Conditional Prob & Bayes Theorem

⇒ Whatsapp Autocomplete

How are
- thing?
- you?
- the?

⇒ Last 30 days

How are	you	27
How are	thing	21
How are	the	10

→ 1k

I am in the city 100

Where are we going 150

10 k Msg

⇒ Conditional Prob →

⇒ Roll 2 dices

D2 ↓ \ D1 →	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Q Prob that Sum = 2

$\frac{1}{36}$

Q Prob that value of Dice1 = 2

$\frac{6}{36}$

Q Prob that Sum is ≤ 5

$= \frac{10}{36}$

Q Probability that value is Dice1 $= 2$ and Sum of D_1 & $D_2 \leq 5$.

$$P(D_1 = 2 \cap D_1 + D_2 \leq 5) = \frac{3}{36}$$

Q Probability that $D_1 + D_2 \leq 5$ given that $D_1 = 2$

$$P(D_1 + D_2 \leq 5 \mid D_1 = 2) = \frac{3}{6}$$

Q Probability that $D_1 = 2$ given that $D_1 + D_2 \leq 5$

$\Rightarrow \frac{3}{10}$

$\Rightarrow P(D_1 + D_2 \leq 5) = \frac{10}{36}$

$P(D_1 = 2) = \frac{6}{36}$

$P(D_1 = 2 \cap D_1 + D_2 \leq 5) = \frac{3}{36}$

$P(D_1 + D_2 \leq 5 \mid D_1 = 2) = \frac{3}{6}$

$= \frac{\text{Possibility where } D_1 = 2 \text{ and } D_1 + D_2 \leq 5}{\text{Possibility with } D_1 = 2}$

$$P(D_1 + D_2 \leq 5 \mid D_1 = 2) = \frac{P(D_1 + D_2 \leq 5 \cap D_1 = 2)}{P(D_1 = 2)}$$

⇒ Event A ⇒ $D_1 + D_2 \leq 5$

Event B ⇒ $D_1 = 2$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

← Conditional Prob Formula

$$P(A \cap B) = P(A|B) \cdot P(B)$$

Multiplication Rule

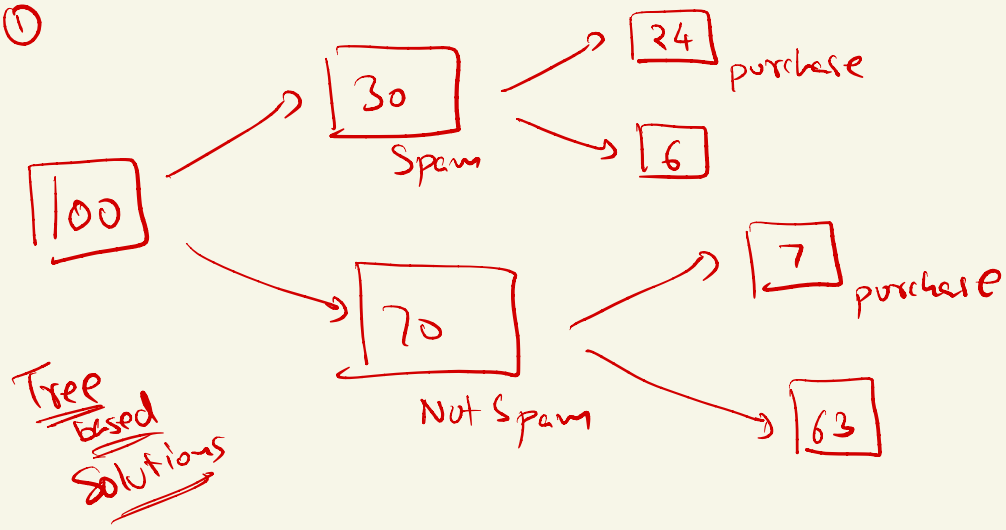
$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$P(A)$ ← Marginal Probability { Prob based on total outcomes)

$P(B)$ ← Marginal Probability

$P(A \cap B)$ ← Joint Probability

It is known that-
30% of emails are spam, and 70% are not spam.
The word “purchase” occurs in 80% of spam emails.
It also occurs in 10% of non-spam emails.
Overall, in what percentage of emails would we see the word “purchase”?

