Sheet #1

Linear Algebra
MATH 202.

مقدار واتجاه

Vector $(\bar{A})$
Magnitude and direction

Ex
- Displacement
- velocity
- Force
- acceleration.

مقدار

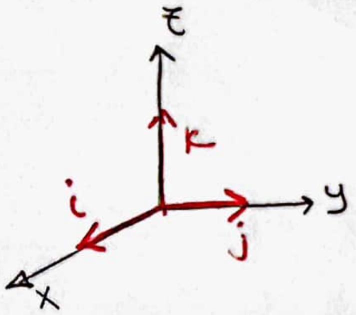
Scalar. $A = |\bar{A}|$
Magnitude only

Ex
- Time
- Temperature
- work
- Mass كتلة
-

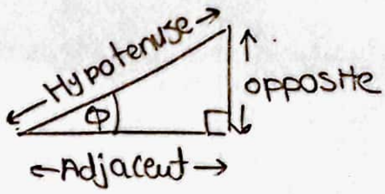
* vector:

$$\bar{A}(a, b, c) = ai + bj + ck = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

→ graphically:



vector → Magnitude $\bar{A} = a\underline{i} + b\underline{j}$ $|\bar{A}| = \sqrt{a^2 + b^2}$
vector → angle.



- unit vector

→ $\bar{A} = \hat{A} = \dfrac{A}{\|\bar{A}\|}$ → to get another vector in the same direction

→ for the opposite direction $\bar{A} = \hat{A} = -\left[\dfrac{A}{\|\bar{A}\|}\right]$

Sheet #1:

1. State which of the following are scalars and which are vectors

a) Specific heat $\longrightarrow$ scalar

b) momentum $\longrightarrow$ vector (mass x Velocity).
قوة دافعة

c) Distance. $\longrightarrow$ Scalar (distance vs. displacement).

d) Speed. $\longrightarrow$ Scalar.

e) Magnetic field inseusity $\longrightarrow$ scalar. شدة المجال المغناطيسي

f) Kinetic Energy. $\longrightarrow$ scalar.
طاقة حركية.

2. An automobile travels 3 miles due north, then 5 miles north east

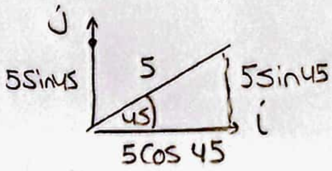
vector

$\rightarrow$ Represent these displacements graphically

$\rightarrow$ Determine these resultant displacement

a) graphically.

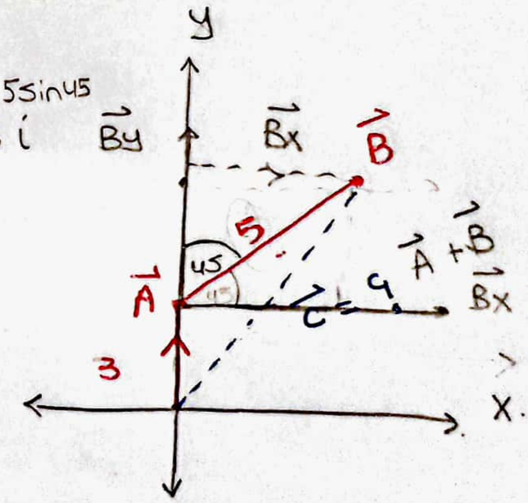
b) Analytically.



$\vec{A} = 3j$

$\vec{B} = \vec{Bx} + \vec{By}$

$= 5\sin 45\, \underline{i} + 5\cos 45\, \underline{j}$



Resultant $\boxed{\bar{C} = \bar{A} + \bar{B}}$

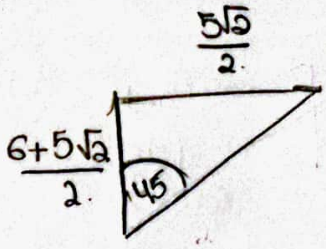
$= (3\underline{j}) + 5\sin 45^\circ \underline{i} + 5\cos 45\, \underline{j}$

