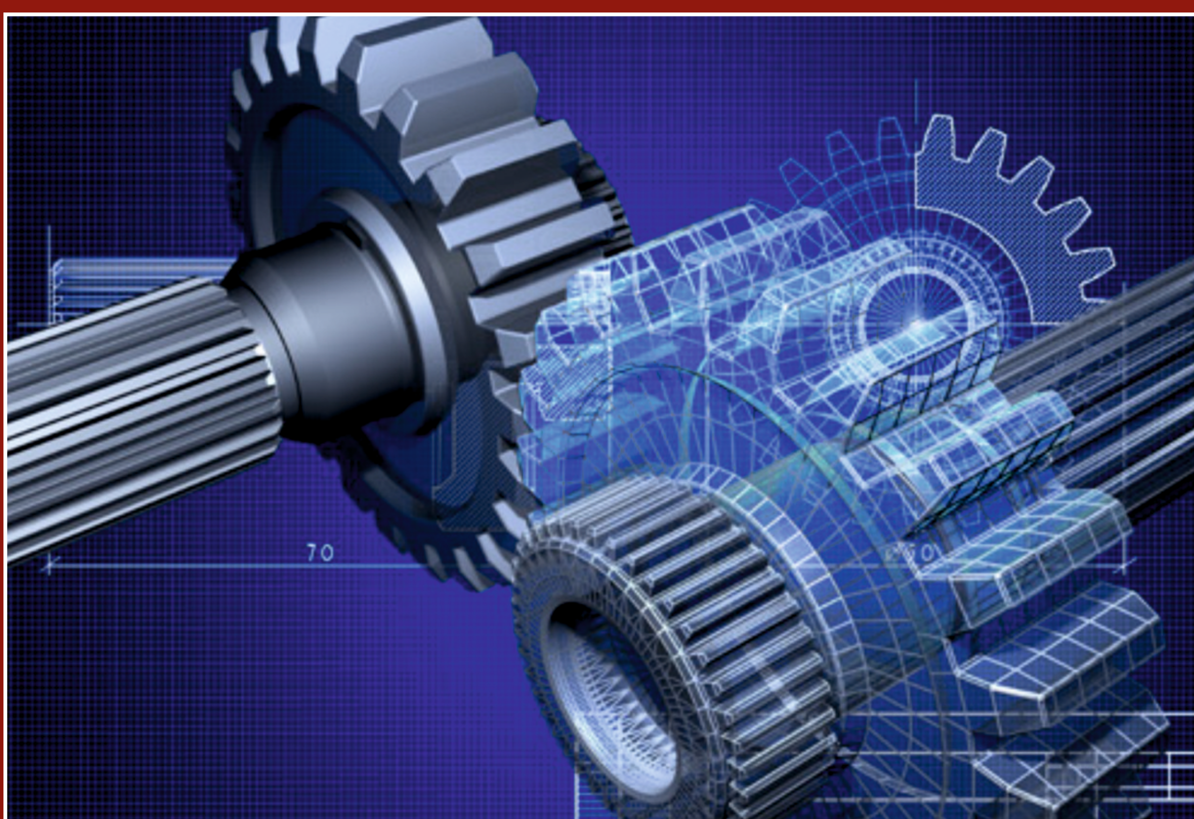
Problems in PHYSICS for IIT-JEE ADVANCED

Fully Solved

Volume I

Mechanics | Waves



McGraw Hill Education

Shashi Bhushan Tiwari

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About The Author

Shashi Bhushan Tiwari is a distinguished academician and Physics guru. He graduated from IIT Kharagpur in year 1995 and has been mentoring students for IIT JEE for more than two decades.

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VOLUME I

Shashi Bhushan Tiwari

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Dedicated

to my beloved wife

Mrs. Kanti Tiwari

PREFACE

In the past decade and a half, the entrance exam for IITs has seen many changes – in structure as well as in design of the question paper. No doubt, it has become more challenging. It requires high level of conceptual clarity and analytical skill, besides promptness and comprehension ability to excel in this exam. There are frequent surprises in terms of problems which require mathematical rigor or in depth understanding of physical conditions.

This book is being presented with a very simple objective – it will test you and nurture you on all parameters which are required to excel in JEE exam.

Every chapter in the book has been divided into three sections –

- LEVEL 1 – This section will test you on all basic fundamentals of the chapter. Problems are not very rigorous though they may be very conceptual.
- LEVEL 2 – This section will develop all necessary skills required to score a high rank in JEE exam. Few problems in this section may appear lengthy but they are the ones which test your confidence and patience. Don't be scared of them.
- LEVEL 3 – This section contains problems that may require exceptional reasoning skill or mathematical ability.

Since difficulty level is quite subjective and may vary from person to person — few problems may appear to you as misplaced in three sections described above. I have judged them to the best of my ability besides taking help from some very bright minds.

I have not tried to include every other problem that is available in this universe. Most of the books available in market have this issue – in the name of being exhaustive, they have become repetitive. Believe me, while solving problems from this book you will not feel like wasting your time in doing similar problems again and again..

Most of the solutions are quite descriptive so that a serious student can understand on his/her own. Diagrams have been included wherever possible to make things lucid.

JEE exam being objective, one may challenge the sanctity of a subjective book. Have no doubts in your mind — pattern of a question paper or type of question will never deter you if you have sound grasp of the subject and have developed right kind of temperament. Physics as a subject is notorious and can be learned only by subjecting yourself to the true rigor and complexity. While doing a subjective problem you cannot make a guess and bluff yourself!

This collection of problems will appear to you as fresh and challenging. Start and enjoy learning physics!

Suggestions are welcome.

CONTENTS

01 BASIC MATHS

Level 1

Q.1 In an experiment mileage of a car was measured to be 24 *kmpl* (Kilometer per liter of fuel consumed). After the experiment it was found that 4 % of the fuel used during the experiment was leaked through a small hole in the tank. Calculate the actual mileage of the car after the tank was repaired.

Q.2 A man is standing at a distance of 500m from a building. He notes that angle of elevation of the top of the building is 3.6°. Find the height of the building. Neglect the height of the man and take $\pi = 3.14$.

Q.3 A Smuggler in a hindi film is running with a bag 0.3 m × 0.2 m × 0.2 m in dimension. The bag is supposed to be completely filled with gold. Do you think than the director of the film made a technical mistake there? Density of gold is 19.6 g/cc.

Q.4 A particle moves along the curve $6y = x^3+2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

Q.5 The area of a regular octagon of side length a is A.

(a) Find the time rate of change of area of the octagon if its side length is being increased at a constant rate of β *m/s*. Is the time rate of change of area of the octagon constant with time?

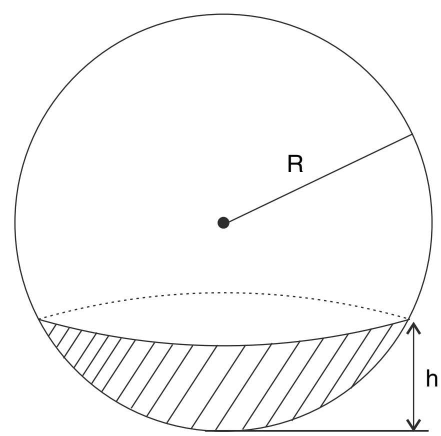
(b) Find the approximate change in area of the octagon as the side length is increased from 2.0 m to 2.001 m.

Level 2

Q.6 Spirit in a bowl evaporates at a rate that is proportional to the surface area of the liquid. Initially, the height of liquid in the bowl is H_0. It becomes $\frac{H_0}{2}$ in time t_0. How much more time will be needed for the height of liquid to become $\frac{H_0}{4}$.

Q.7 Show that the volume of a segment of height h of a sphere of radius R is

$$V = \frac{1}{3}\pi h^2 (3R - h)$$



Q.8 The amount of energy a car expends against air resistance is approximately given by

$$E = K\,ADv^2$$

where E is measured in Joules. K is a constant, A is the cross-sectional area of the car viewed from the front (in m^2), D is the distance traveled (in m), and v is the speed of the car (in m/s). Julie wants to drive from Mumbai to Delhi and get good fuel mileage. For the following questions, assume that the energy loss is due solely to air resistance.

(a) Julie usually drives at a speed of 54 Km/hr. How much more energy will she use if she drives 20% faster?

(b) Harshit drives a very large SUV car, and Julie drives a small car. Every linear dimension of Harshit's car is double that of Julie's car. Find the ratio of energy spent by Harshit's car to

Julie's car when they cover same distance. Speed of Harshit was 10% faster compared to Julie's car.

(c) Write the dimensional formula for K. Will you believe that K depends on density of air?

Q.9 The volume flow rate Q *(in $m^3\ s^{-1}$)* of a liquid through pipe having diameter d is related to viscosity of water 'η' (*unit Pascal. s*) and the pressure gradient along the pipe $\frac{dP}{dx}$ [pressure gradient $\frac{dP}{dx}$ is rate of change of pressure per unit length along the pipe], by a formula of the form

$$Q = k\eta^a d^b \left(\frac{dP}{dx}\right)^c$$

Where K is a dimensionless constant. Find a,b and c.

Q.10. The potential energy (U) of a particle can be expressed in certain case as $U = \frac{A^2}{2mr^2} - \frac{BMm}{r}$

Where m and M are mass and r is distance. Find the dimensional formulae for constants.

Q.11. In the following expression V and g are speed and acceleration respectively. Find the dimensional formulae of a and b

$$\int \frac{VdV}{g - bV^2} = a$$

Q.12 The maximum height of a mountain on earth is limited by the rock flowing under the enormous weight above it. Studies show that maximum height depends on young's modulus (Y) of the rod, acceleration due to gravity (g) and the density of the rock (d).

(a) Write an equation showing the dependence of maximum height (h) of mountain on Y, g and d. It is given that unit of Y is Nm^{-2}.

(b) Take $d = 3 \times 10^3$ kg m^{-3}, $Y = 1 \times 10^{10}$ Nm^{-2} and $g = 10$ ms^{-2} and assume that maximum height of a mountain on the surface of earth is limited to 10 km [height of mount Everest is nearly 8 km]. Write the formula for h.

Q.13 A particle of mass m is given an initial speed V_0. It experiences a retarding force that is proportional to the speed of the particle ($F = aV$). a is a constant.

(a) Write the dimensional formula of constant a.

(b) Using dimensional analysis, derive a formula for stopping time (t) of the particle. Does your formula tell you how 't' depends on initial speed V_0? What can you predict about the constant obtained in the formula?

Q.14 Assume that maximum mass m_1 of a boulder swept along by a river, depends on the speed V of the river, the acceleration due to gravity g, and the density d of the boulder. Calculate the percentage change in maximum mass of the boulder that can be swept by the river, when speed of the river increases by 1%.

Q.15 A massive object in space causes gravitational lensing. Light from a distant source gets deflected by a massive lensing object. This was first observed in 1919 and supported Einstein's general theory of relativity.

The angle θ by which light gets deflected due to a massive body depends on the mass (M) of the body, universal gravitational constant (G), speed of light (c) and the least distance (r) between the lensing object and the apparent path of light. Derive a formula for θ using method of dimensions. Make suitable assumptions.

Q.16 The Casimir effect describes the attraction between two unchanged conducting plates placed parallel to each other in vacuum. The astonishing force (predicted in 1948 by Hendrik Casimir) per unit area of each plate depends on the planck's constant (h), speed of light (c) and separation between the plates (r).

(a) Using dimensional analysis prove that the formula for the Casimir force per unit area on the plates is given by

$F = k\frac{hc}{r^4}$ where k is a dimensionless constant

(b) If the force acting on 1x1 cm plates separated by 1μm is 0.013 dyne, calculate the value of constant k.

Q.17. Scattering of light is a process of absorption and prompt re-emission of light by atoms and molecules. Scattering involving particles smaller than wavelength (λ) of light is known as Rayleigh scattering. Let a_i be amplitude of incident light on a scatterer of volume V. The scattered amplitude at a distance r from the scatterer is a_s. Assume and $a_s \propto a_i$, $a_s \propto \frac{1}{r}$ and $a_s \propto V$.

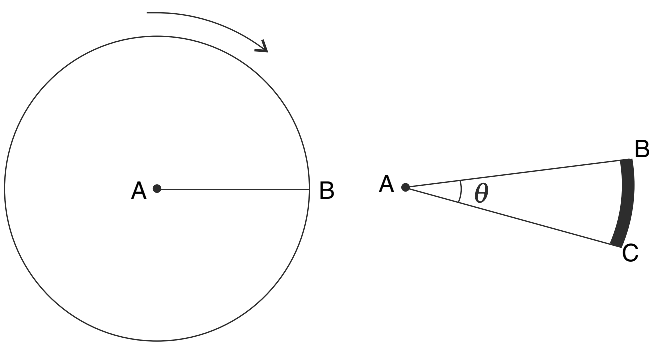
(i) Find the dimensions of the proportionality constant occurring in the expression of a_s

(ii) Assuming that this constant depends on λ, find the dependence of ratio $\frac{a_s}{a_i}$ on λ.

(iii) Knowing that intensify of light I α a^2 find the dependence of $\frac{I_s}{I_i}$ on λ.

Q.18 It is given that $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$. Using methods of dimensions find $\int \frac{dx}{a^2 + x^2}$

Q.19



Two point sources of light are fixed at the centre (A) and circumference (point B) of a rotating turn table. A photograph of the rotating table is taken. On the photograph a point A and an arc BC appear. The angle θ was measured to be $\theta = 10.8° \pm 0.1°$ and the angular speed of the turntable was measured to be ω = (33.3 ± 0.1) revolution per minute. Calculate the exposure time of the camera.

Q.20 The speed (V) of wave on surface of water is given by

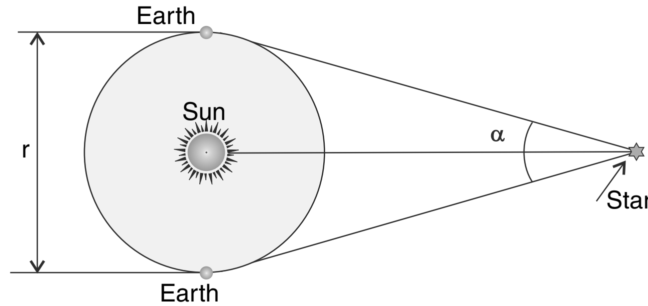
$$V = \sqrt{\frac{a\lambda}{2\pi} + \frac{2\pi b}{\rho\lambda}}$$

where λ is the wavelength of the wave and ρ is density of water. a is a constant and b is a quantity that changes with liquid temperature.

(a) Find the dimensional formulae for a and b.

(b) Surface wave of wavelength 30 mm have a speed of 0.240 ms^{-1}. If the temperature of water changes by 50°C, the speed of waves for same wavelength changes to 0.230 ms^{-1}. Assuming that the density of water remains constant at 1×10^3 kg m^{-3}, estimate the change in value of 'b' for temperature change of 50°C.

Q.21 The line of sight of the brightest star in the sky, Sirius has a maximum parallax angle of $\delta = 0.74 \pm 0.02$ arc second when observed at six month interval. The distance between two positions of earth (at six – month interval) is $r = 3.000 \times 10^{11} m$.



Calculate the distance of Sirius from the Sun with uncertainty, in unit of light year. Given 1 ly = 9.460×10^{15} m. ; $\pi = 3.14$

LEVEL 3

Q.22 You inhale about 0.5 liter of air in each breath and breath once in every five seconds. Air has about 1% argon. Mass of each air particle can be assumed to be nearly 5×10^{-26} kg. Atmosphere can be assumed to be around 20 km thick having a uniform density of 1.2 $kg\ m^{-3}$. Radius of the earth is $R = 6.4 \times 10^6$ m. Assume that when a person breathes, half of the argon atoms in each breath have never been in that person's lungs before. Argon atoms remain in atmosphere for long-long time without reacting with any other substance. Given : one year = 3.2×10^7s

(a) Estimate the number of argon atoms that passed through Newton's lungs in his 84 years of life.

(b) Estimate the total number of argon atoms in the Earth's atmosphere.

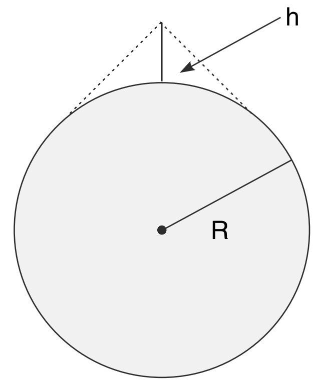
(c) Assume that the argon atoms breathed by Newton is now mixed uniformly through the atmosphere, estimate the number of argon atoms in each of your breath that were once in Newton's lungs.

Q.23 A rope is tightly wound along the equator of a large sphere of radius R. The length of the rope is increased by a small amount ℓ (<< R) and it is pulled away from the surface at a point to make it taut. To what height (h) from the surface will the point rise ?

If the radius of the earth is R=6400 km and ℓ =

10 mm, find the value of h. Does the value surprise you.

[For small θ take $\tan\theta = \theta + \frac{\theta^3}{3}$ and $\sec\theta = 1 + \frac{\theta^2}{2}$. Also take $(2.3)^{\frac{2}{3}} = 1.74$]

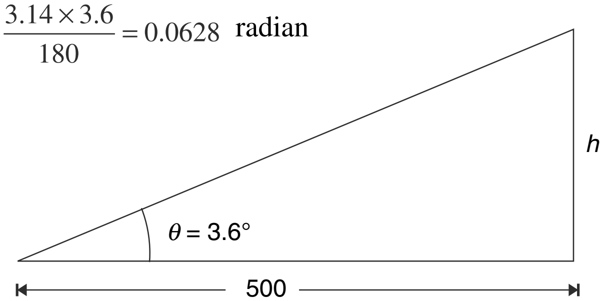


ANSWERS

1. 25 kmpl
2. 31.40 m
3. Yes.
4. $(-4, -\frac{31}{3})$, (4, 11)
5. (a) 4 ($\sqrt{2}$ + 1) $a\ \beta$. No, it is not a constant
   (b) 0.0019 m^2
6. $\frac{t_0}{2}$
8. (a) 44% higher (b) 4.84 (c) $[ML^{-3}]$; Yes
9. $a = -1$; $b = 4$; $c = 1$
10. [A] = $[M^1L^2T^{-1}]$; [B] = $[M^{-1}L^3T^{-2}]$
11. [a] = L; [b] = L^{-1}
12. (a) $h = k\left(\frac{Y}{gd}\right)$; $k = a\ const$
    (b) $h = 0.03\left(\frac{Y}{gd}\right)$
13. (a) [a] = $[M^1T^{-1}]$ (b) $t = k\frac{m}{a}$; $t = \infty$
14. 6%
15. $\theta = k\frac{GM}{cr^2}$
16. (b) $k = 6.5 \times 10^{-3}$
17. (i) $[k] = [L^{-2}]$
    (ii) $\frac{a_s}{a_i} \propto \lambda^{-2}$
    (iii) $\frac{I_s}{I_i} \propto \frac{a_s^2}{a_i^2} \propto \lambda^{-4}$
18. $a\tan^{-1}\left(\frac{x}{a}\right) + c$
19. (0.054 ± 0.003)s
20. (a) [a] = $[M^oL^1T^{-2}]$; [b] = $[M^1L^oT^{-2}]$
    (b) $\Delta b = -0.022\ kg\ s^{-2}$
21. 8.84 ± 0.24 *ly*
22. (a) 3.2×10^{28} (b) 2.5×10^{42}
    (c) 1.5×10^6
23. 5.6 *m*

SOLUTIONS

2. $\theta = 3.6° = \frac{\pi}{180} \times 3.6\ rad = \frac{3.14 \times 3.6}{180} = 0.0628$ radian



$$h = 500 \tan\theta \approx 500.\theta = 500 \times 0.0628\ m = 31.40\ m$$

3. Calculate the mass of gold using – mass = volume × density. The gold in the bag will weigh 235.2 kg. How can a smuggler run with such a heavy bag!

4. As per the question

$$\frac{dy}{dx} = 8$$

$$\Rightarrow \frac{3x^2}{6} = 8$$

$$\Rightarrow x = \pm 4$$

Corresponding Y co ordinates are $\frac{(\pm 4)^3 + 2}{6} = 11, \frac{-31}{3}$

5. **(a)** Area of an octagon is = $A = 2\left(1+\sqrt{2}\right)a^2$ [Prove this yourself]

$$\frac{dA}{dt} = 4\left(\sqrt{2}+1\right)a\frac{da}{dt} = 4\left(\sqrt{2}+1\right)a\beta$$

This expression is not a constant because it depends on side length which itself is time dependent.

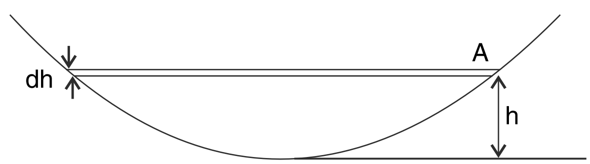
(b) $A = 2\left(1+\sqrt{2}\right)a^2$

$$\frac{dA}{da} = 4\left(\sqrt{2}+1\right)a$$

$$\Delta A \simeq \frac{dA}{da}\Delta a = 4\left(\sqrt{2}+1\right)a\Delta a$$

$$\Delta A = 4\ (\sqrt{2} + 1) \times 2.0 \times 0.001 = 0.0019\ m^2$$

6.



Let the area of cross section of the bowl at height h be A.

Let the height decrease by dh in further interval dt

Volume that evaporates = Adh

As per the question $\frac{Adh}{dt} \alpha A$

$$\Rightarrow \frac{A\ dh}{dt} = -kA$$

Where k is a positive constant. We have placed a negative sign because h is decreasing with time and $\frac{dh}{dt}$ is a negative quantity.

$$dh = -k\ dt$$

$$\int_{H_0}^{h} dh = -k\int_0^t dt$$

$$h - H_0 = -kt$$

$$h = H_0 - kt$$

Now $h = \dfrac{H_0}{2}$ at $t = t_0$

$$\therefore \frac{H_0}{2} = H_0 - kt_0$$

$$\Rightarrow \frac{H_0}{2} = kt_0 \qquad \text{...........(1)}$$

Let height be $\dfrac{H_0}{4}$ at time 't'

$$\frac{H_0}{4} = H_0 - kt$$

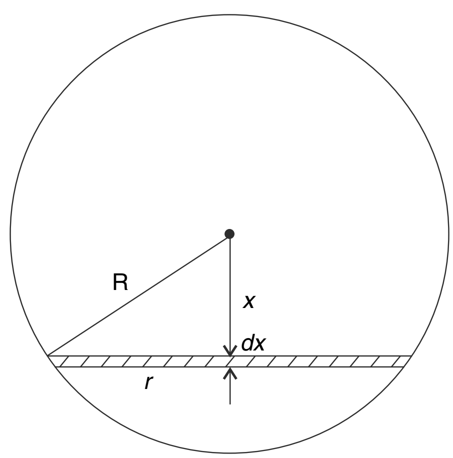
$$\frac{3H_0}{4} = \frac{H_0}{2t_0}t \qquad \left[\text{using (1) } k = \frac{H_0}{2t_0}\right]$$

$$\Rightarrow t = \frac{3}{2}t_0$$

Required answer is $\dfrac{t_0}{2}$

Note: It is easy to see from equation (1) that depth of the liquid decreases at a constant rate. Hence it is obvious that if it takes time t_0 for the level to fall from H_0 to $\dfrac{H_0}{2}$, then it will take $\dfrac{t_0}{2}$ time for level to further fall through $\dfrac{H_0}{4}$.

7. Consider a disc shaped element of thickness dx at a distance x from the centre



$$r^2 = R^2 - x^2$$

Volume of disc element $dV = \pi r^2\, dx = \pi\,(R^2 - x^2)dx$

$\therefore$ Required Volume is

$$V = \int_{x=R-h}^{R} dV$$

$$= \pi \int_{(R-h)}^{R} (R^2 - x^2)dx$$

$$= \left[\pi \left[R^2 x \right]_{R-h}^{R} - \left[\frac{x^3}{3} \right]_{R-h}^{R} \right]$$

$$= \pi \left[R^2 (R - R + h) - \frac{1}{3} R^3 + \frac{1}{3} (R - h)^3 \right]$$

$$= \pi \left[R^2 h - \frac{R^3}{3} + \frac{1}{3} \left(R^3 - h^3 - 3R^2 h + 3Rh^2 \right) \right]$$

$$= \frac{\pi}{3} h^2 [3R - h]$$

8. **(a)** If Julie increases her speed by 20%, then she multiplies her speed by 1.2. Thus the required energy is multiplied by $(1.2)^2 = 1.44$, which is an increase of 44%.

(b) The area of cross section of Harshit's car is 4 times that of Julie's car and his speed is 1.1 times that of Julie's car.

$$\frac{E_{Harshit}}{E_{Julie}} = \frac{4 \times (1.1)^2}{1} = 4.84$$

(c) $[K] = \dfrac{[E]}{[ADV^2]} = \dfrac{[ML^2T^{-2}]}{[L^2 L L^2 T^{-2}]} = [ML^{-3}]$

12. (a) let $h = kY^a g^b d^c$ [k is a contant]

Dimensions on LHS = Dimensions on RHS

$M^0 L^1 T^0 = [M L^{-1} T^{-2}]^a [L^1 T^{-2}]^b [M^1 L^{-3}]^c$

Equating dimensions of M, L and T

$a + c = 0$ (1)

$-a + b - 3c = 1$ (2)

$-2a - 2b = 0$ (3)

Solving $a = 1$, $b = -1$, $c = -1$

$$\therefore h = k\left(\frac{Y}{gd}\right)$$

(b) $h = k\dfrac{Y}{gd}$

$$10 \times 10^3 = \text{k} \cdot \frac{10^{10}}{10 \times 3 \times 10^3}$$

$$\Rightarrow k = \frac{3}{100} = 0.03$$

13. (a) $[a] = \dfrac{[F]}{[V]} = [MT^{-1}]$

(b) let $t = k\, a^x\, m^y V_0^z$

It can be shown that

$x = -1$; $y = 1$; $z = 0$

$\therefore t = k\dfrac{m}{a}$ (a)

As per this expression, the time of motion is independent of initial speed ! This cannot be true. It must grow with V_0. A large initial speed (V_1) will need some non zero time to slow down to V_2. After this the particle will take as much time to halt as it would take when projected with initial speed V_2.

The only way equation (a) can be sensible is by having $k = \infty$.

14. Let mass of the boulder that can be swept by the river be given by.

$m = k\,V^x\,g^y d^z$

$[m] = [V]^x\,[g]^y\,[d]^z$

$[M^1\,L^0\,T^0] = [L^1T^{-1}]^x\,[L^1\,T^{-2}]^y\,[M^1\,L^{-3}]^z$

$= [M^z\,L^{x+y-3z}\,T^{-x-2y}]$

$z = 1\,;\,x + y - 3z = 0\,;\,-x - 2y = 0$

Solving; $x = 6;\,y = -3,\,z = 1$

$\therefore\ m = k\,\mathrm{V}^6\,g^{-3}\,d$

$\therefore \dfrac{\Delta m}{m}\times 100 = 6\dfrac{\Delta V}{V}\times 100$

The percentage change in mass is 6 % when speed changes by 1 %.

15. $[G] = [M^{-1}L^3T^{-3}]$

$[c] = [L^1T^{-1}]$

$[M] = [M^1]$

$[r] = [L^1]$

The angle θ is dimensionless.

Simple observation indicates that mass is present in expression of G and M only. Hence for dimension of mass to be zero we must have expression like $\theta = k\,(GM)^x\,(c)^y\,(r)^z$

$[\theta] = [GM]^x\,[c]^y\,[r]^z$

$[M^0L^0T^0] = [L^3T^{-2}]^x\,[L^1T^{-1}]^y\,[L^1]^z = [M^0\,L^{3x+y+z}\,T^{-2x-2y}]$

$\therefore\ 3x + y + z = 0$

$2x + 2y = 0$

$\Rightarrow y = -x$

and $z = -2x$

Assuming $x = 1$

$$\theta = k\frac{GM}{cr^2}$$

If $x = 2$ $\quad \theta = k\dfrac{G^2M^2}{c^2r^4}$

θ is dimensionless and all expressions with $x = 1,2,3$ are correct. The fact of the matter is that we can also have

$x = \dfrac{1}{2},\dfrac{1}{3}$ etc. All expressions are dimensionally correct. However, the correct expression is

$$\theta = k\frac{GM}{c^2r} \qquad [k = 4]$$

16. (a) Let $F = k\, h^x\, c^y\, r^z$

F is force per unit area

$[F] = [h]^x\, [c]^y\, [r]^z$

$[M^1\, L^{-1}\, T^{-2}] = [M\, L^2\, T^{-1}]^x\, [L^1\, T^{-1}]^y\, [L]^z$

$\left(Note : h = \dfrac{Energy}{frequency}\right)$

$\therefore\ x = 1$

$2x + y + z = -1$

$-x - y = -2$

Solving ; $x = 1;\ y = 1\ ;\ z = -4$

$\therefore F = k\dfrac{hc}{r^4}$

(b) $h = 6.63 \times 10^{-27}$ g cm^2 s^{-1}

$c = 3 \times 10^{10}$ cm s^{-1}

$r = 10^{-4}$ cm

$$\therefore k\frac{6.63\times10^{-27}\times3\times10^{10}}{\left(10^{-4}\right)^4}$$

$$\therefore k = \frac{13}{6.63\times3}\times10^{-2} = 6.5\times10^{-3}$$

17. (i) $a_s \propto \dfrac{Va_i}{r}$

$a_s = k\dfrac{Va_i}{r}$

$\therefore k\dfrac{V}{r}$ is dimensionless

$[k] = [L^{-2}]$

(ii) $k \propto \lambda^{-2}$

$\therefore \dfrac{a_s}{a_i} \propto \lambda^{-2}$

(iii) $\dfrac{I_s}{I_i}\,\alpha\,\dfrac{a_s^2}{a_i^2} \propto \lambda^{-4}$

18. Denominator is $a^2 + x^2$, which means that $[a] = [x]$.

Pretend that x is a length. Then the integral has dimensions $[L^{-1}]$

Therefore, we may expect the answer to have a term like a $\tan^{-1}x$. Also the argument of $\tan^{-1}$ should be dimensionless. Hence, we can replace x by x/a.

Therefore, the integral can be written equal to $a\ \tan^{-1}\left(\dfrac{x}{a}\right) + c$

19. Let the exposure time be t and the uncertainty in its value be Δt

$$\omega = \frac{33.3\times360°}{60} = 200°\text{s}^{-1}$$

$t = \frac{\theta}{\omega} = \frac{10.8}{200} = 0.054s$

And $\left|\frac{\Delta t}{t}\right| = \left|\frac{\Delta\theta}{\theta}\right| + \left|\frac{\Delta\omega}{\omega}\right|$

$\Rightarrow \Delta t = t\left|\frac{\Delta\theta}{\theta}\right| + t\left|\frac{\Delta\omega}{\omega}\right|$

$= \frac{|\Delta\theta|}{\omega} + \frac{|\Delta\omega| t}{\omega}$

$= \frac{0.1}{33.3} + \frac{0.1 \times 0.054}{33.3}$

$= 0.003$

$t = (0.054 \pm 0.003)$s

20. (a) $V^2 = \frac{a\lambda}{2\pi} + \frac{2\pi b}{\rho\lambda}$

$\left[V^2\right] = [a\lambda] = \left[\frac{b}{\rho\lambda}\right]$

$\left[M^0L^1T^{-1}\right]^2 = [a]\left[L^1\right] = \frac{[b]}{\left[ML^{-3}\right]\left[L^1\right]}$

$\therefore$ $[a] = [M^0 L^1 T^{-2}]$

$[b] = [M^1 L^0 T^{-2}]$

(c) Given data says (in SI units)

$(0.240)^2 = \frac{a\lambda}{2\pi} + \frac{2\pi b_1}{10^3 \times 30 \times 10^{-3}}$

$0.24^2 = \frac{a\lambda}{2\pi} + \frac{2\pi b_1}{30}$ ....(1)

Similarly,

$(0.23)^2 = \frac{a\lambda}{2\pi} + \frac{2\pi b_2}{30}$ ....(2)

(2) - (1)

$\frac{2\pi}{30}(b_2 - b_1) = (0.23)^2 - (0.24)^2$

$\Delta b = -\frac{30}{2\pi} - (0.23 + 0.24)(0.24 - 0.23)$

$= -0.022$ kg s^{-2}

21. Required distance = x

$$x = \frac{r/2}{\tan\left(\alpha/2\right)} \simeq \frac{r/2}{\alpha/2} = \frac{r}{\alpha}$$

α = 0.740 arc second

$$= \left(\frac{0.74}{3600}\right)^{\circ} = \frac{0.74}{3600} \times \frac{\pi}{180} \quad \text{radian}$$

$$\therefore x = \frac{r}{\alpha} = \frac{3.000 \times 10^{11}}{\frac{0.74}{3600} \times \frac{\pi}{180}} m$$

$$= \frac{3.000 \times 10^{11} \times 3600 \times 180}{0.74 \times \pi \times 9.460 \times 10^{15}} ly$$

$= 8.84\ ly$

$$x = \frac{r}{\alpha}$$

$$\Delta x = \frac{r}{\alpha^2} \Delta\alpha$$

$$= \frac{3.000 \times 10^{11}}{\left(\frac{0.74}{3600} \times \frac{\pi}{180}\right)^2} \times \frac{\left(\frac{0.02}{3600} \times \frac{\pi}{180}\right)}{9.460 \times 10^{15}} ly$$

$= 0.24\ ly$

22. (a) Number or argon atoms in each breath

$$N = \frac{\textit{Mass of air inhaled}}{\textit{Mass of one particle}} \times \frac{1}{100}$$

$$= \frac{\left(1.2 kgm^{-3}\right)\left(0.5 \times 10^{-3} m^3\right)}{5 \times 10^{-26}} \times \frac{1}{100} = 1.2 \times 10^{20}$$

Number of times Newton breath in his life time is

n = (frequency of breaths) × 84 years

$n = 0.2 \times 84 \times 3.2 \times 10^7 = 5.4 \times 10^8$

Number of argon atom that Newton breathed in his lifetime $= \frac{1}{2} N.n$

[factor $\frac{1}{2}$ is there to account for re-breathed atoms]

$$= \frac{1}{2} \times 1.2 \times 10^{20} \times 5.4 \times 10^8 = 3.2 \times 10^{28}$$

(b) Volume of atmosphere $v = 4\pi R^2. h$.

We have estimated that half a litre (= 0.5×10^{-3} m^3) air has 1.2×10^{20} argon atoms, hence number of argon atoms in atmosphere

$$N_0 = \frac{4\pi R^2 h}{0.5\times10^3}\times1.2\times10^{20}$$

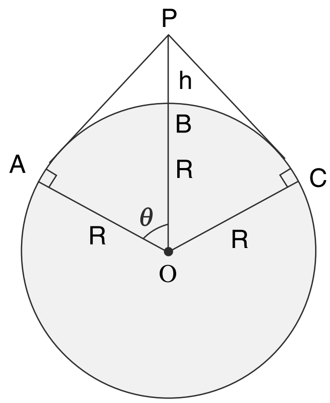
$$= \frac{4\times3.14\times\left(6.4\times10^6\right)^2\times20\times10^3}{5\times10^{-4}}\times1.2\times10^{20} = 2.5\times10^{42}$$

(c) Fraction of argon in atmosphere which must have visited Newton's lungs

$$= \frac{3.2\times10^{28}}{2.5\times10^{42}} = 1.3\times10^{-14}$$

Required answer is $1.3 \times 10^{-14}\, N = 1.5 \times 10^6$

23.



Length AP = arc $AB + \frac{\ell}{2} = R\theta + \frac{\ell}{2}$

$\therefore\ R\tan\theta = R\theta + \frac{\ell}{2}$

$$\Rightarrow R\left[\theta + \frac{\theta^3}{3}\right] \simeq R\theta + \frac{\ell}{2}$$

$$\Rightarrow \frac{R\theta^3}{3} \simeq \frac{\ell}{2} \qquad \Rightarrow \theta \simeq \left(\frac{3\ell}{2R}\right)^{\frac{1}{3}}$$

Now, $\quad OP = R\sec\theta$

$$R + h = R\left[1 + \frac{\theta^2}{2}\right]$$

$$\Rightarrow h = \frac{R}{2}\theta^2 = \frac{R}{2}\left(\frac{3\ell}{2R}\right)^{\frac{2}{3}}$$

For earth $R = 6400$ km $= 6.4 \times 10^6\, m$

And given $l = 10$ mm $= 10^{-2}\, m$

$$h = \frac{6.4\times10^6}{2}\left(\frac{3\times10^{-2}}{2\times6.4\times10^6}\right)^{2/3}$$

$h = 3.2 \times (2.3)^{2/3} = 3.2 \times 1.74 = 5.6$ m

02 KINEMATICS

Level 1

Q. 1. A particle is travelling on a curved path. In an interval Δt its speed changed from v to $2v$. However, the change in magnitude of its velocity was found to be $|\overrightarrow{\Delta V}| = \sqrt{5}\,v$. What can you say about the direction of velocity at the beginning and at the end of the interval (Δt)?

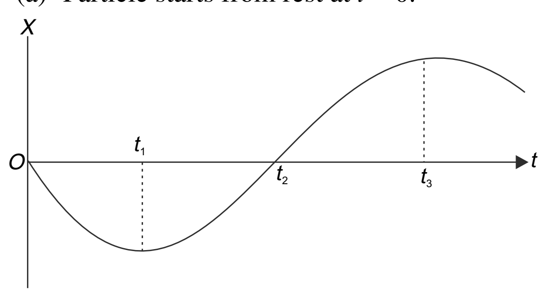
Q. 2. Two tourist A and B who are at a distance of 40 km from their camp must reach it together in the shortest possible time. They have one bicycle and they decide to use it in turn. 'A' started walking at a speed of 5 $km\ hr^{-1}$ and B moved on the bicycle at a speed of 15 $km\ hr^{-1}$. After moving certain distance B left the bicycle and walked the remaining distance. A, on reaching near the bicycle, picks it up and covers the remaining distance riding it. Both reached the camp together.

(a) Find the average speed of each tourist.

(b) How long was the bicycle left unused?

Q. 3. The position time graph for a particle travelling along x axis has been shown in the figure. State whether following statements are true of false.

(a) Particle starts from rest at $t = 0$.



(b) Particle is retarding in the interval 0 to t_1 and accelerating in the interval t_1 to t_2.

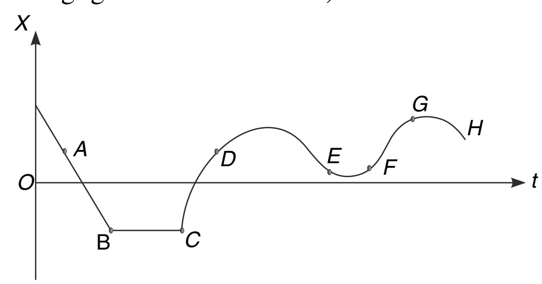
(c) The direction of acceleration has changed once during the interval 0 to t_3

Q. 4. The position time graph for a particle moving along X axis has been shown in the fig. At which of the indicated points the particle has

(i) negative velocity but acceleration in positive X direction.

(ii) positive velocity but acceleration in negative X direction.

(iii) received a sharp blow (a large force for negligible interval of time)?



Q. 5. A particle is moving along positive X direction and is retarding uniformly. The particle crosses the origin at time $t = 0$ and crosses the point $x = 4.0\ m$ at $t = 2\ s$.

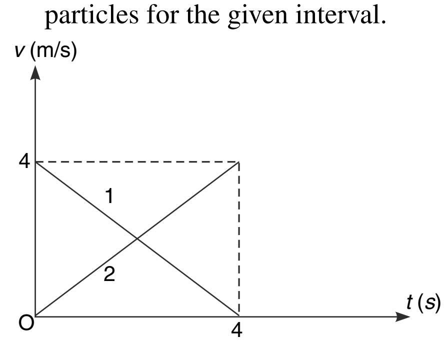
(a) Find the maximum speed that the particle can possess at $x = 0$.

(b) Find the maximum value of retardation that the particle can have.

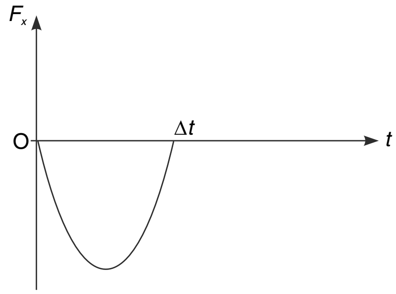
Q. 6. The velocity time graph for two particles (1 and 2) moving along X axis is shown in fig. At time $t = 0$, both were at origin.

(a) During first 4 second of motion what is maximum separation between the particles? At what time the separation is maximum?

(b) Draw position (x) vs time (t) graph for the particles for the given interval.



Q. 7. A ball travelling in positive X direction with speed V_0 hits a wall perpendicularly and rebounds with speed V_0. During the short interaction time (Δt) the force applied by the wall on the ball varies as shown in figure.



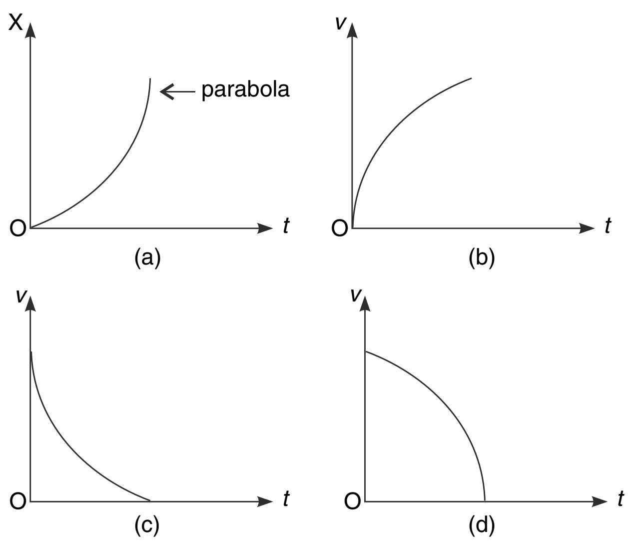
Draw the velocity-time graph for the ball during the interval 0 to Δt

Q. 8. For a particle moving along a straight line consider following graphs A, B, C and D. Here x, v and t are position, velocity and time respectively.

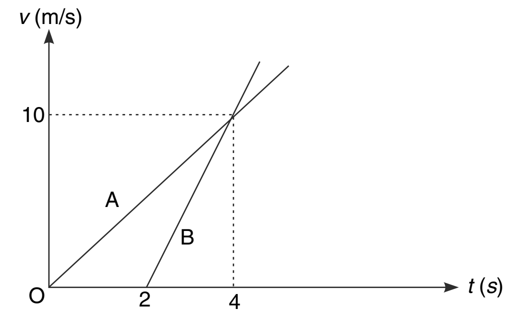
(i) In which of the graphs the magnitude of acceleration is decreasing with time?

(ii) In which of the graphs the magnitude of acceleration is increasing with time?

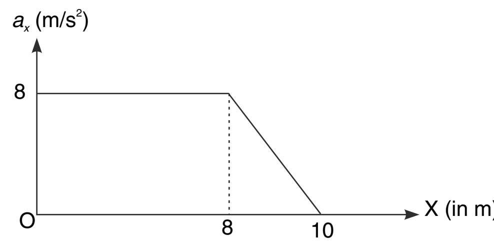
(iii) If the body is definitely going away from the starting point with time, which of the given graphs represent this condition.



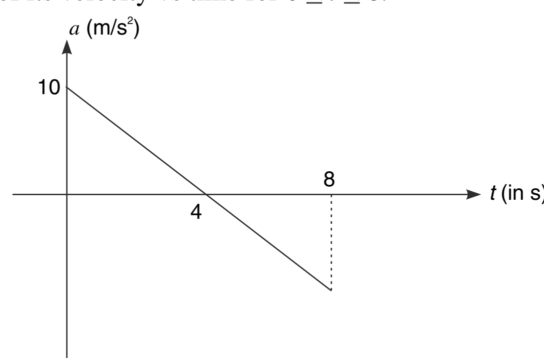
Q. 9. Two particles A and B start from same point and move along a straight line. Velocity-time graph for both of them has been shown in the fig. Find the maximum separation between the particles in the interval $0 < t < 5$ sec.



Q. 10. A particle starts from rest (at $x = 0$) when an acceleration is applied to it. The acceleration of the particle changes with its co-ordinate as shown in the fig. Find the speed of the particle at $x = 10m$.



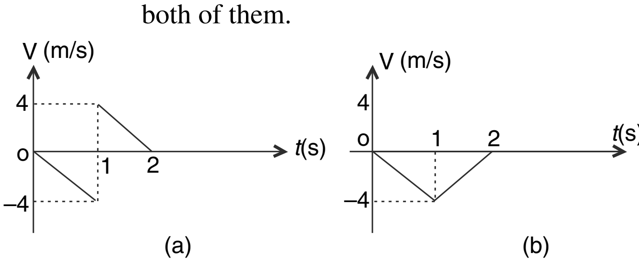
Q. 11. Acceleration vs time graph for a particle moving along a straight line is as shown. If the initial velocity of the particle is $u = 10$ m/s, draw a plot of its velocity vs time for $0 \leq t \leq 8$.



Q. 12. The velocity (V) – time (t) graphs for two particles A and B moving rectilinearly have been shown in the figure for an interval of 2 second.

(a) At $t = 1$ s, which of the two particles (A or B) has received a severe blow?

(b) Draw displacement (X) – time (t) graph for both of them.



Q. 13. A particle starts moving rectilinearly at time $t = 0$ such that its velocity(v) changes with time (t) as per equation –

$v = (t^2 - 2t)\ m/s$ for $0 \le t \le 2\ s$

$= (-t^2 + 6t - 8)\ m/s$ for $2 \le t \le 4\ s$

(a) Find the interval of time between $t = 0$ and $t = 4\ s$ when particle is retarding.

(b) Find the maximum speed of the particle in the interval $0 \le t \le 4\ s$.

Q. 14. Our universe is always expanding. The rate at which galaxies are receding from each other is given by Hubble's law (discovered in 1929 by *E.* Hubble). The law states that the rate of separation of two galaxies is directly proportional to their separation. It means relative speed of separation of two galaxies, presently at separation r is given by $v = Hr$

H is a constant known as Hubble's parameter. Currently accepted value of H is $2.32 \times 10^{-18}\ s^{-1}$

(a) Express the value of H in unit of

$$\frac{Km.\,s^{-1}}{\text{Mega light year}}$$

(b) Find time required for separation between two galaxies to change from r to $2r$.

Q. 15. A stone is projected vertically up from a point on the ground, with a speed of 20 m/s. Plot the variation of followings with time during the entire course of flight –

(a) Velocity

(b) Speed

(c) Height above the ground

(d) distance travelled

Q. 16. A ball is dropped from a height H above the ground. It hits the ground and bounces up vertically to a height $\frac{H}{2}$ where it is caught. Taking origin at the point from where the ball was dropped, plot the variation of its displacement *vs* velocity. Take vertically downward direction as positive.

Q. 17. A helicopter is rising vertically up with a velocity of 5 ms^{-1}. A ball is projected vertically up from the helicopter with a velocity V (relative to the ground). The ball crosses the helicopter 3 second after its projection. Find V.

Q. 18. A chain of length L supported at the upper end is hanging vertically. It is released. Determine the interval of time it takes the chain to pass a point $2L$ below the point of support, if all of the chain is a freely falling body.

Q. 19. Two nearly identical balls are released simultaneously from the top of a tower. One of the balls fall with a constant acceleration of $g_1 = 9.80\ ms^{-2}$ while the other falls with a constant acceleration that is 0.1% greater than g_1. [This difference may be attributed to variety of reasons. You may point out few of them]. What is the displacement of the first ball by the time the second one has fallen 1.0 mm farther than the first ball?

Q. 20. Two projectiles are projected from same point on the ground in x-y plane with y direction as vertical. The initial velocity of projectiles are

$$\vec{V}_1 = V_{x_1}\hat{i} + V_{y1}\hat{j}$$

$$\vec{V}_2 = V_{x2}\hat{i} + V_{y2}\hat{j}$$

It is given that $V_{x1} > V_{x2}$ and $V_{y1} < V_{y2}$. Check whether all of the following statement/s are True.

(a) Time of flight of the second projectile is greater than that of the other.

(b) Range of first projectile may be equal to the range of the second.

(c) Range of the two projectiles are equal if $V_{x1} V_{y1} = V_{x2} V_{y2}$

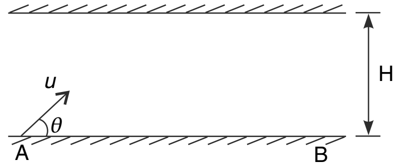
(d) The projectile having greater time of flight can have smaller range.

Q. 21. (a) A particle starts moving at $t = 0$ in x-y plane such that its coordinates (in cm) with time (in sec) change as $x = 3t$ and $y = 4 \sin (3t)$. Draw the path of the particle.

(b) If position vector of a particle is given by $\vec{r} = \left(4t^2 - 16t\right)\hat{i} + \left(3t^2 - 12t\right)\hat{j}$, then find distance travelled in first 4 sec.

Q. 22. Two particles projected at angles θ_1 and θ_2 ($<\theta_1$) to the horizontal attain same maximum height. Which of the two particles has larger range? Find the ratio of their range.

Q. 23. A ball is projected from the floor of a long hall having a roof height of $H = 10\ m$. The ball is projected with a velocity of $u = 25\ ms^{-1}$ making an angle of $\theta = 37°$ to the horizontal. On hitting the roof the ball loses its entire vertical component of velocity but there is no change in the horizontal component of its velocity. The ball was projected from point A and it hits the floor at B. Find distance AB.



Q. 24. In a tennis match Maria Sharapova returns an incoming ball at an angle that is 4° below the horizontal at a speed of 15 *m/s*. The ball was hit at a height of 1.6 *m* above the ground. The opponent, Sania Mirza, reacts 0.2 *s* after the ball is hit and runs to the ball and manages to return it just before it hits the ground. Sania runs at a speed of 7.5 *m/s* and she had to reach 0.8 *m* forward, from where she stands, to hit the ball.

(a) At what distance Sania was standing from Maria at the time the ball was returned by Maria? Assume that Maria returned the ball directly towards Sania.

(b) With what speed did the ball hit the racket of Sania?

$[g = 9.8\ m/s^2]$

Q.25. A player initially at rest throws a ball with an initial speed $u = 19.5\ m/s$ at an angle $\theta = \sin^{-1}\left(\frac{12}{13}\right)$ to the horizontal. Immediately after throwing the ball he starts running to catch it. He runs with constant acceleration (a) for first 2 *s* and thereafter runs with constant velocity. He just manages to catch the ball at exactly the same height at which he threw the ball. Find 'a'. Take $g = 10\ m/s^2$. Do you think anybody can run at a speed at which the player ran?

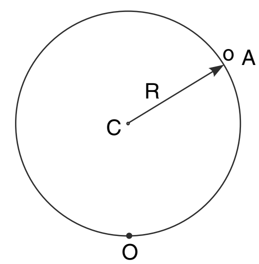
Q. 26. In a cricket match, a batsman hits the ball in air. A fielder, originally standing at a distance of 12 *m* due east of the batsman, starts running 0.6 s after the ball is hits. He runs towards north at a constant speed of 5 *m/s* and just manages to catch the ball 2.4 *s* after he starts running.

Assume that the ball was hit and caught at the same height and take $g = 10\ m/s^2$ $g = 10\ m/s^2$

Find the speed at which the ball left the bat and the angle that its velocity made with the vertical.

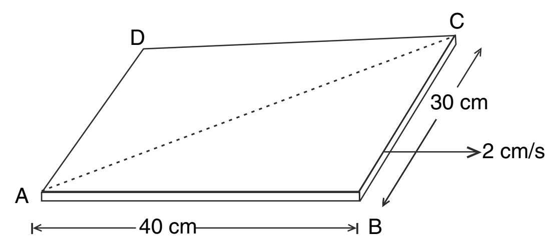
Q. 27. The time of flight, for a projectile, along two different paths to get a given range R, are in ratio 2 : 1. Find the ratio of this range R to the maximum possible range for the projectile assuming the projection speed to be same in all cases.

Q. 28. A boy 'A' is running on a circular track of radius R. His friend, standing at a point O on the circumference of the track is throwing balls at speed $u = \sqrt{gR}$. Balls are being thrown randomly in all possible directions. Find the length of the circumference of the circle on which the boy is completely safe from being hit by a ball.

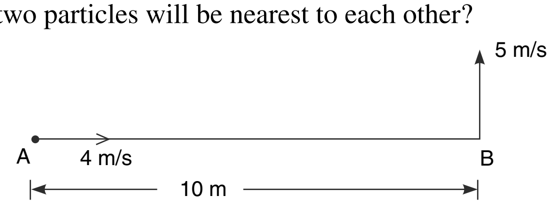


Q. 29. A rectangular cardboard $ABCD$ has dimensions of 40 *cm* × 30 *cm*. It is moving in a direction perpendicular to its shorter side at a constant speed of 2 *cm/s*. A small insect starts at corner A and moves to diagonally opposite corner C. On reaching C it immediately turns back and moves to A. Throughout the motion the insect maintains a constant speed relative to the board. It takes 10 *s* for the insect to reach C starting from A.

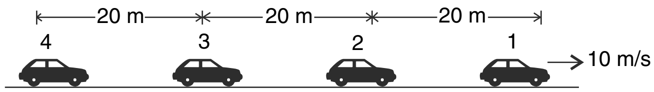
Find displacement and distance travelled by the insect in reference frame attached to the ground in the interval the insect starts from A and comes back to A.



Q. 30. Two particles A and B separated by 10 *m* at time $t = 0$ are moving uniformly. A is moving along line AB at a constant velocity of 4 *m/s* and B is moving perpendicular to the velocity of A at a constant velocity of 5 *m/s*. After what time the two particles will be nearest to each other?



Q. 31. Four cars are moving along a straight road in the same direction. Velocity of car 1 is 10 *m/s*. It was found that distance between car 1 and 2 is decreasing at a rate of 2 *m/s*, whereas driver in car 4 observed that he was nearing car 2 at a speed of 8 *m/s*. The gap between car 2 and 3 is decreasing at a rate of 3 *m/s*.



(a) If cars were at equal separations of 20 *m* at time $t = 0$, after how much time t_0 will the driver of car 2 see for the first time, that another car overtakes him?

(b) Which car will be first to overtake car 1?

Q. 32. Acceleration of a particle as seen from two reference frames 1 and 2 has magnitude 3 m/s^2 and 4 m/s^2 respectively. What can be magnitude of acceleration of frame 2 with respect to frame 1?

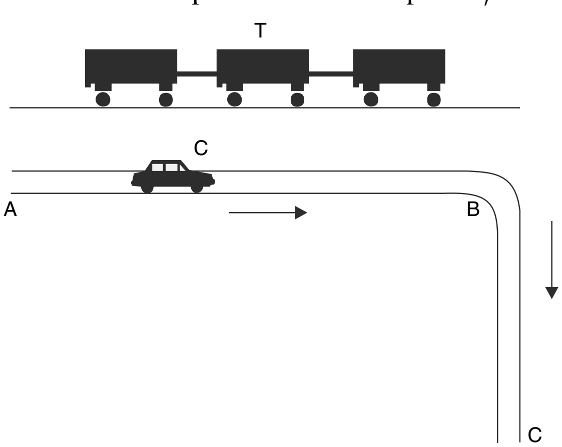
Q. 33. A physics professor was driving a Maruti car which has its rear wind screen inclined at $\theta = 37°$ to the horizontal. Suddenly it started raining with rain drops falling vertically. After some time the rain stopped and the professor found that the rear wind shield was absolutely dry. He knew that, during the period it was raining, his car was moving at a constant speed of $V_c = 20$ *km/hr*.

[tan 37° = 0.75]

(a) The professor calculated the maximum speed of vertically falling raindrops as V_{max}. What is value of V_{max} that he obtained.

(b) Plot the minimum driving speed of the car vs. angle of rear wind screen with horizontal (θ) so as to keep rain off the rear glass. Assume that rain drops fall at constant speed V_r

Q. 34.



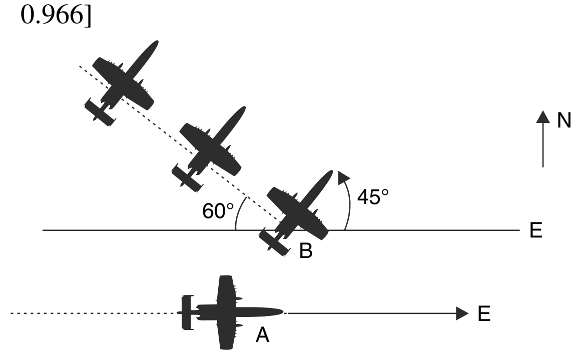
A train(T) is running uniformly on a straight track. A car is travelling with constant speed along section AB of the road which is parallel to the rails. The driver of the car notices that the train is having a speed of 7 *m/s* with respect to him. The car maintains the speed but takes a right turn at B and travels along BC. Now the driver of the car finds that the speed of train relative of him is 13 *m/s*. Find the possible speeds of the car.

35.

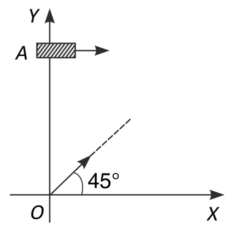
A police car B is chasing a culprit's car A. Car A and B are moving at constant speed V_1 = 108 *km/hr* and V_2 = 90 *km/hr* respectively along a straight line. The police decides to open fire and a policeman starts firing with his machine gun directly aiming at car A. The bullets have a velocity u = 305 *m/s* relative to the gun. The policeman keeps firing for an interval of $T_0 = 20$ *s*. The Culprit experiences that the time gap between the first and the last bullet hitting his car is Δt. Find Δt.

Q. 36. A chain of length L is supported at one end and is hanging vertically when it is released. All of the chain falls freely with acceleration g. The moment, the chain is released a ball is projected up with speed u from a point 2 L below the point of support. Find the interval of time in which the ball will cross through the entire chain.

Q. 37. Jet plane A is moving towards east at a speed of 900 *km/hr*. Another plane B has its nose pointed towards 45° N of E but appears to be moving in direction 60° N of W to the pilot in A. Find the true velocity of B. [sin 60° = 0.866 ; sin 75° = 0.966]



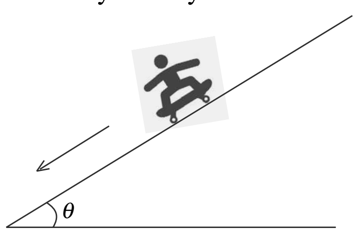
Q. 38. A small cart A starts moving on a horizontal surface, assumed to be x-y plane along a straight line parallel to x-axis (see figure) with a constant acceleration of 4 m/s^2. Initially it is located on the positive y-axis at a distance 9 m from origin. At the instant the cart starts moving, a ball is rolled along the surface from the origin in a direction making an angle 45° with the x-axis. The ball moves without friction at a constant velocity and hits the cart.



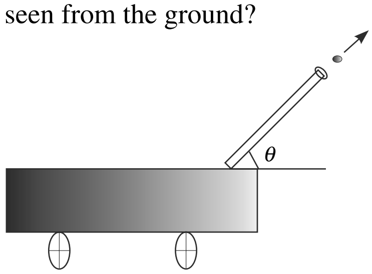
(a) Describe the path of the ball in a reference frame attached to the cart.

(b) Find the speed of the ball.

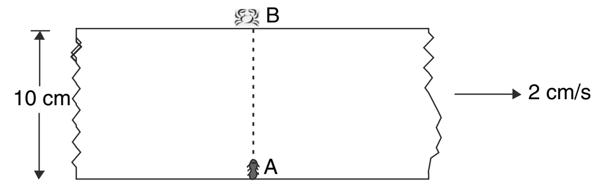
Q. 39. (a) A boy on a skateboard is sliding down on a smooth incline having inclination angle θ. He throws a ball such that he catches it back after time T. With what velocity was the ball thrown by the boy relative to himself ?



(b) Barrel of an anti aircraft gun is rotating in vertical plane (it is rotating up from the horizontal position towards vertical orientation in the plane of the fig). The length of the barrel is $L = \sqrt{2}\ m$ and barrel is rotating with angular velocity $\omega = 2\ rad/s$. At the instant angle θ is 45° a shell is fired with a velocity $2\sqrt{2}\ m/s$ with respect to the exit point of the barrel. The tank recoils with speed 4 m/s. What is the launch speed of the shell as seen from the ground?



Q. 40. long piece of paper is10 cm wide and is moving uniformly along its length with a velocity of 2 cm/s. An ant starts moving on the paper from point A and moves uniformly with respect to the paper. A spider was located exactly opposite to the ant just outside the paper at point B at the instant the ant started to move on the paper. The spider, without moving itself, was able to grab the ant 5 second after it (the ant) started to move. Find the speed of ant relative to the paper.



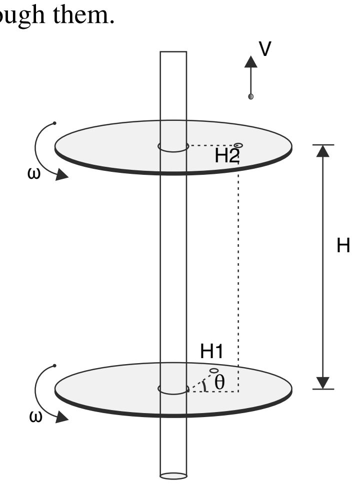
Q. 41. Two particles A and B are moving uniformly in a plane in two concentric circles. The time period of rotation is T_A = 8 minute and T_B = 11 minute respectively for the two particles. At time t = 0, the two particles are on a straight line passing through the centre of the circles. The particles are rotating in same sense. Find the minimum time when the two particles will again fall on a straight line passing through the centre.

Q. 42. A particle moves in xy plane with its position vector changing with time (t) as

$\vec{r} = (\sin t)\hat{i} + (\cos t)\hat{j}$ (in meter)

Find the tangential acceleration of the particle as a function of time. Describe the path of the particle.

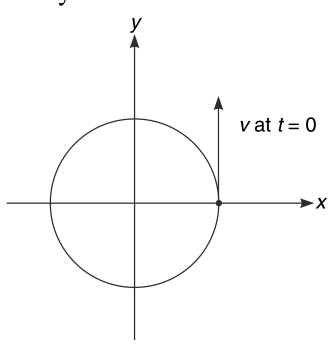
Q. 43. Two paper discs are mounted on a rotating vertical shaft. The shaft rotates with a constant angular speed ω and the separation between the discs is H. A bullet is fired vertically up so that it pierces through the two discs. It creates holes H1 and H2 in the lower and the upper discs. The angular separation between the two holes (measured with respect to the shaft axis) is θ. Find the speed (v) of the bullet. Assume that the speed of the bullet does not change while travelling through distance H and that the discs do not complete even one revolution in the interval the bullet pierces through them.



Q. 44. (a) A car moves around a circular arc subtending an angle of 60° at the centre. The car moves at a constant speed u_0 and magnitude of its instantaneous acceleration is a_0. Find the average acceleration of the car over the 60° arc.

(b) The speed of an object undergoing uniform circular motion is 4 *m/s*. The magnitude of the change in the velocity during 0.5 sec is also 4 *m/s*. Find the minimum possible centripetal acceleration (in m/s^2) of the object.

Q. 45. A particle is fixed to the edge of a disk that is rotating uniformly in anticlockwise direction about its central axis. At time $t = 0$ the particle is on the X axis at the position shown in figure and it has velocity v



(a) Draw a graph representing the variation of the x component of the velocity of the particle as a function of time.

(b) Draw the y-component of the acceleration of the particle as a function of time.

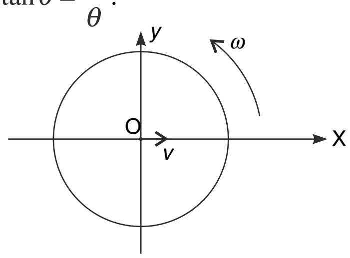
Q. 46. A disc is rotating with constant angular velocity ω in anticlockwise direction. An insect sitting at the centre (which is origin of our co-ordinate system) begins to crawl along a radius at time $t = 0$ with a constant speed V relative to the disc. At time $t = 0$ the velocity of the insect is along the X direction.

(a) Write the position vector $\left(\vec{r}\right)$ of the insect at time 't'.

(b) Write the velocity vector $\left(\vec{v}\right)$ of the insect at time 't'.

(c) Show that the X component of the velocity of the insect become zero when the disc has rotated through an angle θ given by

$$\tan\theta = \frac{1}{\theta}.$$



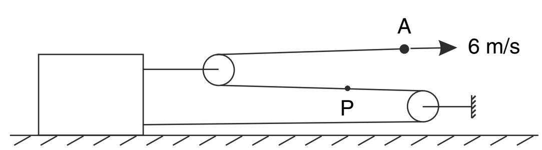
Q. 47. (a) A point moving in a circle of radius R has a tangential component of acceleration that is always n times the normal component of acceleration (radial acceleration). At a certain instant speed of particle is v_0. What is its speed after completing one revolution?

(b) The tangential acceleration of a particle moving in xy plane is given by $a_t = a_0 \cos\theta$. Where a_0 is a positive constant and θ is the angle that the velocity vector makes with the positive direction of X axis. Assuming the speed of the particle to be zero at $x = 0$, find the dependence of its speed on its x co-ordinate.

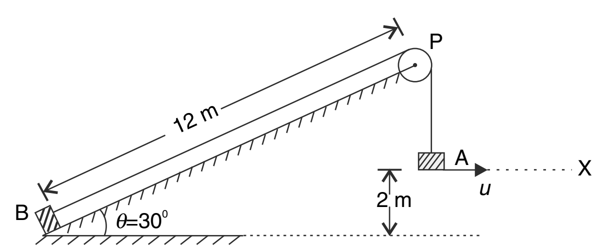
Q. 48. A particle is rotating in a circle. When it is at point A its speed is V. The speed increases to $2V$ by the time the particle moves to B. Find the magnitude of change in velocity of the particle as it travels from A to B. Also, find $\vec{V_A}.\vec{\Delta V}$; where $\vec{V_A}$ is its velocity at point A and $\vec{\Delta V}$ is change in velocity as it moves from A to B.

Q. 49. A particle starts from rest moves on a circle with its speed increasing at a constant rate of . Find the angle through which it 0.8 ms^{-2} would have turned by the time its acceleration becomes 1 ms^2.

Q. 50. In the arrangement shown in the fig, end A of the string is being pulled with a constant horizontal velocity of 6 *m/s*. The block is free to slide on the horizontal surface and all string segments are horizontal. Find the velocity of point P on the thread.



Q. 51. In the arrangement shown in the fig, block A is pulled so that it moves horizontally along the line AX with constant velocity u. Block B moves along the incline. Find the time taken by B to reach the pulley P if $u = 1 m/s$. The string is inextensible.



LEVEL 2

Q. 52. Two friends A and B are running on a circular track of perimeter equal to 40 m. At time $t = 0$ they are at same location running in the same direction. A is running slowly at a uniform speed of 4.5 km/hr whereas B is running swiftly at a speed of 18 km/hr.

(a) At what time t_0 the two friends will meet again?

(b) What is average velocity of A and B for the interval $t = 0$ to $t = t_0$?

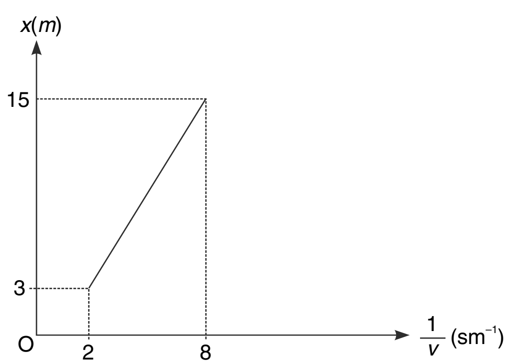
Q. 53. A particle is moving along x axis. Its position as a function of time is given by $x = x(t)$. Say whether following statements are true or false.

(a) The particle is definitely slowing down if

$$\frac{d^2x}{dt^2} > 0 \text{ and } \frac{dx}{dt} < 0$$

(b) The particle is definitely moving towards the origin if $\dfrac{d\left(x^2\right)}{dt} < 0$

Q. 54. Graph of position (x) vs inverse of velocity $\left(\dfrac{1}{v}\right)$ for a particle moving on a straight line is as shown. Find the time taken by the particle to move from $x = 3\ m$ to $x = 15\ m$.



Q. 55. Harshit and Akanksha both can run at speed v and walk at speed $u(u < v)$. They together start on a journey to a place that is at a distance equal to L. Akanksha walks half of the distance and runs the second half. Harshit walks for half of his travel time and runs in the other half.

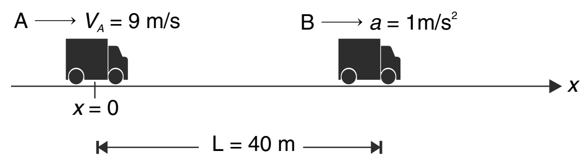
(a) Who wins?

(b) Draw a graph showing the positions of both Harshit and Akanksha versus time.

(c) Find Akanksha's average speed for covering distance L.

(d) How long does it take Harshit to cover the distance?

Q. 56. There are two cars on a straight road, marked as x axis. Car A is travelling at a constant speed of $V_A = 9\ m/s$. Let the position of the Car A, at time $t = 0$, be the origin. Another car B is $L = 40\ m$ ahead of car A at $t = 0$ and starts moving at a constant acceleration of $a = 1\ m/s^2$ (at $t = 0$). Consider the length of the two cars to be negligible and treat them as point objects.



(a) Plot the position–time (x–t) graph for the two cars on the same graph. The two graphs intersect at two points. Draw conclusion from this.

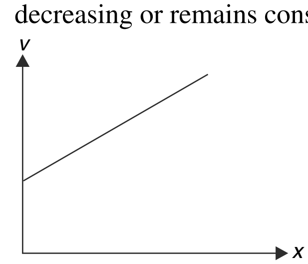
(b) Determine the maximum lead that car A can have.

Q. 57. Particle A is moving with a constant velocity of $V_A = 50\ ms^{-1}$ in positive x direction. It crossed the origin at time $t = 10\ s$. Another particle B started at $t = 0$ from the origin and moved with a uniform acceleration of $a_B = 2\ ms^{-2}$ in positive x direction.

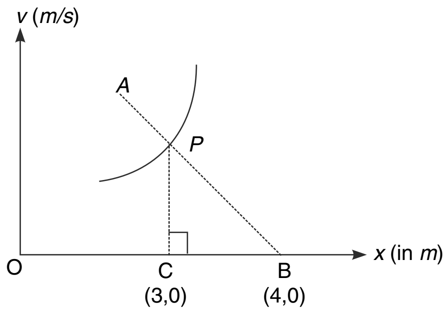
(a) For how long was A ahead of B during the subsequent journey?

(b) Draw the position (x) time (t) graph for the two particles and mark the interval for which A was ahead of B.

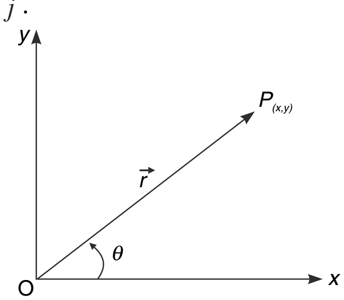
Q. 58. (a) A particle is moving along the x axis and its velocity vs position graph is as shown. Is the acceleration of the particle increasing, decreasing or remains constant?



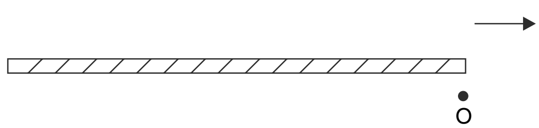
(b) A particle is moving along x axis and its velocity (v) vs position (x) graph is a curve as shown in the figure. Line APB is normal to the curve at point P. Find the instantaneous acceleration of the particle at $x = 3.0\ m$.



Q. 59. A particle has co-ordinates (x, y). Its position vector makes on angle θ with positive x direction. In an infinitesimally small interval of time the particle moves such that length of its position vector does not change but angle θ increases by $d\theta$. Express the change in position vector of the particle in terms of x, y, $d\theta$ and unit vectors $\hat{i}$ and $\hat{j}$.



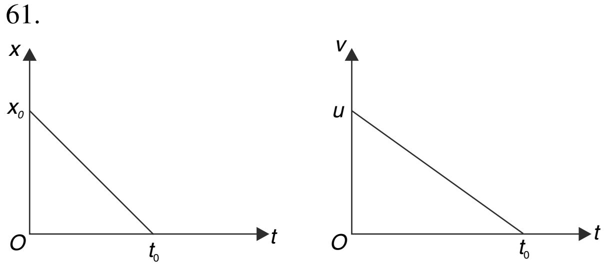
Q. 60. A rope is lying on a table with one of its end at point O on the table. This end of the rope is pulled to the right with a constant acceleration starting from rest. It was observed that last 2 m length of the rope took 5 s in crossing the point O and the last 1m took 2 s in crossing the point O.



(a) Find the time required by the complete rope to travel past point O.

(b) Find length of the rope.

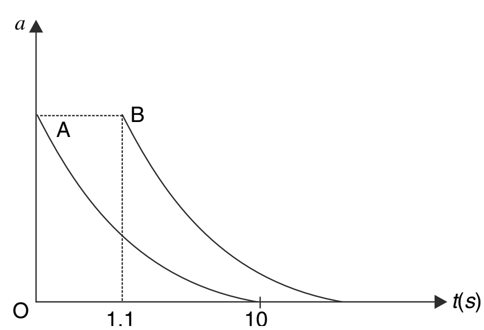
Q. 61.



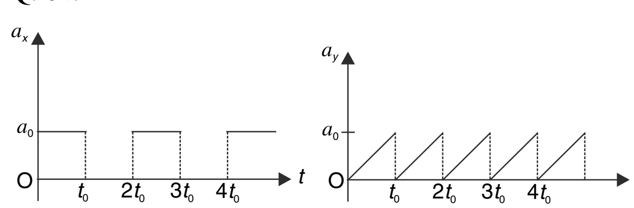
Two particles 1 and 2 move along the x axis. The position (x) - time (t) graph for particle 1 and velocity (v) - time (t) graph for particle 2 has been shown in the figure. Find the time when the two particles collide. Also find the position (x) where they collide. It is given that $x_0 = ut_0$, and that the particle 2 was at origin at $t = 0$.

Q. 62. Two stations A and B are 100km apart. A passenger train crosses station A travelling at a speed of 50 km/hr. The train maintains constant speed for 1 hour 48 minute and then the driven applies brakes to stop the train at station B in next 6 minute. Another express train starts from station B at the time the passenger train was crossing station A. The driver of the express train runs the train with uniform acceleration to attain a peak speed v_0. Immediately after the train attains the peak speed v_0, he applies breaks which cause the train to stop at station A at the same time the passenger train stops at B. Brakes in both the trains cause uniform retardation of same magnitude. Find the travel time of two trains and v_0.

Q. 63. Particle A starts from rest and moves along a straight line. Acceleration of the particle varies with time as shown in the graph. In 10 s the velocity of the particle becomes 60 m/s and the acceleration drops to zero. Another particle B starts from the same location at time $t = 1.1$ s and has acceleration – time relationship identical to A with a delay of 1.1 s. Find distance between the particles at time $t = 15$ s.



Q. 64.



A particle is moving in x–y plane. The x and y components of its acceleration change with time according to the graphs given in figure. At time $t = 0$, its velocity is v_0 directed along positive

y direction. If $a_0 = \frac{v_0}{t_0}$, find the angle that the velocity of the particle makes with x axis at time $t = 4t_0$.

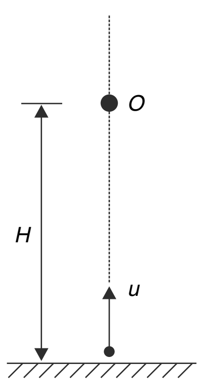
Q. 65. A particle is moving along positive x direction and experiences a constant acceleration of 4 m/s^2 in negative x direction. At time $t = 3$ second its velocity was observed to be 10 m/s in positive x direction.

(a) Find the distance travelled by the particle in the interval $t = 0$ to $t = 3$ s. Also find distance travelled in the interval $t = 0$ to $t = 7.5$ s..

(b) Plot the displacement – time graph for the interval $t = 0$ to 7.5 s.

Q. 66. A bead moves along a straight horizontal wire of length L, starting from the left end with velocity v_0. Its retardation is proportional to the distance that remains to the right end of the wire. Find the initial retardation (at left end of the wire) if the bead reaches the right end of the wire with a velocity $\frac{v_0}{2}$.

Q. 67. A ball is projected vertically up from the ground surface with an initial velocity of $u = 20$ m/s. O is a fixed point on the line of motion of the ball at a height of $H = 15$ m from the ground. Plot a graph showing variation of distance (s) of the ball from the fixed point O, with time (t). [Take $g = 10$ m/s^2]. Plot the graph for the entire time of flight of the ball.



Q. 68. Two bodies 1 and 2 of different shapes are released on the surface of a deep pond. The mass of the two bodies are $m_1 = 1$ kg and $m_2 = 1.2$ kg respectively. While moving through water, the bodies experience resistive force given as $R = bv$, where v is speed of the body and b is a positive constant dependent on shape of the body. For bodies 1 and 2 value of b is 2.5 kg/s and 3.0 kg/s respectively. Neglect all other forces apart from gravity and the resistive force, while answering following questions : [Hint : acceleration = force/mass]

(i) With what speed v_{10} and v_{20} will the two bodies hit the bed of the pond.

[Take $g = 10$ m/s^2]

(ii) Which body will acquire speed equal to half the terminal speed in less time.

Q. 69. A prototype of a rocket is fired from the ground. The rocket rises vertically up with a uniform acceleration of $\frac{5}{4}$ m/s^2. 8 second after the start a small nut gets detached from the rocket. Assume that the rocket keeps rising with the constant acceleration.

(a) What is the height of the rocket at the instant the nut lands on the ground

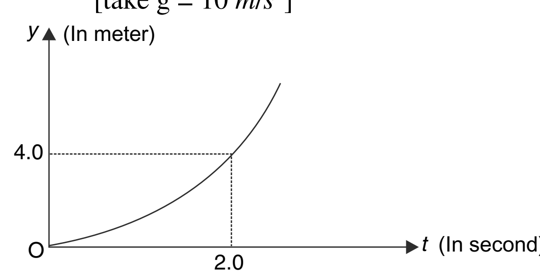
(b) Plot the velocity – time graph for the motion of the nut after it separates from the rocket till it hits the ground. Plot the same velocity–time graph in the reference frame of the rocket. Take vertically upward direction as positive and $g = 10$ m/s^2

Q. 70. An elevator starts moving upward with constant acceleration. The position time graph for the floor of the elevator is as shown in the figure. The ceiling to floor distance of the elevator is 1.5 m. At $t = 2.0$ s, a bolt breaks loose and drops from the ceiling.

(a) At what time t_0 does the bolt hit the floor?

(b) Draw the position time graph for the bolt starting from time $t = 0$.

[take g = 10 m/s^2]



Q. 71. At $t = 0$ a projectile is projected vertically up with a speed u from the surface of a peculiar planet. The acceleration due to gravity on the planet changes linearly with time as per equation $g = \alpha t$ where α is a constant.

(a) Find the time required by the projectile to attain maximum height.

(b) Find maximum height attained.

(c) Find the total time of flight.

Q. 72. A wet ball is projected horizontally at a speed of $u = 10\ m/s$ from the top of a tower $h = 31.25\ m$ high. Water drops detach from the ball at regular intervals of $\Delta t = 1.0\ s$ after the throw.

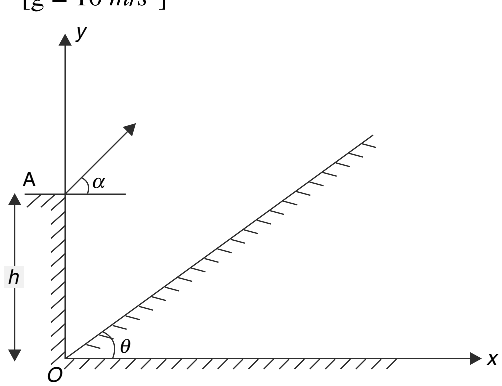
(a) How many drops will detach from the ball before it hits the ground.

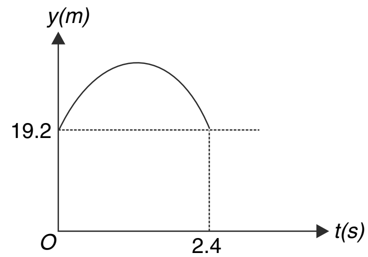
(b) How far away the drops strike the ground from the point where the ball hits the ground?

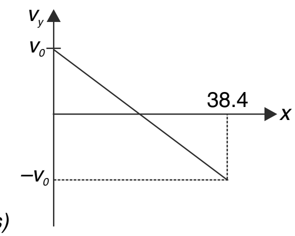
Q. 73. Two stones of mass m and M ($M > m$) are dropped Δt time apart from the top of a tower. Take time $t = 0$ at the instant the second stone is released. Let Δv and Δs be the difference in their speed and their mutual separation respectively. Plot the variation of Δv and Δs with time for the interval both the stones are in flight. [$g = 10\ m/s^2$]

Q. 74. A particle is moving in the xy plane on a sinusoidal course determined by $y = A \sin kx$, where k and A are constants. The X component of the velocity of the particle is constant and is equal to v_0 and the particle was at origin at time $t = 0$. Find the magnitude of the acceleration of the particle when it is at point having x co ordinate $x = \dfrac{\pi}{2k}$.

Q. 75. A ball is projected from a cliff of height $h = 19.2\ m$ at an angle α to the horizontal. It hits an incline passing through the foot of the cliff, inclined at an angle θ to the horizontal. Time of flight of the ball is $T = 2.4\ s$. Foot of the cliff is the origin of the co-ordinate system, horizontal is x direction and vertical is y direction (see figure). Plot of y co-ordinate vs time and y component of velocity of the ball (v_y) vs its x co-ordinate (x) is as shown. x and y are in m and time is in s in the graph. [g = 10 m/s^2]







(a) Find the angle of projection α

(b) Find the inclination (θ) of the incline.

(c) If the ball is projected with same speed but at an angle θ (= inclination of incline) to the horizontal, will it hit the incline above or below the point where it struck the incline earlier?

Q. 76. (i) A canon can fire shells at speed u. Inclination of its barrel to the horizontal can be changed in steps of $\Delta\theta = 1°$ ranging from $\theta_1 = 15°$ to $\theta_2 = 85°$. Let R_n be the horizontal range for projection angle $\theta = n°$.

$$\Delta R_n = |R_n - R_{n+1}|$$

For what value of n the value of ΔR_n is maximum? Neglect air resistance.

(ii) A small water sprinkler is in the shape of a hemisphere with large number of uniformly spread holes on its surface. It is placed on ground and water comes out of each hole with speed u. Assume that we mentally divide the ground into many small identical patches – each having area ΔS. What is the distance of a patch from the sprinkler which receives maximum amount of water ?

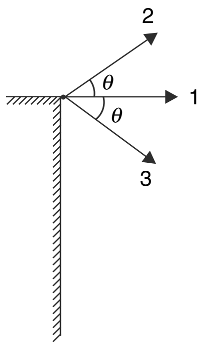
Q. 77. A gun fires a large number of bullets upward. Due to shaking of hands some bullets deviate as much as 1° from the vertical. The muzzle speed of the gun is 150 m/s and the height of gun above the ground is negligible. The radius of the head of the person firing the gun is 10cm. You can assume that acceleration due to gravity is nearly constant for heights involved and its value is g = 10 m/s^2. The gun fires 1000 bullets and they fall uniformly over a circle of radius r. Neglect air resistance.

You can use the fact $\sin\theta \simeq \theta$ when θ is small.

(a) Find the approximate value of r.

(b) What is the probability that a bullet will fall on the person's head who is firing?

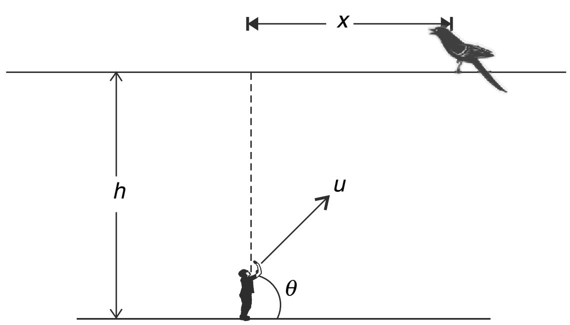
Q. 78. Three stones are projected simultaneously with same speed u from the top of a tower. Stone 1 is projected horizontally and stone 2 and stone 3 are projected making an angle θ with the horizontal as shown in fig. Before stone 3 hits the ground, the distance between 1 and 2 was found to increase at a constant rate u.



(a) Find θ

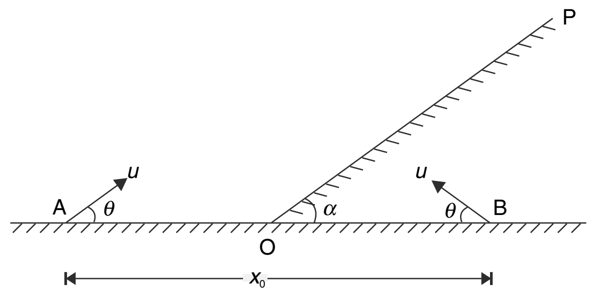
(b) Find the rate at which the distance between 2 and 3 increases.

Q. 79. A horizontal electric wire is stretched at a height $h = 10\ m$ above the ground. A boy standing on the ground can throw a stone at a speed $u = 20\ ms^{-1}$. Find the maximum horizontal distance x at which a bird sitting on the wire can be hit by the stone.



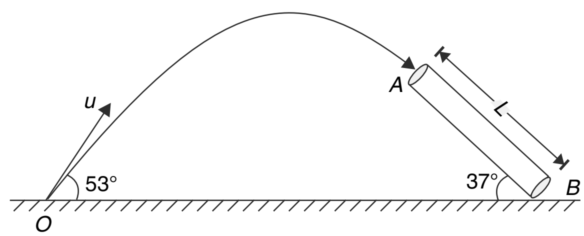
Q. 80. A wall OP is inclined to the horizontal ground at an angle α. Two particles are projected from points A and B on the ground with same speed (u) in directions making an angle θ to the horizontal (see figure). Distance between points A and B is $x_0 = 24\ m$. Both particles hit the wall elastically and fall back on the ground. Time of flight (time required to hit the wall and then fall back on to the ground) for particles projected from A and B are $4\ s$ and $2\ s$ respectively. Both the particles strike the wall perpendicularly and at the same location. [In elastic collision, the velocity component of the particle that is perpendicular to the wall gets reversed without change in magnitude]

(a) Calculate maximum height attained by the particle projected from A.



(b) Calculate the inclination of the wall to the horizontal (α) [$g = 10\ m/s^2$]

Q. 81. AB is a pipe fixed to the ground at an inclination of 37°. A ball is projected from point O at a speed of $u = 20 m/s$ at an angle of 53° to the horizontal and it smoothly enters into the pipe with its velocity parallel to the axis of the pipe. [Take $g = 10\ ms^{-2}$]

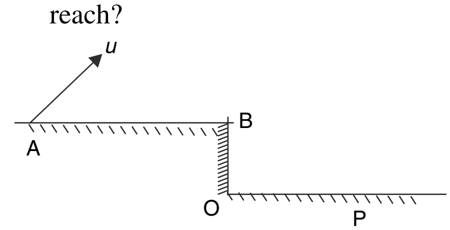


(a) Find the length L of the pipe

(b) Find the distance of end B of the pipe from point O.

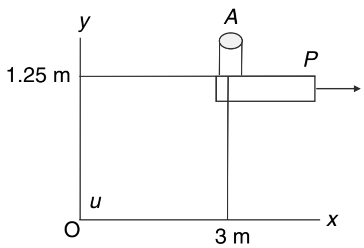
Q. 82. (a) A boy throws several balls out of the window of his house at different angles to the horizontal. All balls are thrown at speed $u = 10\ m/s$ and it was found that all of them hit the ground making an angle of 45° or larger than that with the horizontal. Find the height of the window above the ground [take $g = 10\ m/s^2$]

(b) A gun is mounted on an elevated platform AB. The distance of the gun at A from the edge B is $AB = 960\ m$. Height of platform is $OB = 960\ m$. The gun can fire shells with a velocity of $u = 100\ m/s$ at any angle. What is the minimum distance (OP) from the foot of the platform where the shell of gun can reach?

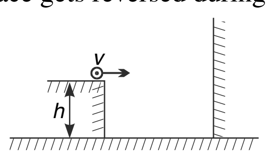


Q. 83 An object A is kept fixed at the point $x = 3\ m$

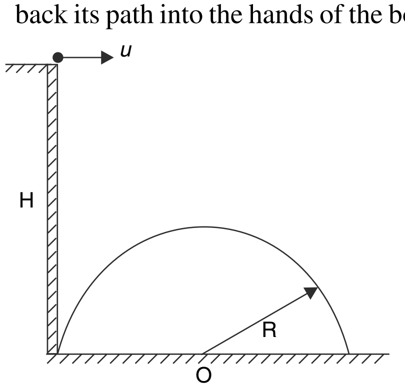
and $y = 1.25\ m$ on a plank P raised above the ground. At time $t = 0$ the plank starts moving along the $+\ x$ direction with an acceleration $1.5\ m/s^2$. At the same instant a stone is projected from the origin with a velocity u as shown. A stationary person on the ground observes the stone hitting the object during its downwards motion at an angle of 45° to the horizontal. All the motions are in x-y plane. Find u and the time after which the stone hits the object. Take $g = 10\ m/s^2$



Q. 84. (a) A particle is thrown from a height h horizontally towards a vertical wall with a speed v as shown in the figure. If the particle returns to the point of projection after suffering two elastic collisions, one with the wall and another with the ground, find the total time of flight. [Elastic collision means the velocity component perpendicular to the surface gets reversed during collision.]



(b) Touching a hemispherical dome of radius R there is a vertical tower of height $H = 4\ R$. A boy projects a ball horizontally at speed u from the top of the tower. The ball strikes the dome at a height $\frac{R}{2}$ from ground and rebounds. After rebounding the ball retraces back its path into the hands of the boy. Find u.



Q. 85. A city bus has a horizontal rectangular roof and a rectangular vertical windscreen. One day it was raining steadily and there was no wind.

(a) Will the quantity of water falling on the roof in unit time be different for the two cases (i) the bus is still (ii) the bus is moving with speed v on a horizontal road ?

(b) Draw a graph showing the variation of quantity of water striking the windscreen in unit time with speed of the bus (v).

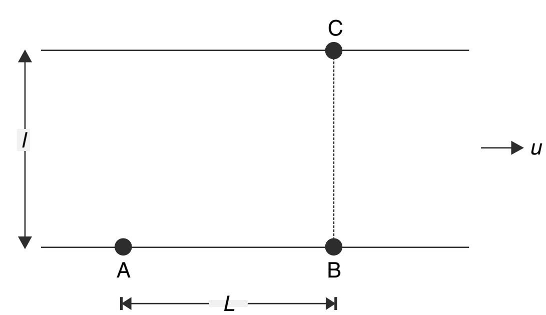
Q. 86. A truck is travelling due north descending a hill of slope angle $\theta = \tan^{-1}(0.1)$ at a constant speed of 90 km/hr. At the base of the hill there is a gentle curve and beyond that the road is level and heads 30° east of north. A south bound police car is travelling at 80 km/hr along the level road at the base of the hill approaching the truck. Find the velocity of the truck relative to police car in terms of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$. Take x axis towards east, y axis towards north and z axis vertically upwards.

Q. 87. Two persons A and B travelling at 60 km/hr^{-1} in their cars moving in opposite directions on a straight road observe an airplane. To the person A, the airplane appears to be moving perpendicular to the road while to the observe B the plane appears to cross the road making an angle of 45°.

(a) At what angle does the plane actually cross the road (relative to the ground).

(b) Find the speed of the plane relative to the ground.

Q. 88.

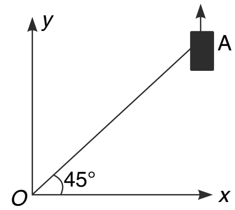


Two friends A and B are standing on a river bank L distance apart. They have decided to meet at a point C on the other bank exactly opposite to B. Both of them start rowing simultaneously on boats which can travel with velocity $V = 5\ km/hr$ in still water. It was found that both reached at C at the same time. Assume that path of both the boats are straight lines. Width of the river is $l = 3.0\ km$ and water is flowing at a uniform speed of $u = 3.0\ km/hr$.

(a) In how much time the two friends crossed the river.

(b) Find L.

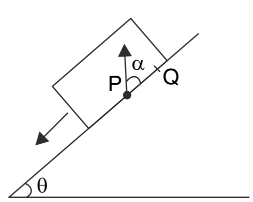
Q. 89. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3} - 1)\ m/s$. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.



(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

(b) Find the speed of the ball with respect to the surface, if $\phi = \dfrac{4\theta}{3}$.

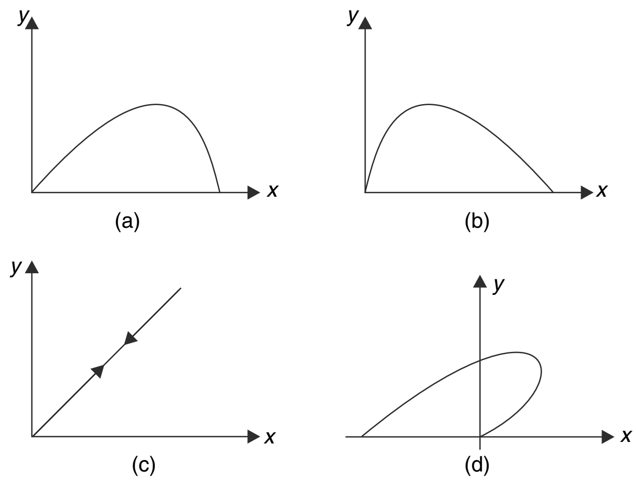
Q. 90. A large heavy box is sliding without friction down a smooth plane having inclination angle θ. From a point P at the bottom of a box, a particle is projected inside the box. The initial speed of the particle with respect to box is u and the direction of projection makes an angle α with the bottom as shown in figure



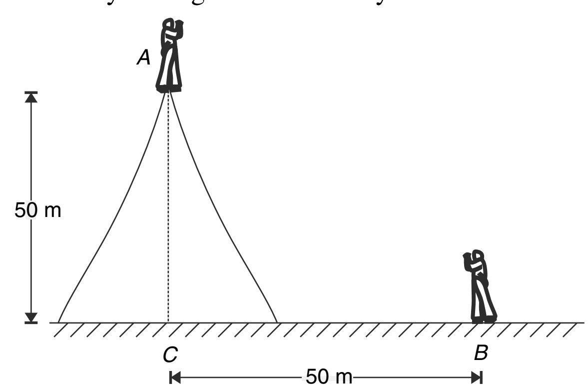
(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance)

(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

Q. 91. A ball is projected in vertical x–y plane from a car moving along horizontal x direction. The car is speeding up with constant acceleration. Which one of the following trajectory of the ball is not possible in the reference frame attached to the car? Give reason for your answer. Explain the condition in which other trajectories are possible. Consider origin at the point of projection.



Q. 92. A boy standing on a cliff 50 m high throws a ball with speed 40 m/s directly aiming towards a man standing on ground at B. At the same time the man at B throws a stone with a speed of 10 m/s directly aiming towards the boy.

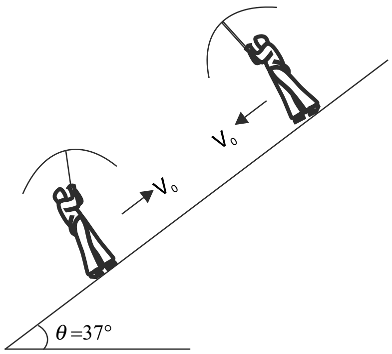


(a) Will the ball and the stone collide? If yes, at what time after projection?

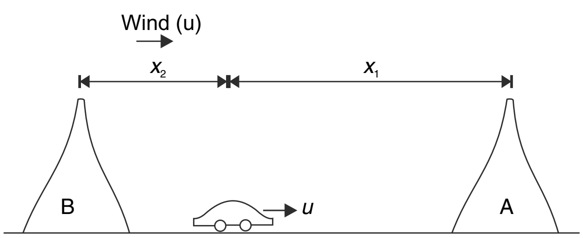
(b) At what height above the ground the two objects collide?

(c) Draw the path of ball in the reference frame of the stone.

Q. 93. A man walking downhill with velocity V_0 finds that his umbrella gives him maximum protection from rain when he holds it such that the stick is perpendicular to the hill surface. When the man turns back and climbs the hill with velocity V_0, he finds that it is most appropriate the hold the umbrella stick vertical. Find the actual speed of raindrops in terms of V_0. The inclination of the hill is $\theta = 37°$.

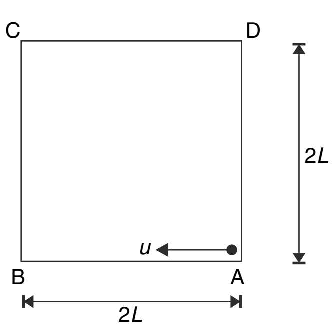


Q. 94. There are two hills A and B and a car is travelling towards hill A along the line joining the two hills. Car is travelling at a constant speed u. There is a wind blowing at speed u in the direction of motion of the car (i.e., from hill B to A). When the car is at a distance x_1 from A and x_2 from B it sounds horn (for very short interval). Driver hears the echo of horn from both the hills at the same time.



Find the ratio $\frac{x_1}{x_2}$ taking speed of sound in still air to be V.

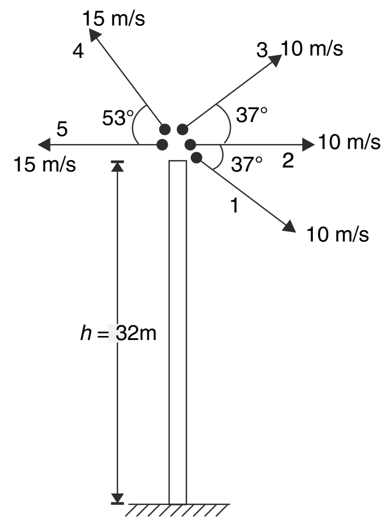
Q. 95. The figure shows a square train wagon $ABCD$ which has a smooth floor and side length of 2 L. The train is moving with uniform acceleration (a) in a direction parallel to DA. A 'ball is rolled along the floor with a velocity u, parallel to AB, with respect to the wagon. The ball passes through the centre of the wagon floor. At the instant it is at the centre, brakes are applied and the train begins to retard at a uniform rate that is equal to its previous acceleration (a)



(a) Will the ball hit the wall BC or wall CD or the corner C?

(b) What is speed of the ball, relative to the wagon at the instant it hits a wall ?

Q. 96. Five particles are projected simultaneously from the top of a tower that is $h = 32\ m$ high. The initial velocities of projection are as shown in figure. Velocity of 2 and 5 are horizontal.



(a) Which particle will hit the ground first?

(b) Separation between which two particles is maximum at the instant the first particle hits the ground?

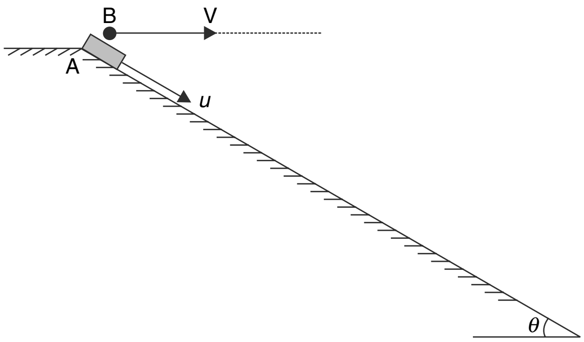
(c) Which two particles are last and last but one to hit the ground? Calculate the distance between these two particles (still in air), at a time 0.3s after the third particle lands on ground.

[$g = 10\ m/s^2,\ \tan 37° = \frac{3}{4}$]

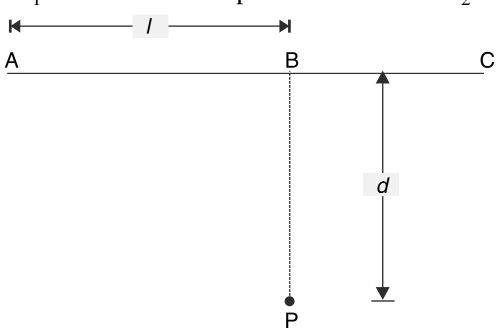
Q. 97. From the top of a long smooth incline a small body A is projected along the surface with speed u. Simultaneously, another small object B is thrown horizontally with velocity $v = 10\ m/s$, from the same point. The two bodies travel in the same vertical plane and body B hits body A on the incline. If the inclination angle of the incline is $\theta = \cos^{-1}\left(\frac{4}{5}\right)$ find

(a) the speed u with which A was projected.

(b) the distance from the point of projection, where the two bodies collide.



Q. 98. A man is on straight road AC, standing at A. He wants to get to a point P which is in field at a distance 'd' off the road (see figure). Distance AB is $l = 50$. The man can run on the road at a speed $v_1 = 5\ m/s$ and his speed in the field is $v_2 = 3\ m/s$.



(a) Find the minimum value of 'd' for which man can reach point P in least possible time by travelling only in the field along the straight line AP.

(b) If value of 'd' is half the value found in (a), what length the man must run on the road before entering the field, in order to reach 'P' in least possible time.

Q. 99. Two particles, A and B are moving in concentric circles in anticlockwise sense in the same plane with radii of the circles being $\gamma_A = 1.0\ m$ and $\gamma_B = 2.0\ m$ respectively. The particles move with same angular speed of $\omega = 4\ rad/s$.

Find the angular velocity of B as observed by A if

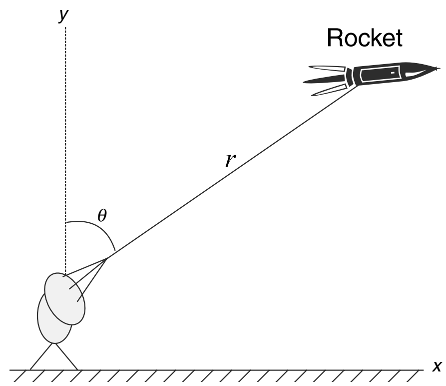
(a) Particles lie on a line passing through the centre of the circle.

(b) Particles lie on two perpendicular lines passing through the centre.

Q. 100. (a) An unpowered rocket is in flight in air. At a moment the tracking radar gives following data regarding the rocket.

r = distance of the rocket from the radar = $4000\ m$, $\dfrac{dr}{dt} = 0, \dfrac{d\theta}{dt}$ = 1.8 deg/sec;

where θ is the angle made by position vector of the rocket with respect to the vertical.



(a) Neglect atmospheric resistance and take $g = 9.8\ m/s^2$ at the concerned height. Neglect height of radar. Calculate the height of the rocket above the ground.

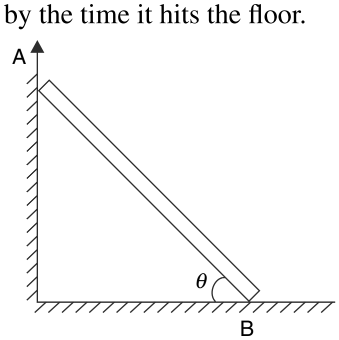
(b) Two points A and B are moving in X - Y plane with constant velocity of $V_A = \left(6\hat{i} - 9\hat{j}\right)\ m/s$ and $V_B = \left(\hat{i} + \hat{j}\right)\ m/s$ respectively. At time $t = 0$ they are 15 m apart and both of them lie on y axis with A lying away on positive Y axis with respect to B. What is the angular velocity of A with respect to B at $t = 1\ s$?

Q. 101. A stone is projected horizontally with speed u from the top of a tower of height h.

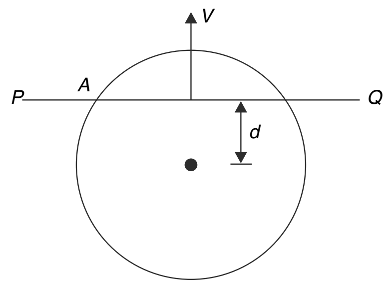
(a) Calculate the radius of curvature of the path of the stone at the point where its tangential and radial accelerations are equal.

(b) What shall be the height (h) of the tower so that radius of curvature of the path is always less than the value obtained in (a) above.

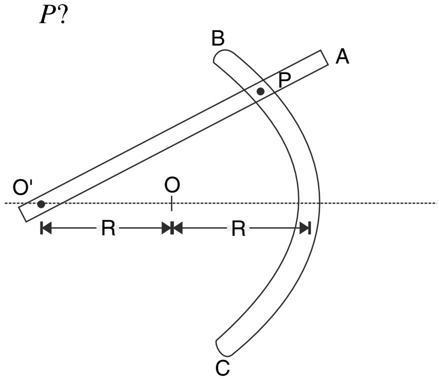
Q. 102. A stick of length $L = 2.0\ m$ is leaned against a wall as shown. It is released from a position when $\theta = 60°$. The end A of the stick remains in contact with the wall and its other end B remains in contact with the floor as the stick slides down. Find the distance travelled by the centre of the stick by the time it hits the floor.



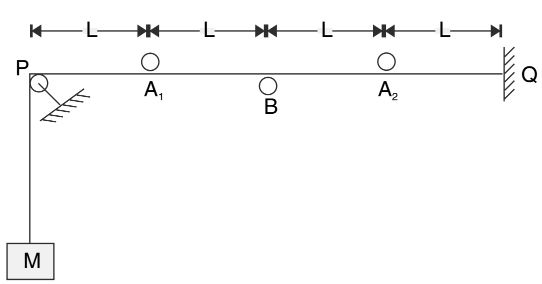
Q. 103. (a) A line PQ is moving on a fixed circle of radius R. The line has a constant velocity v perpendicular to itself. Find the speed of point of intersection (A) of the line with the circle at the moment the line is at a distance $d = R/2$ from the centre of the circle.



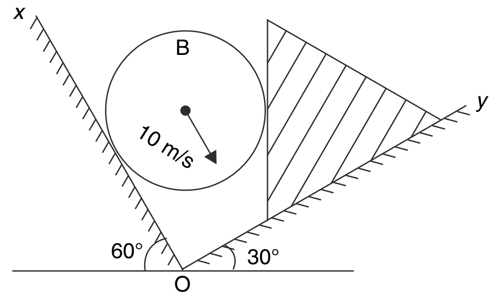
(b) In the figure shown a pin P is confined to move in a fixed circular slot of radius R. The pin is also constrained to remains inside the slot in a straight arm $O'A$. The arm moves with a constant angular speed ω about the hinge O'. What is the acceleration of point P?



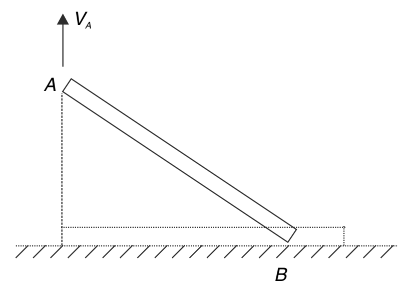
Q. 104. A flexible inextensible cord supports a mass M as shown in figure. A_1, A_2 and B are small pulleys in contact with the cord. At time $t = 0$ cord PQ is horizontal and A_1, A_2 start moving vertically down at a constant speed of v_1, whereas B moves up at a constant speed of v_2. Find the velocity of mass M as a function of time.



Q. 105. In the arrangement shown in the figure A is an equilateral wedge and the ball B is rolling down the incline XO. Find the velocity of the wedge (of course, along OY) at the moment velocity of the ball is 10 m/s parallel to the incline XO.



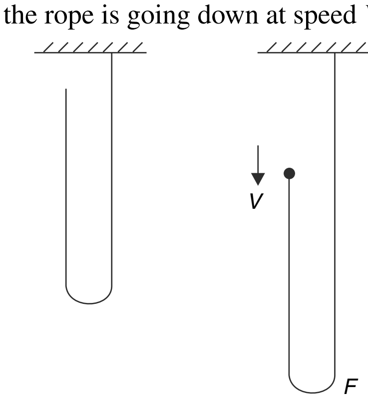
Q. 106. A meter stick AB is lying on a horizontal table. Its end A is pulled up so as to move it with a constant velocity $V_A = 4ms^{-1}$ along a vertical line. End B slides along the floor.



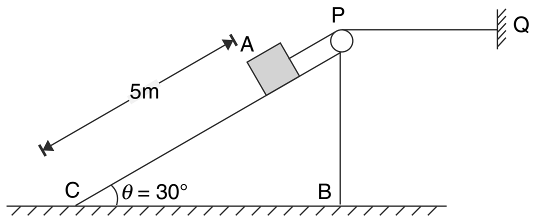
(a) After how much time (t_0) speed (V_B) of end B becomes equal to the speed (V_A) of end A ?

(b) Find distance travelled by the end B in time t_0.

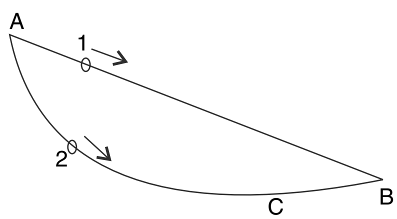
Q. 107. One end of a rope is fixed at a point on the ceiling the other end is held close to the first end so that the rope is folded. The second end is released from this position. Find the speed at which the fold at F is descending at the instant the free end of the rope is going down at speed V.



Q. 108. Block A rests on inclined surface of wedge B which rests on a horizontal surface. The block A is connected to a string, which passes over a pulley P (fixed rigidly to the wedge B) and its other end is securely fixed to a wall at Q. Segment PQ of the string is horizontal and Q is at a large distance from P. The system is let go from rest and the wedge slides to right as A moves on its inclined face. Find the distance travelled by A by the time it reaches the bottom of the inclined surface.

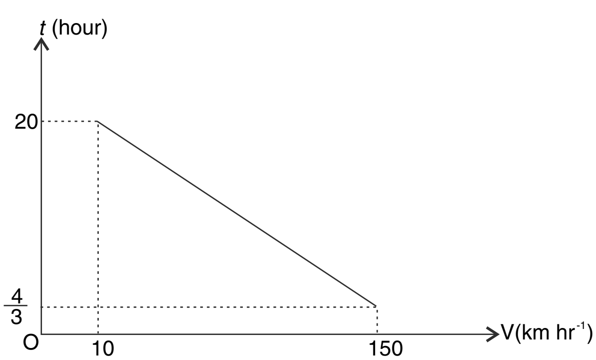


Q. 109. Two frictionless ropes connect points A & B in vertical plane. Bead 1 is allowed to slide along the straight rope AB and bead 2 slides along the curved rope ACB. Which bead will reach B in less time?



LEVEL 3

Q. 110. A car manufacturer usually tells a optimum speed (V_0) at which the car should be driven to get maximum mileage. In order to find the optimum speed for a new model, an engineer of the car company experimented a lot and finally plotted a graph between the extreme time t (defined as number of hours a tank full of petrol lasts) vs the constant speed V at which car was run.



(a) Calculate the optimum speed V_0 for this new model.

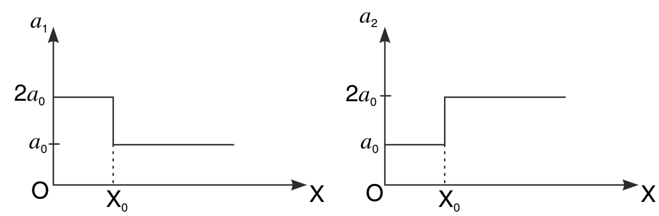
(b) If the fuel tank capacity of this car is 50 litre, what maximum mileage can be obtained from this car?

Q. 111. While starting from a station, a train driver was instructed to stop his train after time T and to cover maximum possible distance in that time.

(a) If the maximum acceleration and retardation for the train are both equal to 'a', find the maximum distance it can cover.

(b) Will the train travel more distance if maximum acceleration is 'a' but the maximum retardation caused by the brakes is '$2a$'? Find this distance.

Q. 112. Two particles 1 and 2 start simultaneously from origin and move along the positive X direction. Initial velocity of both particles is zero. The acceleration of the two particles depends on their displacement (x) as shown in fig.



(a) Particles 1 and 2 take t_1 and t_2 time respectively for their displacement to become x_0. Find $\frac{t_2}{t_1}$.

(b) Which particle will cover $2x_0$ distance in least time? Which particle will cross the point $x = 2x_0$ with greater speed?

(c) The two particles have same speed at a certain time after the start. Calculate this common speed in terms of a_0 and x_0.

Q. 113. A cat is following a rat. The rat is running with a constant velocity u. The cat moves with constant speed v with her velocity always directed towards the rat. Consider time to be $t = 0$ at an instant when both are moving perpendicular to each other and separation between them is L.

(a) Find acceleration of the cat at $t = 0$.

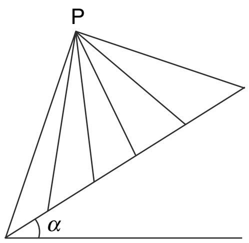
(b) Find the time t_0 when the rat is caught.

(c) Find the acceleration of the cat immediately before it catches the rat.

(d) Draw the path of the rat as seen by the cat.

Q. 114.(a) Prove that bodies starting at the same time $t = 0$ from the same point, and following frictionless slopes in different directions in the same vertical plane, all lie in a circle at any subsequent time.

(b) Using the above result do the following problem. *A* point *P* lies above an inclined plane of inclination angle α. *P* is joined to the plane at number of points by smooth wires, running in all possible directions. Small bodies (in shape of beads) are released from *P* along all the wires simultaneously. Which body will take least time to reach the plane.



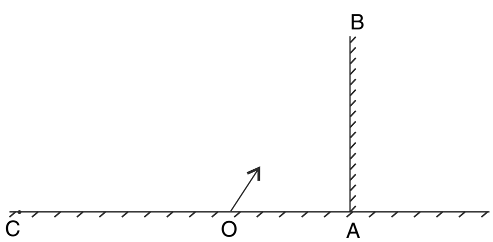
Q. 115. The acceleration due to gravity near the surface of the earth is $\vec{g}$. A ball is projected with velocity $\vec{u}$ from the ground.

(a) Express the time of flight of the ball.

(b) Write the expression of average velocity of the ball for its entire duration of flight.

Express both answers in terms of $\vec{u}$ and $\vec{g}$.

Q. 116. A ball is projected from point *O* on the ground. It hits a smooth vertical wall *AB* at a height *h* and rebounds elastically. The ball finally lands at a point *C* on the ground. During the course of motion, the maximum height attained by the ball is *H*.



(a) Find the ratio $\frac{h}{H}$ if $\frac{OA}{OC}=\frac{1}{3}$

(b) Find the magnitude of average acceleration of the projectile for its entire course of flight if it was projected at an angle of 45° to the horizontal.

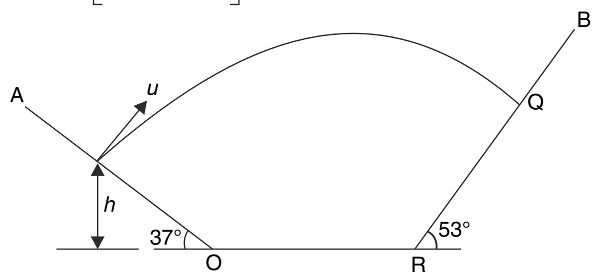
Q. 117. A boy can throw a ball up to a speed of $u = 30\ m/s$. He throws the ball many a times, ensuring that maximum height attained by the ball in each throw is $h = 20\ m$. Calculate the maximum horizontal distance at which a ball might have landed from the point of projection. Neglect the height of the boy. [$g = 10\ m/s^2$]

Q. 118. A valley has two walls inclined at 37° and 53° to the horizontal. A particle is projected from point *P* with a velocity of $u = 20\ m/s$ along a direction perpendicular to the incline wall *OA*. The Particle hits the incline surface RB perpendicularly at Q. Take $g = 10\ m/s^2$ and find:

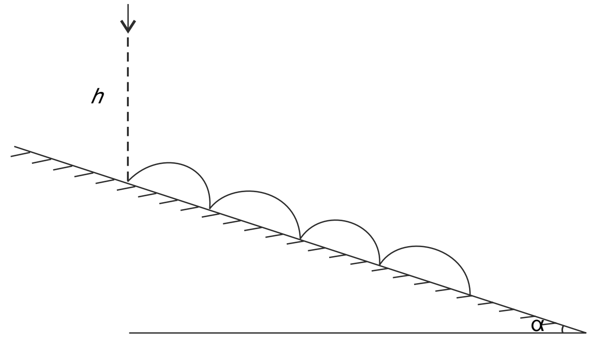
(a) The time of flight of the particle.

(b) Vertical height h of the point *P* from horizontal surface *OR*.

$\left[\tan 37° = \frac{3}{4}\right]$



Q. 119.



A ball is released in air above an incline plane inclined at an angle α to the horizontal. After falling vertically through a distance *h* it hits the incline and rebounds. The ball flies in air and then again makes an impact with the incline. This way the ball rebounds multiple times. Assume that collisions are elastic, i.e., the ball rebound without any loss in speed and in accordance to the law of reflection.

(a) Distance between the points on the incline where the ball makes first and second impact is l_1 and distance between points where the ball makes second and third impact is l_2. Which is large l_1 or l_2?

(b) Calculate the distance between the points on the incline where the ball makes second and fifth impact.

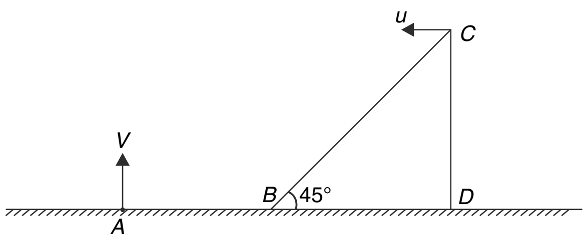
Q. 120. A terrorist '*A*' is walking at a constant speed of 7.5 *km/hr* due West. At time $t = 0$, he was exactly

South of an army camp at a distance of 1 km. At this instant a large number of army men scattered in every possible direction from their camp in search of the terrorist. Each army person walked in a straight line at a constant speed of 6 *km*/*hr*.

(a) What will be the closest distance of an army person from the terrorist in this search operation?

(b) At what time will the terrorist get nearest to an army person?

Q. 121. A large wedge *BCD*, having its inclined surface at an angle $\theta = 45°$ to the horizontal, is travelling horizontally leftwards with uniform velocity $u = 10\ m/s$



At some instant a particle is projected vertically up with speed $V = 20\ m/s$ from point A on ground lying at some distance right to the lower edge B of the wedge. The particle strikes the incline BC normally, while it was falling. [$g = 10\ m/s^2$]

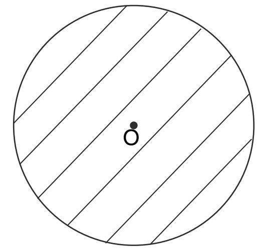
(a) Find the distance *AB* at the instant the particle was projected from *A*.

(b) Find the distance of lower edge *B* of the wedge from point *A* at the instant the particle strikes the incline.

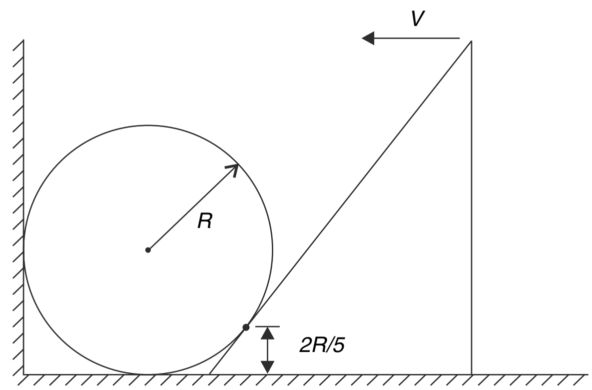
(c) Trace the path of the particle in the reference frame attached to the wedge.

Q. 122. The speed of river current close to banks is nearly zero. The current speed increases linearly from the banks to become maximum ($= V_0$) in the middle of the river. A boat has speed '*u*' in still water. It starts from one bank and crosses the river. Its velocity relative to water is always kept perpendicular to the current. Find the distance through which the boat will get carried away by the current (along the direction of flow) while it crosses the river. Width of the river is *l*.

Q. 123. A water sprinkler is positioned at *O* on horizontal ground. It issues water drops in every possible direction with fixed speed *u*. This way the sprinkler is able to completely wet a circular area of the ground (see fig). A horizontal wind starts blowing at a speed of $\frac{u}{2\sqrt{2}}$. Mark the area on the ground that the sprinkler will now be able to wet.



Q. 124. A cylinder of radius R has been placed in a corner as shown in the fig. A wedge is pressed against the cylinder such that its inclined surfaces touches the cylinder at a height of $\frac{2R}{5}$ from the ground. Now the wedge is pushed to the left at a constant speed $V = 15\ m/s$. With what speed will the cylinder move?

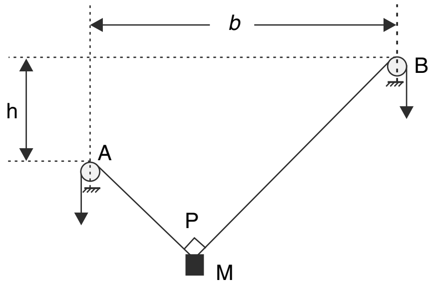


Q. 125. The entrance to a harbour consists of 50 *m* gap between two points *A* and *B* such that *B* is due east of *A*. Outside the harbour there is a 8 km/hr current flowing due east. A motor boat is located 300 m due south of A. Neglect size of the boat for answering following questions-

(a) Calculate the least speed (V_{min}) that the motor boat must maintain to enter the harbour.

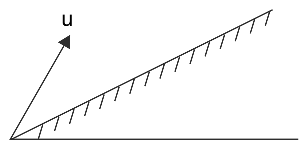
(b) Show that the course it must steer when moving at V_{min} does not depend on the speed of the current.

Q. 126. Two small pegs (*A* and *B*) are at horizontal and vertical separation *b* and *h* respectively. A small block of mass *M* is suspended with the help of two light strings passing over *A* and *B* as shown in fig. The two string are always kept at right angles (i.e., $\angle APB = 90°$). Find the minimum possible gravitation potential energy of the mass assuming the reference level at location of peg *A*. [Hint: the potential energy is minimum when the block is at its lowest position]



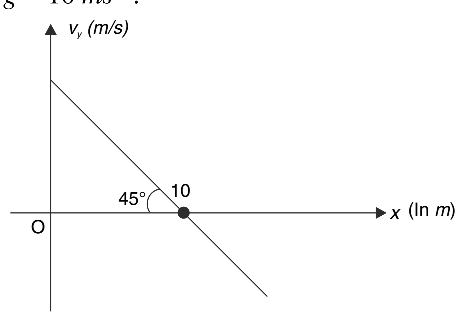
Q. 127. (a) A canon fires a shell up on an inclined plane. Prove that in order to maximize the range along the incline the shell should be fired in a direction bisecting the angle between the incline and the vertical. Assume that the shell fires at same speed all the time.

(b) A canon is used to hit a target a distance R up an inclined plane. Assume that the energy used to fire the projectile is proportional to square of its projection speed. Prove that the angle at which the shell shall be fired to hit the target but use the least amount of energy is same as the angle found in part (a)

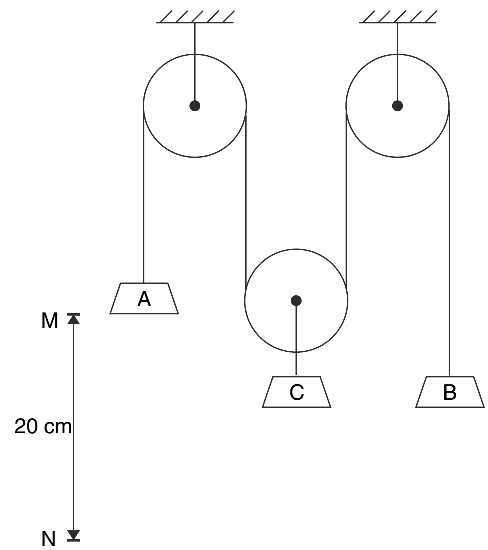


Q. 128. A ball of mass m is projected from ground making an angle θ to the horizontal. There is a horizontal wind blowing in the direction of motion of the ball. Due to wind the ball experiences a constant horizontal force of $\frac{mg}{\sqrt{3}}$ in direction of its motion. Find θ for which the horizontal range of the ball will be maximum.

Q. 129. A projectile is projected from a level ground making an angle θ with the horizontal (x direction). The vertical (y) component of its velocity changes with its x co-ordinate according to the graph shown in figure. Calculate θ. Take $g = 10\ ms^{-2}$.



Q. 130. In the arrangement shown in the figure, the block C begins to move down at a constant speed of 7.5 cm/s at time $t = 0$. At the same instant block A is made to start moving down at constant acceleration. It starts at M and its speed is 30 cm/s when it reaches N ($MN = 20\ cm$). Assuming that B started from rest, find its position, velocity and acceleration when block A reaches N.



Q. 131. A rocket prototype is fired from ground at time $t = 0$ and it goes straight up. Take the launch point as origin and vertically upward direction as positive x direction. The acceleration of the rocket is given by

$$a = \frac{g}{2} - kt^2;\ \ 0 < t \le t_0$$
$$= -g; \qquad t > t_0$$

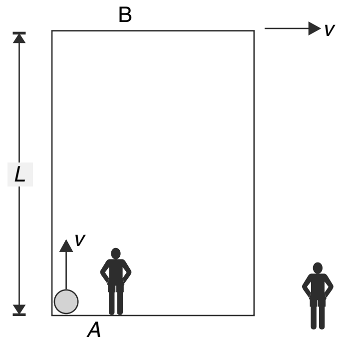
Where $t_0 = \sqrt{\frac{3g}{2k}}$

(a) Find maximum velocity of the rocket during the up journey.

(b) Find maximum height attained by the rocket.

(c) Find total time of flight.

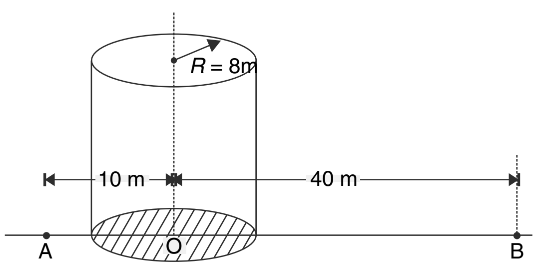
Q. 132. A man standing inside a room of length L rolls a ball along the floor at time $t = 0$. The ball travels at constant speed v relative to the floor, hits the front wall (B) and rebounds back with same speed v. The man catches the ball back at the wall A at time t_0. The ball travelled along a straight line relative to the man inside the room. Another observer standing outside the room found that the entire room was travelling horizontally at constant velocity v in a direction parallel to the two walls A and B.



(a) Find the average speed of the ball in the time interval $t = 0$ to $t = t_0$ as observed by the observer outside the room.

(b) If the room has acceleration in the direction of its velocity draw a sketch of the path of the ball as observed by the observer standing outside. Assume that velocity of room was v at the instant the ball was released.

Q. 133. There is a tall cylindrical building standing in a field. Radius of the cylinder is $R = 8\ m$. A boy standing at A (at a distance of 10 m from the centre of the cylindrical base of the building) knows that his friend is standing at B behind the building. The line joining A and B passes through the centre of the base of the building. Distance between A and B is 50 m. A wants to throw a ball to B but he realizes that the building is too tall and he cannot throw the ball over it. He throws the ball at a speed of 20 m/s such that his friend at B has to move minimum distance to catch it.



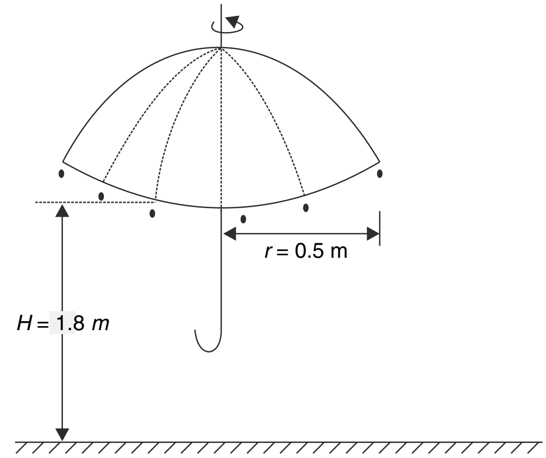
(a) What is the minimum distance that boy at B will have to move to catch the ball?

(b) At what angle to the horizontal does the boy at A throws the ball?

Assume that the ball is released and caught at same height above the ground.

[Take $g = 10\ m/s^2$ and $\sin^{-1}(0.75) \simeq 48.6°$

Q. 134. A wet umbrella is held upright (see figure). The man holding it is rotating it about its vertical shaft at an angular speed of $\omega = 5\ rad\ s^{-1}$. The rim of the umbrella has a radius of $r = 0.5\ m$ and it is at a height of $H = 1.8\ m$ from the floor. The man holding the umbrella gradually increases the angular speed to make it $2\ \omega$. Calculate the area of the floor that will get wet due to water drops spun off the rim and hitting the floor. [$g = 10\ m/s^2$]

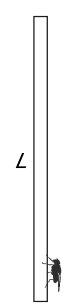


Q. 135. A ball is projected vertically up from ground. Boy A standing at the window of first floor of a nearby building observes that the time interval between the ball crossing him while going up and the ball crossing him while going down is t_1. Another boy B standing on the second floor notices that time interval between the ball passing him twice (during up motion and down motion) is t_2.

(a) Calculate the height difference (h) between the boy B and A.

(b) Assume that the height of boy A from the point of projection of the ball is also equal to h and calculate the speed with which the ball was projected.

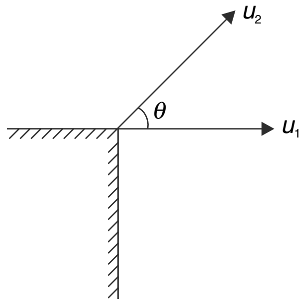
Q. 136. A stick of length L is dropped from a high tower. An ant sitting at the lower end of the stick begins to crawl up at the instant the stick is released. Velocity of the ant relative to the stick remains constant and is equal to u. Assume that the stick remains vertical during its fall, and length of the stick is sufficiently long.



(a) Calculate the maximum height attained by the ant measured from its initial position.

(b) What time after the start the ant will be at the same height from where it started?

Q. 137. Two balls are projected simultaneously from the top of a tall building. The first ball is projected horizontally at speed u_1 = 10 m/s and the other one is projected at an angle $\theta = \tan^{-1}\left(\frac{4}{3}\right)$ to the horizontal with a velocity u_2. [g = 10 m/s^2]



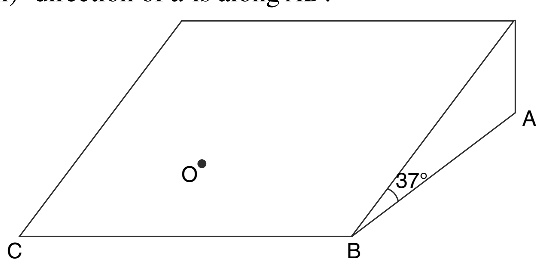
(a) Find minimum value of u_2 (= u_0) so that the velocity vector of the two balls can get perpendicular to each other at some point of time during their course of flight.

(b) Find the time after which velocities of the two balls become perpendicular if the second one was projected with speed u_0.

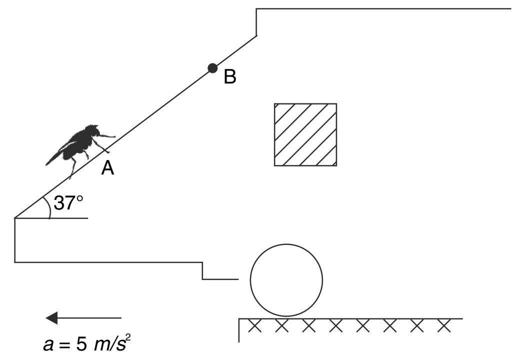
Q. 138. There is a large wedge placed on a horizontal surface with its incline face making an angle of 37° to the horizontal. A particle is projected in vertically upward direction with a velocity of u = 6.5 m/s from a point O on the inclined surface. At the instant the particle is projected, the wedge begins to move horizontally with a constant acceleration of a = 4 m/s^2. At what distance from point O will the particle hit the incline surface if

(i) direction of a is along BC?

(ii) direction of a is along AB?



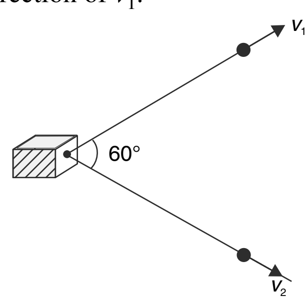
Q. 139. The windshield of a truck is inclined at 37° to the horizontal. The truck is moving horizontally with a constant acceleration of a = 5 m/s^2. At the instant the velocity of the truck is v_0 = 0.77 m/s, an insect jumps from point A on the windshield, with a velocity u = 2.64 m/s (relative to ground) in vertically upward direction. It falls back at point B on the windshield. Calculate distance AB. Assume that the insect moves freely under gravity and g = 10 m/s^2.



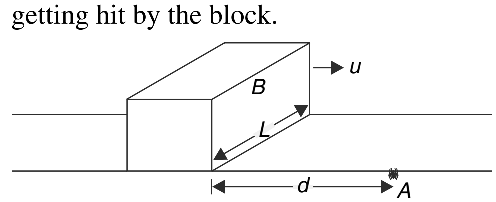
Q. 140. Two persons are pulling a heavy block with the help of horizontal inextensible strings. At the instant shown, the velocities of the two persons are v_1 and v_2 directed along the respective strings with the strings making an angle of 60° between them.

(a) Find the speed of the block at the instant shown.

(b) For what ratio of v_1 and v_2 the instantaneous velocity of the block will be along the direction of v_1.



Q. 141. A heavy block 'B' is sliding with constant velocity u on a horizontal table. The width of the block is L. There is an insect A at a distance d from the block as shown in the figure. The insect wants to cross to the opposite side of the table. It begins to crawl at a constant velocity v at the instant shown in the figure. Find the least value of v for which the insect can cross to the other side without getting hit by the block.



Q. 142. A projectile is thrown from ground at a speed v_0 at an angle α to the horizontal. Consider point of projection as origin, horizontal direction as X axis and vertically upward as Y axis. Let t be the time when the velocity vector of the projectile becomes perpendicular to its position vector.

(a) Write a quadratic equation in t.

(b) What is the maximum angle α for which the distance of projectile from the point of projection always keeps on increasing?

[Hint: Start from the equation you obtained in part (a)]

Q. 143. A projectile is thrown from a point on ground, with initial velocity u at some angle to the horizontal. Show that it can clear a pole of height h at a distance d from the point of projection if

$u^2 \geq g[h + \sqrt{h^2 + d^2}]$

Q. 144. A particle rotates in a circle with angular speed ω_0. A retarding force decelerates it such that angular deceleration is always proportional to square root of angular velocity. Find the mean angular velocity of the particle averaged over the whole time of rotation.

ANSWERS

1. The two velocities are perpendicular.

2. (a) 7.5 km/hr^{-1}

(b) 2 hr 40 min

3. (a) F

(b) T

(c) T

4. (a) E,

(b) D,G

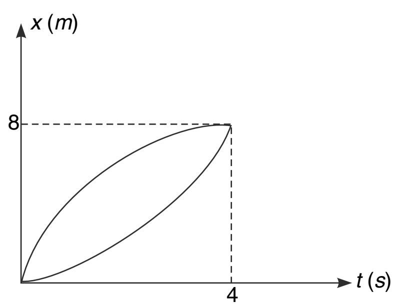
(c) B,C

5. (a) 4 m/s

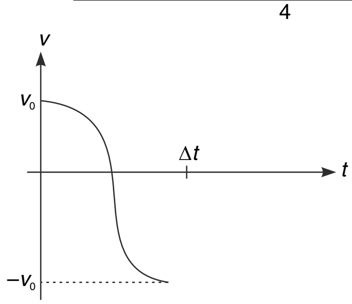
(b) 2 m/s^2

6. (a) $X_{max} = 4\ m$; $t = 2\ s$

(b)



7.



8. (i) B and C

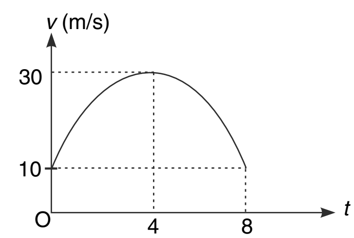
(ii) D

(iii) A, B, C, D

9. 10 m

10. $v = 12\ m/s$

11.



12. (a) particle A

(b) see solution for graph

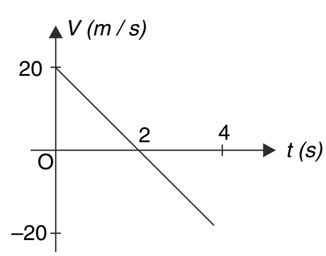
13. (a) $1 < t < 2\ s$ and $3 < t < 4\ s$

(b) 1 m/s

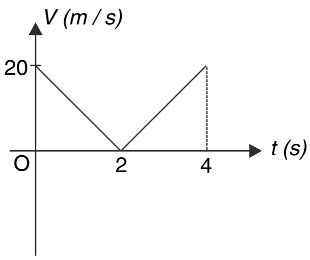
14. (a) 22 $(Km)\ (s^{-1})\ (MLy^{-1})$

(b) $\dfrac{\ln(2)}{H}$

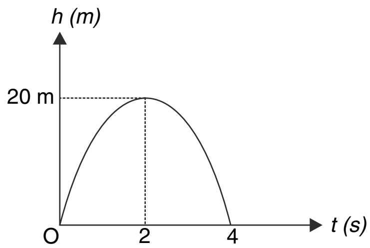
15. (a)



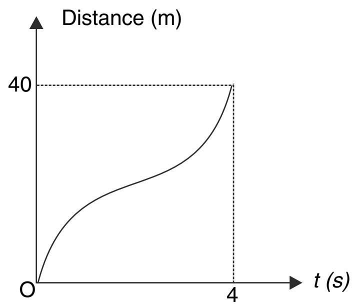
(b)



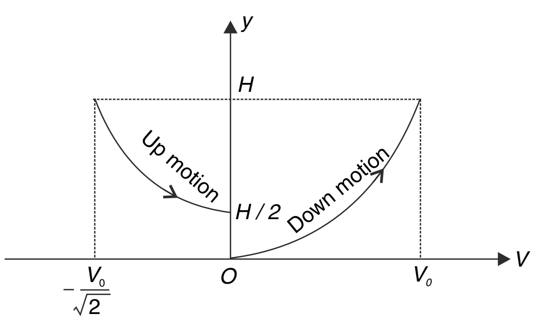
(c)



(d)



16.



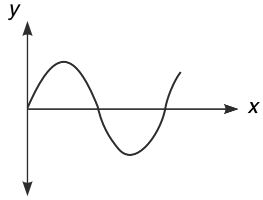
17. $V = 20\ ms^{-1}$

18. $\Delta t = \sqrt{\frac{2L}{g}}\ \left[\sqrt{2}-1\right]$

19. 1 m

20. All statements are true

21. (a)



(b) 40 m

22. The one that is projected at θ_2

$$\frac{R_1}{R_2} = \frac{\tan\theta_2}{\tan\theta_1}$$

23. $20(1 + \sqrt{2})\ m$

24. (a) 12.13 m

(b) 16 m/s

25. $a = 5.19\ m/s^2$

26. $u = 16\ m/s$; $\theta = \tan^{-1}\left(\frac{4\sqrt{2}}{15}\right)$

27. $\frac{4}{5}$

28. $\frac{4}{3}\pi R$

29. Displacement = 40 cm

Distance = $(30\sqrt{5} + 10\sqrt{13})\ cm$

30. $\frac{40}{41}s$

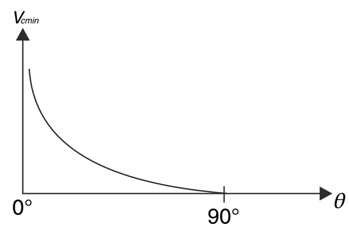
31. (a) $t_0 = 5\ s$

(b) car 4

32. $1\ m/s^2$ to $7\ m/s^2$

33. (a) $V_{max} = 12\ km/hr$

(b)



34. 5 m/s, 12 m/s

35. $\Delta t = 23.33\ s$

36. $\frac{L}{u}$

37. 807 kph

38. (a) Parabolic path

(b) 6 m/s

39. (a) $\frac{1}{2}Tg\cos\theta$ Perpendicular to the incline

(b) $4\sqrt{2}\ ms^{-1}$

40. $2\sqrt{2}\ cms^{-1}$

41. $\frac{88}{3}$ min

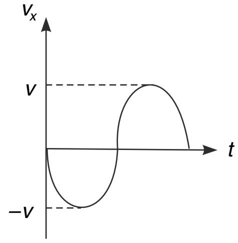
42. $a_t = 0$; path is circular

43. $v = \frac{\omega H}{\theta}$

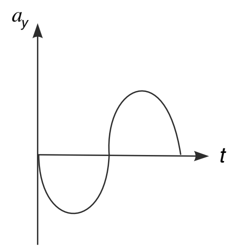
44. (a) $\langle a \rangle = \frac{3}{\pi} a_0$

(b) 8.37 m/s^2

45. (a)



(b)



46. (a) $\vec{r} = vt\,[\cos(\omega t)\,\hat{i} + \sin(\omega t)]\,\hat{j}$

(b) $\overrightarrow{V_p} = V\,[\cos(\omega t) - \omega t \sin(\omega t)]$
$\hat{i} + V\,[\sin(\omega t) + \omega t \cos(\omega t)]\,\hat{j}$

47. (a) $v_0 e^{2\pi n}$

(b) $V = \sqrt{2a_0 x}$

48. $\sqrt{3}\,\upsilon$, zero

49. $\frac{3}{8}$ *rad*

50. 2 *m/s*

51. 1.59 *s*

52. (a) $t_0 = \frac{32}{3} s$;

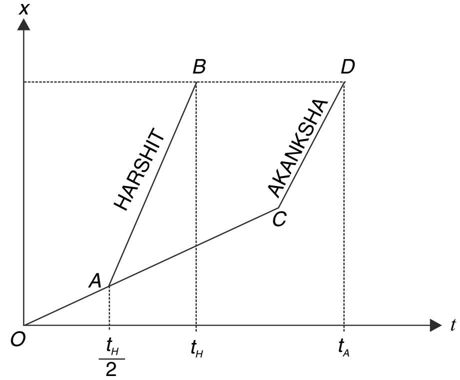
(b) $< V_A > = < V_B > = \frac{15\sqrt{3}}{8\pi} m/s$

53. Both are true

54. 60 *s*

55. (a) Harshit

(b)



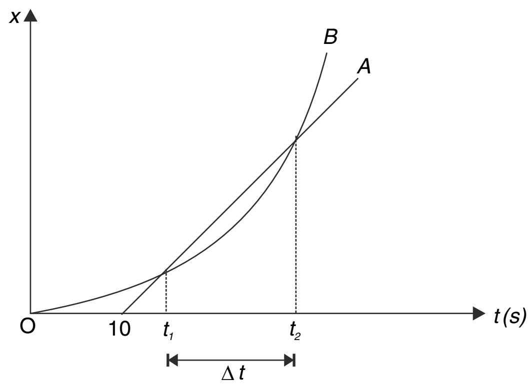
(c) $\frac{2uv}{u+v}$

(d) $\frac{2L}{u+v}$

56. (b) 0.5 *m*

57. (a) $10\sqrt{5}\,s$

(b)



58. (a) Acceleration is increasing

(b) 1 *m/s*^{2}

59. $\vec{\Delta} r = \left(-y\hat{i} + x\hat{j}\right) d\theta$

60. (a) 8.5 *s*

(b) 2.41 *m*

61. $t = (2 - \sqrt{2})t_0$; $x = (\sqrt{2} - 1)x_0$

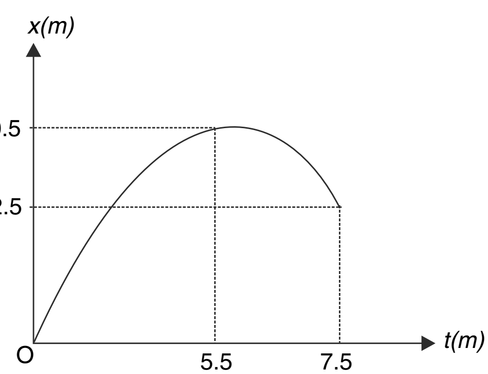
62. 2.2 *hr* ; 90.9 *km/hr*

63. 66 *m*

64. $\theta = \tan^{-1}\left(\frac{3}{2}\right)$

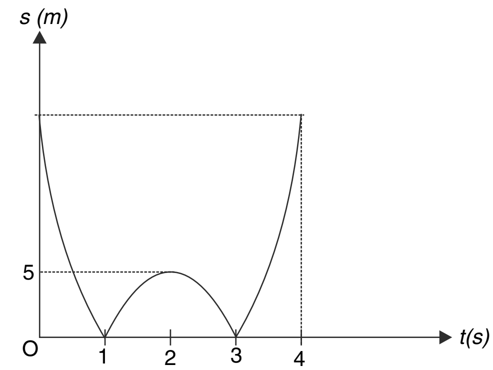
65. (a) 48 *m*, 68.5 *m*

(b)



66. $\frac{3v_0^2}{4L}$

67.

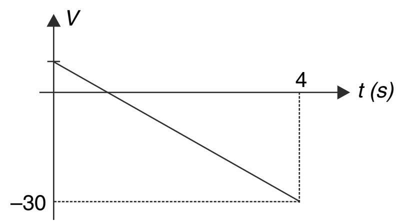


68. (i) $v_{10} = v_{20} = 4 m/s$

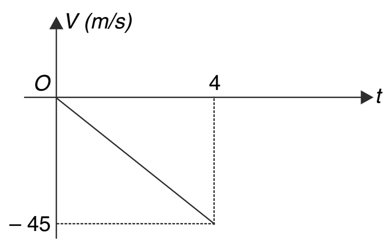
(ii) Both will take same time

69. (a) 90 m

(b)

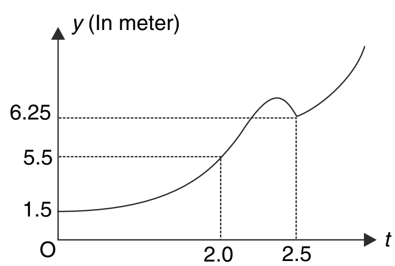


(c)



70. (a) 2.5 s

(b)



71. (a) $t_0 = \sqrt{\dfrac{2u}{\alpha}}$

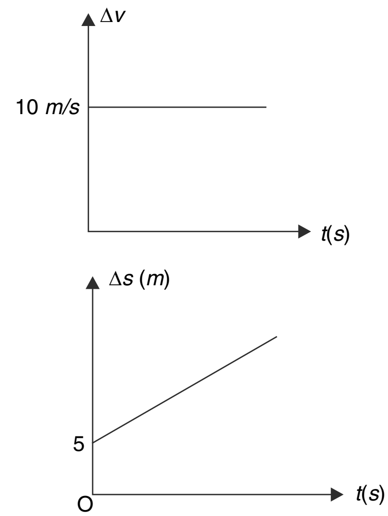
(b) $\dfrac{(2u)^{3/2}}{3\alpha^{1/2}}$

(c) $\sqrt{\dfrac{6u}{\alpha}}$

72. (a) 2

(b) zero

73.



74. $Ak_0v_0^{\;2}$

75. (a) $\alpha = \tan^{-1}\left(\dfrac{3}{4}\right)$

(b) $\theta = \tan^{-1}\dfrac{1}{2}$

(c) The ball will hit at a point lower than the earlier spot.

76. (i) $n = 84°$

(ii) $\dfrac{u^2}{g}$

77. (a) 80 m

(b) 1.6×10^{-3}

78. (a) $\theta = 60°$

(b) $\sqrt{3}\, u$

79. $20\sqrt{2}\, m$

80. (a) 11.25 m

(b) $\tan^{-1}\left(\dfrac{8}{5}\right)$

81. (a) $L = 14.58\, m$

(b) $OB = 41.66\, m$

82. (a) 5 m

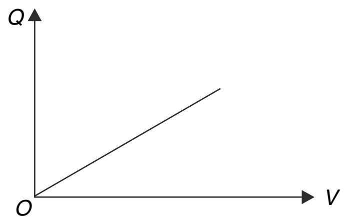
(b) 480 m

83. $u = 7.29\, m/s$, $t = 1\, s$.

84. (a) $2\sqrt{\dfrac{2h}{g}}$

(b) $u = \sqrt{21gR}$

85. (a) No

(b) 

86. $\left(40\hat{i}+158.9\hat{j}-8.9\hat{k}\right)$ $km\ hr^{-1}$

87. (a) $\theta = \tan^{-1}(2)$

(b) $60\sqrt{5}\ kmhr^{-1}$

88. (a) $\frac{3}{4}hr$

(b) 4.5 *km*

89. (a) 45°

(b) 2 *m/s*

90. (a) $\frac{u^2\sin 2\alpha}{g\cos\theta}$

(b) $\frac{u\cos(\alpha+\theta)}{\cos\theta}$

91. (b)

92. (a) yes, $\sqrt{2}\ s$

(b) zero

(c) straight line

93. $\frac{\sqrt{73}}{3}V_0$

94. $\frac{x_1}{x_2}=\frac{v+u}{v-u}$

95. (a) Corner C

(b) *u*

96. (a) particle 1

(b) Particle 2 and 5

(c) particle 3 and 4 ; 50.94 *m*

97. (a) $u = 8\ m/s$,

(b) 18.75 *m*

98. (a) $d_{\min}=\frac{200}{3}$

(b) 25 *m*

99. (a) $\omega = 4\ rad/s$

(b) $\omega = 4\ rad/s$

100. (a) 1600 *m*

(b) $\frac{3}{2}rad/\sec$

101. (a) $R=\frac{2\sqrt{2}u^2}{g}$

(b) $h<\frac{u^2}{2g}$

102. $\frac{\pi}{3}m$

103. (a) $\frac{2v}{\sqrt{3}}$

(b) $4\omega^2R$

104. $v=\frac{dy}{dt}=\frac{2v_1^2t}{\sqrt{L^2+v_1^2t^2}}+\frac{2(v_1+v_2)^2\,t}{\sqrt{L^2+(v_1+v_2)^2\,t^2}}$

105. $\frac{10}{\sqrt{3}}$ *m/s*

106. (a) $t_0=\frac{1}{4\sqrt{2}}s$

(b) $\left(1-\frac{1}{\sqrt{2}}\right)m$

107. *V*/2

108. 10 sin 15°

109. Bead 2

110. (a) 80 $kmhr^{-1}$

(b) 17 kml^{-1}

111. (a) $\frac{1}{4}aT^2$

(b) yes, $\frac{1}{3}aT^2$

112. (a) $\sqrt{2}$

(b) particle 1 will cover $2x_0$ in lesser time. Both will cross $2x_0$ with same speed.

(c) $v = (2+\sqrt{2})\sqrt{a_0x_0}$

113. (a) $\frac{uv}{L}$

(b) $t_0=\frac{vL}{v^2-u^2}$

(c) Zero

(d) The path will be like a spiral

114. (b) Body travelling along a line making an angle $\frac{\alpha}{2}$

with vertical

115. (a) $t = -\dfrac{2\overrightarrow{u}\cdot\overrightarrow{g}}{\left|\overrightarrow{g}\right|^2}$

(b) $\overrightarrow{V_{av}} = \vec{u} - \dfrac{\vec{g}\left(\vec{u}\cdot\vec{g}\right)}{\left|\vec{g}\right|^2}$

116. (a) $\dfrac{16}{25}$

(b) $\sqrt{2}\,g$

117. $40\sqrt{5}\ m$

118. (a) $2.5\ s$

(b) $4.05\ m$

119. (a) $l_2 > l_1$

(b) $72\,h\sin\alpha$

120. (a) $\dfrac{3}{5}km$

(b) 8 min

121. (a) $15\ m$

(b) $15\ m$

(c) parabolic

122. $\dfrac{V_0 l}{2u}$

123. A circle of same size shifted from the original circle by $\Delta X = \dfrac{u^2}{2g}$ in the direction of wind.

124. $20\ m/s$

125. (a) $\sqrt{\dfrac{48}{37}}\ km/hr$

126. $U_{\min} = -\dfrac{1}{2}Mg\ \left[\sqrt{h^2+b^2}-h\right]$

128. $\theta = 60°$

129. $\theta = 45°$

130. Position: 40 *cm* up from starting position

$V_B = 45\ cm/s\ (\uparrow)$

$a_B = 22.5\ cm/s^2\ (\uparrow)$

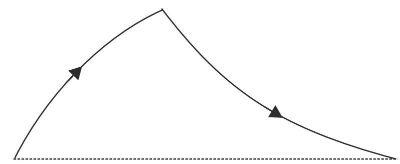
131. (a) $V_{\max} = \sqrt{\dfrac{g^3}{18k}}$

(b) $X_0 = \dfrac{3g^2}{16k}$

(c) $T = \dfrac{3}{2}\sqrt{\dfrac{3g}{2k}}$

132. (a) $\sqrt{2}\,v$

(b) path is as shown



133. (a) $40\ m$

(b) 24.3° or 65.7°

134. $21.2\ m^2$

135. (a) $h = \dfrac{g\,(t_1^2 - t_2^2)}{8}$

(b) $u = \dfrac{g}{2}\sqrt{2t_1^2 - t_2^2}$

136. (a) $H_{\max} = \dfrac{u^2}{2g}$

(b) $\dfrac{2u}{g}$

137. (a) $u_0 = 37.5\ m/s$

(b) $t = 1.5\ m/s$

138. (i) $3.38\ m$

(ii) $2.5\ m$

139. $AB = 0.57\ m$

140. (a) $\dfrac{2}{\sqrt{3}}\sqrt{v_1^2 + v_2^2 - v_1 v_2}$

(b) $\dfrac{v_1}{v_2} = 2$

141. $v_{\min} = \dfrac{uL}{\sqrt{d^2+L^2}}$

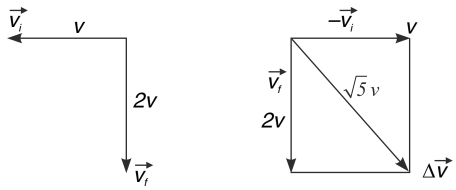
142. (a) $t^2 - \dfrac{3v_0\sin\alpha}{g}t + \dfrac{2v_0^2}{g^2} = 0$

(b) $\sin^{-1}\dfrac{8}{9}$

144. $\dfrac{\omega_0}{3}$

SOLUTIONS

1. The initial and final velocities are perpendicular to each other. (see figure).



2. 'A' must travel 20 km on bicycle & remaining 20 km on foot.

$\therefore$ Time of travel $t = \dfrac{20\ km}{15\ km\,hr^{-1}} + \dfrac{20\,km}{5\,km\,hr^{-1}} = \dfrac{4}{3} + 4 = \dfrac{16}{3}\ hr$.

(a) Average speed $V_{av} = \dfrac{40\,km}{\dfrac{16}{3}hr} = 7.5\ km/hr^{-1}$

(b) Time for with bicycle was unused

$= \dfrac{20\ km}{5\ km\,hr^{-1}} - \dfrac{20\ km}{15\ km\,hr^{-1}}$

$= 4 - \dfrac{4}{3} = \dfrac{8}{3} = 2hr\ 40$ min

3. (a) slope of X-t graph is not zero at $x = 0$. This means velocity is not zero.

(b) Magnitude of slope (= speed) is decreasing in the interval 0 to t_1. This means particle is retarding. In the interval t_1 to t_2 the magnitude of slope is increasing. This means it is accelerating.

(c) During 0 to t_1 velocity is negative but acceleration is positive. From t_1 to t_2 velocity is positive and acceleration is also positive. After this the velocity is positive but acceleration is negative.

4. (i) At indicated point E the slope is negative which means velocity is negative but the speed is decreasing. This means acceleration is opposite to velocity, i.e., in positive direction.

(ii) At D and G velocity is positive (slope is positive) but the slope is decreasing. This means acceleration is negative.

(iii) There is abrupt change in velocity at B and C which means the acceleration is large. This indicates that a large force has acted on the particle.

5. (a) $V_{av} = \dfrac{displacement}{time}$

Also $V_{av} = \dfrac{u+v}{2}$ for uniformly accelerated motion

$\therefore \dfrac{u+v}{2} = \dfrac{4}{2}$

$\dfrac{u+v}{2} = 2\ m/s$

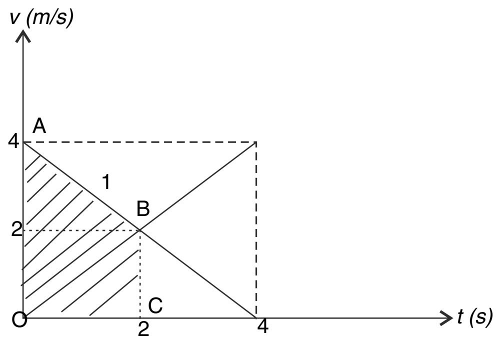
Hence u cannot be greater than 4 m/s

$\therefore u \le 4\ m/s$

(b) The speed of the particle decreases by a maximum of 4 m/s in 2 second.

$\therefore |a|_{\max} = 2 \ m/s^2$

6. (a)

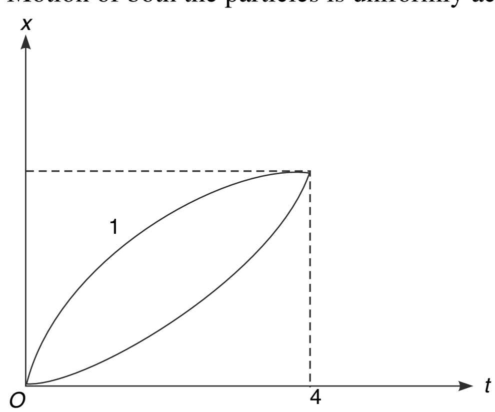


Separation will be maximum at $t = 2\ s$ when $V_1 = V_2$

Maximum separation = area ($OABC$) – area (OBC)

$= \frac{1}{2} \times 2 \times (2+4) - \frac{1}{2} \times 2 \times 2 = 4\ m$

(b) Motion of both the particles is uniformly accelerated /retarded. Hence, $X - t$ graph is parabolic



7. Hint: When the force (i.e., acceleration) is small the velocity time graph has a smaller slope. When force is maximum the slope of the graph becomes maximum.

9. After $t = 4\ s$, velocity of B becomes lesser than velocity of A and distance between the particles start decreasing. Separation is maximum at $t = 4\ s$

$d_{\max} = (X_A \text{ at } t = 4) - (X_B \text{ at } t = 4)$

$= \frac{1}{2} \times 4 \times 10 - \frac{1}{2} \times 2 \times 10 = 10\ m$

10. $V \frac{dv}{dx} = a$

$\int_0^v v dv = \int_{x=0}^{x=10} a dx$

$\frac{v^2}{2}$ = area under the $a - x$ graph

$= \frac{1}{2} \times 8 \times (8+10) = 72 \quad \therefore v = 12\ m/s$

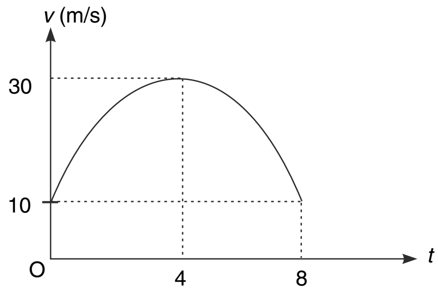
11. Change in velocity = area under a-t graph

For $0 \le t \le 4$

$$v - u = \frac{1}{2} \times 4 \times 10$$
$$v - 10 = 20$$
$$v = 30\ m/s$$

In interval 4 to 8 sec, the velocity decreases by 20 *m/s* and becomes equal to initial velocity (10 *m*/s).

Also, slope of *v-t* graph gives acceleration. Hence slope of *v-t* graph is decreasing from $t = 0$ to $t = 4\ s$. Afterwards, the slope becomes negative with increasing magnitude.



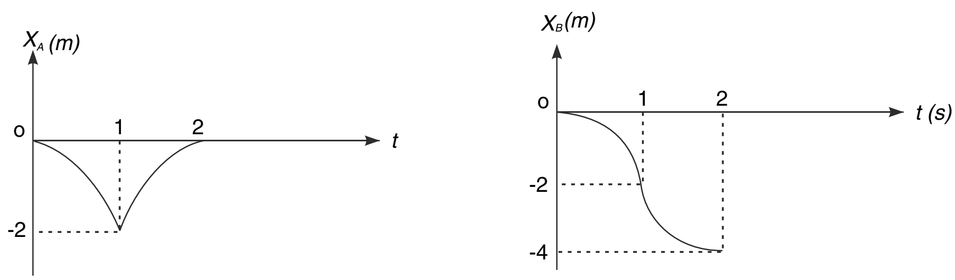
12. (a) Particle A has suffered a sudden change in its velocity at $t = 1\ s$.

It shows that it experienced tremendous acceleration, i.e., a big force.

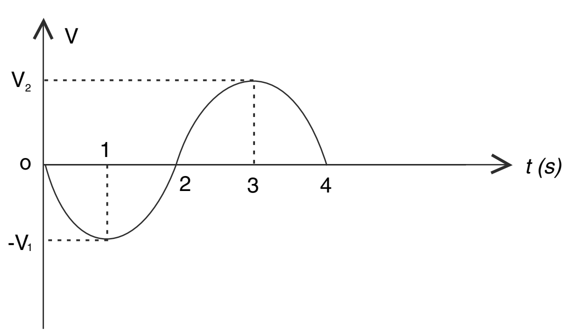
(b) In $o \le t < 1s$ both moved with a constant acceleration of $a = -4\ m/s^2$. For A the acceleration remains $-4\ m/s^2$ in the interval $1 < t \le 2\ s$

For B the acceleration in the later half was $+ 4\ m/s^2$

$x - t$ graph for constant acceleration motion is a parabola.



13. (a) The graph of $v = t^2 - 2t$ is a parabola with $v = 0$ at $t = 0$ and $t = 2\ s$. The graph of $v = -t^2 + 6t - 8$ is also a parabola with $v = 0$ at $t = 2\ s$ and $t = 4\ s$.



Particle is said to be retarding when its speed is decreasing. It is retarding during intervals $1 < t < 2\ s$ and $3 < t < 4\ s$

(b) $V_1 = \left|1^2 - 2(1) = 1\ m/s\right|$

$V_2 = -3^2 + 6(3) - 8 = 1\ m/s$

$\therefore$ Maximum speed = 1 m/s

14. (a) 1 Mega light year = $9.46 \times 10^{21}\ m$

$H.\ (km)\ (s^{-1})\ (Mly^{-1}) = 2.32 \times 10^{-18}\ s^{-1}$

$$\therefore H = \frac{2.32 \times 10^{-18}}{\left[10^3 \times 9.46 \times 10^{21}\right]^{-1}} = 21.95$$

(b) If r is instantaneous separation

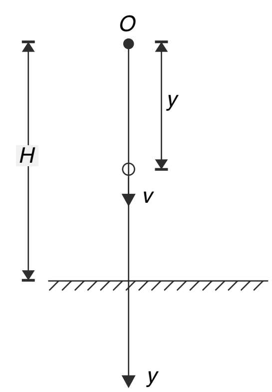
$V = Hr$

$$\frac{dr}{dt} = Hr$$

$$\int_r^{2r} \frac{dr}{r} = H\int_0^{t_0} dt$$

$$\text{In}(2) = Ht_0 \Rightarrow t_0 = \frac{\text{In}(2)}{H}$$

16. Downward journey



$$v^2 = 2gy \quad \text{or,} \quad y = \frac{v^2}{2g}$$

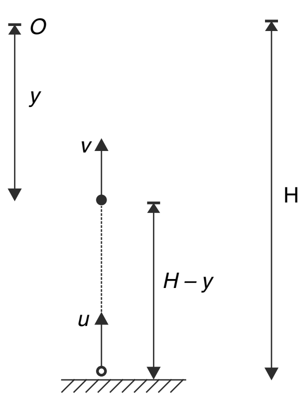
graph of y vs v will be parabolic with v positive.

Ball hits the ground with velocity

$v_0 = \sqrt{2gH}$

It rebounds with speed $u - \dfrac{v_0}{\sqrt{2}} = \sqrt{gH}$

Upward journey

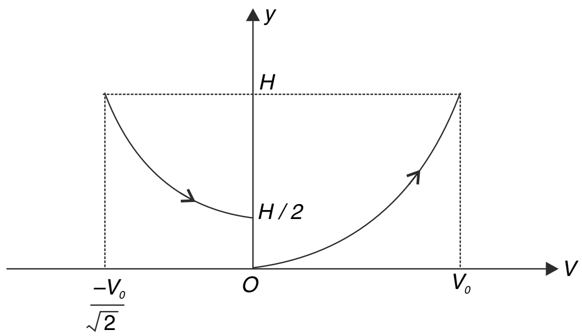


$v^2 = u^2 - 2g\ (H - y)$

$$y = \frac{v^2 - u^2}{2g} + H$$

(with v negative)

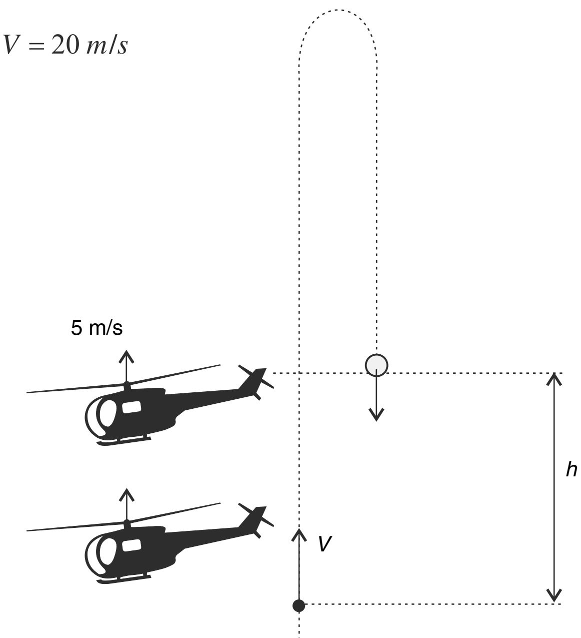
graph is again a parabola



17. In 3 *s* the helicopter rises, by $h = 5 \times 3 = 15$ *m*. It means the ball is at a height of 15 *m* from its initial point of projection after 3 *s*.

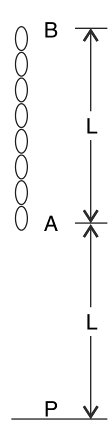
$$y = ut + \frac{1}{2} a\, t^2$$

$$15 = V \times 3 - \frac{1}{2} \times 10 \times 3^2 \Rightarrow V = 20\ m/s$$



18. End '*A*' pass through point P at time t_1 given by

$$L = \frac{1}{2} g\, t_1^{\,2} \Rightarrow t_1 = \sqrt{\frac{2L}{g}}$$



End '*B*' pass through point P at time t_2 given by

$$2L = \frac{1}{2} g\, t_2^{\,2}$$

$$t_2 = 2\sqrt{\frac{L}{g}}$$

$\therefore$ Required interval $\Delta t = t_2 - t_1$

$$= \sqrt{\frac{2L}{g}} \quad \left[\sqrt{2} - 1\right]$$

19. $h = \frac{1}{2}gt^2$

$\therefore\ h_1 = \frac{1}{2}g_1 t^2.$ ......(1)

$\Delta h_1 = \frac{1}{2}t^2\,\Delta g_1$ ......(2)

(2) ÷ (1)

$$\frac{\Delta h_1}{h_1} = \frac{\Delta g_1}{g_1}$$

$$\therefore\ h_1 = \Delta h_1\left(\frac{g_1}{\Delta g_1}\right)$$

$$h_1 = \Delta h_1\left(\frac{1}{\frac{\Delta g_1}{g_1}}\right) = \left(10^{-3}\right)\left(\frac{1}{0.001}\right) \qquad \left[\because \frac{\Delta g_1}{g_1}\times 100 = 0.1\right]$$

= 1.0 m

21. (a) Hint: Eliminate t between the equations to get the trajectory equation**.**

(b) Distance can be obtained if we know the speed.

Velocity $\vec{V} = \frac{d\vec{r}}{dt} = \left(8t-16\right)\hat{i} + \left(6t-12\right)\hat{j}$

Speed $v = \sqrt{\left(8t-16\right)^2 + \left(6t-12\right)^2} = 5(4-t)$

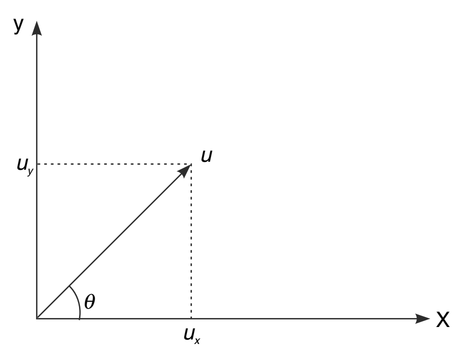
[since speed is positive don't write it as 5 (t – 4)]

Distance = $\int_0^4 v\,dt = 40\ m$

22. The two projectiles attain the same height. It means

$u_{y1} = u_{y2}$.

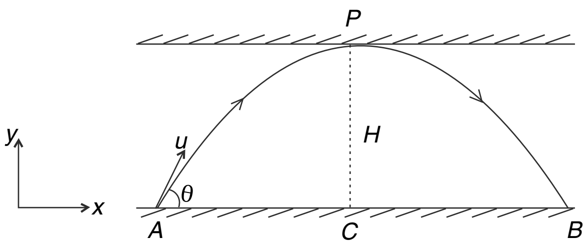
Also $u_x = \frac{u_y}{\tan\theta}$ $\quad \therefore \frac{u_{x1}}{u_{x2}} = \frac{u_{y1}/\tan\theta_1}{u_{y2}/\tan\theta_2} = \frac{\tan\theta_2}{\tan\theta_1}$



Range = $\frac{2u_x u_y}{g}$

$$\therefore \frac{R_1}{R_2} = \frac{u_{x1}}{u_{x2}} = \frac{\tan\theta_2}{\tan\theta_1}$$

23.



$$u_x = u\cos 37° = 25 \times \frac{4}{5} = 20\ m/s$$

$$u_y = u\sin 37° = 10 \times \frac{3}{5} = 15\ m/s$$

Let the ball hit the roof at time 't'

$$H = u_y t - \frac{1}{2} g\, t^2$$

$$10 = 15t - \frac{1}{2} \times 10t^2 \Rightarrow t^2 - 3t + 2 = 0$$

$$\Rightarrow t = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} \Rightarrow t = 1s, 2s$$ [2 s is unacceptable. Why?]

$\therefore AC = (u_x)\ (1s) = 20\ m$

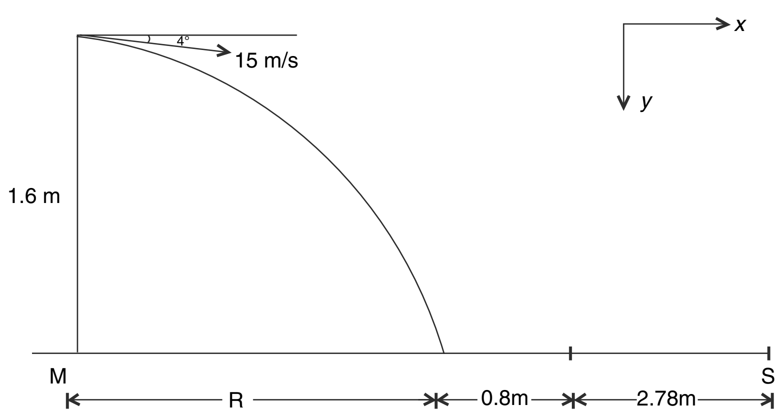
After collision at P, vertical component of velocity is zero. Time of travel from P to B is given by

$$t' = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 10}{10}} = \sqrt{2}s$$

$\therefore CB = u_x t' = 20\sqrt{2}$

$\therefore AB = 20 + 20\sqrt{2} = 20\ (1 + \sqrt{2})\ m$

24.



<u>Time of flight of the ball</u>

$$1.6 = (15\sin 4°)t + \frac{1}{2}gt^2 \quad \text{or } 1.6 \simeq \frac{1}{2} \times 9.8 \times t^2 \quad [\because\ 15\sin 4° \simeq 0]$$

$$\Rightarrow T = 0.57\,s$$

Range $R = (15 \cos 4°)\ (T) \simeq 15 \times 0.57$ [$\because \cos 4° \simeq 1$]

$= 8.55\ m$

Time for which Sania runs $t_0 = 0.57 - 0.2 = 0.37\ s$

Distance run by Sania = $7.5 \times 0.37 = 2.78\ m$.

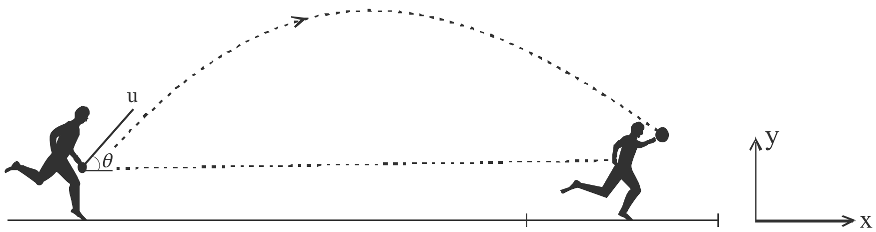
(a) Distance between the two players

$= 8.55 + 0.8 + 2.78 = 12.13\ m$

(b) $V_x \simeq 15$; $V_y \simeq 9.8 \times 0.57 = 5.6$

$\therefore V = \sqrt{V_x^2 + V_y^2} = 16\, m/s$

25.

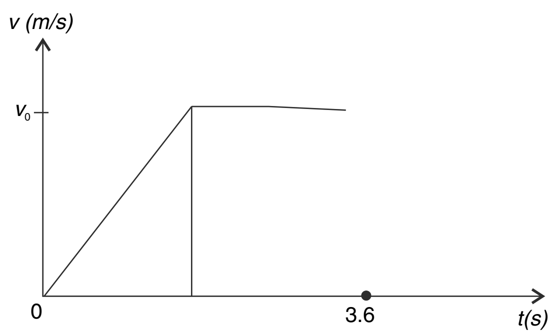


For the ball :

Time of flight, $T = \dfrac{2u_y}{g} = \dfrac{2u\sin\theta}{g} = \dfrac{2\times 19.5}{10} \times \dfrac{12}{13}$

Range, $R = T.u_x = 3.6 \times 19.5 \times \dfrac{5}{15} = 27\, m$

$V - t$ Graph is as shown



$V_0 = at = 2.a$

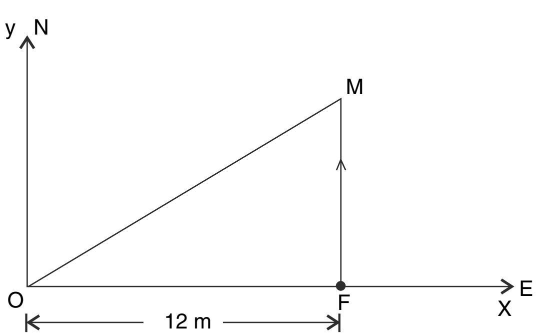
Displacement = area under the graph = 27 m

$\therefore \frac{1}{2} V_0 \left[3.6 + 1.6\right] = 27$

$\frac{1}{2} \times 2a\left[5.2\right] = 27$

$a = 5.19\, m/s^2$

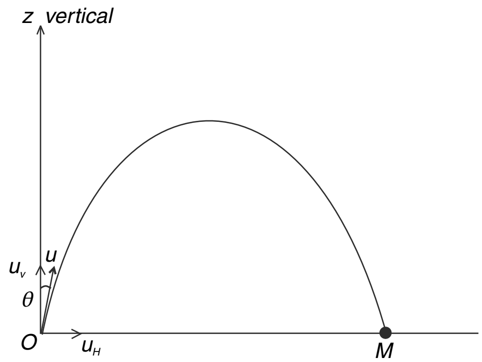
26.



The ball was hit at 0 and caught by the fielder at point M.

Fielder runs for 2.4 s

$\therefore\ FM = 2.4 \times 5 = 12\ m$



Range of the projectile (the ball) = $OM = 12\sqrt{2}\,m$. Time of flight of the projectile $T = 2.4 + 0.6 = 3.0\,s$

$$T = \frac{2u_v}{g}$$

[u_v = vertical component of initial velocity]

$$\therefore 3.0 = \frac{2u_v}{10}$$

$u_v = 15\,m/s$

$R = u_H.\,T$ [u_H= Horizontal component of initial velocity]

$$\therefore u_H = \frac{R}{T} = \frac{12\sqrt{2}}{3} = 4\sqrt{2}\,m/s$$

$\therefore$ Speed of projection

$$u = \sqrt{u_H{}^2 + u_v{}^2} = \sqrt{\left(4\sqrt{2}\right)^2 + 15^2} \simeq 16\,m/s$$

Angle made by initial velocity with vertical

$$\tan\theta = \frac{u_H}{u_v} = \frac{4\sqrt{2}}{15}$$

27. Let u = velocity of projection

Let α_1 & α_2 be two possible angle of projection to get a given range.

Time of flight for two cases are

$$T_1 = \frac{2u\sin\alpha_1}{g}$$

$$T_2 = \frac{2u\sin\alpha_2}{g}$$

Given $\dfrac{T_1}{T_2} = \dfrac{2}{1}$

$$\therefore \frac{2u\sin\alpha_1}{g} \times \frac{g}{2u\sin\alpha_2} = 2$$

$\sin\alpha_1 = 2\sin\alpha_2$ ....(1)

But $\alpha_1 = \dfrac{\pi}{2} - \alpha_2$ [for same range]

$\therefore \cos \alpha_2 = 2 \sin \alpha_2$

$\cot \alpha_2 = 2$

Range $R = \dfrac{u^2 \sin 2\alpha_2}{g} = \dfrac{2u^2 \sin \alpha_2 . \cos \alpha_2}{g}$

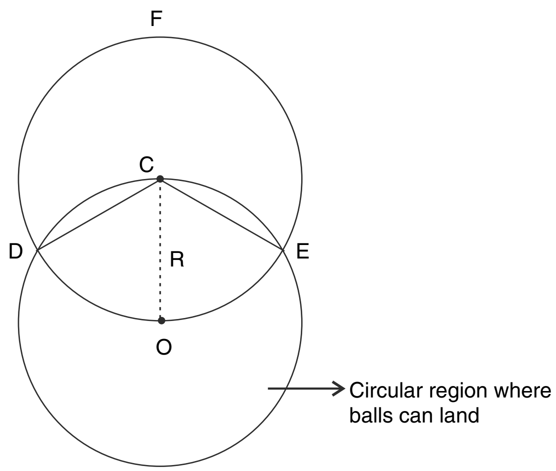
$$= \frac{2u^2}{g} \frac{1}{\sqrt{5}} . \frac{2}{\sqrt{5}} = \frac{4}{5} \frac{u^2}{g}$$

R_{max} corresponds to $\alpha = 45°$

$\therefore R_{max} = \dfrac{u^2}{g}$

Ratio $= \dfrac{4u^2}{5g} \times \dfrac{g}{u^2} = \dfrac{4}{5}$

28. Maximum range of balls $R_{max} = \dfrac{u^2}{g} = R$



From geometry $< DCE = 120°$

$\therefore$ Boy is safe on the arc DFE, i.e, on two third of the circle.

$\therefore$ Required length is $\dfrac{2}{3} . 2\pi R = \dfrac{4}{3} \pi R$

29. Displacement in ground frame is simply the displacement of point A of the cardboard.

Distance travelled is speed multiplied by time.

Speed in reference frame of cardboard $v = \dfrac{50cm}{10s} = 5cms^{-1}$

Speed in ground frame while travelling from A to C

$V_g = \sqrt{2^2 + 5^2 + 2.2.5 \cos\theta}$ where $\cos\theta = \dfrac{4}{5}$

$V_g = \sqrt{45} cms^{-1}$

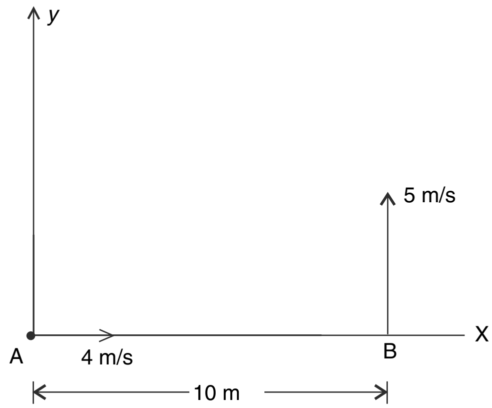
Distance travelled when moving from A to C = $10\sqrt{45} = 30\sqrt{5}\ cm$

While travelling back $V_g = \sqrt{2^2 + 5^2 + 2.2.5 \cos(180 - \theta)} = \sqrt{13}\ cms^{-1}$

Distance travelled while returning = $10\sqrt{13}\ cm$

Total distance = $30\sqrt{5}\ cm + 10\sqrt{13}\ cm$

30.



Let us fix the origin of our coordinate system at the original position of *A*.

X axis is along *AB* and *Y* axis as shown.

At time *t* position of *B* relative to *A* is given by

$$\overrightarrow{r_{BA}} = \overrightarrow{r_B} - \overrightarrow{r_A}$$
$$= 10i + 5tj - 4ti = (10 - 4t)i + 5tj$$

Velocity of *B* relative to A is

$$\overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A} = 5j - 4i$$

The two particles are closest to each other when

$$\overrightarrow{r_{BA}}.\overrightarrow{V_{BA}} = 0 \Rightarrow -4(10 - 4t) + 5t \times 5 = 0 \Rightarrow t = \frac{40}{41} s.$$

32. Acceleration of particle ω *rt* frame 1

$$\overrightarrow{a_{p1}} = \overrightarrow{a_p} - \overrightarrow{a_1} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Acceleration of particle ω *rt* frame 2

$$\overrightarrow{a_{p2}} = \overrightarrow{a_p} - \overrightarrow{a_2} \quad \ldots\ldots\ldots\ldots..(2)$$

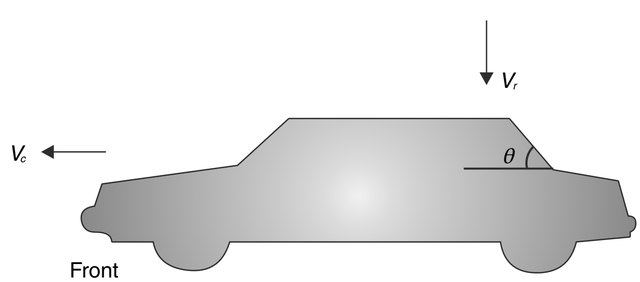
(1)-(2)

$$\overrightarrow{a_{p1}} - \overrightarrow{a_{p2}} = \overrightarrow{a_2} - \overrightarrow{a_1} \Rightarrow \left|\overrightarrow{a_{p1}} - \overrightarrow{a_{p2}}\right| = \left|\overrightarrow{a_2} - \overrightarrow{a_1}\right|$$

$$1 \le \left|\overrightarrow{a_{p1}} - \overrightarrow{a_{p2}}\right| \le 7$$

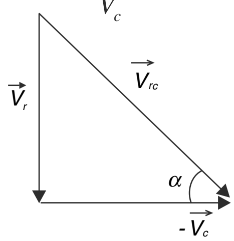
$$1 \le \left|\overrightarrow{a_2} - \overrightarrow{a_1}\right| \le 7.$$

33. (a)



The velocity of rain with respect to the car must make an angle α with the horizontal such that $\alpha \le \theta$

$\tan\alpha = \frac{V_r}{V_c} \le \tan\theta$ ....(1)



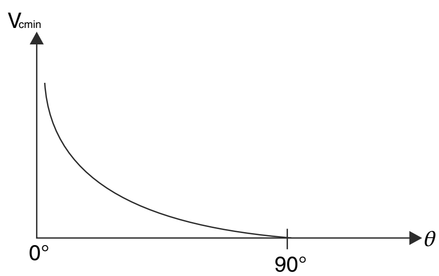
$\Rightarrow V_r \le V_c \tan\theta$

$V_{max} = V_c \tan\theta$

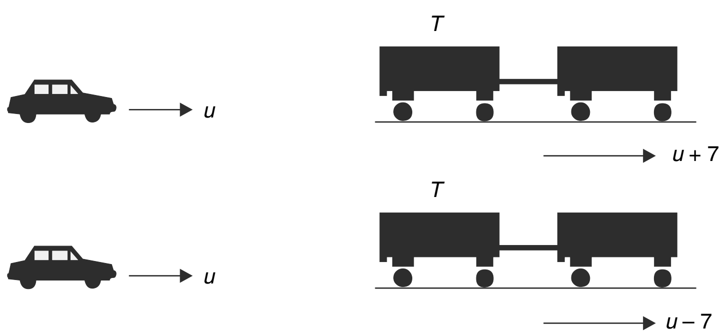
$= 20 \times 0.75 = 12$ *km/hr*

(b) From equation (1) $V_c \ge \frac{V_r}{\tan\theta}$

$V_{c\,min} = \frac{V_r}{\tan\theta}$



34. Let the speed of the car be u towards right. Velocity of train can be $(u + 7)$ (→) or $(u - 7)$ (→)



[Velocity of train can also be considered as $(7 - u)$ (←). But $(7 - u)$ (←) = $(u - 7)$ (→)]

After the car turns, relative speed becomes $\sqrt{u^2 + (u+7)^2}$ or $\sqrt{u^2 + (u-7)^2}$

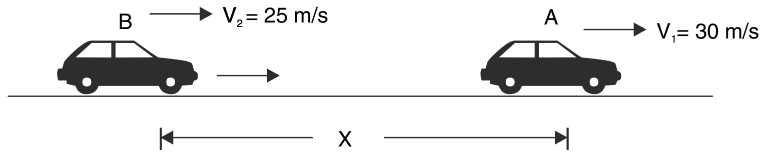
Solving $\sqrt{u^2 + (u+7)^2} = 13$

we get $u = 5$ *m/s*

solving $\sqrt{u^2 + (u-7)^2} = 13$

we get $u = 12$ *m/s*

35.



Let X = distance between the car at the instant first bullet is fired (say at time $t = 0$)

Speed of bullet, relative to ground is

$V_b = u + V_2 = 305 + 25 = 330\ m/s$

Velocity of bullet, relative to car A is

$V_{bA} = 330 - 30 = 300\ m/s$

$\therefore$ Time when the first bullet hits the car A is

$$t_1 = \frac{X}{V_{bA}} = \frac{X}{300} \qquad \text{.......................(1)}$$

Distance between the car when the last bullet is fired (at time $t = T_0 = 20\ s$) is

$X^1 = X + V_{AB}\, T_0 = X + 5 \times 20 = X + 100$

Time when the last bullet hits the car A is

$$t_2 = T_0 + \frac{X}{V_{bA}}$$

$$= 20 + \frac{X+100}{300} \quad \text{.......................(2)}$$

The interval $\Delta t = t_2 - t_1$

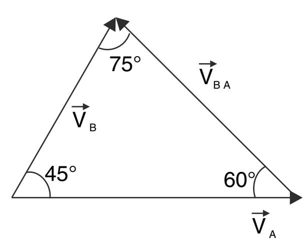
$$= 20 + \frac{100}{30} = 23.33 \sec.$$

36. In the reference frame attached to the chain, the ball appears to be moving up with a constant velocity 'u'.

$\therefore$ Required interval of time is

$$\Delta t = \frac{L}{u}$$

37. $\overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A} \Rightarrow \overrightarrow{V_{BA}} + \overrightarrow{V_A} = \overrightarrow{V_B}$



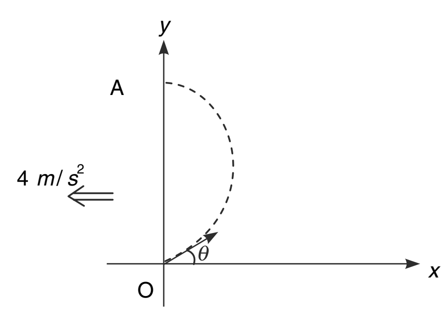
From the fig. $\dfrac{v_B}{\sin 60°} = \dfrac{v_A}{\sin 75°}$

$$v_A = 900\frac{\sin 60°}{\sin 75°} = 900 \times \frac{0.866}{0.966} = 807\ kph.$$

38. With respect to the cart, ball follows a parabolic path.

In this frame it has a constant acceleration of $4\ m/s^2$ in negative X direction.

Due to similarity with projectile motion we can write

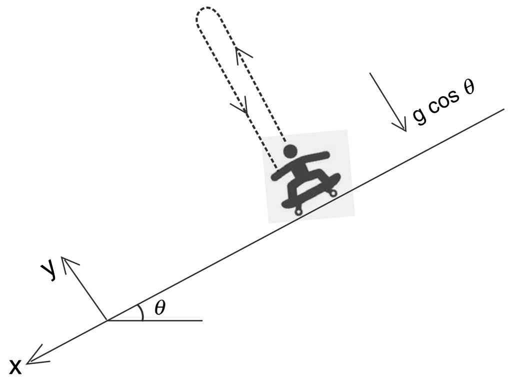


$$\text{Range} = OA = \frac{u^2 \sin 2\theta}{a}$$

$$\Rightarrow 9 = \frac{u^2 \sin 90°}{4} \Rightarrow u = 6\,m/s$$

39. (a) The boy will be able to catch the ball only if he sees the ball moving perpendicular to the incline (i.e., in y direction)

The acceleration of the ball in the reference frame of the boy is $g \cos \theta$ in negative y direction.



[The ball and the boy both have same acceleration (= $g \sin \theta$) in the ground frame. Hence the boy does not see any acceleration in the ball in this direction]

In boy's frame if initial velocity of the ball is V then

$$T = \frac{2V}{g\cos\theta} \quad \therefore V = \frac{1}{2}T.g.\cos\theta$$

(b) The velocity of tip of the barrel

$\omega L = 2\sqrt{2}\ m/s$

Velocity of the shell wrt the body of the gun is vector sum of velocity relative to the exit point and the velocity of the tip of the barrel. This is equal to

$$\sqrt{\left(2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2} = 4\,m/s$$

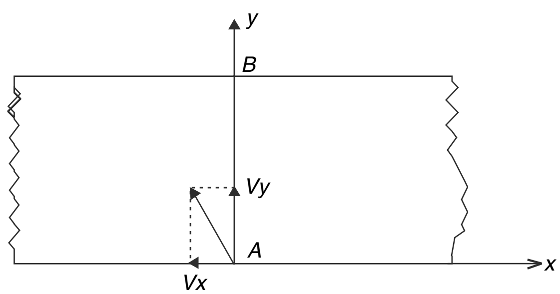
In vertically upward direction.

Velocity relative to ground will be vector sum of this velocity and the recoil velocity of the gun. Resultant velocity is -

$$\sqrt{4^2 + 4^2} = 4\sqrt{2}\,m/s$$

40. In ground frame the ant must move perpendicular to the edge of the paper. Y component of its velocity

$$V_y = \frac{10\,cm}{5\,s} = 2\,cms^{-1}$$



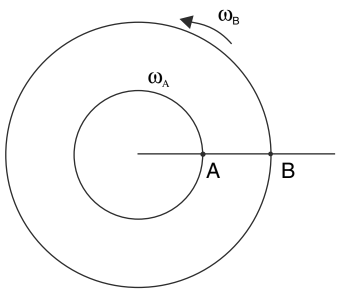
X component of ant's velocity in ground frame must be zero.

$\therefore V_x = 2\ cms^{-1}$

$\therefore$ Speed of ant relative to the paper is

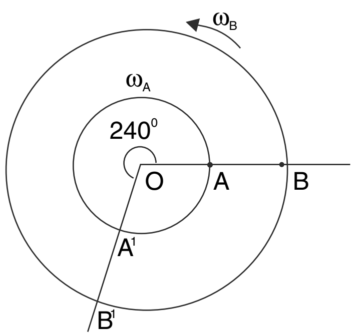
$$V = \sqrt{V_x^{\ 2} + V_y^{\ 2}} = \sqrt{2^2 + 2^2} = 2\sqrt{2}\,cms^{-1}$$

41.



Let the two particles be on a straight line passing through the centre after time t. In this time, A will be one complete rotation ahead of B.

$$\omega_A t = 2\pi + \omega_B t \Rightarrow \left(\frac{2\pi}{T_A} - \frac{2\pi}{T_B}\right)t = 2\pi \Rightarrow \left(\frac{1}{8} - \frac{1}{11}\right)t = 1$$



$$t = \frac{88}{3}\text{ min}.$$

In this time, A has completed $\dfrac{t}{T_A} = \dfrac{88/3}{8}$

$= \dfrac{11}{3}$ rotations = $3 + \dfrac{2}{3}$ rotations.

B has completed $2 + \dfrac{2}{3}$ rotations.