① 

Tree based solutions

$24+7=31$

$\frac{31}{100}$ 31%

② $P(\text{Spam}) = 0.3$ $P(\text{Not Spam}) = 0.7$

$\Rightarrow P(\text{Purch} \mid \text{Spam}) = 0.8$

$P(\text{Purch} \mid \text{Not Spam}) = 0.1$

$P(\text{Purchase}) = ??$

$$P(\text{Purch} \mid \text{Spam}) = \frac{P(\text{Purch} \cap \text{Spam})}{P(\text{Spam})} = 0.8$$

$$\Rightarrow P(\text{Purch} \cap \text{Spam}) = (0.8)(0.3) = 0.24$$

$$P(P \mid \text{Not Spam}) = \frac{P(\text{Purc} \cap \text{Not Spam})}{P(\text{Not Spam})} = 0.1 \Rightarrow$$

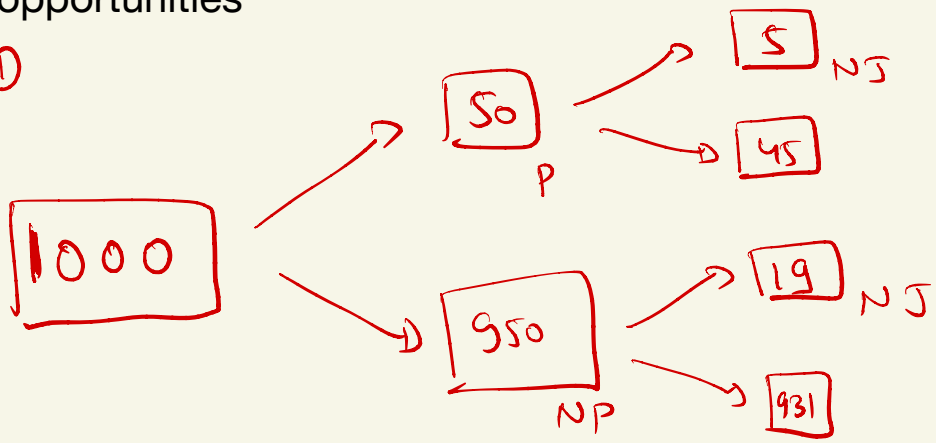
$\Rightarrow P(\text{Purch} \cap \text{Not Spam}) = (0.1)(0.7) = 0.07$

$\Rightarrow P(\text{Purch}) = P(\text{Purch} \cap \text{Not Spam}) + P(\text{Purch} \cap \text{Spam})$

Total emails with purchas = Not spam emails with purch + Spam with purchase

$P(P) = 0.07 + 0.24 = 0.31$

It is known that -
5% of all LinkedIn users are premium users
10% of premium users are actively seeking new job opportunities.
Only 2% of non-premium users are actively seeking new job opportunities.
Overall, what percentage of people are actively seeking new job opportunities

① 

(5+19)
24 out of 1000
$\Rightarrow$ 2.4%

② $P(\text{Prem}) = 0.05$ $P(\text{Non Prem}) = 0.95$

$P(\text{NJ} \mid \text{Prem}) = 0.1$ $P(\text{NJ} \mid \text{Non Prem}) = 0.02$

$P(\text{NJ}) = ??$

$$P(NJ \mid Prem) = \frac{P(NJ \cap Prem)}{P(Prem)} \Rightarrow P(NJ \cap Prem) = 0.1 \times 0.05$$

$$= 0.005$$

$$P(NJ \mid NonPrem) = \frac{P(NJ \cap NonPrem)}{P(NonPrem)} \Rightarrow P(NJ \cap NonPrem) = 0.02 * 0.95$$

