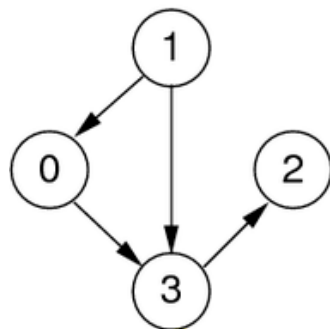
*Undirected graph*

$A[0] = \langle 1, 3 \rangle$

$A[1] = \langle 0, 3 \rangle$

$A[2] = \langle 3 \rangle$

$A[3] = \langle 0, 1, 2 \rangle$



*Directed graph*

$A[0] = \langle 3 \rangle$

$A[1] = \langle 0, 3 \rangle$

$A[2] = \langle \rangle$

$A[3] = \langle \rangle$

## ❖ ... Adjacency List Representation

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### Advantages

- relatively easy to implement in languages like C
- can represent graphs and digraphs
- memory efficient if  $E:V$  relatively small

### Disadvantages:

- one graph has many possible representations  
(unless lists are ordered by same criterion e.g. ascending)

## ❖ ... Adjacency List Representation

Graph initialisation

```
newGraph(V):  
    Input   number of nodes V  
    Output new empty graph  
  
    g.nV = V    // #vertices (numbered 0..V-1)  
    g.nE = 0    // #edges  
    allocate memory for g.edges[]  
    for all i=0..V-1 do  
        g.edges[i]=newList() // empty list  
    end for  
    return g
```

## ❖ ... Adjacency List Representation

Edge insertion:

```
insertEdge(g, (v,w)) :  
    Input   graph g, edge (v,w)  
    Output graph g containing (v,w)  
  
    if not ListMember(g.edges[v],w) then  
        // (v,w) not in graph  
        ListInsert(g.edges[v],w)  
        ListInsert(g.edges[w],v)  
        g.nE=g.nE+1  
    end if
```

## ❖ ... Adjacency List Representation

Edge removal:

```
removeEdge(g, (v,w)) :  
    Input   graph g, edge (v,w)  
    Output graph g without (v,w)  
  
    if ListMember(g.edges[v],w) then  
        // (v,w) in graph  
        ListDelete(g.edges[v],w)  
        ListDelete(g.edges[w],v)  
        g.nE=g.nE-1  
    end if
```

## ❖ ... Adjacency List Representation

Print a list of edges

```
showEdges(g):  
    Input graph g  
  
    for all i=0 to g.nV-1 do  
        for all v in g.edges[i] do  
            if i < v then  
                print i-"v"  
            end if  
        end for  
    end for
```

## ❖ Adjacency List Cost Analysis

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Storage cost:  $O(V+E)$

Cost of operations:

- initialisation:  $O(V)$  (initialise  $V$  lists)
- insert edge:  $O(E)$  (need to check if vertex in list)
- delete edge:  $O(E)$  (need to find vertex in list)

Could sort vertex lists, but no benefit (although no extra cost)

## ❖ Comparison of Graph Representations

Summary of operations above:

	array of edges	adjacency matrix	adjacency list
<i>storage</i> space usage	$E$	$V^2$	$V+E$
initialise	$1$	$V^2$	$V$
insert edge	$E$	$1$	$E$
remove edge	$E$	$1$	$E$

Other operations:

	array of edges	adjacency matrix	adjacency list
disconnected(v)?	$E$	$V$	$1$
isPath(x,y)?	$E \cdot \log V$	$V^2$	$V+E$
copy graph	$E$	$V^2$	$V+E$
destroy graph	$1$	$V$	$V+E$



# Graph ADT

---

- Graph ADT
- Graph ADT (Array of Edges)
- Graph ADT (Adjacency Matrix)
- Graph ADT (Adjacency Lists)
- Example: Graph ADT Client

## ❖ Graph ADT

^ >>

**Data:** set of edges, set of vertices

**Operations:**

- building: create graph, add edge
- deleting: remove edge, drop whole graph
- scanning: check if graph contains a given edge

**Things to note:**

- set of vertices is fixed when graph initialised
- we treat vertices as **ints**, but could be arbitrary **Items**

Will use this ADT as a basis for building more complex operations later.