The two particles meet on the line O $A'B'$

42. $\vec{r} = (\sin t)\hat{i} + (\cos t)\hat{j}$

$\vec{V} = \dfrac{d\vec{r}}{dt} = (\cos t)\hat{i} - (\sin t)\hat{j}$

Speed $V = |\vec{V}| = \sqrt{\cos^2 t + \sin^2 t} = 1$ (= a constant)

Tangential acceleration $a_t = \dfrac{dv}{dt} = 0$

Path

$x = \sin t$

$y = \cos t$

$x^2 + y^2 = 1$

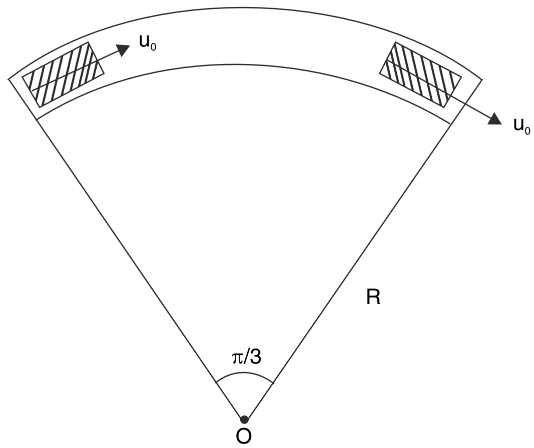
Path of the particle is a circle.

43. Time of travel for the bullet from one disc to the other

$t = \frac{H}{V} \quad \therefore \theta = \omega t$

$\theta = \omega \frac{H}{V} \Rightarrow V = \frac{\omega H}{\theta}$

44. (a)

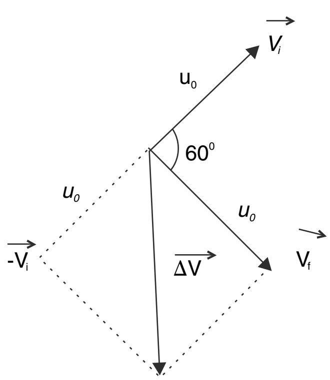


$a_0 = \frac{u_0^2}{R}$

Average acceleration $= \frac{\text{change in velocity}}{\text{time}}$

$\overrightarrow{\Delta V} = \overrightarrow{V_f} + \left(-\overrightarrow{V_i}\right)$

The two velocity to be added have same magnitude and angle between them is 120°



$\therefore \Delta V = u_0$

Time taken $\Delta t = \frac{R.\pi / 3}{u_0} = \frac{\pi R}{3u_0}$

$\therefore \langle a \rangle = \frac{u_0}{\frac{\pi R}{3u_0}} = \frac{3u_0^{\,2}}{\pi R} = \frac{3}{\pi} a_0$

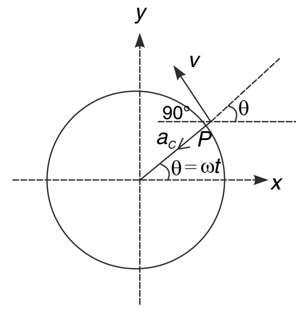
(b) The particle must rotate through an angle of $\frac{\pi}{3}$ for change in its velocity to be 4 m/s. Distance travelled in 0.5 s is 2 m.

$$\frac{\pi}{3}R = 2.$$

$$R = \frac{6}{\pi}$$

$$a = \frac{v^2}{R} = 8.37\, m/s^2$$

45. At any time t, particle is at P and $\omega r = v$



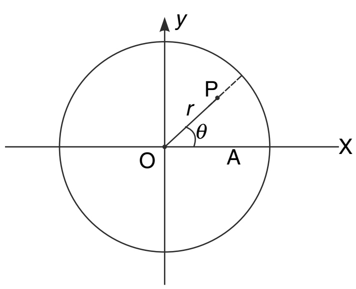
$v_x = -v \sin \omega t$

$v_y = v \cos \omega t$

(b) $\alpha_y = -\alpha_C \sin \omega t$

$$= -\frac{v^2}{r} \sin \omega t$$

46. (a)



At time 't' particle is at point p such that

$r = vt$ and $\theta = \omega t$

$\therefore \vec{r} = Vt\left[\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j}\right]$

(b) velocity $\overrightarrow{V_p} = \dfrac{d\vec{r}}{dt} = V\left[\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j}\right] + Vt\left[-\omega\sin(\omega t)\hat{i} + \omega\cos(\omega t)\hat{j}\right]$

$\overrightarrow{V_p} = V\left[\cos(\omega t) - \omega t\sin(\omega t)\right]\hat{i} + V\left[\sin(\omega t) + \omega t\cos(\omega t)\right]\hat{j}$

(c) $V_x = 0$

$\cos(\omega t) = \omega t \sin(\omega t)$

$\Rightarrow \tan\theta = \dfrac{1}{\theta}$

47. (a) Given $a_t = n\, a_r$

$\Rightarrow \dfrac{dv}{dt} = n\dfrac{v^2}{R}$

Or, $\dfrac{dv}{ds}\dfrac{ds}{dt} = n\dfrac{v^2}{R}$ [s = arc length traversed by the particle = $R\theta$]

Or, $v\dfrac{dv}{ds} = n\dfrac{v^2}{R}$ $\left[\dfrac{ds}{dt} = v\right]$

Or, $\dfrac{dv}{v} = \dfrac{n}{R}ds$

Integrating

$$\int_{v_0}^{v} \frac{dv}{v} = \frac{n}{R}\int_{0}^{2\pi R} ds \quad \text{[One complete rotation means } s = 2\pi R\text{]}$$

$$\therefore \ \ln\frac{v}{v_0} = \frac{n}{R}2\pi R = 2\pi n$$

$$\therefore v = v_0 e^{2\pi n}$$

(b) $a_t = a_0 \cos\theta$

$$\frac{dv}{dt} = a_0\frac{v_x}{v} \qquad \left[\because \ \cos\theta = \frac{v_x}{v}\right]$$

$$v\frac{dv}{dt} = a_0 . \frac{dx}{dt}$$

$$\int_{V=0}^{V} v\, dv = a_0 \int_{X=0}^{X} dx$$

$$\frac{v^2}{2} = a_0 x$$

$$v = \sqrt{2a_0 x}$$

49. Tangential acceleration $a_t = 0.8\ ms^{-2}$

Let Radial acceleration = a_r

$\therefore a_r^2 + a_t^2 = a^2 \Rightarrow a_r^2 + 0.8^2 = 1^2 \Rightarrow a_r = 0.6\, ms^{-2}$

$\Rightarrow \omega^2 r = 0.6$ ....(1)

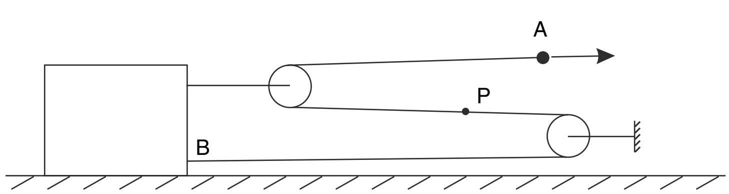
Angular acceleration $\alpha = \frac{a_t}{r} = \frac{0.8}{r}$

Let angular displacement be θ by the time angular speed increases from zero to ω

$\omega^2 = 0^2 + 2\ \alpha\ \theta$

$\omega^2 = 2.\frac{0.8}{r}.\theta \Rightarrow \omega^2 = 1.6\ \theta \Rightarrow \theta = \frac{0.6}{1.6} = \frac{3}{8}$ rad.

50. If the block moves by X when point A moves by X_A then,



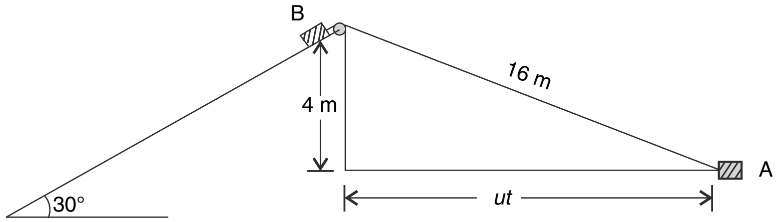
$3X = X_A$ [Because the length of the string is constant]

$$\therefore 3\frac{dX}{dt} = \frac{dX_A}{dt}$$

$\therefore 3V_{\text{block}} = 6 \qquad \therefore V_{\text{block}} = 2\ m/s$

Since length of the string from B to P is fixed, point P will also move with a velocity of 2 m/s ($\leftarrow$)

51. Length of the string = 12 + (12 sin 30° – 2). Final position has been shown in fig. below



$(ut)^2 = 16^2 - 4^2$

$\therefore\ t = \dfrac{\sqrt{252}}{10} = 1.59\ s$

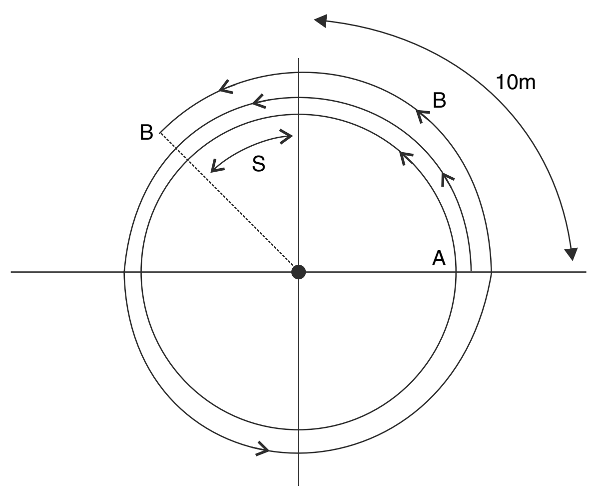
52. (a) Speed of A and B

$U_A = 4.5\ km/hr = 1.25\ m/s$

$U_B = 18\ km/hr = 5\ m/s$

B will complete the circle in 8 sec. At that time A will travel through quarter of the circle.

In next Δt second let A travel through distance 's'. B will meet him if he travels a distance $(10 + s)$ in interval Δt



$\therefore\ 5\ \Delta t = 10 + 1.25\ \Delta t \ \Rightarrow \Delta t = \dfrac{10}{3.75} = \dfrac{8}{3}$ second

$S = 1.25 \times \dfrac{8}{3} = \dfrac{10}{3}\ m$

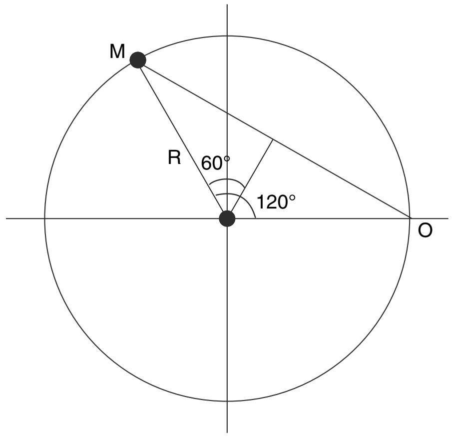
$\therefore$ Both meet after, $t_0 = 8 + \dfrac{8}{3} = \dfrac{32}{3}$ sec

Meeting point is at a distance

$10 + \dfrac{10}{3} = \dfrac{40}{3}\ m$ from the starting point

Displacement of both is same = OM

$O \rightarrow$ starting point, $M \rightarrow$ Meeting point



$OM = R \sin 60° \times 2$ [R = radius of circle $= \dfrac{40}{2\pi}$]

$= \dfrac{40}{2\pi} \cdot \dfrac{\sqrt{3}}{2} \times 2$

$= \dfrac{20\sqrt{3}}{\pi}\ m$

Average velocity for A; $< V_A > = \dfrac{2\sqrt{3}}{\pi \times \left(\dfrac{32}{3}\right)}$

$$= \frac{15\sqrt{3}}{8\pi} m/s \text{ (along } OM)$$

Average velocity for B; $<V_B> = <V_A>$

54. the equation of the straight line shown in the graph is

$$x = \frac{2}{v} - 1$$

But $v = \dfrac{dx}{dt}$

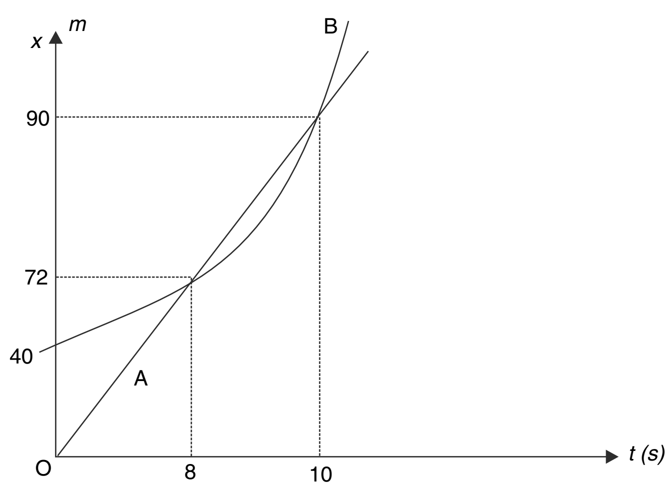
$\therefore \quad x = \dfrac{2dt}{dx} - 1 \qquad \Rightarrow x\,dx = 2\,dt - dx$

$\Rightarrow \int_3^{15} x\,dx = 2\int_0^t dt - \int_3^{15} dx \quad \Rightarrow \dfrac{1}{2}[225 - 9] = 2t - [15 - 3]$

$\Rightarrow 108 + 12 = 2t \qquad \Rightarrow t = 60\ s$

56. (a) $x_B = 40 + \dfrac{1}{2}at^2 = 40 + 0.5t^2$ (parabola)

$x_A = 9t$ (straight line)



The two cars are at same position if $x_B = x_A$

$0.5\,t^2 + 40 = 9\,t$

Solving, $t = 8\ s$, $10\ s$

Conclusion:

First the car A is moving at greater speed. At $t = 8$ sec (when $V_A = 9\ m/s > V_B = 8\ m/s$)

Car A overtakes car B. But soon B will overtake A as its speed keeps on increasing. This happens at $t = 10$ sec ($V_B = 10\ m/s$, $V_A = 9\ m/s$). After this the two cars never meet as speed of B keeps on increasing.

(b) At $t = 8$; $x_A = x_B = 72\ m$.

Car A keeps on taking lead till $V_A > V_B$

After $t = 9$ sec, speed of B exceeds A and the two starts getting closer.

Lead is maximum at $t = 9\ s$

$\Delta L = x_A - x_B$ (at $t = 9$)

$= 9 \times 9 - (0.5 \times 9^2 + 40)$

$= 0.5\ m$

57. $x_B = \frac{1}{2}a_B t^2 = t^2$

$x_A = V_A\ (t - 10) = 50\ (t - 10)$

A and B are at same location if

$x_B = x_A$

$t^2 = 50t - 500\ \therefore\ t^2 - 50t + 500 = 0$

$$t = \frac{50 \pm \sqrt{2500 - 2000}}{2}$$

$$= 25 \pm 5\sqrt{5}$$

$t_1 = 25 - 5\sqrt{5}$ [At this time A crossed 'B']

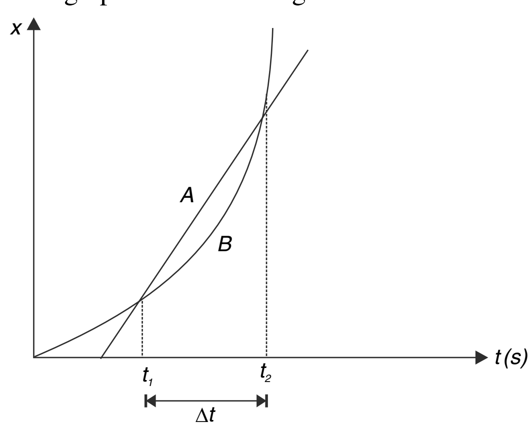
$t_2 = 25 + 5\sqrt{5}$ [B is moving with increasing speed. This is the time when B overtakes A]

$\therefore$ Interval for which A is ahead

$\Delta t = t_2 - t_1 = 10\sqrt{5}\ s$

(b) $x - t$ graph for B is a parabola.

$x - t$ graph for A is a straight line.



58.

(a) $a = v\frac{dv}{dx}$

$\frac{dv}{dx} \rightarrow$ constant

$v \rightarrow$ increasing

$\therefore\ a \rightarrow$ increasing

(b) Slope of line $APB = -\frac{v_0}{1}$

[v_0 = velocity at P]

Let slope of tangent at $P = m$

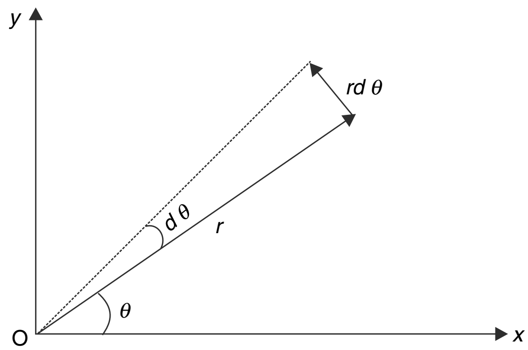
$$m \times \left(-\frac{v_0}{1}\right) = -1$$

$$m = +\frac{1}{v_0}$$

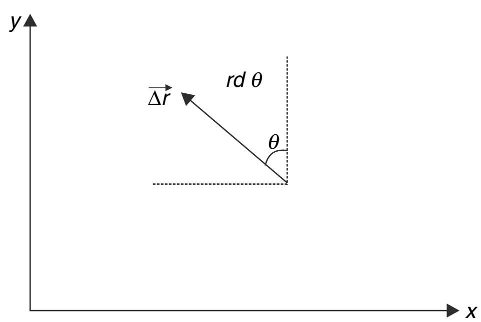
$$\left(\frac{dv}{dx}\right)_P = \frac{1}{v_0}$$

$$\therefore a_P = v_P\left(\frac{dv}{dx}\right)_P = v_0 \cdot \frac{1}{v_0} = 1\, m/s$$

59. The change in position vector $\left(\overrightarrow{\Delta r}\right)$ has length $rd\theta$ where $r = \sqrt{x^2 + y^2}$



Direction of $\overrightarrow{\Delta r}$ is perpendicular to $\vec{r}$



$\therefore\ \overrightarrow{\Delta r} = -rd\theta \sin\theta\ \hat{i}\ + rd\theta \cos\theta\ \hat{j}$

But $r \sin\theta = y$ and $r\cos\theta = x$

$\therefore \overrightarrow{\Delta r} = -yd\theta\,\hat{i} + xd\theta\,\hat{j}$

60. (a) $t =$ time needed for rope to cross point O.

a = acceleration

L = length

$$L = \frac{1}{2}at^2 \qquad \text{............(i)}$$

$$(L-1) = \frac{1}{2}a(t-2)^2 \qquad \text{............(ii)}$$

$$(L-2) = \frac{1}{2}a(t-5)^2 \qquad \text{............(iii)}$$

(i)-(ii) $1 = \frac{1}{2}a\left[t^2 - (t-2)^2\right]$ ....(iv) (ii)-(iii) $1=\frac{1}{2}a\left[(t-2)^2-(t-5)^2\right]$ ....(v)

$\therefore\ t^2-(t-2)^2=(t-2)^2-(t-5)^2$

$\Rightarrow 4t-4=6t-21 \Rightarrow 2t=17$

$t = 8.5\ s$

From (iv) $\frac{1}{2}a\left[8.5^2-6.5^2\right]=1$

$\frac{1}{2}a(15\times 2)=1$

$a=\frac{1}{15}\ m/s^2$

$\therefore L=\frac{1}{2}\times at^2=\frac{1}{2}\times\frac{1}{15}\times 8.5^2$

$=2.41\ m$

61. x co-ordinate of particle 1 at time 't' is

$x_1 = x_0-\left(\frac{x_0}{t_0}\right)t = x_0-ut$ ....(i)

[This is the equation of the straight line given in the question]

For particle 2-

Acceleration = $-\frac{u}{t_0}$ [Slope of $v-t$ graph]

x co-ordinate at time 't' is-

$x_2 = ut-\frac{1}{2}\left(\frac{u}{t_0}\right)t^2$ .... (ii)

Collision occurs when $x_1 = x_2$

$\therefore x_0-ut=ut-\frac{1}{2}\left(\frac{u}{t_0}\right)t^2 \Rightarrow \left(\frac{u}{2t_0}\right)t^2-2ut+x_0=0$

$$\therefore t=\frac{2u\pm\sqrt{4u^2-\frac{2u}{t_0}x_0}}{\frac{u}{t_0}}=\frac{2u\pm\sqrt{4u^2-2u^2}}{\frac{u}{t_0}}$$

$=t_0\left[2\pm\sqrt{2}\right]$

Since t cannot be larger than t_0

$\therefore\ t=(2-\sqrt{2})t_0$

Put this in (i) $x_1 = x_0-(2-\sqrt{2})\,ut_0$

$=(\sqrt{2}-1)x_0$

62. $v-t$ graph for two trains has been shown.

Distance travelled by passenger train in

1 hr 48 min = 50 × 1.8 = 90 km

$\therefore$ Area of ΔPQR = 10 km

$$\frac{1}{2} \times 50 \times t_0 = 10$$

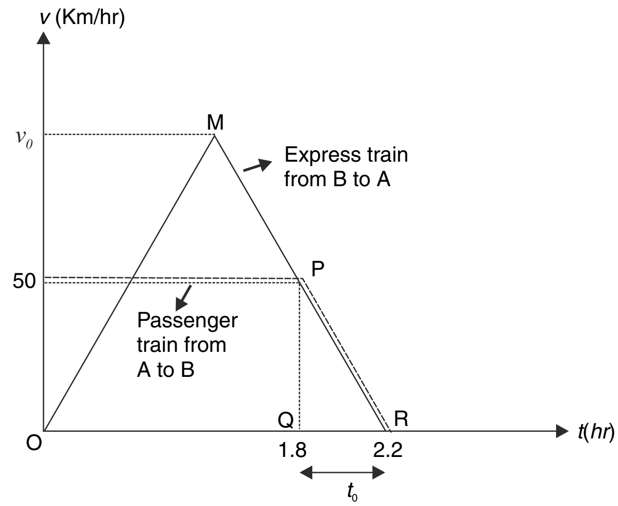
$$t_0 = \frac{2}{5} = 0.4\ hr$$

$\therefore$ Total travel time = 2.2 hr = travel time for express train from B to A

Area of ΔOMR = 100

$$\frac{1}{2} \times 2.2 \times v_0 = 100$$

$$v_0 = \frac{100}{1.1} = 90.9\ km/hr$$



64. Area under a_x vs t graph gives change in v_x

$\therefore \quad v_x = a_0 t_0 + a_0 t_0 = 2a_0 t_0 = 2v_0 \qquad \left[\because\ a_0 = \frac{v_0}{t}\right]$

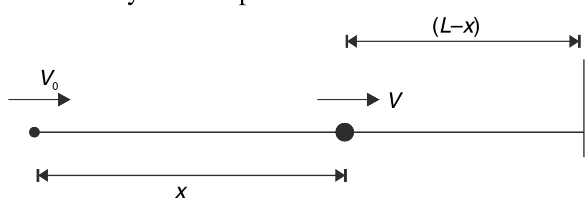
Area under a_y vs t graph = change in v_y

$\therefore \quad v_y - v_0 = \frac{1}{2} \times t_0 \times a_0 \times 4 = 2a_0 t_0 = 2v_0$

$\therefore \quad v_y = 3v_0 \quad \therefore \tan\theta = \frac{3v_0}{2v_0}$

$$\theta = \tan^{-1}\left(\frac{3}{2}\right)$$

66. Let velocity after displacement 'x' be v.



$$v\frac{dv}{dx} = -k(L - x) \qquad [k = a \text{ constant}]$$

$$\int_{v_0}^{v_0/2} v\, dv = -\int_0^L k(L - x)\, dx$$

$$\frac{1}{2}\left[\frac{v_0^2}{4} - v_0^2\right] = -\left[kL^2 - \frac{kL^2}{2}\right] \Rightarrow \frac{3}{4} v_0^2 = kL^2$$

Initial retardation $= kL = \frac{3v_0^2}{6L}$

68. $m\frac{dv}{dt} = mg - bv$

$$v = \frac{mg}{b}\left(1 - e^{-\frac{bt}{m}}\right)$$

$$v = v_0\left(1 - e^{-t/\tau}\right) \qquad \left[\tau = \frac{m}{b}\right]$$

(i) $v_0 = \dfrac{mg}{b}$

$$v_{10} = \frac{1\times10}{2.5} = 4\,m/s$$

$$v_{20} = \frac{1.2\times10}{3.0} = 4\,m/s$$

(ii) The body with smaller value of τ will take lesser time.

For body 1 $\quad \tau_1 = \dfrac{m_1}{b_1} = \dfrac{1}{2.5} = 0.4\,\text{sec}$

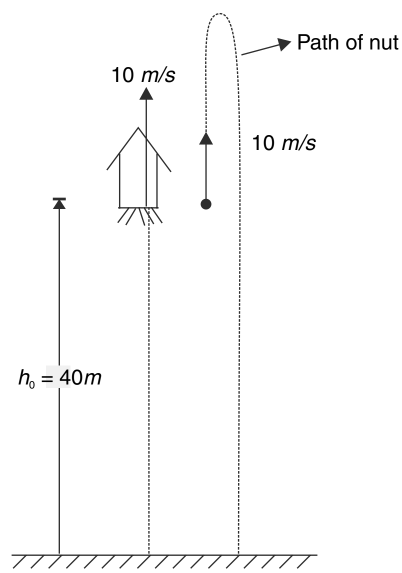
For body 2 $\quad \tau_2 = \dfrac{m_2}{b_2} = \dfrac{1.2}{3.0} = 0.4\,\text{sec}$

$\therefore$ Both will take same time.

69. (a) At the instant the nut gets detached from the rocket, its speed and height are

$$v = 0 + \frac{5}{4}\times 8 = 10\,m/s\,(\uparrow)$$

$$h_0 = 0 + \frac{1}{2}\times\frac{5}{4}\times 8^2 = 40\,m$$



For motion of the nut

$u = 10\,m/s$ ($\uparrow$ is $+ve$)

$h = -40\,m$ ($\downarrow$ is $-ve$)

$a = 10\,m/s^2$

$$\therefore\ h = ut + \frac{1}{2}at^2$$

$$-40 = 10\times t - \frac{1}{2}\times 10\times t^2$$

Solving, $t = 4\,s$

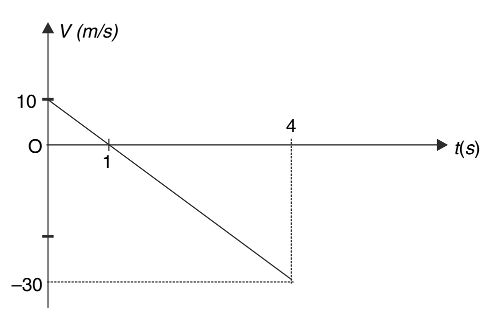
θ Nut will hit the ground in next 4 s.

Further height gained by the rocket in 4 s is

$$h_1 = 10\times 4 + \frac{1}{2}\times\frac{5}{4}\times 4^2$$
$$= 40 + 10 = 50\,m$$

$\therefore$ Required height of the rocket $= h_0 + h_1 = 90\,m$

(b) Velocity of nut in the reference frame of ground is $v = 10 - 10\,t$



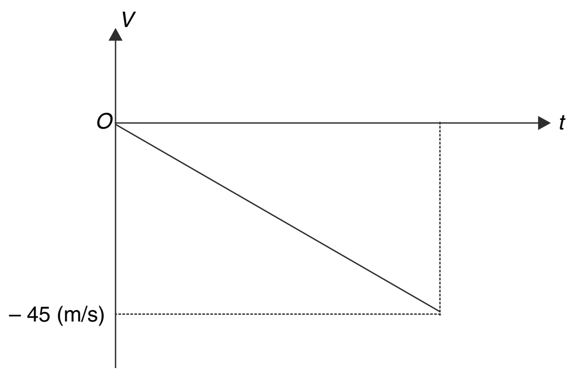
In frame of rocket

Initial velocity $u = 0$

Acceleration $a = 10 + \dfrac{5}{4} = \dfrac{45}{4}\,m/s^2\ (\downarrow)$

$\therefore\ v = 0 + at$

$v = -\frac{45}{4}t$ [– *ve* because acceleration is downward]



70. The elevator moves up with constant acceleration, hence $y - t$ graph must be a parabola.

Let $y = kt^2$

at $t = 2$, $y = 4$

$\therefore\ k = 1 \quad \therefore\ y = t^2$

$\therefore\ \frac{dy}{dt} = 2t = 4\ m/s \quad (at\ t = 2)$

$\frac{d^2y}{dt^2} = 2.0\ m/s^2$

In the reference frame of the elevator the acceleration of bolt is 12 m/s^2 and its initial velocity is zero. Time required for a displacement of 1.5 m in this frame is

$y = \frac{1}{2} \times 12 \times t^2$

$1.5 = \frac{1}{2} \times 12 \times t^2 \quad \Rightarrow t = 0.5\ s$

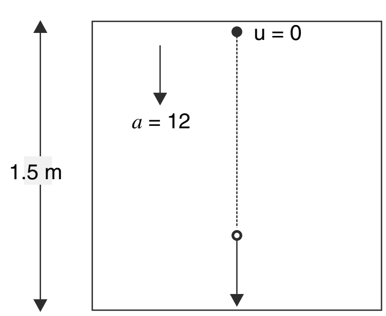
$\therefore$ Bolt hits the floor at $t = 2.5\ s$

In 0.5 second, the displacement of bolt (in reference frame of ground) is

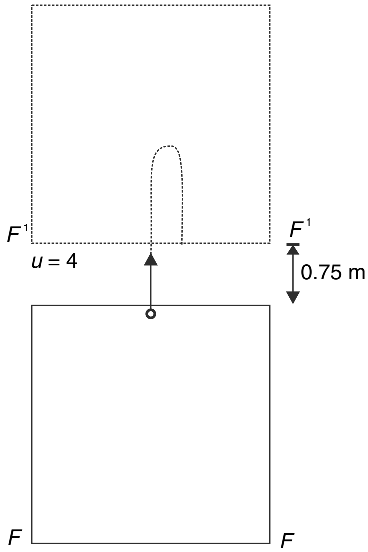
$\Delta y_b = 4 \times 0.5 - \frac{1}{2} \times 10 \times 0.5^2 = 0.75\ m$

The event as seen in reference frame of elevator and ground has been shown in figure.

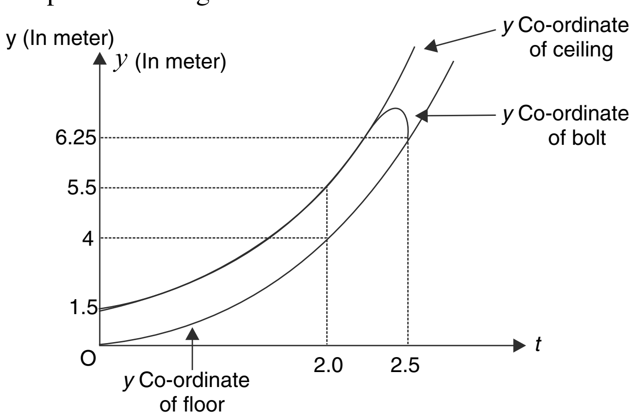
R.F. of elevator



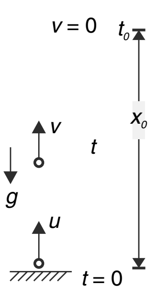
R.F. of ground



Graph in *R.F.* of ground



71. For upward motion



$$\frac{dv}{dt} = -\alpha t$$

$$\int_u^0 dv = -\alpha \int_{t=0}^{t_0} t\,dt$$

$$u = \alpha \frac{t_0^2}{2}$$

$\therefore \quad t_0 = \sqrt{\dfrac{2u}{\alpha}}$ = time for upward motion

Also, velocity at time 't'

$$\int_u^v dv = -\alpha \int_0^t t\, dv$$

$$v = u - \alpha \frac{t^2}{2}$$

$$\therefore \frac{dx}{dt} = u - \alpha \frac{t^2}{2}$$

$$\int_0^{x_0} dx = u\int_0^{t_0} dt - \frac{\alpha}{2}\int_0^t t^2 dt \qquad \text{..................(i)}$$

$$x_0 = ut_0 - \frac{\alpha}{6}t_0^3 = u\sqrt{\frac{2u}{\alpha}} - \frac{\alpha}{6}\frac{2u}{\alpha}\sqrt{\frac{2u}{\alpha}} = \frac{2u}{3}\sqrt{\frac{2u}{\alpha}}$$

Total time of flight

From (i) $\displaystyle\int_0^0 dx = u\int_0^t dt - \frac{\alpha}{2}\int_0^t t^2 dt$

$$\Rightarrow ut - \frac{\alpha}{6}t^3 = 0$$

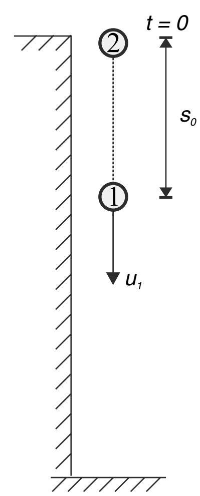
$$\Rightarrow t = \sqrt{\frac{6u}{\alpha}}$$

73. At time $t = 0$, speed of first stone is

$u_1 = 0 + g \times 1 = 10\ m/s$

and separation between the stones is

$$S_0 = 0 + \frac{1}{2} \times g \times 1^2 = 5\ m$$



<u>At time</u> t

Speed of first stone is

$v_1 = 10 + gt = 10 + 10\ t$

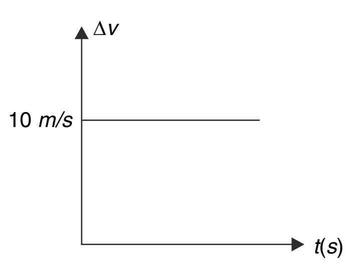
Speed of second stone is

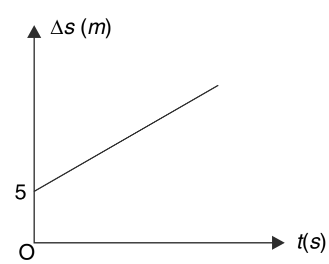
$v_2 = gt = 10\ t$

$\therefore\ \Delta v = v_1 - v_2 = 10$ = constant.

Displacement of first stone from top of the tower is

$$S_1 = 5 + 10t + \frac{1}{2} \times 10 \times t^2$$
$$= 5 + 10t + 5t^2$$





Displacement of second stone from top of the tower is

$$S_2 = \frac{1}{2}gt^2 = 5t^2$$

$$\Delta S = S_1 - S_2 = 10t + 5$$

74. $x = v_0 t$

$$\vec{r} = x\,\hat{i} + y\,\hat{j}$$

$$\vec{v} = \frac{\overrightarrow{dr}}{dt} = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j}$$

$$= \frac{dx}{dt}\,\hat{i} + Akv_0\cos(v_0 tk)\,\hat{j} = v_0\,\hat{i} + Akv_0\cos(v_0 tk)\,\hat{j}$$

$$\frac{\overrightarrow{dv}}{dt} = -Ak^2v_0^2\sin(v_0 tk)\,j = -Ak^2v_0^2\sin(kx)\,j$$

at $x = \dfrac{\pi}{2k}$,

$$\vec{a} = -Ak^2v_0^2\,\hat{j}$$

75. The ball hits the incline at the same horizontal level from which it is projected. (first graph).

Time of flight

$\dfrac{2u_y}{g} = 2.4 \quad \Rightarrow u_y = 12\ m/s \quad$ [= v_0 shown in 2nd graph]

$v_y = 12 - 10\,t$ and $x = u_x t$

Eliminating 't', $v_y = 12 - \dfrac{10x}{u_x}$

Slope of v_y vs x graph

$$\frac{-10}{u_x} = -\frac{2v_0}{38.4}$$

$\therefore\ u_x = 16\ m/s$

(a) $\tan\alpha = \dfrac{u_y}{u_x} = \dfrac{12}{16} = \dfrac{3}{4}$

(b) $\tan\theta = \dfrac{h}{x} = \dfrac{19.2}{38.4} = \dfrac{1}{2}$

(c) $\theta < \alpha$

And θ and α both are less than 45°. So by decreasing the angle of projection the horizontal range of an usual projectile will decrease. Hence the ball will hit the incline at a lower point.

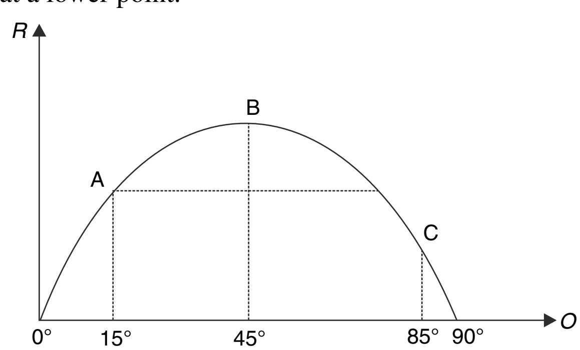
76. (i) Range $R = \dfrac{u^2\sin 2\theta}{g}$

The graph of R vs. θ is as shown.

In the context of the question, the graph ranges from A to B to C.

Slope of the graph is maximum at C.

[graph is symmetrical about θ = 45°, slope at θ = 15° is same as slope at 75°].



It means $\frac{\Delta R}{\Delta \theta}$ is maximum at C. Hence answer is $\theta = 84°$

Range will change by maximum amount when θ is increased from 84° to 85°.

(ii) If we consider small cones of semi vertical angle $\Delta\theta$ with apex at the sprinkler, each of the cone will receive same amount of water. This water spreads on smallest possible area on ground at position where $\frac{\Delta R}{\Delta \theta}$ is minimum. This happens at $\theta = 45°$.

The required distance is $R_{45°} = \frac{u^2}{g}$

77. Minimum Angle of projection with horizontal is $\theta = 90° - 1° = 89°$

For this angle range will be maximum. (look at the graph in solution to previous problem)

$R_{\max}$ = radius of circle (r) $= \frac{u^2 \sin 2\theta}{g} = \frac{(150)^2 \times \sin 178°}{g}$ [$\because$ sin (180 – θ) = sin θ]

But $2° = \frac{2}{180} \times \pi$ radian. Hence, $\sin 2° \cong \frac{2\pi}{180}$

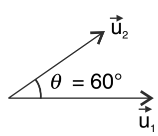
$\therefore \quad r = \frac{150^2 \times 2 \times \pi}{10 \times 180} \cong 80\ m$

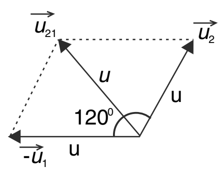
Probability that a bullet fired will fall on person's head = $\frac{\text{Area of head}}{\text{Area of circle of radius r}} = \frac{(0.1)^2}{(80)^2} = 1.6 \times 10^{-6}$

Probability that any of the fired bullets falls on head is

$n = 1.6 \times 10^{-6} \times 1000 = 1.6 \times 10^{-3}$

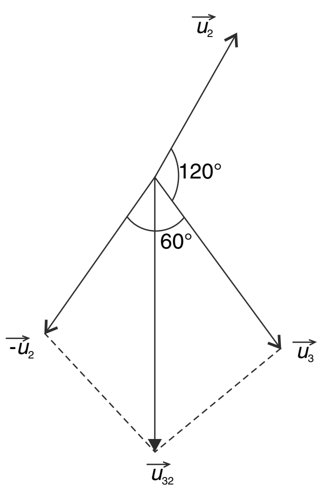
78. (a) For an observer falling down with an acceleration g, all the stones appear to move with constant velocities. The velocity of 2 relative to 1 has magnitude equal to u. It means $\theta = 60°$ (see fig)



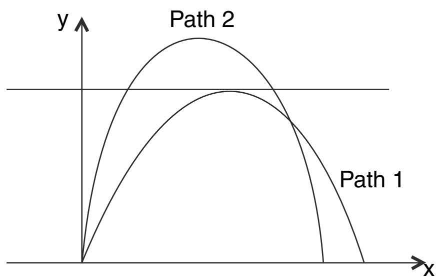


(b) Velocity of 3 relative to 2 will have magnitude

$u_{32} = \sqrt{u^2 + u^2 + 2.u.u.\cos 60°} = \sqrt{3}\,u$



79. If the stone is thrown at an angle of $\theta = 45°$, its range is $R = \frac{u^2}{g} = \frac{20^2}{10} = 40\,m$. In this case the maximum height attained by the stone will be $\frac{(u\sin\theta)^2}{2g} = 10\,m$. But a stone thrown at $\theta > 45°$ can have a path as shown in path 2 in the figure and thereby it can hit the bird sitting at a larger distance. Therefore, we need to calculate the maximum possible x co-ordinate of the projectile for a given height $h = 10\,m$. Trajectory equation is



$$y = x\tan\theta - \frac{g}{2}\left(\frac{x}{u\cos\theta}\right)^2$$

Put $y = 10\,m$, $g = 10\,m/s^2$ and $u = 20\,m/s$

$$10 = x\tan\theta - \frac{5}{400}x^2\sec^2\theta$$

$$10 = x\tan\theta - \frac{1}{80}x^2\left(1+\tan^2\theta\right)$$

$$\Rightarrow 800 = (80\tan\theta)x - x^2 - x^2.\tan^2\theta$$

$$\Rightarrow (x^2)\tan^2\theta - (80x)\tan\theta + (800 + x^2) = 0$$

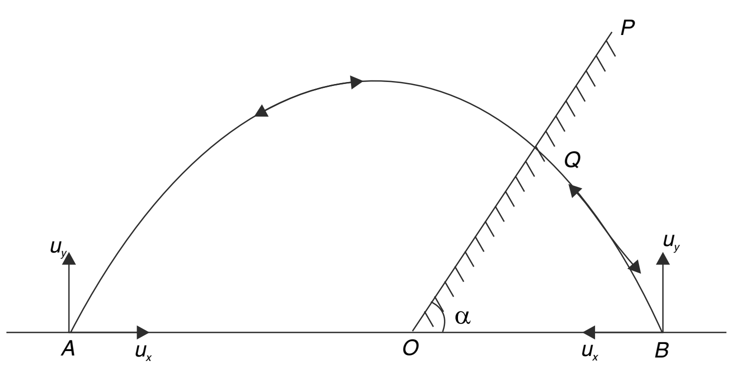
This is a quadratic equation in $\tan\theta$. For $\tan\theta$ to be real we must have

$(80x)^2 - 4(x^2)(800 + x^2) \geq 0$

$$\Rightarrow 1600x^2 - 800x^2 - x^4 \geq 0 \quad \Rightarrow 800 \geq x^2 \Rightarrow 20\sqrt{2} \geq x^2$$

$$\therefore x_{\max} = 20\sqrt{2}\,m$$

80. The path of both the projectiles to the wall, if considered together, makes the complete path of either of the particles in absence of the wall.



In absence of wall, the time of flight will be

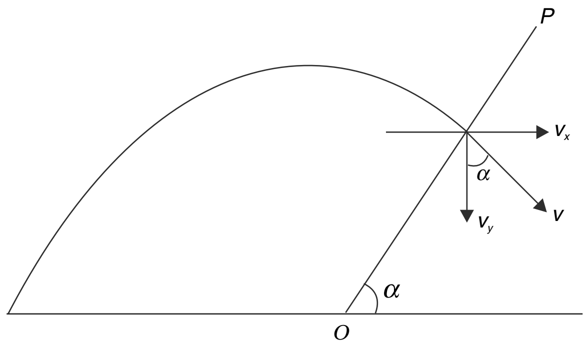
$$T = \frac{4}{2} + \frac{2}{2} = 3\ s$$

$$\Rightarrow \frac{2u_y}{g} = 3 \quad \Rightarrow u_y = 15\,m/s$$

(a) $H_{\max} = \dfrac{u_y^2}{2g} = \dfrac{15^2}{2 \times 10} = 11.25\ m$

(b) x_0 = Range of projectile in absence of wall

$$24 = u_x T \quad \Rightarrow u_x = \frac{24}{3} = 8\ m/s$$



Particle projected from A, hits the wall normally, $2s$ after its projection. It means its velocity vector makes an angle α to the vertical after 2 s.

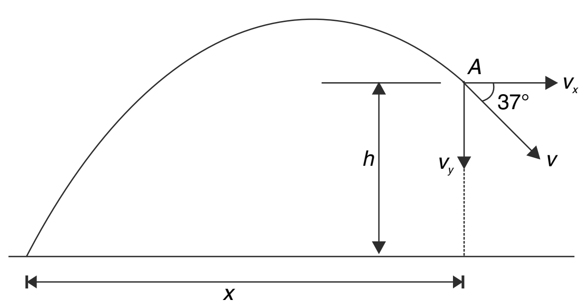
$$\therefore \tan\alpha = \frac{v_x}{|v_y|} = \frac{u_x}{|u_y - gt|} = \frac{8}{|15 - 10 \times 2|}$$

$$\tan\alpha = \frac{8}{5}$$

81. (a) $u_x = 20 \cos 53° = 12\ m/s$

$u_y = 20 \sin 53° = 16\ m/s$

When the ball enters the pipe its velocity vector makes an angle of 37º with the horizontal.



$$\tan 37° = \frac{v_y}{v_x}$$

$$\frac{3}{4} = \frac{v_y}{12}$$

$v_y = 9\ m/s$

$v_y^2 = u_y^2 - 2gh$

$\therefore\ 9^2 = 16^2 - 2 \times 10 \times h \Rightarrow h = 8.75\ m$

Now $\dfrac{h}{L} = \sin 37° \Rightarrow L = \dfrac{8.75 \times 5}{3} = 14.58\ m$

(b) Let the ball be at A at time 't'

$v_y = u_y = gt$

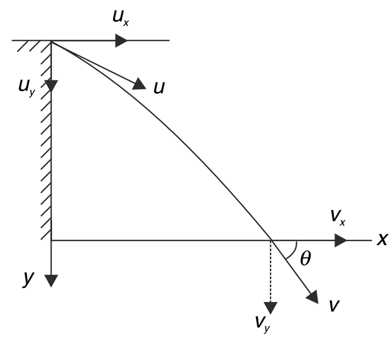
$\Rightarrow -9 = 16 - 10.t \Rightarrow t = 2.5\ s$

$\therefore\ x = u_x.t = 12 \times 2.5 = 30\ m$

$\therefore\ OB = 30 + L \cos 37° = 30 + 14.58 \times \dfrac{4}{5} = 41.66\ m$

82. (a) $v_y^2 = u_y^2 + 2gy$ ....(i)

and $v_x = u_x$ ....(ii)



$\tan\theta = \dfrac{v_y}{v_x}$

As per question minimum value of θ is 45°.

$\Rightarrow$ ratio of minimum v_y and maximum v_x is tan 45° = 1

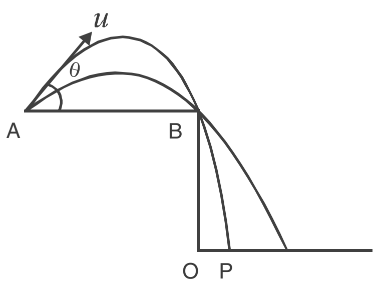
$(v_y)_{\min} = \sqrt{2gy}$ when $u_y = 0$

and$(v_x)_{\max} = u$ when stone is projected horizontally.

$\therefore \dfrac{\sqrt{2gy}}{u} = 1$

$y = 5\ m$

(b)



The shell cannot land in the region OP.

Equation of trajectory for projectile $y = x\tan\theta - \dfrac{gx^2}{2u^2}(1 + \tan^2\theta)$

For point B, $x = 960\ m$ and $y = 0$

$$\tan^2\theta \times \frac{10 \times (960)^2}{2 \times (100)^2} - (\tan\theta)960 + \frac{10 \times 960^2}{2 \times (100)^2} = 0$$

tan θ = 3/4, 4/3

For tanθ = 4/3, OP is smallest. Hence the trajectory equation is- $\dfrac{1}{N}$

$$y = \frac{4}{3}x - \frac{x^2}{2000}\left(\frac{25}{9}\right)$$

For point P $y = -960\ m$

$$-960 = \frac{4}{3}x - \frac{x^2}{720}$$

Solving $x = 1440\ m$

$OP = x - 960 = 480\ m$

84. (a) As the vertical component of velocity does not change during collision, the time of flight is not affected by a collision.

Time to touch the ground = $\sqrt{\dfrac{2h}{g}}$

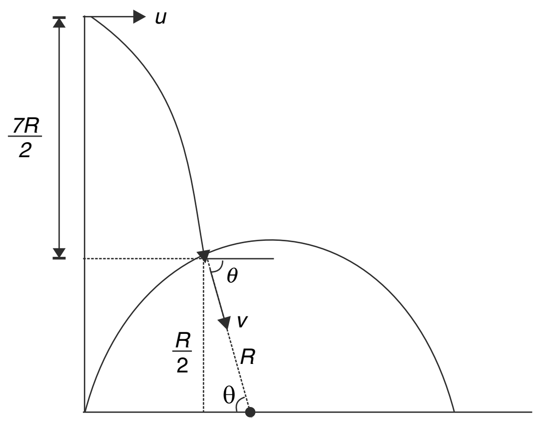
The total time of flight will be twice this time since the particle will bounce back to same height from which it was thrown.

(b) The ball must strike the sphere normally (i.e., along radius) so that it can rebound back along the same path.

$$\sin\theta = \frac{R/2}{R} = \frac{1}{2}$$

$\theta = 30°$

The velocity of the ball at the instant of hitting the sphere is inclined at θ to the horizontal.



$$\therefore \tan\theta = \frac{v_y}{v_x}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{2g\left(\frac{7R}{2}\right)}}{u}$$

$$\therefore u = \sqrt{21gR}$$

85. (a) Hint: The velocity of rain relative to the bus has same vertical component in both cases.

86. Velocity of truck

$$\vec{V}_T = 0\,\hat{i} + 90\cos\theta\,\hat{j} - 90\sin\theta\,\hat{k}$$

Where $\theta = \tan^{-1}(0.1)$

$$\therefore \vec{V}_T = (89.6\,\hat{j} - 8.94\,\hat{k})\ km/hr$$

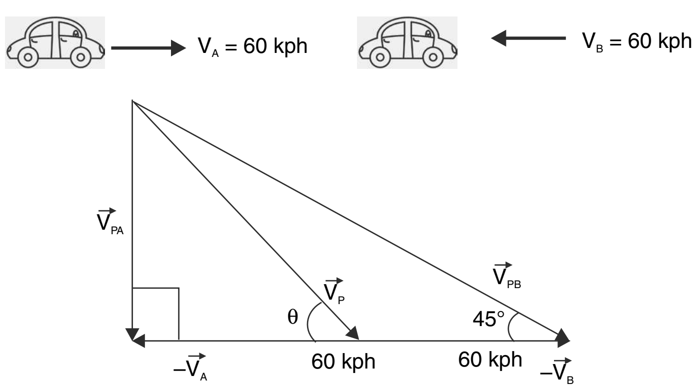
Velocity of police car

$$\because\ \vec{V}_P = -80\sin 30°\,\hat{i} - 80\cos 30°\,\hat{j} + 0\,\hat{k}$$
$$= -40\,\hat{i} - 69.3\,\hat{j}$$

$\therefore$ Relative velocity

$$\vec{V}_{TP} = \vec{V}_T - \vec{V}_P$$
$$= \left(40\,\hat{i} + (158.9)\,\hat{j} - 89\,\hat{k}\right) km/hr$$

87.



From the above relative velocity diagram

$V_{PA} = 120\ kph$

and $\tan\theta = \dfrac{120}{60} = 2$

$V_P = \sqrt{60^2 + 120^2} = 60\sqrt{5}\ kph$

88. V = velocity of boat relative to water.

V_A and V_B = actual velocity of two boats.

From the condition given in the problem it follows that

$V_{Ay} = V_{By}$

$\Rightarrow V\cos\theta = V\cos\theta'$

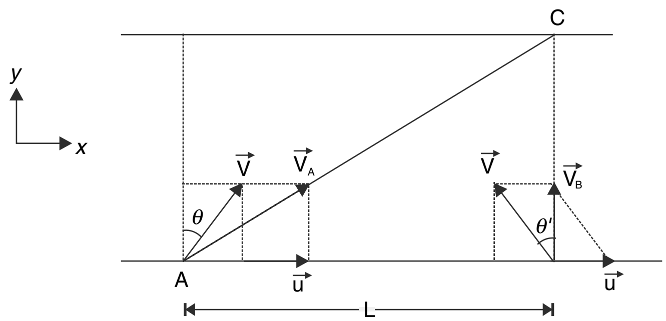
$\Rightarrow \theta = \theta'$

Also, $V\sin\theta' = u$ $[\because V_{Bx} = 0]$

$5\sin\theta' = 3$

$\sin\theta' = 3/5$

$\theta = \theta' = \sin^{-1}\left(\dfrac{3}{5}\right)$



$\therefore V_{Ay} = V_{By} = V\cos\theta = 5\times\dfrac{4}{5} = 4\ km/hr$

time to cross the river $t = \dfrac{3.0\ km}{4.0\ km/hr}$

$= \dfrac{3}{4}\ hr.$

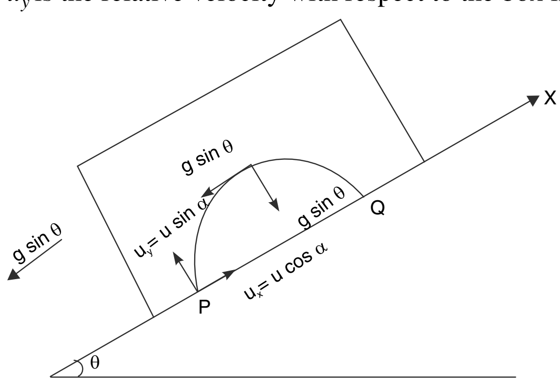
For A, $V_{Ax} = V\sin\theta + u = 5\times\dfrac{3}{5} + 3 = 6\ km/hr$

$\therefore L = V_{Ax}t = 6\times\dfrac{3}{4} = 4.5\ km$

90. (a) Let X direction be along the incline and Y direction be perpendicular to it.

u_x is the relative velocity of particle with respect to the box in x– direction.

u_y is the relative velocity with respect to the box in y – direction



Considering motion in Y direction relative to box

$u_y = +u\sin\alpha$; $a_y = -g\cos\theta$

$s_y = u_y t + \dfrac{1}{2}a_y t^2 \Rightarrow 0 = (u\sin\alpha)t - \dfrac{1}{2}g\cos\theta\times t^2$

$\Rightarrow t = \dfrac{2u\sin\alpha}{g\cos\theta}$

Considering motion in X direction relative to box

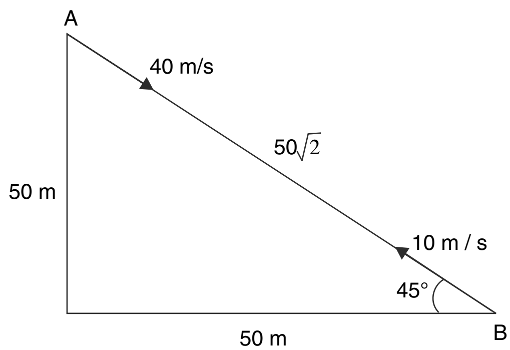
$u_x = + u \cos \alpha; a_x = 0$

$$s_x = u_y t + \frac{1}{2} a_y t^2 \Rightarrow s_x = u\cos\alpha \times \frac{2u\sin\alpha}{g\cos\theta} = \frac{u^2\sin 2\alpha}{g\cos\theta}$$

(b) For the observer on ground, to see the horizontal displacement equal to zero, the horizontal velocity of the particle (as observed from the ground) shall be zero.

$$U = \frac{u\cos(\alpha+\theta)}{\cos\theta}$$

92. (a) In reference frame of the stone, the path of the ball will be a straight line. This is because, relative acceleration of the two objects will be $g\ (\downarrow) - g\ (\downarrow) = 0$



In reference frame attached to the stone, the ball travels a straight line path of length $50\sqrt{2}\ m$ with a velocity of 50 *m*/*s*.

$\therefore$ Time for collision $t = \frac{50\sqrt{2}}{50} = \sqrt{2}$ sec

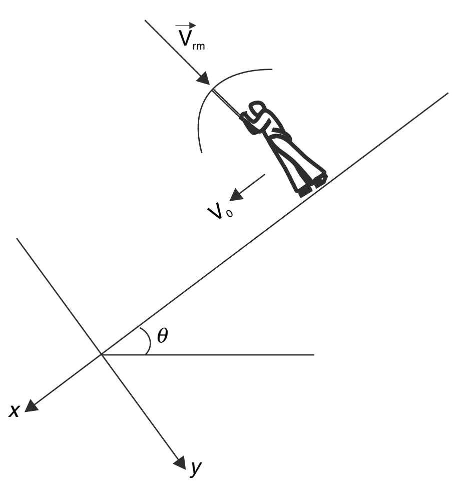
(b) Time of flight for stone

$$T = \frac{2 \times 10 \times \sin 45°}{10} = \sqrt{2}\text{ sec}$$

$\therefore$ The two objects collide just before hitting the ground.

(c) Straight line.

93. Take *x* axis along the incline and *y* direction perpendicular to it.



Velocity of rain relative to the man is perpendicular to the incline in this case (i.e., along the umbrella stick. This keeps canopy perpendicular to the rainfall and provides maximum safety).

$$\vec{V}_{rm} = \vec{V}_r - \vec{V}m$$
$$= \left(V_x\hat{i} + V_y\hat{j}\right) - V_0\hat{i} = \left(V_x - V_0\right)\hat{i} + V_y\hat{j}$$

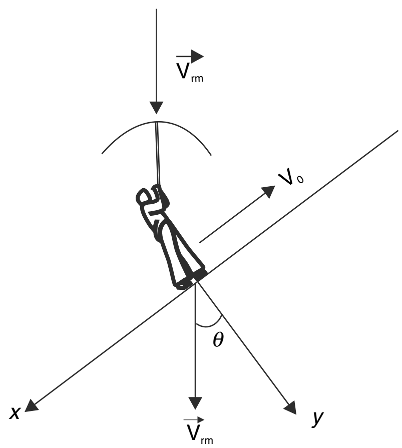
Since $\vec{V}_{rm}$ has no x component

$\therefore V_x = V_0$

When the man is walking up, $\vec{V}_{rm}$ is directed vertically downward.

$$\vec{V}_{rm} = \left(V_x\hat{i} + V_y\hat{j}\right) - \left(-V_0\hat{i}\right)$$
$$= V_0\hat{i} + V_y\hat{j} + V_0\hat{i}$$
$$= 2V_0\hat{i} + V_y\hat{j}$$

From diagram

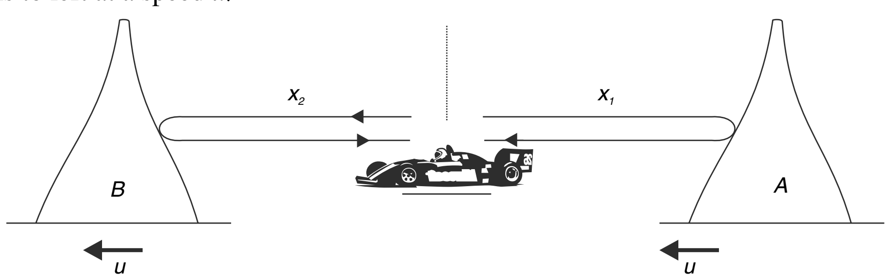


$$\tan\theta = \frac{2V_0}{V_y}$$

$$\frac{3}{4} = \frac{2V_0}{V_y} \Rightarrow V_y = \frac{8V_0}{3}$$

$$\therefore V_r = \sqrt{V_x^2 + V_y^2} = \frac{\sqrt{73}}{3}V_0$$

94. In the reference frame of the car, there is no wind (the air is still). In this frame the sound wave travels at speed v and hill A travels to left at a speed u.



Sound has constant speed in this reference frame while approaching the hill A and after getting reflected from it.

Time for sound to travel from car to hill A; $t_1 = \dfrac{x_1}{v+u}$

Time for sound to travel back from hill to car $t_1 = \dfrac{x_1}{v+u}$

$\therefore$ Time after which echo is heard $T_1 = 2t_1 = \dfrac{2x_1}{v+u}$.

If sound reaches hill B in time t_1' then $x_2 + ut_1' = vt_1'$

$$\therefore t_1' = \frac{x_2}{v-u}$$

Same amount of time is needed for the sound to return back.

Time after which echo is heard

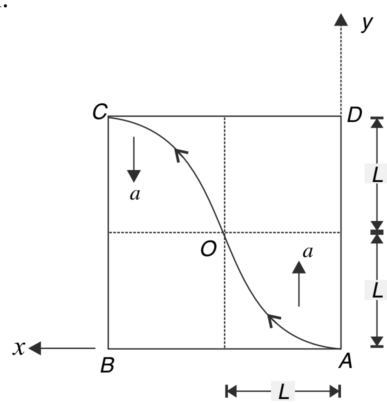
$$T_2 = \frac{2x_2}{v-u}$$

$$T_1 = T_2$$

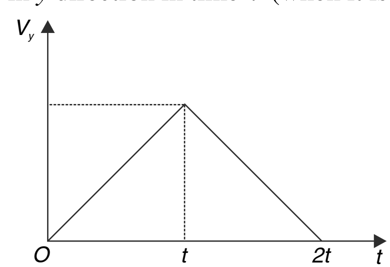
$$\frac{x_1}{v+u} = \frac{x_2}{v-u}$$

$$\frac{x_1}{x_2} = \frac{v+u}{v-u}$$

95. In the reference frame of the wagon, the ball has an acceleration a in + y direction. After brakes are applied the ball has same acceleration in $-y$ direction.



The $V_y - t$ graph for the ball will be as shown. If the ball covers a displacement y in y direction in time 't' (when it is at centre O) then it will again travel same distance y in next interval 't'. This is evident from area under v – t graph. In x direction the ball keeps moving uniformly. It means that it will take equal amount of time for the ball to cover first and second half of displacements each equal to L.



Hence, without any calculation we can say that the ball will hit the corner C.

At that moment $V_y = 0$ and $V_x = u$

96. (a) Particle 1 will hit the ground first. Let it hit the ground after time t.

$$32 = (10 \sin 37°)t + \frac{1}{2} \times g \times t^2$$

$$32 = 6\,t + 5\,t^2 \Rightarrow t = 2\ s$$

(b) Relative acceleration of any two particles is zero.

Hence, particles with maximum initial relative speed will be at maximum separation. Particle 2 and 5 have maximum relative speed

$U_{25} = 25\ m/s$

Separation between them at $t = 2\ s$ will be

$25 \times 2 = 50\ m$

(c) Particle 1, followed by 2 and 5 will land on ground. Time at which 2 and 5 land on ground is

$$t_2 = \sqrt{\frac{2 \times 32}{10}} = \sqrt{6.4} = 2.53\ s$$

Time at which separation between 3 and 4 is to be calculated is $t' = 2.33 + 0.3 = 2.83$

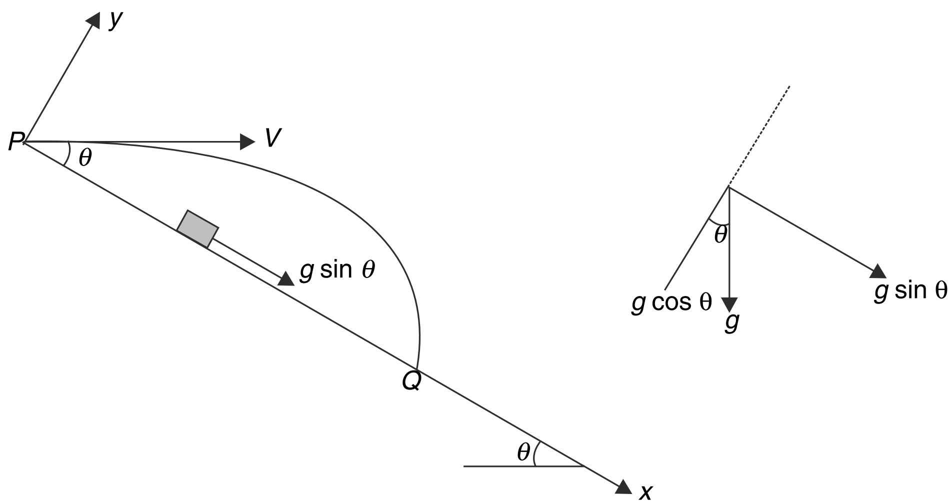
[you can check that both 3 & 4 have time of flight greater than 2.83 s]

Relative velocity of 3 and 4 is

$$V_{34} = \sqrt{6^2 + 17^2} = 18.0\ m/s$$

$\therefore$ Required separation = 2.83 × 18 = 50.94 m

97.



(a) If we fix our co-ordinate axes along the incline (x) and perpendicular to it (y), the acceleration of both objects in x direction is same equal to $a_x = g \sin \theta$.

$\therefore$ The two bodies will collide if the x component of their initial velocities is equal.

$\Rightarrow u = v \cos \theta$

$$\therefore\ u = 10 \times \frac{4}{5} = 8\ m/s$$

(b) The time of flight of the projectile can be calculated by consideration of its motion in y direction.

$$O = (v \sin\theta)t - \frac{1}{2}(g\cos\theta)t^2$$

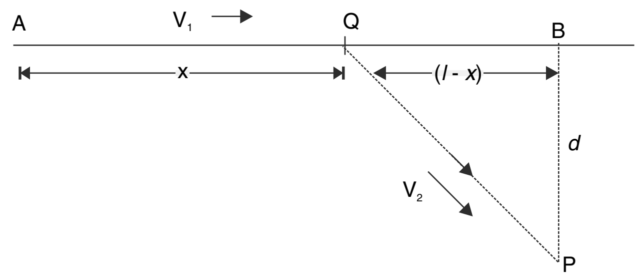
$$\Rightarrow t = \frac{2v\sin\theta}{g\cos\theta} = \frac{2 \times 10 \times 3}{10 \times 4} = \frac{3}{2}\text{sec}$$

$$\therefore \text{Distance } PQ = ut + \frac{1}{2}(g\sin\theta)t^2$$

$$= 8 \times \frac{3}{2} + \frac{1}{2} \times 10 \times \frac{3}{5} \times \left(\frac{3}{2}\right)^2$$

$$= 18.75\ m$$

98. Let the man travel a distance 'x' on road in order to reach 'P' in minimum time



$$t = \frac{x}{v_1} + \frac{\sqrt{d^2 + (l-x)^2}}{v_2} \quad \text{.....................(A)}$$

Time to travel from A to P depends on x.

This time (t) is minimum when

$$\frac{dt}{dx} = 0$$

$$\Rightarrow \frac{1}{v_1} + \frac{1}{v_2}\frac{\ell - x}{\sqrt{d^2 + (\ell - x)^2}} = 0$$

Solving, $\quad x = l \pm \dfrac{v_2 d}{\sqrt{v_1^2 - v_2^2}}$

The positive sign in unacceptable as x cannot be larger than l

$$\therefore \quad x = l - \frac{v_2 d}{\sqrt{v_1^2 - v_2^2}}$$

(a) If $\dfrac{v_2 d}{\sqrt{v_1^2 - v_2^2}} \geq l$

$$\Rightarrow d \geq \ell \frac{\sqrt{v_1^2 - v_2^2}}{v_2} = 50 \times \frac{4}{3} = \frac{200}{3}\, m$$

In this case x is negative or zero.

But it makes no sense to have $x < 0$.

Hence, man will need to walk along straight line AP in order to cover the distance in minimum time.

(b) $\therefore$ If $l > \dfrac{v_2 d}{\sqrt{v_1^2 - v_2^2}}$ then x is positive.

$$x = l - \frac{v_2 d}{\sqrt{v_1^2 - v_2^2}}$$

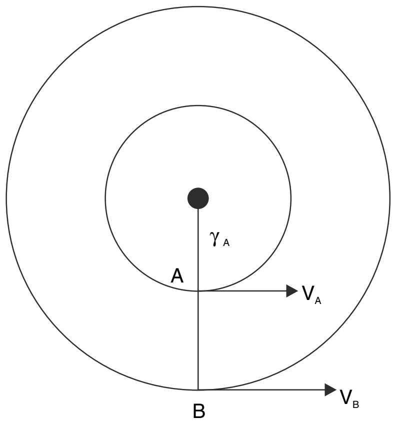
$$= 50 - \frac{3 \times \frac{100}{3}}{\sqrt{5^2 - 3^2}}$$

$$= 50 - 25 = 25\, m$$

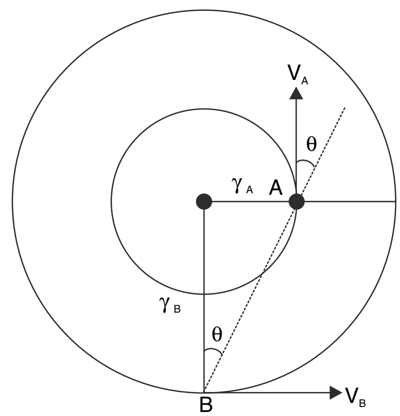
Putting this in equation (A)

$$t_{\min} = \frac{25}{5} + \frac{\sqrt{\left(\frac{100}{3}\right)^2 + (50-25)^2}}{3} = \frac{170}{9}\, s$$

99. (a) $\omega_{BA} = \dfrac{V_B - V_A}{\gamma_B - \gamma_A} = \dfrac{\omega(\gamma_B - \gamma_A)}{(\gamma_B - \gamma_A)} = \omega$



(b) $\tan\theta = \dfrac{\gamma_A}{\gamma_B}$.



ω_{BA} = (velocity of B wrt A perpendicular to AB) / AB

$$\omega_{BA} = \frac{V_B\cos\theta + V_A\sin\theta}{AB}$$

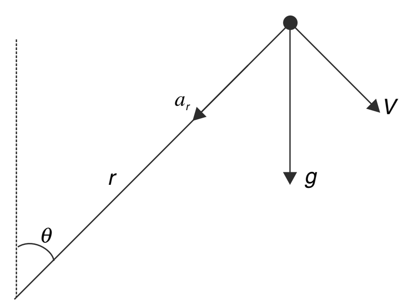
$$= \frac{\omega\gamma_B\cdot\dfrac{\gamma_B}{\sqrt{\gamma_A^2+\gamma_B^2}} + \omega\gamma_A\cdot\dfrac{\gamma_A}{\sqrt{\gamma_A^2+\gamma_B^2}}}{\sqrt{\gamma_A^2+\gamma_B^2}} = \omega$$

The questions can be answered by simple observation as well. Imagine yourself as particle A. You can see that particle B rotates around you and completes one revolution in the same time as you complete your rotation.

100. (a) Since $\dfrac{dr}{dt} = 0$, the velocity of the rocket is perpendicular to r.

$$v = \omega r = \left(\frac{d\theta}{dt}\right).r = 1.8 \times \frac{\pi}{180} \times 4000$$

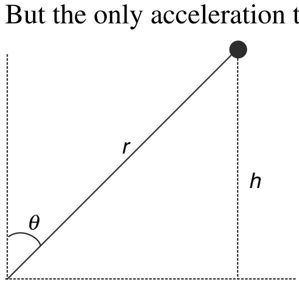
$$= 125.6\ m/s$$



The radial acceleration of the rocket is

$$a_r = \frac{v^2}{r} = \frac{(125.6)^2}{4000} = 3.94\ m/s^2$$

But the only acceleration that the rocket is having is $g(\downarrow)$



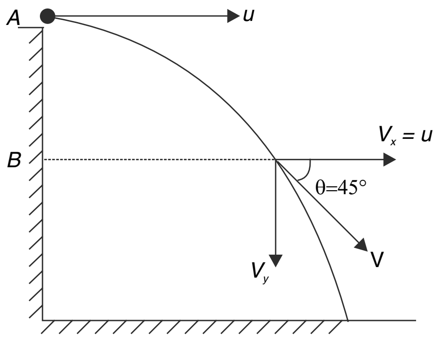
$\therefore\ g\cos\theta = a_r$

$9.8\cos\theta = 3.94$

$\cos\theta = 0.4$

Height $h = r\cos\theta = 4000 \times 0.4 = 1600\ m$

101.



(a) Tangential and radial accelerations will be equal at a point where $\theta = 45°$ [i.e. $v_x = v_y$]

$$a_t = a_r = \frac{g}{\sqrt{2}}$$

Speed of stone at this point

$v = \sqrt{2}\, u$

$$\therefore\ \frac{v^2}{R} = \frac{g}{\sqrt{2}} \Rightarrow R = \frac{2\sqrt{2}\, u^2}{g} \qquad \text{............(i)}$$

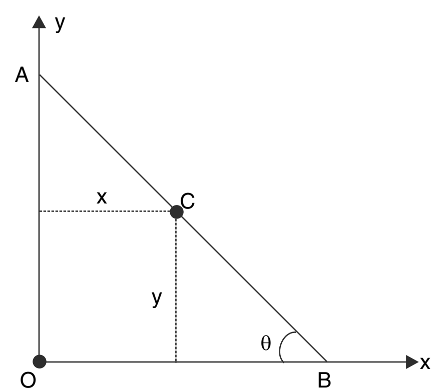
(b) If height of tower is less than AB, then R will always be less then value given by (i)

$$v_y^2 = 0^2 + 2gh$$

$\dfrac{u^2}{2g} = h =$ height AB for which v_y becomes equal to u

$$\therefore\ h < \frac{u^2}{2g}$$

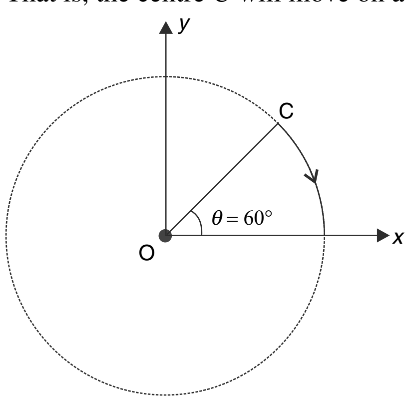
102. In the given co-ordinate system



$$x = \frac{L}{2}\cos\theta;\ y = \frac{L}{2}\sin\theta$$

$$\therefore\ x^2 + y^2 = \left(\frac{L}{2}\right)^2$$

That is, the centre C will move on a circle of radius $\dfrac{L}{2} = \dfrac{2}{2} = 1.0\ m$



The centre C will rotate through an angle $\theta = 60°$ on the circle of radius 1.0 m.

$$\therefore \text{ Distance travelled } S = \frac{2\pi R}{6} \quad [R = 1.0\ m]$$

$$= \frac{\pi}{3}\ m$$

103. (a) Let speed of point A be V_A. Then

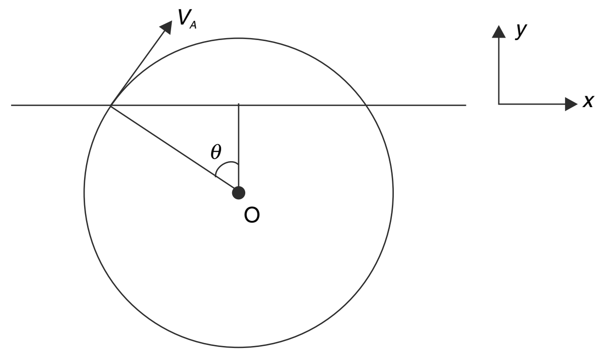
$V_A \sin\theta = V$ [$\because$ point is always on the rod]

$\therefore V_A = \dfrac{V}{\sin\theta}$

At the instant given

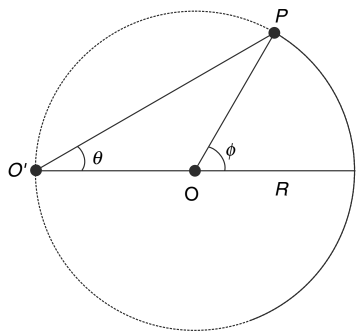
$$\cos\theta = \frac{d}{R} = \frac{1}{2}$$

$\therefore \sin\theta = \dfrac{\sqrt{3}}{2} \quad \therefore V_A = \dfrac{2V}{\sqrt{3}}$



(b) See the figure.

It is clear from the geometry of the figure that



$\phi = 2\theta$

$\therefore \dfrac{d\phi}{dt} = 2\dfrac{d\theta}{dt}$

But $\dfrac{d\theta}{dt} = \omega$ = angular velocity of P figure about O'

And $\dfrac{d\phi}{dt} = \omega_0$ = angular velocity of P figure about O

$\therefore \omega_0 = 2\omega$

$\therefore$ Angular velocity of P about O is constant.

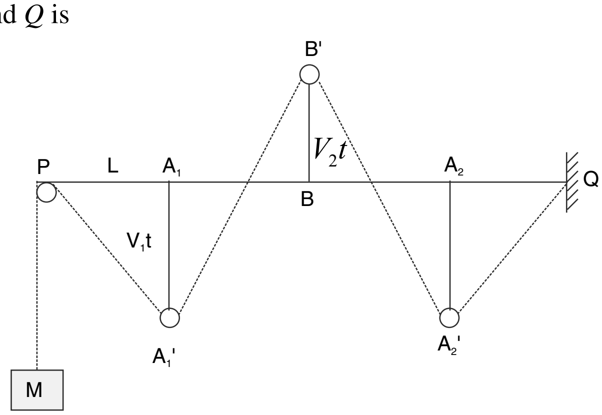
$\therefore$ Tangential acceleration $a_t = 0$

And, Radial (Centripetal) acceleration $a_r = \omega_0^2 R = 4\omega^2 R$

$\therefore$ Acceleration of P is $4\omega^2 R$ directed towards O.

104. The position at time t has been shown in figure. The shape of string has been shown by dotted lines.

Length of string between P and Q is



$PA_1' + A_1'B' + B'A_2' + A_2'Q$

$$= \left[\sqrt{L^2 + (v_1 t)^2} + \sqrt{L^2 + (v_1 + v_2)^2 t^2} \right] \times 2$$

Increase in length of string between PQ must be equal to distance(y) through which M moves up

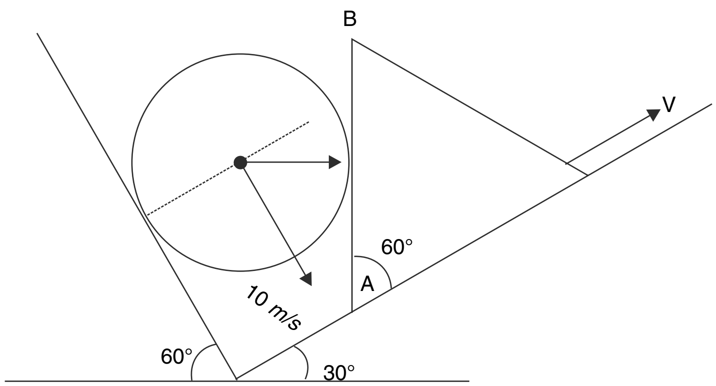
$$\therefore \; y = 2\left[\sqrt{L^2 + v_1^2 t^2} + \sqrt{L^2 + (v_1 + v_2)^2 t^2} \right] - 4L$$

Hence velocity of M is

$$v = \frac{dy}{dt} = \frac{2v_1^2 t}{\sqrt{L^2 + v_1^2 t^2}} + \frac{2(v_1 + v_2)^2 t}{\sqrt{L^2 + (v_1 + v_2)^2 t^2}}$$

This is the required answer.

105. Wall AB of the wedge is vertical. For ball to remain in contact with the wedge the velocity component of the ball perpendicular to the wall AB must be equal to velocity component of the wedge in horizontal direction (i.e., perpendicular to wall AB)



$\therefore$ 10 cos 60° = V cos 30°

$$5 = V\frac{\sqrt{3}}{2}$$

$$\therefore V = \frac{10}{\sqrt{3}}\, m/s$$

106. (a) Length of the stick is fixed. Hence velocity component of end A along the length of the stick = velocity component of end B along the length of the stick.

$\therefore \; V_B \cos\theta = V_A \sin\theta$

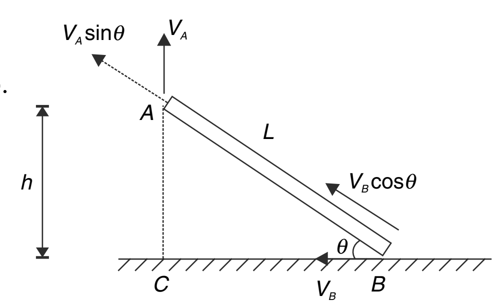
Initially, θ is small (which means $\cos\theta$ is large and $\sin\theta$ is small). Therefore, V_B is smaller than V_A.

As θ increases, V_B will increase.

when $V_A = V_B$

$\cos\theta = \sin\theta \Rightarrow \tan\theta = 1$

$\Rightarrow \theta = 45°$

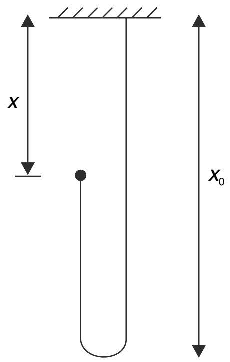


$$\therefore \; h = L\sin 45° = \frac{1}{\sqrt{2}} \qquad [\because L = 1\,m]$$

$$\therefore \; t_0 = \frac{h}{V_A} = \frac{1}{4\sqrt{2}}\, s$$

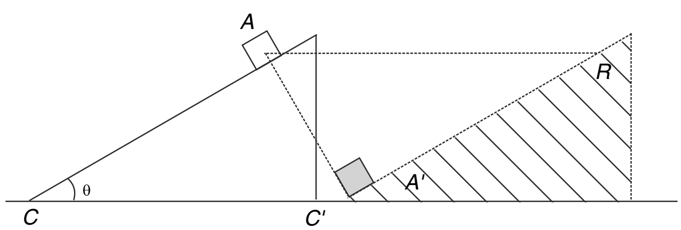
(b) $BC = L\cos 45° = \frac{1}{\sqrt{2}}$ $\therefore$ Distance travelled by end $B = \left(1 - \frac{1}{\sqrt{2}}\right)$

107. $2(x_0 - x) + x = L$ = length of the rope



$$\therefore \quad 2\frac{dx_0}{dt} - \frac{dx}{dt} = 0 \quad \therefore \quad \frac{dx_0}{dt} = \frac{1}{2}\left(\frac{dx}{dt}\right) = \frac{1}{2}V$$

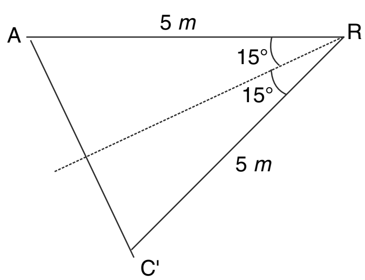
108.



Because length of string does not change

$\therefore AR = RA' = 5\ m$

$\angle ARC' = \theta = 30°$



Distance travelled by A

$= AC' = [5 \sin 15°] \times 2 = 10 \sin 15°$

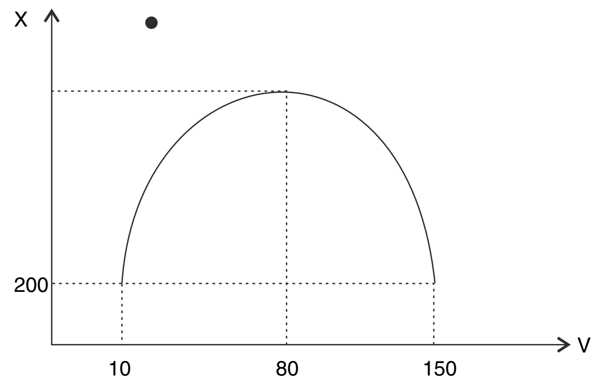
110. Equation of the line given in graph is

$$t = -\frac{2}{15}V + \frac{64}{3} \qquad ....(1)$$

When car moves at constant speed V for time t, it will cover a distance

$$X = Vt = -\frac{2}{15}V^2 + \frac{64}{3}V \qquad ....(2)$$

Graph of X vs V is parabolic as shown



X is maximum for $V = 80\ km/hr^{-1}$ [$\because$ parabola is symmetric curve and vertex will lie midway between $V = 10$ and $V = 150$]

For $V = 80$, value of X is

$$X = -\frac{2}{15}\times 80^2 + \frac{64}{3}\times 80$$

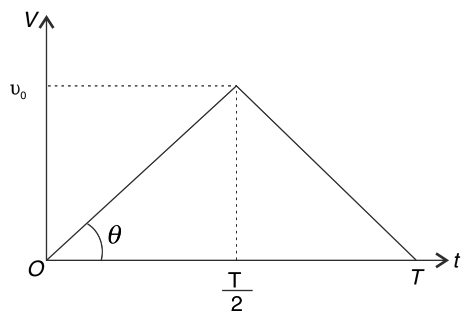
$$= \frac{2560}{} = 853.3\,km$$

This is the maximum distance that the car can travel on a tank full of petrol.

Maximum mileage $= \dfrac{853.3\ km}{50\ litre}$

$= 17\ km/l^{-1}$

111. (a) To cover maximum possible distance, the train shall accelerate for half the time and retard for the remaining half with both acceleration and retardation equal to a. $V - t$ graph for the case is as shown.



$\tan\theta = a$

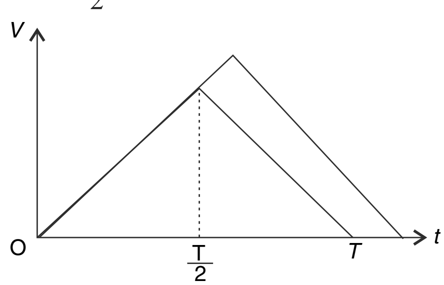
$\Rightarrow V_0 = a.\dfrac{T}{2}$

Distance travelled = area of the triangular graph

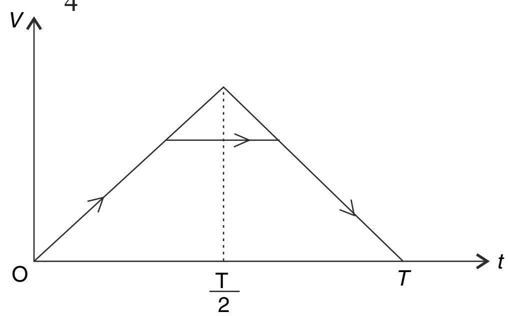
$$= \frac{1}{2}\times T\times a\frac{T}{2} = \frac{1}{4}aT^2$$

You can plot the graph for any other case and show that it is not possible to cover a distance greater then $\frac{1}{4}aT^2$ and also come to rest at $t = T$. For example –

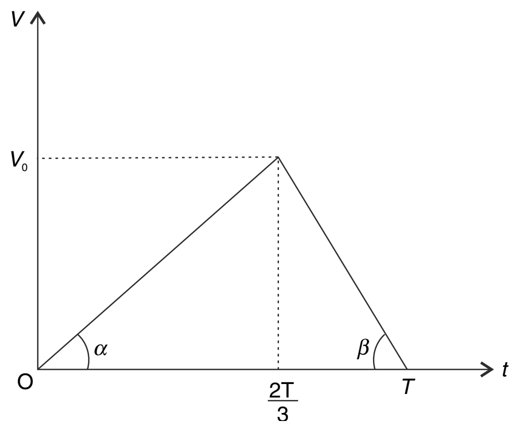
(i) If train accelerates for $t > \dfrac{T}{2}$, it cannot be stopped at $t = T$



(ii) If train is accelerated for $t < \dfrac{T}{2}$, then moved with constant velocity and then braked, the distance will be less then $\frac{1}{4}aT^2$



(b) Again Maximum distance is covered when train is allowed to travel with maximum acceleration for largest amount of time



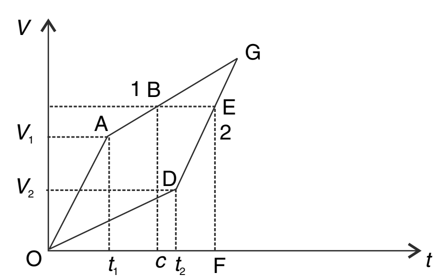
tan $\alpha = a$; tan $\beta = 2a$

$\therefore V_0 = a.\frac{2T}{3} = \frac{2}{3}aT$

Distance $= \frac{1}{2} \times T \times 2\frac{aT}{3} = \frac{1}{3}aT^2$

112. (a) $\frac{\text{For particle 1}}{X_0 = \frac{1}{2}(2a_0)t_1^2}$ ....(1)

$\frac{\text{For particle 2}}{X_0 = \frac{1}{2}a_0 t_2^2}$ ....(2)



(b) $V - t$ graph for motion of both particles is shown.

Obviously, particle 1 will cover $2X_0$ distance in lesser time.

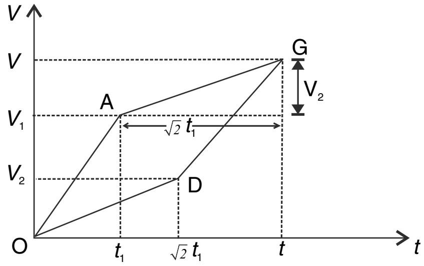
Area $OABCO = 2X_0$

Area $ODEFO = 2X_0$

Area under $a - x$ graph gives $\frac{V^2 - u^2}{2} = \frac{V^2}{2}$ where V = find speed; $u = 0$ = initial speed. For both 1 and 2 this area is same for their displacement $2X_0$

Hence, both will cross $2X_0$ with same speed.

(c) In above graph *OAGD* is a parallelogram. *V* is the common speed acquired at time *t*.

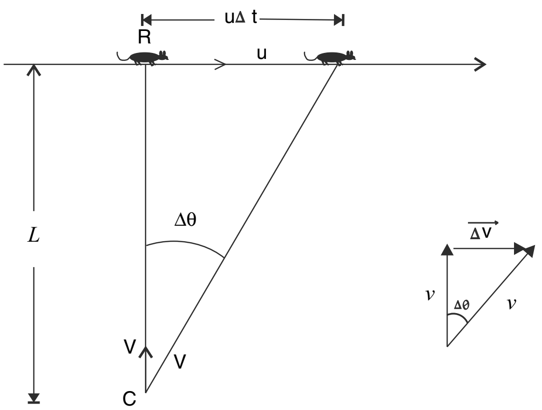


$V = V_1 + V_2$

$= 2a_0 t_1 + \sqrt{2}\ a_0\ t_1 = \left(2+\sqrt{2}\right)a_0 t_1 = \left(2+\sqrt{2}\right)a_0\sqrt{\dfrac{X_0}{a_0}} \qquad \left[\text{from (1) } t_1 = \sqrt{\dfrac{X_0}{a_0}}\right]$

$\therefore\ V = \left(2+\sqrt{2}\right)\sqrt{a_0 X_0}$

113. (a)



In a small interval, the rat moves a distance $u\,\Delta t$. The direction of velocity of cat changes towards the rat, by an angle $\Delta\theta = \dfrac{u\,\Delta t}{L}$ $\quad[\text{for } \Delta t \to 0]$

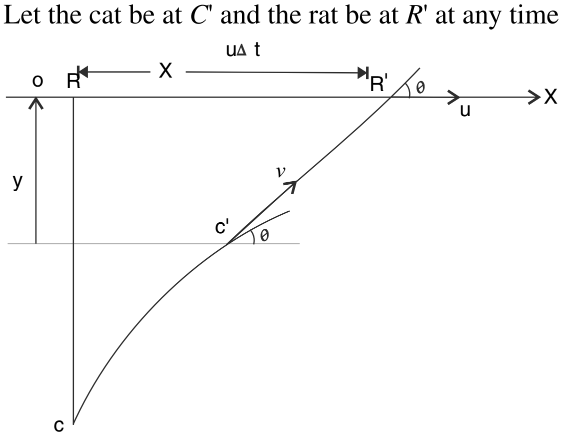
$\therefore$ Change in velocity of cat has magnitude

$\Delta v = \upsilon\,\Delta\theta = \dfrac{u\,\upsilon\,\Delta t}{L}$

$\dfrac{\Delta v}{\Delta t} = \dfrac{u\,\upsilon}{L}$

Acceleration is $\lim\limits_{\Delta t \to 0}\dfrac{\Delta v}{\Delta t} = \dfrac{uv}{L}$

(b) Let the cat be at *C*' and the rat be at *R*' at any time *t*



Relative velocity of approach is $v - u\cos\theta$

$\therefore$ Cat catches the rat at the instant when–

$$\int_0^{t_0} (v - u\cos\theta)dt = L \Rightarrow vt_0 - u\int_0^{t_0} \cos\theta dt = L \qquad \text{..................(1)}$$

Also $\quad ut_0 = \int_0^{t_0} v\cos\theta\, dt$

$$\frac{u}{v}t_0 = \int_0^{t_0} \cos\theta\ dt \qquad \text{..................(2)}$$

From (1) and (2)

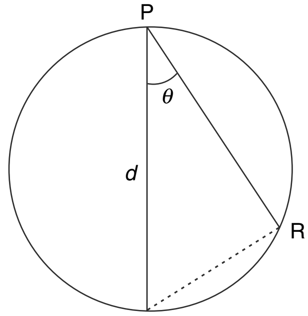
$$vt_0 - \frac{u^2 t_0}{v} = L$$

$$t_0 = \frac{vL}{v^2 - u^2}$$

(c) Zero

(d) The path will be like a spiral

114. (a) The acceleration is largest along vertical direction (= g) and the body falling along this path will cover maximum distance in any time t



$$PQ = \frac{1}{2}gt^2 = d(say)$$

Naturally P is the top point and PQ the diameter of any such circle.

Now consider a body moving at an angle θ to the vertical. Distance travelled in time t will be

$PR = \frac{1}{2}g\cos\theta\, t^2 \quad [\because$ acceleration along $PR = g\cos\theta]$

$= d\cos\theta$

It can be easily proved that $d\cos\theta$ is the length of cord of the circle with diameter d and P as top point; the cord making an angle θ with PQ.

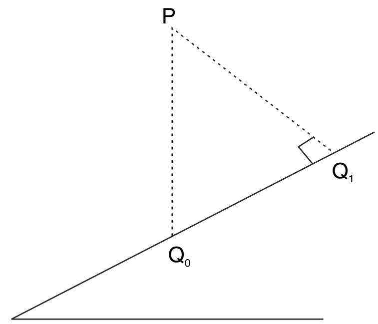
Hence the point following any path lies on such a circle.

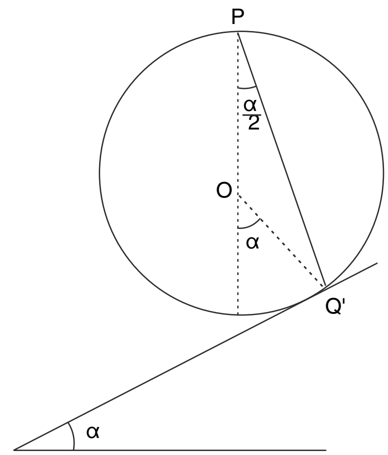
(b) Now we consider two possible paths

PQ_0 (vertically down)

and – (PQ_1 perpendicular to incline)

Acceleration along PQ_0 is maximum (= g) but path involved is also large. Acceleration along PQ_1, is less but path involved is shortest. We assume that path of least time lies somewhere in between PQ_0 and PQ_1.





The problem is solved easily by using the result proved in part (*a*). Bodies starting from *P* at the same time and travelling in different directions, always form a circle that grows with time and *P* as its top most point. After some time the circle with touch the inclined plane, with the plane tangential to the circle at the contact point *Q*' (see fig). Thus, body travelling along *PQ*' reaches the incline plane before any other such body.

It is easy to see that $< OPQ' = \dfrac{\alpha}{2}$

$\therefore$ Body travelling along line PQ' making an angle $\dfrac{\alpha}{2}$ with vertical reaches the plane in least time.

115. (a) Displacement at time *t* is

$$\vec{r} = \vec{u}t + \frac{1}{2}\vec{g}t^2$$

At the end of flight $\vec{r}$ is perpendicular to $\vec{g}$

$$\therefore \quad \vec{r}.\vec{g} = o \Rightarrow \left(\vec{u}.\vec{g}\right)t + \frac{1}{2}\left(\vec{g}.\vec{g}\right)t^2 = o$$

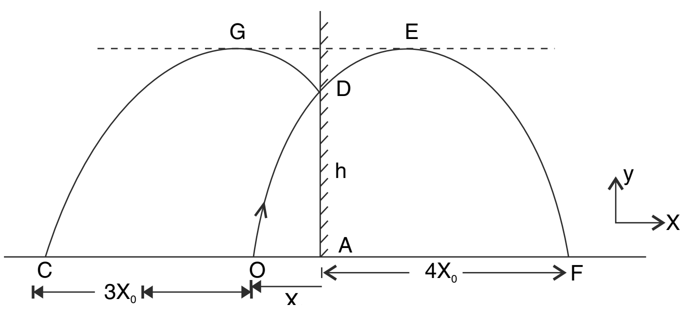
$$\Rightarrow t = -2\frac{\vec{u}.\vec{g}}{\left|\vec{g}\right|^2} \qquad \left[\because \vec{g}.\vec{g} = \left|\vec{g}\right|^2\right]$$

This is time of flight.

(b) $\vec{V}_{av} = \dfrac{\vec{r}}{t} = \vec{u} + \dfrac{1}{2}\vec{g}t$

$$\vec{V}_{av} = \vec{u} + \frac{1}{2}\vec{g}\left(-\frac{2\vec{u}.\vec{g}}{\left|\vec{g}\right|^2}\right) = \vec{u} - \frac{\vec{g}\left(\vec{u}.\vec{g}\right)}{\left|\vec{g}\right|^2}$$

116. In elastic collision, the horizontal velocity component will get inverted. There is no change in vertical motion of the projectile ($\because$ wall is smooth)



(a) In absence of wall, path of projectile would have been *DEF*. After collision, the path become *DGC* (mirror image of DEF, with wall as mirror)

Time of flight $T = \dfrac{2u_y}{g}$

Time of flight from O to D is $t = \dfrac{T}{5} = \dfrac{2u_y}{5g}$

[$\because$ Horizontal displacement is $\dfrac{1}{5}$ of the range of a usual projectile]

$$\therefore h = u_y t - \frac{1}{2}gt^2 = u_y.\frac{2u_y}{5g} - \frac{1}{2}g.\left(\frac{2u_y}{5g}\right)^2$$

$$= \frac{2u_y^2}{2g}\left[\frac{4}{5} - \frac{4}{25}\right] = \frac{16}{25}H \qquad \left[\text{Because} \frac{u_y^2}{2g} = H\right]$$

$$\therefore \frac{h}{H} = \frac{16}{25}$$

(b) $\overrightarrow{a_{av}} = \dfrac{\overrightarrow{\Delta V}}{\Delta t}$

The y component of average acceleration is certainly $-g$

Change in X component of velocity is

$\Delta V_x = -u_x - u_x = -2u_x$

$\therefore$ X Component of average acceleration is

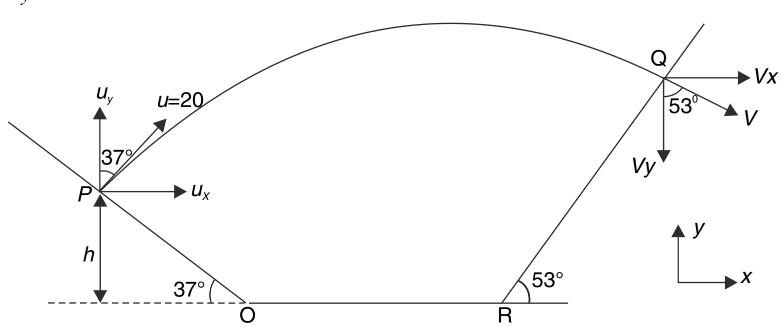
$$(a_{av})_x = -\frac{2u_x}{T} = -\frac{u_x}{u_y}.g \qquad \left[\because T = \frac{2u_y}{g}\right]$$

$$= -g\left[\because \frac{u_x}{u_y} = \tan 45° = 1\right]$$

$$\therefore \ |a_{av}| = \sqrt{g^2 + g^2} = \sqrt{2}\,g$$

118. At P : $u_x = 20 \sin 37° = 12$ m/s

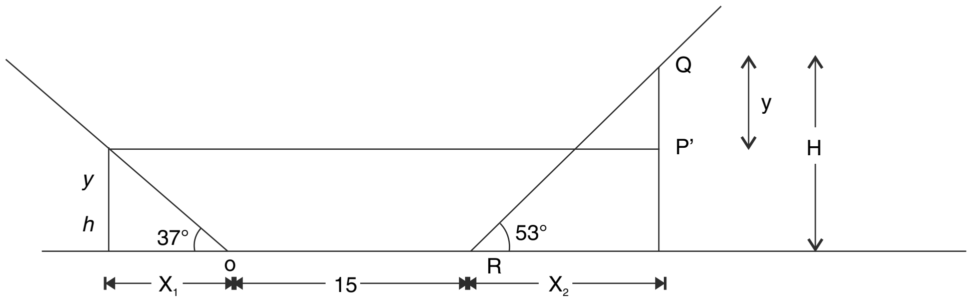
$u_y = 20 \cos 37° = 16$ m/s



At Q :

$V_x = u_x = 12$ m/s

$$\frac{V_x}{V_y} = \tan 53° \quad \Rightarrow V_y = \frac{12 \times 3}{4} = 9\,m/s$$



Time of flight = time interval in which the vertical velocity component charges from 16 m/s (↑) to 9 m/s (↓)

$V_y = u_y - gt$

$\Rightarrow -9 = 16 - 10\,t$

$\Rightarrow t = 2.5\ s$

Height of Q above P is

$$y = u_y t - \frac{1}{2}gt^2$$

$$y = 16\times 2.5 - \frac{1}{2}\times 10\times 2.5^2 = 8.75\,m$$

Horizontal distance covered by the projectile from P to Q is

$X = 12 \times 2.5 = 30\ m$

$X_1 + X_2 + 15 = 30$

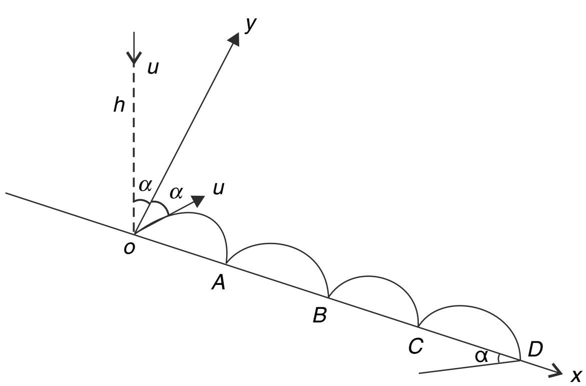
$X_1 + X_2 = 15$

$$\frac{4}{3}h + \frac{3}{4}H = 15$$

$$\frac{4}{3}h + \frac{3}{4}(h + y) = 15$$

Substituting the value of y and solving we get $h = 4.05\ m$

119.



Speed of ball just before striking the incline, $u = \sqrt{2gh}$.

After first hit the velocity is u making an angle α with y axis.

$u_x = u \sin\alpha$; $u_y = u \cos\alpha$

$a_x = g \sin\alpha$; $a_y = -g \cos\alpha$

Time of flight from O to A is obtained by the fact that from O to A displacement in y direction is 0

$$0 = u\cos\alpha.T - \frac{1}{2}g\cos\alpha.T^2$$

$$\therefore\ T = \frac{2u}{g}$$

Just after each impact, the y component of velocity remains $u_y = u\cos\alpha$. Hence, time of flight between any two impacts is same = T.

Required distance

= $OD - OA$

$$= \left[u_x(4T) + \frac{1}{2}a_x(4T)^2\right] - \left[u_xT + \frac{1}{2}a_xT^2\right]$$

$$= 3u_xT + \frac{15}{2}a_xT^2$$

Substituting for u_x, T and a_x

$AD = 72\ h \sin\alpha$

120. Let $\vec{V}_a$ be velocity of an army person and $\vec{V}_T$ be velocity of the terrorist. With respect to the terrorist, velocity of an army person will be

$$\vec{V}_{aT} = \vec{V}_a + \left(-\vec{V}_T\right)$$

$\overrightarrow{AO}$ Represents $\left(-\vec{V}_T\right)$

$\overrightarrow{OB_1}, \overrightarrow{OB_2}$ … represent $\vec{V}_a$ in various possible directions.

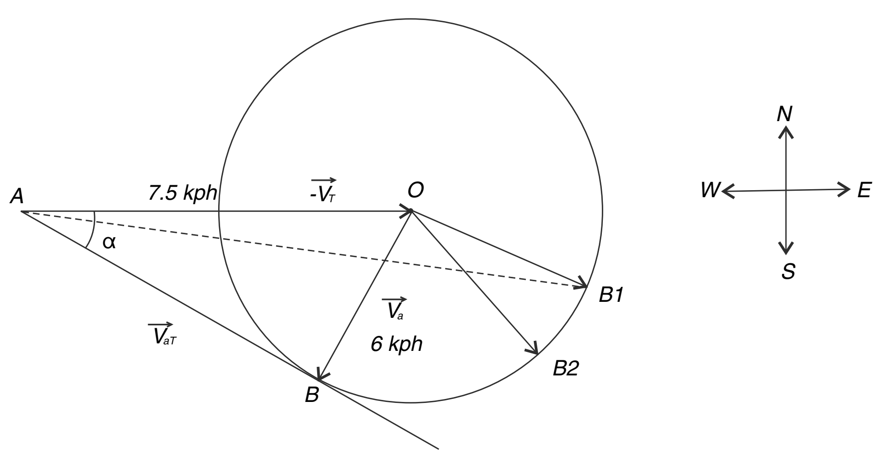
$\overrightarrow{AB_1}, \overrightarrow{AB_2}$ .. are relative velocities for various possible directions of $\vec{V}_a$. When AB gets tangential to the circle shown, we get maximum value of angle α. This is the case for which an army person will get closest to the terrorist. For this case

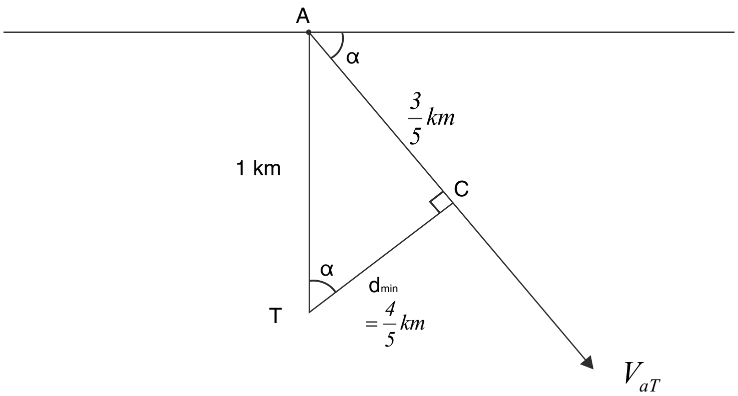
$$\sin\alpha = \frac{6}{7.5} = \frac{3}{5}$$

Terrorist sees one army person walking along a direction $\alpha = \sin^{-1}\left(\frac{3}{5}\right)$ south of East (with respect to himself) who gets nearest to him.

$$\frac{d_{\min}}{1\,km} = \cos\alpha$$

$$d_{\min} = 1 \times \frac{4}{5} = \frac{4}{5}\,km$$





(b)

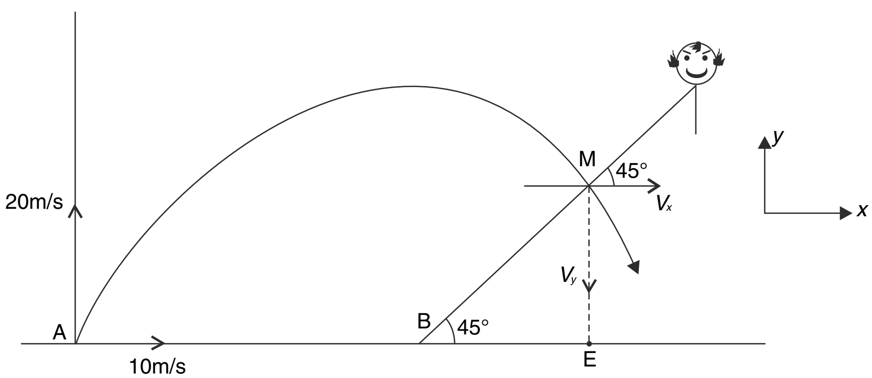
Time required

$$t = \frac{3/5\ km}{\sqrt{7.5^2 - 6^2}} = \frac{0.6}{4.5} hr$$
$$= 8 \min$$

121. In the reference frame of the wedge, initial velocity of the particle is

$\vec{V} = 10\hat{i} + 20\hat{j}\ m/s$

Hence, the motion in this frame is that of a usual projectile



This projectile hits the wedge normally, means its velocity at the instant of hit is inclined at 45^0 to the horizontal.

$\therefore\ \tan 45° = \left|\frac{V_y}{V_x}\right|$

10 = 10 t – 20

$\therefore$ 10 = 3 sec $\left[\because \left|V_y\right| = gt - 20 \text{ as the particle is going down}\right]$

Height of projectile at this instant

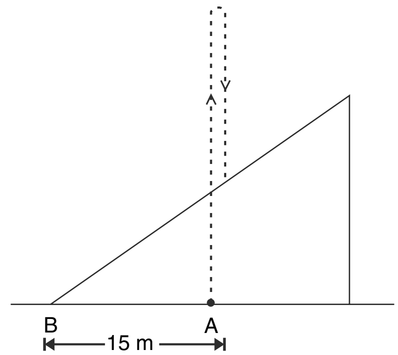
$$ME = 20 \times 3 - \frac{1}{2} \times 10 \times 3^2 = 15\, m$$

$$\frac{ME}{BE} = \tan 45° \quad \Rightarrow \quad BE = ME = 15\, m$$

And AE = 10 × 3 = 30 m

(a) $AB = 30 - 15 = 15\ m$

(b)



B will be 15 m to the left of A.

(c) Path is parabolic as shown above.

122.

For $y \le \frac{l}{2}$

$$V_x = \frac{2V_0}{l}y \quad \text{and} \quad V_y = u$$

$$\therefore \quad \frac{dy}{dx} = \frac{ul}{2V_0 y}$$

$$\Rightarrow \int_0^{l/2} y\,dy = \frac{ul}{2V_0}\int_{x=0}^{x_0} dx$$

$$\left(\frac{l}{2}\right)^2 = \frac{ul}{V_0}x_0 \qquad \Rightarrow \quad x_0 = \frac{V_0 l}{4u}$$

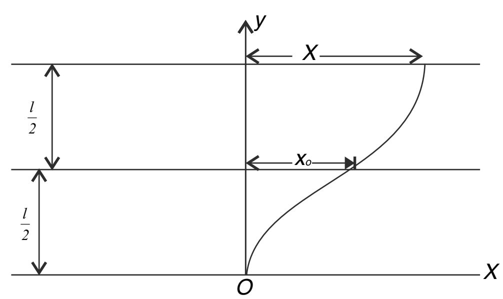
For $y \ge \frac{l}{2} \quad \Rightarrow \quad V_x = 2V_0\left(1 - \frac{y}{l}\right)$

$$\therefore \quad \frac{dy}{dx} = \frac{ul}{2V_0(l-y)}$$

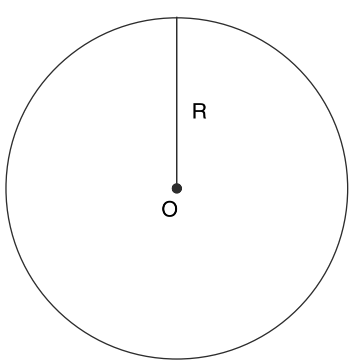
$$2V_0\int_{l/2}^{l}(l-y)\,dy = ul\int_{x_0}^{x} dx$$

$$\frac{l^2V_0}{4} = ul\left[x - \frac{V_0 l}{4u}\right]$$

$$\Rightarrow \quad x = \frac{V_0 l}{2u}$$



123.



The sprinkler wets a circle of radius R where R = maximum range of a projectile fired at speed u.

$$R = \frac{u^2}{g} = \frac{u}{\sqrt{2}}T \quad [T = \text{time of flight}] \qquad \text{..........................(i)}$$

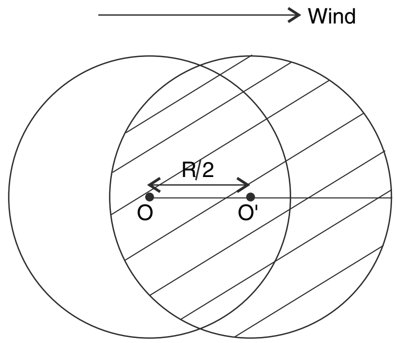
When there is wind, the velocity of wind gets added to the velocity of the drops. This will not change the time of flights because the vertical velocity component remains unaffected.

Let the wind be along X direction. Due to wind there will be additional shift (ΔX) of each drop along X direction.

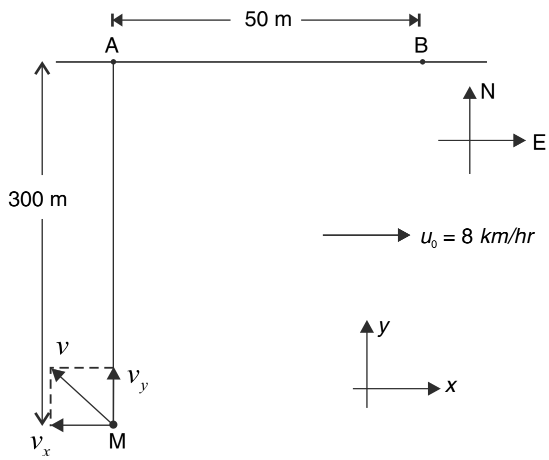
For the drop whose vertical velocity is $\frac{u}{\sqrt{2}}$ (i.e., those drops which fall on the circumference of the circle shown above)

$$\Delta X = \left(\frac{u}{2\sqrt{2}}\right)(T) = \frac{R}{2} \qquad \text{[Using (1)]}$$

The region which will get wet is again a circle of radius R, shifted from the original circle by $\Delta X = {}^{R}/_{2}$ (see fig)



125. (a) The situation has been shown in fig.



Let $\vec{v}$ = velocity of boat relative to water

$$= -v_x\hat{i} + v_y\hat{j}$$

Resultant velocity component in x direction is

$$V_x = u_0 - v_x$$

Resultant velocity component in y direction is

$$V_y = v_y \quad \left[u_0\hat{i} = \text{velocity of current}\right]$$

Time taken to travel 300 m in Y direction is

$$t = \frac{300}{v_y}$$

To enter the harbour, drift along X direction must be between O and 50 m

$$\therefore\ 0 < (u_0 - v_x)t \le 50$$

$$\therefore\ u_0 - v_x \le \frac{50}{(300/v_y)} = \frac{v_y}{6} \qquad \text{................(a)}$$

And $0 < u_0 - v_x$

$\Rightarrow u_0 > v_x$ ....(b)

Now speed of motor boat is

$$v = \sqrt{v_x^2 + v_y^2}$$

But $v_x^2 + v_y^2 \geq v_x^2 + 6^2 \left(u_0 - v_x\right)^2$ ....(c)

[$\because$ $v_y \geq 6(u_0 - v_x)$ from (a)]

$$\Rightarrow v_x^2 + v_y^2 \geq \left(v_x\sqrt{37} - \frac{36u_0}{\sqrt{37}}\right)^2 + \frac{36u_0^2}{37}$$

$\therefore$ $v_x^2 + v_y^2$ is minimum if $v_x\sqrt{37} - \dfrac{36u_0}{\sqrt{37}} = 0$

$$\Rightarrow v_x = \frac{36}{37}u_0 \quad \text{.................(d)}$$

[This condition is consistent with (b)]

$$\therefore \left(v_x^2 + v_y^2\right)_{\min} = \frac{36u_0^2}{37}$$

$$\therefore v_{\min} = \sqrt{\left(v_x^2 + v_y^2\right)_{\min}} = \sqrt{\frac{36}{37}}u_0$$

$$= \frac{6}{\sqrt{37}} \times 8 = \sqrt{\frac{48}{37}} km/hr$$

(b) For $v_{\min}$ we have

$$v_x = \frac{36}{37}u_0$$

$$v_y = \sqrt{v_{\min}^2 - v_x^2}$$

$$= \sqrt{\frac{36}{37}u_0^2 - \left(\frac{36}{37}\right)^2 u_0^2}$$

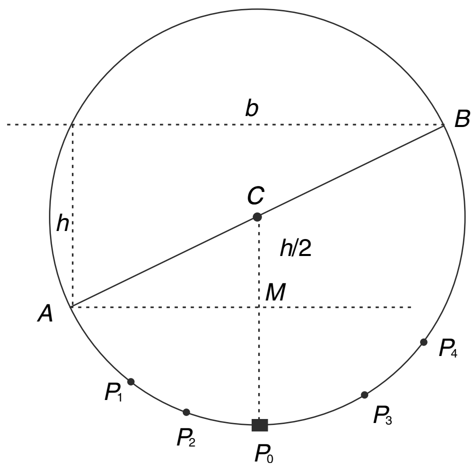
$$= u_0\sqrt{\frac{36}{37}\left(1 - \frac{36}{37}\right)} = \frac{6u_0}{37}$$

$\therefore$ If θ is angle made by $\vec{v}$ with y direction.

$$\tan\theta = \frac{v_x}{v_y} = 6\left(\text{independent of } u_0\right)$$

126. Given $\angle APB = 90°$

It means that A, P and B lie on a semicircle with AB as diameter.



$AB = 2R = \sqrt{h^2 + b^2}$

The two strings will form a right angle if the block lies on any of the points P_1, P_2, P_3, P_4 etc. But the *PE* will be minimum when the block is at the lowest point P_0 (CP_0 is vertical line through the centre)

$$MP_0 = R - \frac{h}{2} \quad \left[\because CM = \frac{h}{2}\right]$$

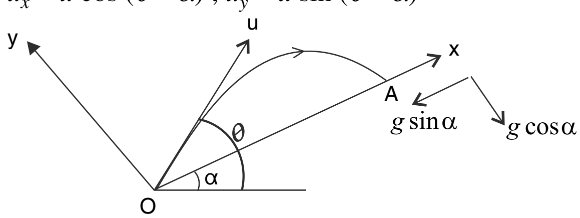
$$= \frac{\sqrt{h^2 + b^2}}{2} - \frac{h}{2} = \frac{1}{2}\left(\sqrt{h^2 + b^2} - h\right)$$

$$\therefore U_{\min} = -Mg(MP_0) = \frac{Mg}{2}\left[\sqrt{h^2 + b^2} - h\right]$$

127. Let the *X* and *Y* axes be as shown.

$a_x = -g \sin \alpha$; $a_y = -g \cos \alpha$

$u_x = u \cos (\theta - \alpha)$; $u_y = u \sin (\theta - \alpha)$



If time of flight is *T*.

$$Y = u_y t + \frac{1}{2} a_y t^2$$

$$0 = u \sin(\theta - \alpha) T - \frac{1}{2} g \cos \alpha T^2$$

$$\therefore T = \frac{2u \sin(\theta - \alpha)}{g \cos \alpha}$$

Range along the incline = *OA* (= *R* say)

$$R = u_x T + \frac{1}{2} a_x T^2$$

$$= \frac{2u^2 \sin(\theta - \alpha) \cos(\theta - \alpha)}{g \cos \alpha} - \frac{1}{2} g \sin \alpha \frac{4u^2 \sin^2(\theta - \alpha)}{g^2 \cos^2 \alpha}$$

$$= \frac{2u^2 \sin(\theta - \alpha)}{g \cos\alpha}\left[\cos(\theta - \alpha) - \frac{\sin\alpha \sin(\theta - \alpha)}{\cos\alpha}\right]$$

$$= \frac{2u^2 \sin(\theta - \alpha)}{g \cos^2\alpha}\left[\cos(\theta - \alpha)\cos\alpha - \sin(\theta - \alpha)\sin\alpha\right]$$

$$R = \frac{2u^2 \sin(\theta - \alpha).\cos\theta}{g\cos^2\alpha} \qquad \ldots\ldots\ldots\ldots..(1)$$

R will be maximum when $\frac{dR}{d\theta} = 0$

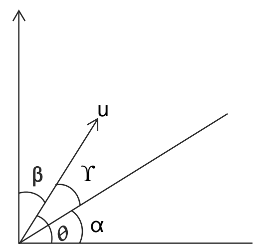
$$\Rightarrow \frac{d}{d\theta}\left[\sin(\theta - \alpha).\cos\theta\right] = 0$$

$$\Rightarrow \cos(\theta - \alpha)\cos\theta - \sin\theta\sin(\theta - \alpha) = 0$$

$$\Rightarrow \cos(2\theta - \alpha) = 0$$

$$\Rightarrow 2\theta - \alpha = \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\alpha}{2} + \frac{\pi}{4}$$



In this case one can show that $\beta = \gamma = \frac{\pi}{4} - \frac{\alpha}{2}$

(b) From (1)

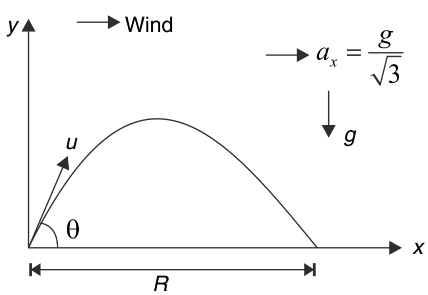
$$R = \frac{2u^2 \sin(\theta - \alpha)\cos\theta}{g\cos^2\alpha}$$

We want u^2 to be minimum for a given R.

This is possible if sin $(\theta - \alpha)$. cos θ is maximum, which is same as the condition in part (a).

Hence, least energy will be needed when the shell is fired exactly bisecting the angle between the vertical and the incline.

128. $a_x = \frac{g}{\sqrt{3}}; a_y = -g$



The time of flight depends only on vertical component of velocity and acceleration.

$$\therefore \quad T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$$

Horizontal range $R = u_x T + \frac{1}{2}a_x T^2$

$$\Rightarrow R = u\cos\theta\cdot\frac{2u\sin\theta}{g}+\frac{1}{2}\frac{g}{\sqrt{3}}\cdot\left(\frac{2u\sin\theta}{g}\right)^2=\frac{2u^2}{g}\left[\cos\theta\sin\theta+\frac{1}{\sqrt{3}}\sin^2\theta\right]$$

For R to be maximum $\frac{dR}{d\theta}=0$

$$\Rightarrow -\sin^2\theta+\cos^2\theta+\frac{2}{\sqrt{3}}\sin\theta\cos\theta=0$$

$$\Rightarrow \frac{1}{\sqrt{3}}\sin 2\theta=-\cos 2\theta \Rightarrow \tan 2\theta=-\sqrt{3}\Rightarrow 2\theta=\frac{2\pi}{3}\Rightarrow\theta=\frac{\pi}{3}$$

129. From graph

$u_y = 10\ m/s$

v_y becomes zero when $x=\frac{\text{Range}}{2}$

$\therefore$ Range = 20 m $\therefore$ $\frac{2u_xu_y}{g}=20$

$\therefore$ $u_x = 10\ m/s$ $\therefore$ $\tan\theta=\frac{u_y}{u_x}=1$ $\therefore$ $\theta = 45°$

130. For A

$V^2 = u^2 + 2as$

$(30)^2 = 0 + 2 \times a \times 20 \Rightarrow a = 22.5\ cm/s^2$

And $V = u + at$

$30 = 0 + 22.5 \times t \Rightarrow t=\frac{4}{3}s$

In time $\frac{4}{3}s$, block C descends by $7.5\times\frac{4}{3}=10\,cm$

Now length of the string

$X_A + 2X_C + X_B$ = constant ……..(i)

$\therefore$ $\Delta X_A + 2\Delta X_C + \Delta X_B = 0$ $\therefore$ $\Delta X_B = -[\Delta X_A + 2\Delta X_C]$

$= -[20 + 2 \times 10] = -40\ cm$

$\Rightarrow X_B$ decreases by 40 cm. Hence, B goes up by 40 cm.

From (i)

$$\frac{dX_B}{dt}=-\left[\frac{dX_A}{dt}+2\frac{dX_C}{dt}\right] \quad \text{……….. (ii)}$$

$= -[30 + 2 \times 7.5] = -45.0\ cm/s$

$-ve$ sign indicates X_B decreases with time, i.e., B moves up.

Differentiating (ii) once more

$$\frac{d^2X_B}{dt^2}=-\frac{d^2X_A}{dt^2}+2\frac{d^2X_C}{dt^2}$$

$$a_B = \frac{d^2X_B}{dt^2} = -[22.5+0] = -22.5\,cm/s^2$$

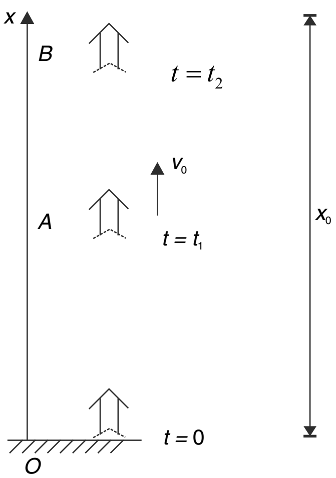
Negative sign once again indicates upward acceleration.

131. Initially, acceleration is positive. It becomes zero at time $t_1 = \sqrt{\frac{g}{2k}}$

At this time velocity v_0 of the rocket is maximum.

For $t > t_1$ the acceleration becomes negative (i.e., downwards) and the rocket retards. Finally, it stops at B (at $t = t_2$). Thereafter it falls back.

$a = \frac{g}{2} - kt^2$ [for interval $0 < t \le t_0$]



$$\frac{dv}{dt} = \frac{g}{2} - kt^2$$

$$\int_0^v dv = \frac{g}{2}\int_0^t dt - k\int_0^t t^2\,dt$$

$$v = \frac{g}{2}t - \frac{kt^3}{3} \qquad \text{..............(i)}$$

If v becomes zero at time t

$$0 = \frac{g}{2}t - \frac{k}{3}t^3$$

$$\Rightarrow t\left[\frac{g}{2} - \frac{k}{3}t^2\right] = 0 \Rightarrow t = 0$$

And $t = \sqrt{\frac{3g}{2k}}$ $\therefore$ $t_2 = t_0 = \sqrt{\frac{3g}{2k}}$

Actually, much before t_0 the acceleration of the rocket has turned negative [at time t_1 it becomes zero and thereafter it becomes negative]. Velocity decreases after t_1 and becomes zero at t_0.

(a) Maximum velocity during up journey is

$$V_{\max} = V_0 = \frac{g}{2}t - \frac{kt^3}{3} \qquad \left[t = t_1 = \sqrt{\frac{g}{2k}}\right]$$

$$= \frac{g}{2} \cdot \sqrt{\frac{g}{2k}} - \frac{k}{3} \cdot \frac{g}{2k}\sqrt{\frac{g}{2k}}$$

$$= \frac{g}{3}\sqrt{\frac{g}{2k}} = \sqrt{\frac{g^3}{18k}}$$

(b) From (i)

$$V = \frac{g}{2}t - \frac{kt^3}{3}$$

$$\frac{dx}{dt} = \frac{g}{2}t - \frac{k}{3}t^3$$

$$\int_0^{x_0} dx = \frac{g}{2}\int_0^{t_0} t\,dt - \frac{k}{3}\int_0^{t_0} t^3\,dt$$

$$x_0 = \frac{g}{2}\frac{t_0^2}{2} - \frac{k}{12}t_0^4$$

$$= \frac{t_0^4}{4}\left[g - \frac{k}{3}t_0^2\right]$$

$$= \frac{3g}{8k}\left[g - \frac{k}{3}\frac{3g}{2k}\right]$$

$$= \frac{3g^2}{16k}$$

(c) Time to fall down from height x_0 (acceleration being g downward) will be given by

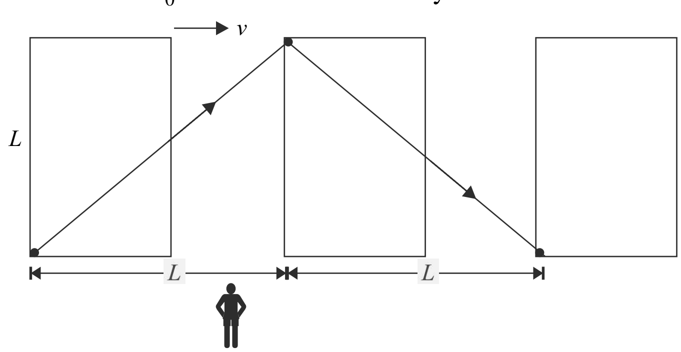
$$\frac{1}{2}gt^2 = \frac{3g^2}{16k} \Rightarrow t = \sqrt{\frac{3}{8}\frac{g}{k}}$$

$\therefore$ Total time of flight $T = t_0 + t = \sqrt{\frac{3}{2}\frac{g}{k}} + \sqrt{\frac{3}{8}\frac{g}{k}}$

$$= \frac{3}{2}\sqrt{\frac{3}{2}\frac{g}{k}}$$

132. (a) The path of the ball as observed by the observer outside the room has been shown in the figure.

The time t_0 can be calculated easily in the reference frame of the room.



$$t_0 = \frac{2L}{V}$$

Displacement of room in this interval

$$= V \cdot \frac{2L}{V} = 2L$$

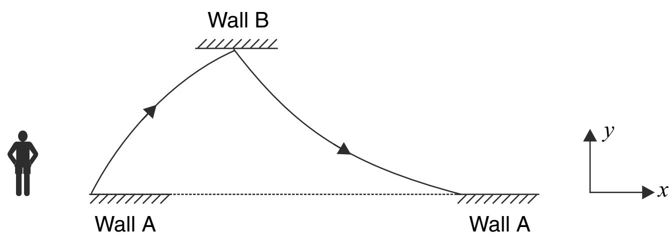
Distance travelled by the ball in reference frame outside the room is

$$s = 2 \times \sqrt{L^2 + L^2} = 2\sqrt{2}L$$

$$\therefore \text{ Average speed } = \frac{s}{t_0} = \frac{2\sqrt{2}L}{\frac{2L}{V}} = \sqrt{2}\,V$$

(b) The speed of ball remains constant in y direction but it increases continuously in x direction.

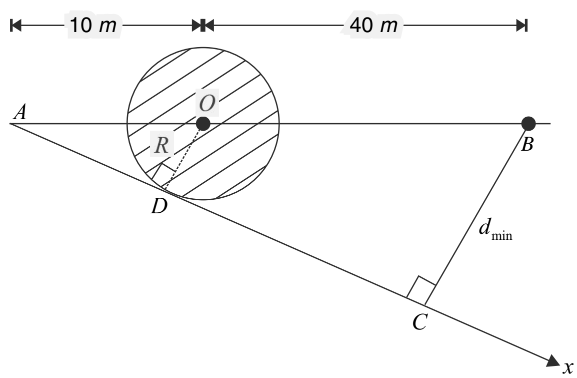
The path will be as shown.



133. (a) The figure shows the top view of the situation. A will throw the ball so that it passes tangentially (just missing) to the building.

The ball can fall on line AX at different locations for different angles of projection.

Boy at B will have to move a minimum distance if the ball lands at C such that $BC \perp AX$.



In similar, triangles

ΔAOD and ΔABC

$$\frac{d_{\min}}{50\,m} = \frac{R}{10\,m} \quad [R = 8\ m]$$

$\therefore\ d_{\min} = 40\ m$

(b) Now $AC^2 = AB^2 - BC^2$

$= 50^2 - 40^2$

$\therefore\ AC = 30\ m$

Therefore, the horizontal range of the projected ball must be 30 m.

Note that the ball moves in a vertical plane passing through the line AX.

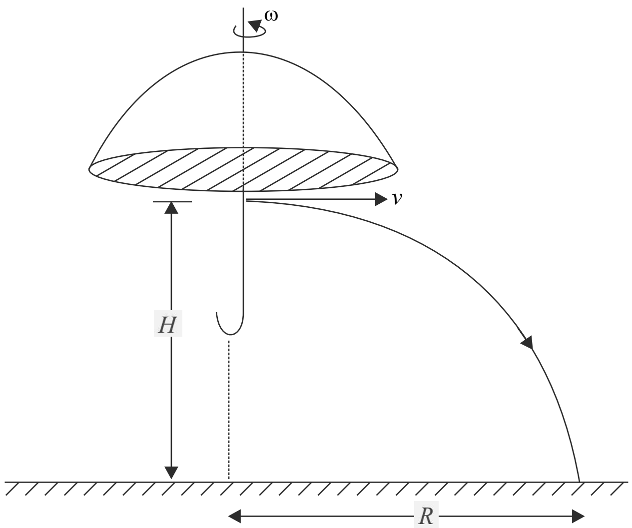
$\because \quad R = \dfrac{u^2 \sin 2\theta}{g} \quad \therefore \quad 30 = \dfrac{(20)^2 \times \sin 2\theta}{10}$

$\Rightarrow \quad \sin 2\theta = \dfrac{3}{4} \quad \therefore \quad 2\theta = \sin^{-1}\left(\dfrac{3}{4}\right)$

$2\theta = 48.6°$ or $131.4°$

$\therefore \theta = 24.3°$ or $65.7°$

134. A water drop leaves the rim at a horizontal velocity $v = \omega r$.



Time of flight $T = \sqrt{\dfrac{2H}{g}} = \sqrt{\dfrac{2 \times 1.8}{10}} = 0.6\,s$

Horizontal range of drops

$R = v\mathrm{T} = \omega r \times 0.6$

For $\omega = 5$ $rad\ s^{-1}$

$R_1 = 5 \times 0.5 \times 0.6 = 1.5\ m$

For $\omega = 10$ $rad\ s^{-1}$

$R_2 = 10 \times 0.5 \times 0.6 = 3.0\ m$

Locus of drops when $\omega = 5$ $rad\ s^{-1}$ will be circle of radius

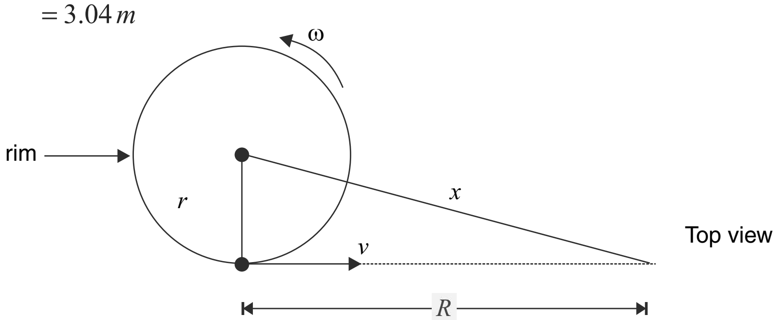
$x_1 = \sqrt{R_1^2 + r^2} = \sqrt{1.5^2 + 0.5^2}$

$= 1.58\ m$

Similarly, locus of drops when $\omega = 10$ $rad\ s^{-1}$ will be a circle of radius.

$x_2 = \sqrt{R_2^2 + r^2} = \sqrt{3^2 + 0.5^2}$

$= 3.04\ m$



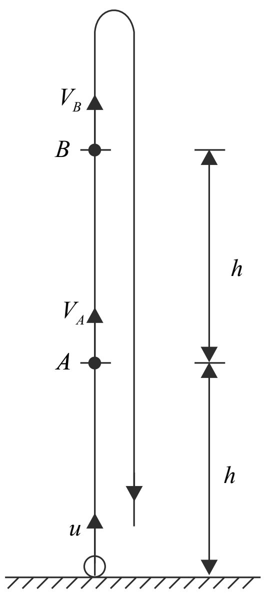
Hence, area of the floor that is wet is

$$A = \pi\left(x_2^2 - x_1^2\right)$$

$$= 3.14\ [9.25 - 2.5]$$

$$= 21.2\ m^2$$

135.



(a) $\frac{2V_A}{g} = t_1 \quad \Rightarrow \quad V_A = \frac{gt_1}{2}$

$\frac{2V_B}{g} = t_2 \quad \Rightarrow \quad V_B = \frac{gt_2}{2}$

$\therefore \quad V_B^2 = V_A^2 - 2gh \Rightarrow \frac{g^2}{4}(t_1^2 - t_2^2) = 2gh$

$\therefore \quad h = \frac{g(t_1^2 - t_2^2)}{8}$

(b) Now $V_A^2 = u^2 - 2gh$

$$\therefore \quad u^2 = V_A^2 + 2gh$$

$$= \frac{g^2}{4}t_1^2 + \frac{g^2}{4}(t_1^2 - t_2^2)$$

$$= \frac{g^2}{4}\left(2t_1^2 - t_1^2\right)$$

$$u = \frac{g}{2}\sqrt{2t_1^2 - t_2^2}$$

136. In reference frame of the ground

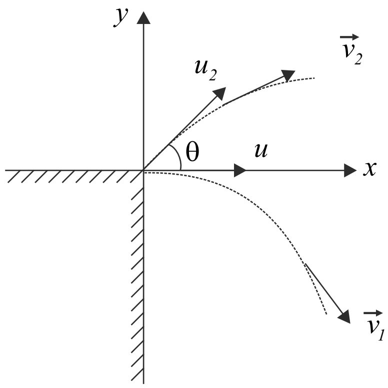
Initial velocity of the ant = u (↑)

Acceleration of the ant = g (↓)

(a) $\therefore \quad H_{\max} = \frac{u^2}{2g}$

(b) $t = \frac{2u}{g}$

137. (a)



After time 't' the velocity of two balls will become

$\vec{V}_1 = 10\hat{i} - gt\,\hat{j} = 10\hat{i} - 10t\,\hat{j}$

$\vec{V}_2 = (u_2\cos\theta)\hat{i} + (u_2\sin\theta - gt)\hat{j}$

$= \left(\frac{3}{5}u_2\right)\hat{i} + \left(\frac{4}{5}u_2 - 10t\right)\hat{j}$

For the two velocities to be perpendicular, $\vec{V}_1 \cdot \vec{V}_2 = 0$

$\therefore \quad 10 \cdot \frac{3}{5}u_2 - 10\,t\left(\frac{4}{5}u_2 - 10t\right) = 0$

$\Rightarrow 3u_2 - t\,(4u_2 - 50t) = 0$

$\Rightarrow 50t^2 - (4u_2)t + 3u_2 = 0$ ....(i)

For this quadratic equation to have real solutions

$D \geq 0$

$(4u_2)^2 - 4 \times 50 \times 3u_2 \geq 0$

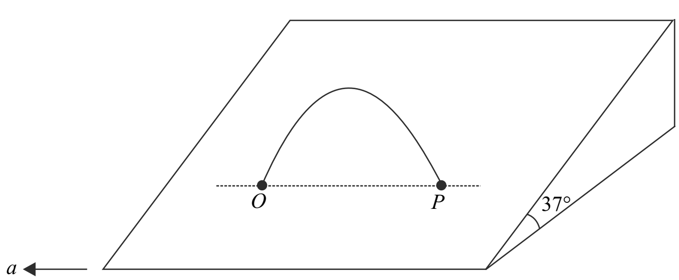
$\Rightarrow \quad u_2 \geq \frac{75}{2}\ m/s$

$\therefore \quad u_0 = \frac{75}{2} = 37.5\ m/s$

Solution of equation (i) [when $D = 0$] is

$t = \frac{4u_2}{2 \times 50} = \frac{4 \times 75}{100 \times 2} = 1.5\ s$

138. (i) When a is directed horizontally along BC, the particle will hit the wedge at same height. In ground frame path of the particle will be a straight line. In reference frame attached the wedge, the particle will have a horizontal acceleration along CB equal to $a = 4\ m/s^2$: apart from its vertical acceleration g (↓).

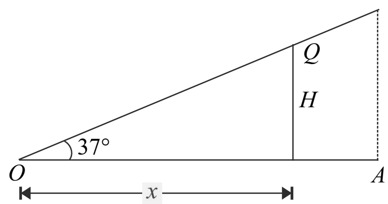


Time of flight; $T = \dfrac{2u}{g} = 1.3\ s$

$$OP = \frac{1}{2}aT^2 = \frac{1}{2} \times 4 \times 1.3^2 = 3.38\ m$$

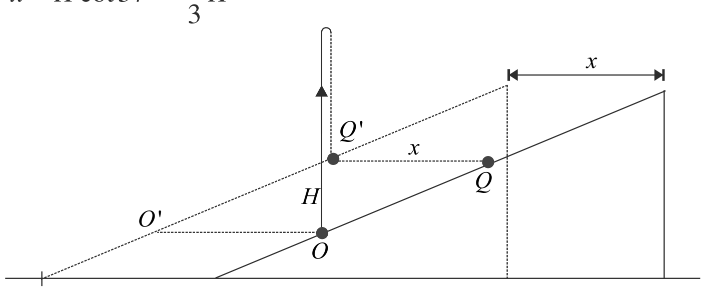
(ii) In this case the particle will hit the incline at a height above its point of projection.

Let the particle strike the surface at a height H above its point of projection.



Two points (O & Q for example) separated by height H (and lying on the line of greatest slope) are separated horizontally by

$$x = H \cot 37° = \frac{4}{3}H$$



O and Q are two points on the wedge along the line of greatest slope. Particle is projected from O. The particle lands back at point Q which has moved a distance X (to position Q') by that time.

$\therefore \quad X = \dfrac{1}{2}at^2 = 2t^2 \quad \therefore H = \dfrac{3}{4}x = \dfrac{3}{2}t^2$

For vertical motion of the particle from O to the top and then back to Q' we can write

$$H = ut - \frac{1}{2}gt^2$$

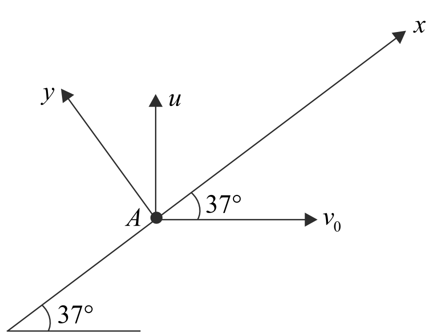
$\Rightarrow \quad \dfrac{3}{2}t^2 = 6.5t - 5t^2 \Rightarrow 6.5t = 6.5 \Rightarrow t = 1.0\,s \quad H = ut - \dfrac{1}{2}gt^2$

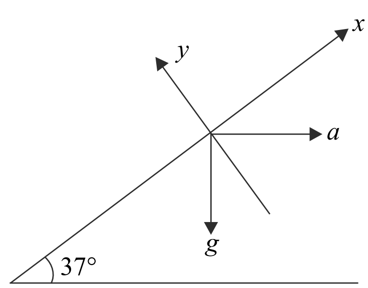
$\therefore x = 2t^2 = 2.0\ m$

$H = 1.5\ t^2 = 1.5\ m$

$\therefore OQ = \sqrt{x^2 + H^2} = \sqrt{4 + (1.5^2)} = 2.5\ m$

139. We will study the motion of the insect in the reference frame of the truck.





Initial velocity of the insect in this reference frame will be vector sum of two velocities

$u = 2.64\ m/s\ (\uparrow)$ and $v_0 = 0.77\ m/s\ (\rightarrow)$

We will take x and y direction as shown.

Components of initial velocity

$u_x = 2.64 \sin 37° + 0.77 \cos 37°$

$$= 2.64 \times \frac{3}{5} + 0.77 \times \frac{4}{5} = \frac{7.92 + 3.08}{5}$$

$$= \frac{11}{5}\ m/s = 2.2\ m/s$$

$u_y = 2.64 \cos 37° - 0.77 \sin 37°$

$$= 2.64 \times \frac{4}{5} - 0.77 \times \frac{3}{5} = \frac{10.56 - 2.31}{5}$$

$$= \frac{8.25}{5}\ m/s = 1.65\ m/s$$

Components of acceleration

$$a_x = -(g \sin 37° - a \cos 37°) = -\left(10 \times \frac{3}{5} - 5 \times \frac{4}{5}\right) = -2\ m/s^2$$

$$a_y = -(g \cos 37° - a \sin 37°) = -\left(10 \times \frac{4}{5} + 5 \times \frac{3}{5}\right) = -11\ m/s^2$$

Consider motion in y direction

$$y = u_y t + \frac{1}{2} a_y t^2$$

To get time of flight put $y = 0$

$$1.65t - \frac{1}{2} \times 11 \times t^2 = 0 \quad \Rightarrow \quad t = \frac{1.65 \times 2}{11} = 0.3\ s$$

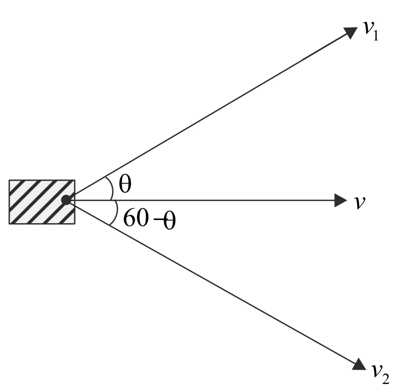
Now consider motion in x direction

$$AB = u_x t + \frac{1}{2} a_x t^2$$

$$= 2.2 \times 0.3 - \frac{1}{2} \times 2 \times 0.3^2$$

$$= 0.66 - 0.09 = 0.57\ m$$

140. Let the velocity of the block be v in a direction making an angle θ with the direction of v_1.



For a string to be taut, the objects at the two ends must have same velocity component along the length of the string.

$\therefore\ v \cos\theta = v_1$ ....(1)

$v \cos(60 - \theta) = v_2$

$$\Rightarrow v\left[\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right] = v_2$$

using (1) $\cos\theta = \frac{v_1}{v}$ and $\sin\theta = \frac{\sqrt{v^2 - v_1^2}}{v}$

$$\therefore\quad v\left[\frac{1}{2}\frac{v_1}{v} + \frac{\sqrt{3}}{2}.\frac{\sqrt{v^2 - v_1^2}}{v}\right] = v_2$$

$$v_1 + \sqrt{3}\sqrt{v^2 - v_1^2} = 2v_2$$

$$3\left(v^2 - v_1^2\right) = 4v_2^2 + v_1^2 - 4v_1v_2$$

$$\Rightarrow 3v^2 = 4v_2^2 + 4v_1^2 - 4v_1v_2$$

$$\therefore v = \frac{2}{\sqrt{3}}\sqrt{v_1^2 + v_2^2 - v_1v_2} \quad \text{.............(2)}$$

(b) For $\theta = 0°$

$v = v_1$ [from (1)]

Putting into (2) we get

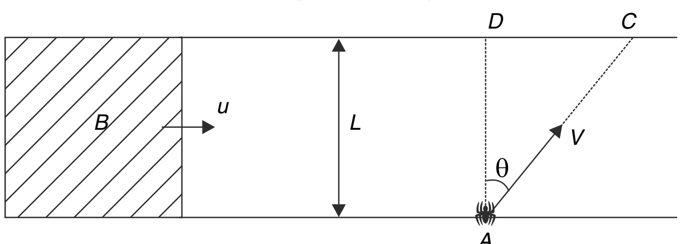
$$v_1 = \frac{2}{\sqrt{3}}\sqrt{v_1^2 + v_2^2 - v_1v_2}$$

$$\frac{3v_1^2}{4} = v_1^2 + v_2^2 - v_1v_2$$

$$\Rightarrow\ v_1^2 - 4v_1v_2 + 4v_2^2 = 0 \quad \Rightarrow (v_1 - 2v_2)^2 = 0$$

$$\Rightarrow\ v_1 = 2v_2 \qquad \Rightarrow\ \frac{v_1}{v_2} = 2$$

141. Let the insect move with speed v along the direction shown in the figure.



Time to cross is $t = \frac{L}{v\cos\theta}$

$DC = v \sin\theta .t = L\frac{\sin\theta}{\cos\theta} = L\tan\theta.$

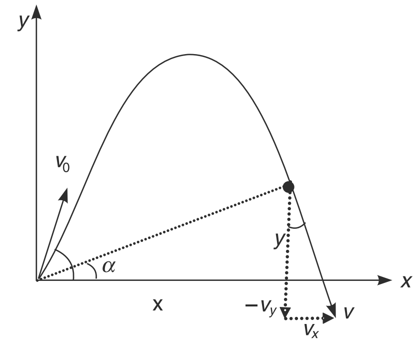
The block will just miss the insect if distance travelled by the block in time t is just less than $d + DC$

$$\Rightarrow\ u.t = d + L\tan\theta \quad \Rightarrow u.\frac{L}{v\cos\theta} \quad [= d + L\tan\theta]$$

$$\Rightarrow\ v = \frac{uL}{d\cos\theta + L\sin\theta}$$

The maximum value of $d\cos\theta + L\sin\theta$ is $\sqrt{d^2 - L^2}$

$\therefore \quad v_{min} = \dfrac{uL}{\sqrt{d^2 + L^2}}$



142. $x = v_0 t \cos\alpha$, $y = v_0 t \sin\alpha - \dfrac{g}{2}t^2$,

$v_x = v_0 \cos\alpha$, $v_y = v_0 \sin\alpha - gt$

The stone is at the greatest distance from the origin when its velocity is perpendicular to its position vector.

The condition for this is that the dot product of velocity vector and position vector is zero.

$\dfrac{y}{x} = \dfrac{v_x}{v_y}$,

This yields a quadratic equation for time t at which this happens;

$$t^2 - \frac{3v_0 \sin\alpha}{g}t + \frac{2v_0^2}{g^2} = 0$$

If this is not to happen, the discriminant of this equation must be negative i.e.,

$$\left(\frac{3v_0 \sin\alpha}{g}\right)^2 < 4\left(\frac{2v_0^2}{g^2}\right).$$

Thus, for the stone to the permanently moving away from the thrower, we must have $\sin\alpha < \sqrt{8/9} = 0.94$. i.e., $\alpha < 70.5°$.

143. Method 1 : Trajectory equation

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

$$y = x\tan\theta - \frac{gx^2}{2u^2}\left(1 + \tan^2\theta\right)$$

$$\left(\frac{gx^2}{2u^2}\right)\tan^2\theta - x\tan\theta + \left(y + \frac{gx^2}{2u^2}\right) = 0$$

Putting $x = d$ & $y = h$

$$\frac{\left(gd^2\right)}{2u^2}\tan^2\theta - d\tan\theta + \left(h + \frac{gd^2}{2u^2}\right) = 0$$

If projectile clears the pole then roots of above equation must be real i.e. $D \geq 0$

$$d^2 - 4\left(\frac{gd^2}{2u^2}\right)\left(h + \frac{gd^2}{2u^2}\right) \geq 0$$

$$1 - \left(\frac{g}{u^2}\right)\left(\frac{2u^2h + gd^2}{u^2}\right) \geq 0$$

$u^4 - (2gh)\,u^2 - g^2d^2 \geq 0$ …………(i)

Let, $u^4 - (2gh)\,u^2 - g^2d^2 = 0$

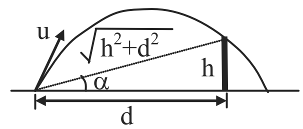
$$u_0{}^2 = \frac{2gh + \sqrt{4g^2h^2 + 4g^2d^2}}{2}$$

$$u_0{}^2 = g\left[h + \sqrt{h^2 + d^2}\right]$$

If $u^2 \geq u_0^2$ then expression (i) is greater than or equal to zero.

i.e. if $u^2 \geq g\left[h + \sqrt{h^2 + d^2}\right]$

then projectile will clear the pole.



Method 2 :

$$\sin\ \alpha = \frac{h}{\sqrt{h^2 + d^2}}$$

Maximum range of projectile on inclined plane is

$$R_{max} = \frac{u^2}{g(1 + \sin\alpha)}$$

If $R_{max} \geq \sqrt{h^2 + d^2}$; then projectile will clear the pole

$$\frac{u^2}{g(1 + \sin\alpha)} \geq \sqrt{h^2 + d^2}$$

$$u^2 = g\left[h + \sqrt{h^2 + d^2}\right]$$

144. Mean (average) angular velocity = $\dfrac{\text{Total angular displacement } (\theta)}{\text{Total time taken } (\mathrm{T})}$

Given $\dfrac{d\omega}{dt} = -k\sqrt{\omega}$ [where k is a constant and – *ve* sign corresponds to retardation]

$$\therefore \int_{\omega_0}^{\omega} \frac{d\omega}{\sqrt{\omega}} = -\int_0^t k dt$$

$$\Rightarrow \sqrt{\omega_0} - \sqrt{\omega} = +\frac{kt}{2}$$

$$\Rightarrow \omega = \left(\sqrt{\omega_0} - \frac{kt}{2}\right)^2$$

$$\therefore \omega = 0 \text{ ; when } \sqrt{\omega_0} - \frac{kt}{2} = 0 \quad [\text{Particle stops}]$$

$$\therefore T = \frac{2\sqrt{\omega_0}}{k}$$

Now $\omega = \left(\sqrt{\omega_0} - \dfrac{kt}{2}\right)^2$

$$\therefore \frac{d\theta}{dt} = \left(\sqrt{\omega_0} - \frac{kt}{2}\right)^2$$

$$\int_0^{\theta} d\theta = \int_0^T \left(\sqrt{\omega_0} - \frac{kt}{2}\right)^2 dt$$

[θ = angular displacement when particle stops, i.e., $t = T$]

$$\therefore \theta = \left[\frac{\left[\sqrt{\omega_0} - \frac{kt}{2}\right]^3}{-\frac{3k}{2}}\right]_0^T$$

$$= -\frac{2}{3k}\left[\left(\sqrt{\omega_0} - \sqrt{\omega_0}\right)^3 - \left(\sqrt{\omega_0}\right)^3\right] \qquad \left[\because T = \frac{2\sqrt{\omega_0}}{k}\right]$$

$$= \frac{2}{3k}\omega_0^{3/2}$$

$$\therefore \quad < \omega > \ = \frac{\theta}{T} = \frac{\omega_0}{3}$$

03 NEWTON'S LAWS

LEVEL 1

Q. 1. Let $\vec{u}$ be the initial velocity of a particle and $\vec{F}$ be the resultant force acting on it. Describe the path that the particle can take if

(a) $\vec{u} \times \vec{F} = 0$ and $\vec{F}$ = constant

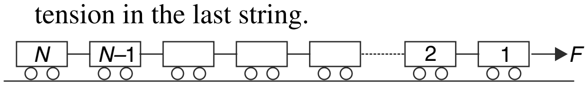
(b) $\vec{u}.\vec{F} = 0$ and $\vec{F} =$ constant

In which case can the particle retrace its path.

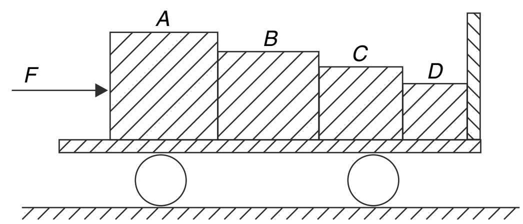
Q. 2. A ball is projected vertically up from the floor of a room. The ball experiences air resistance that is proportional to speed of the ball. Just before hitting the ceiling the speed of the ball is 10 *m/s* and its retardation is 2*g*. The ball rebounds from the ceiling without any loss of speed and falls on the floor 2*s* after making impact with the ceiling. How high is the ceiling? Take $g = 10\ m/s^2$.

Q. 3. A small body of super dense material, whose mass is half the mass of the earth (but whose size is very small compared to the size of the earth), starts from rest at a height *H* above the earth's surface, and reaches the earth's surface in time *t*. Calculate time *t* assuming that *H* is very small compared to the radius of the earth. Acceleration due to gravity near the surface of the earth is *g*.

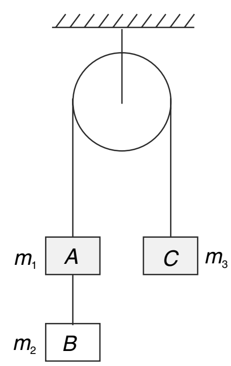
Q. 4. *N* identical carts are connected to each other using strings of negligible mass. A pulling force *F* is applied on the first cart and the system moves without friction along the horizontal ground. The tension in the string connecting 4th and 5th cart is twice the tension in the string connecting 8th and 9th cart. Find the total number of carts (*N*) and tension in the last string.



Q. 5. A toy cart has mass of 4 *kg* and is kept on a smooth horizontal surface. Four blocks *A, B, C* and *D* of masses 2 *kg*, 2 *kg*, 1 *kg* and 1 *kg* respectively have been placed on the cart. A horizontal force of $F = 40\ N$ is applied to the block *A* (see figure). Find the contact force between block *D* and the front vertical wall of the cart.



Q. 6. (i) Three blocks *A, B* and *C* are placed in an ideal Atwood machine as shown in the figure. When the system is allowed to move freely it was found that tension in the string connecting *A* to *C* was more than thrice the tension in the string connecting *A* and *B*. The masses of the three blocks *A, B* and *C* are m_1, m_2 and m_3, respectively. State whether the following statements are true or false [All masses have finite non zero values and the system has a non zero acceleration].



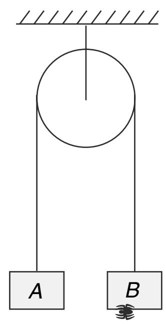
(a) m_3 can have any finite value

(b) $m_1 > 2m_2$

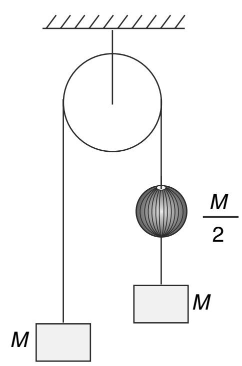
(ii) In an Atwood machine the sum of two masses is a constant. If the string can sustain a tension equal to $\left(\frac{24}{30}\right)$ of the weight of the sum of two masses, find the least acceleration of the masses. The string and pulley are light.

(iii) A load of w newton is to be raised vertically through a height h using a light rope. The greatest tension that the rope can bear is ηw ($\eta > 1$). Calculate the least time of ascent if it is required that the load starts from rest and must come to rest when it reaches a height h.

Q. 7. In the arrangement shown in the figure the system is in equilibrium. Mass of the block A is M and that of the insect clinging to block B is m. Pulley and string are light. The insect loses contact with the block B and begins to fall. After how much time the insect and the block B will have a separation L between them.



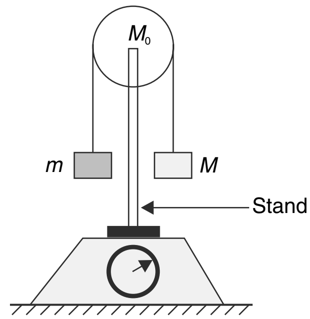
Q. 8. Two blocks of equal mass, M each, are connected to two ends of a massless string passing over a massless pulley. On one side of the string there is a bead of mass $\frac{M}{2}$.



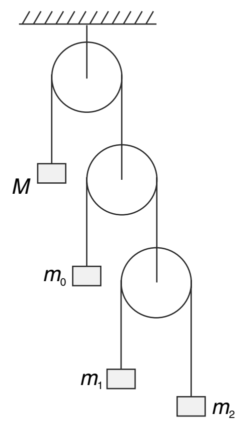
(a) When the system is released from rest the bead continues to remain at rest while the two blocks accelerate. Find the acceleration of the blocks.

(b) Find the acceleration of the two blocks if it was observed that the bead was sliding down with a constant velocity relative to the string.

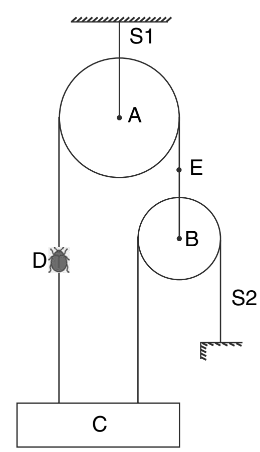
Q. 9. A pulley is mounted on a stand which is placed over a weighing scale. The combined mass of the stand and the pulley is M_0. A light string passes over the smooth pulley and two masses m and M ($> m$) are connected to its ends (see figure). Find the reading of the scale when the two masses are left free to move.



Q. 10. In the given arrangement, all strings and pulleys are light. When the system was released it was observed that M and m_0 do not move. Find the masses M and m_0 in terms of m_1 and m_2. Find the acceleration of all the masses if string is cut just above m_2.



Q.11 The system shown in the fig. is in equilibrium. Pulleys A and B have mass M each and the block C has mass $2M$. The strings are light. There is an insect (D) of mass $M/2$ sitting at the middle or the right string. Insect does not move.



(a) Just by inspection, say if the tension in the string S1 is equal to, more than or less than 9/2 Mg.

(b) Find tension in the string $S2$, and $S1$.

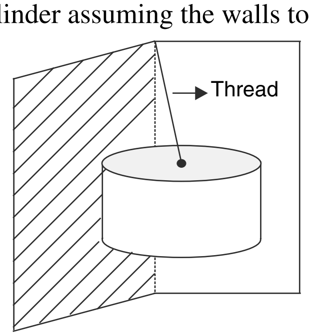
(c) Find tension in $S2$ if the insect flies and sits at point E on the string.

Q. 12. A block slides down a frictionless plane inclined at an angle θ. For what value of angle θ the horizontal component of acceleration of the block is maximum? Find this maximum horizontal acceleration.

Q. 13. A tall elevator is going up with an acceleration of $a = 4\ m/s^2$. A 4 kg snake is climbing up the vertical wall of the elevator with an acceleration of a. A 50 g insect is riding on the back of the snake and it is moving up relative to the snake at an acceleration of a. Find the friction force between the elevator wall and the snake. Assume that the snake remains straight.

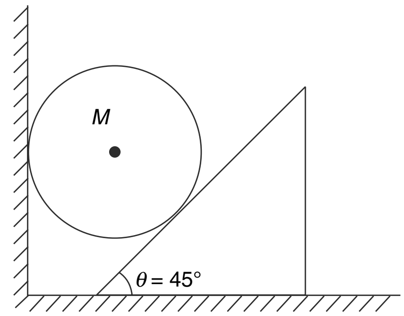
Q. 14. Due to air drag the falling bodies usually acquire a constant speed when the drag force becomes equal to weight. Two bodies, of identical shape, experience air drag force proportional to square of their speed ($F_{drag} = kv^2$, k is a constant). The mass ratio of two bodies is 1 : 4. Both are simultaneously released from a large height and very quickly acquire their terminal speeds. If the lighter body reaches the ground in 25 s, find the approximate time taken by the other body to reach the ground.

Q. 15. A cylinder of mass M and radius r is suspended at the corner of a room. Length of the thread is twice the radius of the cylinder. Find the tension in the thread and normal force applied by each wall on the cylinder assuming the walls to be smooth.



Q. 16. A rod of mass M and length L lies on an incline having inclination of $\theta = 37°$. The coefficient of friction between the rod and the incline surface is $\mu = 0.90$. Find the tension at the mid point of the rod.

Q. 17. A ball of mass M is in equilibrium between a vertical wall and the inclined surface of a wedge. The inclination of the wedge is $\theta = 45°$ and its mass is very small compared to that of the ball. The coefficient of friction between the wedge and the floor is μ and there is no friction elsewhere. Find minimum value of μ for which this equilibrium is possible.



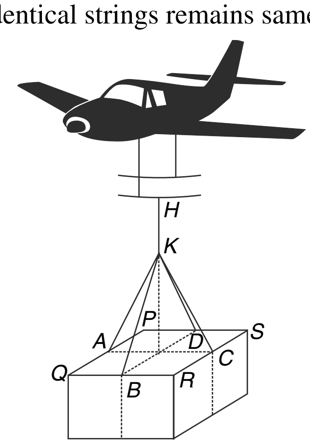
Q.18 A helicopter of mass $M = 15000\ kg$ is lifting a cubical box of mass $m = 2000\ kg$. The helicopter is going up with an acceleration of $a = 1.2\ m/s^2$. The four strings are tied at mid points of the sides of the square face $PQRS$ of the box. The strings are identical and form a knot at K. Another string KH connects the knot to the helicopter. Neglect mass of all strings and take g = 10 m/s^2. Length of each string AK, BK, CK and DK is equal to side length of the cube.

(a) Find tension T in string AK.

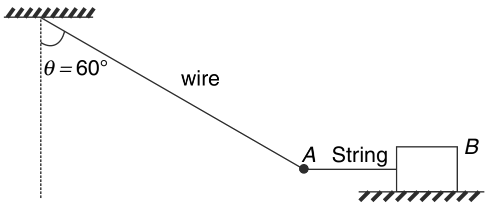
(b) Find tension T_0 in string KH.

(c) Find the force (F) applied by the atmosphere on the helicopter. Assume that the atmosphere exerts a negligible force on the box.

(d) If the four strings are tied at P, Q, R and S instead of A, B, C & D, how will the quantities T, T_0 and F change? Will they increase or decrease? Assume that length of the four identical strings remains same.



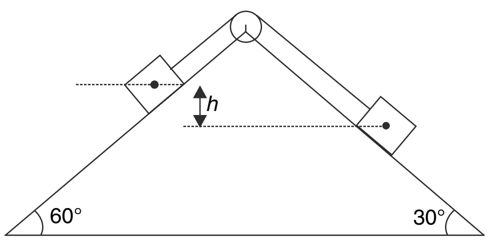
Q. 19. A pendulum has a bob connected to a light wire. Bob '*A*' is in equilibrium in the position shown. The string is horizontal and is connected to a block *B* resting on a rough surface. The block *B* is on verge of sliding when $\theta = 60°$.



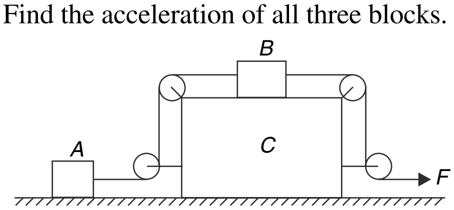
(a) Is equilibrium possible if θ were 70°?

(b) With θ = 60°, calculate the ratio of tension in the pendulum wire immediately after the string is cut to the tension in the wire before the string is cut.

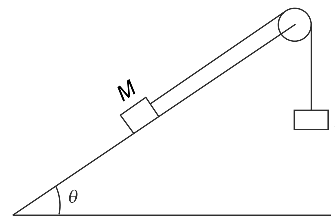
Q. 20. Two blocks of equal mass have been placed on two faces of a fixed wedge as shown in figure. The blocks are released from position where centre of one block is at a height *h* above the centre of the other block. Find the time after which the centre of the two blocks will be at same horizontal level. There is no friction anywhere.



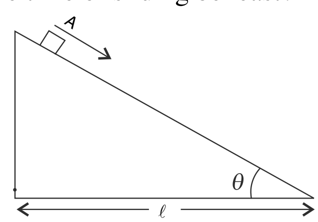
Q. 21. In the system shown in the figure, all surfaces are smooth. Block *A* and *B* have mass *m* each and mass of block *C* is 2 *m*. All pulleys are massless and fixed to block *C*. Strings are light and the force *F* applied at the free end of the string is horizontal. Find the acceleration of all three blocks.



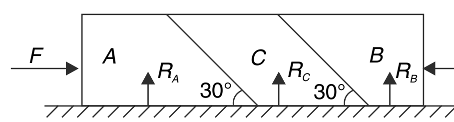
Q. 22. A particle of mass *M* rests on a rough inclined plane at an angle θ to the horizontal $\left(\sin\theta = \frac{4}{5}\right)$. It is connected to another mass *m* as shown in fig. The pulley and string are light. The largest value of *m* for which equilibrium is possible is *M*. Find the smallest value of *m* for which equilibrium is possible.



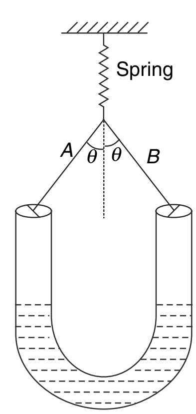
Q. 23. A small body *A* starts sliding down from the top of a wedge (see fig) whose base is equal to ℓ. The coefficient of friction between the body and wedge surface is $\mu = 1.0$. At what value of angle θ will the time of sliding be least?



Q. 24. Three blocks *A*, *B* and *C* each of mass *m* are placed on a smooth horizontal table. There is no friction between the contact surfaces of the blocks as well. Horizontal force *F* is applied on each of *A* and *B* as shown. Find the ratio of normal force applied by the table on the three blocks (i.e., $R_A : R_B : R_C$). Take $F = \frac{mg}{2\sqrt{3}}$



Q. 25. A *U* shaped container has uniform cross sectional area *S*. It is suspended vertically with the help of a spring and two strings *A* and *B* as shown in the figure. The spring and strings are light. When water (density = *d*) is poured slowly into the container it was observed that the level of water remained unchanged with respect to the ground. Find the force constant of the spring.

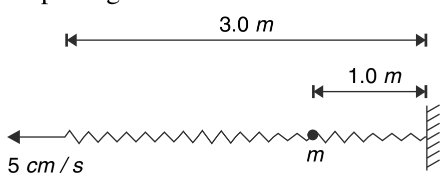


Q. 26. A uniform light spring has unstretched length of 3.0 *m*. One of its end is fixed to a wall. A particle of mass $m = 20\ g$ is glued to the spring at a point 1.0 m away from its fixed end. The free end of the spring is pulled away from the wall at a constant speed of 5 *cm/s*.

Assume that the spring remains horizontal (i.e., neglect gravity). Force constant of spring = 0.6 N/*cm*.

(a) With what speed does the particle of mass *m* move?

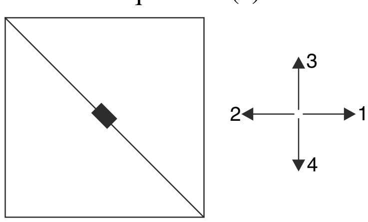
(b) Find the force applied by the external agent pulling the spring at time 2.0 s after he started pulling.



Q. 27. It was observed that a small block of mass *m* remains in equilibrium at the centre of a vertical square frame, which was accelerated. The block is held by two identical light strings as shown. [Both strings are along the diagonal]

(a) Which of 1, 2, 3 & 4 is/are possible direction/s of acceleration of the frame for block to remain in equilibrium inside it?

(b) Find the acceleration of the frame for your answers to question (a).



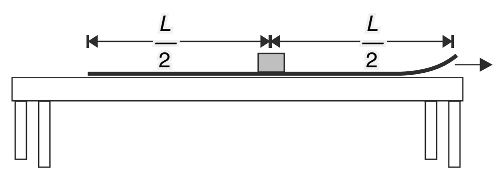
Q.28 In an emergency situation while driving one has tendency to jam the brakes, trying to stop in shortest distance. With wheels locked, the car slides and steering get useless. In ABS system the electronic sensors keep varying the brake pressure so as to keep the wheels rolling (without slipping) while ensuring that the friction remains limiting.

Your friend has an old car with good brakes. He boasts saying that all the four wheels of his car get firmly locked and stop rotation immediately after the brakes are applied. You know that your new car which has a computerized anti lock braking system (*ABS*) is much safer. How will you convince your friend?

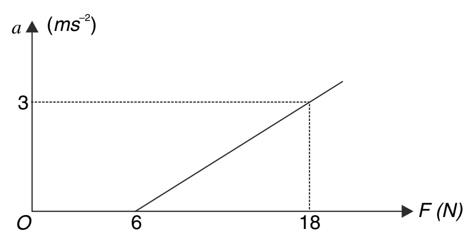
In a typical situation, car without *ABS* needs 20 *m* as minimum stopping distance. Under identical conditions, what minimum distance a car with *ABS* would need to stop? Coefficient of kinetic friction between tyre and road is 25% less than the coefficient of static friction.

Q. 29. Starting from rest a car takes at least '*t*' second to travel through a distance *s* on a flat concrete road. Find the minimum time that will be needed for it to climb through a distance '*s*' on an inclined concrete road. Assume that the car starts from rest and inclination of road is $\theta = 5°$ with horizontal. Coefficient of friction between tyres and the concrete road is $\mu = 1$.

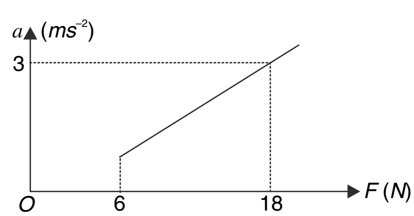
Q. 30. A table cloth of length *L* is lying on a table with one of its end at the edge of the table. A block is kept at the centre of the table cloth. A man pulls the end of the table cloth horizontally so as to take it off the table. The cloth is pulled at a constant speed V_0. What can you say about the coefficient of friction between the block and the cloth if the block remains on the table (i.e., it does not fall off the edge) as the cloth is pulled out.



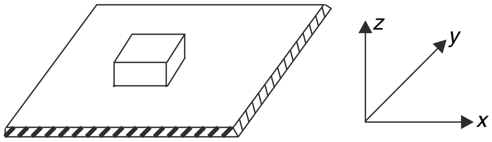
Q. 31. A block rests on a horizontal surface. A horizontal force *F* is applied to the block. The acceleration (*a*) produced in the block as a function of applied force (*F*) has been plotted in a graph (see figure). Find the mass of the block.



Q. 32. Repeat the last problem if the graph is as shown below.

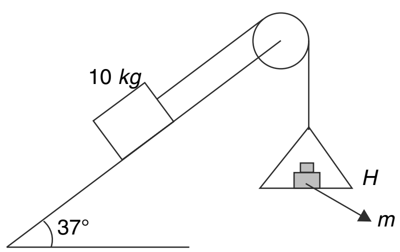


Q. 33. A solid block of mass $m = 1\ kg$ is resting on a horizontal platform as shown in figure. The z direction is vertically up. Coefficient of friction between the block and the platform is $\mu = 0.2$. The platform is moved with a time dependent velocity given by $\vec{V} = \left(2t\hat{i} + t\hat{j} + 3t\hat{k}\right)\text{m/s}$. Calculate the magnitude of the force exerted by the block on the platform. Take g = 10 m/s^2

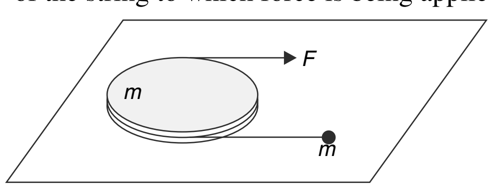


Q. 34. In the system shown in the figure, the string is light and coefficient of friction between the 10 kg block and the incline surface is $\mu = 0.5$. Mass of the hanger, H is 0.5 kg. A boy places a block of mass m on the hanger and finds that the system does not move. What could be values of mass m?

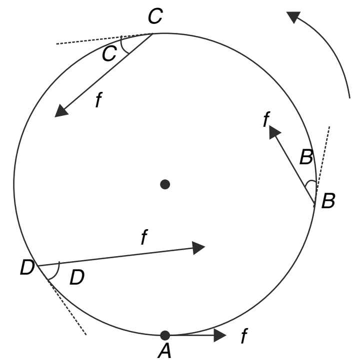
$\tan 37° = \frac{3}{4}$ *and* $g = 10\ \text{m/s}^2$



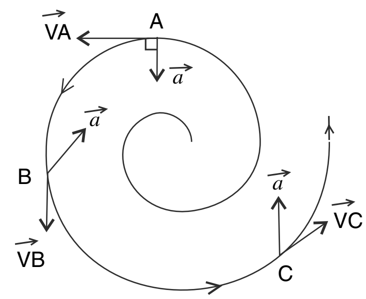
Q. 35. A disc of mass m lies flat on a smooth horizontal table. A light string runs halfway around it as shown in figure. One end of the string is attached to a particle of mass m and the other end is being pulled with a force F. There is no friction between the disc and the string. Find acceleration of the end of the string to which force is being applied.



Q. 36. (a) A car starts moving (at point A) on a horizontal circular track and moves in anticlockwise sense. The speed of the car is made to increase uniformly. The car slips just after point D. The figure shows the friction force (f) acting on the car at points A, B, C and D. The length of the arrow indicates the magnitude of the friction and it is given that $\angle D > \angle B > \angle C$. At which point ($A$, B, C or D) the friction forces represented is certainly wrong ?

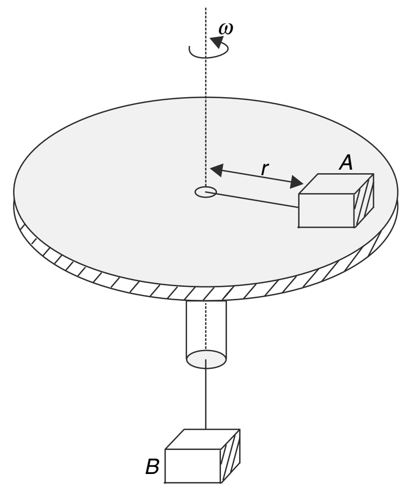


(b) A particle is moving along an expanding spiral (shown in fig) such that the normal force on the particle [i.e., component of force perpendicular to the path of the particle] remains constant in magnitude. The possible direction of acceleration $\left(\vec{a}\right)$ of the particle has been shown at three points A, B and C on its path. At which of these points the direction of acceleration has been represented correctly.



(c) A particle is moving in XY plane with a velocity. $\vec{v} = 4\hat{i} + 2t\hat{j}\ \text{ms}^{-1}$. Calculate its rate of change of speed and normal acceleration at $t = 2\ s$.

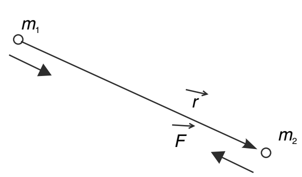
Q. 37. (i) A spinning disk has a hole at its centre. The surface of the disk is horizontal and a small block A of mass $m = 1\ kg$ is placed on it.



Block A is tied to a light inextensible string, other end of which passes through the hole and supports another block B of mass $M = 2\ kg$. The coefficient of friction between A and the disk surface is 0.5. It was observed that the disk is spinning with block A remaining at rest relative to the disk. Block B was found to be stationary. It was estimated that length of horizontal segment of the string (r) was anywhere between 1.0 m to 1.5 m. With this data what estimate can be made about the angular speed (ω) of the disk. [$g = 10\ m/s^2$]

(ii) A spring has force constant equal to $k = 100\ Nm^{-1}$. Ends of the spring are joined to give it a circular shape of radius $R = 20\ cm$. Now the spring is rotated about its symmetry axis (perpendicular to its plane) such that the circumference of the circle increases by 1%. Find the angular speed (ω). Mass of one meter length of the spring is $\lambda = 0.126\ gm^{-1}$.

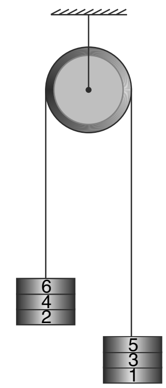
Q. 38. Two particles of mass m_1 and m_2 are in space at separation $\vec{r}$ [vector from m_1 to m_2]. The only force that the two particles experience is the mutual gravitational pull. The force applied by m_1 on m_2 is $\vec{F}$. Prove that $\mu \frac{d^2\vec{r}}{dt^2} = \vec{F}$ Where $\mu \frac{m_1 m_2}{m_1 + m_2}$ is known as reduced mass for the two particle system.



Q. 39. Six identical blocks – numbered 1 to 6 – have been glued in two groups of three each and have been suspended over a pulley as shown in fig. The pulley and string are massless and the system is in equilibrium. The block 1, 2, 3, and 4 get detached from the system in sequence starting with block 1. The time gap between separation of two consecutive block (i.e., time gap between separation of 1 and 2 or gap between separation of 2 and 3) is t_0. Finally, blocks 5 and 6 remain connected to the string.

(a) Find the final speed of blocks 5 and 6.

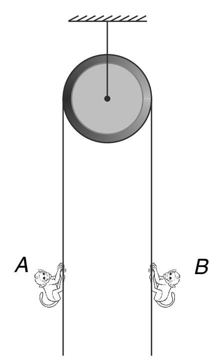
(b) Plot the graph of variation of speed of block 5 with respect to time. Take $t = 0$ when block 1 gets detached.



Q. 40. Two monkeys A and B are holding on the two sides of a light string passing over a smooth pulley. Mass of the two monkeys are $m_A = 8\ kg$ and $m_B = 10\ kg$ respectively [$g = 10\ m/s^2$]

(a) Monkey A holds the string tightly and B goes down with an acceleration $a_r = 2\ m/s^2$ relative to the string. Find the weight that A feels of his own body.

(b) What is the weight experienced by two monkeys if A holds the string tightly and B goes down with an acceleration $a_r = 4\ m/s^2$ relative to the string.



LEVEL 2

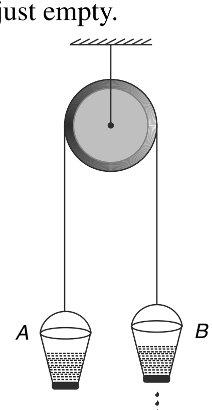
Q. 41. Two strange particles A and B in space, exert no force on each other when they are at a separation greater than $x_0 = 1.0\ m$. When they are at a distance less than x_0, they repel one another along the line joining them. The repulsion force is constant and does not depend on the distance between the particles. This repulsive force produces an acceleration of 6 ms^{-2} in A and 2 ms^{-2} in B when the particles are at separation less than x_0. In one experiment particle B is projected towards A with a velocity of 2 ms^{-1} from a large distance so as to hit A head on. The particle A is originally at rest and the system of two particles do not experience any external force.

(a) Find the ratio of mass of A to that of B.

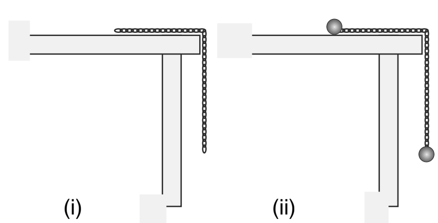
(b) Find the minimum distance between the particles during subsequent motion.

(c) Find the final velocity of the two particles.

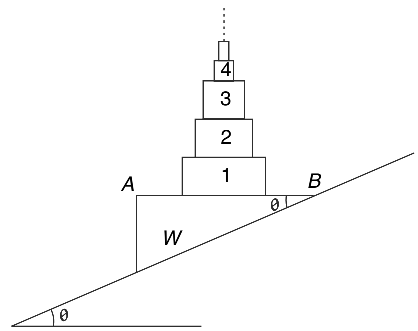
Q. 42. A light string passing over a smooth pulley holds two identical buckets at its ends. Mass of each empty bucket is M and each of them holds M mass of sand. The system was in equilibrium when a small leak developed in bucket B (take this time as $t = 0$). The sand leaves the bucket at a constant rate of μ kg/s. Assume that the leaving sand particles have no relative speed with respect to the bucket (it means that there is no impulsive force on the bucket like leaving exhaust gases exert on a rocket). Find the speed (V_0) of the two bucket when B is just empty.



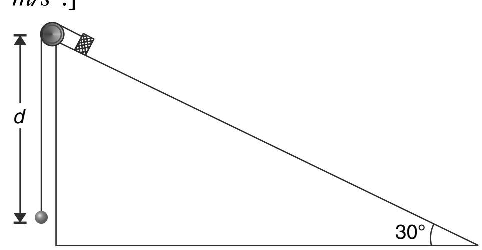
Q. 43. A chain is lying on a smooth table with half its length hanging over the edge of the table [fig(i)]. If the chain is released it slips off the table in time t_1. Now, two identical small balls are attached to the two ends of the chain and the system is released [fig(ii)]. This time the chain took t_2 time to slip off the table. Which time is larger, t_1 or t_2?



Q. 44. A triangular wedge W having mass M is placed on an incline plane with its face AB horizontal. Inclination of the incline is θ. On the flat horizontal surface of the wedge there lies an infinite tower of rectangular blocks. Blocks 1, 2, 3, 4 ………. have masses $M, \frac{M}{2}, \frac{M}{4}, \frac{M}{8}$ ....respectively. All surfaces are smooth. Find the contact force between the block 1 and 2 after the system is released from rest. Also find the acceleration of the wedge.



Q. 45. In the system shown in fig, mass of the block is $m_1 = 4$ kg and that of the hanging particle is $m_2 = 1$ kg. The incline is fixed and surface is smooth. Block is initially held at the top of the incline and the particle hangs a distance $d = 2.0$ m below it. [Assume that the block and the particle are on same vertical line in this position]. System is released from this position. After what time will the distance between the block and the particle be minimum ? Find this minimum distance. [$g = 10$ m/s^2.]



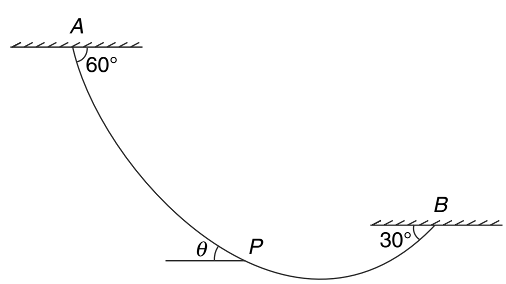
Q. 46. A uniform chain of mass $M = 4.8$ kg hangs in vertical plane as shown in the fig.

(a) Show that horizontal component of tension is same throughout the chain.

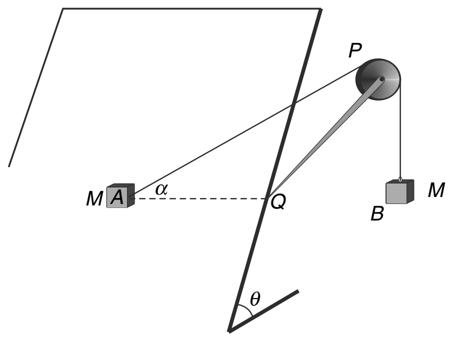
(b) Find tension in the chain at point P where the chain makes an angle $\theta = 15°$ with horizontal.

(c) Find mass of segment AP of the chain.

[Take $g = 10$ m/s^2; cos 15° = 0.96, sin 15° = 0.25]

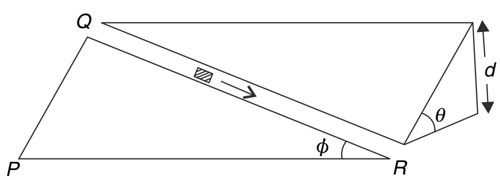


Q. 47. Block A of mass M is placed on an incline plane, connected to a string, passing over a pulley as shown in the fig. The other end of the string also carries a block B of mass M. The system is held in the position shown such that triangle APQ lies in a vertical plane with horizontal line AQ in the plane of the incline surface.

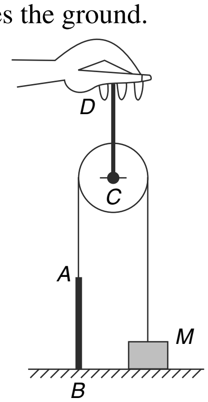


Find the minimum coefficient of friction between the incline surface and block A such that the system remains at rest after it is released. Take $\theta = \alpha = 45°$.

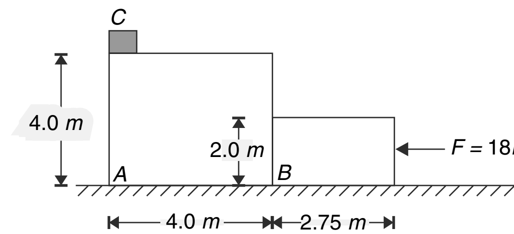
Q. 48. Figure shown a fixed surface inclined at an angle θ to the horizontal. A smooth groove is cut on the incline along QR forming an angle ϕ with PR. A small block is released at point Q and it slides down to R in time t. Find t.



Q. 49. In the system shown in the figure AB and CD are identical elastic cords having force constant K. The string connected to the block of mass M is inextensible and massless. The pulley is also massless. Initially, the cords are just taut. The end D of the cord CD is gradually moved up. Find the vertical displacement of the end D by the time the block leaves the ground.



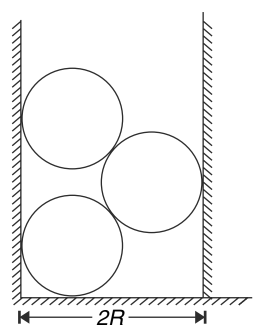
Q. 50. Blocks A and B have dimensions as shown in the fig. and their masses are 8 kg and 1 kg respectively. A small block C of mass 0.5 kg is placed on the top left corner of block A. All surfaces are smooth. A horizontal force $F = 18\ N$ is applied to the block B at time $t = 0$. At what time will the block C hit the ground surface? Take $g = 10\ m/s^2$.



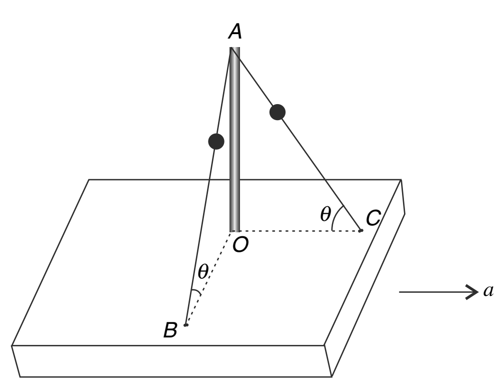
Q. 51. Three identical smooth balls are placed between two vertical walls as shown in fig. Mass of each ball is m and radius is $r = \frac{5R}{9}$ where $2R$ is separation between the walls.

(a) Force between which two contact surface is maximum? Find its value.

(b) Force between which two contact surface is minimum and what is its value?



Q. 52. A horizontal wooden block has a fixed rod OA standing on it. From top point A of the rod, two wires have been fixed to points B and C on the block. The plane of triangle OAB is perpendicular to the plane of the triangle OAC. There are two identical beads on the two wires. One of the wires

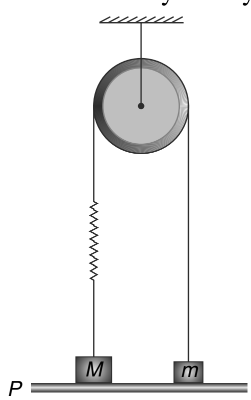


is perfectly smooth while the other is rough. The wooden block is moved with a horizontal acceleration (a) that is perpendicular to the line OB and it is observed that both the beads do not slide on the wire. Find the minimum coefficient of friction between the rough wire and the bead.

Q. 53. In the arrangement shown in the fig. the pulley, the spring and the thread are ideal. The spring is stretched and the two blocks are in contact with a horizontal platform P. When the platform is gradually moved up by 2 cm the tension in the string becomes zero. If the platform is gradually moved down by 2 cm from its original position one of the blocks lose contact with the platform. Given $M = 4\ kg$; $m = 2\ kg$.

(a) Find the force constant (k) of the spring

(b) If the platform continues to move down after one of the blocks loses contact, will the other block also lose contact? Assume that that the platform moves very slowly.

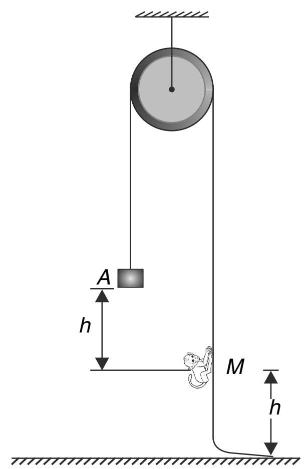


Q. 54. In the arrangement shown in the fig. a monkey of mass M keeps itself as well as block A at rest by firmly holding the rope. Rope is massless and the pulley is ideal. Height of the monkey and block A from the floor is h and $2h$ respectively [$h = 2.5\ m$]

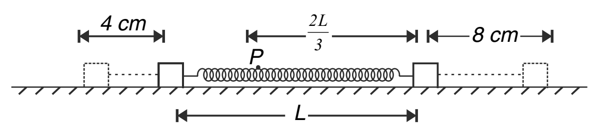
(a) The monkey loosens its grip on the rope and slides down to the floor. At what height from the ground is block A at the instant the money hits the ground?

(b) Another block of mass equal to that of A is stuck to the block A and the system is released. The monkey decides to keep itself at height h above the ground and it allows the rope to slide through its hand. With what speed will the block strike the ground?

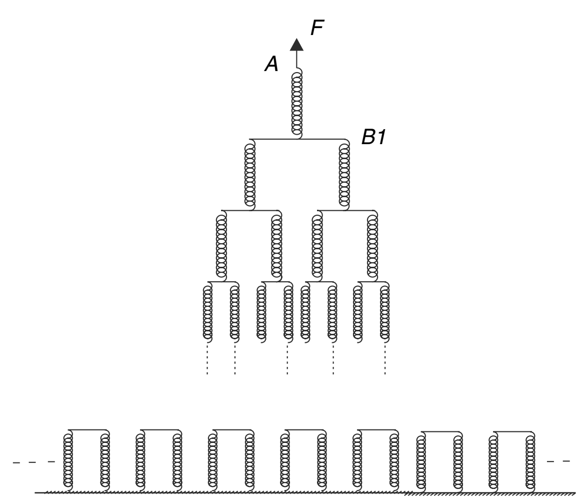
(c) In the situation described in (b), the monkey decides to prevent the block from striking the floor. The monkey remains at height h till the block crosses it. At the instant the block is crossing the monkey it begins climbing up the rope. Find the minimum acceleration of the monkey relative to the rope, so that the block is not able to hit the floor. Do you think that a monkey can climb with such an acceleration? ($g = 10\ ms^{-2}$)



Q. 55. An ideal spring is in its natural length (L) with two objects A and B connected to its ends. A point P on the unstretched spring is at a distance $\frac{2L}{3}$ from B. Now the objects A and B are moved by 4 cm to the left and 8 cm to the right respectively. Find the displacement of point P.



Q. 56. The fig. shows an infinite tower of identical springs each having force constant k. The connecting

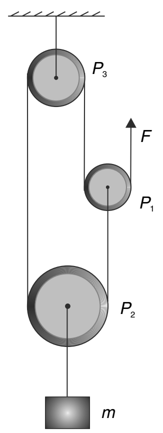


bars and all springs are massless. All springs are relaxed and the bottom row of springs is fixed to horizontal ground. The free end of the top spring is pulled up with a constant force F. In equilibrium, find

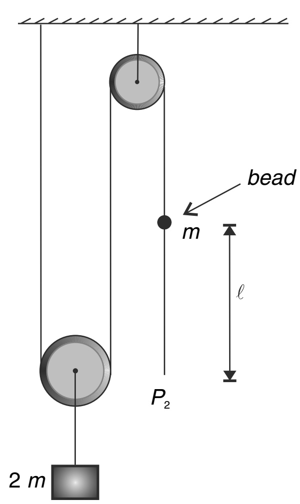
(a) The displacement of free end A of the top spring from relaxed position.