$$= 0.019$$

$$\boxed{P(NJ) = P(NJ \cap Prem) + P(NJ \cap Non\ Prem)}$$

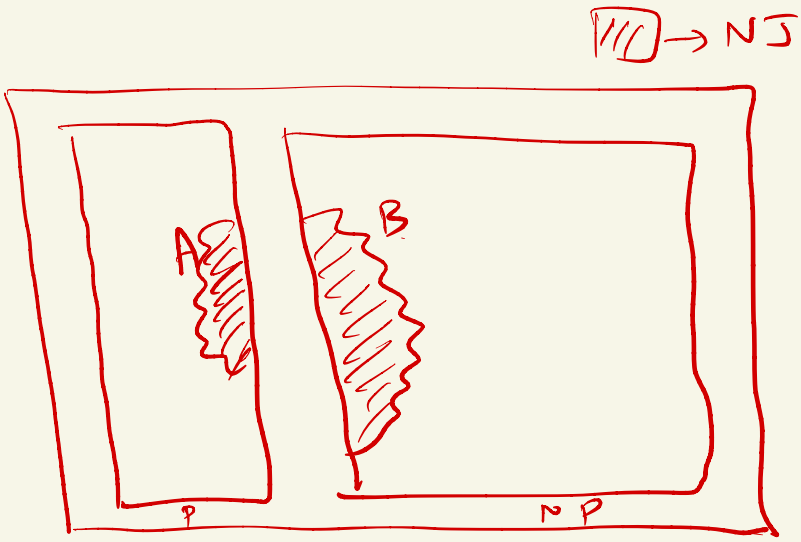
$$= 0.005 + 0.019$$

$$= 0.024$$

Law of Total Probability

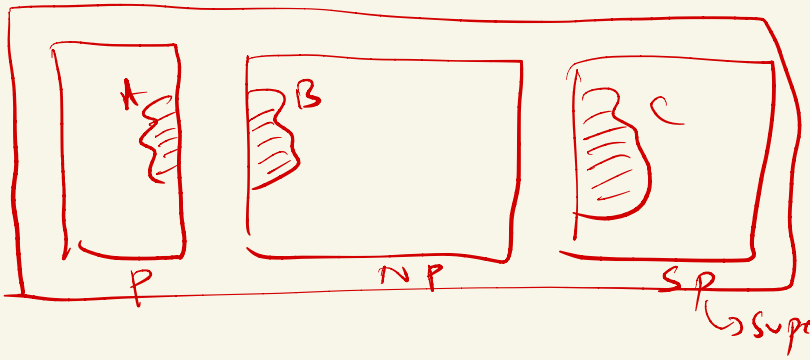
$$\boxed{P(C) = P(C \cap A) + P(C \cap B)}$$

$$\boxed{P(C) = P(C|A)\,P(A) + P(C|B)\,P(B)}$$



Total NJ = A + B

NJ = NJ ∩ P + NJ ∩ NP



NJ = A + B + C

⇒ Conditional Prob ⇒ $P[A|B] = \frac{P(A \cap B)}{P(B)}$

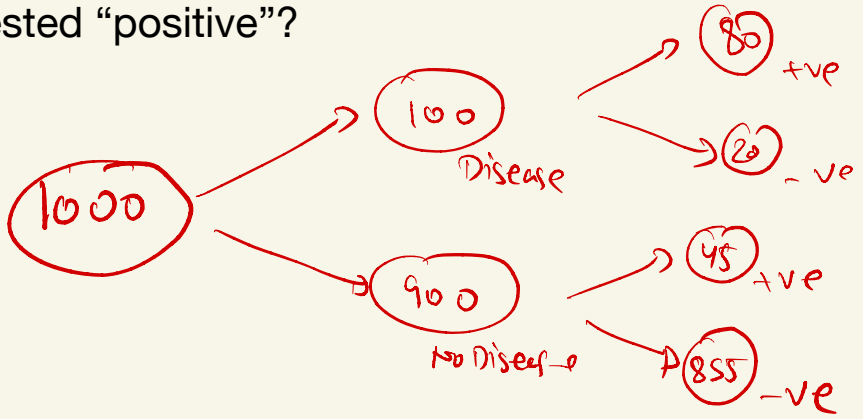
Multiplication Rule ⇒ $P(A \cap B) = P(A|B) \times P(B)$

Law of Total Probability ⇒ $P(C) = P(C|A)\,P(A) + P(C|B)\,P(B)$

$= P(C \cap A) + P(C \cap B)$

$\Rightarrow P(C) = P(C|A_1)\,P(A_1) + P(C|A_2)\,P(A_2) + \cdots$

A disease affects 10% of the population. Among those who have the disease, 80% get “positive” test result Among those who don’t have the disease, 5% get “positive” test result. Overall, what percentage of people tested “positive”?



Tootal +ve = 80 + 45 = 125

What is P(+ve | Disease) ?