## ❖ ... Graph ADT

### Graph ADT interface **Graph.h**

```
// graph representation is hidden
typedef struct GraphRep *Graph;

// vertices denoted by integers 0..N-1
typedef int Vertex;

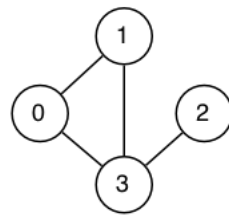
// edges are pairs of vertices (end-points)
typedef struct Edge { Vertex v; Vertex w; } Edge;

// operations on graphs
Graph newGraph(int V); // new graph with V vertices
void insertEdge(Graph, Edge);
void removeEdge(Graph, Edge);
bool adjacent(Graph, Vertex, Vertex);
// is there an edge between two vertices?
void freeGraph(Graph);
```

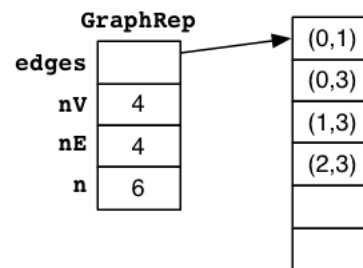
## ❖ Graph ADT (Array of Edges)

Implementation of **GraphRep** (array-of-edges representation)

```
typedef struct GraphRep {
    Edge *edges; // array of edges
    int nV;      // #vertices (numbered 0..nV-1)
    int nE;      // #edges
    int n;       // size of edge array
} GraphRep;
```



*Undirected graph*



## ❖ ... Graph ADT (Array of Edges)

Implementation of graph initialisation (array-of-edges)

```
Graph newGraph(int V) {
    assert(V >= 0);
    Graph g = malloc(sizeof(GraphRep));
    assert(g != NULL);
    g->nV = V; g->nE = 0;
    // allocate enough memory for edges
    g->n = Enough;
    g->edges = malloc(g->n*sizeof(Edge));
    assert(g->edges != NULL);
    return g;
}
```

How much is enough? ... No more than  $V(V-1)/2$  ... Much less in practice (sparse graph)

$$\frac{4 \times 3}{2} = 7 \times 3 = 21$$

## ❖ ... Graph ADT (Array of Edges)

Some useful utility functions:

```
// check if two edges are equal
bool eq(Edge e1, Edge e2) {
    return ( (e1.v == e2.v && e1.w == e2.w)
            || (e1.v == e2.w && e1.w == e2.v) );
}

// check if vertex is valid in a graph
bool validV(Graph g, Vertex v) {
    return (g != NULL && v >= 0 && v < g->nV);
}

// check if an edge is valid in a graph
bool validE(Graph g, Edge e) {
    return (g != NULL && validV(e.v) && validV(e.w));
}
```

## ❖ ... Graph ADT (Array of Edges)

### Implementation of edge insertion (array-of-edges)

```
void insertEdge(Graph g, Edge e) {  
    // ensure that g exists and array of edges isn't full  
    assert(g != NULL && g->nE < g->n && isValidE(g,e));  
    int i = 0; // can't define in for (...)  
    for (i = 0; i < g->nE; i++)  
        if (eq(e,g->edges[i])) break; // 已经有了  
    if (i == g->nE) // edge e not found  
        g->edges[g->nE++] = e;  
}
```

number  
of edges

## ❖ ... Graph ADT (Array of Edges)

Implementation of edge removal (array-of-edges)

```
void removeEdge(Graph g, Edge e) {  
    // ensure that g exists  
    assert(g != NULL && validE(g,e));  
    int i = 0;  
    while (i < g->nE && !eq(e,g->edges[i]))  
        i++;  
    if (i < g->nE) // edge e found  
        g->edges[i] = g->edges[--g->nE];  
}
```



## ❖ ... Graph ADT (Array of Edges)

Implementation of edge check (array-of-edges)

```
bool adjacent(Graph g, Vertex x, Vertex y) {  
    assert(g != NULL && validV(g,x) && validV(g,y));  
    Edge e;  
    e.v = x; e.w = y;  
    for (int i = 0; i < g->nE; i++) {  
        if (eq(e,g->edges[i])) // edge found  
            return true;  
    }  
    return false; // edge not found  
}
```

## ❖ ... Graph ADT (Array of Edges)

Re-implementation of edge insertion (array-of-edges)

```
void insertEdge(Graph g, Edge e) {  
    // ensure that g exists  
    assert(g != NULL && validE(g,e));  
    int i = 0;  
    for (i = 0; i < g->nE; i++)  
        if (eq(e,g->edges[i])) break;  
    if (i == g->nE) { // edge e not found  
        if (g->n == g->nE) { // array full expand  
            g->edges = realloc(g->edges, 2*g->n);  
            assert(g->edges != NULL);  
            g->n = 2*g->n;  
        }  
        g->edges[g->nE++] = e;  
    }  
}
```

## ❖ ... Graph ADT (Array of Edges)

---

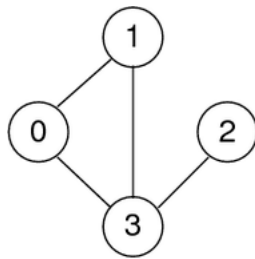
Implementation of graph removal (array-of-edges)

```
void freeGraph(Graph g) {  
    assert(g != NULL);  
    free(g->edges); // free array of edges  
    free(g);        // remove Graph object  
}
```