(b) The displacement of the top bar $B1$ from the initial relaxed position.

Q. 57. In the system shown in the fig. there is no friction and string is light. Mass of movable pulley P_2 is M_2. If pulley P_1 is massless, what should be value of applied force F to keep the system in equilibrium?



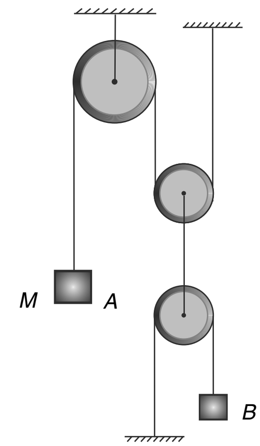
Q. 58. In the system shown in the fig., the bead of mass m can slide on the string. There is friction between the bead and the string. Block has mass equal to twice that of the bead. The system is released from rest with length l of the string hanging below the bead. Calculate the distance moved by the block before the bead slips out of the thread. Assume the string and pulley to be massless.



Q. 59. In the arrangement shown in the fig. all pulleys are mass less and the strings are inextensible and light. Block A has mass M.

(a) If the system stays at rest after it is released, find the mass of the block B.

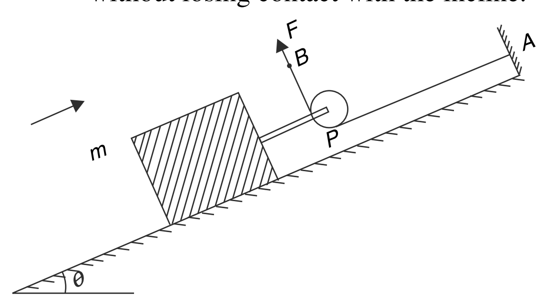
(b) If mass of the block B is twice the value found in part (a) of the problem, calculate the acceleration of block A.



Q. 60. In the fig. shown, the pulley and string are mass less and the incline is frictionless. The segment AP of the string is parallel to the incline and the segment PB is perpendicular to the incline. End of the string is pulled with a constant force F.

(a) If the block is moving up the incline with acceleration while being in contact with the incline, then angle θ must be less then θ_0. Find θ_0

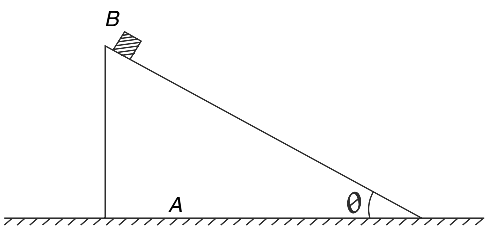
(b) If $\theta = \frac{\theta_0}{2}$ find the maximum acceleration with which the block can move up the plane without losing contact with the incline.



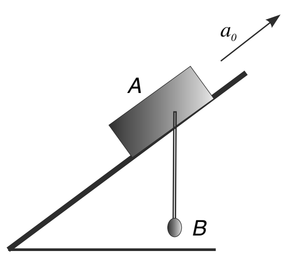
Q.61. A triangular wedge A is held fixed and a block B is released on its inclined surface, from the top. Block B reaches the horizontal ground in time t. In another experiment, the wedge A was free to slide on the horizontal surface and it took t' time for the block B to reach the ground surface after it was released from the top. Neglect friction and assume that B remains in contact with A.

(a) Which time is larger t or t'? Tell by simple observation.

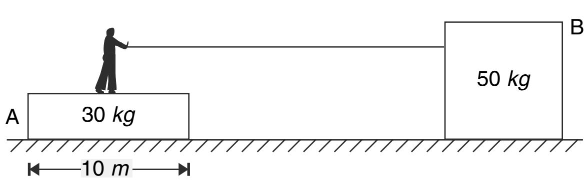
(b) When wedge A was free to move, it was observed that it moved leftward with an acceleration $\frac{g}{4}$ and one of the two measured times (t & t') was twice the other. Find the inclination θ of the inclined surface of the wedge.



Q. 62. A block A is made to move up an inclined plane of inclination θ with constant acceleration a_0 as shown in figure. Bob B, hanging from block A by a light inextensible string, is held vertical and is moving along with the block. Calculate the magnitude of acceleration of block A relative to the bob immediately after bob is released.



Q. 63. A 50 *kg* man is standing at the centre of a 30 *kg* platform A. Length of the platform is 10 m and coefficient of friction between the platform and the horizontal ground is 0.2. Man is holding one end of a light rope which is connected to a 50 *kg* box B. The coefficient of friction between the box and the ground is 0.5. The man pulls the rope so as to slowly move the box and ensuring that he himself does not move relative to the ground. If the shoes of the man does not slip on the platform, calculate how much time it will take for the man to fall off the platform. Assume that rope remains horizontal, and coefficient of friction between shoes and the platform is 0.6.

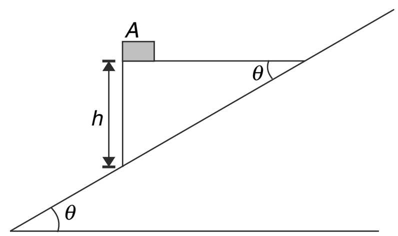


Q. 64. A wedge is placed on the smooth surface of a fixed incline having inclination θ with the horizontal. The vertical wall of the wedge has height h and there is a small block A on the edge of the horizontal surface of the wedge. Mass of the wedge and the small block are M and m respectively.

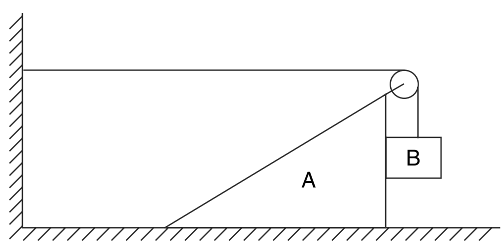
(a) Find the acceleration of the wedge if friction between block A and the wedge is large enough to prevent slipping between the two.

(b) Find friction force between the block and the wedge in the above case. Also find the normal force between the two.

(c) Assuming there is no friction between the block and the wedge, calculate the time in which the block will hit the incline.



Q. 65. In the system shown in figure, all surfaces are smooth, pulley and strings are massless. Mass of both A and B are equal. The system is released from rest.



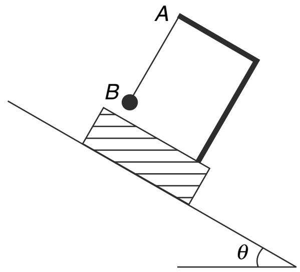
(a) Find the $\vec{a}_A \cdot \vec{a}_B$ immediately after the system is released. $\vec{a}_A$ and $\vec{a}_B$ are accelerations of block A and B respectively.

(b) Find $\vec{a}_A$ immediately after the system is released.

Q.66. A block is placed on an incline having inclination θ. There is a rigid L shaped frame fixed to the block. A plumb line (a ball connected to a thread) is attached to the end A of the frame. The system is released on the inline. Find the angle that the plumb line will make with vertical in its equilibrium position relative to the block when

(a) the incline is smooth

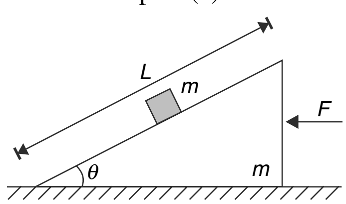
(b) there is friction and the acceleration of the block is half its value when the incline is smooth



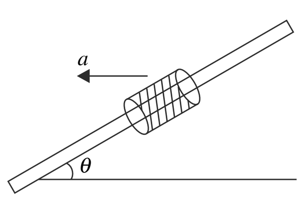
Q. 67. A wedge of mass m is placed on a horizontal smooth table. A block of mass m is placed at the mid point of the smooth inclined surface having length L along its line of greatest slope. Inclination of the inclined surface is $\theta = 45°$. The block is released and simultaneously a constant horizontal force F is applied on the wedge as shown.

(a) What is value of F if the block does not slide on the wedge?

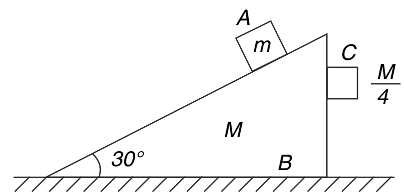
(b) In how much time the block will come out of the incline surface if applied force is 1.5 times that found in part (a)



Q. 68. A rod is kept inclined at an angle θ with the horizontal A sleeve of mass m can slide on the rod. If the coefficient of friction between the rod and the sleeve is μ, for what values of horizontal acceleration a of the rod, towards left, the sleeve will not slide over the rod?



Q. 69. In the arrangement shown in figure, a block A of mass m has been placed on a smooth wedge B of mass M. The wedge lies on a horizontal smooth surface. Another block C of mass $\frac{M}{4}$ has been placed in contact with the wedge B as shown. The coefficient of friction between the block C and the vertical wedge wall is $\mu = \frac{3}{4}$. Find the ratio $\frac{m}{M}$ for which the block C will not slide with respect to the wedge after the system is released?

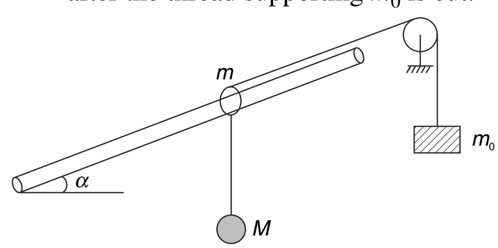


Q. 70. A smooth rod is fixed at an angle α to the horizontal. A small ring of mass m can slide along the rod. A thread carrying a small sphere of mass M is attached to the ring. To keep the system in equilibrium, another thread is attached to the ring which carries a load of mass m_0 at its end (see figure). The thread runs parallel to the rod between the ring and the pulley.

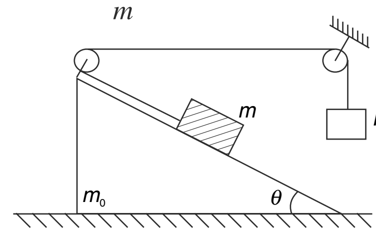
All threads and pulley are massless.

(a) Find m_0 so that system is in equilibrium.

(b) Find acceleration of the sphere M immediately after the thread supporting m_0 is cut.

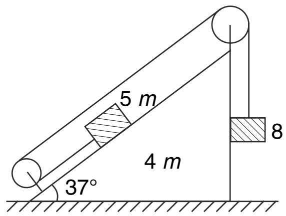


Q. 71. In the system shown in figure all surfaces are smooth and string and pulleys are light. Angle of wedge $\theta = \sin^{-1}\left(\frac{3}{5}\right)$. When released from rest it was found that the wedge of mass m_0 does not move. Find $\frac{M}{m}$.

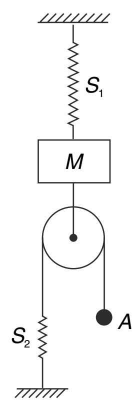


Q. 72. In the last problem take $M = m$ and $m_0 = 2\,m$ and calculate the acceleration of the wedge.

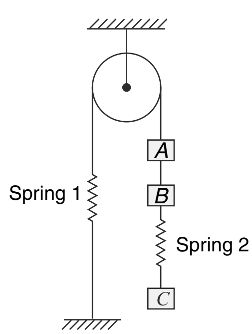
Q. 73. In the system shown in the figure all surfaces are smooth, pulley and string are massless. The string between the two pulleys and between pulley and block of mass 5 m is parallel to the incline surface of the block of mass 4 m. The system is released from rest. Find the acceleration of the block of mass 4 m. $\left[\tan 37^\circ = \dfrac{3}{4}\right]$



Q. 74. In the system shown in figure, the two springs S_1 and S_2 have force constant k each. Pulley, springs and strings are all massless. Initially, the system is in equilibrium with spring S_1 stretched and S_2 relaxed. The end A of the string is pulled down slowly through a distance L. By what distance does the block of mass M move?



Q. 75. The system shown in figure is in equilibrium. Pulley, springs and the strings are massless. The three blocks A, B and C have equal masses. x_1 and x_2 are extensions in the spring 1 and spring 2 respectively.

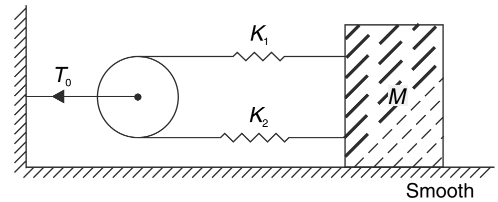


(a) Find the value of $\left|\dfrac{d^2x_2}{dt^2}\right|$ immediately after spring 1 is cut.

(b) Find the value of $\left|\dfrac{d^2x_1}{dt^2}\right|$ and $\left|\dfrac{d^2x_2}{dt^2}\right|$ immediately after string AB is cut.

(c) Find the value of $\left|\dfrac{d^2x_1}{dt^2}\right|$ and $\left|\dfrac{d^2x_2}{dt^2}\right|$ immediately after spring 2 is cut.

Q. 76. In the figure shown, the pulley, strings and springs are mass less. The block is moved to right by a distance x_0 from the position where the two springs are relaxed. The block is released from this position.

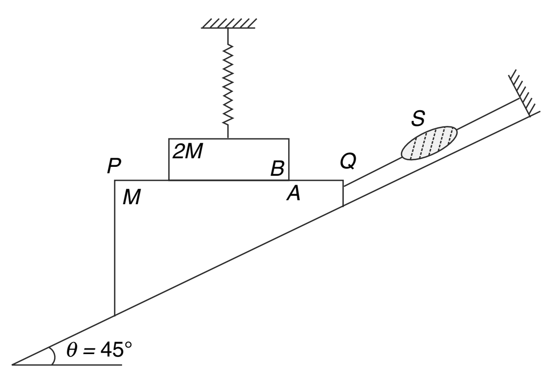


(a) Find the acceleration of the block immediately after it is released.

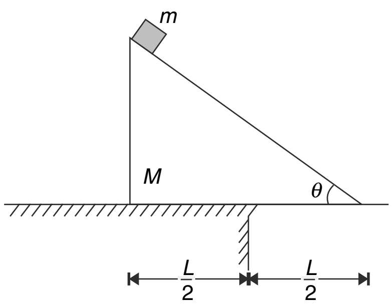
(b) Find tension (T_0) in the support holding the pulley to the wall, immediately after the block is released.

Assume no friction.

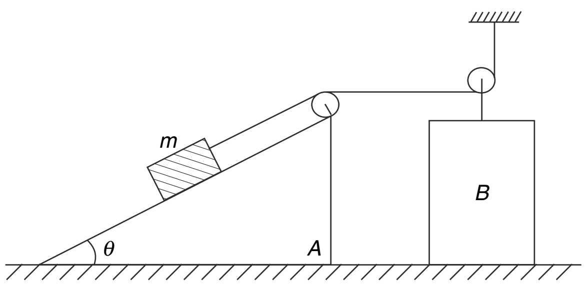
Q.77. The system shown in figure is in equilibrium. Surface PQ of wedge A, having mass M, is horizontal. Block B, having mass $2M$, rests on wedge A and is supported by a vertical spring. The spring balance S is showing a reading of $\sqrt{2}\,Mg$. There is no friction anywhere and the thread QS is parallel to the incline surface. The thread QS is cut. Find the acceleration of A and the normal contact force between A and B immediately after the thread is cut.



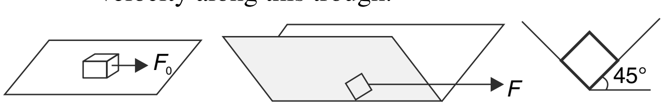
Q.78. A triangular wedge of mass M lies on a smooth horizontal table with half of its base projecting out of the edge of the table. A block of mass m is kept at the top of the smooth incline surface of the wedge and the system is let go. Find the maximum value of $\frac{M}{m}$ for which the block will land on the table. Take $\theta = 60°$.



Q.79. In the system shown in the figure all surfaces are smooth and both the pulleys are mass less. Block on the incline surface of wedge A has mass m. Mass of A and B are $M = 4\,m$ and $M_0 = 2\,m$ respectively. Find the acceleration of wedge A when the system is released from rest.

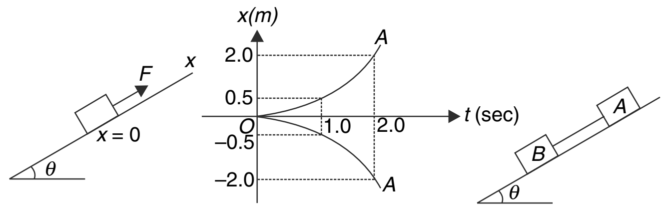


Q.80. A block of mass m requires a horizontal force F_0 to move it on a horizontal metal plate with constant velocity. The metal plate is folded to make it a right angled horizontal trough. Find the horizontal force F that is needed to move the block with constant velocity along this trough.

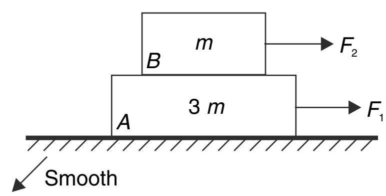


Q.81. Block A of mass $m_A = 200\,g$ is placed on an incline plane and a constant force $F = 2.2\,N$ is applied on it parallel to the incline. Taking the initial position of the block as origin and up along the incline as x direction, the position (x) time (t) graph of the block is recorded (see figure (b)). The same experiment is repeated with another block B of mass $m_B = 500\,g$. Same force F is applied to it up along the incline and its position – time graph is recorded (see figure (b)). Now the two blocks are connected by a light string and released on the same incline as shown in figure (c). Find the tension in the string.

$$\left[\tan\theta = \frac{3}{4}; g = 10\,m/s^2\right]$$



Q.82. Block B of mass m has been placed on block A of mass 3 m as shown. Block A rests on a smooth horizontal table. F_1 is the maximum horizontal force that can be applied on the block A such that there is no slipping between the blocks. Similarly, F_2 is the maximum horizontal force that can be applied on the block B so that the two blocks move together without slipping on each other. When F_1 and F_2 both are applied together as shown in figure.

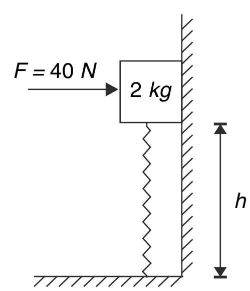


(a) Find the friction force acting between the blocks.

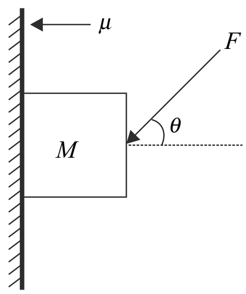
(b) Acceleration of the two blocks.

(c) If F_2 is decreased a little, what will be direction of friction acting on B.

Q. 83. (i) In the arrangement shown in the figure the coefficient of friction between the 2 kg block and the vertical wall is $\mu = 0.5$. A constant horizontal force of 40 N keeps the block pressed against the wall. The spring has a natural length of 1.0 m and its force constant is $k = 400\ Nm^{-1}$. What should be the height h of the block above the horizontal floor for it to be in equilibrium. The spring is not tied to the block.



(ii) A block of mass M is pressed against a rough vertical wall by applying a force F making an angle of θ with horizontal (as shown in figure). Coefficient of friction between the wall and the block is $\mu = 0.75$.

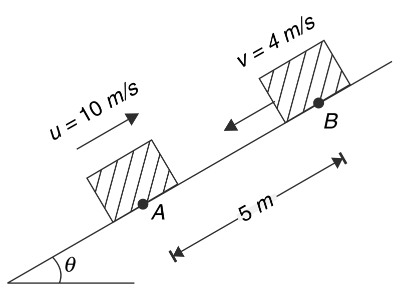


(a) If $F = 2\ Mg$, find the range of values of θ so that the block does not slide

[Take tan 37° = 0.75; sin 24° = 0.4]

(b) Find the maximum value of θ above which equilibrium is not possible for any magnitude of force F.

Q. 84. A block is projected up along a rough incline with a velocity of $\upsilon = 10\ m/s$. After 4 s the block was at point B at a distance of 5 m from the starting point A and was travelling down at a velocity of $v = 4\ m/s$.

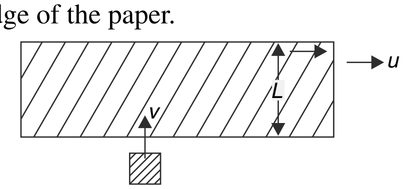


(a) Find time after projection at which the block came to rest.

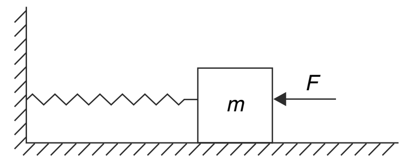
(b) Find the coefficient of friction between the block and the incline.

Take $g = 10\ m/s^2$

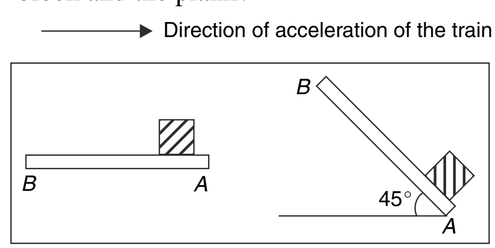
Q. 85. A long piece of paper is being pulled on a horizontal surface with a constant velocity υ along its length. Width of the paper is L. A small block moving horizontally, perpendicular to the direction of motion of the paper, with velocity v slides onto the paper. The coefficient of friction between the block and the paper is μ. Find maximum value of v such that the block does not cross the opposite edge of the paper.



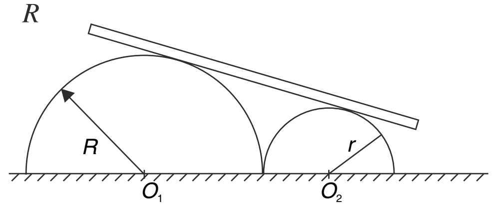
Q. 86. A block of mass $m = 1\ kg$ is kept pressed against a spring on a rough horizontal surface. The spring is compressed by 10 cm from its natural length and to keep the block at rest in this position a horizontal force (F) towards left is applied. It was found that the block can be kept at rest if $8\ N \le F \le 18\ N$. Find the spring constant (k) and the coefficient of friction (μ) between the block and the horizontal surface.



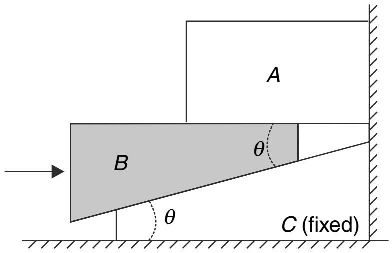
Q. 87. An experimenter is inside a uniformly accelerated train. Train is moving horizontally with constant acceleration a_0. He places a wooden plank AB in horizontal position with end A pointing towards the engine of the train. A block is released at end A of the plank and it reaches end B in time t_1. The same plank is placed at an inclination of 45° to the horizontal. When the block is released at A it now climbs to B in time t_2. It was found that $\frac{t_2}{t_1} = 2^{5/4}$. What is the coefficient of friction between the block and the plank?



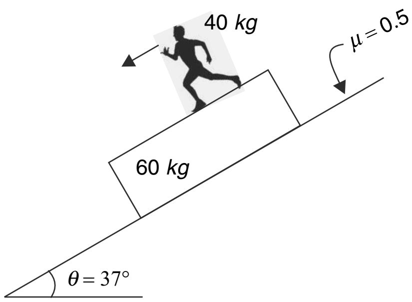
Q. 88. Two hemispheres of radii R and r ($< R$) are fixed on a horizontal table touching each other (see figure). A uniform rod rests on two spheres as shown. The coefficient of friction between the rod and two spheres is μ. Find the minimum value of the ratio $\frac{r}{R}$ for which the rod will not slide.



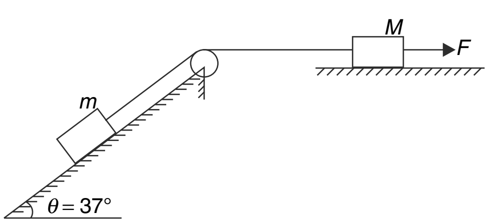
Q. 89. In order to lift a heavy block A, an engineer has designed a wedge system as shown. Wedge C is fixed. A horizontal force F is applied to B to lift block A. Wedge B itself has negligible mass and mass of A is M. The coefficient of friction at all surfaces is μ. Find the value of applied force F at which the block A just begins to rise.



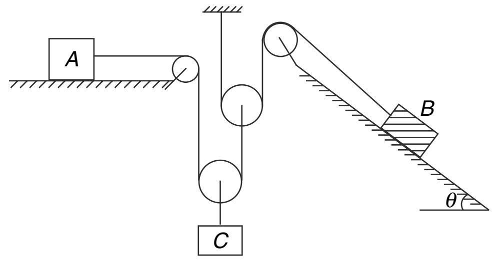
Q.90. A 60 *kg* platform has been placed on a rough incline having inclination $\theta = 37°$. The coefficient of friction between the platform and the incline is μ = 0.5. A 40 *kg* man is running down on the platform so as to keep the platform stationary. What is the acceleration of the man? It is known that the man cannot manage to go beyond an acceleration of 7 m/s^2. $\left[\sin 37° = \frac{3}{5}\right]$



Q. 91. In the system show in figure, mass of the block placed on horizontal surface is M = 4 *kg*. A constant horizontal force of F = 40 N is applied on it as shown. The coefficient of friction between the blocks and surfaces is μ = 0.5. Calculate the values of mass m of the block on the incline for which the system does not move. $\left[\sin 37° = \frac{3}{5}; g = 10 m/s^2\right]$



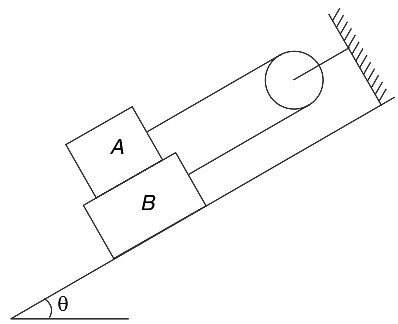
Q. 92. In the arrangement shown in the figure, block A of mass 8 *kg* rests on a horizontal table having coefficient of friction μ = 0.5. Block B has a mass of 6 *kg* and rests on a smooth incline having inclination angle $\theta = \sin^{-1}\left(\frac{2}{5}\right)$. All pulleys and strings are mass less. Mass of block C is M. [g = 10 m/s^2]



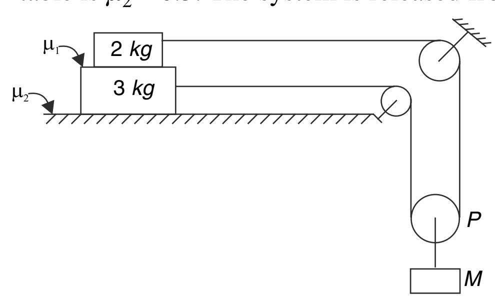
(a) Find value of M for which block B does not accelerate

(b) Find maximum value of M for which A does not accelerate.

Q.93. In the arrangement shown in figure, pulley and string are light. Friction coefficient between the two blocks is μ whereas the incline is smooth. Block A has mass m and difference in mass of the two blocks is Δm. Find minimum value of μ for which the system will not accelerate when released from rest.



Q. 94. In the arrangement shown in figure pulley P can move whereas other two pulleys are fixed. All of them are light. String is light and inextensible. The coefficient of friction between 2 *kg* and 3 *kg* block is μ_1 = 0.75 and that between 3 *kg* block and the table is μ_2 = 0.5. The system is released from rest



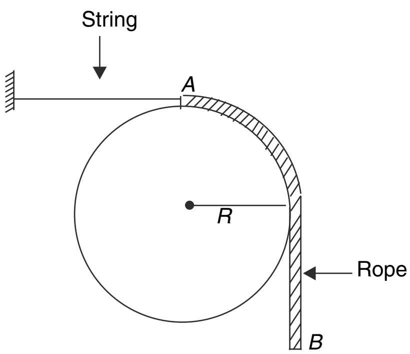
(i) Find maximum value of mass M, so that the system does not move. Find friction force between 2 *kg* and 3 *kg* blocks in this case.

(ii) If M = 4 *kg*, find the tension in the string attached to 2 *kg* block.

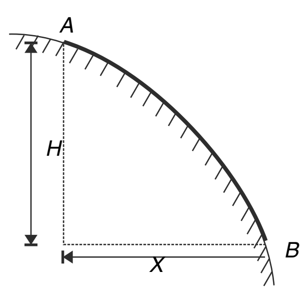
(iii) If $M = 4\ kg$ and $\mu_1 = 0.9$, find friction force between the two blocks, and acceleration of M.

(iv) Find acceleration of M if $\mu_1 = 0.75$, $\mu_2 = -0.9$ and $M = 4\ kg$.

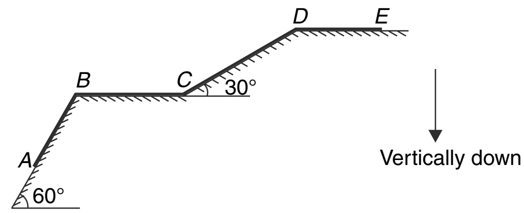
Q. 95. A rope of length $\left(\dfrac{\pi}{2}+1\right)R$ has been placed on a smooth sphere of radius R as shown in figure. End A of the rope is at the top of the sphere and end B is overhanging. Mass per unit length of the rope is λ. The horizontal string holding this rope in place can tolerate tension equal to weight of the rope. Find the maximum mass (M_0) of a block that can be tied to the end B of the rope so that the string does not break.



Q.96. A uniform rope has been placed on a sloping surface as shown in the figure. The vertical separation and horizontal separation between the end points of the rope are H and X respectively. The friction coefficient (μ) is just good enough to prevent the rope from sliding down. Find the value of μ.



Q.97. A uniform rope $ABCDE$ has mass M and it is laid along two incline surfaces (AB and CD) and two horizontal surfaces (BC and DE) as shown in figure. The four parts of the rope AB, BC, CD and DE are of equal lengths. The coefficient of friction (μ) is uniform along the entire surface and is just good enough to prevent the rope from sliding.

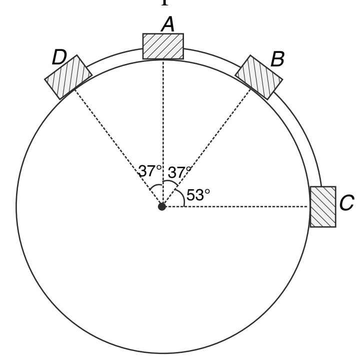


(a) Find μ

(b) x is distance measured along the length of the rope starting from point A. Plot the variation of tension in the rope (T) with distance x.

(c) Find the maximum tension in the rope.

Q. 98. (i) Four small blocks are interconnected with light strings and placed over a fixed sphere as shown. Blocks A, B and C are identical each having mass $m = 1\ kg$. Block D has a mass of $m' = 2\ kg$. The coefficient of friction between the blocks and the sphere is $\mu = 0.5$. The system is released from the position shown in figure.

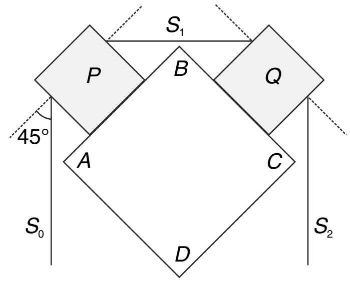


(a) Find the tension in each string. Which string has largest tension?

(b) Find the friction force acting on each block.

$$\left[\text{Take}\tan 37° = \frac{3}{4};\ g = 10\ m/s^2\right]$$

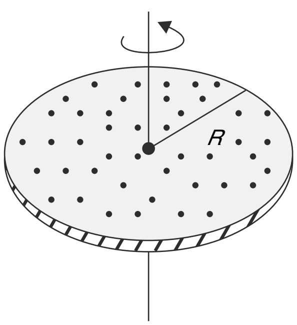
(ii) A fixed square prism $ABCD$ has its axis horizontal and perpendicular to the plane of the figure. The face AB makes 45° with the vertical. On the upper faces AB and BC of the prism there are light bodies P and Q respectively. The two bodies (P and Q) are connected using a string S_1 and strings S_0 and S_2 are hanging from P and Q respectively. All strings are mass less, and inextensible. String S_1 is horizontal and the other two strings are vertical. The coefficient of friction between the bodies and the prism is μ_0. Assume that P and Q always remain in contact with the prism.



(a) If tension in S_0 is T_0, find the minimum tension (T_1) in S_1 to keep the body P at rest.

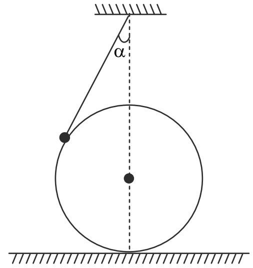
(b) A mass M_0 is tied to the lower end of string S_0 and another mass m_2 is tied to S_2. Find the minimum value of m_2 so as to keep P and Q at rest.

Q. 99. A metal disc of radius R can rotate about the vertical axis passing through its centre. The top surface of the disc is uniformly covered with dust particles. The disc is rotated with gradually increasing speed. At what value of the angular speed (ω) of the disc the 75% of the top surface will become dust free. Assume that the coefficient of friction between the dust particles and the metal disc is $\mu = 0.5$. Assume no interaction amongst the dust particles.



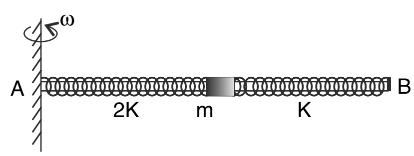
Q.100. In the last question, the axis of the disc is tilted slightly to make an angle θ with the vertical. Redo the problem for this condition and check the result by putting $\theta = 0$ in your answer.

Q. 101.

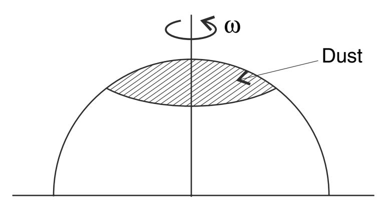


A sphere of mass M is held at rest on a horizontal floor. One end of a light string is fixed at a point that is vertically above the centre of the sphere. The other end of the string is connected to a small particle of mass m that rests on the sphere. The string makes an angle $\alpha = 30°$ with the vertical. Find the acceleration of the sphere immediately after it is released. There is no friction anywhere.

Q. 102. A light rod AB is fitted with a small sleeve of mass m which can slide smoothly over it. The sleeve is connected to the two ends of the rod using two springs of force constant $2k$ and k (see fig). The ends of the springs at A and B are fixed and the other ends (connected to sleeve) can move along with the sleeve. The natural length of spring connected to A is ℓ_0. Now the rod is rotated with angular velocity ω about an axis passing through end A that is perpendicular to the rod. Take $\frac{k}{m\omega^2} = \eta$ and express the change in length of each spring (in equilibrium position of the sleeve relative to the rod) in terms of ℓ_0 and η.



Q. 103. A metallic hemisphere is having dust on its surface. The sphere is rotated about a vertical axis passing through its centre at angular speed $\omega = 10\ rad\ s^{-1}$. Now the dust is visible only on top 20% area of the curved hemispherical surface. Radius of the hemisphere is $R = 0.1\ m$. Find the coefficient of friction between the dust particle and the hemisphere [$g = 10\ ms^{-2}$].



Q. 104. Civil engineers bank a road to help a car negotiate a curve. While designing a road they usually ignore friction. However, a young engineer decided to include friction in his calculation while designing a road. The radius of curvature of the road is R and the coefficient of friction between the tire and the road is μ.

(a) What should be the banking angle (θ_0) so that car travelling up to a maximum speed V_0 can negotiate the curve.

(b) At what speed (V_1) shall a car travel on a road banked at θ_0 so that there is no tendency to skid. (No tendency to skid means there is no static friction force action on the car).

(c) The driver of a car travelling at speed (V_1) starts retarding (by applying brakes). What angle (acute, obtuse or right angle) does the resultant friction force on the car make with the direction of motion?

Q. 105. A turn of radius 100 m is banked for a speed of 20 m/s

(a) Find the banking angle

(b) If a vehicle of mass 500 kg negotiates the curve find the force of friction on it if its speed is – (i) 30 m/s (ii) 10 m/s

Assume that friction is sufficient to prevent skidding and slipping.

[Take tan 22° = 0.4, sin 22° = 0.375, cos 22° = 0.93, $g = 10\ ms^{-2}$]

Q. 106. A horizontal circular turning has a curved length L and radius R. A car enters the turn with a speed V_0 and its speed increases at a constant rate f. If the coefficient of friction is μ,

(a) At what time t_0, after entering the curve, will the car skid? (Take it for granted that it skids somewhere on the turning)

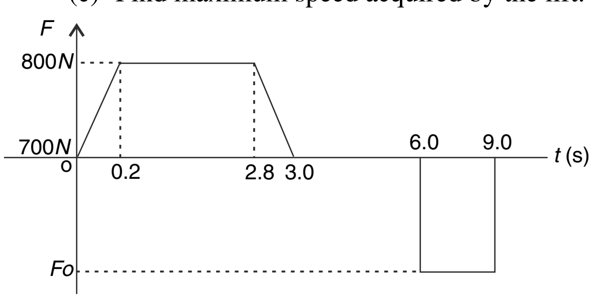
(b) At a time t ($< t_0$) what is the force of friction acting on the car?

Q. 107. A 70 kg man enters a lift and stands an a weighing scale inside it. At time t = 0, the lift starts moving up and stops at a higher floor at t = 9.0 s. During the course of this journey, the weighing scale records his weight and given a plot of his weight vs time. The plot is shown in the fig. [Take $g = 10\ m/s^2$]

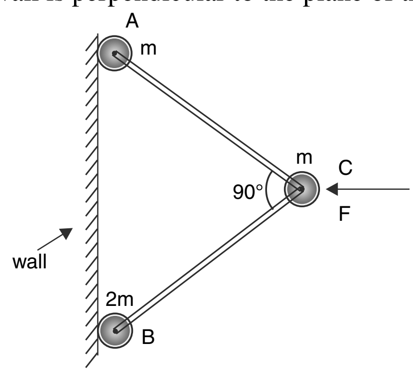
(a) Find F_0

(b) Find the magnitude of maximum acceleration of the lift.

(c) Find maximum speed acquired by the lift.



Q.108. Three small discs are connected with two identical massless rods as shown in fig. The rods are pinned to the discs such that angle between them can change freely. The system is placed on a smooth horizontal surface with discs A and B touching a smooth wall and the angle ACB being 90°. A force F is applied to the disc C in a direction perpendicular to the wall. Find acceleration of disc B immediately after the force starts to act. Masses of discs are $m_A = m$; $m_B = 2\ m$; $m_C = m$ [wall is perpendicular to the plane of the fig.]

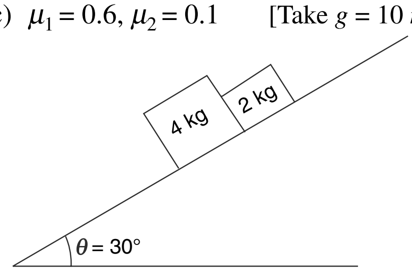


Q. 109. Figure shows two blocks in contact placed on an incline of angle $\theta = 30°$. The coefficient of friction between the block of mass 4 kg and the incline is μ_1, and that between 2 kg block and incline is μ_2. Find the acceleration of the blocks and the contact force between them if –

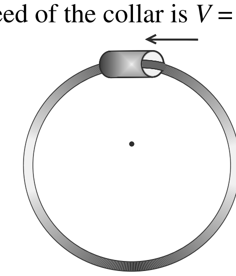
(a) $\mu_1 = 0.5, \mu_2 = 0.8$

(b) $\mu_1 = 0.8, \mu_2 = 0.5$

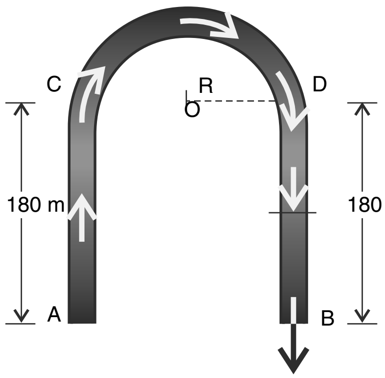
(c) $\mu_1 = 0.6, \mu_2 = 0.1$ [Take $g = 10\ m/s^2$]



Q. 110. A small collar of mass m = 100 g slides over the surface of a horizontal circular rod of radius R = 0.3 m. The coefficient of friction between the rod and the collar is $\mu = 0.8$. Find the angle made with vertical by the force applied by the rod on the collar when speed of the collar is $V = 2\ m/s$.



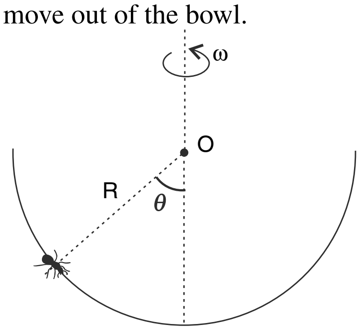
Q. 111. A flat race track consists of two straight section *AC* and *DB* each of length 180 *m* and one semi circular section *DC* of radius $R = 150$ *m*. A car starting from rest at *A* has to reach *B* in least possible time (the car may cross through point *B* and need not stop there). The coefficient of friction between the tyres and the road is $\mu = 0.6$ and the top speed that the car can acquire is 180 *kph*. Find the minimum time needed to move from A to B under ideal conditions. Braking is not allowed in the entire journey [$g = 10\ m/s^2$]



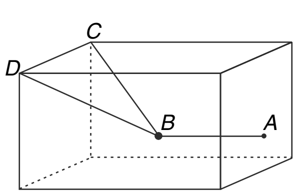
Q. 112. A small insect is climbing slowly along the inner wall of a hemispherical bowl of radius R. The insect is unable to climb beyond $\theta = 45°$. Whenever it tries to climb beyond $\theta = 45°$, it slips.

(a) Find the minimum angular speed ω with which the bowl shall be rotated about its vertical radius so that the insect can climb upto $\theta = 60°$.

(b) Find minimum ω for which the insect can move out of the bowl.



Q. 113.



A room is in shape of a cube. *A* heavy ball (*B*) is suspended at the centre of the room tied to three inextensible strings as shown. String *BA* is horizontal with A being the centre point of the wall. Find the ratio of tension in the string *BA* and *BC*.

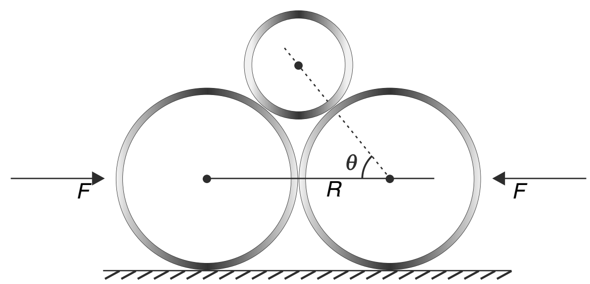
Q.114. Two identical smooth disc of radius *R* have been placed on a frictionless table touching each other. Another circular plate is placed between them as shown in figure. The mass per unit area of each object is σ, and the line joining the centers of the plate and the disc is θ

(a) Find the minimum horizontal force F_0 that must be applied to the two discs to keep them together.

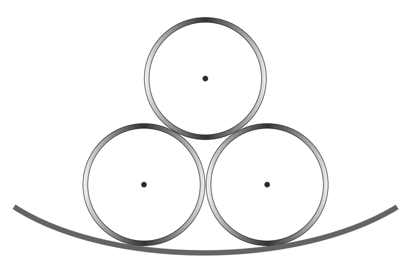
(b) Angle θ can be changed by changing the size of the circular plate. Find F_0 when $\theta \to 0$.

$$\left[\text{use } \cos\theta = 1 - \frac{\theta^2}{2} \text{ and } \sin\theta = \theta \text{ for small } \theta\right]$$

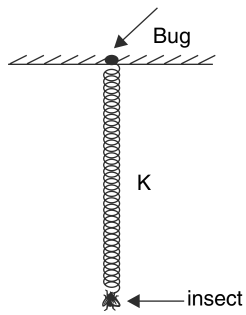
(c) Find F_0 when $\theta \to \pi/2$. Explain the result.



Q. 115. Three identical smooth cylinders, each of mass *m* and radius *r* are resting in equilibrium within a fixed smooth cylinder of radius *R* (only a part of this cylinder has been shown in the fig). Find the largest value of *R* in terms of *r* for the small cylinders to remain in equilibrium.



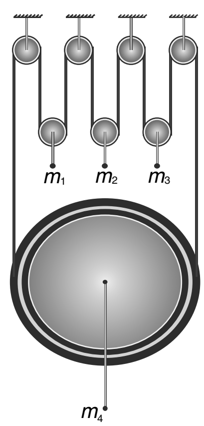
Q. 116. A massless spring of force constant *K* and natural length ℓ_0 is hanging from a ceiling. An insect of mass *m* is sitting at the lower end of the spring and the system is in equilibrium . The insect starts slowing climbing up the spring so as to eat a bug sitting on the ceiling. Assume that insect climbs without slipping on the spring and $K = \dfrac{mg}{\ell_0}$. Find the length of the spring when the insect is at $\dfrac{1}{4}$th of its original distance from the bug.



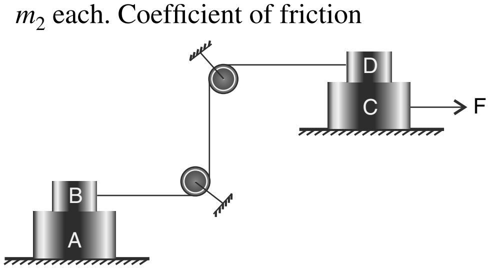
Q. 117. In the system shown in fig., all pulleys are mass less and the string is inextensible and light.

(a) After the system is released, find the acceleration of mass m_1

(b) If $m_1 = 1\ kg$, $m_2 = 2\ kg$ and $m_3 = 3\ kg$ then what must be value of mass m_4 so that it accelerates downwards?



Q. 118. In the system shown in fig., block A and C are placed on smooth floors and both have mass equal to m_1. Blocks B and D are identical having mass m_2 each. Coefficient of friction

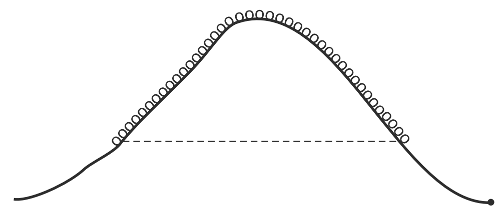


Between A and B and that between C and D are both equal to μ. String and pulleys are light. A horizontal force F is applied on block C and is gradually increased.

(a) Find the maximum value of F (call it F_0) so that all the four blocks move with same acceleration.

(b) Will the value of F_0 increase or decrease if another block (E) of mass m_2 is placed above block D and coefficient of friction between E and D is μ?

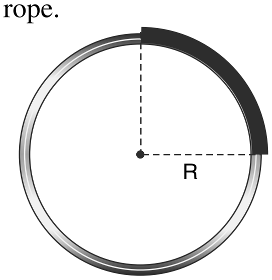
Q. 119. A chain with uniform mass per unit length lies in a vertical plane along the slope of a smooth hill. The two end of the chain are at same height. If the chain is released from this position find its acceleration.



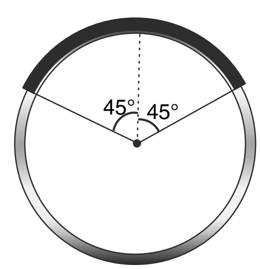
Q. 120. A uniform rope of length $\dfrac{\pi R}{2}$ has been placed on fixed cylinder of radius R as shown in the fig. One end of the rope is at the top of the cylinder. The coefficient of friction between the rope and the cylinder is just enough to prevent the rope from sliding. Mass of the rope is M.

(a) At what position, the tension in the rope is maximum?

(b) Calculate the value of maximum tension in the rope.

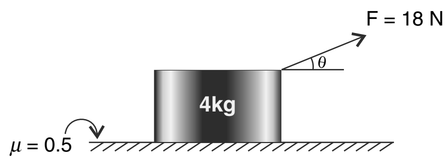


Q. 121. In the last problem, the rope is placed on the cylinder as shown. Find maximum tension in the rope.



Q. 122. A 4 kg block is placed on a rough horizontal surface. The coefficient of friction between the block and the surface is $\mu = 0.5$. A force $F = 18\ N$ is applied on the block making an angle θ with the horizontal. Find the range of values of θ for which the block can start moving.

$$\left[\begin{array}{c}\text{Take } g = 10 \text{ m/s}^2, \tan^{-1}(2) = 63° \\ \sin^{-1}\left(\dfrac{10}{9\sqrt{1.25}}\right) = 84°\end{array}\right]$$



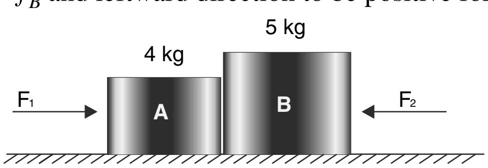
Q. 123. Two rectangular blocks A and B are placed on a horizontal surface at a very small separation. The masses of the blocks are $m_A = 4\ kg$ and $m_B = 5\ kg$. Coefficient of friction between the horizontal surface and both the blocks is $\mu = 0.4$. Horizontal forces F_1 and F_2 are applied on the blocks as shown. Both the forces vary with time as

$F_1 = 15 + 0.5\ t$

$F_2 = 2t$

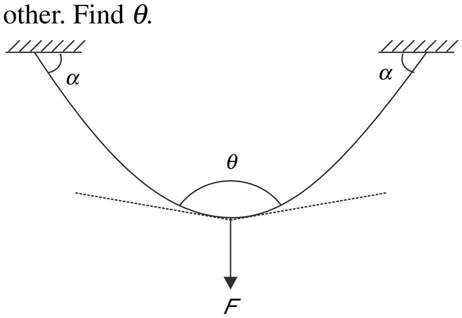
Where 't' is time in second.

Plot the variation of friction force acting on the two blocks (f_A and f_B) vs time till the motion starts. Take rightward direction to be positive for f_B and leftward direction to be positive for f_A.



LEVEL 3

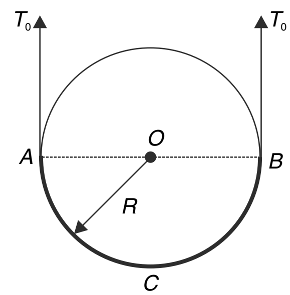
Q. 124. A rope of mass m is hung from a ceiling. The centre point is pulled down with a vertical force F. The tangent to the rope at its ends makes an angle α with horizontal ceiling. The two tangents at the lower point make an angle of θ with each other. Find θ.



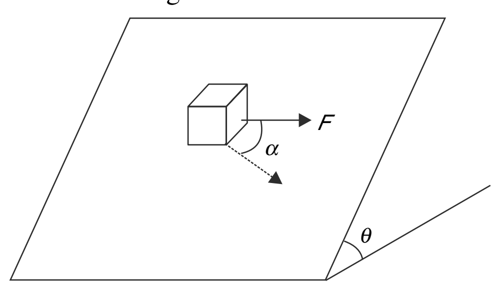
Q. 125. A smooth cylinder is fixed with its axis horizontal. Radius of the cylinder is R. A uniform rope (ACB) of linear mass density λ (kg/m) is exactly of length πR and is held in semicircular shape in vertical plane around the cylinder as shown in figure. Two massless strings are connected at the two ends of the rope and are pulled up vertically with force T_0 to keep the rope in contact with the cylinder.

(a) Find minimum value of T_0 so that the rope does not lose contact with the cylinder at any point.

(b) If T_0 is decreased slightly below the minimum value calculated in (a), where will the rope lose contact with the cylinder.

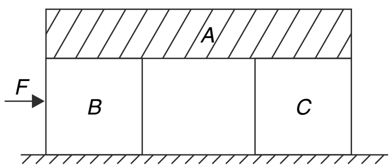


Q. 126. A block of mass m placed on an incline just begins to slide when inclination of the incline is made $\theta_0 = 45°$. With inclination equal to $\theta = 30°$, the block is placed on the incline. A horizontal force (F) parallel to the surface of the incline is applied to the block. The force F is gradually increased from zero. At what angle α to the force F will the block first begin to slide?



Q. 127. In the last problem if it is allowed to apply the force F in any direction, find the minimum force F_{min} needed to move the block on the incline.

Q. 128. A block A has been placed symmetrically over two identical blocks B and C. All the three blocks have equal mass, M each, and the horizontal surface on which B and C are placed is smooth. The coefficient of friction between A and either of B and C is μ. The block A exerts equal pressure on B and C. A horizontal force F is applied to the block B.

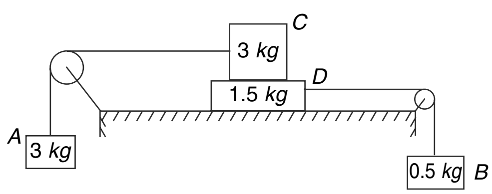


(a) Find maximum value of F so that A does not slip on B or C and the three blocks move together.

(b) If F is increased beyond the maximum found in (a) where will we see slipping first- at contact of A and B or at the contact of A and C.

(c) If F is kept half the maximum found in (a), calculate the ratio of friction force between A and B to that between A and C. Does this ratio change if F is decreased further?

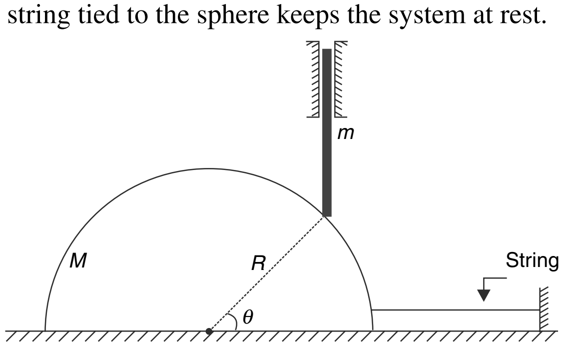
Q. 129. In the arrangement shown in the figure the coefficient of friction between the blocks C and D is $\mu_1 = 0.7$ and that between block D and the horizontal table is $\mu_2 = 0.2$. The system is released from rest. [Take g = 10 ms $^{-2}$] Pulleys and threads are massless.



(a) Find the acceleration of the block C.

(b) Block B is replaced with a new block. What shall be the minimum mass of this new block so that block C and D accelerate in opposite direction?

Q. 130. A hemisphere of mass M and radius R rests on a smooth horizontal table. A vertical rod of mass m is held between two smooth guide walls supported on the sphere as shown. There is no friction between the rod and the sphere. A horizontal string tied to the sphere keeps the system at rest.



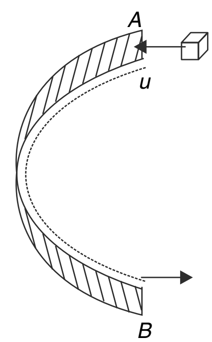
(a) Find tension in the string.

(b) Find the acceleration of the hemisphere immediately after the string is cut.

Q. 131. A semicircular ring of radius R is fixed on a smooth horizontal table. A small block is projected with speed u so as to enter the ring at end A. Initial velocity of the block is along tangent to the ring at A and it moves on the table remaining in contact with the inner wall of the ring. The coefficient of friction between the block and the ring is μ.

(a) Find the time after which the block will exit the ring at B.

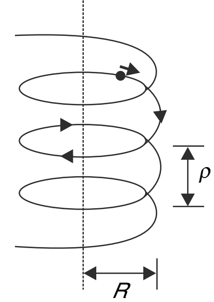
(b) With what speed will the block leave the ring at B.



Q. 132. A long helix made of thin wire is held vertical. The radius and pitch of the helix are R and ρ respectively. A bead begins to slide down the helix.

(a) Find the normal force applied by the wire on the bead when the speed of the bead is v.

(b) Eventually, the bead acquires a constant speed of v_0. Find the coefficient of friction between the wire and the bead.

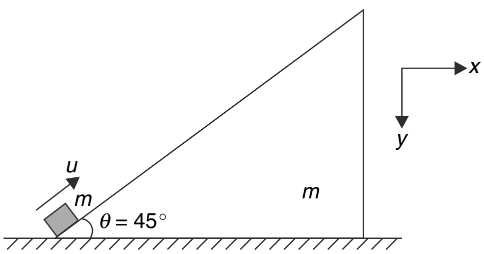


Q. 133. A wedge of mass m is kept on a smooth table and its inclined surface is also smooth. A small block of mass m is projected from the bottom along the incline surface with velocity u. Assume that the block remains on the incline and take $\theta = 45°$, $g = 10\ m/s^2$.

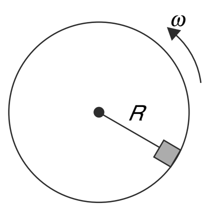
(a) Find the acceleration of the wedge and the x and y components of acceleration of the block.

(b) Draw the approximate path of the block as observed by an observer on the ground. At what angle does the block hit the table?

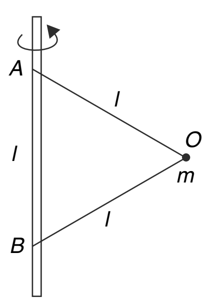
(c) Calculate the radius of curvature of the path of the block when it is at the highest point.



Q. 134. A cylinder with radius R spins about its horizontal axis with angular speed ω. There is a small block lying on the inner surface of the cylinder. The coefficient of friction between the block and the cylinder is μ. Find the value of ω for which the block does not slip, i.e., stays at rest with respect to the cylinder.



Q. 135. A particle of mass m is attached to a vertical rod with two inextensible strings AO and BO of equal lengths l. Distance between A and B is also l. The setup is rotated with angular speed ω with rod as the axis.



(a) Find the values of ω for which the particle remains at point B.

(b) Find the range of values of ω for which tension (T_1) in the string AO is greater than mg but the other string remains slack

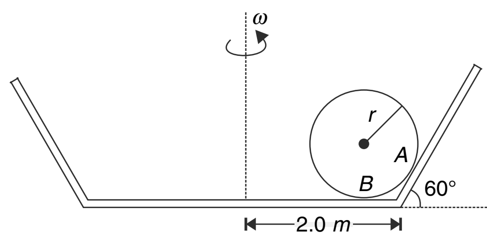
(c) Find the value of ω for which tension (T_1) in string AO is twice the tension (T_2) in string BO

(d) Assume that both strings are taut when the string AO breaks. What will be nature of path of the particle moment after AO breaks ?

Q. 136. A sphere of mass m and radius $r = \sqrt{3}m$ is placed inside a container with flat bottom and slant sidewall as shown in the figure. The sphere touches the slant wall at point A and the floor at point B. It does not touch any other surface. The container, along with the sphere, is rotated about the central vertical axis with angular speed ω. The sphere moves along with the container, i.e., it is at rest relative to the container. The normal force applied by the bottom surface and the slant surface on the sphere are N_1 and N_2 respectively. There is no friction.

(a) Find the value of ω above which N_2 becomes larger than N_1

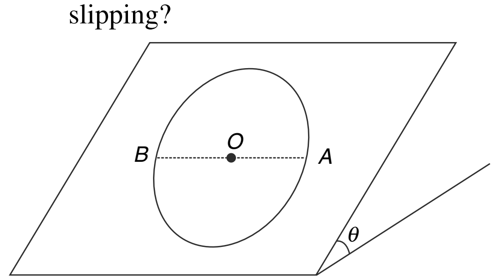
(b) Find the value of ω above which the sphere leaves contact with the floor.



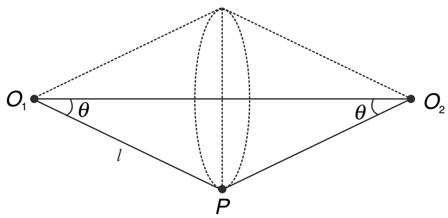
Q. 137. A car is being driven on a tilted ground. The ground makes an angle θ with the horizontal. The driven drives on a circle of radius R. The coefficient of friction between the tires and the ground is μ.

(a) What is the largest speed for which the car will not slip at point A? Assume that rate of change of speed is zero.

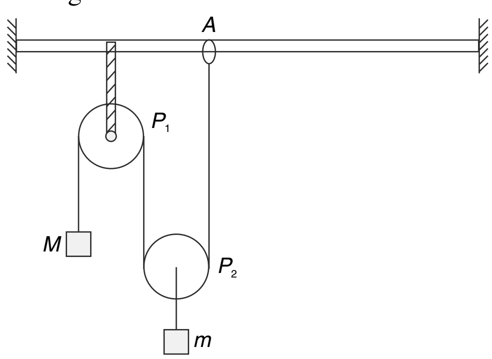
(b) What is the largest constant speed with which the car can be driven on the circle without slipping?



Q. 138. A particle P is attached to two fixed points O_1 and O_2 in a horizontal line, by means of two light inextensible strings of equal length l. It is projected with a velocity just sufficient to make it describe a circle, in a vertical plane, without the strings getting slack and with the angle $< O_2O_1P = <O_1O_2P = \theta$. When the particle is at its lowest point, the string O_2P breaks and the subsequent path of the particle was found to be a circle of radius $l \cos \theta$. Find θ.



Q. 139. The arrangement shown in figure is in equilibrium with all strings vertical. The end A of the string is tied to a ring which can be slid slowly on the horizontal rod. Pulley P_1 is rigidly fixed but P_2 can move freely. A mass m is attached to the centre of pulley P_2 through a thread. Pulleys and strings are mass less.



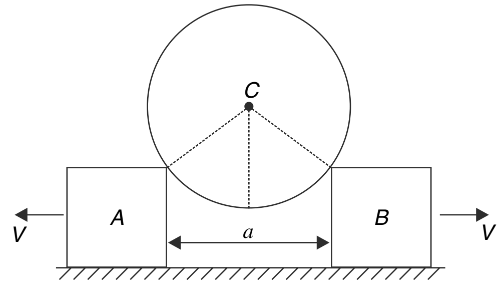
(a) Which block will move up as A is moved slowly to the right?

(b) Will the block of mass m have horizontal displacement?

(c) Is it possible, for a particular position of A, that M has no acceleration but m does have an acceleration? If this happens when string from P_2 to A makes an angle θ with vertical, find the acceleration of m at the instant.

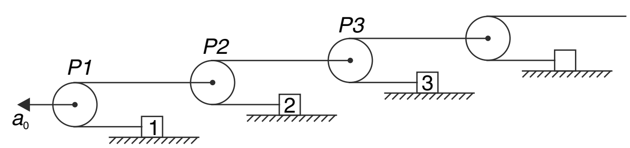
Q. 140. A smooth spherical ball of mass $M = 2\ kg$ is resting on two identical blocks A and B as shown in the figure. The blocks are moved apart with same horizontal velocity $V = 1\ m/s$ in opposite directions (see figure).

(a) Find the normal force applied by each of the blocks on the sphere at the instant separation between the blocks is $a = \sqrt{2}R$; $R = 1.0\ m$ being the radius of the ball.

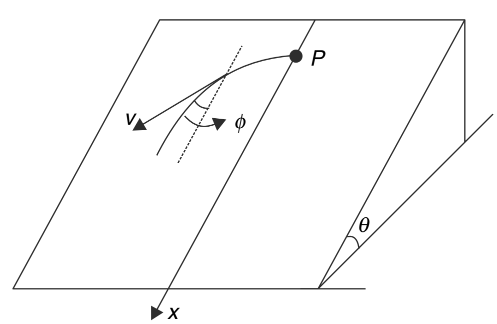


(b) How much force must be applied on each of the two blocks (when $a = \sqrt{2}R$) so that they do not have any acceleration. Assume that the he horizontal surface is smooth.

Q. 141. In the figure all pulleys ($P1$, $P2$, $P3$ …….) are massless and all the blocks (1,2,3 …..) are identical, each having mass m. The system consist of infinite number of pulleys and blocks. Strings are light and inextensible and horizontal surfaces are smooth. Pulley $P1$ is moved to left with a constant acceleration of a_0. Find the acceleration of block1. Assume the strings to remain horizontal.



Q. 142. A small disc P is placed on an inclined plane forming an angle θ with the horizontal and imparted an initial velocity v_0. Find how the velocity of disc depends on the angle ϕ which its velocity vector makes with the x axis (see figure). The coefficient of friction is $\mu = \tan \theta$ and initially $\phi_0 = \dfrac{\pi}{2}$.



ANSWERS

1. (a) straight line
(b) Parabolic

2. 20 m

3. $\sqrt{\frac{4H}{3g}}$

4. $N = 12$; Tension $= \frac{F}{N} = \frac{F}{12}$

5. 16 N

6. (i) (a) True
(b) True
(ii) $\frac{g}{5}$
(iii) $\sqrt{\frac{2\eta h}{(\eta-1)g}}$

7. $t = \sqrt{\frac{2(M-m)L}{Mg}}$

8. (a) $a = \frac{g}{4}$
(b) $a = \frac{g}{5}$

9. $\frac{4Mmg}{M+m} + M_0 g$

10. $m_0 = \frac{4m_1m_2}{m_1+m_2}$; $M = \frac{8m_1m_2}{m_1+m_2}$
All masses will fall down with acceleration g

11. (a) More than 9/2 Mg
(b) Tension in $S2 = Mg/2$, Tension in $S1 = 5\ Mg$
(c) Tension in $S2$ = Mg/6

12. $\theta = 45°$, $g/2$

13. 73.1 N

14. 12.5 s

15. $T = \sqrt{2}\,Mg$; $N = \frac{Mg}{\sqrt{2}}$

16. Zero.

17. $\mu_{min} = 1$

18. (a) 6467 N
(b) 22400 N
(c) 190400 N
(d) T_0 and F do not change. T will increase.

19. (a) No
(b) 1 : 4

20. $2\sqrt{\frac{2h}{g}}$

21. $a_A = a_B = \frac{F}{2m}$; $a_c = 0$

22. $3M/5$

23. $\theta = 62.5°$

24. $R_A : R_B : R_C = 3 : 1 : 2$

25. $K = 2$ Sdg

26. (a) $\frac{5}{3} cms^{-1}$
(b) 6 N

27. (a) 2, 4
(b) In both cases acceleration of the frame must be 'g'.

28. 15 m

29. $\frac{6t}{\sqrt{36-\pi}}$

30. $\mu \le \frac{v_0^2}{gL}$

31. 4 kg

32. 4.8 kg

33. $\sqrt{174}\ N$

34. $1.5\ kg \le m \le 9.5\ kg$

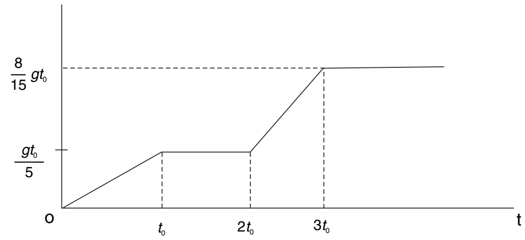
35. $\frac{5F}{m}$

36. (a) At C
(b) At C
(c) $\sqrt{2}\ m/s^2$ and $\sqrt{2}\ m/s^2$

37. (i) $\sqrt{15} < \omega \sqrt{16.67}$ rad/s
(ii) 500 $rad\ s^{-1}$

39. (a) $\frac{8gt_0}{15}$

(b)



40. (a) 80 N

(b) $\frac{640}{9}N$ for both

41. (a) $\frac{m_A}{m_B}=\frac{1}{3}$

(b) $X_{\min} = 0.75\ m$

(c) $V_A'=\left(\frac{3}{2}+\frac{3}{\sqrt{2}}\right)ms^{-1};V_B'=\left(\frac{3}{2}-\frac{1}{\sqrt{2}}\right)ms^{-1}$

42. $V_0=\frac{Mg}{\mu}\left[4\ell n\left(\frac{4}{3}\right)-1\right]$

43. t_2

44. $N_{12}=\frac{Mg\cos^2\theta}{1+2\sin^2\theta}$

$a=\frac{3g\sin\theta}{1+2\sin^2\theta}$

45. 1.0 s, 1.0 m

46. (b) $T_P = 21.65\ N$

(c) 3.05 kg

47. $\frac{\sqrt{5-2\sqrt{2}}}{\sqrt{2}-1}$

48. $t=\frac{1}{\sin\theta\sin\phi}\sqrt{\frac{2d}{g}}$

49. $\frac{5Mg}{2K}$

50. 2.9 s

51. (a) Force between the wall and the middle ball is maximum. It is 4 mg

(b) Force between upper ball and wall is least. It is $\frac{4}{3}$ mg.

52. $\mu_{\min}=\frac{\sin\theta}{\sqrt{\cos^2\theta+\tan^2\theta}}$

53. (a) $K = 2.5\ N/cm$

(b) No

54. (a) The block is at height $h = 2.5\ m$

(b) $V = 5\sqrt{2}\ m/s$

(c) 25 m/s^2 (↑)

55. Zero

56. (a) $\frac{2F}{K}$

(b) $\frac{F}{K}$

57. With pulley P_1 having zero mass, equilibrium is not possible

58. $\frac{l}{3}$

59. (a) M

(b) $\frac{g}{3}$

60. (a) $\theta_0=\frac{\pi}{4}$

(b) $a_{\max}=g\left[\cos\left(\frac{\pi}{8}\right)-\sin\left(\frac{\pi}{8}\right)\right]$

61. (a) $t > t^1$

(b) $\theta=\tan^{-1}\left(\frac{1}{12}\right)$

62. $a_0 \cos\theta$

63. $t=\sqrt{\frac{10}{3}}s$

64. (a) $g\sin\theta$

(b) $f=\frac{1}{2}mg\sin 2\theta$

$N = mg\cos^2\theta$

(c) $t=\sqrt{\frac{2h\left(M+m\sin^2\theta\right)}{\left(M+m\right)g\sin^2\theta}}$

65. (a) $\overrightarrow{a_A} \cdot \overrightarrow{a_B} = 0$ immediately after release

(b) $\vec{a}_A = \frac{g}{2}(\leftarrow)$

66. (a) θ

(b) $\tan^{-1}\left(\frac{\sin\theta \cdot \cos\theta}{2 - \sin^2\theta}\right)$

67. (a) $2\,mg$

(b) $t = \sqrt{\frac{3L}{\sqrt{2}g}}$

68. $\frac{g(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)} \le a \le \frac{g(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$

69. $\frac{m}{M} = \frac{20}{3\sqrt{3} - 4} = 16.7$

70. (a) $(M + m)\, g \sin\alpha$

(b) $\frac{(M+m)g\sin^2\alpha}{m + M\sin^2\alpha}$

71. $\frac{M}{m} = \frac{1}{5}$

72. $a_0 = \frac{48g}{199}$

73. $\frac{44g}{205}$

74. $\frac{2L}{5}$

75. (a) $\left|\frac{d^2x_2}{dt^2}\right| = \frac{3g}{2}$

(b) $\left|\frac{d^2x_1}{dt^2}\right| = 2g;\qquad \left|\frac{d^2x_2}{dt^2}\right| = 2g$

(c) $\left|\frac{d^2x_1}{dt^2}\right| = \frac{g}{2};\qquad \left|\frac{d^2x_2}{dt^2}\right| = \frac{3g}{2}$

76. (a) $\frac{4k_1k_2x_0}{(k_1 + k_2)M}$

(b) $\frac{4k_1k_2x_0}{k_1 + k_2}$

77. $a = \frac{g}{\sqrt{2}}$; $N_{AB} = 0$

78. 3

79. $\frac{6g}{47}$

80. $\sqrt{2}\,F_0$

81. $T = 0.49\ N$

82. (a) Zero

(b) $\frac{F_1}{3m} = \frac{F_2}{m}$

(c) To right

83. (a) $\theta \le 13°$

(b) 37°

84. (a) $\frac{13}{7}s$

(b) 0.18

85. $V_{\max} = \frac{-u^2}{2} + \sqrt{\frac{u^4}{4} + (2\mu gL)^2}$

86. $k = 130\ Nm^{-1}$; $\mu = 0.5$

87. $\mu = \left(\frac{3a_0 - 4g}{4a_0 + 3g}\right)$

88. $\left(\frac{r}{R}\right)_{\min} = \frac{\sqrt{1+\mu^2} - \mu}{\sqrt{1+\mu^2} + \mu}$

89. $F = \frac{Mg}{1-\mu^2}\left[\mu + \frac{\mu\cos\theta + \sin\theta}{\cos\theta - \mu\sin\theta}\right]$

90. $5\ m/s^2 \le a \le 7\ m/s^2$

91. $2\ kg \le m \le 30\ kg$

92. (a) $\frac{960}{95}kg$

(b) $\frac{480}{61}kg$

93. $\mu_{\min} = \frac{\Delta m}{2m}\tan\theta$

94. (i) 2.5 *kg*; 12.5 *N*

(ii) $\dfrac{50}{3}N$

(iii) $\dfrac{40}{3}N, a = \dfrac{5}{3}\ m/s^2$

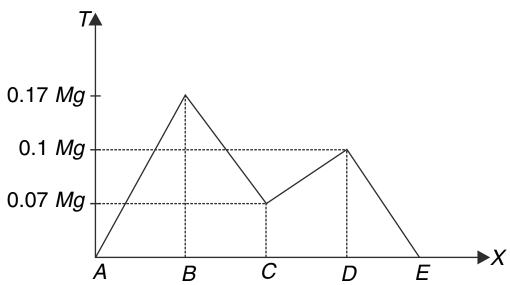
(iv) $\dfrac{5}{6}m/s^2$

95. $M_0 = \lambda R\left(\dfrac{\pi}{2} - 1\right)$

96. $\dfrac{H}{x}$

97. (a) $\mu = \dfrac{\sqrt{3}+1}{\sqrt{3}+5} = 0.4$

(b)



(c) $T_{max} = 0.17\ Mg$

98. (i) (a) $T_{BC} = 10\ N$; $T_{AB} = 12\ N$; $T_{AD} = 7\ N$

(b) $f_C = 0$; $f_B = 4N$; $f_A = 5N$; $f_D = 5N$

(ii) (a) $T_1 = \left(\dfrac{1-\mu_0}{1+\mu_0}\right) T_0$

(b) $m_2 = \left(\dfrac{1-\mu_0}{1+\mu_0}\right)^2 M_0$

99. $\omega = \sqrt{\dfrac{g}{R}}$

100. $\omega = \sqrt{\dfrac{g}{R}(\cos\theta - 2\sin\theta)}$

101. $\dfrac{\sqrt{3}mg}{3m+4M}$

102. $x = \dfrac{l_0}{3\eta - 1}$

103. 2.45

104. (a) $\theta_0 = \tan^{-1}\left(\dfrac{\dfrac{V_0^2}{Rg} - \mu}{1 + \dfrac{\mu V_0^2}{Rg}}\right)$

(b) $V_1 = \sqrt{rg\tan\theta_0}$

(c) Obtuse

105. (a) 22°

(b) (i) 2315 *N*, 1389 *N*

106. (a) $t_0 = \dfrac{\left[R^2\left(\mu^2 g^2 - f^2\right)\right]^{1/4} - V_0}{f}$

(b) $m\sqrt{\dfrac{(V_0 + ft)^4}{R^2} + f^2}$

107. (a) 93.3 N

(b) $\dfrac{10}{7}\ m/s^2$

(c) 4 *m/s*

108. $\dfrac{F}{5m}$

109. (a) contact force = 0, acceleration of 4 *kg* block is 0.7 m/s^2 and that of other block is zero

(b) contact force = 1.4 *N,* acceleration of both = 0

(c) Contact force = 5.74 *N,* acceleration of both = 1.27 m/s^2

110. $\theta = \cos^{-1}\left(\dfrac{3}{\sqrt{41}}\right)$

111. 30.1 *s*

112. (a) $\sqrt{\dfrac{g}{R}\dfrac{2\left(\sqrt{3}-1\right)}{\sqrt{3}\left(\sqrt{3}+1\right)}}$

(b) $\sqrt{\dfrac{g}{R}}$

113. $\dfrac{2}{\sqrt{3}}$

114. (a) $F_0 = \dfrac{\sigma\pi R^2 g\left(1-\cos\theta\right)^2}{2\sin\theta.\cos\theta}$

(b) $F_0 = 0$

(c) ∞

115. $R = r(1 + 2\sqrt{7})$

116. $\frac{5l_0}{4}$

117. (a) $a_1 = g\left(1 - \frac{4M}{m_1}\right)$

(b) $m_4 > \frac{18}{11} kg$

118. $F_0 = 2\mu m_2 g \left(\frac{m_2 + m_1}{2m_2 + m_1}\right)$; increase

119. Zero

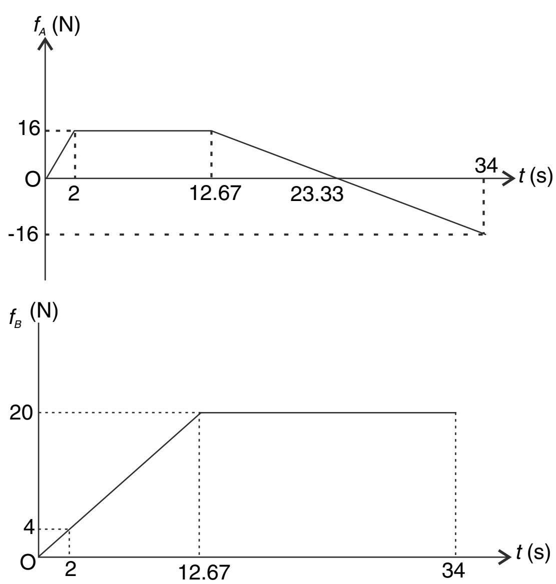
120. (a) $\theta = 45°$ from vertical diameter.

(b) $T_{max} = 2\left(\frac{\sqrt{2} - 1}{\pi}\right)Mg$

121. Zero

122. $21° < \theta < 33°$

123.



124. $\theta = 2\tan^{-1}\left[\left(1 + \frac{mg}{F}\right)\cot\alpha\right]$

125. (a) $T_0 = 2\lambda Rg$

(b) At the lowest point

126. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

127. $F_{min} = \frac{mg}{2\sqrt{2}}\left(\sqrt{3} - 1\right)$

128. (a) $F_{max} = \frac{3}{4}\mu Mg$

(b) Between A and B

(c) 2, No

129. (a) $2\ ms^{-1}$

(b) $2.1\ kg$

130. (a) $T = N\cos\theta$

(b) $\frac{mg}{M\tan\theta + m\cot\theta}$

131. (a) $t = \frac{R}{\mu u}\left[e^{\pi\mu} - 1\right]$

(b) $V = \frac{u}{e^{\pi\mu}}$

132. (a) $mg\cos\theta\sqrt{1 + \left(\frac{v^2\cos\theta}{Rg}\right)^2}$

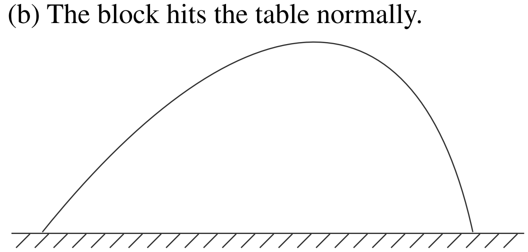
(b) $\frac{\tan\theta}{\sqrt{1 + \left(\frac{v^2\cos\theta}{Rg}\right)^2}}$ where $\tan\theta = \frac{\rho}{2\pi R}$

133. (a) $\vec{a}_{wedge} = \frac{g}{3}\hat{i}$

$a_{x\ block} = \frac{-g}{3}$

$a_{y\ block} = \frac{2g}{3}$

(b) The block hits the table normally.



(c) $\frac{3u^2}{16g}$

134. $\omega \geq \sqrt{\frac{g\sqrt{1 + \mu^2}}{R\mu}}$

135. (a) $\omega > \sqrt{\frac{g}{l}}$

(b) $\sqrt{\frac{g}{l}} < \omega \le \sqrt{\frac{2g}{l}}$

(c) $\sqrt{\frac{6g}{l}}$

(d) parabolic

136. (a) $\sqrt{\frac{g}{\sqrt{3}}}$

(b) $\sqrt{\sqrt{3}g}$

137. (a) $[g^2 R^2 (\mu^2 \cos^2 \theta - \sin^2 \theta)]^{1/4}$

(b) $\sqrt{gR(\mu \cos\theta - \sin\theta)}$

138. $\theta = \tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

139. (a) Block with mass M will move up.

(b) yes

(c) $g(1 - \cos\theta)$

140. (a) $(10\sqrt{2} - 8)\,N$

(b) $(5 - 4\sqrt{2})N$

141. $\frac{3a_0}{2}$

142. $\frac{v_0}{1+\cos\phi}$

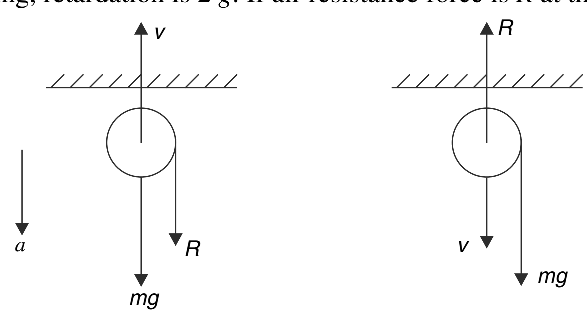
SOLUTIONS

1. (a) Initial velocity is parallel to $\vec{F}$ or anti parallel to $\vec{F}$. Hence particle moves in a straight line and speed may increase or decrease.

(b) Path is parabolic with speed increasing.

In case (a) the particle may retrace its path.

2. Just before striking the ceiling, retardation is 2 g. If air resistance force is R at this instant, then



$$ma = mg + R$$
$$m(2g) = mg + R$$
$$\Rightarrow R = mg$$

After impact, the air resistance force will be upward but its magnitude will remain mg. This is because speed has not changed.

$\therefore$ After impact net force on the ball = 0

$\therefore$ Ball will fall down with constant speed

$\therefore H = (10\ m/s)(2\ s) = 20\ m.$

3. Attraction of the earth will produce an acceleration of g in the body. According to Newton's third law the falling body applies equal and opposite force on the earth. This will produce an acceleration of $\frac{g}{2}$ in the earth since mass of the earth is twice that of the body.

Relative acceleration of approach = $\frac{3g}{2}$

$\therefore t = \sqrt{\frac{2H}{3g/2}} = \sqrt{\frac{4H}{3g}}$

4. Acceleration $a = \frac{F}{Nm}$

Where m = mass of each cart.

Let, T_1 = tension between 4th and 5th cart

Considering motion of last $(N-4)$ carts we get $(N-4)\,ma = T_1$ ….. (1)

Similarly, tension between 8th and 9th cart (T_2) can be written considering the motion of last $(N-8)$ carts

$(N-8)\,ma = T_2$ ……… (2)

As per the question $T_1 = 2T_2$

$(N-4)\,ma = 2\,(N-8)\,ma$

$N-4 = 2N-16$

$N = 12$

Tension in the last string $T = ma = \frac{F}{N}$

5. Acceleration of the entire system is

$a = \frac{F}{m_A + m_B + m_C + m_D + m_{cart}} = \frac{40}{2+2+1+1+4}$

Why does the cart accelerate? It is because of the force that D applies on it.

$\therefore N_D = m_{cart}\,.\,a = 4 \times 4 = 16\,N$

6. (i) If $m_1 + m_2$ is accelerating down $(m_1 + m_2)g - T_1 = (m_1 + m_2)a$

$\therefore T_1 = (m_1 + m_2)\,(g-a)$ And $m_2g - T_2 = m_2a$

$\therefore T_2 = m_2\,(g-a)$ $\because T_1 > 3T_2$

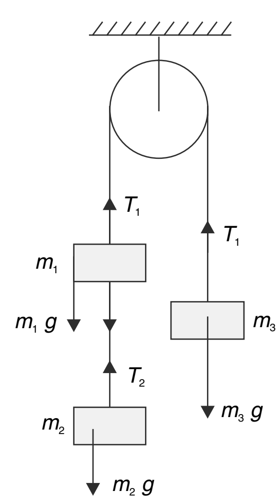
$\therefore (m_1 + m_2)\,(g-a) > 3\,m_2\,(g-a)$

$m_1 + m_2 > 3\,m_2$

$m_1 > 2\,m_2$

The same result is obtained even if the system is moving in opposite direction.

The ratio of tension T_1 & T_2 is independent of mass m_3.



(ii) Let the heavier mass be x.

The other mass = $M - x$

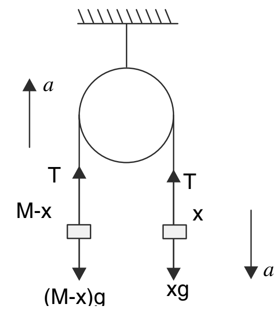
$xg - T = xa$ ....(i)

$T - (M - x)\, g = (M - x)\, a$ ....(ii)

Eliminating x between the two equations we get

$$T - mg + \frac{T}{(g-a)} = Ma - \frac{Ta}{g-a} \Rightarrow T\left[1 + \frac{g}{g-a} + \frac{a}{g-a}\right] = Ma + Mg$$

$$\therefore T = \frac{M\left(g^2 - a^2\right)}{2g}$$



Smaller 'a' means higher T. For $a = 0$, $T = \dfrac{Mg}{2}$

Which is higher than permissible limit.

If a increases above zero, T decreases.

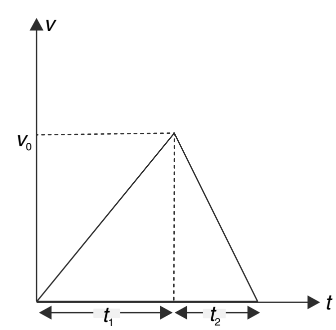
T must decrease to $\left(\dfrac{24}{50}mg\right)$ or below it .

$$T \le \frac{24}{50}Mg$$

$$\frac{M\left(g^2 - a^2\right)}{2g} \le \frac{24Mg}{50} \Rightarrow 1 - \frac{a^2}{g^2} \le \frac{24}{25}$$

$$\Rightarrow \frac{a^2}{g^2} \ge \frac{1}{25} \Rightarrow a \ge \frac{g}{5}$$

(iii) The load will travel through a distance h (rest to rest) in minimum time if it moves with maximum possible acceleration (a_1) and then retards with maximum possible retardation (a_2). Obviously,



$$a_1 = \frac{(\eta - 1)w}{M} = (\eta - 1)g$$

And $a_2 = g$ when tension in the rope = 0.

The $v - t$ graph for the situation has been shown in the figure.

$v_0 = a_1 t_1 = a_2 t_2$

$\Rightarrow (\eta - 1)\, g t_1 = g t_2 \Rightarrow (\eta - 1) t_1 = t_2$

Area under $v - t$ graph = h

$$\therefore \frac{1}{2}\,(t_1 + t_2)\, v_0 = h$$

$$\frac{1}{2}\,[t_1 + (\eta - 1)t_1]\,(\eta - 1)\, g t_1 = h$$

$$\Rightarrow t_1^2 = \frac{2h}{g\eta(\eta - 1)} \Rightarrow t_1 = \sqrt{\frac{2h}{g\eta(\eta - 1)}}$$

$$t_{\min} = t_1 + t_2 = t_1 + (\eta - 1)t_1 = \eta t_1$$

$$= \sqrt{\frac{2\eta h}{g(\eta - 1)}}$$

7. Mass of A; $m_A = M$

Mass of B; $m_B = M - m$

Acceleration of B after the insect falls is

$$a_B = \left(\frac{m_A - m_B}{m_A + m_B}\right) g(\uparrow)$$

$$= \frac{mg}{2M - m}$$

Acceleration of the insect = $g\ (\downarrow)$

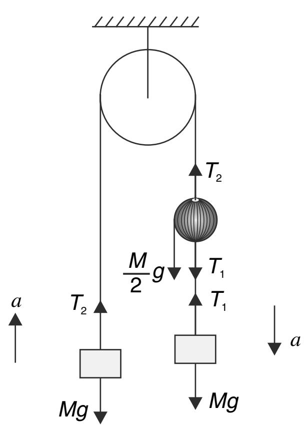
The two objects separate with a relative acceleration of $a = g + \dfrac{mg}{2M - m} = \dfrac{2Mg}{2M - m}$

$$\therefore \frac{1}{2} a t^2 = L$$

$$\left(\frac{Mg}{2M - m}\right) t^2 = L$$

$$t = \sqrt{\frac{(2M - m)L}{Mg}}$$

8. (a) The friction force between the bead and the string causes the tension to change in the string on the two sides of the bead.



For the bead $T_2 = T_1 + \dfrac{Mg}{2}$ .....(1)

For the blocks $T_2 - Mg = Ma$ .....(2)

$Mg - T_1 = Ma$ .....(3)

(2) + (3) $T_2 - T_1 = 2Ma$

Using (1) $\dfrac{Mg}{2} = 2Ma \Rightarrow a = \dfrac{g}{4}$

(b) In this case acceleration of the bead is same as acceleration of the two blocks.

In above solution equation (1) changes to

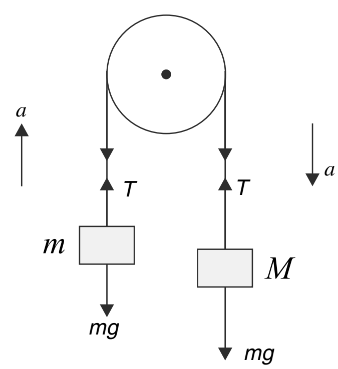
$$\frac{Mg}{2} + T_1 - T_2 = \frac{M}{2}a$$

The other two equations remain same.

Solving the three equations we get

$$a = \frac{g}{5}$$

9.



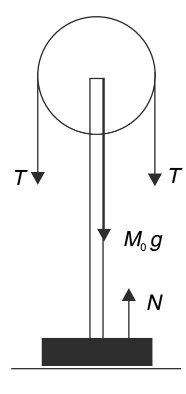
For motion of the two masses we can write :

$Mg - T = Ma$.....(1)

$T - mg = ma$..... (2)

Solving $T = \dfrac{2Mmg}{M+m}$

Now we consider the equilibrium of the stand and pulley system



N = normal force between the stand and the scale = Reading of the scale $N = 2T + M_0 g$

$$N = \frac{4Mmg}{M+m} + M_0 g$$

10 Tension in string connecting m_1 and m_2 is

$$T = \frac{2m_1 m_2}{m_1 + m_2} g$$

$\therefore\ m_0 g = 2T$ [for equilibrium of m_0]

$$\Rightarrow \boxed{m_0 = \frac{4m_1 m_2}{m_1 + m_2}}$$

For equilibrium of M

$Mg = 4T$

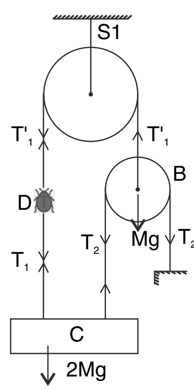
$$\boxed{M = \frac{8m_1 m_2}{m_1 + m_2}}$$

11. (a) Weight of the system = 9/2 Mg.

The support of the string S2 applies a downward force (T_2)

$\therefore$ Tension in S1 is $\frac{9}{2}Mg + T_2$

(b)



For pulley B, $2T_2 + Mg = T_1'$ ....(1)

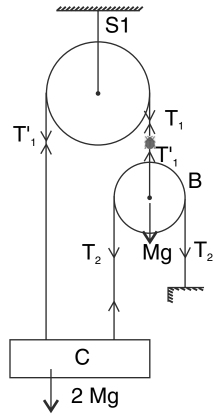
For insect $T_1' = \frac{M}{2}g + T_1$ ....(2)

For C $T_1 + T_2 = 2\,Mg$ ....(3)

Solving $T_2 = \frac{Mg}{2}$

Tension in S1 $= \frac{9}{2}Mg + T_2 = 5Mg$

(c)



The equations are

$2T_2 + Mg = T'_1$ ....(1)

$T'_1 + \dfrac{M}{2} g = T_1$ ....(2)

$T_1 + T_2 = 2Mg$ ....(3)

(1) + (2) + (3)

$$3T_2 = \frac{Mg}{2}$$

$$T_2 = \frac{Mg}{6}$$

12. $a = g \sin\theta$

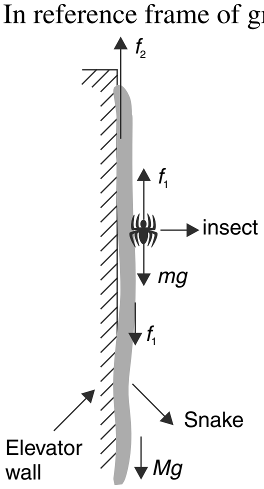
$a_x = g \sin\theta \cos\theta = \dfrac{1}{2} g \sin 2\theta$

$\therefore\ (a_x)_{\max} = \dfrac{g}{2}$

When $2\theta = 90°$

$\theta = 45°$

13. In reference frame of ground acceleration of snake is 8 m/s^2 (↑) and that of the insect is 12 m/s^2 (↑).



For insect

$f_1 - mg = m(12)$

[f_1 = friction between insect and snake]

$\therefore\ f_1 = \left(\frac{50}{1000} kg\right) \times 22 = 1.1N$

For snake

$f_2 - Mg - f_1 = M\ (8)$

[f_2 = friction between snake and elevator]

$f_2 = 4 \times 18 + 1.1 = 73.1N$

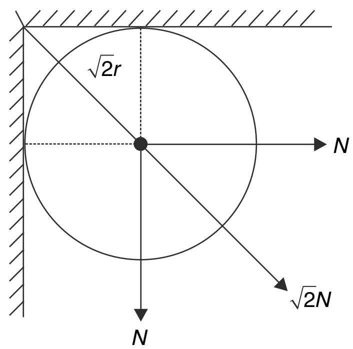
14. $kV_T^2 = mg$

$\therefore\ \frac{V_{T1}}{V_{T2}} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\Rightarrow \frac{h/25}{h/t} = \frac{1}{2}$ [h = height from which bodies have been dropped]

$\Rightarrow t = 12.5\ s$

15. The thread makes an angle of $\sin^{-1}\left(\frac{\sqrt{2}\,r}{2\,r}\right) = 45°$ with the vertical.



If N is normal force by each wall on the cylinder

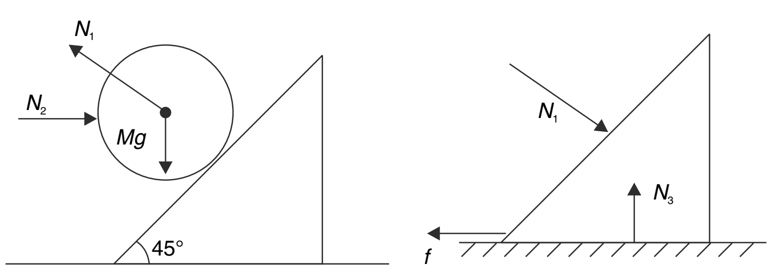
$\sqrt{2}N = \frac{T}{\sqrt{2}}$

And $Mg = \frac{T}{\sqrt{2}}$

$\Rightarrow T = \sqrt{2}Mg$

And $N = \frac{Mg}{\sqrt{2}}$

17.



For Ball $\frac{N_1}{\sqrt{2}} = N_2$ And $\frac{N_1}{\sqrt{2}} = Mg \Rightarrow N_1 = \sqrt{2}\,Mg$

For Wedge $N_3 = \frac{N_1}{\sqrt{2}} = Mg$

$\therefore$ Wedge will not move if

$$f_{\max} \geq \frac{N_1}{\sqrt{2}}$$

$$\mu Mg \geq \frac{\sqrt{2}Mg}{\sqrt{2}} \Rightarrow \mu \geq 1$$

18. Considering the helicopter + Box as our system:

$F - (M + m)g = (M + m)a$

$\therefore F = (M + m)(g + a)$

$= 17000 \times 11.2 = 190400\ N$

For helicopter alone

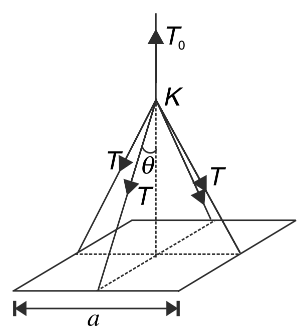
$F - Mg - T_0 = Ma$

$T_0 = F - Mg - Ma$

$= 190400 - 15000 \times 11.2$

$= 22400\ N$

And $T_0 = 4T \cos\theta$ ...(1)



$$T_0 = 4T\frac{\sqrt{a^2 - \left(\frac{a}{2}\right)^2}}{a}$$

$T_0 = 2\sqrt{3}T$

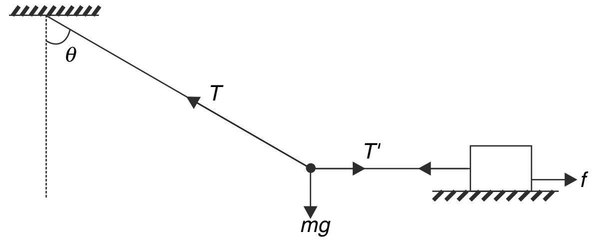
$\therefore T = \frac{22400}{2\sqrt{3}} = 6467\ N$

If the strings are fixed at $PQRS$, value of $\cos\theta$ in equation (1) will decrease.

$\therefore T$ will increase

T_0 and F will not change

19. (a)



$T \cos \theta = mg$ .... (1)

and $T \sin \theta = T'$ .... (2)

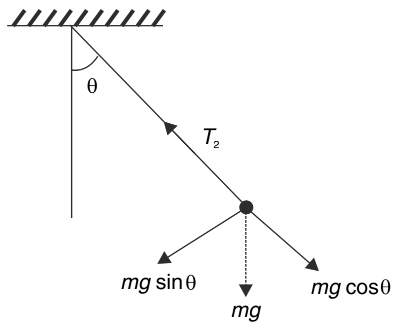
[$T' = f_{max}$ when the block is about to slide]

When θ increases, T will increase (equation.1)

From (2) T' will increase and the block will slide.

(b) Before string is cut tension in wire is [equation … (1)]

$$T_1 = \frac{mg}{\cos 60°} = 2\, mg$$



After the string is cut, tension in wire is

$$T_2 = mg \cos 60° = \frac{mg}{2} \quad \therefore \frac{T_2}{T_1} = \frac{1}{4}$$

20. Acceleration of the two blocks

$$a = \frac{mg \sin 60° - mg \sin 30°}{3m} = \left(\frac{\sqrt{3}-1}{4}\right)g$$

If the two blocks move a distance x along respective inclines in time t

$$x = \frac{1}{2}at^2$$

The centre of two blocks will be at equal height if

$x \sin 60° + x \sin 30° = h$

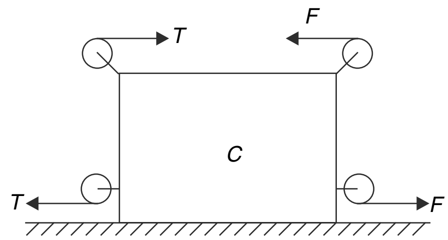
$$\therefore\ x\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = h$$

$$\frac{1}{2}\left(\frac{\sqrt{3}-1}{4}\right)gt^2\left(\frac{\sqrt{3}+1}{4}\right) = h$$

$$\therefore\ t^2 = \frac{16h}{(\sqrt{3}-1)(\sqrt{3}+1)g} = \frac{8h}{g}$$

$$\therefore t = 2\sqrt{2}\sqrt{\frac{h}{g}}$$

21. Block 'C' will not move as net horizontal force on it is zero.



Horizontal forces on C have been shown in the fig.

$\therefore$ Accelerations of A and B are same equal to

$$a = \frac{F}{2m}$$

22. Hint: When m is maximum possible take limiting friction to be down the incline and when m is minimum take the limiting friction to be up the incline.

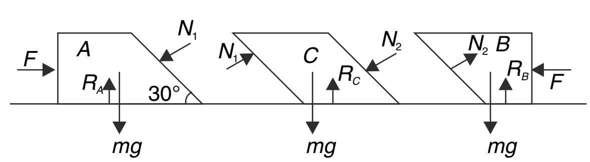
23. $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2l\sec\theta}{g(\sin\theta - \mu\cos\theta)}}$

$$= \sqrt{\frac{2l}{g(\sin\theta\cos\theta - \mu\cos^2\theta)}}$$

This is minimum when the $z = \sin\theta\cos\theta - \mu\cos^2\theta$ is maximum.

$\frac{dz}{d\theta} = 0$ gives $\theta = 62.5°$

24.



For A: $F = N_1 \sin 30° = \frac{N_1}{2}$ … (1)

And $R_A = mg + N_1 \cos 30° = mg + 2\left(\frac{mg}{2\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{2}$

$$= \frac{3}{2}mg \ \ldots (2)$$

For B: $F = N_2 \sin 30° = \frac{N_2}{2}$

$\Rightarrow N_2 = 2F = \frac{mg}{\sqrt{3}}$ …(3)

And $R_B = mg - N_2 \cos 30° = mg - \frac{mg}{\sqrt{3}}.\frac{\sqrt{3}}{2} = \frac{mg}{2}$ ....(4)

For C: $N_1 \sin 30° = N_2 \sin 30°$

$\Rightarrow N_1 = N_2 = \frac{mg}{\sqrt{3}}$ [from (3)] ....(5)

And $R_C + N_1 \cos 30° = N_2 \cos 30° + mg$

$R_C = mg$ ....(6)

$\therefore R_A : R_B : R_C = \frac{3}{2} : \frac{1}{2} : 1 = 3 : 1 : 2$

25. Mass of water added when the level of water rises by x (wrt container)

$m = 2Sxd$

The spring stretches by x so that the level of water does not change wrt ground.

$\therefore Kx = 2Sxdg$

$\therefore K = 2Sdg$

26. (a) Displacement (and hence speed) of each point on the spring will be proportional to its distance from the wall.

$\therefore$ Speed of the particle. $V = 5 \times \frac{1}{3} = \frac{5}{3} cm/s$

(b) The free end moves by 10 cm in 2.0 s

$\therefore$ Tension in the spring = $Kx = 0.6 \times 10 = 6\ N$

27. Hint : The resultant of mg and the pseudo force ma must be along the diagonal or they must add to zero.

28. $x = \frac{u^2}{2\mu g}$

For Car without ABS $20 = \frac{u^2}{2\mu_k . g}$

For Car with ABS $x = \frac{u^2}{2\mu_s g}$

$\therefore \frac{x}{20} = \frac{\mu_k}{\mu_s}$

$x = 20 \times 0.75 = 15\ m$

29. On flat road

$S = \frac{1}{2} at^2$

$S = \frac{1}{2} \mu g t^2$ [$\because a_{\max} = \mu g$]

$S = \frac{1}{2} g t^2$ $\because$ [$\mu = 1$]

On inclined road

$a' = g\ (\mu \cos \theta - \sin \theta)$

$= g\ (\cos \theta - \sin \theta)$

$= g\ (1 - \theta)$ [for small angle $\cos \theta \approx 1, \sin \theta \approx \theta$]

$\therefore\ S = \frac{1}{2} a' t'^2$

$\frac{1}{2} g t^2 = \frac{1}{2} g(1-\theta) t'^2 \Rightarrow \quad t' = \frac{t}{\sqrt{1-\frac{\pi}{36}}} \qquad \left[\because 5^\circ = \frac{\pi}{36} rad\right]$

$= \frac{6t}{\sqrt{36-\pi}}$

30. Time required to pull out the cloth is

$$t_0 = \frac{L}{v_0}$$

If we observe the motion of the block in the reference frame of the cloth, it has initial velocity v_0 ($\leftarrow$) and acceleration μg ($\rightarrow$).

Note that friction between the block and the cloth is kinetic friction since the cloth moves at speed v_0 but initial velocity of the block is zero.

In the reference frame of the cloth the displacement of the block (taking left as positive) should exceed $\frac{L}{2}$ at or before t_0.

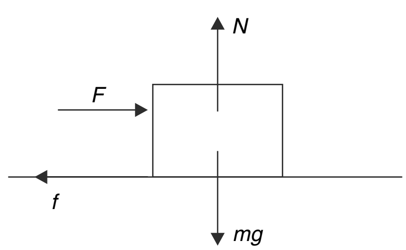
$$\therefore\ v_0 t_0 - \frac{1}{2}\mu g t_0^2 \geq \frac{L}{2}$$

$$\therefore\ L - \frac{1}{2}\mu g\left(\frac{L}{v_0}\right)^2 \geq \frac{L}{2}$$

$$\therefore\ \frac{L}{2} \geq \frac{1}{2}\mu g \frac{L^2}{v_0^2}$$

$$\frac{v_0^2}{gL} \geq \mu$$

31. Block starts to move when $F \geq 6N$



If means $f_{max} = 6N$

When $F = 18N$, acceleration is $3ms^{-2}$.

$\therefore\ F - f_{max} = ma$

[Once the motion starts the friction remains constant at $6N$]

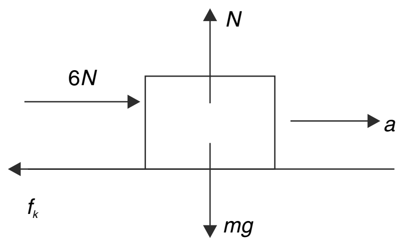
$\therefore\ 18 - 6 = m \times 3 \Rightarrow m = 4\ kg$

32. In this case $\mu_s > \mu_k$

Block does not accelerate till $F = 6\ N$.

$\therefore\ \mu_s N = 6 \Rightarrow \mu_s\ mg = 6N$

Once F becomes slightly larger than $6N$, the block has an acceleration $a = 0.5\ ms^{-2}$



$\therefore$ $6 - \mu_k . mg = m. (0.5)$ …....(1)

When $F = 18N$

$18 - \mu_k mg = m (3)$ …....(2)

Subtracting equation – (1) from (2) gives 12 = (2.5) (m)

$\therefore\ m = \dfrac{12}{2.5} = 4.8\ kg$

33. Acceleration of the platform

$$\vec{a}_P = \frac{d\vec{v}}{dt} = 2\hat{i} + \hat{j} + 3\hat{k}$$

Horizontal and vertical acceleration

$a_H = \sqrt{4+1} = \sqrt{5}\ m/s^2$

$a_v = 3\ m/s^2$

Normal force on the block

$N = m (g + a_v) = 1 \times 13 = 13N$

Maximum possible friction = μN

$= \mu m (g + a_v)$

Maximum acceleration that friction can provide

$a_{max} = \mu(g + a_v) = 0.2 \times 13 = 2.6\ m/s^2$

$\because\ a_{max} > a_H$

$\therefore$ Block will not slide on the platform.

$\therefore$ Value of friction force on the block

$f = ma_H = \sqrt{5}\ N$

$\therefore$ Force by the platform on the block is

$F = \sqrt{N^2 + f^2} = \sqrt{169 + 5} = \sqrt{174}\ N$

34. Component of weight of 10 kg block down the incline is = 10 g sin 37° = 60 N

Maximum friction on the block is μ10g cos 37° = 40 N

The block on the incline will not slide down until weight of the hanger + weight in it does not remain below 60 – 40 = 20 N. It means the additional mass in the hanger shall not be less than 1.5 kg.
The block on the incline will not slide up until weight of the hanger + weight in it does not exceed 60 + 40 = 100 N. It means the additional mass in the hanger should not exceed 9.5 kg.

35. For pulley $2F = ma$

For particle $F = ma'$

Hence, $a = 2a' = 2\ F/m$

Acceleration of the point of application of the force is = $2a + a' = 5a/2 = 5F/m$

36. (a) The inclination of friction force with the tangent will go on increasing because the tangential acceleration is constant but the radial acceleration goes on increasing

(b) Normal force (i.e., normal acceleration) of the particle is constant.

$$\therefore \frac{v^2}{R} = Const.$$

[v = *speed*

R = *radius of curvature of the path*]

R is increasing, hence v is increasing.

$\therefore$ Component of acceleration along the tangent must be in the direction of velocity. Correct representation is at C.

Representation B means speed is decreasing and A means speed is not changing.

(c) $\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{j}$

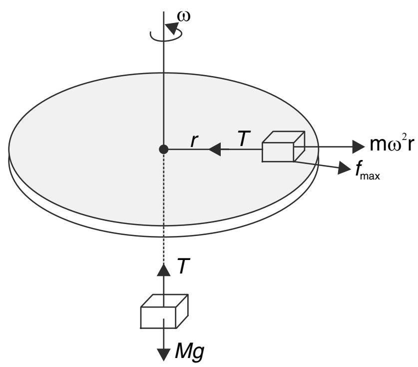
Rate of change of speed = tangential acceleration =

$$\frac{\vec{a}.\vec{v}}{v} = \frac{4t}{\sqrt{16+4t^2}}$$

At $t = 2s$, tangential acceleration $a_t = \sqrt{2}\ m/s^2$

$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{2^2 - \left(\sqrt{2}\right)^2} = \sqrt{2}\ ms^{-2}$$

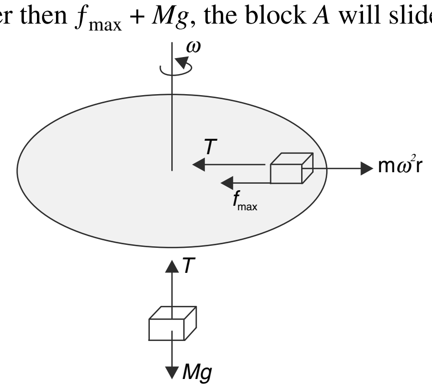
37. (i) If value of $m\omega^2 r$ (centrifugal force) + $f_{\max}$ is less than T (= Mg), block A will slide radially inward.



$\therefore m\,(\omega^2 r)_{\min} + f_{\max} = Mg$

$\therefore (\omega^2 r)_{\min} = 20 - 0.5 \times 10 = 15N$ ....(1)

Also, if value of $m\omega^2 r$ is higher then $f_{\max} + Mg$, the block A will slide radially outward.



$\therefore\ m\,(\omega^2 r)_{max} = Mg + f_{max}$

$(\omega^2 r)_{max} = 20 + 5 = 25\ N$ ....(2)

If $r = 1.0\ m$

$\omega^2_{min} = 15$ from (1)

And $\omega^2_{max} = 25$ from (2)

If $r = 1.5m$

$$\omega^2_{\min} = \frac{15}{1.5} = 10$$

$$\omega^2_{\max} = \frac{25}{1.5} = 16.67$$

Because r can be anywhere between 1.0 and 1.5 m, hence to be absolutely sure that A does not slip it is required that

$15 < \omega^2 < 16.67$

$\sqrt{15} < \omega < \sqrt{16.67}$

(ii) Increase in length of the spring

$$\Delta l = (2\pi R) \times \frac{1}{100} = \frac{2 \times 3.14 \times 20}{100}\,cm$$

$= 1.26\ cm$

$\therefore$ Tension in the spring $T = k\Delta l$

$$= 100 \times \frac{1.26}{100} = 1.26N$$

If we consider a small element on the spring having angular width $\Delta\theta$, the centripetal force to its is provided by the tension.

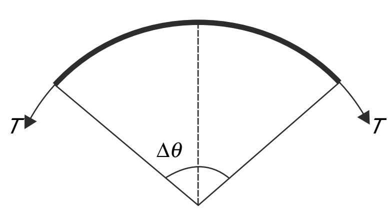
$$2T.\sin\frac{\Delta\theta}{2} = \lambda(R\Delta\theta)\omega^2.R$$

For $\Delta\theta \to 0$

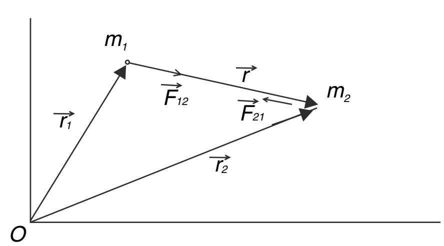
$$2T\,\frac{\Delta\theta}{2} = \lambda R^2 \Delta\theta.\omega^2$$

$$\omega^2 = \frac{T}{\lambda R^2} = \frac{1.26}{0.126 \times 10^{-3} \times (0.2)^2}$$

$$\Rightarrow \omega = \frac{10^2}{0.2} = 500\ \ rad\ s^{-1}$$



38.



Let $\overrightarrow{F_{12}}$ = force on m_1 due to m_2

$\overrightarrow{F_{21}}$ = force on m_2 due to m_1

$$m_1 \frac{d^2 \overrightarrow{r_1}}{dt^2} = \overrightarrow{F_{12}}$$

$$m_2 \frac{d^2 \overrightarrow{r_2}}{dt^2} = \overrightarrow{F_{21}}$$

$$\frac{d^2 \overrightarrow{r_2}}{dt^2} - \frac{d^2 \overrightarrow{r_1}}{dt^2} = \frac{\overrightarrow{F_{21}}}{m_2} - \frac{\overrightarrow{F_{12}}}{m_1}$$

$$\frac{d^2 \left(\overrightarrow{r_2} - \overrightarrow{r_1}\right)}{dt^2} = \frac{\overrightarrow{F_{21}}}{m_2} + \frac{\overrightarrow{F_{21}}}{m_1} \quad \left[\because \vec{F}_{12} = -\vec{F}_{21}\right]$$

$$\therefore \frac{d^2 \vec{r}}{dt^2} = \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\vec{F}_{21}$$

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$\therefore \mu \frac{d^2 \vec{r}}{dt^2} = \overrightarrow{F_{21}}$$

39. (a) Let mass of each block be m.

Acceleration after block 1 gets detached is

$$a_1 = \left(\frac{3m - 2m}{3m + 2m}\right) g$$

$$= \frac{g}{5}$$

Speed acquired in time t_0 $V_1 = 0 + a_1 t_0 = \frac{g t_0}{5}$

At this moment block 2 gets detached and the masses on two sides become equal. The speed remains constant for next interval t_0

Then block 3 falls. Acceleration becomes $a_2 = \left(\frac{2m - m}{2m + m}\right) g = \frac{g}{3}$

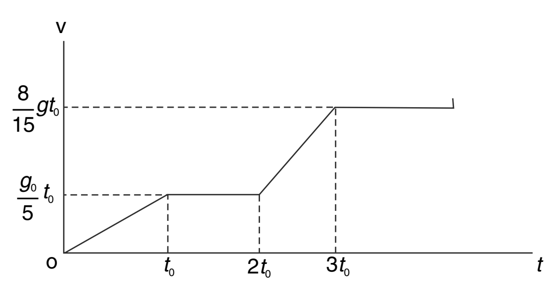
Speed at time t_0 after block 3 falls is

$$V_2 = V_1 + a_2 t_0 = \frac{g t_0}{5} + \frac{g t_0}{3} = \frac{8 g t_0}{15}$$

Now block 4 falls and once again the acceleration becomes zero.

$\therefore$ final speed for 5 and 6 is $V_2 = \frac{8 g t_0}{15}$

(b)



40. (a) Net force on B in downward direction = 20 N

Tension in string = 80 N

A will remain at rest and will feel his true weight.

(b) Hint: Let acceleration of A be 'a' in downward direction. Then acceleration of the rope where B is holding it will be 'a'. This means the acceleration of B is (4-a) in downward direction(rope is slipping from his hand). Write equation of motion for both the monkeys and solve.

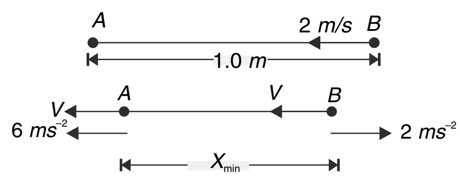
41. (a) From Newton's third law the particles exert equal and opposite force on one another.

$\therefore m_A a_A = m_B a_B$

$$\frac{m_A}{m_B} = \frac{a_B}{a_A} = \frac{2}{6} = \frac{1}{3}$$

(b) Till $x = x_0$, A does not move and B moves uniformly.

The two particles will be nearest when both have same velocity. Let this happen at a time t_1 after B reaches at a separation $x_0 = 1.0\ m$ to A.



$$V_A = V_B$$
$$0 + 6t_1 = 2 - 2t_1$$
$$\Rightarrow t_1 = \frac{1}{4} s$$

$$\therefore V_A = V_B = V \text{ (say) } = 6 \times \frac{1}{4} = \frac{3}{2} ms^{-1}$$

Let displacement of A and B be X_A and X_B by time t_1

$$X_A = \frac{1}{2} \times 6 \times t_1^2 = \frac{1}{2} \times 6 \times \left(\frac{1}{4}\right)^2 = \frac{3}{16} m$$

$$X_B = 2t_1 - \frac{1}{2} \times 2 \times t_1^2 = \frac{2}{4} - \left(\frac{1}{4}\right)^2 = \frac{7}{16}$$

$\therefore X_{\min} = 1.0 + X_A - X_B = 0.75$ m

(c) The particles continue to repel till distance between them becomes 1.0 m. Let this happen a time t after they attain minimum separation.

$$\left(\frac{3}{2}t + \frac{1}{2} \times 6 \times t^2\right) - \left(\frac{3}{2}t + \frac{1}{2} \times 2 \times t^2\right) = 0.25$$

$$\Rightarrow t = \frac{1}{\sqrt{8}}$$

$\therefore$ Final velocities are

$$V_A' = \frac{3}{2} + 6.\frac{1}{\sqrt{8}} = \left(\frac{3}{2} + \frac{3}{\sqrt{2}}\right) ms^{-1}$$

$$V_B' = \frac{3}{2} - 2.\frac{1}{\sqrt{8}} = \left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right) ms^{-1}$$

42. Time required to empty the bucket B

$$t_0 = \frac{M}{\mu}$$

Mass of B at time 't'

$$= 2M - \mu t$$

$\therefore$ Acceleration at time t is $a = \left(\frac{m_A - m_B}{m_A + m_B}\right)g$

$$= \frac{\left[2M - (2M - \mu t)\right]g}{2M + 2M - \mu t}$$

$$= \frac{\mu t}{4M - \mu t} g$$

$\therefore$ If v is the speed of the buckets at time 't'

$$\frac{dv}{dt} = \left(\frac{\mu t}{4M - \mu t}\right)g$$

$$\int_o^{v_o} dv = -\int_o^{t_o} g \frac{(4M - \mu t)}{4M - \mu t} dt + 4Mg\int_o^{t_o} \frac{dt}{4M - \mu}$$

$$V_o = -gt_o - \frac{4Mg}{\mu}\left[\ell n(4M - Mt)\right]_o^{t_o}$$

$$= -gt_o - \frac{4Mg}{\mu}\left[\ell n(4M - \mu t_o) - \ell n(4M)\right]$$

$\because \mu t_o = M$

$$\therefore V_0 = -gt_o - \frac{4Mg}{\mu}\left[\ell n(3M) - \ell n(4M)\right]$$

$$= -gt_o + \frac{4Mg}{\mu}\ell n\left(\frac{4}{3}\right)$$

$$= \frac{Mg}{\mu}\left[4\ell n\left(\frac{4}{3}\right) - 1\right] \quad \left[\because t_o = \frac{M}{\mu}\right]$$

43. Let the mass of the chain be m and its length be L

The mass of unit length of the chain= m/L

When a part of the chain of length x is hanging, the acceleration of the chain will equal to

$$a = \frac{(m/L)xg}{m} = \frac{gx}{L}$$

Initially, when $x = L/2$, the acceleration equals $g/2$, but then it increases.

After identical masses M are attached to the ends of the chain, then acceleration at the moment when a length x of the chain is hanging will be

$$a' = \frac{(m/L)xg + Mg}{m + 2M} = \frac{gx}{L}\left[\frac{(m) + \frac{ML}{x}}{m + 2M}\right]$$

To decide whether a is larger or a' let us assume

$a > a'$

$$1 > \left[\frac{(m) + \frac{ML}{x}}{m + 2M}\right] \qquad \Rightarrow m + 2M > m + \frac{ML}{x}$$

$\Rightarrow 2x > L$, which is true.

Hence, the chain without the balls will slip off faster.

44. Let the acceleration of wedge be 'a' down the incline.

Sum of masses of all rectangular blocks

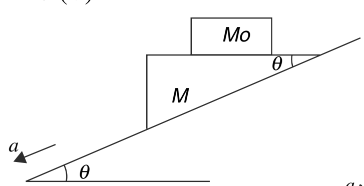
$$= M_o = M + \frac{M}{2} + \frac{M}{4} + \frac{M}{8} + \ldots\ldots\ldots\ldots\ldots$$

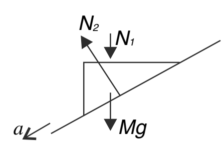
$$= M\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\ldots\ldots\right]$$

$$= M\frac{1}{\left(1 - \frac{1}{2}\right)} = 2M$$

For finding a we can treat the system as shown below.

Acceleration of M_o will be $a \sin\theta\,(\downarrow)$

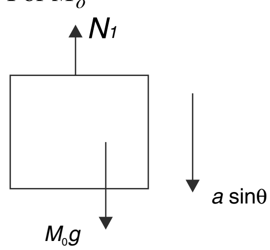




For wedge $N_2 = (N_1 + Mg)\cos\theta$ ....(1)

And $(N_1 + Mg)\sin\theta = Ma$ ....(2)

For M_o



$M_o\, g - N_1 = M_o\, a \sin\theta$

$2Mg - N_1 = 2\,M\,a \sin\theta$--------------(3)

Multiply (3) with $\sin\theta$ and add to (2)

$3Mg \sin\theta = Ma\,(1 + 2\sin^2\theta)$

$$\therefore a = \frac{3g\sin\theta}{1 + 2\sin^2\theta}$$

Acceleration of rectangular blocks

$$= a\sin\theta(\downarrow) = \frac{3g\sin^2\theta}{1 + 2\sin^2\theta}(\downarrow)$$

Mass of all rectangular blocks except 1 is M. Replace blocks 2, 3, 4, 5, ………… with a single block of mass M

Considering motion of this mass M,

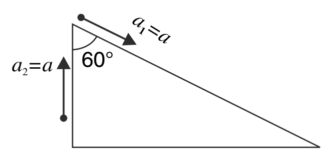
$$Mg - N_{12} = Ma \sin \theta$$

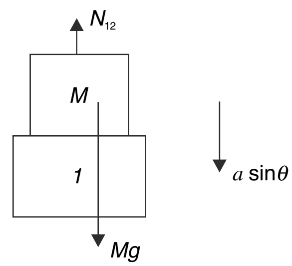
$$\therefore N_{12} = Mg - \frac{3Mg\sin^2\theta}{1+2\sin^2\theta}$$

$$= \frac{Mg\left[1-\sin^2\theta\right]}{\left[1+2\sin^2\theta\right]} = \frac{Mg\cos^2\theta}{1+2\sin^2\theta}$$

45. Acceleration of the system

$$a = \frac{m_1 g\sin 30° - m_2 g}{m_1 + m_2} = \frac{4\times 10\times\frac{1}{2} - 1\times 10}{4+1} = 2\ m/s^2$$



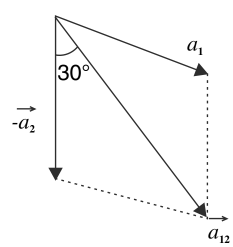


Acceleration of block 1 wrt particle 2 is

$$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2$$

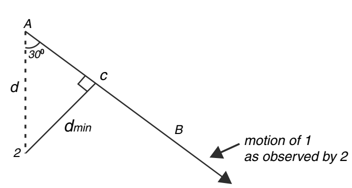
$$a_{12} = \sqrt{a^2 + a^2 + 2.a.a\cos 60°}$$

$$= \sqrt{3}\, a = 2\sqrt{3}\ m/s^2$$



$\vec{a}_{12}$ makes an angle of 30° with the vertical.

This means that to particle 2, block 1 appears to move in a direction making an angle of 30° to the vertical (Along line AB)



Drop perpendicular from 2 on to the line AB. Length of this perpendicular is the desired minimum distance.

$$\therefore d_{\min} = d\sin 30° = 2\times\frac{1}{2} = 1.0\ m$$

Time required to reach this position is calculated as

$$\frac{1}{2}a_{12}t^2 = (AC)$$

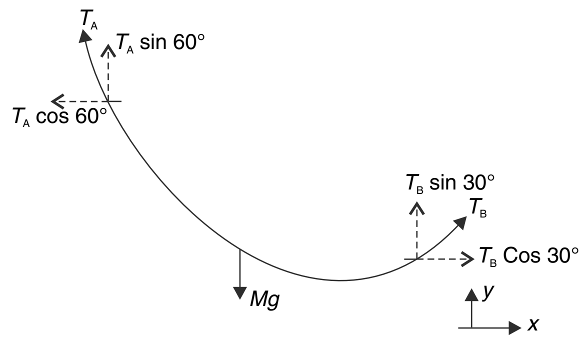
$$\frac{1}{2} \times 2\sqrt{3} \times t^2 = 2\cos 30°$$

$t^2 = 1$

$t = 1.0\ s.$

46. (a) If we consider the horizontal equilibrium of any segment of the chain, one can easily show that horizontal component of tension is same everywhere.

(b) Considering equilibrium of entire chain



$T_A \cos 60° = T_B \cos 30°$ = (X component of tension everywhere)

$\therefore\ T_A = \sqrt{3}\ T_B$ ---------------- (1)

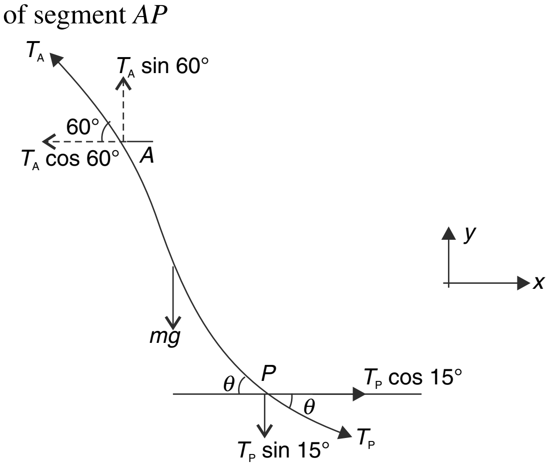
And $T_A \sin 60° + T_B \sin 30° = Mg$

$$\frac{\sqrt{3}}{2} T_A + \frac{T_A}{2\sqrt{3}} = \text{Mg [using (1)]}$$

$$\therefore T_A = \frac{\sqrt{3}Mg}{2}$$

And $T_B = \dfrac{Mg}{2}$

Considering equilibrium of segment AP



Horizontal component of tension must be same everywhere

$\therefore\ T_P \cos 15° = T_A \cos 60°$

$\therefore\ T_P \times 0.96 = \dfrac{\sqrt{3}}{4} Mg$ [$M = 4.8\ kg,\ g = 10\ m/s^2$]

$T_P = 21.65\ N$

(c) Considering vertical equilibrium of AP

$mg + T_P \sin 15° = T_A \sin 60°$

$$mg = \frac{3}{4}Mg - 21.65 \times \sin 15°$$

$$m = \frac{3}{4} \times 4.8 - \frac{21.65}{10} \times 0.25$$

$$= \frac{14.4 - 2.165}{4} = 3.05\ kg$$

47. Fig. (a) shows component of tension (acting on A) along horizontal and vertical. Fig. (b) shows forces along the line of greatest slope (XX') and perpendicular to the incline. In equilibrium

$T = mg$ [considering block B] -------(1)

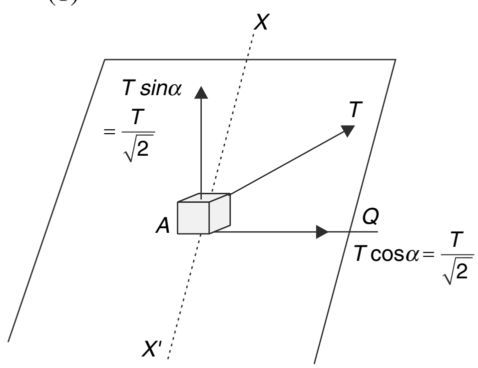


Fig (a)

$$N = \frac{Mg}{\sqrt{2}} - \frac{T}{2} = Mg\left[\frac{\sqrt{2}-1}{2}\right] \qquad \text{------------(2)}$$

Forget about the friction for a moment.

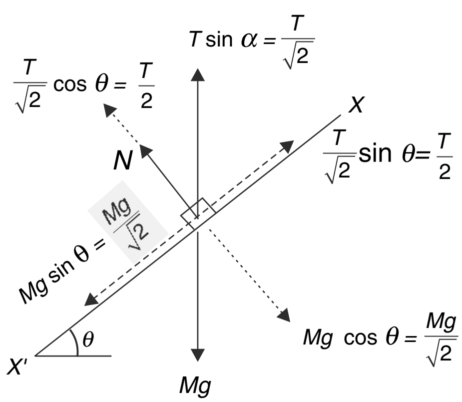


Fig (b)

Unbalanced force along XX' =

$$\frac{Mg}{\sqrt{2}} - \frac{T}{2} = Mg\left[\frac{\sqrt{2}-1}{2}\right] \qquad \text{...............(3)}$$

And there is a force along $AQ = \dfrac{T}{\sqrt{2}} = \dfrac{Mg}{\sqrt{2}}$ ....(4)

The friction will balance the resultant of the force given by (3) and (4)

The resultant has been shown in fig. (c)

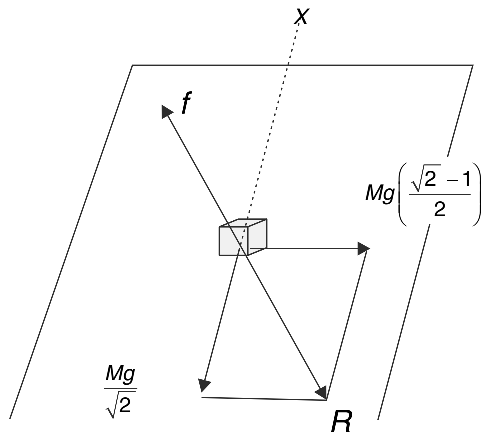


Fig (c)

$$\therefore f = Mg\sqrt{\left(\frac{\sqrt{2}-1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{\sqrt{5-2\sqrt{2}}}{2}Mg$$

$f \le \mu N$

$$\frac{\sqrt{5-2\sqrt{2}}}{2}Mg \le \mu\left(\frac{\sqrt{2}-1}{2}\right)Mg$$

$$\Rightarrow \frac{\sqrt{5-2\sqrt{2}}}{\sqrt{2}-1} \le \mu$$

49. Block will move up when tension in the string becomes larger than Mg.

Tension in string = Tension in cord AB

$\Rightarrow Mg = Kx$, [x_1= extension in AB] $\Rightarrow x_1 = \frac{Mg}{K}$

If AB stretches by x_1, the pulley moves up by $\frac{x_1}{2}$.

Tension in cord CD = twice the tension in string

$\therefore$ $Kx_2 = 2Mg$ [x_2 = extension in CD]

$$x_2 = \frac{2Mg}{K}$$

$\therefore$ End D moves up by a distance

$$= x_2 + \frac{x_1}{2}$$

$$= \frac{2Mg}{K} + \frac{Mg}{2K}$$

$$= \frac{5Mg}{2K}$$

50. Mass of $A = 8\ kg$

Mass of $B = 1\ kg$

Acceleration of $(A + B)$; $a = \dfrac{F}{9\ kg} = 2\ m/s^2$.

We will solve the problem in the reference frame attached to the blocks $(A + B)$. In this frame acceleration of C is $2.0\ m/s^2\ (\rightarrow)$

Time required for C to travel a distance of $4.0\ m$ is given as

$$4.0 = \frac{1}{2} \times a \times t_1^2 \qquad \Rightarrow t_1 = 2.0 \sec$$

$[\because a = 2\ m/s^2]$

Velocity of C at this time is

$u = at_1 = 2 \times 2 = 4\ m/s\ (\rightarrow)$

Now, the block C falls off the edge of A. Its motion in our reference frame has horizontal acceleration of $2\ m/s^2\ (\rightarrow)$ and a vertical acceleration of $(g = 10\ m/s^2)$. Time needed to fall through a height of $2.0\ m$ is given by

$$2.0 = \frac{1}{2} g t_2^2$$

$\Rightarrow t_2 = \sqrt{0.4} = 0.63$ sec

Horizontal distance travelled in this time interval is

$$x = ut + \frac{1}{2} at^2$$

$$= 4 \times 0.63 = \frac{1}{2} \times 2 \times (0.63)^2 = 2.92\ m$$

This is larger than width of block B. It means C will fall off the edge of A and land directly on the ground.

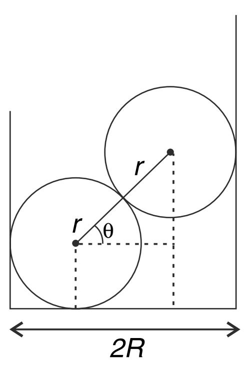
Time of fall can be directly calculated as

$$4.0 = \frac{1}{2} \times g \times t_3^2 \Rightarrow t_3 = \sqrt{0.8} = 0.9 \text{ sec}$$

$\therefore$ Total time needed to hit the ground is

$t_1 + t_3 = 2 + 0.9 = 2.9$ sec

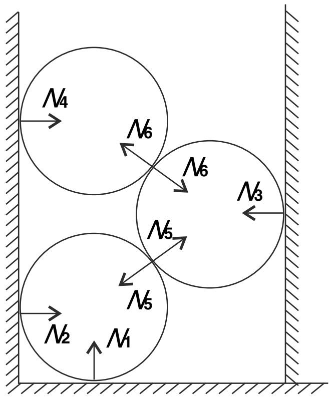
51.



$$r + r + 2\, r \cos \theta = 2\, R$$

$$\therefore 1 + \cos\theta = \frac{R}{r}$$

$\cos\theta = \frac{9}{5} - 1 = \frac{4}{5}$ $\Rightarrow \sin\theta = \frac{3}{5}$



Various contact forces are as shown.

Considering all 3 balls together

$N_1 = 3\,mg$ ....(1)

And $N_2 + N_4 = N_3$ ....(2)

Vertical equilibrium of lowest ball

$N_5 \sin\theta + mg = N_1$

$\therefore N_5 . \frac{3}{5} = 2\,mg$

$N_5 = \frac{10}{3} mg$

Vertical equilibrium of top ball

$N_6 . \sin\theta = mg$

$\Rightarrow N_6 = \frac{5}{3} mg$

Horizontal equilibrium of top ball

$N_4 = N_6 \cos\theta = \frac{5}{3} mg \times \frac{4}{5} = \frac{4}{3} mg$

Horizontal equilibrium of middle ball

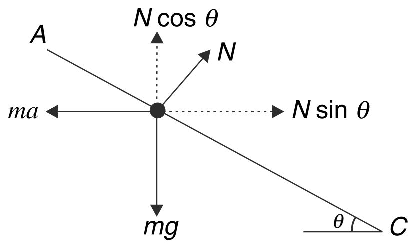
$N_3 = (N_6 + N_5)\cos\theta = \left(\frac{5}{3} + \frac{10}{3}\right)\frac{4}{5} mg = 4mg$

From (1) $N_2 = 4mg - \frac{4}{3} mg = \frac{8}{3} mg$

Hence N_3 is largest and its value is 4 mg

N_4 is smallest and its value is $\frac{4}{3} mg$

52. Bead on wire AC can remain at rest relative to the wire even if the wire is smooth. This is not possible with the other bead (It will become clear when we draw the force diagram). Let's first study the equilibrium of bead on AC in the reference frame of the wooden block.



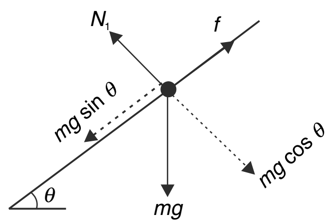
ma = *pseudo force*

$N \sin \theta = ma$

$N \cos \theta = mg$

$\Rightarrow \tan\theta = \dfrac{a}{g} \Rightarrow a = g\tan\theta$ ....(1)

This must be the acceleration of the block. Now we will study the equilibrium of the other bead in reference frame attached to the block.



There are two other force perpendicular to the plane of the fig.

(1) Pseudo force *ma* that is perpendicularly into the plane of the fig.

(2) Another component of normal force N_2 that is perpendicular to the plane of the fig.

(coming out towards you)

$N_1 = mg \cos \theta$

$N_2 = ma = mg \tan \theta$

And f = mg sin θ

But $f \le \mu\sqrt{N_1^2 + N_2^2}$

$\therefore mg \sin \theta \le \mu\, mg\, \sqrt{\cos^2\theta + \tan^2\theta}$

$$\frac{\sin\theta}{\sqrt{\cos^2\theta + \tan^2\theta}} \le \mu$$

53. (a) As per the problem, the string loses tension if platform goes up by2 *cm*.

It means the initial stretch in the spring was 4 *cm* (Why?)

At the instant block of mass *m* loses contact, the stretch in the spring is 8 *cm*.

$\therefore k\,(8\ cm) = mg$

$\therefore k = \dfrac{2\times10}{8}\dfrac{N}{cm} = 2.5\ N/cm$

(b) No

The block of mass *m* will remain in equilibrium. It will gradually move up so that the spring does not stretch further.

54. (a) Mass of A = mass of monkey.

Tension is actually the friction force between the hands of the monkey and the rope. Since forces on the monkey and the block are identical hence they will have same acceleration

$\therefore$ When monkey falls through 'h' the block also falls through 'h'

Hence, the block is at height h from the floor at the moment the monkey hits the ground.

(b) To keep the monkey at rest, tension $T = mg$

$\therefore$ Downward acceleration of the block is given by

$2\,Ma = 2\,Mg - T$

$\therefore a = \frac{g}{2} = 5\ m/s^2$

$\therefore$ Speed of the block at the instant it hits the ground is

$V^2 = 0 + 2a\,(2h)$

$V = \sqrt{2 \times 5 \times 2 \times 2.5} = 5\sqrt{2}\ m/s$

(c) At the instant the block crosses the monkey its downward velocity is

$V_o = \sqrt{2\,ah} = \sqrt{gh} = 5\ m/s$

After this the block must retard with $a = 5\ m/s^2$ so as to stop after moving through a further distance h.

Tension in the rope increases to T' such that

$T' - 2\,Mg = 2M\,\frac{g}{2}$

$T' = 3\,Mg$

$\therefore$ Upward acceleration of the monkey will be given by

$T' - Mg = Ma$

$\therefore a = 2\,g$

Rope on the side of the monkey will have acceleration $\frac{g}{2}$ in downward direction.

$\therefore$ Acceleration of the monkey relative to the rope is

$= 2g + \frac{g}{2} = 5\frac{g}{2}(\uparrow)$

$= 25\ m/s^2\ (\uparrow)$

56. Let the top spring stretch by x.

Since net force on bar $B1$ must be zero ($\because$ it has zero mass), hence the two spring connected below $B1$ will get stretched by $\frac{x}{2}$ each.

Similarly, one can argue that the next layer of four springs will have extension of $\frac{x}{4}$ each and so on

(a) Displacement of top point A will be

$$x_A = x + \frac{x}{2} + \frac{x}{4} + \frac{x}{8} + \ldots\ldots\ldots\ldots\ldots\infty$$

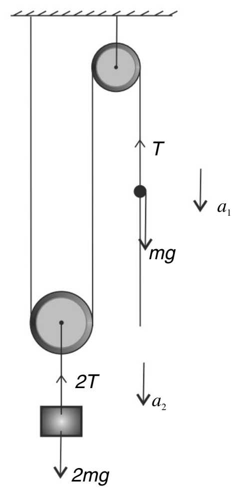
$$= x\left[\frac{1}{1 - \frac{1}{2}}\right]$$

$$= 2x = 2\frac{F}{K}$$

(b) Displacement of $B1$

$$X_{B1} = 2x - x = x = \frac{F}{K}$$

58.



Friction between the bead and the thread = tension in the string carrying the bead = T (say)

<u>*For bead*</u>

$mg - T = ma_1$ ....(1)

<u>*For block*</u>

$2\,mg - 2T = 2\,m\,a_2$

$\Rightarrow mg - T = ma_2$ ....(2)

From (1) and (2)

$a_1 = a_2 = a$ (say)

The end of the string below the bead will move up with acceleration of $2a$ as the block goes down with acceleration of a.

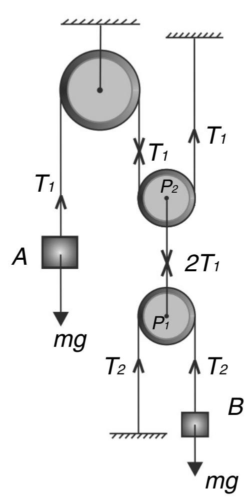
Hence relative acceleration of the bead and the end of the string = $3a$

$\therefore l = \frac{1}{2}(3a)t^2$ ....(3)

The block falls through

$s = \frac{1}{2}at^2 = \frac{l}{3}$ [from (3)]

59.



(a) For equilibrium

$T_1 = Mg$

And $T_2 = mg$

But $2T_2 = 2T_1$ (for equilibrium of pulley P_1)

$\therefore m = M$

(b) When $m = 2M$, let acceleration of A be a (↑).

Acceleration of pulley P_2 will be $\frac{a}{2}(\downarrow)$

Acceleration of P_1 = acceleration of $P_2 = \frac{a}{2}(\downarrow)$.

Acceleration of B = 2 × acceleration of $P_2 = a$ (↓).

For pulley P_1; $2T_1 = 2T_2 \Rightarrow T_1 = T_2$ ....(1)

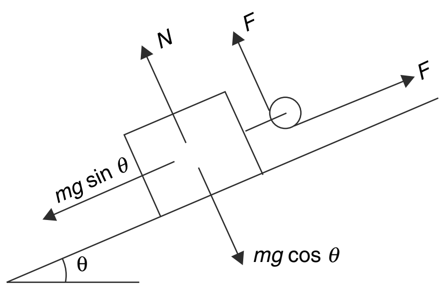
For A; $T_1 - Mg = Ma$ ....(2)

For B; $2Mg - T_2 = 2Ma$ ....(3)

Solving (1), (2) and (3)

$a = \frac{g}{3}$

60. (a)



For block to accelerate up the incline

$F > mg \sin\theta$

For block to remain in contact

$F < mg \cos\theta$

This is possible only if $\theta < 45°$

$\therefore \theta_o = 45° = \frac{\pi}{4}$

(b) If $\therefore \theta_o = \frac{\theta_o}{2} = \frac{\pi}{8}$, maximum allowed value of F

so that block does not lose contact is

$F = mg \cos\left(\frac{\pi}{8}\right)$

$\therefore ma_{\max} = mg\cos\left(\frac{\pi}{8}\right) - mg\sin\left(\frac{\pi}{8}\right)$

$$\therefore a_{\max} = g\left[\cos\left(\frac{\pi}{8}\right) - \sin\left(\frac{\pi}{8}\right)\right]$$

61. <u>When *A* is fixed</u>

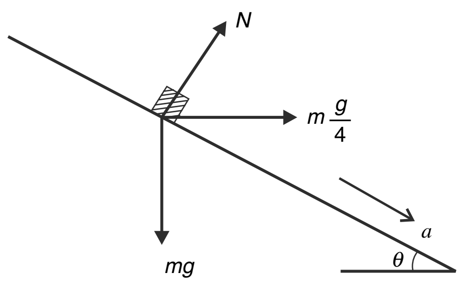
$S = \frac{1}{2}(g\sin\theta)t^2$ [S = length of incline] ----- (1)

<u>When *A* is free to move</u>

In reference frame attached to *A*. The acceleration of *B* will be given by

$$ma = mg\sin\theta + \frac{mg}{4}\cos\theta$$

$$a = g\left(\sin\theta + \frac{1}{4}\cos\theta\right)$$



$$\therefore S = \frac{1}{2}g\left(\sin\theta + \frac{1}{4}\cos\theta\right)t'^2$$

$= \frac{1}{2}g\left(\sin\theta + \frac{1}{4}\cos\theta\right)\left(\frac{t}{2}\right)^2$ [$\because t' = \frac{t}{2}$] -----(2)

From (1) and (2)

$$\sin\theta = \left(\sin\theta + \frac{1}{4}\cos\theta\right)\frac{1}{4}$$

$$3\sin\theta = \frac{1}{4}\cos\theta$$

$$\tan\theta = \frac{1}{12}$$

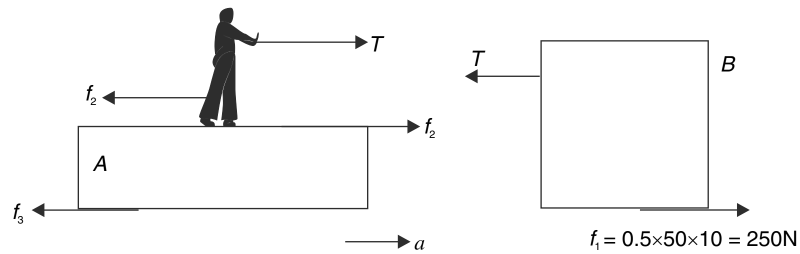
62. Let the horizontal and vertical directions be *X* and *Y* directions respectively.

$\vec{a}_A = a_o \cos\theta\,\hat{i} + a_o \sin\theta\,\hat{j}$

$\vec{a}_B = a_o \sin\theta\,\hat{j}$ (since length of string is constant)

$\Rightarrow \vec{a}_{AB} = a_o \cos\theta\,\hat{i}$

63.



For block B to move slowly $T = f_1 = 250\ N$.

Maximum possible friction on shoes of the man

$f_{2\,max} = 0.6 \times 500 = 300N$

For man to remain at rest relative to the ground

$f_2 = T = 250\ N$

Acceleration of platform A is

$30.\ a = f_2 - f_3$

$30.\ a = 250 - 0.2 \times 800$

$a = 3\ m/s^2$

Man will fall off the platform once he moves $5m$.

Hence, time required is given by

$$\frac{1}{2}at^2 = 5$$

$$\frac{1}{2}\times 3\times t^2 = 5$$

$$t = \sqrt{\frac{10}{3}}s$$

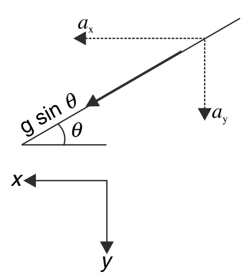
64. (a) In this case the mass $(M + m)$ will slide down the incline with acceleration

$a = g \sin\theta$

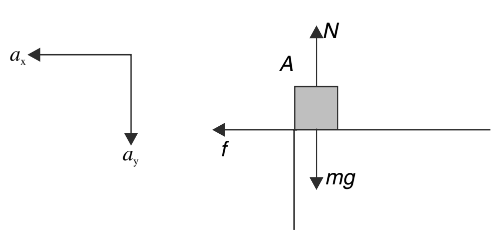
(b) The horizontal and vertical components of the above acceleration are

$a_x = g \sin\theta . \cos\theta$

$a_y = g \sin\theta \sin\theta$



For motion of block A



$f = ma_x$

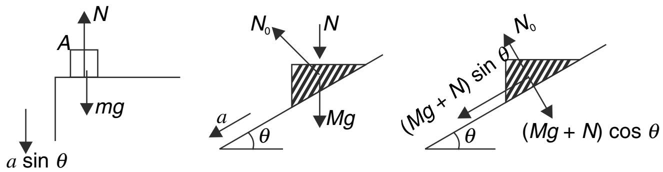
$= mg \sin\theta . \cos\theta$

$= \frac{1}{2} mg \sin 2\theta$

And $mg - N = ma_y$

$N = mg - mg \sin^2\theta = mg. \cos^2\theta$

(c) In case of no friction, the wedge slides down the incline with acceleration a and the small block will move vertically down with acceleration $a \sin\theta$.



For Block: $mg - N = ma \sin\theta$ ….. (1)

For Wedge: $N_o = (Mg + N) \cos\theta$

and $(Mg + N) \sin\theta = Ma$ ….. (2)

Solving (1) and (2)

$$a = \frac{(M+m)g\sin\theta}{M + m\sin^2\theta}$$

$\therefore$ acceleration of the block = $a \sin\theta$

$$= \frac{(M+m)g\sin^2\theta}{M+m\sin^2\theta}$$

Now, for displacement of the block

$$h = \frac{1}{2}\left(\frac{(M+m)g\sin^2\theta}{M+m\sin^2\theta}\right)t^2$$

$$t = \sqrt{\frac{2h(M+m\sin^2\theta)}{(M+m)g\sin^2\theta}}$$

65. (a) Immediately after release none of the blocks have any speed.

$\vec{a}_A$ is towards ($\leftarrow$)

$\vec{a}_B$ is ($\downarrow$)

$\therefore \vec{a}_A.\vec{a}_B = a_A.a_B.\cos 90° = 0$

(b) Immediately after release, the constraint that length of string is constant means that

$|\vec{a}_A| = |\vec{a}_B| = a(say)$

For B:

$Mg - T = Ma$ ….. (1) [Normal force between the blocks is zero]

For A:

$T = Ma$ …..(2)

$\therefore$ (1) + (2) gives

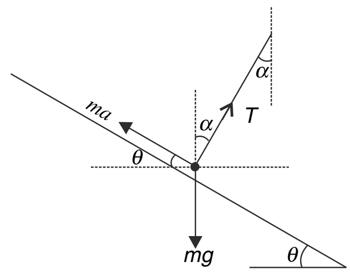
$a = g/2$

66. (a) The block will slide down with an acceleration $a = g \sin \theta$

For the ball to have same acceleration, the thread shall be perpendicular to the incline. Tension will balance $mg \cos \theta$ and the ball will have a resultant force of $mg \sin \theta$ down the incline.

It means that the thread makes an angle θ to the vertical.

(b) The acceleration of the block is $a = \frac{1}{2} g \sin\theta$. In reference frame of the block there is a pseudo force (= ma) on the ball.



Taking equilibrium in horizontal and vertical direction –

$T \sin \alpha = ma \cos \theta$ .....(i)

$T \cos \alpha + ma \sin \theta = mg$

$\Rightarrow T \cos \alpha = mg - ma \sin \theta$ ..... (ii)

Taking ratio of (i) and (ii)

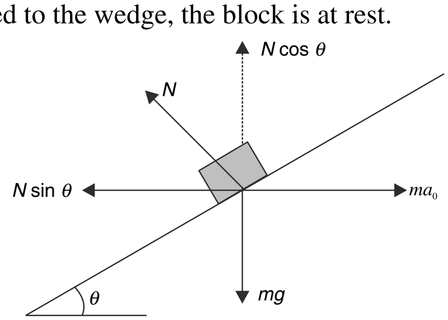
$$\tan\alpha = \frac{a\cos\theta}{g - a\sin\theta} = \frac{\frac{1}{2} g \sin\theta . \cos\theta}{g - \frac{1}{2} g \sin^2\theta}$$

$$\tan\alpha = \frac{\sin\theta . \cos\theta}{2 - \sin^2\theta}$$

$$\alpha = \tan^{-1}\left(\frac{\sin\theta . \cos\theta}{2 - \sin^2\theta}\right)$$

67. (a) Let acceleration of wedge be a_o.

In the reference frame attached to the wedge, the block is at rest.



$N \sin \theta = ma_o$ ..... (1)

$N \cos \theta = mg$ ..... (2)

(1) ÷ (2) gives $\tan\theta = \frac{a_o}{g}$

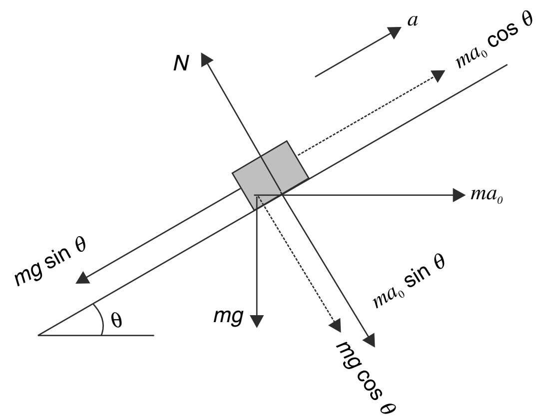
$\therefore a_o = g \tan \theta = g \tan 45° = g$

$\therefore$ Force on wedge $F_o = 2m.g$

(b) $F = 1.5 \times 2\ mg = 3\ mg$

Let a_o = acceleration of wedge

a = acceleration of block relative to wedge (up along the incline)

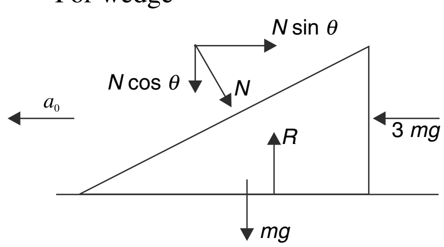


$N = mg\cos\theta + ma_o \sin\theta$ ……..(1)

$ma = ma_o \cos\theta - mg\sin\theta$

$a = a_o \cos\theta - g\sin\theta$ ……..(2)

For wedge



$3mg - N\sin\theta = ma_o$

$\therefore N\sin\theta = 3mg - ma_o$….. (3)

Eliminating N between (1) and (3) and writing

$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$, we get

$$a_o = \frac{5}{3}g$$

Putting in (2), we get

$$a = \frac{\sqrt{2}g}{3}$$

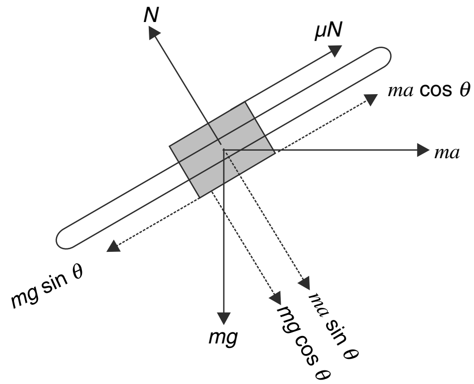
The block will fall off the wedge after moving through a distance $\frac{L}{2}$ up the incline.

$$\therefore \frac{L}{2} = \frac{1}{2}at^2$$

$$\Rightarrow t^2 = \frac{3L}{\sqrt{2}g}$$

$$t = \sqrt{\frac{3L}{\sqrt{2}g}}$$

68. Figure shows the free body diagram of the sleeve in a reference frame attached to the rod when a is small, the rod has a tendency to slide down, hence friction is up the rod.



For the minimum value of a for which the sleeve does not slide, friction will take its maximum possible value, i.e., μN

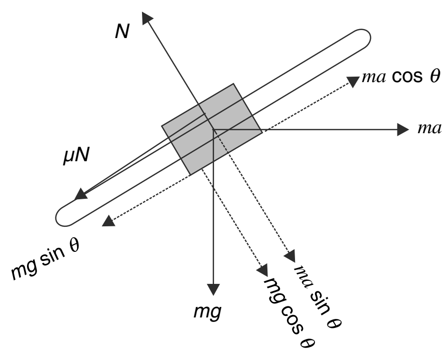
$N = m\,(a \sin\theta + g\cos\theta)$

and $\mu N + ma\cos\theta = mg\sin\theta$

$\mu m\,(a\sin\theta + g\cos\theta) + ma\cos\theta = mg\sin\theta$

$$\Rightarrow\ a = g\frac{(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}$$

This is the minimum value of a for which the sleeve does not slide.



When a increases the sleeve has a tendency to move up.
Thus friction is directed down the rod.
a is maximum when friction is μN
$N = m\,(a\sin\theta + g\cos\theta)$
and mg $\sin\theta + \mu N = ma\cos\theta$
or
$mg\sin\theta + \mu m\,(a\sin\theta + g\cos\theta) = ma\cos\theta$

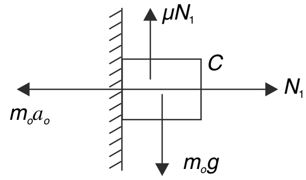
or $a = \dfrac{g(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}$

$$\therefore\ g\frac{(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)} \le a \le g\frac{(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}$$

69. Let a_o = acceleration of B (towards right)

a = acceleration of A wrt B

FBD of C in the reference frame attached to B is shown in figure



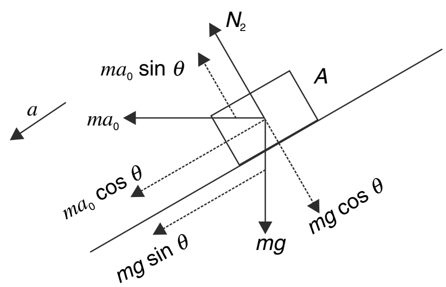
$m_o = \frac{M}{4} =$ mass of C.

$N_1 = m_o a_o$ ….. (1)

and $\mu N_1 = m_o g$ …..(2)

or $a_o = \frac{g}{\mu}$ ….. (3)

FBD of A is shown

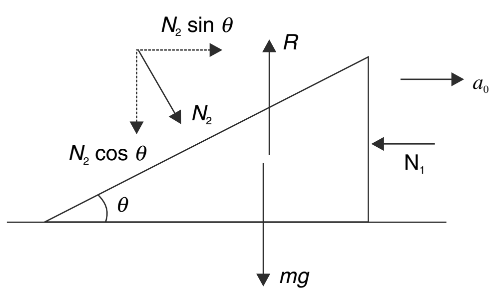


$N_2 = mg \cos\theta - ma \sin\theta$ ……(4)

and $ma = mg \sin\theta + ma_o \cos\theta$

or $a = g \sin\theta + a_o \cos\theta$ ……(5)

FBD of B is also shown



$N_2 \sin\theta - N_1 = Ma_o$……(6)

Using (i)

$N_2 \sin\theta = (M + m_o)a_o$

Eliminating N_2 using (4) we get

$$a_o = \frac{mg\sin 2\theta}{2\left(M + m_o + m\sin^2\theta\right)}$$

From (3) $\frac{g}{\mu} = \frac{mg\sin 2\theta}{2\left(M + m_o + m\sin^2\theta\right)}$

or $2M + 2m_o \times 2m \sin^2 \theta = \mu m \sin 2\theta$

or $2M + \dfrac{M}{2} + 2m\sin^2\theta = \dfrac{3}{4}m\sin 2\theta$

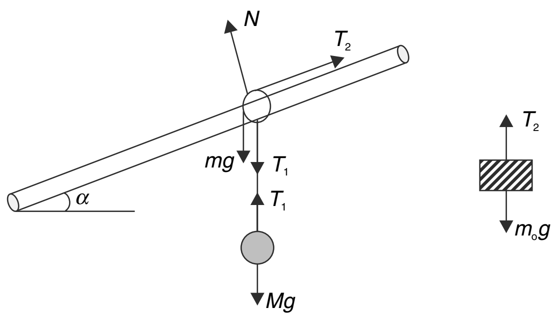
or $\dfrac{5}{2}M = m\left(\dfrac{3}{4}\sin 60° - 2\sin^2 30°\right)$

or $\dfrac{5}{2}M = m\left(\dfrac{3}{4}\dfrac{\sqrt{3}}{2} - \dfrac{2}{4}\right)$

or $\dfrac{5}{2}M = m\left(\dfrac{3\sqrt{3}}{8} - \dfrac{1}{2}\right)$

or $\dfrac{m}{M} = \dfrac{20}{3\sqrt{3}-4} = \dfrac{20}{1.196} = 16.7$

70. (a) Equilibrium



$T_1 = Mg$ and $T_2 = m_o g$

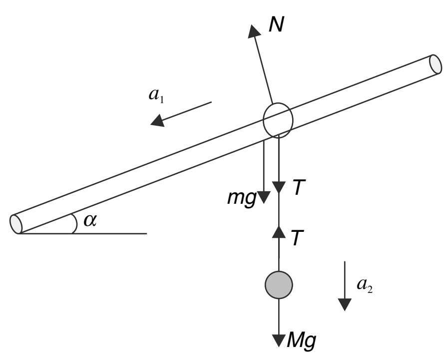
For ring:

$T_2 = (T_1 + mg) \sin \alpha$

$\therefore m_o g = (M + m)g \sin \alpha$

$\therefore m_o = (M + m) \sin \alpha$

(b) Relation between a_1 and a_2 is



$a_1 \sin \alpha = a_2$ ……... (1)

For ring

$(T + mg) \sin \alpha = ma_1$ ……... (2)

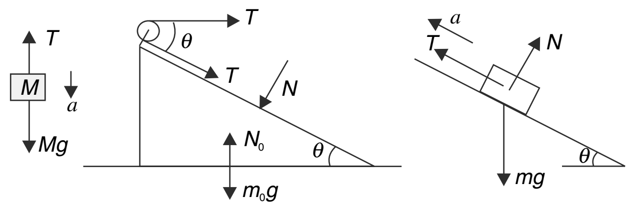
For sphere

$Mg - T = ma_2$ ……... (3)

Solving (1), (2) and (3)

$$a_2 = \frac{(M+m)g\sin^2\alpha}{m+M\sin^2\alpha}$$

71.



For M: $Mg - T = Ma$ .... (1)

For m: $T - mg\sin\theta = ma$ .... (2)

and $N = mg\cos\theta$ .... (3)

For m_o: $T + T\cos\theta = N\sin\theta$ .... (4)

[In horizontal direction]

Eliminating a between (1) and (2)

$$g - \frac{T}{M} = \frac{T}{m} - g\sin\theta$$

$$\Rightarrow T\left(\frac{1}{m}+\frac{1}{M}\right) = g(1+\sin\theta)$$

$$\Rightarrow T = \frac{mMg}{m+M}(1+\sin\theta)$$

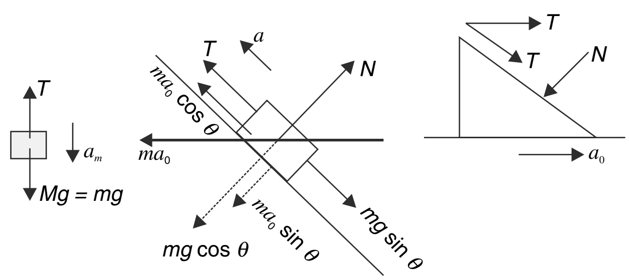
Put this value of T and value of N from equation (3) into equation (4)

$$\frac{mMg(1+\sin\theta)}{m+M}(1+\cos\theta) = mg\cos\theta.\sin\theta$$

$$\therefore \frac{M}{m+M} = \frac{\cos\theta.\sin\theta}{(1+\sin\theta)(1+\cos\theta)} = \frac{\frac{4}{5}.\frac{3}{5}}{\left(1+\frac{3}{5}\right)\left(1+\frac{4}{5}\right)} = \frac{1}{6}$$

$$\Rightarrow \frac{M}{m} = \frac{1}{5}$$

72.



Let acceleration of m wrt wedge be $a\ (\nwarrow)$ and that of the wedge be as $a_o\ (\rightarrow)$.

Acceleration of M is $a_M\ (\downarrow) = a + a_o$ …(1)

For M: $mg - T = ma_M$ …(2)

For m in reference frame of wedge

$$T - mg\frac{3}{5} + ma_o.\frac{4}{5} = ma \qquad \text{… (3)}$$

and $N = mg.\frac{4}{5} + ma_o\frac{3}{5}$ … (4)

For wedge in horizontal direction

$$T + T.\frac{4}{5} - N.\frac{3}{5} = 2m.a_o \qquad \text{… (5)}$$

(2) + (2) $\frac{2}{5}mg + \frac{4}{5}ma_o = m(a + a_o) + ma$

$\therefore$ $2g + 4a_o = 10a + 5a_o$

$\Rightarrow 10a + a_o = 2g$ … (A)

Eliminating N between (4) and (5) gives.

$45T - 12\ mg = 59\ ma_o$

Substituting for T from (2)

$45\ (mg - ma_M) - 12mg = 59ma_o$

$45g - 45(a + a_o) - 12g = 59a_o$

$45a + 104\ a_o = 33\ g$ … (B)

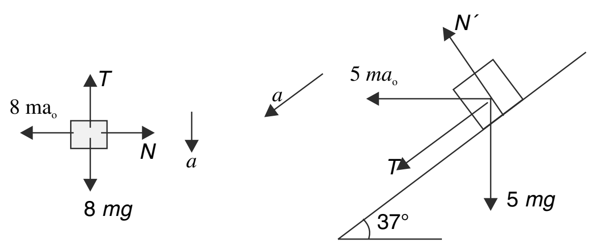
Solving (A) and (B)

$$a_o = \frac{48g}{199}$$

73. Let a_o = acceleration of 4 m towards right

a = acceleration of 5 m and 8 m wrt the block of mass 4 m.

We will consider motion of block of mass 8m and 5m in the reference frame attached to the block of mass 4 m.



For 4 m: $8\ mg - T = 8\ ma$ …....(1)

and $N = 8\ ma_o$ …....(2)

For 5 m:

Perpendicular to incline

$$N' + 5ma_o\frac{3}{5} = 5mg.\frac{4}{5}$$

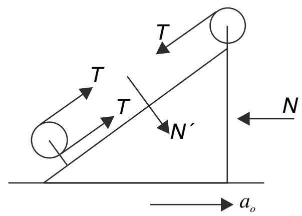
$\Rightarrow N' = 4\ mg - 3\ ma_o$ …..... (3)

Down the incline

$$T + 5ma_o.\frac{4}{5} + 5mg.\frac{3}{5} = 5ma$$

$T = 5\ ma - 4\ ma_o - 3\ mg$ .... (4)

For block of mass 4 m:



We will write equation of motion in horizontal direction. Relevant forces are shown in figure.

$$T.\frac{4}{5} + N' \frac{3}{5} - N = 4ma_o$$

$4T + 3N' - 5N = 20\ ma_o$ .... (5)

Substituting for T, N and N' using equations (1), (2) and (3)

$4[8\ mg - 8\ ma] + 3\ [4\ mg - 3\ ma_o] - 5\ [8\ ma_o] = 20\ ma_o$
$\Rightarrow 69\ a_o + 32a = 44\ g$ .... (6)

Eliminating T between (1) and (4)

$13\ a - 4a_o = 11\ g$ .... (7)

Solving (6) and (7)

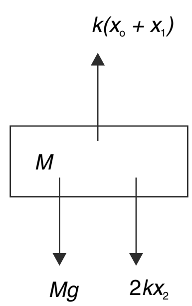
$$a_o = \frac{44g}{205}$$

74. Initial extension in $S1$ be x_o

$$kx_o = Mg$$

Let extension in $S2$ be x_2 and further extension in $S1$ be x_1 as point A moves down by L.

$2x_1 + x_2 = L$ ...... (1)



For equilibrium of M

$kx_o + kx_1 = Mg + 2kx_2$

But $kx_o = Mg$

$\therefore x_1 = 2x_2$ ......(2)

Solving (1) and (2)

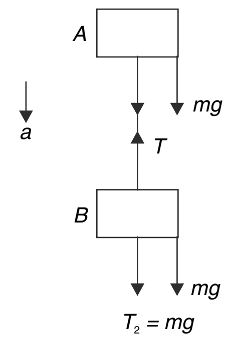
$$x_2 = \frac{L}{5}$$

$$x_1 = \frac{2L}{5}$$

75. In equilibrium, tension in spring 1 is $T_1 = 3mg$ and tension in spring 2 is $T_2 = mg$.

(a) When spring 1 is cut, there will be no immediate change in tension in spring 2.

$\therefore a_c = 0$



Let a be acceleration of A and B

$2ma = T_2 + mg + mg$

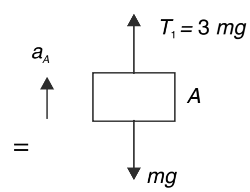
$$a = \frac{3}{2}g(\downarrow)$$

$$\frac{d^2x_2}{dt^2} = a_C - a_B = 0 - \frac{3}{2}g(\downarrow)$$

$$\left|\frac{d^2x_2}{dt^2}\right| = \frac{3g}{2}$$

(b) When AB is cut, tension in both springs remain unchanged.

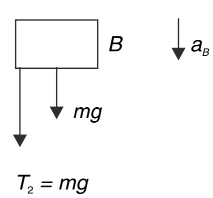
For A



$ma_A = 3mg - mg$

$a_A = 2g\ (\uparrow)$

For B



$ma_B = mg + mg$

$a_B = 2g\ (\downarrow)$

For C

$a_C = 0$

$$\therefore \left|\frac{d^2x_1}{dt^2}\right| = a_A = 2g$$

$$\left|\frac{d^2x_2}{dt^2}\right| = |a_C - a_B| = 2g$$

(c) When spring 2 is cut, ($A + B$) together move up.

$2ma = 3mg - mg - mg$

$$a = \frac{g}{2}(\uparrow)$$

$a_C = g\ (\downarrow)$

$$\therefore \left|\frac{d^2x_1}{dx^2}\right| = |a_A| = \frac{g}{2}$$

$$\left|\frac{d^2x_2}{dt^2}\right| = |a_A - a_C| = \frac{3g}{2}$$

76. Let stretch in 1st spring be x_1

stretch in 2nd spring be x_2

$$x_o = \frac{x_1 + x_2}{2} \qquad \text{...... (1)}$$

Also $k_1x_1 = k_2x_2$ ...... (2)

Solving (1) and (2)

$$x_1 = \left(\frac{k_2}{k_1 + k_2}\right)(2x_0)$$

$\therefore$ Tension in springs

$$T = k_1x_1 = \frac{2k_1k_2x_0}{k_1 + k_2}$$

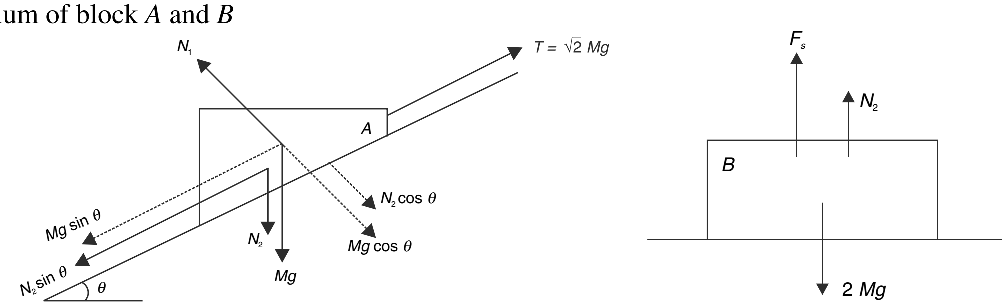
(a) $Ma = 2T$

$$a = \frac{4k_1k_2x_0}{(k_1 + k_2)M}$$

(b) $T_0 = 2T = \dfrac{4k_1k_2x_0}{k_1 + k_2}$

77. Let's consider

Equilibrium of block A and B



For A:

$Mg \sin\theta + N_2 \sin\theta = \sqrt{2}\, Mg$

$$\frac{Mg}{\sqrt{2}} + \frac{N_2}{\sqrt{2}} = \sqrt{2}Mg$$

$\therefore N_2 = Mg$

For B:

$F_S + N_2 = 2Mg$

$\therefore F_S = Mg$

Immediately, after the string is cut, tension in the spring (F_S) will not change.

Let acceleration of A be 'a' down the incline.

Acceleration of B will be $a\sin\theta = \frac{a}{\sqrt{2}}$ in vertically downward direction.

We will use the same force diagram as drawn above (Now $T = 0$). For motion of A.

$(N_2 + Mg)\sin\theta = Ma$

$N_2 + Mg = \sqrt{2}\, Ma$ ……... (1)

For motion of B

$2Mg - N_2 = 2M \,.\, a\sin\theta$

$2Mg - N_2 = 2\sqrt{2}\, Ma$ ……... (2)

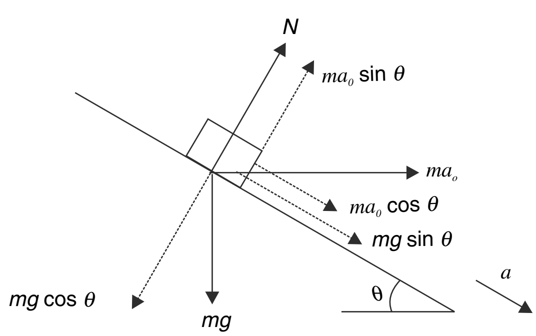
From (1) and (2)

$N_2 = 0$

and $a = \frac{g}{\sqrt{2}}$

78. Let a_0 = acceleration of the wedge

a = acceleration of the block relative to the wedge. We consider motion of the block in reference frame attached to the wedge

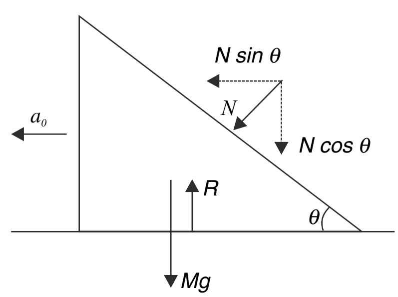


$N + ma_0 \sin\theta = mg\cos\theta$ ....(1)

$ma_0 \cos\theta + mg\sin\theta = ma$

$\Rightarrow a_0\cos\theta + g\sin\theta = a$ ....(2)

For motion of the wedge



$N\sin\theta = Ma_0$ .... (3)

Eliminating N between (1) and (3)

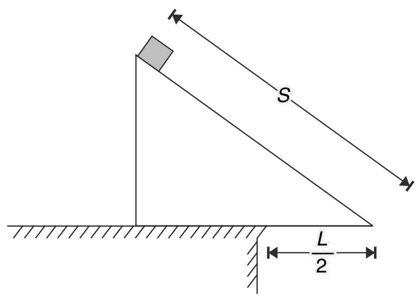
$$a_0 = \frac{mg\sin\theta\cos\theta}{\left(M + m\sin^2\theta\right)}$$

Putting this in (2) gives

$$a = \frac{\left(M+m\right)g\sin\theta}{\left(M + m\sin^2\theta\right)}$$

$$\therefore \ \frac{a}{a_0} = \frac{M+m}{m}$$

For the block to remain on the table, it is required that by the time displacement of the block relative to the wedge becomes equal to 'S', the horizontal displacement of the wedge must have become larger than $\frac{L}{2}$.



$$\Rightarrow \frac{1}{2}at^2 = S \qquad \text{.......... (1)}$$

and $\frac{1}{2}a_0t^2 \geq \frac{L}{2}$ .... (2)

For limiting case let's take the ratio [(1) $\div$ (2)]

$$\frac{a}{a_0} = \frac{2S}{L}$$

$$\Rightarrow \frac{M+m}{m} = \frac{2S}{L}$$

$$\frac{M}{m} + 1 = \frac{2}{\cos\theta} = 4 \quad \left[\cos 60° = \frac{1}{2}\right]$$

$$\therefore \frac{M}{m} = 3$$

79. Let a = acceleration of m relative to the wedge A

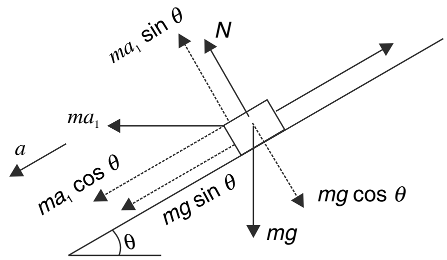
a_1= acceleration of wedge A towards right

a_2 = acceleration of Block B towards left

A careful observation will tell you that

$a = a_1 + a_2$ ....(1)

Consider motion of m in reference frame of wedge A

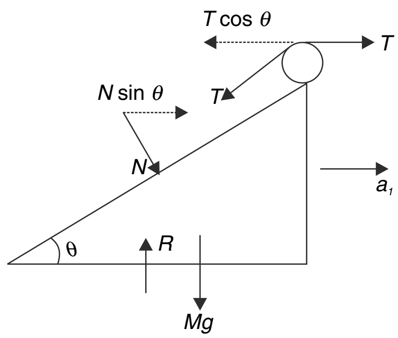


$ma_1 \sin\theta + N = mg \cos\theta$ .... (2)

$ma_1 \cos\theta + mg \sin\theta - T = ma$ .... (3)

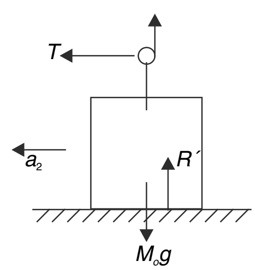
Consider motion of wedge *A*

We will write equation for horizontal motion only



$T + N \sin\theta - T \cos\theta = Ma_1$ ....(4)

For motion of Block *B*



The equation for horizontal motion is

$T = M_0 a_2$ ....(5)

First let's eliminate *N* between 2 and 4

$ma_1 \sin^2\theta - T(1 - \cos\theta) = mg \cos\theta.\sin\theta - Ma_1$

$\Rightarrow a_1 [m \sin^2\theta + M] - T(1 - \cos\theta) = mg \sin\theta \cos\theta$

Put $\sin\theta = \frac{3}{5}; \cos\theta = \frac{4}{5}$ and $M = 4\,m$

$$ma_1\left[\frac{9}{25} + 4\right] - \frac{T}{5} = \frac{12}{25}mg$$

$\Rightarrow 109\,ma_1 - 5\,T = 12\,mg$ .... (6)

Put value of *T* from (5) into (3) & (6)

$ma_1 \cos\theta + mg \sin\theta - M_0 a_2 = ma$

$$\frac{4}{5}a_1 + \frac{3}{5}g - 2a_2 = a$$

$4\,a_1 - 10\,a_2 = 5\,a - 3\,g$ ……….. (7)

And $109\,a_1 - 10\,a_2 = 12\,g$ ……….. (8)

Solving (1), (7) and (8)

$4a_1 - 10\,a_2 = 5\,(a_1 + a_2) - 3\,g$

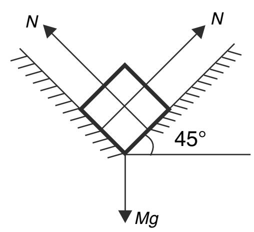
$\Rightarrow a_1 + 15\,a_2 = 3\,g$

Multiply this equation with (2) and add it to equation (8) multiplied by 3

$$a_1 = \frac{6g}{47}$$

80. If friction coefficient is μ.

$F_0 = \mu mg$



When the block is in the trough

$N\cos 45° + N\cos 45° = Mg$

$\therefore \sqrt{2}\,N = Mg$

$$\therefore\ N = \frac{Mg}{\sqrt{2}}$$

$\therefore$Friction force on the block is

$f = \mu N + \mu N$ [for two contact surfaces]

$= \sqrt{2}\,\mu\,Mg$

$\therefore$ Required force $F = \sqrt{2}\,\mu\,Mg = \sqrt{2}\,F_0$

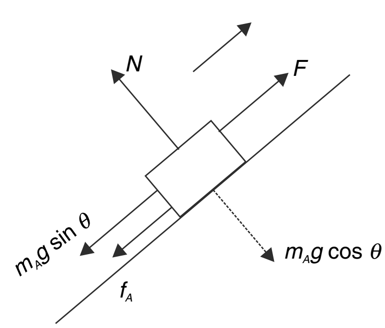
81. Block A moved up along the incline with acceleration a_A such that

$$X_A = 0 + \frac{1}{2}a_A t^2$$

$$2 = \frac{1}{2}a_A \times 2^2$$

$\therefore\ a_A = 1.0\ m/s^2$

Force equation for block A



$F - m_A g \sin\theta - \mu_A m_A g \cos\theta = m_A.a_A$

$$2.2 - 0.2\times 10\times\frac{3}{5} - \mu_A\times 0.2\times 10\times\frac{4}{5} = 0.2\times 1$$

$$\Rightarrow \mu_A = 0.5$$

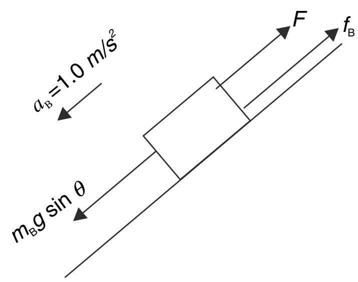
For B

$$X_B = 0 + \frac{1}{2}a_B t^2$$

$$-2 = \frac{1}{2}\times a_B\times 2^2$$

$$\Rightarrow a_B = -1.0\ ms^{-2}$$

Force equation for B

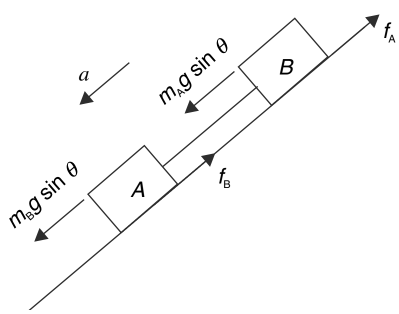


$m_B g \sin\theta - F - \mu_B.\ m_B g \cos\theta = m_B a_B$

$\Rightarrow 0.5\times 10\times 0.6 - 2.2 - \mu_B\times 0.5\times 10\times 0.8 = 0.5\times 1$

$$\Rightarrow \mu_B = \frac{3}{40} = 0.075$$

With both blocks together



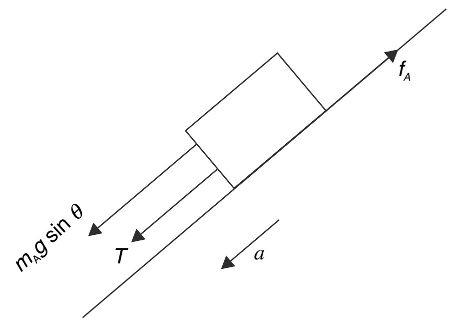
$(m_B + m_A)g \sin\theta - \mu_A m_A g \cos\theta - \mu_B m_B \cos\theta = (m_A + m_B)a$

$\Rightarrow 0.7\times 10\times 0.6 - 0.5\times 0.2\times 10\times 0.8 - \dfrac{3}{40}\times 0.5\times 10\times 0.8 = 0.7a$

$\Rightarrow 4.2 - 0.8 - 0.3 = 0.7a$

$\Rightarrow a = 4.43\ m/s^2$

Considering only A



$m_A g \sin\theta + T - f_A = m_A a$
$1.2 + T - 0.8 = 0.2 \times 4.43$
$T = 0.49\ N$

82. Let coefficient of friction between the blocks be μ.

$F_1 = 4\ m\ (\mu g)$

$\therefore\ \mu g$ is the maximum acceleration that friction can impart to block B.

Similarly, when force is applied to B, friction can impart a maximum acceleration of $\dfrac{\mu g}{3}$ to the block A.

$\therefore\ F_2 = 4m\left(\dfrac{\mu g}{3}\right)$

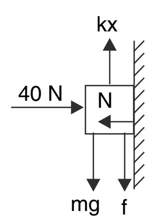
(a) With both F_1 and F_2 acting, friction will adjust itself to zero, as the two blocks will have equal accelerations

$a = \dfrac{F_2}{m} = \dfrac{F_1}{3m} = \dfrac{4}{3}\mu g$

(b) $a = \dfrac{4}{3}\mu g = \dfrac{F_1}{3m} = \dfrac{F_2}{m}$

83. (i) Consider the case when the spring is compressed and the block is in equilibrium.

$kx = mg + f$



$(kx)_{max} = mg + f_{max}$

$400\ x_{max} = 20 + 0.5 \times 40$

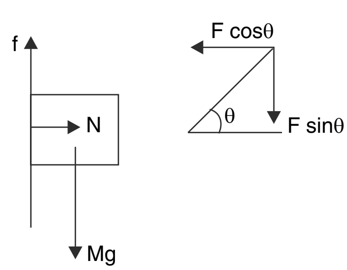
$x_{max} = 0.1\ m$

In this position $h = 0.9\ m$

Above this height block can be kept anywhere.

If compression in the spring is more than 0.1 m, the block will slide up.

(ii)



$N = F\cos\theta$

$F\sin\theta + Mg = f$

But $f \le \mu N$

$\therefore F\sin\theta + Mg \le \mu F\cos\theta$

$Mg \le F(\mu\cos\theta - \sin\theta)$ ....(1)

$\Rightarrow 0.5 \le \mu\cos\theta - \sin\theta$

$\Rightarrow 0.5 \le \sqrt{\mu^2+1}\sin(\alpha-\theta)$ [where $\alpha = \tan^{-1}\mu = \tan^{-1} 0.75 = 37°$]

$\Rightarrow 0.5 \le 1.25\sin(\alpha-\theta)$

$\Rightarrow 0.4 \le \sin(\alpha-\theta)$

$\Rightarrow \sin 24° \le \sin(\alpha-\theta)$

$\Rightarrow 24° \le 37° - \theta \Rightarrow \theta \le 13°$

(b) From equation (1), F will have a finite positive value only if

$\mu\cos\theta - \sin\theta > 0$

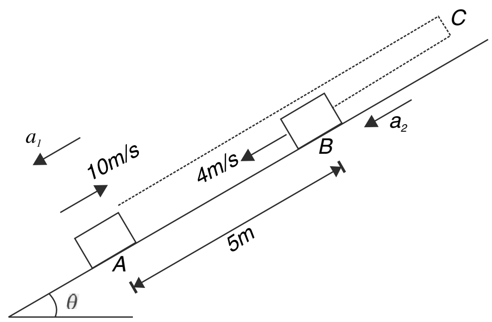
$\mu > \tan\theta$

$0.75 > \tan\theta$

$37° > \theta$

84. Retardation while going up will be $a_1 = g\sin\theta + \mu g\cos\theta = x + y$ [say]

Acceleration while coming down $a_2 = g\sin\theta - \mu g\cos\theta = x - y$



Let time for up journey = t_1

Time for down journey (C to B) = t_2

$AC = S_1$ and $CB = S_2$

$V_C = V_A - a_1 t_1 \Rightarrow 0 = 10 - (x+y)t_1$

$$\Rightarrow \frac{10}{x+y} = t_1$$

And $V_B = V_C + a_2 t_2 \Rightarrow 4 = 0 + (x-y)t_2$

$$\frac{4}{x-y} = t_2$$

Given $t_1 + t_2 = 4$

$$\therefore \frac{10}{x+y} + \frac{4}{x-y} = 4 \quad ....(1)$$

$$V_C^2 = V_A^2 - 2a_1 S_1 \Rightarrow \frac{10}{2(x+y)} = S_1$$

Similarly, $\dfrac{4^2}{2(x-y)} = S_2$

Given $S_1 - S_2 = 5m$

$\Rightarrow \dfrac{50}{x+y} - \dfrac{8}{x-y} = 5 \ldots(2)$

Let $\dfrac{1}{x+y} = a$ and $\dfrac{1}{x-y} = b$

$\therefore$ Equation (1) and (2) can be written as

$10a + 4b = 4 \Rightarrow 5a + 2b = 2$

And $50a - 8b = 5$

Solving these two equations we get

$a = \dfrac{13}{70}$ and $b = \dfrac{15}{28}$

$\therefore$ $13x + 13y = 70$

And $15x - 15y = 28$

Solving these two equations we get

$x = 3.62$ and $y = 1.76$

(a) $t_1 = \dfrac{10}{x+y} = 10 \cdot a = 10 \cdot \dfrac{13}{70} = \dfrac{13}{7} s$

(b) $x = 3.62 \Rightarrow g \sin\theta = 3.62 \Rightarrow \sin\theta = 0.362$

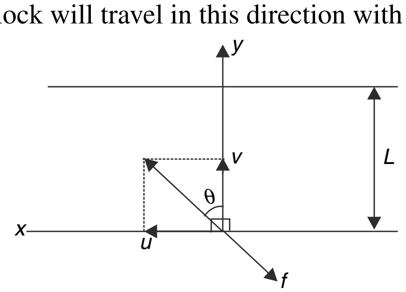
$\therefore \cos\theta = \sqrt{1-(0.362)^2} = 0.94$

$y = 1.76 \Rightarrow \mu g \cos\theta = 1.76$

$\therefore \mu = \dfrac{1.76}{10 \times 0.94} = 0.18$

85. We will observe the motion of the block in the reference frame attached to the moving paper.

In this frame, the initial velocity of the block is $\sqrt{v^2+u^2}$ in a direction making an angle θ with Y direction, as shown. The block will travel in this direction with friction force exactly opposite to velocity.



We will consider the motion of the block in y direction only.

Initial velocity $u_y = v$

y component of friction = $\mu mg.\cos\theta$ [along – ve y]

$\therefore$ y component of acceleration = $\mu g \cos\theta$

$= \mu g \dfrac{v}{\sqrt{u^2+v^2}}$ [along – *ve y*]

The maximum *v* can be obtained by assuming that the *y* component of velocity of the block becomes zero just when the block travels through a distance *L* in *y* direction.

$v_y^2 = u_y^2 - 2a_y .L$

$\Rightarrow O = v^2 - 2\mu g \dfrac{v}{\sqrt{u^2+v^2}}.L$

$v = \dfrac{2\mu g L}{\sqrt{u^2+v^2}}$

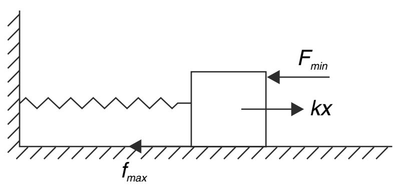
$v^2 (u^2 + v^2) = 4\mu^2 g^2 L^2$

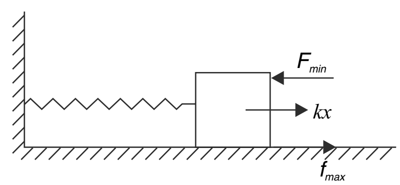
$\Rightarrow v^4 + u^2 . v^2 - 4\mu^2 g^2 L^2 = 0$

$\therefore\ v^2 = \dfrac{-u^2 \pm \sqrt{u^4 + 16\mu^2 g^2 L^2}}{2}$

$\therefore\ v^2 = -\dfrac{u^2}{2} + \sqrt{\dfrac{u^4}{4} + (2\mu g L)^2}$

86. Minimum force (F_{min}) is needed when the force and limiting friction (acting to left) together balance the spring force (*kx*).





$F_{min} + f_{max} = kx$

$8 + f_{max} = kx$ ....(1)

Maximum force that can be applied without moving the block can be calculated as

$F_{max} = f_{max} + kx$

$18 = f_{max} + kx$ ....(2)

The block will begin to move towards left if F is increased beyond this value.

Solving (1) and (2)

$kx = 13N$

$\therefore\ k\ (0.1) = 13 \Rightarrow k = 130\ N/m$

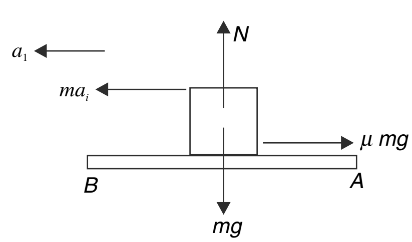
$f_{max} = 5N$

$\therefore\ \mu mg = 5$

$\mu \times 1 \times 10 = 5$

$\mu = 0.5$

87. When the plank is horizontal

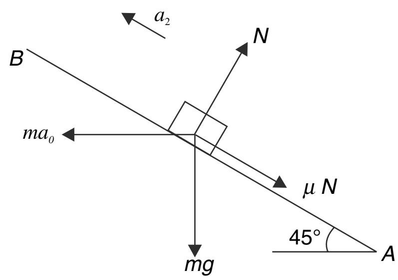


Let acceleration of block in reference frame of train be a_1.

$$ma_1 = ma_0 - \mu mg$$

$a_1 = a_0 - \mu g$ ....(1)

When the plank is inclined, let acceleration in reference frame of train be a_2.



$$N = \frac{mg}{\sqrt{2}} + \frac{ma_0}{\sqrt{2}}$$

and $ma_2 = \dfrac{ma_0}{\sqrt{2}} - \dfrac{mg}{\sqrt{2}} - \mu N$

$$a_2 = \frac{a_0}{\sqrt{2}} - \frac{g}{\sqrt{2}} - \mu\left(\frac{g}{\sqrt{2}} + \frac{a_0}{\sqrt{2}}\right)$$

$$a_2 = \left[(a_0 - g) - \mu(a_0 + g)\right]\frac{1}{\sqrt{2}} \qquad \text{..........(2)}$$

Let time to travel from A to B in horizontal position = t_1

Time in inclined position = t_2

$2^{5/4}\, t_1 = t_2$

$$2^{5/4}\sqrt{\frac{2L}{a_1}} = \sqrt{\frac{2L}{a_2}}$$

$4\sqrt{2}\, a_2 = a_1$ ....(3)

Substituting for a_1 and a_2 from (1) and (2) in equation (3)

$4\,[(a_0 - g) - \mu\,(a_0 + g)] = a_0 - \mu g$

$$\Rightarrow \mu = \left(\frac{3a_0 - 4g}{4a_0 + 3g}\right)$$

88.

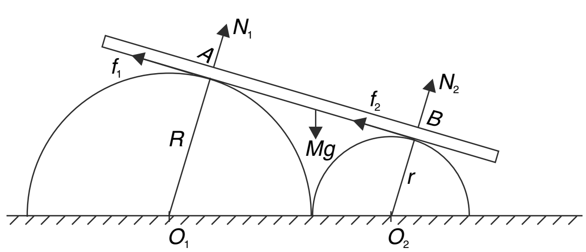


Figure (a)

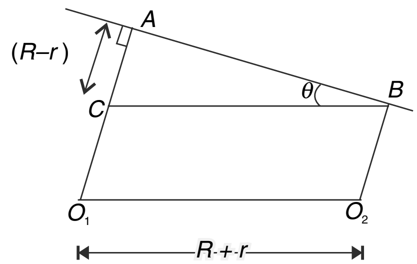


Figure (b)

In figure (b)

$$\sin\theta = \frac{R-r}{R+r} = \frac{1-\eta}{1+\eta}\left[\text{where } \eta = \frac{r}{R}\right] \quad .... (1)$$

The equilibrium of rod gives

$N_1 + N_2 = Mg \cos\theta$ .... (2)

and $\mu(N_1 + N_2) = mg \sin\theta$ .... (3)

Assuming the friction to be at its limiting value.

(3) ÷ (2) gives

$\tan\theta = \mu$

$$\therefore \sin\theta = \frac{\mu}{\sqrt{1+\mu^2}}$$

Put in (1)

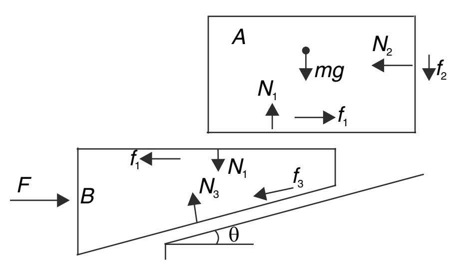
$$\frac{1-\eta}{1+\eta} = \frac{\mu}{\sqrt{1+\mu^2}}$$

$$\sqrt{1+\mu^2} - \eta\sqrt{1+\mu^2} = \mu + \mu\eta$$

$$\therefore \frac{\sqrt{1+\mu^2} - \mu}{\sqrt{1+\mu^2} + \mu} = \eta$$

If the ratio η is decreased, θ will increase and the rod will begin to slide.

