Automotive active suspensions Part 1: basic principles

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Abstract: Automotive suspension design is a compromise brought about by the conflicting demands of ride and handling. The past few years have seen the introduction of increasingly sophisticated, electronically controlled, components into automotive suspensions which redefine the boundaries of the compromise.

The paper has been written in two parts. This first part reviews the compromises which are required in the design of a conventional passive suspension. It then goes on to show how those compromises can be changed by the inclusion of active components.

The second part discusses the hardware employed which ranges from simple switched dampers, through semi-active dampers, and low bandwidth/soft active suspensions, to high bandwidth/stiff active suspensions. The benefits to be derived from each of the technologies will be assessed, together with their strengths and weaknesses.

Keywords: active, dampers, dynamics, control, ride, suspension, adaptive, semi-active

NOTATION

$\mathbf{A}$	state matrix
$\boldsymbol{b}_1$	control input vector
$\boldsymbol{b}_2$	road input vector
c	damper rate = 1.3 kN s/m
c_a	skyhook damping rate = 3 kN s/m
c_s	body to sprung mass damping rate = 1.3 kN s/m
$\mathbf{C}$	output matrix
$\boldsymbol{d}_1$	control output vector
$\boldsymbol{d}_2$	Road output vector
f_n	natural frequency (Hz)
F_a	actuator force (N)
F_s	skyhook damper force (N)
g	acceleration due to gravity = 9.81 m/s^2
G	road roughness constant (m*cycle)
H_A	acceleration transfer function
H_S	suspension displacement transfer function
H_T	dynamic tyre force transfer function
H_v	complex frequency response of vehicle
J	performance index
k	spring rate = 20 kN/m
k_t	tyre stiffness = 250 kN/m
k_n	state feedback gains
m	unsprung mass = 50 kg
M	quarter car body mass = 400 kg
n	spatial frequency cycle/m
$\mathbf{P}_r$	Riccati matrix
$\mathbf{Q}$	performance weighting matrix
s	Laplace operator
$S(n)$	road power spectrum (m^3/cycle)
$S(\omega)$	road power spectrum, observed by moving car (m^3/rad)
$S_v(\omega)$	vehicle power spectrum (m^3/rad)
u	control input
$\boldsymbol{u}$	control input vector
v_x	vehicle speed (m/s)
w	road power spectrum constant
$\boldsymbol{x}$	state vector
$\boldsymbol{y}$	response vector
z	body vertical displacement (m)
z_{mes}	estimated height of road
z_r	vertical road displacement (m)
z_{rel}	relative damper displacement
z_{sky}	skyhook damper displacement
z_t	unsprung mass vertical displacement
Γ	white noise function
$\mathbf{\Gamma}_r$	control input weighting matrix (body acceleration)
ς	damping ratio
τ	tyre force weighting factor
ω	frequency (rads/s)
ω_c	filter cut-off frequency
Ω	suspension displacement weighting factor

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1 INTRODUCTION

As a precursor to understanding active suspension systems it is worth reflecting on the fundamental requirements which must be met by all suspensions; why provide a suspension at all? If the civil engineers responsible for building highways were capable of constructing perfectly flat roadways there would be little need for the vehicle engineers to provide a suspension, as the primary function of a suspension is to isolate the occupants of a vehicle from the accelerations generated in the vehicle by roadway irregularities. These irregularities arise as a result of construction tolerances, and the degradation which occurs in the roadway due to use, and exposure to the elements.

While the original need to isolate the vehicle from roadway irregularities was to reduce the discomfort experienced by the occupants, there is an equal need to protect the roadway from the impact of the vehicles using it, as this can lead to rapid deterioration and high maintenance costs.

When a suspension is isolating a vehicle from roadway irregularities, it must also allow the vehicle to follow intentional, long wavelength, features in the roadway, where it is following contours in the landscape. Thus a suspension is a low pass filter which enables a vehicle to follow the long wavelength features that are designed into the roadway, while isolating it from the short wavelength features which arise as a result of construction tolerances and wear and tear.

In order to provide maximum isolation in a vehicle it is necessary to allow the wheels to follow the vertical profile of the road while the vehicle body remains at a fixed height in space. In practice the body cannot remain at a fixed height in space as this produces a requirement for infinite suspension travel. Consequently the degree of isolation produced by a suspension is a function of the amount of suspension travel available.

2 AUTOMOTIVE SUSPENSION CONSTRAINTS

The simplest representation of a motor car suspension is a quarter car with a spring and damper connecting the body to a single wheel, which is in turn connected to the ground via the tyre spring, Fig. 1. The mass representing the wheel, tyre, brakes and part of the suspension linkage mass, is referred to as the unsprung mass.

In order to isolate the vehicle from irregularities in the road, the suspension is required to act as a filter. However, while acting as a filter it is also required to carry the static load of the vehicle. This creates a static deflection in the spring which has to be taken into account in the suspension design. It is also required to accommodate changes which occur in the static deflection due to changes in load, unless some form of levelling mechanism is employed.

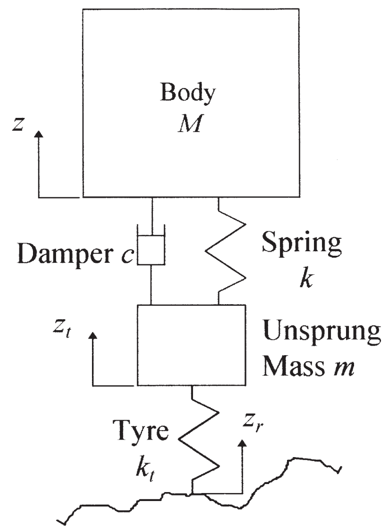

Fig. 1 Quarter car

Further demands are made on the suspension working space by the inertia loads, which have to be reacted by the suspension when the vehicle negotiates bends, or when braking and accelerating.

In order to manoeuvre a motor car, it is necessary for the tyres to generate side forces. The side force is a function of the slip angle (the angle between the direction in which the tyre is travelling and the direction it would be travelling without slip, Fig. 2) and the static load on the tyre. Therefore, on a bumpy road, side force is affected by dynamic changes in tyre load, reaching a limit when the tyre has insufficient contact with the road to generate the side forces commanded by the steering input, at which point the car leaves the road. Therefore any motor car suspension design must also seek to minimize changes in dynamic tyre force.