$= \underbrace{\frac{5\sqrt{2}}{2}}_{a}\, \underline{i} + \underbrace{\frac{6+5\sqrt{2}}{2}}_{b}\, \underline{j}$ $\rightarrow$ Magnitude, Direction

→ for Magnitude (Modulus):

$$\boxed{|\vec{C}| = \sqrt{a^2+b^2}} = \sqrt{\left(\frac{5\sqrt{2}}{2}\right)^2 + \left(\frac{6+5\sqrt{2}}{2}\right)^2} = 7.43$$

→ for direction (Angle):



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{المقابل}}{\text{المجاور}}$$

$$\tan\theta = \frac{\frac{5\sqrt{2}}{2}}{\frac{6+5\sqrt{2}}{2}} = 0.54$$

to get angle ∴ $\tan^{-1}$ for both sides.

$$\tan^{-1}(\tan\theta) = \tan^{-1}(0.54)$$

$$\hookrightarrow \theta = \tan^{-1}(0.54) = 28.41^\circ$$

$$= \underbrace{28^\circ}_{\text{degrees}}\ \underbrace{24'}_{\text{minutes}}\ \underbrace{44''}_{\text{seconds}}$$

to determine latitude and longitude

لتحديد خطوط الطول ودوائر العرض.

$\boxed{3}$ let $A = \langle 1, 3, -2, 6 \rangle$

$B = \langle 4, -3, 3, 1 \rangle$

$C = \langle 2, 1, 5, 0 \rangle$

be three vectors in $\mathbb{R}^4$ → four dimensional (has 4 points).

Evaluate the following vectors

a) $\bar{A}+\bar{B} = \langle 1, 3, -2, 6 \rangle + \langle 4, -3, 3, 1 \rangle$

$= \langle 5, 0, 1, 7 \rangle$

b) $\bar{A}+\bar{B}-\bar{C} = \langle 5, 0, 1, 7 \rangle - \langle 2, 1, 5, 0 \rangle$

$= \langle 3, -1, -4, 7 \rangle$

c) $7\bar{A} - 2\bar{B} - 3\bar{C}$

$7\bar{A} = 7\langle 1, 3, -2, 6\rangle$

$= \boxed{\langle 7, 21, -14, 42\rangle}$

$2\bar{B} = 2\langle 4, -3, 3, 1\rangle$

$= \boxed{\langle 8, -6, 6, 2\rangle}$

$3\bar{C} = 3\langle 2, 1, 5, 0\rangle = \boxed{\langle 6, 3, 15, 0\rangle}$

$7\bar{A} - 2\bar{B} - 3\bar{C} = \langle 7, 21, -14, 42\rangle - \langle 8, -6, 6, 2\rangle - \langle 6, 3, 15, 0\rangle$

$= \langle -7, 24, -35, 40\rangle$

d) $2\bar{A} + \bar{B} - 3\bar{C}$

$2\bar{A} = 2\langle 1, 3, -2, 6\rangle = \boxed{\langle 2, 6, -4, 12\rangle}$

$= \langle 2, 6, -4, 12\rangle + \langle 4, -3, 3, 1\rangle - \langle 6, 3, 15, 0\rangle$

$= \langle 0, 0, -16, 13\rangle \longrightarrow$ #

4. Find a vector $\bar{B}$ of length 8 which in the opposite direction of the vector $\bar{A} = \langle 3, 2, -6\rangle$.