89.



For A:

$N_2 = f_{1\max}$

$N_2 = \mu N_1$ ….......(1)

and $N_1 = f_{2\max} + Mg$

$N_1 = \mu N_2 + Mg$

$$N_1 = \mu^2 N_1 + Mg$$

$$\therefore\ N_1 = \frac{Mg}{1-\mu^2} \qquad \ldots\ldots\ldots(2)$$

For B:

$N_3 \cos\theta = f \sin\theta + N_1$

$$N_3\left(\cos\theta - \mu\sin\theta\right) = \frac{mg}{1-\mu^2} \qquad \ldots\ldots\ldots(3)$$

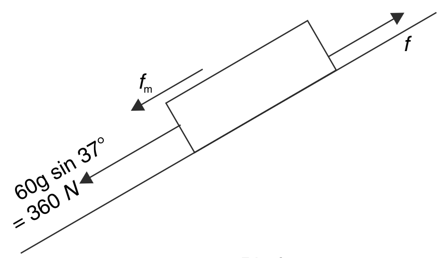
and $F = f_1 + f_3 \cos\theta + N_3 \sin\theta$

$= \mu N_1 + \mu N_3 \cos\theta + N_3 \sin\theta$

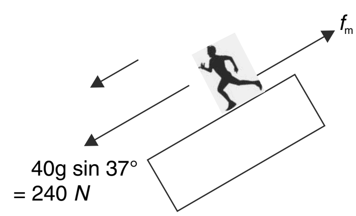
Substituting for N_3 from equation (3) we get

$$F = \frac{\mu Mg}{1-\mu^2} + \frac{\mu\cos\theta + \sin\theta}{\left(\mu\cos\theta - \mu\sin\theta\right)}\frac{Mg}{\left(1-\mu^2\right)}$$

90.



Platform



Man

Normal reaction by the incline on the platform is

$N = 100\ g \cos 37° = 800\ N$

$\therefore$ Maximum possible friction force on the platform (by incline) is

$f_{max} = \mu N = 400\ N$

The maximum friction by the man on the platform in downward direction can be $f_m = 40\ N$. This means that the man is experiencing friction force of 40 N up the plane.

Acceleration of man in this case $a = \dfrac{240-40}{40} = 5\ m/s^2$

If f_m is less than this value man will have higher acceleration.

If $f_m = 0$; acceleration of man $a = 6\ m/s^2$

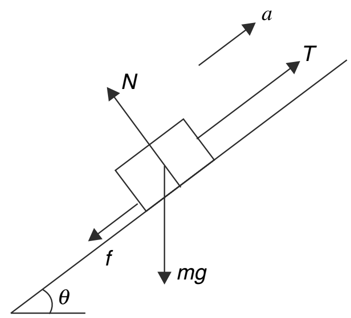
If acceleration of man is larger than 6 m/s^2, the friction f_m reverses direction.

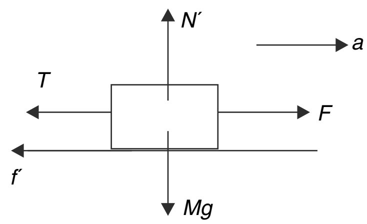
When $a = 7\ m/s^2$

$f_m = 40\ N$

Friction on platform by incline will adjust itself to 320 N.

91. Consider the case when m moves up the incline with acceleration 'a'.





For m

$$N = m\cos 37^{o} = \frac{4}{5}mg$$

$T - mg \sin 37° - \mu N = ma$

$$T - \frac{3}{5}mg - \frac{2}{5}mg = ma \qquad .....(1)$$

For M

$F - T - f' = Ma$ [put $\mu = \frac{1}{2}$]

$$40 - T - \frac{1}{2}Mg = Ma \qquad .....(2)$$

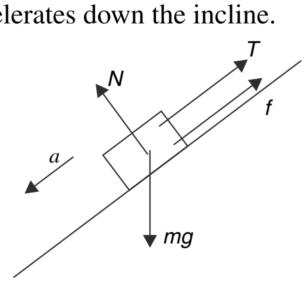
(1) + (2)

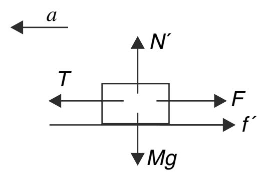
$$40 - mg - \frac{Mg}{2} = (m + M)a$$

For $a > 0$ $mg + \frac{Mg}{2} < 40$

$m < 2$

Similarly consider the case when m accelerates down the incline.





$mg \sin 37° - T - \mu.mg \cos 37° = ma$

$\frac{3}{5}mg - T - \frac{2}{5}mg = ma$

and $T - 40 - \mu Mg = Ma$

$T - 40 - \frac{Mg}{2} = Ma$ ….. (4)

(3) + (4) $\frac{mg}{5} - 40 - \frac{Mg}{2} = (M+m)a$

for $a > 0$ $\frac{mg}{5} > 40 + \frac{Mg}{2}$

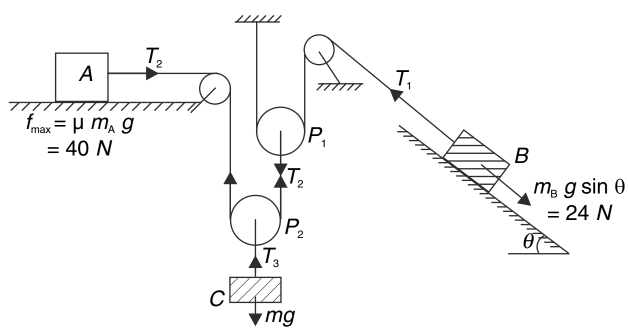
— $> 4 + 2$

$m > 30\ kg$

$\therefore$ No motion happens when

$2\ kg \le m \le 30\ kg$

92.



(a) For $a_B = 0$, $T_1 = 24\ N$

$\therefore T_2 = 48\ N$ and $T_3 = 96\ N$

$$a_A = \frac{T_2 - f_{\max}}{m_A} = \frac{48 - 40}{8} = 1\ m/s^2$$

If B does not move, P_1 does not move.

$\therefore$ Acceleration of C, $a_C = \frac{a_A}{2} = \frac{1}{2} m/s^2 (\downarrow)$

$\therefore\ Mg - T_3 = M.\frac{1}{2}$

$\therefore M\ (9.5) = 96$

$$M = \frac{960}{95}\,kg$$

(b) If A does not move

$T_{2\max} = 40\ N$

$\therefore\ T_1 = 20\ N$ and $T_3 = 80\ N$

With $T_1 = 20N$, B will accelerate down the incline.

$$a_B = \frac{24-20}{6} = \frac{2}{3}\ m/s^2\ (\searrow)$$

$$\therefore \text{ acceleration of } P_1 = \frac{1}{3}\ m/s^2\ (\uparrow)$$

$$\therefore \text{ acceleration of } P_2 = \frac{1}{6}\ m/s^2\ (\uparrow)$$

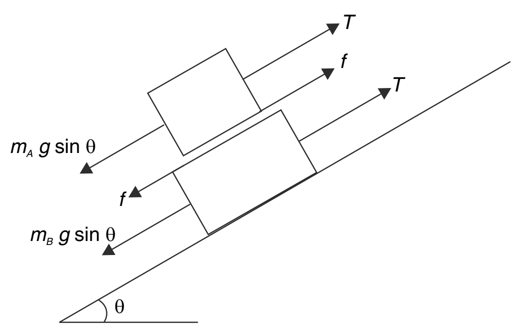
For C

$$T_3 - Mg = M.\frac{1}{6}$$

$$\therefore\ 80 = \frac{61}{6}M$$

$$\therefore\ M = \frac{480}{61}\,kg$$

93. If $m_A > m_B$, the direction of forces on the two blocks is as shown



There will be no acceleration if

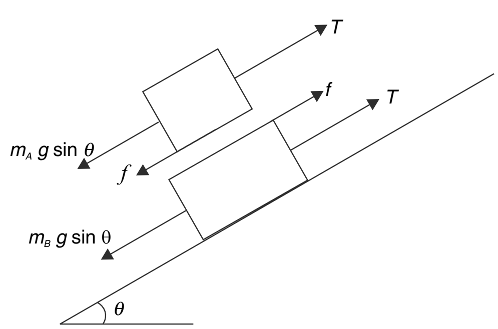
$m_A g \sin\theta < m_B g \sin\theta + f + f$

With friction at its peak value we get

$(m_A - m_B) g \sin\theta < 2\,\mu m_A g \cos\theta$

$$\Rightarrow \frac{\Delta m}{2m}\tan\theta < \mu$$

If $m_A < m_B$

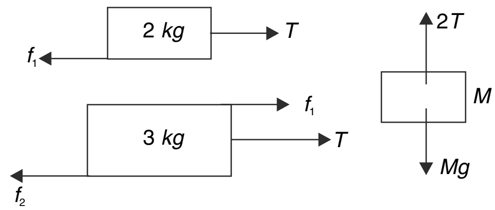


There will be no acceleration if

$m_B g \sin\theta < m_A g \sin\theta + 2f$

$$\therefore \frac{\Delta m}{2m}\tan\theta < \mu$$

94. (i)



$f_{1\max} = 0.75 \times 20 = 15\ N$

$f_{2\max} = 0.5 \times 50 = 25\ N$

When $2T > 25\ N$ the system will move

$\therefore$ For no motion $2T \le 25\ N$

$Mg \le 25\ N$

$M \le 2.5\ kg$

When $M = 2.5\ kg$

$f_1 = T = 12.5N$

(ii) If we assume that 2 *kg* and 3 *kg* block are moving together, then

$2T - f_{2\max} = 5a$ ………(i)

and $4g - 2T = 4a$ ……… (ii)

Solving (i) and (ii) $a = \frac{5}{3}\ m/s^2$

And $T = \frac{50}{3}N$

Considering only 2 *kg* block

$T - f_1 = 2 \times a$

$\Rightarrow f_1 = \frac{50}{3} - \frac{10}{3} = \frac{40}{3} = 13.33\ N$

This value of f_1 is possible (as $f_{1\max} = 15N$).

Hence, our assumption that both the 2 *kg* and 3 *kg* blocks are moving together is correct.

(iii) f_1 will still be $\frac{40}{3}N$

acceleration of M will also be same i.e.,

$a = \frac{5}{3}\ m/s^2$

(iv) $f_{2\max} = 0.9 \times 50 = 45\ N$

The 3 *kg* block can experience a maximum possible rightward force

$= T_{\max} + f_{\max} = 20 + 15 = 35\ N$

But friction f_2 can go upto 45 N.

Hence, 3 kg block will not move.

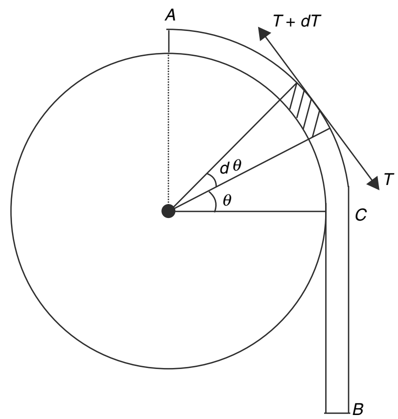
Let acceleration of M be a, then acceleration of 2 kg block will be $2a$.

$\therefore\ 40 - 2T = 4.a \quad \ldots(1)$

$T - 15 = 2.(2a) \quad \ldots(2)$

Solving, $a = 5/6\ m/s^2$

95. Let's calculate the tension in the string when no block is attached to the rope.



$$dT = \lambda\,(Rd\theta)g\cos\theta$$

$$\int_{T_C}^{T_A} dT = \lambda Rg\int_0^{\pi/2}\cos\theta\, d\theta$$

$$T_A - T_C = \lambda Rg$$

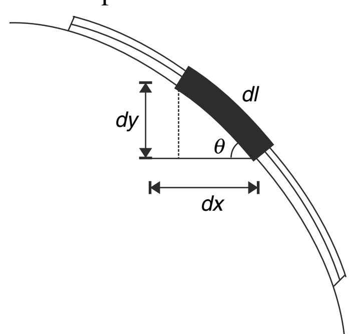
$$T_A - \lambda Rg = \lambda Rg$$

$$T_A = 2\,\lambda\, Rg$$

$$\therefore\ M_0 g = \lambda\left(\frac{\pi}{2}+1\right)Rg - 2\lambda Rg$$

$$\Rightarrow\ M_0 = \lambda R\left(\frac{\pi}{2}-1\right)$$

96. Consider a small segment of length dl on the rope.



The tangential component of weight

$= \lambda\, dl\, g.\sin\theta = \lambda g\,(dl\sin\theta) = \lambda g\, dy$

For complete rope, sum of tangential component of weight is

$$W_t = \lambda g\int_0^H dy = \lambda gH$$

Maximum friction force on the segment

$= \mu (\lambda\, dl\, g \cos \theta) = \mu\lambda g\, (dl \cos \theta) = \mu\lambda g\, dx$

Sum of friction on the complete rope

$$f_{\max} = \mu\lambda g \int_0^x dx = \mu\lambda g x$$

Rope will be in equilibrium if

$$f_{\max} \geq W_t$$
$$\mu\lambda g x \geq \lambda g H$$
$$\mu \geq \frac{H}{x}$$

97. (a) Let mass of each segment of the rope be

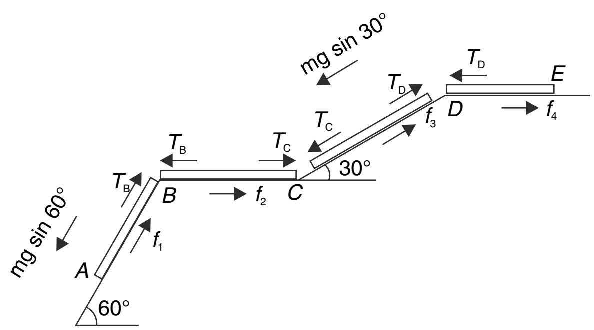
$$m = \frac{M}{4}$$

For rope to just remain in equilibrium

$$mg \sin 60° + mg \sin 30° = \mu\, mg \cos 60° + \mu\, mg + \mu\, mg \cos 30° + \mu\, mg$$

$$\Rightarrow \mu = \frac{\sqrt{3}+1}{\sqrt{3}+5} = 0.4$$

(b)



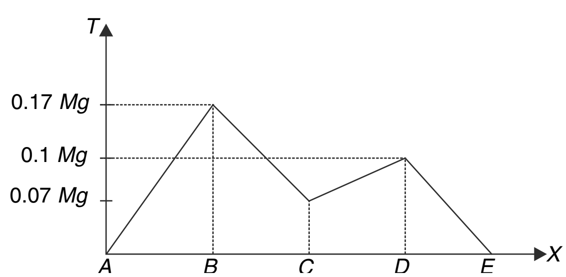
$T_B = mg \sin 60° - f_1 = 0.67\, mg = 0.17\, Mg$ $\quad [f_1 = \mu\, mg \cos 60°]$

$T_C = T_B - f_2 = 0.27\, mg = 0.07\, Mg$ $\quad [f_2 = \mu\, mg]$

$T_D = T_C + mg \sin 30° - f_3 = 0.4\, mg = 0.1 Mg$ $\quad [f_3 = \mu\, mg \cos 30°]$

$T_E = T_D - f_4 = 0$ $\quad [f_4 = \mu\, mg]$

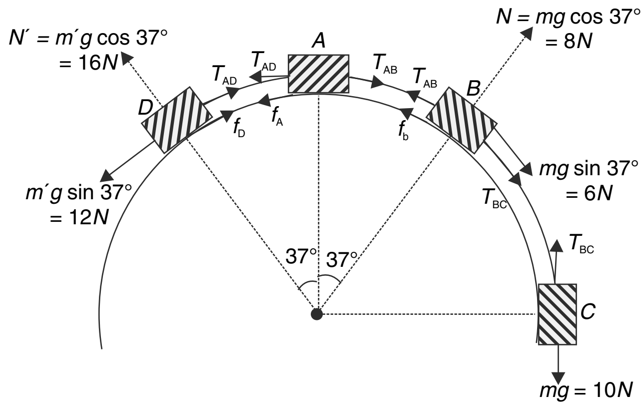
$\therefore\ T_D = \mu\, mg$



Maximum tension is at B

$T_{\max} = T_B = 0.17 Mg$

98.



$f_C = 0$

$f_{B\max} = \mu\, mg \cos 37° = 4\ N$

$f_{A\max} = \mu\, mg = 5\ N$

$f_{D\max} = \mu\, m'g \cos 37° = 8\ N$

If we look at the system as a whole, with no friction, it will move to right. The net driving force will be 10 + 6 – 12 = 4N. Friction is strong enough to prevent the blocks from moving.

Start with block C. There is no friction

$\therefore\ T_{BC} = 10N$

For B: $mg \sin 37° + T_{BC} = 16N$

Friction will assume its peak value of 4N but a tension $T_{AB} = 12\ N$ is required to maintain the block in equilibrium.

For A: $T_{AB} = 12\ N$ is balanced by friction $f_{A\max} = 5\ N$ and $T_{AD} = 7\ N$

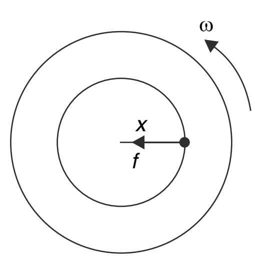
For D: It has tendency to slide down.

Friction is up along the tangent

$f_D = 12 - 7 = 5\ N$

Note that friction on D is not at its peak.

99. 75% of the surface becoming dust free means that only $\left(\frac{1}{4}\right)$th of the disc surface will remain covered with dust. This means all particles beyond $r > \frac{R}{2}$ will fly away.



For a dust particle at a distance x from the centre

$$N = mg$$

$$\text{and } f = m\omega^2 x$$

$$\text{But } f \le \mu N$$

$$\Rightarrow m\omega^2 x \le \mu\, mg$$

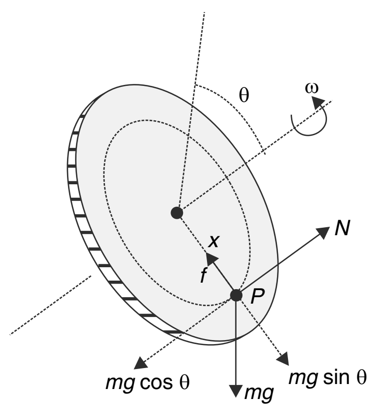
$$\Rightarrow \omega^2 \le \frac{\mu g}{x}$$

We want for $x \ge \frac{R}{2}$ the dust particles should fly away and for $x < \frac{R}{2}$ the friction should be able to provide the necessary centripetal force

$$\therefore\ \omega^2 = \frac{\mu g}{R/2} = \frac{g}{R} \quad \left[\because \mu = \frac{1}{2}\right]$$

$$\therefore\ \omega = \sqrt{\frac{g}{R}}$$

100. Consider a dust particle P rotating in a circle of radius x. We are considering the particle at the lowest position of the circular motion. This is the position where a particle has maximum chance of leaving the disc, if friction fails to provide the necessary centripetal force.



$N = mg \cos\theta$

and $f - mg \sin\theta = m\omega^2 x$

$f = mg \sin\theta + m\omega^2 x$

[If you consider the particle at the top most point friction force required will be least equal to $f = m\omega^2 x - \text{mg} \sin\theta$]

But $f \le \mu N$

$\therefore\ mg \sin\theta + m\,\omega^2 x \le \mu\, mg \cos\theta$

$$\Rightarrow\ \omega \le \sqrt{\frac{g(\mu\cos\theta - \sin\theta)}{x}}$$

putting $x = \frac{R}{2}$ [for 75% dust particles to fly off]

$$\omega = \sqrt{\frac{g(\mu\cos\theta - \sin\theta)}{\frac{R}{2}}}$$

$$= \sqrt{\frac{2g}{R}\left(\frac{1}{2}\cos\theta - \sin\theta\right)}$$

For $\theta = 0°$

$$\omega = \sqrt{\frac{2g}{R}\left(\frac{1}{2}\cos 0° - \sin 0°\right)}$$

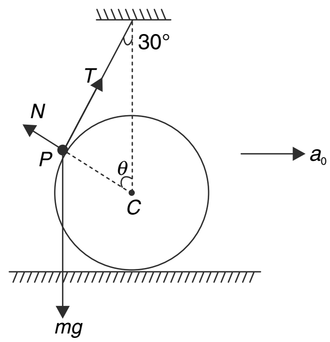
$$= \sqrt{\frac{g}{R}}$$

This matches with the result obtained in the previous problem.

101. Since the particle is small, the string will be tangential to the sphere. $\theta = 60°$ (see figure).

Let acceleration of the sphere be a_0 immediately after release

The particle will have its initial acceleration (a) along normal to the string towards PC. And, component of a_0 in the direction $\overrightarrow{PC}$ must be equal to a



$$\therefore\ a = a_0 \cos(90 - \theta) = a_0 \sin\theta = \frac{\sqrt{3}}{2} a_0$$

Force on the particle has been shown in the figure. The equation of motion along PC will be

$$mg\cos\theta - N = ma \Rightarrow \frac{1}{2}mg - N = \frac{\sqrt{3}}{2}ma_0 \qquad \text{...........(i)}$$

Sphere experiences a force N along PC. It has a horizontal component = $N \sin\theta$

$$\therefore\ N\frac{\sqrt{3}}{2} = Ma_0 \Rightarrow N = \frac{2}{\sqrt{3}}Ma_0 \qquad \text{...........(ii)}$$

From (i) and (ii) $\frac{1}{2}mg = \left(\frac{\sqrt{3}m}{2} + \frac{2M}{\sqrt{3}}\right)a_0$

$$a_0 = \frac{\sqrt{3}mg}{3m + 4M}$$

102. Let the spring connected to A get stretched by x. The other spring gets compressed by x.

In the reference frame attached to the rod, forces acting on the sleeve are –

$2\,kx$ towards A

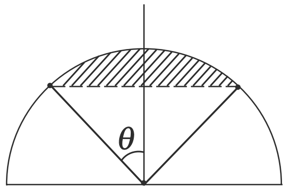
kx towards A

$m\omega^2(\ell_0 + x)$ towards B.

$\therefore\ 2kx + kx = m\omega^2(\ell_0 + x)$

$$x = \frac{m\omega^2 \ell_0}{3k - m\omega^2} = \frac{\ell_0}{3\eta - 1}$$

103.



The solid angle subtended by the spherical cap at the centre of the hemisphere is $\Omega = 2\pi\,(1 - \cos\theta)$

Surface area of the cap

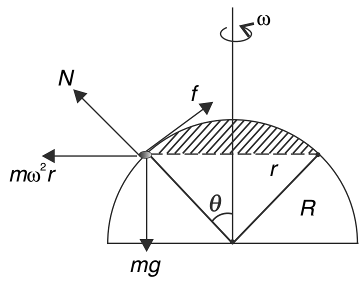
$s = \Omega R^2 = 2\pi R^2 (1 - \cos\theta)$

If $s = (0.2)(2\pi R^2)$ then

$0.2 = 1 - \cos\theta$

$\Rightarrow \cos\theta = 0.8 \Rightarrow \theta = 37°$

A dust particle at an angle $\theta > 37°$ will slide. A particle at $\theta = 37°$ is in limiting equilibrium in the reference frame of the hemisphere.



$N = mg\cos\theta - m\omega^2 r\sin\theta$

And $\mu N = mg\sin\theta + m\omega^2 r\cos\theta$

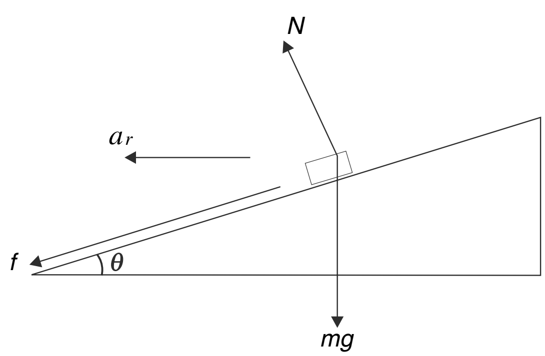
$\Rightarrow \mu\,(mg\cos\theta - m\omega^2 r\sin\theta) = mg\sin\theta + m\,\omega^2 r\cos\theta$

Where $r = R\sin\theta = 0.1 \times 0.6 = 0.06$

$$\therefore\ \mu = \frac{g\sin\theta + \omega^2 r\cos\theta}{g\cos\theta - \omega^2 r\sin\theta}$$

$$= \frac{10 \times 0.6 + 10^2 \times 0.06 \times 0.8}{10 \times 0.8 - 10^2 \times 0.06 \times 0.6} = \frac{0.6 + 0.48}{0.8 - 0.36} = 2.45$$

104.



As the speed of car increases, the friction force (in direction shown) increases in strength. For a car travelling at speed V

$$N\sin\theta + f\cos\theta = \frac{mV^2}{R} \qquad \text{........(1)}$$

$$N\cos\theta - f\sin\theta = mg \qquad \text{........(2)}$$

(1) ÷ (2)

$$\frac{V^2}{Rg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}$$

As V increases, f also increases. But maximum allowed value of f is μN

$$\therefore \frac{V_0^2}{Rg} = \frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta - \mu N\sin\theta}$$

$$\therefore \frac{V_0^2}{Rg} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}$$

$$\Rightarrow \tan\theta = \frac{\frac{V_0^2}{Rg} - \mu}{1 + \frac{\mu V_0^2}{Rg}}$$

$$\therefore \theta_0 = \tan^{-1}\left(\frac{\frac{V_0^2}{Rg} - \mu}{1 + \frac{\mu V_0^2}{Rg}}\right)$$

(b) For friction to be zero

$$N\sin\theta = \frac{mV_1^2}{R}$$

and $N\cos\theta = mg$

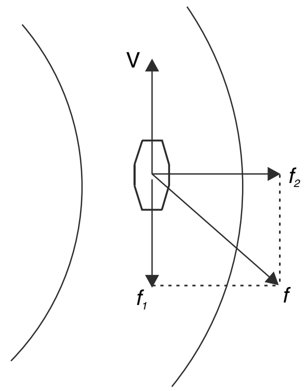
$$\Rightarrow \tan\theta_0 = \frac{V_1^2}{Rg}$$

$$\therefore V_1 = \sqrt{Rg\tan\theta_0}$$

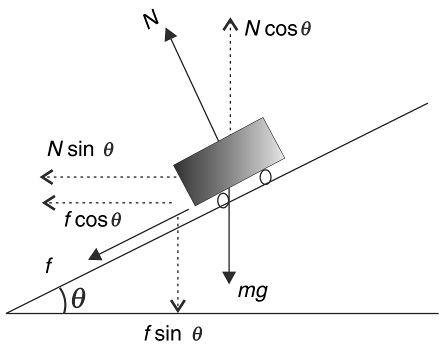
$$= \sqrt{Rg\left(\frac{\frac{V_0^2}{Rg} - \mu}{1 + \frac{\mu V_0^2}{Rg}}\right)}$$

(c) Since car is retarding, there is a component of friction (f_1) opposite to the direction of motion. There is another component of friction (f_2) opposite to that shown in previous fig . At very low speed $N\sin\theta$ exceeds the necessary centripetal force – and friction is directed so as to reduce the net force towards centre.

Resultant friction makes obtuse angle with the direction of motion.



105. (a) Banking angle is given by



$\tan\theta = \dfrac{V^2}{Rg}$ where V = correct speed = 20 m/s

$\therefore \tan\theta = \dfrac{400}{100\times 10} = 0.4$

$\therefore\ \theta = \tan^{-1}(0.4) = 22°$

(a) <u>when $V = 30\ m/s$</u>

Friction is directed inwards.

In horizontal direction $N\sin\theta + f\cos\theta = \dfrac{mV^2}{R}$

In vertical direction $N\cos\theta = f\sin\theta + mg$

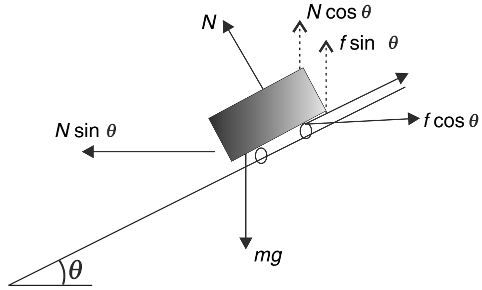
Eliminating N between two equations

$$f(\tan\theta.\sin\theta + \cos\theta) = \frac{mV^2}{R} - mg\tan\theta$$

$$\Rightarrow f[0.4\times 0.375 + 0.93] = \frac{500\times 900}{100} - 500\times 10\times 0.4$$

$\Rightarrow f = 2315\ N$

(ii) When $V = 10\ m/s$



The vehicle has a tendency to slip in and friction is directed upwards. Thus,

$$N\sin\theta - f\cos\theta = \frac{mV^2}{R}$$

And $N\cos\theta + f\sin\theta = mg$

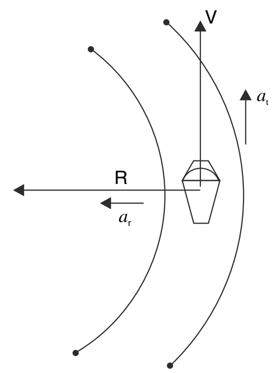
Again eliminating N we get

$$f\left[\tan\theta.\sin\theta + \cos\theta\right] = mg\tan\theta - \frac{mV^2}{R}$$

Putting $V = 10\ m/s$ and $\theta = 22°$ we get

$f = 1398\ N$

106. (a) The speed of car increases uniformly at a rate f.



It means, tangential acceleration of the car is $a_t = f$ and motion is non – uniform circular. At time t, its speed is

$$V = V_0 + ft$$

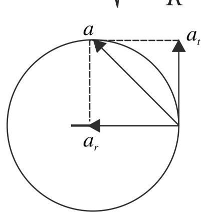
Thus, radial (centripetal) acceleration at time t is

$$a_r = \frac{V^2}{R} = \frac{(V_0 + ft)^2}{R}$$

The resultant acceleration of the car is

$$a = \sqrt{a_r^2 + a_t^2}$$

$$= \sqrt{\frac{(V_0 + ft)^4}{R^2} + f^2}$$



This acceleration is provided by friction force, f which has a maximum value

$f = \mu mg$

If mass times acceleration exceeds this maximum value of friction the car skids

$\therefore$ Car skids if

$$m\sqrt{\frac{(V_0+ft)^4}{R^2}+f^2} \geq \mu mg$$

$$\Rightarrow \frac{(V_0+ft)^4}{R^2}+f^2 \geq \mu^2 g^2$$

$$\Rightarrow (V_0 + ft) \geq [R^2(\mu^2 g^2 - f^2)]^{1/4}$$

$$\Rightarrow t \geq \frac{\left[R^2\left(\mu^2 g^2 - f^2\right)\right]^{1/4} - V_0}{f}$$

$$\therefore\ t_0 = \frac{\left[R^2\left(\mu^2 g^2 - f^2\right)\right]^{1/4} - V_0}{f}$$

(c) For $t < t_0$, friction adjusts itself so as to provide the necessary acceleration a

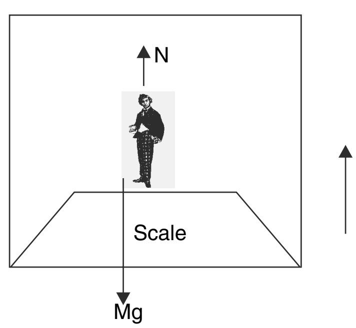
$\therefore f = ma$

$$= m\sqrt{\frac{(V_0+ft)^4}{R^2}+f^2}$$

The direction of friction is in the direction of a as shown in fig.

107. (a) Total impulse of all the force acting on the man is zero in 0 to 9.0 s interval.

$$\therefore \int_0^g (N - Mg)\,dt = 0$$



Integration is easily obtained by area under the graph.

$$\frac{1}{2}\times 100\times[3+2.6]-(F_0)\times 3 = 0$$

$F_0 = 93.3\ N$

(b) Maximum change in weight = 100 N

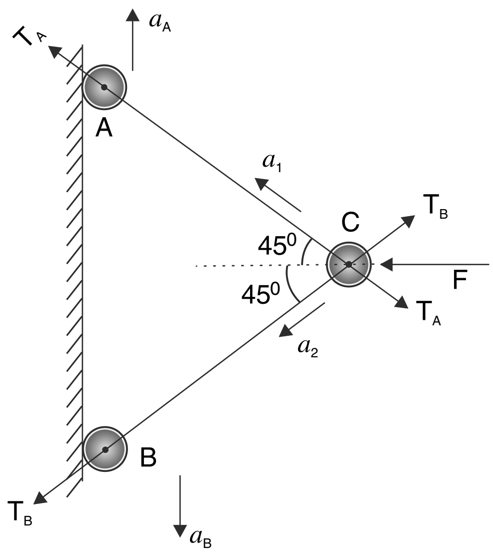
$\therefore 70 \times a_{max} = 100$

$$a_{max} = \frac{10}{7}\ m/s^2$$

(c) $70 \times V_{\max} = \frac{1}{2} \times 100 \times [3 + 2.6]$

$\Rightarrow V_{\max} = 4\ m/s$

108.



Component of acceleration of C can be assumed along the two rods (the rods are $\perp_r$) as a_1 and a_2.
Since rods are rigid

$$a_1 = \frac{a_A}{\sqrt{2}}$$

$$a_2 = \frac{a_B}{\sqrt{2}}$$

For A $\frac{T_A}{\sqrt{2}} = ma_A$ ………..(1)

For B $\frac{T_B}{\sqrt{2}} = 2ma_B$ ………..(2)

For C $\frac{F}{\sqrt{2}} - T_A = ma_1$ ………..(3)

And $\frac{F}{\sqrt{2}} - T_B = ma_2$ ………..(4)

Using (2) and (4)

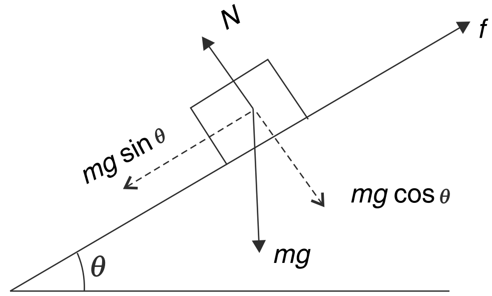
$$\frac{F}{\sqrt{2}} - 2\sqrt{2}ma_B = ma_2$$

$$\frac{F}{\sqrt{2}} - 2\sqrt{2}ma_B = m\frac{a_B}{\sqrt{2}} \quad \left(\because a_2 = \frac{a_B}{\sqrt{2}}\right)$$

$$\therefore \frac{F}{\sqrt{2}} = ma_B\left(\frac{1}{\sqrt{2}} + 2\sqrt{2}\right)$$

$$\frac{F}{5m} = a_B$$

109.



(a) If the 4 *kg* block is placed on the incline separately, then driving force on it is

$$mg\sin\theta = 4\times 1\times\frac{1}{2} = 20\ N$$

The opposing force is friction, which can attain a maximum value of

$f = \mu N = 0.5 \times mg \cos 30°$

$= 0.5 \times 4 \times 10 \times 0.86$

$= 17.2 N$

Therefore, 4 *kg* block will accelerate down if allowed to move separately.

For 2 *kg* block (if allowed to move separately).

Driving force = $mg \sin 30° = 2\times 10\times\frac{1}{2} = 10\ N$

Opposing force = $\mu mg \cos 30°$

$= 0.8 \times 2 \times 10 \times 0.86 = 13.76\ N$

$\therefore$ 2 *kg* block has no tendency to slide down. When placed together on the incline, 4 *kg* block will accelerate down but 2 *kg* block will remain stationary behind it.

$\therefore$ Contact force = zero.

Acceleration of 4 *kg* block is given by

$ma = mg \sin 30° - \mu mg \cos 30°$

$\Rightarrow a = g (\sin 30° - \mu \cos 30°)$

$$= 10\left[\frac{1}{2} - 0.5\times 0.86\right]$$

$= 0.7\ m/s^2$

(b) $\mu_1 = 0.8$, $\mu_2 = 0.5$

In this case (if allowed to move separately) driving force on 4 *kg* block = $mg \sin 30° = 20\ N$.

Opposing force on 4 *kg* block = $\mu_1 mg \cos 30° = 0.8 \times 4 \times 10 \times 0.86$

$= 27.52\ N$

Driving force on 2 *kg* block = 10 N

Opposing force on 2 *kg* block = $\mu_2 mg \cos 30° = 8.6\ N$

The 4 *kg* block has no tendency to move but 2 *kg* block has. Since 2 *kg* block is placed behind 4 *kg* block, it tries to push the 4 *kg* block. Whether motion will take place depends on the magnitude of driving force and opposing force on the combined system.

Net driving force = 20 + 10 = 30 *N*

Net opposing force = 27.52 + 8.6 = 36.12 *N*

$\therefore$ Motion will not take place and acceleration of both = 0.

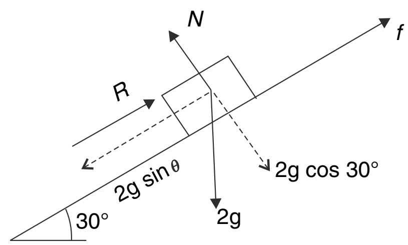
Since, the 2 *kg* block is pushed against the 4 *kg* block, there will be normal contact force (Say R) between them.

To know the contact force (R) consider the equilibrium of 2 *kg* block (see fig.)

$R + \mu N = 2g \sin 30°$

$\Rightarrow R = 2\,g \sin 30° - \mu 2g \cos 30° = 1.4\,N$

2 *kg* block applies equal and opposite force on 4 *kg* block



(c) $\mu_1 = 0.6;\ \mu_2 = 0.1$

In this case (if blocks are allowed to move separately)

Driving force on 4 *kg* = 20 *N*

Opposing force on 4 *kg* = 0.6 × 4 × 10 × 0.86 = 20.64 *N*

Driving force on 2 *kg* = 20 *N*

Opposing force on 2 *kg* = 0.1 × 2 × 10 × 0.86 = 1.72 *N*

Net driving force = 30 *N*

Net opposing force = 22.36 *N*

4 *kg* block when left alone has no tendency to move. It is pushed by 2 *kg* block and motion takes place.

$$a = \frac{\text{Net driving force– opposing force}}{\text{Total mass}}$$

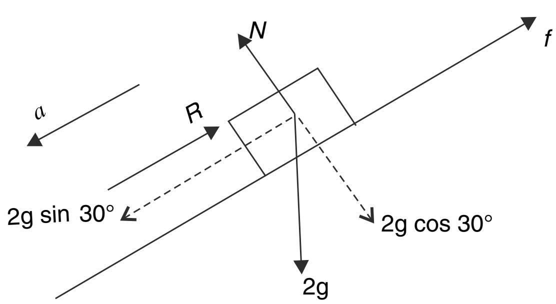
$$= \frac{30\,N - 22.36\,N}{6\,kg} = 1.27\ m/s^2$$

To know the contact force we can consider the free body diagram of either blocks. Here we consider the 2 *kg* block.

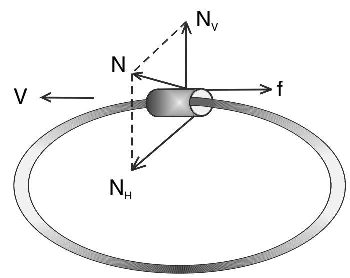
$2g \sin 30° - R - \mu\,(2g) \cos 30° = 2a$

$$\Rightarrow R = 20\left(\frac{1}{2} - 0.1 \times 0.86\right) - 2 \times 1.27$$

= 8.28 – 2.54 = 5.74 *N*



110.



Vertical normal reaction $N_v = mg = 0.1 \times 10 = 1\ N$

Horizontal normal $N_H = \frac{mv^2}{R} = \frac{0.1 \times 2^2}{0.3} = \frac{4}{3} N$

Resultant normal force

$$N = \sqrt{N_v^2 + N_H^2} = \frac{5}{3} N$$

Friction $f = \mu N = \frac{4}{3} N$

Resultant force by rod

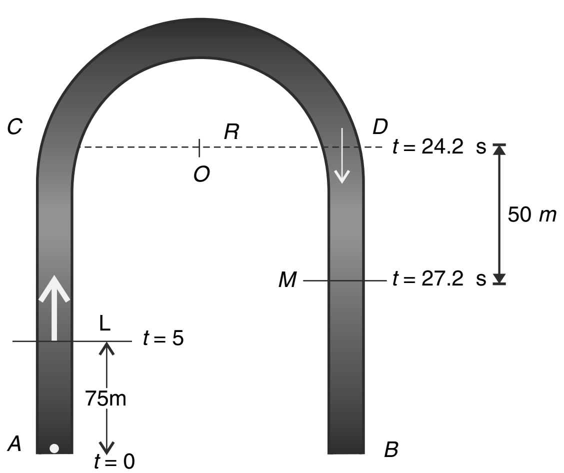
$$R = \sqrt{N^2 + f^2} = \frac{\sqrt{41}}{3} N$$

Angle made by R with vertical

$$\cos \quad = \text{—} = \frac{1}{\sqrt{41}} = \frac{3}{\sqrt{41}}$$

$$\theta = \cos^{-1}\left(\frac{3}{\sqrt{41}}\right)$$

111.



Maximum speed allowed on the circular portion is

$V_0 = \sqrt{\mu Rg} = \sqrt{0.6 \times 150 \times 10} = 30\ m/s$

The car will accelerate with maximum acceleration $a = \mu g = 6\ m/s^2$

It acceleration for 5 seconds to acquire a speed of 30 m/s and covers a distance $\frac{1}{2} \times 6 \times 5^2 = 75\ m$ in the period.

Car moves from L to C to D with this constant speed.

Time needed to move from L to D is

$= \frac{105 + \pi \times 150}{30} = 19.2\ s$

Hence, car reaches D at $t = 24.2\ s$. After this it accelerate to acquire the top speed of 50 m/s (= 180 kph)

Time required for acceleration $= \frac{50 - 30}{6} = 3.3\ s$

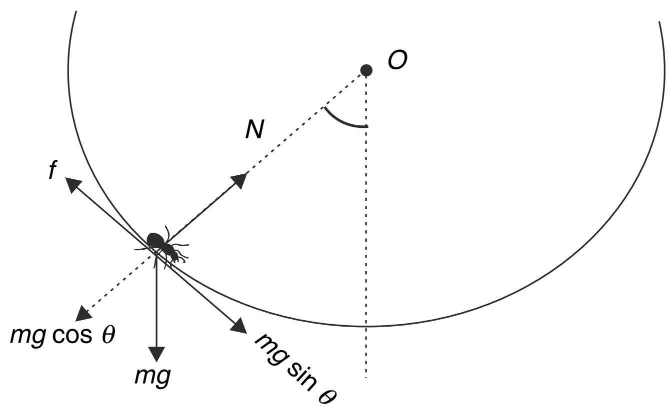
Distance covered in this period $= \frac{50^2 - 30^2}{2 \times 6} = 50\ m$

Remaining distance of 130 m is covered at a speed of 50 m/s.

Time required for this is $= \frac{130}{50} = 2.6\ s$

$\therefore$ Total time = 24.2 + 3.3 + 2.6 = 30.1 Sec.

112. For stationary bowl –



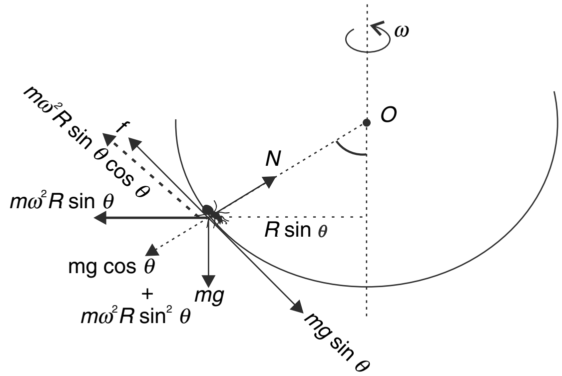
Insect can move upto $\theta = 45°$

$\Rightarrow f_{max} = mg \sin \theta$ [for $\theta = 45°$]

$\mu mg \cos \theta = mg \sin \theta$

$\Rightarrow \mu = 1.0$

For rotating bowl



In reference frame of bowl

$N = mg\cos\theta + m\omega^2 R\sin^2\theta$

[Insect moves slowly]

If insect can just reach $\theta = 60°$

$\mu N + m\omega^2 R\sin\theta\cos\theta = mg\sin\theta$

$mg\cos\theta + m\omega^2 R\sin^2\theta + m\omega^2\sin\theta\cos\theta = mg\sin\theta$ [$\because\ \mu = 1$]

$\omega^2 R\sin\theta(\sin\theta + \cos\theta) = g(\sin\theta - \cos\theta)$

$$\omega^2 R\frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right) = g\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$$

$$\Rightarrow \omega^2 R = g\frac{2\left(\sqrt{3}-1\right)}{\sqrt{3}\left(\sqrt{3}+1\right)}$$

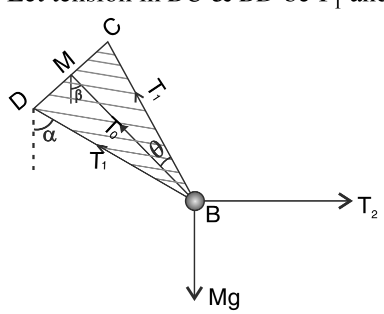
$$\omega = \sqrt{\frac{g}{R}\frac{2\left(\sqrt{3}-1\right)}{\sqrt{3}\left(\sqrt{3}+1\right)}}$$

(b) for $\theta = 90°$

$\omega^2 R = g$

$$\omega = \sqrt{\frac{g}{R}}$$

113. Let tension in BC & BD be T_1 and that in string BA be T_2.



The string CB and DB make an angle of $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ with vertical because the diagonal of a cube makes $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ angle with a side.

Line BM makes $\beta = 45°$ with vertical.

$$CB = \frac{\sqrt{3}\,a}{2}$$

$$CM = \frac{a}{2}$$

$\therefore \sin\theta = \dfrac{1}{\sqrt{3}}$

Resultant of tension in CB and DB is along BM equal to

$T_0 = 2T_1\cos\theta = 2T_1\sqrt{\dfrac{2}{3}}$

Vertical component of T_0 balance Mg and its horizontal component is equal to T_2.

$\therefore\ T_0 \cos\beta = Mg$

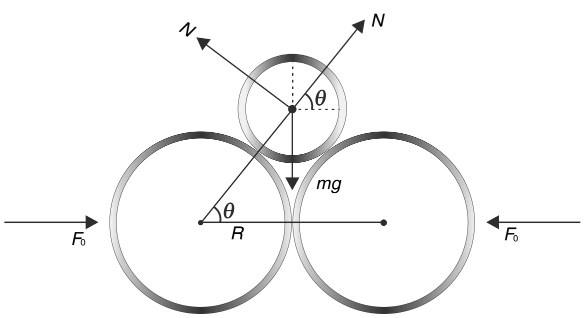
$T_0 \sin\beta = T_2$

$\Rightarrow 2\sqrt{\dfrac{2}{3}}\,T_1\sin\beta = T_2$

$\Rightarrow 2\sqrt{\dfrac{2}{3}}\,T_1\dfrac{1}{\sqrt{2}} = T_2$

$\Rightarrow \dfrac{T_2}{T_1} = \dfrac{2}{\sqrt{3}}$

114.



Radius of the circular plate is $r = R\,(\sec\theta - 1)$

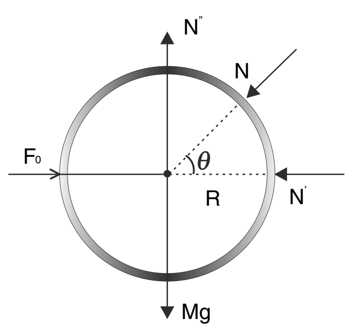
Mass of the plate is m = $\sigma\pi r^2$

$= \sigma\pi R^2\,(\sec\theta - 1)^2$

For equilibrium of the plate -

$2\,N\sin\theta = \sigma\pi R^2\,(\sec\theta - 1)^2\,g$

$\Rightarrow N = \dfrac{\sigma\pi R^2 g\left(\sec\theta - 1\right)^2}{\sin\theta}$



For two discs to remain in contact, normal force between them; $N' \geq 0$.

If the discs are just in contact $N' = 0$

For horizontal equilibrium

$$F_0 = N\cos\theta = \frac{\sigma\pi R^2 g\cos\theta(\sec\theta - 1)^2}{\sin\theta}$$

$$F_0 = \frac{\sigma\pi R^2 g(1-\cos\theta)^2}{2\sin\theta.\cos\theta}$$

(b) When $\theta \to 0, \cos\theta \simeq 1 - \frac{\theta^2}{2}; \sin\theta \simeq \theta$

$$\Rightarrow F_0 = \frac{\sigma\pi R^2 g\frac{\theta^4}{4}}{2.\theta\left[1-\theta^2/2\right]} \to 0$$

(c) When

$$\theta \to \pi/2; F_0 \propto \frac{1}{\cos\theta} \to \infty$$

When $\theta \to \pi/2$, the circular plate will become massive. Even through N points almost vertically, its horizontal component still approaches infinity due to large magnitude of N.

115. O = centre of larger cylinder.

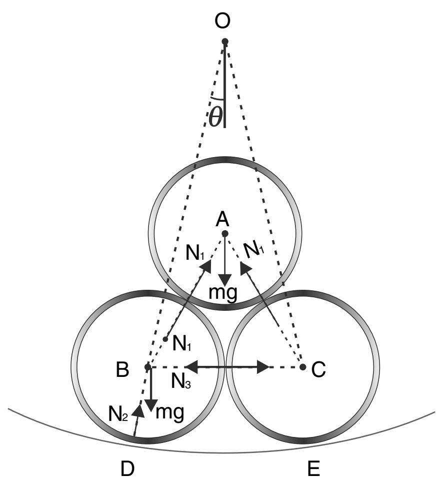
DO will pass through centre of lower cylinder (B)

N_1 = Contact force between A and B (and A and C)

N_2 = Contact force between B and the large cylinder.

N_3 = Contact force between B and C.

<u>For the vertical equilibrium of A</u>



$2N_1 \cos 30° = mg \Rightarrow \sqrt{3}\, N_1 = mg$ ....(1)

<u>For horizontal equilibrium of B</u>

$N_1 \sin 30° = N_2 \sin\theta$

$\frac{mg}{2\sqrt{3}} = N_2 \sin\theta$ ....(2)

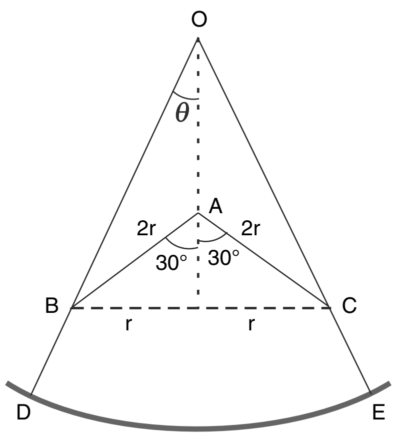
For vertical equilibrium of whole system

$2N_2 \cos \theta = 3\ mg$

$\therefore N_2 \cos\theta = \frac{3}{2}mg$ ....(3)

(2) ÷ (3)

$\tan\theta = \frac{1}{3\sqrt{3}} \Rightarrow \frac{r}{\sqrt{(R-r)^2 - r^2}} = \frac{1}{3\sqrt{3}}$



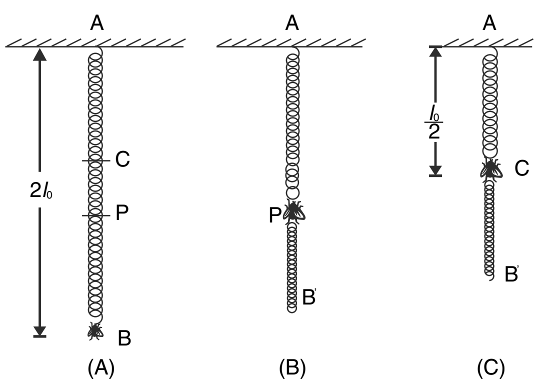
$\Rightarrow 27r^2 = (R - r)^2 - r^2$

$\Rightarrow (R - r)^2 = 28r^2$

$R - r = 2\sqrt{7}\ r$

$\therefore R = r\,(1 + 2\sqrt{7})$

116.



Initial extension in the spring is $\frac{mg}{k} = \ell_0$ $\left[\because k = \frac{mg}{\ell_0}\right]$

$\therefore$ Initial distance between the insect and the bug is $2\ell_0$

As the insect climbs up (slowly), the tension in the segment of the spring above the insect does not change (tension in segment AP = mg) and the segment of the spring below the insect becomes relaxed. It means the length of segment AP of the spring remains unchanged when the insect moves from B to P.

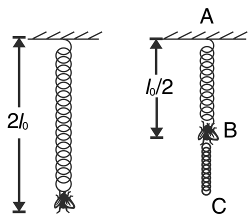
When the insect is at $\frac{1}{4}$th the original distance, it is at point C such that $AC = \frac{2\ell_0}{4} = \frac{\ell_0}{2}$

Looking at fig (A) and (B) one can realize that segment AC is $\frac{1}{4}$th of the spring (i.e., its elongated length = $\frac{\ell_0}{2}$). The natural length of segment $AC = \frac{\ell_0}{4}$.

Hence, length of the spring when the insect is at C is

$$= AC + CB = \frac{\ell_0}{2} + \frac{3\ell_0}{4} = \frac{5\ell_0}{4}$$

Alter: Let initial stretch in the spring be x_0



$kx_0 = mg$

$$\Rightarrow x_0 = \frac{mg}{mg/\ell_0} = \ell_0$$

$\therefore$ Initial distance between the insect and the bug = $2\ell_0$

(c) When insect is at B such that

$$AB = \frac{2\ell_0}{4} = \frac{\ell_0}{2}$$

Only the portion AB of the spring takes the load and the length BC remains unstretched.

In other words part AB behave as a separate spring supporting the insect.

Let natural length of segment AB be ℓ_1

Then force constant of AB is $k_1 = \frac{\ell_0}{\ell_1}k$

Extension in part $AB = \frac{\ell_0}{2} - \ell_1$

$$\therefore k_1\left(\frac{\ell_0}{2} - \ell_1\right) = mg$$

or, $\frac{\ell_0}{\ell_1}k\left(\frac{\ell_0}{2} - \ell_1\right) = mg$

or, $mg\left(\frac{\ell_0}{2} - \ell_1\right) = mg\ell_1 \quad [\because \ell_0 k = mg]$

or, $\frac{\ell_0}{2} = 2\ell_1$

or, $\ell_1 = \dfrac{\ell_0}{4}$

$\therefore$ Total length of spring = $AB + BC$

$$= \frac{\ell_0}{2} + \left[\ell_0 - \frac{\ell_0}{4}\right]$$

$$= \frac{5\ell_0}{4}$$

117. (a) Let T be the tension in the string and a_1, a_2, a_3, a_4 be acceleration of the four masses in downward direction.

The string has fixed length, which implies that sum of the displacements (and hence acceleration) of all four masses must be zero. In other words,

$a_1 + a_2 + a_3 + a_4 = 0$ ………(1)

<u>For m_1</u>

$m_1a_1 = m_1g - 2\,T$

$$\Rightarrow a_1 = g - \frac{2T}{m_1} \qquad \text{………(2)}$$

Similar equation can be written for a_2, a_3 and a_4 put all these in (1)

$$4g - 2T\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3} + \frac{1}{m_4}\right) = 0$$

$$\Rightarrow 2T = \frac{4g}{\dfrac{1}{m_1} + \dfrac{1}{m_2} + \dfrac{1}{m_3} + \dfrac{1}{m_4}} = 4mg$$

$$\left[\text{Where } \frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3} + \frac{1}{m_4}\right]$$

From (2)

$$a_1 = g - \frac{4mg}{m_1}$$

$$= g\left(1 - \frac{4m}{m_1}\right)$$

(b) Similarly, $a_4 = g\left(1 - \dfrac{4m}{m_4}\right)$

for a_4 to be positive

$$\frac{4m}{m_4} < 1$$

$$\frac{4}{m_4} < \frac{1}{m}$$

$$\frac{4}{m_4} < \frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3} + \frac{1}{m_4}$$

$$\frac{3}{m_4} < \frac{1}{1} + \frac{1}{2} + \frac{1}{3}$$

$$\frac{3}{m_4} < \frac{11}{6}$$

$$m_4 > \frac{18}{11}\ kg$$

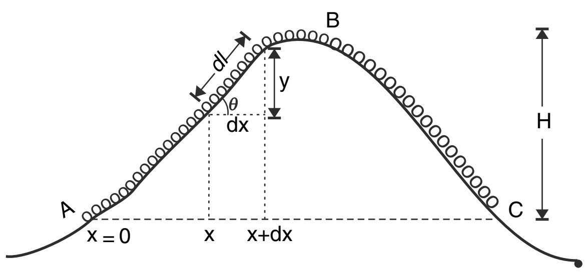
118. If F is increased beyond a limit, block D will slip over C. This is because friction force acting on D is responsible to accelerate $D + B + A$. Hence, for all the four block to move together –

Acceleration of the system produced by $F \leq$ acceleration produced by friction on D in the system $D + B + A$

$$\therefore \frac{F}{2m_1 + 2m_2} \leq \frac{\mu m_2 g}{m_1 + 2m_2}$$

$$\therefore F_0 = 2\mu m_2 g\left(\frac{m_1 + m_2}{2m_2 + m_1}\right)$$

119. Let's calculate the net force of gravity along the slope on the part AB of the chain



consider the small segment of the chain between

X and $X + dX$.

Gravitational force along the slope is

$\lambda d\ell g \sin\theta$ [λ = mass per unit length]

$\therefore$ Net gravity force along the slope on part AB is

$$\lambda g\int dl \sin\theta = \lambda g\int dy = \lambda gH$$

Thus, the force does not depend on the profile of the hill. It only depends on height of B from A.

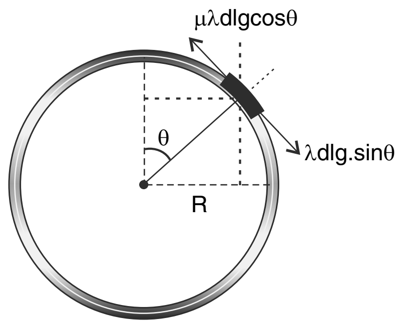
For portion BC also the force will be same. Hence, net force on the chain is zero.

Acceleration of the chain in zero.

120. From last question

$$\mu = \frac{H}{X} = \frac{R}{R} = 1.0$$

(a) At some lower point in the rope, if we take a small segment, the tangential component of weight is higher then friction on the segment. As we move up, friction increases and tangential component of the weight decrease. Tension will be increasing as we go up from bottom till the point where tangential component of weight on an element equal the friction force on it. After this point the friction on a segment gets larger than tangential component of weight. Tension starts decreasing.



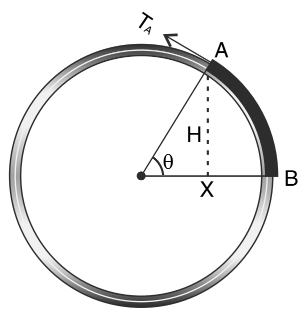
Tension is maximum where

$\mu\lambda dl\, g \cos\theta = \lambda dl\, \mathrm{g} \sin\theta$ [$\because$ $\mu = 1$]

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = 45°$

(b)



For segment *AB* of the rope (refer to last question)

$$w_t = \lambda g H = \lambda g R \sin 45° = \lambda g \frac{R}{\sqrt{2}}$$

and $f = \mu\lambda g X = 1\lambda g R\,(1 - \cos 45°) = \lambda g R \left(1 - \frac{1}{\sqrt{2}}\right)$

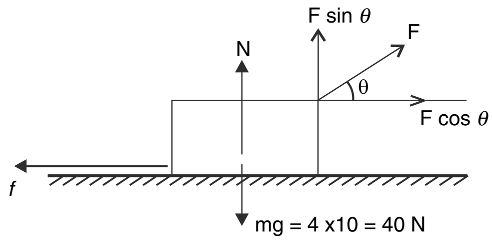
$\therefore$ Tension at *A* is

$$T_A = W_t - f = \lambda g R\,(\sqrt{2} - 1)$$

$$= \frac{M}{\frac{\pi R}{2}} g R\left(\sqrt{2} - 1\right)$$

$$= 2\left(\frac{\sqrt{2} - 1}{\pi}\right) Mg$$

122.



$N = 40 - F \sin \theta$ ....(1)

The block can move if

$F \cos \theta > f_{max}$

$F \cos \theta > \mu N$

$F \cos \theta > 0.5\ (40 - F \sin \theta)$

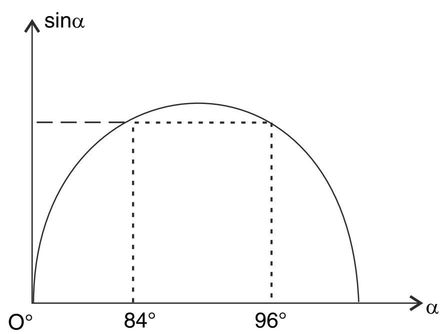
$18\ [\cos \theta + 0.5 \sin \theta] > 20$

$\Rightarrow \cos \theta + 0.5 \sin \theta > \dfrac{10}{9}$

$$\Rightarrow \sqrt{1^2 + 0.5^2}\left[\frac{1}{\sqrt{1^2 + 0.5^2}}\cos\theta + \frac{0.5}{\sqrt{1^2 + 0.5^2}}\sin\theta\right] > \frac{10}{9}$$

$$\Rightarrow \sqrt{1.25}\sin(\theta + \delta) > \frac{10}{9}$$

$$\left[\text{Where } \delta = \tan^{-1}\left(\frac{1}{0.5}\right) = \tan^{-1}(2) = 63^{\circ}\right]$$



$\therefore \sin (\theta + \delta) > \dfrac{10}{9.\sqrt{1.25}}$

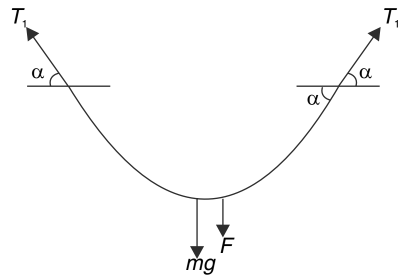
$\therefore 84° < \theta + \delta < 96°$

$\Rightarrow 84° - 63° < \theta < 96 - 63°$

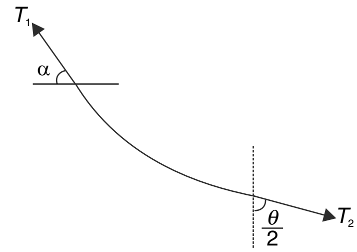
$21° < \theta < 33°$

124. For equilibrium of rope

$2T_1 \sin \alpha = F + mg$ ....(1)



Consider the equilibrium of half the rope



$$T_1 \cos\alpha = T_2 \sin\left(\frac{\theta}{2}\right) \qquad \text{.........(2)}$$

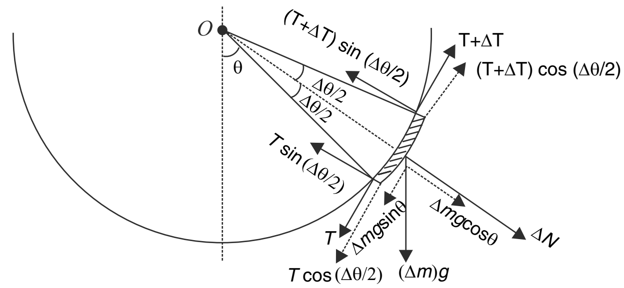
And also, $2T_2 \cos\left(\frac{\theta}{2}\right) = F \qquad \text{.........(3)}$

(2) ÷ (3) $\frac{1}{2}\tan\left(\frac{\theta}{2}\right) = \frac{T_1 \cos\alpha}{F}$

Using (1) $\tan\left(\frac{\theta}{2}\right) = \frac{(F+mg)\cos\alpha}{\sin\alpha . F}$

$\therefore\ \theta = 2\tan^{-1}\left[\left(\frac{F+mg}{F}\right)\cot\alpha\right]$

125. Consider an element of the rope having angular width Δθ.



Length of the element $= R\Delta\theta$

Mass of the element = $\lambda R \Delta\theta$

Tension at lower end of the element $= T$

Tension at upper end of the element = $T + \Delta T$

For tangential equilibrium of the element

$$(T+\Delta T)\cos\left(\frac{\Delta\theta}{2}\right)-T\cos\left(\frac{\Delta\theta}{2}\right)=\lambda R\Delta\theta.g\sin\theta$$

For small angle $\Delta\theta$, $\cos\left(\frac{\Delta\theta}{2}\right)\simeq 1$

$\therefore\ T + \Delta T - T = \lambda Rg\ \Delta\theta \sin\theta$

$\Delta T = \lambda Rg\ \Delta\theta.\ \sin\theta$

For $\Delta\theta \to 0$

$dT = \lambda Rg \sin\theta d\theta$ ....(1)

For equilibrium in radial direction

$$\Delta N + \lambda Rg\ \Delta\theta \cos\theta = T\sin\left(\frac{\Delta\theta}{2}\right)+(T+\Delta T)\sin\left(\frac{\Delta\theta}{2}\right)$$

$$\Delta N = T\frac{\Delta\theta}{2}+(T+\Delta T)\frac{\Delta\theta}{2}-\lambda Rg\Delta\theta.\cos\theta$$

[$\because$ for small $\Delta\theta$; $\sin\left(\frac{\Delta\theta}{2}\right)\to\frac{\Delta\theta}{2}$]

$\Delta T\Delta\theta = 0$ [product of very small quantities]

$\Delta N = T\Delta\theta - \lambda Rg\Delta\theta\ .\cos\theta$

For rope to remain in contact

$\Delta N \geq 0$ at all points

$\Rightarrow T \geq \lambda Rg \cos\theta$ at all points

At bottom (i.e., at $\theta = 0°$)

$T_1 \geq \lambda Rg$ ....(2)

From equation (1)

$dT = \lambda Rg \sin\theta\, d\theta$

$$\therefore \int_{T_1}^{T_0} dT = \lambda Rg\int_0^{\pi/2}\sin\theta\, d\theta$$

$T_0 - T_1 = \lambda Rg$

$\therefore\ T_0 = T_1 + \lambda Rg$

$\therefore\ T_0 \geq 2\lambda Rg$ [using (2)]

126. $\mu = \tan\theta_0 = 1$

$\therefore$ Maximum friction on the block $f_{max} = \mu mg\cos\theta = mg\ \frac{\sqrt{3}}{2}$

Block will begin to slide when resultant of $mg \sin\theta\left(=\frac{mg}{2}\right)$ and F will become just larger than f_{max}.

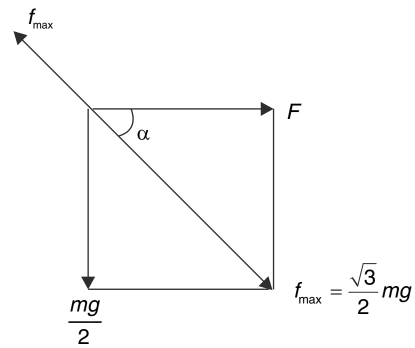
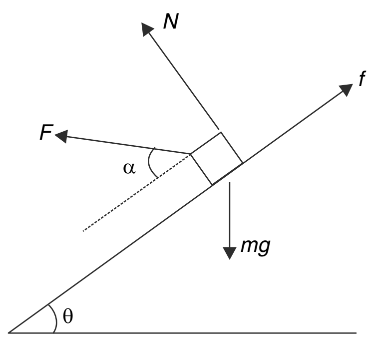


Fig. In the plane of the incline

From figure $\sin\alpha = \dfrac{mg/2}{\frac{\sqrt{3}}{2}mg}$

$$\sin\alpha = \frac{1}{\sqrt{3}}$$

127. Force shall be applied in the vertical plane through the line of greatest slope. Let the direction of the force make an angle α with the incline plane.



$N = mg\cos\theta - F\sin\alpha$

Block just moves if

$F\cos\alpha + mg\sin\theta = \mu N$

$F\cos\alpha + mg\sin\theta = \mu(mg\cos\theta - F\sin\alpha)$

$$\therefore\ F = \frac{mg(\mu\cos\theta - \sin\theta)}{\cos\alpha + \mu\sin\alpha}$$

F is minimum when $\cos\alpha + \mu\sin\alpha$ is maximum

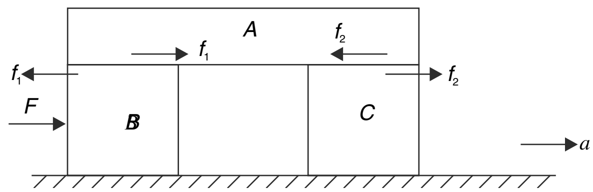
Maximum value of $\cos\alpha + \mu\sin\alpha$ is $\sqrt{1+\mu^2}$

$$\therefore\ F_{min} = \frac{mg(\mu\cos\theta - \sin\theta)}{\sqrt{1+\mu^2}}$$

$\theta = 30°$ and $\mu = 1$

$$\therefore\ F_{min} = \frac{mg\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)}{\sqrt{2}} = \frac{mg}{2\sqrt{2}}\left(\sqrt{3}-1\right)$$

128.



(a) Let the common acceleration be $a = \dfrac{F}{3M}$

For A $f_1 - f_2 = M.a$

For C $f_2 = Ma$

Adding $f_1 = 2Ma$

f_1 is larger than f_2.

$$f_1 = 2Ma = \frac{2F}{3} \le \mu \frac{Mg}{2}$$

$$\therefore\ F \le \frac{3}{4}\mu Mg$$

$$\therefore\ F_{\max} = \frac{3}{4}\mu Mg$$

(b) Since $f_1 > f_2$

The slippage will take place at surface between A and B.

(c) For $F = \dfrac{3}{8}\mu Mg$

$$a = \frac{\mu g}{8}$$

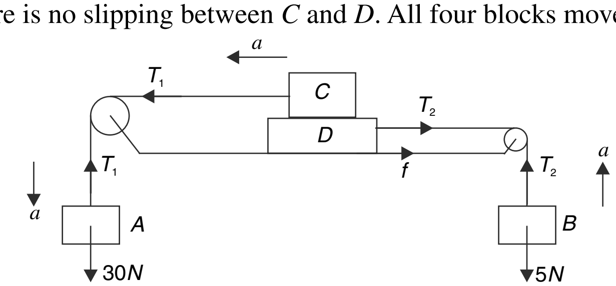
$$\therefore\ f_2 = Ma = \frac{\mu Mg}{8}$$

$f_1 - f_2 = Ma$

$$\therefore\ f_1 = 2Ma = \frac{\mu Mg}{4}$$

$$\therefore\ \frac{f_1}{f_2} = 2$$

129. (a) Let's assume that there is no slipping between C and D. All four blocks move with acceleration a.



For A: $30 - T_1 = 3a$ .... (1)

For B: $T_2 - 5 = 0.5a$ ....(2)

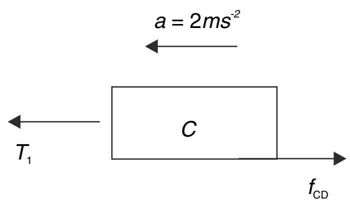
For $(C + D)$ $T_1 - T_2 - f = 4.5a$

$T_1 - T_2 - 0.2 \times 45 = 4.5a$ ....(3)

(1) + (2) + (3) $16 = 8a$

$\therefore\ a = 2\ ms^{-2}$

We must check whether C can have this acceleration without the friction force on it exceeding its peak limit.



From (1) $T_1 = 24\ N$

For C $T_1 - f_{CD} = 3.a$

$\therefore\ f_{CD} = 24 - 6 = 18\ N$

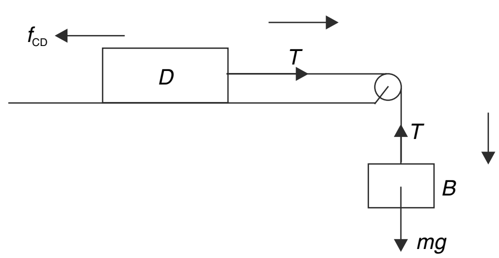
Peak value of $f_{CD} = \mu N = 0.7 \times 30 = 21\ N$

Hence, friction is within limit. Our assumption that all four are moving together is correct.

(b) If C and D move in opposite direction friction between them will be kinetic.

$f_{CD} = \mu N = 21\ N$

For $(B + D)$



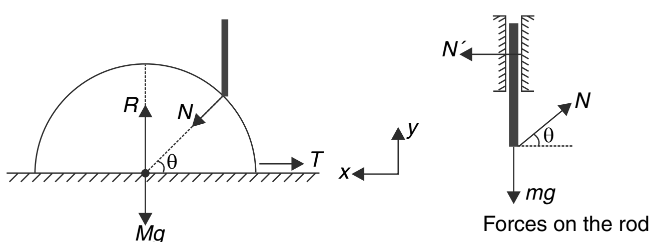
The block D will move to right (and B will move downward) if

$mg \geq f_{CD}$

$mg \geq 21$

$\therefore\ m \geq 2.1\ kg$

130.



(a) Forces on the hemisphere

Let R = Normal force by ground on the hemisphere

N = Normal force by the rod on the sphere

N' = Normal force by the wall on the rod

T = Tension in string.

For hemisphere

$R = Mg + N \sin \theta$ .... (1)

and $T = N \cos \theta$ .... (2)

For rod

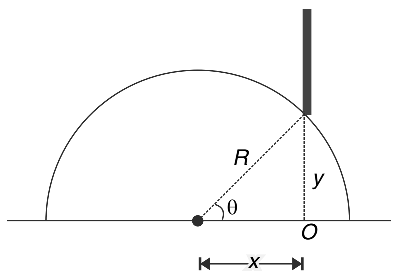
$N' = N \cos \theta$ .... (3)

And $N \sin \theta = mg$ ……..... (4)

Put value of N from (4) into (2)

$$T = \frac{mg}{\sin\theta}.\cos\theta = mg\cot\theta$$

(b)



From the figure

$y^2 + x^2 = R^2$

$$2y\frac{dy}{dt} + 2x\frac{dx}{dt} = 0$$

$$y\frac{dy}{dt} = -x\frac{dx}{dt}$$

[– *ve* sign indicates that as x increases, y will decrease i.e., the rod will move down]

We will discard the negative sign. Differentiating once again

$$\left(\frac{dy}{dt}\right)\left(\frac{dy}{dt}\right) + y.\frac{d^2y}{dt^2} = \left(\frac{dx}{dt}\right)\left(\frac{dx}{dt}\right) + x.\frac{d^2x}{dt^2}$$

Just after the string is cut, velocity of the rod as well as hemisphere is zero.

$$\Rightarrow \frac{dy}{dt} = \frac{dx}{dt} = 0$$

$$\therefore \quad \frac{d^2y}{dt^2} = \frac{x}{y}\frac{d^2x}{dt^2}$$

$a_y = (\cot \theta)\, a_x$ ………..(1)

a_y = downward acceleration of the rod

a_x = leftward acceleration of the hemisphere

Using the force diagram in the part (a) we can write

For hemisphere: $N \cos \theta = Ma_x$ ………..(2)

For rod $mg - N\sin\theta = ma_y$ …..........(3)

Solving (1), (2) and (3)

$$a_x = \frac{mg}{M\tan\theta + m\cot\theta}$$

131. Let the speed be v after time t when the block has moved through a distance S.

Normal reaction of the ring is $N = \frac{mv^2}{R}$

$$\therefore\ m\frac{dv}{dt} = -\mu N$$

$$\frac{dv}{dt} = -\frac{\mu v^2}{R}$$

$$\Rightarrow \int_u^v \frac{dv}{v^2} = -\frac{\mu}{R}\int_0^t dt$$

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{\mu}{R}t$$

$$\Rightarrow \frac{1}{v} = \frac{1}{u} + \frac{\mu}{R}t \qquad \text{…..........(1)}$$

$$\Rightarrow \frac{dt}{ds} = \frac{1}{u} + \frac{\mu}{R}t$$

$$\int_0^t \frac{dt}{\frac{1}{u} + \frac{\mu}{R}t} = \int_0^{\pi R} ds$$

$$\ln\left[\frac{1}{u} + \frac{\mu t}{R}\right] - \ln\left[\frac{1}{u}\right] = \frac{\mu}{R}\pi R$$

$$\ln\left[1 + \frac{\mu u}{R}t\right] = \pi\mu$$

$$1 + \frac{\mu u}{R}t = e^{\pi\mu}$$

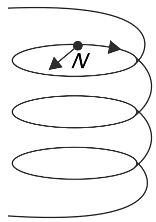
$$\therefore\ t = \frac{R}{\mu u}\left[e^{\pi\mu} - 1\right] \qquad \text{…..........(2)}$$

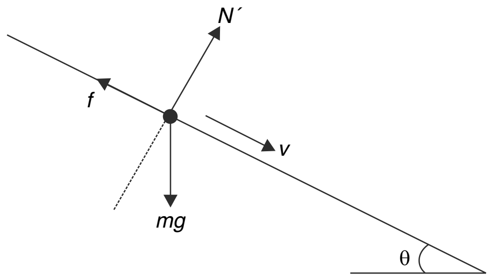
Put value of t from equation (2) into (1)

$$v = \frac{Ru}{R + \mu ut} = \frac{Ru}{R + R\left[e^{\pi\mu} - 1\right]}$$

$$v = \frac{u}{e^{\pi\mu}}$$

132. The helix wire is inclined to the horizontal at an angle given by





$\tan\theta = \dfrac{\rho}{2\pi R}$ ....(1)

The normal force applied by the wire on the bead can be thought to have two components.

N = directed horizontally towards axis of helix. (In the second fig. given above, this force is normal to the plane of the drawing pointing towards you)

N' = directed perpendicular to the velocity in the plane of above figure.

Force equations are (when the bead acquires constant speed)

$f = mg\sin\theta$ ....(2)

$N' = mg\cos\theta$ ....(3)

$$N = \frac{mv_H^2}{R}$$

$\Rightarrow N = \dfrac{mv^2\cos^2\theta}{R}$ ....(4)

Where v_H = Horizontal component of velocity = $v\cos\theta$

The resultant of two components of normal forces is

$$N_0 = \sqrt{N^2 + N'^2}$$

$$= \sqrt{(mg\cos\theta)^2 + \left(\frac{mv^2\cos^2\theta}{R}\right)^2}$$

$$= m\cos\theta\sqrt{g^2 + \frac{v^4}{R^2}\cos^2\theta}$$

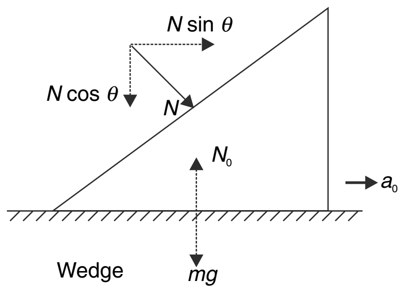
Now friction $f = \mu N$

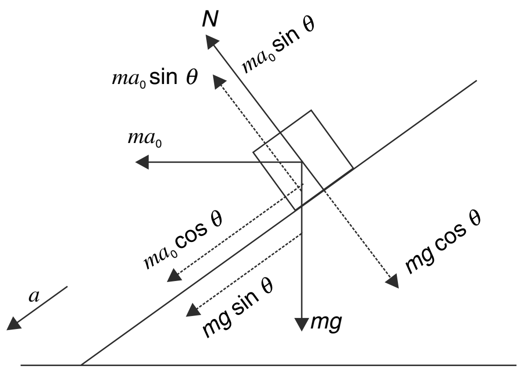
$\therefore$ from (2)

$$\mu mg\cos\theta\sqrt{1 + \frac{v^4}{R^2 g^2}\cos^2\theta} = mg\sin\theta$$

$$\therefore\ \mu = \frac{\tan\theta}{\sqrt{1+\left(\dfrac{v^2\cos\theta}{Rg}\right)^2}}$$

133. Force diagram of the wedge and that of block in the reference frame of the wedge are as shown.





For wedge: $N \sin\theta = ma_0$ ………… (1)

For block: (a = acceleration of block relative to wedge)

$N + ma_0 \sin\theta = mg\cos\theta$ ………… (2)

and $N\sin\theta = ma_0$ …………(3)

Solving (1), (2) and (3)

$$a_0 = \frac{g\sin\theta\cos\theta}{1+\sin^2\theta} = \frac{g\dfrac{1}{\sqrt{2}}\cdot\dfrac{1}{\sqrt{2}}}{1+\left(\dfrac{1}{\sqrt{2}}\right)^2} = \frac{g}{3}$$

$$a = \frac{2g\sin\theta}{1+\sin^2\theta} = \frac{2g\dfrac{1}{\sqrt{2}}}{1+\left(\dfrac{1}{\sqrt{2}}\right)^2} = \frac{2\sqrt{2}\,g}{3}$$

For block

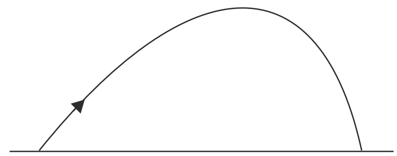
$a_x = a\cos\theta\,(\leftarrow) + a_0\,(\rightarrow) = a\cos\theta\,(\leftarrow) - a_0\,(\leftarrow)$

$$= \frac{2g}{3} - \frac{g}{3} = \frac{g}{3}(\leftarrow)$$

$$a_y = a\sin\theta = \frac{2g}{3}(\downarrow)$$

(b) Due to a_x the horizontal velocity of the block decreases. It is easy to show that $v_x = 0$ when the block lands on the table.

The block strikes the table normally. Path is as shown.



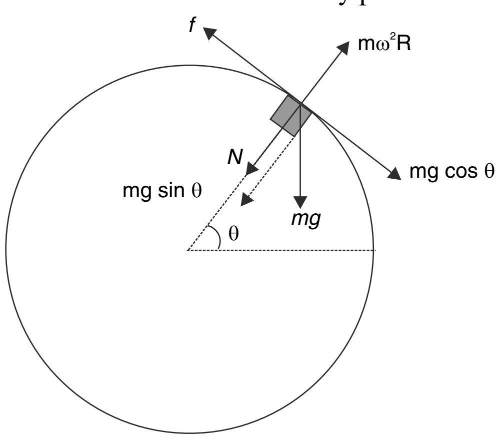
(c) At highest point velocity is horizontal (equal to that of wedge).

Yourself show that $v_x = \dfrac{u\cos\theta}{2} = \dfrac{u}{2\sqrt{2}}$

$$\therefore\ a_y = \frac{v_x^2}{R}$$

$$R = \frac{v_x^2}{a_y} = \frac{u^2}{8\times\frac{2g}{3}} = \frac{3u^2}{16g}$$

134. Consider the block at any position θ shown in the figure. [In reference frame of cylinder]



$N = m\omega^2 R - mg\sin\theta$ .... (1)

$f = mg\cos\theta$ .... (2)

$\because\ f \le \mu N$

$\therefore\ mg\cos\theta \le \mu\, m\omega^2 R - \mu mg\sin\theta$

$\therefore\ g[\cos\theta + \mu\sin\theta] \le \mu\omega^2 R$ [for all value of 'θ']

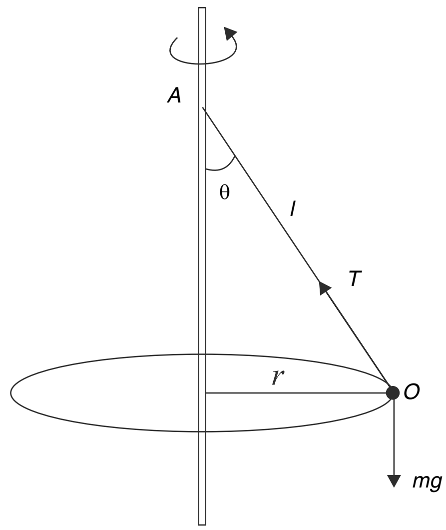
$\because$ maximum value of $\cos\theta + \mu\sin\theta$ will be $\sqrt{1+\mu^2}$

$$\therefore\ g\sqrt{1+\mu^2} \le \mu\omega^2 R$$

$$\therefore\ \omega \ge \sqrt{\frac{g\sqrt{1+\mu^2}}{R\mu}}$$

135.

(a) Consider a conical pendulum with one string (AO) only.



$r = l \sin\theta$ ……..(1)

$T \sin\theta = m\omega^2 r$

$T \sin\theta = m\omega^2 l \sin\theta$

$\Rightarrow T = m\omega^2 l$ ……... (2)

and $T \cos\theta = mg$

$\Rightarrow \cos\theta = \dfrac{mg}{m\omega^2 l} = \dfrac{g}{\omega^2 l}$ ……..(3)

For a conical pendulum to be possible

$\theta > 0°$

$\Rightarrow \cos\theta < 1 \Rightarrow \dfrac{g}{\omega^2 l} < 1$

$\sqrt{\dfrac{g}{l}} < \omega$

Hence, particle will stay at B if $\omega \le \sqrt{\dfrac{g}{l}}$

(b) When $\omega > \sqrt{\dfrac{g}{l}}$, the particle begins to rotate in a circle and the tension in string becomes greater than mg.

The second string will get taut only when $\theta = 60°$

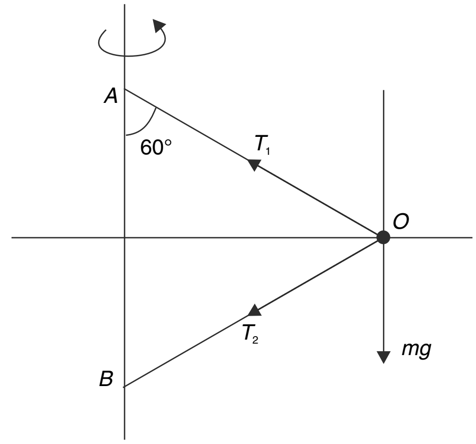
From (3) $\omega^2 = \dfrac{g}{l\cos 60°} = \dfrac{2g}{l}$

$\omega = \sqrt{\dfrac{2g}{l}}$

If $\omega > \sqrt{\dfrac{2g}{l}}$, there is tension in BO

$\therefore$ Answer is $\sqrt{\dfrac{g}{l}} < \omega \le \sqrt{\dfrac{2g}{l}}$

(c)



$T_1 \cos 60° = T_2 \cos 60° + mg$

$T_1 - T_2 = 2mg$ .... (4)

and $(T_1 + T_2) \sin 60° = m\omega^2 r$

$$\frac{\sqrt{3}}{2}(T_1 + T_2) = m\omega^2 \frac{\sqrt{3}}{2} l$$

$\Rightarrow T_1 + T_2 = m\omega^2 l$ .... (5)

For, $T_1 = 2T_2$, equation (4) and (5) give

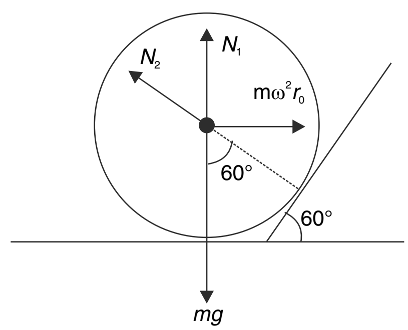
$T_2 = 2\ mg$

and $3T_2 = m\omega^2 l$

$\Rightarrow 6mg = m\omega^2 l$

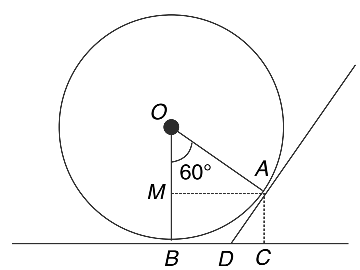
$$\Rightarrow \omega = \sqrt{\frac{6g}{l}}$$

136. Draw the free body diagram in reference frame of the container.



r_0 = radius of circular path of the centre of mass of the body

= 1.0 m



$BC = AM = r\sin 60° = \frac{\sqrt{3}}{2}r$

$AC = MB = r(1 - \cos 60°) = \frac{r}{2}$

$CD = AC \,.\, \cot 60° = \frac{r}{2}\cdot\frac{1}{\sqrt{3}}$:

$BD = BC - DC = \frac{\sqrt{3}}{2}r - \frac{r}{2\sqrt{3}} = \frac{2r}{2\sqrt{3}} = \frac{r}{\sqrt{3}} = 1.0$

$\therefore\ r_0 = 2.0 - BD$

$= 2 - 1 = 1.0\ m$

$N_2 \cos 60° + N_1 = mg$

$\frac{N_2}{2} + N_1 = mg$ .... (1)

and $N_2 \sin 60° = m\omega^2 r_0$

$N_2\frac{\sqrt{3}}{2} = m\omega^2$ $[r_0 = 1]$

$N_2 = \frac{2}{\sqrt{3}}m\omega^2$ .... (2)

Put in (1) $N_1 = mg - \frac{m\omega^2}{\sqrt{3}}$

(a) For $N_2 > N_1$

$\frac{2}{\sqrt{3}}m\omega^2 > mg - \frac{m\omega^2}{\sqrt{3}}$

$\frac{3}{\sqrt{3}}m\omega^2 > mg$

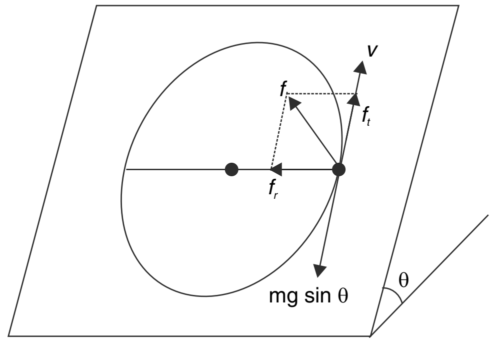
$\Rightarrow\ \omega > \sqrt{\frac{g}{\sqrt{3}}}$

(b) For $N_1 = 0$

$\frac{m\omega^2}{\sqrt{3}} = mg$

$\Rightarrow\ \omega = \sqrt{\sqrt{3}g}$

137. (a) Normal force on the car $N = mg\cos\theta$.



The friction has two roles to play. It has a component along the tangent (f_t) which balances mg sin θ, so that there is no tangential acceleration and it provides the necessary centripetal force also.

$\therefore\ f_t = mg \sin \theta$

$$f_r = \frac{mv^2}{R}$$

$\therefore$ friction

$$f = \sqrt{f_t^2 + f_r^2} = m\sqrt{(g \sin \theta)^2 + \left(\frac{v^2}{R}\right)^2}$$

But $f \le \mu N$

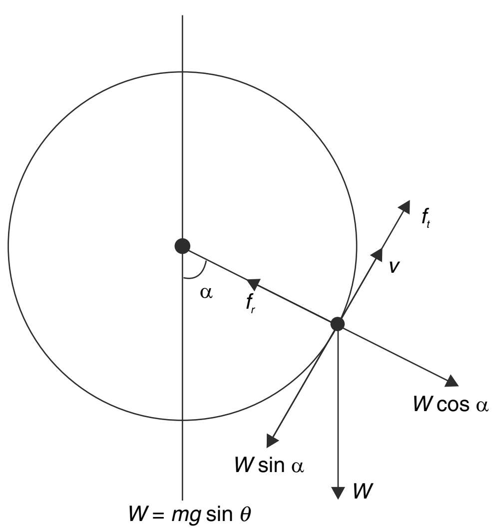
$$\Rightarrow\ m\sqrt{(g\sin\theta)^2 + \left(\frac{v^2}{R}\right)^2} \le \mu mg \cos\theta$$

$$\Rightarrow\ g^2 \sin^2\theta + \frac{v^4}{R^2} \le \mu^2 g^2 \cos^2\theta$$

$$\Rightarrow\ \frac{v^4}{R^2} \le g^2\left[\mu^2 \cos^2\theta - \sin^2\theta\right]$$

$\Rightarrow v \le [g^2R^2\ [\mu^2 \cos^2\ \theta - \sin^2\ \theta]]^{1/4}$

(b) For a given speed, the friction force will be largest at the bottom most point. To prove this consider the car at an angle α (as shown).



$$f_r = W\cos\alpha + \frac{mv^2}{R}$$

$f_t = W \sin \alpha$

$$\therefore\ f^2 = W^2 + \frac{m^2v^4}{R^4} + \frac{2mv^2}{R}W\cos\alpha$$

Obviously, f is maximum when $\cos \alpha = 1$; i.e., when $\alpha = 0°$
and, maximum friction for a given speed v is ,

$$f^2 = \left(W + \frac{mv^2}{R}\right)^2$$

$$\Rightarrow\ f = mg\sin\theta + \frac{mv^2}{R}$$

But $f \le \mu\, mg \cos \theta$

$$\therefore\ mg\sin\theta + \frac{mv^2}{R} \le \mu mg\cos\theta$$

$$\Rightarrow\ v \le \sqrt{Rg\left[\mu\cos\theta - \sin\theta\right]}$$

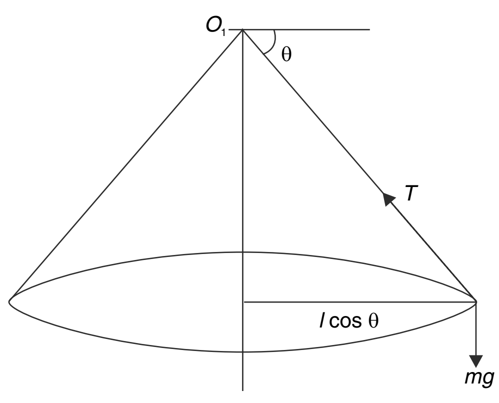
138. Particle goes in a vertical circle of radius

$R = l \sin \theta$

It can be proved that, in this case also, minimum speed at the lowest point to complete the circle is [Do it yourself].

$$u = \sqrt{5gR} = \sqrt{5gl\sin\theta} \qquad \text{………. (1)}$$

After one string breaks, the particle will go in a horizontal circle (forming a conical pendulum) of radius $r = l \cos \theta$



$$T\cos\theta = \frac{mu^2}{r}$$

$T \sin \theta = mg$

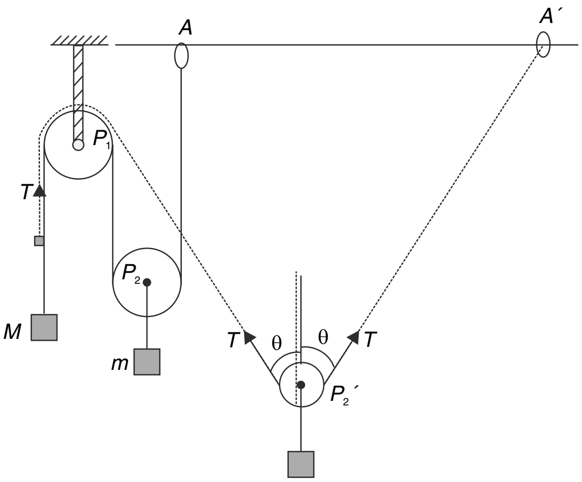
$$\Rightarrow\ \tan\theta = \frac{gr}{u^2}$$

$$\Rightarrow u^2 = \frac{gl\cos^2\theta}{\sin\theta} \quad \text{.......... (2)}$$

From (1) and (2) $\dfrac{gl\cos^2\theta}{\sin\theta} = 5gl\sin\theta$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{5}}$$

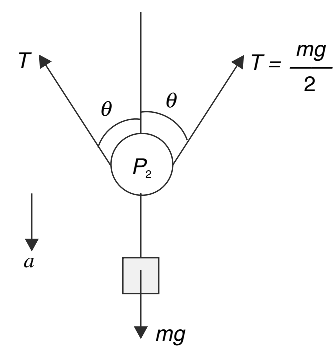
139. For equilibrium $M = \dfrac{m}{2}$



For horizontal equilibrium of pulley P_2 it is necessary that the two segments ($P_1P_2´$ and $P_2´A´$) of the string have equal inclination to the vertical.

Hence, P_2 will move horizontally as well as vertically.

Yes, it is possible that M does not have acceleration (i.e., it moves with constant velocity) and m has an acceleration.



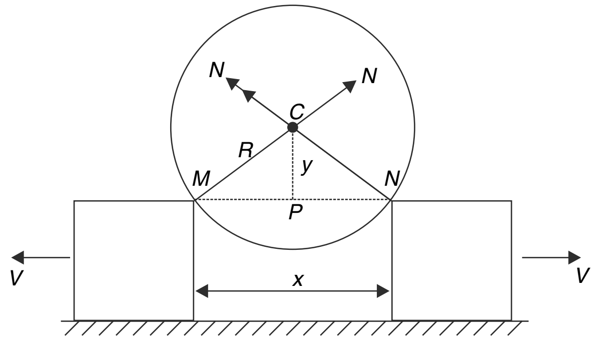
$mg - 2T\cos\theta = ma$

$$mg - 2.\frac{mg}{2}\cos\theta = ma$$

[$\because$ If M does not have acceleration $T = Mg = \dfrac{mg}{2}$]

$\therefore a = g(1 - \cos\theta)$

140.



Let $CP = y$ and $MN = x$

$$\therefore\ y^2 + \left(\frac{x}{2}\right)^2 = R^2$$

$$2y\frac{dy}{dt} + \frac{1}{4}.2x\frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{4y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{4y}(2v) \qquad \left[\because \frac{dx}{dt} = 2v\right]$$

$$\frac{dy}{dt} = -\frac{x}{2y}v \qquad \text{..........(1)}$$

When $x = a = \sqrt{2}\,R$

$$MP = \frac{a}{2} = \frac{R}{\sqrt{2}}$$

$$\therefore y = \frac{R}{\sqrt{2}}$$

At this moment velocity of centre of the sphere is

$$\frac{dy}{dt} = -\frac{\sqrt{2}R}{2\frac{R}{\sqrt{2}}}.v = -v$$

[– sign indicates that y is decreasing]

Differentiating (1) once again

$$\frac{d^2y}{dt^2} = -v\left[\frac{y\frac{dx}{dt} - x\frac{dy}{dt}}{y^2}\right]$$

$\therefore$ Acceleration of centre C when

$$y = \frac{R}{\sqrt{2}};\ x = \sqrt{2}R;\ \frac{dx}{dt} = 2v$$

$$\frac{dy}{dt} = -v$$

$$a = \frac{d^2y}{dt^2} = -v\left[\frac{\frac{R}{\sqrt{2}}.2v - \sqrt{2}R(-v)}{\left(\frac{R}{\sqrt{2}}\right)^2}\right]$$

$$= -\frac{v^2}{R}\left(4\sqrt{2}\right)$$

$$= -\frac{1^2}{1} \times 4\sqrt{2} = -4\sqrt{2}\ m/s^2$$

– sign indicates that the acceleration is downward.

Force equation for the ball

$Mg - 2N\cos 45° = M.a$

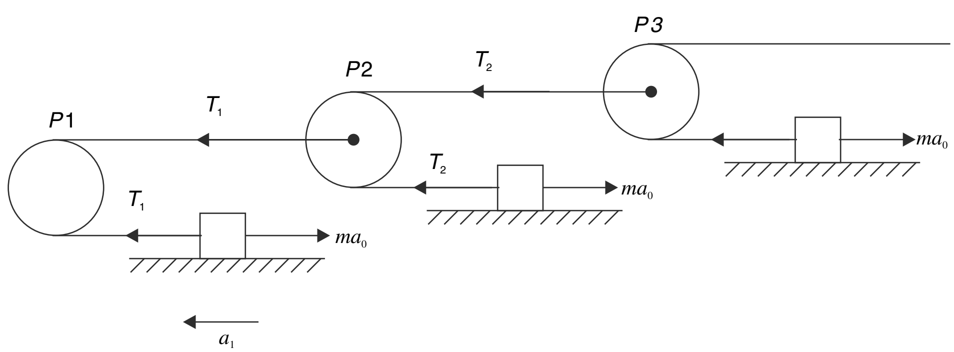
$2 \times 10 - 2 \times 4\sqrt{2} = N\sqrt{2}$

$\therefore$ N = $10\sqrt{2} - 8$

(b) $\frac{N}{\sqrt{2}} = 5 - 4\sqrt{2}$

141. Let's consider the problem in a reference frame moving to left with an acceleration of a_0.

In this frame P1 is at rest and all the blocks experience a force ma_0 towards right.



Clearly $T_1 \propto a_0$ .... (1)

In this frame let the acceleration of block 1 be a_1 ($\leftarrow$). Pulley $P2$ will move towards right with an acceleration of a_1.

If an observer is located in reference frame attached to $P2$ [which is moving to left with an acceleration of $(a_0 - a_1)$], he will similarly conclude that

$T_2 \propto (a_0 - a_1)$ .... (2)

$$\frac{T_1}{T_2} = \frac{a_0}{a_0 - a_1}$$

But pulley is massless

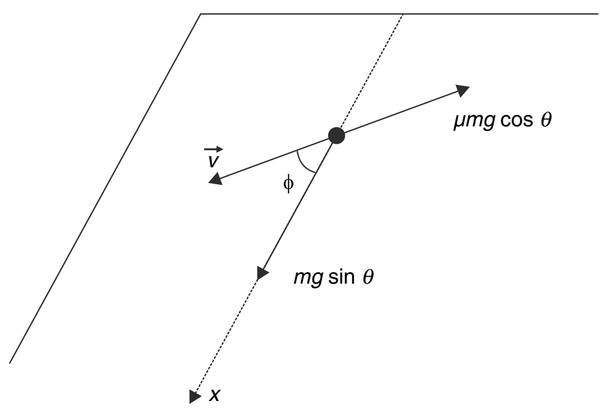
$$\therefore\ T_1 = 2T_2$$

$$\therefore\ \frac{a_0}{a_0 - a_1} = 2$$

$$a_1 = \frac{a_0}{2}$$

In ground frame, acceleration of block 1 will be $\frac{3a}{2}(\leftarrow)$

142. Normal reaction on the particle is always $N = mg\cos\theta$



Friction force on the particle is $f = \mu mg\cos\theta$ and is always opposite the direction of instantaneous velocity.

Another force on the particle in the plane of the incline is $mg\sin\theta$ which is always directed along x direction. Force on the particle in tangential direction (i.e., in direction of its velocity) is

$F_t = mg\sin\theta\,.\cos\phi - \mu mg\cos\theta$

Acceleration in tangential direction is

$a_t = g\sin\theta(\cos\phi - 1)$ [$\because$ $\mu = \tan\theta$] .... (a)

Similarly, acceleration of disc along x – direction is

$a_x = g\sin\theta - (\mu g\cos\theta)\cos\phi$

$= g\sin\theta(1 - \cos\phi)$ .... (b)

$\therefore\ a_t + a_x = 0$ [Adding (a) and (b)]

Integrating, we get

$v + v_x = c$ [c is a constant)

But $v_x = v\cos\phi$

$\therefore\ v + v\cos\phi = c$

Initially $\phi = \dfrac{\pi}{2}; v = v_0$

$\therefore\ c = v_0$

$$\therefore\ v = \frac{v_0}{1+\cos\phi}$$

04 WORK - POWER - ENERGY

Level 1

Q. 1. (i) The cause of increases in kinetic energy when a man starts running without his feet slipping on ground is asked to two students. Their answers are–

Harshit: Cause of increase in kinetic energy is work done by friction force. Without friction the man cannot run.

Akanksha: Cause of increase in kinetic energy is work done by internal (muscle) forces of the body.

Who is right?

(ii) An inextensible rope is hanging from a tree. A monkey, having mass m, climbs to a height h grabbing the rope tightly. The monkey starts from rest and ends up hanging motionlessly on the rope at height h.

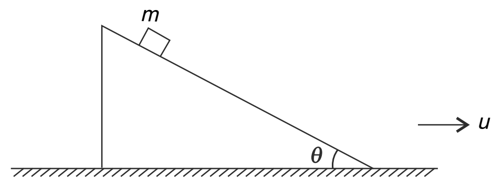
(a) How much work is done by gravity on the monkey?

(b) How much work is done by the rope on the monkey?

(c) Using work – energy theorem, explain the increase in mechanical energy of the monkey.

Q. 2. A man of mass M jumps from rest, straight up, from a flat concrete surface. Centre of mass of the man rises a distance h at the highest point of the motion. Find the work done by the normal contact force (between the man's feet and the concrete floor) on the man.

Q. 3. A block of mass $m = 10$ *kg* is released from the top of the smooth inclined surface of a wedge which is moving horizontally toward right at a constant velocity of $u = 10$ *m/s*. Inclination of the wedge is $\theta = 37°$. Calculate the work done by the force applied by the wedge on the block in two seconds in a reference frame attached to -



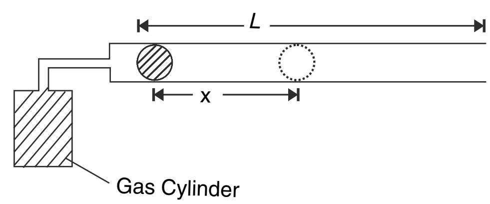
(a) the ground (b) the wedge.

[Take $g = 10$ ms^{-2}]

Q. 4. In an industrial gun, when the trigger is pulled a gas under pressure is released into the barrel behind a ball of mass m. The ball slides smoothly inside the barrel and the force exerted by the gas on the ball varies as

$$F = F_o\left(1-\frac{x}{L}\right)$$

Where L is length of the end of the barrel from the initial position of the ball and x is instantaneous displacement of the ball from its initial position. Neglect any other force on the ball apart from that applied by the gas. Calculate the speed (V) of the ball with which it comes out of the gun.



Q. 5. A particle of mass 3 *kg* takes 2 second to move from point A to B under the action of gravity and another constant force

$\vec{F} = \left(12\hat{i} - 3\hat{j} + 21\hat{k}\right)N$, where the unit vector $\hat{k}$ is in the direction of upward vertical. The position vector of point B is $\vec{r}_B = \left(15\hat{i} - 7\hat{j} - 6\hat{k}\right)m$ and velocity of the particle when it reaches B is

$$\vec{V}_B = \left(12\hat{i} + \hat{j} - 4\hat{k}\right)m/s.$$

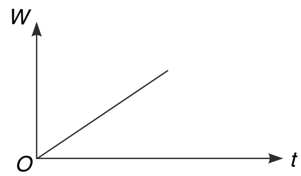
(a) Find the velocity, $\vec{V}_A$ of the particle when it was at A.

(b) Find position vector, $\vec{r}_A$ of point A.

(c) Find work done by the force $\vec{F}$ as the particle moves from A to B.

(d) Find change in gravitational potential energy of the particle as it moves from A to B.

Q. 6. A particle can move along a straight line. It is at rest when a force (F) starts acting on it directed along the line. Work done by the force on the particle changes with time(t) according to the graph shown in the fig. Can you say that the force acting on the particle remains constant with time?



Q. 7. A particle is moving on a straight line and all the forces acting on it produce a constant power P calculate the distance travelled by the particle in the interval its speed increase from V to $2V$.

Q. 8. Work done and power spent by the motor of an escalator are W and P respectively when it carries a standing passenger from ground floor to the first floor. Will the work and power expended by the motor change if the passenger on moving escalator walks up the staircase at a constant speed?

Q. 9. (i) A block is connected to an ideal spring on a horizontal frictionless surface. The block is pulled a short distance and released. Plot the variation of kinetic energy of the block vs the spring potential energy.

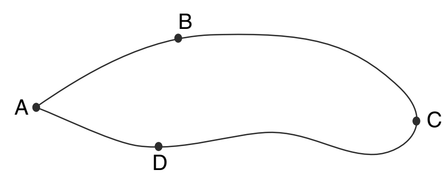
(ii) A ball of mass 200 g is projected from the top of a building 20 m high. The projection speed is 10 m/s at an angle $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ from the horizontal. Sketch a graph of kinetic energy of the ball against height measured from the ground. Indicate the values of kinetic energy at the top and bottom of the building and at the highest point of the trajectory, specifying the heights on the graph. Neglect air resistance and take $g = 10\ m/s^2$

Q. 10. A car of mass $m = 1600\ kg$, while moving on any road, experiences resistance to its motion given by $(m + nV^2)$ newton; where m and n are positive constants. On a horizontal road the car moved at a constant speed of 40 m/s when the engine developed a power of 53 KW. When the engine developed an output of 2 KW the car was able to travel on a horizontal road at a constant speed of 10 m/s.

(a) Find the power that the engine must deliver for the car to travel at a constant speed of 40 m/s on a horizontal road.

(b) The car is able to climb a hill at a constant speed of 40 m/s with its engine working at a constant rate of 69 KW. Calculate the inclination of the hill (in degree)

Q. 11. A particle moves along the loop $A–B–C–D–A$ while a conservative force acts on it. Work done by the force along the various sections of the path are – $W_{A \to B} = -50\ J$; $W_{B \to C} = 25\ J$; $W_{C \to D} = 60\ J$. Assume that potential energy of the particle is zero at A. Write the potential energy of particle when it is at B and D.



Q. 12. A moving particle of mass m is acted upon by five forces $\vec{F_1}, \vec{F_2}, \vec{F_3}, \vec{F_4}$ and $\vec{F_5}$. Forces $\vec{F_2}$ and $\vec{F}_3$ are conservative and their potential energy functions are U and W respectively. Speed of the particle changes from V_a to V_b when it moves from position a to b. Which of the following statement is/are true –

(a) Sum of work done by $\vec{F_1}, \vec{F_4}$ and $\vec{F_5}$ = $U_b - U_a + W_b - W_a$

(b) Sum of work done by $\vec{F_1}, \vec{F_4}$ and $\vec{F_5}$ =

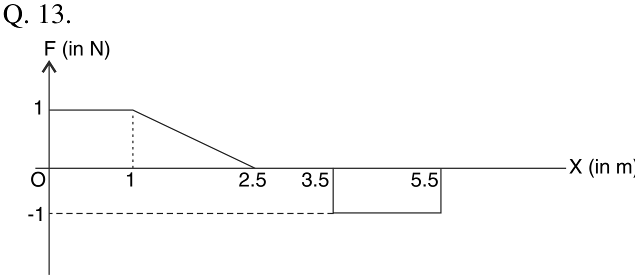
$$U_b - U_a + W_b - W_a + \frac{1}{2}m\left(V_b^2 - V_a^2\right)$$

(c) Sum of work done by all five forces =

$$\frac{1}{2}m\left(V_b^2 - V_a^2\right)$$

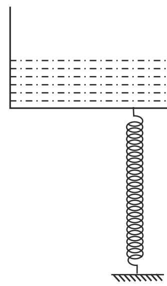
(d) Sum of work done by $\vec{F_2}$ and $\vec{F_3}$ = $(U_b + W_b) - (U_a + W_a)$.

Q. 13.

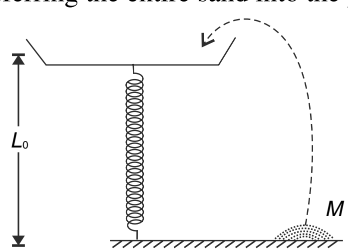


The given graph represents the total force in x direction being applied on a particle of mass $m = 2\ kg$ that is constrained to move along x axis. What is the minimum possible speed of the particle when it was at $x = 0$?

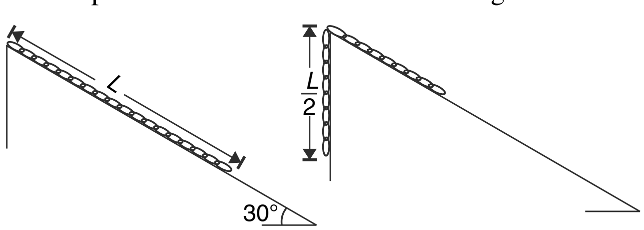
Q. 14. A vertical spring supports a beaker containing some water in it. Water slowly evaporates and the compression in the spring decreases. Where does the elastic potential energy stored in the spring go?



Q. 15. A pan of negligible mass is supported by an ideal spring which is vertical. Length of the spring is L_o. A mass M of sand is lying nearby on the floor. A boy lifts a small quantity of sand and gently puts it into the pan. This way he slowly transfers the entire sand into the pan. The spring compresses by $\frac{L_o}{2}$. Assume that height of the sand heap on the floor as well as in the pan is negligible. Calculate the work done by the boy against gravity in transferring the entire sand into the pan.



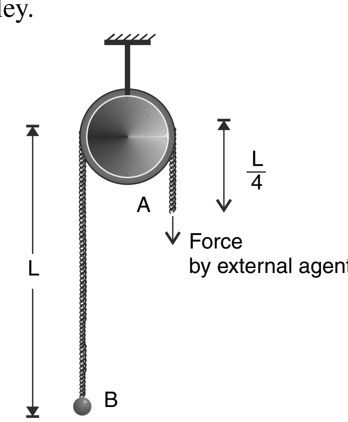
Q. 16. A snake of mass M and length L is lying on an incline of inclination 30°. It craws up slowly and overhangs half its length vertically. Assume that the mass is distributed uniformly along the length of the snake and its hanging part as well as the part on the incline both remain straight.



(a) Find the work done by the snake against gravity (W_g)

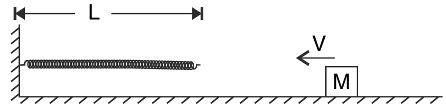
(b) Will the answer to part (a) be different if the snake were of half the length but of same mass.

Q. 17. A uniform rope of linear mass density λ (kg/m) passes over a smooth pulley of negligible dimension. At one end B of the rope there is a small particle having mass one fifty of the rope. Initially the system is held at rest with length L of the rope on one side and length $\frac{L}{4}$ on the other side of the pulley (see fig). The external agent begins to pull the end A downward. Find the minimum work that the agent must perform so that the small particle will definitely reach the pulley.



Q. 18. A particle of mass $m = 100\ g$ is projected vertically up with a kinetic energy of 20 J form a position where its gravitational potential energy is – 50 J. Find the maximum height to which the particle will rise above its point of projection. [$g = 10\ m/s^2$]

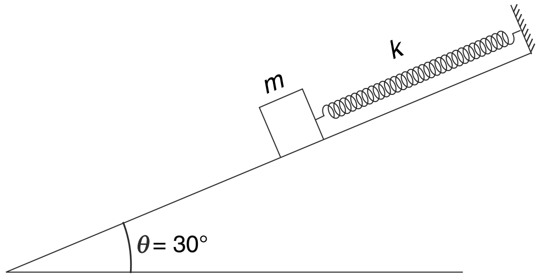
Q. 19. A physics student writes the elastic potential energy stored in a spring as $U = \frac{1}{2}KL^2 + \frac{1}{2}Kx^2$, where L is the natural length of the spring, x is extension or compression in it and K is its force constant. A block of mass M travelling with speed V hits the spring and compresses it.



Find the maximum compression caused.

Q. 20. A block of mass $m = 4\ kg$ is kept on an incline connected to a spring (see fig). The angle of the incline is $\theta = 30°$ and the spring constant is

$K = 80\ N/m$. There is a very small friction between the block and the incline. The block is released with spring in natural length. Find the work done by the friction on the block till the block finally comes to rest.[$g = 10\ m/s^2$]

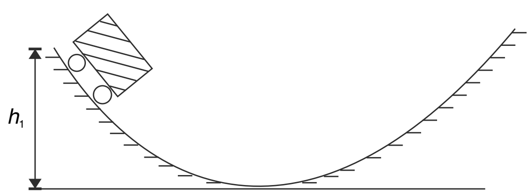


Q. 21. A body is projected directly up a plane which is inclined at an angle θ to the horizontal. It was found that when it returns to the starting point its speed is half its initial speed.

(a) Was dissipation of mechanical energy of the body, due to friction, higher during ascent or descent?

(b) Calculate the coefficient of friction (μ) between the body and the incline.

Q. 22. A tanker filled with water starts at rest and then rolls, without any energy loss to friction, down a valley. Initial height of the tanker is h_1. The tanker, after coming down, climbs on the other side of the valley up to a height h_2. Throughout the journey, water leaks from the bottom of the tanker. How does h_2 compare with h_1?



Q. 23. A stone with weight W is thrown vertically upward into air with initial speed u. Due to air friction a constant force R acts on the stone, throughout its flight. Find –

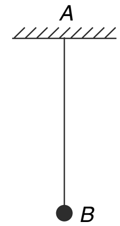
(a) the maximum height reached and

(b) speed of stone on reaching the ground.

Q. 24. A mass $m = 0.1\ kg$ is attached to the end B of an elastic string AB with stiffness $k = 16\ N/m$ and natural length $l_0 = 0.25\ m$. The end A of the string is fixed. The mass is pulled down so that AB is $2l_0 = 0.5\ m$ and then released.

(a) Find the velocity of the mass when the string gets slack for the first time.

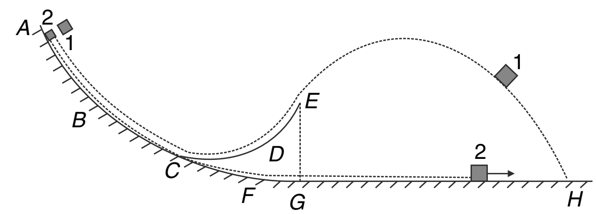
(b) At what distance from A the mass will come to rest for the first time after being released.



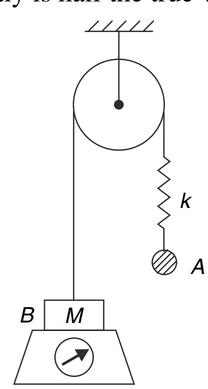
Q. 25. Two blocks 1 and 2 start from same point A on a smooth slide at the same time. The track from A to B to C is common for the two blocks. At C the track divides into two parts. Block 1 takes the route C–D–E and gets airborne after E. Block 2 moves along $CFGH$. Point E is vertically above G and the stretch GH is horizontal. Block 1 lands at point H.

(a) Where is the other block at the time block 1 lands at H? Has it already crossed H or yet to reach there?

(b) Which block will reach at H with higher speed ?

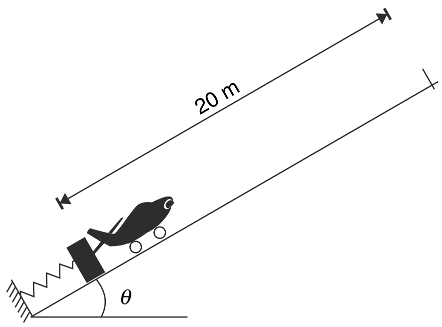


Q. 26 In the arrangement shown in the figure, block B of mass M rests on a weighing scale. Ball A is released from a position where spring is in its natural length and the scale shows the correct weight of block B. Find the mass of ball A so that the minimum reading shown by the scale subsequently is half the true weight of B.



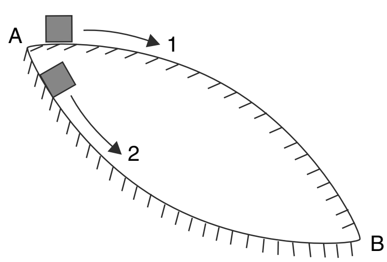
Q. 27 In an aircraft carrier warship the runway is a 20 m long strip inclined at $\theta = 20°$ to the horizontal. The launcher is effectively a large spring that pushes an aircraft of mass $m = 2000$

kg for first 5 *m* of the 20 *m* long runway. The jet engine of the plane produces a constant thrust of 6×10^4 *N* for the entire length of the runway. The plane needs to have a speed of 180 *kph* at the end of the runway. Neglect air resistance and calculate the spring constant of the launcher. [sin 20° = 0.3 and $g = 10\ m/s^2$]



Q. 28 A block of mass *M* is placed on a horizontal surface having coefficient of friction μ. A constant pulling force $F = \frac{Mg}{2}$ is applied on the block to displace it horizontally through a distance *d*. Find the maximum possible kinetic energy acquired by the block.

Q. 29 A small block is made to slide, starting from rest, along two equally rough circular surfaces from *A* to *B* through path 1 and 2. The two paths have equal radii. The speed of the block at the end of the slide was found to be V_1 and V_2 for path 1 and 2 respectively. Which one is larger V_1 or V_2?



Q. 30 A particle can move along x axis under influence of a conservative force. The potential energy of the particle is given by $U = 5x^2 - 20x + 2$ joule where *x* is co-ordinate of the particle expressed in meter.

The particle is released at $x = -3\ m$

(a) Find the maximum kinetic energy of the particle during subsequent motion.

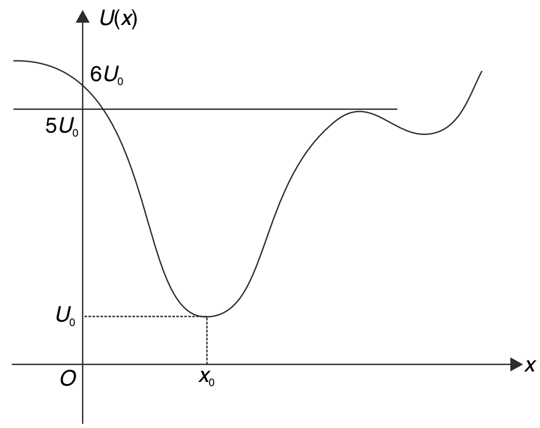
(b) Find the maximum *x* co-ordinate of the particle.

Q. 31 A particle is constrained to move along *x* axis under the action of a conservative force. The potential energy of the particle varies with position *x* as shown in the figure.

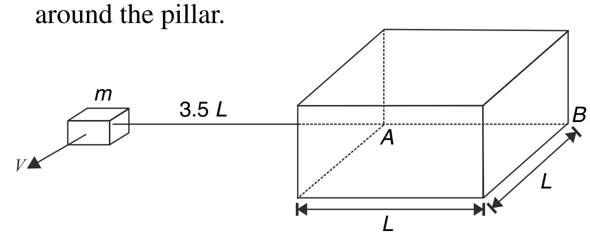
When the particle is at $x = x_0$, it is given a kinetic energy (*k*) such that $0 < k < 4U_0$

(a) Does the particle ever reach the origin?

(b) Qualitatively describe the motion of the particle.



Q. 32 A pillar having square cross section of side length *L* is fixed on a smooth floor. A particle of mass *m* is connected to a corner *A* of the pillar using an inextensible string of length 3.5 *L*. With the string just taut along the line *BA*, the particle is given a velocity *v* perpendicular to the string. The particle slides on the smooth floor and the string wraps around the pillar.



(a) Find the time in which the particle will hit the pillar.

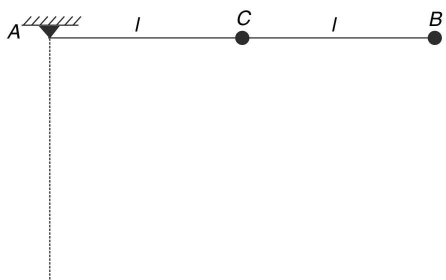
(b) Find the tension in the string just before the particle hits the pillar.

Neglect any energy loss of the particle.

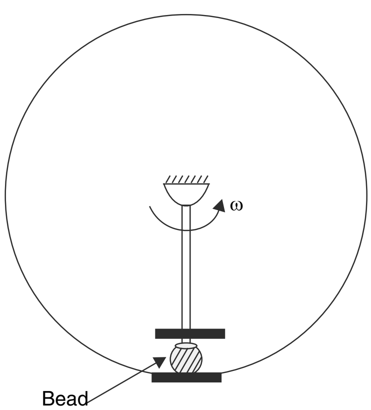
Q. 33 (i) A simple pendulum consist of a small bob of mass *m* tied to a string of length *L*. Show that the total energy of oscillation of the pendulum is $E \simeq \frac{1}{2} mg\, L\theta_0^2$ when it is oscillating with a small angular amplitude θ_0. Assume the gravitational potential energy to be zero of the lowest position of the bob.

(ii) Three identical pendulums *A, B* and *C* are suspended from the ceiling of a room. They are swinging in semicircular arcs in vertical planes. The string of pendulum *A* snaps when it is vertical and it was found that the bob fell on the floor with speed V_1. The string of B breaks when it makes an angle of 30° to the vertical and the bob hits the floor with speed V_2. The string of pendulum C was cut when it was horizontal and the bob falls to the floor hitting it with a speed V_3. Which is greatest and which is smallest among V_1, V_2 and V_3?

Q. 34 *AB* is a mass less rigid rod of length $2l$. It is free to rotate in vertical plane about a horizontal axis passing through its end A. Equal point masses (m each) are stuck at the centre C and end $B$of the rod. The rod is released from horizontal position. Write the tension in the rod when it becomes vertical.



Q. 35 A rigid mass less rod of length L is rotating in a vertical plane about a horizontal axis passing through one of its ends. At the other end of the rod there is a mass less metal plate welded to the rod. This plate supports a heavy small bead that can slide on the rod without friction. Just above the bead there is another identical metal plate welded to the rod. The bead remains confined between the plates. The gap between the plates is negligible compared to L. The angular speed of the rod when the bead is at lowest position of the circle is $\omega = 2\sqrt{\frac{g}{L}}$. How many times a clink of the bead hitting a metal plate is heard during one full rotation of the rod ?



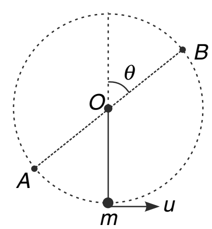
Q. 36 A child of mass m is sitting on a swing suspended by a rope of length L. The swing and the rope have negligible mass and the dimension of child can be neglected. Mother of the child pulls the swing till the rope makes an angle of $\theta_0 = 1$ radian with the vertical. Now the mother pushes the swing along the arc of the circle with a force $F = \frac{mg}{2}$ and releases it when the string gets vertical. How high will the swing go?

[Take cos(1 radian) $\simeq$ 0.5]

Q. 37. A particle of mass m is suspended by a string of length l from a fixed rigid support. Particle is imparted a horizontal velocity $u = \sqrt{2gl}$. Find the angle made by the string with the vertical when the acceleration of the particle is inclined to the string by 45°?

Q. 38 A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. Calculate the power delivered to the particle by the force acting on it.

Q. 39 A ball is hanging vertically by a light inextensible string of length L from fixed point O. The ball of mass m is given a speed u at the lowest position such that it completes a vertical circle with centre at O as shown. Let AB be a diameter of circular path of ball making an angle θ with vertical as shown. (g is acceleration due to gravity)



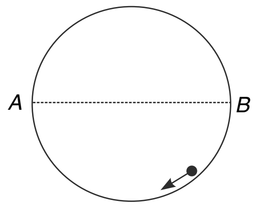
(a) Let T_A and T_B be tension in string when ball is at A and B respectively, then find $T_A - T_B$.

(b) Let $\vec{a}_A$ and $\vec{a}_B$ be acceleration of ball when it is at A and B respectively, then find the value of $|\vec{a}_A + \vec{a}_B|$.

Q. 40 A ball suspended by a thread swings in a vertical plane so that the magnitude of its total acceleration in the extreme position and lowest position are equal. Find the angle θ that the thread makes with the vertical in the extreme position.

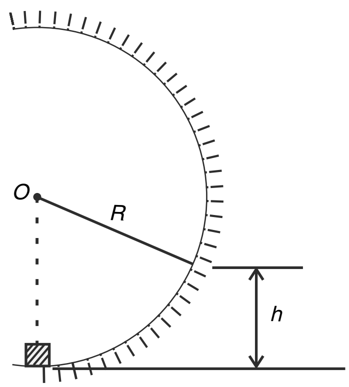
Q. 41 A particle of mass m oscillates inside the smooth surface of a fixed pipe of radius R. The axis of the pipe is horizontal and the particle moves from B

to A and back. At an instant the kinetic energy of the particle is K (say at position of the particle shown in the figure). What is the force applied by particle on the pipe at this instant?



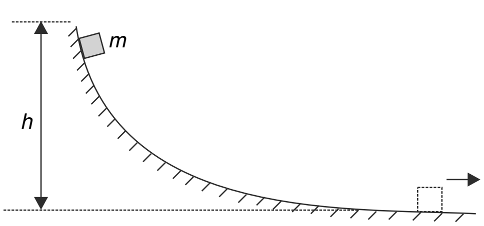
LEVEL 2

Q. 42.

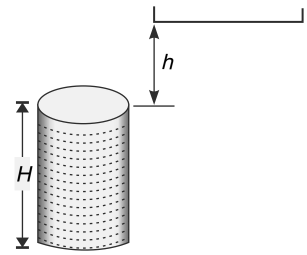


(i) There is a vertical loop of radius R. A small block of mass m is slowly pushed along the loop from bottom to a point at height h. Find the work done by the external agent if the coefficient of friction is μ. Assume that the external agent pushes tangentially along the path.

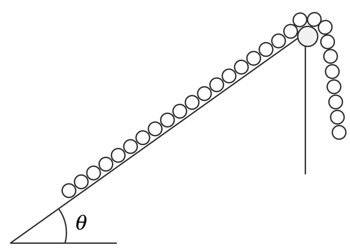
(ii) A block of mass m slides down a smooth slope of height h, starting from rest. The lower part of the track is horizontal. In the beginning the block has potential energy $U = mgh$ which gets converted into kinetic energy at the bottom. The velocity at bottom is $v = \sqrt{2gh}$. Now assume that an observer moving horizontally with velocity $v = \sqrt{2gh}$ towards right observes the sliding block. She finds that initial energy of the block is $E = mgh + \frac{1}{2}mv^2$ and the final energy of the block when it reaches the bottom of the track is zero. Where did the energy disappear?



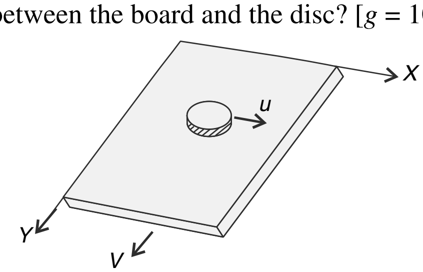
Q. 43. A completely filled cylindrical tank of height H contains water of mass M. At a height h above the top of the tank there is another wide container. The entire water from the tank is to be transferred into the container in time t_0 such that level of water in tank decreases at a uniform rate. How will the power of the external agent vary with time?



Q. 44. A uniform chain of mass m_0 and length l rests on a rough incline with its part hanging vertically as shown in the fig. The chain starts sliding up the incline (and hanging part moving down) provided the hanging part equals η times the chain length ($\eta < 1$). What is the work performed by the friction force by the time chain slides completely off the incline. Neglect the dimension of pulley and assume it to be smooth.



Q. 45. A large flat board is lying on a smooth ground. A disc of mass $m = 2\ kg$ is kept on the board. The coefficient of friction between the disc and the board is $\mu = 0.2$. The disc and the board are moved with velocity $\vec{u} = 2\hat{i}\ \ ms^{-1}$ and $\vec{V} = 2\hat{j}\ ms^{-1}$ respectively [in reference frame of the ground]. Calculate the power of the external force applied on the disc and the force applied on the board. At what rate heat is being dissipated due to friction between the board and the disc? [$g = 10\ \text{ms}^{-2}$]



Q. 46. A car can pull a trailer of twice its mass up a certain slope at a maximum speed V. Without the trailer the maximum speed of the car, up the same slope is 2 V. The resistance to the motion is proportional to mass and square of speed. If the car (without trailer) starts to move down the same slope, with its engine shut off, prove that eventually it will acquire a constant speed. Find this speed.

Q. 47 Force acting on a particle in a two dimensional XY space is given as $\vec{F} = \frac{3(X\hat{i} + Y\hat{j})}{(X^2 + Y^2)^{3/2}}$. Show that the force is conservative.

Q. 48. In a two dimensional space the potential energy function for a conservative force acting on a particle of mass m = 0.1 kg is given by $U = 2\,(x + y)$ joule (x and y are in m). The particle is being moved on a circular path at a constant speed of $V = 1\ ms^{-1}$. The equation of the circular path is $x^2 + y^2 = 4^2$.

(a) Find the net external force (other than the conservative force) that must be acting on the particle when the particle is at (0, 4).

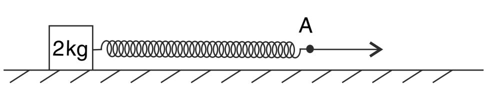
(b) Calculate the work done by the external force in moving the particle from (4, 0) to (0, 4).

Q. 49. A particle of mass m moves in xy plane such that its position vector, as a function of time, is given by $\vec{r} = b(kt - \sin kt)\hat{i} + b(kt + \cos kt)\hat{j}$; where b and k are positive constants.

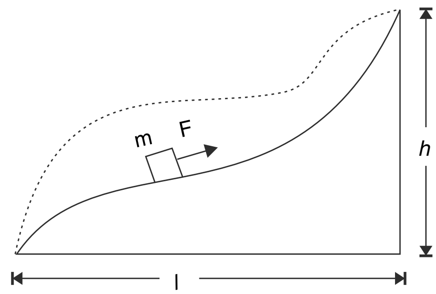
(a) Find the time t_0 in the interval $o \le t \le \frac{\pi}{k}$ when the resultant force acting on the particle has zero power.

(b) Find the work done by the resultant force acting on the particle in the interval $t_o \le t \le \frac{\pi}{k}$

Q.50. A block of mass 2 kg is connect to an ideal spring and the system is placed on a smooth horizontal surface. The spring is pulled to move the block and at an instant the speed of end A of the spring and speed of the block were measured to be 6 m/s and 3 m/s respectively. At this moment the potential energy stored in the spring is increasing at a rate of 15 J/s. Find the acceleration of the block at this instant.

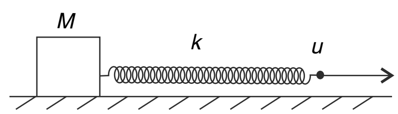


Q. 51. A body of mass m is slowly hauled up a rough hill as shown in fig by a force F which acts tangential to the trajectory at each point. Find the work performed by the force, if the height of hill is h, the length of its base l and coefficient of friction between the body and hill surface is μ. What is the work done if body is moved along some alternative path shown by the dotted line, friction coefficient being same.

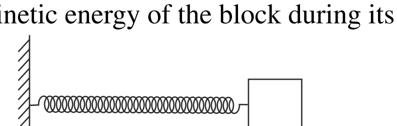


Q. 52. In previous problem what is the work done by $\vec{F}$ if the body started at rest at the base and has a velocity v on reaching the top?

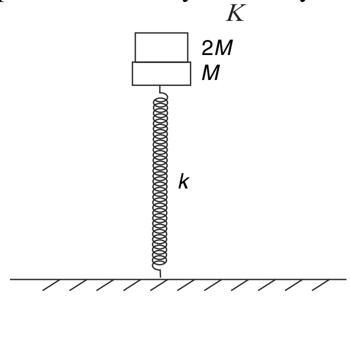
Q. 53. A block of mass M is placed on a horizontal smooth table. It is attached to an ideal spring of force constant k as shown. The free end of the spring is pulled at a constant speed u. Find the maximum extension (x_o) in the spring during the subsequent motion.



Q. 54. A spring block system is placed on a rough horizontal floor. Force constant of the spring is k. The block is pulled to right to give the spring an elongation equal to x_0 and then it is released. The block moves to left and stops at the position where the spring is relaxed. Calculate the maximum kinetic energy of the block during its motion.



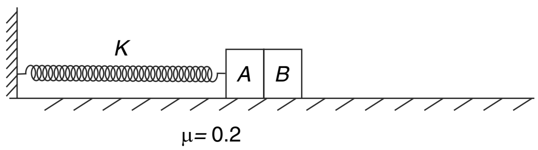
Q. 55. In the fig shown, a block of mass M is attached to the spring and another block of mass 2M has been placed over it. The system is in equilibrium. The block are pushed down so that the spring compresses further by $\frac{9Mg}{K}$. System is released.



(a) At what height above the position of release, the block of mass $2M$ will lose contact with the other block?

(b) What is maximum height attained by $2M$ above the point of release?

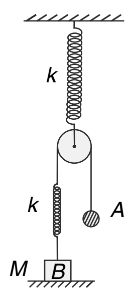
Q. 56. Block A and B are identical having 1 kg mass each. A is tied to a spring of force constant k and B is placed in front of A (touching it). Block 'B' is pushed to left so as to compress the spring by $0.1\,m$ from its natural length. The system is released from this position. Coefficient of friction for both the blocks with horizontal surface is $\mu = 0.2$.



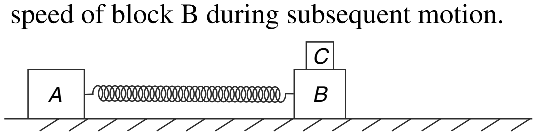
(a) Take $k = \dfrac{200}{3}\ N/m$. Kinetic energy of the system comprising of the two blocks will be maximum after travelling through a distance x_0 from the initial position. Find x_0. Find the contact force between the two blocks when they come to rest.

(b) Take $k = 100\ N/m$. What distance (x_1) will the block travel together, after being released, before B separates from A.

Q. 57. In the arrangement shown in the fig. string, springs and the pulley are mass less. Both the springs have a force constant of k and the mass of block B resting on the table is M. Ball A is released from rest when both the springs are in natural length and just taut. Find the minimum value of mass of A so that block B leaves contact with the table at some stage.



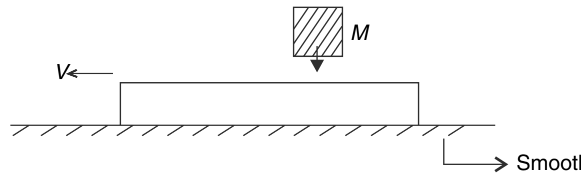
Q. 58. Two block A and B are connected to a spring (force constant $k = 480\ N/m$) and placed on a horizontal surface. Another block C is placed on B. The coefficient of friction between the floor and block A is $\mu_1 = 0.5$, whereas there is no friction between B and the floor. Coefficient of friction between C and B is $\mu_2 = 0.85$. Masses of the blocks are $M_A = 50\ kg$; $M_B = 28\ kg$ and $M_C = 2\ kg$. The system is held at rest with spring compressed by $x_0 = 0.5\ m$. After the system is released, find the maximum speed of block B during subsequent motion.



Q. 59. A plank is moving along a smooth surface with a constant speed V. A block of mass M is gently placed on it. Initially the block slips and then acquires the constant speed (V) same as the plank. Throughout the period, a horizontal force is applied on the plank to keep its speed constant.

(a) Find the work performed by the external force.

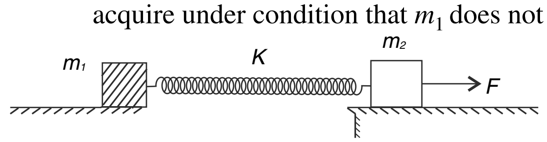
(b) Find the heat developed due to friction between the block and the plank.



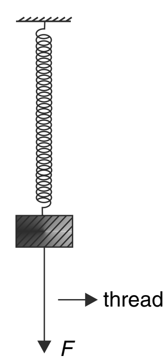
Q. 60. A block of mass m_1 is lying on the edge of a rough table. The coefficient of friction between the block and the table is μ. Another block of mass m_2 is lying on another horizontal smooth table. The two block are connected by a horizontal spring of force constant K. Block of mass m_2 is pulled to the right with a constant horizontal force F.

(a) Find the maximum value of F for which the block of mass m_1 does not fall off the edge.

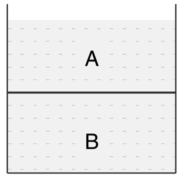
(b) Calculate the maximum speed that m_2 can acquire under condition that m_1 does not fall.



Q. 61. A vertical spring supports a block in equilibrium. The spring is designed to break when extension in it crosses a limit. There is a light thread attached to the block as shown. The thread is pulled down with a force F which gradually increases from zero. The spring breaks when the force becomes F_0. Instead of gradually increasing the force, if the thread were pulled by applying a constant force, for what minimum value of the constant force the spring will break?



Q. 62 Two liquid A & B having densities 2ρ and ρ respectively, are kept in a cylindrical container separated by a partition as shown in figure. The height of each liquid in the container is h and area of cross section of the container is A. Now the partition is removed. Calculate change in gravitational potential energy (ΔU) of the system



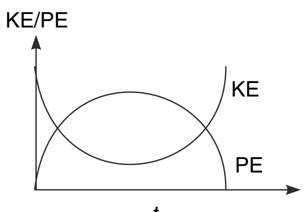
(a) assuming that the two liquids mix uniformly.

(b) Assuming that the two liquids are immiscible.

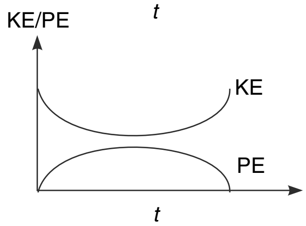
What do you conclude from the sign of ΔU in the above two cases?

Q. 63. A particle is projected at an angle θ = 30º with the horizontal. Two students A and B have drawn the variation of kinetic energy and gravitational potential energy of the particle as a function of time taking the point of projection as the reference level for the gravitational potential energy. Who is wrong and why?

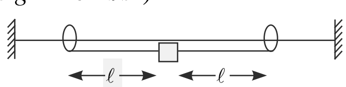
(a)



(b)



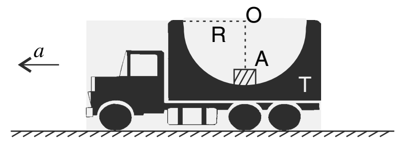
Q. 64. Two small rings each of mass 'm' are connected to a block of same mass 'm' through inextensible light strings. Rings are constrained to move along a smooth horizontal rod. Initially system is held at rest (as shown in figure) with the strings just taut. Length of each string is 'ℓ'. The system is released from the position shown. Find the speed of the block (v) and speed of the rings (u) when the strings make an angle of θ = 60º with vertical. (Take g = 10 m/s^2)



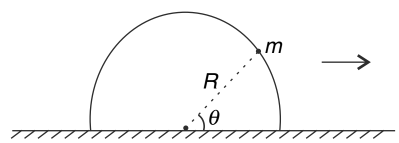
Q. 65. A toy truck T at rest, has a hemispherical trough of radius R in it [O is the centre of the hemisphere]. A small block A is kept at the bottom of the trough. The truck is accelerated horizontally with an acceleration a.

(i) Find the minimum value of a for which the block is able to move out of the trolley.

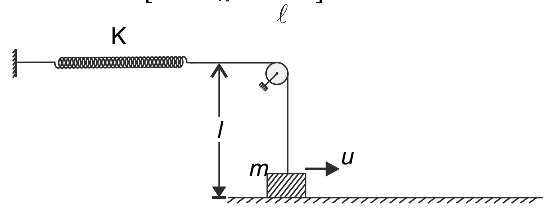
(ii) If magnitude of a is twice the minimum value found in (i), find the maximum height (measured from its original level at the bottom of the trough) to which the block will rise.



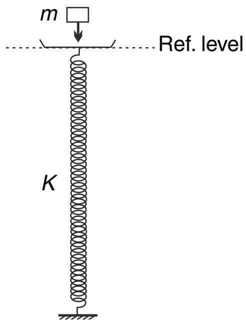
Q. 66. A semicircular wire frame of radius R is standing vertical on a horizontal table. It is pulled horizontally towards right with a constant acceleration. A bead of mass m remain in equilibrium (relative to the semicircular wire) at a position where radius makes an angle θ_0 with horizontal. There is no friction between the wire and the bead. The bead is displaced a little bit in upward direction and released. Calculate the speed of the bead relative to the wire at the instant it strikes the table. Assume that all throughout the semicircular wire keeps moving with constant acceleration.



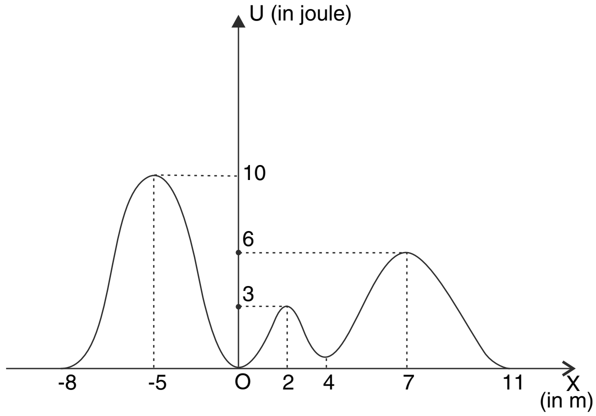
Q. 67. A ideal spring of force constant k is connected to a small block of mass m using an inextensible light string (see fig). The pulley is mass less and friction coefficient between the block and the horizontal surface is $\mu = \frac{1}{\sqrt{3}}$. The string between the pulley and the block is vertical and has length l. Find the minimum velocity u that must be given to the block in horizontal direction shown, so that subsequently it leaves contact with the horizontal surface. [Take $k = \frac{2mg}{\ell}$]



Q. 68. A light spring is vertical and a mass less pan is attached to it. Force constant of the spring is k. A block of mass m is gently dropped on the pan. Plot the variation of spring potential energy, gravitation potential energy and the total potential energy of the system as a function of displacement (x) of the block. For gravitational potential energy take reference level to be the initial position of the pan.



Q. 69. A particle of mass m = 1.0 kg is free to move along the x axis. It is acted upon by a force which is described by the potential energy function represented in the graph below. The particle is projected towards left with a speed v, from the origin. Find minimum value of v for which the particle will escape far away from the origin.



Q. 70. A particle of mass m = 1 kg is free to move along x axis under influence of a conservative force. The potential energy function for the particle is

$$U = a\left[\left(\frac{x}{b}\right)^4 - 5\left(\frac{x}{b}\right)^2\right]\text{joule}$$

Where b = 1.0 m and a = 1.0 J. If the total mechanical energy of the particle is zero, find the co-ordinates where we can expect to find the particle and also calculate the maximum speed of the particle.

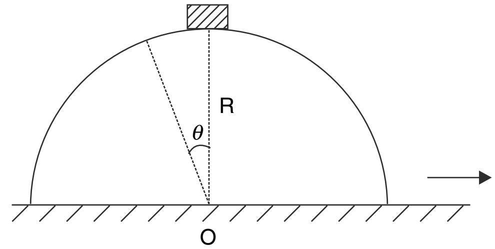
Q. 71. A particle of mass m moves under the action of a central force. The potential energy function is given by $U(r) = mkr^3$

Where k is a positive constant and r is distance of the particle from the centre of attraction.

(a) What should be the kinetic energy of the particle so that it moves in a circle of radius a_0 about the centre of attraction?

(b) What is the period of this circular motion?

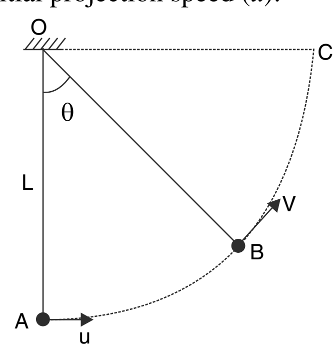
Q. 72. A small block is placed on the top of a smooth inverted hemispherical bowl of radius R.



(a) The bowl is given a sudden impulse so that it begins moving horizontally with speed V. Find minimum value of V so that the block immediately loses contact with the bowl as it begins to move.

(b) The bowl is given a constant acceleration 'a' in horizontal direction. Find maximum value of 'a' so that the block does not lose contact with the bowl by the time it rotates through an angle $\theta = 1°$ relative to the bowl. You can make suitable mathematical approximations justified for small value of angle θ.

Q. 73 A pendulum bob is projected form its lowest position with velocity (u), in horizontal direction, that is just enough to make the string horizontal (position OC). At angular position θ, at point B, the speed (V) of the bob was observed to be half its initial projection speed (u).

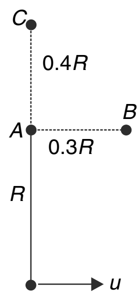


(a) Find θ

(b) Plot variation of magnitude of tangential acceleration with θ.

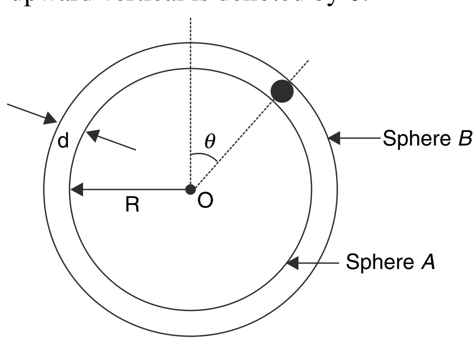
(c) Let the travel time from A to B be t_1 and that from B to C be t_2. Looking at the graph obtained in part (b), tell which is larger $-t_1$ or t_2?

Q. 74. A small ball is attached to an end of a light string of length R. It is suspended in vertical plane supported at point A. B and C are two nails (of negligible thickness) at a horizontal distance 0.3 R from A and a vertical distance 0.4 R above A respectively. The ball is given a horizontal velocity $u = \sqrt{5gR}$ at its lowest point. Subsequently, after the string hitting the nails, the nails become the centre of rotation. Assume no loss in kinetic energy when the string hits the nails. It is known that the string will break if tension in it is suddenly increased by 200% or more.



Will the string break during the motion? If yes, where? What is tension in the string at the instant the string breaks?

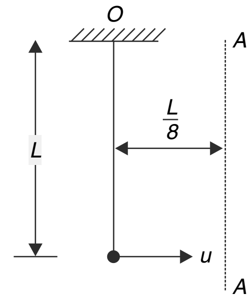
Q. 75. A spherical ball of mass m is kept at the highest point in space between two fixed concentric spheres A and B (see figure). The smaller sphere has a radius R and the space between the two spheres has a width d. The ball has diameter just less than d. All surfaces are frictionless. The ball is given a gentle push (towards the right). The angle made by the radius vector of the ball with upward vertical is denoted by θ.



(a) Express the total normal reaction force exerted by the spheres as a function of θ.

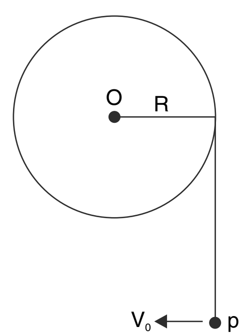
(b) Let N_A and N_B denote the magnitudes of normal reaction forces on the ball exerted by the spheres A and B respectively. Sketch the variations of N_A and N_B as function of cos θ in the range of $0 \leq \theta \leq \pi$ by drawing two separate graphs.

Q. 76. A particle is suspended vertically from a point O by an inextensible mass less string of length L. A vertical line AB is at a distance of $\frac{L}{8}$ from O as shown in figure. The particle is given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find u.



Q. 77 A simple pendulum has a bob of mass m and string of length R. The bob is projected from lowest position giving it a horizontal velocity just enough for it to complete the vertical circle. Let the angular displacement of the pendulum from its initial vertical position be represented by θ. Plot the variation of kinetic energy (kE) of the bob and the tension (T) in the string with θ. Plot the graph for one complete rotation of the pendulum.

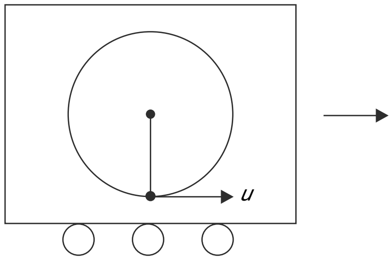
Q. 78 A light thread is tightly wrapped around a fixed disc of radius R. A particle of mass m is tied to the end P of the thread and the vertically hanging part of the string has length πR. The particle is imparted a horizontal velocity $V = \sqrt{\frac{4\pi gR}{3}}$. The string wraps around the disc as the particle moves up. At the instant the velocity of the particle makes an angle of $\theta = 60°$ with horizontal, calculate.



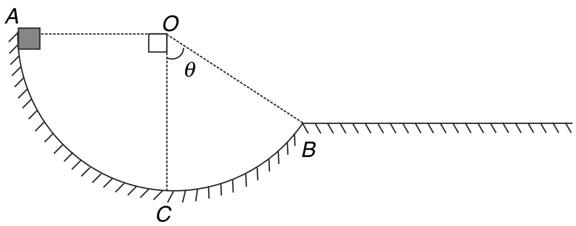
(a) speed of the particle

(b) tension in the string

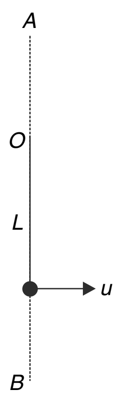
Q. 79 An experimenter is inside a train. He observes that minimum speed at lowest position needed by a pendulum bob to complete a vertical circle is 10 m/s. Calculate the minimum speed (u) needed at the lowest position so as to complete the vertical circle when the train is moving horizontally at an acceleration of a = 7.5 m/s^2. Find the maximum tension in the string during the motion. [$g = 10\ m/s^2$].



Q. 80 A track (ACB) is in the shape of an arc of a circle. It is held fixed in vertical plane with its radius OA horizontal. A small block is released on the inner surface of the track from point A. It slides without friction and leaves the track at B. What should be value of θ so that the block travels the largest horizontal distance by the time it returns to the horizontal plane passing through B?



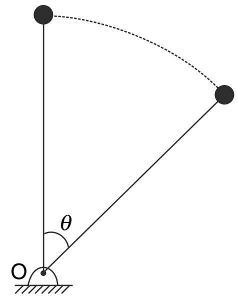
Q. 81 Bob of a simple pendulum of length L is projected horizontally with a speed of $u = \sqrt{4gL}$, from the lowest position. Find the distance of the bob from vertical line AB, at the moment its tangential acceleration becomes zero.



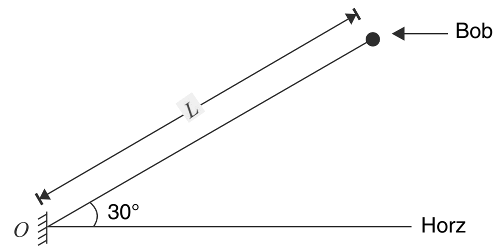
Q. 82 A light rigid rod has a bob of mass m attached to one of its end. The other end of the rod is pivoted so that the entire assembly can rotate freely in a vertical plane. Initially, the rod is held vertical as shown in the figure. From this position it is allowed to fall.

(a) When the rod has rotated through $\theta = 30°$, what kind of force does it experience– compression or tension?

(b) At what value of θ the compression (or tension) in the rod changes to tension (or compression)?



Q. 83 A pendulum has length $L = 1.8\ m$. The bob is released from position shown in the figure. Find the tension in the string when the bob reaches the lowest position. Mass of the bob is 1 kg.



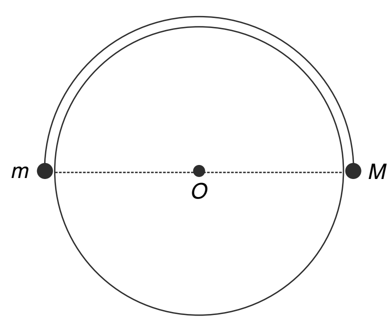
Q. 84 A small body of mass m lies on a horizontal plane. The body is given a velocity v_0, along the plane.

(a) Find the mean power developed by the friction during the whole time of motion, if friction coefficient is $\mu = 0.3$; $m = 2.0\ kg$ and $v_0 = 3\ m/s$.

(b) Find the maximum instantaneous power developed by the friction force, if the friction coefficient varies as $\mu = \alpha x$, where α is a constant and x is distance from the starting point.

Q. 85 Two particles of masses M and m ($M > m$) are connected by a light string of length πR.

The string is hung over a fixed circular frame of radius R.



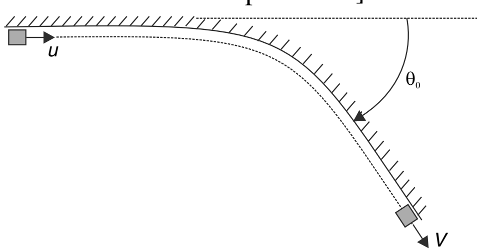
Initially the particles lie at the ends of the horizontal diameter of the circle (see figure). Neglect friction.

(a) If the system is released, and if m remains in contact with the circle, find the speed of the masses when M has descended through a distance $R\theta$ ($\theta < \pi$).

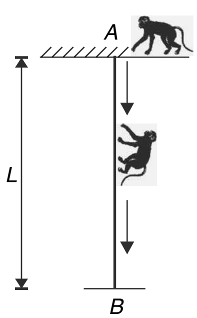
(b) Find the reaction force between the frame and m at this instant.

(c) Prove that m_1 will certainly remain in contact with the frame, just after the release, if $3m > M$.

Q. 86 A small object is sliding on a smooth horizontal floor along a vertical wall. The wall makes a smooth turn by an angle θ_0. Coefficient of friction between the wall and the block is μ. Speed of the object before the turn is u. Find its speed (V) just after completing the turn. Does your answer depend on shape of the curve? [The turn is smooth and there are no sharp corners.]

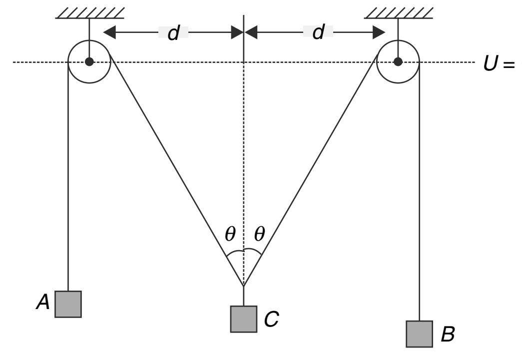


Q. 87 AB is a vertically suspended elastic cord of negligible mass and length L. Its force constant is $k = \frac{4mg}{L}$. There is a massless platform attached to the lower end of the cord. A monkey of mass m starts from top end A and slides down the cord with a uniform acceleration of $\frac{g}{2}$. Just before landing on the platform, the monkey loses grip on the cord. After landing on the platform the monkey stays on it. Calculate the maximum extension in the elastic cord.



LEVEL 3

Q. 88 In the arrangement shown in the fig. all the three blocks have equal mass m. The length of the strings connecting A to C and B to C is L each. Assume the gravitational potential energy of any mass at the level of the pulleys to be zero. Neglect dimension of the pulley and treat the strings to be massless. Distance between the pulleys is $2d$.

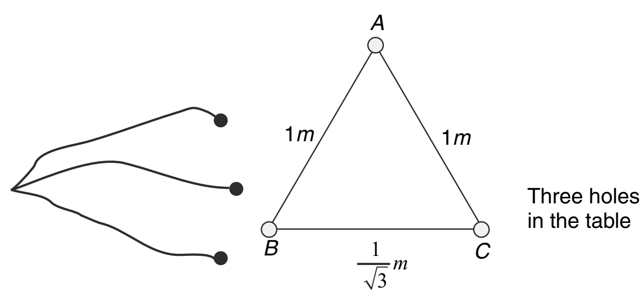


(a) Write the potential energy of the system as a function of angle θ.

(b) Knowing that potential energy of the system will be maximum or minimum in equilibrium position, find value of θ for equilibrium.

(c) Tell if the equilibrium is stable or unstable.

Q. 89 Three identical masses are attached to the ends of light strings, the other ends of which are connected together as shown in the figure. Each of the three strings has a length of 3 m. The three masses are dropped through three holes in a table and the system is allowed to reach equilibrium.



(a) What is total length of the strings lying on the table in equilibrium?

(b) Select a point K inside the ΔABC such that $AK + BK + CK$ is minimum, use the result obtained in (a) and the fact that potential energy of the system will be minimum when it is in equilibrium.

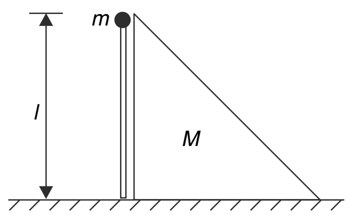
Q. 90 A particle of mass m is attached to an end of a light rigid rod of length a. The other end of the rod is fixed, so that the rod can rotate freely in vertical plane about its fixed end. The mass m is given a horizontal velocity u at the lowest point.

(a) Prove that when the radius to the mass makes an angle θ with the upward vertical the horizontal component of the acceleration of the mass (measured in direction of u) is

$[g\,(2 + 3\cos\theta) - u^2/a]\sin\theta$

(b) If $4ag < u^2 < 5ag$, show that there are four points at which horizontal component of acceleration is zero. Locate the points.

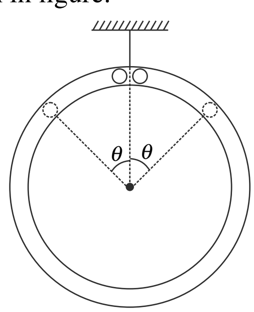
Q. 91 A weightless rod of length l with a small load of mass m at one of its end is held vertical with its lower end hinged on a horizontal surface. The load touches a wedge of mass M in this position. A slight jerk towards right sets the system in motion (see figure), with rod rotating freely in vertical plane about its lower end. There is no friction.



(a) For what mass ratio $\dfrac{M}{m}$ will the rod form an angle $\theta = \pi/3$ with the vertical at the moment the load separates from the wedge?

(b) What is speed of the wedge at that moment? Neglect friction.

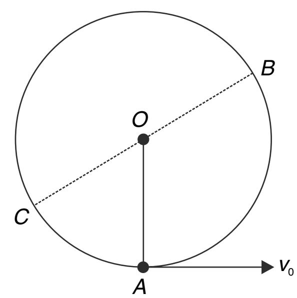
Q. 92 A tube of mass M hangs from a thread and two balls of mass m slide inside it without friction (see figure). The balls are released simultaneously from the top of the ring and slide down on opposite sides. θ defines the positions of balls at any time as shown in figure.



(a) Show that ring will start to rise if $m \geq \dfrac{3M}{2}$.

(b) If $M = 0$, find the angle θ at which the tube begins to rise.

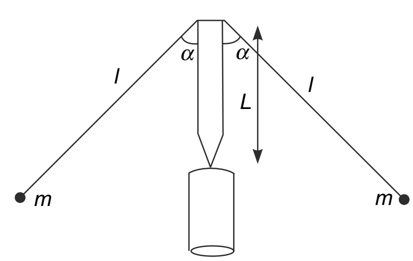
Q. 93 A heavy particle is attached to one end of a light string of length l whose other end is fixed at O. The particle is projected horizontally with a velocity v_0 from its lowest position A. When the angular displacement of the string is more than 90°, the particle leaves the circular path at B. The string again becomes taut at C such that B, O, C are collinear. Find v_0 in terms of l and g.



Q. 94. The teeter toy consists of two identical weights hanging from a peg on dropping arms as shown. The arrangement is surprisingly stable. Let us consider only oscillatory motion in the vertical plane. Consider the peg and rods (connecting the weights to the peg) to be very light. The length of each rod is l and length of the peg is L. In the position shown the peg is vertical and the two weights are in a position lower than the support point of the peg. Angle α that the rods make with the peg remains fixed.

(a) Assuming the zero of gravitational potential energy at the support point of the peg evaluate the potential energy (U) when the peg is tilted to an angle θ to the vertical. The tip of the peg does not move.

(b) Knowing that U shall be minimum in stable equilibrium position prove that $\theta = 0$ is the stable equilibrium position for the toy if the two weights are in a position lower than the support point of the peg.



ANSWERS

1. (i) Akanksha is right.

(ii) (a) $-mgh$

(b) 0

(c) internal (muscle) forces of the body perform work

2. Zero

3 (a) 960 J

(b) zero

4. $V=\sqrt{\frac{F_o L}{m}}-$

5. (a) $\vec{V_A}=\left(4\hat{i}+3\hat{j}+2\hat{k}\right)m/s$

(b) $\vec{r_A}=\left(-\hat{i}+3\hat{j}-4\hat{k}\right)m$

(c) $W=138J$

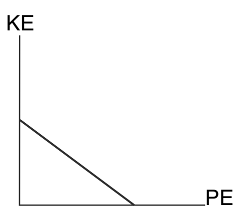
(d) $\Delta U_g=-60\ J$

6. No, the force is decreasing with time.

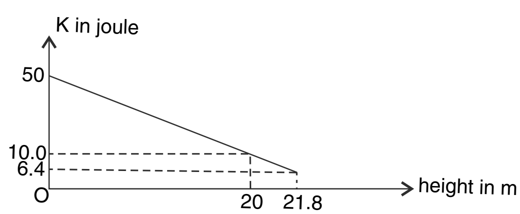
7. $x=\dfrac{7mv^3}{3P}$

8. The power will not change but work done will decrease.

9. (i)



(ii)



10. (a) 53 KW

(b) $\theta=1.43°$

11. $U_B=50\ J$; $U_D=-35\ J$

12. (b), (c)

13. 0.5 m/s

14. When a small amount of water evaporates, the spring relaxes a little bit. Water remaining in the beaker gains gravitational potential energy. Therefore, the spring energy gets converted into the gravitational potential energy of beaker + water.

15. $\frac{3}{4}Mg\,L_o$

16. (a) $\dfrac{MgL}{16}$ (b) Yes.

17. $\dfrac{\lambda Lg}{4}$

18. 20 m

19. $\sqrt{\frac{M}{K}}.V$

20. $-5\ J$

21. (a) Same in both

(b) $\mu=\frac{3}{5}\tan\theta$

22. $h_2=h_1$

23. (a) $\dfrac{Wu^2}{2g(W+R)}$

(b) $v=u\left(\dfrac{W-R}{W+R}\right)^{1/2}$

24. (a) $\sqrt{5}\ m/s$

(b) zero

25. (a) Block 2 has already crossed H.

(b) Both reach H with same speed.

26. $\dfrac{M}{4}$

27. $k=2.096\times10^5\ N/m$

28. $Mgd\left[\dfrac{\sqrt{\mu^2+1}}{2}-\mu\right]$

29. V_1

30. (a) $K_{max}=125\ J$

(b) $X_{max}=7m$

31. (a) No

(b) Oscillations about x_0

32. (a) $t=\dfrac{4\pi L}{v}$

(b) $T=\dfrac{2mv^2}{L}$

33. (i) $V_1 = V_2 = V_3$

34. Tension in segment $AC = \frac{28}{5} mg$

Tension in Segment $BC = \frac{17}{5} mg$

35. 2

36. The swing gets horizontal

37. $\tan^{-1} 2$

38. $mk^2 r^2 t$

39. (a) $6\, mg \cos\theta$

(b) $g\sqrt{4 + 12\cos^2\theta}$

40. $2\tan^{-1}\left(\frac{1}{2}\right)$

41. $\frac{3K}{R}$

42. (i) $mgh + \mu mg\sqrt{2Rh - h^2}$

43. $P = \frac{Mg}{t_0}\left(h + H\frac{t}{t_0}\right)$

44. $-\frac{l(1-\eta)[\eta - (1-\eta)\sin\theta}{2} m_0 g$

45. $4\sqrt{2}\ W,\ 4\sqrt{2}\ W\ ;\ 8\sqrt{2}\ W$

46. $u = \sqrt{5}\, V$

48. (a) $\left(2\hat{i} + 1.975\hat{j}\right) N$

(b) Zero

49. (a) $t_o = \frac{\pi}{4k}$

(b) $W = \left(\frac{1 + 2\sqrt{2}}{2}\right) mb^2 k^2$

50. $2.5\ m/s^2$

51. $mg\left(h + \mu\ell\right)$. Work done is path independent and will be same for the alternative path

52. $\frac{1}{2} mv^2 + mg\left(h + \mu\ell\right)$

53. $x_o = \sqrt{\frac{M}{k}}\ u$

54. $K_{\max} = \frac{1}{8} kx_o^2$

55. (a) $\frac{12Mg}{k}$

(b) $\frac{24Mg}{k}$

57. $m = \frac{M}{2}$

58. $4\ m/s$

59. (a) MV^2

(b) $\frac{1}{2} MV^2$

60. (a) $\frac{\mu m_1 g}{2}$

(b) $\frac{\mu m_1 g}{2} \frac{1}{\sqrt{m_2 k}}$

61. $\frac{F_0}{2}$

62. (a) $\frac{1}{2}\rho A h^2 g$

(b) $-\rho A h^2 g$

The positive sign of ΔU means external work will be required to mix the two liquids uniformly.

ΔU is negative in second case which means the heavier liquid will automatically move to lower side.

63. A is wrong. Under given conditions the two curves cannot touch.

64. $u = \sqrt{\frac{g\ell}{5}},\ v = \sqrt{\frac{3g\ell}{5}}$

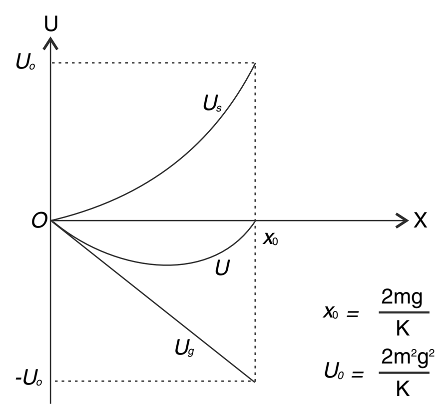
65. (i) $a_{\min} = g$

(ii) $2R$

66. $\sqrt{2gR\left(\frac{1 + \cos\theta_0}{\sin\theta_0}\right)}$

67. $\mu = 2\sqrt{gl}$

68.



69. $2\sqrt{3}\ m/s$

70. $-\sqrt{5} \le x \le \sqrt{5};\ v_{\max} = 5/\sqrt{2}\ ms^{-1}$

71. (a) $\frac{3}{2}km\,a_0^3$

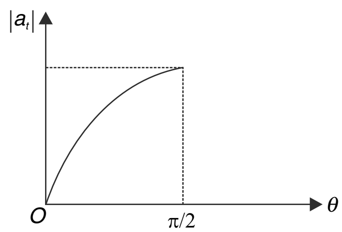
(b) $\frac{2\pi}{\sqrt{3ka_0}}$

72. (a) $\sqrt{Rg}$

(b) $\frac{60g}{\pi}$

73. (a) $\theta = \cos^{-1}\left(\frac{1}{4}\right)$

(b)

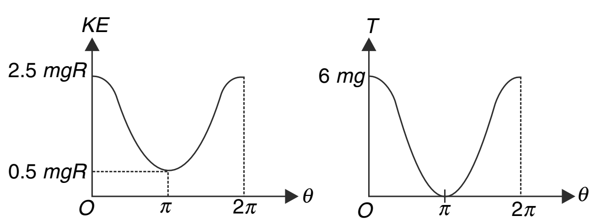


(c) $t_1 > t_2$

74. The string will break on hitting the second nail at C. $T = 8.6\,mg$

76. $\sqrt{gL\left(2+\frac{3\sqrt{3}}{2}\right)}$

77.



78. (a) $\sqrt{\sqrt{3}gR}$

(b) $mg\left(\sqrt{3}+\frac{1}{2}\right)$

79. $u = \sqrt{115}\ m/s$

80. $\theta = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

81. $\frac{5\sqrt{5}}{27}L$

82. (a) Compression

(b) $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

83. $35\ N$

84. (a) –9 Watt

(b) $-\frac{1}{2}mv_0^2\sqrt{\alpha g}$

85. (a) $\sqrt{2gR\frac{(M\theta - m\sin\theta)}{M+m}}$

(b) $\frac{mg}{M+m}\left[(M+3m)\sin\theta - 2M\theta\right]$

86. $V = ue^{-\mu\theta_0}$; No the answer does not depend on the shape of the curve.

87. $\frac{L}{8}\left(2+\sqrt{19}\right)$

88. (a) $U = -2mgL + mgd\left(\frac{2-\cos\theta}{\sin\theta}\right)$

(b) $\theta = 60°$

(c) Stable

89. (a) $\frac{1}{2}\left[1+\sqrt{\frac{11}{3}}\right]$

90. The four points are represented by –

$\theta = 0$, π, $\cos^{-1}\left(\frac{u^2-2ag}{3ag}\right)$ and

$\left[2\pi - \cos^{-1}\left(\frac{u^2-2ag}{3ag}\right)\right]$,

92. $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

93. $\frac{(4+3\sqrt{2})}{2}gl$

94. (a) $2\,mg\cos\theta\,[L - l\cos\alpha]$

SOLUTIONS

1. (i) While walking the contact point of the shoes do not move relative to the ground, hence no work is done by the friction. Internal forces of a system can always cause a change in *KE*. The muscle forces perform work which results in change in *KE*.

(ii) Similar reasoning can be given for increase in *PE* of the monkey.

3. (a) The reference frame attached to the wedge is inertial.

In frame attached to the wedge acceleration of the block is

$$a = g\sin\theta = 10.\sin 37° = 10\cdot\frac{3}{5} = 6\ ms^{-2}$$

Direction of the acceleration is down the incline. Displacement in 2 s in this frame is

$$X_w = \frac{1}{2}\times a\times t^2 = \frac{1}{2}\times 6\times 2^2 = 12\,m.(\searrow)$$

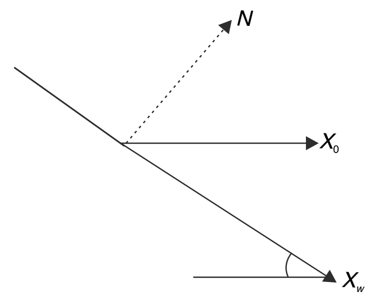
Displacement of the wedge $X_0 = 10\times 2 = 20\,m(\rightarrow)$

Displacement of the block in the ground frame is vector sum of X_w *and* X_o The component of resultant displacement perpendicular to the incline (i.e., in the direction of normal force) is $X_0 \sin\theta$

Work done by N ($= mg\cos\theta$) is

$W_N = \text{mg}.\cos\theta.\ X_0 \sin\theta$

$$= 10\times 10\times\frac{4}{5}\times 20\times\frac{3}{5} = 960\,J$$



(b) In the frame attached to the wedge, there in no displacement in the direction of N.

$\therefore\ W_N = o$

4. Work done by the force on the ball is

$$W = \int_{x=o}^{x=L} F dx = F_o\int_o^L\left(1-\frac{x}{L}\right)dx = F_o\left[L-\frac{L}{2}\right] = \frac{F_o L}{2}$$

Work energy theorem

$$\frac{1}{2}mV^2 = \frac{F_o L}{2}$$

$$V = \sqrt{\frac{F_0 L}{m}}$$

5. (a) let position vector of be $\bar{r}_A$

Position vector of B, $\bar{r}_B = 15\,\hat{i} + 7\,\hat{j} - 6\,\hat{k}$

Net force on particle $= \vec{F} + m\bar{g}$

$\therefore$ Acceleration $\vec{a} = \frac{1}{3}\left(12\hat{i} - 3\hat{j} + 21\hat{k}\right) + (g)\left(-\hat{k}\right)$

$= 4\hat{i} - \hat{j} - 3\hat{k}$ [Taking $g = 10\ m/s^2$]

$\therefore\ \vec{V}_B = \vec{V}_A + \vec{a}\ t$

$$\begin{bmatrix} \overline{V_2} = \text{velocity at } B \\ \overline{V_1} = \text{velocity at } A \end{bmatrix}$$

$$12\hat{i} + \hat{j} - 4\hat{k} = \overrightarrow{V_A} + \left(4\hat{i} - \hat{j} - 3\hat{k}\right) \times 2$$

$$\Rightarrow \overrightarrow{V_A} = \left(4\hat{i} + 3\hat{j} + 2\hat{k}\right)\ m/s$$

(b) Displacement $\overrightarrow{AB} = \overrightarrow{V_A}\, t + \frac{1}{2}\vec{a}\, t^2$

$$= \left(4\hat{i} + 3\hat{j} + 2\hat{k}\right) \times 2 + \frac{1}{2} \times \left(4\hat{i} - \hat{j} - 3\hat{k}\right) \times 4$$

$$= 16\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\therefore \vec{r}_A = \vec{r}_B - \overrightarrow{AB}$$

$$= -\hat{i} + 3\hat{j} - 4\hat{k}$$

(c) $W = \vec{F} \cdot \overrightarrow{AB} = 138\ J$

(d) Change in potential energy = $mg \times$ (*Vertical displacement*)

$$= mg\,(-2) = -60J$$

$\left[\because +ve\ \ z-\text{axis is along upward vertical, } Z_2 - Z_1 = -2\ m.\right]$

6. $P = \dfrac{dW}{dt} =$ Slope of the graph = a constant.

With increase in speed of the particle, the force must decrease so that $P = Fv$ = a constant.

7. $Fv = P$

$$mv\frac{dv}{dx}v = P$$

$$m\,v^2 dv = P\,dx$$

$$\int_v^{2v} v^2 dv = \frac{P}{m}\int_0^x dx$$

Integrating and simplifying gives

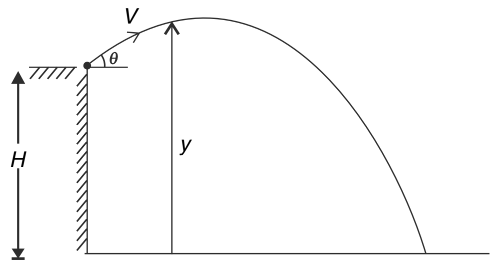
$$x = \frac{7mv^3}{3P}$$

8. If the passenger moves up an escalator at a constant speed, the average force that he exerts on the staircase remains unaltered equal to his weight. Therefore the force with which the motor must drive the staircase will also remain the same. The power which has to be exerted by the motor remains the same. However a walking man will reach the top of the escalator sooner and therefore the work done by the motor of the escalator in raising a moving man will be less than that done in raising a stationary one (the part of the work is done by the man).

9. (ii) Mechanical energy at the time of projection

$$E = \frac{1}{2}mV^2 + mgH$$

$$= \frac{1}{2} \times 0.2 \times 10^2 + 0.2 \times 10 \times 20 = 10 + 40 = 50\ J$$



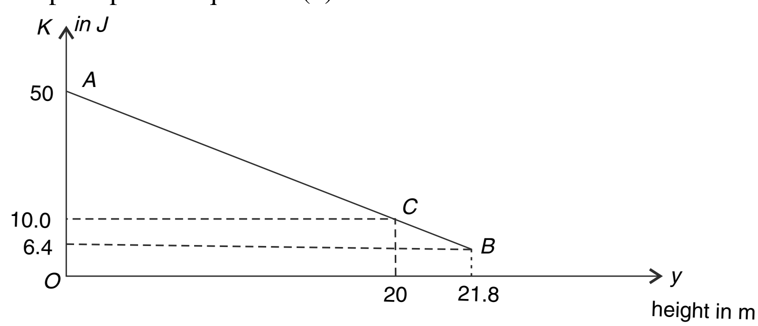
Kinetic energy at height y is

$K = E - mgy$

$K = 50 - 2y$ ………………….(i)

$$y_{\max} = H + \frac{V^2 \sin^2 \theta}{2g} = 20 + \frac{10^2 \times 9}{2 \times 10 \times 25} = 20 + 1.8 = 21.8\ m$$

Graph is plot of equation (1)



point A ⟶ bottom of the building

point B ⟶ highest point in trajectory

point C ⟶ top of the building

If we follow the motion of the ball, the graph starts at C, moves to B and then moves from B to A.

10. Resistance force $F_R = (m + nV^2)$

Power $= F_R.V$

$\therefore (m + nV^2)\ V = P$

For $V = 40$ m/s

$[m + n\ (40)^2] \times 40 = 53 \times 10^3$

$\Rightarrow m + 1600\ n = 1325$ …………(1)

For $V = 10$ m/s

$[m + n\ (10)^2] \times 10 = 2 \times 10^3$

$m + 100n = 200$ …………(2)

(1) - (2)

$1500\ n = 1125$

$n = 0.75$

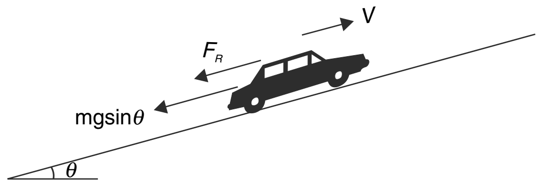
put in (2) $m = 125$

$\therefore F_R = 125 + 0.75\ V^2$

(a) $P = F_R.\text{V}$

$= [125 + 0.75\ (40)^2]\ 40 = 53000\ W = 53\ KW$

(b)



$(mg \sin \theta + F_R)V = 69 \times 10^3$

$mg \sin\theta . V + F_R V = 69000$

From last part of the question

$F_A V = 53000\ W$ for $V = 40\ m/s$

$\therefore\ mg \sin \theta = \dfrac{69000 - 53000}{40} = 400$

$\therefore \sin \theta = \dfrac{400}{1600 \times 10} = \dfrac{1}{40}$

θ in radian will be approximately $\left(\dfrac{1}{40}\right)$

In degree $\theta = \dfrac{180}{\pi} \times \dfrac{1}{40} = 1.43°$

11. Work done by a conservative force in a closed path is zero

$\therefore\ W_{AB} + W_{BC} + W_{CD} + W_{DA} = 0$

$-50 + 25 + 60 + W_{DA} = 0$

$\Rightarrow W_{DA} = -35\ J$

$U_B - U_A = -W_{A \to B}$

$U_B - O = 50\ J \Rightarrow U_B = 50\ J$

And $U_D - U_A = -W_{A \to D}$

$U_D - O = -(+35) \Rightarrow U_D = -35\ J$

13. Work done by the force in position of the particle changing from $x = 0$ to $x = 5.5\ m$ is

W = area under the given graph

$= \dfrac{1}{2} \times 1 \times (1 + 2.5) - 1 \times 2.0$

$= 1.75 - 2.0 = -0.25\ J$

From work – energy theorem

$K_f - K_i = -0.25$

$\therefore\ K_i = K_f + 0.25$

Since $K_f \geq 0$

$\therefore\ K_i \geq 0.25$

$\dfrac{1}{2} \times 2 \times u^2 \geq 0.25$

$\Rightarrow u > 0.5$ m/s

15. If the height of the pan at an instant is l and the boy transfers a small mass m of the sand into it, he performs a work = mg l (against gravity).

This work done increases the gravitational potential energy of the sand mass which loses a part of it in compressing the spring.

Therefore, total work done by the boy will be finally found as gravitational potential energy of the sand plus the elastic potential energy of the spring.

As per the question.

$$K\frac{L_o}{2} = Mg \qquad \text{..................(i)}$$

Work done $W = Mg\frac{L_o}{2} + \frac{1}{2}K\left(\frac{L_o}{2}\right)^2$

$$= Mg\frac{L_o}{2} + \frac{1}{8}KL_o^2$$

$$= Mg\frac{L_o}{2} + \frac{1}{4}MgL_o \qquad \text{[using (1)]}$$

$$= \frac{3}{4}MgL_o$$

16. (a) Initially, the *COM* of the snake is $\frac{L}{2}\sin 30° = \frac{L}{4}$ vertically below the top of the incline.

In final position, the COM is h below the top point, where

$$h = \frac{\frac{M}{2}\frac{L}{4} + \frac{M}{2}\cdot\frac{L}{4}\sin 30°}{M} = \frac{3L}{16}$$

$\therefore$ Centre of mass rises by $\Delta h = \frac{L}{4} - \frac{3L}{16} = \frac{L}{16}$

$\therefore$ Work done against gravity $W_g = Mg\ \Delta h = \frac{MgL}{16}$

(b) If length were $\frac{L}{2}$

$$W_g = \frac{MgL}{32}$$

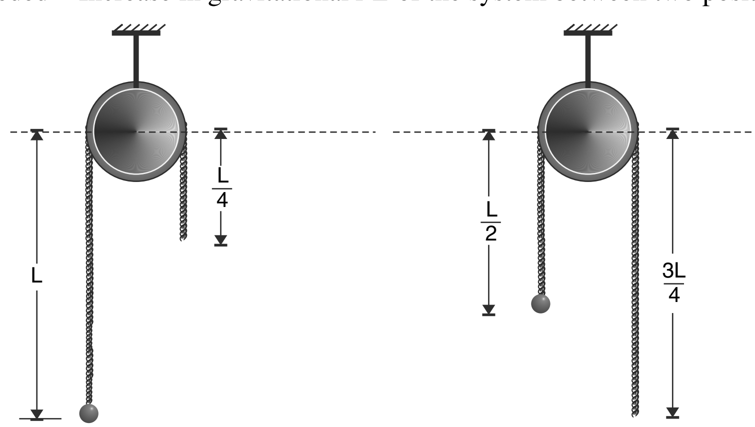
17. Total mass of the system

$$\frac{5L\lambda}{4} + \frac{L\lambda}{4} = \frac{3}{2}L\lambda$$

The external agent must ensure that more than half the mass of the system gets to the right of the pulley. After that he can leave the system. Ball will accelerate to the pulley.

Take *PE* to be zero at the horizontal level of the centre of the pulley.

Minimum work needed = Increase in gravitational *PE* of the system between two positions shown in fig. below.



mass on both sides equal

$$W_{\min} = \left[-\frac{\lambda L}{2}\frac{L}{4}\cdot g - \frac{\lambda 3L}{4}\cdot\frac{3L}{8}g - \frac{\lambda}{4}\frac{L}{2}g\right] - \left[-\lambda L\;\frac{L}{2}g - \frac{\lambda L}{4}\frac{L}{8}g - \frac{\lambda}{4}L\,g\right] = \frac{\lambda Lg}{4}$$

18. Increase in gravitational PE = Loss in KE

$mgh = 20\ J$

$$h = \frac{20}{0.1\times 10} = 20\ m$$

19. Hint: Change in spring $PE = \frac{1}{2}Kx^2$, when the spring compresses or stretches by x.

20. The block will finally come to rest when the extension in the spring is X_0 such that

$KX_0 = mg\sin\theta$

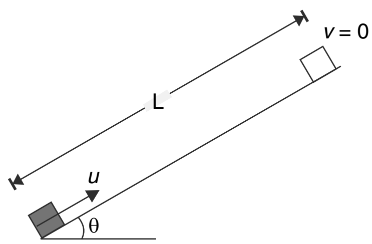
$$X_0 = \frac{4\times 10\times\frac{1}{2}}{80} = \frac{1}{4}m$$

The friction will dissipate all the energy possessed by the system initially, which is

$E = mgh = 4 \times 10 \times (X.\sin\theta) = 40\times\frac{1}{8} = 5\,J$

$\therefore\ W_f = -5\ J$

21. (a) The friction force has same magnitude during up and down journey.



$\therefore$ Work done by friction while going up is same as work done by it while coming down.

(b) Total loss of mechanical energy $= \frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{3}{8}mu^2$

$\therefore\ (\mu mg\cos\theta)(2L) = \frac{3}{8}mu^2$ .... (1)

Retardation during up journey = $g(\sin\theta + \mu\cos\theta)$

$\therefore\ u^2 = 2g(\sin\theta + \mu\cos\theta).\ L$

Using (1) $u^2 = 2gL\sin\theta + \frac{3}{8}u^2$

$\Rightarrow\ \frac{5u^2}{8} = 2gL\sin\theta$ ....(2)

From (2) and (1)

$$\frac{\tan\theta}{\mu} = \frac{5}{3}$$

$\therefore\quad \mu = \frac{3}{5}\tan\theta$

22. Hint: There is no change in speed of the tanker due to leakage of water.

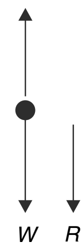
23. (a) Let the stone reach a height h

Forces on stone are

Gravity W and

Air resistance R

Both act opposite to displacement during the upward motion.



$\therefore$ Work done on stone

$W_o = W_g + W_R$

$= -Wh - Rh$ [both W and R are constant force]

$= -(W + R)h$

Change in kinetic energy of stone in moving from ground to top of its path is

$\Delta k = k_f - k_i$

$= 0 - \frac{1}{2}mu^2 = -\frac{1}{2}mu^2$

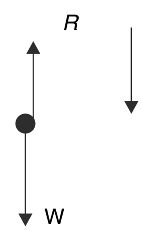
From work energy theorem

$W_o = \Delta k$

$\Rightarrow -(W + R)h = -\frac{1}{2}mu^2$

$\Rightarrow h = \frac{1}{2}\frac{mu^2}{(W+R)} = \frac{Wu^2}{2g(W+R)} \quad \left[\because m = \frac{W}{g}\right]$

(b) For downward motion



$W_g = Wh$ (positive)

$W_R = -Rh$ (negative)

$\Delta k = k_f - k_i$

$= \frac{1}{2}mv^2 - 0$

[$K_i = KE$ at the top = 0]

$\therefore$ From work energy theorem

$Wh - Rh = \frac{1}{2}mv^2$

$\Rightarrow v^2 = \frac{2(W-R)h}{m}$

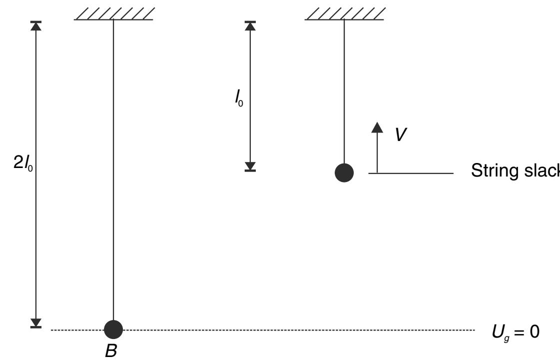
$= \frac{2(W-R)gh}{W}$

Substituting for h

$v = u\left(\frac{W-R}{W+R}\right)^{1/2}$

24. Consider zero of gravitational PE at initial position of the mass.

The string becomes slack when it regains its natural length.



Mechanical energy conservation:

$$\frac{1}{2}k(l_0)^2 = \frac{1}{2}mv^2 + mg\,l_0$$

$$\Rightarrow \frac{1}{2}\times 16\times .0.25^2 = \frac{1}{2}\times 0.1\times v^2 + 0.1\times 10\times 0.25$$

$$\Rightarrow \quad v = \sqrt{5}\ m/s$$

Let the mass come to rest at height h above the initial position.

Energy conservation

$$\frac{1}{2}k\,l_0^2 = mgh$$

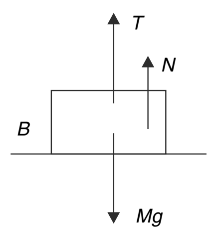
$$\frac{1}{2}\times 16\times 0.25^2 = 0.1\times 10\times h$$

$$h = 0.5\ m$$

Therefore, the mass will come to rest at point A.

26. When ball A is at its lowest position, let the stretch in the spring be x. At this instant tension acting on B is kx.

$\therefore\ N = Mg - T$



For N to be equal to $\frac{Mg}{2}$,

$$T = \frac{Mg}{2}$$

$\therefore$ The spring must stretch by x such that

$kx = \frac{Mg}{2}$ (when A comes to rest)

Energy Conservation:

Loss in PE of A= gain in spring PE

$$mgx = \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = \frac{2mg}{k}$$

$$kx = 2mg$$

$$\Rightarrow \frac{Mg}{2} = 2mg$$

$$\therefore \quad m = \frac{M}{4}$$

27. $W_{\text{spring}} + W_{\text{thrust}} = \frac{1}{2}mv^2 + mgh$

$\frac{1}{2}k(5)^2 + 6\times 10^4\times 20 = \frac{1}{2}\times 2000\times (50)^2 + 2000\times 10\times (20\sin 20°)$ [180 kph = 50 m/s; g = 10 m/s^2]

$$\frac{25}{2}k = 2.5\times 10^6 + 0.12\times 10^6$$

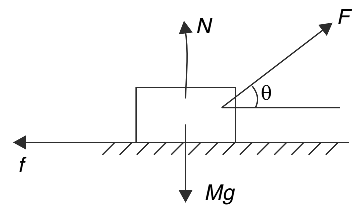
$$k = \frac{2\times 2.62}{25}\times 10^6$$

$=2.096\times 10^5\ N/m$

28. $N = mg - F\sin\theta$

$\therefore f = \mu N = \mu\,(Mg - F\sin\theta)$

WE Theorem:



$$W_F + W_f = \frac{1}{2}mv^2$$

$$\therefore\ \frac{1}{2}mv^2 = (F\cos\theta)\cdot d - \mu(Mg - F\sin\theta)\cdot d$$

$$= Fd\,(\mu\sin\theta + \cos\theta) - \mu Mgd$$

Maximum value of $(\mu\sin\theta + \cos\theta)$ is $\sqrt{\mu^2+1}$

When $\tan\theta = \mu$

$$\therefore\ (KE)_{\max} = \sqrt{\mu^2+1}\ Fd - \mu Mgd$$

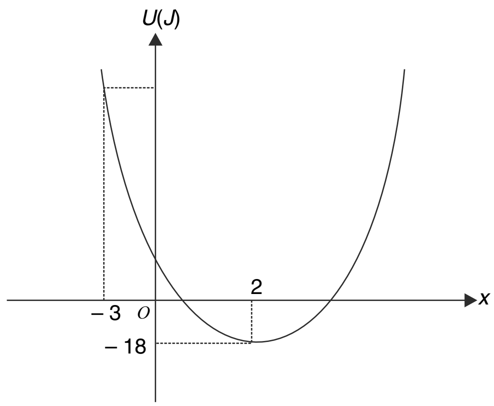
$$= \sqrt{\mu^2+1}\,\frac{Mg}{2}d - \mu Mgd$$

$$= Mgd\left[\frac{\sqrt{\mu^2+1}}{2} - \mu\right]$$

29. Hint : Along path 1 the normal force is less and friction force is small. Along path 2 the normal force is high.

30. (a) $U = 5x^2 - 20x + 2$

The variation of U with x is as shown.



The *PE* is minimum when $\dfrac{dU}{dx} = 0$

$\Rightarrow 10x - 20 = 0$

$\Rightarrow x = 2m$

At $x = 2m$, $U = 5\,(2)^2 - 20\,(2) + 2 = -\,18\ J$

At $x = -3m$, $U = 5\,(-\,3)^2 - 20\,(-\,3) + 2 = 107\ J$

When particle is released at $x = -3$, it experiences a force in positive x direction. Its *KE* is maximum at $x = 2\ m$

$k_{\max} + (-\,18) = 107$

$\Rightarrow k_{\max} = 125\ J$

(b) Total energy of the particle is $107J$. When it is at rest its its $KE = 0$.