In summary, any motor car suspension design must seek to minimize acceleration in the body and the dynamic tyre load, while operating within the constraints of the allowable suspension movement.

3 PASSIVE DYNAMIC PERFORMANCE

The accelerations and displacements experienced in a motor car are determined by the vehicle's dynamic response to the road input. Two forms of input are commonly considered when analysing a vehicle's dynamic response:

1. Discrete events, such as steps and pot holes, which are usually investigated in the time domain.
2. Random events, caused by the general texture of the road's surface, which are usually investigated in the frequency domain.

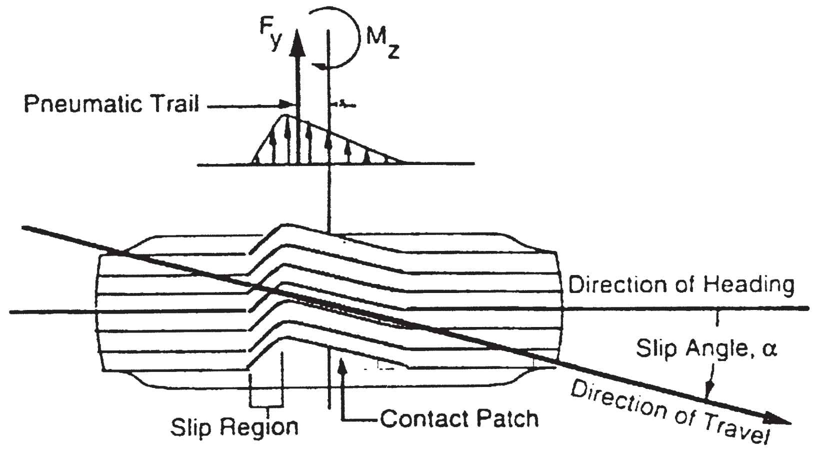

Fig. 2 Rolling tyre side force generation

3.1 Random road profiles

Measured road profiles have been shown to be a random process (**1**) which can be expressed as a power spectrum of the form:

$$S(n) = \frac{G}{n^w} \tag{1}$$

where G is a roughness constant, which is dependent on the quality of the road, and n is the spatial frequency (cycle/m). A range of values has been used for w from $w = 2.0$ to $w = 3.0$, however, the most commonly used value is $w = 2$. As this surface is experienced by a vehicle travelling at a velocity v_x

$$\dot{z}_r = \Gamma \tag{2}$$

where Γ is a white noise function of intensity $2\pi\sqrt{(Gv_x)}$.

In the frequency domain the vehicle will be subjected to an input power spectrum of the form

$$S(\omega) = \frac{4\pi^2 G v_x}{\omega^2} \tag{3}$$

If the vehicle response has a complex transfer function $H_v(\omega)$, the spectral response of the vehicle will be

$$S_v(\omega) = H_v(\omega) S(\omega) H_v^*(\omega) \tag{4}$$

and the mean square response

$$y^2 = \int_0^\infty S_v(\omega)\,d\omega \tag{5}$$

3.2 Quarter car equations of motion

The quarter car model shown in Fig. 1 can be described by a pair of second-order differential equations:

$$M\ddot{z}_b + c(\dot{z} - \dot{z}_t) + k(z - z_t) = 0 \tag{6}$$

$$m\ddot{z}_t + c(\dot{z}_t - \dot{z}) + k(z_t - z) + k_t z_t = k_t z_r \tag{7}$$

In a typical modern motor car, with independent suspension, the sprung mass M is around eight times the unsprung mass m, while the tyre spring stiffness is around 12.5 times the road spring stiffness. With moderate damping rates this leads to two distinct modes of vibration, a body mode around 1.2 Hz and a wheel hop mode around 12 Hz. A fortuitous side effect of these ratios is that the single passive damper imparts similar damping ratios to each of the modes.

3.3 Passive suspension component values

When designing a suspension the two major variables available to the suspension designer are damper rate and spring rate. Fig. 3 shows the effect of varying the damper rate, from the nominal values appropriate to a luxury motor car, when it is driven over a road with a profile given in reference (**1**). At low damping rates there is a large peak in the body acceleration at the body frequency (1.2 Hz), and a smaller one at the wheel hop frequency (12 Hz). Increasing the damping rate reduces the acceleration at body frequencies while increasing it at high frequencies, creating an optimum damping rate for minimum acceleration. The dynamic tyre force and suspension displacement responses also show peaks at body and wheel hop frequencies, both of which reduce as the damping is increased. If the damping rate becomes very high, the body and tyre become locked and both body acceleration and tyre force become large, with a resonant frequency around 4 Hz, as the system has very little damping. Consequently, in order to ensure a good ride, the damping rate has to be a compromise.

Figure 4 shows the trade off which occurs between suspension displacement and body acceleration for a vehicle travelling over a random road of the form given in equation (1). For a constant suspension frequency

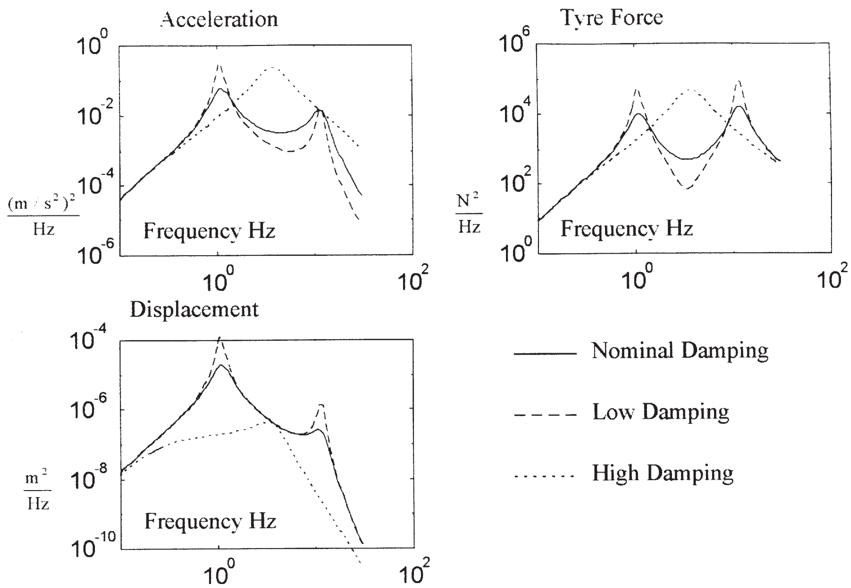

Fig. 3 Power spectral density (PSD) of body acceleration, suspension displacement and dynamic tyre force, for a quarter car with three levels of damping, subjected to a random road input