0.8

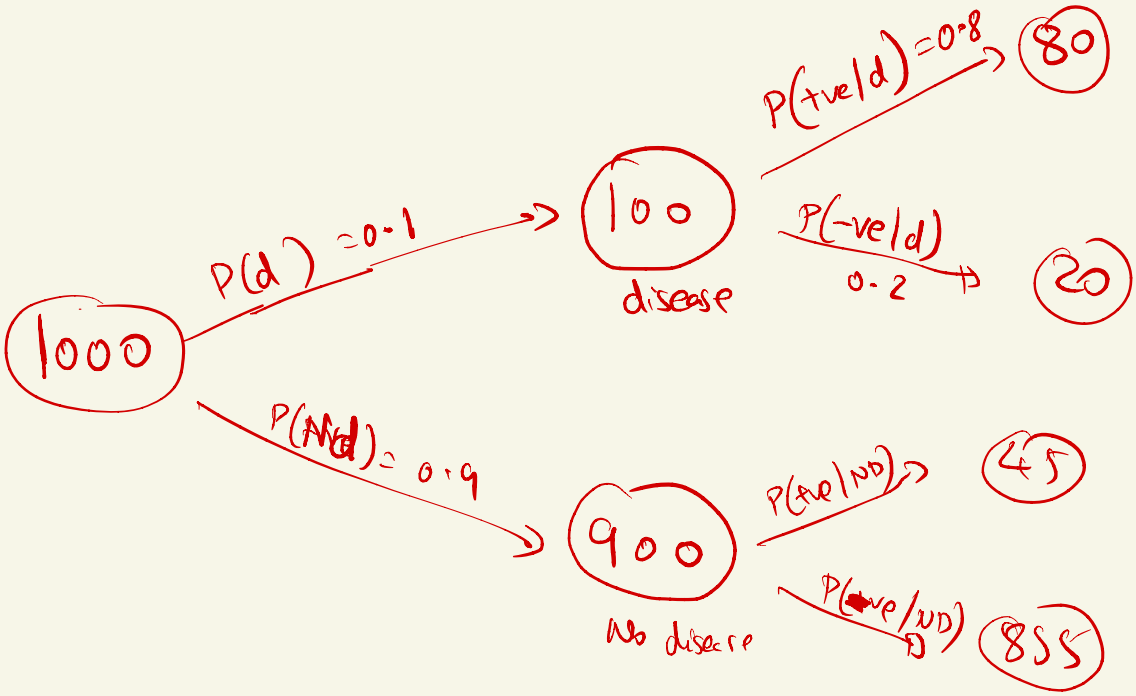
What is P(+ve ∩ Disease)?

⇒ Out of overall 1000 people if you choose a random person what are chances that he/she will have disease and a +ve result

0.08

What is P(+ve ∩ No Disease)?

$\frac{45}{1000} = 0.045$



Q $P(+ve \mid disease) = 0.8$

- What is the $P(disease \mid +ve)$ ←
- Given a person is test +ve what are chances that they have disease ?? ←

$$P(+ve) = \frac{80 + 45}{1000}$$

$$P(disease \mid +ve) = \frac{80}{80+45} = \frac{80}{125} \approx 64\%$$

$$P(disease \mid +ve) = \frac{P(disease \wedge +ve)}{P(+ve)}$$

Bayes theorem

$$P(disease \mid +ve) = \frac{P(+ve \mid disease) * P(disease)}{P(+ve)}$$

$$P(+ve) \Rightarrow \frac{80 + 45}{1000}$$

$$\Rightarrow P(+ve \wedge Dise) + P(+ve \wedge ND)$$

$\Rightarrow P(A \cap B) = P(B \cap A)$

$P(A|B)\,P(B) = P(B|A)\,P(A)$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Bayes Theorem

$\Rightarrow$ Prior, Posterior, Likelihood, marginal

$$\underbrace{P(A|B)}_{\text{Posterior}} = \underbrace{P(A)}_{\text{Prior}} \times \frac{\overbrace{P(B|A)}^{\text{likelihood}}}{\underbrace{P(B)}_{\text{marginal}}}$$

likelihood → prob of evidence given the belief is true

Posterior → Probability after evidence is considered

Prior → Probability before evidence is considered

marginal → Probability of evidence under any circumstance

Posterior = Prior × likelihood over Marginal probability

$$P(\text{Hypothesis}|\text{Evidence}) = P(\text{Hypotheses}) \times \frac{P(\text{Evidence}|\text{Hypo})}{P(\text{Evidence})}$$

Doubts

⇒ Linked Premium vs Non Premium ⇒ Job applicants

⇒ Event A ⇒ Somebody looking for Job

Event B ⇒ Premium user

B^c ⇒ Non Premium user

Total JS = JS who are premium + JS who are no premium

$$P(A) = P(A \cap B) + P(A \cap B')$$

⇒ Assignment ⇒ Role 2 dices ⇒ 36 possibilities

Case where Sum ≤ 5 ⇒ 10

Case where Sum ≥ 3 ⇒ 2

Google Collab Link : https://colab.research.google.com/drive/1L-rOMcOcrzWD34iLtkV8TPly_MovwOGE?usp=sharing#scrollTo=-C4b2WoSGy3G