$\therefore U = 107 \Rightarrow\ 5x^2 - 20x + 2 = 107$

$\Rightarrow x^2 - 4\,x - 21 = 0$

$\Rightarrow x = -3,\ 7\ m \qquad \therefore x_{\max} = 7\ m$

31. Hint: For reaching the origin the total mechanical energy of the particle must be greater than $6U_0$.

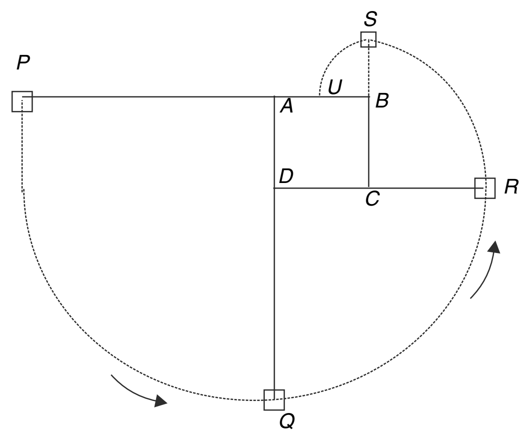
A minima of PE curve is the position of stable equilibrium and the particle performs oscillations about this position.

32. From P to Q the particle rotates in a circle of radius 3.5 L with speed V.

From Q to R it rotates in a circle of radius $2.5L$ (with centre at D) with speed V .

From R to S it rotates in a circle of radius $1.5L$ with centre at C .

From S to U it goes in a circle of radius $0.5L$ with speed V .



The speed of the particle does not change because the string tension force acting on it is always perpendicular to its velocity. The power of the tension force is always zero. In absence of any work done on it the kinetic energy of the particle does not change.

(a) $\therefore\ t = \dfrac{2\pi(3.5L)}{4\cdot V} + \dfrac{2\pi(2.5L)}{4\cdot V} + \dfrac{2\pi(1.5L)}{4.V} + \dfrac{2\pi(0.5L)}{4V}$

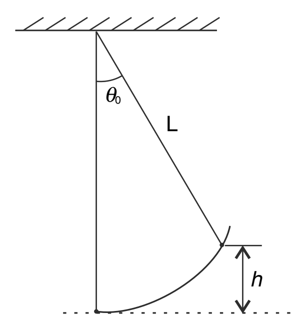
$$= \frac{2\pi}{4V}(3.5 + 2.5 + 1.5 + 0.5)L$$

$$= \frac{4\pi L}{V}$$

(b) Just before hitting the pillar, the particle is moving in a circle of radius 0.5 L with speed V

$$\therefore\ T = \frac{mV^2}{0.5L} = \frac{2mV^2}{L}$$

33. (i) $E = PE$ at extreme position [since KE at extreme is zero]



$$\therefore\ E = mgh = mg\,L\,(1 - \cos\theta_0) = 2\,mg\,L\sin^2\left(\frac{\theta_0}{2}\right)$$

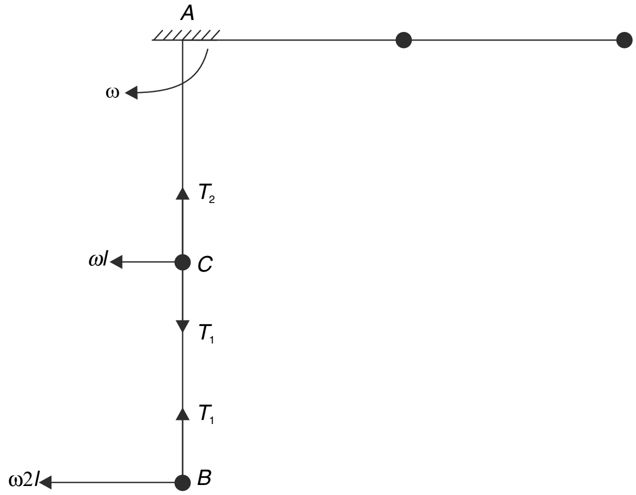
$$\simeq 2\,mg\,L\left(\frac{\theta_0}{2}\right)^2$$

$$\left[\because\ \sin\left(\frac{\theta_0}{2}\right) \simeq \frac{\theta_0}{2}\right]$$

$$= \frac{1}{2}\,mg\,L\,\theta_0^2$$

(ii) The total mechanical energy of the pendulum bob at any location is same. Therefore, energy conservation gives $V_1 = V_2 = V_3$.

34. Energy Conservation gives :



$$\frac{1}{2}m(\omega l)^2+\frac{1}{2}m(2\omega l)^2=mgl+2mgl$$

$$\omega^2=\frac{6g}{5l}$$

Let T_1 be tension in segment BC . For circular motion of mass at B

$T_1-mg=m\omega^2\,(2l)$

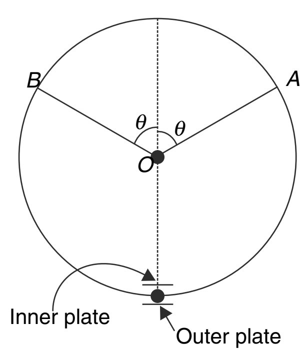
$$T_1=mg+\frac{12\,mg}{5}=\frac{17}{5}mg$$

For circular motion of mass at C

$T_2-T_1-mg=m\omega^2 l$

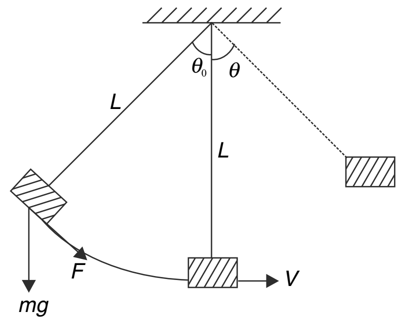
$$T_2=\frac{17}{5}mg+mg+\frac{6\,mg}{5}=\frac{28}{5}mg$$

35. Initially the bead will be in contact with the outer plate so that it receives a normal force towards centre (think of tension in a pendulum). Since velocity at bottom is $v=\omega L=\sqrt{4gL}$ (which is less than $\sqrt{5gL}$), there will be a point (A) where bead will need no normal force (apart from weight) towards centre. Beyond A (up to B) it needs a radially outward force. Hence, at A the bead falls on to the inner plate producing a clink (sound when two metal collide). After B the bead will once again hit the outer plate. This produces the second sound.



36. Let the swing rise to an angle θ

$$F\,(L\theta_0)+mgL(1-\cos\theta_0)=\frac{1}{2}mV^2=mgL(1-\cos\theta)$$



$$\frac{mg}{2}L\theta_0 + mgL(1-\cos\theta_0) = mgL(1-\cos\theta)$$

0.5 + (1 – 0.5) = 1 – cos θ

[$\because$ $\theta_0 = 1$; $\cos\theta_0 = 0.5$]

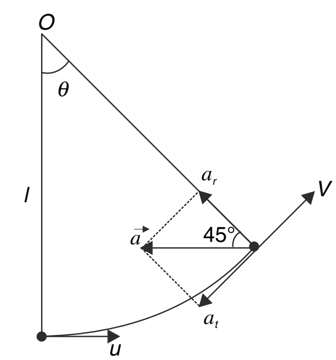
$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = 90°$

$\therefore$ Swing will become horizontal.

37. At position θ,

Radial acceleration of the particle is $a_r = \dfrac{v^2}{l}$ and



Tangential acceleration is $a_t = g \sin\theta$

As per the question $\dfrac{a_r}{a_t} = \tan 45°$

$\Rightarrow \dfrac{v^2}{l} = g\sin\theta$ .... (1)

From conservation of mechanical energy,

$$\frac{1}{2}mV^2 + mgl\,[1-\cos\theta] = \frac{1}{2}mu^2$$

$$\Rightarrow \frac{v^2}{l} = \frac{u^2}{l} - 2g\,[1-\cos\theta]$$

$= 2g - 2g + 2g\cos\theta$

$= 2g\cos\theta$ .... (2)

From (1) and (2). $\sin\theta = 2\cos\theta$

$\Rightarrow \tan\theta = 2 \Rightarrow \theta = \tan^{-1}2$.

38. $a_c = k^2rt^2$

Or, $\dfrac{v^2}{r} = k^2rt^2$

Or, $v = krt$

Therefore, tangential acceleration, $a_t = \dfrac{dv}{dt} = kr$

Or, Tangential force,

$F_t = ma_t = mkr$

Only tangential force performs work.

Power = $F_t\, v = (mkr)\,(krt)$

Or, Power = $mk^2\, r^2\, t$

39. (a) The difference in $K.E.$ at positions A and B is

$$K_A - K_B = \frac{1}{2}mv_A^2 - \frac{1}{2}mv_B^2 = mg\,(2L\cos\theta) = 2mgL\cos\theta \quad ....\ (1)$$

$$T_A = \frac{mv_A^2}{L} + \text{mg}\cos\theta$$

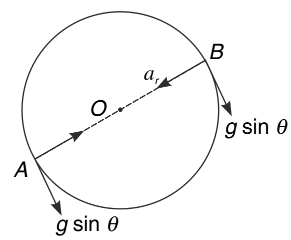
$$T_B = \frac{mv_B^2}{L} - \text{mg}\cos\theta$$

$$\therefore\ T_A - T_B = \frac{mv_A^2 - mv_B^2}{L} + 2\text{ mg}\cos\theta \quad ....(2)$$

from equation (1) and (2)

$T_A - T_B = 6\, mg \cos\theta$

(b) The component of accelerations of ball at A and B are as shown in figure.



$$\Rightarrow\ |\vec{a}_A + \vec{a}_B| = \sqrt{(2g\sin\theta)^2 + \left(\frac{v_A^2}{L} - \frac{v_B^2}{L}\right)^2}$$

$$= \sqrt{4g\sin^2\theta \quad 16g^2\cos^2\theta} = g\sqrt{4 + 12\cos^2\theta}$$

40. In extreme position, total acceleration is equal to tangential acceleration. This is because the radial acceleration is zero. It means total acceleration is $g\sin\theta$.

At mean position (lowest position), total acceleration is the radial acceleration = v^2/ℓ as tangential acceleration is zero.

Conservation of energy gives

$$\frac{1}{2}mv^2 = mg\ell(1-\cos\theta)$$

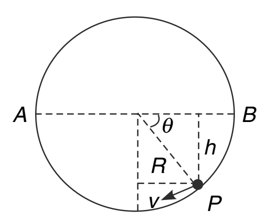
$$\Rightarrow\ \frac{v^2}{\ell} = 2g(1-\cos\theta)$$

$$\Rightarrow\ g\sin\theta = 2g\,(1-\cos\theta)$$

$$\Rightarrow\ 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 4\sin^2\frac{\theta}{2}$$

$$\Rightarrow\ 2\tan\frac{\theta}{2} = 1 \ \Rightarrow\ \theta = 2\tan^{-1}\frac{1}{2}$$

41. Let velocity of particle at point P be v.



From conservation of mechanical energy

$$\frac{1}{2}\,mv^2 = K = \text{mgh}$$

Let N be the normal reaction between the particle and the pipe at this instant. Then

$$N - \text{mg}\sin\theta = \frac{mv^2}{R}$$

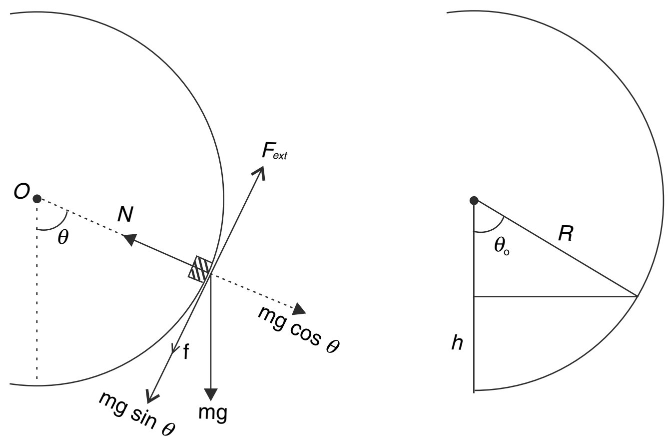
But, $\dfrac{mv^2}{R} = \dfrac{2K}{R}$ and $\sin\theta = \dfrac{h}{R}$

Hence , N = mg $\left(\dfrac{h}{R}\right) + \dfrac{2K}{R} = \dfrac{K}{R} + \dfrac{2K}{R}$

($\because K = mgh$)

Hence, $N = \dfrac{3K}{R}$ = force on the pipe.

42. (i)



Forces acting at position θ have been shown.

$F_{\text{ext}} = mg \sin\theta + f$

$= mg \sin\theta + \mu\, mg \cos\theta$

Work done in small angular displacement $d\theta$ is $dW_{\text{ext}} = F_{\text{ext}}.Rd\theta$

$$\therefore W_{\text{ext}} = \int_0^{\theta_0} \left(mg\sin\theta + \mu mg\cos\theta\right) Rd\theta$$

$$= mgR\left[-\cos\theta\right]_0^{\theta_0} + \mu mgR\left[\sin\theta\right]_0^{\theta_o}$$

$$= mgR\left[1-\cos\theta_0\right] + \mu mgR\left[\sin\theta_0\right]$$

From geometry, $\sin\theta_0 = \dfrac{\sqrt{R^2-(R-h)^2}}{R}$

$$= \frac{1}{R}\sqrt{2Rh-h^2}$$

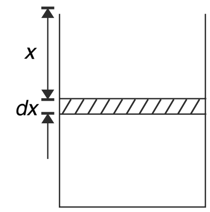
And $\cos\theta_0 = \dfrac{R-h}{R}$

$$\therefore W_{\text{ext}} = mgR\left[1-\frac{R-h}{h}\right] + \mu\, mgR\frac{1}{R}\sqrt{2Rh-h^2}$$

$$= mgh + \mu mg\sqrt{2Rh-h^2}$$

(ii) In the reference frame of the ground, the normal force of the track does not perform any work on the block. But in the reference frame of the moving observer, the normal force does perform negative work on the block.

43. Mass of water in unit length of the cylinder is $= \dfrac{M}{H}$



The water level decreases at a constant rate

$$a = \frac{H}{t_0}$$

Level falls by x at time $t = \dfrac{x}{a}$

Mass of a length dx of water . $= \dfrac{M}{H}dx$

If this dx length of water is transferred to the wide container in time dt then

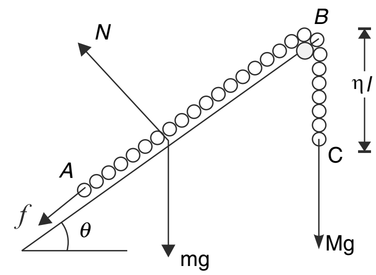
$$dW = \left(\frac{M}{H}dx\right) g\,(x+h)$$

$$\frac{dW}{dt} = \frac{Mg}{H}\left(\frac{dx}{dt}\right)(at+h)$$

$$P = \frac{Mg}{H}\left(\frac{H}{t_o}\right)\left(\frac{H}{t_o}t+h\right) \qquad \left[\because\ a = \frac{dx}{dt} = \frac{H}{t_o}\right]$$

$$= \frac{Mg}{t_o}\left(h + H\frac{t}{t_o}\right)$$

44. $BC = \eta\,\ell$



and $AB = (1-\eta)\,\ell$

Mass of BC, $M = \eta\,\ell.\dfrac{m_o}{\ell} = \eta\, m_o$

Mass of AB, $m = (1-\eta)\,\ell\dfrac{m_o}{\ell} = (1-\eta)\, m_o$

Since chain is on verge of sliding, friction is at its limiting value

$f = \mu N = \mu\, mg \cos\theta$

$\mu = (1-\eta)\, m_o\, g \cos\theta$

Chain will just start sliding if

Net driving force = Net resisting force

$\Rightarrow\ Mg = mg \sin\theta + f$

$\Rightarrow\ \eta m_o g = (1-\eta)\, m_o g \sin\theta + \mu(1-\eta)\, m_o g \cos\theta$

$$\Rightarrow\ \frac{\eta-(1-\eta)\sin\theta}{(1-\eta)\cos\theta} = \mu$$

Now as the chain moves, friction force changes due to change in N.

We consider a situation when $BC = x$

And $AB = l - x$

Mass of $AB = (l-x)\dfrac{m_0}{l}$

$\therefore\ f = \mu N$

$$= \mu\frac{(l-x)m_0 g}{l}cos\theta$$

If the chain moves by a distance dx

$dW_f = -fdx$ [displacement (dx) and friction (f) are oppositely directed.]

$$= -\frac{\mu(l-x)m_0 g\cos\theta}{l}dx$$

$$\therefore W_f = -\frac{\mu m_0 g\cos\theta}{l}\int_{\eta l}^{l}(l-x)dx$$

[$\because$ x = length of BC changes from ηl to l]

$$\therefore W_f = \frac{-\mu m_0 g\cos\theta}{l}\left[lx - \frac{x^2}{2}\right]_{\eta l}^{l}$$

$$= -\frac{\mu m_0 g\cos\theta}{l}\frac{l^2}{2}(1-\eta)^2$$

$$= -\frac{\eta-(1-\eta)\sin\theta}{2(1-\eta)\cos\theta}m_0 g\cos\theta\ .l(1-\eta)^2$$

$$= -\frac{[\eta-(1-\eta)\sin\theta]l(1-\eta)}{2}m_0 g$$

$$= -\frac{l(1-\eta)[\eta-(1-\eta)\sin\theta}{2}m_0 g$$

45. Velocity of disc relative to the board is $\overrightarrow{V_{db}} = \overrightarrow{V_d} - \overrightarrow{V_b} = 2\hat{i} - 2\hat{j}$

$\therefore$ Friction force acting on the board will be along $\overrightarrow{V_{db}}$, while that acting on the disc will be along $-\overrightarrow{V_{db}}$.

Friction force $\mu mg = 0.2 \times 2 \times 10 = 4\ N$

Friction on board $\overrightarrow{f_b} = 4\,\dfrac{(2\hat{i}-2\hat{j})}{2\sqrt{2}} = 2\sqrt{2}\,(\hat{i} - \hat{j})$

$\therefore$ External force on the board $\overrightarrow{F_b} = -2\sqrt{2}(\hat{i}-\hat{j})$

Power of this force $P_b = \overrightarrow{F_b}\ .\ \vec{V} = 4\sqrt{2}\ W$

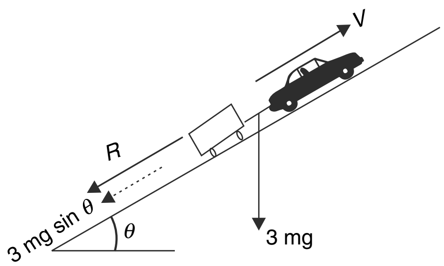
Friction force on the disc $\overrightarrow{f_d} = \dfrac{4(-2\hat{i} + 2\hat{j})}{2\sqrt{2}} = 2\sqrt{2}(-\hat{i} + \hat{j})$

External force on the disc $\overrightarrow{f_d} = 2\sqrt{2}\,(\hat{i}-\hat{j})$

Power $P_d = \overrightarrow{F_d}.\vec{u} = 4\sqrt{2}\ \ W$

Heat dissipated $= P_b + P_d = 8\sqrt{2}\ \ Js^{-1}$

46. The important thing to notice is that car engine can develop a certain maximum power (P_{max})
When car moves up with trailer the net force against the motion is $F = 3\ mg\ \sin\theta + R$
Where R = resistance to motion = $k(3m)V^2 \Rightarrow$ [k = a constant]



$\therefore\ F = 3\ mg\ \sin\theta + 3kmV^2$

$\therefore\ P_{max} = F.V = (\ 3mg\ \sin\theta + 3\ kmV^2)V$

When moving without trailer

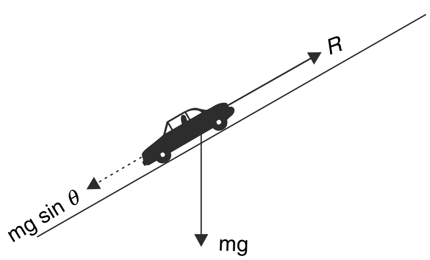
$P_{max} = 2V\ (mg\ \sin\ \theta + km\ (2V)^2)$

$\quad = 2V\ (mg\ \sin\ \theta + 4\ km\ V^2)$

$\therefore\ (3\ mg\ \sin\theta + 3\ km\ V^2)\ V = 2V\ (mg\ \sin\ \theta + 4km\ V^2)$

$\Rightarrow\ k = \dfrac{g\sin\theta}{5V^2}$ ……………………. (a)

When car moves down the slope, with engine shut off, initially it accelerated due to mg sin θ, R being small due to small speed. But as the speed grows, R increases and eventually R becomes equal to mg sin θ. After this car experience no acceleration and moves with constant speed.



Let this constant speed be u.

Then $kmu^2 = mg\ \sin\ \theta$

$u^2 = \dfrac{g\sin\theta}{k} = 5V^2 \qquad \left[\text{from } (a)\right]$

$\therefore\ u = \sqrt{5}\,V$

47. Work done by the force for a displacement is given by performing following integration with limit changing from initial to final position

$W = \int \vec{F}.\vec{dr}$

$= \int \dfrac{3(X\hat{i} + Y\hat{j})}{(X^2 + Y^2)^{3/2}}\ (dX\ \hat{i}\ + dY\ \hat{j}\ )$

$= 3\int \dfrac{XdX + YdY}{(X^2 + Y^2)^{3/2}}$

Taking : $X^2 + Y^2 = t$

$2XdX + 2YdY = dt$

$\Rightarrow\ XdX + YdY = \dfrac{dt}{2}$

$\Rightarrow \; W = 3\int \frac{dt/2}{t^{3/2}} = \frac{3}{2}\int \frac{dt}{t^{3/2}}$

This integral can be evaluated easily if the co ordinates of initial and final position are known. In order to evaluate the integral we need not know the path taken by the particle. It means work done is path independent. Hence the force is conservative.

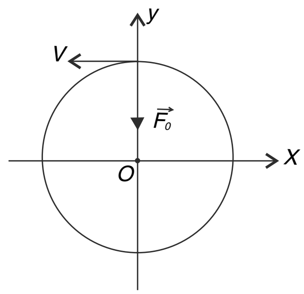
48. (a) $U = 2\,(x + y)$

$$F_x = -\frac{\partial U}{\partial x} = -2$$

$$F_y = -\frac{\partial U}{\partial y} = -2$$

Conservative force is $\overrightarrow{F_c} = -2\hat{i} - 2\hat{j}$

To keep the particle moving uniformly on a circle, net force is centripetal force. When the particle is at (0, 4) net force acting on it is



$$\overrightarrow{F_o} = \frac{mV^2}{R}\left(-\hat{j}\right) = -\frac{0.1\times 1^2}{4}\hat{j}$$

$$= -\left(0.025\,N\right)\hat{j}$$

$$\overrightarrow{F_{ext}} + \overrightarrow{F_c} = \overrightarrow{F_o}$$

$$\overrightarrow{F_{ext}} = -0.025\hat{j} - \left(-2\hat{i} - 2\hat{j}\right)$$

$$= \left(2\hat{i} + 1.975\hat{j}\right)N$$

(b) $W_{\text{ext}} = \Delta K + \Delta U$

$= 0 + U\text{ (at 4, 0)} - U\text{ (at 0, 4)}$

$= 8 - 8 = 0.$

49. $\vec{r} = b\left(kt - \sin kt\right)\hat{i} + b\left(kt + \cos kt\right)\hat{j}$

$$\vec{V} = \frac{d\vec{r}}{dt} = \left(bk - bk\cos kt\right)\hat{i} + \left(bk - bk\sin kt\right)\hat{j}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \left(bk^2\sin kt\right)\hat{i} - \left(bk^2\cos kt\right)\hat{j}$$

(a) When power of the force is zero

$$\vec{F}.\vec{V} = 0$$

$$m\,\vec{a}.\vec{V} = 0 \Rightarrow \vec{a}\cdot\vec{V} = 0$$

$\Rightarrow bk^2 \sin kt\,(bk - bk\cos kt) - bk^2\cos kt\,(bk - bk\sin kt) = 0$

$\Rightarrow b^2k^3 \sin kt = b^2k^3 \cos kt)$

$\Rightarrow \tan kt = 1$

$\therefore\ tk = \dfrac{\pi}{4};\ t = \dfrac{\pi}{4k}$

$\therefore t_o = \dfrac{\pi}{4k}$

(b) Velocity at $t_o = \dfrac{\pi}{4k}$

$$\vec{V_1} = \left(bk - bk\cos\frac{\pi}{4}\right)\hat{i} + \left(bk - bk\sin\frac{\pi}{4}\right)\hat{j}$$

$$V_1^2 = (bk)^2\left[\left(1-\frac{1}{\sqrt{2}}\right)^2 + \left(1-\frac{1}{\sqrt{2}}\right)^2\right]$$

$$= (bk)^2\left(3-2\sqrt{2}\right)$$

Velocity at $t = \dfrac{\pi}{k}$

$$\vec{V_2} = bk(1-\cos\pi)\hat{i} + bk(1-\sin\pi)\hat{j} = 2b\ k\hat{i}$$

$\therefore V_2 = 2bk$

$$W = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2 = \frac{1}{2}m(bk)^2\left[4-\left(3-2\sqrt{2}\right)\right] = \frac{\left(1+2\sqrt{2}\right)}{2}m\,b^2k^2$$

50. Elongation in the spring is $x = x_A - x_B$

Where x_A= displacement of end A measured from original natural length position

x_B =displacement of block measured from original position.

$$\frac{dx}{dt} = \frac{dx_A}{dt} - \frac{dx_B}{dt} = V_A - V_B = 6 - 3 = 3 \text{ m/s}$$

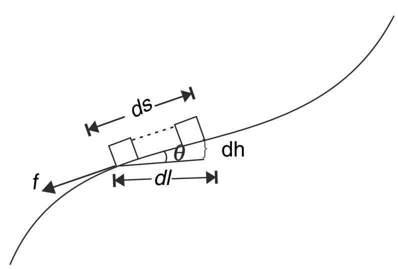
Potential energy in the spring $U = \dfrac{1}{2}k\,x^2$

$$\frac{dU}{dt} = k\,x \cdot \frac{dx}{dt}$$

$$\Rightarrow 15 = (kx)(3) \Rightarrow kx = 5\ N$$

$\therefore$ acceleration of the block $a = \dfrac{5}{2} = 2.5\ m/s^2$

51. Work done by friction can be calculated as–



Consider a small displacement ds of the body at an intermediate point where slope angle of hill is θ. (see fig)

Force of friction is $f = \mu mg \cos\theta$ (down the slope)

$\therefore$ Work done by friction in small displacement ds is $dW_f = -f\,ds = -\mu mg\,(ds\cos\theta)$

$= -\mu\, mg\, d\ell$ $\quad$ [from figure $d\ell = ds\cos\theta$]

$$\therefore W_f = -\mu mg \int_{base}^{top} d\ell = -\mu mg\ell$$

Since the body is hauled slowly, at each point its velocity is nearly zero.

$\therefore \Delta k = 0$

But $W_N + W_{mg} + W_f + W_F = \Delta k = 0$ ……………….(a)

$\Rightarrow W_F = -(W_N + W_{mg} + W_f)$

$= (0 - mgh - \mu mg\ell) = mg\,(h + \mu\ell)$

52. Body acquires a velocity means it is accelerated during the course of its motion.

However, W_N ,W_{mg} and W_f remain same as in previous question.

$\therefore$ From equation (a) of previous solution.

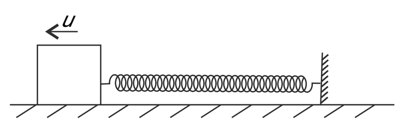
$$W_N + W_{mg} + W_f + W_F = \Delta k = \frac{1}{2}mv^2 \qquad \left[\because \Delta k = k_f - k_i = \frac{1}{2}mv^2 - 0\right]$$

$$\Rightarrow W_F = \frac{1}{2}mv^2 - [-mgh - \mu mg\ell]$$

$$= \frac{1}{2}mv^2 + mg(h + \mu\ell)$$

53. Observe the system from a reference frame moving to right with velocity u. In this frame the free end of the spring remains as rest (as if tied to a wall) and the initial velocity of the block is u to left. Hence, maximum extension (or compression) is given by

$$\frac{1}{2}kx_0^2 = \frac{1}{2}Mu^2 \Rightarrow x_o = \sqrt{\frac{M}{k}}\,u$$



54. Let the kinetic friction force on the block be f

From work energy theorem:

$W_f = \Delta U + \Delta K$

$$-fx_0 = \left(0 - \frac{1}{2}kx_0^2\right) + 0$$

$$\therefore f = \frac{1}{2}kx_0$$

As the block is released from extreme position it accelerates because the spring force exceeds the friction.

At position where extension in the spring is $\frac{x_0}{2}$, the friction force equals the spring force. After this the friction force will exceed the spring force and the block will retard.

$\therefore$ Speed is maximum when extension $x = \frac{x_0}{2}$. Again using work energy theorem gives-

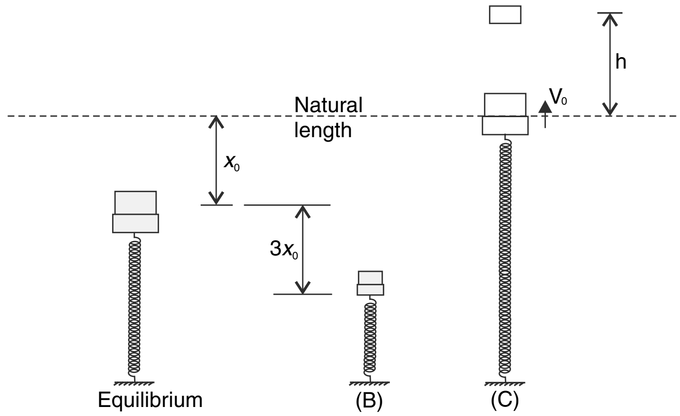
$W_f = \Delta U + \Delta K$

$$-f\frac{x_0}{2} = \left[\frac{1}{2}k\left(\frac{x_0}{2}\right)^2 - \frac{1}{2}kx_0^2\right] + [K_{max}]$$

$$\therefore -\frac{kx_0^2}{4} = -\frac{3}{8}kx_0^2 + K_{max}$$

$\therefore\ K_{max} = \frac{1}{8}kx_0^2$

55. (a)



In equilibrium, the spring is compressed by $x_0 = \frac{3Mg}{K}$ [fig. (A)]

The system is released from position shown in fig (B). Compression in the spring at this position is $4x_0$. The upper block will lose contact with the lower block when the spring acquires its natural length position. [Why?]

Hence, answer to first part of the question is $4x_0 = \frac{12Mg}{k}$

(b) Let's apply law of conservation of mechanical energy to get the speed of the block when the spring reaches its natural length.

$$\frac{1}{2}k(4x_0)^2 = \frac{1}{2}3M\,V_0^2 + 3Mg(4x_0)$$

$$72\frac{M^2g^2}{k} = \frac{3}{2}MV_0^2 + \frac{36M^2g^2}{k}$$

$$V_0 = \sqrt{\frac{24M}{k}}\,.\,g$$

Further height attained by the block of mass 2M is given by $h = \frac{V_0^2}{2g} = \frac{12Mg}{k}$

$\therefore$ Height attained above point of release is $H = 4x_0 + h = \frac{24Mg}{k}$

56. (a) Initial PE is spring $U = \frac{1}{2}k\,x^2$

$$= \frac{1}{2}\times\frac{200}{3}\times(0.1)^2 = \frac{1}{3}J = 0.33J$$

Work done by friction when the two blocks get displaced by x is

$W_f = f.x = -\,0.2 \times 2 \times 10 \times x = -\,4x$

Till the spring returns to natural length the work done by friction will be

$W_f = -\,4 \times 0.1 = -\,0.4J$

But energy available for dissipation is 0.33 J only.

$\therefore$ Blocks will stop before the spring returns to its natural length.

KE will be increasing till the spring force exceeds friction.

It will be maximum where

$kx = \mu(2mg)$ [x = compression in spring]

$$\frac{200}{3}x = 0.2\times2\times1\times10$$

$$x = 0.06\ m$$

$\therefore\ x_0 = 0.1 - 0.06 = 0.04\ m = 4\ cm$

let x = compression when the two blocks come to rest.

$$\frac{1}{2}k\left[0.1^2 - x^2\right] = \mu(2mg).(0.1 - x)$$

Solving for x we get

$x = 0.02\ m = 2\ cm$

In this position, spring force on block A is

$$kx = \frac{200}{3}\times\ 0.02\ = \frac{4}{3}\ N$$

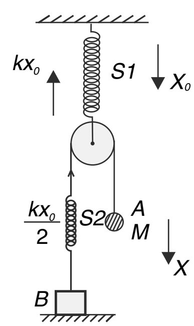
The maximum static friction on A can be

$\mu mg = 0.2 \times 1 \times 10 = 2\ N$

Hence, friction on A will adjust to $\frac{4}{3}N$ and balance the spring force. There will be no contact force between A and B.

(b) As in (a) you can check that the blocks will cross the natural length position of the spring. Just when the spring starts getting stretched retardation of A will exceed the retardation of B and the two blocks will get separated.

57.



When spring $S1$ Stretches by x_0, tension in it is kx_0

At this instant tension in $S2$ shall be $\frac{kx_0}{2}$

It means $S2$ is stretched by $\frac{x_0}{2}$.

When $S1$ and $S2$ stretch by x_0 and $\frac{x_0}{2}$, the ball A will fall through a distance

$$x = 2x_0 + \frac{x_0}{2} = \frac{5x_0}{2} \qquad ....(1)$$

[we are assuming that B does not move]

If A falls through x before coming to rest,

$$x_0 = \frac{2x}{5}$$

$$\therefore \text{ Spring force on } B = \frac{kx_0}{2} = \frac{kx}{5}$$

B will just leave the table at this instant if

$$\frac{kx}{5} = Mg \qquad \text{..................(2)}$$

When ball A falls through x (before coming to rest) principle of conservation of energy says loss in PE of A = Gain in spring PE

$$mg\,x = \frac{1}{2}kx_0^2 + \frac{1}{2}k\left(\frac{x_0}{2}\right)^2$$

$$mg\,x = \frac{1}{2}k\left(\frac{2x}{5}\right)^2 + \frac{1}{2}k\left(\frac{x}{5}\right)^2$$

$\Rightarrow kx = 10\,mg$

From (2) $5\,Mg = 10\,mg$

$$\therefore m = \frac{M}{2}$$

58. Spring force is maximum when the system is released

$F_{S\max} = kx_0 = 240\ N$

The limiting friction on A can be

$F_{lA} = \mu_1 M_A\, g = 250\ N$

Hence, block A remains fixed and does not move at all.

The limiting friction on C can be

$F_{lc} = \mu_2 M_C\, g$

Thus maximum acceleration that friction can provide to C is

$a_{c\max} = \mu_2 g = 8.5\ m/s^2$

Just after the release spring force is maximum and it will cause maximum acceleration in B. Let us assume that there is no slipping between B and C. In that case the maximum acceleration is

$$a_{c\max} = \frac{240}{28+2} = 8.0\ m/s^2$$

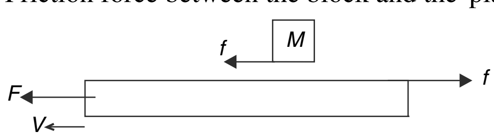
Friction can easily provide this acceleration to block C hence it will not slip over B.

Speed is maximum when the spring acquires its natural length.

$$\frac{1}{2}(M_B + M_C)v_{\max}^2 = \frac{1}{2}k\,x_0^2$$

$\Rightarrow v_{\max} = 4\ m/s$

59. (a) Friction force between the block and the plank $= f$



Acceleration of the block $- a = \dfrac{f}{M}$.

If the slipping stops in time 't'

$V = 0 + at$

$$V = \frac{ft}{M} \qquad \text{..................(1)}$$

For the plank external force F = friction (f)

Power of the external force $P = FV$

Work done by external force in time 't'

$W = Pt = FVt = ftV = MV^2$ [$\because ft = MV$ from (1)]

(b) The work done by the external force can be interpreted as

W = Heat produced + gain in kE of the block.

$$\therefore \text{ Heat} = MV^2 - \frac{1}{2}MV^2 = \frac{1}{2}MV^2$$

Note: In the reference frame of the plank the block has initial velocity V and final velocity zero. It has lost $\frac{1}{2}MV^2$ amount of KE. This energy has been lost as heat.

60. Maximum friction on m_1 is $f_{1\max} = \mu m_1 g$

$\therefore$ Extension in the spring needed to move m_1 is $x_0 = \frac{\mu m_1 g}{k}$

Therefore, F shall be such that extension in the spring does not exceed x_0. Let F_0 be the force for which the block of mass M_2 moves through x_0 before coming to rest.

WE theorem

$$F_0 \; x_0 = \frac{1}{2}k \; x_0^2$$

$$F_0 = \frac{k \; x_0}{2} = \frac{k}{2}\frac{\mu m_1 g}{k} = \frac{\mu m_1 g}{2}$$

(a) Speed of m_2 is maximum when it reaches the equilibrium position.

$kx = f_0$

$$x = \frac{F_0}{k} = \frac{\mu m_1 g}{2k}$$

WE Theorem

$$F_0 . x = \frac{1}{2}k x^2 + \frac{1}{2}m_2 V^2$$

$$\therefore \frac{1}{2}m_2 V^2 = \left(\frac{\mu m_1 g}{2}\right)\left(\frac{\mu m_1 g}{2k}\right) - \frac{1}{2}k\left(\frac{\mu m_1 g}{2k}\right)^2$$

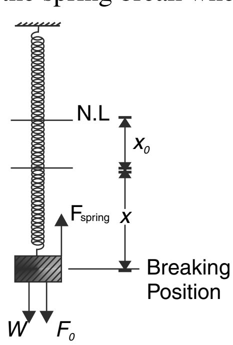
$$\frac{1}{2}m_2 V^2 = \frac{\mu^2 m_1^{\ 2} g^2}{8k}$$

$$\therefore V^2 = \frac{\mu^2 m_1^{\ 2} g^2}{4m_2 k}$$

$$V = \frac{\mu m_1 g}{2} \frac{1}{\sqrt{k \, m_2}}$$

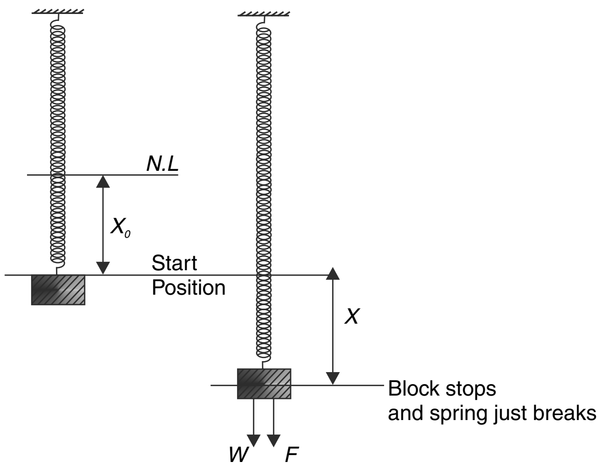
61. In equilibrium the spring is stretched by $x_0 = \frac{W}{K}$, where W = weight of the block.

Let the spring break when it is stretched further by x



$k(x + x_0) = F_0 + W$ ....(1)

When a constant force is applied, the block starts to accelerate and it gains speed. It will not come to rest at position where $k(x + x_0) = W + F$. It will be having a speed at that point and will move further down. The spring can break only if the spring stretches further by x before the block stops



Applying work energy theorem between starting position and the position where the block stops.

$$Fx + Wx = \frac{1}{2}k(x + x_0)^2 - \frac{1}{2}kx_0^2$$

$$(F + W)x = \frac{1}{2}k\left[x^2 + 2x x_0\right]$$

From (1) $kx + kx_0 = F_0 + W$ and $kx_0 = W$

$$\therefore \quad 2F + 2W = F_0 + W + W \Rightarrow F = \frac{F_0}{2}$$

62. (a) Assuming the reference level at the bottom of the container

$$U_i = 2\rho Ahg.\frac{3h}{2} + \rho Ahg.\frac{h}{2} = \frac{7}{2}\rho Ah^2 g$$

If the liquids mix uniformly the entire mixture will have a density of $\frac{3}{2}\rho$ and the final PE will be

$$U_f = \frac{3}{2}\rho A 2h.g.h = 3\rho Ah^2 g$$

Change in PE, $\Delta U = U_f - U_i = \frac{1}{2}\rho Ah^2 g$

(b) In this case, liquid A will settle at the bottom and B will be above it. Final PE will be

$$U_f = \rho Ahg.\frac{3h}{2} + 2\rho Ahg.\frac{h}{2} = \frac{5}{2}\rho Ah^2 g$$

Change in PE, $\Delta U = U_f - U_i = \frac{5}{2}\rho Ah^2 g - \frac{7}{2}\rho Ah^2 g = -\rho Ah^2 g$

The positive sign of ΔU in first case tells us that some external work will be required to mix the two liquids uniformly.

In second case ΔU is negative. The heavier liquid will automatically move to lower side as this decreases the overall *PE*.

63. $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left[v_{0x}^{\ 2} + \left(v_{0y} - gt\right)^2\right] = \frac{1}{2}mv_0^2 + \frac{1}{2}mg^2t^2 - mv_0 \sin\theta\, gt$

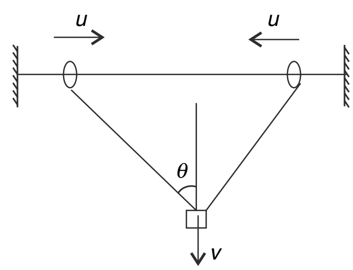
$U = mgh = mg\left[v_0 \sin\theta.t - \frac{1}{2}gt^2\right] = mv_0 gt \sin\theta - \frac{1}{2}mg^2t^2$

$K = U \Rightarrow g^2t^2 - 2v_0 gt \sin\theta + \frac{v_0^2}{2} = 0$

Discriminant of this equation is – ve for $\theta = 30°$

$\therefore$ the two curves do not touch

64. $v \cos\theta = u \sin\theta$



$v = \sqrt{3}u$ ....(i)

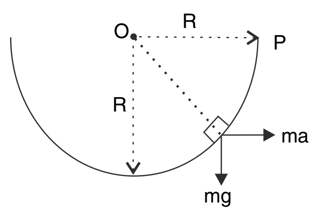
$mg\, l \cos\theta = mu^2 + \frac{mv^2}{2}$ ....(ii)

$u = \sqrt{\frac{g\ell}{5}}, v = \sqrt{\frac{3g\ell}{5}}$

65. (i) In reference frame of the truck there are two forces on the block which perform work on it –

The pseudo force $ma(\rightarrow)$ and

Gravitational force $mg(\downarrow)$



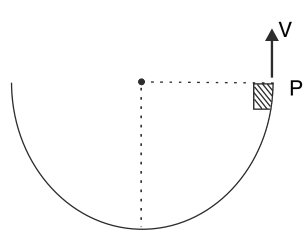
Applying work energy theorem for the situation when the block just manages to reach the top of the hemisphere (change in $KE = 0$)

$maR - mgR = 0$

$a = g$

(ii) Once again applying work energy theorem in reference frame of the truck

$maR - mgR = \frac{1}{2}mv^2$



Where v is vertically upward velocity of the block in reference frame of the truck at the instant it leaves the truck.

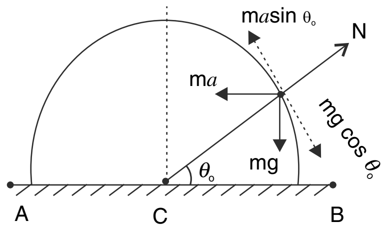
Given $a = 2g$

Hence, $mgR = \frac{1}{2}mv^2$

Which means the block will rise further by R after leaving the truck.

Therefore, answer is $2R$

66. In a reference frame attached to the wire, the force on the bead has been shown in the fig.



For equilibrium $ma \sin\theta_0 = mg \cos\theta_0$ ....(1)

If θ increases a little bit the tangential force $ma \sin \theta$ will rise and $mg \cos \theta$ will decrease. Therefore, the bead will rise on the ring and then it will fall on other side to hit the table at end A.

Work done by the pseudo force = $ma\ [R + R \cos\theta_0]$

work done by mg is = $mg\ R \sin \theta_0$

$\therefore \frac{1}{2}mv^2 = maR[1+\cos\theta_0] + mgR\sin\theta_0$

$v^2 = 2aR\ (1 + \cos\ \theta_0) + 2gR \sin\ \theta_0$

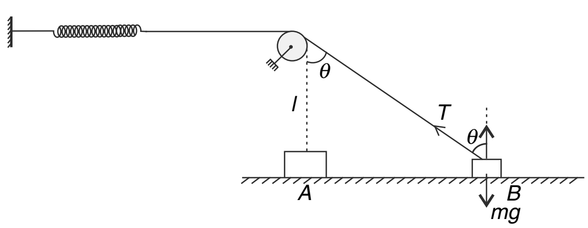
$$= 2R\left[\frac{g\cos\theta_0}{\sin\theta_0}(1+\cos\theta_0) + g\sin\theta_0\right]$$

$$= 2gR\left[\frac{\cos\theta_0 + \cos^2\theta_0 + \sin^2\theta_0}{\sin\theta_0}\right]$$

$$= 2gR\left[\frac{1+\cos\theta_0}{\sin\theta_0}\right]$$

$$\therefore\ v = \sqrt{2gR\left(\frac{1+\cos\theta_0}{\sin\theta_0}\right)}$$

67.



The block will leave the horizontal surface if it can reach a point B where

$T\cos\theta = mg$.................(i) $\begin{bmatrix} T = \text{tension in string} \\ = \text{tension in spring} \end{bmatrix}$

But $T = kx = kl\ [\sec\theta - 1]$ [x = extension in spring = $l(\sec\ \theta - 1)$]

Using (i) $\therefore\ k\ell(\sec\theta - 1)\cos\theta = mg$ [at B]

$\Rightarrow\ 2mg(1-\cos\theta) = mg \ \Rightarrow\ 1-\cos\theta = \dfrac{1}{2}$

$\Rightarrow\ \theta = 60°$

From work energy theorem

Work done by friction = $\Delta k + \Delta U$

$$-\mu\, mg\,\ell \tan 60° = -\frac{1}{2}mu^2 + \frac{1}{2}k\left[\ell(\sec 60° - 1)\right]^2$$

$$\therefore\ \frac{1}{2}mu^2 = \frac{1}{2}k\ell^2 + \sqrt{3}\,\mu mg\ell. \quad = \frac{1}{2}\frac{2mg}{\ell}.\ell^2 + \sqrt{3}\,\frac{1}{\sqrt{3}}mg\ell = 2mg\ell.$$

$\therefore\ u = 2\sqrt{g\ell}$

68. The maximum compression in the spring can be calculated as

$$\frac{1}{2}kx_0^2 = mgx_0 \ \Rightarrow x_0 = \frac{2mg}{k}$$

$$U_g = -mgx;\ (U_g)_{\max} = -mg\left(\frac{2mg}{k}\right) = -\frac{2m^2g^2}{k}$$

And $U_s = \dfrac{1}{2}kx^2; (U_s)_{\max} = \dfrac{1}{2}k\left(\dfrac{2mg}{k}\right)^2 = \dfrac{2m^2g^2}{k}$

$$U = U_g + U_s = -mgx + \frac{1}{2}kx^2$$

Graph has been shown in the answer.

69. $\dfrac{1}{2}mv^2 = 6 \quad \Rightarrow\ v^2 = 12 \ \Rightarrow\ v = 2\sqrt{3}\ m/s$

Note that energy to be given is 6 *J* and NOT 10 *J*. Why?

70. Putting $a = 1$ and $b = 1$

$U = x^4 - 5x^2$

$$\frac{dU}{dx} = 4x^3 - 10x$$

$$\frac{dU}{dx} = 0;\ \text{when}\ x = 0,\ x = \sqrt{\frac{5}{2}},\ \ x = -\sqrt{\frac{5}{2}}$$

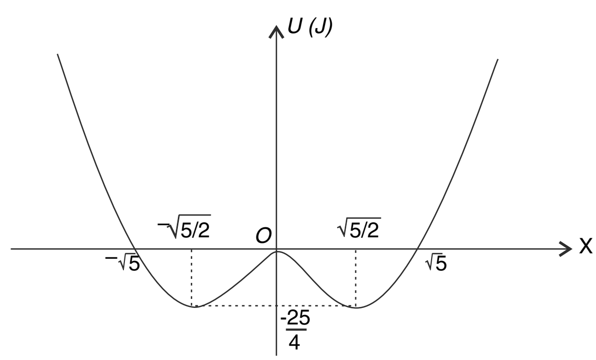
At $x = 0$; $U = 0$

at $x = 0$; $U = 0$

at $x = \sqrt{\dfrac{5}{2}}$; $U = \dfrac{25}{4} - \dfrac{25}{2} = -\dfrac{25}{4}$

$x = -\sqrt{\dfrac{5}{2}}$; $U = -\dfrac{25}{4}J = U_{\min}$

Graph of U vs x is as shown in the fig.



$U = 0$ at $x = \pm\sqrt{5}$

Since $K + U = 0$

$\therefore$ particle will remain between $-\sqrt{5}\,m$ to $+\sqrt{5}\,m$

$$U_{\min} = -\frac{25}{4}\,J \quad \begin{bmatrix} \text{When particle is at} \\ x = \pm\sqrt{\dfrac{5}{2}} \end{bmatrix}$$

$$\frac{1}{2}mv_{\max}^2 = \frac{25}{4}$$

$$\therefore V_{max} = \frac{5}{\sqrt{2}}\ m/s$$

71. (a) Force on the particle, $F = -\dfrac{dU}{dr} = -3mkr^2$

The – *ve* sign indicates attractive force.

For the motion to be circular

$$\frac{mv^2}{r} = 3mk\,r^2$$

$$\Rightarrow v = r\sqrt{3kr} \qquad \ldots\ldots\ldots\ldots\ldots\ (i)$$

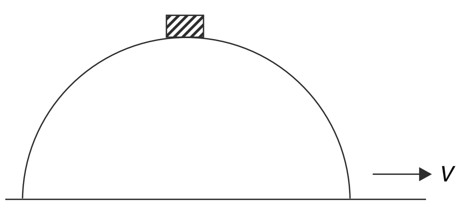
For $r = a_0$

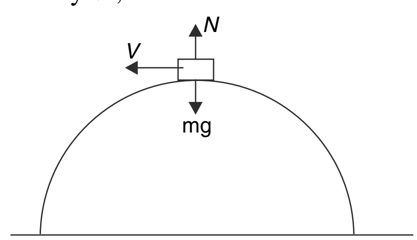
$$v = a_0\sqrt{3ka_0}$$

$$\therefore KE = \frac{1}{2}mv^2 = \frac{3}{2}kma_0^3$$

(b) Time period, $T = \dfrac{2\pi a_0}{v} = \dfrac{2\pi a_0}{a_0\sqrt{3ka_0}} = \dfrac{2\pi}{\sqrt{3ka_0}}$

72. (a) In reference frame of bowl, the block acquires a velocity V, in horizontal direction.



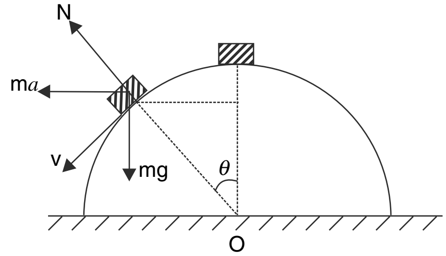


Block will lose contact ($N = 0$) if the force mg is not good enough to provide sharp enough turning to the path of the block.

$$\frac{mV^2}{R} \geq mg$$

$$V \geq Rg$$

(b) In reference frame of the bowl, we use work energy theorem for motion of the block



$$\frac{1}{2}mV^2 = maR\sin\theta + mgR(1-\cos\theta)$$

For small θ, $\sin\theta = \theta$ and $\cos\theta = 1$

$$\therefore \frac{mV^2}{R} = 2ma\theta \qquad \text{................} (1)$$

Also $\frac{mV^2}{R} = mg\cos\theta - ma\sin\theta - N \simeq mg - ma\theta - N$

$$\therefore N = mg - ma\,\theta - \frac{mV^2}{R}$$

$$= mg - ma\theta - 2ma\theta$$

For $N \geq 0$

$mg \geq 3ma\theta$

$$\Rightarrow \frac{g}{3\theta} \geq a$$

$$\therefore \frac{g}{3\theta} = \frac{g}{3\frac{\pi}{180}} = \frac{60g}{\pi}$$

73. (a) Energy conservation

$$\frac{1}{2}mV^2 + mgL(1-\cos\theta) = \frac{1}{2}mu^2$$

But as per question $\frac{1}{2}mu^2 = mgL$

and $\frac{1}{2}mV^2 = \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{1}{4}mgL$

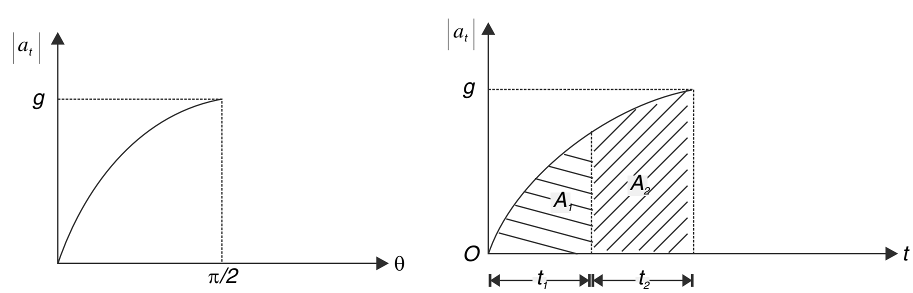
$$\therefore \frac{1}{4}mgL + mgL(1-\cos\theta) = mgL$$

$$\Rightarrow \frac{1}{4} + 1 - \cos\theta = 1$$

$$\Rightarrow \cos\theta = \frac{1}{4}$$

$$\theta = \cos^{-1}\left(\frac{1}{4}\right)$$

(b) $|a_t| = g\sin\theta$ → increases with time. The graph is as shown below.



(c) Area A_1 under the graph gives change in magnitude of velocity from A to B

$\therefore\ A_1 = \dfrac{V}{2}$

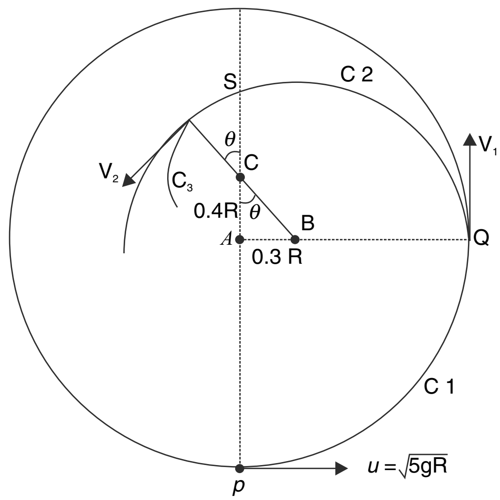
Similarly, A_2 = change in magnitude of velocity from B to C .

$\therefore\ A_2 = \dfrac{V}{2}$

For $A_1 = A_2$, looking at the graph one can easily say that

$t_1 > t_2$

74. Particle is projected from P and follows a circular path $C1$ till it reaches Q .



At Q (just before the thread hits the nail at B) its speed is V_1 .

$$\frac{1}{2}mV_1^2 + mgR = \frac{1}{2}m(5gR)$$

$$\Rightarrow\ V_1 = \sqrt{3gR}$$

Tension at this moment is $T_1 = \dfrac{mV_1^2}{R} = 3mg$

Just after the thread hits the nail at B, particle starts moving on circle $C2$ (centered at B)

Tension at Q just after the thread hits the nail at B is

$$T_1^1 = \frac{mV_1^2}{0.7R} = \frac{3mg}{0.7} = \frac{30}{7}mg$$

% change in tension is $\dfrac{T_1^1 - T_1}{T_1} \times 100 = 42.8\%$

The particle will be at point S when the thread hits the nail at C .

Energy Conservation between S and Q

$$\frac{1}{2}mV_2^2 = mg(0.4R + 0.2R\cos\theta) = \frac{1}{2}mV_1^2$$

$$[\because BC = \left(\sqrt{0.3^2 + 0.4^2}\right)R = 0.5R;\ CS = 0.7\,R - 0.5\,R = 0.2\,R]$$

$$\therefore\ \frac{mV_2^2}{R} = \frac{mV_1^2}{R} - 2mg(0.4 + 0.2\cos\theta)\ \left[\cos\theta = \frac{4}{5}\right]$$

$= 3mg - 2mg\left(0.4 + 0.2 \times \frac{4}{5}\right) = 1.88\,mg$

Tension just before the string hits the nail C is

$T_2 + mg\cos\theta = \frac{mV_2^2}{0.7R}$

$T_2 = \frac{1.88\ mg}{0.7} - mg.\frac{4}{5} = 1.88\ mg$

Just after the string hits the nail, particle moves on a circle of radius $0.2R$

$\therefore T_2^1 = \frac{mV_2^2}{0.2R} - mg\ \cos\theta$

$T_2^1 = \frac{1.88mg}{0.2} - mg \times 0.8$

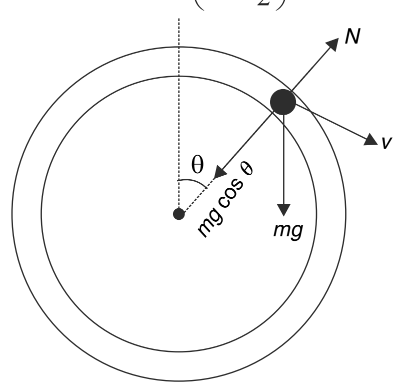
$= 8.6\ mg$

% change in tension

$\frac{T_2^1 - T_2}{T_2}\times 100 = \frac{8.6 - 1.88}{1.88}\times 100 = 357\%$

Hence, the string breaks.

75. (a) The centre of the ball moves in a circle of radius $\left(R + \frac{d}{2}\right)$.



From figure $\frac{mv^2}{R + \frac{d}{2}} = mg\ \cos\theta - N$ .... (a)

From conservation of energy $mg\left(R + \frac{d}{2}\right)(1 - \cos\theta) = \frac{1}{2}mv^2$ .... (b)

Eliminating v^2 from (a) and (b) we get

$N = mg(3\cos\theta - 2)$ .... (c)

Initially, for small θ, $(3\cos\theta - 2) > 0$ and N is positive.

This means ball is in contact with the sphere A which exerts an outward normal force.

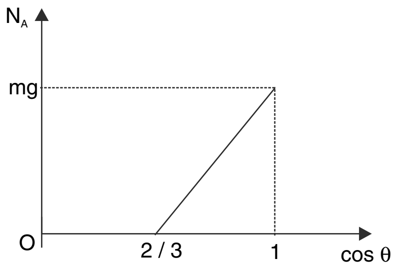
When $\cos\theta = \frac{2}{3}$; $N = 0$ and ball loses contact with A.

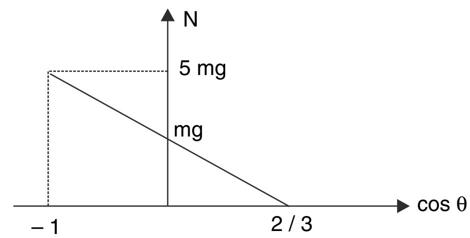
Naturally, it comes in contact with B and direction of N gets reversed. In fact, this is envisaged in the equation (c) as N is < 0 for $\theta > \cos^{-1}\frac{2}{3}$

$\therefore\ N_A = mg(3\cos\theta - 2)$ for $0 \le \theta \le \cos^{-1}\frac{2}{3}$

$N_B = mg(2 - 3\cos\theta)$ for $\cos^{-1}\frac{2}{3} \le \theta \le \pi$

(b) The two graphs look as shown in figure





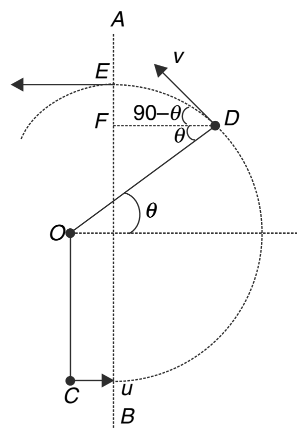
Note that when $\theta = 0°$ $(\cos\theta = 1)$

$N_A = mg$

And, when $\theta = \pi$ $(\cos\theta = -1)$

$N_B = 5\ mg$

76. Let the string slack at point D (see figure).



Particle moves in a circular path from C to D. At D it leaves the circular path and follows a parabolic path (a projectile). It crosses the line AB, when it is moving horizontally. It means while crossing AB at point E it is at the top of the parabolic path.

At D, string tension = 0

$\therefore\ mg\sin\theta = \frac{mv^2}{L}$ ....(a)

[v = velocity at D]

From conservation of energy

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgL.(1+\sin\theta)$

$v^2 = u^2 - 2gL(1 + \sin\theta)$ ....(b)

$DF = \frac{1}{2}\times$ Horizontal range of projectile (starting from D at an angle $(90 - \theta)$ with horizontal)

$$=\frac{1}{2}\frac{v^2 \sin 2(90°-\theta)}{g}$$ [From formula for range]

From geometry of figure

$$DF = L\cos\theta - \frac{L}{8}$$

$$\therefore\ L\left(\cos\theta - \frac{1}{8}\right) = \frac{v^2 \sin 2\theta}{2g}$$

$$\Rightarrow\ \cos\theta - \frac{1}{8} = \frac{v^2}{gL}(\sin\theta.\cos\theta)$$

$$\Rightarrow\ \cos\theta - \frac{1}{8} = \sin^2\theta.\cos\theta\ \ [\because \text{ from (a), } \frac{v^2}{gL} = \sin\theta\,]$$

$$\Rightarrow\ \cos\theta - \frac{1}{8} = (1-\cos^2\theta)\cos\theta$$

$$\Rightarrow\ \cos\theta - \frac{1}{8} = \cos\theta - \cos^3\theta \Rightarrow\ \cos^3\theta = \frac{1}{8}$$

$$\Rightarrow\ \cos\theta = \frac{1}{2}\ \Rightarrow\ \boldsymbol{\theta} = 60°$$

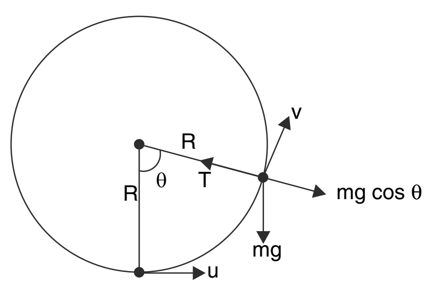
Now from (a) $v^2 = gL\sin\theta = gL\sin 60° = \frac{\sqrt{3}}{2}gL$

Substituting in (b) $u^2 = v^2 + 2gL\,(1 + \sin\theta)$

$$= \frac{\sqrt{3}}{2}gL + 2gL\left(1+\frac{\sqrt{3}}{2}\right) = gL\left(2+\frac{3\sqrt{3}}{2}\right)$$

$$\therefore u = \sqrt{gL\left(2+\frac{3\sqrt{3}}{2}\right)}$$

77. At position θ, let the speed be V and tension T.



Energy conservation $\frac{1}{2}mV^2 + mgR(1-\cos\theta) = \frac{1}{2}mu^2$

$$\frac{1}{2}mV^2 + mgR(1-\cos\theta) = \frac{1}{2}m(5gR) \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

$[\because\ u = \sqrt{5gR}\ ]$

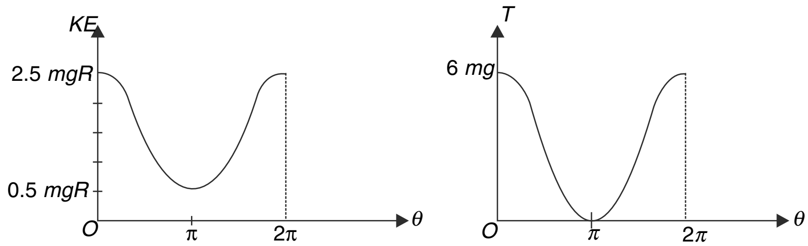
$\therefore$ Kinetic energy is $K = \frac{3}{2}mgR + mgR\cos\theta = mgR\left[\frac{3}{2} + \cos\theta\right]$

Equation for force in radial direction $T - mg\cos\theta = \frac{mV^2}{R}$

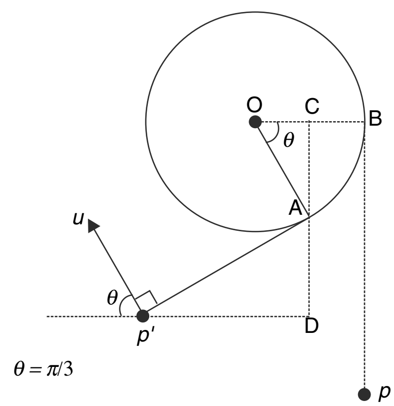
$T = mg\cos\theta + (3mg + 2mg\cos\theta)$ [using (1) to get the value of $\frac{mV^2}{R}$]

$T = 3mg(1 + \cos\theta)$ .... (2)

The graphical plots are



78. The desired position is shown.



Arc $(BA) = \frac{\pi}{3}R$

$AP' = \pi R - \frac{\pi R}{3} = \frac{2}{3}\pi R$

The vertical height difference between p' and p is

$h = \pi R - CD$

$= \pi R - \left[R\sin\theta + \frac{2}{3}\pi R\cos\theta\right]$

$= \pi R - \frac{R\sqrt{3}}{2} - \frac{\pi R}{3}$

$= \frac{2\pi R}{3} - \frac{\sqrt{3}}{2}R$

$\therefore \frac{1}{2}mV^2 + mgh = \frac{1}{2}mV_0^2$

$$V^2 = V_0^2 - 2gh = \frac{4\pi Rg}{3} - 2g\left[\frac{2\pi R}{3} - \frac{\sqrt{3}R}{2}\right]$$

$$V = \sqrt{\sqrt{3}gR}$$

(b) Writing equation for centripetal force $T - mg\cos\theta = \frac{mV^2}{R}$

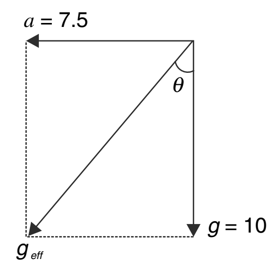
$$T = \frac{mg}{2} + \sqrt{3}\,mg$$

79. When train is at rest

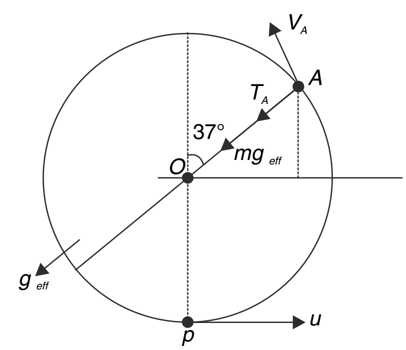
$U_{\min} = \sqrt{5gL}$ [L = length of pendulum]

$$\Rightarrow\ L = \frac{U_{\min}^2}{5g} = \frac{10^2}{5\times 10} = 2m$$

When the train is moving horizontally at an acceleration 'a', we can assume that inside the train effective acceleration due to gravity is



$$g_{eff} = \sqrt{g^2 + a^2} = \sqrt{10^2 + 7.5^2} = 12.5\ m/s^2$$



$$\tan\theta = \frac{a}{g} = \frac{7.5}{10} = \frac{3}{4}$$

$\theta = 37°$

In accelerated train, minimum tension will be at point A.

$$mg_{eff} + T_A = \frac{mV_A^2}{L}$$

In limiting case $T_A = 0$

$$\frac{mV_A^2}{L} = mg_{eff} \qquad \text{..............(1)}$$

Applying energy conservation between points P and A

$$\frac{1}{2}mV_A^2 + mg_{eff}L(1+\cos 37°) = \frac{1}{2}mu^2$$

$$u^2 = V_A^2 + 2g_{eff}L(1+\cos 37°)$$

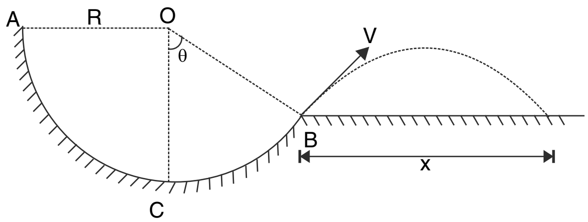
$$= L\,g_{eff} + \frac{18}{5}Lg_{eff}$$

$$= \frac{23}{5}Lg_{eff}$$

$$= \frac{23}{5} \times 2 \times 12.5$$

$$u = \sqrt{115}\, m/s$$

80.



v = speed with which the block leaves the surface at B

$$\frac{1}{2} mV^2 = mgR\cos\theta$$

$v^2 = 2gR \cos\theta$ .... (1)

Range of the projectile released from B is

$$x = \frac{v^2 \sin 2\theta}{g} = 2R\cos\theta . \sin 2\theta \qquad \text{[from (1)]}$$

$$= 4\, R\, cos^2\, \theta . \sin\theta$$

x is maximum when

$$\frac{dx}{d\theta} = 0 \qquad \Rightarrow -2\cos\theta \sin^2\theta + \cos^3\theta = 0$$

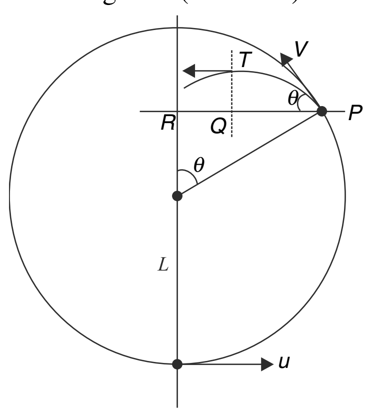
$\Rightarrow \cos\theta\, [\cos^2\theta - 2\sin^2\theta] = 0$

$\cos\theta = 0$ is not acceptable [It will give zero range]

$\therefore \cos^2\theta - 2\sin^2\theta = 0$

$$\tan\theta = \frac{1}{\sqrt{2}}$$

81. Let the particle leave the circular path at an angle θ (as shown)



Energy conservation

$$\frac{1}{2} mV^2 = \frac{1}{2} mu^2 - mgL\,[1 + \cos\theta]$$

$$\Rightarrow \frac{V^2}{L} = \frac{u^2}{L} - 2g\left[1+\cos\theta\right] \quad \text{..............} \ (1)$$

Centripetal force [with tension becoming zero]

$$\frac{mV^2}{L} = mg\cos\theta \qquad \text{................} \ (2)$$

From (1) and (2) $g \cos \theta = 4g - 2g - 2g \cos \theta$

$$\cos\theta = \frac{2}{3}$$

From (2) $V = \sqrt{\frac{2}{3}gL}$

After this the particle goes in a parabolic path. Tangential acceleration becomes zero when it is at the top most point of its trajectory (at point T)

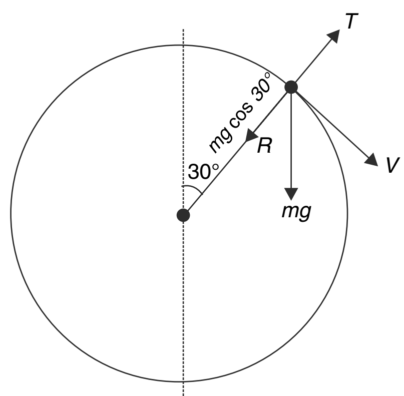
Required distance is $RQ = L \sin \theta$ – Half the range of projectile.

$$= L\,.\frac{\sqrt{5}}{3} - \frac{2V^2 \sin\theta\cos\theta}{2g}$$

$$= \frac{\sqrt{5}}{3}L - \frac{2L}{3}\frac{\sqrt{5}}{3}\cdot\frac{2}{3}$$

$$= \frac{5\sqrt{5}}{27}L$$

82. (a)



When $\theta = 30°$

Let the force exerted by the rod on the bob be T as shown.

Then $mg \cos \theta - T = \dfrac{mV^2}{R}$ .... (i)

Where R = length of the rod

From conservation of energy

$$mgR = mgR \cos 30° + \frac{1}{2}mV^2$$

$$\Rightarrow 2mgR\left(1 - \frac{\sqrt{3}}{2}\right) = mV^2$$

$$\Rightarrow \frac{mV^2}{R} = mg\left(2 - \sqrt{3}\right)$$

From (i) $mg\cos\theta - T = \dfrac{mV^2}{R}$

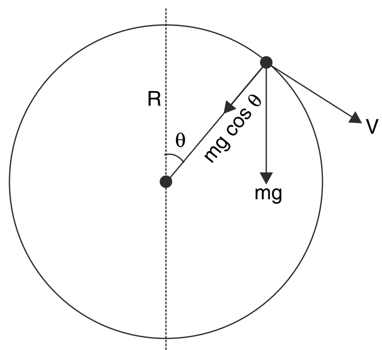
$\Rightarrow T = mg\dfrac{\sqrt{3}}{2} - mg\left(2-\sqrt{3}\right)$

$= mg\left(\dfrac{\sqrt{3}}{2} + \sqrt{3} - 2\right) > 0$

$\therefore$ Rod exerts a force T on the bob radially outwards.

$\therefore$ Force exerted on the rod by the bob is radially inward, i.e., compressive.

(b)



At the point where compression changes to tension, we have

$T = 0$

$\therefore \dfrac{mV^2}{R} = mg\cos\theta$

$\Rightarrow V^2 = gR\cos\theta$ …........... (i)

From Energy conservation

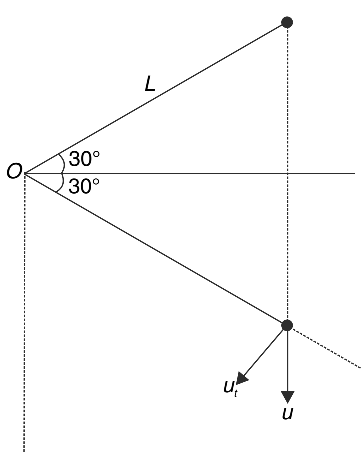
$\dfrac{1}{2}mV^2 = mgR\left(1-\cos\theta\right)$

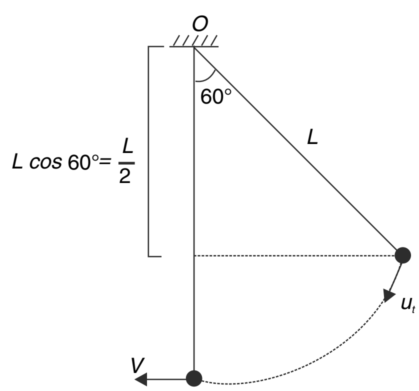
$\Rightarrow gR\cos\theta = 2gR\,(1-\cos\theta)$

$\Rightarrow \cos\theta = 2/3 \Rightarrow \theta = \cos^{-1}\left(\dfrac{2}{3}\right)$

83. The bob will experience free fall for a distance of $L = 1.8\ m$.
Speed of the bob just before the string gets taut is

$u = \sqrt{2gL} = \sqrt{2\times 10\times 1.8} = 6\,m/s$





As the string gets taut, the radial component of velocity $u_r \left[= u\cos 60° = \frac{u}{2} \right]$ vanishes.
Only the tangential component will remain.

$$u_t = u\cos 30° = \frac{\sqrt{3}u}{2} = 3\sqrt{3}\, m/s$$

Energy conservation

$$\frac{1}{2}mV^2 = \frac{1}{2}mu_t^2 + mg\frac{L}{2}$$

$V^2 = 27 + 10 \times 1.8 = 45$

$$V = 3\sqrt{5}\, m/s$$

$$\therefore\ T - mg = \frac{mV^2}{L}$$

$$T = 10 + \frac{45}{1.8} = 35N$$

84. (a) Mean or average power $< p > = \frac{\text{Work done}\,(W)}{\text{time}\,(t)}$

Time of motion can be calculated as

$v = u + at$

$\Rightarrow 0 = v_0 - \mu g t$ $\quad$ [$\because f = \mu mg\ \therefore\ a = \mu g$]

$$\Rightarrow\ t = \frac{v_0}{\mu g}$$

Work done by friction = loss in kinetic energy [whole kinetic energy is lost in doing work against friction]

$\therefore$ Work done by friction, $W = -\frac{1}{2}mV_0^2$

$$\therefore\ < p > = -\frac{mV_0^2 \mu g}{2V_0} = -\frac{1}{2}mV_0\ \mu g = -\frac{1}{2}\times 2\times 3\times 0.3\times 10 = -9 \text{ watt}$$

(b) When body is at a distance x from the starting point, retarding friction force is

$f = \mu mg$

$$\therefore\ \frac{dv}{dt} = -\mu g$$

$$\therefore\ \frac{dv}{dx}\frac{dx}{dt} = -\mu g$$

$$\therefore\ v\,dv = -\mu g\,dx$$

$$\int_{v_0}^{v} v\,dv = -\alpha g\int_0^x x\,dx$$

$$\Rightarrow\ \frac{v^2 - v_0^2}{2} = -\alpha g\frac{x^2}{2}$$

$$\therefore\ v^2 = v_0^2 - \alpha g x^2$$

$\Rightarrow \quad v = \sqrt{v_0^2 - \alpha g x^2}$

Instantaneous power of friction force is

$$p = \vec{f} \cdot \vec{v}$$

$$= f\, v \cos 180°$$

$$= -\mu\, mg \sqrt{v_0^2 - \alpha g x^2}$$

$$= -\alpha\, mg\; x\sqrt{v_0^2 - \alpha g x^2}$$

p is maximum when $\dfrac{dp}{dx} = 0$

$$\Rightarrow \; -\alpha\, mg\left[\sqrt{v_0^2 - \alpha g x^2} + \frac{x(-2\alpha g x)}{2\sqrt{v_0^2 - \alpha g x^2}}\right] = 0$$

$$\Rightarrow \; v_0^2 - \alpha g x^2 - \alpha g x^2 = 0$$

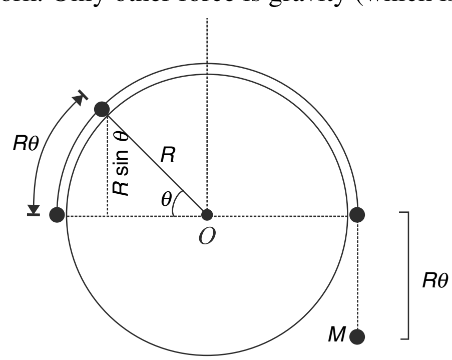
$$\Rightarrow \; x^2 = \frac{v_0^2}{2\alpha g}$$

$$\Rightarrow \; x = \frac{v_0}{\sqrt{2\alpha g}}$$

$$\therefore \; p_{\max} = -\alpha\, mg\left(\frac{v_0}{\sqrt{2\alpha g}}\right)\sqrt{v_0^2 - \alpha g \frac{v_0^2}{2\alpha g}}$$

$$= -\frac{1}{2} m v_0^2 \sqrt{\alpha g}$$

85. (a) When M descends through $R\theta$, radius vector of m rotates through θ as shown in figure. The normal contact force does not perform any work. Only other force is gravity (which is conservative).



$\therefore$ loss in PE = gain in KE

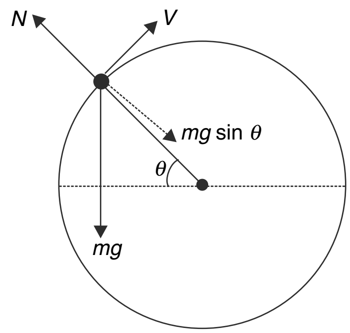
Loss in PE of M–gain in PE of $m = \dfrac{1}{2}Mv^2 + \dfrac{1}{2}mv^2$

(Since speed of both M and m will be same.)

$$\therefore \; MgR\theta - mg(R\sin\theta) = \frac{1}{2}(M + m)\, v^2$$

$$\therefore\ v = \sqrt{2gR\frac{(M\theta - m\sin\theta)}{M+m}}$$

(b) Consider the motion of m at this instant.



$$\frac{mv^2}{R} = mg\sin\theta - N$$

$$\Rightarrow\ N = mg\sin\theta - \frac{mv^2}{R}$$

$$= mg\sin\theta - \frac{2mgR(M\theta - m\sin\theta)}{(M+m)R}$$

$$= mg\left[\frac{(M+m)\sin\theta - 2M\theta + 2m\sin\theta)}{M+m}\right]$$

$$= \frac{mg}{M+m}\left[(M+3m)\sin\theta - 2M\theta\right]$$

(c) Just after release

$\theta \to 0$

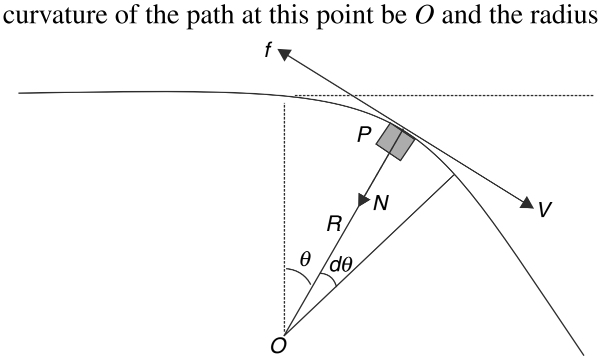
$\sin\theta = \theta$

$$\therefore\ N = \frac{mg}{M+m}\left[(M+3m)\theta - 2M\theta\right] = \frac{mg}{M+m}\left[(3m-M)\theta\right]$$

For contact to be maintained $N > 0$

$\Rightarrow\ 3m > M$

86. Consider the object at point P on the curve. Let its velocity at this point be V making an angle θ with the original direction. Let the centre of curvature of the path at this point be O and the radius of curvature be R.



The normal force is $N = \dfrac{mV^2}{R}$

Kinetic friction force on the object is $f = \mu N = \frac{\mu m V^2}{R}$

Work done by the friction in small angular displacement $d\theta$ of the block is $dW_f = -f(Rd\theta) = -\mu m V^2 d\theta$

From work energy theorem, work done by the friction is equal to change in KE of the object

$$K = \frac{1}{2} m V^2$$

$$dK = mV\,dV$$

$$\therefore\ mVdV = -\mu m V^2 d\theta$$

$$\Rightarrow\ \frac{dV}{V} = -\mu\, d\theta \quad \Rightarrow \int_u^V \frac{dV}{V} = -\mu \int_0^{\theta_0} d\theta$$

$$\Rightarrow\ \ln V - \ln u = -\mu\theta_0 \Rightarrow\ \ln\left(\frac{V}{u}\right) = -\mu\theta_0$$

$$\Rightarrow\ \frac{V}{u} = e^{-\mu\theta_0}$$

$$\Rightarrow\ V = ue^{-\mu\theta_0}$$

Answer does not depend on the shape of the curve.

87. Monkey slides down with a constant acceleration of $\frac{g}{2}$. It means the tension in the elastic cord just before the monkey lands on the platform is $\frac{mg}{2}$.

Extension in the cord by this time = x_0

Then $kx_0 = \frac{mg}{2}$

$$\frac{4mg}{L} \cdot x_0 = \frac{mg}{2} \Rightarrow\ x_0 = \frac{L}{8}$$

Elastic potential energy stored in the cord

$$U_0 = \frac{1}{2} k (x_0)^2 = \frac{1}{2} \frac{4\,mg}{L} \left(\frac{L}{8}\right)^2 = \frac{mgL}{32}$$

Kinetic Energy of monkey just at the time of hitting the platform

$$K_0 = \frac{1}{2} m V^2 = \frac{1}{2} m \left[2 \cdot \frac{g}{2} \cdot \frac{9L}{8}\right]$$

[$\because$ monkey accelerates over a length of $L + \frac{L}{8} = \frac{9L}{8}$]

$$\therefore\ K_0 = \frac{9}{16} mgL$$

Let the cord stretch further by x.

Applying energy conservation

Loss in KE + loss in gravitational PE = gain in spring PE

$$K_0 + mg\,x = \frac{1}{2} k (x + x_0)^2 - U_0$$

$$\frac{9}{16} mgL + mgx = \frac{1}{2} k \left(x^2 + x_0^2 + 2xx_0\right) - U_0 \quad \left[U_0 = \frac{1}{2} k x_0^2\right]$$

$$\therefore \frac{9}{16}mgL + mgx = \frac{4mg}{2L}\left(x^2 + 2x_0 \cdot x\right)$$

$$\frac{9}{16}L + x = \frac{2}{L}x^2 + \frac{4}{L}\frac{L}{8}.x \qquad \left[\because\ x_0 = \frac{L}{8}\right]$$

$$\frac{9L+16x}{16} = \frac{4x^2 + xL}{2L}$$

$$\Rightarrow 9L^2 + 16\,Lx = 32\,x^2 + 8L.x$$

$$\Rightarrow 32x^2 - 8\,Lx - 9\,L^2 = 0$$

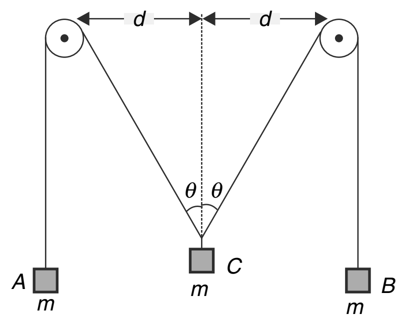
$$\therefore\ x = \frac{8L \pm \sqrt{64L^2 + 4\times 32\times 9L^2}}{64}$$

$$= \frac{L}{8} + \frac{\sqrt{19}}{8}L \qquad \text{[–ve sign is unacceptable]}$$

$$\therefore\ x = \frac{L}{8}\left(1+\sqrt{19}\right)$$

$\therefore$ Maximum extension $= x + x_0 = \dfrac{L}{8}(2+\sqrt{19})$

88.



(a) PE of the system in position shown is

$U = -\,mg\,d\cot\theta - 2\,mg\,[L - d\,\text{cosec}\,\theta]$

$= -\,2mgL - mgd\,[\cot\theta - 2\,\text{cosec}\,\theta]$

$$= -2mg\,L - mgd\left[\frac{\cos\theta - 2}{\sin\theta}\right]$$

$$= -2mg\,L + mgd\left[\frac{2-\cos\theta}{\sin\theta}\right]$$

(b) U is maximum or minimum when

$$\frac{dU}{d\theta} = 0 \;\Rightarrow\; mgd\cdot\frac{\sin\theta[0+\sin\theta]-(2-\cos\theta)\cos\theta}{\sin^2\theta} = 0$$

$$\Rightarrow\ \sin\theta[0+\sin\theta]-(2-\cos\theta)\cos\theta = 0$$

$$\Rightarrow\ \sin^2\theta + \cos^2\theta - 2\cos\theta = 0$$

$$\Rightarrow\ \cos\theta = \frac{1}{2} \;\Rightarrow\; \theta = 60°$$

$\therefore$ Equilibrium is at $\theta = 60°$

You can verify this using simple force diagram.

(c) If $\dfrac{d^2U}{d\theta^2} > 0$, equilibrium is stable

If $\dfrac{d^2U}{d\theta^2} < 0$, equilibrium is unstable

$$\frac{dU}{d\theta} = mgd\left(\frac{1-2\cos\theta}{\sin^2\theta}\right)$$

$$\therefore\ \frac{d^2U}{d\theta^2} = mgd\cdot\left[\frac{\sin^2\theta(2\sin\theta)-(1-2\cos\theta)(2\sin\theta\cos\theta)}{\sin^4\theta}\right]$$

At $\theta = 60°$

$$\therefore\ \frac{d^2U}{d\theta^2} = mgd\cdot\left[\frac{2\cdot\frac{3}{4}\cdot\frac{\sqrt{3}}{2}-\left(1-2\cdot\frac{1}{2}\right)\left(2\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)^4}\right]$$

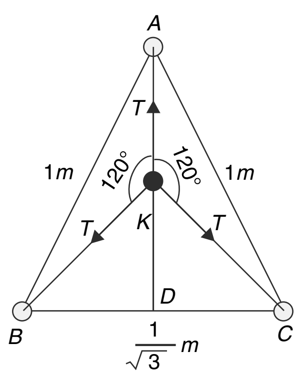
$$\therefore\ \frac{d^2U}{d\theta^2} > 0$$

$\therefore$ $\theta = 60°$ is position of stable equilibrium

89. (a) If m is mass of each particle, tension in each string is $T = mg$.

For equilibrium of the knot it is necessary that the three strings form 120° angle with each other. The situation has been shown in figure.

A, B and C are holes and K is the knot.



$$KD = \frac{1}{2\sqrt{3}}\tan 30° = \frac{1}{6}m$$

$$AD = \sqrt{1^2-\left(\frac{1}{2\sqrt{3}}\right)^2} = \sqrt{\frac{11}{12}} = \frac{1}{2}\sqrt{\frac{11}{3}}\ m$$

$$\therefore\ AK = \frac{1}{2}\sqrt{\frac{11}{3}}-\frac{1}{6}$$

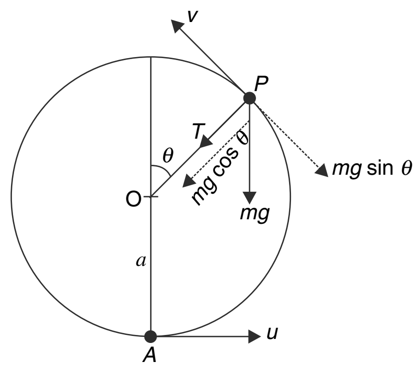
And $BK = CK = \dfrac{1}{2\sqrt{3}\sin 60^\circ} = \dfrac{1}{3}m$

$\therefore$ length of string on the table is

$$BK + CK + AK = \frac{1}{3} + \frac{1}{2}\sqrt{\frac{11}{3}} - \frac{1}{6} + \frac{1}{3} = \frac{1}{2}\left[1 + \sqrt{\frac{11}{3}}\right]m$$

(b) The point K shown in the figure above is the desired point. The equilibrium position is the one with the lowest potential energy of masses, that is the one with the most string hanging below the table. In other words, in equilibrium least length of the strings will be lying on the table.

90. Let P be the point where rod makes an angle θ with the upward vertical. v is velocity of mass at this point.



T is force exerted y rod on the particle towards centre. By conservation of mechanical energy

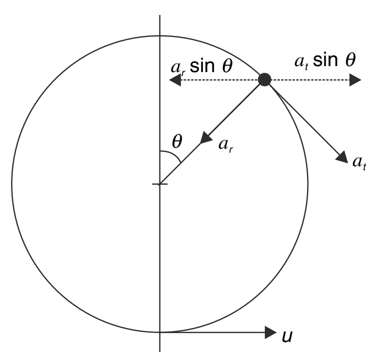
$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\,a(1+\cos\theta)$$

$\Rightarrow\; v^2 = u^2 - 2ag\,(1 + \cos\theta)$ .... (a)

At point P

Radial acceleration $a_r = \dfrac{v^2}{a} = \dfrac{u^2}{a} - 2g(1+\cos\theta)$

Tangential acceleration $a_t = g\sin\theta$ (in the direction shown)



Resolving a_r and a_t in horizontal directions, we get the horizontal component of acceleration (in the direction of u) as

$a_H = a_t\cos\theta - a_r\sin\theta$

$$= g\sin\theta\cdot\cos\theta - \frac{u^2}{a}\sin\theta + 2g(1+\cos\theta)\sin\theta$$

$$= \left[g(3\cos\theta + 2) - \frac{u^2}{a} \right] \sin\theta$$

(b) When $a_H = 0$

$$\sin\theta \left[g\left(3\cos\theta + 2\right) - \frac{u^2}{a} \right] = 0$$

$\Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$ or π

Also when $g(3\cos\theta + 2) - \dfrac{u^2}{a} = 0$

$$\cos\theta = \frac{u^2 - 2ag}{3ag} \qquad \text{..............} \text{(b)}$$

Since $4\, ag < u^2 < 5\, ag$; $(u^2 - 2\, ag) < 3\, ag$ and a real value of θ exists satisfying (b)

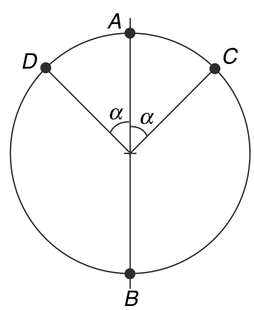
$$\therefore \ \theta = \cos^{-1}\left(\frac{u^2 - 2ag}{3ag}\right)$$

and $\theta = 2\pi - \cos^{-1}\left(\dfrac{u^2 - 2ag}{3ag}\right)$

Thus, horizontal acceleration is zero at four points given by

$\theta = 0, \pi$, $\cos^{-1}\left(\dfrac{u^2 - 2ag}{3ag}\right)$ and $\left[2\pi - \cos^{-1}\left(\dfrac{u^2 - 2ag}{3ag}\right)\right]$

Figure shows the four points



$\alpha = \cos^{-1}\left(\dfrac{u^2 - 2ag}{3ag}\right)$

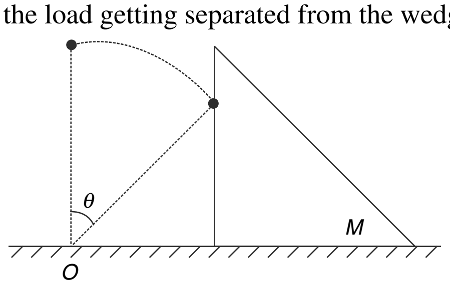
Note: If $u^2 > 5ag$, then

$$\cos\theta \neq \frac{u^2 - 2ag}{3ag}$$

As $\dfrac{u^2 - 2ag}{3ag} > 1$

Thus, only two solutions result, $\theta = 0°$ and π.

91. What is the physical condition for the load getting separated from the wedge?



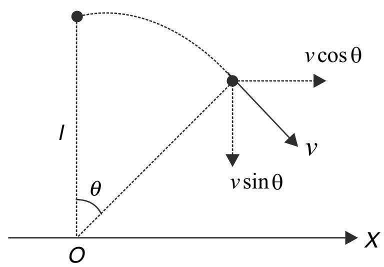
As the mass m moves in a vertical circle, it pushes the wedge towards right. At any moment, the horizontal component of displacement of m and M are equal. This means, the horizontal component of velocity of m is equal to the velocity of M. However, at a certain value of θ, the horizontal component of acceleration of m becomes zero and thereafter becomes negative. From this point onwards the horizontal component of velocity of m starts decreasing. But the velocity acquired by M cannot decrease as there are no retarding forces on it. Thus, the two masses get separated at the moment the horizontal acceleration of m changes from positive to negative (i.e., becomes zero).

$\therefore$ Condition when contact is lost is

a_x = horizontal acceleration of $m = 0$

At the moment contact is lost, let the velocity of m be v.

Velocity of M will be equal to horizontal component of velocity of m .



$\therefore$ Velocity of M is $v_x = v\cos\theta$

As there are no dissipative forces, mechanical energy of the system is conserved.

$\therefore$ loss in P.E of m = gain in K.E of m + gain in K.E. of M

$$\therefore\ mgl(1-\cos\theta)=\frac{1}{2}mv^2+\frac{1}{2}M(v\cos\theta)^2$$

$$\Rightarrow\ v^2=\frac{2\,mgl(1-\cos\theta)}{m+M\cos^2\theta} \qquad \text{............ (a)}$$

Now the mass m has two accelerations

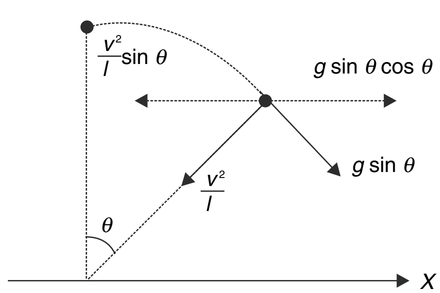
Radial $a_r=\dfrac{v^2}{l}$

Tangential $a_t = g\sin\theta$

[Note that contact force N, between m and M is zero at the instant contact is lost. Therefore, force in tangential direction is only mg $\sin\theta$.]

The acceleration of mass m in horizontal direction is

$$a_x=g\sin\theta\cdot\cos\theta-\frac{v^2}{l}\sin\theta \qquad \text{[See figure.]}$$



For loosing contact $a_x = 0$

$$\therefore\ g\sin\theta\cdot\cos\theta-\frac{2mg(1-\cos\theta)\sin\theta}{m+M\cos^2\theta}=0 \qquad \text{[using (a)]}$$

$\Rightarrow 2\,mg - 2mg\cos\theta - mg\cos\theta - Mg\cos^3\theta = 0$ [$\sin\theta = 0$ is not acceptable]

It is given that this happens when $\theta = \pi/3$

$$\therefore\ 2mg - 3mg \times \cos\frac{\pi}{3} = Mg(\cos\pi/3)^3$$

$$\Rightarrow\ 2m - 3m \times \frac{1}{2} = M \times \frac{1}{8}$$

$$\Rightarrow\ \frac{1}{2}m = \frac{M}{8}$$

$\Rightarrow M = 4m$

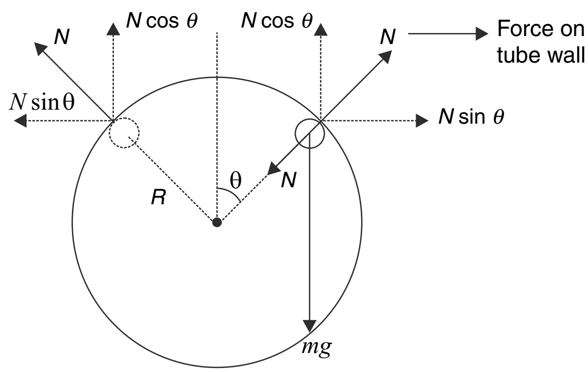
(b) from (a)

$$v^2 = \frac{2mgl(1-\cos\pi/3)}{m+4m(\cos\pi/3)^2} = \frac{gl}{2}$$

$\therefore$ Speed of M at the instant is

$$v_x = v\cos\theta = \sqrt{\frac{gl}{2}}\cos\pi/3 = \frac{1}{2}\sqrt{\frac{gl}{2}}$$

92. Why at all the tube will rise? As the balls slide down, initially they are in contact with inner wall of the tube and at a certain point they leave the inner circle and get in touch with outer circle, so that normal reaction is inward on the balls.



The balls exert an equal and opposite force on the outer wall of tube (see figure)

When $2N\cos\theta > Mg$ (weight of tube), the tube will begin to rise.

Considering the dynamics of ball

$$\frac{mv^2}{R} = mg\cos\theta + N \quad \text{.............. (a)}$$

From conservation of energy

$$\frac{mv^2}{2} = mgR\,(1-\cos\theta) \quad \text{.............. (b) [For each ball]}$$

From (a) and (b), eliminating v^2, we get

$N = mg\,(2 - 3\cos\theta)$

For tube to rise

$2\,N\cos\theta \geq Mg$

$2\,m\cos\theta\,(2 - 3\cos\theta) \geq M$ ....(c)

For the tube to just start rising, the maximum value of

$Z = \cos\theta\,(2 - 3\cos\theta)$ must satisfy the above inequality.

Z is maximum if

$$\frac{dz}{d\theta} = -2\sin\theta + 3 \times 2\cos\theta \cdot \sin\theta = 0$$

$\Rightarrow \cos\theta = \frac{1}{3}$

$\therefore Z_{max} = \frac{1}{3} \times \left(2 - 3 \times \frac{1}{3}\right) = \frac{1}{3}$

$\therefore$ from (c) $2m\frac{1}{3} \geq M$

$\Rightarrow m \geq \frac{3M}{2}$ for the tube to rise.

(b) Equation (c) gives the value of θ.

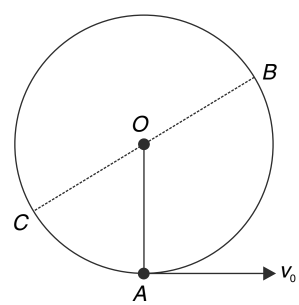
When $M = 0$,

$2m\cos\theta(2 - 3\cos\theta) = 0$ [In limiting case]

$\Rightarrow \cos\theta = 2/3$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

93.



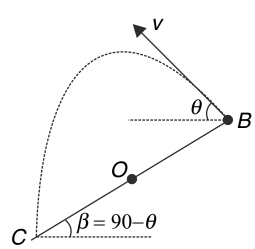
Let v = speed at B.

Tension in the string at $B = 0$

$\therefore mg\cos\theta = \frac{mv^2}{l}$

$\Rightarrow v^2 = gl\cos\theta$ .... (a)

After B, the motion of particle is that of a projectile. One can consider it a projectile thrown down the incline BOC. The particle hits the incline at C.



$\therefore$ Range along the incline $= BO + OC = 2l$

Incline angle $\beta = 90 - \theta$

For such a projectile, you can prove that the range along the incline will be

$$\frac{2gl\cos\theta \cdot \sin 90^\circ \cos\theta}{g\sin^2\theta} = 2l\cot^2\theta$$

$\therefore\ 2l = 2l\cot^2\theta$

$\therefore\ \cot^2\theta = 1$

Or, $\theta = 45°$

Now we can use conservation of energy between points A and B, which gives

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgl(1+\cos\theta)$$

$$\Rightarrow\ v_0^2 = v^2 + 2gl(1+\cos\theta)$$

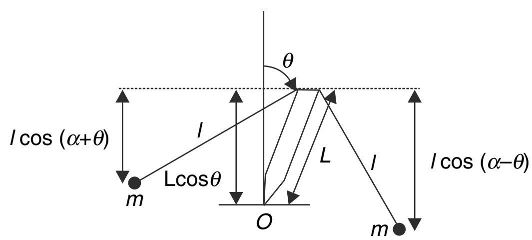
Using (a)

$$v_0^2 = 3gl\cos\theta + 2gl$$

$$= gl\left[3\times\frac{1}{\sqrt{2}} + 2\right] \quad [\because\ \theta = 45°]$$

$$= \frac{(4+3\sqrt{2})}{2}gl$$

94. As shown in sketch. If we take the zero of gravitational potential energy at the pivot (point O), we have
$U(\theta)$ = mg [$L\cos\theta - l\cos(\alpha+\theta)$] + mg [$L\cos\theta - l\cos(\alpha-\theta)$]
= 2 mg cos θ [$L - l\cos\alpha$]



For equilibrium,

$$\frac{dU}{d\theta} = -2mg\sin\theta\ (L - l\cos\alpha) = 0$$

$\Rightarrow$ sin $\theta = 0$ or $\theta = 0$, as we expect from symmetry.

To investigate the stability of the equilibrium position, we must examine the second derivative of the potential energy. We have

$$\frac{d^2U}{d\theta^2} = -2mg\cos\theta\ [L - l\cos\alpha]$$

At equilibrium, $\left.\frac{d^2U}{d\theta^2}\right|_{\theta=0} = -2mg\alpha\ [L - l\cos\alpha]$

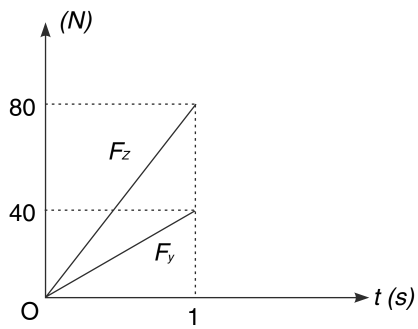
For the second derivative to be positive, we have $L - l\cos\alpha < 0$ or $L < l\cos\alpha$.

In order for the teeter toy to be stable, the weights must hang below pivot.

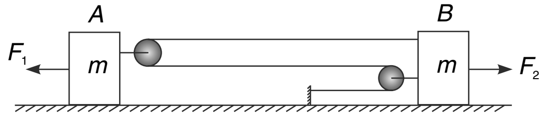
05 MOMENTUM AND CENTER OF MASS

LEVEL 1

Q. 1. A particle is acted upon by a force for 1 second whose X component remains constant at F_x = 30 N but y and z components vary with time as shown in the graph. Calculate the magnitude of change in momentum of the particle in 1 s. What angle does the change in momentum ($\Delta \vec{P}$)make with X axis?



Q. 2. Two block A and B of equal mass are connected using a light inextensible string passing over two light smooth pulleys fixed to the blocks (see fig). The horizontal surface is smooth. Every segment of the string (that is not touching the pulley) is horizontal. When a horizontal force F_1 is applied to A the magnitude of momentum of the system, comprising of $A + B$, changes at a rate R. When a horizontal force F_2 is applied to B (F_1 not applied) the magnitude of momentum of the system $A + B$ once again changes at the rate R. Which force is larger - F_1 or F_2?



Q. 3. A particle of mass m = 1 kg is moving in space in X direction with a velocity of 10 ms^{-1}. A 4 N force acting in Y direction is applied on it for a time interval of 5.0 s. Later a 5 N force was applied on it in Z direction for 4.0 s

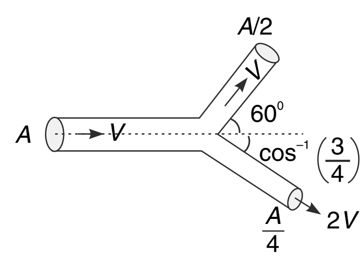
(a) Calculate the total work done by both the forces.

(b) Which force performed greater work?

Q. 4. An observer O_1 standing on ground finds that momentum of a projectile of mass 2 kg changes with time as $\vec{P}_{01} = \left(4t\ \hat{i} + 20t\,\hat{k}\right) kg\ m/s$ Acceleration due to gravity is $\vec{g} = \left(10\hat{k}\right)$ m/s^2 and there is a wind blowing in horizontal direction. Another observer O_2 driving a car observes that momentum of the same projectile changes with time as -

$\vec{P}_{02} = \left(8t\,\hat{i} - 16t^2\ \hat{j} + 20t\,\hat{k}\right) kg\,m/s$. Find the acceleration of the car at $t = \frac{1}{8}s$

Q. 5. Water flows through a tube assembly as shown in the fig. Speed of flow (marked as V and 2V), cross sectional area (A, A/2 and A/4) and the angles between segments has been shown in fig. Calculate the force applied by the water flow on the tube. Take density of water to be ρ.

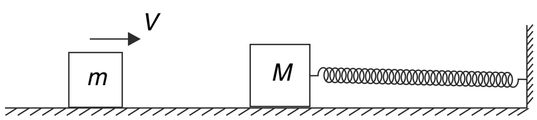


Q. 6. A man is running along a road with speed u. On his chest there is a paper of mass m and area S. There is a wind blowing against the man at speed V. Density of air is ρ. Assume that the air molecules after striking the paper come to rest relative to the man. Find the minimum coefficient of friction between the paper and the chest so that the paper does not fall?

Q. 7. Two particles A and B of mass 2 m and m respectively attract each other by mutual gravitational force and no other force acts on them. At time $t = 0$, A was observed to be at rest and B was moving away from A with a speed u. At a later time t it was observed that B was moving towards A with speed u. Assume no collision has taken place by then. Find work done by the gravitational force in the time interval 0 to t.

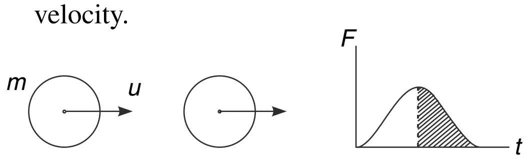
Q. 8. (i) A block of mass m moving towards right with a velocity V strikes (head on) another block of mass M which is at rest connected to a spring. The coefficient of restitution for collision between the blocks is $e = 0.5$.

Find the ratio $\frac{M}{m}$ for which the subsequent compression in the spring is maximum. There is no friction.



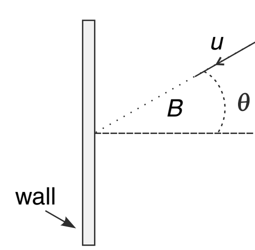
(ii) Ball A collides head on with another identical ball B at rest. Find the coefficient of restitution if ball B has 80% of the total kinetic energy of the system after collision.

Q. 9. A ball having mass m and velocity u makes a head on collision with another ball. After collision velocity of the ball of mass m was found to be V in the direction of its original motion. The interaction force between the two balls during their collision has been shown in the graph. The area of the shaded part of the graph is same as the area of the not shaded part. Find the velocity of the balls at the instant they were having equal velocity.

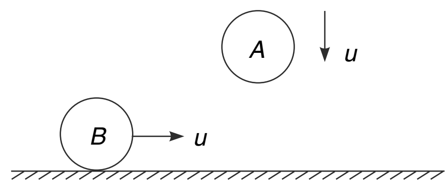


Q. 10. Ball A is about to hit a wall at an angle of incidence of $\theta = 30°$. But before hitting the wall it made a head on collision with another identical ball B. The ball B then collides with the wall. The coefficient of restitution for collision between two balls is $e_1 = 0.8$ and that between a ball and the wall is $e_1 = 0.6$. Find the final velocity of ball B. Initial velocity of A was $u = 5\ ms^{-1}$. Neglect friction.

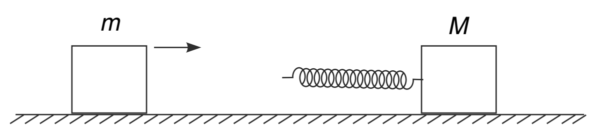
$$\left[\tan^{-1}\left(\frac{2.25}{2.34}\right) = 44°\right]$$



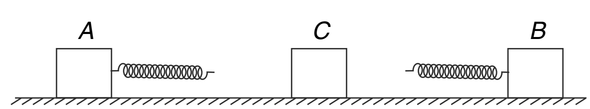
Q. 11. Two identical balls A and B are moving as shown in the fig. Ball A hits a smooth floor head on with a velocity u and at the same instant ball B strikes A head on with a horizontal velocity u. The collision between A and B is perfectly inelastic whereas the coefficient of restitution for collision between A and the floor is $e = 0.5$. At what time the two balls will collide again? Assume friction to be absent everywhere.



Q. 12. Two blocks of mass m and M are lying on a smooth table. A spring is attached with the block of mass M (see fig). Block of mass m is given a velocity towards the other block. Find the value of $\frac{M}{m}$ for which the kinetic energy of the system will never fall below one third of the initial kinetic energy imparted to the block of mass m.



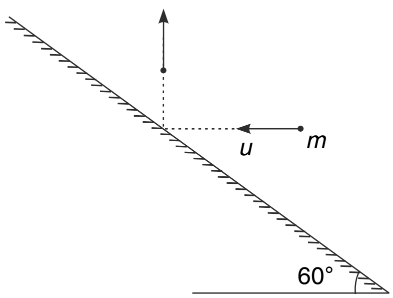
Q. 13. Two identical blocks A and B have two identical springs fixed to them (see fig). Mass of each block is M and force constant of each spring is K. The two blocks have been placed on a smooth table. Another block C of mass m ($<< M$) is placed between A and B and is held close to A so as to compress the spring attached to A by X_0. From this position the system is released. C moves to push B and then is back to push A. The sequence continues until all interactions between the blocks cease. Find the speeds eventually acquired by A and B.



Q. 14 A particle of mass m is flying horizontally at velocity u. It strikes a smooth inclined surface and its velocity becomes vertical.

(a) Find the loss in kinetic energy of the particle due to impact if the inclination of the incline is 60° to the horizontal.

(b) Can the particle go vertically up after collision if inclination of the incline is 30°?

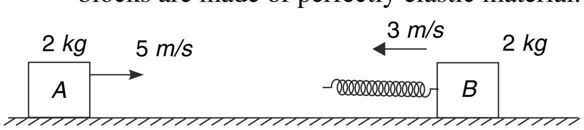


Q. 15. A block A of mass 2 kg is moving to right with a speed of 5 m/s on a horizontal smooth surface. Another block B of mass 2 kg, with a mass less spring of force constant K =200 N/m attached to it, is moving to left on the same surface with a speed of 3 m/s. Block A collides with the spring attached to B. Calculate

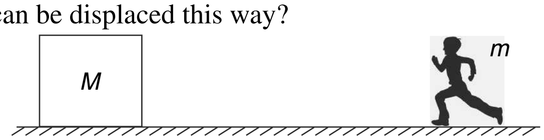
(a) the final velocity of the block A.

(b) the minimum kinetic energy of the system of two blocks during subsequent motion.

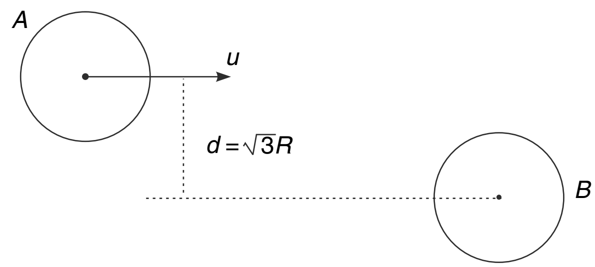
(c) Repeat part (b) if there is no spring and the two blocks collide head on. Assume that the blocks are made of perfectly elastic material.



Q. 16. A box of mass M is at rest on a horizontal surface. A boy of mass m ($< M$) wants to push the box by applying a horizontal force on it. The boy knows that he will not be able to push the box as the coefficient of friction μ between his shoes and ground is almost equal to that between the box and the ground. He decides to run, acquire a speed u and then bang into the box. After hitting the box, the boy keeps pushing as hard as possible. What is the maximum distance through which the box can be displaced this way?



Q. 17. A smooth ball A travels towards another identical ball B with a velocity u. Ball B is at rest and the impact parameter d is equal to $\sqrt{3}\,R$ where R is radius of each ball. Due to impact the direction of motion of ball A changes by 30°. Find the velocity of B after the impact. It is given that collision is elastic.

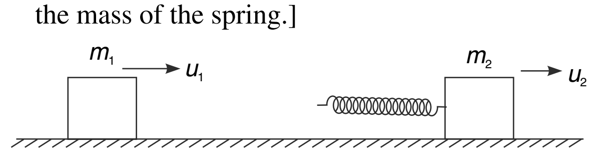


Q. 18. (a) Two identical balls are moving along X axis and undergo an elastic collision. Plot the position time graph for the two balls.

(b) Consider five identical balls moving along X axis. What is the maximum number of collisions that is possible? Assume that more than two balls do not collide at the same time and collisions are elastic.

Q. 19. Two particles of mass m each are attached to the end of a mass less spring. This dumb-bell is moving towards right on a smooth horizontal surface at speed V with the spring relaxed. Another identical dumb-bell is moving along the same line is opposite direction with the same speed. The two dumb-bells collide head on and collision is elastic. Assuming collisions to be instantaneous, how many collisions will take place?

Q. 20. Two blocks of mass m_1 and m_2 are moving along a smooth horizontal floor. A non-ideal spring is attached at the back of mass m_2. Initial velocities of the blocks are u_1 and u_2 as shown; with $u_1 > u_2$. After collision the two blocks were found to be moving with velocities V_1 and V_2 respectively. Find the ratio of impulse (on each block) during the deformation phase of the spring and that during its restoration phase. [By non ideal spring we mean that it does not completely regain its original shape after deformation. You can neglect the mass of the spring.]



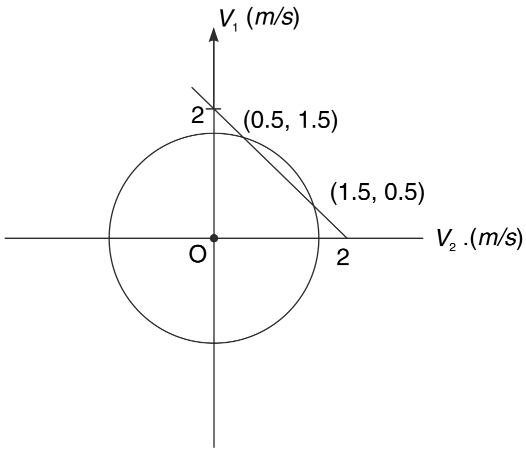
Q. 21. A ball moving with velocity V_0, makes a head on collision with another identical ball at rest. The velocity of incident ball and the other ball after collision is V_1 and V_2 respectively.

(a) Using momentum conservation write an equation having V_1 and V_2 as unknowns. Plot a graph of V_1 vs V_2 using this equation.

(b) Assuming the collision to be elastic write an equation for kinetic energy. Plot a graph of V_1 vs V_2 using this equation.

(c) The intersection point of the above two graphs gives solution. Find V_1 and V_2.

(d) In a particular collision, the plot of graphs mentioned above is as shown in figure

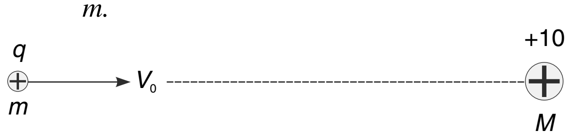


Find V_1 and V_2 for this collision. Also write the percentage loss in kinetic energy during the collision.

Q. 22. A particle having charge $+q$ and mass m is approaching (head on) a free particle having mass M and charge 10 q. Initially the mass m is at large distance and has a velocity V_0, whereas the other particle is at rest.

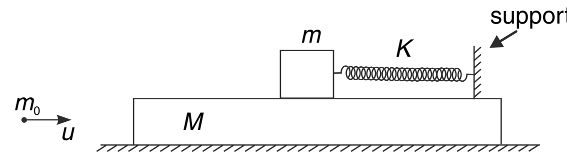
(a) Find the final velocity of the two particles when $M = 20\ m$.

(b) Find the final state of the two particles if $M = m$.

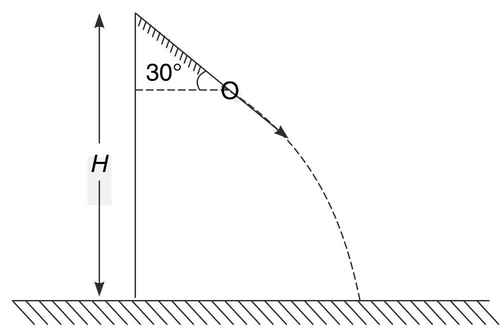


Q. 23. In the system shown in fig. block of mass M is placed on a smooth horizontal surface. There is a mass less rigid support attached to the block. Block of mass m is placed on the first block and it is connected to the support with a spring of force constant K. There is no friction between the blocks. A bullet of mass m_0, moving with speed u hits the block of mass M and gets embedded into it. The collision is instantaneous. Assuming that m always stays over M, calculate the maximum extension in the spring caused during the subsequent motion.

$K = 8960\ N/m$; $u = 400\ m/s$



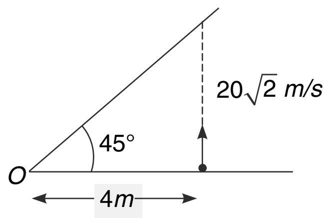
Q. 24. Starting from a height H, a ball slips without friction, down a plane inclined at an angle of 30° to the horizontal (fig.). After leaving the inclined plane it fall under gravity on a parabolic path and hits the horizontal ground surface. The impact is perfectly elastic (It means that there is no change in horizontal component of ball's velocity and its vertical velocity component gets inverted. There is no change in speed due to collision). Will the ball rise to a height equal to H or less than H after the impact?



Q. 25. Hailstones are observed to strike the surface of the frozen lake at an angle of 30° with the vertical and rebound at 60° with the vertical. Assuming the contact to be smooth, find the coefficient of restitution.

Q. 26. A ball of mass m approaches a heavy wall of mass M with speed 4 m/s along the normal to the wall. The speed of wall before collision is 1m/s towards the ball. The ball collides elastically with the wall. What can you say about the speed of the ball after collision? Will it be slightly less than or slightly higher than 6 m/s ?

Q. 27. A particle is thrown upward with speed $20\sqrt{2}$ m/s. It strikes the inclined surface as shown in the figure. Collision of particle and inclined surface is perfectly inelastic. What will be maximum height (in m) attained by the particle from the ground ($g = 10\ m/s^2$)



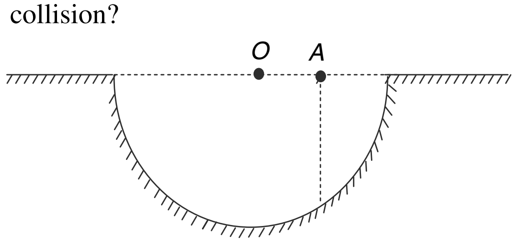
Q.28. 2n identical cubical blocks are kept in a straight line on a horizontal smooth surface. The separation between any two consecutive blocks is same. The odd numbered blocks 1, 3, 5,.....($2n$–1) are given velocity v to the right whereas blocks 2, 4, 6,......$2n$ are given velocity v to the left. All collisions between blocks are perfectly elastic. Calculate the total number of collisions that will take place.

Q.29. A small ball with mass $M = 0.2$ kg rests on top of a vertical column with height $h = 5\ m$. A bullet with mass $m = 0.01$ kg, moving with velocity $v_0 = 500\ m/s$, passes horizontally through the center of the ball. The ball reaches the ground at a horizontal distance $s = 20\ m$ from the column. Where does the bullet reach the ground? What part of the kinetic energy of the bullet was converted into heat when the bullet passed through the ball? Neglect resistance of the air. Assume that $g = 10\ m/s^2$.

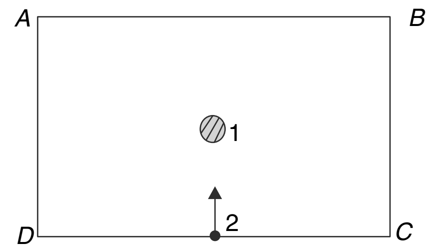
Q. 30. Figure shows a circular frictionless track of radius R, centred at point O. A particle of mass M is released from point A ($OA = R/2$). After collision with the track, the particle moves along the track.

(a) Find the coefficient of restitution e.

(b) What will be value of e if the velocity of the particle becomes horizontal just after collision?

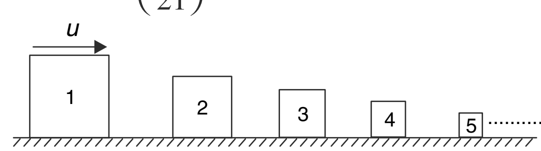


Q. 31. A rectangular billiard table has dimensions $AB = 4\sqrt{3}$ feet and $BC = 2$ feet. Ball 1 is at the centre of the table. Ball 2 moving perpendicular to CD hits ball 1. After the collision ball 2 itself goes straight into the hole at A. Prove that ball 1 will fall into the hole at C. Assume that the balls are identical and their dimensions are too small compared to the dimensions of a hole. All collisions are elastic

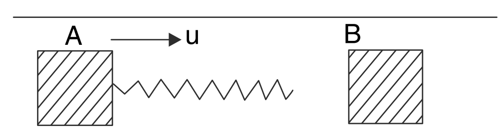


Q. 32. Blocks shown in figure have been placed on a smooth horizontal surface and mass of $(n + 1)^{\text{th}}$ block is $\frac{1}{20}$ times the mass of n^{th} block (where n = 1, 2, 3, 4,.....). The first block is given an initial velocity u towards the second block. All collision are head on elastic collisions. If u = 10 m/s then how many blocks must be kept so that the last one acquires speed equal to or greater than the escape speed (= 11.0 km s^{-1})

[**Take** $\log_{10}\left(\frac{40}{21}\right) = 0.28$ and $\log_{10}(11) = 1.04$]



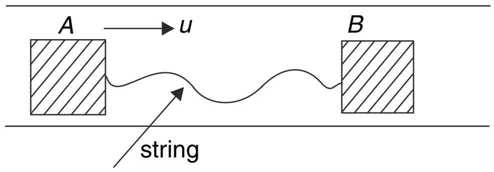
Q. 33. There is a long narrow and smooth groove in a horizontal table. Two identical blocks A and B each of mass m are placed inside the groove at some separation. An ideal spring is fixed to A as shown. Block A is given a velocity u to the right and it interacts with B through the spring.



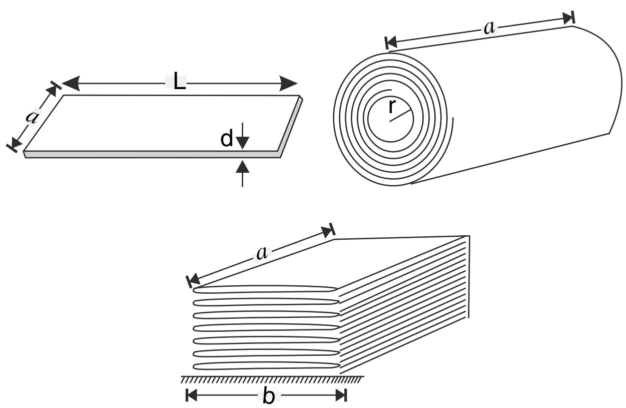
(a) What will be final state of motion of the two blocks?

(b) During their course of interaction what is the minimum kinetic energy of the system?

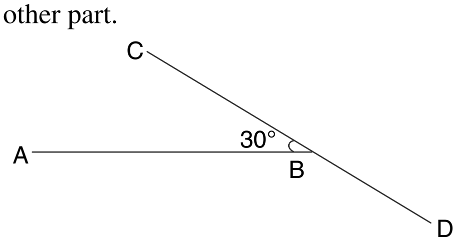
(c) The spring is removed and the two blocks are tied using a mass less string. Now A is set into motion with speed u. What will be the final state of motion of the two blocks in this case ? How much kinetic energy is lost by the system? Where goes this energy?



Q. 34. A carpet lying on ground has length L, width a and a small thickness d. It is rolled over a light cylindrical pipe of radius $r = \frac{L}{100\sqrt{\pi}}$ and kept on a level ground. Increase in gravitational potential energy of the carpet is ΔU_1 (compared to its initial position when it was lying flat). In another experiment the carpet was folded to give it a shape of a cuboid (see figure) having width b. When this is placed on level ground its gravitational potential energy is ΔU_2 higher than its initial position (flat on ground). It is given that $d = 10^{-4} L$. Find b for which $\Delta U_1 = \Delta U_2$. [Take $\sqrt{\frac{\pi}{2}} = 1.25$]



Q. 35. Two identical thin rods are welded as shown in the fig. B is midpoint of rod CD. Now the system is cut into two parts through its center of mass M. The part AM weights 4 kg. Find the mass of the other part.



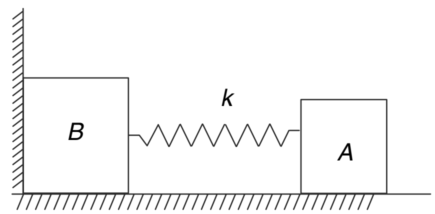
Q. 36. (i) A regular polygon has 2016 sides and r is the radius of the circle circumscribing the polygon. Particles of equal mass are placed at 2015 vertices of the polygon. Find the distance of the centre of mass of the particle system from the centre of the polygon.

(ii) In the last problem you have been asked to remove any one particle from the system so that the centre of mass of the remaining 2014 particles lies farthest from the geometrical centre of the polygon. Which particle will you remove?

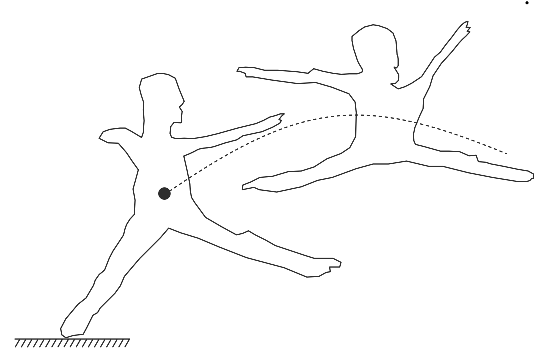
Q. 37. Two identical block A and B each having mass m, are connected with a spring of force constant k. The floor is smooth and A is pushed so as to compress the spring by x_0. The system is released from this position

(a) Calculate the maximum speed of the centre of mass of the system during subsequent motion.

(b) What is acceleration of the centre of mass at the instant it acquires half its maximum speed?



Q. 38. A dancer leaps off the floor with her centre of mass having a velocity of 5 m/s making an angle of $\theta = 37°$ to the horizontal. At the top of the trajectory the dancer has her legs stretched so that the centre of mass gets closer to head by a vertical distance of 0.25 m. By how much does the head rises vertically from its initial position? $\left[\sin 37° = \frac{3}{5}\right]$.

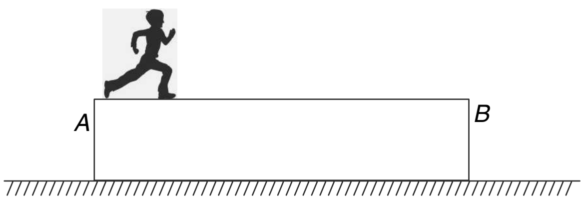


Q. 39. In order to make a jump straight up, a 60 kg player starts the motion crouched down at rest. He pushes hard against the ground, raising his centre of mass by a height $h_0 = 0.5$ m. Assume that his legs exert a constant force F_0 during this motion. At this point, where his centre of mass has gone up by h_0 his feet leave the ground and he has an upward velocity of v. Centre of mass of his body rises further by $h = 0.8\ m$ before falling down [Take $g = 10\ m/s^2$]

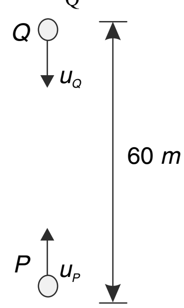
(a) Find v.

(b) Find the normal force applied by the ground on his feet just before he left the ground.

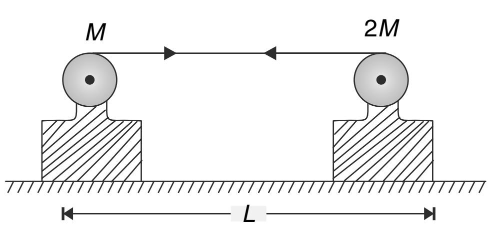
Q. 40. A platform is kept on a rough horizontal surface. At one end A of the platform there is a man standing on it. The man runs towards the end B and the platform is found to be moving. In which direction will the platform be moving after the man abruptly comes to rest on the platform at B?



Q. 41. Two particles P and Q have mass 1 kg and 2 kg respectively. They are projected along a vertical line with velocity $u_p = 20\ m/s$ and $u_Q = 5\ m/s$ when separation between them was 60 m. P was projected vertically up while Q was projected vertically down. Calculate the maximum height attained by the centre of mass of the system of two particles, measured from the initial position of P. Assume that the particles do not collide and that the ground is far below their point of projection [$g = 10\ m/s^2$]

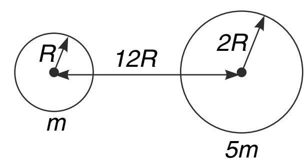


Q. 42. Two small motors are kept on a smooth table at a separation L. The motors have mass M and $2M$ and are connected by a light thread. The motors begin to wrap the thread and thereby move closer to each other. The tension in the thread is maintained constant at F. Find the time after which the two motors will collide. Neglect the dimensions of the motors and their stands.



Q. 43. Consider a uniform rectangular plate. If a straight line is drawn, passing through its centre of mass (in the plane of the plate), so as to cut the plate in two parts – the two parts obtained are of equal mass irrespective of the orientation of the line. Can you also say that a straight line passing through the centre of mass of a triangular plate, irrespective of its orientation, will also divide the triangle into two pieces of equal mass?

Q. 44. Two spherical bodies of masses m and $5\ m$ and radii R and $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only then find the distance covered by smaller sphere just before collision.



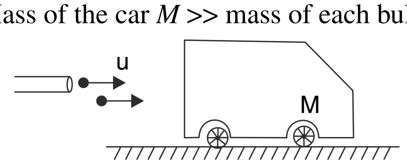
Q. 45. A shell is fired vertically upward with a speed of 60 m/s. When at its maximum height it explodes into large number of fragments. Assume that the fragments fly in every possible direction and all of them have same initial speed of 25 m/s
[Take $g = 10\ m/s^2$]

(a) Prove that after the explosion all the fragments will lie on an expanding sphere. What will be speed of the centre of the sphere thus formed – one second after explosion?

(b) Find the radius of the above mentioned sphere at the instant the bottom of the sphere touches the ground.

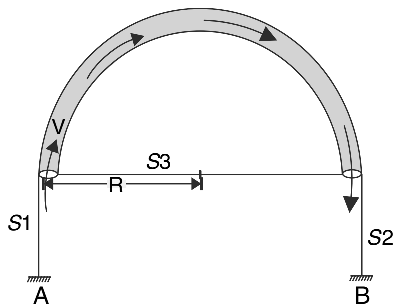
LEVEL 2

Q. 46. A car of mass M is free to move on a frictionless horizontal surface. A gun fires bullets on the car. The bullets leave the stationary gun with speed u and mass rate $b\ kg\ s^{-1}$. The bullets hit the vertical rear surface of the car while travelling horizontally and collisions are elastic. If the car starts at rest find its speed and position as a function of time. Mass of the car $M >>$ mass of each bullet.



Q. 47. (i) Liquid of density ρ flows at speed υ along a flexible pipe bent into a semicircle of radius

R. The cross sectional area of the pipe is A and its cross sectional radius is small compared to R. Three strings $S1$, $S2$ and $S3$ keep the pipe in place. $S3$ ties the two ends of the pipe and the other two string have their ends secured at A and B. Strings $S1$and $S2$ are perpendicular to the string $S3$. The entire system is in horizontal plane. Find the tension in the three strings.



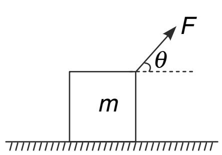
(ii) A car of mass M is moving with a velocity V_0 on a smooth horizontal surface. Bullets, each of mass m, are fired horizontally perpendicular to the velocity of the car with a speed u relative to the car. After firing n bullets it was found that the car was travelling with velocity V_0 in a direction opposite to its original direction of motion. Assume that $mu << MV_0$ and also that $nm << M$. Find n in terms of other given parameters.

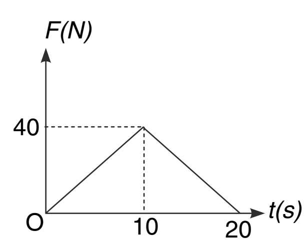
Q. 48. A block of mass m = 4.4 kg lies on a horizontal rough surface. The coefficient of friction between the block and the surface is μ = 0.5. A force F starts acting on the block making an angle $\theta = 37°$ to the horizontal. The force changes with time as shown in the graph.

(a) At what time the block begins to move?

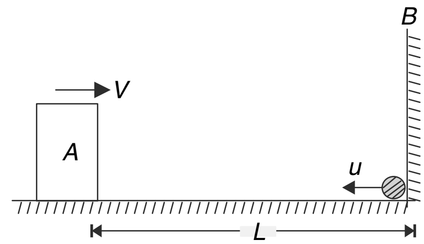
(b) Calculate the maximum speed attained by the block.

$$\left[\tan 37° = \frac{3}{4}\ ;\ g = 10ms^{-2}\right]$$

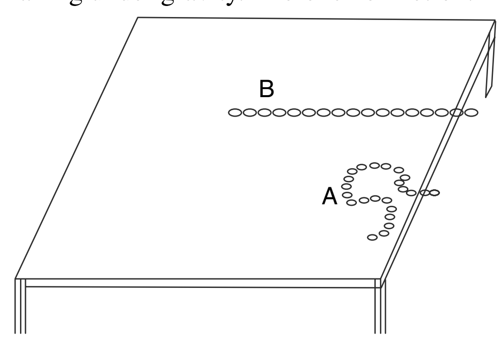




Q. 49. A heavy block A is made to move uniformly along a smooth floor with velocity $V = 0.01$ *m/s* towards left. A ball of mass m = 50 g is projected towards the block with a velocity of u = 100 *m/s*. The ball keeps bouncing back and forth between the block A and fixed wall B. Each of the collisions is elastic. After the ball has made 1000 collisions with the block and wall each, the distance between the block and the wall was found to be L = 1.2 *m*. Calculate the average force being experienced by the block due to collision at this instant. All collision are instantaneous.



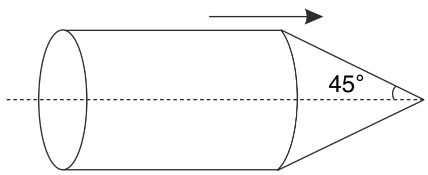
Q. 50. A chain (A) of length L is coiled up on the edge of a table. Another identical chain (B) is placed straight on the table as shown. A very small length of both the chains is pushed off the edge and it starts falling under gravity. There is no friction.



(a) Find the acceleration of the chain B at the instant $\frac{L}{2}$ length of it is hanging. Assume no kinks in the chain so that the entire chain moves with same speed.

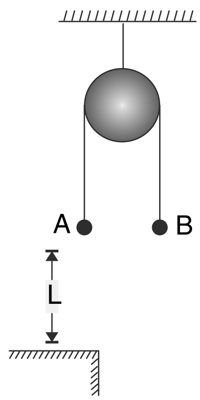
(b) For chain A assume that velocity of each element remains zero until it is jerked into motion with a velocity equal to that of the falling section. Find acceleration of the hanging section at the instant a length l_0 has slipped off the table and its speed is known to be v_0 at the instant.

Q. 51 To understand the effect of air resistance on the motion of a bullet let's consider a bullet of the shape shown in the fig. The bullet is flying horizontally. The cross section of cylindrical part is A and the conical part has a semi vertical angle of 45°. Assume that the bullet is fired with initial velocity u and moves in a gaseous medium in which molecules are at rest (do you think this assumption is necessary?). Collisions of the molecules with the bullet are elastic. Take mass of bullet to be M, density of gaseous medium as ρ and disregard gravity.



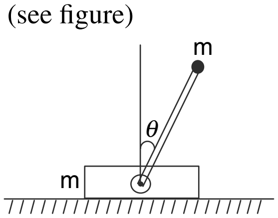
(a) Consider two bullets one small and other large, made of same material. Which will experience larger retardation due to air resistance?
(b) Write the speed of bulled after time t.
(c) Write distance travelled by the bullet in time t.

Q. 52. Two particle A and B, of mass $3m$ and $2m$ respectively, are attached to the ends of a light inextensible string which passes over a smooth fixed pulley of negligible mass. After the system is released and A falls through a distance L, it hits a horizontal inelastic table so that its speed is immediately reduced to zero. Assume that B never hits the table or the pulley. Find



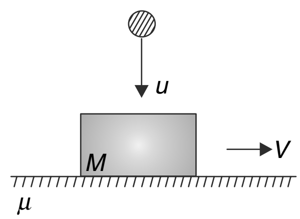
(a) the time for which A is resting on the table after the first collision and before it is jerked off,
(b) the difference between the total kinetic energy of the system immediately before A first hits the table and total kinetic energy immediately after A starts moving upwards for the first time. Explain the loss in kinetic energy.

Q. 53. A light rod of length L is hinged to a plank of mass m. The plank is lying on the edge of a horizontal table such that the rod can swing freely in the vertical plane without any hindrance from the table. A particle of mass m is attached to the end of the rod and system is released from $\theta = 0°$ position (see figure)



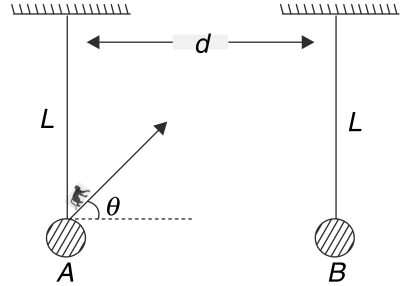
(a) Assume that friction between the plank and the table is large enough to prevent it from slipping and calculate the smallest normal force applied by the plank on the table.
(b) Assume that friction is absent everywhere and calculate the speed of the plank when the rod makes $\theta = 180°$.

Q. 54. A block of mass M = 5 kg is moving on a horizontal table and the coefficient of friction is $\mu = 0.4$. A clay ball of mass m = 1 kg is dropped on the block, hitting it with a vertical velocity of u = 10 m/s. At the instant of hit, the block was having a horizontal velocity of v = 2 m/s. After an interval of Δt, another similar clay ball hits the block and the system comes to rest immediately after the hit. Assume that the clay balls stick to the block and collision is momentary. Find Δt. Take g = 10 m/s^2.



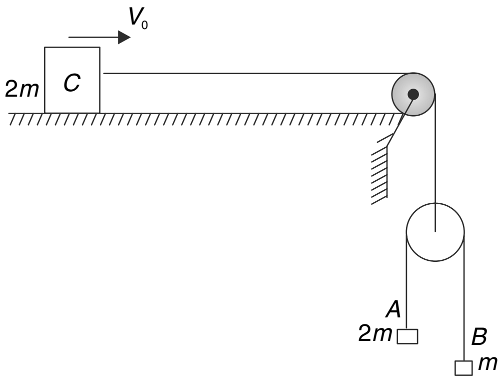
Q. 55. Vertical strings of same length L support two balls A and B of mass $2m$ each. There is a small monkey of mass m sitting on ball A. Suddenly, the monkey jumps off the ball A at an angle θ = 45° to the horizontal and lands exactly on the ball B. Thereafter, the monkey and the ball B just manage to complete the vertical circle.

(a) Find distance d between the two string and the speed with which the monkey jumped of the ball A.
(b) Find the impulse of the string tension on ball A during the small period when the monkey interacted with the ball to jump off it.

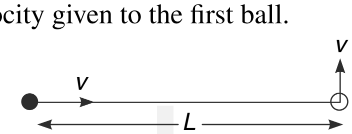


Q. 56. In the shown figure, pulleys and strings are ideal and horizontal surface is smooth. The block C (mass $2m$) is given a horizontal velocity of

$V_0 = 3$ *m/s* towards right and the entire system is let go. Find the velocity of three blocks, just after the strings regain tension. Mass of *A* and *B* are $2m$ and *m* respectively and take $g = 10$ m/s^2.



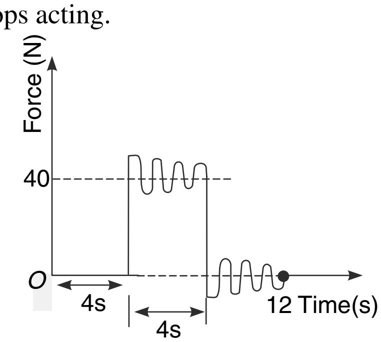
Q. 57. Two identical small balls are interconnected with a light and inextensible thread having length *L*. The system is on a smooth horizontal table with the thread just taut. Each ball is imparted a velocity *v*, one towards the other ball and the other in a direction that is perpendicular to the velocity given to the first ball.



(a) After how much time the thread will become taut again?

(b) Calculate the kinetic energy of the system after the string gets taut.

Q. 58. A particle of mass 1 kg is moving with a velocity of 200 m/s. An impulsive force of 4 s duration acts on the particle in a direction opposite to its motion. The force fluctuates a little bit around 40 N magnitude and then it dies out in next 4s showing small fluctuations. An oscilloscope records the force as shown. The two oscillating components in the graph are identical except that one is mirror image of the other. Find the magnitude of velocity of particle after the force stops acting.

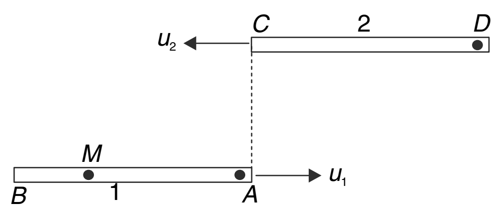


Q. 59. A moving particle of mass *m* collides elastically with a stationary particle of mass $2m$. After collision the two particles move with velocity $\vec{v}_1$ and $\vec{v}_2$ respectively. Prove that $\vec{v}_2$ is perpendicular to $(2\vec{v}_1 + \vec{v}_2)$

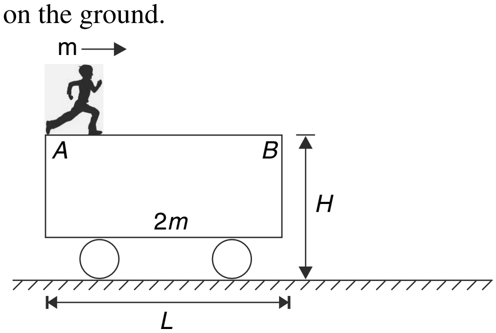
Q. 60. Two identical carts are moving on parallel smooth tracks with velocities $u_1 = 10$ ms^{-1} and $u_2 = 15$ ms^{-1}. The empty carts (with drivers) have mass 3m each. Each cart has a sack of mass *m* kept at end *A* and end *D* (see figure). At the instant the carts being to cross, the sack in cart 1 is the thrown perpendicular (relative to cart 1) with some unknown velocity and it lands on cart 2 at its end *D* after a time t_0. Immediately after the sack lands into cart 2, the original sack in cart 2 is thrown perpendicularly (relative to cart 2) towards cart 1 in identical fashion. The sack lands on cart 1 at point *M*, a time t_0 after the throw. Assume that the carts are constrained to move in straight lines.

(i) Find length *BM* if length of each cart is *L*

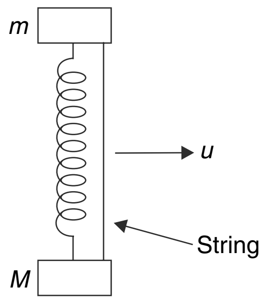
(ii) Find the velocity of cart 1 after the sack thrown from cart 2 lands on it.



Q. 61. A man of mass *m* is standing on the flat top of a cart of mass $2m$. The length and height of the cart is *L* and *H* respectively and it is at rest on a smooth horizontal ground. The man starts running from end *A*, speeds up and jumps out of the cart at point *B* with a velocity *u* relative to the cart in horizontal direction. Calculate the total horizontal distance covered by the man by the time he lands on the ground.



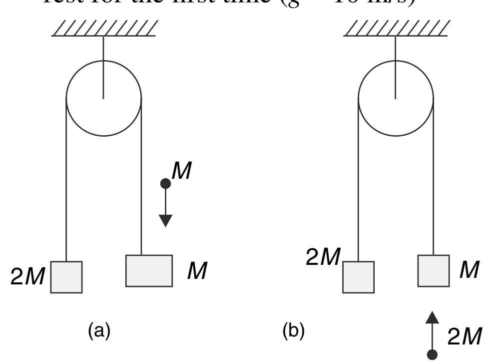
Q. 62. Two blocks of masses $m = 2$ *kg* and $m = 8$ *kg* are connected to a spring of force constant $K = 1$ *kN/m*. The spring is compressed by 20 *cm* and the two blocks are held in this position by a string. The system is placed on a horizontal smooth surface and given a velocity $u = 3$ m/s perpendicular to the spring. The string snaps while moving. Find the speed of the block of mass m when the spring regains its natural length.



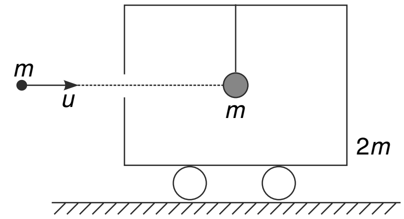
Q. 63. Two blocks of mass M and $2M$ are connected to the two ends of a light, inextensible string passing over an ideal pulley as shown in figure. The system is released from rest.

(a) One second after the system is released, a particle of mass M hits the block of mass M and sticks to it. The particle hits the block with a speed of 10 m/s while travelling downward. Find the total distance travelled by block of mass $2M$ after it is released.

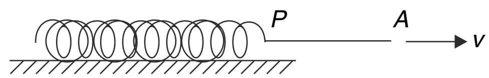
(b) One second after the system is released, a particle of mass $2M$ hits the block of mass M and sticks to it. The particle hits the block with a speed of 10 m/s while travelling in upward direction. Find distance travelled by the block of mass $2M$ after it is released to the time it comes to rest for the first time ($g = 10$ m/s)^2



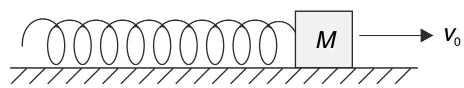
Q. 64. A toy car of mass 2 m is at rest on a smooth horizontal surface. A small bob of mass m is suspended by a mass less string of length L from the roof of the car. A horizontally flying bullet of mass m enters into the car through a small window and sticks to the bob. Speed of the bullet is u. Find minimum value of u (call it u_0) for which the bob can touch the roof of the car.



Q. 65. A heap of rope is lying on a horizontal surface. One free end A of the rope is pulled horizontally with a constant velocity v. Assume that the heap does not move and the moving part of the rope remains straight and horizontal (i.e. there is no sag). Mass per unit length of the rope is λ. Find the tension at point P where the straightened part of the rope meets the heap. How much force the external agent must apply at end A?



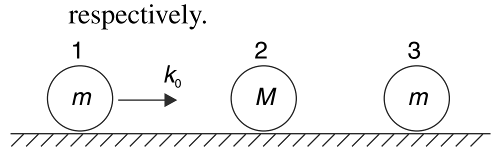
Q. 66. In the last problem, the free end A of the rope is tied to a block of mass M and the block is given a horizontal velocity v_0 (see figure). Calculate the following quantities at the instant the block is at a distance x from the right end of the heap (here 'heap' means the coiled part of the rope that is not moving).



(i) Speed of the block.

(ii) Tension force applied by the rope on the block.

Q. 67. (i) Three balls 1, 2 and 3 lie on a smooth horizontal table. Ball 1 is given a velocity towards ball 2. Kinetic energy given to ball 1 is k_0. It collides with 2 and in turn ball 2 hits ball 3. All collisions are head on elastic. Masses of the balls are m, M and m respectively.

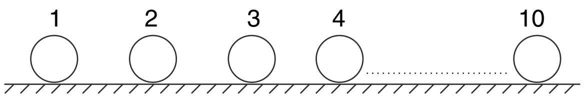


(a) Calculate the kinetic energy (k_3) of ball 3 after ball 2 hits it.

(b) Draw the variation of k_3 as a function of M.

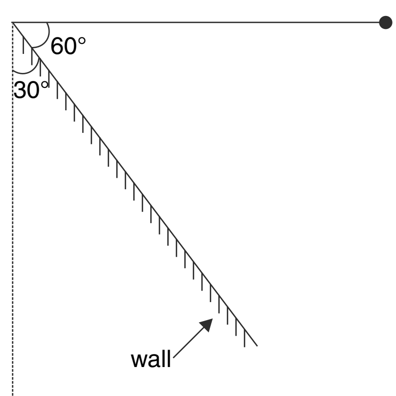
(ii) Consider 10 balls laid on a smooth surface with masses $m, \frac{m}{2}, \frac{m}{4}, \frac{m}{8} \ldots\ldots \frac{m}{512}$ and first ball is pushed towards the second with kinetic energy k_0. [All collisions are elastic and head on]. The kinetic energy acquired by the last ball is k_{10}. In a separate experiment the 10^{th} ball is pushed towards 9^{th} ball with kinetic energy k_0. This time the kinetic energy acquired by 1^{st} ball is k_1.

Compare k_{10} and k_1.

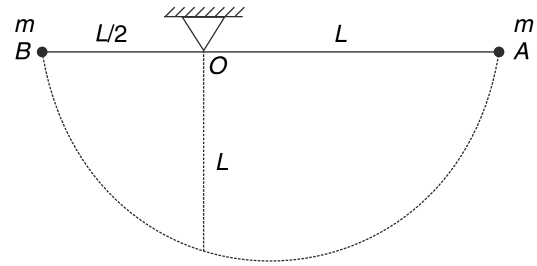


Q. 68. A simple pendulum is suspended from a peg on a wall which is inclined at an angle of 30° with the vertical. The pendulum is pulled away from the wall to a horizontal position (with string just taut) and released. The bob repeatedly bounces off the wall, the coefficient of restitution being $e = \dfrac{2}{\sqrt{5}}$.

Find the number of collisions of the bob with the wall, after which the amplitude of oscillation (measured from the wall) becomes less than 30°.



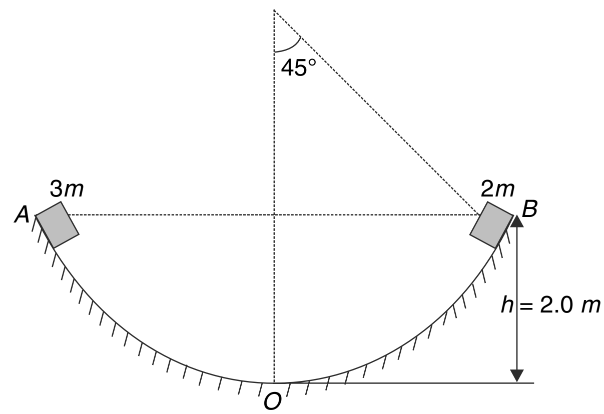
Q. 69. Two particles A and B, having same mass m are tied to a common point of suspension O. A is tied with the help of an inextensible string of length L and B is tied using an elastic string of unstretched length $\dfrac{L}{2}$. The two particles are released from horizontal positions as shown in figure. The particles have been released at a time gap so that both the string and the elastic cord become vertical simultaneously. It was observed that the length of the cord became equal to that of the string at this moment and the two particles collided. The particles got stuck together and their velocity just after the collision was observed to be $\dfrac{\sqrt{gL}}{2}$.



(a) In which direction will the combined mass move immediately after collision – right or left?

(b) Find tension in the string immediately after the collision.

Q. 70. A smooth track, fixed to the ground, is in the shape of a quarter of a circle. Two small blocks of mass $3m$ and $2m$ are released from the two edges A and B of the circular track. The masses slide down and collide at centre O of the track. Vertical height of A and B from O is $h = 2m$. Collision is elastic. Find the maximum height (above O) attained by the block of mass $2m$ after collision.

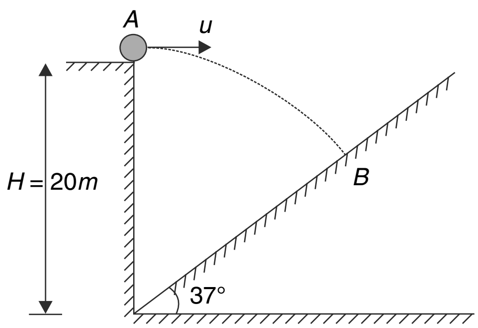


Q. 71. A man stands on a frictionless horizontal ground. He slides a 10 *kg* block on the surface with a speed of 3 *m/s* relative the ground, towards a vertical massive wall. The wall itself it moving towards the man at a constant speed of 2 *m/s*. The block makes a perfectly head on elastic collision with the wall, rebounds and reaches back to the man 3 second after the throw. At the moment the block was thrown, the wall was at a distance of 10 *m* from the man.

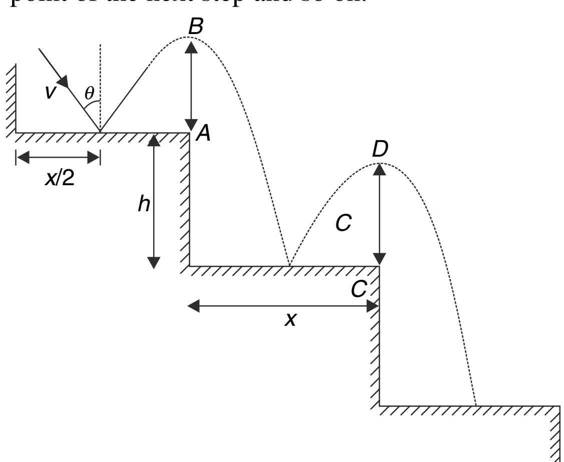
(a) Find the mass of the man.

(b) Find the ratio of work done by the man in throwing the block to the work done by the wall on the block.

Q. 72. A ball is projected from point A in horizontal direction with a velocity of $u = 28$ *m/s*. It hits the incline plane at point B and rebounds. Show that whatever be the coefficient of restitution between the ball and the incline, the ball will always hit the incline for the second time at a point above B (i.e., it will not hit the incline below B). Assume the incline to be smooth and take $g = 10\ m/s^2$ $\left[\sin 37° = \frac{3}{5}\right]$



Q. 73. A staircase has each step of height h and width x. A ball strikes the centre point of a step with velocity v making an angle θ with the vertical. It rebounds and strikes the centre point of the next step. Once again it rebounds and hits the centre point of the next step and so on.



Assume that there is no friction between the ball and the steps and coefficient of restitution is e.

(a) Show that each time after hitting a step, the ball climbs to the same height (i.e., heights like AB and CD shown in figure are equal).

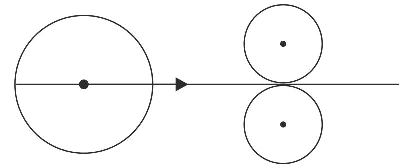
(b) Find h and x.

Q. 74. Two identical discs are initially at rest in contact on a horizontal table. A third disc of same mass but of double radius strikes them symmetrically and comes to rest after the impact.

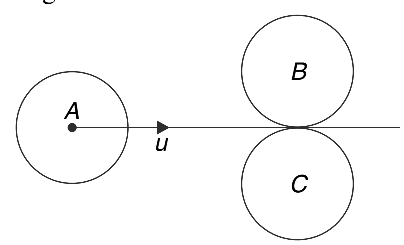
(a) Find the coefficient of restitution for the impact.

(b) Find the minimum kinetic energy of the system (as a percentage of original kinetic energy before collision) during the process of collision.

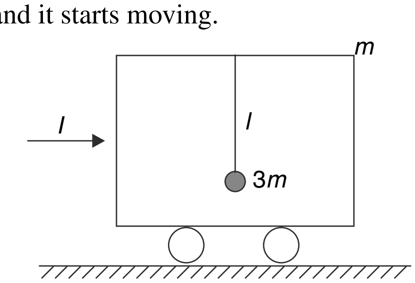
Treat the collision to be instantaneous.



Q. 75. On a billiard table two balls B and C are at rest touching each other. A third ball A, travelling with speed u, strikes the two balls elastically (see fig.). Somehow, A hits B first and within a fraction of a second hits ball C. You may assume that B and C are placed symmetrically with respect to the line of motion of A and that all the balls are identical. What angle does the final velocity of A make with its original direction of motion.



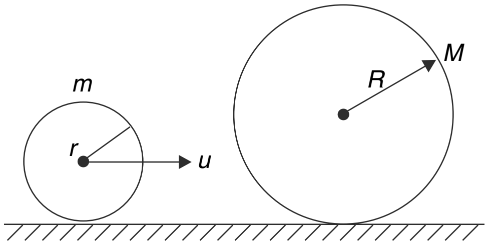
Q. 76. A toy car of mass m is placed on a smooth horizontal surface. A particle of mass $3m$ is suspended inside the car with the help of a string of length l. Initially everything is at rest. A sudden horizontal impulse $I = 2m\sqrt{gl}$ is applied on the car and it starts moving.



(a) Find the maximum angle θ_0 that the string will make with the vertical subsequently.

(b) Find tension in the string when it makes angle θ_0 with the vertical.

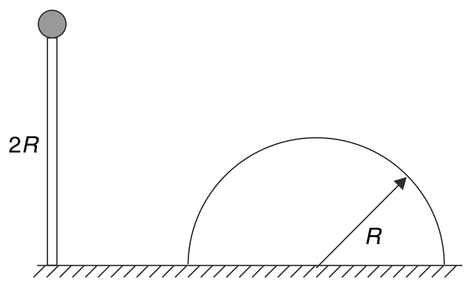
Q. 77. A smooth ball of mass M and radius R is lying on a smooth horizontal table. A smaller ball of radius r and mass m travelling horizontally on the table with velocity u hits the larger ball. Collision is elastic. During the interaction of the balls the larger ball does not lose contact with the table at any instant.



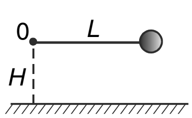
(i) Calculate the velocity of the balls after collision.

(ii) Calculate the maximum possible interaction force between the balls during collision.

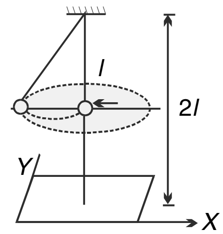
Q. 78. A light rigid rod has a small ball of mass m attached to its one end. The other end is hinged on a table and the rod can rotate freely in vertical plane. The rod is released from vertical position and while falling the ball at its end strikes a hemisphere of mass m lying freely on the table. The collision between the ball and the hemisphere is elastic. The radius of hemisphere and length of the rod are R and $2R$ respectively. Find the velocity of the hemisphere after collision.



Q. 79. (i) O is a fixed peg at a height H above a perfectly inelastic smooth horizontal plane. A light inextensible string of length L (> H) has one end attached to O and the other end is attached to a heavy particle. The particle is held at the level of O with string horizontal and just taut and released from rest. Find the height of the particle above the plane when it comes to rest for the first time after the release.



(ii) The bob of a pendulum has mass m and the length of pendulum is l. It is initially at rest with the string vertical and the point of suspension at a height $2l$ above the floor. A particle P of mass $\frac{m}{2}$ moving horizontally along –ve x-direction with velocity $\sqrt{2gl}$ collides with the bob and comes to rest. The bob swings and when it comes to rest for the first time, another particle Q of mass m moving horizontally along y direction collides with the bob and sticks to it. It is observed that the bob now moves in a horizontal circle.



(a) Find tension in string just before the second collision.

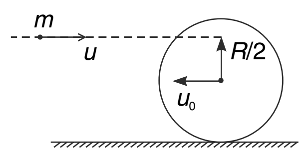
(b) Find the height of the circular path above the floor.

(c) Find the time period of the circular motion.

(d) The string breaks during the circular motion at time $t = 0$. At what time the bob will hit the floor?

Q. 80. A billiard ball collides elastically with an identical stationary ball. The collision is not head on. Show that the directions of motion of the two balls are at right angles after the collision. Solve the problem in centre of mass frame as well as in lab frame.

Q. 81.

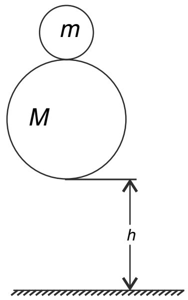


A heavy ball of radius R is travelling on a smooth horizontal surface with a velocity of u_0 towards left. A horizontally moving small ball of mass m strikes it at a height $\frac{R}{2}$ above the centre while travelling with velocity u towards right.

(a) After collision the small ball moves in vertically upwards direction with velocity u. Prove that this can happen only if $u > \sqrt{3}u_0$

(b) Find the velocity of small ball after collision if the collision is elastic and the balls are smooth.

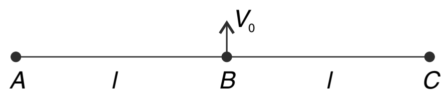
Q. 82. Two elastic balls of masses M and m ($M >> m$) are placed on top of each other with a small gap between them. The balls are dropped on to the ground with the bottom of the lower ball at height h above the ground. The lower ball has a radius R and the upper ball has negligible dimension.



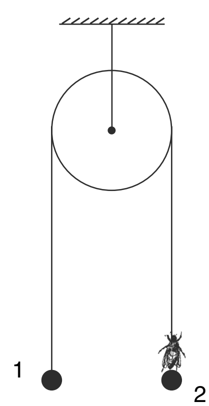
(a) Up to what height the ball of mass m will bounce above the ground ?

(b) Does the result obtained above violates the low of conservation of mechanical energy?

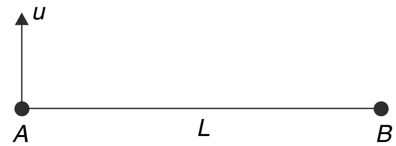
Q. 83. Three identical particles are placed on a horizontal smooth table, connected with strings as shown. The particle B is imparted a velocity $V_0 = 9\ m/s$ in horizontal direction perpendicular to the line ABC. Find speed of particle A when it is about to collide with C.



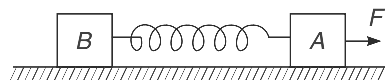
Q. 84. A light inextensible string, passing over a pulley, supports two particles 1 and 2 at its ends. An insect of mass m is sitting on particle 2 and the system is in equilibrium. The sum of masses of particles and the insect is M. Now the insect crawls a distance x up relative to the string. Find the displacement of centre of mass of the system of two particles and the insect. In which direction does the centre of mass move and why?



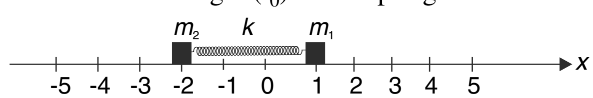
Q. 85. Two particles (A and B) of masses m and 2m are joined by a light rigid rod of length L. The system lies on a smooth horizontal table. The particle (A) of mass m is given a sharp impulse so that it acquires a velocity u perpendicular to the rod. Calculate maximum speed of particle B during subsequent motion. By what angle θ will the rod rotate by the time the speed of particle B become maximum for the first time?



Q. 86. Two blocks A and B, each of mass m, are connected by a spring of force constant K. Initially, the spring is in its natural length. A horizontal constant force F starts acting on block A at time t=0 and at time t , the extension in the spring is seen to be ℓ. What is the displacement of the block A in time t?



Q. 87. Two blocks of mass m_1 and m_2 are connected to the ends of a spring. The spring is held compressed and the system is placed on a smooth horizontal table. The block of mass $m_1 = 2\ kg$ is kept at x = 1 cm mark and the other block is at x = 2 cm mark. The system is released from this position. It was observed that at the instant m_1 was at x = 5 cm mark its velocity was zero and at that moment m_2 was located at x = –4 cm. Find mass m_2 and unstretched length (l_0) of the spring.



Q. 88. Two particles having masses m_1 and m_2 are moving with velocities $\vec{V}_1$ and $\vec{V}_2$ respectively. $\vec{V}_0$ is velocity of centre of mass of the system.

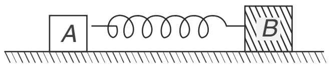
(a) Prove that the kinetic energy of the system in a reference frame attached to the centre of mass of the system is $KE_{cm} = \frac{1}{2}\mu V^2_{rel}$. Where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ and V_{rel} is the relative speed of the two particles.

(b) Prove that the kinetic energy of the system in ground frame is given by

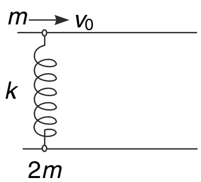
$$KE = KE_{cm} + \frac{1}{2}(m_1 + m_2)V_0^2$$

(c) If the two particles collide head on find the minimum kinetic energy that the system has during collision.

Q. 89. Two blocks A and B of mass m and 2m respectively are connected by a light spring of force constant k. They are placed on a smooth horizontal surface. Spring is stretched by a length x and then released. Find the relative velocity of the blocks when the spring comes to its natural length.



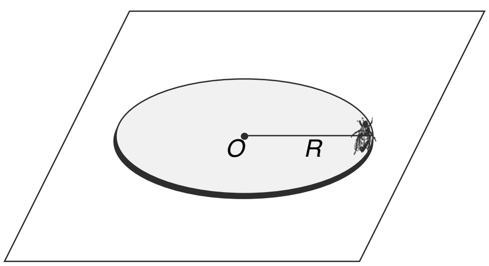
Q. 90. Two ring of mass m and $2m$ are connected with a light spring and can slide over two frictionless parallel horizontal rails as shown in figure. Ring of mass m is given velocity 'v_0' in horizontal direction as shown. Calculate the maximum stretch in spring during subsequent motion.



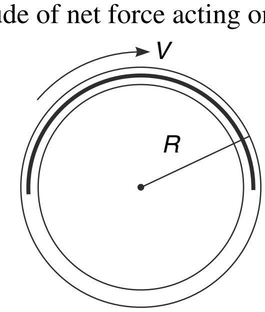
Q. 91. A disc of mass M and radius R is kept flat on a smooth horizontal table. An insect of mass m alights on the periphery of the disc and begins to crawl along the edge.

(a) Describe the path of the centre of the disc.

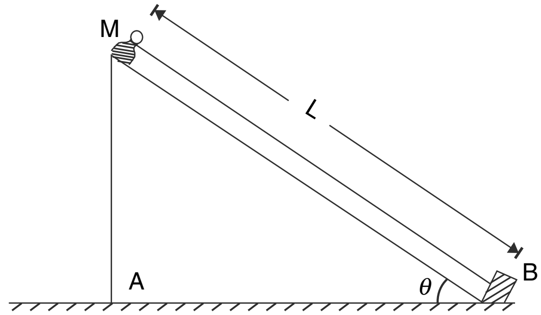
(b) For what value of $\frac{m}{M}$ the centre of the disc and the insect will follow the same path?



Q. 92. A metal wire having mass M is bent in the shape of a semicircle of radius R and is sliding inside a smooth circular grove of radius R present in a horizontal table. The wire just fits into the groove and is moving at a constant speed V. Find the magnitude of net force acting on the wire.

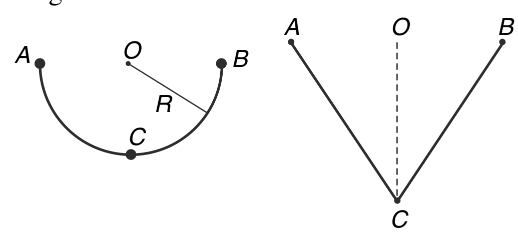


Q. 93. A triangular wedge (A) has inclined surface making an angle $\theta = 37°$ to the horizontal. A motor (M) is fixed at the top of the wedge. Mass of the wedge plus motor system is $3m$. A small block (B) of mass $m = 1$kg is placed at the bottom of the incline and is connected to the motor using a light string. The motor is switched on and it slowly hauls block B through a distance $L = 2.0$ meter along the incline. Calculate the work done by the string tension force on the wedge plus motor system. All surfaces are frictionless.



Q. 94. An ice cream cone of mass M has base radius R and height h. Assume its wall to be thin and uniform. When ice cream is filled inside it (so as to occupy the complete conical space) its mass becomes 5 M. Find the distance of the centre of mass of the ice cream filled cone from its vertex.

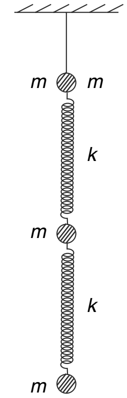
Q. 95. A flexible rope is in the shape of a semicircle ACB with its centre at O. Ends A and B are fixed. Radius of the semicircle is R. The midpoint C is pulled so that the rope acquires V shape as shown in the figure.



(a) Make a guess whether the centre of mass of the rope moves closer to O or moves away from it when it is pulled?

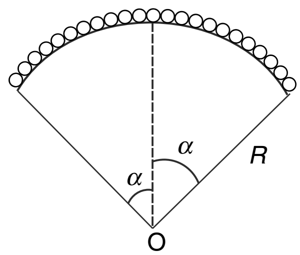
(b) Calculate the shift in position of the centre of mass of the rope.

Q. 96. Three small balls of equal mass (m) are suspended from a thread and two springs of same force constant (K) such that the distances between the first and the second ball and the second third ball are the same. Thus the centre of mass of the whole system coincides with the second ball. The thread supporting the upper ball is cut and system starts a free fall. Find the distance of the centre of mass of

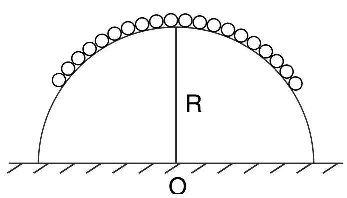


the system from the second ball when both the springs acquire their natural length in the falling system.

Q. 97. (a) A uniform chain is lying in form of on arc of a circle of radius R. The arc subtends an angle of 2α at the centre of the circle. Find the distance of the centre of mass of the chain from the centre of the circle.



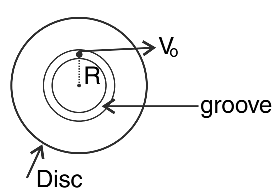
(b) A uniform chain of length $\frac{\pi R}{2}$ is lying symmetrically on the top of a fixed smooth half cylinder (see figure) of radius R. The chain is pulled slightly from one side and released. It begins to slide. Find the speed of the chain when its one end just touches the floor. What is speed of centre of mass of the chain at this instant?



(c) In part (b) assume that the half cylinder is not fixed and can slide on the smooth floor. Find the displacement of the cylinder by the time one end of the chain touches the floor. Mass of cylinder is equal to that of the chain.

For part (b) and (c) assume that the chain remains in contact with the cylinder all the while.

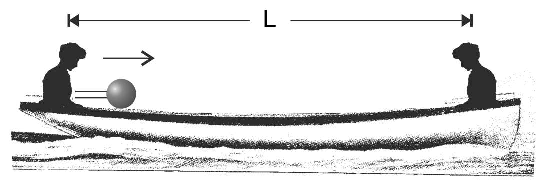
Q. 98. A small body of mass m is at rest inside a narrow groove carved in a disc. Groove is a circle of radius R concentric to the disc. Mass of the disc is also m . The disc lies on a smooth horizontal floor. The small body is given a sharp impulse so that it acquires a tangential velocity V_o at time $t = 0$.



(a) The velocity of the centre of the disc becomes zero for the first time at time t_0. Find t_0.

(b) Find speed of the small body at time $\frac{t_0}{3}$.

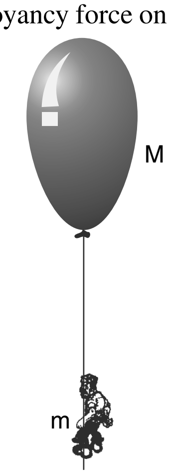
Q. 99. Laila and Majnu are on a boat for a picnic. The boat is initially at rest. Laila has a big watermelon which she throws towards Majnu. The man catches the melon and eats half of it. He throws back the remaining half to Laila. She eats the half of the melon that she receives & throws the remaining part to Majnu. Majnu again eats half of what he receives and returns the remaining part back to Laila. This continues till the melon lasts. The two are sitting at the two ends of the boat which has a length L. Combined mass of the boat and the two lovers is M_0 and the mass of the water melon is M. Assume that the boat can move horizontally on water without any resistive force. Find the displacement of the boat when the watermelon gets finished.



Q. 100. A hot air balloon (mass M) has a passenger (mass m) and is stationary in the mid air. The passenger climbs out and slides down a rope with constant velocity u relative to the balloon.

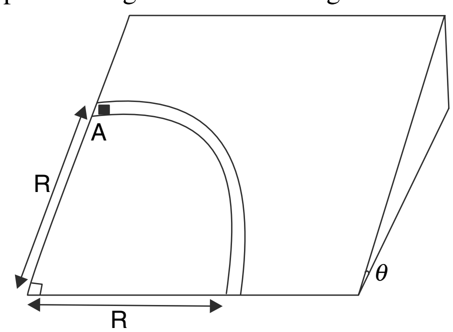
(a) Show that when the passenger is sliding down, there is no change in mechanical energy (kinetic + gravitational potential energy) of the system (Balloon + passenger). Calculate the speed of balloon.

(b) Calculate the power of the buoyancy force on the system when the man is sliding. For easy calculation, assume that volume of man is negligible compared to the balloon.



(c) If buoyancy force is doing positive work, where is this work done lost? You have proved that sum of kinetic and potential energy of the system remains constant.

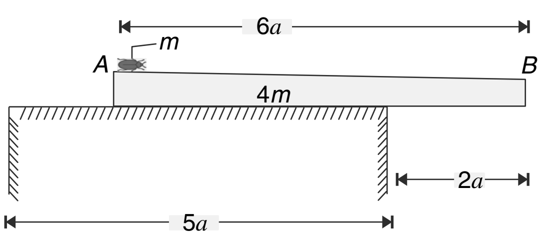
Q.101. A wooden wedge of mass 10 m has a smooth groove on its inclined surface. The groove is in shape of quarter of a circle of radius R = 0.55 m. The inclined face makes an angle $\theta = \cos^{-1}\left(\frac{\sqrt{11}}{5}\right)$ with the horizontal. A block 'A' of mass m is placed at the top of the groove and given a gentle push so as to slide along the groove. There is no friction between the wedge and the horizontal ground on which it has been placed. Neglect width of the groove.



(a) Find the magnitude of displacement of the wedge by the instant the block A reaches the bottom of the groove.

(b) Find the velocity of the wedge at the instant the block A reaches the bottom of the groove.

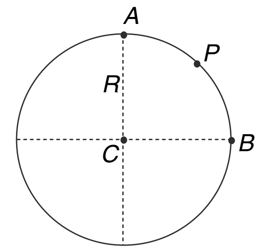
Q. 102. A uniform bar AB of length 6a has been placed on a horizontal smooth table of width 5 a as shown in the figure. Length 2a of the bar is overhanging. Mass of the bar is 4m. An insect of mass m is sitting at the end A of the bar. The insect walks along the length of the bar to reach its other end B.



(a) Will the bar topple when the insect reaches end B of the bar?

(b) After the insect reaches at B, another insect of mass M lands on the end A of the bar. Find the largest value of M which will not topple the bar.

Q. 103. A disc of mass M and radius R lies on a smooth horizontal table. Two men, each of mass $\frac{M}{2}$, are standing on the edges of two perpendicular radii at A and B.

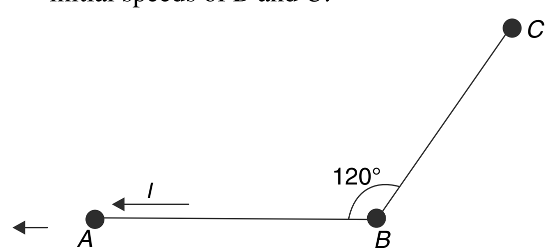


Find the displacement of the centre of the disc if

(a) The two men walk radically relative to the disc so as to meet at the centre C.

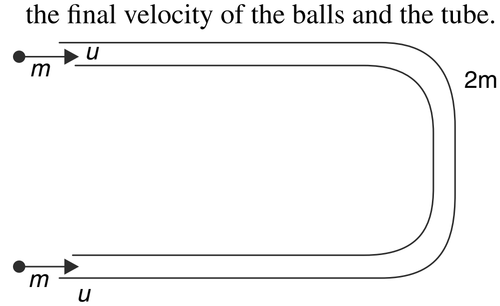
(b) The two men walk along the circumference to meet at the midpoint(P) of the are AB.

Q. 104. There particles A, B and C have masses m, 2m and m respectively. They lie on a smooth horizontal table connected by light inextensible strings AB and BC. The string are taut and <ABC = 120°. An impulse is applied to particle A along BA so that it acquires a velocity u. Find the initial speeds of B and C.

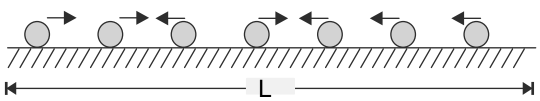


LEVEL 3

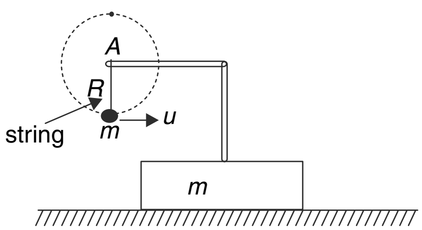
Q. 105. A smooth hollow U shaped tube of mass 2m is lying at rest on a smooth horizontal table. Two small balls of mass m, moving with velocity u enter the tube simultaneously in symmetrical fashion. Assume all collisions to be elastic. Find the final velocity of the balls and the tube.



Q. 106. There are 40 identical balls travelling along a straight line on a smooth horizontal table. All balls have equal speed v and each one is travelling to right or left. All collisions between the balls is head on elastic. At some point in time all balls will have fallen off the table. The time at which this happens will definitely depend on initial positions of the balls. Over all possible initial positions of the balls; what is the longest amount of time that you would need to wait to ensure that the table has no more balls? Assume that length of the table is L.



Q. 107. A small ball of mass m is suspended from the end A of a L shaped mass less rigid frame which is fixed to a block of mass m. The block is placed on a smooth table. The ball is given a horizontal impulse so as to impart it a velocity of u. The ball beings to rotate in a circle of radius R about the point A, while the block and the frame slide on the table. Find the tension in the string, to which the ball is attached, at the instant the ball is at the top most position. The rod does not interfere with the string during the motion.



Q. 108. A heavy rope of mass m and length $2L$ is hanged on a smooth little peg with equal lengths on two sides of the peg. Right part of the rope is pulled a little longer and released. The rope begins to slide under the action of gravity. There is a smooth cover on the peg (so that the rope passes through the narrow channel formed between the peg and the cover) to prevent the rope from whiplashing.

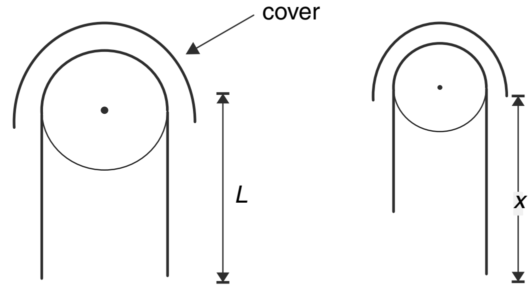
(a) Calculate the speed of the rope as a function of its length (x) on the right side.

(b) Differentiate the expression obtained in (a) to find the acceleration of the rope as a function of x.

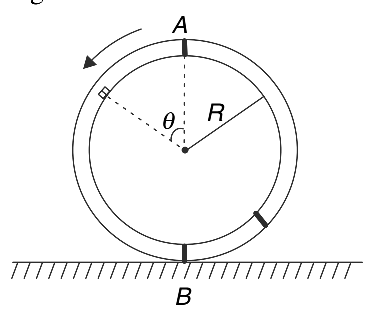
(c) Write the rate of change of momentum of the rope as a function of x. Take downward direction as positive

(d) Find the force applied by the rope on the peg as a function of x.

(e) For what value of x, the force found in (d) becomes zero? What will happen if there is no cover around the peg?



Q. 109. Two thin rings of slightly different radii are joined together to make a wheel (see figure) of radius R. There is a very small smooth gap between the two ring. The wheel has a mass M and its centre of mass is at its geometrical centre. The wheel stands on a smooth surface and a small particle of mass m lies at the top (A) in the gap between the rings. The system is released and the particle begins to slide down along the gap. Assume that the ring does not lose contact with the surface.

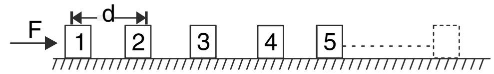


(a) As the particle slides down from top point A to the bottom point B, in which direction does the centre of the wheel move?

(b) Find the speed of centre of the wheel when the particle just reaches the bottom point B. How much force the particle is exerting on the wheel at this instant?

(c) Find the speed of the centre of the wheel at the moment the position vector of the particle with respect to the centre of the wheel makes an angle θ with the vertical. Do this calculation assuming that the particle is in contact with the inner ring at desired value of θ.

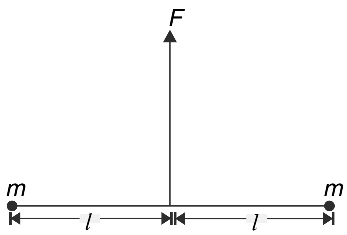
Q. 110. A large number of small identical blocks, each of mass m, are placed on a smooth horizontal surface with distance between two successive blocks being d. A constant force F is applied on the first block as shown in the figure.



(a) If the collisions are elastic, plot the variation of speed of block 1 with time.

(b) Assuming the collisions to be perfectly inelastic, find the speed of the moving blocks after n collisions. To what value does this speed tend to if n is very large.

Q. 111. Two small balls, each of mass m are placed on a smooth table, connected with a light string of length $2l$, as shown in the figure. The midpoint of the string is pulled along y direction by applying a constant force F. Find the relative speed of the two particles when they are about to collide. If the two masses collide and stick to each other, how much kinetic energy is lost.

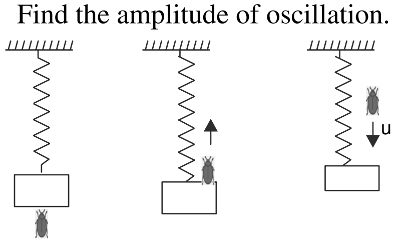


Q. 112. A block of mass M is tied to a spring of force constant K and the system is suspended vertically. Consider three situations shown in fig. (a), (b) and (c).

(a) In fig. (a), an insect of mass M is clinging to the block and the system is in equilibrium. The insect leaves the block and falls. Find the amplitude of resulting oscillations.

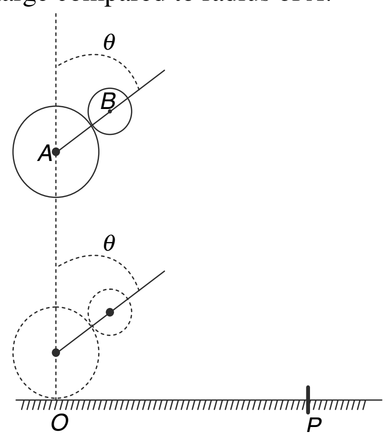
(b) In fig. (b), an insect of mass M is resting on the top of the block and the system is in equilibrium. The insect suddenly jumps up with a sped $u = g\sqrt{\dfrac{M}{K}}$ and the block starts oscillating. Find amplitude of oscillation assuming that the insect never falls back on the block.

(c) In fig. (c), an insect of mass M falls on the block that is in equilibrium. The insect hits the block with velocity $u = g\sqrt{\dfrac{M}{K}}$ while moving downwards and sticks to the block. Find the amplitude of oscillation.

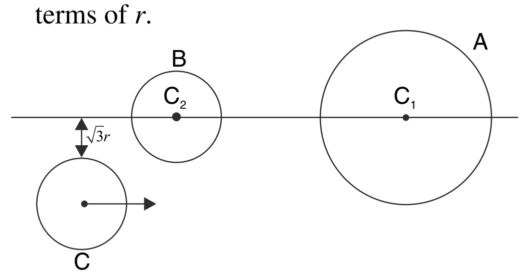


(a) (b) (c)

Q. 113. A massive ball (A) is dropped from height h on a smooth horizontal floor. A smaller ball (B) is also dropped simultaneously. Initially ball B was just touching ball A (see fig.). Radii of both balls is much smaller than h. Ball A hits the floor, rebounds and immediately hits B. Motion of both the balls is vertical before the collision of two balls. All collision are elastic and there is no friction. Ball B lands at point P on the ground after colliding with A. Find OP, assuming that it is large compared to radius of A.



Q. 114. Disc A of radius R is lying flat on a horizontal surface. Disc B is also at rest. Disc C, which is identical to B is traveling along the surface with its velocity parallel to the line joining the centre C_1 and C_2 of the discs A and B. The distance between the line C_1C_2 and the line of motion of centre of disc C is $\sqrt{3}.r$, where r is radius of both B and C. Impact of C with B is completely elastic. Subsequently it is observed that both B and C just miss hitting the disc A. Find the radius (R) of A in terms of r.



Q. 115. A mass m moving with speed u in x direction collides elastically with a stationary mass $2m$. After the collision, it was found that both masses have equal x components of velocity. What angle does the velocity of mass $2m$ make with the x axis?

Q. 116. A ball of mass M collides elastically with another stationary ball of mass m. If $M > m$, find the maximum angle of deflection of M.

Q. 117. A tennis ball is lying on a rigid floor. A steel ball is dropped on it from some height. The steel ball bounces vertically after hitting the ball on the floor. Is it possible that the tennis ball will also bounce?

ANSWERS

1. $\Delta p = 10\sqrt{29}$ Ns; $\theta = \cos^{-1}\left(\frac{3}{\sqrt{29}}\right)$

2. F_2

3. (a) 400 J

   (b) Both performed equal work.

4. $\vec{a}_{car} = -2\hat{i} + 2\hat{j}$

5, $\left(\frac{\sqrt{7}-\sqrt{3}}{4}\right)\rho A V^2$

6. $\mu_{\min} = \frac{mg}{\rho S (V+u)^2}$

7. mu^2

8. (i) $\frac{M}{m} = 1$

   (ii) 1/3

9. $\frac{u+V}{2}$

10. 3.24 ms^{-1}making an angle of 44° with the normal to the wall

11. A time $\frac{u}{g}$ after first collision

12. $\frac{M}{m} \le 2$

13. $V = \sqrt{\frac{K}{2M}}x_0$

14. (a) $KE_{\text{loss}} = \frac{mu^2}{3}$

    (b) No

15. (a) 3 m/s towards left

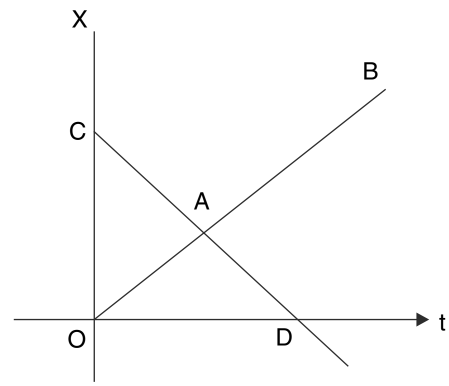
    (b) $K_{\min} = 2J$

    (c) $K_{\min} = 2J$

16. $\frac{m^2u^2}{2\mu g\left(M^2 - m^2\right)}$

17. $\frac{u}{2}$

18. (a)



$O - A - D \rightarrow$ *Ball* 1.

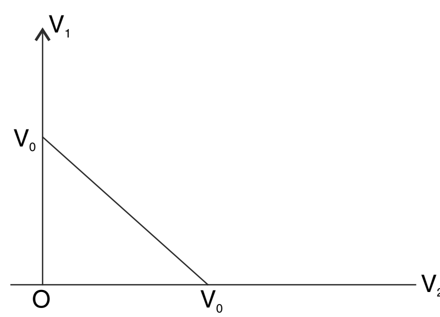
$C - A - B \rightarrow$ *Ball* 2.

(b) 10

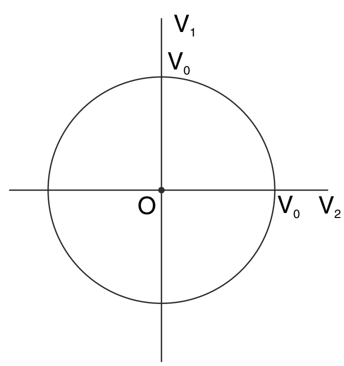
19. 2

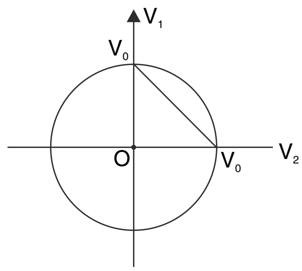
20. $\frac{u_1 - u_2}{V_2 - V_1}$

21. (a)



(b)



(c) 

$V_1 = 0, V_2 = V_0$

(d) $V_1 = 0.5$ *m/s* ; $V_2 = 1.5$ *m/s*; % loss in KE = 37.5%

22. (a) The heavier particle moves with a velocity of $\frac{2V_0}{21}$ in the direction of $\vec{V}_0$.

Lighter particle moves with velocity $\frac{19V_0}{21}$ opposite to $\vec{V}_0$.

The incident particle comes to rest. The other particle moves with $\vec{V}_0$.

23. 0.1*m*

24. less than *H*

25. 1/3

26. Slightly less than 6 *m/s*

27. 13 *m*

28. $\frac{n(n+1)}{2}$

29. 100 *m*, 92.8%

30. (a) $e = 0$ (b) $\frac{1}{3}$

32. 12

33. (a) *A* will be at rest and *B* will have a velocity *u*

(b) $\frac{mu^2}{4}$

(c) Both will be travelling with velocity $\frac{u}{2}$. Loss in $KE = \frac{mu^2}{4}$

34. $\frac{L}{260}$

35. $\frac{20}{3}kg$

36. (i) $\frac{r}{2015}$

(ii) A particle next to the blank vertex.