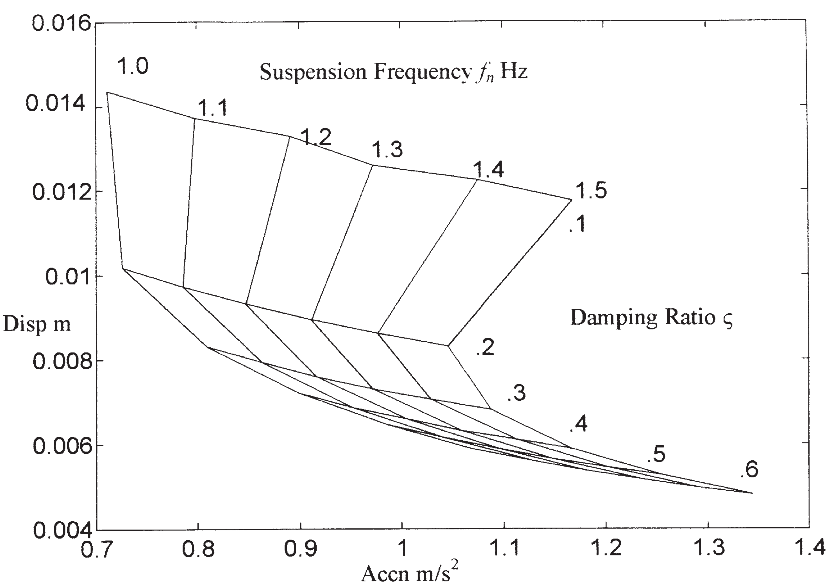

Fig. 4 Variation of r.m.s. body accleration and suspension displacement in a quarter car subjected to a random road input, with suspension frequency and damping ratio

$$f_n = \frac{1}{2\pi}\sqrt{\left(\frac{k}{M}\right)} \tag{8}$$

and the damping ratio

$$\varsigma = \frac{c}{2\sqrt{(kM)}} \tag{9}$$

It will be noted that the acceleration is a minimum for values of $\varsigma = 0.2$.

When the vehicle is subjected to a discrete input, such as encountering a kerb, the lightly damped suspension produces the lowest peak acceleration, Fig. 5, but continues to oscillate much longer after the initial impact. When subjected to a load change, such as occurs during braking and cornering manoeuvres, a high damping rate is beneficial (Fig. 6) as it slows down the response of the suspension and reduces both transient tyre forces and suspension displacements. During a cornering manoeuvre, variations in vertical tyre force will affect the side forces generated by the tyre, producing an undesirable interaction between the vertical and lateral dynamics of the vehicle.

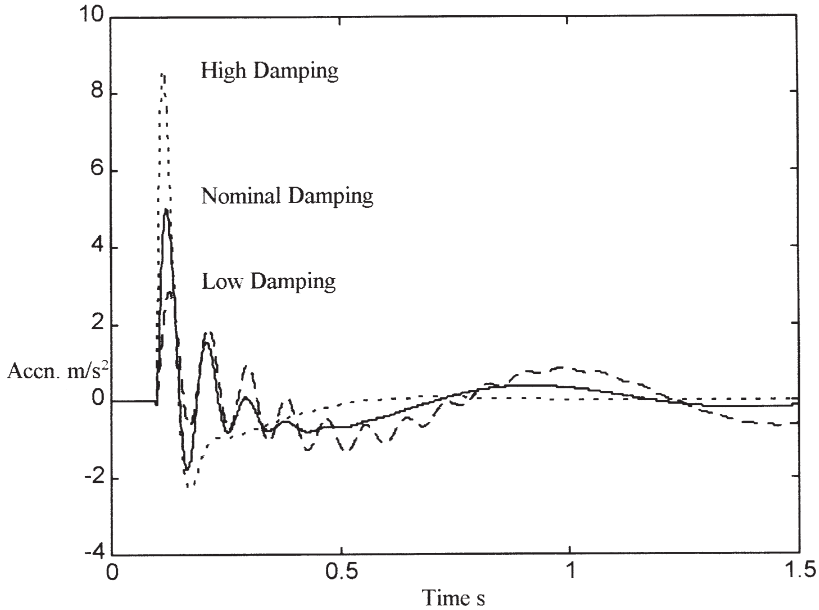

Fig. 5 Body acceleration of a quarter car with three levels of damping, when subjected to a 50 mm step input

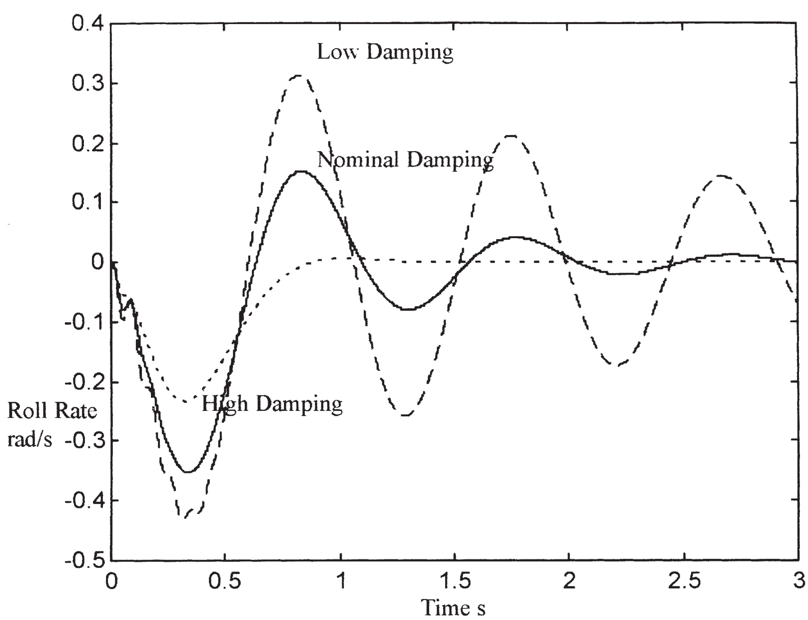

Fig. 6 Roll rate of a vehicle with three levels of damping, subjected to a step steering input

Similarly, with most suspensions, geometrical changes occur when the suspension is displaced which give rise to changes in tyre force. These are usually a combination of small changes in steer angle and camber, which again lead to undesirable interactions between the vertical and lateral motions of the vehicle (**2**).