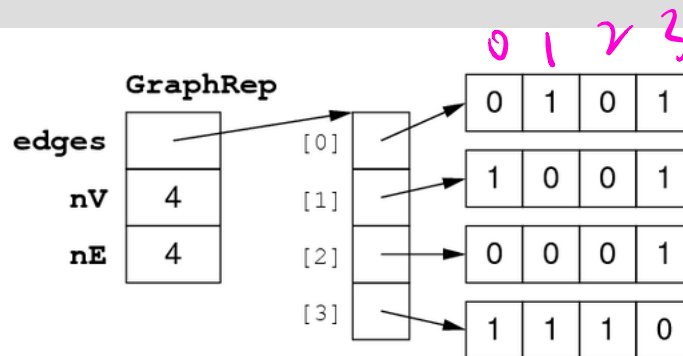
## ❖ Graph ADT (Adjacency Matrix)

Implementation of **GraphRep** (adjacency-matrix representation)

```
typedef struct GraphRep {
    int **edges; // adjacency matrix
    int nV;      // #vertices
    int nE;      // #edges
} GraphRep;
```



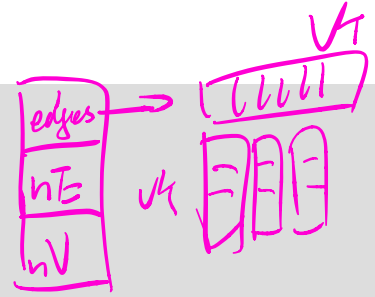
*Undirected graph*



## ❖ ... Graph ADT (Adjacency Matrix)

Implementation of graph initialisation (adjacency-matrix)

```
Graph newGraph(int V) {
    assert(V >= 0);
    Graph g = malloc(sizeof(GraphRep));
    assert(g != NULL);
    g->nV = V; g->nE = 0;
    // allocate array of pointers to rows
    g->edges = malloc(V * sizeof(int *));
    assert(g->edges != NULL);
    // allocate memory for each column and initialise with 0
    for (int i = 0; i < V; i++) {
        g->edges[i] = calloc(V, sizeof(int));
        assert(g->edges[i] != NULL);
    }
    return g;
}
```



Standard library function `calloc(size_t nelems, size_t nbytes)`

- allocates a memory block of size `nelems*nbytes`
- and sets all bytes in that block to `zero`

## ❖ ... Graph ADT (Adjacency Matrix)

Implementation of edge insertion (adjacency-matrix)

```
void insertEdge(Graph g, Edge e) {  
    assert(g != NULL && validE(g,e));  
  
    if (!g->edges[e.v][e.w]) { // edge e not in graph  
        g->edges[e.v][e.w] = 1;  
        g->edges[e.w][e.v] = 1;  
        g->nE++;  
    }  
}
```

## ❖ ... Graph ADT (Adjacency Matrix)

Implementation of edge removal (adjacency-matrix)

```
void removeEdge(Graph g, Edge e) {  
    assert(g != NULL && validE(g,e));  
  
    if (g->edges[e.v][e.w]) {    // edge e in graph  
        g->edges[e.v][e.w] = 0;  
        g->edges[e.w][e.v] = 0;  
        g->nE--;  
    }  
}
```

## ❖ ... Graph ADT (Adjacency Matrix)

Implementation of edge check (adjacency matrix)

```
bool adjacent(Graph g, Vertex x, Vertex y) {  
    assert(g != NULL && validV(g,x) && validV(g,y));  
    return (g->edges[x][y] != 0);  
}
```

Note: all operations, except creation, are  $O(1)$



## ❖ ... Graph ADT (Adjacency Matrix)

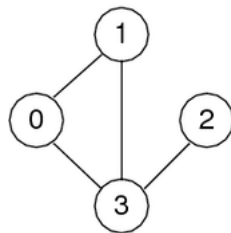
Implementation of graph removal (adjacency matrix)

```
void freeGraph(Graph g) {  
    assert(g != NULL);  
    for (int i = 0; i < g->nV; i++)  
        // free one row of matrix  
        free(g->edges[i]);  
    free(g->edges); // free array of row pointers  
    free(g);        // remove Graph object  
}
```