* Solu:

to get a vector $\bar{B}$ in the opposite direction of A

$$\hat{B} = -\hat{A} = -\left[\frac{\bar{A}}{\|\bar{A}\|}\right]$$

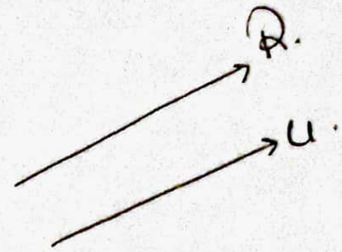
$$\|\bar{A}\| = \sqrt{3^2 + 2^2 + (-6)^2} = 7$$

$$\therefore \hat{B} = -\frac{\langle 3, 2, -6\rangle}{7} = \langle \frac{-3}{7}, \frac{-2}{7}, \frac{6}{7}\rangle$$

for length 8 $\therefore$ $\vec{B} = 8.\langle \frac{-3}{7}, \frac{-2}{7}, \frac{6}{7}\rangle = \langle \frac{-24}{7}, \frac{-16}{7}, \frac{48}{7}\rangle$

5) Find a unit vector $\underline{\underline{u}}$ parallel to the resultant $\underline{\underline{R}}$ of vectors $r_1 = 2\underline{i} + 4\underline{j} - 5\underline{k}$

$r_2 = -i - 2j + 3k$.



$\therefore R \parallel u$

$\therefore R = u$

*solu.

$\bar{R} = \bar{r}_1 + r_2$

$= 2i + 4j - 5k + (-i - 2j + 3k)$.

$= i + 2j - 2k = \langle 1, 2, -2 \rangle$

unit vector $\hat{R} = \dfrac{\bar{R}}{\|\bar{R}\|} = \dfrac{\langle 1, 2, -2 \rangle}{3} = \langle \frac{1}{3}, \frac{2}{3}, \frac{-2}{3} \rangle$.

$\|\bar{R}\| \to \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{9} = 3$

$\to$ Since R parallel to u

therefore $\hat{R} = \hat{U} = \langle \frac{1}{3}, \frac{2}{3}, \frac{-2}{3} \rangle$.

6) Given the radius vector

$r_1 = 3\underline{i} + 2\underline{j} - \underline{k}$

$r_2 = 3\underline{i} - 4j + 9k$.

$r_3 = -i + 2j - 2k$

Find the magnitudes of

a) r_3

b) $r_1 + r_2 + r_3$

c) $r_1 - r_2 + 4r_3$.

*solu.

a) $r_3 = -i + 2j - 2k = \langle -1, 2, -2 \rangle$

$\|\bar{r}_3\| = \sqrt{(-1)^2 + (2)^2 + (-2)^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = \underline{\underline{3}}$.

b) $\bar{r}_1 + \bar{r}_2 + \bar{r}_3 = 3i + 2j - 1k$
$\qquad + 3i - 4j + 9k$
$\qquad + (-i + 2j - 2k)$

$= 5i + 0j + 6k = \langle 5, 0, 6 \rangle$.

$\| \bar{r}_1 + \bar{r}_2 + \bar{r}_3 \| = \sqrt{25 + 0 + 36} = \sqrt{61}$.

c) $\bar{r}_1 - \bar{r}_2 + 4\bar{r}_3 = \langle 3, 2, -1 \rangle - \langle 3, -4, 9 \rangle + 4\langle -1, 2, -2 \rangle$.

$= \langle 3, 2, -1 \rangle - \langle 3, -4, 9 \rangle + \langle -4, 8, -8 \rangle$

$= \langle -4, 14, -18 \rangle$

$\| \bar{r}_1 - \bar{r}_2 + 4\bar{r}_3 \| = \sqrt{(-4)^2 + (14)^2 + (-18)^2} = 2\sqrt{134} = \underline{23.15}$

7) Find a unit vector u parallel to the resultant R. of vectors

$a = \langle 2, -4, 5 \rangle$
$b = \langle 1, 2, 3 \rangle$.

$\bar{R} = a + b = \langle 2, -4, 5 \rangle + \langle 1, 2, 3 \rangle = \langle 3, -2, 8 \rangle$

$\hat{R} = \dfrac{\bar{R}}{\|\bar{R}\|} = \dfrac{\langle 3, -2, 8 \rangle}{\sqrt{9 + 4 + 64}} = \left\langle \dfrac{3}{\sqrt{77}}, \dfrac{-2}{\sqrt{77}}, \dfrac{8}{\sqrt{77}} \right\rangle$

$\sqrt{9+4+64} \rightarrow \sqrt{77}$