The springs are required to carry the static load of the vehicle, as well as react the dynamic loads induced by cornering and braking. Therefore, when the spring rate is increased:

(a) the change in ride height caused by changes in load will be reduced; and
(b) dynamic suspension displacements induced while cornering, accelerating and braking will be reduced.

If the damping rate is also increased, to maintain the optimum damping ratio, this will be accompanied by an increase in high-frequency transmission, making the ride less comfortable, and a small reduction in dynamic spring deflection.

Roll angles can be reduced while the vehicle is cornering by fitting an anti-roll bar. When both wheels on an axle with an anti-roll bar are subjected to the same road input, it does not affect the ride. However, when the vehicle is subjected to single wheel inputs the anti-roll bar adds to the local wheel stiffness, and increases the input to the vehicle.

4 INVARIANT POINTS

4.1 Invariant equations

While finite suspension movement places an easily identifiable constraint on suspension performance, whether passive or active, a more subtle constraint is applied by the invariant equation (**3**). Consider a quarter car in which the suspension is a force generator F_a. The equations of motion are:

$$M\ddot{z} = F_a \tag{10}$$

$$m\ddot{z}_t = -F_a + k_t(z_r - z_t) \tag{11}$$

Adding equations (10) and (11) the suspension force is eliminated, to form a dynamic equation which is independent of the suspension force

$$M\ddot{z} + m\ddot{z}_t = k_t(z_r - z_t) \tag{12}$$

This is the invariant equation.

From Section 3 it is seen that the road input can be approximated to a white noise velocity input $\dot{z}_r$. Consequently it is convenient to consider the performance of the system using the following transfer functions:

body acceleration

$$H_A(s) = \frac{\ddot{z}(s)}{\dot{z}_r(s)} \tag{13}$$

suspension displacement

$$H_S(s) = \frac{z(s) - z_t(s)}{\dot{z}_r(s)} \tag{14}$$

tyre deflection

$$H_T(s) = \frac{z_t(s) - z_r(s)}{\dot{z}_r(s)} \tag{15}$$

Using the Laplace transfer function of the invariant equation (12) and substituting the transfer functions in equations (13), (14) and (15), it is possible to derive the following set of equations relating body acceleration, suspension displacement and tyre force:

$$MH_A(s) + (k_t + ms^2)H_T(s) = -ms \tag{16}$$

$$Ms^2 H_S(s) + [k_t + (M + m)s^2]H_T(s) = -(M + m)s \tag{17}$$

$$-s^2(k_t + ms^2)H_S(s) - (k_t + (M + m)s^2 H_A(s) = -k_t s \tag{18}$$

Once one of the transfer functions H_A, H_S or H_T has been specified, the other two are automatically defined by the constraint equations (16), (17) and (18). This indicates that there are restrictions on the trade-off between acceleration, suspension displacement and tyre force which are possible with even the most sophisticated control algorithm.

4.2 Invariant points

Substituting $s = j\omega$ into equations (16), (17) and (18) it is possible to find frequencies at which H_A, H_S and H_T are only dependent on M, m and k_t, and are therefore independent of the suspension. These are

$$\left.\begin{aligned} H_A(\omega) &= \frac{\sqrt{(mk_t)}}{M} \\ \omega_1 &= \sqrt{\left(\frac{k_t}{m}\right)} \end{aligned}\right\} \tag{19}$$

On most passenger cars the body frequency $\omega_b \gg \omega_t$ the wheel hop frequency, so that ω_1 is approximately the wheel hop frequency.

$$\left.\begin{aligned} H_S(\omega_2) &= \frac{M + m}{m}\sqrt{\left(\frac{M + m}{k_t}\right)} \\ \omega_2 &= \sqrt{\left(\frac{k_t}{M + m}\right)} \end{aligned}\right\} \tag{20}$$

The invariant point in the suspension displacement occurs at ω_2 which is the frequency at which the vehicle bounces on the tyre if the suspension becomes locked. The only invariant point in the tyre force transfer function occurs at $\omega = 0$.

The invariant points are shown in Fig. 7, which compares the transfer functions of a passive and active suspension

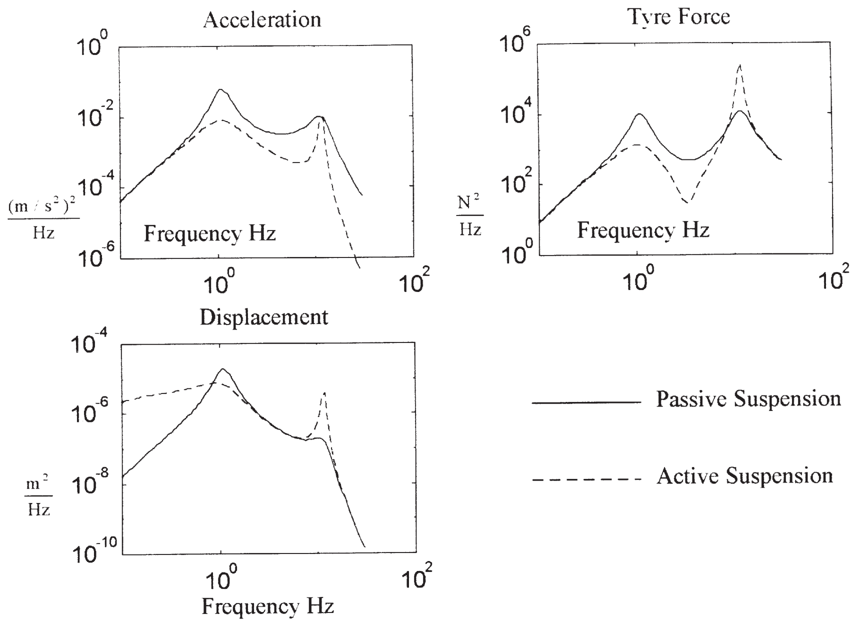

Fig. 7 PSD of body accleration, suspension displacement and dynamic tyre force for a quarter car with passive and active suspensions, subjected to a random road input, showing invariant points

applied to the same base vehicle parameters. They can also be seen in Fig. 3, Section 3.3, where the effects of varying damping rate are described.