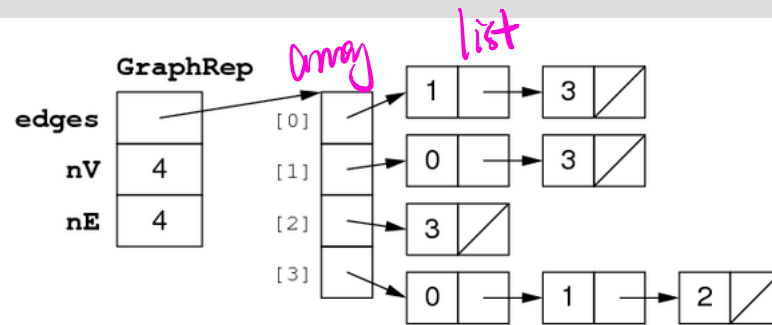
## ❖ Graph ADT (Adjacency Lists)

Implementation of **GraphRep** (adjacency-lists representation)

```
typedef struct GraphRep {
    Node **edges; // array of lists
    int    nV;    // #vertices
    int    nE;    // #edges
} GraphRep;
```



*Undirected graph*



## ❖ ... Graph ADT (Adjacency Lists)

Assume that we have a linked list implementation

```
typedef struct Node {  
    Vertex v;  
    struct Node *next;  
} Node;
```

with operations like **inLL**, **insertLL**, **deleteLL**, **freeLL**, e.g.

```
bool inLL(Node *L, Vertex v) {  
    while (L != NULL) {  
        if (L->v == v) return true;  
        L = L->next;  
    }  
    return false;  
}
```

check ✓ 是否存在

## ❖ ... Graph ADT (Adjacency Lists)

Implementation of graph initialisation (adjacency lists)

```
Graph newGraph(int V) {  
    assert(V >= 0);  
    Graph g = malloc(sizeof(GraphRep));  
    assert(g != NULL);  
    g->nV = V; g->nE = 0;  
    // allocate memory for array of lists  
    g->edges = malloc(V * sizeof(Node *));  
    assert(g->edges != NULL);  
    for (int i = 0; i < V; i++)  
        g->edges[i] = NULL;  
    return g;  
}
```

array  
→ Null  
→ Null  
|

## ❖ ... Graph ADT (Adjacency Lists)

Implementation of edge insertion/removal (adjacency lists)

```
void insertEdge(Graph g, Edge e) {
    assert(g != NULL && validE(g,e));
    if (!inLL(g->edges[e.v], e.w)) { // edge e not in graph
        g->edges[e.v] = insertLL(g->edges[e.v], e.w);
        g->edges[e.w] = insertLL(g->edges[e.w], e.v);
        g->nE++;
    }
}

void removeEdge(Graph g, Edge e) {
    assert(g != NULL && validE(g,e));
    if (inLL(g->edges[e.v], e.w)) { // edge e in graph
        g->edges[e.v] = deleteLL(g->edges[e.v], e.w);
        g->edges[e.w] = deleteLL(g->edges[e.w], e.v);
        g->nE--;
    }
}
```

## ❖ ... Graph ADT (Adjacency Lists)

Implementation of edge check (adjacency lists)

```
bool adjacent(Graph g, Vertex x, Vertex y) {  
    assert(g != NULL && validV(g,x) && validV(g,y));  
    return inLL(g->edges[x], y);  
}
```

Note: all operations, except creation, are  $O(E)$

edges 遍历

## ❖ ... Graph ADT (Adjacency Lists)

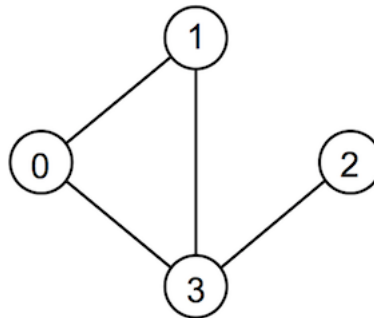
Implementation of graph removal (adjacency lists)

```
void freeGraph(Graph g) {  
    assert(g != NULL);  
    for (int i = 0; i < g->nV; i++)  
        freeLL(g->edges[i]); // free one list  
    free(g->edges); // free array of list pointers  
    free(g); // remove Graph object  
}
```

## ❖ Example: Graph ADT Client

A program that uses the graph ADT to

- build the graph depicted below
- print all the nodes that are incident to vertex 1 in ascending order





## ❖ ... Example: Graph ADT Client

```
#include <stdio.h>
#include "Graph.h"

#define NODES 4
#define NODE_OF_INTEREST 1

int main(void) {
    Graph g = newGraph(NODES);
    Edge e;

    while (scanf("%d %d", &(e.v), &(e.w)) == 2)
        insertEdge(g,e);

    for (Vertex v = 0; v < NODES; v++) {
        if (adjacent(g, v, NODE_OF_INTEREST))
            printf("%d\n", v);
    }

    freeGraph(g);
    return 0;
}
```