37. (a) $\frac{1}{2}\sqrt{\frac{k}{m}}x_0$

(b) $\frac{\sqrt{3}kx_0}{4m}$

38. 0.2 *m*

39. (a) 4 *m/s*

(b) 1560*N*

40. (To right)

41. 40.56 *m*

42. $t = \sqrt{\frac{4ML}{3F}}$

43. No

44. 7.5*R*

45. (a) 10 *m/s*

(b) 100 *m*

46. $v = \frac{u}{1 + \frac{M}{2bt}}$

47. (i) $T_{s1} = T_{s2} = T_{s3} = \rho A v^2$

(ii) $\frac{\pi M V_0}{mu}$

48. (a) 5 *s*

(b) 25 ms^{-1}

49. $F \simeq 1200\ N$

50. (a) $\frac{g}{2}$

(b) $g - \frac{v_0^2}{l_0}$

51. (a) Smaller bullet

(b) $\frac{Mu}{M + \rho Aut}$

(c) $\frac{M}{\rho A}\ell n\left[1 + \frac{\rho Au}{M}t\right]$

52. (a) $\sqrt{\frac{8}{5}\frac{L}{g}}$

(b) $\frac{mgL}{5}$

53. (a) $\frac{2mg}{3}$

(b) $2\sqrt{gL}$

54. $\Delta t = \frac{1}{12}s$

55. (a) $d = 90\,L$ $u = \sqrt{90gL}$

(b) $m\sqrt{45gL}$

56. $V_A = \frac{36}{7} m/s$; $V_B = \frac{30}{7} m/s$; $V_C = \frac{33}{7} m/s$

57. (a) $\frac{L}{v}$

(b) $\frac{3}{4}\ mv^2$

58. 40 *m/s*

60. (i) $\frac{L}{5}$

(ii) $5\ m/s^{-1}$

61. $\frac{2}{3}\left[L + \mu\sqrt{\frac{2H}{g}}\right]$

62. 5 *m/s*

63. (a) $\frac{5}{3} m$

(b) $\frac{96}{9} = 10.67 m$

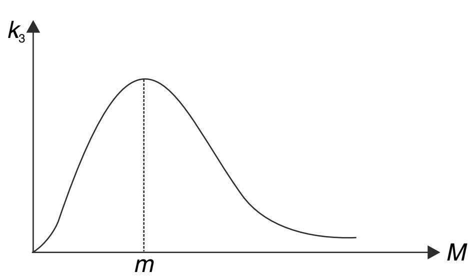
65. $T_P = \lambda v^2$; $F_{ext} = \lambda v^2$

66. (i) $v = \frac{Mv_0}{M + \lambda x}$

(ii) $T = \frac{M^3 {v^2}_0 \lambda}{(M + \lambda x)^3}$

67. (i) (a) $16k_0 \frac{m^2 M^2}{(M+m)^4}$

(b)



(ii) $k_{10} = k_1$

68. 4

69. (a) Left

(b) Zero

70. 2.96 *m*

71. (a) 90 *kg*

(b) $\frac{1}{4}$

73. (b) $h = \frac{v^2 \cos^2\theta\left(1 - e^2\right)}{2g}$

$x = (e+1)\frac{v^2}{2g}\sin 2\theta$

74. (a) $\frac{9}{16}$

(b) 36 %

75. 120°

76. (a) $\theta_0 = 60°$

(b) $T = \frac{2mg}{\sqrt{3}}$

77. (i) $v_m = \left(\frac{m - M}{M + m}\right)u; v_M = \left(\frac{2mu}{M + m}\right)$

(ii) $Mg\left(\frac{R + r}{R - r}\right)$

78. $\frac{4\sin\theta\sqrt{gR(1 - \sin\theta)}}{1 + \sin^2\theta}$ where $\sin\theta = \frac{1}{\sqrt{5}}$

79. (i) $\frac{H^5}{L^4}$

(ii) (a) $\frac{3}{4} mg$

(b) $\frac{5l}{4}$

(c) $2\pi\sqrt{\frac{3l}{4g}}$

(d) $\sqrt{\frac{5l}{2g}}$

81. (b) $\sqrt{u^2 + 3u_0^2 + 3uu_0}$

82. (a) $2R + 9h$

83. 6 *m/s*

84. $\frac{mx}{M}$

85. $\frac{2u}{3}$, 180°

86. $\frac{Ft^2}{4m} + \frac{l}{2}$

87. $m_2 = 4\ kg$; $l_0 = 6\ cm$

88. (c) $\frac{1}{2}(m_1 + m_2)V_0^2$

89. $v_r = \sqrt{\frac{3k}{2m}}x$

90. $\sqrt{\frac{2m}{3k}}\ v_0$

91. (a) A circle of radius $\frac{mR}{M+m}$

(b) $\frac{m}{M} = 1$

92. $\frac{2}{\pi} = \frac{MV^2}{R}$

93. 1.92 J

94. $\frac{11h}{15}$

95. (a) Closer to O

(b) 0.03 R

96. $\frac{mg}{3K}$

97. (a) $\frac{R\sin\alpha}{\alpha}$

(b) $V = 2\sqrt{\frac{(\sqrt{2}-1)\text{gR}}{\pi}}$; $V_{cm} = \frac{4\sqrt{2}}{\pi}\sqrt{\frac{(\sqrt{2}-1)\text{gR}}{\pi}}$

(c) $\frac{R}{\pi}$

98. (a) $t_0 = \frac{2\pi R}{V_0}$

(b) $\frac{V_0}{2}$

99. $\frac{2ML}{3(M_0 + M)}$

100. (a) $\frac{mu}{M+m}$

(b) $mg.\ u$

101. (a) 6 cm

(b) 0.18 m/s

102. (a) No

(b) 85 m

103. (a) $\frac{R}{2\sqrt{2}}$

(b) $\frac{(\sqrt{2}-1)R}{2\sqrt{2}}$

104. $v_B = \frac{2\sqrt{31}}{11}u,\ v_c = \frac{4u}{11}$

105. $v_{ball} = 0$; $v_{tube} = u$

106. $\frac{L}{v}$

107. $T = \frac{mu^2}{R} - 13\ mg$

108. (a) $v = \sqrt{\frac{g}{L}}(x-L)$

(b) $a = \frac{g}{L}(x-L)$

(c) $\frac{dp}{dt} = 2mg\left(\frac{x-L}{L}\right)^2$

(d) $F = mg\left[1 - 2\left(\frac{x-L}{L}\right)^2\right]$

(e) $x = L + \frac{L}{\sqrt{2}}$

109. (a) First moves to right and then to left

(b) $v_w = 2m\sqrt{\frac{gR}{M(M+m)}}$

(c) $v_w = \sqrt{\frac{2m^2 gR\cos^2\theta(1-\cos\theta)}{(M+m)^2 + Mm\text{co}^2\theta}}$

110. (a) See the solution for the graph

(b) $\sqrt{\frac{n}{n+1}\frac{Fd}{m}};\sqrt{\frac{Fd}{m}}$

111. $2\sqrt{\frac{Fl}{m}};Fl$

112. (a) $\frac{Mg}{K}$ (b) $\sqrt{2}\frac{Mg}{K}$

(c) $\frac{\sqrt{6}}{2}\frac{Mg}{K}$

113. $16h\ \sin 2\theta\left[\frac{1}{2} + \cos 2\theta\right]$

114. $R = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)r$

115. 45°

116. $\sin^{-1}(m/M)$

117. Yes.

SOLUTIONS

1.

$$\Delta\vec{P} = \int \vec{F}dt = \hat{i}\int F_x dt + \hat{j}\int F_y dt + \hat{k}\int F_z dt$$

$$= \hat{i}(30\times 1) + \hat{j}\left(\frac{1}{2}\times 40\times 1\right) + \hat{k}\left(\frac{1}{2}\times 80\times 1\right)$$

$$= 30\hat{i} + 20\hat{j} + 40\hat{k}$$

[Area under $F_y - t$ graph gives $\int F_y dt$]

$\therefore\ |\Delta p| = \sqrt{30^2+20^2+40^2} = 10\sqrt{29}\ \ N-s$

If angle made by $\Delta\vec{P}$ with x axis is θ

Then , $\cos\theta = \dfrac{30}{10\sqrt{29}} = \dfrac{3}{\sqrt{29}}$

2. Rate of change of momentum = Net external force.

External forces are (F_1 or F_2) and the force applied by the support on the string.

When F_1 is applied, the support exerts a force (T_1) on the string in the direction of F_1

$\therefore\ R = F_1 + T_1$

When F_2 is applied

$R = F_2 - T_2$

[T_2 = force by the support on the string]

$\therefore\ F_2 > F_1$

3. (a) $\Delta\ \vec{p}$ = Impulse

$m\vec{V} - m\vec{u} = 4\times 5\hat{j} + 5\times 4\hat{k}$

$1\times\vec{V} - 1\times 10\hat{i} = 20\hat{j} + 20\hat{k}$

$\vec{V} = 10\hat{i} + 20\hat{j} + 20\hat{k}$

$\therefore V = \sqrt{10^2+20^2+20^2} = 30\ \text{ms}^{-1}$

Work energy theorem gives–

$$W = \frac{1}{2}mV^2 - \frac{1}{2}mu^2 = \frac{1}{2}\times 1\times\left(30^2 - 10^2\right) = 400\ J$$

(b) Let the velocity be $\vec{V}_1$ after the first force stops acting.

$m\vec{V}_1 - m\vec{u} = 20\hat{j}$

$\vec{V}_1 = 10\hat{i} + 20\hat{j}$

$V_1 = 10\sqrt{5}\text{ms}^{-1}$

$\therefore W_1 = \frac{1}{2}mV_1^2 - \frac{1}{2}mu^2$

$= \frac{1}{2}\times 1\times(500-100) = 200J$

$\therefore W_1 = W_2 = 200J$

4. For O_1 force on the projectile is

$$\vec{F}_{01}=\frac{\vec{d}p_{01}}{dt}=\left(4\hat{i}+20\hat{k}\right)N$$

This is real force on the projectile due to gravity and blowing wind.

For O_2 force on the projectile is –

$$\vec{F}_{02}=\frac{\vec{d}p_{02}}{dt}=\left(8\hat{i}-32t\,\hat{j}+20\hat{k}\right)N$$

$$\vec{F}_{real}+\vec{F}_{pseudo}=8\hat{i}-32t\,\hat{j}+20\hat{k}$$

$$\left(4\hat{i}+20\hat{k}\right)+\vec{F}_{pseudo}=8\hat{i}-32t\,\hat{j}+20\hat{k}$$

$$\therefore \vec{F}_{pseudo}=4\hat{i}-32t\,\hat{j}$$

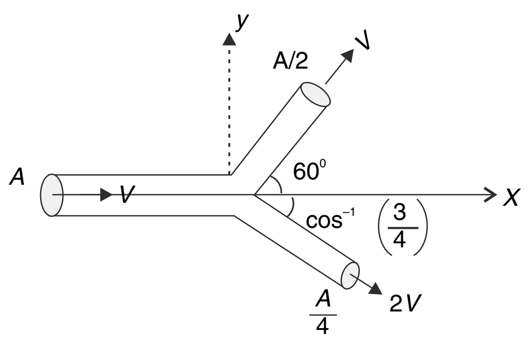
$$\text{At}=\frac{1}{8}s$$

$$\vec{F}_{pseudo}=4\hat{i}-4\hat{j}$$

$$\therefore -m\vec{a}_{car}=4\hat{i}-4\hat{j}$$

$$\vec{a}_{car}=-2\hat{i}+2\hat{j}$$

5.



Let mass of water that gets divided into two streams in time interval Δt be Δm.

$\Delta m = AV\rho\,\Delta t$

Change in momentum of water in X and Y direction is

$$\Delta P_x=\frac{A}{2}V\Delta t.\rho.V.\cos 60°+\frac{A}{4}2V\Delta t\;\rho.2V\left(\frac{3}{4}\right)-AV\Delta t.V=O$$

$$\Delta P_y=\frac{A}{2}V\Delta t\rho V\sin 60°-\frac{A}{4}2V.\rho.\Delta t2V\cdot\left(\frac{\sqrt{7}}{4}\right)$$

$$=-\left(\frac{\sqrt{7}-\sqrt{3}}{4}\right)\rho AV^2\Delta t$$

$\therefore$ Force on water is along negative y direction

$$Fy=\frac{\Delta P_y}{\Delta t}=-\left(\frac{\sqrt{7}-\sqrt{3}}{4}\right)\rho AV^2$$

$\therefore$ Force on tube, $F_y=\left(\dfrac{\sqrt{7}-\sqrt{3}}{4}\right)\rho AV^2$

6. Volume of air striking the paper in unit time is $= S\,(V + u)$

Mass of air striking the paper in unit time = $\rho S\,(V + u)$

In reference frame of man, the air molecules strike at a speed $(V + u)$ and come to rest.

$\therefore$ Rate of charge of momentum for air particles = $\rho S\,(V + u)\,(V + u)$

$\therefore$ Force on paper due to air = $\rho S\,(V + u)^2$

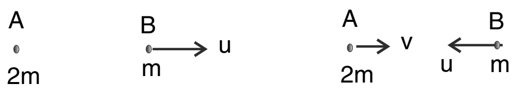
For the paper to not fall, friction on it must balance its weight

$$f_{\max} \geq mg$$

$$\mu\,\rho S\,(V + u)^2 \geq mg$$

$$\mu \geq \frac{mg}{\rho S(V+u)^2}$$

7. Momentum conservation gives $2\,mv - mu = mu \;\Rightarrow v = u$



$$W = K_f - K_i = \left[\frac{1}{2}(2m)u^2 + \frac{1}{2}mu^2\right] - \frac{1}{2}mu^2 = mu^2$$

8. (i) The compression in the spring will be maximum when maximum KE is transferred to M. This happens when $M = m$

(ii) $K_B = 4.K_A \Rightarrow V_B = 2V_A$ (after collision)

$$mu = mV_A + mV_B$$

$$\Rightarrow u = 3V_A \Rightarrow V_A = \frac{u}{3} \text{ and } V_B = \frac{2u}{3}$$

$$\therefore e = -\left(\frac{\frac{u}{3} - \frac{2u}{3}}{u - 0}\right) = \frac{1}{3}$$

9. The not shaded area in the graph = Impulse during the period of deformation

Shaded area = impulse during the period of restitution

$\therefore$ $(Impulse)_{\text{deformation}} = (Impulse)_{\text{Restitution}}$

$m\,(V_0 - u) = m\,(V - V_0)$ [V_0 = Common velocity when the balls have maximum deformation]

$$V_0 - u = V - V_0$$

$$\therefore V_0 = \frac{u+V}{2}$$

10. For collision of A and B

$V_B - V_A = e_1\,(u_A - u_B)$

$V_B - V_A = 0.8 \times 5 = 4$ ....(1)

and $mV_B + mV_A = mu$

$V_B + V_A = 5$ ....(2)

Solving (1) and (2) we get

$$V_B = 4.5\ ms^{-1}\ ;\ V_A = 0.5\ ms^{-1}$$

Now B collides with the wall.

Velocity component parallel to the wall remains unchanged.

$V_{11} = V_B \sin 30° = 2.25\ m/s$

Velocity component perpendicular to the wall becomes

$$V_{\perp} = e_2 V_B \cos 30° = 0.6 \times 4.5 \times \frac{\sqrt{3}}{2} = 2.34\ m/s$$

If final velocity makes an angle α with normal to the wall

$$\tan\alpha = \frac{2.25}{2.34} \Rightarrow \alpha \simeq 44° \text{ and } V = \sqrt{2.25^2 + 2.34^2} = 3.24 m/s$$

11. Applying momentum conservation in horizontal direction, it is easy to see that the two balls will have horizontal component of velocity $V_x = \frac{u}{2}$

[Collision between the balls is perfectly inelastic].

Due to collision with the floor, ball A acquires a vertical velocity = 0.5 u

Both balls are traveling in horizontal direction with same velocity after collision.

Hence, they collide next where A falls back on to the floor.

$$t = \frac{2(0.5u)}{g} = \frac{u}{g}$$

12. KE of the system is minimum when compression in the spring is maximum. This happens when both the blocks have same velocity (Say V). If initial velocity of mass m is u then–

$$(M+m)V = mu \Rightarrow V = \frac{mu}{M+m}$$

According to the problem

$$\frac{1}{2}(M+m)\left(\frac{mu}{M+m}\right)^2 \ge \frac{1}{3}\cdot\frac{1}{2}mu^2$$

$$\Rightarrow \frac{m}{M+m} \ge \frac{1}{3} \Rightarrow \frac{M}{m} \le 2$$

13. Initial momentum = 0

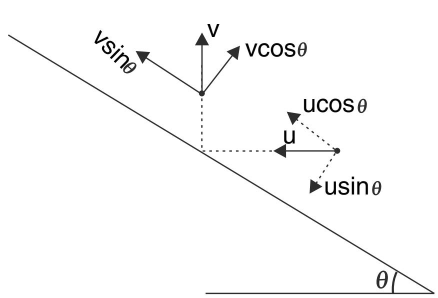
$\therefore$ Final momentum = 0

Hence, A and B will have equal and opposite velocity (C will have negligible momentum due to its small mass. Its KE is also negligible)

$$\therefore \frac{1}{2}MV^2 + \frac{1}{2}MV^2 = \frac{1}{2}KX_0^2$$

$$\therefore V = \left(\sqrt{\frac{K}{2M}}\right)X_0$$

14. (a) Since the incline surface is smooth, the velocity component of the particle parallel to the incline will not change.



$\therefore\ V\sin\theta = u\cos\theta$ ....(1)

For $\theta = 60°$, $V = \dfrac{u}{\sqrt{3}}$

Loss in $KE = \dfrac{1}{2}mu^2 - \dfrac{1}{2}m\left(\dfrac{u}{\sqrt{3}}\right)^2 = \dfrac{mu^2}{3}$

(b) *From (1)* $\tan\theta = \dfrac{u}{V}$

Since, $u > V$ $\therefore$ $\tan\theta > 1$ $\therefore \theta < 45°$

Therefore, particle cannot go vertically up if $\theta < 45°$.

15. The collision process is quite similar to collision between two elastic bodies except for one difference that in presence of the spring the time duration of the collision gets prolonged.

(a) In head on elastic collision of two equal masses exchange of velocities takes place. Hence, final velocity of the block A will be 3 *m/s* towards left.

(b) KE will be least when the spring is compressed the most. In this position both the blocks will have same velocity (say V).

$mV + mV = 5\,m - 3\,m \Rightarrow V = 1\ m/s$

$\therefore K_{\min} = \dfrac{1}{2}\times 2\times 1^2 + \dfrac{1}{2}\times 2\times 1^2 = 2J$

(c) Answer will be same as in (b) due to the reason given above.

16. Speed immediately after collision

$(M + m)\,V = mu$

$V = \dfrac{mu}{M+m}$

Retarding force on $(M + m)$ is $\mu Mg - \mu mg$ after collision

$\therefore$ Retardation $a = \dfrac{\mu g(M-m)}{(M+m)}$

Displacement (s) before stopping can be calculates as $2\,as = V^2$

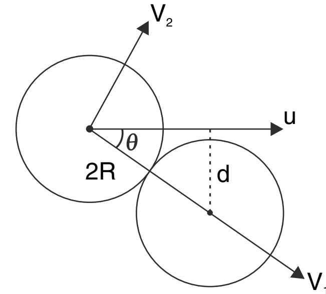
$2.\dfrac{\mu(M-m)g}{(M+m)}.s = \dfrac{m^2u^2}{(M+m)^2}$

$s = \dfrac{m^2u^2}{2\mu g\left(M^2 - m^2\right)}$

17. B will move along the line of impact.

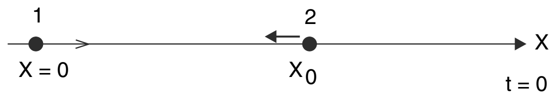
$\sin\theta = \dfrac{\sqrt{3}}{2}$

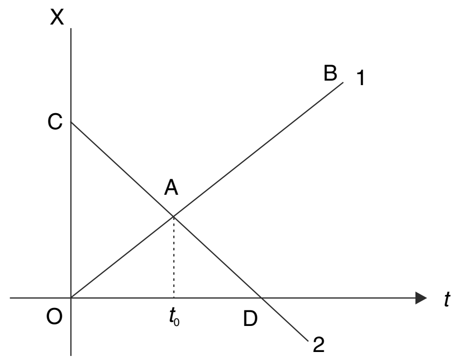
According to the problem ball A moves perpendicular to its original direction of motion. For momentum to remain conserved along the line of impact the other ball must move with velocity



$u\cos\theta = \dfrac{u}{2}$

18. (a) Let the position of two particles at time $t = 0$ be as shown in figure.

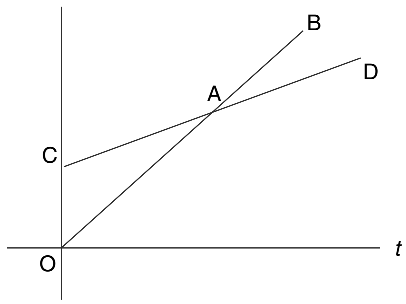


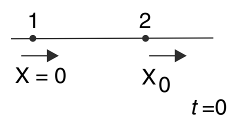


Collision occurs at $t = t_0$

$X - t$ graph for ball 1 is $O - A - D$ and for ball 2 it is $C - A - B$

The graph may appear slightly different depending on initial position & direction of motion. The figure below shows another situation.



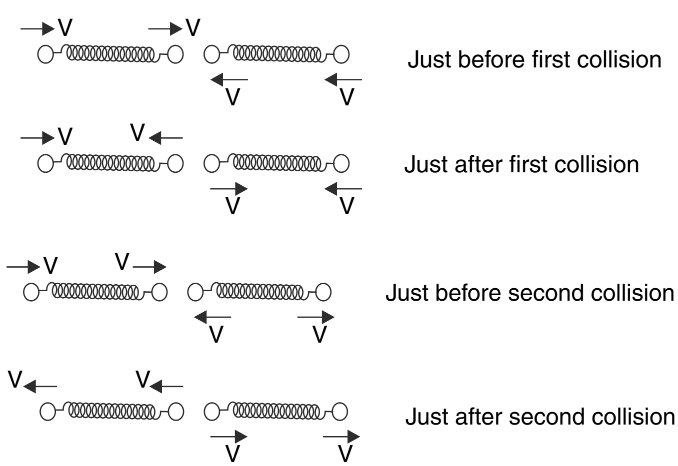


Ball 1 $\rightarrow$ OAD, Ball 2 $\rightarrow$ CAB

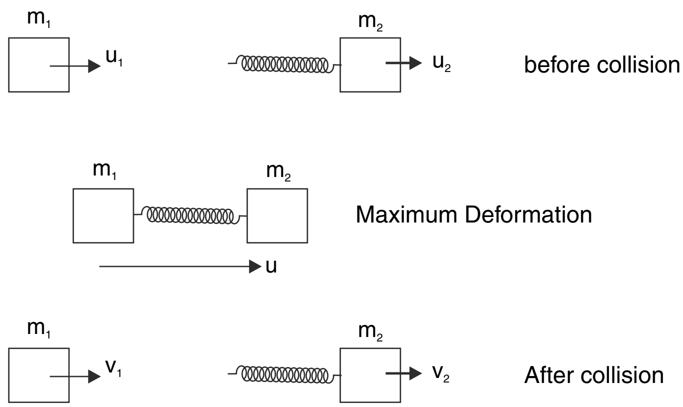
(b) Any intersection on $X - t$ graph represents a collision. In elastic collision, the $X - t$ graph gets exchanged [as in (a)]. Therefore, the lines ($X - t$ graph) representing the motion of 5 balls will not change even after a collision takes place.

5 lines can intersect at maximum number of points given by $^5C_2 = 10$

19. There will be two collisions. The sequence of events has been shown below.



20.



Impulse on m_2 during deformation phase is given by-

$J_D = m_2\, u - m_2\, u_2$

Where u = Common velocity in maximum deformation state

Impulse on m_2 during restoration phase is given by-

$$J_R = m_2 V_2 - m_2 u$$

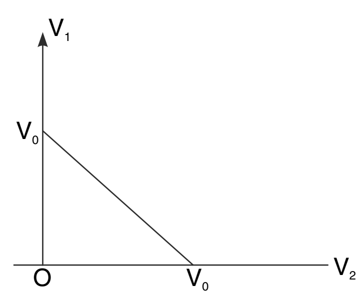
$$\therefore \frac{J_D}{J_R} = \frac{u - u_2}{V_2 - u}$$

But $u = \dfrac{m_1 u_1 + m_2 u_2}{m_1 + m_2} = \dfrac{m_1 V_1 + m_2 V_2}{m_1 + m_2}$

$$\therefore \frac{J_D}{J_R} = \frac{\dfrac{m_1 u_1 + m_2 u_2}{m_1 + m_2} - u_2}{V_2 - \dfrac{m_1 V_1 + m_2 V_2}{m_1 + m_2}} = \frac{u_1 - u_2}{V_2 - V_1}$$

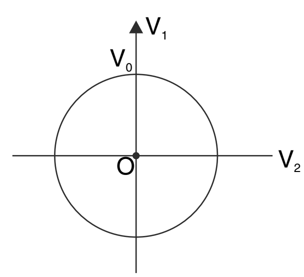
21. (a) $mV_0 = mV_1 + mV_2$

$V_1 + V_2 = V_0$ ………………..(1)

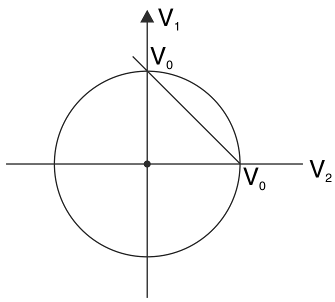


(b) $\frac{1}{2} m V_1^2 + \frac{1}{2} m V_2^2 = \frac{1}{2} m V_0^2$

$V_1^2 + V_2^2 = V_0^2$



(c)



Two solution are $V_1 = 0$, $V_2 = V_0$

And $V_1 = V_0$; $V_2 = 0$ [This is just not possible]

(d) $V_1 + V_2 = 2$ ....(i)

$$\frac{1}{2}mV_1^2 + \frac{1}{2}mV_2^2 = \eta\frac{1}{2}mV_0^2 \qquad [\eta < 1]$$

$$V_1^2 + V_2^2 = \eta V_0^2$$

Radius of circle = $\sqrt{\eta}V_0$

$$\sqrt{0.5^2 + 1.5^2} = \sqrt{\eta}.2$$

$$\eta = 0.625$$

Percentage loss in $KE = (1 - 0.625) \times 100 = 37.5\%$

The two solutions are –

$V_1 = 1.5$ *m/s*; $V_2 = 0.5$ *m/s* (This is not possible)

$V_1 = 0.5$ *m/s*; $V_2 = 1.5$ *m/s*

22 The problem can be solved as a usual elastic collision problem. The interaction time of the two particles is prolonged but eventually the kinetic energy will be equal to initial *KE*.

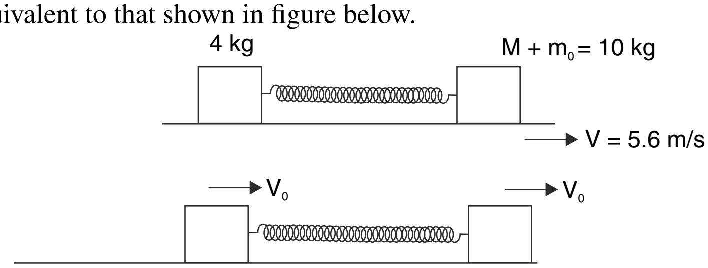
23. After collision velocity of $(M + m_0)$ system is given by momentum conservation.

$(M + m_0)\,V = m_0\,u$

$10.\ V = 0.14 \times 400$

$V = 5.6$ m/s

Now the system is equivalent to that shown in figure below.



Extension is maximum when both blocks have same velocity

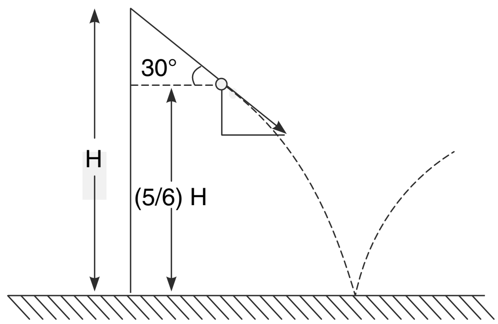
$(4 + 10)\,V_0 = 10 \times 5.6 \Rightarrow V_0 = 4$ m/s

Energy conservation

$$\frac{1}{2}Kx^2 + \frac{1}{2}(14)V_0^2 = \frac{1}{2} \times 10 \times 5.6^2$$

Solving $x = 0.1$ *m*

24. Once the ball starts moving, its horizontal component of velocity never becomes zero. The impact does not affect the horizontal velocity. Therefore, kinetic energy of the ball never becomes zero after the impact. It will never reach height H.



25. Since there is no friction, components of velocity before and after collision parallel to the plane are equal, So

v sin 60° = u sin30°............. (1)

Components of velocity normal to the plane, after and before collision, are related to each other as

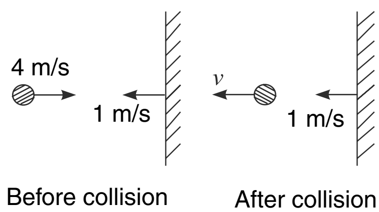
v cos 60° = e u (cos30°).........(2)

Taking ratio of (2) and (1)

e cot 30° = cot60°

$$e = \frac{1}{3}$$

26.



If the mass of the wall is infinite its velocity will not change. If v be the velocity of ball after collision then

Relative velocity of separation = relative velocity of approach

$\therefore\ v - 1 = 4 + 1$

or v = 6 *m/s* (away from the wall)

If mass of the wall is huge but finite, its speed will decrease a little bit and ball will have its speed reduced (compared to 6 *m/s*) by same amount.

27. Speed just before collision is given by

$v^2 = (20\sqrt{2})^2 - 2 \times 10 \times 4 = 720$

$v^2 = \sqrt{720}$ *m/s*

After inelastic collision the particle will have a velocity along the incline surface and its velocity perpendicular to the incline will be lost.

Velocity along the incline = $\sqrt{720} \times \frac{1}{\sqrt{2}}\ m/s = 6\sqrt{10}\ m/s$

Horizontal and vertical component of this velocity is

$v_x = 6\sqrt{5}$ *m/s*

$v_y = 6\sqrt{5}$ *m/s*

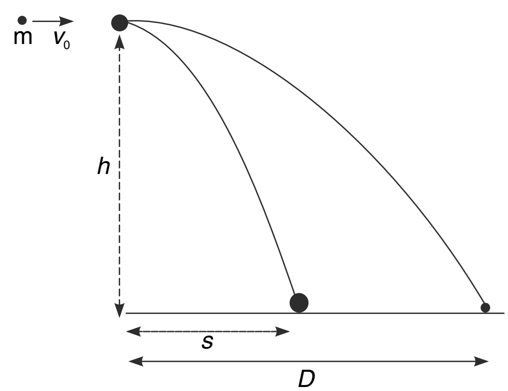
Further height attained after collision will be given by –

$$h = \frac{v_y^2}{2g} = \frac{\left(6\sqrt{5}\right)^2}{20} = 9\,m$$

Height attained from ground = 9 + 4 = 13 m

28. Initially there will be n collisions. Exchange of velocity takes place between two colliding blocks. The two extreme blocks will move out and never encounter any other collision. Remaining $(2n–2)$ blocks will further have $(n–1)$ collisions and so on. This way total no of collisions = $n + (n-1) + (n-2) + (n-3) + \ldots\ldots + 3 + 2 + 1 = \frac{n(n+1)}{2}$

29.



Let v = horizontal component of the velocity of the bullet just after collision

V = horizontal component of the velocity of the ball just after collision

As no horizontal force acts on the system (ball + bullet), the horizontal component of momentum of this system before collision and after collision must be the same:

$$mv_o = mv + MV$$

$$v = v_0 - \frac{M}{m}V$$

After collision both the ball and the bullet continue a free motion in the gravitational field with initial horizontal velocities v and V, respectively. The time of flight for both is same equal to:

$$t = \sqrt{\frac{2h}{g}}$$

The distances passed by the ball and bullet during time t are: $S = Vt$ and $D = vt$, respectively.

Thus, $V = \frac{s}{t}; V = s\sqrt{\frac{g}{2h}}$

Therefore, $v = v_0 - \frac{M}{m}s\sqrt{\frac{g}{2h}}$

And $D = v_0\sqrt{\frac{2h}{g}} - \frac{M}{m}s$ = 100 m

The kinetic energy of the system just before collision is: $E_0 = \frac{mv_0^2}{2}$.

Immediately after the collision the total kinetic energy of the system is equal to:

$$E_m = \frac{mv^2}{2},\ E_M = \frac{MV^2}{2}$$

The difference is converted into heat,

$\Delta E = E_0 - (E_m + E_M)$.

Fraction converted into heat is:

$$\frac{\Delta E}{E_0} = 1 - \frac{E_m + E_M}{E_0}$$

$$= \frac{M}{m}\frac{s^2}{v^2{}_0}\frac{g}{2h}\left(2\frac{v_0}{s}\sqrt{\frac{2h}{g}} - \frac{M+m}{m}\right).$$

$= 92.8\%$.

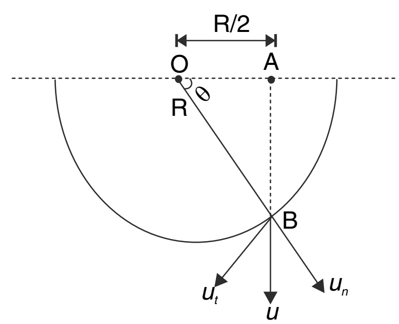
30. $\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

(a) After collision the particle moves along the track. This means there is no normal component of velocity. Hence $e = 0$

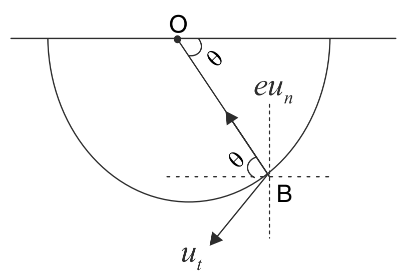
(b) Just before collision, the components of velocity along the normal and along the tangent are

$u_n = u \sin\theta$

$u_t = u \cos\theta$



During collision u_t does not change. The normal component of velocity becomes eu_n along $\overrightarrow{B}O$.



Just after collision

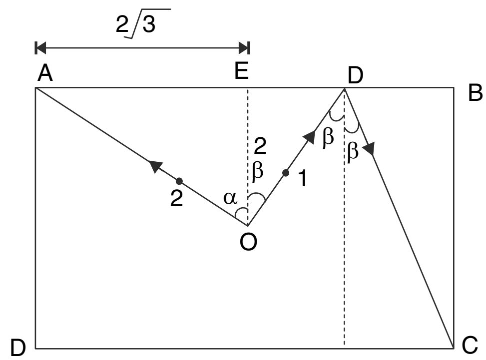
Question says that velocity of the particle is horizontal after collision, which means

$u_t \cos\theta = eu_n \sin\theta$

$u\cos^2\theta = eu\sin^2\theta \Rightarrow e = \cot^2\frac{\pi}{3} = \frac{1}{3}$

31. Path of the two balls after collision has been shown in figure

$$\tan\alpha = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \alpha = 60°$$



Since it is elastic collision between two identical balls both of them will travel at right angle to each other

$\therefore\ \beta = 30°$

$\therefore ED = 2\tan 30° = \dfrac{2}{\sqrt{3}}$ and $DB = 4\tan 30° = \dfrac{4}{\sqrt{3}}$

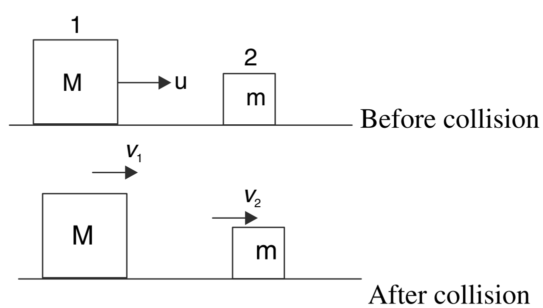
$\therefore EB = \dfrac{6}{\sqrt{3}} = 2\sqrt{3}$

$\Rightarrow$ The ball 1 will fall into the hole at C.

32. It can be proved using law of conservation of momentum and energy that speed of blocks after collision in the shown figure will be -

$v_1 = \dfrac{(M-m)u}{M+m}$ and $V_2 = \dfrac{2Mu}{(M+m)}$

Now $V_2 = \dfrac{2u}{1+\dfrac{m}{M}} = \dfrac{2u}{1+\dfrac{1}{20}} = \left(\dfrac{40}{21}\right)u$



When block 2 hits block 3, the velocity of 3 will become

$v_3 = \dfrac{40}{21}.\dfrac{40u}{21} = \left(\dfrac{40}{21}\right)^2 u$

$\therefore$ speed of n^{th} block after it gets a hit is

$$v_n = \left(\frac{40}{21}\right)^{n-1} u$$

If $v_n \geq 11000\ m/s$

$$\left(\frac{40}{21}\right)^{n-1} \times 10 \geq 11000$$

$$(n-1)\log\left(\frac{40}{21}\right) \geq \log 11 + \log(100)$$

$(n - 1) \times 0.28 \geq 1.04 + 2$

$(n - 1) \geq 10.86$

$\therefore\ n \geq 11.86$

Since n must be an integer

$n = 12$

33. (a) It is just an elastic collision between two blocks of equal mass.

(b) First B will acquire the speed of A and A will come to rest after collision. Now after B moves through a distance equal to the length of the string, the string gives a jolt to both the blocks and thereafter both will move with same speed. Conservation of momentum requires that both the blocks travel with a speed = $\frac{u}{2}$.

34. Let outer radius of the cylinder formed be R

Equating volume $\pi\ (R^2 - r^2)\ a = L.a.d$

$$R^2 = \frac{Ld}{\pi} + r^2 = \frac{L^2}{10^4\pi} + \frac{L^2}{10^4\pi}$$

$$\therefore R = \frac{\sqrt{2}L}{100\sqrt{\pi}} = \sqrt{2}r$$

If the height of the cuboid is h then $\frac{Ld}{b} = h$

$$\therefore h = \frac{L^2}{10^4.b}$$

The centre of mass of cylinder is at a height R from the ground and the height of COM of cuboid is $\frac{h}{2}$

$\therefore$ if $\Delta U_1 = \Delta U_2$ then $R = \frac{h}{2}$

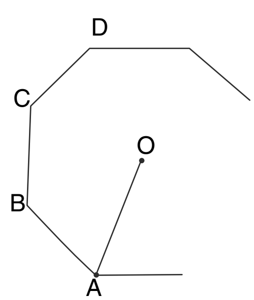
$$\frac{\sqrt{2}L}{100\sqrt{\pi}} = \frac{L^2}{2.10^4 b}$$

$$\therefore b = \frac{\sqrt{\pi}L}{2\sqrt{2}.100} = \frac{1.25L}{200} = \frac{L}{160}$$

36. (i) If particles were present at all vertices, the COM would have been at the centre O. Assume that a particle from A has been removed. The COM of remaining system lies somewhere on the line AO produced. Let the COM be at a distance x from O

$2015\ mx = mr\ [OA = r]$

$$\Rightarrow x = \frac{r}{2015}$$



37. (a) The normal force of wall on B is the reason for acceleration of the COM. Just when B is about to leave the wall (i.e. when the spring is relaxed) let the speed of A be v.

$$\frac{1}{2}mv^2 = \frac{1}{2}kx_0^2 \Rightarrow v = \sqrt{\frac{k}{m}}x_0$$

Speed of COM is $v_0 = \frac{v}{2} = \frac{1}{2}\sqrt{\frac{k}{m}}x_0$

This is the final maximum speed.

(b) Let compression in the spring be x when speed of A is $\frac{v}{2}$

$$\frac{1}{2}kx^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{2}kx_0^2$$

$$kx^2 + \frac{1}{4}kx_0^2 = kx_0^2$$

$$x^2 = \frac{3x_0^2}{4}; x = \frac{\sqrt{3}x_0}{2}$$

$\therefore$ Normal force by the wall = $\frac{\sqrt{3}kx_0}{2}$

$$a_{cm} = \frac{\sqrt{3}kx_0}{2(2m)} = \frac{\sqrt{3}kx_0}{4m}$$

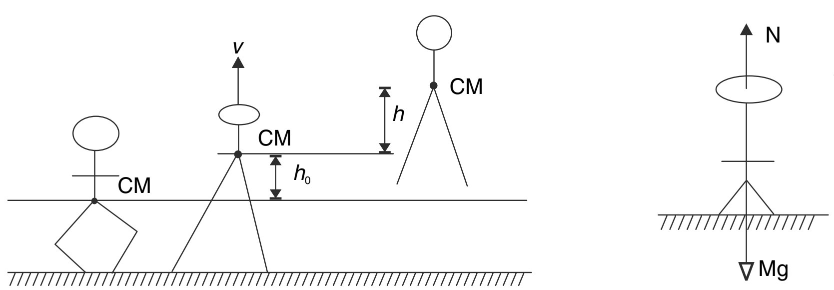
38. The COM follows a parabolic trajectory like a projectile.

Maximum height gained by the COM is

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{5^2 \times \left(\frac{3}{5}\right)^2}{2\times 10} = 0.45 \text{ m}$$

$\therefore$ Rise in position of head = 0.45 – 0.25 = 0.2 m

39. (a)



To rise to a height h any object must be thrown up with a speed

$$v = \sqrt{2gh} = \sqrt{2\times 10\times 0.8} = 4 \ m/s$$

(b) The centre of mass (COM) acquires a velocity of 4 m/s during a displacement of h_0 = 0.5 m

$\therefore v^2 = O^2 + 2ah_0$

$$a = \frac{v^2}{2h_0} = \frac{4^2}{2\times 0.5} = 16\, m/s^2$$

Net force on the mass $N - Mg = Ma \Rightarrow N = Ma + Mg = 60 \times 16 + 60 \times 10 = 1560\ N$

40. As the man moves to right, the platform moves to left. Friction (By horizontal surface) on the platform is towards right. Thus the momentum of the system (Man + platform) increases towards right. When the man comes to rest, the system will have a net momentum towards right.

41. $$a_{cm} = \frac{F_{ext}}{mass} = \frac{m_p g + m_Q g}{m_p + m_Q} = g(\downarrow)$$

$$u_{cm} = \frac{m_p u_p + m_Q u_Q}{m_p + m_Q} = \frac{1 \times 20 - 2 \times 5}{1+2} = \frac{10}{3}\text{ m/s}(\uparrow)$$

Initial height of COM above P is

$$h_0 = \frac{1 \times 0 + 2 \times 60}{3} = 40m$$

Further height attained

$$h = \frac{u^2_{cm}}{2g} = \frac{\left(\frac{10}{3}\right)^2}{2 \times 10} = \frac{5}{9}m$$

Total height $H = 40 + 0.56 = 40.56\ m$

42. COM is at a distance x_1 from the motor of mass M

$$x_1 = \frac{2ML}{3M} = \frac{2L}{3}$$

Acceleration of M is $a = \frac{F}{M}$

The collision will take place when mass M moves through a distance x_1

$$\therefore \frac{1}{2}at^2 = x_1 \Rightarrow \frac{1}{2}\left(\frac{F}{M}\right)t^2 = \frac{2L}{3}$$

$$\therefore t = \sqrt{\frac{4ML}{3F}}$$

43. A line through COM does not necessarily divide a mass distribution into two halves of equal masses. Fact of the matter is that

$$\left[\sum m_i \vec{r}_i\right]_{\text{one half}} = \left[\sum m_i \vec{r}_i\right]_{\text{other half}}$$

$$\left[\sum m_i\right]_{\text{one half}} \neq \left[\sum m_i\right]_{\text{other half}}$$

44. Hint – The sum of distance moved by the two balls = $9R$

Since COM will not move $m_1\, x_1 = m_2\, x_2$

45. (a) The COM will fall down with acceleration 'g'. In the COM frame every particle will move uniformly with speed v = 25 m/s. Therefore, all the particles will keep moving away from the COM at a constant speed of 25 m/s. They all will lie on a sphere whose radius will be increasing at a rate of 25 m/s.

Centre of the sphere falls down with acceleration g. Speed of centre after 1 sec = 10 *m/s*

(b) The particle which was ejected in vertically downward direction will hit the ground first. Height of explosion

$$h = \frac{60^2}{2 \times 10} = 180\text{ m}$$

If t = time required for sphere to touch the ground after explosion then

$$180 = 25.t + \frac{1}{2} \times 10t^2$$

$$\Rightarrow t^2 + 5t - 36 = 0$$

$$\Rightarrow t = 4s$$

Height (above point of explosion) of the particle which was emitted vertically up is

$$h^1 = 25 \times 4 - \frac{1}{2} \times 10 \times 4^2 = 20 \text{ m}$$

$\therefore$ Diameter of the sphere = $h + h^1$ = 200 m

$\therefore$ Radius of sphere = 100 m

46. Let the speed of the car at time t be v. Let dm mass of bullets hit the car in a small time interval dt.

Velocity of approach = velocity of separation

$u - v = v_b + u \Rightarrow v_b = 2u - v$

v_b is velocity of bullet after the hit in direction opposite to its original direction. We are assuming that change in speed of car due to impact of one bullet is negligible.

Momentum transferred to the car in interval dt is

$dp = 2(u - v)\, dm$

$\therefore$ Force on car $F = \frac{dp}{dt} = 2(u - v)\frac{dm}{dt}$

The bullets do not hit the car at the rate at which they leave the gun.

$$\frac{dm}{dt} = \frac{b(u-v)}{u}$$

$$\therefore M\frac{dv}{dt} = \frac{2(u-v)^2}{u} b \Rightarrow \int_0^v \frac{dv}{(u-v)^2} = \frac{2b}{Mu}\int_0^t dt$$

$$\Rightarrow \frac{1}{u-v} - \frac{1}{u} = \frac{2bt}{Mu}$$

$$\Rightarrow v = \frac{\left(\frac{2bt}{M}\right)u}{1+\frac{2bt}{M}} = \frac{u}{\frac{M}{2bt}+1}$$

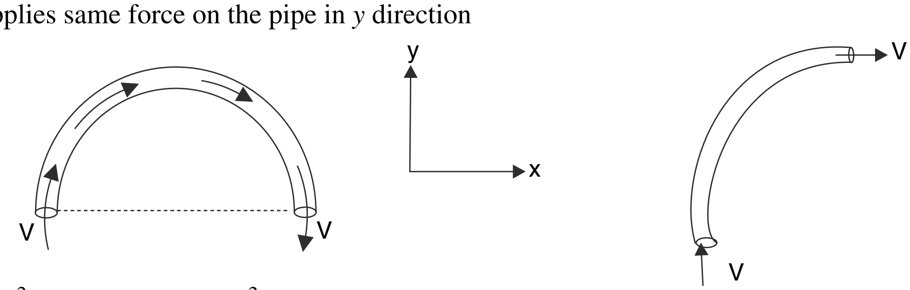
As $t \to \infty$; $v \to u$

47. (i) Force on water inside the pipe in y direction = rate of change of y component of momentum of water

= (mass of water entering / leaving the pipe in unit time) × ($2v$)

= (ρAv) ($2v$) = 2 ρAv^2(in – ve y direction)

Water applies same force on the pipe in y direction



$F_y = 2\rho Av^2$ $\therefore$ $T_{s1} + T_{s2} = 2\ \rho Av^2$

From symmetry $T_{s1} = T_{s2}$

$\therefore$ $T_{s1} = T_{s2} = \rho Av^2$

Now consider a quarter of the pipe.

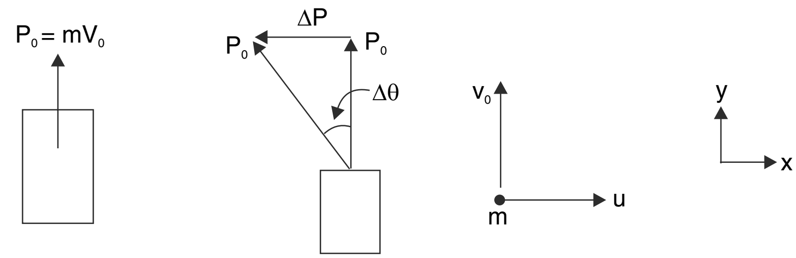
Change in x component of momentum of water in this segment in unit time is the x component of force

$F_X = (\rho Av)\,(v) = 2\ \rho Av^2$

Tension in $s3$ balances this force

$\therefore$ $T_{S3} = \rho Av^2$

(ii)



Let's assume that the car is travelling in y direction and a bullet is fired in x direction (wrt the car)

Change in momentum of the car is $\Delta P = mu$ in a direction perpendicular to original momentum P_0

$\Delta P = P_0 \Delta\theta$

where $\Delta\theta$ = angular change in direction of motion

$mu = MV_0\,\Delta\theta \Rightarrow \Delta\theta = \dfrac{mu}{MV_0}$

After n firings the car will deviate from original direction by

$n\Delta\theta = \pi$

$\dfrac{nmu}{MV_0} = \pi$

$\therefore n = \dfrac{\pi MV_0}{mu}$

48. (a) For $0 < t < 10\ s$

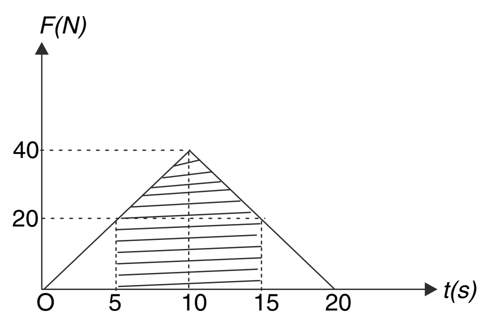
$F = 4t$

If the block begins to move at time t_0, then

$F \cos\theta = \mu\,(\text{mg} - F\sin\theta)$

$$4t_0 \times \frac{4}{5} = \frac{1}{2}\left(4.4 \times 10 - 4t_o \times \frac{3}{5}\right)$$

$\Rightarrow t_0 = 5s$



(b) The block accelerates in the interval $5 < t < 15$ s

Hence speed is maximum at $t = 15\ s$

Δp = Impulse

$$\therefore mv_{\max} = \int_5^{15} F\cos\theta dt - \int_5^{15} \mu\left(mg - F\sin\theta\right)dt$$

$$\therefore mv_{\max} = \cos\theta\int_5^{15} Fdt - \mu mg\int_5^{15} dt + \mu\sin\theta\int_5^{15} Fdt$$

$$= \left(\cos\theta + \mu\sin\theta\right)\int_5^{15} Fdt - \mu mg \times 10$$

$$= \left(\frac{4}{5} + \frac{1}{2} \times \frac{3}{5}\right)(\text{shaded area}) - \frac{1}{2} \times 4.4 \times 10 \times 10$$

$$= \frac{11}{10} \times \left[20 \times 10 + \frac{1}{2} \times 20 \times 10\right] - 220$$

$$\Rightarrow 4.4v_{\max} = 110$$

$$\Rightarrow v_{\max} = \frac{110}{4.4} = 25ms^{-1}$$

49. Block move uniformly at speed $V = 0.01$ *m/s*.

After each collision with A the speed of the ball will increase by $2V$ [Collision with fixed wall B does not cause change in speed]

After $n = 1000$ collisions, speed of the balls is

$u_n = u + 2nV = 100 + 2 \times 1000 \times 0.01 = 120$ *m/s*

Relative speed of approach = V+u

Relative speed of separation = u′ -V = V+u

Notice that speed of the ball changes marginally during each collision but the change becomes significant after a large number of collisions. Change in momentum of the ball during n^{th} collision with the body is

$\Delta P = m\,[u + 2nV] + m\,[u + 2\,(n-1)V] = m\,[2u + 4\,nV - 2V]$

For n as large as 1000, we can neglect $2V$

$\therefore \Delta P = m\left[2u + 4nV\right]$

$= \dfrac{50}{1000} \times \left[2 \times 100 + 4 \times 1000 \times 0.01\right]$

$= 12 kg\ m/s$

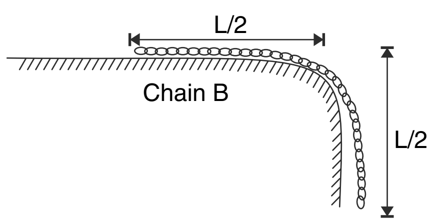
At this instant frequency of collision of the ball with body A is $f = \dfrac{120\,m/s}{1.2\,m} = 100\,s^{-1}$

$\therefore$ Force $= \Delta P f$

$\simeq 1200\ N$

50. (a) Acceleration of B is

$$a_B = \frac{\text{Weight of hanging part}}{\text{Mass of chain}} = \frac{\lambda \frac{L}{2} g}{\lambda L} = \frac{g}{2}$$



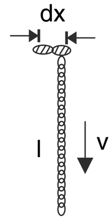
(b) As the falling chain jerks off an element on the table, a thrust force acts on the falling section in upward direction $F_{th} = \dfrac{\lambda dxv}{dt} = \lambda v^2$

[Here we have assumed that a segment of dx length acquires speed v in time dt]

$\therefore F_{th} = \dfrac{dp}{dt} = \dfrac{\lambda dxv}{dt} = \lambda v^2$

$\therefore$ Equation of motion for falling section is

$\lambda l \dfrac{dv}{dt} = \lambda \ell g - \lambda v^2$



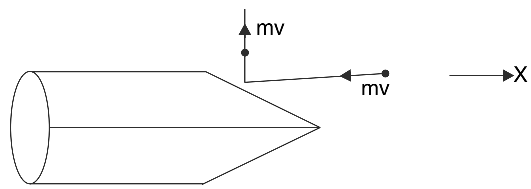
When $l = l_0 .\ v = v_0$

$\therefore \dfrac{dv}{dt} = g - \dfrac{v_0^2}{l_0}$

51. Let's observe the collision of one molecular (mass = m) in the reference frame attached to the bullet. In this frame, the bullet is at rest and molecules are moving towards it at velocity V (= velocity of bullet in ground frame)

From symmetry, we understand that there will not be any transfer of momentum perpendicular to the X direction. We need to consider momentum transfer along X direction only.

For one molecule $\Delta\rho = mV$



In unit time, the bullet sweeps through a volume AV and mass of an molecules in this volume is ρAV.

$\therefore$ Force on bullet = rate of momentum transfer = $(\rho AV)V = \rho AV^2$

$\therefore$ Retardation = $\dfrac{\rho AV^2}{M}$ ....(1)

$M = d.V_0$ [d = density of bullet V_0 = volume of bullet]

For smaller bullet volume is small and hence retardation will be higher. [Ans to (a)]

$$M\frac{dV}{dt} = -\rho AV^2$$

$$\therefore \int_u^V \frac{dV}{V^2} = -\frac{\rho A}{M}\int_0^t dt$$

$$\frac{1}{V} - \frac{1}{u} = \frac{\rho A}{M}t$$

$$\therefore \frac{1}{V} = \frac{1}{u} + \frac{\rho At}{M} \text{....................(i)}$$

$$\therefore V = \frac{Mu}{M + \rho Aut} \text{.................(ii)}$$

$$\text{Now } \frac{dx}{dt} = \frac{Mu}{M + \rho Aut}$$

$$\therefore \int_0^x dx = Mu\int_0^t \frac{dt}{M + \rho\, Aut}$$

$$x = \frac{Mu}{\rho Au}\ell n\left[1 + \frac{\rho Au}{M}t\right]$$

$$x = \frac{M}{\rho A}\ell n\left[1 + \frac{\rho Au}{M}t\right]$$

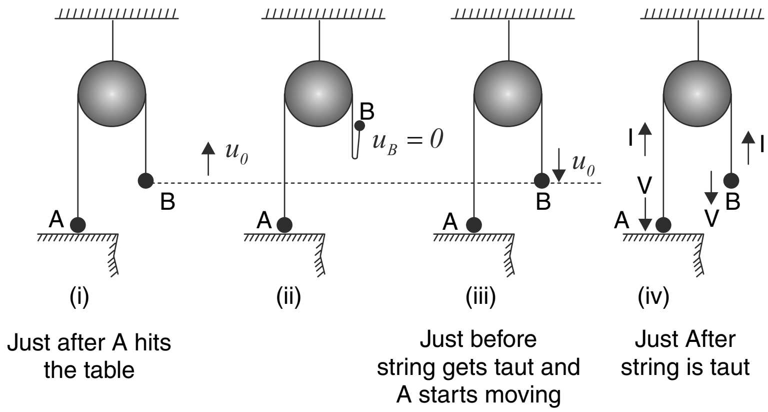
52. Acceleration before A hits the table is

$$a = \left(\frac{3m - 2m}{3m + 2m}\right)g = \frac{g}{5}$$

Speed of A just before hitting the table is

$$u_0 = \sqrt{2aL} = \sqrt{\frac{2}{5}gL}$$

This is also the speed of B at this moment. Now A comes to rest and string becomes slack. Now B moves up with retardation g. B stops and then starts falling down. When B acquires speed u_0 and comes to



position shown in fig. (iii), the string is about to regain tension. As soon as the string becomes taut, the speed of both A and B will becomes same, say V.

The impulse, I applied by the string causes sudden change in momentum of A and B.

For A $\;-I = 3mV$ ....(i)

For B $\;-I = 2mV - 2mu_0$ ....(ii)

From (i) and (ii) $V = \frac{2}{5}u_0$

The required time = time of flight of a particle projected vertically with speed u_0

$$= \frac{2u_0}{g} = \frac{2}{g}\sqrt{\frac{2}{5}gL} = \sqrt{\frac{8}{5}\frac{L}{g}}$$

(a) Loss in kinetic energy is

$$\Delta k = \frac{1}{2}(5m)u_0^2 - \frac{1}{2}(5m)V^2$$

$$= \frac{5m}{2}u_0^2\left[1 - \frac{4}{5}\right] \qquad \left[\therefore V = \frac{2}{5}u_0\right]$$

$$= \frac{1}{2}mu_0^2 = \frac{mgL}{5} \qquad \left[\therefore u_0 = \sqrt{\frac{2}{5}gL}\right]$$

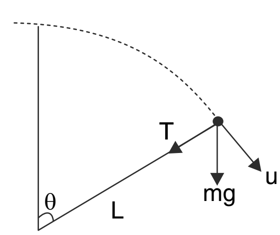
53. (a) Let the speed of the particle be u at position θ. Energy conservation gives–

$$\frac{1}{2}mu^2 = mgL(1 - \cos\theta)$$

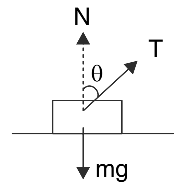
And the equation for centripetal force is $T + mg\cos\theta = \frac{mu^2}{L}$

$T = 2mg\,(1 - \cos\,\theta) - mg\cos\,\theta$

$T = mg\,(2 - 3\cos\,\theta)$



For the plank normal force will be minimum when $T\cos\,\theta$ is maximum



That is when $\cos\theta\,(2-3\cos\theta)$ is maximum

$\Rightarrow -\sin\theta\,(2-3\cos\theta)+\cos\theta\,(3\sin\theta)=0$

$\Rightarrow 2\sin\theta = 6\sin\theta\cos\theta$

$$\Rightarrow \cos\theta = \frac{1}{3}$$

$$\therefore N_{min} = mg - mg\cos\theta\,(2-3\cos\theta)$$

$$= mg - mg.\frac{1}{3}\left(2-3.\frac{1}{3}\right)=\frac{2mg}{3}$$

(b) Momentum conservation tells us that the velocity of the particle and the plank must be equal and opposite in horizontal direction when the rod gets vertical.

$$\therefore \frac{1}{2}mu^2+\frac{1}{2}mu^2 = 2mgL \Rightarrow u=\sqrt{4gL}$$

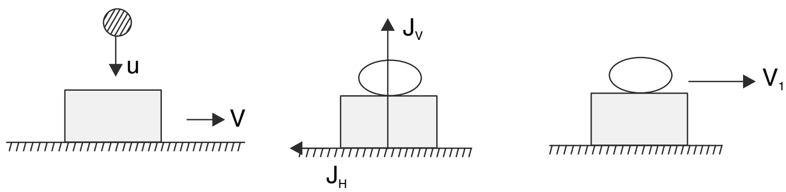
54. During the interaction period of the clay ball and the block, the vertical impulse (due to normal force) applied by ground is

$J_v = \int N\,dt =$ change in momentum in

vertical direction = $1 \times 10 = 10$ *kg m/s* ....(i)

Horizontal impulse of ground friction during the same period is

$J_H = \int \mu\,N\,dt = 0.4\ \times 10 = 4\ kg\ m/s$ ....(ii)



Velocity (V_1) of the (block + ball) system just after the impact is given by

$(M+m)V_1 = MV - J_H$

$$V_1 = \frac{5\times 2-4}{6} = 1\ m/s$$

Let the velocity of $(M + m)$ be reduced to V_2 [due to friction] in interval Δt.

At this point another clay ball hits the block.

J_V and J_H given by (i) and (ii) remain same for the second impact

$\therefore\ 0 = (M+m)V_2 - J_H$

$$\therefore V_2 = \frac{4}{6}=\frac{2}{3}m/s$$

During the interval between two impact, friction causes a retardation of

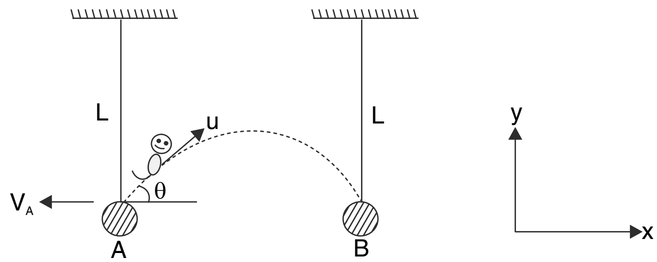
$a = \mu g = 4\ m/s^2$

$\therefore\ V_2 = V_1 - a\Delta t$

$$\Rightarrow \Delta t = \frac{1-\frac{2}{3}}{4} = \frac{1}{12}\text{sec}.$$

55. (a) Let the monkey jump off A with a velocity having horizontal and vertical components u_x and u_y respectively. A will recoil to left with velocity

$$V_A = \frac{mu_x}{2m} = \frac{u_x}{2}$$



When the monkey lands on B, it imparts a horizontal velocity to B

For (monkey + B) system

$V_B = \frac{mu_x}{3m} = \frac{u_x}{3}$ [What happens to vertical velocity?]

This velocity is just sufficient for completing the circle

$$\therefore \frac{u_x}{3} = \sqrt{5gL} \Rightarrow u_x = 3\sqrt{5gL}$$

$\therefore$ Time of flight from A to B for the monkey

$$T = \frac{d}{u_x} = \frac{d}{3\sqrt{5gL}}$$

$$\therefore \frac{2u_y}{g} = \frac{d}{3\sqrt{5gL}}$$

$$\therefore u_y = \frac{d}{6}\sqrt{\frac{g}{5L}}$$

$$\therefore \tan\theta = \frac{u_y}{u_x} = \frac{d}{6}\sqrt{\frac{g}{5L}} \times \frac{1}{3\sqrt{5gL}} = \frac{d}{90}\frac{1}{L}$$

If $\theta = 45°$; $d = 90\,L$

u can be calculated, $u = \sqrt{90gL}$

Impulse of tension $= mu_y = m\sqrt{45gL}$

56. Before the strings get taut, block C moves with constant velocity of 3 *m/s* and the blocks A and B (along with the pulley) fall with an acceleration g.

The strings recover tension, when distance travelled by C = distance travelled by the falling pulley.

$$\therefore \frac{1}{2} \times gt^2 = 3t$$

$$\Rightarrow t = \frac{6}{10} = 0.6s$$

Speed of A and B at this instant is 6 *m/s*. During the short interval in which tension is regained, let the impulse of string tension on C be $\int T\,dt = I(\rightarrow)$

Impulse on A and B both will be $\frac{I}{2}(\uparrow)$

For (C)

$I = 2mV_C - 2m.3$

$\dfrac{I}{m} = 2V_C - 6$......................(i) [V_C =velocity of C after string regains tension]

For A $-\dfrac{I}{2} = 2mV_A + 2m.6$

$\therefore \dfrac{I}{m} = 24 - 4V_A$...............(ii)

For B $-\dfrac{I}{2} = mV_B - m.6$

$\dfrac{I}{m} = 12 - 2V_B$.................(iii)

And after the strings are taut, velocity of A and B relative to the falling pulley must be equal and opposite.

$\therefore V_A - V_C = -(V_B - V_C)$

$\therefore V_A + V_B = 2V_C$........................(iv)

From (i) and (ii) $2V_C - 6 = 24 - 4V_A$

$\therefore V_C + 2V_A = 15$ ....(v)

From (i) and (iii) $2V_C - 6 = 12 - 2V_B$

$\therefore V_C + V_B = 9$ ....(vi)

Solving (iv), (v) and (vi)

$V_C = \dfrac{33}{7}$ m/s, $V_A = \dfrac{36}{7}$ m/s, $V_B = \dfrac{30}{7}$ m/s,

57. (a) We will study the motion of second particle in the reference frame attached to the first particle. The velocity of second particle makes an angle of 45° with the initial line joining the two particles (see fig 2). The thread is loose before the distance between particles again becomes L. Fig.3 shows the situation just before the string gets taut.

Required time is $t = \dfrac{L\sqrt{2}}{v\sqrt{2}} = \dfrac{L}{v}$

(b) In the reference frame of ground, velocities just before the string gets taut, has been shown in fig.4. The velocity component for the two particles along the string will be same for both particles after the string is taut. Fig.5 shows the situation immediately after the string gets taut.

Fig. (1)

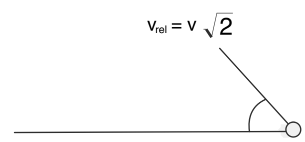


Fig. (2)

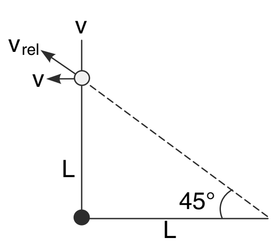


Fig. (3)

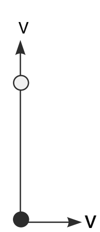


Fig. (4)

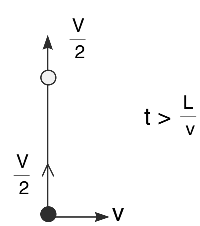


Fig. (5)

Now, total K.E. $= \frac{1}{2}\frac{mv^2}{4} + \frac{1}{2}mv^2\left(1^2 + \frac{1}{2^2}\right) = \frac{3}{4}mv^2$

58. The total impulse of the force is $40N \times 4s = 160\ Ns$ because the impulse due to two fluctuating components will add to zero.

$\therefore mv = 1 \times 200 - 160 = 40 \Rightarrow v = 40$ m/s

59. Momentum conservation

$$m\vec{v} = m\vec{v}_1 + 2m\vec{v}_2$$

$$\vec{v} = \vec{v}_1 + 2\vec{v}_2$$

Taking dot product of the equation with itself gives

$\vec{v}.\vec{v} = \vec{v}_1.\vec{v}_1 + 4\vec{v}_1.\vec{v}_2 + 4\vec{v}_2.\vec{v}_2$.......(i)

Conservation of kinetic energy gives

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2$$

$\vec{v}.\vec{v} = \vec{v}_1.\vec{v}_1 + 2\vec{v}_2.\vec{v}_2$.................(ii)

(i) – (ii) gives

$$4\vec{v}_1.\vec{v}_2 + 2\vec{v}_2.\vec{v}_2 = 0$$

$$\Rightarrow \vec{v}_2\left(2\vec{v}_1 + \vec{v}_2\right) = 0$$

$\therefore \vec{v}_2$ is perpendicular to $\left(2\vec{v}_1 + \vec{v}_2\right)$

60. (i) Relative to cart 1, cart 2 has a velocity of 25 m/s (←).

According to the problem

$25\ t_0 = L$ .... (i)

As soon as the first sack lands on cart 2, its velocity will change

$5\ mV_2 = 4m \times 15 - m \times 10$

[$\because$ the sack brings a momentum $10\ m$ (→) with itself]

$\therefore\ V_2 = 10\ m/s$

The second sack is thrown when the carts are in position shown.

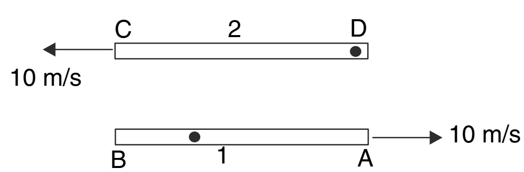
Relative velocity = $20\ ms^{-1}$

The time of flight of the sack = t_0

$\therefore\ 20t_0 = AM$

$$\therefore AM = 20 \times \frac{L}{25} = \frac{4L}{5}$$

$$\therefore BM = \frac{L}{5}$$



(ii) The second sack brings in a momentum $m \times 10$ (←) with itself

$4\ mV_1 = 3m \times 10 - m \times 10$

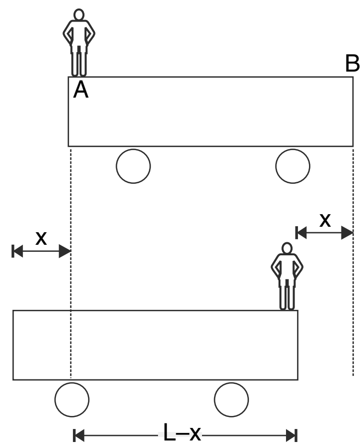
$V_1 = 5ms^{-1}$

61. Let displacement of the cart be x ($\leftarrow$) by the time man reaches the edge B.

Since centre of mass of the system (Man + Cart) will remain at rest hence,

$$m(L-x) = 2x$$

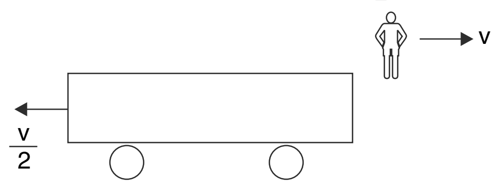
$$\Rightarrow x = \frac{L}{3}$$



$\therefore$ Horizontal displacement of man $= L - \frac{L}{3} = \frac{2L}{3}$

Let the man jump out with absolute velocity v ($\rightarrow$)

For momentum to remain conserved the velocity of cart must be $\frac{v}{2}(\leftarrow)$



A per question $u(\rightarrow) = v(\rightarrow) - \frac{v}{2}(\leftarrow)$

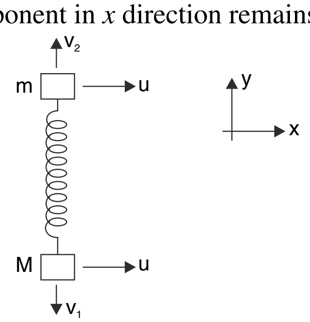
$$u = \frac{3v}{2} \therefore v = \frac{2u}{3}$$

Time of flight for the man from B to ground is $t = \sqrt{\frac{2H}{g}}$.

Horizontal distance covered $= vt = \frac{2u}{3}\sqrt{\frac{2H}{g}}$

$\therefore$ Total horizontal distance travelled from the start $= \frac{2L}{3} + \frac{2u}{3}\sqrt{\frac{2H}{g}}$

62. After the string snaps, the velocity component in x direction remains unchanged.



For conserving momentum in y direction

$mv_2 = Mv_1$

$2v_2 = 8v_1 \Rightarrow \quad v_2 = 4v_1$

Energy conservation

$$\frac{1}{2}m\left(u^2 + v_2^2\right) + \frac{1}{2}M\left(u^2 + v_1^2\right)$$

$$= \frac{1}{2}kx^2 + \frac{1}{2}mu^2 + \frac{1}{2}Mu^2$$

$$\therefore mv_2^2 + Mv_1^2 = kx^2$$

$$2\left(4v_1\right)^2 + 8\left(v_1^2\right) = 1000 \times \left(0.2\right)^2$$

$$\therefore 40v_1^2 = 40$$

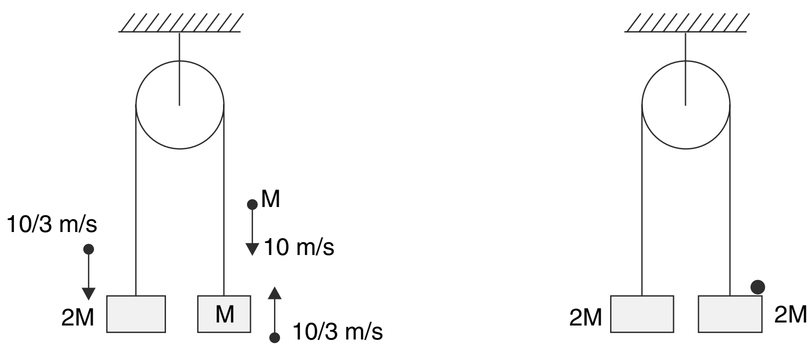
$$v_1 = 1m/s$$

$$\therefore v_m = \sqrt{u^2 + v_2^2} = \sqrt{3^2 + 4^2} = 5m/s$$

63. (a) Acceleration of the two blocks for 1 sec after start is $a = \dfrac{\left(2M - M\right)g}{\left(2M + M\right)} = \dfrac{g}{3}$

$\therefore$ speed of the system after 1 sec is $u = at = \dfrac{10}{3} \times 1 = \dfrac{10}{3} m/s$

At this moment, a particle of mass M travelling with speed = 10 m/s strikes.



By applying momentum conservation to the system comprising of the two blocks and the striking particle, it is easy to see that the whole system will come to rest after collision. Thereafter the system will remain in equilibrium.

[Alternately, one can consider the effect of impulsive string tension on two masses separately to arrive at the result that there will be no motion after impact.]

$\therefore$ Distance travelled by 2M = distance travelled in 1s before collision

$$= 0 + \frac{1}{2}at^2 = \frac{1}{2} \times \frac{10}{3} \times 1^2 = \frac{5}{3}m$$

(b) As before, speed of the system and distance travelled by 2M, after 1 sec will be

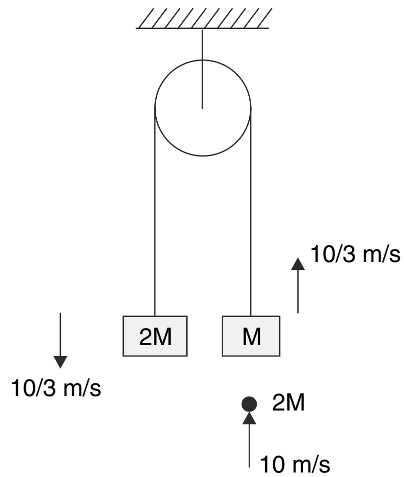
$$u = \frac{10}{3}m/s; \qquad s = \frac{5}{3}m$$

This time, immediately after collision, the string will become loose.

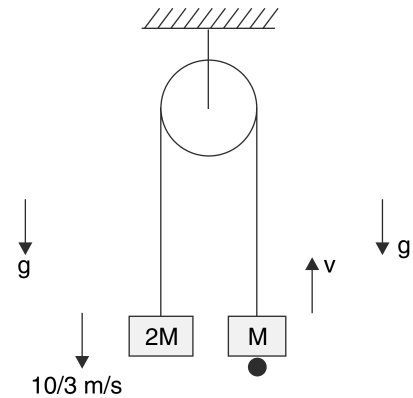
Let speed of (M + 2M) just after collision be v.

$$3Mv = M \times \frac{10}{3} + 2M \times 10 \Rightarrow v = \frac{70}{9}m/s$$

String will get taut after the two blocks travel through same distance S_1 in time t.



Just before Collision



Just after Collision

$$S_1 = \frac{10}{3}t + \frac{1}{2}\times 10\times t^2 = \frac{70}{9}t - \frac{1}{2}\times 10\times t^2$$

$$\Rightarrow t = \frac{4}{9}s \text{ and } S_1 = \frac{200}{81}m.$$

Speed of block of mass 2M just before the string gets taut is

$$v_1 = \frac{10}{3} + 10\times\frac{4}{9} = \frac{70}{9}m/s$$

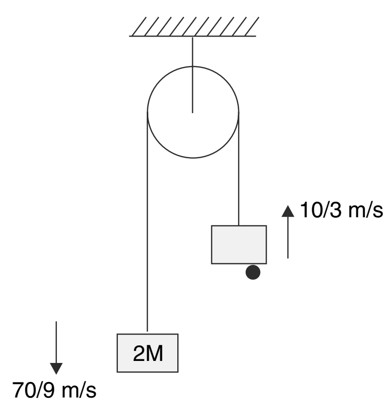
Speed of (M + 2M) just before string is taut is

$$v_2 = \frac{70}{9} - 10\times\frac{4}{9} = \frac{10}{3}m/s$$

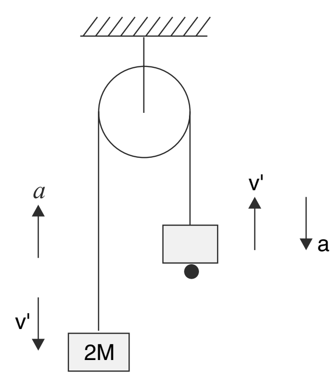
Speed just after string is taut (by momentum conservation) is

$$5M.v' = 2M\times\frac{70}{9} + 3M\times\frac{10}{3}$$

$$v' = \frac{46}{9}m/s$$



Just before string is taut



Just after the string is taut

Now the system retards with acceleration

$$a = \left(\frac{3M-2M}{3M+2M}\right)g = \frac{1}{5}g = 2m/s^2$$

System will stop after travelling a distance S_2.

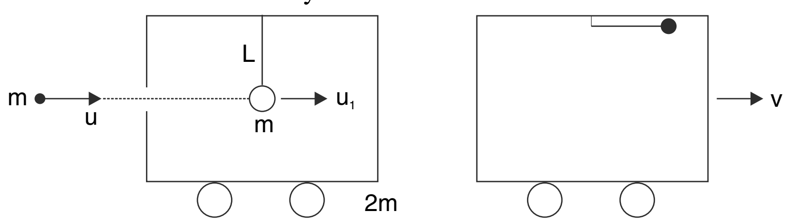
$$0^2 = \left(\frac{46}{9}\right)^2 - 2\times 2\times S_2$$

$$\Rightarrow S_2 = \frac{529}{81}m$$

$\therefore$ Total distance travelled by $2M$ before coming to rest

$$= \frac{5}{3} + \frac{200}{81} + \frac{529}{81} = \frac{864}{81} = 10.67$$

64. The bob just manages to touch the roof. It means that velocity of bob (+ bullet) relative to the car is zero when the string gets horizontal. Let the common velocity be v.



Momentum conservation in horizontal direction gives

$(m + m + 2m)v = mu_0 \Rightarrow v = \dfrac{u_0}{4}$.......(i)

During collision of the bullet and the bob, there is loss in mechanical energy.

Let velocity of (bullet + bob) be u_1 just after collision.

$(m + m)u_1 = mu_0 \Rightarrow u_1 = \dfrac{u_0}{2}$

KE after collision

$$K = \frac{1}{2}(2m)\left(\frac{u_0}{2}\right)^2 = \frac{mu_0^2}{4}$$

This will be equal to mechanical energy of the system when the string becomes horizontal.

$$\therefore\ \frac{1}{2}(4m)v^2 + 2mgL = \frac{mu_0^2}{4}$$

Using (i) $2m\left(\dfrac{u_0}{4}\right)^2 + 2mgL = \dfrac{mu_0^2}{4}$

$$\Rightarrow u_0 = 4\sqrt{gL}$$

65. In time 'dt' a small length of the rope is jerked out to move with speed v.

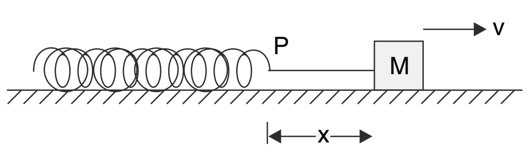
It means a mass λdx acquired a speed v in an interval dt.

$$F = \frac{dp}{dt} = \frac{(\lambda dx)(v)}{dt} = \lambda v\frac{dx}{dt} = \lambda v^2$$

This force is actually the tension in the rope at point P which sets the next element into motion.

Since moving part of the rope has no acceleration hence the external agent must apply force equal to λv^2.

66.



(a) The block plus the x length of the rope is moving at the instant.

From momentum conservation

$(M + \lambda x)v = Mv_0$

$$\Rightarrow v = \frac{Mv_0}{M + \lambda x}$$

Note that the heap contains loose rope and it does not apply force on the moving part. Only the moving part applies an impulse to next small segment of the rope to bring it to motion. [Actually if you take the moving rope and next small element, which will be brought to motion next, in your system then the remaining heap is not applying force on this system].

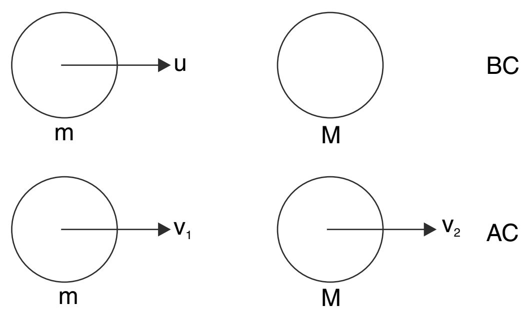
(b) $\frac{dx}{dt} = v$ [Because this is the rate at which the rope is unwinding]

$$\therefore a = \frac{dv}{dt} = v\frac{dv}{dx} = -\frac{Mv_0\lambda}{(M+\lambda x)^2}\frac{Mv_0}{M+\lambda x}$$

$$= -\frac{M^2 v_0^2 \lambda}{(M+\lambda x)^3}$$

$$\therefore T = Ma = \frac{M^3 v_0^2 \lambda}{(M+\lambda x)^3}$$

67. (a) Collision between 1 and 2



$mv_1 + Mv_2 = \text{mu}$ ….. (1)

Elastic Collision $-\left(\frac{v_1 - v_2}{u - 0}\right) = 1$

$\therefore\ v_2 - v_1 = u$ ….. (2)

Solving (1) and (2) $v_2 = \left(\frac{2mu}{M+m}\right)$

Fraction of KE transferred to 2 is

$$\eta = \frac{\frac{1}{2}Mv_2^2}{\frac{1}{2}mu^2} = \frac{4mM}{(M+m)^2} \text{.............(i)}$$

$\therefore$ After 2 collides with 3, the KE of 3 will be

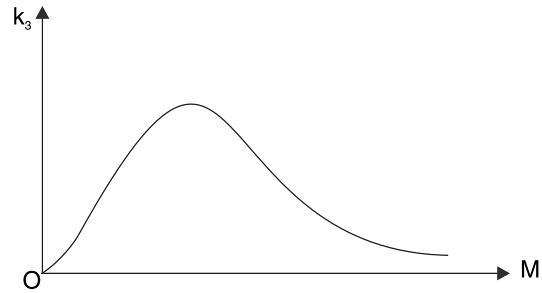
$$k_3 = \frac{4mM}{(M+m)^2}\cdot\frac{1}{2}Mv_2^2$$

$$= \frac{1}{2}mu^2\frac{(16m^2M^2)}{(M+m)^4} = k_0.16\frac{m^2M^2}{(M+m)^4}$$

(b) For $M \to 0$; $k_3 \to 0$

For $M \to \infty$; $k_3 \to 0$

You can prove that k_3 is maximum when $M = m$. Hence, graph is as shown.



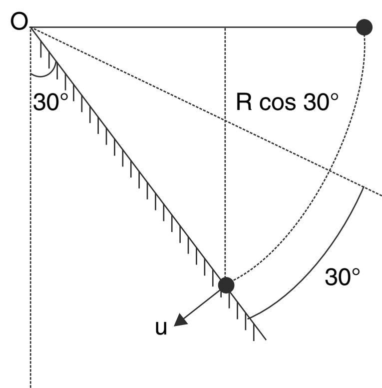
(c) For $m = m_0$; $M = \dfrac{m_0}{2}$

$$\eta = \frac{8}{9}$$

For $m = m_0$; $M = 2m_0$ also $\eta = \dfrac{8}{9}$.

$\therefore$ In both cases the KE of last ball will be same.

68.



u = speed of bob just before first collision.

$$\frac{1}{2}mu^2 = mgR\cos 30° \Rightarrow u = \sqrt{\sqrt{3}gR}$$

After 'n' hits the speed of bob will become

$$v_n = e^n u = \left(\frac{2}{\sqrt{5}}\right)^n \sqrt{\sqrt{3}gR}$$

$$\frac{1}{2}mv_n^2 \le mgR\left[\cos 30° - \cos 60°\right]$$

$$\therefore \left(\frac{4}{5}\right)^n \sqrt{3}gR \le 2gR\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$$

$$\left(\frac{4}{5}\right)^n \le \frac{1}{\sqrt{3}}\left(\sqrt{3} - 1\right) = 0.423$$

Smallest value of n satisfying this equation is 4.

69. (a) The ball tied to elastic cord will have lesser speed, when it reaches the bottom.

$\therefore$ Combined mass will move to left.

(b) Energy conservation for particle B and elastic cord system

$$\frac{1}{2}m\left(\frac{\sqrt{gL}}{2}\right)^2+\frac{1}{2}k\left(\frac{L}{2}\right)^2=mgL$$

$$mgL+kL^2=8mgL$$

$$\therefore k=\frac{7mg}{L}$$

Speed of A just before collision $=\sqrt{2gL}$.

$\therefore$ Momentum Conservation gives–

$$2mv=m\sqrt{2gL}-\frac{m\sqrt{gL}}{2}$$

$$v=\frac{\sqrt{gL}}{\sqrt{2}}-\frac{\sqrt{gL}}{4}=0.46\sqrt{gL}$$

Centripetal force

$$=\frac{2mv^2}{L}=2\times(0.46)^2\times mg=0.42mg$$

Tension in elastic cord $=k.\frac{L}{2}=3.5mg$

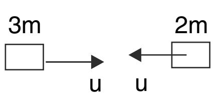
Tension in cord > mg + centripetal force

$\therefore$ Particle will not go in circle.

String will have no tension.

70. Speed of both the blocks when they reach O is

$$u=\sqrt{2gh}=\sqrt{2\times10\times2}=2\sqrt{10}\text{ m/s}$$



Just before collision

Just after collision

Momentum Conservation

$2\,mv_1+3\,mv_2=3mu-2mu$

$2v_1+3v_2=u$ ....…. (1)

Elastic Collision

$$\frac{v_1-v_2}{-u-(u)}=-1$$

$v_1-v_2=2u$ ….. (2)

Solving (1) and (2)

$$v_1=\frac{7u}{5}=\frac{7}{5}\sqrt{2gh}$$

When mass 2m reaches B after collision, its speed will be given by

$$v_0 = \sqrt{v_1^2 - 2gh} = \sqrt{\frac{48}{25}gh}$$

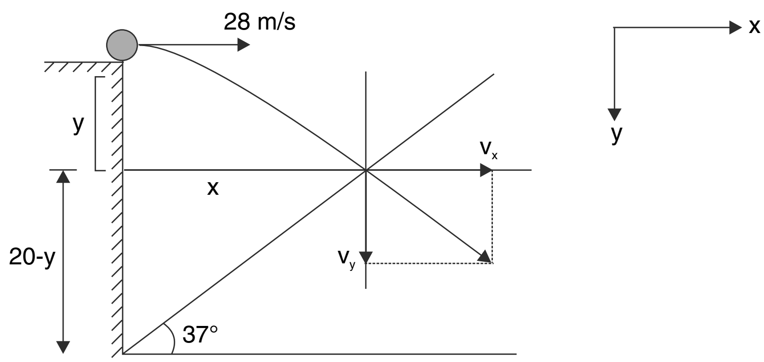
After this it will move like a projectile projected at 45° to the horizontal.

$$H_{\max} = \frac{v_0^2 \sin^2 45°}{2g} = \frac{12h}{25}$$

$\therefore$ Maximum height above O is

$$h + \frac{12h}{25} = \frac{37h}{25} = \frac{37}{25} \times 2 = 2.96m$$

72.



Let the ball hit the incline at time t.

$x = 28\ t$ and

$$y = \frac{1}{2}gt^2 = 5t^2$$

$$\frac{20-y}{x} = \tan 37° \therefore \frac{20-5t^2}{28t} = \frac{3}{4}$$

$80 - 20t^2 = 84t \Rightarrow t^2 + 4.2\ t - 4 = 0$

Solving t = 0.8s

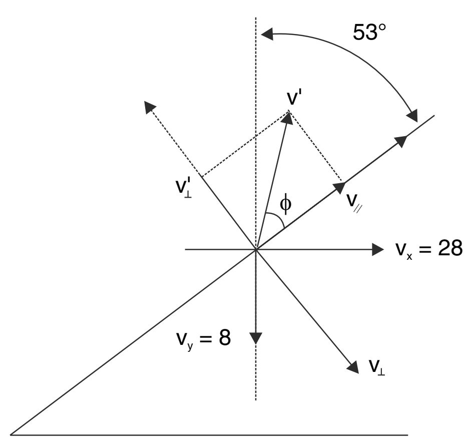
Just before hitting the incline, the velocity components of the ball are

$v_x = 28$ m/s; $v_y = 10 \times 0.8 = 8$ m/s

If we resolve this velocity parallel to the incline and perpendicular to it, we get the components as

$v_{11} = 28 \cos 37° - 8\cos 53° = 17.6$ m/s

and $v_\perp = 28 \sin 37° + 8 \sin 53° = 23.2$ m/s



Just after impact

v_{11} = 17.6 m/s

v_1 = 23.2. e

Resultant velocity after impact (v') makes an angle ϕ with the incline. If $\phi < 53°$, the ball will move up the incline.

$$\tan\phi = \frac{v_1'}{v_{11}} = \frac{23.2e}{17.6} = 1.32e$$

Since $\tan 53° = \frac{4}{3} = 1.33$, therefore, for any value of e (0 to 1.0), tan $\phi <$ tan 53°

$\therefore$ The velocity v' remains to the left of the vertical line and hence the ball will move up the incline.

73. (a) During collision the horizontal component of the velocity of the ball does not change. So it will travel equal horizontal distances in equal intervals. It means time of flight between two collisions is same.

This is possible only if vertical velocity component after each collision remains same. Height like AB and CD depend on this component of velocity only. Hence AB = CD

(b) Before first collision, the horizontal and vertical components of ball's velocity are

$v_x = v \sin\theta$; $v_y = v\cos\theta$

During collision v_x does not change.

After first collision $v_{1y} = ev_y$

Just before second collision, the vertical velocity component becomes (u_{2y})

$$u_{2y}^2 = v_{1y}^2 + 2gh, \quad \text{i.e.,} \quad u_{2y} = \left(ev_y\right)^2 + 2gh$$

Just after second collision, the vertical velocity becomes $v_{2y} = eu_{2y}$

But this must be equal to v_{1y}

i.e.,

$$v_{2y}^2 = v_{1y}^2$$

$$e^2 u_{2y}^2 = v_{1y}^2$$

$$e^2\left[e^2 v_y^2 + 2gh\right] = e^2 v_y^2$$

$$\therefore h = \frac{v_y^2\left(1-e^2\right)}{2g}$$

$$\therefore h = \frac{v^2\cos^2\theta\left(1-e^2\right)}{2g} \qquad \text{................(i)}$$

Time of flight between two collisions can be calculated as

$$h = -\left(ev_y\right)t + \frac{1}{2}gt^2$$

$$\Rightarrow gt^2 - 2ev_y t - 2h = 0$$

$$\therefore t = \frac{2ev_y \pm \sqrt{4e^2 v_y^2 + 8gh}}{2g}$$

$$t = \frac{ev_y + \sqrt{e^2 v_y^2 + 2gh}}{g} \left[-ve \text{ sign is not acceptable}\right]$$

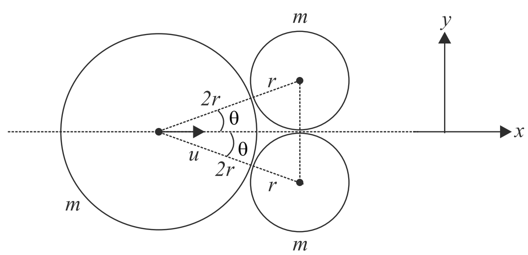
Substituting for h from (1)

$$t = \left(\frac{e+1}{g}\right) v_y$$

$$= (e+1)\frac{v_y}{g} = (e+1)\frac{v\cos\theta}{g}$$

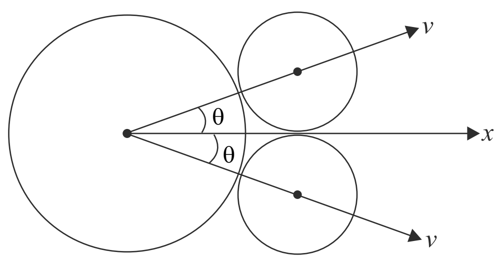
$$\therefore x = v\sin\theta t = (e+1)\frac{v^2}{2g}\sin 2\theta$$

74. (a)



Just before Collision

$$\sin\theta = \frac{r}{3r} = \frac{1}{3}$$



Just after impact

The two smaller discs will move along the respective lines of impact.

Momentum Conservation along x gives -

$mu = mv\cos\theta + mv\cos\theta$

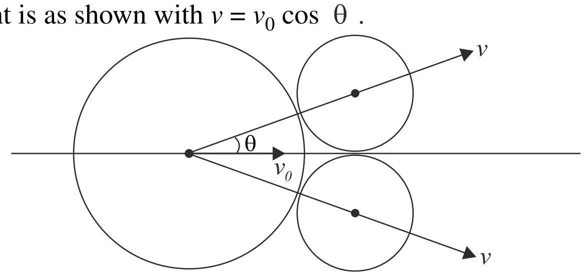
$$\therefore \quad v = \frac{u}{2\cos\theta} = \frac{3u}{2\sqrt{8}}$$

Coefficient of restitution

$$e = -\left(\frac{v-0}{0-u\cos\theta}\right) = \frac{9}{16}$$

(b) The KE is least, when deformation is maximum. This occurs when relative velocity along the line of impact becomes zero.

Situation at this moment is as shown with $v = v_0 \cos\theta$.



$\therefore$ Momentum Conservation along x

$2\,mv\cos\theta + mv_0 = mu \quad \therefore\ 2\,v_0\cos^2\theta + v_0 = u$

$$\Rightarrow \left(2\times\frac{8}{9}+1\right)v_0 = u$$

$$\Rightarrow v_0 = \frac{9u}{25}$$

$$\therefore v = \frac{9u}{25}\times\frac{\sqrt{8}}{3} = \frac{3\sqrt{8}u}{25}$$

$$\therefore KE_{\min} = \frac{1}{2}mv^2\times 2 + \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}m.\frac{9\times 8}{625}u^2\times 2 + \frac{1}{2}m.\frac{81}{625}u^2$$

$$= \frac{1}{2}mu^2\left[\frac{144+81}{625}\right] = \left(\frac{1}{2}mu^2\right)\frac{9}{25}$$

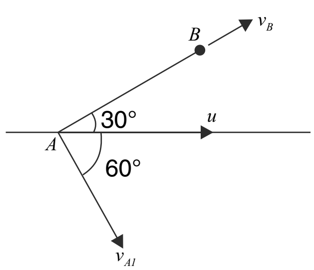
$$\therefore \frac{KE_{\min}}{\frac{1}{2}mu^2} = \frac{9}{25}$$

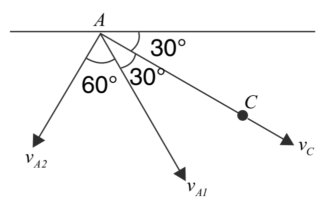
In percentage this is equal to $\frac{9}{25}\times 100 = 36\%$

75. In an oblique elastic collision of two identical balls, the two will move perpendicular to each other after collision. The figures show the centers of the balls. After first collision, B moves along the line of impact and A moves perpendicular to that.

Then A hits C will velocity v_{A1} which makes 30° angle with line of impact. C goes along the impact and A is deflected perpendicular to that.

v_{A2} makes 120° with original direction of motion.





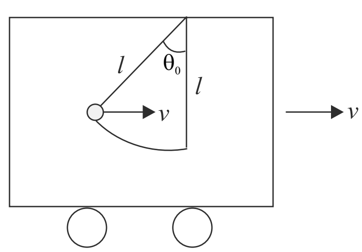
76. (a) Immediately after the impulse, velocity

of the car is $v_0 = \frac{I}{m} = 2\sqrt{gl}$.

The string will make maximum angle with the vertical when there is no relative motion between the particle and the car and both move horizontally with common velocity v.

$mv + 3mv = mv_0 \Rightarrow v = \frac{v_0}{4}$

Energy Conservation



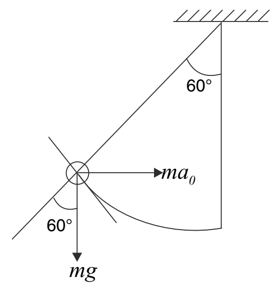
$$\frac{1}{2}(3m)v^2 + \frac{1}{2}mv^2 = mgl(1-\cos\theta)$$

$$\Rightarrow 1-\cos\theta_0 = \frac{v_0^2}{8gl}$$

$$\Rightarrow \cos\theta_0 = 1 - \frac{4gl}{8gl}$$

$$\Rightarrow \theta_0 = 60°$$

(b) In the reference frame attached to the car, the particle has no radial acceleration when at maximum deflection.

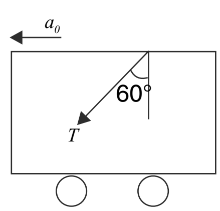


If a_0 is acceleration of the car towards left at this moment, then

$ma_0 \cos 30° = \text{mg} \cos 60°$

$$a_0 = \frac{g}{\sqrt{3}}$$

If we consider the car only then,



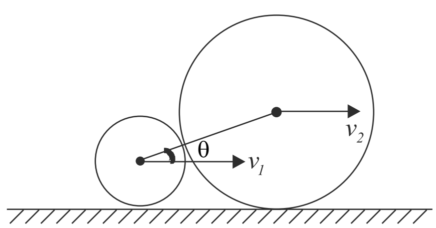
$T \sin 60° = ma_0$

$$T\frac{\sqrt{3}}{2} = \frac{mg}{\sqrt{3}} \text{ i.e., } T = \frac{2mg}{3}$$

77. (a) Momentum Conservation along horizontal direction gives

$mv_1 + Mv_2 = mu$ … (1)

Since collision is elastic



$$\frac{u\cos\theta - 0}{v_1\cos\theta - v_2\cos\theta} = -1 \quad \text{… (2)}$$

$$\Rightarrow v_2 - v_1 = u$$

Solving (1) and (2)

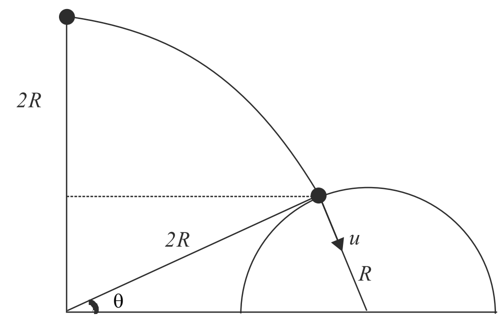
$$v_1 = \frac{(m-M)u}{M+m}; v_2 = \frac{2mu}{M+m}$$

For ball of mass M to remain in contact with the table

$F \sin\theta \le Mg$ i.e., $F \le \dfrac{Mg}{\sin\theta}$

$$F \le Mg\left(\frac{R+r}{R-r}\right)$$

78.

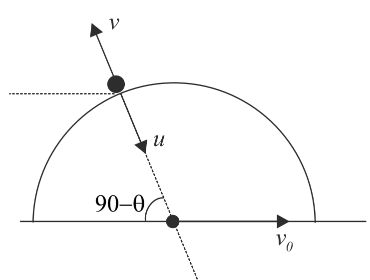


$$\tan\theta = \frac{R}{2R} = \frac{1}{2}$$

u = speed of ball just before collision.

$\dfrac{1}{2}mu^2 = mg2R(1-\sin\theta)$ i.e., $u = 2\sqrt{gR(1-\sin\theta)}$

Let v_0 and v be velocities of the hemisphere and the ball just after collision.



Relative velocity of approach = relative velocity of separation

$u = v_0 \sin\theta + v$ … (1)

Momentum conservation in horizontal direction gives

$mv_0 - mv\sin\theta = mu\sin\theta$

$v_0 - v\sin\theta = u\sin\theta$ … (2)

From (1) and (2)

$v_0 - (u - v_0\sin\theta)\sin\theta = u\sin\theta$

$v_0[1 + \sin^2\theta] = 2u\sin\theta$

$v_0 = \dfrac{2u\sin\theta}{1+\sin^2\theta} = \dfrac{4\sin\theta\sqrt{gR(1-\sin\theta)}}{1+\sin^2\theta}$ where $\sin\theta = \dfrac{1}{\sqrt{5}}$

79. (i) When the bob hits the floor the string makes an angle α with the horizontal.

$$\sin\alpha = \frac{H}{L}$$

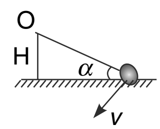
Velocity component parallel to the

horizontal surface is $v_x = v\sin\alpha = v\dfrac{H}{L}$; where $v = \sqrt{2gH}$

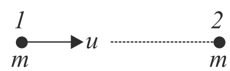
During collision the velocity component perpendicular to the horizontal surface is lost and the particle slides along the surface with velocity v_x. After it reaches a point on left side of the vertical line through O where its distance from O is L , the string suddenly gets taut and the particle swings up like a pendulum. Just when the string gets taut the velocity is perpendicular to the string equal to $v_x \sin\alpha$ (the component along the length of the string is lost). If the height attained is h then from conservation of energy it follows that –

$$mgh = \frac{1}{2}m\left(v_x \sin\alpha\right)^2$$

$$h = \frac{1}{2g}\left(v\sin^2\alpha\right)^2 = \frac{2gH}{2g}\left(\frac{H}{L}\right)^4 = \frac{H^5}{L^4}$$



80. The problem is solved easily in centre of mass frame.



$$v_{cm} = \frac{mu}{m+m} = \frac{u}{2}(\rightarrow)$$

v_{cm} remains unchanged even after collision, due to conservation of momentum.

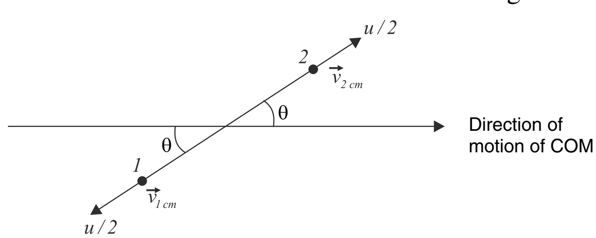
Before collision, the velocities of the two particles in COM frame is as shown below.

COM frame

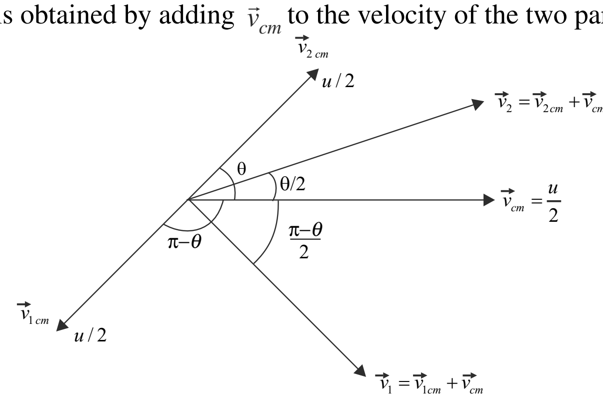
After collision, the two particles must move with equal and opposite momentum in the COM frame (because momentum of the system always remains zero in the COM frame)

To keep the kinetic energy conserved, the two particles will have same speed, $\dfrac{u}{2}$ in the

COM frame. After collision the situation in COM frame is as shown in figure.



The velocity in lab frame is obtained by adding $\vec{v}_{cm}$ to the velocity of the two particles in COM frame.



From the figure, angle between $\vec{v}_1$ and $\vec{v}_2$ is $\frac{\theta}{2}+\frac{\pi-\theta}{2}=\frac{\pi}{2}$

Here I am presenting a neat solution in lab frame.

Momentum Conservation gives

$$m\vec{u}=m\vec{v}_1+m\vec{v}_2$$

i.e, $\vec{u}=\vec{v}_1+\vec{v}_2$

Taking dot product of this equation with itself

$$\vec{u}.\vec{u}=\vec{v}_1.\vec{v}_1+2\vec{v}_1.\vec{v}_1+\vec{v}_2\ \vec{v}_2 \quad ....(i)$$

Conservation of kinetic energy

$$\frac{1}{2}mu^2=\frac{1}{2}mv_1^2+\frac{1}{2}mv_2^2$$

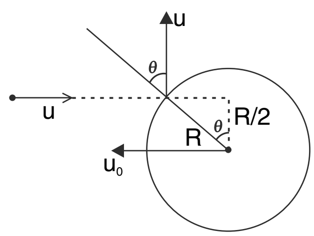
$$\Rightarrow \vec{u}.\vec{u}=\vec{v}_1.\vec{v}_1+\vec{v}_2.\vec{v}_2 \quad ....(ii)$$

Subtract equation (ii) from (i)

$$\vec{v}_1.\vec{v}_2=0$$

$$\Rightarrow \vec{v}_1\perp\vec{v}_2$$

81. (a)



Since the bigger ball is heavy, its velocity will not change much due to collision.

| Relative velocity of separation along radius |

= e | Relative velocity of approach along radius |

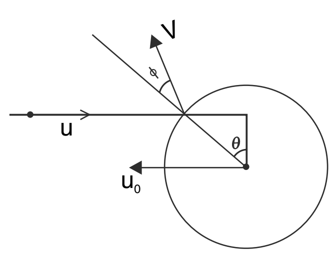
$\therefore$ $u\cos\theta-u_0\sin\theta=e\,(u\sin\theta+u_0\sin\theta)$

$$\therefore e=\frac{u\cos\theta-u_0\sin\theta}{(u+u_0)\sin\theta}$$

$$e=\frac{u\frac{1}{2}-u_0\frac{\sqrt{3}}{2}}{(u+u_0)\frac{\sqrt{3}}{2}}=\frac{u-\sqrt{3}u_0}{\sqrt{3}(u+u_0)}$$

e must be positive $\therefore u>\sqrt{3}u_0$

(b)



Let the velocity of small ball be V in the direction shown. Since collision is elastic

$\therefore\ V \cos \phi - u_0 \sin \theta = u \sin \theta + u_0 \sin \theta$

$\therefore\ V \cos \phi = (u + 2u_0) \sin \theta$ ....(a)

Velocity component of the ball along the tangent remains unchanged.

$\therefore\ u \cos \theta = v \sin \phi$ ....(b)

From (a) and (b)

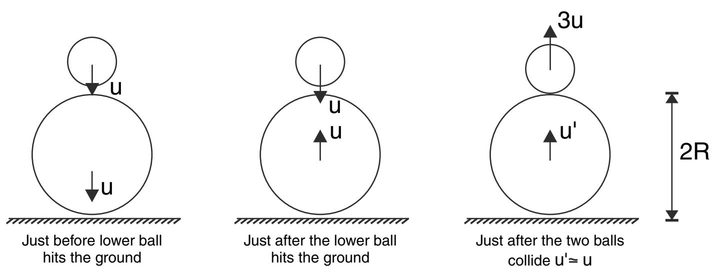
$V^2 \sin^2 \phi + V^2 \cos^2 \phi = (u \cos \theta)^2 + [(u + 2u_0) \sin \theta]^2$

$\Rightarrow V^2 = u^2 \cos^2\theta + u^2 \sin^2\theta + 4u_0^2 \sin^2\theta + 4\, uu_0 \sin^2 \theta$

$$= u^2 + 4u_0^2\left(\frac{\sqrt{3}}{2}\right)^2 + 4uu_0\left(\frac{\sqrt{3}}{2}\right)^2 = u^2 + 3u_0^2 + 3uu_0$$

$$\therefore V = \sqrt{u^2 + 3u_0^2 + 3uu_0}$$

82. $u = \sqrt{2gh}$



During collision between the balls, speed of M does not change much ($M >>$ m).

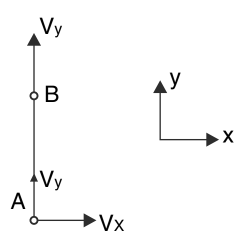
And, relative speed of separation = relative speed of approach.

$\Rightarrow$ The smaller bounces off with speed $3u$.

$\therefore$ Height attained by the ball above the ground is

$$H = 2R + \frac{(3u)^2}{2g} = 2R + 9h \left[\because u^2 = 2gh\right]$$

83.



Particle A and C will collide when the strings BA and BC become parallel to y axis. At this time B will be moving along y with velocity V_y (say).

Component of velocity of A & C along y axis will also be V_y. Particle A has a component of velocity along X axis as well (say V_x).

Momentum conservation along y direction gives $mV_0 = 3mV_y$

$$\Rightarrow V_y = \frac{V_0}{3} = 3 \text{ m/s}$$

Energy conservation

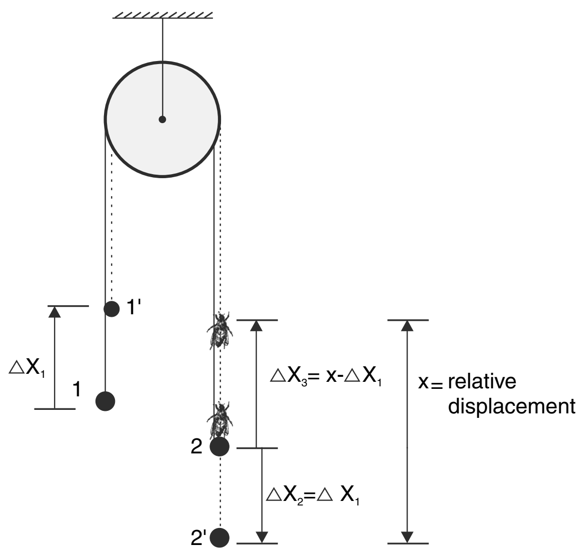
$$\frac{1}{2}mV_0^2 = \frac{1}{2}m\left(\frac{V_0}{3}\right)^2 + \frac{1}{2}(2m)\left(\left(\frac{V_0}{3}\right)^2 + V_x^2\right)$$

$$\left[\therefore \text{Speed of A and C} = \sqrt{V_y^2 + V_y^2} = \sqrt{\left(\frac{V_0}{3}\right)^2 + V_x^2}\right]$$

Solving, $V_x = 3\sqrt{3}$ m/s

$\therefore$ Speed of A, $V_A = \sqrt{\left(3\sqrt{3}\right)^2 + 3^2} = 6$ m/s

84. Let the mass 1 move up by ΔX_1. Mass 2 will go down by the same distance because the string is inextensible. The insect goes up by $(x - \Delta X_1)$



$$\Delta X_{cm} = \frac{m_1\Delta X_1 + m_2\Delta X_2 + m_3\Delta X_3}{M}$$

$$= \frac{m_1\Delta X_1 - m_2\Delta X_1 + m(x - \Delta X_1)}{M}$$

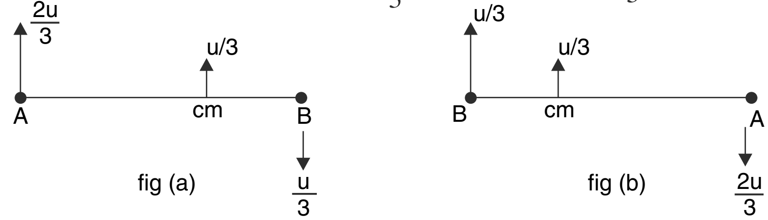
$$= \frac{(m_1 - m_2 - m)\Delta X_1 + mx}{M}$$

$$= \frac{mx}{M} \qquad \left[\begin{array}{l}\therefore \text{For equilibrium} \\ m_2 + m_1 = m\end{array}\right]$$

The impulses of impulsive tension during the crawling of the insect are responsible for moving the COM up.

85. $V_{cm} = \dfrac{mu}{3m} = \dfrac{u}{3}$

In COM frame, the system rotates with speed of $A = \dfrac{2u}{3}$ and speed of $B = \dfrac{u}{3}$



The two figures show the velocities of A and B in COM frame.
In ground frame B will have maximum velocity when it is in position indicated in fig (b)

$\therefore \left(V_B\right)_{\max} = \dfrac{2u}{3}$

86. Acceleration of the COM of the system is

$$a_0 = \frac{F}{2m}$$

Displacement of COM in time t is

$$x_0 = \frac{1}{2}a_0 t^2 = \frac{Ft^2}{4m}$$

Let x_1 and x_2 be the displacement of block A and B respectively.

$$x_0 = \frac{mx_1 + mx_2}{2m} = \frac{x_1 + x_2}{2}$$

$$\therefore x_1 + x_2 = \frac{Ft^2}{2m} \quad \text{...................(i)}$$

And $x_1 - x_2 = l$ .... (ii)

Solving (i) and (ii)

$$x_1 = \frac{Ft^2}{4m} + \frac{l}{2}$$

87. let x_1 = displacement of m_1 to right and
x_2 = displacement of m_2 to left
$m_2 x_2 = m_1 x_1$
(So that COM does not move)
When m_1 is at $x = 5$; $x_1 = 4$ cm
And m_2 is at $x = -4$ cm; $x_2 = 2$ cm
$\therefore$ $m_2 \times 2 = 2 \times 4 \Rightarrow m_2 = 4$ kg
At $x = 5$; m_1 is at rest. At that time m_2 must also be at rest (to keep the linear momentum conserved). This is the time when extension in the spring is maximum.
Initial compression in the spring = Maximum extension in the spring
[Since kinetic energy is zero at both instances]

$\Rightarrow \ell_0 - 3 = 9 - \ell_0$

$\left[\ell_0 = \text{unstretched length of the spring}\right]$

$\Rightarrow \ell_0 = 6$ cm

88. (a) $\vec{V}_0 = \dfrac{m_1\vec{V}_1 + m_2\vec{V}_2}{m_1 + m_2}$

Velocity of the two particles in COM frame

$$\vec{V}_{1cm} = \vec{V}_1 - \vec{V}_0 = \frac{m_2\left(\vec{V}_1 - \vec{V}_2\right)}{m_1 + m_2}$$

$$\vec{V}_{2cm} = \vec{V}_2 - \vec{V}_0 = \frac{m_1\left(\vec{V}_2 - \vec{V}_1\right)}{m_1 + m_2}$$

$$\therefore KE_{cm} = \frac{1}{2}m_1\left|V_{1cm}\right|^2 + \frac{1}{2}m_2\left|V_{2cm}\right|^2$$

$$= \frac{1}{2}\left(\frac{m_1 m_2^2}{m_1 + m_2} + \frac{m_2 m_1^2}{m_1 + m_2}\right)\left|\vec{V}_1 - \vec{V}_2\right|^2$$

$$= \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} V_{rel}^2 = \frac{1}{2}\mu V_{rel}^2$$

(b) $KE_{cm} + \frac{1}{2}(m_1 + m_2)V_0^2$

$$= \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}\left(\vec{V}_1 - \vec{V}_2\right)^2 + \left(\frac{m_1 + m_2}{2}\right)\left(\frac{m_1\vec{V}_1 + m_2\vec{V}_2}{m_1 + m_2}\right)^2$$

$$= \frac{1}{2(m_1 + m_2)}\left[m_1 m_2\left(V_1^2 + V_2^2 - 2\vec{V}_1.\vec{V}_2\right) + m_1^2 V_1^2 + m_2^2 V_2^2 + 2m_1 m_2 \vec{V}_1.\vec{V}_2\right]$$

$$= \frac{1}{2(m_1 + m_2)}\left[m_1 V_1^2 (m_2 + m_1) + m_2 V_2^2 (m_1 + m_2)\right]$$

$$= \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 V_2^2 = KE \text{ in ground frame}$$

(c) Using the above result when $V_{rel} = 0$, the KE is minimum.

$$\therefore KE_{\min} = \frac{1}{3}(m_1 + m_2)V_0^2$$

89. The result obtained in the last problem will help us produce an easy solution to this problem.

In the COM frame (a reference frame travelling with the velocity of the centre of mass of the system) the initial mechanical energy of the system is equal to the spring potential energy as velocity of both blocks is zero.

This energy gets converted into KE when the spring regains its natural length.

$$\frac{1}{2}\mathrm{kx}^2 = \frac{1}{2}\mu \mathrm{v_r^2} \quad .....(i)$$

Here, μ = reduced mass of the blocks = $\frac{(\mathrm{m})(2\mathrm{m})}{\mathrm{m} + 2\mathrm{m}} = \frac{2}{3}\mathrm{m}$

and v_r = relative velocity of the two.

Substituting in Equation (1), we get

$$\mathrm{kx}^2 = \mu v_r^2$$

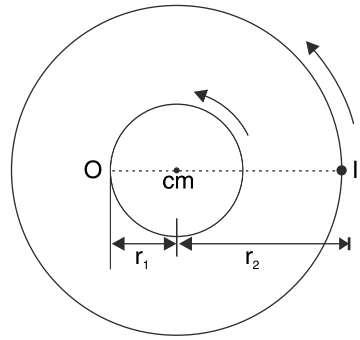
$$\mathrm{kx}^2 = \frac{2m}{3}v_r^2 \Rightarrow v_r = \sqrt{\frac{3k}{2m}}x$$

90. Maximum expansion in spring is given by

$$\frac{1}{2}\mathrm{kx_{max}^2} = \frac{1}{2}\mu \mathrm{v_0^2} \qquad [\mu = \text{Reduced mass}]$$

$$\Rightarrow \mathrm{x_{max}} = \sqrt{\frac{\mu}{\mathrm{k}}}.\ \mathrm{v_0} = \sqrt{\frac{2\mathrm{m}}{3\mathrm{k}}}\ v_0$$

91. (a)



Distance of COM of the system (disc + Insect) from the centre (O) of the disc is

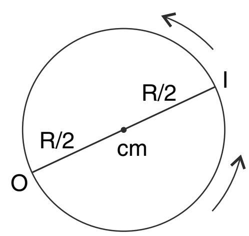
$$r_1 = \frac{mR}{M+m}$$

Distance of insect (I) from COM is

$$r_2 = \frac{MR}{M+m}$$

The COM of the system remains at rest and both the insect and the centre (O) of the disc move in circle of radii r_2 and r_1 respectively. The angular speed of both is same so that O, CM and I always remain on a straight line.

(b) For $m = M$, both are equidistant from the COM. They will move along the same circle

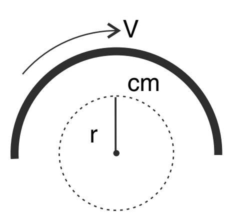


92. The COM moves in a circle of radius $r = \frac{2R}{\pi}$

Angular speed of COM = angular speed of the wire $\omega = \frac{V}{R}$

$\therefore$ Net force F = $Ma_{cm} = M.\omega^2 r$

$$= M\left(\frac{V}{R}\right)^2 \cdot \left(\frac{2R}{\pi}\right) = \frac{2}{\pi}\frac{MV^2}{R}$$



93. Let the wedge move by a distance x towards right by the time the block reaches the top of the incline.

Actual horizontal displacement of the block will be $2\cos 37° - x = 1.6 - x$

COM of the entire system will remain undisplaced

$$\Rightarrow m(1.6 - x) = 3mx$$

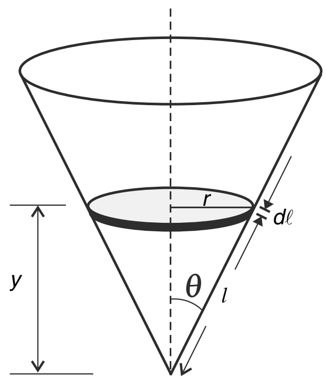
$$\Rightarrow x = \frac{1.6}{4} = 0.4m$$

Work done by tension on the wedge + motor system is

$$W_T = (T\cos\theta)x = mg\sin 37°.\cos 37°.x$$

$$= 1\times 10\times\frac{3}{5}\times\frac{4}{5}\times 0.4 = 1.92J.$$

94. The cream is like a solid cone of mass 4M and height h. Its COM is at a distance of $\frac{3h}{4}$ from the apex.
The biscuit cone is hollow and its COM is at a distance $\frac{2h}{3}$ from the apex. Here I will derive the result for an empty cone and doing the derivation for a solid cone is being for students.
We divide our hollow cone into multiple rings as shown in the fig.



$r = y\tan\theta$ and $\ell = y\sec\theta$

$$d\ell = (dy)\sec\theta$$

$\therefore$ Mass of the ring element is

$$dm = \sigma(2\pi r d\ell)$$

$$dm = 2\pi\sigma . y\tan\theta\sec\theta\, dy$$

$$\therefore y_{cm} = \frac{\int y dm}{\sigma.\pi R\ell} = \frac{2\pi\tan\theta\sec\theta.\sigma\int_o^h y^2 dy}{\sigma.\pi R\ell}$$

$$= \frac{2}{3}\tan\theta.\sec\theta\frac{h}{R}\frac{h}{\ell}.h$$

Put $\frac{h}{R} = \cot\theta$ and $\frac{h}{\ell} = \cos\theta$ to get $y_{cm} = \frac{2h}{3}$

The cone can be replaced with a particle of mass M at a distance $\frac{2h}{3}$ from the apex and the cream can be replaced with another point mass 4M kept at a distance $\frac{3h}{4}$ from the apex.

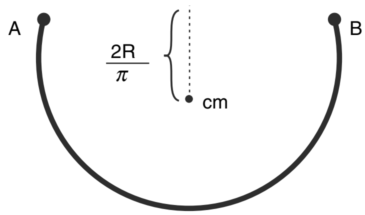
$$\therefore y_{cm} = \frac{4M.\frac{3h}{4} + M.\frac{2h}{3}}{5M} = \frac{11h}{15}$$

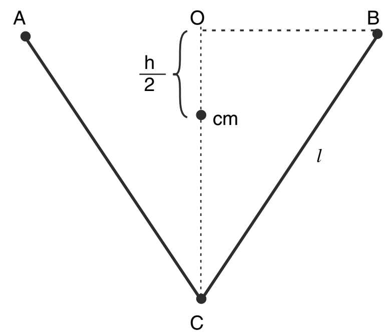
95. (a) The centre of mass gets closer to O.

(b) For semicircle, the distance of COM from O is

$$y_1 = \frac{2R}{\pi} = 0.64R$$

For V shaped rope $l = BC = \frac{\pi R}{2}$





$$h = OC = \sqrt{\ell^2 - R^2}$$

$$= \sqrt{\frac{\pi^2}{4} - 1}\,R = 1.21R$$

Distance of COM from O is

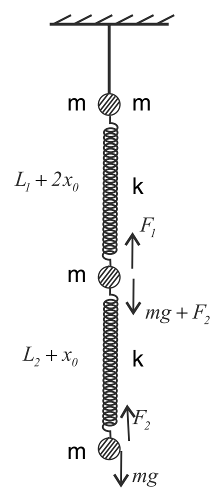
$$y_2 = \frac{h}{2} = 0.61R$$

$\therefore$ Shift $\Delta y = y_1 - y_2 = 0.03\,R$

96. The two spring have same force constant but different tensions. Therefore, they must have different natural lengths. Let the natural lengths of the upper and lower springs be L_1 and L_2 respectively.

Spring forces in equilibrium are $F_2 = mg$

$$\Rightarrow Kx_2 = mg \Rightarrow x_2 = \frac{mg}{K} = x_0\,(\text{say})$$



And $F_1 = \text{mg} + F_2 \Rightarrow F_1 = 2mg$

$$\therefore x_1 = \frac{2mg}{K} = 2x_0$$

Given, $L_1 + 2x_0 = L_2 + x_0 \Rightarrow L_2 = L_1 + x_0$

When springs are in natural length, distance of centre of mass from the top ball will be

$$x_{cm} = \frac{0 + mL_1 + m(L_1 + L_2)}{3m}$$

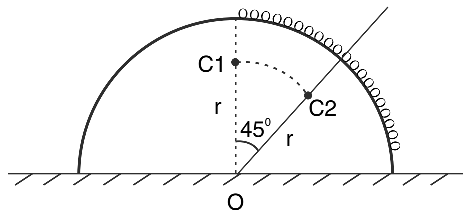
$$= \frac{2L_1 + L_2}{3} = \frac{2L_1 + L_1 + x_0}{3} = L_1 + \frac{x_0}{3}$$

$\therefore$COM is at a distance of $\frac{x_0}{3}$ (i.e., $\frac{mg}{3K}$) from the second ball

97. (b) Distance of COM of the chain from O is

$$r = \frac{R\sin\alpha}{\alpha} = \frac{R\sin 45^\circ}{\frac{\pi}{4}} = \frac{2\sqrt{2}R}{\pi}$$

The COM of the chain moves on a circle of radius r as the chain slides.



$C1$ and $C2$ are initial and final positions of COM of the chain

Energy conservation

$$\frac{1}{2}mV^2 = mgr\left[1-\cos 45°\right]$$

$$\therefore V^2 = 2gr\left(1-\frac{1}{\sqrt{2}}\right) = 2g\frac{2\sqrt{2}R}{\pi}\frac{\sqrt{2}-1}{\sqrt{2}}$$

$$V = \sqrt{\frac{4\left(\sqrt{2}-1\right)gR}{\pi}}$$

Angular speed of COM and the chain must be same

$$\therefore V_{cm} = \frac{Vr}{R} = \frac{2\sqrt{2}}{\pi}\sqrt{\frac{4\left(\sqrt{2}-1\right)gR}{\pi}}$$

(c) COM of the system (chain + Half cylinder) will suffer no displacement along the horizontal direction.

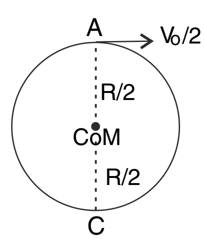
If X is displacement of the half cylinder towards left, then $m\,(r \sin 45° - X) = mX$

$$\therefore X = \frac{r}{2\sqrt{2}} = \frac{R}{\pi}$$

98. The COM of the (disc + body) system moves in a straight line with velocity $\frac{V_0}{2}$.

In COM frame, the body and the centre of the disc always have equal and opposite velocity. In this frame, the body (A) and the centre of the disc (C) move in circle of radius $\frac{R}{2}$ with angular speed given by –

$$\omega = \frac{V_0/2}{R/2} = \frac{V_0}{R}$$

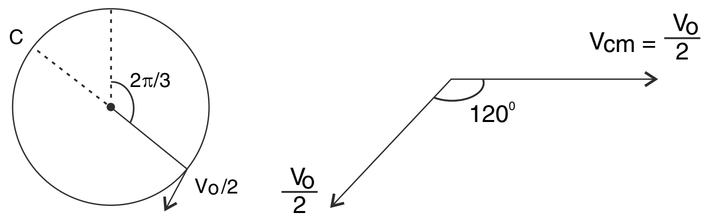


(a) The velocity of the centre (C) will become zero in ground frame after it has completed one rotation in the COM frame.

$$\therefore t_0 = \frac{2\pi R/2}{V_0/2} = \frac{2\pi R}{V_0}$$

(b) In time $\frac{t_0}{3}$, the body A rotates through $\frac{2\pi}{3}$ radian in COM frame.

Its velocity in this frame is as shown.



In ground frame velocity of *A* is obtained by adding the velocity of COM to this velocity.

Resultant of two vectors also has a magnitude of $\frac{V_0}{2}$.

99. Mass of water melon eaten by Majnu is

$$m_m = \frac{M}{2} + \frac{M}{2}.\frac{1}{2}.\frac{1}{2} + \frac{M}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2} + ...$$

$$= \frac{M}{2} + \frac{M}{8} + \frac{M}{32} + ... + ...\infty \text{ terms}$$

$$= \frac{\frac{M}{2}}{1-\frac{1}{4}} = \frac{2M}{3}$$

Mass eaten by Laila

$$m_L = \frac{M}{3}$$

So, a mass $\frac{2M}{3}$ has effectively moved from

left end of the boat to its right end. Let the boat move to the left by x

$$\frac{2M}{3}(L-x) = \left(M_0 + \frac{M}{3}\right)x \quad \therefore \frac{\frac{2}{3}ML}{M_0+M} = x$$

$$x = \frac{2ML}{3(M_0+M)}$$

100. **(a)** Let man move down with velocity V_m (wrt ground) and the balloon go up with velocity V_b (wrt ground)

$V_m + V_b = u \Rightarrow V_b = u - V_m$

Momentum conservation gives –

$mV_m = M(u - V_m)$

$$\therefore V_m = \frac{Mu}{M+m}$$

And $V_b = \frac{mu}{M+m}$

As COM of the system does not move, we have –

$$mX_m(\downarrow) = M \quad X_b(\uparrow) \quad ....(a)$$

Change in $PE = -mgX_m + MgX_b = 0$ [using (a)]

Both man and balloon move with constant speed

$\therefore$ there is no change in KE.

$\therefore$ KE + PE = const

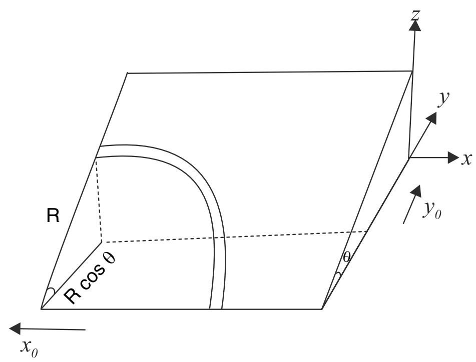
(b) $F_B = (M + m)g$

$$P_B = F_B.V_b = \left(M+m\right)g.\frac{mu}{M+m} = mgu$$

(c) Heat is generated in rubbing.

101. COM of the block + wedge system will not get displaced in x and y direction.

If x_0 and y_0 are components of displacement of the wedge in the shown directions, then $m(R - x_0) = 10\,mx_0$



$$\Rightarrow x_0 = \frac{R}{11} \quad \text{...........(i)}$$

And $m\,[R\cos\theta - y_0] = 10\,my_0$

$$\Rightarrow y_0 = \frac{R\cos\theta}{11} = \frac{R}{11}\frac{\sqrt{11}}{5} = \frac{\sqrt{11}}{55}R \quad \text{.........(ii)}$$

$\therefore$ Displacement of wedge is

$$r = \sqrt{x_0^2 + y_0^2} = \frac{6R}{55} = \frac{6 \times 0.55}{55} = 0.06 \text{ m}$$

$$= 6 \text{ cm}$$

(b) Let the velocity of block and wedge at the instant block leaves the wedge be

$\vec{v}_b = -v_1\hat{j} - v_2\hat{k}$ and $\vec{v}_w = v_0.\hat{j}$

Momentum conservation along y direction

$mv_1 = 10\,mv_0 \Rightarrow v_1 = 10v_0$ .... (i)

Velocity of block relative to wedge

$$\vec{v}_{bw} = \vec{v}_b - \vec{v}_w = -\left(v_0 + v_1\right)\hat{j} - v_2\hat{k}$$

This velocity makes an angle

$\theta = \cos^{-1}\dfrac{\sqrt{11}}{5}$ with horizontal

$$\therefore \tan\theta = \frac{v_2}{v_0 + v_1}$$

$$\sqrt{\frac{14}{11}} = \frac{v_2}{11v_0} \Rightarrow v_2 = \sqrt{154}v_0 \quad ....(ii)$$

Mechanical energy conservation

$$\frac{1}{2}m\left(v_1^2 + v_2^2\right) + \frac{1}{2} \times 10mv_0^2 = mgR\sin\theta$$

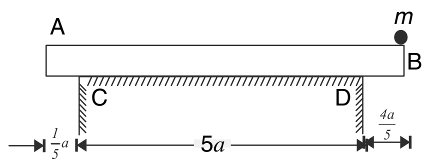
$$\left(10v_0\right)^2 + 154v_0^2 + 10v_0^2 = 2 \times 10 \times 0.55 \times \frac{\sqrt{14}}{5}$$

$$264v_0^2 = 8.23 \quad \therefore v_0 = 0.18 \text{ m/s}$$

102. (a) Since there is no external force, in horizontal direction on the system, its COM will not move.

Let the bar move a distance x towards left by the time 1st insect reaches B.

Then $4mx = m\left(6a - x\right) \Rightarrow \frac{6}{5}a = x$



$\therefore$ Length hanging out of the table (on right) is $2a - \frac{6a}{5} = \frac{4a}{5}$

Length hanging out on left $= \frac{a}{5}$

The bar will topple if the vertical line through centre of mass of the system passes outside the table. The COM of the system lies at a distance

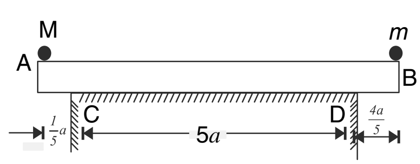
$\frac{m \times 0 + 4m \times 3a}{5m}$ from end B

$= \frac{12}{5}a > \frac{4a}{5}$

The bar will not topple.

(b) When the second insect with large mass sits at end A, the bar has a tendency to topple about C (see figure). If M increases COM of the system shifts to left. M is maximum (for not toppling) when COM is at C

Distance of COM from $A = \frac{M \times 0 + 4\text{m} \times 3a + m \times 6a}{M + 5\text{m}}$



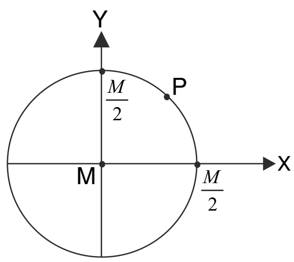
$$\Rightarrow \frac{a}{5} = \frac{18ma}{M + 5\text{m}}$$

$\Rightarrow M + 5\text{m} = 90\text{ m}$

$\Rightarrow m = 85\text{ m}$

103. (a) The initial co-ordinates of COM of the system is $x = \dfrac{\frac{M}{2}R}{2M} = \dfrac{R}{4}$

$$y = \frac{\frac{M}{2}.R}{2M} = \frac{R}{4}$$

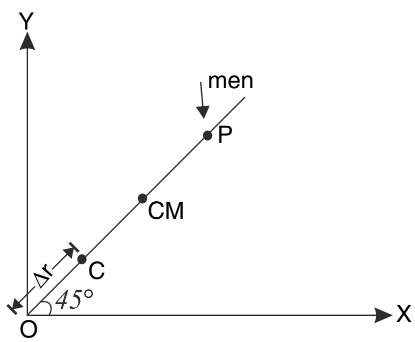


When the men are at centre (C), the centre must have shifted to the original position of the COM.

$\therefore$ Displacement of centre of the disc

$$= \sqrt{x^2 + y^2} = \frac{R}{2\sqrt{2}}$$

(b) In our co-ordinate system (whose origin is fixed to the table at original position of the centre of the disc), initial distance of COM from origin is $r_{CM} = \dfrac{R}{2\sqrt{2}}$



Let the centre of the disc translate by Δr. The distance of point P from origin is $R + \Delta r$

$$\therefore \frac{M\Delta r + M(R + \Delta r)}{2M} = \frac{R}{2\sqrt{2}}$$

$$\therefore [2\Delta r + R]\sqrt{2} = R$$

$$2\sqrt{2}\Delta r = -R(\sqrt{2} - 1)$$

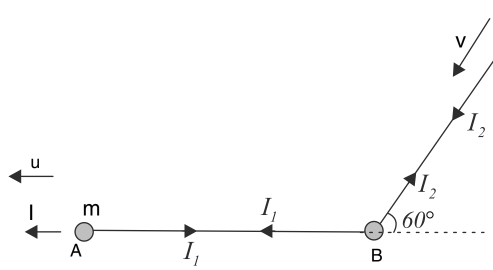
$$\Rightarrow \Delta r = -\frac{R(\sqrt{2} - 1)}{2\sqrt{2}}$$

$\therefore$ Centre moves by a distance $\dfrac{R(\sqrt{2} - 1)}{2\sqrt{2}}$

in direction opposite to that shown in the figure.

104. Let the impulse of tension in the string AB and BC be I_1 and I_2 respectively.

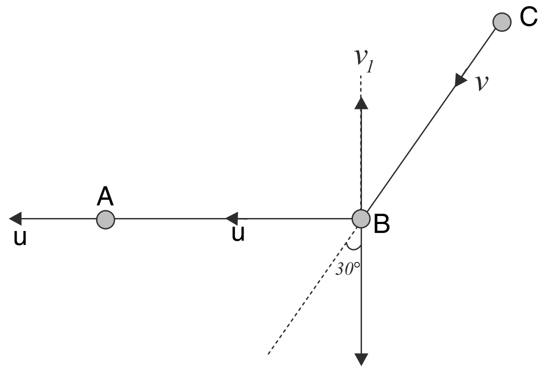
Particle C receives an impulse along CB. There is no other force on it. Therefore, it will start moving along CB. Let its initial velocity be v



For : C $\quad mv = I_2$ ……….(i)

For : A $\quad mu = I - I_1$ …….(ii)

Where I = external impulse applied



Component of velocity of B in direction – BA must be u

Let its velocity component perpendicular to BA be v_1

Velocity component of B along CB is $-v_1 \cos 30° + u \sin 30°$

This must be equal to v (= velocity of C)

$\therefore -\frac{\sqrt{3}}{2}v_1 + \frac{u}{2} = v$...............(iii)

Let's write impulse momentum theorem for B in two perpendicular directions $-BA$ and perpendicular to it.

$I_1 - I_2 \cos 60° = 2mu$

$I_1 - \frac{I_2}{2} = 2mu$

$2I_1 - I_2 = 4mu$ ....(iv)

And $I_2 \cos 30° = 2mv_1$

$\frac{\sqrt{3}}{2}I_2 = 2mv_1$ ....(v)

We need to solve the set of equations (i), (iii) and (v).

(i) and (v) gives

$\sqrt{3}mv = 4mv_1 \Rightarrow \sqrt{3}v = 4v_1$

put in (iii)

$$-\frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{4}v\right)+\frac{u}{2}=v$$

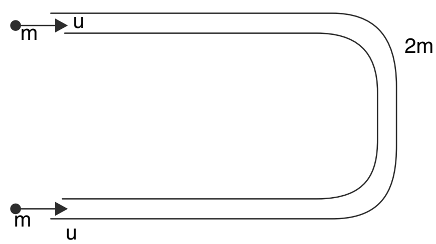
$$\frac{11}{8}v=\frac{u}{2}\Rightarrow v=\frac{4u}{11}$$

and $v_1=\frac{\sqrt{3}v}{4}=\frac{\sqrt{3}u}{11}$

Speed of B is

$$v_B=\sqrt{u^2+v_1^2}=\frac{2\sqrt{31}}{11}u$$

105. It is like 2 m mass hitting another 2 m mass (at rest) head on.

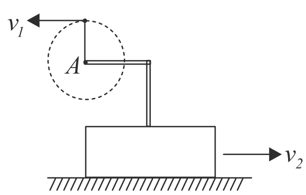


106. During each elastic collision the balls will exchange velocities. It is given that all balls are identical (the two colliding balls have no names), we could say that the balls simply pass through each other moving independent of each other. If a ball was at the edge it will take longest time to fall off the other edge

$$\therefore t_{\max}=\frac{L}{v}$$

107. Let v_1 = velocity of ball at the top

v_2 = velocity of the block at this instant



Momentum conservation gives:

$$mv_2 - mv_1 = mu$$

$v_2 - v_1 = u$ ....(i)

Energy conservation gives

$$\frac{1}{2}mv_1^2+\frac{1}{2}mv_2^2+mg(2R)=\frac{1}{2}mu^2$$

$v_1^2+v_2^2=u^2-4gR$...............(ii)

Squaring (i) $v_2^2+v_1^2-2v_1v_2=u^2$

$\Rightarrow u^2 - 4gR - 2v_1 v_2 = u^2$

$\Rightarrow 2v_1 v_2 = -4gR$ [This product can be negative !]

Now, $(v_2 + v_1)^2 = v_2^2+v_1^2+4v_1v_2$

$(v_2 + v_1)^2 = u^2 - 4gR - 8gR$

$(v_2 + v_1)^2 = u^2 - 12gR$

$v_2 + v_1 = \pm\sqrt{u^2 - 12gR}$

Negative sign is not acceptable since $v_1 + v_2$ is velocity of ball relative to point A, which cannot be towards right.

$\therefore v_2 + v_1 = \sqrt{u^2 - 12gR}$ ....(iii)

Solving (i) and (iii) we get v_2 and v_1

At the top, string is vertical. Therefore, there is no acceleration of the block (i.e., point A). With respect to point A, the ball is rotating in a circle of radius R with speed

$$v_1\ (\leftarrow) - v_2(\rightarrow) = v_1 + v_2(\leftarrow) = \sqrt{u^2 - 12gR}$$

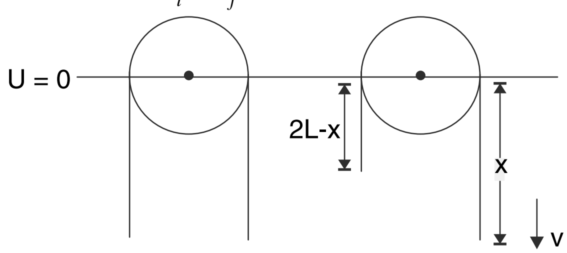
$$\therefore T + mg = \frac{m\left(u^2 - 12gR\right)}{R}$$

$$\therefore T = \frac{mu^2}{R} - 13mg$$

108. (a) We will apply conservation of mechanical energy.

[since peg is small, we will neglect the semicircular part of rope lying over it]

Loss in PE = $U_i - U_f$



$$= -\lambda Lg\frac{L}{2} \times 2 - \left[-\lambda xg\frac{x}{2} - \lambda(2L - x)g\left(\frac{2L - x}{2}\right)\right]$$

$$= -\lambda L^2 g + \frac{\lambda x^2 g}{2} + \frac{\lambda g}{2}\left(4L^2 + x^2 - 4Lx\right)$$

$$= \lambda L^2 g + \lambda x^2 g - 2\lambda gLx$$

$$\therefore \frac{1}{2}(2\lambda L)v^2 = \lambda g\left[L^2 + x^2 - 2Lx\right]$$

$$v^2 = \frac{g}{L}(x - L)^2$$

$$v = \sqrt{\frac{g}{L}}(x - L)$$

(b) $2v\dfrac{dv}{dt} = \dfrac{g}{L}2(x - L)\dfrac{dx}{dt}$

$$a = \frac{dv}{dt} = \frac{g}{L}(x - L)$$

(c) $P = \lambda xv - \lambda(2L - x)\,v = 2\lambda v\,(x - L)$

$$\frac{dp}{dt} = 2\lambda v \frac{dx}{dt} + 2\lambda(x-L)\frac{dv}{dt}$$

$= 2\lambda v^2 + 2\lambda\,(x-L)a$

$$= 2\lambda \frac{g}{L}(x-L)^2 + 2\lambda(x-L)g\frac{(x-L)}{L}$$

$$= \frac{4\lambda g}{L}(x-L)^2 = \frac{2mg}{L^2}(x-L)^2$$

(d) $mg - F = \dfrac{dp}{dt}$

F = force applied by the peg on the rope

$$\therefore F = mg - \frac{dp}{dt} = mg - \frac{4\lambda g}{L}(x-L)^2$$

$$= mg\left[1 - 2\left(\frac{(x-L)^2}{L^2}\right)\right]$$

(e) $F = 0$ for

$$\frac{x-L}{L} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x = L + \frac{L}{\sqrt{2}}$$

109. (a) The particle starts moving down maintaining its contact with the inner ring. The centre of the ring moves to right (the normal force applied by the particle on the ring has a rightward component). Later the particle loses contact with the inner ring and gets into contact with the outer ring. Now the ring experiences a horizontal force towards left. It will come to rest and then start moving towards left. This can also be seen knowing that the COM of the system shall not get displaced horizontally.

(b) v_p = velocity of particle at B

v_w = velocity of wheel when particle is at B

$mv_p\,(\rightarrow) = Mv_w\,(\leftarrow)$ [Momentum conservation in horizontal direction]

Energy conservation:

$$\frac{1}{2}mv_p^2 + \frac{1}{2}Mv_w^2 = mg.2R$$

$$\frac{1}{2}m\left(\frac{Mv_w}{m}\right)^2 + \frac{1}{2}Mv_w^2 = 2mgR \quad \therefore v_w = 2m\sqrt{\frac{gR}{M(M+m)}}$$

(c) Let v = speed of particle with respect to the wheel

v_w = speed of the wheel

When particle is in contact with the inner wheel, the wheel is moving to right.

Note: velocity of particle in ground frame is

$$\vec{v}_p = \vec{v} + \vec{v}_w$$

$$v_p = \sqrt{v^2 + v_w^2 + 2vv_w + \sin(180-\theta)}$$

$$= \sqrt{v^2 + v_w^2 + 2vv_w \sin\theta}$$

Velocity component of the particle in horizontal direction

$v_x = v \cos \theta - v_w (\rightarrow)$

Momentum conservation in Horizontal direction gives –

$m (v \cos \theta - v_w) = Mv_w$

$$v = \frac{(M+m)v_w}{m\cos\theta}$$

Energy conservation

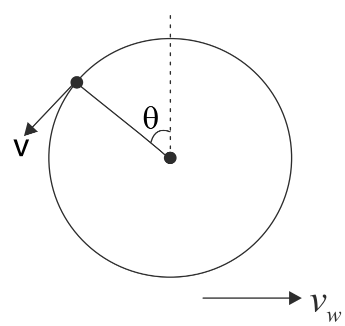
$$\frac{1}{2}mv^2 + \frac{1}{2}Mv_w^2 = mgR[1-\cos\theta]$$

$$\frac{(M+m)^2}{m\cos^2\theta}v_w^2 + Mv_w^2 = 2mgR[1-\cos\theta]$$

$$v_w^2\left[\frac{(M+m)^2 + Mm\cos^2\theta}{m\cos^2\theta}\right] = 2mgR[1-\cos\theta]$$

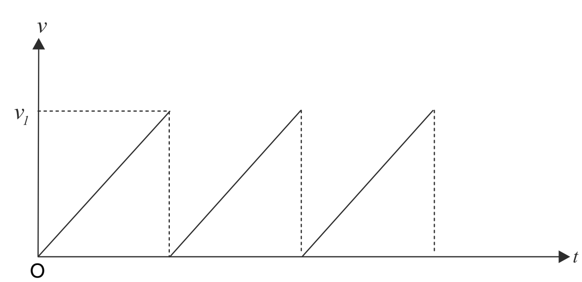
$$v_w^2 = \frac{2m^2gR\cos^2\theta(1-\cos\theta)}{(M+m)^2 + Mm\cos^2\theta}$$

$$v_w = \sqrt{\frac{2m^2gR\cos^2\theta(1-\cos\theta)}{(M+m)^2 + Mm\cos^2\theta}}$$



110. (a) Speed of block 1 grows linearly from zero to v_1 before hitting the block 2.

$$v_1^2 = 0^2 + 2\frac{F}{m}d \Rightarrow v_1\sqrt{\frac{2Fd}{m}}$$



Block 1 hits block 2 with speed v_1. Block 1 comes to rest and 2 starts moving with velocity v_1. Block 1 will again begin to accelerate as the force is still acting on it. Block 2 will move a distance d, hit block 3 and come to rest. By the time block 1 covers distance d and is about to hit block 2, it will once again have velocity equal to v_1. It will hit 2 and come to rest. The process continues like this.

(b) Block 1 will hit 2 and both will stick. 1 + 2 will hit 3 and 1 + 2 + 3 will move together. The process continues. Let $u_{1,2,3......,n}$ represent the velocity of 1 + 2 + 3 + ...+ n immediately after the nth block (is hit and) starts moving.

Let $v_{1,2,3,......,n}$ represent the velocity of 1 + 2 + 3 + ...+ n just before the moving system is about to hit the $(n+1)^{\text{th}}$ block.

$u_1 = 0$

$$v_1 = u_1^2 + 2\frac{F}{m}d \Rightarrow v_1 = \sqrt{\frac{2Fd}{m}}$$

Momentum conservation gives $2\,mu_{12} = mv_1$

$$\Rightarrow u_{12} = \sqrt{\frac{Fd}{2m}}$$

$$v_{12} = u_{12}^2 + 2\frac{F}{2m}d \Rightarrow v_{12} = \sqrt{\frac{3Fd}{2m}}$$

Again momentum conservation gives

$$u_{123} = \frac{2}{3}\sqrt{\frac{3}{2}\frac{Fd}{m}} = \sqrt{\frac{2Fd}{3m}}$$

$$v_{123}^2 = u_{123}^2 + 2.\frac{F}{3m}.d \Rightarrow v_{123} = \sqrt{\frac{4}{3}\frac{Fd}{m}}$$

After 'n' collision $(n+1)$ blocks will be moving together

$$\therefore u_{123..................(n+1)} = \sqrt{\frac{n}{n+1}\frac{Fd}{m}}$$

If n is large $\dfrac{n}{n+1} \rightarrow 1$

$$\therefore u_{123.........(n+1)} \rightarrow \sqrt{\frac{Fd}{m}}$$

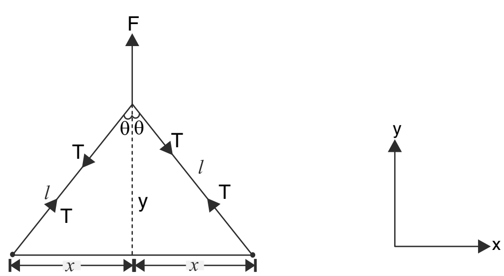
111. Position of the balls at a later time has been shown.

$$2T\cos\theta = F \Rightarrow T = \frac{F}{2\cos\theta} \quad ....(i)$$

$x = l\sin\theta$

$dx = l\cos\theta\, d\theta$ ....(ii)

For motion of a ball in x direction



$$T\sin\theta = m\frac{dv_x}{dt} \Rightarrow v_x\frac{dv_x}{dx} = \frac{T}{m}\sin\theta$$

$$\Rightarrow v_x dv_x = \frac{T}{m}\sin\theta\ l\cos\theta\ d\theta\ \left[\text{using (ii)}\right]$$

$$= \frac{Fl}{2m}\sin\theta\ d\theta\ \left[\text{using (i)}\right]$$

$$\therefore \int_0^{v_x} v_x dv_x = -\frac{Fl}{2\text{m}}\int_{\pi/2}^{0}\sin\theta\ d\theta$$

[we have placed negative sign as θ is decreasing]

$$\frac{v_x^2}{2} = \frac{F\ell}{2m} \Rightarrow v_x = \sqrt{\frac{Fl}{m}}$$

Relative velocity of approach before collision

$$= 2v_x = 2\sqrt{\frac{Fl}{m}}$$

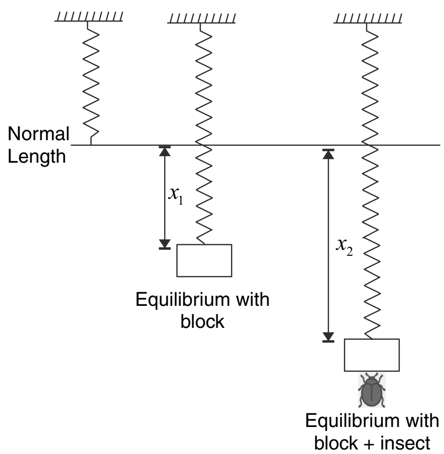
[The two balls will always have zero relative velocity in y direction]

It follows from momentum conservation that after collision the two balls will lose all their velocity in x direction.

$\therefore$ loss in $KE = 2 \times \frac{1}{2} m v_x^2 = Fl$

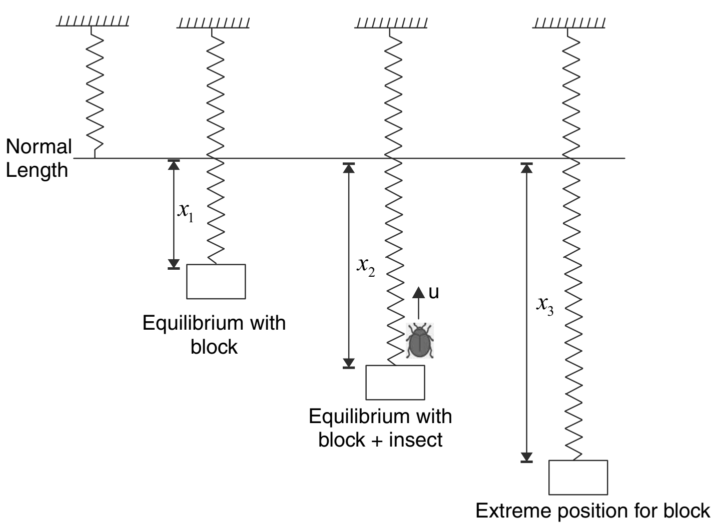
112. (a) Amplitude of oscillation

$$A = x_2 - x_1 = \frac{2Mg}{K} - \frac{Mg}{K} = \frac{Mg}{K}$$



(b) As the insect jumps the block recoils with same speed (Momentum conservation)

Speed of recoil $v = u = g\sqrt{\frac{M}{K}}$



Applying energy conservation

$$\frac{1}{2}Kx_3^2 - Mg\left(x_3 - x_2\right) = \frac{1}{2}Kx_2^2 + \frac{1}{2}Mv^2$$

$$\frac{1}{2}Kx_3^2 - Mgx_3 + \frac{2M^2g^2}{K} = \frac{2M^2g^2}{K} + \frac{M^2g^2}{2K}$$

$$x_3^2 - \frac{2Mg}{K}x_3 - \frac{M^2g^2}{K^2} = 0$$

$$x_3 = \frac{\frac{2Mg}{K} \pm \sqrt{\frac{4M^2g^2}{K^2} + \frac{4M^2g^2}{K^2}}}{2}$$

$$= \frac{Mg}{K}\left[1 + \sqrt{2}\right]$$

[Negative sign is not acceptable]

$\therefore$ Amplitude $A = x_3 - x_1 = \sqrt{2}\,\frac{Mg}{K}$

(c) The spring is stretched by x_1 when the insect strikes. Speed after the hit

$$v = \frac{u}{2} = \frac{g}{2}\sqrt{\frac{M}{K}}$$

Let stretch in spring be x_4 at extreme position

Energy conservation:

$$\frac{1}{2}Kx_1^2 + \frac{1}{2}2Mv^2 = \frac{1}{2}Kx_4^2 - 2Mg\left(x_4 - x_1\right)$$

$$\frac{1}{2}\frac{M^2g^2}{K} + \frac{M^2g^2}{4K} = \frac{1}{2}Kx_4^2 - 2Mgx_4 + \frac{2M^2g^2}{K}$$

$$x_4^2 - 4\frac{Mg}{K}x_4 + \frac{5}{2}\frac{M^2g^2}{K^2} = 0$$

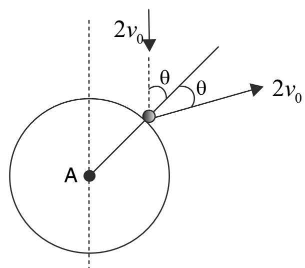
$$\therefore x_4 = \frac{\frac{4Mg}{K} \pm \sqrt{(16-10)\frac{M^2g^2}{K^2}}}{2}$$

$$= \frac{Mg}{K}\left[2 + \frac{\sqrt{6}}{2}\right]$$

Amplitude $A = x_4 - x_2 = \frac{\sqrt{6}}{2}\frac{Mg}{K}$

113. Speed of both balls just before A collides with floor is $v_0 = \sqrt{2gh}$

Ball A rebounds with velocity v_0



Before colliding with B, A sees it moving with velocity $2v_0$ ($\downarrow$)

$\therefore$ relative velocity of approach = relative velocity of separation

In reference frame of A, the velocity of ball B has been shown in the figure. After collision it is $2v_0$ making an angle 2θ with vertical. Note that the true velocity of B has a vertical component equal to $2v_0 \cos 2\theta + v_0$

$$x = \frac{2v_x v_y}{g} = \frac{2[2v_0 \sin 2\theta][2v_0 \cos 2\theta + v_0]}{g}$$

$$= \frac{8v_0^2}{g} \sin 2\theta \left(\frac{1}{2} + \cos 2\theta \right)$$

$$= 16h \sin 2\theta \left(\frac{1}{2} + \cos 2\theta \right)$$

114. The line of impact makes an angle of 60° with the line C_1C_2

$$\sin\theta = \frac{\sqrt{3}r}{2r} = \frac{\sqrt{3}}{2}$$

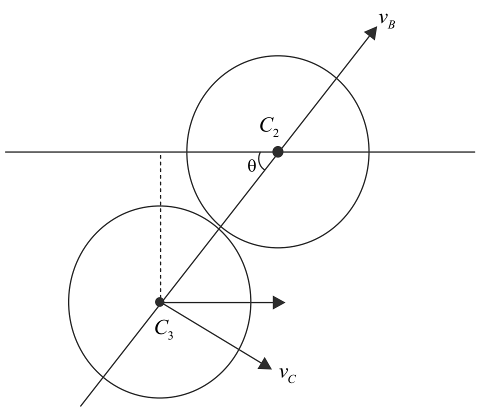


Figure 1

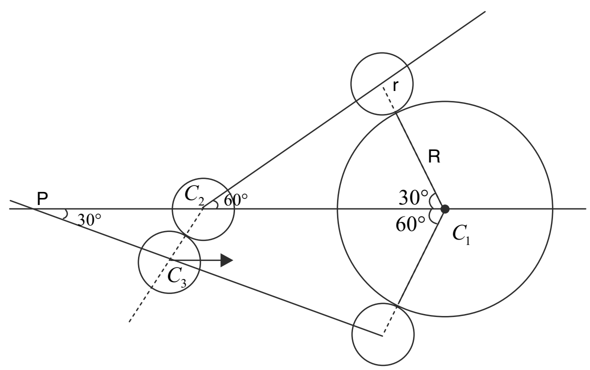


Figure 2

Disc B will move along this line of impact and C will move perpendicular to this. From figure 2

$$C_1C_2 = \frac{(R+r)}{\cos 30°} = \frac{2(R+r)}{\sqrt{3}}$$

$$PC_1 = \frac{(R+r)}{\cos 60°} = 2(R+r)$$

$$\therefore PC_2 = 2(R+r) - \frac{2(R+r)}{\sqrt{3}}$$

But $PC_2 = \dfrac{2r}{\sin 30°}$ $\quad$ [In ΔPC_2C_3]

$$= 4r$$

$$\therefore 4r = 2(R+r)\left[1 - \frac{1}{\sqrt{3}}\right]$$

$$\frac{2\sqrt{3}r}{\sqrt{3}-1} = R + r$$

$$R = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \right) r$$

115. 

$v_{cm} = \frac{u}{3}$

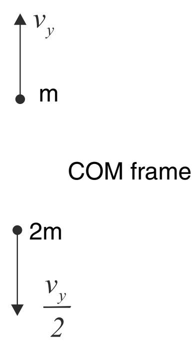
In COM frame, before collision, the situation appears as shown below–

In lab frame, the x component of velocity of the two particles is $v_x = v_{cm} = \frac{u}{3}$

[Because x component of momentum = mu]

Since COM is moving along x, the velocity components in y direction are same in lab frame as well as COM frame.

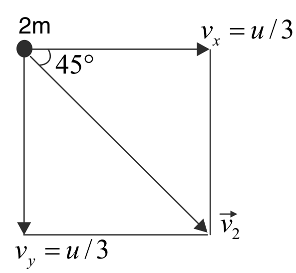
After collision, particles travel along y in COM frame. Kinetic energy is conserved in COM frame also.



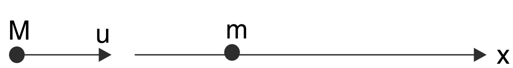
$$\frac{1}{2}mv_y^2 + \frac{1}{2}2m\left(\frac{v_y}{2}\right)^2 = \frac{1}{2}m\left(\frac{2u}{3}\right)^2 + \frac{1}{2}2m\left(\frac{u}{3}\right)^2$$

$$\frac{3v_y^2}{2} = \frac{2}{3}u^2 \Rightarrow v_y = \frac{2}{3}u$$

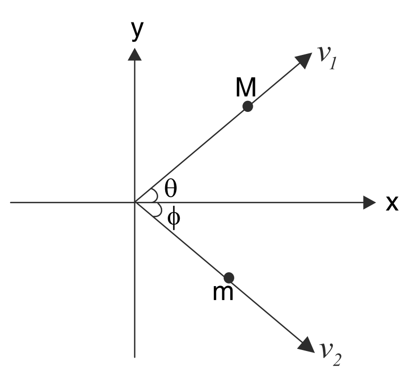
$\therefore$ In lab frame the situation will appear as shown below.



116.



Before collision



After collision

Momentum conservation along x:

$Mv_1 \cos\theta + mv_2 \cos\phi = Mu$

$\Rightarrow mv_2 \cos\phi = Mu - Mv_1 \cos\theta$ ....(i)

Momentum conservation along y:

$mv_2 \sin\phi = Mv_1 \sin\theta$ ....(ii)

Squaring and adding equation (i) & (ii) so as to eliminate ϕ

$m^2v_2^2 = M^2u^2 + M^2v_1^2 - 2M^2uv_1\cos\theta$

$K^2v_2^2 = u^2 + v_1^2 - 2uv_1\cos\theta$ ....(iii)

[Where $K = \dfrac{m}{M}$]

Kinetic energy conservation:

$\frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}Mu^2$

$\Rightarrow Kv_2^2 = u^2 - v_1^2$ ....(iv)

Putting the value of v_2^2 in (iii)

$K\left(u^2 - v_1^2\right) = u^2 + v_1^2 - 2uv_1\cos\theta$

$\Rightarrow (1+k)v_1^2 - (2u\cos\theta)v_1 + (1-K)u^2 = 0$

For v_1 to be real, discriminant of this quadratic equation must be zero or positive

$4u^2\cos^2\theta - 4(1-K^2)u^2 \geq 0$

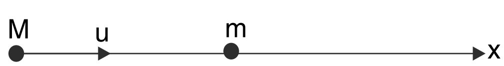
$\cos^2\theta - 1 + K^2 \geq 0$

$K^2 \geq \sin^2\theta$

$K \geq \sin\theta$

$\sin^{-1}\left(\dfrac{m}{M}\right) \geq \theta$

$\therefore \theta_{\max} = \sin^{-1}\left(\dfrac{m}{M}\right)$

Alternate: 

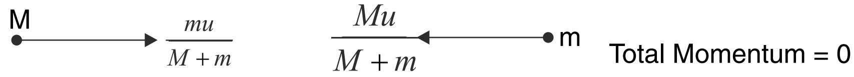
$v_{cm} = \dfrac{Mu}{M+m}$

In COM frame,

velocity of $M = u - \dfrac{Mu}{M+m} = \dfrac{mu}{M+m}$

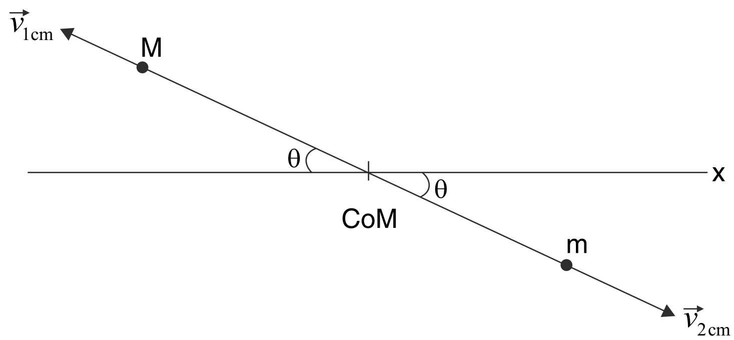
velocity of $m = 0 - v_{cm} = -\dfrac{Mu}{M+m}$

<u>Before collision in COM frame</u>



After collision, in COM frame, the two balls will continuous to move with same speed (otherwise KE will not be conserved) in opposite directions

<u>After collision in COM frame</u>



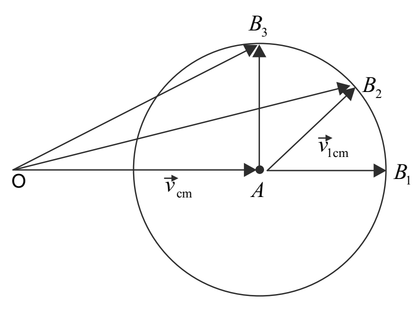
θ can assume any value.

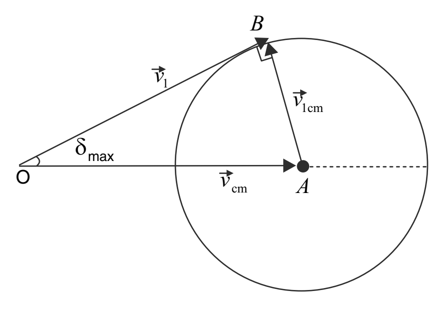
Velocity $\vec{v}_1$ of mass M in lab frame can be obtained by $\vec{v}_1 = \vec{v}_{1cm} + \vec{v}_{cm}$

Let $\vec{O}A$ represent $\vec{v}_{cm}$. Keeping the tail of $\vec{v}_{1cm}$ at A we can rotate it so that its tip, B moves on a circle of radius

$\left|\vec{v}_{1cm}\right| = \dfrac{mu}{M+m}$. (This is because θ can assume any value.) Vector $\vec{O}B$ gives $\vec{v}_1$. The angle of

deflection is largest when OB is tangent to the circle





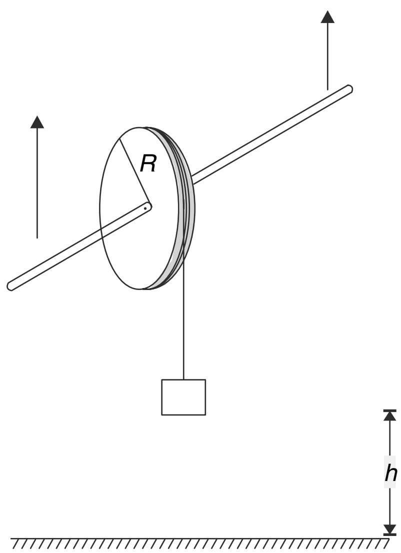
$$\therefore \delta_{\max} = \sin^{-1}\left(\frac{v_{1cm}}{v_{cm}}\right) = \sin^{-1}\left(\frac{m}{M}\right)$$

117. As the steel ball falls on the tennis ball, it compresses it. The tennis ball gets deformed and air pressure inside it increases. As soon as the steel ball comes to rest the tennis ball begins to expand and it pushes the steel ball up. The moment the steel ball separates from the tennis ball the upper point of the tennis ball has same speed as the steel ball but the lower point of the ball (in contact with the floor) is at rest. Therefore, the COM of the tennis ball will have a nonzero velocity and it will bounce.

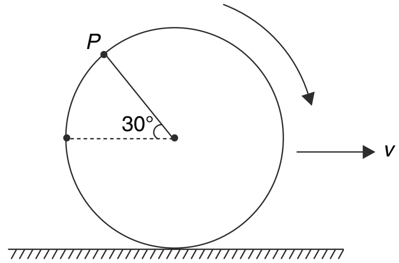
06 ROTATIONAL MOTION

Level 1

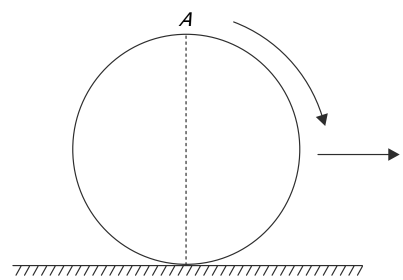
Q. 1. The pulley of radius R can rotate freely about its axle as shown in the figure. A thread is tightly wrapped around the pulley and its free end carries a block of mass m. When the block is at a height h above the ground the system is released (i.e., the pulley is made free to rotate & the block is allowed to fall) and at the same instant the axle is moved up keeping it horizontal all the time. When the block hits the floor the axle has gone up by a distance $2h$. Find the angle by which the pulley must have rotated by this time.



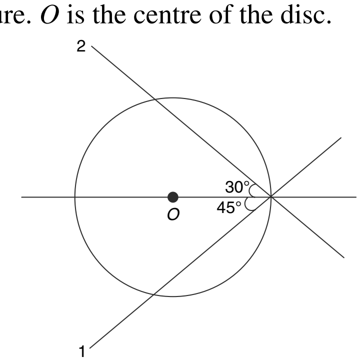
Q. 2. A disc is rolling without sliding on a horizontal surface. Velocity of the centre of the disc is v. Find the maximum relative speed of any point on the circumference of the disc with respect to point P.



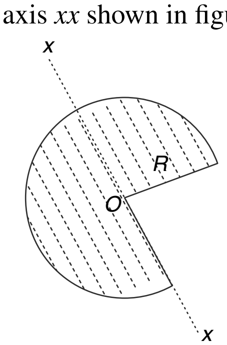
Q. 3. A ring is rolling, without slipping on a horizontal surface with constant velocity. Speed of point A (at the top) is v_A. After an interval T, the speed of point A again becomes v_A. During what fraction of the interval T speed of point A was greater than $\frac{\sqrt{3}}{2} v_A$.



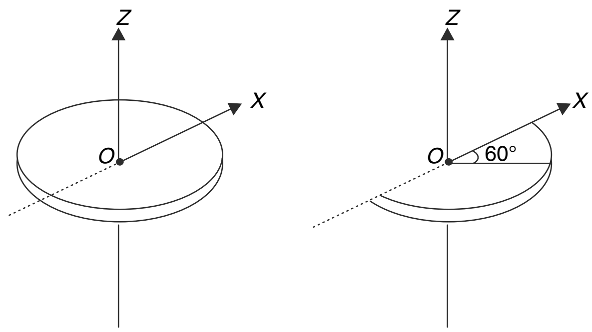
Q. 4. Calculate the ratio of moment of inertia of a thin uniform disc about axis 1 and 2 marked in the figure. O is the centre of the disc.



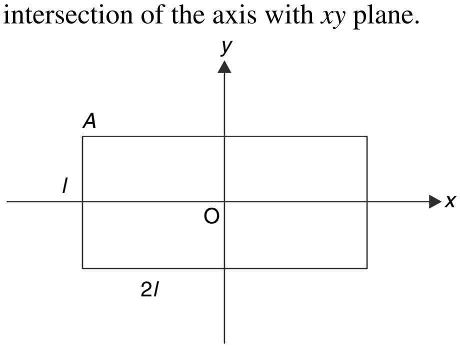
Q. 5. A uniform circular disc has a sector of angle 90° removed from it. Mass of the remaining disc is M. Write the moment of inertia of the remaining disc about the axis xx shown in figure (Radius is R)



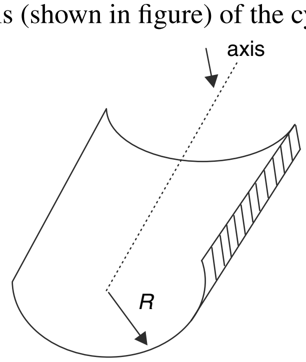
Q. 6. An Indian bread "Roti" is a uniform disc of mass M and radius R. Before eating a person usually folds it about its diameter (say about x axis). After folding it a sector of angle 60° is removed from it. Find the moment of inertia of the remaining "Roti" about Z-axis.



Q. 7. A uniform rectangular plate has side length ℓ and 2ℓ. The plate is in $x - y$ plane with its centre at origin and sides parallel to x and y axes. The moment of inertia of the plate about an axis passing through a vertex (say A) perpendicular to the plane of the figure is I_0. Now the axis is shifted parallel to itself so that moment of inertia about it still remains I_0. Write the locus of point of intersection of the axis with xy plane.



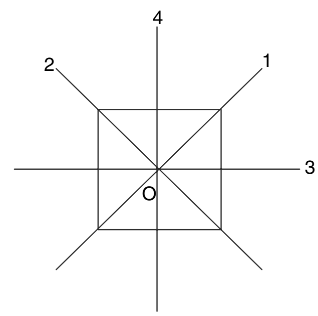
Q. 8. A thin semi circular cylindrical shell has mass M and radius R. Find its moment of inertia about a line passing through its centre of mass parallel to the axis (shown in figure) of the cylinder.



Q. 9. Consider a uniform square plate shown in the figure. I_1, I_2, I_3 and I_4 are moment of inertia of the plate about the axes 1, 2, 3 and 4 respectively. Axes 1 and 2 are diagonals and 3 and 4 are lines passing through centre parallel to sides of the square. The moment of inertia of the plate about an axis passing through centre and perpendicular to the plane of the figure is equal to which of the followings.

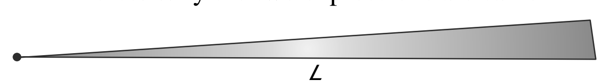
(a) $I_3 + I_4$ (b) $I_1 + I_3$

(c) $I_2 + I_3$ (d) $\frac{1}{2}(I_1 + I_2 + I_3 + I_4)$

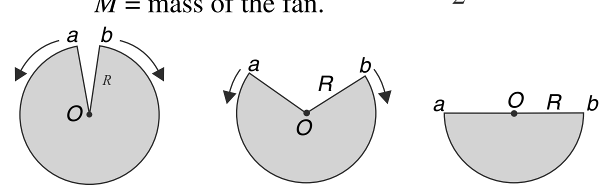


Q. 10. An asteroid in the shape of a uniform sphere encounters cosmic dust. A thin uniform layer of dust gets deposited on it and its mass increases by 2%. Find percentage change in its moment of inertia about diameter.

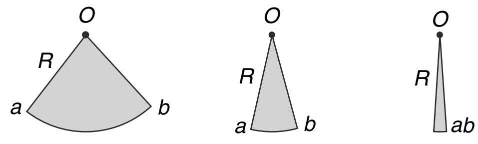
Q. 11. (i) Consider an infinitesimally thin triangular strip having mass M and length L. Find the moment of inertia of the strip about on axis passing through its tip and perpendicular to the plane. Compare the result with moment of inertia of a uniform disc of mass M and radius L about an axis passing through its centre and perpendicular to the plane the disc. Why the two expressions are same?



(ii) A circular fan made of paper is in shape of a disc of radius R. The fan can be folded (various stages shown in figure (a) through (f)) to the shape of a thin stick. The moment of inertia of the circular fan about an axis passing through centre O and perpendicular to the plane of the figure is $I_0 = \frac{1}{2}MR^2$ where M = mass of the fan.



(a) (b) (c)



(d) (e) (f)

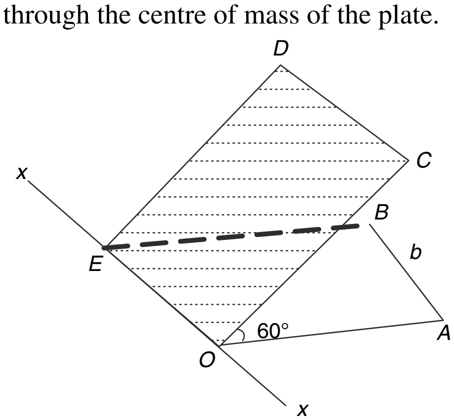
(a) How does the moment of inertia (I), about an axis perpendicular to the plane of the figure passing through O, change as the fan is folded through stage a to b to c to d to e?

(b) When the fan is completely folded in the shape of a stick (fig. (f)), write its moment of inertia about the above mentioned axis.

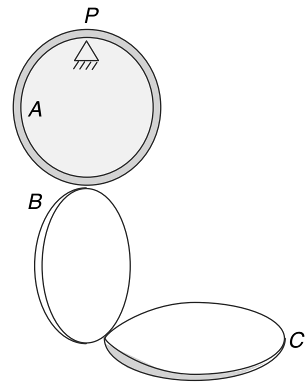
Note : Moment of inertia of a uniform rod about an axis through its end and perpendicular to it is $\frac{ML^2}{3}$.

Q. 12. A uniform rectangular plate has moment of inertia about its longer side, equal to I. The moment of inertia of the plate about an axis in its plane, passing through the centre and parallel to the shorter sides is also equal to I. Find its moment of inertia about an axis passing through its centre and perpendicular to its plane.

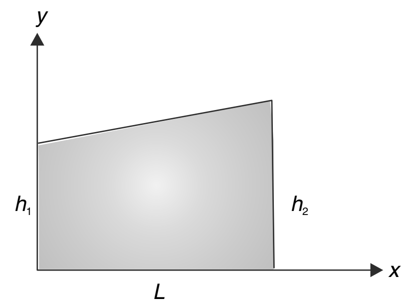
Q. 13. A uniform rectangular plate has been bent as shown in the figure. The two angled parts of the plate are of identical size. The moment of inertia of the bent plate about axis xx is I. Find its moment of inertia about an axis parallel to xx and passing through the centre of mass of the plate.



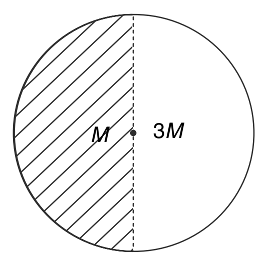
Q. 14. Three identical rings each of mass M and radius R are welded together with their planes mutually perpendicular to each other. Ring A is vertical and B is also vertical in a plane perpendicular to A. Ring C is in horizontal plane. Find moment of Inertia of this system about a horizontal axis perpendicular to the plane of the figure passing through point P (top point of ring A)



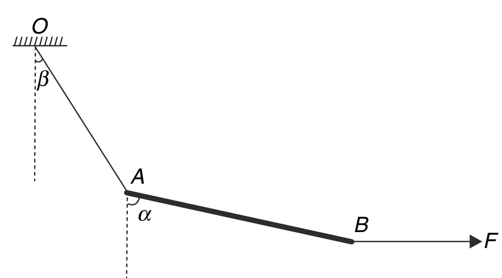
Q. 15. Determine the moment of inertia of the shaded area about y axis. The mass of the shaded area is M.



Q. 16. Two uniform semicircular discs, each of radius R, are stuck together to form a disc. Masses of the two semicircular parts are M and $3M$. Find the moment of inertia of the circular disc about an axis perpendicular to its plane and passing through its centre of mass.

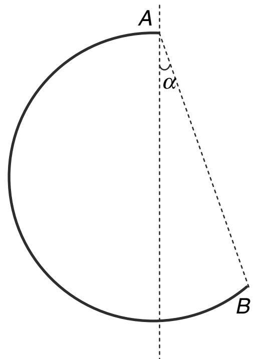


Q. 17. A stick AB of mass M is tied at one end to a light string OA. A horizontal force $F = Mg$ is applied at end B of the stick and its remains in equilibrium in position shown. Calculate angles α and β.

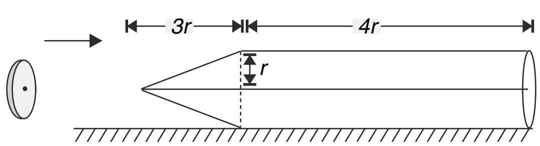


Q. 18. When brakes are applied on a moving car, the car dips to the front. Why ? [That is try to show that front wheels are more pressed as compared to rear ones when the brakes are applied]. Assume that centre of mass of the car is equidistant from the front and rear wheels.

Q. 19. A uniform wire has been bent in shape of a semi circle. The semicircle is suspended about a horizontal axis passing through one of its ends, so that the semicircular wire can swing in vertical plane. Find the angle α that the diameter of the semicircle makes with vertical in equilibrium.

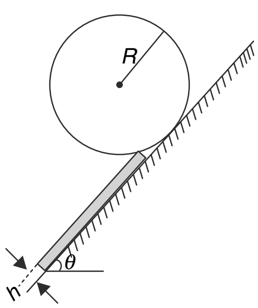


Q. 20. A uniform cylindrical body of radius r has a conical nose. The length of the cylindrical and conical parts are $4r$ and $3r$ respectively. Mass of the conical part is M. The body rests on a horizontal surface as shown. A ring of radius $\frac{r}{2}$ is to be tightly fitted on the nose of the body. What is maximum permissible mass of the ring so that the body does not topple?



Q. 21. There is a step of height h on an incline plane. The step prevents a ball of radius R from rolling down.

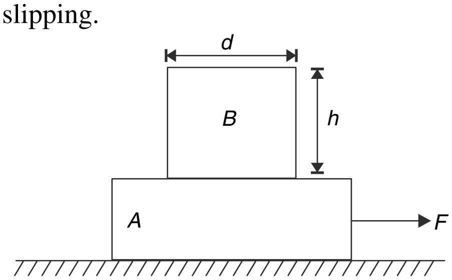
(a) If the inclination (θ) of the incline is increased gradually, at what value of θ the ball will just manage to climb the step?



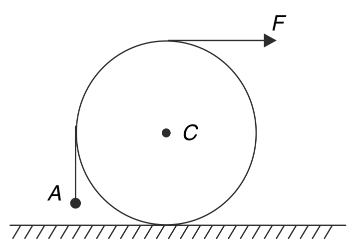
(b) Does the gravitational potential energy of the ball increases or decreases as it climbs the step?

Q. 22. The centre of mass of an inhomogeneous sphere is at a distance of 0.3 R from its geometrical centre. R is the radius of the sphere. Find the maximum inclination (θ) of an incline plane on which this sphere can be placed in equilibrium. Assume that friction is large enough to prevent slipping.

Q. 23. Rectangular block B, having height h and width d has been placed on another block A as shown in the figure. Both blocks have equal mass and there is no friction between A and the horizontal surface. A horizontal time dependent force $F = kt$ is applied on the block A. At what time will block B topple? Assume that friction between the two blocks is large enough to prevent B from slipping.



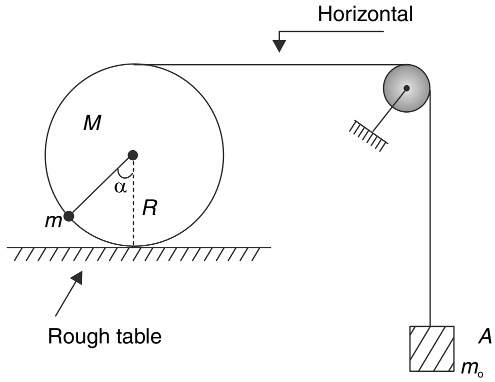
Q. 24. A cylinder C rests on a horizontal surface. A small particle of mass m is held in equilibrium connected to an overhanging string as shown. The other end of the mass less string is being pulled horizontally by a force F as shown. Find F.



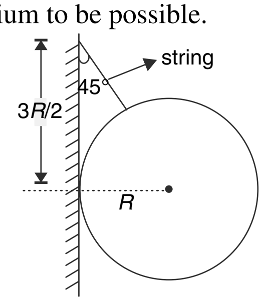
Q. 25. A hollow cylindrical pipe of mass M and radius R has a thin rod of mass m welded inside it, along its length. A light thread is tightly wound on the surface of the pipe. A mass m_0 is attached to the end of the thread as shown in figure. The system stays in equilibrium when the cylinder is placed such that $\alpha = 30°$. The pulley shown in figure is a disc of mass $\frac{M}{2}$.

(a) Find the direction and magnitude of friction force acting on the cylinder.

(b) Express mass of the rod 'm' in terms of m_0



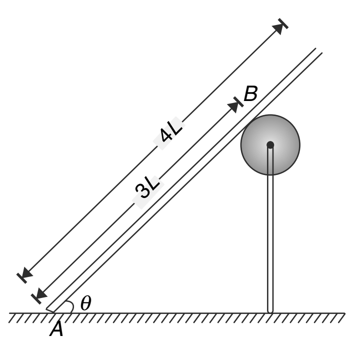
Q. 26. A sphere of radius R is supported by a rope attached to the wall. The rope makes an angle $\theta = 45°$ with respect to the wall. The point where the rope is attached to the wall is at a distance of $\frac{3R}{2}$ from the point where the sphere touches the wall. Find the minimum coefficient of friction (μ) between the wall and the sphere for this equilibrium to be possible.



Q. 27. A uniform rod has mass M and length $4L$. It rests in equilibrium with one end on a rough horizontal surface at A. At point B, at a distance $3L$ from A, it is supported by a fixed smooth roller. The rod just remains in equilibrium when $\theta = 30°$

(a) Find the normal force applied by the horizontal surface on the rod at point A.

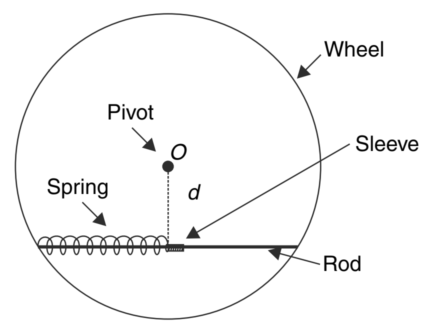
(b) Find the coefficient of friction between the rod and the surface.



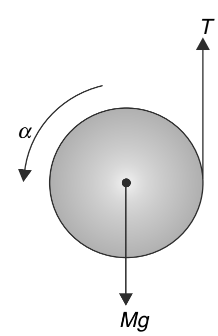
Q. 28. A wheel is mounted on frictionless central pivot and it can rotate freely in the vertical plane. There is a horizontal light rod fixed to the wheel below the pivot. There is a small sleeve of mass m which can slide along the rod without friction. The sleeve is connected to a light spring. The other end of the spring is fixed to the rim as shown. The sleeve is at the centre of the rod and the spring is relaxed. Now the wheel is held at rest and the sleeve is moved towards left so as to compress the sprig by some distance. The sleeve and the wheel are released simultaneously from this position.

(a) Is it possible that the wheel does not rotate as the sleeve perform SHM on the rod ?

(b) Find the value of spring constant k for situation described in (a) to be possible. The distance of rod from centre of the wheel is d.

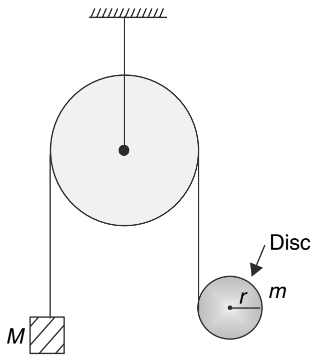


Q. 29. A string is wrapped around a cylinder of mass M and radius R. The string is pulled vertically upward to prevent the centre of mass from falling as the string unwinds. Assume that the cylinder remains horizontal throughout and the thread does not slip. Find the length of the string unwound when the cylinder has reached an angular speed ω.



Q. 30. A mass less string is wrapped around a uniform disc of mass m and radius r. The string passes over a mass less pulley and is tied to a block of mass M at its other end (see figure). The system is released from rest. Assume that the string does not slip with respect to the disc.

(a) Find the acceleration of the block for the case $M = m$

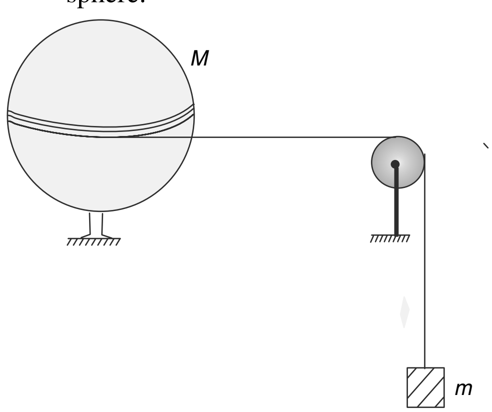


(b) Find $\frac{M}{m}$ for which the block can accelerate upwards.

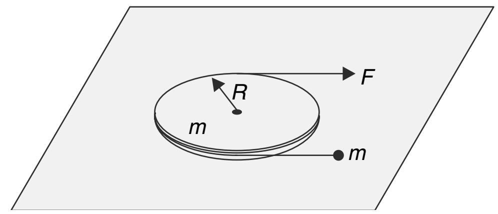
Q. 31. A solid uniform sphere of mass M and radius R can rotate about a fixed vertical axis. There is no frictional torque acting at the axis of rotation. A light string is wrapped around the equator of the sphere. The string has exactly 6 turns on the sphere. The string passes over a light pulley and carries a small mass m at its end (see figure). The string between the sphere and the pulley is always horizontal. The system is released from rest and the small mass falls down vertically. The string does not slip on the sphere till 5 turns get unwound. As soon as 5^{th} turn gets unwound completely, the friction between the sphere and the string vanishes all of a sudden.

(a) Find the angular speed of the sphere as the string leaves it.

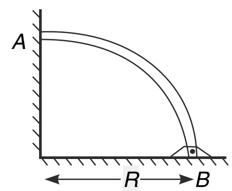
(b) Find the change in acceleration of the small mass m after 5 turns get unwound from the sphere.



Q. 32. A disc of mass m and radius R lies flat on a smooth horizontal table. A mass less string runs halfway around it as shown in figure. One end of the string is attached to a small body of mass m and the other end is being pulled with a force F. The circumference of the disc is sufficiently rough so that the string does not slip over it. Find acceleration of the small body.



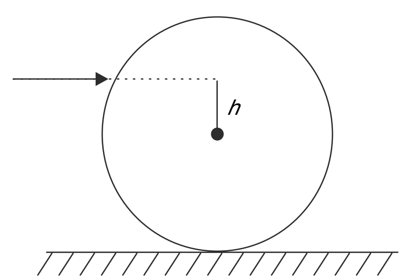
Q. 33. A uniform quarter circular thin rod of mass M and radius R is pivoted at a point B on the floor. It can rotate freely in the vertical plane about B. It is supported by a smooth vertical wall at its other free end A so that it remains at rest. Find the reaction force of wall on the rod.



Q. 34. A ball is rolling without sliding down an incline. Is the force applied by the ball on the incline larger than or less than its (ball's) own weight ?

Q. 35. A solid sphere of mass M and radius R is covered with a thin shell of mass M. There is no friction between the inner wall of the shell and the sphere. The ball is released from rest, and then it rolls without slipping down an incline that is inclined at an angle θ to the horizontal. Find the acceleration of the ball.

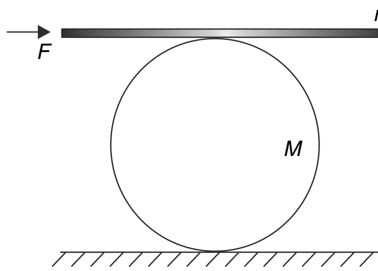
Q. 36. A homogeneous solid sphere of radius R is resting on a horizontal surface. It is set in motion by a horizontal impulse imparted to it at a height h above the centre. If h is greater than h_0, the velocity of the sphere increases in the direction of its motion after the start. If $h < h_0$, the velocity decreases after the start. Find h_0



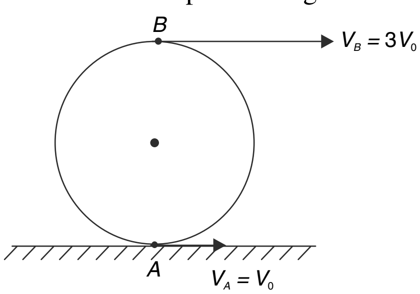
Q. 37. A boy pushes a cylinder of mass M with the help of a plank of mass m as shown in figure. There is no slipping at any contact. The horizontal component of the force applied by the boy on the plank is F. Find

(a) The acceleration of the centre of the cylinder

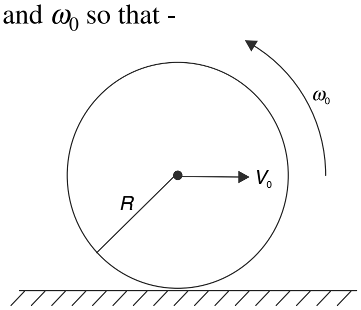
(b) The friction force between the plank and the cylinder



Q. 38. (i) A solid sphere of radius R is released on a rough horizontal surface with its top point having thrice the velocity of its bottom point A ($V_A = V_0$) as shown in figure. Calculate the linear velocity of the centre of the sphere when it starts pure rolling.



(ii) Solid sphere of radius R is placed on a rough horizontal surface with its centre having velocity V_0 towards right and its angular velocity being ω_0 (in anticlockwise sense). Find the required relationship between V_0 and ω_0 so that -



(a) the slipping ceases before the sphere loses all its linear momentum.

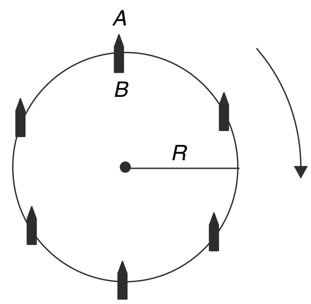
(b) the sphere comes to a permanent rest after some time.

(c) the velocity of centre becomes zero before the spinning ceases.

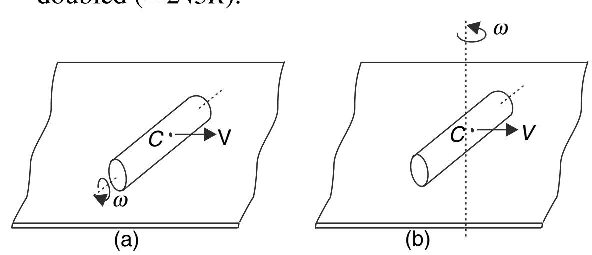
Q. 39. A thin pencil of mass M and length L is being moved in a plane so that its centre (i.e. centre of mass) goes in a circular path of radius R at a constant angular speed ω. However, the orientation of the pencil does not change in space. Its tip (A) always remains above the other end (B) in the figure shown

(a) Write the kinetic energy of the pencil.

(b) Find the magnitude of net force acting on the pencil.

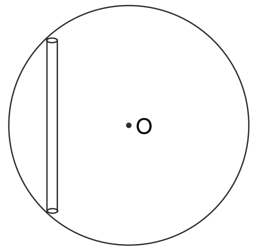


Q. 40. In figure (a) there is a uniform cylinder of mass M and radius R. Length of the cylinder is $L = \sqrt{3}R$. The cylinder is rolling without sliding on a horizontal surface with its centre moving at speed V. In figure (b) the same cylinder is moving on a horizontal surface with its centre moving at speed V and the cylinder rotating about a vertical axis passing through its centre. [Place your pencil on the table and give a sharp blow at its end. Look at the motion of the pencil. This is how the cylinder is moving]. The angular speed is $\omega = \dfrac{V}{R}$. Write the kinetic energy of the cylinder in two cases. In which case, the kinetic energy would have been higher if length of the cylinder were doubled ($= 2\sqrt{3}R$).



Q. 41. There is a fixed hollow cylinder having smooth inner surface. Radius of the cylinder is $R = 4m$. A uniform rod of $M = 4kg$ and length $L = 4m$ is released from vertical position inside the cylinder as shown in the figure. Convince yourself that the rod will perform pure rotation about the axis of the cylinder passing through O.

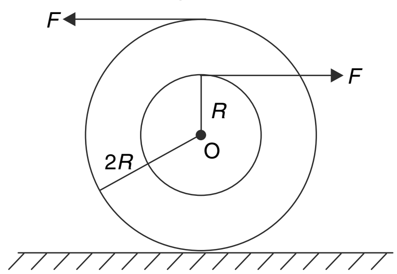
Fid the angular speed of the rod when its becomes horizontal.



Q. 42. A disc shaped body has two tight windings of light threads - one on the inner rim of radius $R = 1m$ and the other on outer rim of radius $2R$ (see figure). It is kept on a horizontal surface and the ends of the two threads are pulled horizontally in opposite directions with force of equal magnitude $F = 20N$. Mass of the body and its moment of inertia about an axis through centre O and perpendicular to the plane of the figure are $M = 4kg$ and $I = 8kg - m^2$ respectively. Find the kinetic energy of the body 2 seconds after the forces begin to act, if

(i) the surface is smooth,

(ii) the surface is rough enough to ensure rolling without sliding.



Q. 43. A uniform square plate has mass M and side length a. It is made to oscillate in vertical plane in two different ways shown in figure (A) and (B). In figure (A), the plate is hinged at its upper corners with the help of two mass less rigid rods each of length a. The rods can rotate freely about both ends.

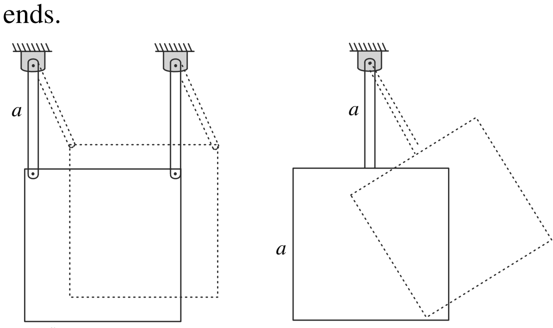
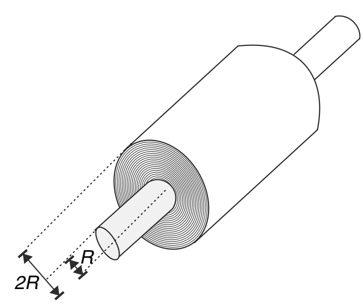


fig. (a) fig. (b)

In figure (B) the plate is rigidly connected at the centre of its top edge to a mass less rod of length a. The rod can rotate about its upper end only. In both cases the plate is pushed from its equilibrium position so that centre of mass of the plate acquires a speed V. In which case will the centre of mass of the plate rise to a greater height. There is no friction

Q. 44. A thin carpet of mass $2m$ is rolled over a hollow cylinder of mass m. The cylinder wall is thin and radius of the cylinder is R. The carpet rolled over it has outer radius $2R$ (see figure). This roll is placed on a rough horizontal surface and given gentle push so that the carpet begins to roll and unwind. Friction is large enough to prevent any slipping of the carpet on the floor. Also assume that the carpet does not slip on the surface of the cylinder. The entire carpet is laid out on the floor and the hollow cylinder rolls out with speed V. Find V.