5 CLOSED-LOOP CONTROL SYSTEMS

In order to exert real-time control on a suspension it is essential to provide some form of closed-loop control scheme. Before examining closed-loop controllers on discontinuous, semi-active, devices (see Part 2 of this paper) it is easier to consider how they operate on continuous devices.

5.1 Linear quadratic regulator

As a first step to investigating closed-loop controllers, it is necessary to consider the design of a linear quadratic regulator (**3**–**5**) for a quarter car, in which the suspension consists of an actuator capable of exerting an equal and opposite force u on the sprung and unsprung masses, Fig. 8.

The equations of motion of the system are

$$M\ddot{z} = u \tag{21}$$

and

$$m\ddot{z}_t + k_t(z_t - z_r) = -u \tag{22}$$

In a passive suspension

$$u = c(\dot{z}_t - \dot{z}) + k(z_t - z) \tag{23}$$

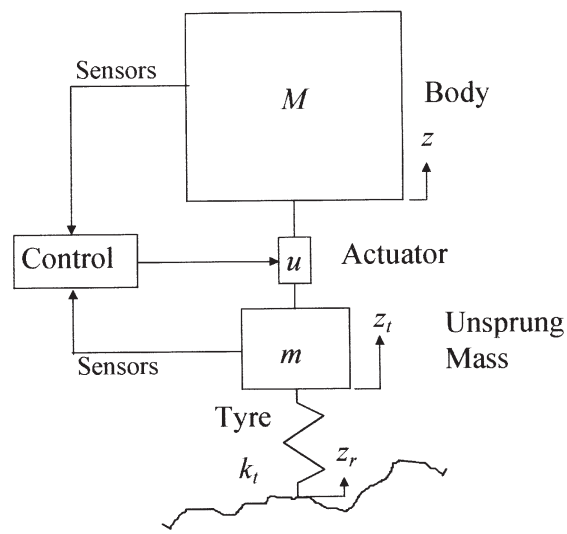

Fig. 8 Active suspension with state feedback

In order to design a linear regulator for the system, the equations have to be arranged in state space form.

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \boldsymbol{b}_1\boldsymbol{u} + \boldsymbol{b}_2 z_r \tag{24}$$

$$\boldsymbol{y} = \mathbf{C}\boldsymbol{x} + \boldsymbol{d}_1\boldsymbol{u} + \boldsymbol{d}_2 z_r \tag{25}$$

The state vector

$$\boldsymbol{x}^T = [\dot{z} \;\; z \;\; \dot{z}_t \;\; z_t] \tag{26}$$

and the output vector

$$\boldsymbol{y}^{\mathrm{T}} = \left[\frac{u}{M} \quad (z_{\mathrm{t}} - z) \quad \frac{k_{\mathrm{t}}(z_{\mathrm{t}} - z_{\mathrm{r}})}{m}\right] \tag{27}$$

gives body acceleration, suspension displacement and dynamic tyre force.

In matrix form the equations become

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\dfrac{k_{\mathrm{t}}}{m} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\boldsymbol{b}_1 = \begin{bmatrix} \dfrac{1}{M} \\ 0 \\ \dfrac{-1}{m} \\ 0 \end{bmatrix}$$

$$\boldsymbol{b}_2 = \begin{bmatrix} 0 \\ 0 \\ \dfrac{k_{\mathrm{t}}}{m} \\ 0 \end{bmatrix} \tag{28}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & \dfrac{k_{\mathrm{t}}}{m} \end{bmatrix}$$

$$\boldsymbol{d}_1 = \begin{bmatrix} \dfrac{1}{m} \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{d}_2 = \begin{bmatrix} 0 \\ 0 \\ \dfrac{-k_{\mathrm{t}}}{m} \end{bmatrix}$$

The linear regulator is designed to minimize a quadratic cost function of the form

$$J = 0.5\int_0^{\infty} (\Gamma_{\mathrm{r}} u^2 + \Omega(z - z_{\mathrm{t}})^2 + \tau(z_{\mathrm{t}} - z_{\mathrm{r}})^2)\,\mathrm{d}t \tag{29}$$

where the weighting factors Γ_{r}, Ω and τ apply to body acceleration, suspension displacement and tyre force respectively.

In order to formulate this into a standard regulator problem it is necessary to redefine the state vector as

$$\boldsymbol{x}^{\mathrm{T}} = [\dot{z} \;\; (z - z_{\mathrm{r}}) \;\; \dot{z}_{\mathrm{t}} \;\; (z_{\mathrm{t}} - z_{\mathrm{r}})] \tag{30}$$

with an initial unit road input

$$\boldsymbol{x}_0^{\mathrm{T}} = [0 \;\; -1 \;\; 0 \;\; -1] \tag{31}$$

Equation (29) can be written in the form in which it is used in the regulator problem

$$J = 0.5\int_0^{\infty} (\Gamma_{\mathrm{r}} u^2 + \boldsymbol{x}^{\mathrm{T}}\mathbf{Q}\boldsymbol{x})\,\mathrm{d}t \tag{32}$$

where

$$\mathbf{Q} = \begin{bmatrix} \Omega + \tau & -\Omega & 0 & 0 \\ -\Omega & \tau & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{33}$$