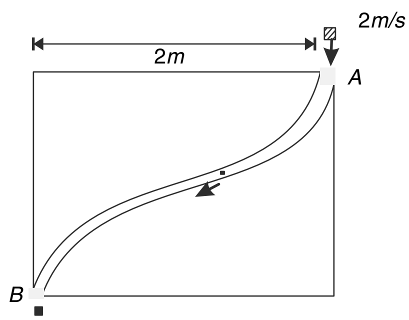


Q. 45. A uniform rod of mass M is moving in a plane and has a kinetic energy of $\frac{4}{3}MV^2$ where V is speed of its centre of mass. Find the maximum and minimum possible speed of the end point of the rod.

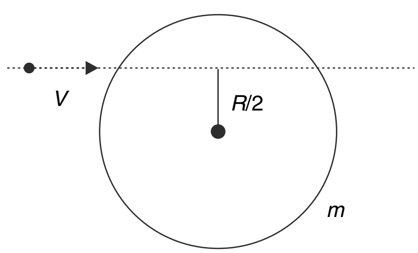
Q. 46. The propeller of a small airplane is mounted in the front. The propeller rotates clockwise if seen from behind by the pilot. The plane is flying horizontally and the pilot suddenly turns it to the right. Will the body of the plane have a tendency to get inclined to the horizontal? If yes, does the nose of the plane veer upward or downward? Why?

Q. 47. A massive star is spinning about its diameter with an angular speed $\omega_0 = \frac{\pi}{1000}$ rad/day. After its fuel is exhausted, the star collapses under its own gravity to form a neutron star. Assume that the volume of the star decreases to 10^{-12} times the original volume and its shape remains spherical. Assuming that density of the star is uniform, find the angular speed of the neutron star.

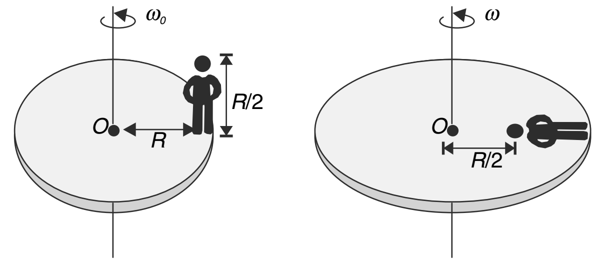
Q. 48. A square plate of side length $2m$ has a groove made in the shape of two quarter circles joining at the centre of the plate. The plate is free to rotate about vertical axis passing through its centre. The moment of inertia of the plate about this axis is $4\ kg - m^2$. A small block of mass $1\ kg$ enters the groove at end A travelling with a velocity of $2m/s$ parallel to the side of the square plate. The block move along the frictionless groove of the horizontal plate and comes out at the other end B with speed V. Find V assuming that width of the groove is negligible.



Q. 49. A disc of mass m and radius R lies flat on a smooth horizontal table. A particle of mass m, moving horizontally along the table, strikes the disc with velocity V while moving along a line at a distance $\frac{R}{2}$ from the centre. Find the angular velocity acquired by the disc if the particle comes to rest after the impact.



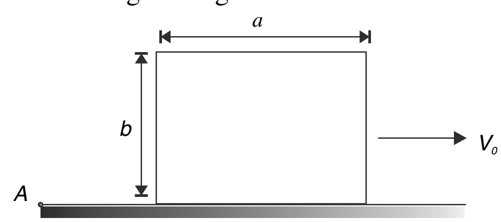
Q. 50. A disc of mass M and radius R is rotating with angular velocity ω_0 about a vertical axis passing through its centre (O). A man of mass $\frac{M}{2}$ and height $\frac{R}{2}$ is standing on the periphery. The man gradually lies down on the disc such that his head is at a distance $\frac{R}{2}$ from the centre and his feet touching the edge of the disc. For simplicity assume that the man can be modelled as a thin rod of length $\frac{R}{2}$. Calculate the angular speed (ω) of the platform after the man lies down.



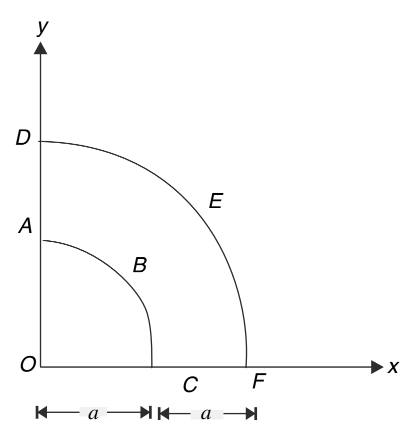
Q. 51. A uniform block of mass M and dimensions as shown in the figure is placed on a rough horizontal surface and given a velocity V_0 to the right. A is a point on the surface to the left of the block.

(a) Write the angular momentum of the block about point A just after it begins to move

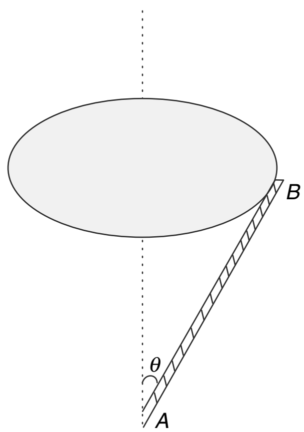
(b) Due to friction the block stops. What happened to its angular momentum about point A? Which torque is responsible for change in angular momentum of the block?



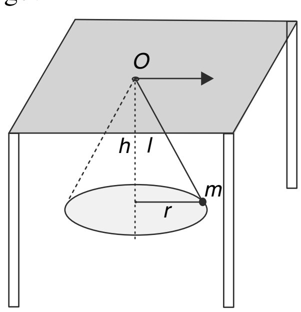
Q. 52. ABCFED is a uniform plate (shown in figure). ABC and DEF are circular arcs with common centre at O and having radii a and $2a$ respectively. This plate is lying on a smooth horizontal table. A particle of mass half the mass of the plate strikes the plate at point A while travelling horizontally along the x direction with velocity u. The particle hits the plate and rebounds along negative x with velocity $\frac{u}{2}$. Find the velocity of point D of the plate immediately after the impact. [Take $\frac{28}{9\pi} \simeq 1$]



Q. 53. A uniform rod of mass m and length L is fixed to an axis, making an angle θ with it as shown in the figure. The rod is rotated about this axis so that the free end of the rod moves with a uniform speed 'v'. Find the angular momentum of the rod about the axis. Is the angular momentum of the rod about point A constant?



Q. 54. A mass m in attached to a mass less string and swings in a horizontal circle, forming a conical pendulum, as shown in the figure. The other end of the string passes through a hole in the table and is dragged slowly so as to reduce the length l. The string is slowly drawn up so that the depth h shown in the figure becomes half. By what factor does the radius (r) of the circular path of the mass m change?

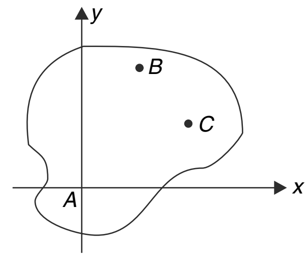


LEVEL 2

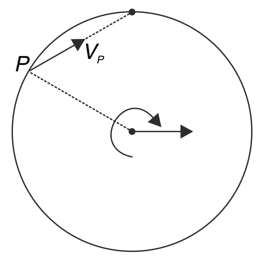
Q. 55. A flat rigid body is moving in $x - y$ plane on a table. The plane of the body lies in the $x - y$ plane. At an instant it was found that some of the velocity components of its three particles A, B and C were $V_{Ax} = 4m/s$, $V_{Bx} = 3m/s$ and $V_{cy} = -2m/s$, respectively. At the instant the three particles A, B and C were located at (0,0) (3,4) , (4,3) (all in meter) respectively in a co-ordinate system attached to the table.

(a) Find the velocity of A, B and C

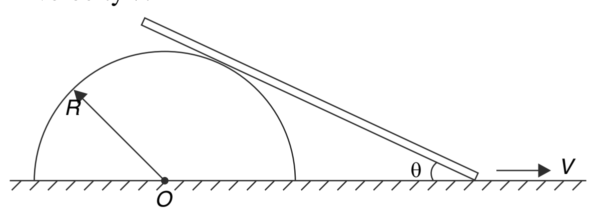
(b) Find the angular velocity of the body.



Q. 56. A wheel is rolling without sliding on a horizontal surface. Prove that velocities of all points on the circumference of the wheel are directed towards the top most point of the wheel.



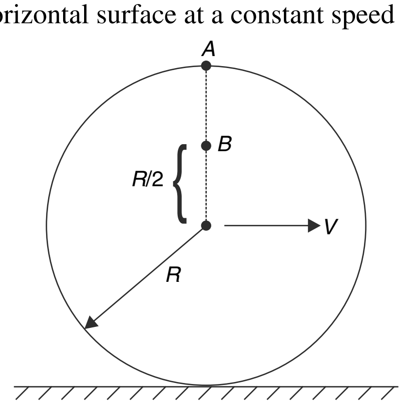
Q. 57. There is a fixed half cylinder of radius R on a horizontal table. A uniform rod of length $2R$ leans against it as shown. At the instant shown, $\theta = 30°$ and the right end of the rod is sliding with velocity v.



(a) Calculate the angular speed of the rod at this instant.

(b) Find the vertical component of the velocity of the centre of the rod at this instant.

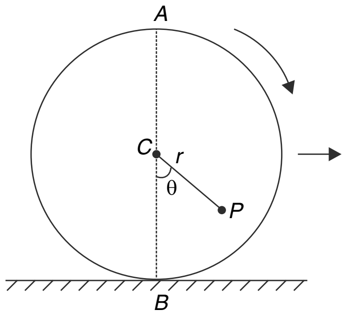
Q. 58. A disc of radius R is rolling without sliding on a horizontal surface at a constant speed of v



(a) What is speed of points A and B on the vertical diameter of the disc ? Given $AB = \frac{R}{2}$

(b) After what time, for the first time, speed of point A becomes equal to present speed (i.e., the speed at the instant shown in the figure) of point B?

Q. 59. A uniform disc of radius $R = 2\sqrt{3}\ m$ is moving on a horizontal surface without slipping. At some instant its angular velocity is $\omega = 1$ rad/s and angular acceleration is $\alpha = \sqrt{3}\ rad/s^2$.

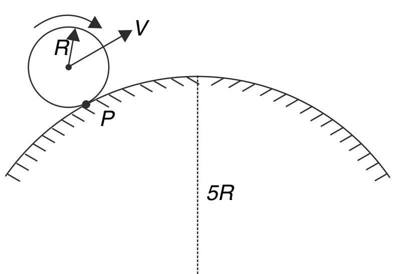


(a) Find acceleration of the top point A.

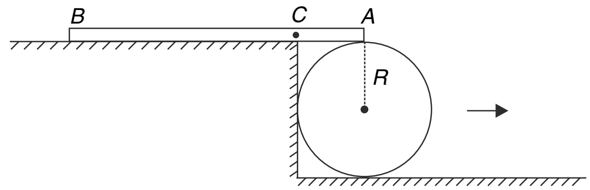
(b) Find acceleration of contact point B.

(c) Find co - oridnates (r, θ) for a point P which has zero acceleration.

Q. 60. A convex surface has a uniform radius of curvature equal to $5R$. A wheel of radius R is rolling without sliding on it with a constant speed v. Find the acceleration of the point (P) of the wheel which is in contact with the convex surface.



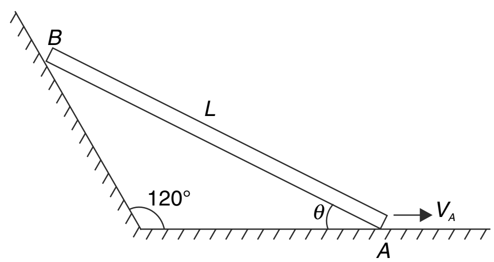
Q. 61. AB is a non uniform plank of length $L = 4R$ with its centre of mass at C such that $AC = R$. It is placed on a step with its one end A supported by a cylinder of radius R as shown in figure. The centre of mass of the plank is just outside the edge of the step. The cylinder is slowly rolled on the lower step such that there is no slipping at any of its contacts. Calculate the distance through which the centre of the cylinder moves before the plank loses contact with the horizontal surface of the upper step.



Q. 62. A wheel of radius R is rolling without sliding uniformly on a horizontal surface. Find the radius of curvature of the path of a point on its circumference when it is at highest point in its path.

Q. 63. A wall is inclined to a horizontal surface at an angle of 120° as shown. A rod AB of length L = 0.75 m is sliding with its two ends A and B on the horizontal surface and on the wall respectively. At the moment angle $\theta = 20°$ (see figure), the velocity of end A is $v_A = 1.5\ m/s$ towards right. Calculate the angular speed of the rod at this instant.

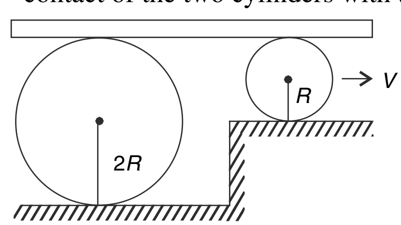
[Take cos 40° = 0.766]



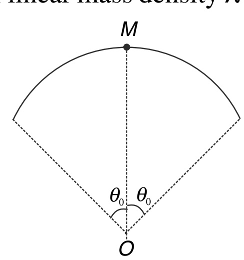
Q. 64. In the figure the plank resting on two cylinders is horizontal. The plank is pulled to the right such that the centre of smaller cylinder moves with a constant velocity v. Friction is large enough to prevent slipping at all surfaces. Find-

(a) The velocity of the centre of larger cylinder.

(b) The ratio of accelerations of the points of contact of the two cylinders with the plank.



Q. 65. A wire of linear mass density λ (kg/m) is bent into



an arc of a circle of radius R subtending an angle $2\theta_0$ at the centre. Calculate the moment o inertia of this circular arc about an axis passing through its midpoint (M) and perpendicular to its plane.

Q. 66. A metallic plate has been fabricated by welding two semicircular discs - D_1 and D_2 of radii R and $\frac{R}{2}$ respectively (fig. a). O and O' are the centre of curvature of the two discs and each disc has a mass $6m$. The plate is in xy plane. Now the plate is folded along the y axis so as to bring the part OAB in yz plane. (fig. b). The plate is now set free to be able to rotate freely about the z – axis. A particle of mass m, moving with a velocity v in the xy plane along the line $x = d$ hits the plate and sticks to it ($d < R$). Just before collision speed of the particle was v.

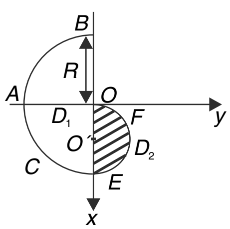


Fig (a)

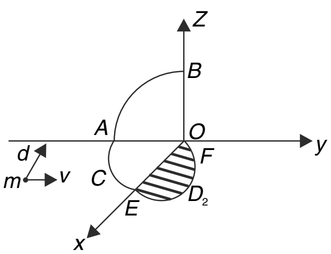


Fig (b)

(a) Find the moment of inertia of inertia of the plate about z axis.

(b) Find the angular speed of the plate after collision.

Q. 67. There is a square plate of side length a. It is divided into nine identical squares each of side $\frac{a}{3}$ and the central square is removed (see fig. (i)). Now each of the remaining eight squares of side length $\frac{a}{3}$ are divided into nine identical squares and central square is removed from each of them (see fig. (ii)).

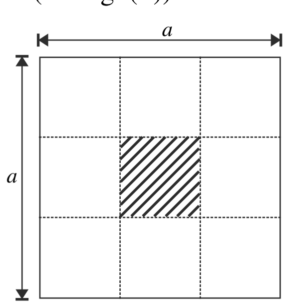


Fig. (i)

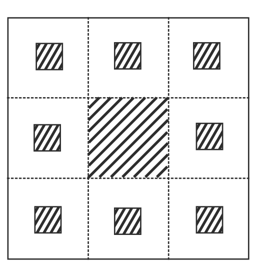
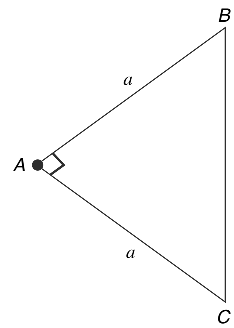


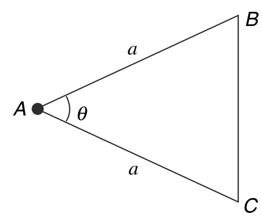
Fig. (ii)

Mass of the plate with one big and eight small holes is M. Find its moment of inertia about an axis passing through its centre and perpendicular to its plane.

Q. 68. ABC is an isosceles triangle right angled at A. Mass of the triangular plate is M and its equal sides are of length a. Find the moment of inertia of this plate about an axis through A perpendicular to the plane of the plate. Use the expression of moment of inertia for a square plate that you might have studied.



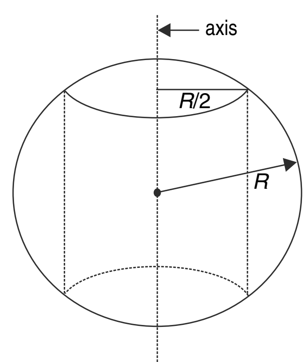
Q. 69. The triangular plate described in the last question has angle $<A = \theta$. Now find its moment of inertia about an axis through A perpendicular to the plane of the plate.



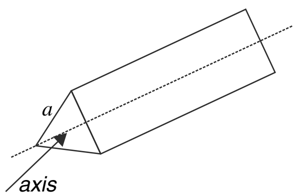
Q. 70. A thin uniform spherical shell of radius R is bored such that the axis of the boring rod passes through the centre of the sphere. The boring rod is a cylinder of radius $\frac{R}{2}$. Take the mass of the sphere before boring to be M.

(a) Find the mass of the leftover part

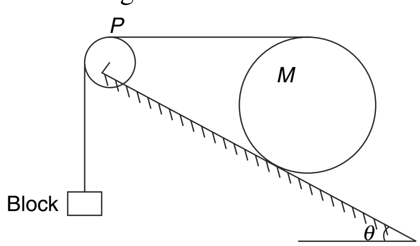
(b) Find the moment of inertia of the leftover part about the axis shown.



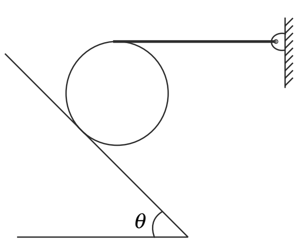
Q. 71. Consider an equilateral prism as shown in the figure. The mass of the prism is M and length of each side of its cross section is a. Find the moment of inertia of such a prism about the central axis shown.



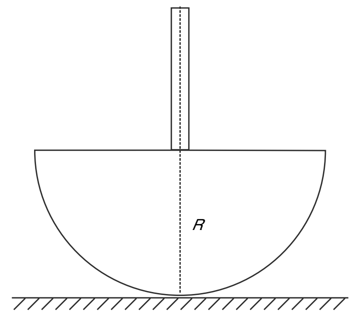
Q. 72. In the arrangement shown in figure the cylinder of mass M is at rest on an incline. The string between the cylinder and the pulley (P) is horizontal. Find the minimum coefficient of friction between the incline and the cylinder which can keep the system in equilibrium. Also find the mass of the block. Assume no friction between the pulley (P) and the string.



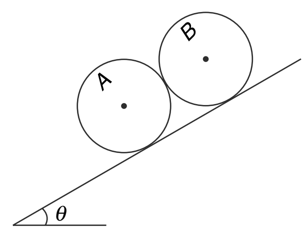
Q. 73. A horizontal stick of mass m has its right end attached to a pivot on a wall, while its left end rests on the top of a cylinder of mass m which in turn rests on an incline plane inclined at an angle θ. The stick remains horizontal. The coefficient of friction between the cylinder and both the plane and the stick is μ. Find the minimum value of μ as function of θ for which the system stays in equilibrium.



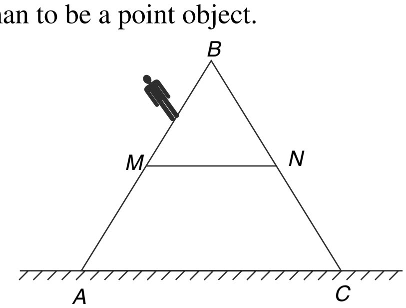
Q. 74. Consider the object shown in the figure. It consist of a solid hemisphere of mass M and radius R. There is a solid rod welded at its centre. The object is placed on a flat surface so that the rod is vertical. Mass of the rod per unit length is $\frac{M}{2R}$. What is the maximum length of the rod that can be welded so that the object can perform oscillations about the position shown in diagram? Note : Centre of mass of a solid hemisphere is at a distance of $\frac{3R}{8}$ from its base.



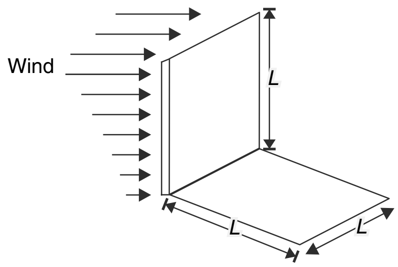
Q. 75. Two cylinders A and B have been placed in contact on an incline. They remain in equilibrium. The dimensions of the two cylinders are same. Which cylinder has larger mass?



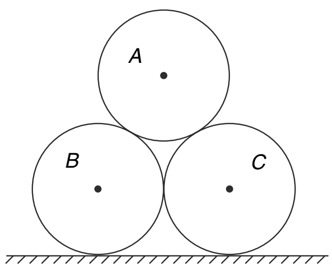
Q. 76. The ladder shown in the figure is light and stands on a frictionless horizontal surface. Arms AB and BC are of equal length and M and N are their mid points. Length of MN is half that of AB. A man of mass M is standing at the midpoint of BM. Find the tension in the mass less rod MN. Consider the man to be a point object.



Q. 77. A uniform metal sheet of mass M has been folded to give it L shape and it is placed on a rough floor as shown in figure. Wind is blowing horizontally and hits the vertical face of the sheet as shown. The speed of air varies linearly from zero at floor level to v_0 at height L from the floor. Density of air is ρ. Find maximum value of v_0 for which the sheet will not topple. Assume that air particles striking the sheet come to rest after collision, and that the friction is large enough to prevent the sheet from sliding.

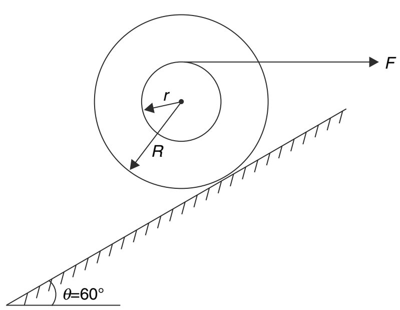


Q. 78. Three identical cylinders have mass M each and are placed as shown in the figure. The system is in equilibrium and there is no contact between B and C. Find the normal contact force between A and B.



Q. 79. A spool is kept in equilibrium on an incline plane as shown in figure. The inner and outer radii of the spool are in ratio $\frac{r}{R} = \frac{1}{2}$. The force applied on the thread (wrapped on part of radius r) is horizontal. Find the angle that the force applied by the incline on the spool makes with the vertical.

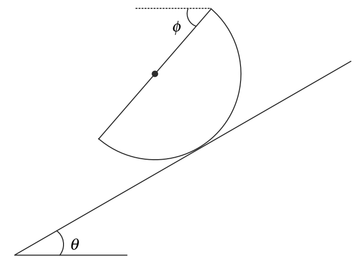
[Take $\tan^{-1}\left(\frac{\sqrt{3}}{5}\right) \simeq 19°$]



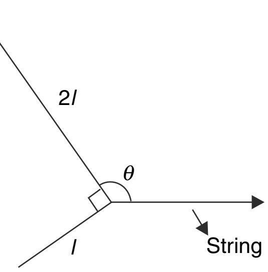
Q. 80. A uniform hemisphere placed on an incline is on verge of sliding. The coefficient of friction between the hemisphere and the incline is $\mu = 0.3$.

Find the angle ϕ that the circular base of the hemisphere makes with the horizontal.

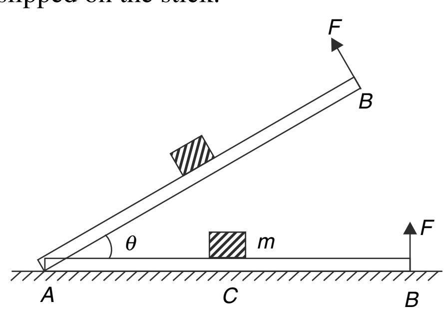
Given $\sin(\tan^{-1} 0.3) \simeq 0.29$ and $\sin^{-1}(0.77) \simeq 50°$



Q. 81. A L shaped, uniform rod has its two arms of length l and $2l$. It is placed on a horizontal table and a string is tied at the bend. The string is pulled horizontally so that the rod slides with constant speed. Find the angle θ that the longer side makes with the string. Assume that the rod exerts uniform pressure at all points on the table.



Q. 82. A uniform meter stick AB of mass M is lying in state of rest on a rough horizontal plane. A small block of mass m is placed on it at its centre C. A variable force F is applied at the end B of the stick so as to rotate the stick slowly about A in vertical plane. The force F always remains perpendicular to the length of the stick. The stick is raised to $\theta = 60°$ and it was observed that neither the end A slipped on the ground nor the block of mass m slipped on the stick.



(a) F_1 is force applied by the stick on the block. Plot the variation of F_1 with θ $(0 \le \theta \le 60°)$.
(b) What must be the minimum coefficient of friction between the block and the stick.
(c) f is the friction force acting at end A of the stick. Plot variation of f *vs* θ $(0° \le \theta \le 60°)$.

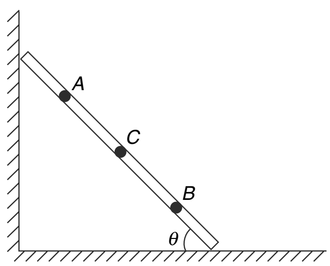
Q. 83. A ladder of mass M and length L stays at rest against a smooth wall. The coefficient of friction between the ground and the ladder is μ.

(a) Let F_{wall}, W and F_g be the force applied by wall, weight of the ladder and force applied by ground on the ladder. Argue to show that the line of action of these three forces must intersect.

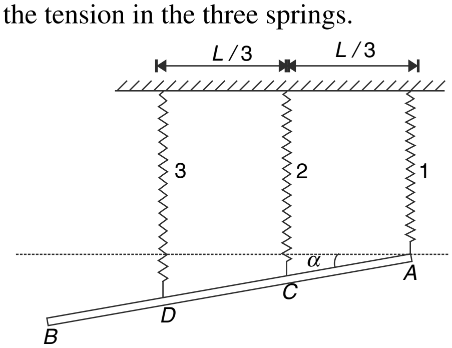
(b) Using the result obtained in (a) show that line of action of F_g makes an angle $\tan^{-1}(2\tan\theta)$ with the horizontal ground where θ is the angle made by the ladder with the ground.

(c) Find the smallest angle that the ladder can make with the ground and not slip.

(d) You climb up the ladder, your presence makes the ladder more likely to slip. Where are you at A or B? C is the centre of mass of the ladder.



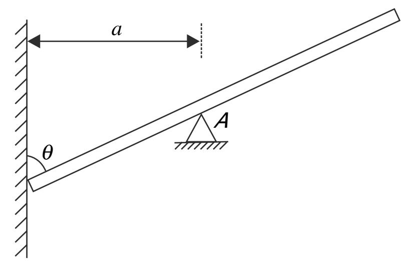
Q. 84. A uniform rod AB has mass M and length L. It is in equilibrium supported in vertical plane by three identical springs as shown in figure. The springs are connected at A, C and D such that $AC = CD = \frac{L}{3}$. Assume that the springs are very stiff and the angle α made by the rod with the horizontal in equilibrium position is very small. (All springs are nearly vertical). Calculate the tension in the three springs.



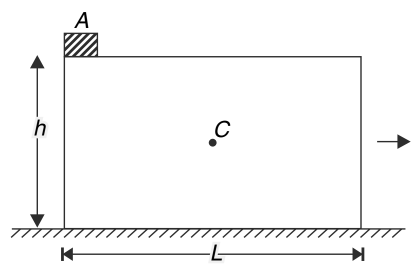
Q. 85. A uniform rod of length b can be balanced as shown in figure. The lower end of the rod is resting against a vertical wall. The coefficient of friction between the rod and the wall and that between the rod and the support at A is μ. Distance of support from the wall is a.

(a) Find the ratio $\frac{a}{b}$ if the maximum value of θ is θ_1.

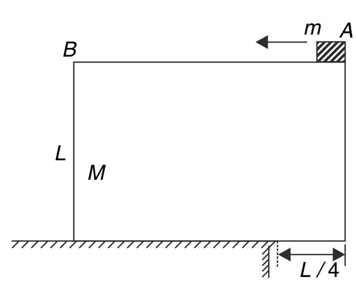
(b) Find the ratio $\frac{a}{b}$ if the minimum value of θ is θ_2.



Q. 86. A uniform rectangular block is moving to the right on a rough horizontal floor (the block is retarding due to friction). The length of the block is L and its height is h. A small particle (A) of mass equal to that of the block is stuck at the upper left edge. Coefficient of friction between the block and the floor is $\mu = \frac{2}{3}$. Find the value of h (in terms of L) if the normal reaction of the floor on the block effectively passes through the geometrical centre (C) of the block.



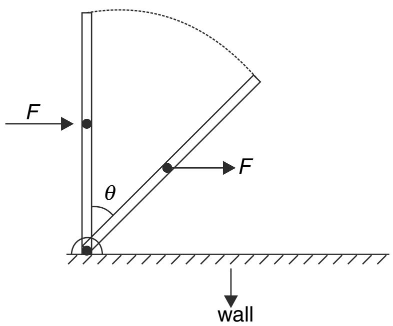
Q. 87. A uniform cubical block of mass M and side length L is lying on the edge of a rough table with $\frac{1}{4}th$ of its edge overhanging. When a small block of mass m is placed on its top surface at the right edge (see fig.), the cube is on verge of toppling. The block of mass m is given a sharp horizontal impulse so that it acquires a velocity towards B. The small block moves on the top surface and falls on the other side. What is maximum coefficient of friction between the small block and the cube so that the cube does not rotate as the block moves over it. Assume that the friction between the cube and the table is large enough to prevent sliding of the cube on the table.



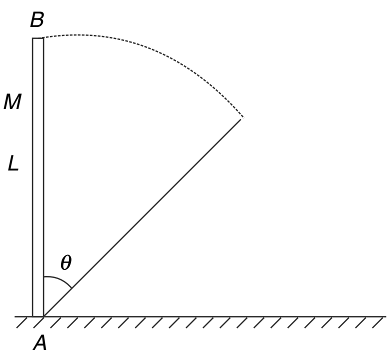
Q. 88. A uniform rod of mass M and length L is hinged at its end to a wall so that it can rotate freely in a horizontal plane. When the rod is perpendicular to the wall a constant force F starts acting at the centre of the rod in a horizontal direction perpendicular to the rod. The force remains parallel to its original direction and acts at the centre of the rod as the rod rotates. (Neglect gravity).

(a) With what angular speed will the rod hit the wall ?

(b) At what angle θ (see figure) the hinge force will make a 45° angle with the rod ?



Q. 89. A rod of mass $M = 5kg$ and length $L = 1.5\ m$ is held vertical on a table as shown. A gentle push is given to it and it starts falling. Friction is large enough to prevent end A from slipping on the table.



(a) Find the sum of linear momentum of all the particles of the rod when it rotates through an angle $\theta = 37°$.

(b) Find the friction force and normal reaction force by the table on the rod, when $\theta = 37°$.

(c) Find value of angle θ when the friction force becomes zero.

$$[\tan 37° = \frac{3}{4} \text{ and } g = 10\ m/s]$$

Q. 90. (a) In the system shown in figure 1, the uniform rod of length L and mass m is free to rotate in vertical plane about point O. The string an pulley are mass less. The block has mass equal to that of the rod. Find the acceleration of the block immediately after the system is released with rod in horizontal position.

(b) System shown in figure 2 is similar to that in figure 1 apart from the fact that rod is mass less and a block of mass m is attached to the centre of the rod with the help of a thread. Find the acceleration of both the blocks immediately after the system is released with rod in horizontal position.

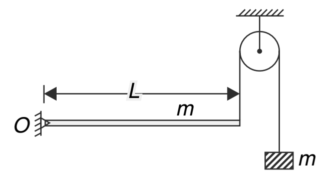


Fig.(1)

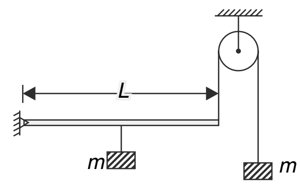
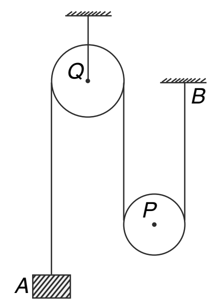


Fig.(2)

Q. 91.

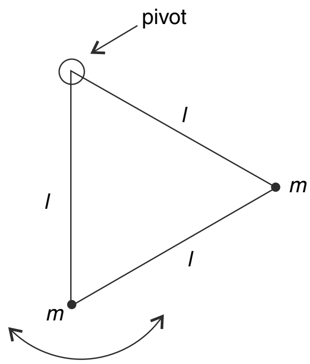


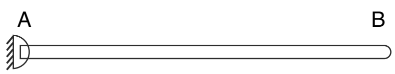
A light thread is wrapped tightly a few turns around a disc P of mass M. One end of the thread is fixed to the ceiling at B. The other end of the thread is passed over a mass less pulley (Q) and carries a block of mass M. All segment of the thread (apart from that on the pulley and disc) are vertical when the system is released. Find the acceleration of block A. On which object – the block A or the ceiling at B – does the thread exert more force ?

Q. 92. An equilateral triangle is made from three mass less rods, each of length l. Two point masses m are attached to two vertices. The third vertex is hinged and triangle can swing freely in a vertical plane as shown. It is released the position shown with one of the rods vertical. Immediately after the system is released, find –

(a) tensions in all three rods (specify tension or compression),

(b) accelerations of the two masses



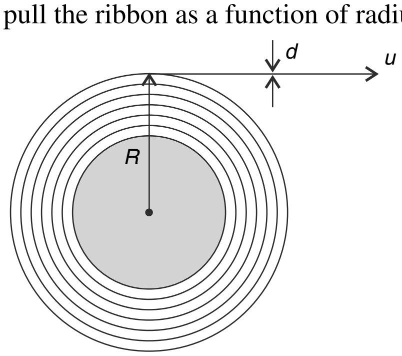
Q. 93. 

A rod of mass M and length L is hinged about its end A so that it can rotate in vertical plane. When the rod is released from horizontal position it takes t_0 time for it to become vertical.

(a) A particle of mass M is stuck at the end B of the rod and the rod is once again released from its horizontal position. Will it take more time or less time (than t_0) for the rod to become vertical from its horizontal position.

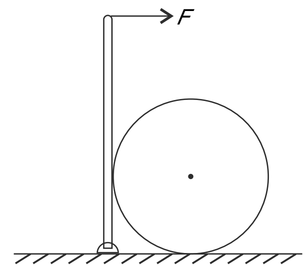
(b) At what distance x from end A shall the particle of mass M be stuck so that it takes minimum time for the rod to become vertical from its horizontal position.

Q. 94. A disc is free to rotate about an axis passing through its centre and perpendicular to its plane. The moment of inertia of the disc about its rotation axis is I. A light ribbon is tightly wrapped over it in multiple layers. The end of the ribbon is pulled out at a constant speed of u. Let the radius of the ribboned disc be R at any time and thickness of the ribbon be d ($<< R$). Find the force (F) required to pull the ribbon as a function of radius R.

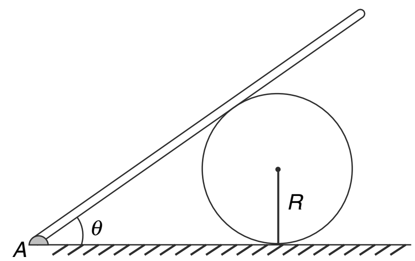


Q. 95. A uniform rod of mass M and length L is hinged at its lower end on a table. The rod can rotate freely in vertical plane and there is no friction at the hinge. A ball of mass M and radius $R = \frac{L}{3}$ is placed in contact with the vertical rod and a horizontal force F is applied at the upper end of the rod.

(a) Find the acceleration of the ball immediately after the force starts acting.

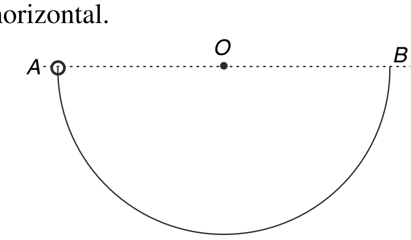


(b) Find the horizontal component of hinge force acting on the rod immediately after force F starts acting.

Q. 96. 

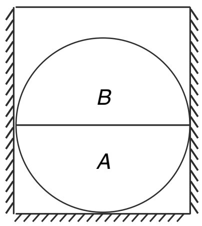
A ring of mass M and radius R is held at rest on a rough horizontal surface. A rod of mass M and length $L = 2\sqrt{3}R$ is pivoted at its end A on the horizontal surface and is supported by the ring. There is no friction between the ring and the rod. The ring is released from this position. Find the acceleration of the ring immediately after the release if $\theta = 60°$. Assume that friction between the ring on the horizontal surface is large enough to prevent slipping of the ring.

Q. 97. A uniform semicircular wire is hinged at 'A' so that it can rotate freely in vertical plane about a horizontal axis through 'A'. The semicircle is released from rest when its diameter AB is horizontal.

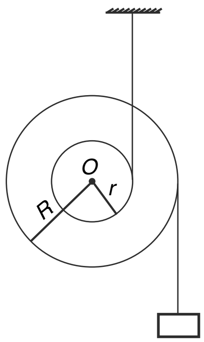


Find the hinge force at 'A' immediately after the wire is released.

Q. 98. A uniform solid hemisphere A of mass M radius R is joined with a thin uniform hemispherical shell B of mass M and radius R (see fig.). The sphere thus formed is placed inside a fixed box as shown. The floor , as well as walls of the box are smooth. On slight disturbance, the sphere begins to rotate. Find its maximum angular speed (ω_0) and maximum angular acceleration (α_0) during the subsequent motion. Do the walls of the box apply any force on the sphere while it rotates?



Q. 99.



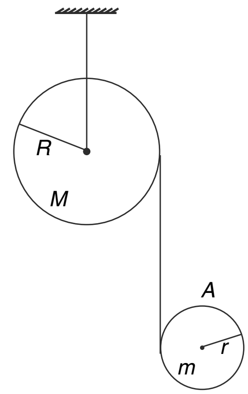
In the arrangement shown, the double pulley has a mass M and the two mass less threads have been tightly wound on the inner (radius = r) and outer circumference (radius $R = 2r$). The block shown has a mass 4 M. The moment of inertia of the double pulley system about a horizontal axis passing through its centre and perpendicular to the plane of the figure is $I = \frac{Mr^2}{2}$.

(a) Find the acceleration of the center of the pulley after the system is released.

(b) Two seconds after the start of the motion the string holding the block breaks. How long after this the pulley will stop ascending?

Q. 100. A thread is tightly wrapped on two pulleys as shown in figure. Both the pulleys are uniform disc with upper one having mass M and radius R being free to rotate about its central horizontal axis. The lower pulley has mass m and radius r and it is released from rest. It spins and falls down. At the instant of release a small mark (A) was at the top point of the lower pulley.

(a) After what minimum time (t_0) the mark will again be at the top of the lower pulley?



(b) Find acceleration of the mark at time t_0.

(c) Is there any difference in magnitude of acceleration of the mark and that of a point located on the circumference at diametrically opposite end of the pulley.

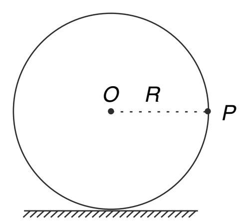
Q. 101. A point mass $m = 1$ kg is attached to a point P on the circumference of a uniform ring of mass M = 3kg and radius $R = 2.0$ m. The ring is placed on a horizontal surface and is released from rest with line OP in horizontal position (O is centre of the ring). Friction is large enough to prevent sliding. Calculate the following quantities immediately after the ring is released-

(a) angular acceleration (α) of the ring,

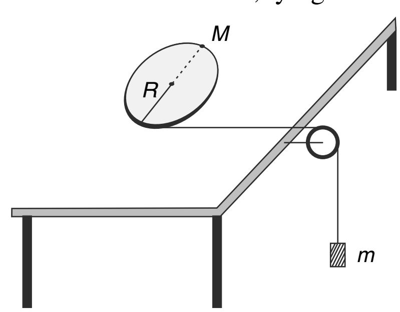
(b) normal reaction of the horizontal surface on the ring and

(c) the friction force applied by the surface on the ring.

[Take g = 10 m/s^2]



Q. 102. A light thread has been tightly wrapped around a disc of mass M and radius R. The disc has been placed on a smooth table, lying flat as shown.

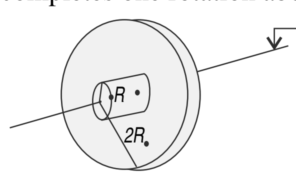


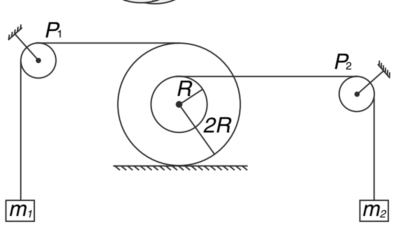
The other end of the string has been attached to a mass m as shown. The system is released from rest. If $m = M$, which point of the disc will have zero acceleration, immediately after the system is released?

Q. 103. A spool has the shape shown in figure. Radii of inner and outer cylinders are R and $2\ R$ respectively. Mass of the spool is $3\ m$ and its moment of inertia about the shown axis is $2mR^2$. Light threads are tightly wrapped on both the cylindrical parts. The spool is placed on a rough surface with two masses $m_1 = m$ and $m_2 = 2m$ connected to the strings as shown. The string segment between spool and the pulleys P_1 and P_2 are horizontal. The centre of mass of the spool is at its geometrical centre. System is released from rest.

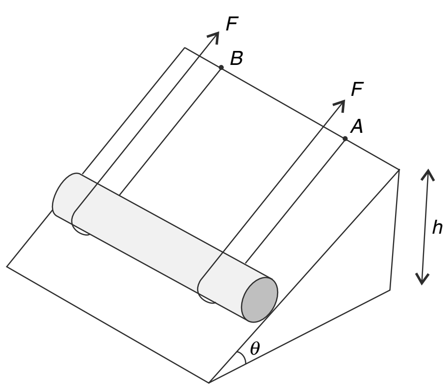
(a) What is minimum value of coefficient of friction between the spool and the table so that it does not slip?

(b) Find the speed of m_1 when the spool completes one rotation about its centre.





Q. 104.



A heavy uniform log of mass M is pulled up an incline surface with the help of two parallel ropes as shown in figure. The ropes are secured at point A and B. The height of the incline is h and its inclination is θ.

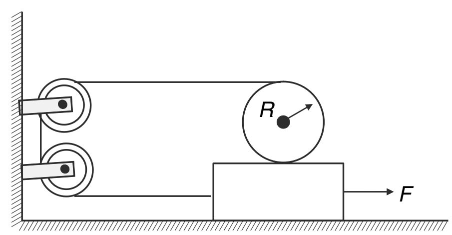
(a) Find the minimum force F_0 needed to roll the log up the incline.

(b) Find the work done by the force in moving the log from the bottom to the top of the incline if the applied force is $F = 2F_0$

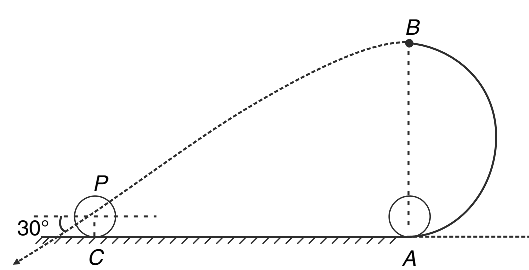
Q. 105. In the figure shown, the light thread is tightly wrapped on the cylinder and masses of plank and cylinder are same each equal to m. An external agent begins to pull the plank to the right with a constant force F. The friction between the plank and the cylinder is large enough to prevent slipping. Assume that the length of the plank is quite large and the cylinder does not fall off it for the time duration concerned.

(a) Find the acceleration of the cylinder. (Hint : don't write any equations)

(b) Find the kinetic energy of the system after time t.



Q. 106. A disc of radius $r = 0.1\ m$ is rolled from a point A on a track as shown in the figure. The part AB of the track is a semi-circle of radius R in a vertical plane. The disc rolls without sliding and leaves contact with the track at its highest point B. Flying through the air it strikes the ground at point C. The velocity of the center of mass of the disc makes an angle of 30° below the horizontal at the time of striking the ground. At the same instant, velocity of the topmost point P of the disc is found to be 6 m/s. (Take $g = 10\ m/s^2$).

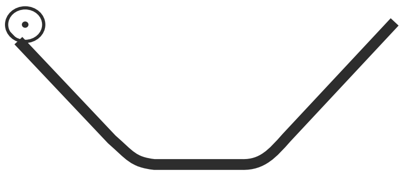


(a) Find the value of R.

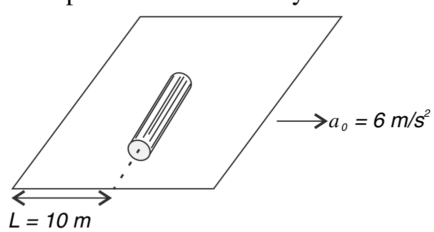
(b) Find the velocity of the center of mass of the disc when it strikes the ground.

(c) Find distance AC.

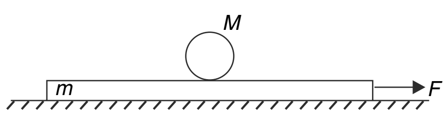
Q. 107. A trough has two identical inclined segments and a horizontal segment. A ball is released on the top of one inclined part and it oscillates inside the trough. Friction is large enough to prevent slipping of the ball. Time period of oscillation is T. Now the liner dimension of each part of the trough is enlarged four times. Find the new time period of oscillation of the ball.



Q. 108. A uniform cylinder is lying on a rough sheet of paper as shown in fig. The strip is pulled horizontally to the right with a constant acceleration of $a_0 = 6\ m/s^2$. Initially the cylinder is located at a distance of $L = 10\ m$ from the left end of the strip. Find the velocity of the centre of the cylinder at the instant it moves off the edge of the strip. Assume that the cylinder does not slip.

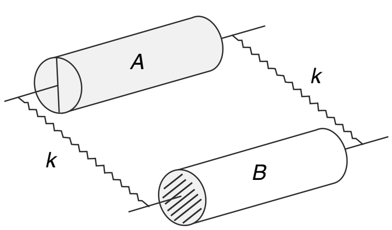


Q. 109.



A hollow pipe of mass $M = 6\ kg$ rests on a plate of mass $m = 1.5\ kg$. The thickness of the pipe is negligible. The coefficient of friction at all contacts is $\mu = 0.2$. The system is initially at rest. A horizontal force F of magnitude $25N$ is applied on the plate as shown in figure. Will the cylinder slide on the plate? Find the acceleration of the centre of the cylinder.

Q. 110.

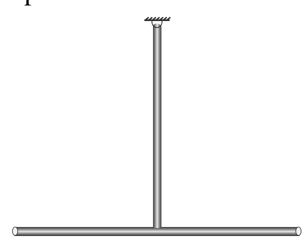


A hollow cylindrical pipe A has mass M and radius R. With the help of two identical springs (each of force constant k) it is connected to solid cylinder B having mass M and radius R. The springs are connected symmetrically to the axle of the cylinders. Moment of inertia of the two Bodies A and B about their axles are $I_A = MR^2$ and $I_B = \frac{1}{2}MR^2$ respectively. Cylinders are pulled apart so as to stretch the springs by x_0 and released. During subsequent motion the cylinders do not slip.

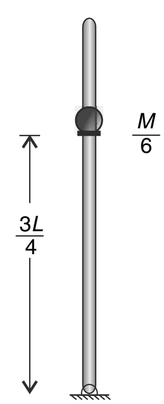
(a) Find acceleration of the centre of mass of the system immediately after it is released.

(b) Find the distance travelled by cylinder A by the time it comes to rest for the first time after being released.

Q. 111. Two identical uniform thin rods have been connected at right angles to form a 'T' shape. One end of a rod is connected to the centre of the other rod. Length of each rod is L. The upside down 'T' can swing like a pendulum about a horizontal axis passing through the top end (see fig.). Axis is perpendicular to plane of the fig. The speed of the meeting point of the two rods is $u = 2\sqrt{gL}$ when it is at its lowest position. Calculate the angular acceleration of the 'T' shaped object when it is at extreme position of its oscillation.



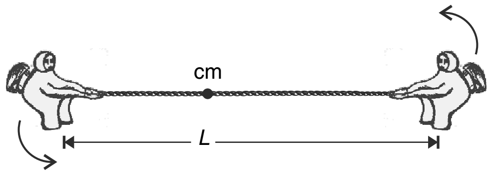
Q. 112. A uniform rod of mass M and length L is hinged at its lower end so as to rotate freely in the vertical plane of the fig. There is a small tight fitting bead of mass $\frac{M}{6}$ on the rod at a distance $\frac{3L}{4}$ from the hinged end. A small mass less pin welded to the rod supports the bead. The system is released from the vertical position shown. It was observed that the bead just begins to slide on the rod when the rod becomes horizontal.



(a) Find the normal contact force between the rod and the bead when the rod gets horizontal. What is the direction of this force?

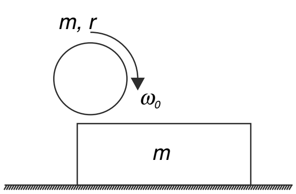
(b) Find the coefficient of friction between the bead and rod.

Q. 113



Two astronauts having mass of 75 kg and 50 kg are connected by a rope of length L = 10m and negligible mass. They are in space, orbiting their centre of mass at an angular speed of $\omega_0 = 5\ rad/s$. The centre of mass itself is moving uniformly in space at a velocity of 10 m/s. By pulling on the rope, the astronauts shorten the distance between them to $\frac{L}{2} = 5m$. How much work is done by the astronauts in shortening the distance between them ? Assuming that the astronauts are athletic and each of them can generate a power of 500 watt, is it possible for the two astronauts to reduce the distance between them to 5 m, within a minute?

Q. 114. In the figure shown a plank of mass m is lying at rest on a smooth horizontal surface. A cylinder of same mass m and radius r is rotated to an angular speed w_0 and then gently placed on the plank. It is found that by the time the slipping between the plank and the cylinder cease, 50% of total kinetic energy of the cylinder and plank system is lost. Assume that plank is long enough and μ is the coefficient of friction between the cylinder and the plank.

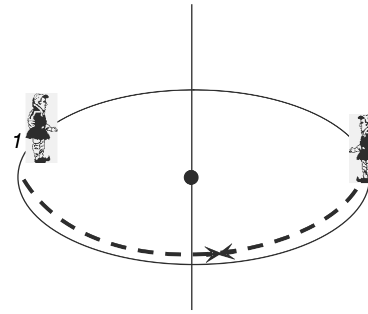


(a) Find the final velocity of the plank.

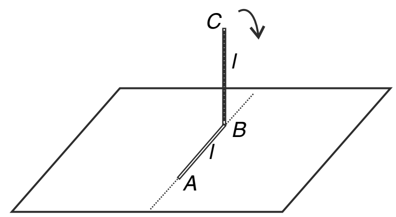
(b) Calculate the magnitude of the change in angular momentum of the cylinder about its centre of mass.

(c) Distance moved by the plank by the time slipping ceases between cylinder and plank.

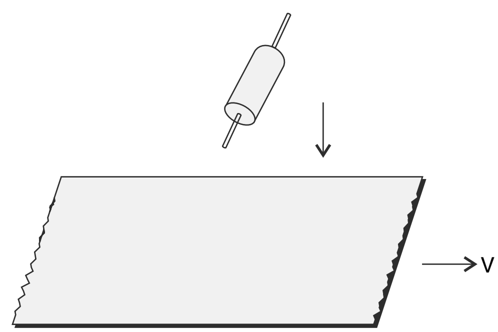
Q. 115. A horizontal turn table of mass 90kg is free to rotate about a vertical axis passing through its centre. Two men – 1 and 2 of mass 50kg and 60kg respectively are standing at diametrically opposite point on the table. The two men start moving towards each other with same speed (relative to the table) along the circumference. Find the angle rotated by table by the time the two men meet. Treat the men as point masses.



Q. 116. A L shaped uniform rod has both its sides of length l. Mass of each side is m. The rod is placed on a smooth horizontal surface with its side AB horizontal and side BC vertical. It tumbles down from this unstable position and falls on the surface. Find the speed with which end C of the rod hits the surface.



Q.117.

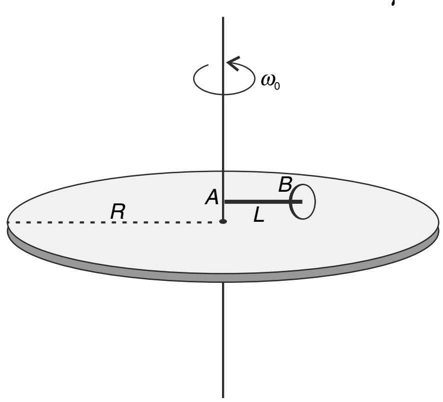


A flat horizontal belt is running at a constant speed V. There is a uniform solid cylinder of mass M which can rotate freely about an axle passing through its centre and parallel to its length. Holding the axle parallel to the width of the belt, the cylinder is lowered on to the belt. The cylinder begins to rotate about its axle and eventually stops slipping. The cylinder is, however, not allowed to move forward by keeping its axle fixed. Assume that the moment of inertia of the cylinder about its axle is $\frac{1}{2}MR^2$ where M is its mass and R its radius and also assume that the belt continues to move at constant speed. No vertical force is applied on the axle of the cylinder while holding it.

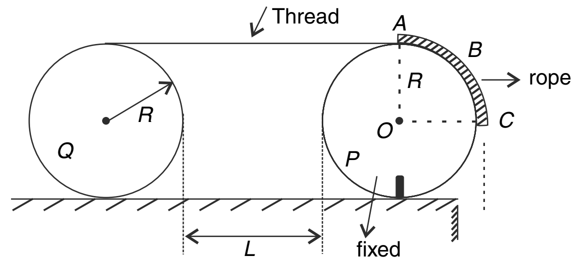
(a) Calculate the extra power that the motor driving the belt has to spend while the cylinder gains rotational speed. Assume coefficient of friction = μ.

(b) Prove that 50% of the extra work done by the motor after the cylinder is placed over it, is dissipated as heat due to friction between the belt and the cylinder.

Q.118. A uniform disc of mass M and radius R is rotating freely about its central vertical axis with angular speed ω_0. Another disc of mass m and radius r is free to rotate about a horizontal rod AB. Length of the rod AB is L (< R) and its end A is rigidly attached to the vertical axis of the first disc. The disc of mass m, initially at rest, is placed gently on the disc of mass M as shown in figure. Find the time after which the slipping between the two discs will cease. Assume that normal reaction between the two discs is equal to mg. Coefficient of friction between the two discs is μ.



Q.119. P is a fixed smooth cylinder of radius R and Q is a disc of mass M and radius R. A light thread is tightly wound on Q and its end is connected to a rope ABC. The rope has a mass m and length $\frac{\pi R}{2}$ and is initially placed on the cylinder with its end A at the top. The system is released from rest. The rope slides down the cylinder as the disc rolls without slipping. The initial separation between the disc and the cylinder was $L = \frac{\pi R}{2}$ (see fig). Find the speed with which the disc will hit the cylinder. Assume that the rope either remains on the cylinder or remains vertical; it does not fly off the cylinder.

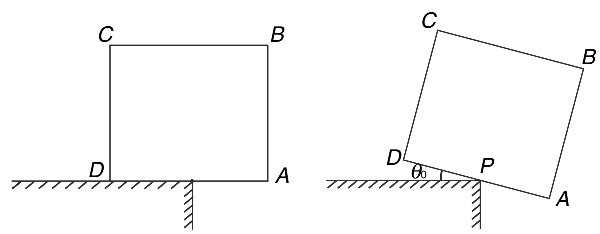


Q.120. A uniform cube of mass M and side length a is placed at rest at the edge of a table. With half of the cube overhanging from the table, the cube begins to roll off the edge. There is sufficient friction at the edge so that the cube does not slip at the edge of the table. Find -

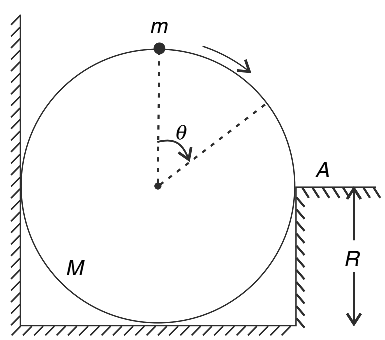
(a) the angle θ_0 through which the cube rotates before it leaves contact with the table.

(b) the speed of the centre of the cube at the instant it breaks off the table.

(c) the rotational kinetic energy of the cube at the instant its face AB becomes horizontal.



Q.121. A uniform frictionless ring of mass M and radius R, stands vertically on the ground. A wall touches the ring on the left and another wall of height R touches the ring on right (see figure).There is a small bead of mass m positioned at the top of the ring. The bead is given a gentle push and it being to slide down the ring as shown. All surfaces are frictionless.

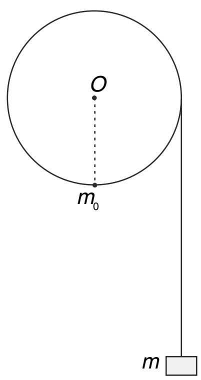


(a) As the bead slides, up to what value of angle θ the force applied by the ground on the ring is larger than Mg?

(b) Write the torque of force applied by the bead on the ring about point A as function of θ.

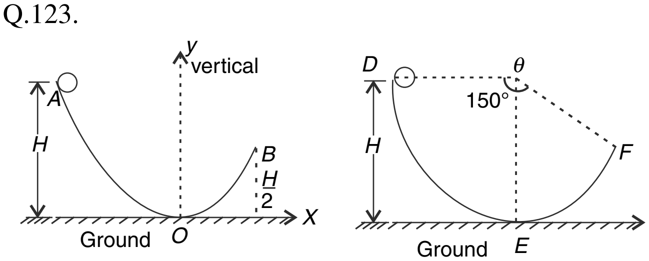
(c) What is the maximum possible value of torque calculated in (b)? Using this result tell what is the largest value of $\frac{m}{M}$ for which the ring never rises off the ground ?

Q.122.



A uniform disc shaped pulley is free to rotate about a horizontal axis passing through the centre of the pulley. A light thread is tightly wrapped over it and supports a mass m at one of its end. A small particle of mass $m_0 = \sqrt{2}m$ is stuck at the lowest point of the disc and the system is released from rest. Will the particle of mass m_0 climb to the top of the pulley?

Q.123.



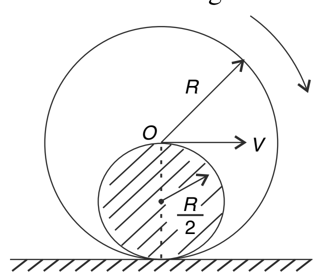
AOB is a frictionless parabolic track in vertical plane. The equation of parabolic track can be expressed as $y = \frac{3}{2H}x^2$ for co-ordinate system shown in the figure. The end B of the track lies at $y = \frac{H}{2}$. When a uniform small ring is released on the track at A it was found to attain a maximum height of h_1, above the ground after leaving the track at B. There is another track DEF which is in form of an arc of a circle of radius H subtending angle an angle of 150° at the centre. The radius of the track at D is horizontal. The same ring is released on this track at point D and it rolls without sliding. The ring leaves the track at F and attains a maximum height of h_2 above the ground. Find the ratio $\frac{h_1}{h_2}$.

Q.124. A uniform sphere of radius R has a spherical cavity of radius $\frac{R}{2}$ (see figure). Mass of the sphere with cavity is M. The sphere is rolling without sliding on a rough horizontal floor [the line joining the centre of sphere to the centre of the cavity remains in vertical plane]. When the centre of the cavity is at lowest position, the centre of the sphere has horizontal velocity V. Find:

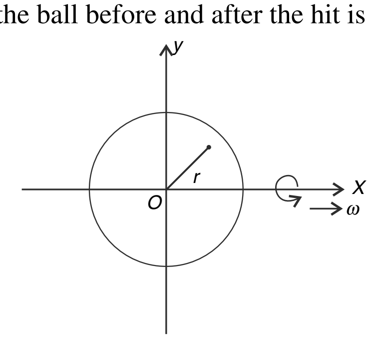
(a) The kinetic energy of the sphere at this moment.

(b) The velocity of the centre of mass at this moment.

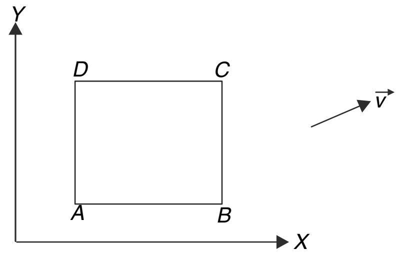
(c) The maximum permissible value of V (in the position shown) which allows the sphere to roll without bouncing



Q. 125. A uniform ball of mass M and radius R can rotate freely about any axis through its centre. Its angular velocity vector is directed along positive x axis. A bullet is fired along negative Z direction and it pierces through the ball along a line that is at a perpendicular distance r ($\leq R$) from the centre of the ball. The bullet passes quickly and its net effect is that it applies an impulse on the ball. Mass of the bullet is m and its velocity charges from u to v ($\leq u$) as it passes through the ball. As a result the ball stops rotating about X axis and begins to rotate about y axis. The angular speed of the ball before and after the hit is ω. Find r.



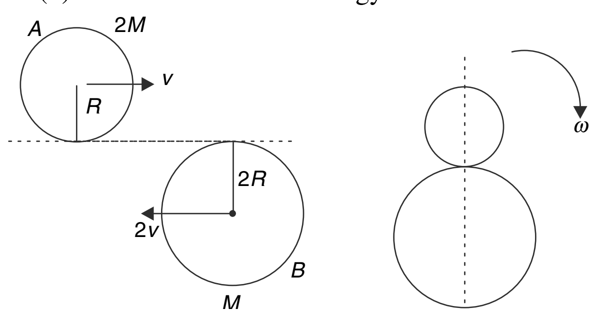
Q. 126. A uniform square plate $ABCD$ has mass M side length a. It is sliding on a horizontal smooth surface with a velocity of $\vec{v} = v_0\left(4\hat{i} + 2\hat{j}\right)$. There is no rotation. Vertex A of the plate is suddenly fixed by a nail. Calculate the velocity of centre of the plate immediately after this.



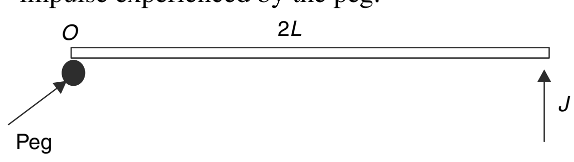
Q. 127. Two discs A and B are moving with their flat circular surface on a smooth horizontal surface. Mass, radius and velocity of the two discs are – $m_A = 2M$, $m_B = M$, $r_A = R$, $r_B = 2R$, $v_A = v$, and $v_B = 2v$. The velocities of the two discs are oppositely directed so that they just cannot avoid collision and stick to each other (see figure)

(a) Find the angular speed of the composite system after collision

(b) Find loss in kinetic energy due to collision



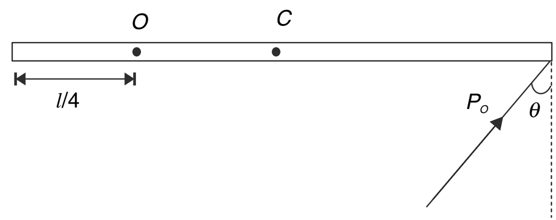
Q. 128. A uniform rod of mass M and length $2L$ lies on a smooth horizontal table. There is a smooth peg O fixed on the table. One end of the rod is kept touching the peg as shown in the figure. An impulse J is imparted to the rod at its other end. The impulse is horizontal and perpendicular to the length of the rod. Find the magnitude of impulse experienced by the peg.



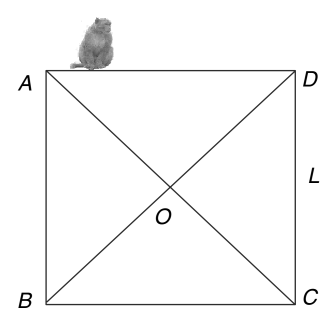
Q. 129. A uniform rod of mass M and length ℓ is hinged at point O and is free to rotate on a horizontal smooth surface. Point O is at a distance of $\frac{\ell}{4}$ from one end of the rod. A sharp impulse $P_0 = 2\sqrt{130}\ kg\ m/s$ is applied along the surface at one end of the rod as shown in figure $\left[\tan\theta = \frac{9}{7}\right]$

(a) Find the angular speed of the rod immediately after the hit

(b) Find the impulse on the rod due to the hinge.



Q. 130. Four thin rods of length $L = 1.0\ m$ each are joined to form a square $ABCD$. The opposite vertices of the square are joined by mass less rods AC and BD. This square frame is mounted on a horizontal axis through its centre so that the frame can rotate freely in the vertical plane. Masses of rods AB and BC are $m = 2kg$ each and the rod AD and DC have mass $M = 4kg$ each. A monkey of mass m_o =12 kg is at rest on the horizontal rod AD and keeps the system in equilibrium. The monkey takes a sudden jump and rises to a height H from its initial position. Calculate minimum value of H so that the square frame is able to complete a rotation about its central axis. Assume no further contact between monkey and the frame.

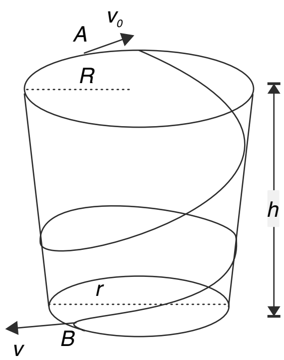


Q. 131. A frustum has been mounted with its axis vertical. It has a height h and radii of its upper and lower cross sections are R and r respectively. A particle is projected with horizontal velocity v_0 along its upper brim. The particle spirals down the inner surface and leaves the lower face at point B. The inner wall of the frustum is smooth.

(a) Find the vertical component of velocity of the particle as it leaves the frustum at B.

(b) Find minimum value of h for which the particle will never come out of the frustum.

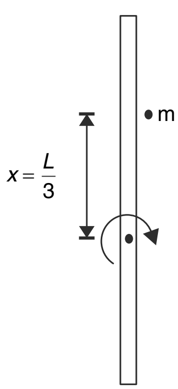
Take $r = \frac{R}{2}$ for solving this part of the problem.



Q. 132. A uniform thin stick of mass $M = 24kg$ and length L rotates on a friction less horizontal plane, with its centre of mass stationary. A particle of mass m is placed on the plane at a distance $x = \frac{L}{3}$ from the centre of the stick . This stick hits the particle elastically

(a) Find the value of m so that after the collision, there is no rotational motion of the stick

(b) For what minimum of x can we get a value of 'm' so that the rod has no rotational motion after elastic collision?



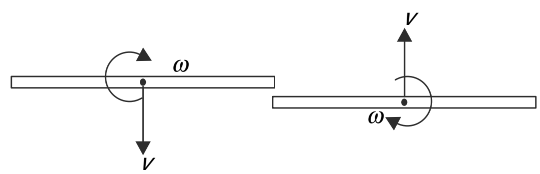
Q. 133. A uniform rod of length L is rotating in a horizontal plane about a vertical axis passing through one of its ends. At a distance x ($< L$) from the axis there is a fixed vertical pole. The rod hits the pole and its direction of motion is reversed. Find x if it is known that during the impact the axis of rotation imparts no impulse to the rod. Does your answer depend on coefficient of restitution?

[NOTE : If you hit a lamp post with a rod, the hand holding the rod gets hurt as long as the impact misses the so called sweet spot of the rod (and hits either above or below the sweet spot). After solving the above problem you know where the sweet spot is ! You may assume that during the impact the rod is rotating about its holding hand. And if you play cricket, you know that there is a sweet spot in your bat too ! If the ball hits way above or below the spot you get stung.]

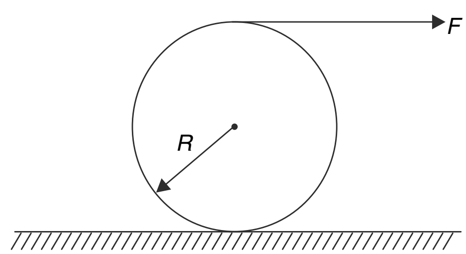
Q. 134. Two identical thin rods are moving on a smooth table, as shown. Both of them are rotating with angular speed ω, in clockwise sense about their centres. Their centres have velocity V in opposite directions. The rods collide at their edge and stick together. Length of each rod is L.

(a) For what value of $\frac{V}{\omega L}$ there will be no motion after collision ?

(b) If the ratio $\frac{V}{\omega L}$ is half the value found in (a) above, what fraction of kinetic energy is lost in the collision?



Q. 135. Light thread is tightly wound on a uniform solid cylinder of radius R. The cylinder is placed on a smooth horizontal table and the thread is pulled horizontally as shown, by applying a constant force F. How much length of the thread is unwound from the cylinder by the time its kinetic energy becomes equal to K.



Q. 136. A horizontal disc of radius R and mas 20 M is pivoted to rotate freely about a vertical axis through its centre. A small insect A of mass M and another small insect B of mass $m = \frac{M}{4}$ are initially at diametrically opposite points on the periphery of the disc. The whole system is imparted an angular speed ω_0. Insect A walks along the diameter with constant velocity v relative to the disc unit it reaches B which remains at rest on the disc. A then eats B and returns to its starting point along the original path with same speed v relative to the disc.

(a) Find the angular speed of the disc when A reaches the centre after eating B.

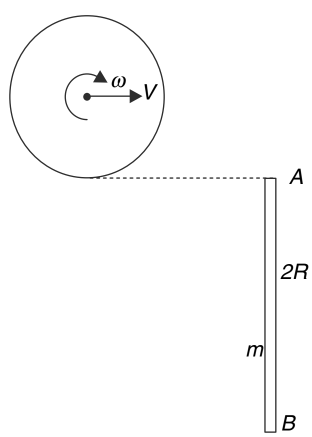
(b) Plot approximately, the variation of angular speed of the disc with time for the entire journey of the insect A.

Q.137. A disc of mass m and radius R is moving on a smooth horizontal surface with the flat circular face on the surface. It is spinning about its centre with angular peed ω and has a velocity V (see figure). It just manages to hit a stick AB at its end A. The stick was lying free on the surface and stick to the disc. [The combined object becomes like a badminton racket]. Mass and length of the stick are m and $2R$ respectively.

(a) Calculate the angular speed of the combined object assuming $V = R\omega$

(b) Calculate loss in kinetic energy. Why is energy lost?

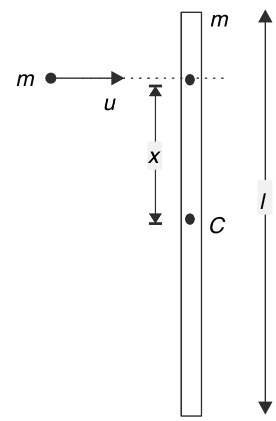
(c) If $V = \eta(R\omega)$, loss in kinetic energy is minimum. Find η. [Assume ω is given]



Q. 138. A uniform rod of mass m and length ℓ has been placed on a smooth table A particle of mass m, travelling perpendicular to the rod, hits it at a distance $x = \dfrac{\ell}{\sqrt{6}}$ from the centre C of the rod. Collision is elastic.

(a) Find the speed of the centre of the rod and the particle after the collision.

(b) Do you think there is a chance of second Collision? If yes, how is the system of particle and stick moving after the second collision?

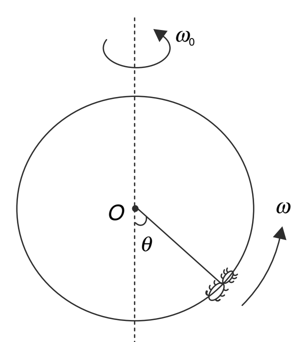


Q. 139. A ring is made to rotate about its diameter at a constant angular speed of ω_0. A small insect of mass m walks along the ring with a uniform angular speed ω relative to the ring (see figure). Radius of the ring is R.

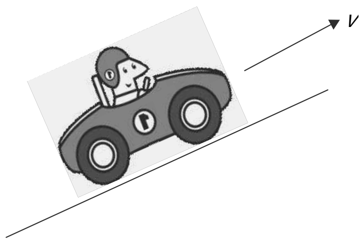
(a) Find the external torque needed to keep the ring rotating at constant speed as the insect walks. Express your answer as a function of θ. For what value of θ is this torque maximum?

[given your answer for $0 \le \theta \le 90°$]

(b) Find the component of force perpendicular to the plane of the ring, that is applied by the ring on the insect. For what value of θ is this force maximum? Argue quantitatively to show that indeed the force should be maximum for this value of θ. [Give your answer for $0° \le \theta \le 90°$]



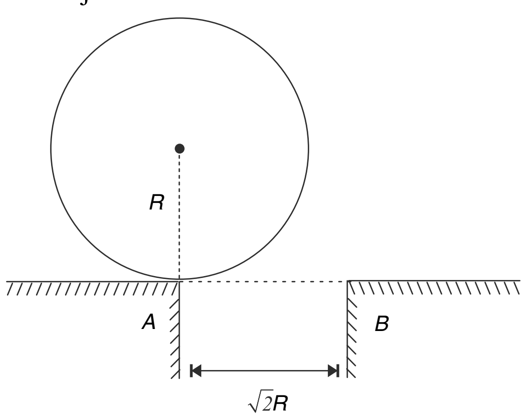
Q. 140. A small car took off a ramp at a speed of 30 *m/s*. Immediately after leaving the ramp, the driver applied brakes on all the wheels. The brakes retarded the wheels uniformly to bring them to rest in 2 second. Calculate the angle by which the car will rotate about its centre of mass in the 2 second interval after leaving the ramp. Radius of each wheel is $r = 0.30\ m$. Moment of inertia of the car along with the driver, about the relevant axis through its centre of mass is $I_M = 80 kgm^2$ and the moment of inertia of each pair of wheels about their respective axles is $0.3 kg\ m^2$. Assume that the car remained in air for more than 2 second . Also assume that before take- off the wheels rolled without sliding.



Q. 141. A disc of radius R stands at the edge of a table. If the given a gentle push and it beings to fall. Assume that the disc does not slip at A and it rotates about the point as it falls. The falling disc hits the edge of another table placed at same height as the first one at a horizontal distance of $\sqrt{2}\,R$. Imagine that the disc hits the edge B and rotates (up) about the edge

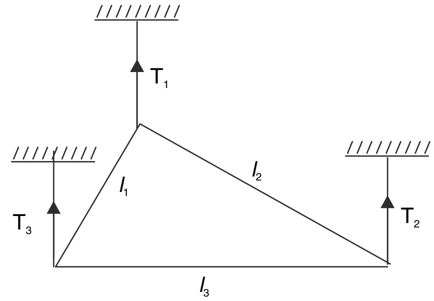
(a) Find the speed of the centre of the disc at the instant just before it hits the edge B.

(b) Find the angular speed of the disc about B just after the hit.

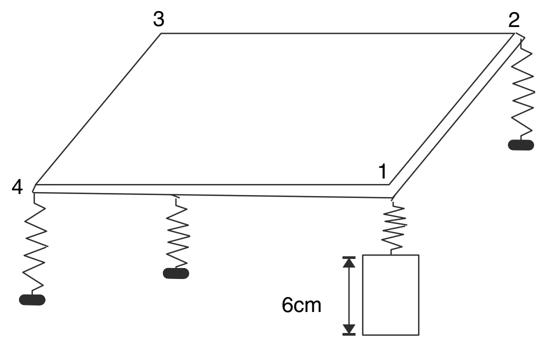


LEVEL 3

Q. 142. A uniform triangular plate is kept horizontal suspended with the help of three vertical threads as shown. The sides of the plate have length l_1, l_2 and l_3. Find tension T_1, T_2 and T_3 in the three threads. Mass of the plate is M.



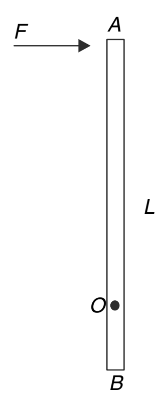
Q. 143. A rigid large uniform square platform is resting on a flat horizontal ground supported at its vertices by four identical spring. At vertex 1 a wooden block, 6 cm high, is inserted below the spring. Calculate the change in height of the centre of the platform. Assume change in height to be small compared to dimension of the platform.



Q. 144. A uniform rod of mass M and length L is placed freely on a horizontal table. A horizontal force F is applied perpendicular to the rod at one of its ends. The force F is increased gradually from zero and it is observed that when its value becomes F_0, the rod just begins to rotate about point O

(i) Find length AO

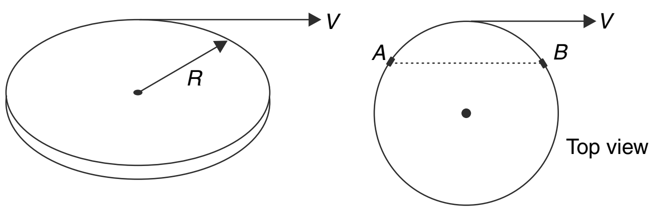
(ii) Find F_0



Q. 145. A ring of mass M and radius R lies flat on a horizontal table. A light thread is wound around it and its free end is pulled with a constant velocity v.

(a) Two small segment A and B (see fig.) in the ring are rough and have a coefficient of friction μ with the table. Rest of the ring is smooth. Find the speed with which the ring moves.

(b) Find the speed of the ring if coefficient of friction is μ everywhere; for all points on the ring.

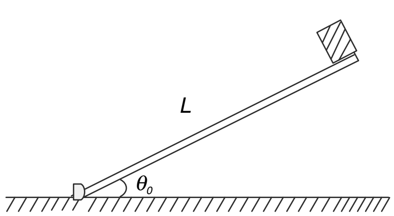


Q. 146. A uniform stick of length L is pivoted at one end on a horizontal table. The stick is held forming an angle θ_0 with the table. A small block of mass m is placed at the other end of the stick and it remains at rest. The system is released from rest

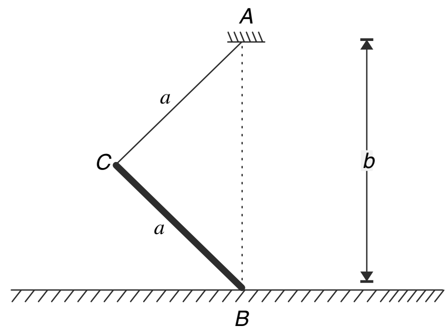
(a) Prove that the stick will hit the table before the block if $\cos\theta_0 \geq \sqrt{\frac{2}{3}}$

(b) Find the contact force between the block and the stick immediately before the system is released. Take $\theta_0 = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

(c) Find the contact force between the block and the stick immediately after the system is released if $\theta_0 = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$.



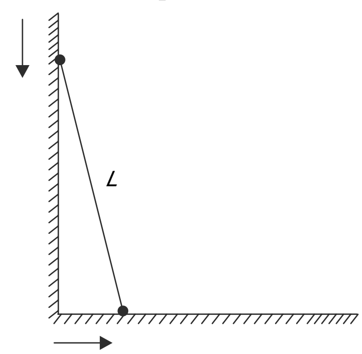
Q. 147. A uniform rod BC with length a is attached to a light string AC. End A of the string is fixed to the ceiling and the end B of the rod is on a smooth horizontal surface. B is exactly below point A and length AB is b ($a < b < 2a$). The system is released from rest and the rod begins to slide. Find the speed of the centre of the rod when the string becomes vertical.



Q. 148. A dumb – bell has a rigid mass less stick and two point masses at its ends. Each mass is m and length of the stick is L. The dumb- bell leans against a frictionless wall, standing on a frictionless ground. It is initially held motionless, with its bottom end an infinitesimal distance from the wall. It is released from this position and its bottom end slides away from the wall where as the top end slides down along the wall.

(a) Show that centre of mass of the dumb-bell moves along a circle.

(b) When the dumb-bell loses contact with the wall what is speed of the centre of mass?



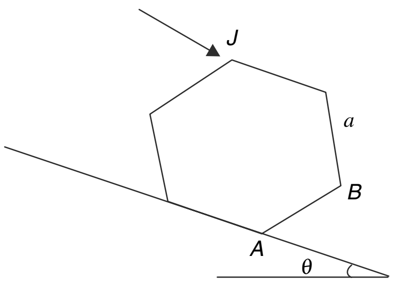
Q. 149. A hexagonal pencil of mass M and sides length a has been placed on a rough incline having inclination θ. Friction is large enough to prevent sliding. If at all the pencil moves, during one full rotation each of its 6 edges, in turn, serve as instantaneous axis of rotation.

(a) Show that for $\theta > 30°$ the pencil cannot remain at rest.

(b) For inclination of incline θ (< 30°) the pencil will not roll on its own. A sharp impulse J is given to the pencil parallel to the incline at its upper edge (see figure). Friction does not allow the pencil to slide but it begins to rotate about the edge through A with initial angular speed ω_0. Find ω_0. Moment of inertia of the pencil about its edge is I.

(c) Find minimum value of J so that the pencil will turn about A; and B will land on the incline.

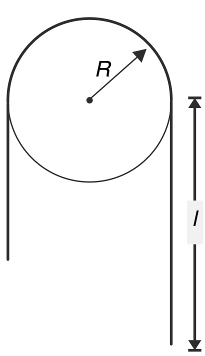
(d) If kinetic energy acquired by the pencil just after the impulse is K_0, find its kinetic energy just before edge B lands on the incline



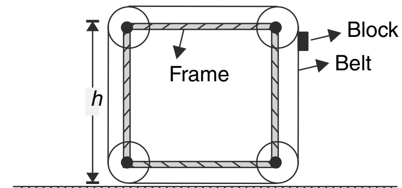
Q. 150. A rope of length L and mass per unit length λ passes over a disc shaped pulley of mass M and radius R. The rope hangs on both sides of the pulley and the length of larger hanging part is l. The pulley can rotate about a horizontal axis passing through its centre. The system is released from rest and it begins to move. The pulley has no friction at its axle and the rope has large enough friction to prevent it from slipping on the pulley.

(a) Find the acceleration of the rope immediately after it is released.

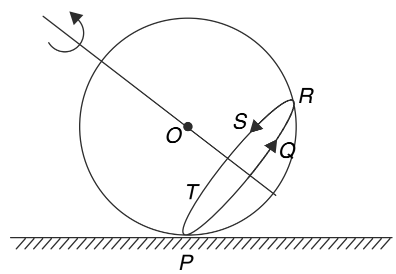
(b) Find the horizontal component of the force applied by the axle on the pulley immediately after the system is released



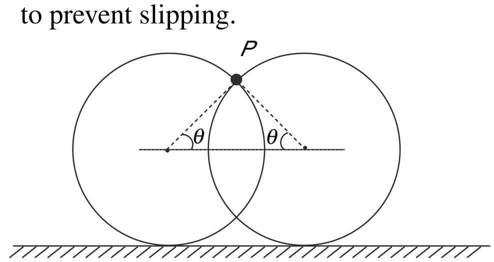
Q. 151. A toy is made of a rectangular wooden frame with four small wheels at its vertices. A tight fitting belt of negligible mass runs around the frame passing over the wheels. Mass of the complete toy is M. Now a small block of mass m is stuck at the top of the right vertical segment of the belt and the system is released. Height of the toy is h. Find the speed of the block when it is about to hit the ground. Assume no slipping anywhere and neglect the dimension of the wheels.



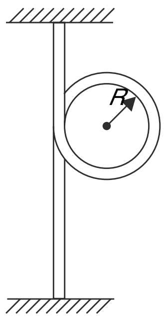
Q. 152. Consider an idealized case of rolling of a solid ball in which the point P does not rotate in a vertical plane. But it rotates on circular path $PQRSTP$ when observed from the centre of the ball. The radius of circular path $PQRSTP$ is half the radius of the ball. The ball rolls without sliding with its centre moving with speed v_0 in direction perpendicular to the plane of the figure calculate the kinetic energy of the rolling ball. Mass of the ball and its radius are M and R respectively.



Q. 153. Two identical rings, each of mass M and radius R, are standing on a rough horizontal surface. The rings overlap such that the horizontal line passing through their centre makes an angle of $\theta = 45°$ with the radius through their intersection point P. A small object of mass m is placed symmetrically on the rings at point P and released. Calculate the acceleration of the centre of the ring immediately after the release. There is no friction between the small object and the rings. The friction between the small object and the rings, and the friction between the rings and the ground is large enough to prevent slipping.



Q. 154. A uniform rope tightly wraps around a uniform thin ring the mass M and radius R. The mass of the segment of the rope around the ring (i.e. , mass of the length $2\pi R$ of the rope) is also M. The ends of the rope are fixed one above the other and it is taut. The ring is let go. Find its acceleration. Assume no slipping and thickness of the rope to be negligible.

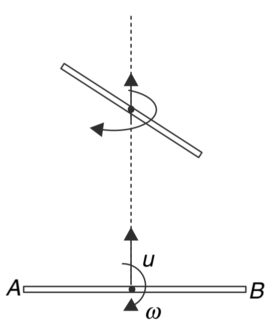


Q. 155. A uniform stick AB has length L. It is tossed up from horizontal position such that its centre receives a velocity $u = \pi\sqrt{gL}$ in vertically upward direction and the stick gets an angular velocity. The stick lands back to its point of projection in horizontal position. During its course of flight its angular velocity remained constant and the stick made one complete rotation. Stick rotates in vertical plane.

(a) Calculate the angular velocity (ω) imparted to the stick.

(b) Calculate the maximum height, above the point of projection, to which the end B of the stick rises.

[Take solution of equation $\cos x = 2x$ to be $x = 0.45$ and $\sin(0.45) = 0.43$]

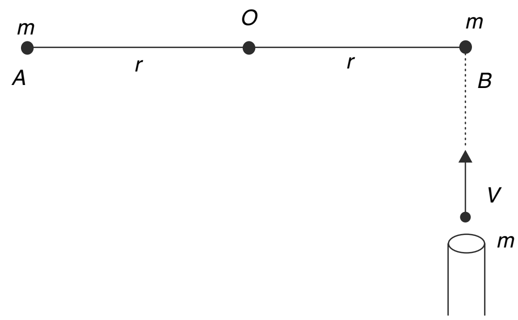


Q. 156. A uniform rod of mass $4m$ and length $2r$ can rotate in horizontal plane about a vertical axis passing through its centre O. Two small balls each of mass m are attached to its ends. A fixed gun fires identical balls with speed v in horizontal direction. The firing is being done at suitable intervals so that the fired balls either hit the ball at end A or B while moving in the direction of velocity of A or B. All collisions are elastic.

(i) Initial angular velocity of the rod is zero and its angular velocity after n^{th} collision is ω_n. Write ω_{n+1} in terms of ω_n

(ii) Solve the above equation to get ω_n

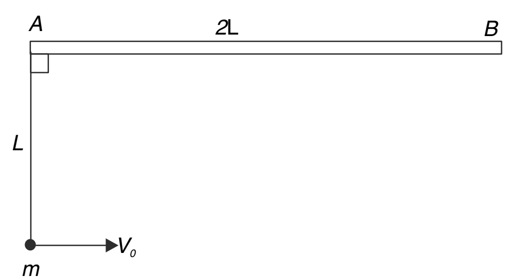
(iii) Find the limiting value of ω.



Q. 157. A uniform rod of mass m and length $2L$ on a smooth horizontal surface. A particle of mass m is connected to a string of length L whose other end is connected to the end 'A' of the rod. Initially the string is held taut perpendicular to the rod and the particle is given a velocity v_0 parallel to the initial position of the rod.

(a) Calculate the acceleration of the centre of the rod immediately after the particle is projected.

(b) The particle strikes the centre of the rod and sticks to it. Calculate the angular speed of the rod after this.



Q. 158. Two boys support by the ends a uniform rod of mass M and length $2L$. The rod is horizontal. The two boys decided to change the ends of the rod by throwing the rod into air and catching it. The boys do not move from their position and the rod remained horizontal throughout its flight. Find the minimum impulse applied by each boy on the rod when it was thrown.

Q. 159 A uniform rod of mass m and length l pivoted at one of its top end is hanging freely in vertical plane. Another identical rod moving horizontally with velocity v along a line passing through its lower end hits it and sticks to it. The two rods were perpendicular during the hit and later also they remain perpendicularly connected to each other. Find the maximum angle turned by the two-rod system after collision.

ANSWERS

1. $\frac{3h}{R}$
2. $2v$
3. $\frac{1}{3}$
4. $\frac{3}{2}$
5. $\frac{1}{4}MR^2$
6. $\frac{MR^2}{3}$
7. $x^2 + y^2 = \frac{5\ell^2}{4}$
8. $MR^2\left(1 - \frac{4}{\pi^2}\right)$
9. a, b, c, d
10. $\frac{10}{3}\%$
11. (i) $\frac{ML^2}{2}$

    (ii) (a) MOI does not change

    (b) $I = I_0 = \frac{1}{2}MR^2$
12. $5\frac{I}{4}$
13. $\frac{7I}{16}$
14. $29\ MR^2$
15. $\frac{ML^2}{6}\left(\frac{h_1 + 3h_2}{h_1 + h_2}\right)$
16. $2MR^2\left(1 - \frac{8}{9\pi^2}\right)$
17. $\alpha = \tan^{-1}(2)\ ;\ \beta = 45°$
19. $\alpha = \tan^{-1}\left(\frac{2}{\pi}\right)$
20. $\frac{29}{6}M$
21. (a) $\cos\theta = \frac{R - h}{R}$

    (b) Decreases
22. $\sin^{-1}(0.3)$
23. $t = \frac{2d.g.m}{kh}$
24. $F = \frac{mg}{2}$
25. (a) $f = m_0 g$ towards left

    (b) $m = 4m_0$
26. $\mu_{\min} = \frac{1}{2}$
27. (a) $\frac{Mg}{2}$

    (b) $\frac{1}{\sqrt{3}}$
28. (a) Yes

    (b) $k = \frac{mg}{d}$
29. $\frac{R^2\omega^2}{4g}$

30. (a) $\frac{g}{2}$

(b) $\frac{M}{m} < \frac{1}{3}$

31. (a) $\omega = \sqrt{\frac{100\pi mg}{R(5m+2M)}}$

(b) $g\left(\frac{2M}{5m+2M}\right)$

32. $a = \frac{F}{4m}$

33. $N = \left(1 - \frac{2}{\pi}\right)\text{Mg}$

34. Less than weight

35. $a = \frac{3g\sin\theta}{4}$

36. $h_0 = \frac{2R}{5}$

37. (a) $\frac{4F}{3M+8m}$

(b) $\frac{3MF}{3M+8m}$

38. (i) $\frac{12V_0}{7}$

(ii) (a) $V_0 > \frac{2}{5}\omega_0 R$

(b) $V_0 = \frac{2}{5}\omega_0 R$

(c) $V_0 < \frac{2}{5}\omega_0 R$

39. (a) $\frac{1}{2}M(\omega R)^2$

(b) $M\omega^2 R$

40. $\frac{3}{4}MV^2$ in both cases. In case (b) the Kinetic energy will be higher if length was doubled.

42. (i) 100 J
(ii) 33.33 J

43. Case (b)

44. $V = \sqrt{5gR}$

45. $V_{max} = (\sqrt{5} + 1)\ V$; $V_{min} = (\sqrt{5} - 1)\ V$

46. The nose of the plane tends to veer downward.

47. $\pi \times 10^5$ rad/s

48. $\frac{6}{5} m/s$

49. $\frac{V}{R}$

50. $\omega = \frac{24}{19}\omega_0$

51. (a) $\frac{MV_0 b}{2}$

(b) Normal reaction produces a torque

52. $\frac{3u}{4}$

53. $\frac{1}{3}mvL\sin\theta$, No

54. $\left(\frac{1}{2}\right)^4$

55. (a) $\vec{V}_A = 4i - 3j\,; \vec{V}_B = 3i - \frac{9}{4}j;$

$\vec{V}_C = \frac{13}{4}i - 2j$ all in unit of m/s

(b) $\omega = \frac{1}{4}$ rad/s

57. (a) $\omega = \frac{v}{2\sqrt{3}R}$

(b) $v_y = \frac{v}{4}(\downarrow)$

58. (a) $V_A = 2V$, $V_B = \frac{3V}{2}$

(b) $t = \frac{2R}{V}\cos^{-1}\left(\frac{3}{4}\right)$

59. (a) $\sqrt{156}\ ms^{-2}$

(b) $2\sqrt{3}\ ms^{-2}$

(c) $(3, \pi/6)$

60. $\dfrac{5v^2}{6R}$

61. R

62. $4R$

63. 2.26 rad/s

64. (a) v

(b) $\dfrac{1}{2}$

65. $I = 4\lambda R^3\,(\theta_0 - \sin\theta_0)$

66. (a) $\dfrac{9}{2}mR^2$

(b) $\omega = \dfrac{2vd}{11R^2}$

67. $\dfrac{91}{486}Ma^2$

68. $\dfrac{Ma^2}{3}$

69 $\dfrac{1}{2}Ma^2\left(1-\dfrac{2}{3}\sin^2\dfrac{\theta}{2}\right)$

70. (a) $\dfrac{\sqrt{3}}{2}M$

(b) $\dfrac{3\sqrt{3}}{8}MR^2$

71. $\dfrac{Ma^2}{12}$

72. $\mu_{\text{min}} = \dfrac{\sin\theta}{1+\cos\theta}$; $m_{\text{block}} = \dfrac{M\sin\theta}{1+\cos\theta}$

73. $\dfrac{3\sin\theta}{1+\cos\theta}$

74. Less than $\sqrt{\dfrac{3}{2}}R$

75. $M_A < M_B$

76. $\dfrac{\sqrt{3}}{4}Mg$

77. $(v_0)_{\text{max}} = \dfrac{1}{L}\sqrt{\dfrac{3Mg}{\rho}}$

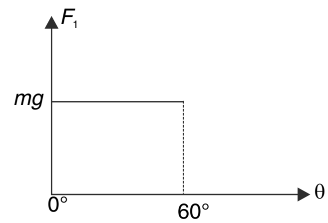
78. $\dfrac{Mg}{2}$

79. 41°

80. 50°

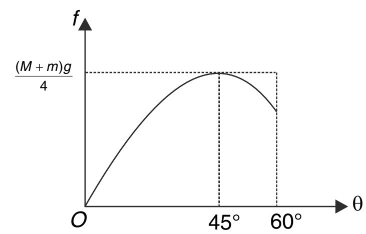
81. $\theta = \pi - \tan^{-1}\left(\dfrac{1}{4}\right)$

82. (a)



(b) $\mu_{\text{min}} = \sqrt{3}$

(c)



83. (b) $\alpha = \tan^{-1}(2\tan\theta)$

(c) $\theta_{\text{min}} = \tan^{-1}\left(\dfrac{1}{2\mu}\right)$

(d) Slippage is more likely when at *A*.

84. $T_1 = \dfrac{Mg}{12}; T_2 = \dfrac{Mg}{3}; T_3 = \dfrac{7Mg}{12}$

85.

(a) $\dfrac{a}{b} = \dfrac{1}{2}\sin^2\theta_1\left[\left(1-\mu^2\right)\sin\theta_1 - 2\mu\cos\theta_1\right]$

(b) $\dfrac{a}{b} = \dfrac{1}{2}\sin^2\theta_2\left[\left(1-\mu^2\right)\sin\theta_2 + 2\mu\cos\theta_2\right]$

86. $h = \dfrac{L}{2}$

87. $\mu_{\text{max}} = \dfrac{1}{2}$

88. (a) $\sqrt{\dfrac{3F}{ML}}$

(b) $\theta = \tan^{-1}\left(\dfrac{1}{10}\right)$

89. (a) 7.5 kg m/s

(b) $f = 9\ N$, $N = 24.5\ N$

(c) $\cos^{-1}\left(\dfrac{2}{3}\right)$

90. (a) $\dfrac{3g}{8}$

(b) $\dfrac{2g}{5}$ and $\dfrac{g}{5}$

91. $\dfrac{4g}{11}$; Thread exerts more force on A

92. (a) $\dfrac{5}{4}mg, \dfrac{mg}{2}, \dfrac{3}{4}mg$

(b) $\dfrac{\sqrt{3}}{4}g$

93. (a) More time

(b) $\dfrac{\sqrt{21}-3}{6}$

94. $F = \dfrac{Idu^2}{2\pi R^4}$

95. (a) $\dfrac{3F}{4M}$

(b) $\dfrac{7F}{8}$

96. $a = \dfrac{g}{4\sqrt{3}}$

97. $Mg\sqrt{\dfrac{1}{4}+\dfrac{1}{\pi^2}}$

98. $\omega_0 = \sqrt{\dfrac{15}{32}\dfrac{g}{R}}$; $\alpha_0 = \dfrac{15}{128}\dfrac{g}{R}$; Yes

99. (a) $\dfrac{6g}{11}$

(b) $\dfrac{18}{11}s$

100. (a) $t_0 = \sqrt{\dfrac{2\pi(2m+3M)r}{Mg}}$

(b) $a = \dfrac{2g}{2m+3M}\sqrt{M^2+(4\pi M+M+m)^2}$

(c) Yes.

101. (a) $\dfrac{5}{8} rad/s^2$

(b) $\dfrac{155}{4}N$

(c) 5 N

102. A point at a distance $\dfrac{R}{2}$ from centre.

103. $\mu_{min} = \dfrac{1}{g}$

104. (a) $F_0 = \dfrac{Mg\sin\theta}{4}$

(b) 2 Mgh

105. (a) Zero

(b) $\dfrac{F^2t^2}{3m}$

106. (a) 1 m

(b) 12 m/s

(c) 3.6 $\sqrt{3}$ m

107. 2T

108. $t = \sqrt{5}\ s$.

109. No; $\dfrac{10}{9} m/s^2$

110. (a) $\dfrac{kx_0}{6M}$

(b) $\dfrac{6x_0}{7}$

111. $\dfrac{\sqrt{935}}{34}\dfrac{g}{L}$

112. (a) $\dfrac{2Mg}{123}$ Vertically down

(b) $\mu = 22.5$

113. 112.5 KJ ; No.

114. (a) $\frac{r\omega_0}{4}$

(b) $\frac{1}{4}mr^2\omega_0$

(c) $\frac{r^2\omega_0^2}{32\mu g}$

115. 5.8º

116. $\sqrt{3gl}$

117. (a) $P = \mu mg.V$

118. $\frac{MR^2 L\omega_0}{2\mu g\left[MR^2 + mL^2\right]}$

119. $V = \sqrt{\left(\frac{8+3\pi^2}{\pi}\right)\left(\frac{mgR}{3M+8m}\right)}$

120. (a) $\cos^{-1}\left(\frac{6}{11}\right)$

(b) $\sqrt{\frac{3ga}{11}}$

(c) $\frac{Mga}{11}$

121. (a) $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

(b) $mgR\ (2\cos\theta - 3\cos^2\theta)$

(c) $\tau_{\max} = \frac{mgR}{3}; \left(\frac{m}{M}\right)_{\max} = 3$

122. No.

123. $\frac{h_1}{h_2} = \frac{14}{11}$

124. (a) $\frac{31}{40}MV^2$

(b) $\frac{15V}{14}$

(c) $V \le \sqrt{14gR}$

125. $\frac{2\sqrt{2}}{5}\frac{MR^2\omega}{m(u-v)}$

126. $v_c = \frac{3v_0}{4}\left(\hat{i} - \hat{j}\right)$

127. (a) $\omega = \frac{2v}{3R}$

(b) $kE_{\text{loss}} = Mv^2$

128. $\frac{J_0}{2}$

129. (a) $\omega = \frac{36P_0\cos\theta}{7m\ell} = \frac{36}{m\ell}$

(b) $P = P_0\sqrt{\sin^2\theta + \left(\frac{2}{7}\cos\theta\right)^2} = \sqrt{85}N - s$

130. $4(\sqrt{2} - 1)m$

131. (a) $\sqrt{2gh - v_0^2\left(\frac{R^2}{r^2} - 1\right)}$

(b) $h_{\min} = \sqrt{\frac{3}{2g}}v_0$

132. (a) $m = 72\ kg$

(b) $x > \frac{L}{2\sqrt{3}}$

133. $\frac{2\ell}{3}$

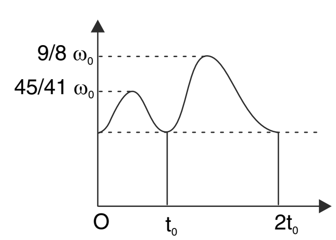
134. (a) $\frac{1}{6}$

(b) $\frac{49}{52}$

135. $\frac{2K}{3F}$

136. (a) $\frac{9}{8}\omega_0$

(b)



137. (a) $\omega_0 = \frac{9}{17}\omega$

(b) $\Delta E = \frac{5}{68} mV^2 + \frac{7}{34} mR^2\omega^2 - \frac{3}{17} mV R\omega$

(c) $\eta = \frac{6}{5}$

138. (a) Both have speed $\frac{u}{2}$

(b) The stick is at rest. Particle moves in original direction with speed

139. (a) $\tau = mR^2 \omega_0 \omega \sin 2\theta$; τ is maximum for $\theta = 45°$

(b) $F_\perp = 2mR\omega_0 \omega \cos\theta$; $\theta = 0°$

140. $\frac{3}{4}$ rad $\simeq 43°$

141. (a) $v = \sqrt{\frac{2\sqrt{2}\left(\sqrt{2}-1\right)}{3} gR}$

(b) $\omega = \frac{1}{3}\sqrt{\frac{2\sqrt{2}\left(\sqrt{2}-1\right)}{3} \frac{g}{R}}$

142. $T_1 = T_2 = T_3 = \frac{Mg}{3}$

143. 1.5 *cm*

144. (i) $\frac{L}{\sqrt{2}}$

(ii) $F_0 = \mu Mg\,(\sqrt{2} - 1)$

145. (a) $\frac{v}{2}$

(b) $\frac{v}{2}$

146. (b) *mg*

(c) zero

147. $v = \sqrt{g\left(a - \frac{b}{2}\right)}$

148. (b) $\sqrt{\frac{gL}{6}}$

149. (b) $\omega_0 = \frac{\sqrt{3}Ja}{I}$

(c) $\omega = \sqrt{\frac{2Mga}{I}\left(1 - \cos\left(30° - \theta\right)\right)}$

(d) $K = K_0 + Mga \sin\theta$

150. (a) $\frac{\lambda g\left(2\ell + \pi R - L\right)}{\left(\lambda L + \frac{M}{2}\right)}$

(b) $\frac{2\lambda^2 Rg\left(2\ell + \pi R - L\right)}{\left(\lambda L + \frac{M}{2}\right)}$

151. $2\sqrt{\frac{mgh}{2m + M}}$

152. $\frac{13}{10} Mv_0^2$

153. $\frac{mg}{4M + m}$

154. $\frac{g}{2}$

155. (a) $\omega = \sqrt{\frac{g}{L}}$

(b) $h_B = 5.04L$

156. (i) $\omega_{n+1} = \frac{6v}{13r} + \frac{7}{13}\omega_n$

(ii) $\omega_n = \frac{v}{r}\left[1 - \left(\frac{7}{13}\right)^{n-1}\right]$

(iii) $\frac{v}{r}$

157. (a) $\frac{v_0^2}{5L}$

(b) $\frac{3}{2}\frac{v_0}{L}$

158. $\frac{M}{2}\sqrt{\frac{\pi Lg}{3}}$

159. $\cos^{-1}\left\{\frac{3}{\sqrt{10}}\left(1 - \frac{v^2}{5g\ell}\right)\right\} + \cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$

SOLUTIONS

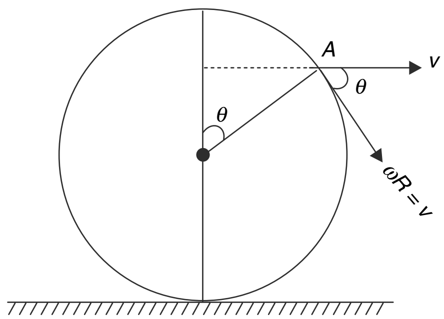
1. Total length of thread unwrapped from the pulley = $3h$

$$R\theta = 3h \Rightarrow \theta = \frac{3h}{R}$$

2. In reference frame attached to the centre of the disc, the velocity of point P is along the tangent. A point which is diametrically opposite to P has same velocity in opposite direction. This point has maximum speed relative to P. This maximum speed is equal to $2v$.

3. Let the Speed of centre of the ring be v. Then $v_A = 2v$

After the ring rotates through an angle θ, speed of point A is given by vector sum of v and $\omega R = v$ as shown in figure

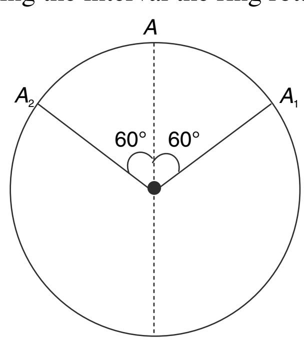


$$v_\theta = \sqrt{v^2 + v^2 + 2vv\cos\theta} = 2v\cos\left(\frac{\theta}{2}\right)$$

When $v_\theta = \frac{\sqrt{3}}{2} v_A = \sqrt{3}v \Rightarrow \sqrt{3}v = 2v\cos\left(\frac{\theta}{2}\right)$

$$\Rightarrow \cos\frac{\theta}{2} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60°$$

Speed of the point remains above $\frac{\sqrt{3}}{2} v_A$ during the interval the ring rotates from A to A_1 and the from A_2 to A



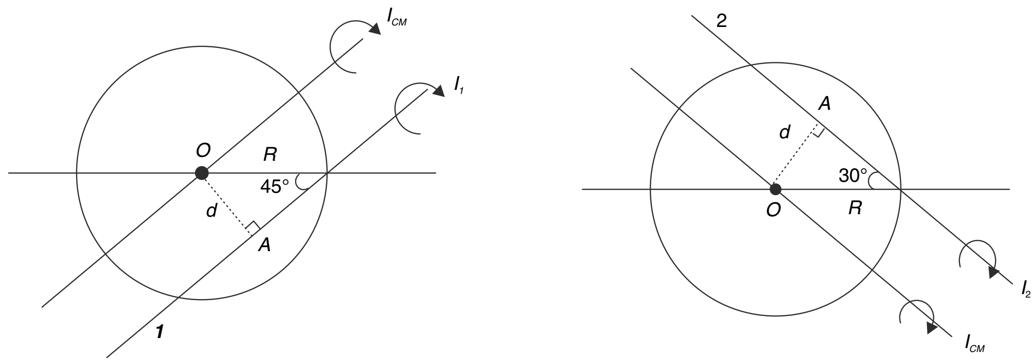
$\therefore$ The required time interval is -

$$t = \frac{120°}{360°} T = \frac{T}{3}$$

$\therefore$ Required answer is $\frac{t}{T} = \frac{1}{3}$

4. $I_1 = I_{CM} + Md^2 = \dfrac{MR^2}{4} + M\left(\dfrac{R}{\sqrt{2}}\right)^2$

$= \dfrac{3}{4}MR^2$

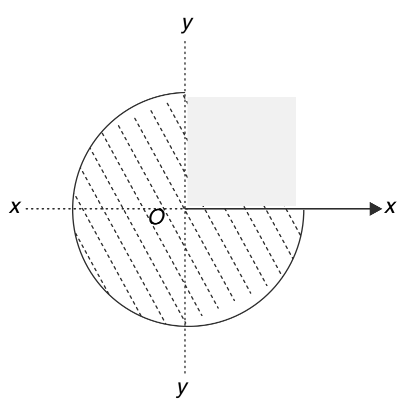


$I_2 = I_{CM} + Md^2 = \dfrac{MR^2}{4} + M\left(\dfrac{R}{\sqrt{2}}\right)^2 = \dfrac{MR^2}{2}$

$\therefore \dfrac{I_1}{I_2} = \dfrac{3}{4} \times \dfrac{2}{1} = \dfrac{3}{2}$

5. It is important to notice that the MOI (Moment of Inertia) of the disc about the Z axis will be $I_{zz} = \dfrac{1}{2}MR^2$

But $I_{xx} = I_{yy}$ $\therefore I_{xx} = \dfrac{1}{2}I_{zz} = \dfrac{1}{4}MR^2$



6. On folding about x axis the MOI about z axis does not change. If one third part is removed the MOI of the remaining part will be $\dfrac{2}{3}\dfrac{MR^2}{2} = \dfrac{1}{3}MR^2$

7. $I_0 = I_{CM} + Md^2$

$M(x^2 + y^2) = I_0 - I_{CM}$ [= a constant]

$\Rightarrow M\left(x^2 + y^2\right) = M\left(\ell^2 + \dfrac{\ell^2}{4}\right)$

$\Rightarrow x^2 + y^2 = \dfrac{5\ell^2}{4}$

8. The COM is at a distance of $d=\dfrac{2R}{\pi}$ from the cylinder axis

MOI about the axis of cylinder is $I = MR^2$

Using $I_{CM} = I - Md^2$

$$I_{CM}=MR^2-M\left(\frac{2R}{\pi}\right)^2=MR^2\left[1-\frac{4}{\pi^2}\right]$$

9. Hint: Using perpendicular axes theorem and symmetry one can show that $I_1=I_2=I_3=I_4=\dfrac{I_z}{2}$

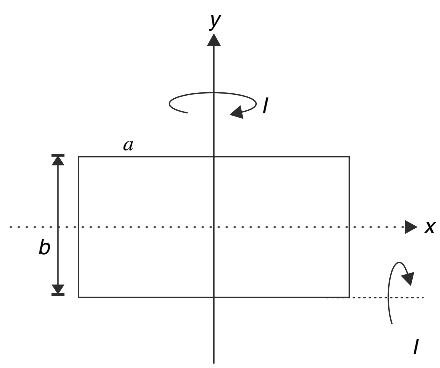
10. $I_0=\dfrac{2}{5}MR^2$ and $\Delta I=\dfrac{2}{3}mR^2$ Where $m = 0.02M$

$$\therefore \Delta I=\frac{0.04}{3}MR^2$$

$$\therefore \frac{\Delta I}{I_0}\times 100=\frac{0.04}{3}\times\frac{5}{2}\times 100=\frac{10}{3}\%$$

11. Hint: One can cut large number of triangular strips (as given in question) out of a disc of radius L and mass M. If all such strips are pasted one over another we get the strip described in the problem.

12. According to the problem $I=\dfrac{Ma^2}{12}$ and $I=\dfrac{Mb^2}{3}$



$$\therefore \frac{a^2}{12}=\frac{b^2}{3}\Rightarrow a=2b$$

$$\therefore I_z=\frac{Ma^2}{12}+\frac{Mb^2}{12}=I+\frac{I}{4}=\frac{5I}{4}$$

14. $I = I_A + I_B + I_C$

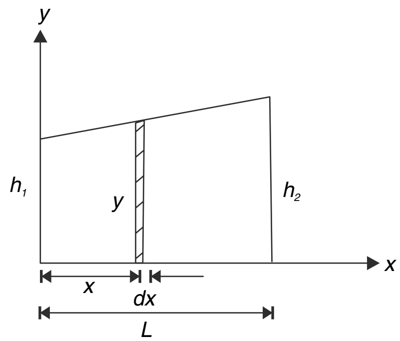
$$=\left(2MR^2\right)+\left[\frac{MR^2}{2}+M\left(3R\right)^2\right]+\left[\frac{MR^2}{2}+M\left(16R^2+R^2\right)\right]$$

$$=\frac{58}{2}MR^2=29MR^2$$

15. $y=h_1+\dfrac{h_2-h_1}{L}.x$

Area of strip of width dx is - $dA=\left[h_1+\dfrac{h_2-h_1}{L}x\right]dx$

Mass per unit area is $\sigma = \dfrac{M}{\frac{1}{2}L(h_1+h_2)} = \dfrac{2M}{L(h_1+h_2)}$



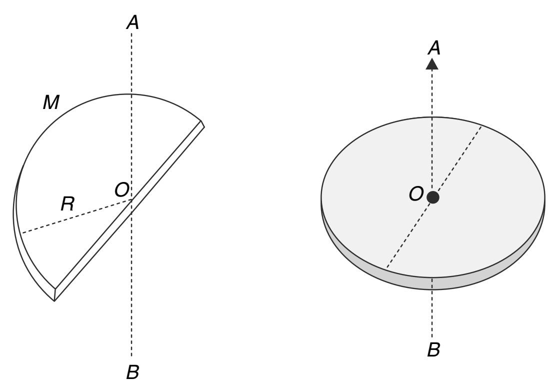
MOI of strip about y axis is $dI = \sigma dA\, x^2$

$$\therefore I = \int dI = \sigma\left[\int_0^L h_1 x^2 dx + \frac{h_2-h_1}{L}\int_0^L x^3 dx\right]$$

$$= \sigma\left[h_1\frac{L^3}{3} + \frac{h_2-h_1}{L}\frac{L^4}{4}\right] = \sigma L^3\left[\frac{h_1}{3}+\frac{h_2-h_1}{4}\right] = \frac{\sigma L^3}{12}[h_1+3h_2]$$

$$= \frac{ML^2}{6}\left(\frac{h_1+3h_2}{h_1+h_2}\right)$$

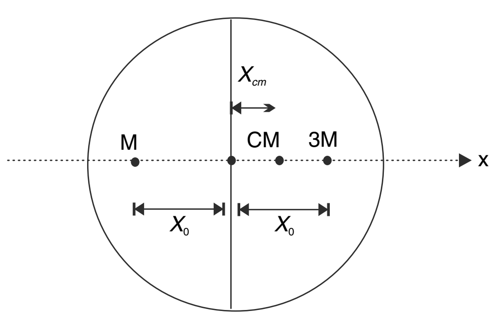
16. The *MOI* of a uniform semicircular disc about axis *AOB* shown in figure is $\frac{1}{2}MR^2$ (why?) and distance of its COM from *O* is $\dfrac{4R}{3\pi}$.



The *MOI* of the composite disc about *AOB* is

$$I_0 = \frac{1}{2}MR^2 + \frac{1}{2}(3M)R^2 = 2MR^2$$

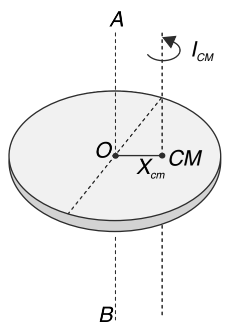
Let's locate the COM of the composite disc. In the figure shown $x_0 = \dfrac{4R}{3\pi}$



$$X_{cm} = \frac{3M.\frac{4R}{3\pi} - M\frac{4R}{3\pi}}{4M} = \frac{2R}{3\pi}$$

MOI of the composite disc about an axis through COM and parallel to *AOB* is

$$I_{CM} = I_0 - 4M\left(\frac{2R}{3\pi}\right)^2$$



$$\therefore I_{CM} = 2MR^2 - \frac{16MR^2}{9\pi^2}$$

$$= 2MR^2\left(1 - \frac{8}{9\pi^2}\right)$$

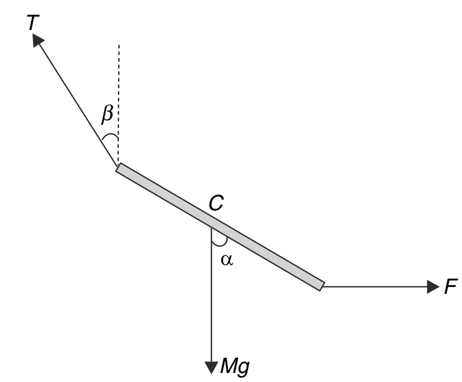
17. Horizontal and vertical equilibrium of the stick requires -

$F = T\sin\beta \Rightarrow Mg = T\sin\beta$ ....(i)

and mg = $T\cos\beta$ ....(ii)

Solving (i) and (ii)

$\tan\beta = 1 \Rightarrow \beta = 45°$



$T = \sqrt{2}\,Mg$

Rotational equilibrium requires torque about C to be zero ($\tau_c = 0$)-

$$T\cos\beta\frac{\ell}{2}\sin\alpha - T\sin\beta\frac{\ell}{2}\cos\alpha = F\frac{\ell}{2}\cos\alpha$$

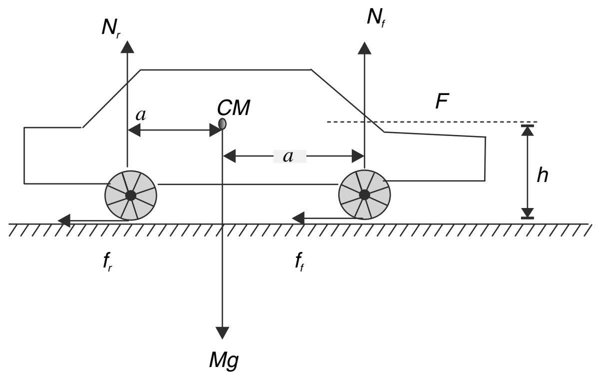
$$T\sin\alpha - T\cos\alpha = \sqrt{2}F\cos\alpha \quad \left[\because \cos\beta = \sin\beta = \frac{1}{\sqrt{2}}\right]$$

$$\sqrt{2}Mg\left(\sin\alpha - \cos\alpha\right) = \sqrt{2}Mg.\cos\alpha$$

$$\therefore \tan\alpha = 2 \Rightarrow \alpha = \tan^{-1}(2)$$

18. For rotational equilibrium, resultant torque about centre of mass of the car = 0

$\therefore N_f a = N_r a + (f_f + f_r)\, h$



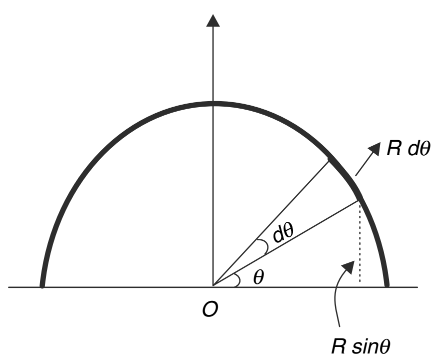
$$N_f = N_r + \left(f_f + f_r\right)\frac{h}{a}$$

$$\therefore N_f > N_r$$

19. In equilibrium the COM of the semicircle must lie on the vertical line through A. This will result in no torque of weight about A, hence equilibrium.

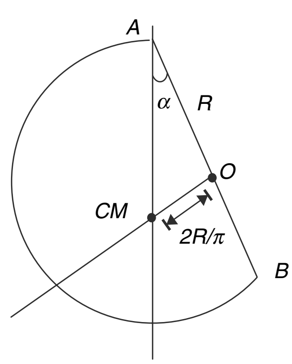
First we need to find the position of COM of a semicircular ring. For this consider an element of angular width dθ as shown.

$$y_{cm} = \frac{\int y dm}{\int dm} = \frac{\int_0^{\pi}(R\sin\theta)(\lambda R d\theta)}{\lambda \pi R}$$



[where λ is linear mass density and $\int dm$ = mass of the ring = $\lambda\pi R$]

$$\therefore y_{cm} = \frac{2R}{\pi}$$



From symmetry $x_{cm} = 0$

For equilibrium of the suspended ring $\tan\alpha = \dfrac{2R/\pi}{R}$

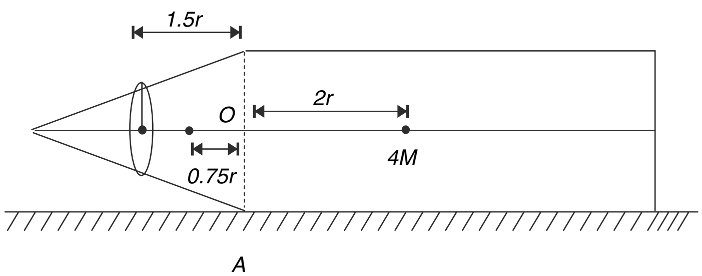
$$\Rightarrow \alpha = \tan^{-1}\left(\frac{2}{\pi}\right)$$

20. In critical case the COM of the system will lie at the centre of the circular base of the conical part (Just above A ; at O)

Volume of cone $= \dfrac{1}{3}\pi r^2(3r) = \pi r^3$

Volume of cylinder $= \pi r^2(4r) = 4\pi r^3$

$\therefore$ Mass of cylinder is $4M$



COM of the cone is at a distance of $\dfrac{h}{4} = \dfrac{3r}{4}$ from O.

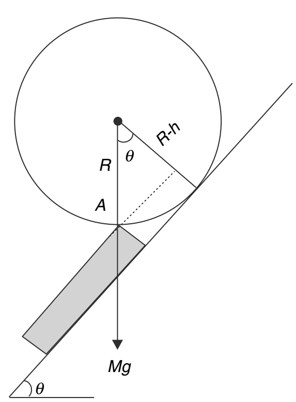
Let mass of the ring $= m$

$m \times 1.5r + M \times 0.75r = 4M \times 2r$

$$\therefore m = \frac{29}{6}M$$

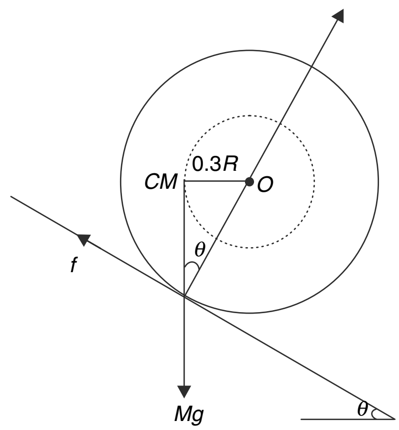
21. (a) The ball will begin to climb the step if the line of action of its weight (Mg) passes to the left of point A. This will cause an unbalanced torque about A.

In critical case (see figure) $\cos\theta = \dfrac{R-h}{R}$



(b) In the position shown, the COM of the ball is vertically above point A at a distance R. As the ball climbs, its centre of mass will lower from this position. (Otherwise why will the ball come down?) In the position shown the centre of mass is at highest position.

22. For rotational equilibrium of the sphere, it is necessary that vertical line (along which force Mg acts) through COM pass through contact point. This will ensure no torque about the contact point. Situation for maximum θ is shown in figure. Obviously, $\sin\theta = 0.3$.

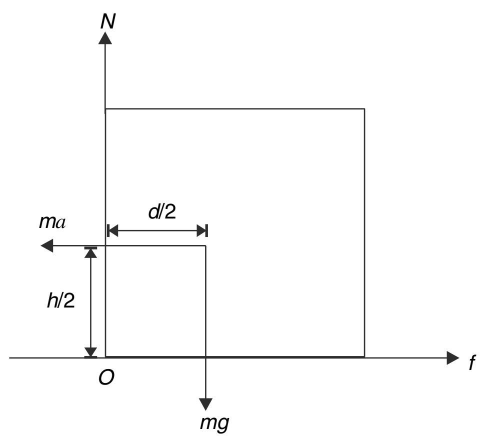


If θ is less than $\sin^{-1}$ (0.3) the sphere can be rotated so that vertical line through COM passes through contact point. If θ is larger than $\sin^{-1}$ (0.3), the line of Mg will pass from right of contact point and will produce an unbalanced torque.

23. Acceleration $a = \dfrac{F}{2m} = \dfrac{kt}{2m}$

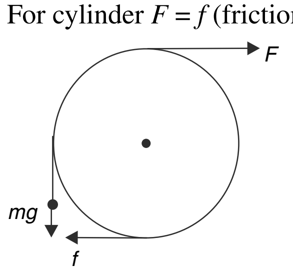
The force diagram of block B in reference frame attached to A is shown in figure. The normal force passes through left edge at the instant the block is about to topple.

The block will be on verge of toppling when $ma\dfrac{h}{2} = mg\dfrac{d}{2}$



$$m\frac{kt}{2m}\frac{h}{2} = mg\frac{d}{2} \quad \therefore t = \frac{2dgm}{kh}$$

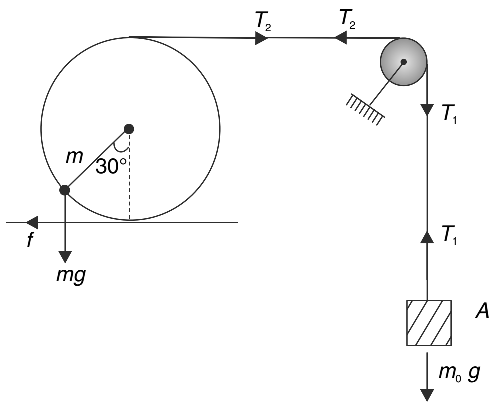
24. For cylinder $F = f$ (friction)



And $FR + fR = mg\,R$

$$\Rightarrow 2F = mg \quad \Rightarrow F = \frac{mg}{2}$$

25 For equilibrium of m_0 ; $T_1 = m_0\,g$



For rotation equilibrium of pulley

$$T_2 = T_1 = m_0 g$$

For translational equilibrium of cylinder + rod in horizontal direction

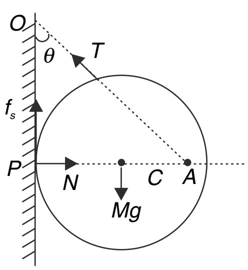
$f = T_2 = m_0 g$ (towards left)

For rotational equilibrium of cylinder + rod

$mg\,R\sin 30° = f\,R + T_2\,R$

$$\frac{mg}{2} = 2m_0 g \quad \Rightarrow m = 4m_0$$

26. Net torque about $A = 0$



$$f_s\frac{3R}{2} = Mg.\frac{R}{2}$$

$\therefore f_s = \dfrac{Mg}{3}$........................(i) $\quad \left[\therefore PA = \dfrac{3R}{2}\tan 45° = \dfrac{3R}{2}\right]$

Horizontal equilibrium $N = \dfrac{T}{\sqrt{2}}$..............(ii)

Vertical equilibrium $\dfrac{T}{\sqrt{2}} = Mg - f_s$

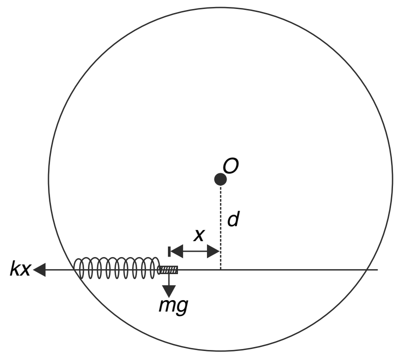
$$T = \frac{2\sqrt{2}}{3}Mg \quad \text{.............(iii)}$$

Put in (ii) $N = \frac{2}{3}Mg$

$$f_s \le \mu_s N \Rightarrow \frac{Mg}{3} \le \mu_s.\frac{2}{3}Mg$$

$$\Rightarrow \frac{1}{2} \le \mu_s$$

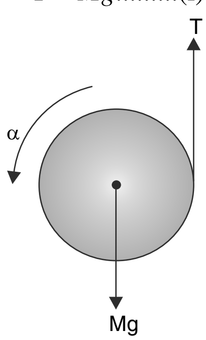
28. When spring is compressed by x the torque of weight of the sleeve is anticlockwise. But the torque of spring force on the wheel is clockwise.



These two torques can balance each other if $k\ x\ d\ =\ mg.\ x \quad \Rightarrow k = \frac{mg}{d}$

29. Translational equilibrium requires

$T = Mg$.........(i)



For rotation

$$T = I\alpha \quad \Rightarrow T.R = \frac{1}{2}MR^2.\alpha$$

$$\Rightarrow Mg = \frac{1}{2}MR\alpha \quad \text{[using (i)]}$$

$$\Rightarrow \alpha = \frac{2g}{R}$$

Using

$$\omega^2 = \omega_0^2 + 2\alpha\theta\,; \omega^2 = 0 + 2.\frac{2g}{R}\theta$$

$$\Rightarrow \theta = \frac{\omega^2 R}{4g}$$

$\Rightarrow$ The cylinder rotates through angle $\theta = \dfrac{\omega^2 R}{4g}$ by the time it acquires angular speed ω.

$\therefore$ Length of string unwound $R\omega = \dfrac{\omega^2 R^2}{4g}$

30. **(a)** For $M = m$ acceleration of the two objects must be same. Because both experience a downward force mg and an upward force T

Considering any one of them $mg - T = ma$...(i)

Rotation of the disc $Tr = \dfrac{1}{2}mr^2\alpha$

$\Rightarrow T = \dfrac{1}{2}mr\alpha$ But $r\alpha = a + a$

$\therefore T = \dfrac{1}{2}m(2a) \Rightarrow T = ma$...........(ii)

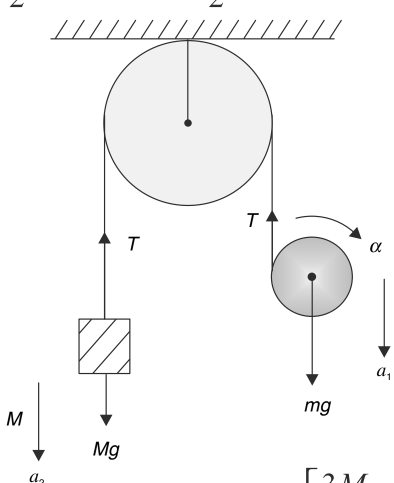
Solving (i) and (ii) $a = \dfrac{g}{2}$

(b) $a_1 + a_2 = r\alpha$ ....(i)

$Mg - T = Ma_2$ ....(ii)

$mg - T = ma_1$ ....(iii)

$T.r = \dfrac{1}{2}mr^2.\alpha \Rightarrow T = \dfrac{1}{2}m(a_1 + a_2)$ ....(iv)



Solving (ii), (iii) and (iv) $a_2 = g\left[\dfrac{3M - m}{3M + m}\right]$

For a_2 to be negative $m > 3M \Rightarrow \dfrac{1}{3} > \dfrac{M}{m}$

31. For sphere $TR = \dfrac{2}{5}MR^2.\alpha$

$TR = \dfrac{2}{5}MR(R\alpha)$ But $a = R\alpha$ = acceleration of m

$\therefore T = \frac{2}{5} Ma$.................(i)

For the block $mg - T = ma$ ....(ii)

(i) + (ii) gives $\frac{mg}{m + \frac{2}{5}M} = a \Rightarrow a = \frac{5mg}{5m + 2M}$

$$\therefore \alpha = \frac{a}{R} = \frac{5mg}{R(5m + 2M)}$$

Angular displacement of sphere while it rotates 5 turns = $\theta = 2\pi \times 5$ rad

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\omega^2 = 0 + \frac{10mg}{R(5m + 2M)} 10\pi = \frac{100\pi mg}{R(5m + 2M)}$$

$$\omega = \sqrt{\frac{100\pi mg}{R(5m + 2M)}}$$

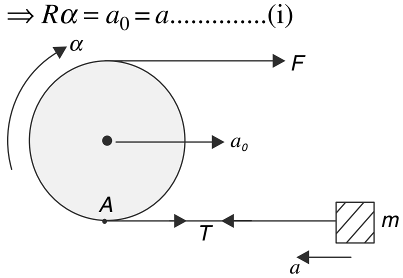
(b) After completion of 5 turns acceleration of m will become g

$$\therefore \Delta a = g\left[1 - \frac{5m}{5m + 2M}\right]$$

$$\Delta a = g\left[\frac{2M}{5m + 2M}\right]$$

32. For no slipping condition of thread acceleration of point A on the circumference of the disc = acceleration of the small body

$\Rightarrow R\alpha = a_0 = a$..............(i)



Other equations are

$T = ma$ ....(ii)

$F + T = ma_0$ ....(iii)

And $(F - T)R = \frac{1}{2}mR^2.\alpha \quad \Rightarrow F - T = \frac{1}{2}mR\alpha$.......(iv)

Using (i) in (iv) $F - T = \frac{1}{2}m(a + a_0)$

Using (ii) in this $F = \frac{3}{2}ma + \frac{1}{2}ma$ ....(v)

From (ii) and (iii) $F = ma_0 - ma$.........(vi)

Solving (v) and (vi) $a = \dfrac{F}{4m}$

33. Let the normal reaction of the wall be N.

Note that the force Mg will be acting at COM of the rod which is located at a horizontal distance of $\dfrac{2R}{\pi}$ from the centre

$$\text{N.R} = \left(R - \frac{2R}{\pi}\right)\text{Mg}$$

$$\text{N} = \left(1 - \frac{2}{\pi}\right)\text{Mg}$$

34. Friction $f < mg \sin\theta$

Normal reaction $N = mg \cos\theta$

$\therefore$ Total force $F = \sqrt{f^2 + N^2} < mg$

35. The solid ball will not rotate. It will only translate. We can replace it with a point mass placed at the centre of the shell. Torque about the point of contact is $\tau = 2\, MgR \sin\theta$

Moment of inertia about the rotation axis through point of contact is

$$I = \left(\frac{2}{3}MR^2 + MR^2\right) + MR^2 = \frac{8}{3}MR^2$$

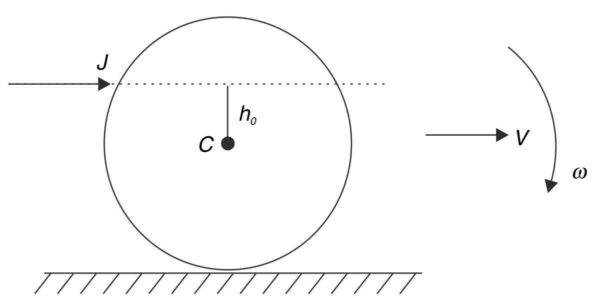
$$\therefore \alpha = \frac{\tau}{I} = \frac{3g\sin\theta}{4R}$$

$$\therefore a = R\alpha = \frac{3g\sin\theta}{4}$$

You may also proceed by assuming a friction force on the ball and then writing equation of translation and rotation about centre of mass. In an altogether different approach, you can use energy conservation to arrive at the same result

Write the total mechanical energy of the rolling ball and take it to be a constant. Differentiate this equation with respect to time to get the acceleration.

36. For $h = h_0$ the sphere starts pure rolling. For $h > h_0$, the angular speed acquired by the sphere is high such that $\omega R > V$. In this case the contact point has a backward velocity. Friction acts in forward direction and the sphere accelerates.



For $h < h_0$, $\omega R < V$

The contact point has a forward velocity and the friction acts in backward direction. The sphere decelerates.

If linear impulse = J

$J = MV$...........(i)

Angular impulse about centre = Jh

$$Jh = \frac{2}{5}MR^2\omega$$

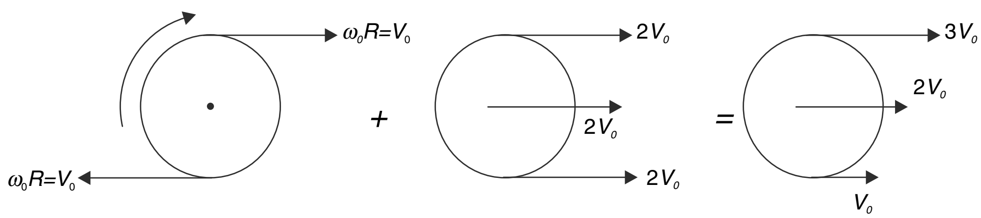
$$\therefore R\omega = \frac{5}{2}\frac{Jh}{RM} = \frac{5}{2}\frac{MV.h}{RM} = \frac{5}{2}V\frac{h}{R}$$

For pure rolling

$$V = R\omega \quad \therefore h = \frac{2}{5}R \quad \Rightarrow h_0 = \frac{2}{5}R$$

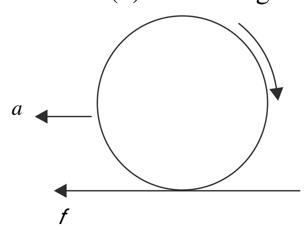
37. Hint : The acceleration of the plank will be twice that of the cylinder

38. (i) The initial velocity of the centre of the sphere will be $2V_0$ and its angular speed is $\omega_0 = \frac{V_0}{R}$



Because contact point is sliding forward, the kinetic friction will act backward.

This friction will produce a linear retardation (a) and an angular acceleration (α)



$$a = \frac{f}{F} \text{ and } \alpha = \frac{fR}{\frac{2}{5}MR^2} = \frac{5f}{2MR}$$

Velocity at time 't' is

$$V = 2V_0 - at = 2V_0 - \frac{ft}{M}$$

Angular velocity at time 't' $\omega = \frac{V_0}{R} + \frac{5ft}{2MR}$

If pure rolling starts at time 't' $V = \omega R$

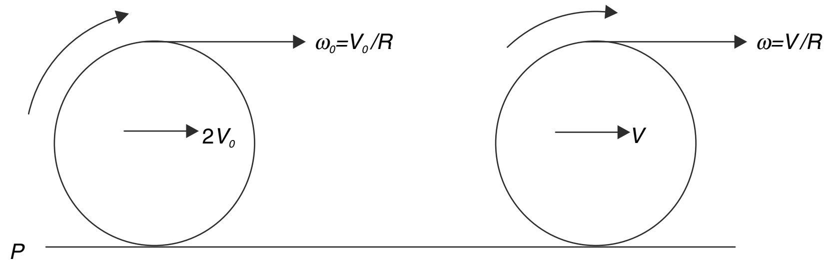
$$2V_0 - \frac{ft}{M} = V_0 + \frac{5ft}{2M}$$

$$\therefore V_0 = \frac{7}{2}\frac{ft}{M} \Rightarrow t = \frac{2V_0 M}{7f}$$

At this time speed is -

$$V = 2V_0 - \frac{ft}{M} = 2V_0 - \frac{2V_0}{7} = \frac{12V_0}{7}$$

ALTERNATE METHOD :



Angular momentum is conserved about a point (like P) on the ground $L_f = L_i$

$$\frac{2}{5}MR^2.\omega + MVR = \frac{2}{5}MR^2.\omega_0 + M(2V_0)R$$

$$\Rightarrow 7V = 12V_0 \Rightarrow V = \frac{12}{7}V_0$$

(ii) This problem can be solved by proceeding in a manner similar to the previous one.

39. Hint. The motion is translational. There is no rotation.

40. In figure (a) $KE = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}MV^2 + \frac{1}{2} \times \frac{1}{2}MR^2\left(\frac{V}{R}\right)^2 = \frac{3}{4}MV^2$$

In figure (b) *MOI* about rotation axis

$$I = M\left(\frac{L^2}{12} + \frac{R^2}{4}\right) = M\left(\frac{3R^2}{12} + \frac{R^2}{4}\right)$$

$$= \frac{MR^2}{2}$$

$$\therefore KE = \frac{1}{2}MV^2 + \frac{1}{2}\left(\frac{MR^2}{2}\right)\left(\frac{V}{R}\right)^2$$

$$= \frac{3}{4}MV^2$$

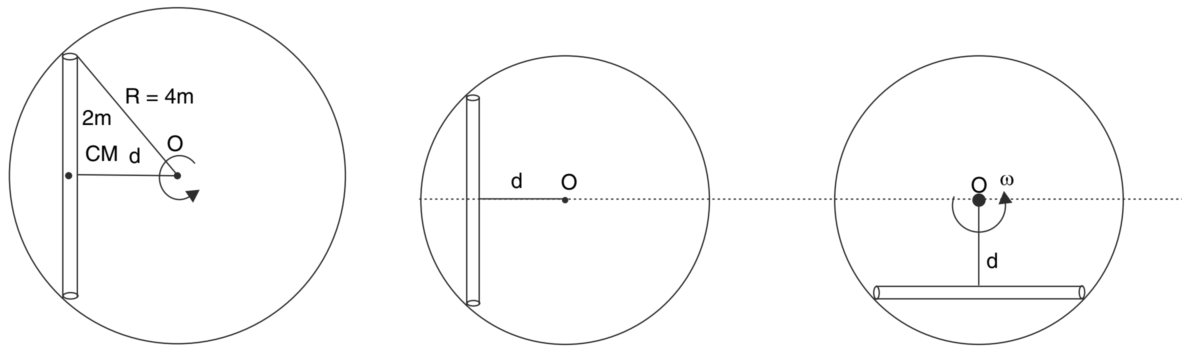
Kinetic energy in case (a) does not depend on the length of the cylinder. It will increase in case (b) if the length is increased.

41. Moment of Inertia of the rod about an axis through O is

$$I = I_{CM} + Md^2 = \frac{ML^2}{12} + Md^2$$

$$= \frac{4 \times 4^2}{12} + 4 \times \left(\sqrt{4^2 - 2^2}\right)^2$$

$$= \frac{160}{3}\,\text{kg} - m^2$$



$$d=\sqrt{4^2-2^2}=2\sqrt{3}$$

The rod is in pure rotation about O. Applying conservation of mechanical energy $\frac{1}{2}I\omega^2 = Mgd$

$$\Rightarrow \frac{1}{2}\times\frac{160}{3}\times\omega^2 = 4\times10\times2\sqrt{3}$$

$$\omega^2 = 6\sqrt{3} \Rightarrow \omega = 3.2\ rad/s$$

42. (i) Net force on the body = 0

However, it has a net torque acting on it.

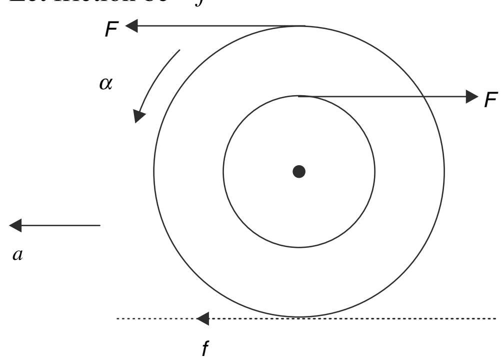
$\tau = F.2R - FR = FR = 20 \times 1 = 20\ N-m$

$\therefore \alpha = \frac{\tau}{I} = \frac{20}{8} = \frac{5}{2}\ rad/s^2$

Angular speed at $t = 2s$ is $\omega = \alpha t = 5\ rad/s$

$\therefore K = \frac{1}{2}I\omega^2 = \frac{1}{2}\times 8\times 5^2 = 100J$.

(ii) Let friction be $= f$



$f = Ma \Rightarrow f = 4a$ ....(i)

And $I\alpha = F.2R - FR - f.2R$

$\Rightarrow 8.\alpha = 20 \times 1 - 2f \Rightarrow 4\alpha = 10 - f$

But $2\ R.\alpha = a \Rightarrow 2\alpha = a$

$\therefore\ 10 - f = 2a$ ....(ii)

Solving (i) and (ii) $a = \frac{10}{6}\, m/s^2 = \frac{5}{3}\, m/s^2$

$$\alpha = \frac{a}{2R} = \frac{5}{6}\, rad/s^2$$

Hence at $t = 2s; V = at = \frac{10}{3}\, m/s$ and $\omega = \alpha t = \frac{5}{3}\, rad/s$

$$\therefore K = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2} \times 4 \times \left(\frac{10}{3}\right)^2 + \frac{1}{2} \times 8 \times \left(\frac{5}{3}\right)^2 = 33.33J$$

43. In case A, motion is translational, kinetic energy given to the plate is $K_A = \frac{1}{2}MV^2$

In case B, the plate is in pure rotation about a horizontal axis through the upper end of the connecting rod.

$$\therefore K_B = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}\left[I_{CM} + Md^2\right]\left(\frac{V}{d}\right)^2$$

d = distance of COM from the upper hinge $= a + \frac{a}{2} = \frac{3a}{2}$

$$\therefore K_B = \frac{1}{2}\left[\frac{Ma^2}{6} + M\left(\frac{3a}{2}\right)^2\right]\left(\frac{2V}{3a}\right)^2$$

$$= \frac{1}{2}MV^2\left(\frac{29}{27}\right)$$

Hence, the plate has been given more *KE* in case (B)

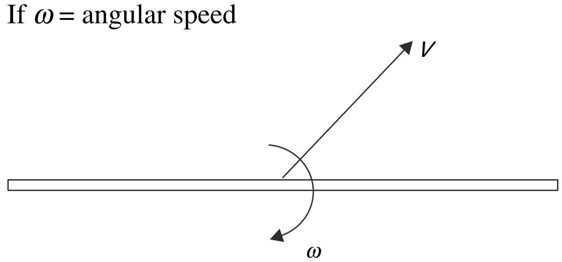
Therefore, COM will rise to a greater height in case (B).

44. Energy conservation :

$$3m.g.2R = mgR + \frac{1}{2}mV^2 + \frac{1}{2}\left(mR^2\right)\left(\frac{V}{R}\right)^2$$

$$5gR = V^2 \Rightarrow V = \sqrt{5gR}$$

45. If ω = angular speed



$$\frac{1}{2}MV^2 + \frac{1}{2}\left(\frac{1}{12}ML^2\right)\omega^2 = \frac{4}{3}MV^2$$

$$= \frac{1}{24}(L\omega)^2 = \frac{5}{6}MV^2 \therefore (\omega L)^2 = 20V^2$$

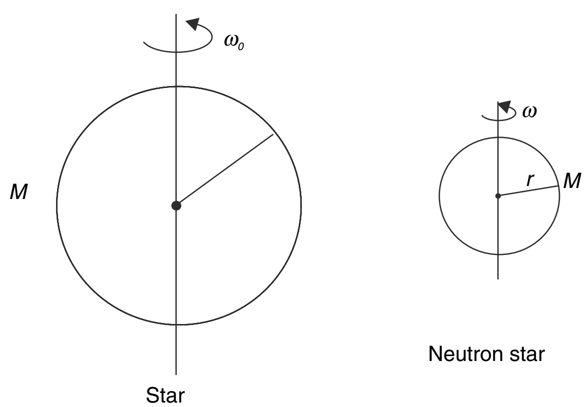
$$\Rightarrow \omega L = 2\sqrt{5}V \Rightarrow \frac{\omega L}{2} = \sqrt{5}V$$

The velocity of one end of the rod is given by vector sum of its velocity of COM and $\frac{\omega L}{2} = \sqrt{5}V$ (that is perpendicular to length).

Speed of end point can range from ($\sqrt{5}\,V - V$) to ($\sqrt{5}\,V + V$) depending on the direction of velocity of the COM of the rod.

46. The change in angular momentum of the propeller is to the right $\left(\overrightarrow{\Delta L}\right)$ if the plane turns to right. To conserve the angular momentum the entire body of the plane will have a tendency to rotate in vertical plane. The plane will tilt with its nose down. This will create an angular momentum in horizontal direction towards left for the plane.

47. $$\frac{V}{V_0} = 10^{-12} \Rightarrow \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = 10^{-12} \Rightarrow \frac{r}{R} = 10^{-4}$$



Angular momentum conservation gives -

$$\frac{2}{5}MR^2\omega_0 = \frac{2}{5}Mr^2\omega$$

$$\Rightarrow \omega = \omega_0\left(\frac{R}{r}\right)^2 = \frac{\pi}{10^3} \times \left(10^4\right)^2 = \pi \times 10^5 \ rad/s$$

48. Let ω = angular speed of square plate when the block moves out

Angular momentum conservation about rotation axis of the plate gives

$1 \times 2 \times 1 = -1 \times V \times 1 + 4\omega$

$$\Rightarrow \omega = \frac{V+2}{4} \text{...............(i)}$$

Energy conservation

$$\frac{1}{2}\times 1\times 2^2=\frac{1}{2}\times 1\times V^2+\frac{1}{2}\times 4\times \omega^2$$

$V^2+4\omega^2=4$..................(ii)

From (i) $V^2+4\left(\frac{V+2}{4}\right)^2=4$

$$5V^2+4V-12=0\Rightarrow V=\frac{6}{5}\text{ m/s}$$

49. Impulse to the particle = mV (←)

Impulse to the disc = mV (→)

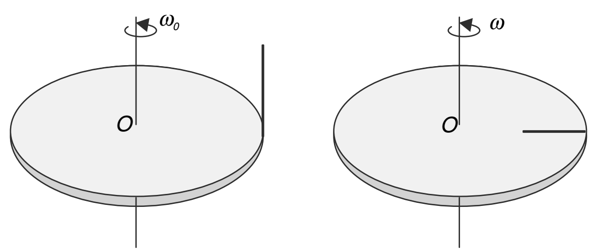
Angular impulse about centre $=mV\frac{R}{2}$

$\therefore \frac{1}{2}mR^2.\omega=mV\frac{R}{2}$

$\therefore \omega=\frac{V}{R}$

50. No external force has torque about the rotation axis. Weight of the man is a force that is parallel to the rotation axis. Hence, angular momentum of the system (platform + man) remains conserved.

$I_0\,\omega_0=I\omega$

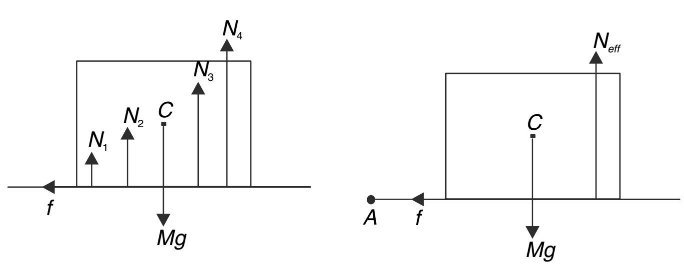


$$\left[\frac{1}{2}MR^2+\frac{M}{2}.R^2\right]\omega_0=\left[\frac{1}{2}MR^2+\frac{1}{12}\frac{M}{2}\left(\frac{R}{2}\right)^2+\frac{M}{2}\left(\frac{3R}{4}\right)^2\right]\omega$$

[Note that MOI of the rod lying on disc can be written using $I=I_{CM}+md^2$]

Simplifying gives $\omega=\frac{24}{19}\omega_0$

51. (a) $L_A=MV_0\frac{b}{2}$



(b) About the centre (C) of the block there is no rotation. Mg has no torque about C. Friction has a clockwise torque about C. The torque of normal reaction balances this. The effective normal force is to the right of C. If you look about point A, N_{eff} has higher torque than torque due to Mg since distance of N_{eff} is greater

This produces an anticlockwise torque which eats away the angular momentum of the block

52. The centre of mass of the plate has co-ordinates given by (prove this yourself)

$$x = \frac{28a}{9\pi} \simeq a; y = \frac{28a}{9\pi} \simeq a$$

The line of impact passes through COM hence there will be no rotation.

Momentum Conservation gives

$$\frac{mu}{2} = -\frac{m}{2}\frac{u}{2} + mV \quad \therefore V = \frac{3u}{4}$$

All point on the plate will have this velocity only.

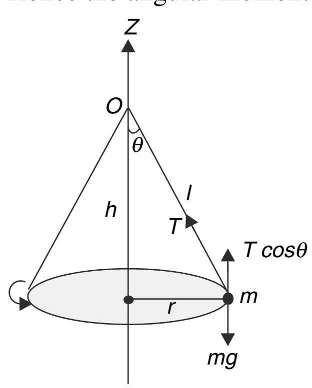
53. Angular momentum about the axis is

$$L = I\omega = \frac{1}{3}mL^2 \sin^2\theta \frac{v}{L\sin\theta}$$

$$\Rightarrow L = \frac{1}{2}mLv\sin\theta$$

The angular momentum about point A is not a constant because it keeps changing direction.

54. The two forces on the mass are mg and T (tension). T has no torque about O. Torque of mg about z – axis is zero. Hence the angular momentum of the mass about z axis will remain conserved as the string is dragged up



$\therefore L_z = mr^2\omega = \text{const}$ ....(i)

For motion of the mass

$T\cos\theta = mg$............(ii)

T $\sin\theta = m\omega^2$r...........(iii)

(iii) ÷ (ii)

$$\tan\theta = \frac{\omega^2 r}{2}$$

$$\tan\theta = \frac{\omega^2}{g}\ell\sin\theta \qquad [\because r = \ell\sin\theta]$$

$$\Rightarrow \omega = \sqrt{\frac{g}{\ell\cos\theta}} = \sqrt{\frac{g}{h}}$$

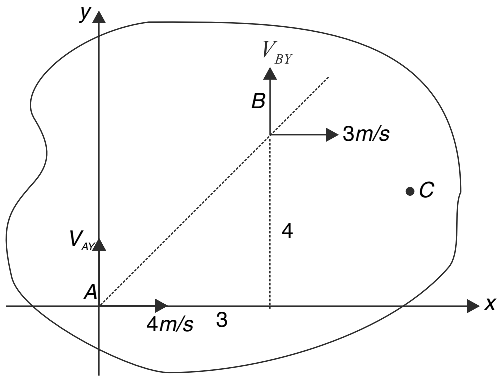
Putting in (i) $mr^2\sqrt{\frac{g}{h}=}$ a const

$$\Rightarrow r^4 \propto h \Rightarrow r \propto h^{\frac{1}{4}}$$

$\therefore$ When h becomes half, r becomes $\left(\frac{1}{2}\right)^{\frac{1}{4}}$ times its original value.

LEVEL 2

55. Two particles of a rigid body must have same velocity component along the line joining them.



Velocity of 'A' along AB = velocity of 'B' along AB

$$4\times\frac{3}{5}+V_{Ay}\ \times\frac{4}{5}=3\times\frac{3}{5}+V_{By}\ \times\frac{4}{5}$$

$$12+4V_{Ay}\ =9+4V_{By}$$

$$\frac{3}{4}=V_{By}\ -V_{Ay}\ \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Similarly, for A and C

$4V_{cx} - 3V_{Ay} = 22$............. (2)

And for B and C

$V_{By} + V_{Cx} = 1$ ....,....... (3)

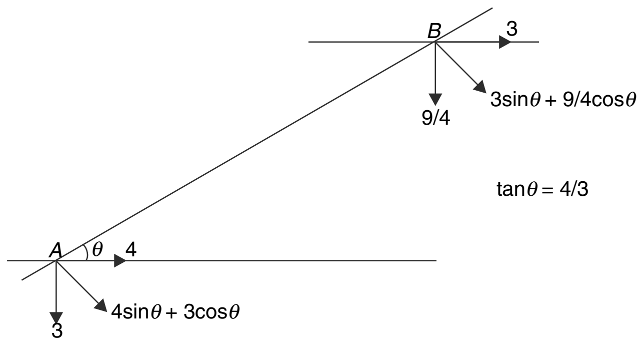
Solving we get $V_{Ay} = -3$ m/s; $V_{By}-\frac{9}{4}m/s$; $V_{cx}=\frac{13}{4}m/s$

$$\therefore \vec{V}_A = 4i-3j;\ \vec{V}_B = 3i-\frac{9}{4}j;\ \vec{V}_C = \frac{13}{4}i-2j$$

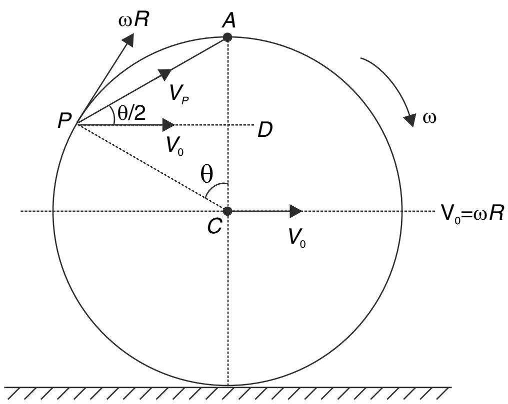
(b) Angular velocity = $\dfrac{\text{Velocity of } B \perp \text{ to } AB - \text{velocity of } A \perp \text{ to } AB}{AB}$

$$=\frac{\left(3\times\frac{4}{5}+\frac{9}{4}\times\frac{3}{5}\right)-\left(4\times\frac{4}{5}+3\times\frac{3}{5}\right)}{5}$$

$$=-\frac{1}{4}\text{rad/s}\,;\omega=\frac{1}{4}\text{rad/s}$$



56.



For a pure rolling motion $V_0 = \omega R$

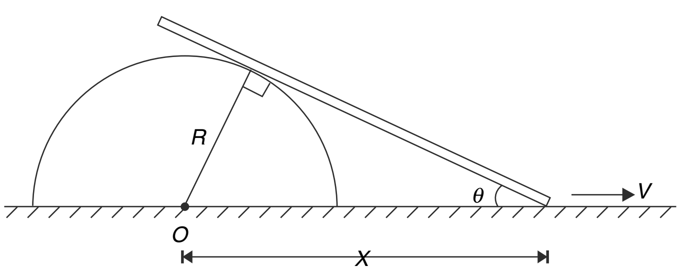
Velocity of a point like P will be along a line bisecting the direction of V_0 and ωR at P. One can easily show that

$\angle APC + \angle PCA + \angle CAP = 180°$

$\Rightarrow PAC$ is a triangle [with $\angle APD = \theta/2$]

$\therefore V_P$ is along $\overrightarrow{PA}$

57. (a)



From geometry, $x=\dfrac{R}{\sin\theta}$

$$\therefore \frac{dx}{dt}=R\frac{d}{dt}\left(\frac{1}{\sin\theta}\right)\Rightarrow V=-R\frac{\cos\theta}{\sin^2\theta}\frac{d\theta}{dt}$$

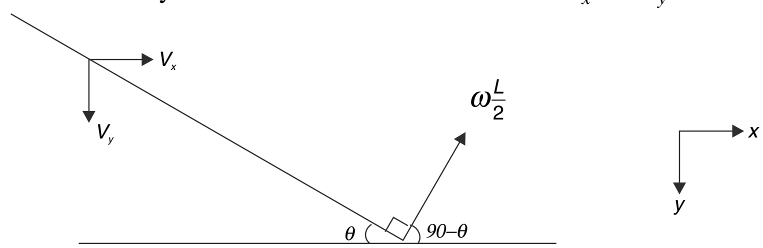
$$-\frac{d\theta}{dt} = \omega = \text{ angular speed of the rod}$$

$$\therefore \omega = \frac{v}{R}\frac{\sin^2\theta}{\cos\theta}$$

For $\theta = 30°$

$$\omega = \frac{v}{R}.\frac{\left(\frac{1}{2}\right)^2}{\frac{\sqrt{3}}{2}} = \frac{v}{2\sqrt{3}R}$$

(b) Let the components of velocity of the centre of mass of the rod be v_x and v_y

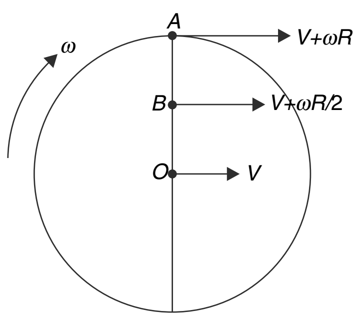


The velocity of end A of the rod with respect to the centre will be $\frac{\omega L}{2} = \omega R$ in the direction shown. For resultant velocity of end A to be along x it is necessary that

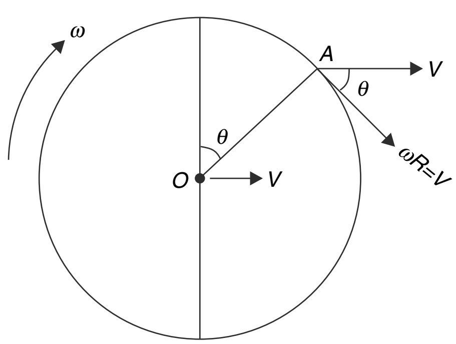
$$v_y = \omega R\cos\theta = \frac{v}{2\sqrt{3}}\frac{\sqrt{3}}{2} = \frac{v}{4}$$

58. (a) $v_A = v + \omega R = 2v$ and

$$v_B = v + \frac{\omega R}{2} = \frac{3v}{2}$$



(b) Let speed of A become equal to $\frac{3v}{2}$ after the disc has rotated through an angle θ about its centre.



Speed of point A is $v_A = \sqrt{v^2 + v^2 + 2vv\cos\theta}$

$\frac{3v}{2} = v\sqrt{2(1+\cos\theta)} \Rightarrow \frac{3}{2} = 2\cos\frac{\theta}{2}$

$\Rightarrow \theta = 2\cdot\cos^{-1}\left(\frac{3}{4}\right)$

Time $t = \frac{\theta}{\omega} = \frac{\theta R}{v} = \frac{2R}{v}\cos^{-1}\left(\frac{3}{4}\right)$

59. (a) With respect to COM, the acceleration of A is

$a_{ACM} = a_{\text{radial}} + a_{\text{ tangential}}$

$= \omega^2 R(\downarrow) + R\alpha(\rightarrow) = 2\sqrt{3}(\downarrow) + 6(\rightarrow)$

$\therefore a_A = a_{ACM} + a_{CM}$

$= 2\sqrt{3}(\downarrow) + 6(\rightarrow) + R\alpha(\rightarrow)$

$= 2\sqrt{3}(\downarrow) + 6(\rightarrow) + 6(\rightarrow)$

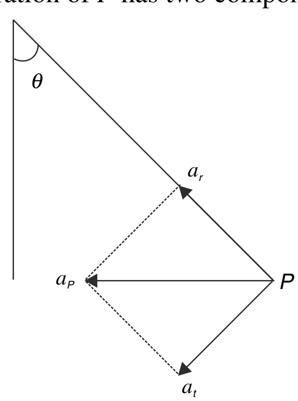
$= 2\sqrt{3}(\downarrow) + 12(\rightarrow)$

$a_A = \sqrt{(2\sqrt{3})^2 + 12^2} = \sqrt{156}\ \text{ms}^{-2}$

(b) $a_{BCM} = \omega^2 R(\uparrow) + R\alpha(\leftarrow) = 2\sqrt{3}(\uparrow) + 6(\leftarrow)$

$\therefore a_B = a_{BCM} + a_{CM} = 2\sqrt{3}(\uparrow)$

(c) With respect to the COM the acceleration of P has two components –



$a_r = \omega^2 r = r$ and $a_t = r\alpha = \sqrt{3}r$

The resultant acceleration of P must be horizontal ($\leftarrow$) equal to 6 ms^{-2}

$\Rightarrow a_r\cos\theta = a_t\sin\theta \quad \Rightarrow \frac{1}{\sqrt{3}} = \tan\theta$

$\Rightarrow \theta = \frac{\pi}{6}$

and $\sqrt{a_r^2 + a_t^2} = 6 \Rightarrow 2r = 6 \Rightarrow r = 3m$

60. The centre of the wheel is moving with constant speed on a circular path of radius 6 R. Hence it has a centripetal acceleration of $a_c = \dfrac{v^2}{6R}$ directed towards the centre of curvature of the convex surface.

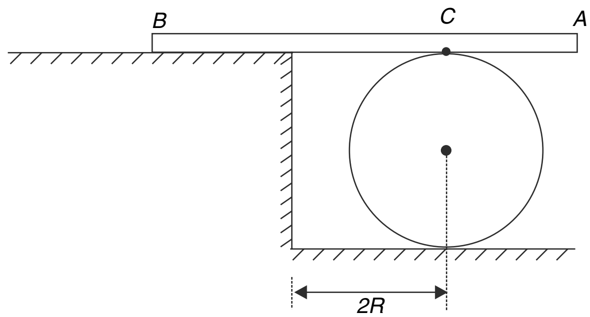
With respect to the centre of the wheel the contact point has acceleration equal to $\dfrac{v^2}{R}$ directed towards the centre of the wheel.

$\therefore$ Acceleration of the contact point in reference frame of ground is $a_P = \dfrac{v^2}{R} - \dfrac{v^2}{6R} = \dfrac{5v^2}{6R}$

61. The plank will lose contact with the horizontal surface after the centre of mass reaches the top point of the cylinder. (Situation is shown in figure)

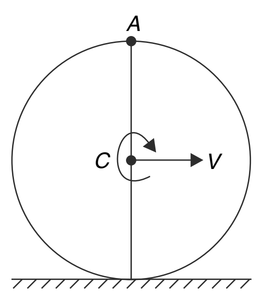
For this the displacement of plank relative to the centre of the sphere shall be equal to R. It means the centre will move a distance R and the plank will move through a distance 2R.

Note that the speed of the plank is twice that of the centre of the sphere.



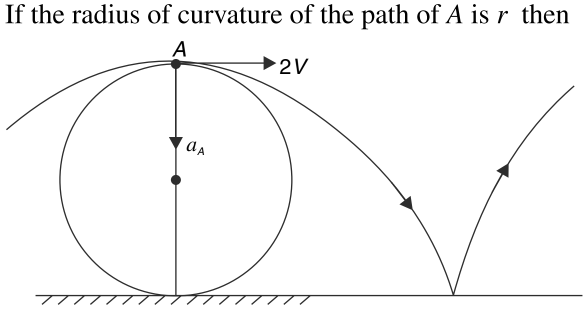
62. Radius of curvature of the path is the radius of the circle that matches up with the path locally at a point.

Let's first find the acceleration of point A with respect to the COM of the wheel. An observer at COM sees that point A is spinning about it. Acceleration of A is $\dfrac{v^2}{R}(\downarrow)$ with respect to COM. Because the COM does not have any acceleration hence acceleration of point A with respect to the ground is $a_A = \dfrac{v^2}{R}(\downarrow)$.



For a person on ground, speed of point A is $v_A = v + \omega R = 2v$.

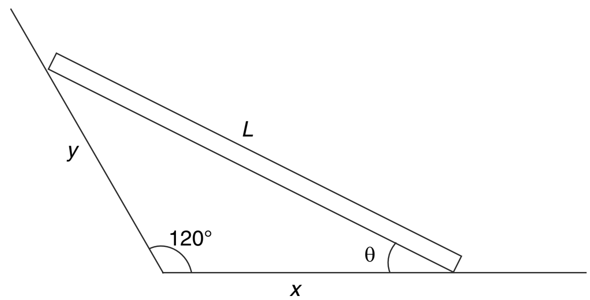
If the radius of curvature of the path of A is r then



$\therefore a_A = \frac{v_A^2}{r}$ i.e., $\frac{v^2}{R} = \frac{(2v)^2}{r}$

$\Rightarrow \boxed{r = 4R}$

63.



Sine rule :

$$\frac{x}{\sin(60-\theta)} = \frac{y}{\sin\theta} = \frac{L}{\sin 120°}$$

$$\therefore x = \frac{2L}{\sqrt{3}}\sin(60-\theta)$$

$$\therefore \frac{dx}{dt} = \frac{2L}{\sqrt{3}}\cos(60-\theta)\left(-\frac{d\theta}{dt}\right)$$

Note $-\frac{d\theta}{dt} = \omega$ = angular speed.

[θ is decreasing, hence a negative sign]

When $\theta = 20°$, $\frac{dx}{dt} = 1.5\text{m/s}$

$$\therefore \omega = \frac{1.5 \times 1.732}{2 \times 0.75 \times 0.766} = 2.26 \text{ rad/s}$$

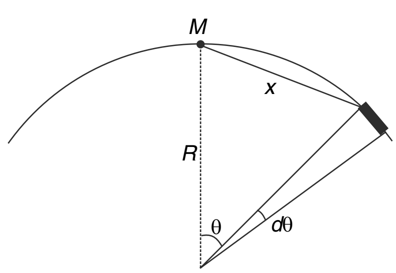
64. (a) The top points of two cylinders have same speed as that of the plank. By looking at the motion of smaller cylinder we can tell that this speed must be $2v$. In pure rolling the speed of centre will be half the speed of the top point. Hence speed of the centre of the bigger cylinder is v.

(b) The acceleration of two points is vertically downward with magnitudes:

$$a_{big} = \frac{v^2}{2R} \text{ and } a_{small} = \frac{v^2}{R}$$

The required ratio is 1/2

65. Consider an element as shown



Mass of the element $dm = \lambda R d\theta$

Distance of this element from point M is $x = 2R\sin\left(\frac{\theta}{2}\right)$

$\therefore dI = x^2 dm = 4\lambda R^3 \sin^2\frac{\theta}{2} d\theta$

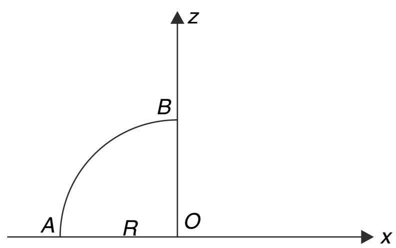
$$\therefore\ I = \int_{-\theta_0}^{\theta_0} dI = 4\lambda R^3 \int_{-\theta_0}^{\theta_0} \sin^2\frac{\theta}{2} d\theta$$

$$= 2\lambda R^3 \int_{-\theta_0}^{\theta_0} (1-\cos\theta) d\theta$$

$$= 2\lambda R^3 [\theta - \sin\theta]_{-\theta_0}^{\theta_0}$$

$$= 4\lambda R^3 (\theta_0 - \sin\theta_0)$$

66. (a) Let us first calculate the moment of inertia of the plate about the z – axis.

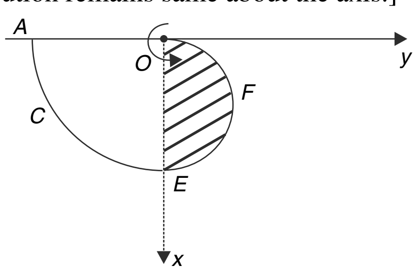


For part AOB : Mass = 3 m and Radius = R

and part AOB is one quarter of a circular disc of which z – axis is a diameter.

$\therefore I_1 = \frac{3mR^2}{4}$

[$\because$ Moment of inertia of a disc of mass m and radius R about its diameter is $\frac{mR^2}{4}$. In the quarter of a disc mass distribution remains same about the axis.]



For part $ACEO$ Moment of inertia about z axis is : $I_2 = \frac{3mR^2}{2}$

Similarly for part $EFOE$

$$I_3 = \frac{3}{2}(6\text{m})\left(\frac{R}{2}\right)^2 = \frac{9}{4}mR^2$$

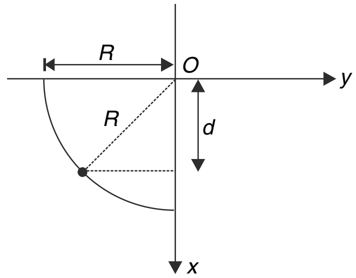
[$\because$ *MOI* of a disc about a tangent perpendicular to its plane is $\frac{3}{2}mR^2$]

$\therefore$ Moment of Inertia of the plate about z – axis is

$I = I_1 + I_2 + I_3$

$$= \left(\frac{3}{4} + \frac{3}{2} + \frac{9}{4}\right)mR^2 = \frac{9}{2}mR^2$$

(b) Angular momentum is conserved about the z – axis.



Moment of Inertia after the particle sticks to the plate

$$I = \frac{9}{2}mR^2 + mR^2 = \frac{11}{2}mR^2$$

$$\therefore \frac{11}{2}mR^2\omega = mvd$$

$$\Rightarrow \omega = \frac{2vd}{11R^2}$$

67. Area of original plate $A_0 = a^2$

Area of bigger hole $A_1 = \left(\frac{a}{3}\right)^2 = \frac{a^2}{9}$

Area of each of 8 smaller holes $A_2 = \left(\frac{a}{9}\right)^2 = \frac{a^2}{81}$

Area of plate with holes ; $A = A_0 - A_1 - 8A_2 = \frac{64}{81}a^2$

Mass of area $A = \left(\frac{64}{81}a^2\right)$ is M

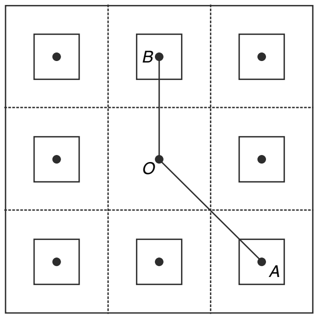
$\therefore$ Mass of area A_0 $(= a^2)$ is $\frac{81M}{64} = M_0$

Mass of area $A_1\left(= \frac{a^2}{9}\right)$ is $\frac{9M}{64} = M_1$

Mass of area $A_2\left(= \frac{a^2}{81}\right)$ is $\frac{M}{64} = M_2$

Required *MOI* is

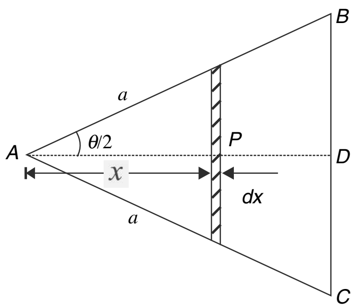
$$I = \frac{1}{6}M_0a^2 - \frac{1}{6}M_1\left(\frac{a}{3}\right)^2 - 4.\left[\frac{1}{6}M_2\left(\frac{a}{9}\right)^2 + M_2\left(\frac{a}{3}\right)^2\right] - 4.\left[\frac{1}{6}M_2\left(\frac{a}{9}\right)^2 + M_2\left(\frac{\sqrt{2}a}{3}\right)^2\right]$$



$$I = \frac{1}{6}\frac{81}{64}Ma^2 - \frac{1}{6}\frac{9}{64}\frac{1}{9}Ma^2 - \frac{4}{6}.\frac{55}{81}.\frac{1}{64}Ma^2 - \frac{4}{6}\frac{109}{81}\frac{1}{64}.Ma^2$$
$$= \frac{Ma^2}{6\times 64}\left[81-1-\frac{220}{81}-\frac{436}{81}\right] = \frac{Ma^2}{6\times 64}\times\frac{5824}{81} = \frac{91}{486}Ma^2$$

68. A square plate can be folded in the shape of described triangular plate without changing the mass distribution about the said axis.

69.



$$h = AD = a\cos\left(\frac{\theta}{2}\right)$$

Consider a strip as shown.

$$dm = \sigma 2x\tan\left(\frac{\theta}{2}\right)dx \text{ where } \sigma = \frac{M}{\frac{1}{2}.2a.\sin\frac{\theta}{2}.a.\cos\frac{\theta}{2}} = \frac{2M}{a^2\sin\theta} \backslash$$

$$dm = \frac{4M}{a^2}\frac{\tan\frac{\theta}{2}}{\sin\theta}.xdx$$

MOI of strip about an axis through *P* perpendicular to the plane of the figure is

$$= \frac{(dm)\cdot\left(2x\tan\frac{\theta}{2}\right)^2}{12} = \frac{1}{3}(dm)x^2\tan^2\frac{\theta}{2}$$

MOI about axis through *A* is

$$dI = \frac{1}{3}dm\cdot x^2\tan^2\frac{\theta}{2} + dm\cdot x^2 = dm\cdot x^2\left(\frac{1}{3}\tan^2\frac{\theta}{2}+1\right)$$

$$dI = \frac{4M}{a^2}\frac{\tan\frac{\theta}{2}}{\sin\theta}\left(\frac{1}{3}\tan^2\frac{\theta}{2}+1\right)x^3dx$$

$$\therefore I = \frac{4M}{a^2}\frac{\tan\frac{\theta}{2}}{\sin\theta}\left(\frac{1}{3}\tan^2\frac{\theta}{2}+1\right)\int_0^h x^3 dx$$

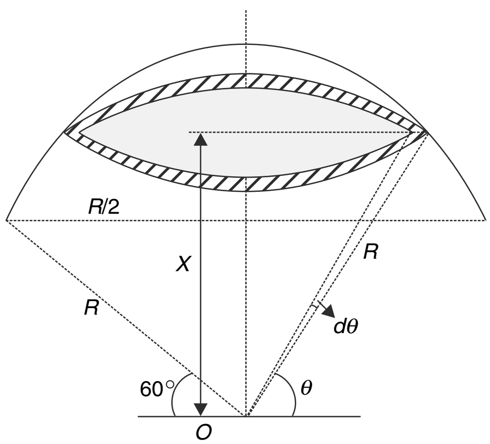
$$= \frac{4M}{a^2}\frac{\tan\frac{\theta}{2}}{\sin\theta}\left(\frac{1}{3}\tan^2\frac{\theta}{2}+1\right)\frac{\left(a\cos\frac{\theta}{2}\right)^4}{4} \quad \left[\because h = \cos\frac{\theta}{2}\right]$$

$$= Ma^2\cdot\frac{\sin\frac{\theta}{2}}{2\cdot\sin\frac{\theta}{2}\cdot\cos^2\frac{\theta}{2}}\cdot\cos^4\frac{\theta}{2}\left(\frac{\sin^2\frac{\theta}{2}+3\cos^2\frac{\theta}{2}}{3\cos^2\frac{\theta}{2}}\right)$$

$$= \frac{1}{2}Ma^2\left(\frac{3\sin^2\frac{\theta}{2}+3\cos^2\frac{\theta}{2}-2\sin^2\frac{\theta}{2}}{3}\right)$$

$$= \frac{1}{2}Ma^2\left(1-\frac{2}{3}\sin^2\frac{\theta}{2}\right)$$

70. (a) We will calculate the surface area of the end caps which got removed due to boring.



Area of the ring element shown

$dA = 2\pi\,(R\cos\theta)\,R.d\theta = 2\pi R^2\cos\theta.\,d\theta$

Area of one cap removed

$$A = 2\pi R^2\int_{\theta=60^\circ}^{\theta=90^\circ}\cos\theta d\theta = 2\pi R^2\left[\sin\theta\right]_{60^\circ}^{90^\circ} = \left(1-\frac{\sqrt{3}}{2}\right)2\pi R^2$$

Area of both caps. $2A = (2-\sqrt{3})\,2\pi R^2$

$\therefore$ Mass of remaining sphere

$$= M - M\frac{\left(2-\sqrt{3}\right)2\pi R^2}{4\pi R^2}$$

$$= M\left[1-\frac{2-\sqrt{3}}{2}\right] = \frac{\sqrt{3}}{2}M$$

(b) To calculate the *MOI*, we can take ring elements in the leftover part. We will add the *MOI* of each ring element.

Let mass per unit area be $\sigma = \dfrac{M}{4\pi R^2}$

MOI of a ring element

$dI = dm\,(R\cos\theta)^2 = \sigma.\,dA.\,(R\cos\theta)^2$

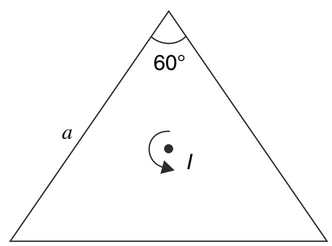
$$= \frac{M}{4\pi R^2}\cdot 2\pi R^2 \cos\theta d\theta \cdot R^2\cos^2\theta = \frac{MR^2}{2}\cdot\cos^3\theta d\theta$$

$$\therefore I = 2\times\frac{MR^2}{2}\int_{\theta=0}^{\theta=60^\circ}\cos^3\theta.d\theta$$

$$= MR^2\int_0^{60^\circ}\cos^3\theta d\theta = \frac{3\sqrt{3}}{8}MR^2$$

71. Note: This solution introduces you to a very different method of calculation of MOI based on use of dimensions and parallel axes theorem.

The moment of inertia will be same as the moment of inertia of a triangular plate about an axis passing through its centroid and perpendicular to its plane.

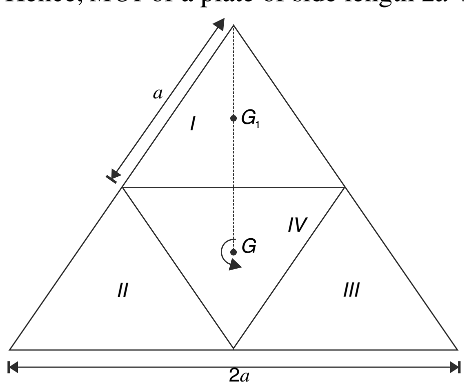


Let the moment of inertia of such a plate of mass *M* and side length *a* about the central axis be *I*. If the triangle were of side length 2*a*, its mass would have been 4*M* ($\because$ Mass $\propto$ area)

Since *MOI* $\propto$ (mass) (side length)^2

$\propto$ (4 *M*) (2*a*)^2

Hence, *MOI* of a plate of side length 2*a* will be 16*I* about a similar axis, passing through *G*.



16*I* = (*MOI* of part I about *G*) + (MOI of part II about *G*) + (*MOI* of part III about *G*) + (*MOI* of part IV about G)

$16I = 3I_1 + I$ [since, first three terms are equal due to symmetry and the fourth term is *I* that we are trying to calculate]

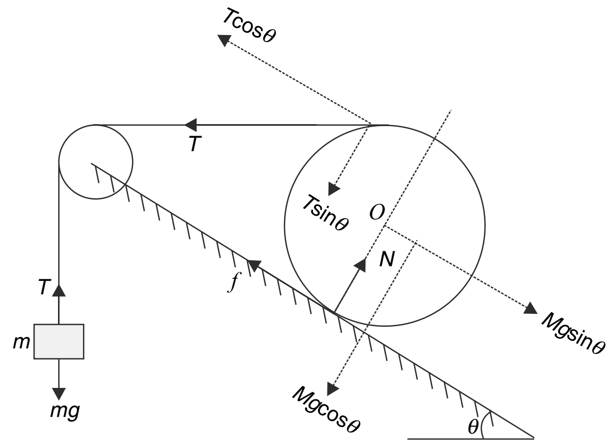
$15I = 3I_1 \Rightarrow 15I = 3[I_{G1} + M\,(GG_1)^2]$

$$5I = I + M\left(\frac{a}{\sqrt{3}}\right)^2 \text{ [you can show that } GG_1 = \frac{a}{\sqrt{3}}\text{]}$$

$$\therefore\ I = \frac{1}{12} Ma^2$$

Note :This method is very neat as compared to usual method by integration. As an additional exercise try to calculate the *MOI* of a uniform rod of mass *M* and length *L* about an axis passing through its centre and perpendicular to its length by using this method.

72. Let the mass of the block be *m*



For equilibrium of the block, $T = mg$...............(1)

For rotational equilibrium of the cylinder, net torque about its centre must be zero.

$\therefore\ T \cdot R = F \cdot R.$ [R = radius of cylinder]

$\Rightarrow mg = f$.........................(2)

For translational equilibrium of the cylinder

$N = Mg \cos\theta + T \sin\theta$

$\Rightarrow\ N = Mg \cos\theta + mg \sin\theta$...................... (3)

and $f + T\cos\theta = Mg \sin\theta$...................... (4)

$\Rightarrow f(1 + \cos\theta) = Mg \sin\theta$

$$\Rightarrow f = \frac{Mg\sin\theta}{(1+\cos\theta)} \Rightarrow mg = \frac{Mg\sin\theta}{1+\cos\theta}$$

$$m = \frac{M\sin\theta}{1+\cos\theta}\text{............(5)}$$

Putting this in (3)

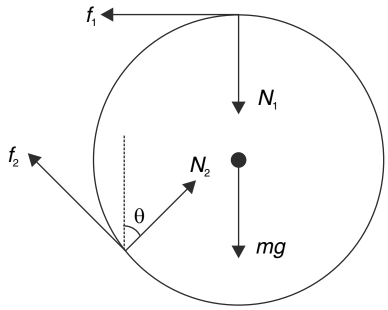
$$N = Mg\cos\theta + \frac{Mg\sin^2\theta}{1+\cos\theta} = \frac{Mg(\cos\theta+1)}{(1+\cos\theta)} = Mg$$

$$\therefore f = mg \le \mu N$$

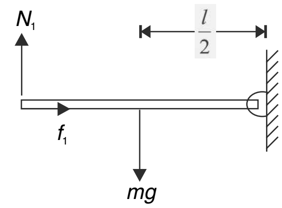
$$\therefore \frac{Mg\sin\theta}{1+\cos\theta} \le \mu Mg$$

$$\frac{\sin\theta}{1+\cos\theta} \le \mu$$

73.



Cylinder



Stick

For cylinder

$f_1 + f_2 \cos\theta = N_2 \sin\theta$ .... (1)

and $N_2 \cos\theta + f_2 \sin\theta = mg + N_1$ .... (2)

Rotational equilibrium: $f_1 R = f_2 R$

$\Rightarrow f_1 = f_2 = f$ (say)

Equation (1) and (2) become

$f + f\cos\theta = N_2 \sin\theta$ .... (3)

and $N_2 \cos\theta + f\sin\theta = mg + N_1$ .... (4)

For stick $mg\frac{l}{2} = N_1 l$

$\Rightarrow N_1 = \frac{mg}{2}$.................(5)

From (3) $f = \left(\frac{\sin\theta}{1+\cos\theta}\right)N_2$..........(6)

$\therefore \left(\frac{\sin\theta}{1+\cos\theta}\right)N_2 \leq \mu N_2$

$\therefore \frac{\sin\theta}{1+\cos\theta} \leq \mu$................(A)

Substituting for N_2 from (6) into (4)

$f\left(\frac{1+\cos\theta}{\sin\theta}\right)\cos\theta + f\sin\theta = mg + N_1$

$f\left[\frac{\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta}\right] = 3N_1 \qquad \left[\because N_1 = \frac{mg}{2}\right]$

$\therefore f = \left(\frac{3\sin\theta}{1+\cos\theta}\right)N_1$

$\therefore \frac{3\sin\theta}{1+\cos\theta}.N_1 \leq \mu N_1$

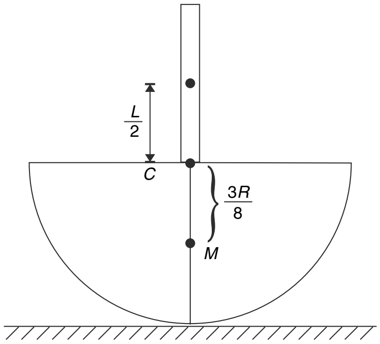
$\therefore \frac{3\sin\theta}{1+\cos\theta} \leq \mu$.....................(B)

Note : $f_1 = f_2$ does not mean that $N_1 = N_2$

From A and B

$\mu \geq \frac{3\sin\theta}{1+\cos\theta}$

74. The object will perform oscillations if it is in stable equilibrium.

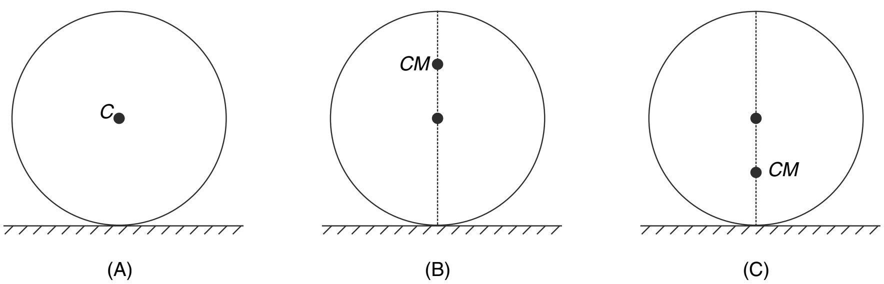


It will be in stable equilibrium if its COM lies below centre of the sphere, C. Let the length of the rod be L. For COM of the system to lie below C

$$M\frac{3R}{8} > \left(\frac{M}{2R}L\right)\frac{L}{2}$$

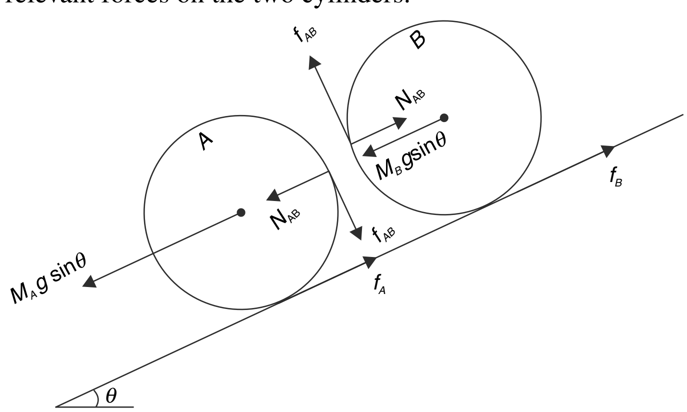
$$\Rightarrow \frac{3R^2}{2} > L^2 \Rightarrow L < \sqrt{\frac{3}{2}}\cdot R$$

Note :For better understanding, consider three spheres : (A) a uniform sphere (B) a non uniform sphere whose COM is above the geometrical centre (C) a non uniform sphere whose COM is below its geometrical centre.



In (A), the COM is always above contact point, hence force mg causes no torque about the point of contact. The sphere is in neutral equilibrium. In (B), if the sphere rotates a little, the COM moves sideways and produces a torque that tends to rotate the sphere farther away. The sphere is in unstable equilibrium. In (C), if the sphere moves a little, the COM moves sideways and this produces a restoring torque. Hence this is position of stable equilibrium.

75. The diagram shows the relevant forces on the two cylinders.



f_{AB} = friction between the two cylinders;

f_A = friction between A and the incline; f_B = friction between B and the incline

For rotational equilibrium of the cylinders

$f_A = f_{AB}$ [for A]

$f_B = f_{AB}$ [for B]

$\therefore f_A = f_B = f_{AB}$

For translational equilibrium of ($A + B$)

$f_A + f_B = (M_A + M_B)$ g sinθ

$$\therefore f_A = f_B = f_{AB} = \left(\frac{M_A + M_B}{2}\right) g \sin\theta$$

For A to be in equilibrium

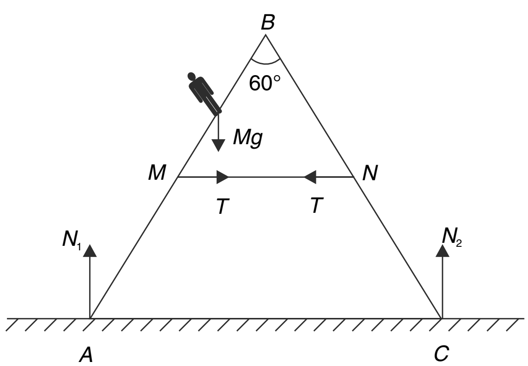
$$N_{AB} + M_A g \sin\theta = \left(\frac{M_A + M_B}{2}\right) g \sin\theta$$

$$N_{AB} = \left(\frac{M_B - M_A}{2}\right) g \sin\theta$$

For the two cylinders to be in contact

$N_{AB} > O \Rightarrow M_B > M_A$

76.



From geometry, it is easy to see that ΔABC is equilateral.

For vertical equilibrium of entire system we must have $N_1 + N_2 = Mg$..............(1)

Now consider rotational equilibrium of (arm AB + man) system.

$\tau_B = 0$

$$N_1 L \cos 60° = T\frac{L}{2}\sin 60° + Mg\frac{L}{4}\cos 60°$$

$$N_1 = \frac{\sqrt{3}}{2}T + \frac{Mg}{4} \text{...............(2)}$$

For arm BC we get; $\tau_B = 0$

$$N_2 L \cos 60° = T\frac{L}{2}\sin 60°$$

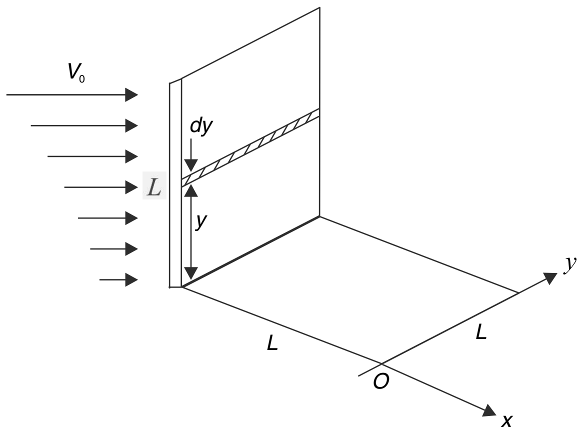
$$N_2 = \frac{\sqrt{3}}{2}T \text{.............(3)}$$

Substituting the values of N_1 and N_2 from (2) and (3) into equation (1) we get

$$\frac{\sqrt{3}}{2}T+\frac{Mg}{4}+\frac{\sqrt{3}}{2}T=Mg$$

$$\sqrt{3}T=\frac{3}{4}Mg\Rightarrow T=\frac{\sqrt{3}}{4}Mg$$

77.



The x co-ordinate of COM of the sheet

$$x_{cm}=\frac{-\frac{M}{2}\cdot\frac{L}{2}-\frac{M}{2}.L}{M}=\frac{-3}{4}L$$

Torque of weight about y axis

$$\tau_g=Mg\cdot\left(\frac{3}{4}L\right)............(1)$$

Let's calculate torque due to wind about y axis.