The optimal control input

$$u = [k_1 \;\; k_2 \;\; k_3 \;\; k_4]\boldsymbol{x} \tag{34}$$

and is computed from

$$\boldsymbol{u} = -\frac{\boldsymbol{b}_1^{\mathrm{T}}\mathbf{P}_{\mathrm{r}}\boldsymbol{x}}{\Gamma_{\mathrm{r}}} \tag{35}$$

where $\mathbf{P}_{\mathrm{r}}$ is the positive definite symmetric matrix obtained as the solution to the algebraic matrix Riccati equation.

$$\mathbf{P}_{\mathrm{r}}\mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{P}_{\mathrm{r}} - \frac{\mathbf{P}_{\mathrm{r}}\boldsymbol{b}_1\boldsymbol{b}_1^{\mathrm{T}}\mathbf{P}_{\mathrm{r}}}{\Gamma_{\mathrm{r}}} + \mathbf{Q} = 0 \tag{36}$$

If the feedbacks are rearranged in the form

$$\begin{aligned} u = {} & (k_1 + k_3)\dot{z} + k_3(\dot{z}_{\mathrm{t}} - \dot{z}) + k_2(z - z_{\mathrm{t}}) \\ & + (k_4 + k_2)(z_{\mathrm{t}} - z_{\mathrm{r}}) \end{aligned} \tag{37}$$

the coefficients can be related to springs and dampers; $(k_1 + k_3)$ provides body damping to an absolute datum, and is a term frequently referred to as skyhook damping; k_3 is a conventional damping term between the body and the unsprung mass; k_2 is the road spring between the body and the unsprung mass; and $(k_4 + k_2)$ modifies the stiffness of the tyre.

In practice it is found that setting $k_4 = -k_2$ has a small effect on the performance of the vehicle. This removes the need to measure the height of the vehicle above the road, the most difficult of the states to measure, and gives rise to the following equations of motion:

$$\ddot{z} = -\frac{\dot{z}c_{\mathrm{a}}}{M} - \frac{(\dot{z} - \dot{z}_{\mathrm{t}})c_{\mathrm{s}}}{M} - \frac{(z - z_{\mathrm{t}})k}{M} \tag{38}$$

$$\ddot{z}_{\mathrm{t}} = \frac{\dot{z}c_{\mathrm{a}}}{m} + \frac{(\dot{z} - \dot{z}_{\mathrm{t}})c_{\mathrm{s}}}{m} + \frac{(z - z_{\mathrm{t}})k}{m} - \frac{(z_{\mathrm{t}} - z_{\mathrm{r}})k_{\mathrm{t}}}{m} \tag{39}$$

5.2 Skyhook damping

Skyhook damping is a widely used concept in the annals of active suspension research (**6–8**). On a simple mass spring and damper model, Fig. 9, the difference between the skyhook damper and a conventional damper is that the damper is not connected to the input. This means that while the transfer function of the conventional, relative damper is given by

$$\frac{z}{z_r} = \frac{cs + k}{ms^2 + cs + k} \tag{40}$$

which at high frequencies tends to

$$\frac{z}{z_r} \approx \frac{c}{Ms} \tag{41}$$

and has an attenuation of 20 db per decade, the skyhook damper has a transfer function

$$\frac{z}{z_r} = \frac{k}{Ms^2 + cs + k} \tag{42}$$

which at high frequencies tends to

$$\frac{z}{z_r} \approx \frac{k}{Ms^2} \tag{43}$$

and has twice the rate of attenuation at 40 db per decade.

As the damper appears in the high-frequency transmission path of the conventional suspension, equation (41), increasing the damper rate, to reduce the resonance peak, will cause an increase in high-frequency transmission. This causes the ride to be a compromise between the amplitude at resonance and high-frequency transmissibility. With the skyhook damper the damping term does not appear in the high-frequency transmission path so that the damping rate can be increased to very high levels without adverse affects on the high-frequency transmission. The active suspension responses shown in Fig. 7 are for a suspension with skyhook damping.

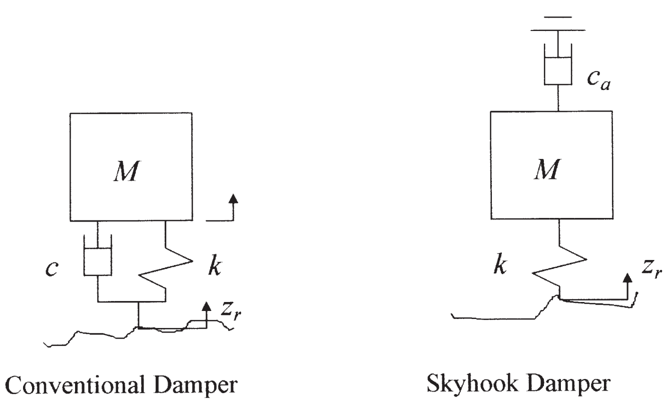

Fig. 9 Conventional and skyhook dampers

While it is possible to achieve good isolation from the road with a skyhook damper, the effects on suspension displacement are not necessarily beneficial. The suspension displacement transfer function of the conventional suspension is

$$\frac{z - z_r}{z_r} = \frac{Ms^2}{Ms^2 + cs + k} \tag{44}$$

It has been shown in Section 3.1 that a typical road profile can be approximated to a white noise on the rate of change on road height, $\dot{z}_r$, giving

$$\frac{z - z_r}{\dot{z}_r} = \frac{Ms}{Ms^2 + cs + k} \tag{45}$$

At low frequencies this becomes

$$\frac{z - z_r}{\dot{z}_r} \approx \frac{Ms}{k} \tag{46}$$

and when $\omega = 0$

$$\frac{z - z_r}{\dot{z}_r} = 0 \tag{47}$$

However, the skyhook damper response to a velocity input is

$$\frac{z - z_r}{\dot{z}_r} = \frac{Ms + c}{Ms^2 + cs + k} \tag{48}$$

so that when $\omega = 0$

$$\frac{z - z_r}{\dot{z}_r} = \frac{c}{k} \tag{49}$$

which implies a suspension offset to zero frequency road inputs. The physical interpretation can be seen by rearranging equation (49) as

$$(z - z_r)k = c\dot{z}_r \tag{50}$$

Thus when the vehicle is running up a slope at velocity $\dot{z}_r$ the damper generates a force $c\dot{z}_r$ which has to be opposed by a spring force $(z - z_r)k$ which involves a suspension offset.

Equation (50) can be rearranged in terms of suspension natural frequency ω_n and damping ratio ς to give the suspension offset

$$(z - z_r) = \frac{2\varsigma\dot{z}_r}{\omega_n} \tag{51}$$

A typical value for ω_n is 6.3 rad/s, and values of around 0.5 for ς in a skyhook damping system are quoted. Applying these figures to a vehicle travelling up or down a 1:20 slope at 20 m/s, it is found that a 160 mm suspension offset is required to counteract the damper force. This is unacceptable for most road vehicles, with typical suspension travel of ±90 mm, and even off-road vehicles with ±150 mm travel would quickly run into difficulties.

It has already been shown that by definition a skyhook damper applies a force to the car which is proportional to the vertical velocity of the body

$$F_s = c_a \dot{z} \tag{52}$$

It is possible to implement such a damper by measuring the vertical velocity of the car body and feeding the resulting signal into a force generator placed between the body and the unsprung mass, Fig. 10. However, low-cost transducers, capable of measuring the skyhook velocity, are not readily available. Schemes have been proposed to overcome this (**5**, **7**) which generate the skyhook velocity of the body by integrating the vertical acceleration. Simple integration is not a practical proposition, as any offset in the accelerometer signal will cause the integrator to drift. The offset could be caused by something as simple as the car travelling along a section of road which is not flat, or thermal drift in the accelerometer. Offset drift can be avoided by carrying out a pseudo-integration with a low pass filter of the form shown in equation (53)

$$\dot{z}_{sky} = \frac{\ddot{z}}{s + \omega_c} \tag{53}$$

Provided the cut-off frequency of the filter $\omega_c \ll \omega_n$ the suspension frequency, the approximation to a skyhook

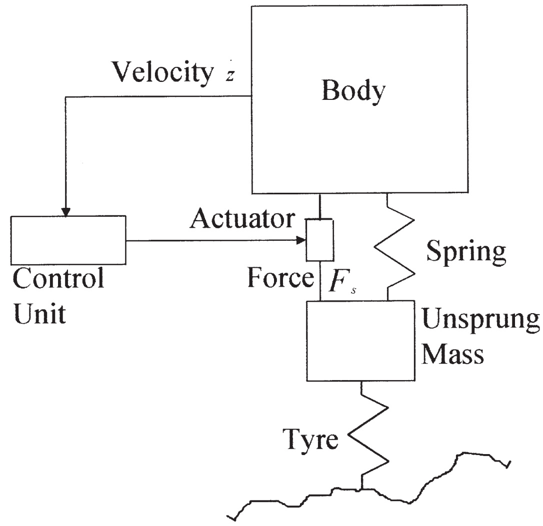

Fig. 10 Skyhook damper implementation

damper is very good. It also has a beneficial effect on the ramp response as when

$$\ddot{z} = 0 \qquad \dot{z}_{sky} = 0$$

Figure 11 shows the ramp response of a vehicle with a skyhook damper generated by the pseudo-integrator in equation (53). It is apparent that increasing $\omega_c \to \omega_n$ causes the offset to be reduced more quickly, at the expense of the transient performance. This is hardly surprising as the transfer function of the system with the pseudo-integrator is

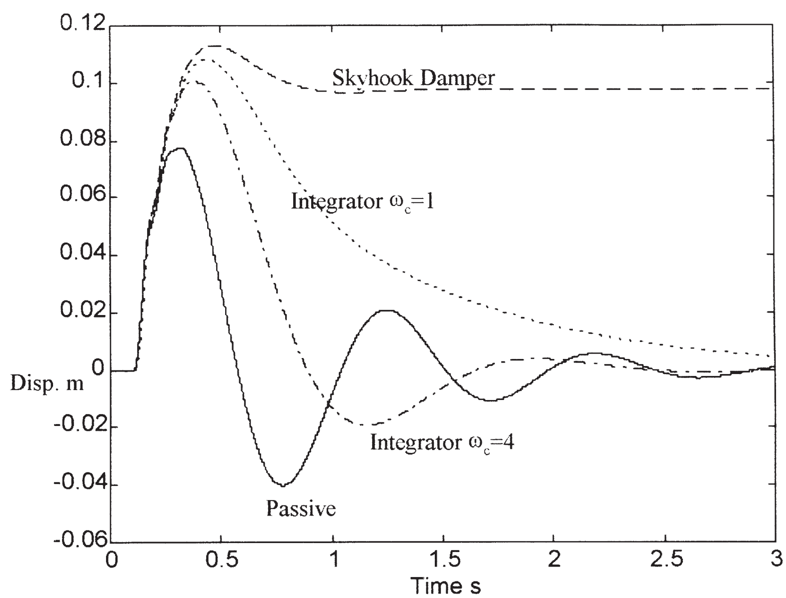

Fig. 11 Suspension displacement of a quarter car negotiating a ramp. Vertical velocity 0.1 m/s with passive suspension, skyhook damper and skyhook damper with pseudo-integrators set at 1 rad/s and 4 rad/s

$$\frac{z(s)}{z_r(s)} = \frac{k}{Ms^2 + cs^2/(s+\omega_c) + k} \tag{54}$$

at the resonant frequency $\omega_n = \sqrt{(k/M)}$, the response of the perfect skyhook damper is

$$\frac{z(\omega_n)}{z_r(\omega_n)} = -\frac{jk}{c\omega_n} \tag{55}$$

a perfectly damped response with the force leading the input by 90°. The amplitude response is

$$\left|\frac{z(\omega_n)}{z_r(\omega_n)}\right| = \frac{-k}{c\omega_n} \tag{56}$$

while the pseudo-skyhook damper has a response

$$\frac{z(\omega_n)}{z_r(\omega_n)} = \frac{k(\omega_c - j\omega_n)}{c\omega_n^2} \tag{57}$$

in which the phase lead decreases as $\omega_c \to \omega_n$ reducing the effective damping. The amplitude response is

$$\left|\frac{z(\omega_n)}{z_r(\omega_n)}\right| = \frac{k\sqrt{(\omega_c^2 + \omega_n^2)}}{c\omega_n^2} \tag{58}$$

which becomes larger as ω_c^2 increases.

One method which has been proposed to allow the suspension offset to return to zero quickly on the ramp transient, without the attendant increase in the resonance peak, is to include a low-frequency conventional damping component in the controller (**8**);

$$\dot{z}_{rel} = \frac{\dot{z} - \dot{z}_r}{s + \omega_c} \tag{59}$$

Combining equations (53) and (59)

$$\dot{z}_{mes} = \dot{z}_{sky} + \dot{z}_{rel} \tag{60}$$

$$\dot{z}_{mes} = \frac{zs^2}{s+\omega_c} + \frac{(z - z_r)s}{s+\omega_c} = zs - \frac{z_r s}{s+\omega_c} \tag{61}$$

As the relative damping component is applied at low frequencies the impact on high-frequency transmission is small, while Fig. 12 shows it to have quite a significant effect on the transient response. From equation (61) it is apparent that the relative velocity term

$$\frac{(z - z_r)s}{s+\omega_c}$$

can be derived by high pass filtering the suspension displacement. This is probably the most convenient way to derive the signal as active suspensions invariably measure the displacement across the suspension.

While the single pole filter in equation (53) successfully removes the offset generated by a skyhook damper on a ramp, a direct current (d.c.) signal on the accelerometer will still create an offset. Direct current acceleration signals can be generated in a number of ways such as the accelerometer being inclined at an angle to the vertical, thermal drift in the accelerometer, or the vehicle being driven on a banked, or even a heavily cambered, road. The offsets generated by these d.c. signals can be overcome by making the pseudo-integrator into a band-pass filter

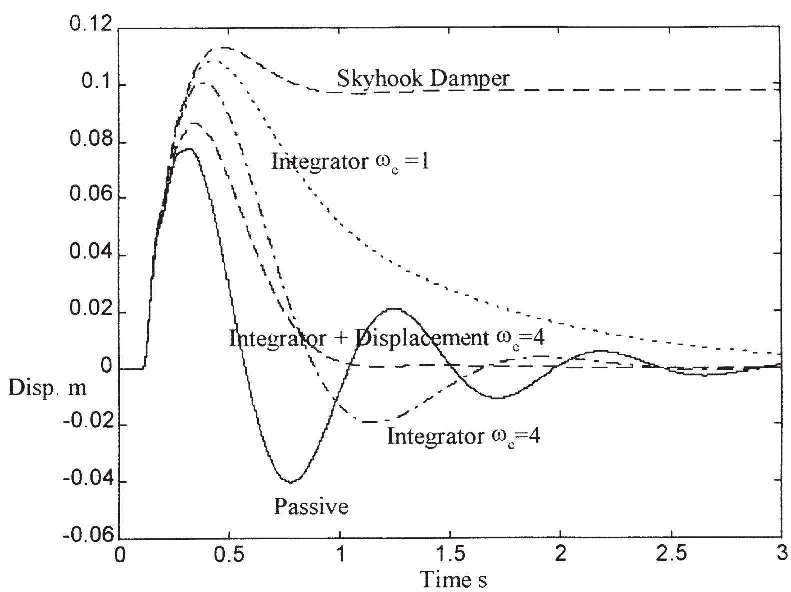

Fig. 12 Suspension displacement of a quarter care negotiating a ramp. Vertical velocity 0.1 m/s with passive suspension, skyhook damper, skyhook damper with pseudo-integrators and skyhook damper with pseudo-integrator plus displacement

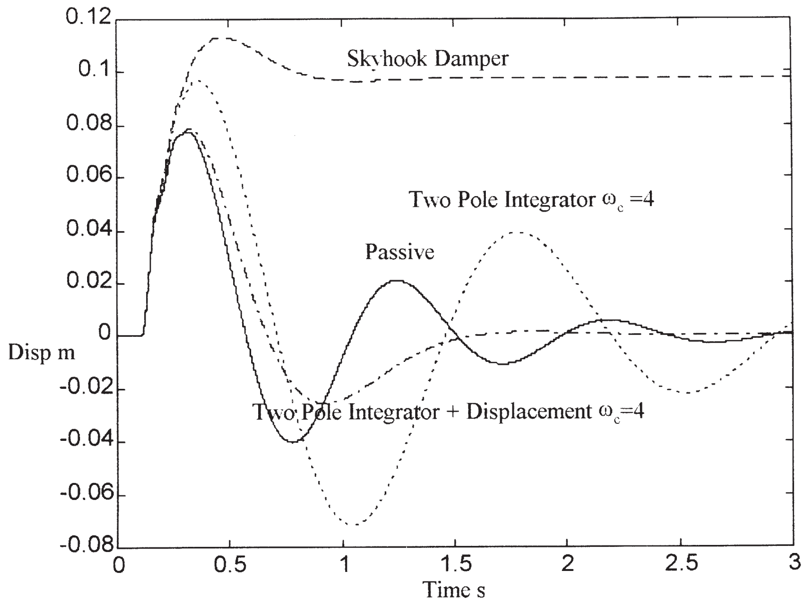

Fig. 13 Suspension displacement of a quarter car negotiating a ramp. Vertical velocity 0.1 m/s with passive suspension, skyhook damper, skyhook damper with two pole pseudo-integrator and two pole pseudo-integrator plus displacement

$$\dot{z}_{\text{sky}} = \frac{\ddot{z}s}{s^2 + 2\varsigma\omega_c s + \omega^2} \tag{62}$$

Applying the filter in equation (62) on its own has a transient response which is even less satisfactory than equation (53) on its own, Fig. 13. The response, however, can be improved by including

$$\dot{z}_{\text{rel}} = (z - z_r)\frac{2\varsigma\omega_c s^2 + \omega_c^2 s}{s^2 + 2\varsigma\omega_c s + \omega_c^2} \tag{63}$$

Figure 13 shows this to be as effective as the equivalent filter applied to the single pole example.

6 CONCLUSIONS

Suspension design is a compromise between ride and handling; by including an active element in the suspension it is possible to reach a better compromise than is possible using purely passive elements. Part 1 of the paper has investigated the theoretical considerations which need to be taken into account when designing an active suspension. The second part of the paper considers the practical considerations.

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