Speed of air at height y is $v=\frac{v_0}{L}y$

Mass of air striking a strip of width dy in unit time is $=\rho\cdot(Ldy)v=\rho Ldy\cdot\frac{v_0}{L}y=\rho v_0 y\cdot dy$

Force on strip of width dy

dF = rate of transfer of momentum by the air $=\rho v_0\, ydy\cdot\frac{v_0}{L}y=\rho\frac{v_0^2}{L}y^2dy$

Torque about y axis due to this force $d\tau=ydF=\frac{\rho v_0^2}{L}y^3dy.$

$\therefore$ Total torque

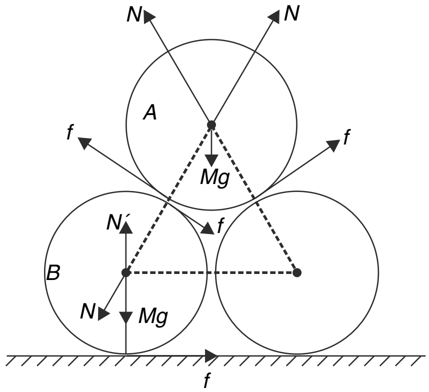
$$\tau_{air}=\int d\tau=\frac{\rho v_0^2}{L}\int_0^L y^3dy=\frac{\rho v_0^2}{4}L^3.......(2)$$

The sheet will not topple if

$$\tau_{air}\le\tau_g\Rightarrow\frac{\rho v_0^2L^3}{4}\le Mg\frac{3}{4}L$$

$$v_0^2\le\frac{3Mg}{\rho L^2}\Rightarrow v_0\le\sqrt{\frac{3Mg}{\rho L^2}}$$

78. For rotational equilibrium of B, the friction force on it due to A must be same as ground friction force on it.



For vertical equilibrium of A

$2N\cos 30° + 2f\sin 30° = Mg$

$\sqrt{3}N + f = Mg$ ....(1)

For Horizontal equilibrium of B

$N\cos 60° = f + f\cos 30°$

$$\frac{N}{2} = f + \frac{\sqrt{3}f}{2}$$

$$f = \frac{N}{2+\sqrt{3}} \text{........(2)}$$

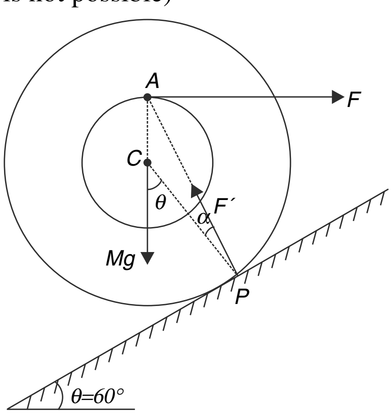
Solving (1) and (2)

$$\sqrt{3}N + \frac{N}{2+\sqrt{3}} = Mg$$

$$N = \frac{2+\sqrt{3}}{2\sqrt{3}+3+1}Mg = \frac{2+\sqrt{3}}{2\left(\sqrt{3}+2\right)}Mg = \frac{Mg}{2}$$

Solution to this question does not require any knowledge of torque. It has been deliberately included here to test your conviction and faith on your self knowledge.

79. For equilibrium, the force applied by the incline (F') must pass through point A, (otherwise rotational equilibrium is not possible)



Let $<CPA = \alpha$

then $<CAP = \theta - \alpha = 60° - \alpha$

Applying sine rule :

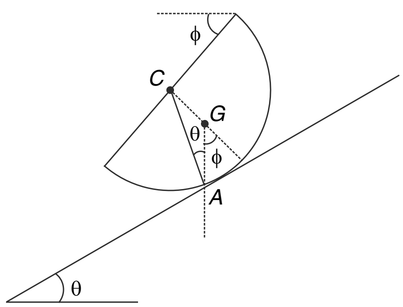
$$\frac{AC}{\sin\alpha} = \frac{PC}{\sin(60° - \alpha)} \therefore \frac{r}{\sin\alpha} = \frac{R}{\sin(60° - \alpha)}$$

$\therefore$ sin (60° – α) = 2 sin α

$$\Rightarrow \frac{\sqrt{3}}{2}\cos\alpha - \frac{1}{2}\sin\alpha = 2\sin\alpha \Rightarrow \tan\alpha = \frac{\sqrt{3}}{5}$$

$\therefore$ Required answer is $60° - \tan^{-1}\left(\frac{\sqrt{3}}{5}\right) \simeq 41°$

80.

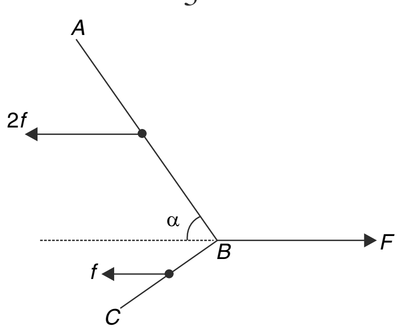


$C \rightarrow$ Centre of circular base; $G \rightarrow$ Centre of mass $\left[CG = \frac{3R}{8}\right]$
$A \rightarrow$ Contact point
CA is $\perp r$ to the inline.
The normal contract force (along AC), weight (along GA) and friction all pass through point A.
tan θ = μ = 0.3 ; $\theta = \tan^{-1}$ (0.3)
In ΔACG

$$\frac{CG}{\sin\theta} = \frac{CA}{\sin(\pi - \phi)} \Rightarrow \frac{3R}{8\sin\theta} = \frac{R}{\sin\phi}$$

$$\Rightarrow \sin\phi = \frac{8 \times 0.29}{3} = 0.77 \Rightarrow \phi = 50°$$

81.



Let friction on arm BC be f. Then friction on arm AB is $2f$.
$\therefore$ $F = 3f$ for uniform motion.
Net torque about B must be zero.

$$\therefore 2fl\sin\alpha = f \cdot \frac{l}{2}\sin(90 - \alpha)$$

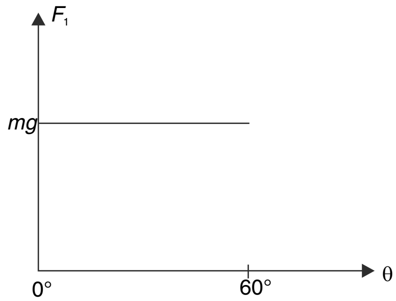
$$\Rightarrow \tan\alpha = \frac{1}{4} \Rightarrow \alpha = \tan^{-1}\left(\frac{1}{4}\right)$$

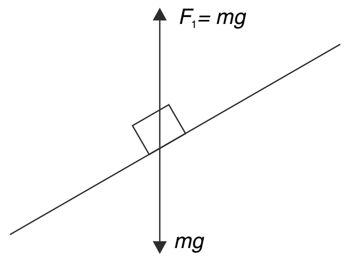
$$\therefore \theta = \pi - \tan^{-1}\left(\frac{1}{4}\right)$$

82. (a) Stick is being rotated slowly. There is no acceleration.

Block remains in equilibrium.

$\therefore$ Force by stick on the block is $F_1 = mg$

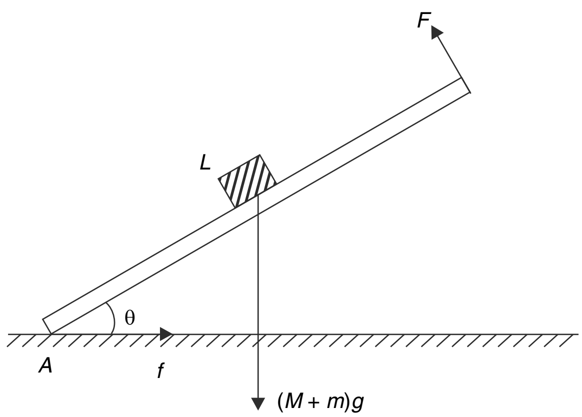




(b) The angle of repose must be $\geq 60°$

$\therefore \mu \geq \tan 60° \Rightarrow \mu \geq \sqrt{3}$

(c) Consider Stick + Block as our system.



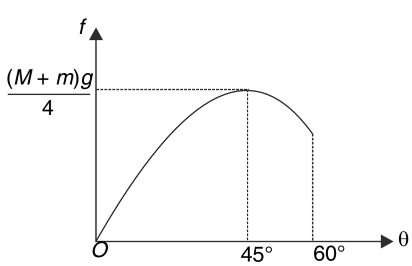
Torque about $A = 0$

$$\therefore FL = (M+m)g\frac{L}{2}\cos\theta \Rightarrow F = \frac{1}{2}(M+m)g\cos\theta$$

For horizontal equilibrium

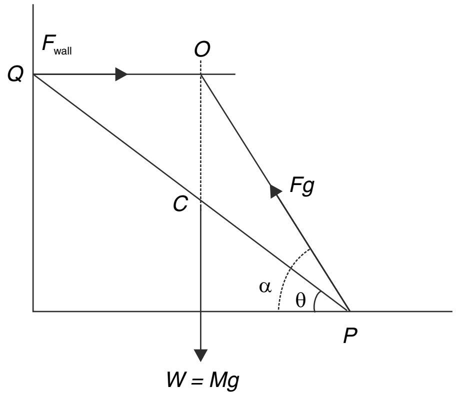
$$f = F\sin\theta \Rightarrow f = \frac{1}{4}(M+m)g\sin 2\theta$$

Plot of f *vs* θ is as shown.



83. (a) The three forces must be concurrent for equilibrium of the ladder. If the three lines (of action of the forces) are not concurrent, then one force would produce a non zero torque about the intersection point of the other two.

(b) The intersection point of F_{wall} and W is O. The force F_g must also pass through point O.



$$\tan\alpha = \frac{L\sin\theta}{\frac{L}{2}\cos\theta} = 2\tan\theta \Rightarrow \alpha = \tan^{-1}(2\tan\theta)$$

(c) Friction is horizontal component of F_g. And vertical component of F_g must be equal to Mg.

$\therefore f = F_g \cos\alpha$

and $N = F_g \sin\alpha = Mg$ [N = normal reaction of ground]

$$\therefore f = \frac{Mg}{\tan\alpha} = \frac{Mg}{2\tan\theta}$$

But $f \le \mu N \Rightarrow \dfrac{Mg}{2\tan\theta} \le \mu Mg$

$$\Rightarrow \tan\theta \ge \frac{1}{2\mu}$$

$$\Rightarrow \theta_{min} = \tan^{-1}\left(\frac{1}{2\mu}\right) \text{.............(A)}$$

(d) If COM is at a distance l from P then (as done in (b)) you can show that

$$\tan\alpha = \frac{L}{l}\tan\theta$$

And $F_g \sin\alpha = Mg$

$F_g \cos\alpha = f$

$$\Rightarrow f = \frac{Mg}{\tan\alpha} = \frac{l \cdot Mg}{L\tan\theta}$$

But $\dfrac{lMg}{L\tan\theta} \le \mu Mg$

$$\Rightarrow \tan\theta \ge \frac{l}{L\mu}$$

If $l > \dfrac{L}{2}$; then $\theta > \theta_{min}$ [found in A]

84. Let the tension in the springs be T_1, T_2 and T_3. The extension in spring 2 is larger than that in spring 1 (say by Δx). Extension in spring 3 is larger than that in 2 by the same amount.

$\therefore\ T_2 - T_1 = T_3 - T_2$ ....(1)

Translational equilibrium

$\therefore\ T_1 + T_2 + T_3 = Mg$.......(2)

Rotational equilibrium ($\tau_A = 0$)

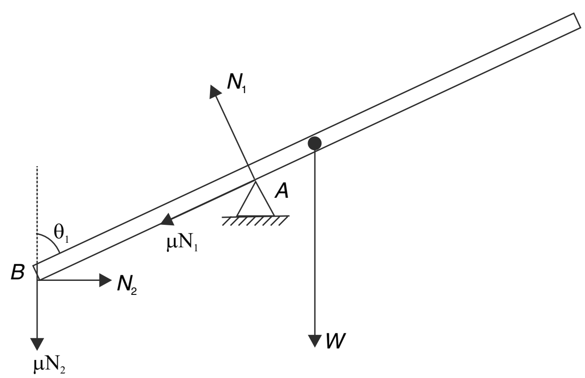
$$T_2\frac{L}{3} + T_3\frac{2L}{3} = Mg\frac{L}{2}$$

$$\Rightarrow T_2 + 2T_3 = \frac{3}{2}Mg\text{........(3)}$$

Solving (1), (2) and (3)

$$T_1 = \frac{Mg}{12}; T_2 = \frac{Mg}{3}; T_3 = \frac{7Mg}{12}$$

85. (a) When the end of the rod touching the wall has a tendency to slide up, forces are as shown.



$N_2 = N_1\cos\theta_1 + \mu N_1\sin\theta_1$ ....(1)

$N_1\sin\theta_1 = \mu N_2 + \mu N_1\cos\theta_1 + W$.........(2)

Taking torque about B

$$N_1\left(\frac{a}{\sin\theta_1}\right) = W\frac{b}{2}\sin\theta_1 \qquad \text{...........(3)}$$

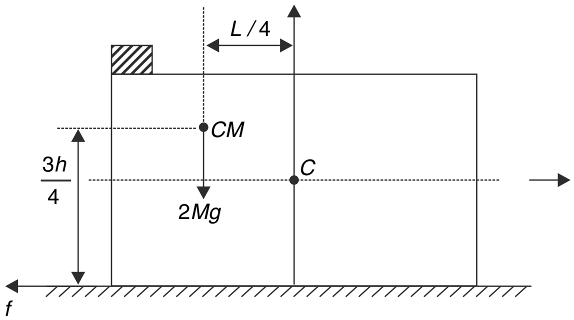
Eliminating N_1 and N_2 we get

$$\frac{2a}{b} = \left[\left(1-\mu^2\right)\sin\theta_1 - 2\mu\cos\theta_1\right]\sin^2\theta_1\text{........(A)}$$

(b) When the end B of the rod has a tendency to slide down, the direction of frictions μN_2 and μN_1 will be opposite. Weight W will pass somewhere between B and A. The equation can be simply obtained by replacing μ with $-\mu$ in equation (A).

$$\frac{2a}{b} = \left[\left(1-\mu^2\right)\sin\theta_2 + 2\mu\cos\theta_2\right]\sin^2\theta_2$$

86. The COM of the block + particle system is at a height of $\frac{3h}{4}$ at a horizontal distance of $\frac{L}{4}$ from the geometric centre (C).



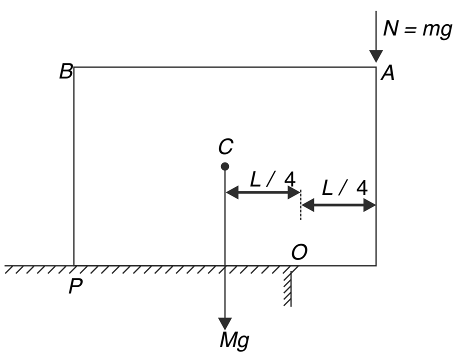
It will be wise to consider torque about COM of the system (why?).

$$\therefore f \cdot \frac{3h}{4} = N \cdot \frac{L}{4} \Rightarrow \mu \cdot 2Mg \cdot \frac{3h}{4} = 2Mg\frac{L}{4}$$

$$\therefore h = \frac{L}{3\mu} = \frac{L}{3 \cdot \frac{2}{3}} = \frac{L}{2}.$$

Note : you can make a gross mistake by balancing torque about C.

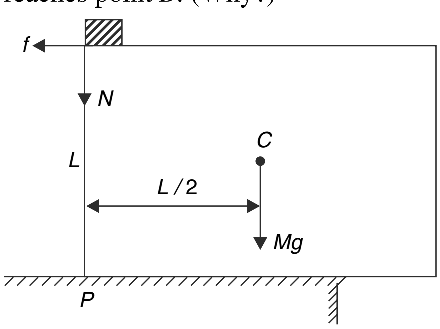
87. With the block at A, the cube is on verge of toppling.



$\therefore$ Torque of Normal force ($N = mg$) by the block about O = torque due to weight of the cube

$$\Rightarrow mg\frac{L}{4} = Mg\frac{L}{4} \therefore m = M$$

Now as the block travels to the left, it exerts a friction on the cube to the left. Now there is no tendency of toppling about O. The cube has tendency to topple about P if the friction is large. This tendency is maximum when the block reaches point B. (Why?)

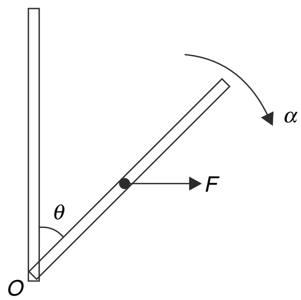


Cube will not topple if

$$fL \le Mg\frac{L}{2} \quad \Rightarrow \mu mgL \le Mg\frac{L}{2}$$

$$\Rightarrow \mu \le \frac{1}{2} \qquad [\because m = M]$$

88. (a)



Toque about O

$$\tau = F\frac{L}{2}\cos\theta$$

$$\therefore \frac{ML^2}{3}\cdot\alpha = F\frac{L}{2}\cos\theta$$

$$\Rightarrow \alpha = \frac{3}{2}\frac{F}{ML}\cos\theta$$

$$\therefore \omega\frac{d\omega}{d\theta} = \frac{3}{2}\frac{F}{ML}\cos\theta$$

$$\int_0^\omega \omega d\omega = \frac{3}{2}\frac{F}{ML}\int_0^\theta \cos\theta d\theta$$

$$\frac{\omega^2}{2} = \frac{3}{2}\frac{F}{ML}\sin\theta \Rightarrow \omega^2 = 3\frac{F}{ML}\sin\theta ......(i)$$

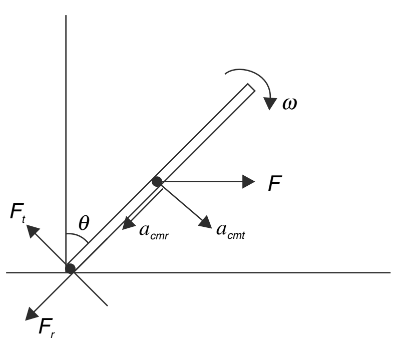
When $\theta = 90°$; $\omega = \sqrt{\frac{3F}{ML}}$

Alternative :

Work done by $F = \Delta k$

$$F\cdot\frac{L}{2} = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \quad \Rightarrow \omega = \sqrt{\frac{3F}{ML}}.$$

At angle θ, the tangential acceleration of COM is $a_{cmt} = \alpha\cdot\frac{L}{2} = \frac{3}{4}\frac{F}{M}\cos\theta$ perpendicular to the rod.



Let F_r and F_t be two components of hinge force along the rod and perpendicular to it.

$$F\cos\theta - F_t = Ma_{CM}$$

$$\therefore \quad F_t = F\cos\theta - \frac{3}{4}F\cos\theta = \frac{1}{4}F\cos\theta$$

And $F_r - F\sin\theta = M\omega^2\dfrac{L}{2}$ [$\therefore$ radial acceleration of COM is $\omega^2\dfrac{L}{2}$]

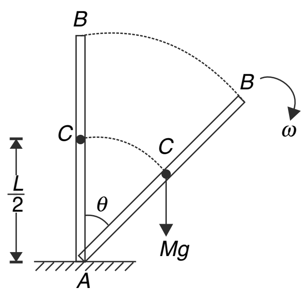
$\therefore$ $F_r = F\sin\theta + \dfrac{3}{2}F\sin\theta = \dfrac{5}{2}F\sin\theta$ [using (1)]

The hinge force makes 45° with the rod when

$F_r = F_t \Rightarrow \dfrac{5}{2}F\sin\theta = \dfrac{F}{4}\cos\theta$

$\Rightarrow \tan\theta = \dfrac{1}{10}$

89.



Mechanical energy conservation

$\dfrac{1}{2}I_A\omega^2 = Mg\dfrac{L}{2}(1-\cos\theta)$

$\dfrac{1}{2}\dfrac{ML^2}{3}\cdot\omega^2 = \dfrac{MgL}{2}\left(1-\dfrac{4}{5}\right)$.......(1)

$\omega = \sqrt{\dfrac{3}{5}\dfrac{g}{L}} = \sqrt{\dfrac{3}{5}\times\dfrac{10}{1.5}} = 2\,rad/s$

$v_{cm} = \dfrac{L}{2}\cdot\omega = 1.5\,m/s$

$\therefore$ Momentum = Mv_{cm} = 7.5 *kg m/s*

Torque about *A*:

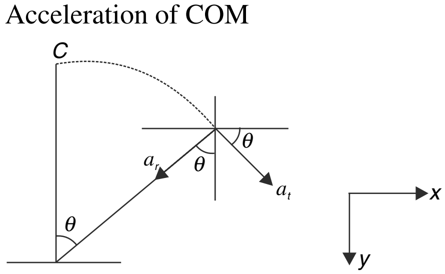
$I_A\cdot\alpha = \tau_A$

$\dfrac{ML^2}{3}\cdot\alpha = Mg\dfrac{L}{2}\sin\theta$..........(2)

$\Rightarrow \alpha = \dfrac{9}{10}\left(\dfrac{g}{L}\right) = 6\,rad/s^2$ $\left[\because \sin\theta = \dfrac{3}{5}\right]$

COM is performing non-uniform circular motion with instantaneous angular acceleration α and angular speed ω.

Acceleration of COM

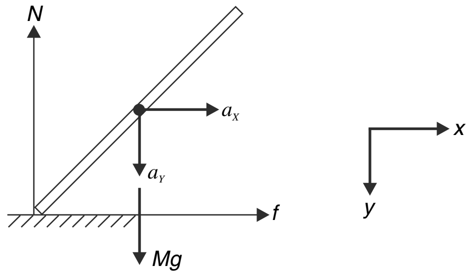


$$a_t = \frac{L}{2}\cdot\alpha = 4.5 m/s^2 \text{ and } a_r = \omega^2\frac{L}{2} = 3 m/s^2$$

$$\therefore a_x = a_t\cos\theta - a_r\sin\theta = 4.5\times\frac{4}{5} - 3\times\frac{3}{5} = 1.8 m/s^2$$

$$a_y = a_t\sin\theta + a_r\cos\theta = 4.5\times\frac{3}{5} + 3\times\frac{4}{5} = 5.1 m/s^2$$

Using Newton's Second Law



$F_x = M_{ax} \Rightarrow f = 5 \times 1.8 = 9N$

$F_y = M_{ay} \Rightarrow Mg - N = M_{ay}$

$\Rightarrow 5 \times 10 - N = 5 \times 5.1 \therefore N = 24.5\ N$

Friction force acting on the rod will be zero at the moment $a_x = 0$

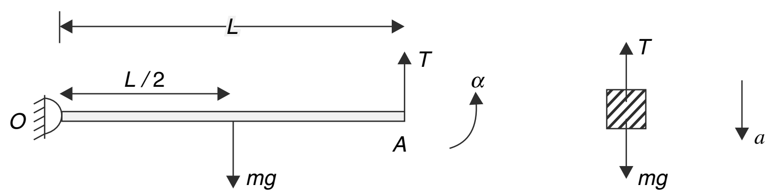
$$\Rightarrow a_t\cos\theta = a_r\sin\theta \quad \Rightarrow \frac{L}{2}\alpha\cdot\cos\theta = \omega^2\frac{L}{2}\cdot\sin\theta$$

$$\Rightarrow \alpha\cos\theta = \omega^2\sin\theta$$

$$\Rightarrow \frac{3g}{2L}\sin\theta\cdot\cos\theta = \frac{3g}{L}(1-\cos\theta) \quad [\text{using (1) and (2)}]$$

$$\Rightarrow \cos\theta = \frac{2}{3}$$

90. (a)



Let angular acceleration of rod = α

Acceleration of block = acceleration of point A of the rod = a

$a = L\alpha$ ....(1)

For block : $mg - T = ma$........(2)

For rod : $TL - mg\frac{L}{2} = \frac{1}{3}mL^2\cdot\alpha$

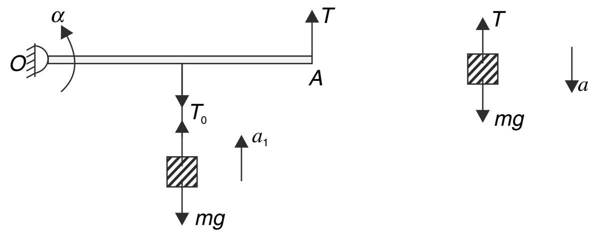
$$T - \frac{mg}{2} = \frac{1}{3}ma\text{.............(3)} \qquad [\therefore L\alpha = a]$$

Solving (2) and (3) $\frac{3}{8}g = a$

(b) $a = L\alpha \Rightarrow a_1 = \frac{L\alpha}{2} = \frac{a}{2}$

Net torque on the rod shall be zero

$\therefore 2T = T_0$



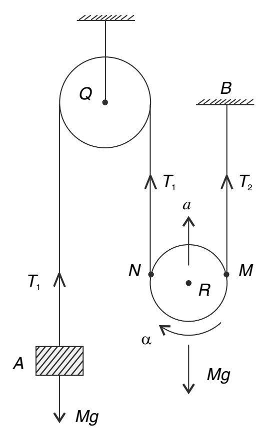
$\therefore mg - T = ma$ .... (1)

$T_0 - mg = m\dfrac{a}{2} \Rightarrow 2T - mg = \dfrac{ma}{2}$......(2)

(1) × 2 + (2) gives:

$mg = \dfrac{5ma}{2} \Rightarrow a = \dfrac{2g}{5}$ and $a_1 = \dfrac{g}{5}$

91.



Let the acceleration and angular acceleration of the disc be a and α.

Point M on the thread (and hence point on the disc touching the thread at M) must have zero acceleration.

$\therefore a = R\alpha$

Point N on the thread (and the point of disc touching it) will have acceleration

$= a + R\alpha = 2a =$ *acceleration of A*

For A $\quad M - T_1 = M(2a)$ ....(1)

For P $\quad T_1 + T_2 - Mg = Ma$.....................(2)

$T_1 R - T_2 R = \dfrac{1}{2} MR^2 \alpha$

$T_1 - T_2 = \dfrac{1}{2} Ma$ ....(3) $[\because R\alpha = a]$

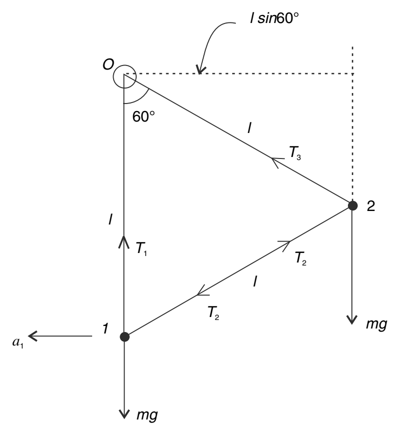
Solving (1), (2) and (3) we get $a = \dfrac{2g}{11}$

$\therefore$ Acceleration of A is $= \dfrac{4g}{11}$

$T_1 = \dfrac{7Mg}{11}$ and $T_2 = \dfrac{6Mg}{11}$

$\therefore T_1 > T_2$

92. Torque about O : $\tau = mg\ell \sin 60^0 = \dfrac{\sqrt{3}}{2} mg\ell$



Moment of inertia about horizontal axis through O, $I = 2m\ell^2$

Use $\tau = I\alpha$

$$2m\ell^2 \cdot \alpha = \frac{\sqrt{3}}{2} mg\ell \quad \Rightarrow \alpha = \frac{\sqrt{3}\, g}{4\,\ell}$$

Acceleration of particle 1

$$a_1 = \alpha\,\ell = \frac{\sqrt{3}}{4} g \text{ (Horizontal)}$$

For motion of particle 1

$$T_2 \cos 30^0 = ma_1 \quad \Rightarrow \frac{\sqrt{3}}{2} T_2 = \frac{\sqrt{3}}{4} mg$$

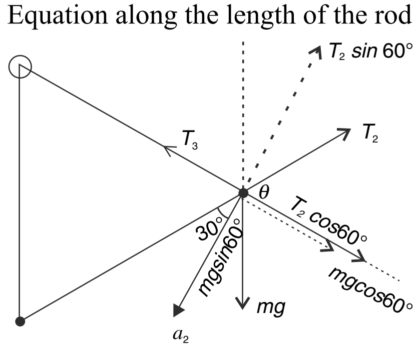
$$\Rightarrow T_2 = \frac{mg}{2} \qquad \text{..........................(1)}$$

And $T_1 = T_2 \sin 30^0 + mg = \dfrac{5}{4} mg$ ....(2)

Acceleration for particle 2

$a_2 = \alpha\ell = \dfrac{\sqrt{3}}{4} g$ (perpendicular to rod connecting the particle to pivot)

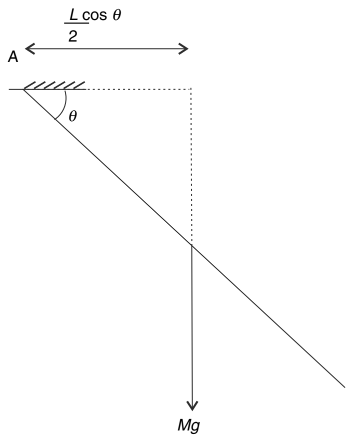
Equation along the length of the rod is $T_3 = mg \cos 60° + T_2 \cos 60° = \dfrac{3}{4} mg$



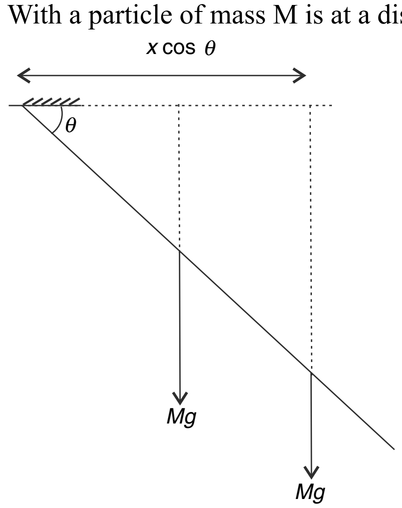
The rod connecting the two particles is under compression. Remaining two rods are under tension.

93. Angular acceleration of the rod at any angular position θ is

$$\alpha_0 = \frac{Mg\frac{L}{2}\cos\theta}{\frac{ML^2}{3}} = \frac{3}{2}\frac{g}{L}\cos\theta$$



With a particle of mass M is at a distance x from A



$$\alpha = \frac{Mg\frac{L}{2}\cos\theta + Mg\,x\cos\theta}{\frac{1}{3}ML^2 + Mx^2} = \frac{3}{2}\left[\frac{L + 2x}{L^2 + 3x^2}\right]g\cos\theta$$

(a) For $x = L$

$$\alpha = \frac{9}{8}\frac{g}{L}\cos\theta$$

For any angle θ, $\alpha_0 > \alpha$

Hence, the rod will take less time when it is alone without a particle.

(b) $\alpha = \frac{3}{2}\left[\frac{L+2x}{L^2 + 3x^2}\right]g\cos\theta$

α is maximum when $\frac{d\alpha}{dx} = 0$

$(L^2 + 3x^2)\cdot 2 - (L + 2x)(6x) = 0$

$2L^2 + 6x^2 - 6xL - 12x^2 = 0$

$6x^2 + 6Lx - 2L^2 = 0 \Rightarrow 3x^2 + 3Lx - L^2 = 0$

$$x = \frac{-3+\sqrt{9+12}}{6} \Rightarrow x = \frac{\sqrt{21}-3}{6}$$

94. $\omega r = u =$ a constant

$$\ell n\omega + \ell nR = \ell nu \Rightarrow \frac{1}{\omega}\frac{d\omega}{dt} + \frac{1}{R}\frac{dR}{dt} = 0$$

$$\therefore \frac{d\omega}{dt} = \frac{\omega}{R}\frac{dR}{dt}$$

Angular acceleration $\alpha = \dfrac{\omega}{R}\dfrac{dR}{dt}$ ---------------(1)

If we consider an interval 'dt' in which radius decreases by dR; then

$2\pi RdR = (udt)d$

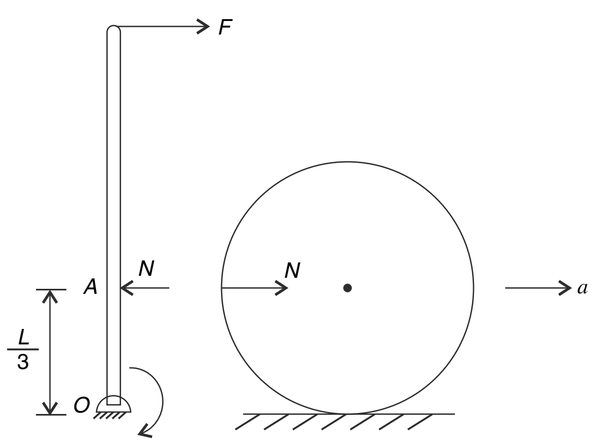
$$\therefore \frac{dR}{dt} = \frac{u.d}{2\pi R}$$

Put in (1) $\alpha = \dfrac{\omega}{R}\cdot\dfrac{ud}{2\pi R} = \dfrac{u^2 d}{2\pi R^3}$

$\therefore$ Torque on disc $\tau = I\alpha$

$$\Rightarrow F.R = I.\frac{u^2 d}{2\pi R^3} \Rightarrow F = \frac{Iu^2 d}{2\pi R^4}$$

95.



For rod $I_0.\alpha = FL - N\dfrac{L}{3}$

$$\therefore \frac{ML^2}{3}\cdot\alpha = FL - \frac{NL}{3}$$

$$M\frac{L\alpha}{3} = F - \frac{N}{3}$$

But $\frac{L\alpha}{3} =$ acceleration of point A = acceleration of the ball (a)

$\therefore\ Ma = F - \frac{N}{3}$ ....(1)

For ball $N = Ma$ ....(2)

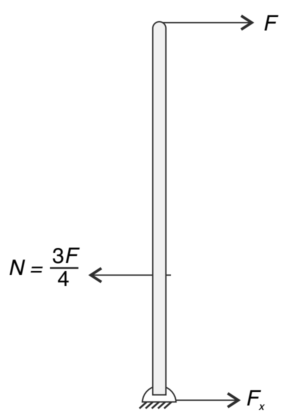
Solving (1) and (2) $a = \frac{3F}{4M}$

$\therefore\ N = \frac{3F}{4}$.

Acceleration of the COM of the rod is

$$a_{cm} = \alpha\frac{L}{2} = \frac{3a}{2} = \frac{9F}{8M}$$

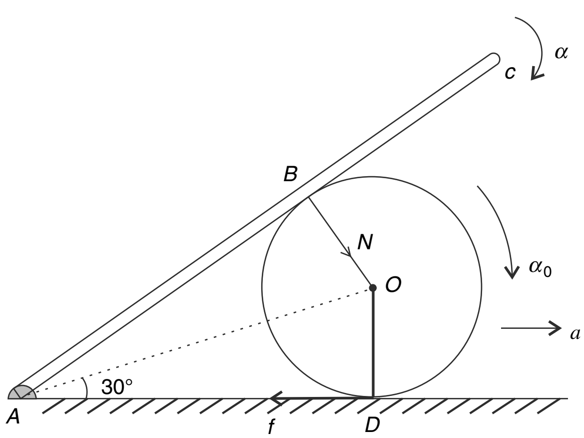
$\therefore$ If hinge force is F_x



$$F - \frac{3F}{4} + F_x = Ma \Rightarrow F - \frac{3F}{4} + F_x = M\frac{9F}{8M}$$

$$F_x = \frac{9F}{8} - \frac{F}{4} = \frac{7F}{8}$$

96.



$AB = R\cdot \cot 30° = \sqrt{3}\,R$

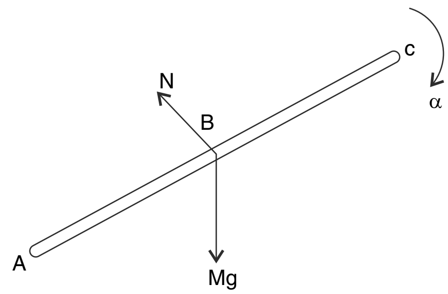
For Ring : $N\cos 30° - f = Ma$ ....(1)

And $fR = MR^2\cdot \alpha_0 \Rightarrow f = MR\alpha_0$ ....(2)

From (1) and (2) $\frac{\sqrt{3}}{2}N = Ma + MR\alpha_0$

$\frac{\sqrt{3}}{2}N = 2Ma \quad [\because a = R\alpha_0]$ ....(3)

For rod : $Mg\,\sqrt{3}R\cos 60^0 - N\,\sqrt{3}R = \frac{1}{3}M\left(2\sqrt{3}R\right)^2\cdot\alpha$



$\frac{Mg}{2} - N = \frac{4}{\sqrt{3}}\,MR\alpha$

$\therefore\ \frac{Mg}{2} - \frac{4Ma}{\sqrt{3}} = \frac{4M}{\sqrt{3}}R\alpha$

$\frac{4M}{\sqrt{3}}\left[R\alpha + a\right] = \frac{Mg}{2}$ ....(4)

Acceleration of point B is $a_B = \sqrt{3}\,R\cdot\alpha$

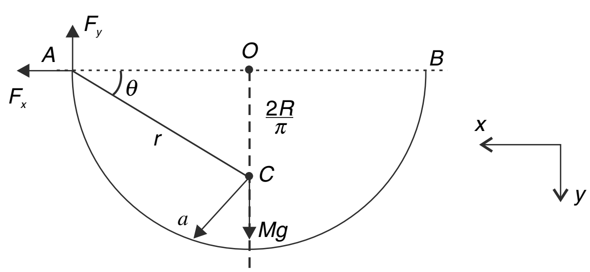
For the rod to remain supported by the ring component of a along the radius must be equal to a_B

$\therefore\ a\sin 60° = \sqrt{3}\,R.\alpha \ \Rightarrow a = 2R\alpha$ ....(5)

Put in (4) $\frac{4}{\sqrt{3}}\left[\frac{a}{2} + a\right] = \frac{g}{2}$

$\therefore\ a = \frac{g}{4\sqrt{3}}$

97.



Moment of inertia about rotation axis

$I_A = 2MR^2$

Torque about A, $\tau_A = MgR$

Angular acceleration, $\alpha = \frac{\tau_A}{I_A} = \frac{g}{2R}$

Acceleration of COM

$$a = r\alpha \qquad \left[AC = r = R\sqrt{1+\frac{4}{\pi^2}}\right]$$

Direction of a is perpendicular to AC

$$a_x = a\sin\theta = r\alpha\sin\theta = \frac{2R}{\pi}\cdot\frac{g}{2R} = \frac{g}{\pi}$$

$$a_y = a\cos\theta = r\alpha\cos\theta = R\cdot\frac{g}{2R} = \frac{g}{2}$$

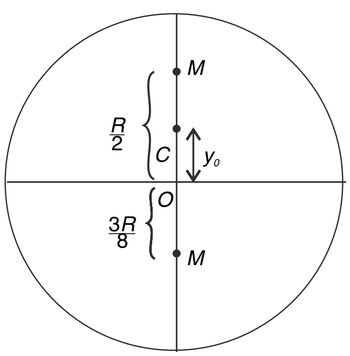
$$\therefore\ F_x = Ma_x = \frac{Mg}{\pi}$$

$$Mg - F_y = Ma_y \ \therefore\ F_y = \frac{Mg}{2}$$

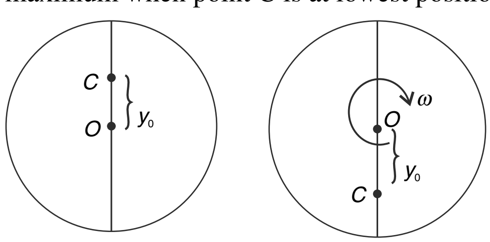
$$\therefore\ F = \sqrt{F_x^2 + F_y^2} = Mg\sqrt{\frac{1}{4}+\frac{1}{\pi^2}}$$

98. COM of a hemispherical shell is at a distance of $\frac{R}{2}$ from the base and for a solid hemisphere it is at a distance $\frac{3R}{8}$ from the base. Hence, the COM of our sphere is at a distance y_0 from the geometrical centre where

$$y_0 = \frac{M\frac{R}{2} - M\frac{3R}{8}}{2M} = \frac{R}{16}$$



On disturbing the sphere, the torque of weight (about centre O) causes the motion. Obviously, the angular speed is maximum when point C is at lowest position.



Using conservation of mechanical energy

Gain in KE = Loss in PE

$$\frac{1}{2}I\omega_0^2 = 2Mg\left(2y_0\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Moment of inertia about a diameter passing through O is -

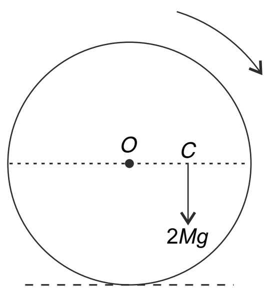
$$I = \frac{2}{3}MR^2 + \frac{2}{5}MR^2 = \frac{16}{15}MR^2$$

$\therefore$ From equation (1) -

$$\frac{1}{2}\frac{16}{15}MR^2\omega_o^2 = 4\,Mg.\frac{R}{16} \quad \Rightarrow \omega_0 = \sqrt{\frac{15}{32}\frac{g}{R}}$$

Maximum angular acceleration will occur when torque due to weight is maximum. This happens when line OC is horizontal.

$$\tau = 2Mg\cdot\frac{R}{16} = \frac{MgR}{8}$$

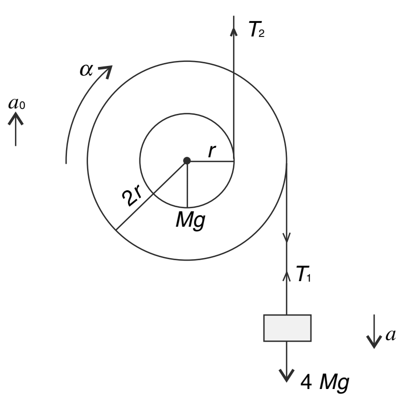


$$I\alpha_0 = \frac{MgR}{8} \Rightarrow \frac{16}{15}MR^2.\alpha_0 = \frac{MgR}{8}$$

$$\alpha_0 = \frac{15}{128}\frac{g}{R}$$

The COM has horizontal acceleration as the sphere spins. This cannot happen in absence of a horizontal force. The walls provide the necessary horizontal force. In absence of the walls, the centre of the ball will oscillate along horizontal direction.

99.



Let α = angular acceleration of the pulley

a_0 = upward acceleration of the centre of the pulley

a = Acceleration of the block in downward direction.

The constraint relations are $a_0 = r\alpha$ and $a = R\alpha - a_0 = 2r\alpha - r\alpha = r\alpha$

For motion of the block : $4\,Mg - T_1 = 4M\,(r\alpha)$ ....(1)

For translation of pulley : $T_2 - T_1 - Mg = Ma_0$ ....(2)

For rotation of the pulley

$T_1\ (2r) - T_2\ (r) = I\alpha$

$$2T_1 - T_2 = \frac{Mr\alpha}{2} \quad \text{-------------(3)} \quad \left[\because\ I = \frac{Mr^2}{2}\right]$$

(1) + (2) + (3)

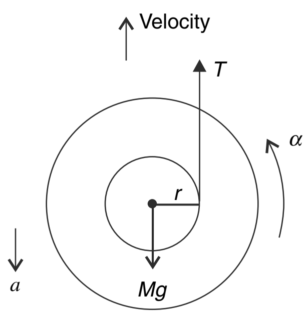
$$3Mg = \left(4+1+\frac{1}{2}\right)Ma_0 \quad [\because\ r\alpha = a_0]$$

$$\frac{6}{11}g = a_0$$

(b) At $t = 2$ second, velocity of pulley

$$V_0 = a_0 t = \frac{12}{11}g$$

We will once again calculate the acceleration of the pulley after the string snaps.



$Mg - T = Ma$ ....(1)

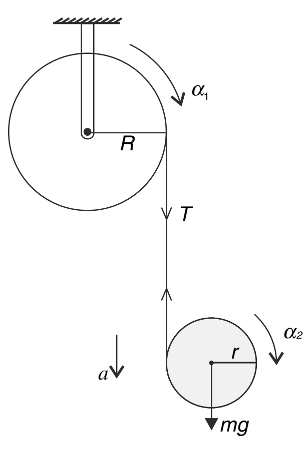
$$Tr = \frac{Mr^2}{2}\cdot\alpha \quad \Rightarrow \quad T = \frac{M}{2}\cdot r\alpha$$

Solving (1) and (2); $a = \dfrac{2g}{3}$

If pulley stops ascending after time 't' it means its upward velocity becomes zero.

$$0 = V_0 - at \ \therefore\ t = \frac{V_0}{a} = \frac{12g}{11}\times\frac{3}{2g} = \frac{18}{11}\ s$$

100.



<u>For lower pulley:</u> $mg - T = ma$ ----------(1)

And $Tr = \frac{1}{2}mr^2 \cdot \alpha_2 \quad \Rightarrow T = \frac{m}{2}\left(r\alpha_2\right)$ -------(2)

For upper pulley : $TR = \frac{1}{2}MR^2 \cdot \alpha_1 \quad \Rightarrow T = \frac{M}{2}\left(R\alpha_1\right)$ -------(3)

But $R\alpha_1 + r\alpha_2 = a$

$$\therefore \frac{2T}{M} + \frac{2T}{m} = g - \frac{T}{m} \qquad \therefore T = \frac{Mmg}{2m+3M}$$

Substituting the value of T in (2), (3) & (1)

$$\alpha_2 = \frac{2Mg}{(2m+3M)r} \qquad \text{and} \qquad \alpha_1 = \frac{2mg}{(2m+3M)R}$$

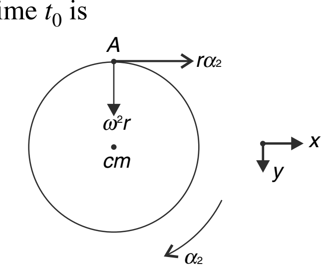
$$a = \frac{2(M+m)g}{(3M+2m)}$$

(a) t_0 = time required by the lower pulley to complete one turn.

$$\therefore \quad \theta = \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow 2\pi = \frac{1}{2}\alpha_2 t_0^2$$

$$\therefore t_0 = \sqrt{\frac{4\pi}{\alpha_2}} = \sqrt{\frac{2\pi(2m+3M)r}{Mg}}$$

(b) Angular speed of lower pulley at time t_0 is



$$\omega^2 = \omega_0^2 + 2\alpha\theta \qquad \Rightarrow \omega = \sqrt{\frac{8\pi Mg}{(2m+3M)r}}$$

Acceleration of mark at A with respect to COM of the pulley is -

$$\vec{a}_{ACoM} = r\alpha_2\hat{i} + \omega^2 r\,\hat{j}$$

$\therefore$ Acceleration of point at A is

$$\vec{a}_A = (r\alpha_2)\hat{i} + \left(\omega^2 r + a\right)\hat{j}$$

$$= \left(\frac{2Mg}{2m+3M}\right)\hat{i} + \left(\frac{8\pi Mg}{2m+3M} + \frac{2(M+m)g}{2m+3M}\right)\hat{j}$$

$$= \frac{2g}{2m+3M}\left[M\hat{i} + (4\pi M + M + m)\hat{j}\right] \qquad = \frac{2g}{2m+3M}\left[M\hat{i} + (4\pi M + M + m)\hat{j}\right]$$

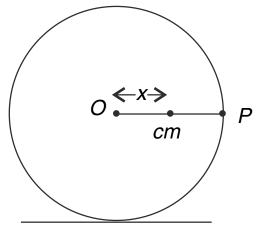
$$\therefore a_A = \frac{2g}{2m+3M}\sqrt{M^2 + (4\pi M + M + m)^2}$$

101. COM of the system is at a distance x from O on the line OP, such that

$$x = \frac{m \cdot R}{m+M} = \frac{1 \times R}{1+3} = \frac{R}{4} = \frac{1}{2}\ m.$$

MOI of the (ring + particle) system about an axis through COM is

$I_{cm} = MR^2 + Mx^2 + m\,(R - x)^2$

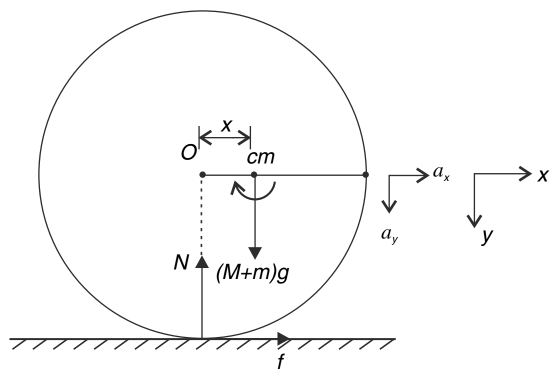


$$= 3R^2 + \ 3\cdot\frac{R^2}{16} + 1\left(R - \frac{R}{4}\right)^2 \quad = 3R^2 + \frac{3R^2}{16} + \frac{9R^2}{16} = \frac{15R^2}{4}$$

Let the acceleration of COM immediately after release be

$\vec{a}_{cm} = a_x\hat{i} + a_y\hat{j}$

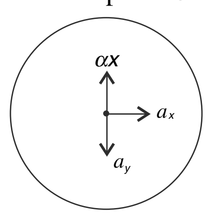
And angular acceleration of the ring be α.



Acceleration of centre point O will be vector sum of three terms

$a_x\,(\rightarrow),\ \alpha.x\,(\uparrow),\ a_y\,(\downarrow)$

Since, there is no acceleration in vertical direction for point O.



$\therefore\ a_y = \alpha\cdot x \Rightarrow a_y = \alpha\dfrac{R}{4}$

For pure rolling $a_x = R\alpha \quad \therefore\ a_y = \dfrac{a_x}{4}$

Equations for translational motion are:

$f = (M + m)a_x \qquad \Rightarrow\ f - 4a_x$ ------------------(1)

$\left(M + m\right)g - N \ = \ \left(M + m\right)a_y \quad \Rightarrow 4g - \ N = 4\cdot\dfrac{a_x}{4}$

$4g - N = a_x$ --------------(2)

Using $\tau_{cm} = I_{cm}.\alpha \qquad \Rightarrow\ N.\,x - fR = \dfrac{15}{4}R^2\alpha$

$$\frac{N}{4} - f = \frac{15}{4}a_x \qquad \text{-------------(3)}$$

Solving (1), (2) and (3)

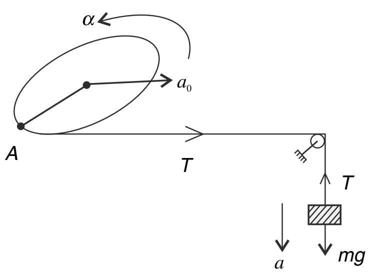
$$a_x = \frac{g}{8}; f = \frac{g}{2}; \quad N = \frac{31}{8}g$$

$$\therefore \alpha = \frac{a_x}{R} = \frac{g}{16} = \frac{5}{8}\, rad/s^2.$$

$$f = 5\ N; \quad N = \frac{155}{4}\ newton$$

You may also solve the problem by considering the ring to be in pure rotation about the contact point.

102. Let a = acceleration of mass m , a_0 = acceleration of COM of the disc
α = angular acceleration of the disc about its COM



Acceleration of point A is: $a_A = a_0 + R\alpha$
But a_A must be equal to a $\therefore a = a_0 + R.\alpha$ ---------(1)
For motion of 'm' $mg - T = ma$ ---------(2)
For translation of disc $T = Ma_0$ ---------(3)
For rotation of disc $TR = \frac{1}{2}MR^2\alpha \Rightarrow T = \frac{1}{2}MR\alpha$ ---------(4)

Finding values of a, a_0 and $R\alpha$ from (2), (3) and (4) and putting in equation (1)

$$g - \frac{T}{m} = \frac{T}{M} + \frac{2T}{M}$$

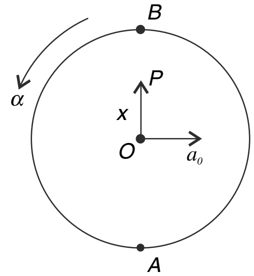
$$\Rightarrow T\left[\frac{3}{M} + \frac{1}{m}\right] = g \qquad \Rightarrow T = \frac{Mmg}{3m + M}$$

from (2) $a = g - \frac{Mg}{3m+M} = \frac{3mg}{3m+M} = \frac{3}{4}g$ $\quad$ (*if m = M*)

from (3) $a_0 = \frac{mg}{3m+M} = \frac{g}{4}$ $\quad$ (*if m = M*)

from (4) $\alpha = \frac{2mg}{(3m+M)R} = \frac{g}{2R}$ $\quad$ (*if m = M*)

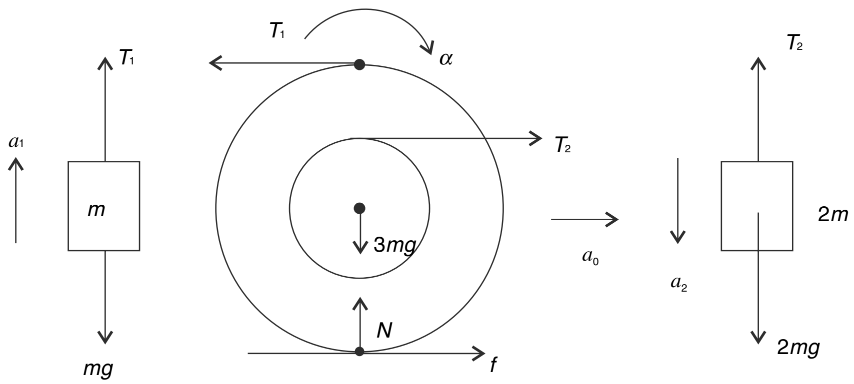
Point P shown in the figure has zero acceleration if



$x\alpha = a_0$

$\dfrac{xg}{2R} = \dfrac{g}{4}; \;\Rightarrow x = \dfrac{R}{2}$

103. Let acceleration of COM of the spool be a_0 towards right, and acceleration of m_2 be a_2 downward and that of m_1 be a_1 upward.



f = friction ; α =Angular acceleration of the spool

$2\,R\alpha = a_0$ ------------------(1)

$R\alpha + a_0 = a_2$ ------------------(2)

$2R\alpha + a_0 = a_1$ ------------------(3)

$T_1 - mg = ma_1$

$T_1 - mg = m\,(a_0 + 2R\alpha) = 2ma_0$ ------------------(4) $\left[\because\; R\alpha = \dfrac{a_0}{2}\right]$

$2mg - T_2 = 2m\,a_2$

$2mg - T_2 = 2m\,[a_2 + R\alpha] = 3ma_0$ ------------------(5)

$T_2 + f - T_1 = 3ma_0$ ------------------(6)

$T_2R - T_1\,2\,R - f\,2R = I\alpha$

$T_2 - 2T_1 - 2f = 2mR\alpha \;[\because I = 2mR^2]$

$T_2 - 2T_1 - 2f = ma_0$ ------------------(7)

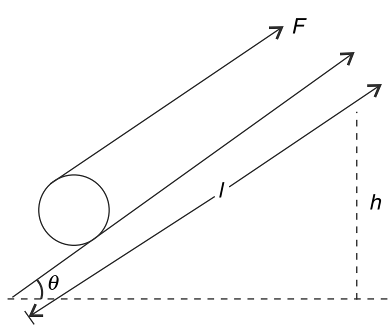
Solving (4), (5), (6) and (7) we get $f = -\dfrac{mg}{3}$

The negative sign indicates that friction is opposite to the direction shown in figure

$\because f \le \mu N \quad \therefore \dfrac{mg}{3} \le \mu\, 3mg$

$\therefore \dfrac{1}{9} \le \mu$

104.



$\ell = \dfrac{h}{\sin\theta}$

Work done by F as the log moves up is $W = F \cdot 2\ell \times 2$

When the log just rolls up, its kinetic energy is zero.

$\therefore\ F_0 \cdot 4\ell = Mgh$

$F_0 \cdot \dfrac{4h}{\sin\theta} = Mgh \ \therefore\ \ F_0 = \dfrac{Mg\sin\theta}{4}$

When $F = 2F_0 = \dfrac{Mg\sin\theta}{2}$

$W_F = 2\,Mgh$

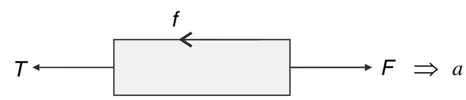
105. (a) If α is the angular acceleration (anticlockwise) of the cylinder with its centre at rest and a is the acceleration of the plank to the right then there will be no slipping between the two if $a = R\alpha$.

This ensures that the length of the thread that unwraps from the cylinder goes to increase the length of the lower segment of the string and that the velocity of the point of contact of the cylinder and that of the plank are equal.

Therefore, the cylinder has no acceleration.

(b) For cylinder $T = f$ and

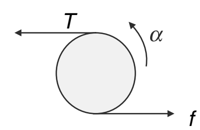
$TR + fr = \dfrac{1}{2}mR^2\alpha$



For the plank

$F - T - f = ma$

If $a = R\alpha$



Solving we get $a = \dfrac{2F}{3m}$

Displacement of the plank in time t is

$S = \dfrac{1}{2}at^2 = \dfrac{1}{2}\dfrac{2F}{3m}t^2 = \dfrac{Ft^2}{3m}$

Work done by the force $W = FS = \dfrac{F^2t^2}{3m}$

As there is no loss of energy due to friction (there is no slipping), this work done is equal to gain in KE of the system.

106. (a) Since the disc was rolling, the velocity of its top point at the instant of leaving the track was zero. It means $v_H = \omega r$

When the disc is in air v_H and ω both do not change. Hence the horizontal component of the velocity of the top point P of the disc at every instant is zero and the vertical component of the velocity of the point P is equal to the vertical component of velocity of the CM of disc.

$\Rightarrow\quad \sqrt{2gh} = \sqrt{2.g.2(R-0.1)} = 6$

$\Rightarrow\quad 4 \times 10\,(R - 0.1) = 36$

$R = 1\ m.$

(b) $\tan 30^\circ = \dfrac{6}{v_x}$

$\Rightarrow v_x = \dfrac{6}{\tan 30^\circ} = 6\sqrt{3}$

$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{36\times 3 + 36}$

$= 6 \times 2 = 12$ m/s.

(c) AC = $v_x t$.

$$= 6\sqrt{3} \times \sqrt{\frac{2\times 2\times 0.9}{10}} = 12\sqrt{3}\times 0.3 = 3.6\sqrt{3}m$$

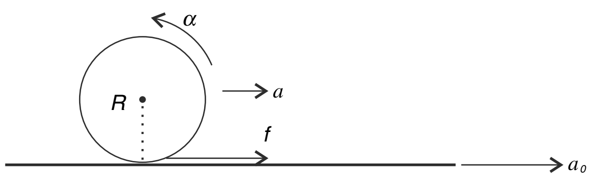
107. Let acceleration on inclined part (while going up as well as while going down) be a. Length of inclined part be ℓ ℓ_1 and that of flat part be ℓ_2

Total time period is given as

$$T = \left[\sqrt{\frac{2\ell_1}{a}} + \frac{\ell_2}{\sqrt{2a\ell_1}} + \sqrt{\frac{2\ell_1}{a}}\right]\times 2.$$

If ℓ_1 and ℓ_2 is made 4 times, the time period doubles.

108.



Let the friction force on the cylinder be f.

Translation : $f = Ma$.............................(1)

Rotation : $fR = I_{cM}\alpha \Rightarrow fR = \frac{1}{2}MR^2\cdot\alpha$

$$\therefore f = \frac{1}{2}MR\alpha ....(2)$$

For no slipping $R\alpha + a = a_0$

$R\alpha + a = 6$ ....(3)

Using (3) in (2) $f = \frac{1}{2}M(6-a)$

Put in (1) $\frac{1}{2}M(6-a) = Ma$

$\therefore 6 - a = 2a \Rightarrow a = 2\ m/s^2$

Let time required for the cylinder to roll off the sheet be t

Acceleration of sheet with respect to cylinder = $a_0 - a = 4\ m/s^2$

$$\therefore 10 = \frac{1}{2}\times 4\times t^2; \Rightarrow t = \sqrt{5}\ s$$

109. Let the friction force between the cylinder and the plate be f_1.

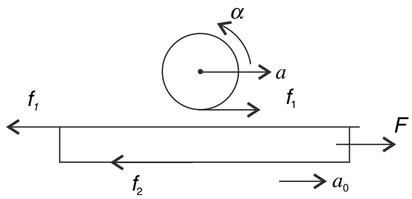
Note that maximum possible value of f_1 is : $f_{1\max} = 6\cdot g\cdot \mu = 60 \times 0.2 = 12\ N$

The friction force between the floor and the plate is

$f_2 = 7.5 \times g \times \mu = 7.5 \times 10 \times 0.2 = 15\ N$

Let acceleration of plate be a_0 and acceleration of cylinder be a. If angular acceleration of cylinder is α, for no slipping we have $a + R\alpha = a_0$ .... (i)

Writing the equation of translation and rotation for pipe :



$f_1 = Ma$ ....(a)

$f_1 R = MR^2 \cdot \alpha \Rightarrow f_1 = M\,(R\alpha)$.......................(b)

(a) and (b) imply that $a = R\alpha$

From (1) $a = R\alpha = \dfrac{a_0}{2}$

$\therefore$ Equation (a) becomes $f_1 = 6\dfrac{a_0}{2} = 3a_0$....................(2)

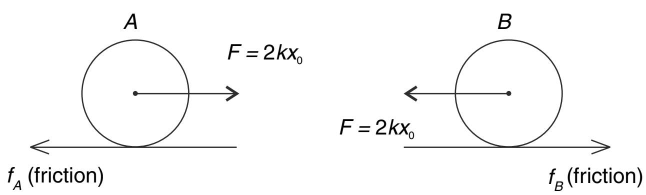
<u>For plate</u> : $25 - 15 - f_1 = 1.5\, a_0$..................(3)

<u>(2) + (3)</u> $a_0 = \dfrac{10}{4.5}\, m/s^2 \Rightarrow a = \dfrac{10}{9}\, ms^{-2}$

Putting in (2) $f_1 = 3 \times \dfrac{10}{4.5} = 6.67\ N$

Since, maximum permissible value of f_1 is $12N$ hence it will adjust itself to $6.67N$ and ensure that the pipe does not slide.

110. (a)



It can be shown that for no slipping condition:

$$f_A = \frac{F}{2} \text{ and } f_B = \frac{F}{3}$$

$$a_{cm} = \frac{f_A - f_B}{2M} = \frac{F}{12M} = \frac{2kx_0}{12M} = \frac{kx_0}{6M}$$

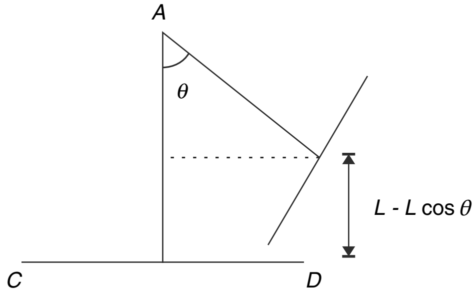
Initially, the acceleration of COM is towards left ($\therefore f_A > f_B$) but once the spring gets compressed the direction of forces will reverse and the acceleration of COM becomes towards right.

(b) Friction is static. It does not dissipate energy. The two cylinder will come to rest simultaneously (why?) when the compression in the springs is x_0.

$$\left|\frac{a_A}{a_B}\right| = \frac{F - f_A}{F - f_B} = \frac{F/2}{2F/3} = \frac{3}{4}$$

$$\therefore \left|\frac{x_A}{x_B}\right| = \frac{3}{4} \therefore x_A = \frac{3}{7}(2x_0) = \frac{6x_0}{7}$$

111. Let angular displacement in extreme position be θ.



Gain in potential energy $\Delta U_{CD} + \Delta U_{AB}$

$$\Delta U = mgL(1-\cos\theta) + mg\frac{L(1-\cos\theta)}{2} = \frac{3}{2}mgL(1-\cos\theta)$$

Moment of inertia about rotation axis is

$$I_A = \frac{mL^2}{3} + \frac{mL^2}{12} + mL^2 = \frac{17}{12}mL^2$$

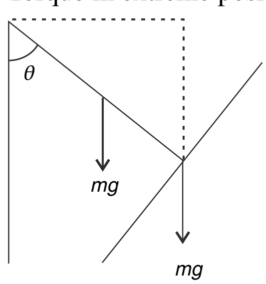
Kinetic energy in lowest position is $k = \frac{1}{2}I_A\omega^2 = \frac{1}{2}\frac{17}{12}mL^2\left(\frac{u}{L}\right)^2 = \frac{17}{24}mu^2 = \frac{17}{24}mgL.$

From energy conservation

$$\frac{3}{2}mgL(1-\cos\theta) = \frac{17}{24}mgL$$

$$\Rightarrow 1-\cos\theta = \frac{17}{36} \Rightarrow \cos\theta = 1-\frac{17}{36} = \frac{19}{36}$$

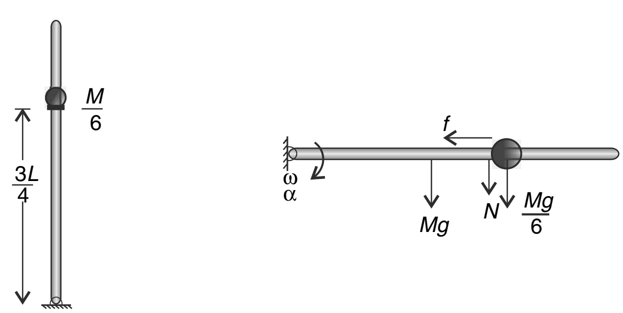
Torque in extreme position



$$\tau = mgL\sin\theta + mg\frac{L}{2}\sin\theta = \frac{3}{2}mgL\sin\theta$$

$$\therefore \alpha = \frac{\tau}{I} = \frac{\frac{3}{2}mgL\sin\theta}{\frac{17}{12}mL^2} = \frac{18}{17}\frac{g}{L}\cdot\frac{\sqrt{36^2-19^2}}{36} = \frac{\sqrt{935}}{34}\frac{g}{L}$$

112. (a)



Let angular speed be ω when rod gets horizontal.

Energy conservation

$$\frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2+\frac{1}{2}\left(\frac{M}{6}\right)\left(\omega\frac{3L}{4}\right)^2=Mg\frac{L}{2}+\frac{M}{6}g\frac{3L}{4}$$

$$\left(\frac{1}{3}+\frac{3}{32}\right)\omega^2L=\left(1+\frac{1}{4}\right)g$$

$$\Rightarrow\frac{41}{96}\omega^2L=\frac{5}{4}g\Rightarrow\omega^2=\frac{120}{41}\frac{g}{L}\text{............(1)}$$

Torque in horizontal position

$$\tau=Mg\frac{L}{2}+\frac{Mg}{6}\cdot\frac{3L}{4}=\frac{5MgL}{8}$$

Angular acceleration in this position is given by $I\alpha=\tau$

$$\left[\frac{ML^2}{3}+\frac{M}{6}\left(\frac{3L}{4}\right)^2\right]\alpha=\frac{5MgL}{8}$$

$$\Rightarrow\frac{41}{96}ML^2\alpha=\frac{5MgL}{8}\quad\therefore\ \alpha=\frac{60}{41}\frac{g}{L}$$

Tangential acceleration of the bead $a_t=\alpha.\frac{3L}{4}=\frac{45}{41}g$

Normal force by the rod on the bead is downward because downward acceleration of the bead is larger than g.

$$N+\frac{M}{6}g=\frac{M}{6}.\frac{45}{41}g\Rightarrow N=\frac{2}{123}Mg$$

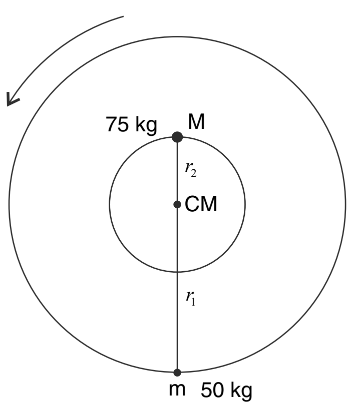
If the bead just begin to slide outward then

$$\mu N=\frac{M}{6}\omega^2\cdot\frac{3L}{4}$$

$$\mu\cdot\frac{2}{123}Mg=\frac{M}{6}\times\frac{3}{3}\times\frac{120}{41}g\quad[\text{using (1)}]$$

$$\mu=\frac{3\times120\times123}{2\times6\times4\times41}=\frac{45}{2}=22.5$$

113.



Distance of the astronauts from COM

$$r_1 = \frac{Mr}{M+m} = \frac{75}{75+50} \times 10 = 6\ m$$
$$r_2 = \frac{mr}{M+m} = \frac{50}{75+50} \times 10 = 4\ m$$

Motion of COM does not change (unless there is external force).

In reference frame attached to the COM, the angular momentum of the system remains conserved. Let the angular speed be ω when distance reduces to 5.0 m.

$[74 \times 2^2 + 50 \times 3^2]\ \omega = [75 \times 4^2 + 50 \times 6^2] \times 5$

$\Rightarrow \omega = 20\ rad/s$

[Or, simply one could have argued that moment of inertia becomes one fourth and hence angular speed becomes 4 times]

Change in KE in COM frame = work done by astronauts.

$$\therefore W = \frac{1}{2} I\omega^2 - \frac{1}{2} I_0\omega_0^2$$
$$= \frac{1}{2}\left(\frac{I_0}{4}\right)(4\omega_0)^2 - \frac{1}{2} I_0\omega_0^2 = \frac{1}{2} I_0\omega_0^2 (4-1)$$
$$= \frac{3}{2} I_0\omega_0^2 = \frac{3}{2} \times \left[75 \times 4^2 + 50 \times 6^2\right] 5^2 = 112.5 KJ$$

114. (a) Let the final velocity of disc and plank are V and V_1 respectively in the direction shown in the figure and angular velocity of the disc be ω.

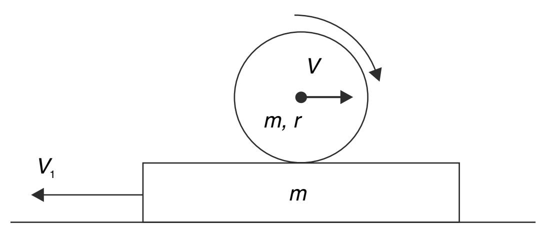
For disc and plank system net force in horizontal direction is zero.

From conservation of momentum-

$0 = mV - mV_1 \Rightarrow V_1 = V$

Since 50% of kinetic energy of system is lost

$$\Rightarrow \frac{1}{2} I\omega^2 + \frac{1}{2} mV^2 + \frac{1}{2} mV_1^2 = \frac{1}{2}\left[\frac{1}{2} I\omega_0^2\right] \text{.........(i)}$$



Also for rolling without slipping

$$r\omega - V = V_1 \Rightarrow r\omega - V = V \Rightarrow \omega = \frac{2V}{r}$$

From (i) $\frac{1}{2} \cdot \frac{mr^2}{2}\left(\frac{2V}{r}\right)^2 + \frac{1}{2} mV^2 + \frac{1}{2} mV^2 = \frac{1}{2}\left[\frac{1}{2} I\omega_0^2\right] \Rightarrow V = \frac{r\omega_0}{4} = V_1$

(b) Change of angular momentum of disc about centre of mass of disc

$$\Delta L_C = I\omega - I\omega_0 = I\left(\frac{2V}{r} - \omega_0\right) = \frac{mr^2}{2}\left(\frac{\omega_0}{2} - \omega_0\right) = -\frac{mr^2\omega_0}{4}$$

(c) Acceleration of plank till slipping ceases = $\frac{f_k}{m} = \frac{\mu mg}{m} = \mu g$

$$V_1^2 = 0^2 + 2as \Rightarrow \left(\frac{r\omega_0}{4}\right)^2 = 2\mu gs \Rightarrow S = \frac{r^2\omega_0^2}{32\mu g}$$

115. Let ω_1, ω_2, & ω_0 be the angular velocity (all with respect to ground) of man 1 in anticlockwise direction, angular velocity of man 2 in clockwise direction and angular velocity of the turntable in anticlockwise direction respectively.

$\omega_1 - \omega_0 = \omega_2 + \omega_0$

From conservation of angular momentum –

$$m_1 r^2 \omega_1 + \frac{1}{2} M r^2 \omega_0 = m_2 r^2 \omega_2$$

$$50\omega_1 + \frac{1}{2} 90\omega_0 = 60\omega_2 \Rightarrow 10\omega_1 + 9\omega_0 = 12\omega_2$$

$$10\omega_1 + 9\omega_0 = 12(\omega_1 - 2\omega_0) \Rightarrow 2\omega_1 = 33\omega_0$$

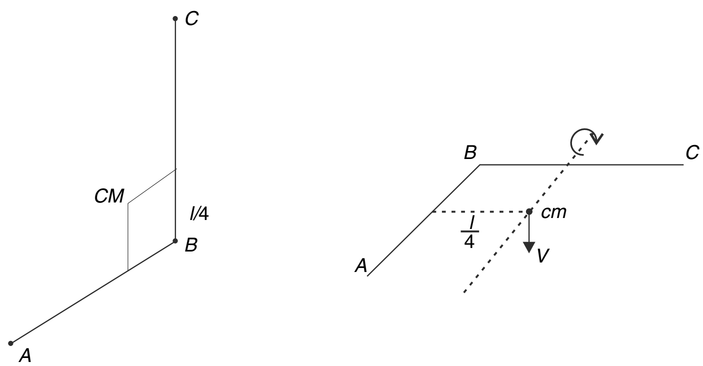
If the angle through which the platform rotates is θ

$$\frac{(90+\theta)}{\omega_1} = \frac{\theta}{\omega_0} \Rightarrow (90+\theta) = \frac{33}{2}\theta$$

$$(90+\theta) = \frac{33}{2}\theta \Rightarrow \theta = \frac{180}{31} = 5.8°$$

116. There is no horizontal force on the rod and its COM must fall vertically. At the moment the rod *BC* is about to hit the surface, velocity of *AB* will go to zero.

Let V and ω be the speed of COM and angular speed of the rod when BC is about to hit the surface. For velocity of part *AB* to be zero we must have -



$$V = \omega \frac{l}{4} \text{....................(1)}$$

Velocity of point C at this moment is $V_c = V + \omega \cdot \frac{3l}{4}$....................(2)

<u>Conservation of Energy</u>

$$\frac{1}{2}(2m)V^2 + \frac{1}{2}I\omega^2 = (2m)g \cdot \frac{l}{4}$$

[I = MOI about the rotation axis = $m\left(\frac{l}{4}\right)^2 + \frac{ml^2}{12} + m\left(\frac{l}{4}\right)^2 = \frac{5ml^2}{24}$

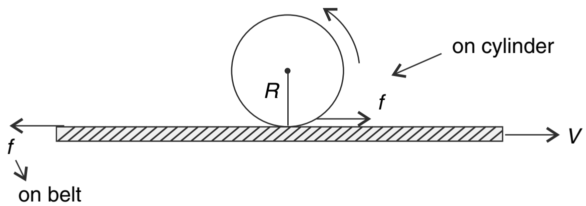
$$\therefore mV^2 + \frac{1}{2}\frac{5ml^2}{24}\left(\frac{4V}{l}\right)^2 = \frac{mgl}{2}$$

$$\frac{8V^2}{3} = \frac{gl}{2}$$

$$\Rightarrow V = \frac{\sqrt{3gl}}{4} \text{ and } \omega = \frac{4V}{l} = \sqrt{\frac{3g}{l}}$$

$$\therefore \text{From (2)} V_c = V + \frac{3}{4}\omega l = \frac{\sqrt{3gl}}{4} + \frac{3}{4}\sqrt{3gl} = \sqrt{3gl}$$

117. (a)



As long as the cylinder slips, friction force is $f = \mu mg$

As the belt keeps moving with constant speed, the extra power developed by the motor is

$P = f \cdot V = \mu mg \cdot V$..........................(1)

(b) Angular acceleration is $\alpha = \dfrac{f \cdot R}{\frac{1}{2}mR^2} = \dfrac{2f}{mR}$

If cylinder stops slipping, it means

$\omega R = V \Rightarrow \alpha t \cdot R = V$ [where t = time taken to stop slipping]

$\therefore t = \dfrac{V}{R\alpha} = \dfrac{mV}{2f}$

Work done by the motor in time t is (using (1))

$W = Pt = fVt = \dfrac{1}{2}mV^2$

Kinetic energy gained by the cylinder

$K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\left(\dfrac{1}{2}mR^2\right)\omega^2 = \dfrac{1}{4}mV^2$

$\therefore$ 50% Work done is spent as heat and remaining 50% is KE of the cylinder.

118. Friction force on the smaller disc is $f = \mu mg \cdot$ Torque due to friction (about axis AB)

$\tau_f = \mu\, mg \cdot r$

$\therefore$ Angular acceleration $\alpha = \dfrac{\tau_f}{I} = \dfrac{\mu mgr}{\frac{mr^2}{2}} = \dfrac{2\mu g}{r}$

Angular speed of the smaller disc after time 't' is $\omega = \alpha t = \dfrac{2\mu gt}{r}$

Torque on larger disc about vertical axis is $\tau = \mu mg\, L$

$\Rightarrow \dfrac{MR^2}{2}\alpha_0 = \mu mgL \Rightarrow \alpha_0 = \dfrac{2\mu mgL}{MR^2}$

Angular speed of larger disc at time t is

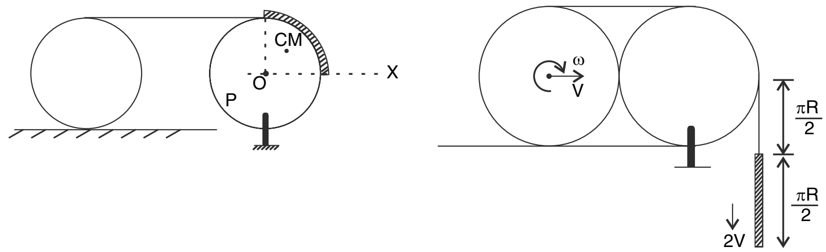
$\omega_0' = \omega_0 - \alpha_0 t = \omega_0 - \dfrac{2\mu mgLt}{MR^2}$

For no slipping $\omega_o' L = \omega r$

$\Rightarrow \omega_0 L - \dfrac{2\mu mgL^2 t}{MR^2} = 2\mu gt$

$\Rightarrow t = \dfrac{\mathrm{MR}^2\omega_0 L}{2\mu g\left[MR^2 + mL^2\right]}$

119.



As the disc moves through a distance $\frac{\pi R}{2}$, a length of thread $\frac{\pi R}{2}$ gets unwind. Thus a point on the rope moves through a distance πR.

Let's apply law of conservation of mechanical energy to the system. Convince yourself that in original position the COM of the rope was at a height of $\frac{2R}{\pi}$ from the line OX.

$$\therefore mg\left[\frac{2R}{\pi}+\frac{3\pi R}{4}\right]=\frac{1}{2}MV^2+\frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2+\frac{1}{2}m(2V)^2$$

$$mgR\left(\frac{8+3\pi^2}{4\pi}\right)=\frac{3}{4}MV^2+2mV^2$$

$$\therefore V^2 = \frac{8+3\pi^2}{\pi}\ \frac{mgR}{(3M+8m)}$$

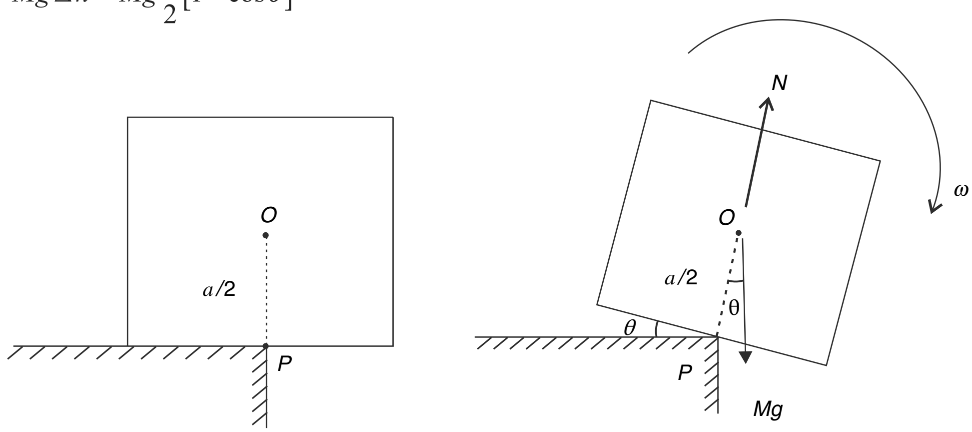
$$V = \sqrt{\left(\frac{8+3\pi^2}{\pi}\right)\left(\frac{mgR}{3M+8m}\right)}$$

120. *MOI* of cube about rotation axis passing through P (lying in the face of the cube) is

$$I=\frac{Ma^2}{6}+M\left(\frac{a}{2}\right)^2=\frac{5Ma^2}{12}$$

Loss in PE when the cube rotates by an angle θ is

$$Mg\,\Delta h = Mg\frac{a}{2}[1-\cos\theta]$$



If angular speed is ω when the cube rotates through θ

$$\frac{1}{2}I\omega^2 = Mg\frac{a}{2}(1-\cos\theta)$$

$$\frac{1}{2}\frac{5Ma^2}{12}\omega^2 = Mg\frac{a}{2}(1-\cos\theta)$$

$$\therefore M\omega^2 a = \frac{12}{5}Mg(1-\cos\theta) \quad \text{....................(1)}$$

If the cube loses contact, the normal force (N) become zero.

$$\therefore Mg\cos\theta = M\omega^2\frac{a}{2} \quad \text{....................(2)} \qquad \left[\omega^2\frac{a}{2} = \textit{centripetal acceleration of COM}\right]$$

$$\therefore Mg\cos\theta = \frac{6Mg}{5}(1-\cos\theta) \Rightarrow \cos\theta_0 = \frac{6}{11}$$

Put in (2) $\omega^2 = \dfrac{12g}{11a}$

$\therefore$ Rotational KE is $KE_R = \dfrac{1}{2}I_{cm}\omega^2 = \dfrac{1}{2}\cdot\dfrac{Ma^2}{6}\cdot\dfrac{12g}{11a} = \dfrac{Mga}{11}$

The rotational KE will not change after the cube leaves the table.

Speed of COM, at the instant of breaking off

$$V_{cm} = \frac{a}{2}\cdot\omega = \frac{a}{2}\sqrt{\frac{12g}{11a}} = \sqrt{\frac{3}{11}ga}$$

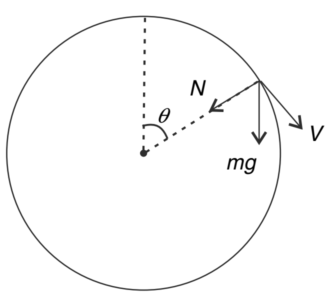
121. (a) Consider motion of the bead.

Energy conservation

$$\frac{1}{2}mV^2 = mgR(1-\cos\theta) \Rightarrow \frac{mV^2}{R} = 2mg(1-\cos\theta)$$

Equation for centripetal force

$N + mg\cos\theta = 2mg(1-\cos\theta)$



$N = mg(2 - 3\cos\theta)$

N is negative (i.e., outwards away from the centre) till $3\cos\theta > 2$

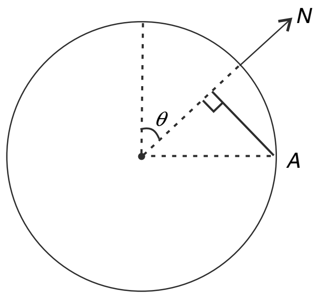
$$\Rightarrow \cos\theta > 2/3 \qquad \Rightarrow \theta < \cos^{-1}\left(\frac{2}{3}\right)$$

Force by the bead on the ring will be radially inwards. This force will have a vertically downward component and therefore the normal force by the ground will exceed Mg.

However, for $\theta > \cos^{-1}\left(\dfrac{2}{3}\right)$ the normal force on the bead will be inward and the force applied by the bead on the ring will be outward.

(b) $\tau_N = N \cdot R\cos\theta$

$= mg\,(2 - 3\cos\theta) \cdot R\cos\theta = mgR\,(2\cos\theta - 3\cos^2\theta)$



(c) τ_N is maximum when $\dfrac{d\tau_N}{d\theta} = 0$

$$\Rightarrow -2\sin\theta + 6\cos\theta\sin\theta = 0 \Rightarrow \cos\theta = \frac{1}{3}$$

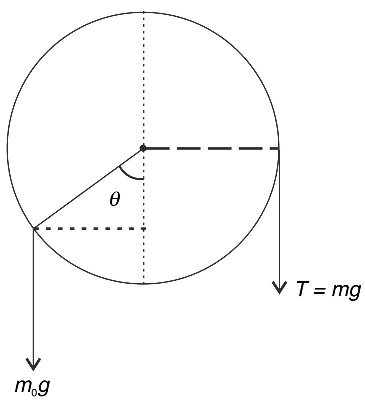
$$\therefore\ \tau_{\max} = mgR\left(2\times\frac{1}{3} - 3\times\frac{1}{9}\right) = \frac{1}{3}mgR$$

The ring will rotate about A (i.e., it will rise) when this torque exceeds the torque due to Mg about A. If $\tau_{\max}$ remains less than torque of Mg the ring will never rise.

$\therefore$ Ring will not rise if

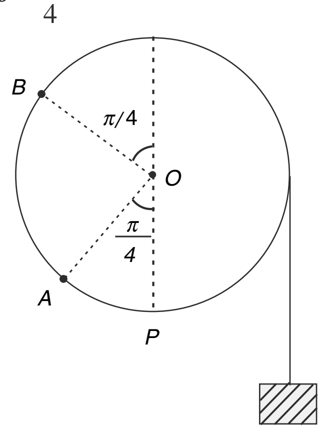
$$MgR \geq \frac{1}{3}mgR \Rightarrow \frac{m}{M} \leq 3$$

122.



For equilibrium : ,$m_0gR\sin\theta = mgR$

$$\sqrt{2}\,m\sin\theta = m \qquad \Rightarrow\ \theta = \frac{\pi}{4}$$



Between P to A the pulley accelerates as torque due to tension will exceed the torque due to $m_0 g$. Between A to B the pulley system retards and beyond B it will once again accelerate.

For m_0 to climb to the top, we need to ensure that it just manages to cross point B. [Note that in no case m_0 can reach the top point with zero speed]

For kinetic energy of the system to be positive when m_0 reaches B we must have

$$mg R\left(\pi-\frac{\pi}{4}\right) > m_0 gR\left(1+\cos\frac{\pi}{4}\right)$$

$$\Rightarrow m.\frac{3\pi}{4} > m_0\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right) \Rightarrow m_0 < \frac{3\pi}{4\left(\sqrt{2}+1\right)}\left(\sqrt{2}m\right)$$

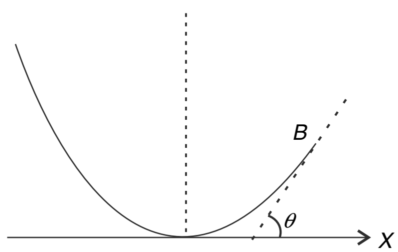
$$\Rightarrow m_0 < (0.975)\sqrt{2}m$$

But it is given that $m_0 = \sqrt{2}\,m$

$\therefore$ m_0 will fail to cross point B.

123. Slope of the parabolic track at B can be obtained as

$$y = \frac{3}{2H}x^2 \Rightarrow \frac{dy}{dx} = \frac{3}{H}x$$



At B ; $\frac{H}{2} = \frac{3}{2H}x^2 \Rightarrow x = \frac{H}{\sqrt{3}}$

$$\therefore \frac{dy}{dx}\Big|_{atB} = \frac{3}{H}\cdot\frac{H}{\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

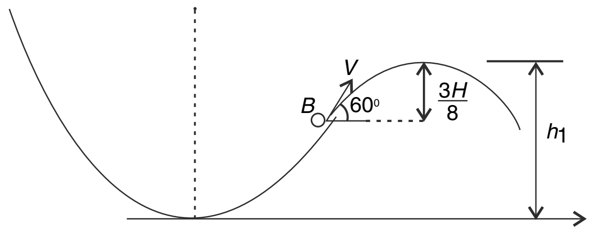
KE of the sliding ring when it reaches B = loss in its potential energy

$$\frac{1}{2}mV^2 = mg\frac{H}{2} \Rightarrow V = \sqrt{gH}$$

After B it will go like a projectile thrown at an angle of 60° to the horizontal.

$\therefore$ Maximum height attained above B is

$$= \frac{V^2\sin^2 60^\circ}{2g} = \frac{gH}{2g}\frac{3}{4} = \frac{3H}{8}$$



$$\therefore h_1 = \frac{H}{2} + \frac{3H}{2} = \frac{7H}{8}$$

On track DEF the ring rolls. Hence, it acquires translational as well rotational kinetic energy. If speed of the centre of the ring is V, when it reaches C, its kinetic energy is mV^2.

$$\therefore mV^2 = mg\frac{H}{2} \Rightarrow V = \sqrt{\frac{gH}{2}}$$

The ring leaves the track at an angle of 60° to the horizontal.

Further height attained $= \dfrac{V^2 \sin^2 60°}{2g} = \dfrac{gH}{2.2g.}\,\dfrac{3}{4} = \dfrac{3H}{16}$

$\therefore$ Height of F above ground = $\dfrac{H}{2}$

$$\therefore h_2 = \frac{H}{2} + \frac{3H}{16} = \frac{11H}{16} \qquad \therefore \frac{h_1}{h_2} = \frac{14}{11}$$

124. Density $\rho = \dfrac{M}{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi\left(\frac{R}{2}\right)^3} = \dfrac{8M}{7\cdot\frac{4}{3}\pi R^3}$

Mass of sphere of radius $\dfrac{R}{2}$ is

$$m_{cavtiy} = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \times \frac{8M}{7\cdot\frac{4}{3}\pi R^3} = \frac{M}{7}$$

$\therefore$ Mass of sphere without cavity = $M_0 = M + \dfrac{M}{7} = \dfrac{8M}{7}$

(a) Kinetic energy = $\left(KE\ of\ a\ sphere\ of\ mass\ \dfrac{8M}{7}\right) - \left(KE\ of\ a\ sphere\ of\ mass\ \dfrac{M}{7}\right)$

[KE can be written as $K = \dfrac{1}{2}I\omega^2$ where $I = MOI$ about an axis through contact point]

$$\therefore KE = \frac{1}{2}\left(\frac{7}{5}M_0R^2\right)\omega^2 - \frac{1}{2}\left(\frac{7}{5}m_{cavity}\left(\frac{R}{2}\right)^2\right)\omega^2$$

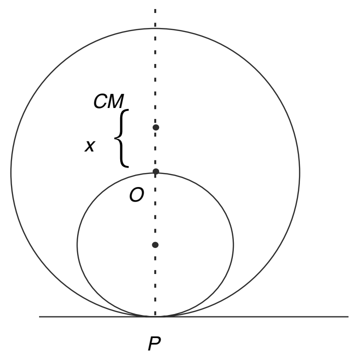
$$= \frac{7}{10}\cdot\frac{8M}{7}\cdot V^2 - \frac{7}{10}\cdot\frac{M}{7}\frac{1}{4}\cdot V^2$$

$$= \left(\frac{4}{5} - \frac{1}{40}\right)MV^2 = \frac{31}{40}MV^2$$

(b) Distance of *COM* from $O = x$

$$x \cdot M = \frac{M}{7} \cdot \frac{R}{2} \;\therefore\; x = \frac{R}{14}$$

The sphere may be assumed to be in pure rotation about an axis through contact point P

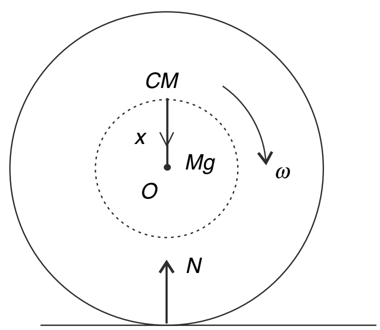


.

$$\therefore V_{CM} = \omega \cdot (R + x) = \frac{V}{R}\left(R + \frac{R}{14}\right) = \frac{15V}{14}$$

The *COM* is moving in a circle of radius x about point O.

In the shown position, acceleration of point is zero (it will not be zero after some time !). We can write the equation of motion for circular motion of *COM* as



$$M\omega^2 x = Mg - N$$

$$\therefore N = Mg - M\left(\frac{V}{R}\right)^2 x = Mg - \frac{MV^2}{14R}$$

For $N \geq 0$; $Mg \geq \dfrac{MV^2}{14R}$

$\sqrt{14gR} \geq V$

Note: (1) Had point O been accelerated, we would have to think about pseudo force

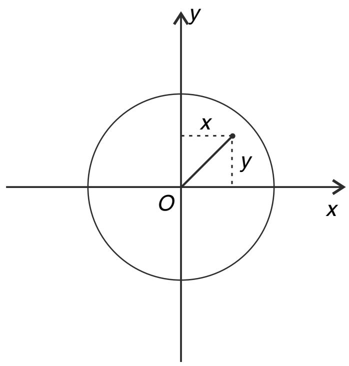
(2) $F_{ext} = M\, a_{cm}$

125. Change in momentum of the bullet

$\Delta p = m\,(u - v)$ = Impulse transferred to the ball.

The angular impulse transferred to the ball about X axis will be:

$(\text{Im})_x = \Delta p \cdot y$



This impulse has vector direction along negative x direction.

For no rotation about x axis we must have -

$$\Delta py = \frac{2}{5}MR^2 \cdot \omega \qquad \text{.............................(1)}$$

For the ball to gain same angular speed about y axis

$$(\text{Im})_y = \Delta px = \frac{2}{5}MR^2\omega \quad \text{.............................(2)}$$

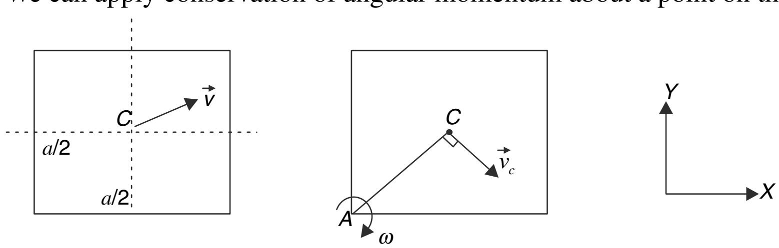
$\therefore$ from (1) and (2) $x = y$

$$x = y = \frac{2MR^2\omega}{5m(u-v)} \qquad [\therefore \Delta p = mu - mv]$$

$$\therefore r = \sqrt{x^2 + y^2}$$

$$r = \frac{2\sqrt{2}}{5}\frac{MR^2\omega}{m(u-v)}$$

126. We can apply conservation of angular momentum about a point on the surface just below A.



$$L_{iA} = M(4v_0)\frac{a}{2}\ (Clockwise) + M(2v_0)\frac{a}{2}\ (Anticlockwise) = Mv_0 a\ (Clockwise)$$

$$L_{fA} = I_A\omega = \frac{2}{3}Ma^2\omega\ \ (Clockwise)$$

$$\therefore \frac{2}{3}Ma^2\omega = Mv_0 a \qquad \therefore \omega = \frac{3}{2}\frac{v_0}{a}$$

Velocity of centre will be :

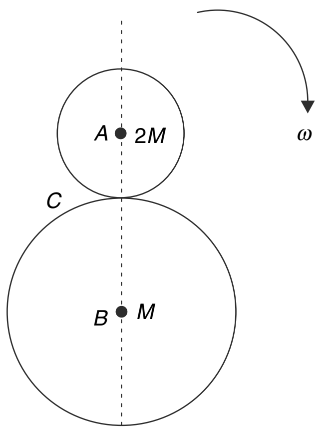
$v_c = \omega\dfrac{a}{\sqrt{2}}$ in the direction perpendicular to AC

$$\therefore v_c = \frac{3v_0}{2\sqrt{2}}$$

$$\therefore \vec{v}_c = \frac{3v_0}{2\sqrt{2}}\left[\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right] = \frac{3v_0}{4}\left(\hat{i} - \hat{j}\right)$$

127. (a) Let's first locate the *COM* of the system. Let it be at a distance x from the centre of disc A

$$x = \frac{M(3R)}{2M + M} = R$$



It means the COM is at point C where the two discs make contact.

Linear momentum of the system is zero. Hence the COM of the composite system will be at rest after collision. The system will rotate about C with angular speed ω after collision. We will apply conservation of angular momentum about C

$$L_{\text{after collision}} = L_{\text{before collision}}$$

$$I_C \cdot \omega = 2MVR + M \cdot 2V \cdot 2R$$

$$\left[\frac{3}{2}(2M)R^2 + \frac{3}{2}(M)(2R)^2\right]\omega = 6MVR$$

$$\Rightarrow 9MR^2\omega = 6MVR \Rightarrow \omega = \frac{2}{3}\frac{V}{R}$$

(b) $$KE_{\text{loss}} = \left[\frac{1}{2}(2M)V^2 + \frac{1}{2}(M)(2V)^2\right] - \frac{1}{2}(I_C)\omega^2$$

$$= 3MV^2 - \frac{1}{2}\left(9MR^2\right)\left(\frac{2V}{3R}\right)^2 = MV^2$$

128. Let the impulse applied by the peg be J_0

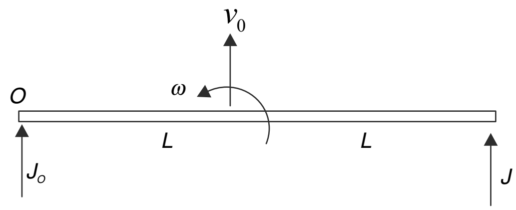
$J + J_0 = MV_0$ ....(i)

Angular impulse about COM = $(J - J_0)L$

MoI about an axis through COM : $I_{CM} = \frac{1}{12}M(2L)^2 = \frac{ML^2}{3}$

$$\therefore \frac{ML^2}{3}\omega = \left(J - J_0\right)L$$

$$\Rightarrow \omega = \frac{3\left(J - J_0\right)}{ML}$$



Velocity of end O of the rod immediately after the impulse is applied is zero.

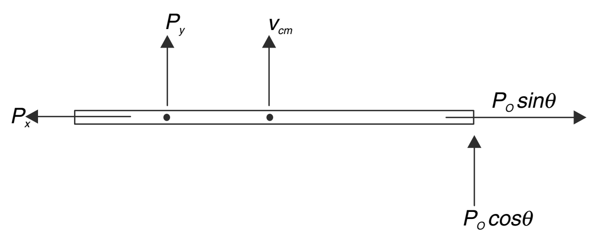
$$\therefore V_0 = \omega L \quad \therefore \frac{J + J_0}{M} = 3\frac{(J - J_0)}{M}$$

$$\Rightarrow J + J_0 = 3J - 3J_0 \Rightarrow J_0 = \frac{J}{2}$$

$$\Rightarrow J + J_0 = 3J - 3J_0 \Rightarrow J_0 = \frac{J}{2}$$

129. Angular impulse = change in angular momentum

$$\Rightarrow \frac{3\ell}{4}P_0\cos\theta = \left[\frac{m\ell^2}{12} + \frac{m\ell^2}{16}\right]\omega \Rightarrow \omega = \frac{36P_0\cos\theta}{7m\ell}$$



Velocity of centre of mass after hit is

$$V_{CM} = \omega\frac{\ell}{4} = \frac{9P_0\cos\theta}{7m}$$

Let the rectangular components of impulse by the hinge be P_x and P_y

$P_x = P_0 \sin\theta$ and $P_0 \cos\theta + P_y = mV_{cm}$

$$\Rightarrow P_y = mV_{cm} - P_0\cos\theta = \frac{2P_0\cos\theta}{7}$$

$\therefore$ Impulse by the hinge has a magnitude

$$P = \sqrt{P_x^2 + P_y^2} = P_0\sqrt{\sin^2\theta + \left(\frac{2}{7}\cos\theta\right)^2}$$

130. COM of rods AB and BC combine will lie on line BO at a distance $\frac{1}{2\sqrt{2}}m$ from B (at point C_1)

COM of $AD + DC$ will lie on DO at a distance $\frac{1}{2\sqrt{2}}$ from D (at point C_2)

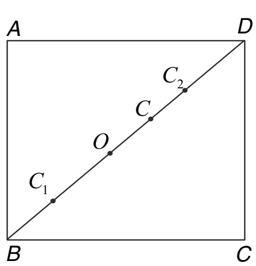


Fig (a)

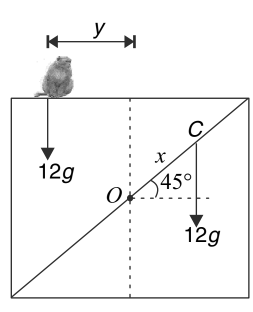


Fig (b)

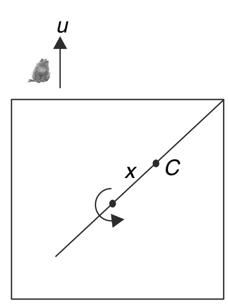


Fig (c)

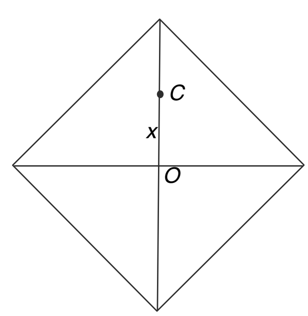


Fig (d)

Distance $OC_1 = \frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} = OC_2$

$\therefore$ COM of the square frame (indicated by C in diagram) will be at a distance x from O where

$$x = \frac{8 \times OC_2 - 4 \times OC_1}{8+4} = \frac{1}{6\sqrt{2}} m$$

For equilibrium, torque about O is zero

$$M_{monkey}\, g\, y = 12\, g\, \frac{x}{\sqrt{2}} \quad \therefore y = \frac{1}{12} m$$

MOI of the square frame about an axis though O perpendicular to the plane of the figure is

$$I = \left(\frac{2 \times 1^2}{12} + 2 \times \left(\frac{1}{2}\right)^2\right) \times 2 + \left[\frac{4 \times 1^2}{12} + 4 \times \left(\frac{1}{2}\right)^2\right] \times 2 = 4 kg\ m^2$$

Let the monkey jump with speed u and angular speed acquired by the frame be ω.

From conservation of angular momentum

$$I\omega = 12 \times u \times y \Rightarrow \omega = \frac{12.\frac{1}{12}}{4} \cdot u = \frac{u}{4}$$

KE acquired by the frame $K = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 4 \left(\frac{u}{4}\right)^2 = \frac{u^2}{8}$

This KE must be sufficient to raise the COM to the highest level shown in figure (d).

$$\therefore 12g\left(x - \frac{x}{\sqrt{2}}\right) = \frac{u^2}{8} \qquad \therefore\ 96g \frac{1}{6\sqrt{2}}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) = u^2$$

$$\therefore u^2 = 8\left(\sqrt{2}-1\right)g$$

If height attained by monkey from its initial position is H

$$H = \frac{u^2}{2g} = 4(\sqrt{2}-1)m$$

131. Forces on the particle are – normal reaction of the frustum wall and weight of the particle.

These forces do not produce any torque about the central vertical axis. Hence, angular momentum of the particle about the central vertical axis is conserved.

Let $v_H =$ horizontal component of velocity at B

$$mv_H r = mv_0 R \Rightarrow v_H = v_0 \frac{R}{r} \text{ ....(i)}$$

Energy conservation gives :

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + mgh \qquad \therefore v^2 = v_0^2 + 2gh$$

$$v_H^2 + v_v^2 = v_0^2 + 2gh \qquad \therefore v_v^2 = v_0^2 + 2gh - \left(v_0 \frac{R}{r}\right)^2 = 2gh - v_0^2\left(\frac{R^2}{r^2} - 1\right)$$

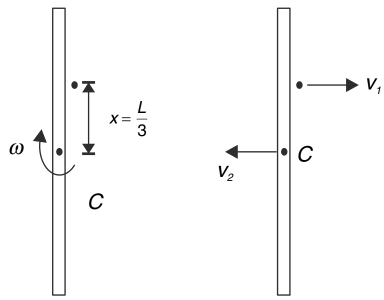
$$\Rightarrow v_V = \sqrt{2gh - v_0^2\left(\frac{R^2}{r^2} - 1\right)}$$

(b) The particle will not leave the frustum if vertical component of its velocity becomes zero before reaching the bottom

$$\Rightarrow 2gh = v_0^2\left(\frac{R^2}{r^2}-1\right)$$

$$\Rightarrow 2gh = 3v_0^2 \Rightarrow h = \sqrt{\frac{3v_0^2}{2g}}$$

132. (a) Let the velocity of particle after collision be V_1 and that of centre of the stick be V_2 (as shown)



Before collision After collision

Momentum conservation gives : $mV_1 = mV_2$ ....(i)

Angular momentum conservation about C: $\frac{ML^2}{12}\omega = mV_1\frac{L}{3} \Rightarrow V_1 = \frac{ML\omega}{4m}$ ....(ii)

Collision is elastic, i.e., $e = 1$

$$\frac{V_1-(-V_2)}{\omega\frac{L}{3}-0} = 1 \quad \Rightarrow V_1 + V_2 = \frac{\omega L}{3}$$

Substituting for V_1 and V_2 from (i) and (ii)

$$\frac{M\omega L}{4m} + \frac{\omega L}{4} = \frac{\omega L}{3} \text{.............(iii)}$$

$$\Rightarrow \frac{M}{m} = \frac{1}{3} \Rightarrow m = 3M$$

(c) In above solution if we replace $\frac{L}{3}$ by x and repeat the steps, we get equation (iii) as $\frac{M}{m}\frac{L^2}{12x}\omega + \frac{L^2\omega}{12x} = \omega x$

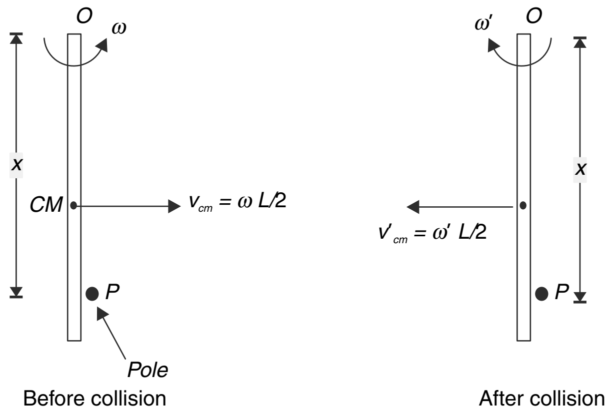
$$\Rightarrow \frac{M}{m}\frac{L^2}{12x} = x - \frac{L^2}{12x}$$

This number shall be greater than zero

$$\therefore x > \frac{L^2}{12x} \Rightarrow \quad x > \frac{L}{\sqrt{12}}$$

133. Since no impulse is applied by the rotation axis, we can apply conservation of angular momentum about P

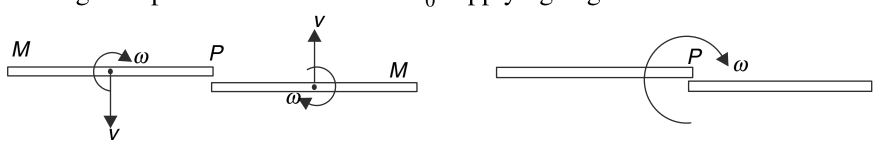
$$\frac{1}{12}ML^2\omega - M\frac{\omega L}{2}\left(x-\frac{L}{2}\right) = M\omega'\frac{L}{2}\left(x-\frac{L}{2}\right) - \frac{1}{12}ML^2\omega'$$



$$\Rightarrow \frac{ML^2}{12}(\omega+\omega') = M\left(x-\frac{L}{2}\right)\frac{L}{2}(\omega'+\omega)$$

$$\Rightarrow \frac{L}{6} = x-\frac{L}{2} \Rightarrow x = \frac{L}{6}+\frac{L}{2} = \frac{2L}{3}$$

134. (a) Linear momentum of the system is zero. Hence, the centre of mass of the system (point P) is at rest after collision. Let angular speed after collision be ω_0. Applying angular momentum conservation about point P gives:



Before collision **after collision**

$L_{BC} = L_{AC}$

$$\frac{ML^2}{12}\omega(CLK) + MV\frac{L}{2}(ACLK) + \frac{ML^2}{12}\omega(CLK) + MV\frac{L}{2}(ACLK) = 2\frac{ML^2}{3}\omega_0(CLK)$$

Where CLK stands for clockwise and $ACLK$ stands for anticlockwise.

$$\Rightarrow \frac{ML^2}{6}\omega - MVL = \frac{2}{3}ML^2\omega_0 \quad \Rightarrow \omega_0 = \frac{\omega}{4} - \frac{3V}{2L}$$

For no motion $\omega_0 = 0 \quad \Rightarrow \frac{V}{\omega L} = \frac{1}{6}$

(b) If $\frac{V}{\omega L} = \frac{1}{12}$; $\omega_0 = \frac{\omega}{8}$

$$KE_{\text{inital}} = \left[\frac{1}{2}\frac{ML^2}{12}\omega^2 + \frac{1}{2}MV^2\right]\times 2 = \left[\frac{ML^2}{12}\omega^2 + M\left(\frac{\omega L}{12}\right)^2\right] = \frac{13}{144}ML^2\omega^2$$

$$KE_{\text{final}} = \frac{1}{2}\cdot\frac{2ML^2}{3}\omega_0^2 = \frac{ML^2}{3}\left(\frac{\omega}{8}\right)^2 = \frac{ML^2\omega^2}{192}$$

$$\Delta KE = \left[\frac{13}{144} - \frac{1}{192}\right]ML^2\omega^2 = \frac{49}{576}ML^2\omega^2$$

$$\therefore \frac{\Delta KE}{KE_{\text{inital}}} = \frac{49}{52}$$

135. $F\Delta t = MV$ [Impulse = change in momentum]

And $R F \Delta t = \frac{1}{2} MR^2 \omega$ [Angular Impulse = change in angular momentum]

$$\Rightarrow MVR = \frac{1}{2} MR^2 \omega \Rightarrow \omega = \frac{2V}{R}$$

$$\Rightarrow \theta = \frac{2x}{R} \quad \Rightarrow \quad R\theta = 2x$$

Where θ = angular displacement of cylinder about its centre and x = displacement of the centre of the cylinder

$R\theta = 2x$ = length of the thread that unwinds

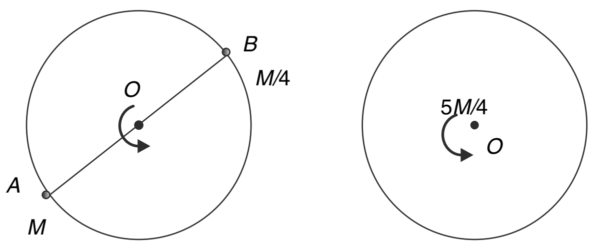
If means point of application of the force moves by $2x + x = 3x$ when the centre of the cylinder moves by x.

$\therefore$ Work done by force = change in kinetic energy

$$F \cdot 3x = K \Rightarrow x = \frac{K}{3F}$$

$\therefore$ Answer is = $2x = \frac{2K}{3F}$

136. (a) The two states have been shown in figure below.



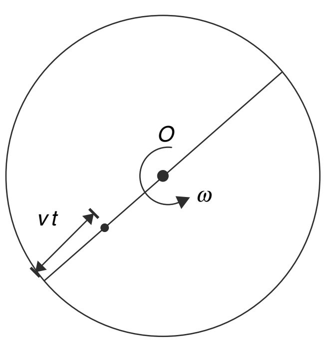
Initial MOI is $I_2 = \frac{1}{2}(20M)R^2 + MR^2 + \frac{M}{4}R^2 = \frac{45MR^2}{4}$

MOI when A reaches the centre after eating B is $I_2 = \frac{1}{2}(20M)R^2 = 10MR^2$

Angular momentum conservation : $I_1\omega_2 = I_1\omega_1$

$$10MR^2\omega_2 = \frac{45MR^2}{4}\omega_0 \Rightarrow \omega_2 = \frac{9}{8}\omega_0 = 1.125\omega_0$$

(b) When A walks to O, the *MOI* decreases and ω increases. After crossing O the *MOI* once again starts increasing and ω decreases.



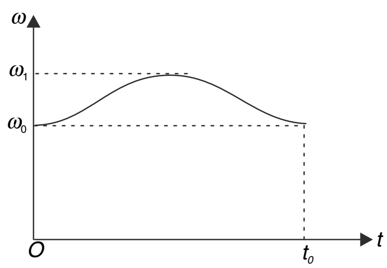
Situation at time t after start is as shown in figure.

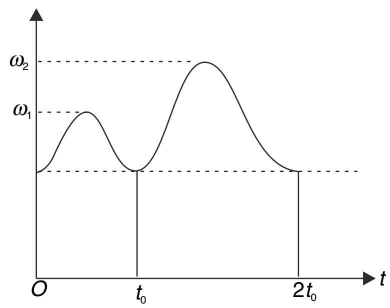
$$I = \frac{1}{2}(20M)R^2 + M(R - vt)^2 + \frac{M}{4}R^2 = \left[41 + 4\left(1 - \frac{v}{R}t\right)^2\right]\frac{MR^2}{4}$$

Angular momentum conservation gives :

$$\left[41 + 4\left(1 - \left(\frac{vt}{R}\right)^2\right)\right]\frac{MR^2}{4}\omega = \frac{45}{4}MR^2\omega_0$$

$$\omega = \frac{45\omega_0}{41 + 4\left(1 - \frac{vt}{R}\right)^2}$$





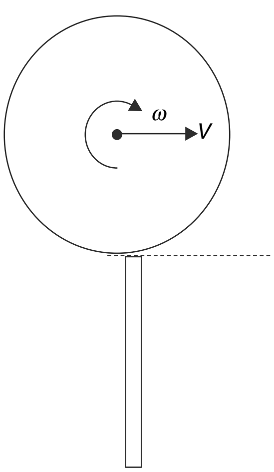
Angular speed when A crosses the centre $\omega_1 = \frac{45}{41}\omega_0 = 1.098\omega_0$

For motion of A from its initial position to B, the variation of ω is as shown in first graph.

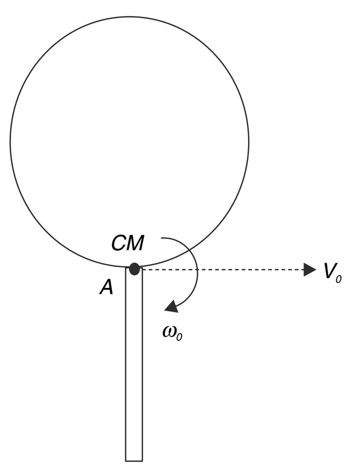
Similarly, one can write the expression of ω when $\left(M + \frac{M}{4} = \frac{5M}{4}\right)$ mass returns back from B to A. When at centre the angular speed is $\omega_2\,(= 1.125\omega_0) > \omega_1$. Hence the graph for complete journey is as shown in the second graph.

137. The COM of the system is at A.

Momentum conservation gives : $2mV_0 = mV \quad \Rightarrow \quad V_0 = \frac{V}{2}$



Before collision



After collision

Angular momentum conservation about A gives -

$$I_A\omega_0 = \frac{1}{2}mR^2\omega + mVR$$

$$\left[\frac{3}{2}mR^2 + \frac{1}{3}m(2R)^2\right]\omega_0 = \frac{mR^2\omega + 2mVR}{2}$$

$$\frac{17}{6}mR^2\omega_0 = \frac{mR^2\omega + 2mVR}{2} \qquad \Rightarrow \omega_0 = \frac{3(R\omega + 2V)}{17.R}$$

(a) If $V = R\omega$ then,

$$\omega_0 = \frac{9}{17}\omega$$

(b) Loss in energy:

$$\Delta E = \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\omega^2\right) - \left[\frac{1}{2}(2m)V_0^2 + \frac{1}{2}I_A\omega_0^2\right]$$

Put $V_0 = \frac{V}{2}$, $I_A = \frac{17}{6}mR^2; \omega_0 = \frac{3(R\omega + 2V)}{17R}$

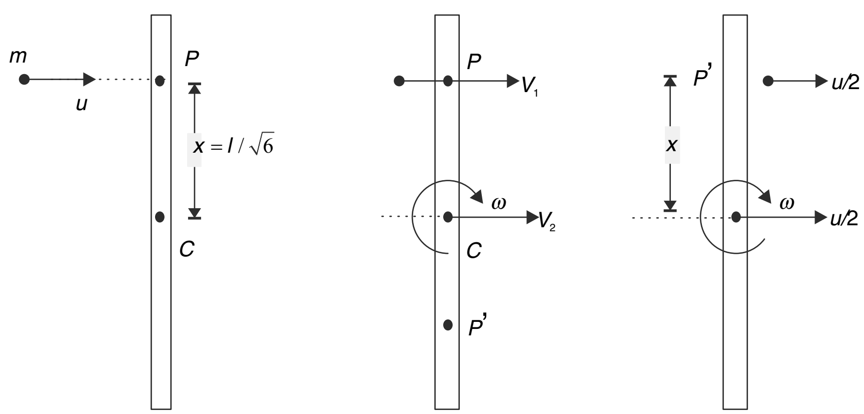
$$\Delta E = \frac{5}{68}mV^2 + \frac{7}{34}mR^2\omega^2 - \frac{3}{17}mVR\omega$$

(c) Loss in kinetic energy is minimum when

$$\frac{d(\Delta E)}{dV} = 0 \ \Rightarrow \frac{5}{34}mV + 0 - \frac{3}{17}mR\omega = 0$$

$$\Rightarrow V = \frac{6}{5}R\omega \therefore \eta = \frac{6}{5}$$

138. Let V_1, V_2 and ω be the velocity of particle, velocity of the centre of the rod and angular speed of the rod immediately after collision.



In the diagrams the first, second and the third one represent the situations just before collision, just after collision and just after second collision.

Momentum conservation

$mu = mV_1 + mV_2 \Rightarrow u = V_1 + V_2$.............(i)

Apply conservation of angular momentum about a fixed point is space that coincides with the initial position of the centre of the rod

$$mux = mV_1x + \frac{m\ell^2}{12}\omega$$

[Note: Angular momentum of the rod about said point is $I_{CM}\vec{\omega} + \vec{R} \times M\vec{V}_2$, where the second term is zero in present context]

$$u\frac{\ell}{\sqrt{6}} = V_1\frac{\ell}{\sqrt{6}} + \frac{\ell^2\omega}{12}$$

$$\Rightarrow u = V_1 + \frac{\omega\ell}{2\sqrt{6}} \text{.......................(ii)}$$

Collision is elastic hence we can use conservation of kinetic energy or the definition of coefficient of restitution (e). Let's use the later.

$$\frac{\text{Relative velocity of point } P \text{ and the particle after collision}}{\text{Relative velocity of point } P \text{ and the particle before collision}} = -1$$

$$\frac{V_2 + \omega x - V_1}{o - u} = -1$$

$$\Rightarrow V_2 - V_1 = u - \frac{\omega\ell}{\sqrt{6}} \text{....................(iii)}$$

Solving (i), (ii) and (iii)

$$V_1 = V_2 = \frac{u}{2} \text{ and } \omega = \frac{\sqrt{6}u}{\ell}$$

Note : For application of conservation of angular momentum one can chose different points as origin. Students may try solving the problem by taking the centre of mass of the entire system as their origin or any other suitable point.

(b) The relative velocity of the centre of the stick is zero relative to the particle after the collision. Hence, the stick will make half a rotation and its point P' will hit the particle. After second collision the stick will come to rest and the particle will move will speed u to the right [This answer is a guess as it satisfies all conservation laws. You may write complete solution for the second collision as we did for the first collision]

139. (a) Angular momentum of the system is

$L = [I_0 + m\,(R \sin \theta)^2]\ \omega_0$ [I_0 = , moment of inertia of the ring]

The angular momentum changes as the insect walks.

$$\frac{dL}{dt} = \frac{d}{dt}\left(mR^2\omega_0 \sin^2\theta\right) = 2mR^2\omega_0 \sin\theta\cos\theta\frac{d\theta}{dt}$$

$\Rightarrow \tau_{ext} = 2mR^2\ \omega_0\ \omega \sin\theta \cos\theta = mR^2\ \omega_0\ \omega \sin 2\theta$ .... (i)

$\Rightarrow \tau_{ext}$ is maximum for $\theta = 45°$

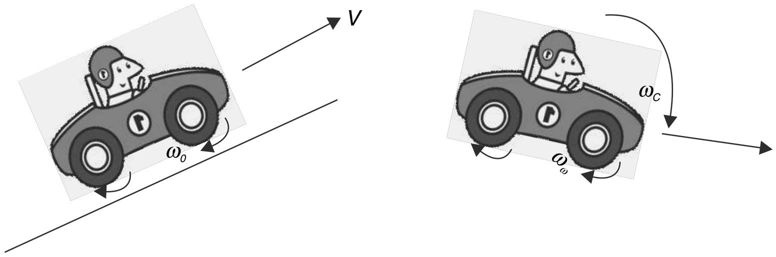
(b) Rate of change of angular momentum of the insect alone about the rotation axis is also given by (i) Angular momentum of the insect changes because of $F_\perp$ - force perpendicular to the plane of the ring applied by the ring on the insect.

$$F_\perp\ R\sin\theta = \frac{dL}{dt}$$

$$F_\perp\ R\sin\theta = 2mR^2\omega_0\omega\sin\theta\cos\theta \qquad \Rightarrow \qquad F_\perp = 2mR\omega_0\omega\cos\theta$$

140. At the time of take – off the angular speed of the wheels is

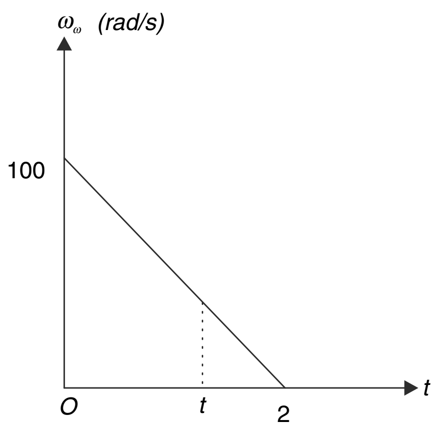
$$\omega_0 = \frac{V}{r} = \frac{30\,m/s}{0.3\,m} = 100\,rad\,s^{-1}$$



At time t (≤ 2), after the take – off the angular speed of the wheels is $\omega_C = (100 - 50t)$ *rad/s*

To keep angular momentum conserved, the car will rotate about its COM. Let the angular speed of car about its COM be ω_C at time 't'

$I_C\,\omega_C + I_w\,\omega_w = I_w \cdot \omega_0$



$$\therefore 80\,\frac{d\theta}{dt} = 0.3 \times 2\left[100 - (100 - 50t)\right]$$

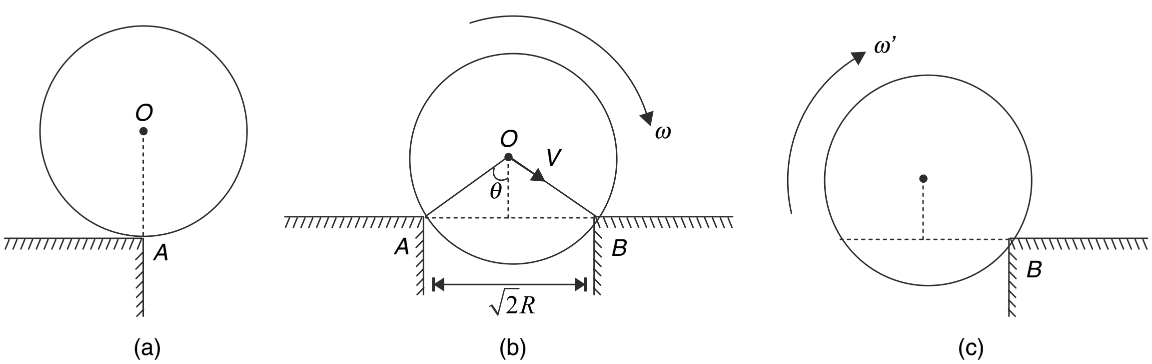
$$\therefore 80\int_0^{\theta} d\theta = 0.6\int_0^{2} 50\,t\,dt$$

$$\Rightarrow \theta = \frac{3}{8} \times 2 = \frac{3}{4}\,\text{rad} = 43°$$

141. Let v = speed of COM before hitting at B

ω = angular speed before hitting at B

$v = \omega R$ [$\therefore$ there is no slipping about A]



Energy conservation

$$\frac{1}{2}mv^2 + \frac{1}{2}I_{CM}\omega^2 = mgR\left(1-\cos 45°\right)$$

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2 = mgR\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\frac{1}{2}v^2 + \frac{1}{4}v^2 = gR\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$$

$$v^2 = \frac{4\left(\sqrt{2}-1\right)}{3\sqrt{2}}gR = \frac{2\sqrt{2}\left(\sqrt{2}-1\right)}{3}gR$$

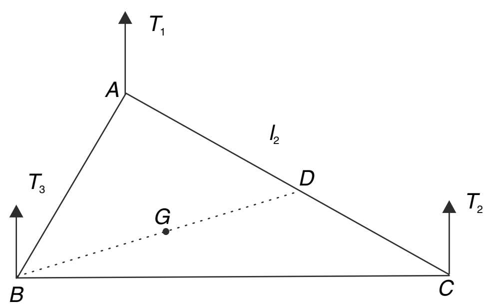
Angular momentum conservation about *B* gives:

$$I_B\omega' = I_{CM}\cdot\omega \quad \Rightarrow \frac{3}{2}mR^2\omega' = \frac{1}{2}mR^2\cdot\omega$$

$$\omega' = \frac{\omega}{3} = \frac{v}{3R}$$

[Note that before impact at *B*, the velocity vector of the *COM* passes through point *B*. Hence, the second term in expression of angular momentum = 0]

142. Considering rotational equilibrium about the median *BD* gives -



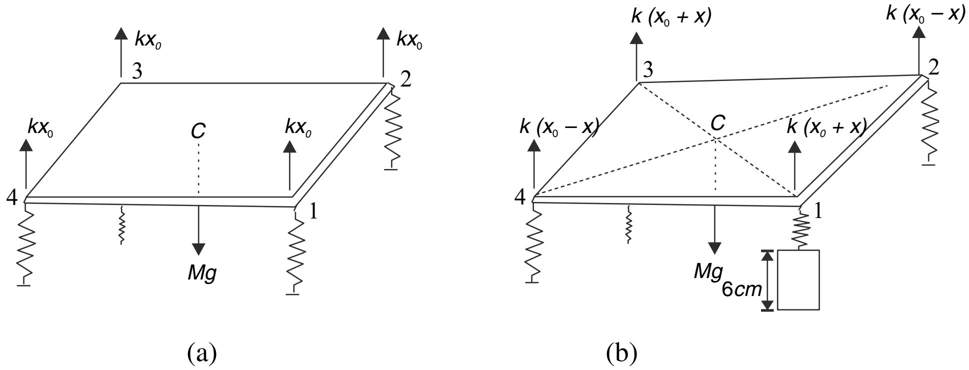
$T_1 = T_2$

Similarly, $T_1 = T_3$

Therefore $T_1 = T_2 = T_3 = \dfrac{Mg}{3}$

143. Originally, each spring is compressed by x_0

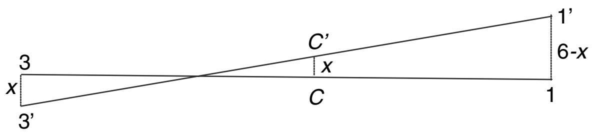
$4kx_0 = Mg$ ....(i)



After insertion of wooden block below spring at 1, let further compression in spring at 1 be x. Spring force at 1 increases by kx. The spring force at 3 must also increases by same amount so that net torque about diagonal 2 – 4 remains zero. Hence, spring at 3 compresses further by x. The resultant of all spring forces must remain Mg. Hence, spring at 2 and 4 must extend by x [i.e. their compression will decreases by x].

Point 2 and 4 both rise by x (from their original position in figure (a)). C must lie on line 2-4. Hence C must also rise by x.

C also lies on the line 1 – 3



Height difference between C' and $3'$ is $2x$

$\therefore$ Height difference between $3'$ and $1'$ will be $4x$

From figure height difference between 3' and 1' is 6.

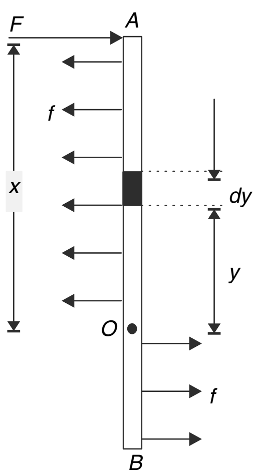
$$\therefore 4x = 6$$

$$\Rightarrow x = 1.5\ cm$$

144. Let $AO = x$

As the rod develops a tendency to rotate about point O, the direction of friction force on segment AO is towards left and that on segment OB is towards right. Friction on segment of length dy is

$$df = \mu\lambda\, dy\, g \qquad \left[\lambda = \frac{M}{L}\right]$$



Torque about O

$d\tau_f = y df = \mu\lambda g\, y\, dy$

$\therefore$ Net torque of friction about O is

$$\tau_f = \mu\lambda g\int_0^x y\,dy + \mu\lambda g\int_0^{L-x} y\,dy = \frac{\mu\lambda g}{2}\left[x^2 + (L-x)^2\right]$$

For the rod to be able to start rotating $F\cdot x = \tau_f$

$$\Rightarrow F = \frac{\mu\lambda g}{2}\left[x + \frac{(L-x)^2}{x}\right] \quad ....(i)$$

Now F is minimum if $\dfrac{dF}{dx} = 0 \Rightarrow x = \dfrac{L}{\sqrt{2}}$

Put $x = \dfrac{L}{\sqrt{2}}$ in (i) to get F_0

$F_0 = \mu\lambda gL\,[\sqrt{2} - 1] = \mu Mg\,(\sqrt{2} - 1)$

[Note : Net friction force on the rod is

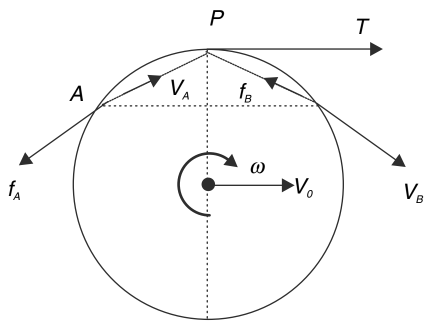
$$f = \mu\lambda g\left[\int_0^{L/\sqrt{2}} dy - \int_0^{L\left(1-\frac{1}{\sqrt{2}}\right)} dy\right]$$

$$= \mu\lambda g\frac{L}{\sqrt{2}} - \mu\lambda gL\left(1 - \frac{1}{\sqrt{2}}\right) = \left(\sqrt{2} - 1\right)\mu Mg$$

This is equal to F_0. Because of this, as $F \geq F_0$ we see that COM of the rod beings to move.

145. (a) For constant speed net force as well as net torque shall be zero.

Refer to problem 56 in this chapter. We have already seen that if velocity of centre $(v_0) = \omega R$ [ω = angular speed], then all points in the ring will have their velocities along a line passing through P.



Friction on an element will be opposite to the direction of instantaneous velocity of the element. Hence, if $v_0 = \omega R$ for our ring, then friction force on elements at A and B will pass through P. The string tension also passes through P. Therefore, all force will have zero torque about P.

The resultant of two friction force is towards left which is balanced by string tension.

Since speed of point $P = v$

$$\Rightarrow v_0 + \omega R = v \Rightarrow 2v_0 = v \Rightarrow v_0 = \frac{v}{2}$$

(b) As seen in the first part of the problem, friction acting on any segment will produce no torque if $v_0 = \omega R$

$$\therefore v_0 = \frac{v}{2}$$

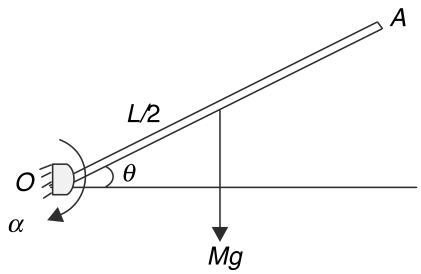
146. (a) Consider only the stick at an angle θ

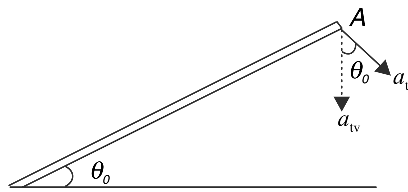
Torque about O, $\tau = Mg\dfrac{L}{2}\cos\theta$

Angular acceleration $\alpha = \dfrac{\tau}{I} = \dfrac{Mg\dfrac{L}{2}\cos\theta}{\dfrac{1}{3}ML^2} = \dfrac{3}{2}\dfrac{g}{L}\cos\theta$

The tangential acceleration of end A is : $a_t = L\alpha = \dfrac{3}{2}g\cos\theta$

Vertical component of tangential acceleration : $a_{tv} = \frac{3}{2} g \cos^2 \theta$





Immediately after the stick is released, if this acceleration happens to be larger than g (= acceleration of the block), the block will lag behind the stick.

$\therefore \; \frac{3}{2} g \cos^2 \theta_0 \geq g \Rightarrow \quad \cos^2 \theta_0 \geq \frac{2}{3} \Rightarrow \; \cos \theta_0 \geq \sqrt{\frac{2}{3}}$

(b) Before the system is released, the block is at rest

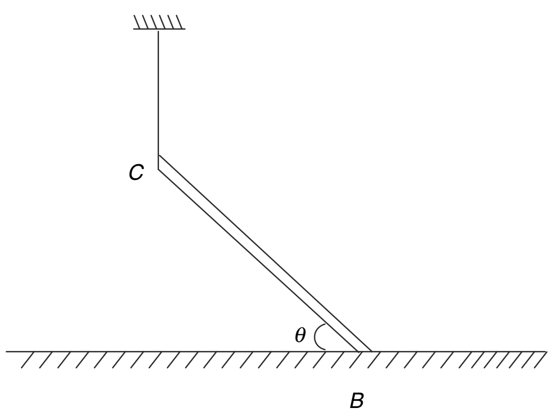
$\therefore$ Contact force = mg

(c) After release, Contact force = 0

Because acceleration of end A has a vertical component = g

147. When the string gets vertical, the centre of the rod has horizontal velocity and the angular speed of the rod will be zero. At this point angle θ has become minimum (i.e. $\frac{d\theta}{dt} = 0$). Velocity of C is horizontal. Velocity of B is also horizontal.

$\therefore$ Rod has a horizontal velocity and no angular velocity.

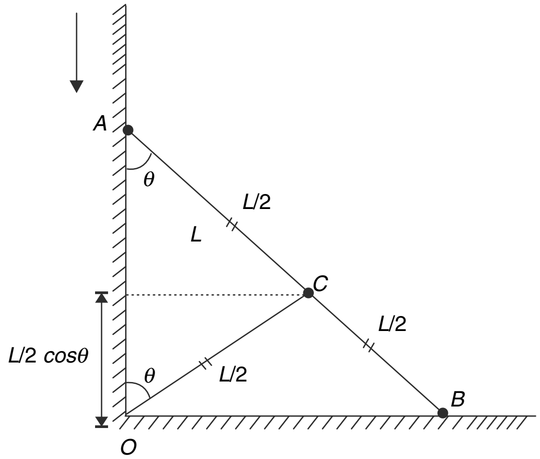


Energy conservation gives :

$$\frac{1}{2} Mv^2 = Mg\frac{b}{4} - Mg\left(\frac{b-a}{2}\right) \qquad \therefore v = \sqrt{g\left(a - \frac{b}{2}\right)}$$

148. (a) The median to the hypotenuse of a right angle triangle has half the length of the Hypotenuse.

$\therefore OC = \frac{L}{2}$ (always)



Which means the centre of mass C goes in a circle of radius $\frac{L}{2}$ with centre at O.

(b) By the time the dumb-bell rotates through angle θ its centre of mass falls through a distance

$$\frac{L}{2}-\frac{L}{2}\cos\theta=\frac{L}{2}(1-\cos\theta)$$

Applying conservation of energy

$$\frac{1}{2}(2m)v^2+\frac{1}{2}I\omega^2=2mg\frac{L}{2}(1-\cos\theta)$$

$$mv^2+\frac{1}{2}\left[m\left(\frac{L}{2}\right)^2\times 2\right]\left(\frac{v}{L/2}\right)^2=mgL(1-\cos\theta)$$

$$v^2+v^2=gL(1-\cos\theta)$$

$$v=\sqrt{\frac{gL(1-\cos\theta)}{2}}=\text{Speed of COM}$$

Note : The COM moving on a circle has rotated by an angle θ, because OC makes on angle θ with the wall. In the same interval the whole dumb-bell has rotated by angle θ about the COM [because the rod now makes an angle θ with the vertical]. Hence angular speed (ω) of rotation of the COM about O is same as the angular speed of rotation of the dumb-bell about its COM $\left(\therefore \frac{L}{2}\omega=v\right)$

The rod will lose contact with the wall at the instant horizontal component of velocity of COM becomes maximum [As long as there is normal force from the wall, the horizontal velocity component keeps on increasing]

$$v_x=v\cos\theta=\sqrt{\frac{gL}{2}(1-\cos\theta)}\cdot\cos\theta$$

v_x is maximum when $\frac{dv_x}{d\theta}=0\Rightarrow\cos\theta=\frac{2}{3}$

Speed of COM at this value of θ is

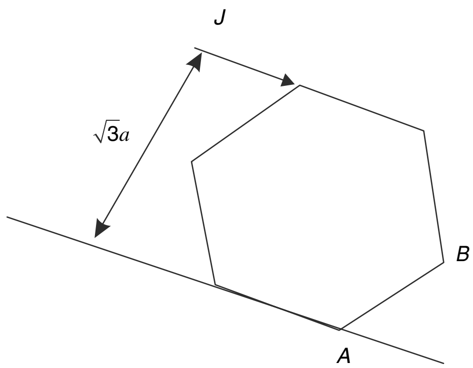
$$v=\sqrt{\frac{gL(1-2/3)}{2}}=\sqrt{\frac{gL}{6}}$$

149. (a) For $\theta > 30°$ the vertical line through *COM* will pass from right of A. Hence, the pencil will topple

(b) Angular Impulse about edge through A $= J\left(\sqrt{3}a\right)$

$\therefore I_A\,\omega_0 = \sqrt{3}\,Ja$

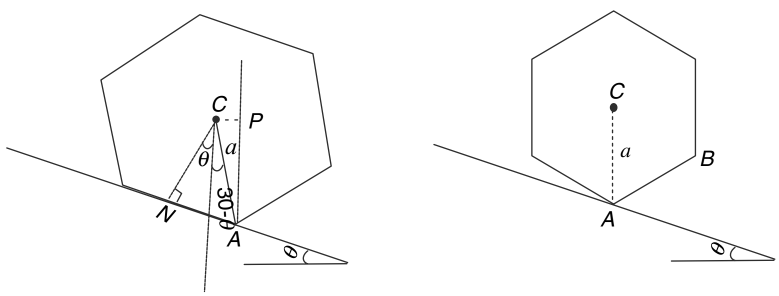
$\Rightarrow \omega_0 = \dfrac{\sqrt{3}\,Ja}{I}$



(c) In shown figure *AP* is vertical height of COM above A. *CN* makes an angle θ with vertical

$\therefore \angle CAP = 30° - \theta$

$\therefore AP = a\cos(30° - \theta)$



The maximum height attained by the *COM* above *A* is *a*.

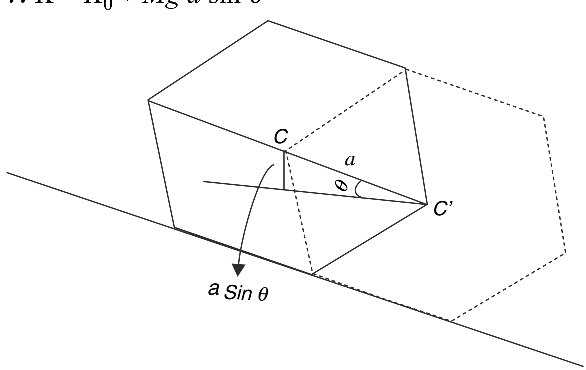
Hence the *KE* of the pencil at the beginning must be enough so that the *COM* can rise to height *a* above *A*.

$\therefore \dfrac{1}{2}I\omega_0^2 = Mga - Mga\,\cos(30° - \theta)$

$\therefore \omega_0 = \sqrt{\dfrac{2Mga}{I}\left[1 - \cos\left(30° - \theta\right)\right]}$

(d) The centre of mass of the pencil falls through a height $a\sin\theta$ during each $\dfrac{1}{6}$ turn

$\therefore K = K_0 + Mg\,a\sin\theta$



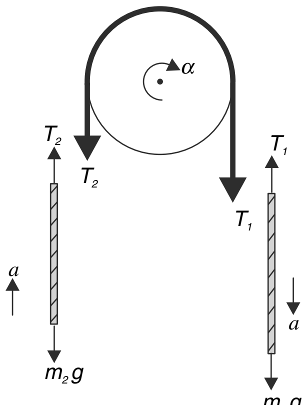
150. Mass of hanging part of the rope on right side: $m_1 = \lambda \ell$

Mass of the rope in contact with the pulley: $m_0 = \lambda \pi R$

Mass of part on left: $m_2 = \lambda [L - l - \pi R]$

$\therefore m_1 g - T_1 = m_1 a$……… (i)

$T_2 - m_2 g = m_2 a$………… (ii)



Consider pulley + part of rope on pulley

$$T_1 R - T_2 R = \left[\frac{1}{2}MR^2 + m_0 R^2\right]\alpha$$

$$T_1 - T_2 = \left(\frac{M}{2} + m_0\right)a \text{................(iii)} \qquad [\because a = R\alpha]$$

(i) + (ii) + (iii)

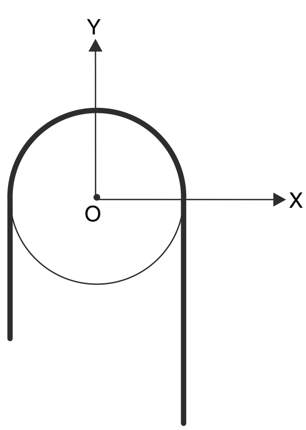
$$(m_1 - m_2)g = \left(m_1 + m_2 + \frac{M}{2} + m_0\right)a$$

$$\therefore a = \frac{(m_1 - m_2)g}{\left(m_1 + m_2 + m_0 + \frac{M}{2}\right)} = \frac{\lambda g[2\ell + \pi R - L]}{\left(\lambda L + \frac{M}{2}\right)}$$

The x co-ordinate of COM of the pulley + rope system is

$$X = \frac{\lambda \ell R - \lambda(L - \ell - \pi R)R}{M + \lambda L}$$

$$\therefore X = \frac{\lambda R[2\ell + \pi R - L]}{M + \lambda L}$$



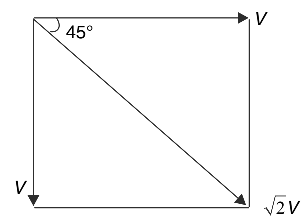
$\therefore$ X component of acceleration of COM is

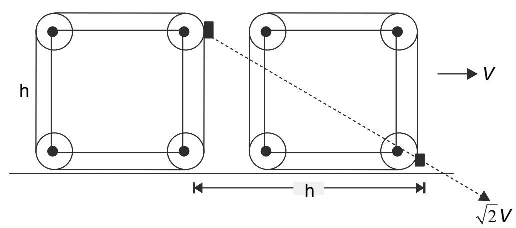
$$a_x = \frac{d^2X}{dt^2} = \frac{2\lambda R}{M+\lambda L}\frac{d^2\ell}{dt^2} = \frac{2\lambda R}{M+\lambda L}\cdot a$$

$$\therefore F_x = (\lambda L + M)a_x = 2\lambda R a$$

$$\therefore F_x = \frac{2\lambda^2 Rg(2\ell + \pi R - L)}{\left(\lambda L + \frac{M}{2}\right)}$$

151. Since there is no slipping the toy travels a horizontal distance h in the time the block travels a vertical distance h.
v = velocity of block with respect to the toy ($\downarrow$)
v = velocity of toy with respect to the ground ($\rightarrow$)
Velocity of block with respect to ground = $\sqrt{2}\,v$





Energy Conservation

$$\frac{1}{2}m\left(\sqrt{2}v\right)^2 + \frac{1}{2}Mv^2 = mgh$$

$$\therefore v = \sqrt{\frac{2mgh}{2m+M}}$$

$\therefore$ Answer is $\sqrt{2}v = 2\sqrt{\dfrac{mgh}{2m+M}}$

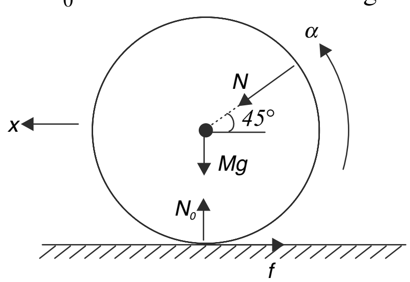
152. ω = angular speed

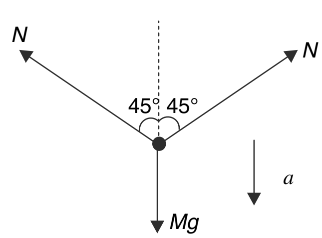
$\dfrac{\omega R}{2} = v_0$ for pure rolling condition

$$KE = \frac{1}{2}Mv_0^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv_0^2 + \frac{1}{2}\frac{2}{5}MR^2\omega^2 = \frac{1}{2}MV_0^2 + \frac{1}{5}M(2v_0)^2 = \frac{13}{10}Mv_0^2$$

153. Let a_0 = acceleration of each ring and α = angular acceleration of each ring





For one ring

$\frac{N}{\sqrt{2}} - f = Ma_0$ .... (i)

and $fR = MR^2 \cdot \alpha \Rightarrow f = Ma_0$ .... (ii)

(i) + (ii) $\frac{N}{\sqrt{2}} = 2Ma_0$ .... (iii)

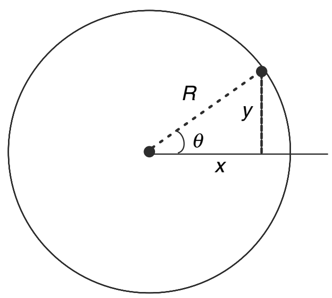
For the small object

$mg - 2 \cdot \frac{N}{\sqrt{2}} = ma \Rightarrow mg - \sqrt{2}\,N = ma$

Using (iii) $mg - 4\,Ma_0 = ma$ .... (iv)

Constraint:

$y^2 + x^2 = R^2 \quad \Rightarrow 2y\frac{dy}{dt} + 2x\frac{dx}{dt} = 0$



Differentiating once again : $\left(\frac{dy}{dt}\right)^2 + y\frac{d^2y}{dt^2} + \left(\frac{dx}{dt}\right)^2 + x\frac{d^2x}{dt^2} = 0$

Immediately after release $\frac{dy}{dt} = \frac{dx}{dt} = 0$

$\therefore y\frac{d^2y}{dt^2} = -x\frac{d^2x}{dt^2}$ [- sign since y is decreasing]

$\therefore ya = xa_0$

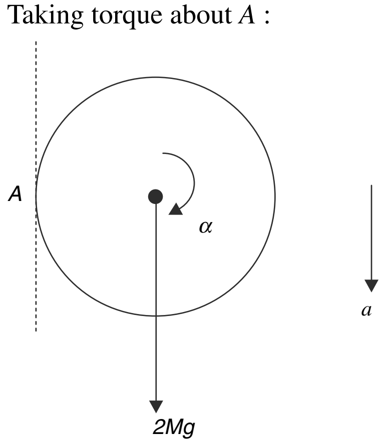
When = $\theta = 45°$, $x = y$

$\therefore a = a_0$ ....(v)

Solving (iv) and (v) $a = a_0 = \frac{mg}{4M + m}$

154. At any given instant the ring and the rope around it is moving. Rest of the rope is not moving at any given instant. The motion is identical to that of a wheel rolling without sliding

Taking torque about A :



$I_A\alpha = \tau_A$

$2\,(2M)R^2\cdot\,\alpha = 2Mg\cdot R\;\therefore \alpha = \dfrac{g}{2R}$

$\therefore$ acceleration of the centre will be $a = R\alpha = \dfrac{g}{2}$.

155. (a) The centre of mass of the stick will move in a straight line with acceleration g (↓)

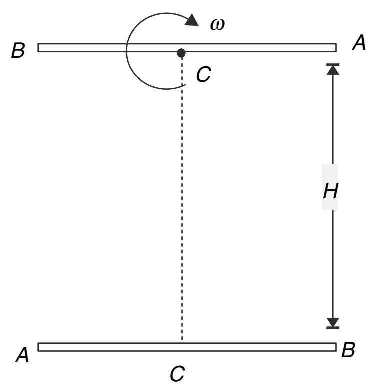
Time of flight $T = \dfrac{2u}{g} = \dfrac{2\pi\sqrt{gL}}{g} = 2\pi\sqrt{\dfrac{L}{g}}$

Angular speed $\omega = \dfrac{2\pi}{T} = \sqrt{\dfrac{g}{L}}$

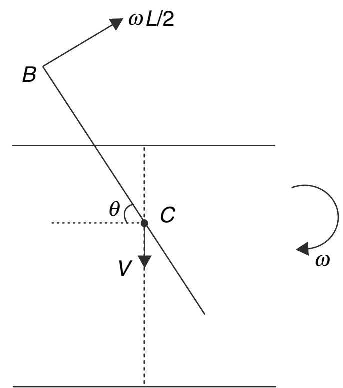
(b) Maximum height attained by the centre is

$$H = \frac{u^2}{2g} = \frac{\pi^2 gL}{2g} = \frac{\pi^2}{2}L$$

As per the question, the stick rotates by 180° by the time it is at the top. After this the centre begins to descend but the end B continues to rise, as it has an upward velocity. The end B stops rising at the instant absolute value of its vertical velocity becomes zero. Let this happen at time 't' after the centre reaches the top.



Stick at top



At time t after the centre begins to fall

$$\omega = \omega t = \sqrt{\frac{g}{L}}t$$

Velocity of end B relative to the centre is $\dfrac{\omega L}{2}$ in a direction perpendicular to the stick. Absolute velocity in vertical direction will be zero if

$$\frac{\omega L}{2}\cos\theta = v \Rightarrow \sqrt{\frac{g}{L}}\cdot\frac{L}{2}\cos\left(\sqrt{\frac{g}{L}}t\right) = gt$$

$$\Rightarrow \cos\left(\sqrt{\frac{g}{L}}t\right) = 2\sqrt{\frac{g}{L}}t$$

Given $\cos x = 2x$ when $x = 0.45$

$$\therefore \sqrt{\frac{g}{L}}t = 0.45 \Rightarrow t = 0.45\sqrt{\frac{L}{g}}$$

Height of the end B at this instant is

$$h_B = H - \frac{1}{2}gt^2 + \frac{L}{2}\sin\theta$$

$$= \frac{\pi^2 L}{2} - \frac{1}{2}g\left(0.45\sqrt{\frac{L}{g}}\right)^2 + \frac{L}{2}\sin(0.45)$$

$$= \left[\frac{3.14^2}{2} - \frac{0.45^2}{2} + \frac{0.43}{2}\right]L = 5.04L$$

156. First collision :

Let velocity of fired ball after collisions be v_1. Since collision is elastic

$r\omega - v_1 = v$ ....(i)

Conservation of angular momentum about O gives -

$$mvr = mv_1 r + 2mr^2\omega_1 + \frac{1}{12}4m(2r)^2\omega_1$$

$$\Rightarrow v - v_1 = \frac{10}{3}r\omega_1 \text{.............(ii)}$$

Solving (i) and (ii) we get $\omega_1 = \dfrac{6v}{13r}$

Second collision:

$v - \omega_1 r = \omega_2 r - v_2$ ....(iii) [elastic collision]

And $mvr + \dfrac{10}{3}mr^2\omega_1 = mv_2 r + \dfrac{10}{3}mr^2\omega_2$ .... (iv)

Solving (iii) and (iv) gives $\omega_2 = \dfrac{6v}{13r} + \dfrac{7}{13}\omega_1$

It means $\omega_{n+1} = \dfrac{6v}{13r} + \dfrac{7}{13}\omega_n$

Say $\omega_{n+1} = a + b\omega_n$

$\therefore\ \omega_1 = a + 0 = a$

$\therefore\ \omega_2 = a + b\omega_1 = a + ab$

$\therefore\ \omega_3 = a + b\omega_2 = a + ab + ab^2$

$$\omega_{n+1} = a + ab + ab^2 + \text{........} + ab^n = a\left[\frac{1-b^n}{1-b}\right]$$

$$= \frac{6v}{13r}\left[\frac{1-\left(\frac{7}{13}\right)^n}{1-\frac{7}{13}}\right] = \frac{v}{r}\left[1-\left(\frac{7}{13}\right)^n\right] \qquad n = 0,1,2,3......$$

$$\text{Or, } \omega_n = \frac{v}{r}\left[1-\left(\frac{7}{13}\right)^{n-1}\right] \qquad n = 1,2,3,......$$

When $n \to \infty$, $\left(\dfrac{7}{13}\right)^{n-1} \to 0$

$$\therefore\ \omega_n \to \frac{v}{r}$$

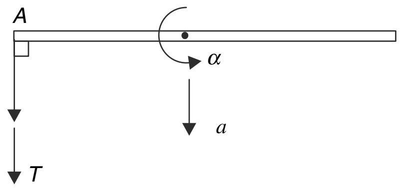
157. (a) Let tension in string immediately after the projection of the particle be T

a = acceleration of *COM* of the rod

α = angular acceleration of the rod

Acceleration of end A is

$a_A = a + L\alpha$

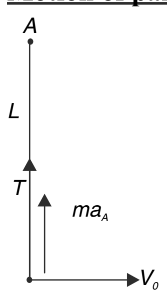


For the rod

$T = ma$ .... (i)

$$TL = \frac{1}{12}m(2L)^2 \cdot \alpha \qquad \Rightarrow T = \frac{1}{3}m \cdot L\alpha \text{..........(ii)}$$

Motion of particle in the reference frame attached to point A



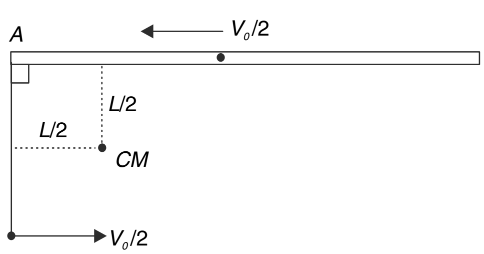
$$T + ma_A = \frac{mv_0^2}{L}$$

$$T + ma + mL\alpha = \frac{mv_0^2}{L}$$

$$T + T + 3T = \frac{mv_0^2}{L} \qquad \therefore T = \frac{mv_0^2}{5L}$$

$$\therefore a = \frac{T}{m} = \frac{v_0^2}{5L}$$

(b) $v_{cm} = \dfrac{v_0}{2}$

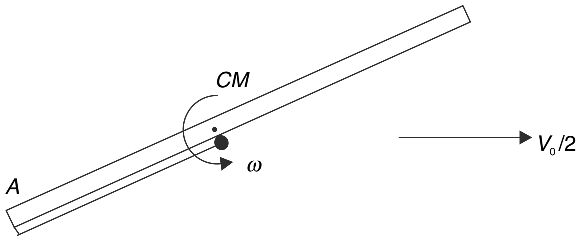


In reference frame attached to the *COM* of the system the particle is initially moving to right with velocity $\dfrac{v_0}{2}$

and the centre of the rod is moving to left with velocity $\frac{v_0}{2}$

Let the angular speed after the particle sticks be ω. Note that the centre of mass continues to move with velocity $\frac{v_0}{2}$

Angular momentum conservation about *COM*



$L_i = L_f$

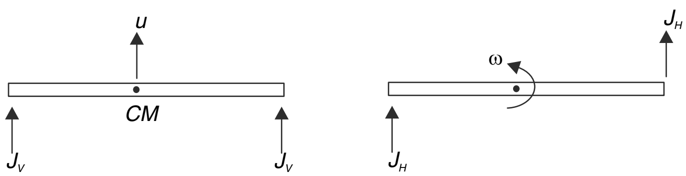
$$m\frac{v_0}{2}\frac{L}{2} + m\frac{v_0}{2}\frac{L}{2} = \frac{1}{12}m(2L)^2 \cdot \omega$$

$$\therefore mv_0 L = \frac{2}{3}mL^2\omega \qquad \therefore \omega = \frac{3}{2}\frac{v_0}{L}$$

158. The vertical component of impulse on the rod will give it an upward velocity (say u) and there must be a horizontal component of impulse which provides an angular speed.

In the first figure J_H is perpendicular to the plane of the figure and in the second figure J_v is perpendicular to the plane of the figure.

The time of flight $t = \frac{2u}{g}$



Angle rotated by the rod in this time = π

$$\therefore \omega t = \pi \qquad \Rightarrow \quad \omega = \frac{\pi g}{2u}$$

Hence, $2J_v = Mu \Rightarrow Jv = \frac{Mu}{2}$

And $2J_H L = \frac{ML^2}{3}\cdot\frac{\pi g}{2u} \qquad \left[I_{CM} = \frac{M(2L)^2}{12} = \frac{ML^2}{3}\right]$

$$\therefore J_H = \frac{ML\pi g}{12u}$$

Impulse applied by one boy has magnitude J given by

$$J^2 = J_H^2 + J_v^2$$

$$J^2 = \frac{M^2L^2\pi^2g^2}{144u^2} + \frac{M^2u^2}{4} \text{.............(i)}$$

J^2 is minimum when $\dfrac{d(J^2)}{du} = 0$

$$\Rightarrow -2\,\frac{M^2L^2\pi^2g^2}{144u^3} + 2\frac{M^2}{4}u = 0$$

$$\Rightarrow u^4 = \frac{L^2\pi^2g^2}{36} \qquad \therefore\ u^2 = \frac{L\pi g}{6}$$

Substituting in (i)

$$J^2_{\min} = \frac{M^2L^2\pi^2g^2}{144\cdot\dfrac{L\pi g}{6}} + \frac{M^2L\pi g}{4\cdot 6} = \frac{M^2\pi Lg}{12}$$

$$J_{\min} = \frac{M}{2}\sqrt{\frac{\pi Lg}{3}}$$

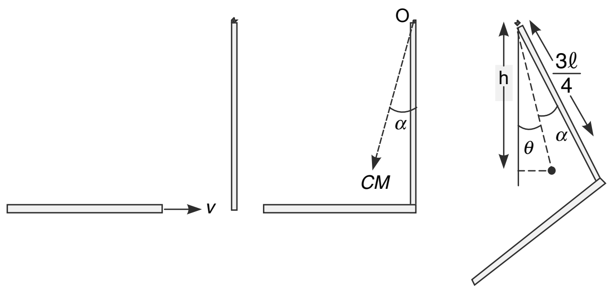
159. Conservation of angular momentum about the point of suspension gives

$$mv\ell = I_0\,\omega \text{..............(1)}$$

$$I_0 = \frac{m\ell^2}{3} + \frac{m\ell^2}{12} + m\left(\frac{\ell^2}{4} + \ell^2\right) = \frac{5}{3}m\ell^2$$

$$\therefore mv\ell = \frac{5m\ell^2}{3}\omega \Rightarrow \omega = \frac{3v}{5\ell}\text{............(2)}$$

ω is the angular velocity of the combined system just after collision.



If O is the reference level, the total energy just after collision and at the maximum displaced position are equal.

$$\frac{1}{2}I_0\omega^2 - mg\ell - mg\frac{\ell}{2} = -2mgh$$

$$\Rightarrow h = \frac{3}{2}\left[\frac{\ell}{2} - \frac{v^2}{10g}\right]\text{.................(3)}$$

From the diagram $\cos\alpha = \dfrac{3\ell/4}{\sqrt{\dfrac{9\ell^2}{16} + \dfrac{\ell^2}{16}}} = \dfrac{3}{\sqrt{10}}$

$$\cos\theta = \frac{h}{x} = \frac{h}{\left(\dfrac{\ell\sqrt{10}}{4}\right)} \Rightarrow h = \frac{\ell\sqrt{10}}{4}\cos\theta\text{...............(4)}$$

$$\therefore \frac{\ell\sqrt{10}}{4}\cos\theta = \frac{3}{4}\left(\ell - \frac{v^2}{5g}\right)$$

$$\cos\theta = \frac{3}{\sqrt{10}}\left(1 - \frac{v^2}{5g\ell}\right),\ \text{max angle} = \theta + \alpha = \cos^{-1}\left\{\frac{3}{\sqrt{10}}\left(1 - \frac{v^2}{5g\ell}\right)\right\} + \cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$$

07 GRAVITATION

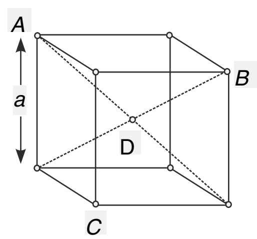
Level 1

Q.1. Two lead balls of mass m and 2 m are placed at a separation d. A third ball of mass m is placed at an unknown location on the line joining the first two balls such that the net gravitational force experienced by the first ball is $\frac{6Gm^2}{d^2}$. What is the location of the third ball?

Q.2. Four identical point masses m each are kept at the vertices A, B, and C of a cube having side length 'a' (see figure). Another identical mass is placed at the center point D of the cube.

(a) Where will you place a fifth identical mass so that the net gravitational force acting on mass at D becomes zero?

(b) Calculate the net gravitational force acting on the mass at D.



Q.3. Two point masses m and M are held at rest at a large distance from each other. When released, they begin moving under their mutual gravitational pull.

(a) Find their relative acceleration (a) when separation between them becomes x.

(b) Integrate the expression of a obtained above to calculate the relative velocity of the two masses when their separation is x.

(c) Write the velocity of centre of mass of the system when separation between them is x.

Q.4. Find the height above the surface of the earth where the acceleration due to gravity reduces by (a) 36% (b) 0.36% of its value on the surface of the earth. Radius of the earth $R = 6400\ km$.

Q.5. An astronaut landed on a planet and found that his weight at the pole of the planet was one third of his weight at the pole of the earth. He also found himself to be weightless at the equator of the planet. The planet is a homogeneous sphere of radius half that of the earth. Find the duration of a day on the planet. Given density of the earth = d_0.

Q.6. A gravity meter can detect change in acceleration due to gravity (g) of the order of 10^{-9} %. Calculate the smallest change in altitude near the surface of the earth that results in a detectable change in g. Radius of the earth $R = 6.4 \times 10^6\ m$.

Q.7. The earth is a homogeneous sphere of mass M and radius R. There is another spherical planet of mass M and radius R whose density changes with distance r from the centre as $\rho = \rho_0 r$.

(a) Find the ratio of acceleration due to gravity on the surface of the earth and that on the surface of the planet.

(b) Find ρ_0.

Q.8. A planet having mass equal to that of the earth ($M = 6 \times 10^{24}\ kg$) has radius R such that a particle projected from its surface at the speed of light ($c = 3 \times 10^8\ ms^{-1}$) just fails to escape.

Assuming Newton's Law of gravitation to be valid calculate the radius and mass density of such a planet. Are the numbers realistic?

Note: The radius that you calculated is known as Schwarzschild radius. Actually we need to use theory of general relativity for solving this problem.

Q.9. Angular speed of rotation of the earth is ω_0. A train is running along the equator at a speed v from west to east. A very sensitive balance inside the train shows the weight of an object as W_1. During the return journey when the train is running at same speed from east to west the balance shows the weight of the object to be W_2. Weight of the object when the train is at rest was shown to be W_0 by the balance. Calculate $W_2 - W_1$.

Q.10. If a planet rotates too fast, rocks from its surface will start flying off its surface. If density of a homogeneous planet is ρ and material is not flying off its surface then show that its time period of rotation must be greater then $\sqrt{\dfrac{3\pi}{G\rho}}$.

Q.11. (a) The angular speed of rotation of the earth is $\omega = 7.27 \times 10^{-5}$ $rad\ s^{-1}$ and its radius is $R = 6.37 \times 10^6$ m. Calculate the acceleration of a man standing at a place at 40° latitude. [cos 40° = 0.77]

(b) If the earth suddenly stops rotating, the acceleration due to gravity on its surface will become $g_0 = 9.82$ ms^{-2}. Find the effective value of acceleration due to gravity (g) at 40° latitude taking into account the rotation of the earth.

Q.12. A planet has radius $\left(\dfrac{1}{36}\right)$ th of the radius of the earth. The escape velocity on the surface of the planet was found to be $\dfrac{1}{\sqrt{6}}$ times the escape velocity from the surface of the earth. The planet is surrounded by a thin layer of atmosphere having thickness h (<< radius of the planet). The average density of the atmosphere on the planet is d and acceleration due to gravity on the surface of the earth is g_e. Find the value of atmospheric pressure on the surface of the planet.

Q.13. Using a telescope for several nights, you found a celestial body at a distance of 2×10^{11} m from the sun travelling at a speed of 60 $km\ s^{-1}$. Knowing that mass of the sun is 2×10^{30} kg, calculate after how many years you expect to see the body again at the same location.

Q.14. A man can jump up to a height of $h_0 = 1$ m on the surface of the earth. What should be the radius of a spherical planet so that the man makes a jump on its surface and escapes out of its gravity? Assume that the man jumps with same speed as on earth and the density of planet is same as that of earth. Take escape speed on the surface of the earth to be 11.2 km/s and radius of earth to be 6400 km.

Q.15. It is known that if the length of the day were T_0 hour, a man standing on the equator of the earth would have felt weightlessness. Assume that a person is located inside a deep hole at the equator at a distance of $\dfrac{R}{2}$ from the centre of the earth. What should be the time period of rotation of the earth for such a person to feel weightlessness? [R = radius of the earth]

Q.16. A small satellite of mass m is going around a planet in a circular orbit of radius r. Write the kinetic energy of the satellite if its angular momentum about the centre of the planet is J.

Q.17. Suppose that the gravitational attraction between a star of mass M and a planet of mass m is given by the expression $F = K\dfrac{Mm}{r^n}$ where K and n are constants. If the orbital speed of the planets were found to be independent of their distance (r) from the star, calculate the time period (T_0) of a planet going around the star in a circular orbit of radius r_0.

Q.18. A near surface earth's satellite is rotating in equatorial plane from west to east. The satellite is exactly above a town at 6:00 A.M today. Exactly how many times will it cross over the town by 6:00 A.M tomorrow. [Don't count its appearance today at 6:00 A.M above the town].

Q.19. Imagine an astronaut inside a satellite going around the earth in a circular orbit at a speed of $\sqrt{\dfrac{gR}{2}}$ where R is radius of the earth and g is acceleration due to gravity on the surface of the earth.

(a) What is weight experienced by the astronaut inside the satellite?

(b) Assume that an alien demon stops the satellite and holds it at rest. What is weight experienced by the astronaut now?

(c) The demon now releases the satellite (from rest). What is weight experienced by the astronaut now?

Q.20. The height of geostationary orbit above the surface of the earth is h. Radius of the earth is R. The earth shrinks to half its present radius (mass remaining unchanged). Now what will be height of a geostationary satellite above the surface of the earth?

Q.21. (a) Estimate the average orbital speed of the earth going around the sun. The average Earth-sun distance is 1.5×10^{11} *m*.

(b) An asteroid going around the sun has an average orbital speed of 15 *km/s*. Is the asteroid farther from the sun or closer to the sun as compared to the Earth? Explain your answer.

Q.22. Assume that the earth is not rotating about its axis and that Scientists have developed an engine which can propel vehicles to very high speed on the surface of the earth. What is the maximum possible speed for any such vehicle running on surface of the earth. Earth is a sphere of radius R = 6400 *km* and acceleration due to gravity on the surface is $g = 10\ m/s^2$.

Q.23. A satellite of Earth is going around in an elliptical orbit. The smallest distance of the satellite from the centre of the earth happens to be $2R$ (where R = radius of the earth). Find the upper limit of the maximum speed of such a satellite.

Q.24. Haley's Comet is going around the Sun in a highly elliptical orbit with a period of 76 *y*. It was closest to the sun in the year 1987 (I was 13 year old then and heard a lot about it on radio). In which year of 21st century do you expect it to have least kinetic energy?

Q.25. A planet goes around the sun in an elliptical orbit. The minimum distance of the planet from the Sun is 2×10^{12} *m* and the maximum speed of the planet in its path is 40 $km\ s^{-1}$. Find the rate at which its position vector relative to the sun sweeps area, when the planet is at a distance 2.2×10^{12} *m* from the sun.

Q.26. To launch a satellite at a height h above the surface of the earth (radius R) a two stage rocket is used . The first stage is used to lift the satellite to the desired height and the second stage is used to impart it a tangential velocity so as to put it in a circular orbital. Assume (incorrectly) that the mass of rocket is negligible and that there is no atmospheric resistance.

(a) If E_1 and E_2 are the energies delivered by the first and the second stage of the rocket.

Calculate the ratio $\dfrac{E_1}{E_2}$.

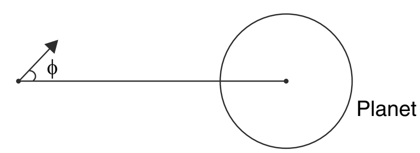
(b) Calculate the time period of the satellite if it is given that $\dfrac{E_1}{E_2} = 1$. Take mass of the earth to be M.

Q.27. A satellite of mass m is going around the earth in a circular orbital at a height $\dfrac{R}{2}$ from the surface of the earth. The satellite has lived its life and a rocket, on board, is fired to make it leave the gravity of the earth. The rocket remains active for a very small interval of time and imparts an impulse in the direction of motion of the satellite. Neglect any change in mass due to firing of the rocket.

(a) Find the minimum impulse imparted by the rocket to the satellite.

(b) Find the minimum work done by the rocket engine. Mass of the earth = M, Radius of the earth = R

Q.28. A small asteroid is at a large distance from a planet and its velocity makes an angle $\phi\ (\neq 0)$ with line joining the asteroid to the centre of the planet. Prove that such an asteroid can never fall normally on the surface of the planet.



LEVEL 2

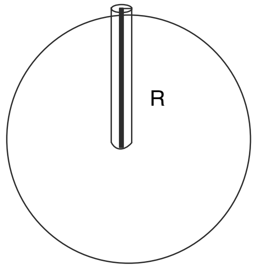
Q.29. Three identical particles, each of mass m, are located in space at the vertices of an equilateral triangle of side length a. They are revolving in a circular orbital under mutual gravitational attraction.

(a) Find the speed of each particle.

(b) Find the acceleration of the centre of mass of a system comprising of any two particles.

(c) Assume that one of the particles suddenly loses its ability to exert gravitational force. Find the velocity of the centre of mass of the system of other two particles after this.

Q.30. Imagine a hole drilled along the radius of the earth. A uniform rod of length equal to the radius (R) of the earth is inserted into this hole. Find the distance of centre of gravity of the rod from the centre .

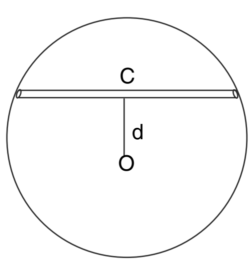


Q.31. A large non rotating star of mass M and radius R begins to collapse under its own gravity and ultimately becomes very small (nearly a point mass). Assume that the density remains uniform inside the sphere in any stage. Plot the variation of gravitational field intensity (well, you can call it acceleration due to gravity) at a distance $\frac{R}{2}$ from the centre vs the radius (r) of the star.

Q.32. At a depth $h_1 = \frac{R}{2}$ from the surface of the earth acceleration due to gravity is g_1. It's value changes by Δg_1 when one moves down further by 1 km. At a height h_2 above the surface of the earth acceleration due to gravity is g_2. It's value changes by Δg_2 when one moves up further by 1 km. If $\Delta g_1 = \Delta g_2$ find h_2. Assume the earth to be a uniform sphere of radius R.

Q.33. Due to rotation of the earth the direction of vertical at a place is not along the radius of the earth and actually makes a small angle ϕ with the true vertical (i.e. with radius). At what latitude (θ) is this angle ϕ maximum ?

Q.34. A tunnel is dug along a chord of non rotating earth at a distance $d = \frac{R}{2}$ [R = radius of the earth] from its centre. A small block is released in the tunnel from the surface of the earth. The block comes to rest at the centre (C) of the tunnel. Assume that the friction coefficient between the block and the tunnel wall remains constant at μ.



(a) Calculate work done by the friction on the block.

(b) Calculate μ.

Q.35. Diameter of a planet is $10d_\circ$, its mean density is $\frac{\rho_\circ}{4}$ and mass of its atmosphere is $10m_\circ$ where $d_\circ$, $\rho_\circ$ and $m_\circ$ are diameter, mean density and mass of atmosphere respectively for the earth. Assume that mean density of atmosphere is same on the planet and the earth and height of atmosphere on both the planets is very small compared to their radius.

(a) Find the ratio of atmospheric pressure on the surface of the planet to that on the earth.

(b) If a mercury barometer reads 76 cm on the surface of the earth, find its reading on the surface of the planet.

Q.36. A particle of mass m is projected upwards from the surface of the earth with a velocity equal to half the escape velocity. (R is radius of earth and M is mass of earth)

(a) Calculate the potential energy of the particle at its maximum.

(b) Write the kinetic energy of the particle when it was at half the maximum height.

Q.37. A uniform spherical planet is rotating about its axis. The speed of a point on its equator is v and the effective acceleration due to gravity on the equator is one third its value at the poles. Calculate the escape velocity for a particle at the pole of the planet. Give your answer in multiple of v.

Q.38. A planet is a homogeneous ball of radius R having mass M. It is surrounded by a dense atmosphere having density $\rho = \frac{\sigma_\circ}{r}$ where $\sigma_\circ$ is a constant and r is distance from the centre of the planet. It is found that acceleration due to gravity is constant throughout the atmosphere of the planet. Find $\sigma_\circ$ in terms of M and R.

Q.39. A projectile is to be launched from the surface of the earth so as to escape the solar system. Consider the gravitational force on the projectile due to the earth and the sun only. The projectile is projected perpendicular to the radius vector of the earth relative to the centre of the sun in the direction of motion of the earth. Find the minimum speed of projection relative to the earth so that the projectile escapes out of the solar system. Neglect rotation of the earth.

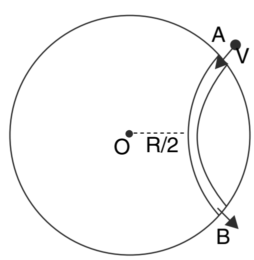
Mass of the sun $M_s = 2 \times 10^{30}$ kg; Mass of the earth $M_e = 6 \times 10^{24}$ kg

Radius of the earth $R_e = 6.4 \times 10^6$ m; Earth-Sun distance $r = 1.5 \times 10^{11}$ m

Q.40. Assume that there is a tunnel in the shape of a circular arc through the earth. Wall of the tunnel is smooth. A ball of mass m is projected into the tunnel at A with speed v. The ball comes out of the tunnel at B and escapes out of the gravity of the earth. Mass and radius of the earth are M and R respectively and radius of the circle shaped tunnel is also R.

(a) Find minimum possible value of v (call it v_0)

(b) If the ball is projected into the tunnel with speed v_0, calculate the normal force applied by the tunnel wall on the ball when it is closest to the centre of the earth. It is given that the closest distance between the ball and the centre of the earth is $\frac{R}{2}$.



Q.41. A celestial body, not bound to sun, will only pass by the sun once. Calculate the minimum speed of such a body when it is at a distance of 1.5×10^{11} m from the sun (this is average distance between the sun & the earth and is known as astronomical unit- $A.U.$)

The mass of the sun is $M \simeq 2 \times 10^{30}$ kg.

$$G = 6.67 \times 10^{-11} \frac{N - m^2}{kg^2}$$

Show that this speed is $\sqrt{2}$ times greater than speed of earth around the sun, assuming circular trajectory

Q.42. A body is projected vertically upward from the surface of the earth with escape velocity. Calculate the time in which it will be at a height (measured from the surface of the earth) 8 time the radius of the earth (R). Acceleration due to gravity on the surface of the earth is g.

Q.43. An astronaut on the surface of the moon throws a piece of lunar rock (mass m) directly towards the earth at a great speed such that the rock reaches the earth.

Mass of the earth = M, Mass of the moon $= \frac{M}{81}$

Radius of the earth = R, Distance between the centre of the earth and the moon = $60R$

(a) In the course of its journey calculate the maximum gravitational potential energy of the rock

(b) Find the minimum possible speed of the rock when it enters the atmosphere of the earth.

Q.44. The radius of the circular path of a geostationary satellite was inadvertently made $\Delta r = 1$ km larger than the correct radius $r = 42000$ km

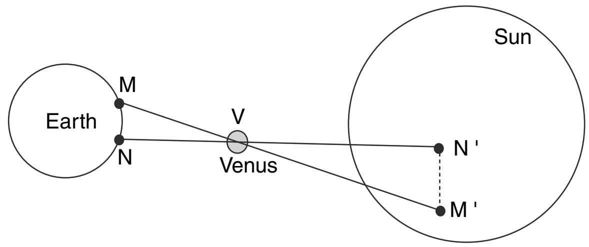
(a) Calculate the difference in angular speed of the satellite and the earth.

(b) If the satellite was exactly above a house on the equator on a particular day, what will be angular separation between the house and the satellite a year later?

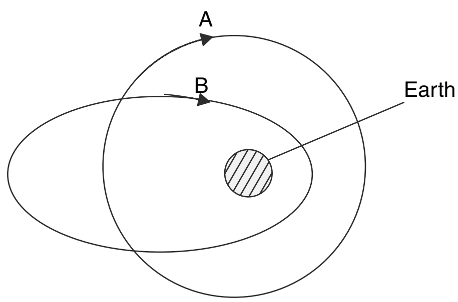
Q.45. A spy satellite S_1, travelling above the equator is taking pictures at quick intervals. The satellite is travelling from west to east and is ready with picture around the whole equator in 8 hours. Another similar satellite S_2, travelling in the same plane is travelling from east to west and is able to take pictures around the whole equator in 6 hours. Find the ratio of radii of the circular paths of the satellite S_1 and S_2.

Q.46. A comet is going around the sun in an elliptical orbit with a period of 64 year. The closest approach of the comet to the sun is 0.8 AU [AU = astronomical unit]. Calculate the greatest distance of the comet form the sun.

Q.47. The astronomical phenomenon when the planet Venus passes directly between the Sun and the earth is known as Venus transit. For two separate persons standing on the earth at points M and N, the Venus appears as black dots at points M' and N' on the Sun. The orbital period of Venus is close to 220 days. Assuming that both earth and Venus revolve on circular paths and taking distance MN = 1000 km, calculate the distance $M'N'$ on the surface of the Sun. [Take $(2.75)^{1/3} = 1.4$]



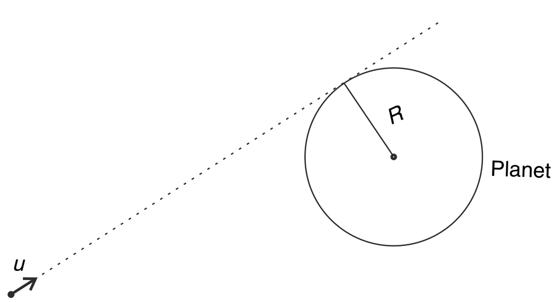
Q.48. Satellite A is following a circular path of radius a around the earth another satellite B follows an elliptical path around the earth. The two satellites have same mechanical energy and their orbits intersect. Find the speed of satellite B at the point where its path intersects with the circular orbit of A. Take mass of earth to be M.



Q.49. A satellite of mass m is orbiting around the earth (mass M, radius R) in a circular orbital of radius $4R$. It starts losing energy slowly at a constant rate $-\frac{dE}{dt} = \eta$ due to friction. Find the time (t) in which the satellite will spiral down to the surface of the earth.

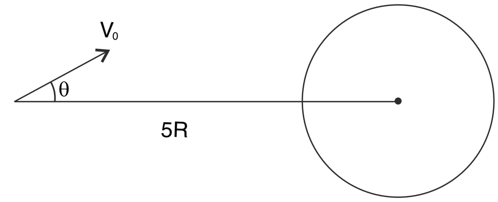
Q.50. Energy of a satellite going around the earth in an elliptical orbit is given by $-\frac{GMm}{2a}$ where M and m are masses of the earth and the satellite respectively and $2a$ is the major axis of the elliptical path. A satellite is launched tangentially with a speed $= \sqrt{\frac{3GM}{5R}}$ from a height $h = R$ above the surface of the earth. Calculate its maximum distance from the centre of the earth

Q.51.



A small asteroid is approaching a planet of mass M and radius R from a large distance. Initially its velocity (u) is along a tangent to the surface of the planet. It fall on the surface making an angle of 30° with the vertical. Calculate u.

Q.52. An asteroid was fast approaching the earth. Scientists fired a rocket which hit the asteroid at a distance of 5 R from the centre of the earth (R = radius of the earth). Immediately after the hit the asteroid's velocity (V_0) was making an angle of $\theta = 30°$ with the line joining the centre of the earth to the asteroid. The asteroid just grazed past the surface of the earth. Find V_0 [Mass of the earth = M]



Q.53. A satellite is orbiting around the earth in a circular orbit. Its orbital speed is V_0. A rocket on board is fired from the satellite which imparts a thrust to the satellite directed radially away from the centre of the earth. The duration of the engine burn is negligible so that it can be considered instantaneous. Due to this thrust a velocity variation ΔV is imparted to the satellite. Find the minimum value of the ratio $\frac{\Delta V}{V_0}$ for which the satellite will escape out of the gravitational field of the earth.

Q.54. In last question assume that circular orbit of the satellite has radius r_0. Find $\frac{\Delta V}{V_0}$ for which the maximum distance of the satellite from the centre of the earth become $2r_0$ after the rocket is fired.

Q.55. A satellite is at a distance r_1 from the centre of the earth at its apogee. The distance is r_2 when it is at perigee. Mass of the earth is M.

(a) Calculate the maximum speed of the satellite in its orbit around the earth.

(b) Estimate the maximum speed of the moon going around the earth. For moon

$r_1 \simeq 400{,}000\ km$ and $r_2 \simeq 360{,}000\ km$

mass of the earth $M = 6 \times 10^{24}\ kg$

Q.56. A satellite is going around the earth in an elliptical orbit and has maximum and minimum distance from the centre of earth equal to $10r$ and r respectively. It was planned to fire on board rocket so as to increase the energy of the satellite by maximum amount. Assume that the rocket is fired for a small time (almost instantaneous) and gives an impulse J to the satellite in forward direction. Take J to be small compared to overall momentum of the satellite.

(a) Show that firing the rocket when the satellite is at perigee (nearest to earth) will result in maximum gain in energy of the satellite.

[The orbit of mass orbiter mission, informally called Mangalyaan, was raised in five steps using this principle; before it was given the escape speed]

(b) Find the impulse J that the rocket must impart to the satellite at perigee so that its maximum distance from earth's centre, during its course of motion in elliptical path, becomes $12r$. Take mass of satellite as m and mass of earth as M. Assume that there is negligible change in mass of the satellite due to firing of the rocket.

Q.58. Imagine a smooth tunnel along a chord of non-rotating earth at a distance $\frac{R}{2}$ from the centre. R is the radius of the earth. A projectile is fired along the tunnel from the centre of the tunnel at a speed $V_o = \sqrt{gR}$ [g is acceleration due to gravity at the surface of the earth].

(a) Is the angular momentum [about the centre of the earth] of the projectile conserved as it moves along the tunnel?

(b) Calculate the maximum distance of the projectile from the centre of the earth during its course of motion.

Q.59. A geostationary satellite is nearly at a height of $h = 6\,R$ from the surface of the earth where R is the radius of the earth. Calculate the area on the surface of the earth in which the communication can be made using this satellite.

LEVEL 3

Q.60. (a) There is an infinite thin flat sheet with mass density σ per unit area. Find the gravitational force, due to sheet, on a point mass m located at a distance x from the sheet.

(b) Consider a large flat horizontal sheet of material density ρ and thickness t, placed on the surface of the earth. The density of the earth is ρ_0. If it is found that gravitational field intensity just between the sheet is larger than field just above it, prove that $\rho_o > \frac{3}{2}\rho$. Assume $t << R$

Q.61. A spaceship is orbiting the earth in a circular orbit at a height equal to radius of the earth ($R_c = 6400$ km) from the surface of the earth. An astronaut is on a space walk outside the spaceship. He is at a distance of $l = 200$ m from the ship and is connected to it with a simple cable which can sustain a maximum tension of10 N. Assume that the centre of the earth, the spaceship and the astronaut are in a line. Mass of astronaut along with all his accessories is 100 kg.

(a) Do you think that a weak cable that can only take a load of 10 N, can prevent him from drifting in space ? Make a guess.

(b) Estimate the tension in the cable.
[Acceleration due to gravity on the surface of Earth = 9.8 m/s^2]

Q.62. Earth is rotating about its axis with angular speed ω_0 and average density of earth is ρ. It is proposed to make a space elevator by placing a long rod with uniform mass density extending from just above the surface for the earth out to a radius nR (R is radius of the earth). Prove that the rod can remain above the same point on the equator all time if, $n^2 + n = \frac{8\pi G\rho}{3\omega^2}$, where ρ is density of the earth

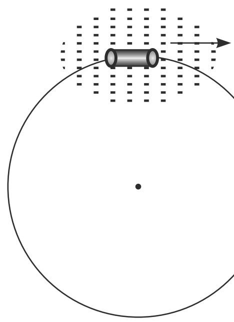
Q.63. A body is projected up from the surface of the earth with a velocity half the escape velocity at an angle of 30° with the horizontal. Neglecting air resistance and earth's rotation, find

(a) the maximum height above the earth's surface to which the body will rise.

(b) will the body move around the earth as a satellite?

Q.64. A near surface earth satellite has cylindrical shape with cross sectional area of $S = 0.5$ m^2 and mass of $M = 10$ kg. It encounters dust which has density of $d = 1.6 \times 10^{-11}$ kg/m^3. Assume that the dust particles are at rest and they stick to the satellite's front face on collision. Take mean density of earth to be $\rho = 5500$ kg/m^3

(a) Find the drag force experienced by the satellite



(b) If the dust extends throughout the orbit, find the change in velocity and radius of the circular path of the satellite in one revolution.

ANSWERS

1. Exactly midway between the two balls *OR* at a distance of $\frac{d}{2\sqrt{2}}$ from the ball of mass m.

2. (a) At the diagonally opposite corner of C on the floor of the cube

(b) $\frac{4Gm^2}{3a^2}$

3. (a) $\frac{G(M+m)}{x^2}$

(b) $\sqrt{\frac{2G(M+m)}{x}}$

(c) zero

4. (a) 1600 km

(b) 11.52 km

5. $T=\sqrt{\frac{9\pi}{2Gd_0}}$

6. 32 μm

7. (a) 1

(b) $\frac{M}{\pi R^4}$

8. $R = 8.9\ mm$; $d = 2\times 10^{30}\ kg\ m^{-3}$

9. $\frac{4W_0\omega_0 v}{g}$

11. (a) 0.026 ms^{-2}

(b) 9.80 ms^{-2}

12. 6 dgh

13. The body will never return to the same location.

14. 2.5 km

15. T_0

16. $\frac{J^2}{2mr^2}$

17. $T_0 = \frac{2\pi r_0}{\sqrt{KM}}$

18. 16

19. (a) Zero

(b) $\frac{mg}{4}$

(c) zero

20. $h+\frac{R}{2}$

21. (a) 30 km/s

(b) Farther

22. $V_0 = \sqrt{gR} \simeq 7.9\ km/s$

23. $\sqrt{gR} \simeq 7.9\,km/s$

24. 2025

25. $4\times 10^{16}\ m^2s^{-1}$

26. (a) $\frac{2h}{R}$

(b) $\sqrt{\frac{27\pi^2 R^3}{2GM}}$

27. (a) $\left(\sqrt{2}-1\right)m\sqrt{\frac{2GM}{3R}}$

(b) $\frac{GMm}{3R}$

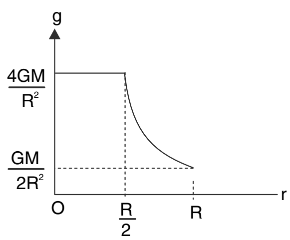
29. (a) $\sqrt{\dfrac{Gm}{a}}$

(b) $\dfrac{\sqrt{3}}{2}\dfrac{Gm}{a^2}$

(c) $\dfrac{1}{2}\sqrt{\dfrac{Gm}{a}}$

30. $\dfrac{2R}{3}$

31.



32. $h_2 = R\left(2^{\frac{1}{3}} - 1\right)$

33. 45°

34. (a) $-\dfrac{3GMm}{8R}$

(b) $\dfrac{\sqrt{3}}{2}$

35. (a) $\dfrac{5}{2}$

(b) 7.6 *cm*

36. (a) $\dfrac{-3GMm}{4R}$

(b) $\dfrac{3GMm}{28R}$

37. $\sqrt{3}v$

38. $\sigma_\circ = \dfrac{M}{2\pi R^2}$

39. 13.6 *km/s*

40. (a) $v_0 = \sqrt{\dfrac{2GM}{R}}$

(b) $\dfrac{27}{4}\dfrac{GMm}{R^2}$

41. 4.2×10^4 *m/s*

42. $t = \dfrac{52}{3}\sqrt{\dfrac{R}{2g}}$

43. (a) $-\dfrac{GMm}{243R}$

(b) $\dfrac{14045}{14337}\dfrac{GMm}{R}$

44. (a) 9.3×10^{-6} *rad/hr*

(b) 4.6°

45. $\dfrac{r_1}{r_2} = \left(\dfrac{3}{4}\right)^{2/3}$

46. 31.2 *AU*

47. 2500 *km*

48. $\sqrt{\dfrac{GM}{a}}$

49. $t = \dfrac{3GMm}{8\eta R}$

50. $2R$

51. $u = \sqrt{\dfrac{2GM}{3R}}$

52. $V_0 = \sqrt{\dfrac{32}{105}\dfrac{GM}{R}}$

53. (1)

54. (1/2)

55. (a) $\sqrt{\dfrac{2GM}{r_2}\left(\dfrac{1}{1+{r_2}/{r_1}}\right)}$

(b) 1.08 *km/s*

56. (b) $J = 0.01m\sqrt{\dfrac{GM}{r}}$

58. (a) NO

(b) $r_{\max} = \left(\dfrac{8+\sqrt{57}}{14}\right)R$

59. $\dfrac{12}{7}\pi R^2$

60. (a) $G2\pi\sigma m$

61. (b) 0.01 *N*

63. (a) $\left(\dfrac{\sqrt{7}-2}{6}\right)R$

(b) No

64. **(a)** $5 \times 10^{-4}\,N$

(b) $\Delta V = 0.25$ *m/s*; $\Delta R = -0.4$ *km*

SOLUTIONS

1. There are two possibilities –

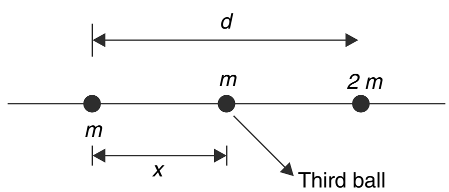
(1) the net force is towards right

(2) the net force is towards left (see figure)

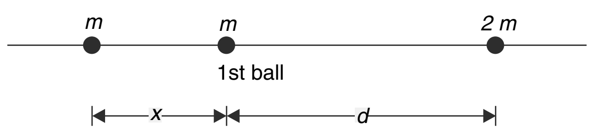
When net force is towards right–

$$\frac{Gm(2m)}{d^2} + \frac{Gmm}{x^2} = \frac{6Gm^2}{d^2}$$

$$\Rightarrow \frac{1}{x^2} = \frac{4}{d^2} \qquad \Rightarrow x = \frac{d}{2}$$



When the net force is to the left

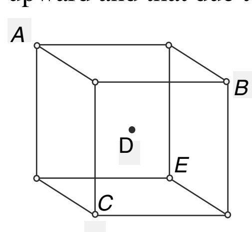


$$\frac{Gmm}{x^2} - \frac{Gm(2m)}{d^2} = \frac{6Gm^2}{d^2}$$

$$\frac{1}{x^2} = \frac{8}{d}$$

$$x = \frac{d}{2\sqrt{2}}$$

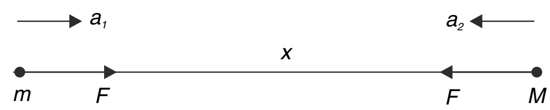
2. (a) The net force on mass at D will be zero if an identical mass is placed at E. In this case the resultant force due to masses at A and B will be vertically upward and that due to masses at C and E will be vertically down.



(b) Required force = Force equal and opposite to that applied by the mass placed at E

$$= \frac{Gm^2}{(\sqrt{3}a/2)^2} = \frac{4Gm^2}{3a^2}$$

3. (a) Force between the two masses $F = \dfrac{GmM}{x^2}$



$\therefore$ Acceleration of m, $a_1 = \dfrac{F}{m} = \dfrac{GM}{x^2}$

Acceleration of M, $a_2 = \dfrac{F}{M} = \dfrac{Gm}{x^2}$

Relative acceleration $\quad a = a_1 + a_2 = \dfrac{G(M+m)}{x^2}$

(b) $\therefore\ a = \dfrac{dv}{dt} = \dfrac{dv}{dx} \cdot \dfrac{dx}{dt} = v\dfrac{dv}{dx}$ $\quad$ [v = relative velocity]

$$-v\frac{dv}{dx} = \frac{G(M+m)}{x^2}$$

[– sign has been placed because v is increasing with decreasing x]

$$\therefore\quad v\,dv = -G(M+m)\frac{dx}{x^2}$$

$$\therefore\quad \int_0^v v\,dv = -G(M+m)\int_\infty^x \frac{dx}{x^2}$$

$$\frac{v^2}{2} = -G(M+m)\left[-\frac{1}{x}\right]_\infty^x = \frac{G(M+m)}{x}$$

$$\therefore\quad v = \sqrt{\frac{2G(M+m)}{x}}$$

(c) The centre of mass has no acceleration as there is no external force.
Hence, velocity of COM is zero throughout.

4. (a) $g' = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$

$g'\ g - 0.36g = 0.64g$

$$\therefore\quad 0.64 = \frac{1}{\left(1+\dfrac{h}{R}\right)^2}$$

$$\Rightarrow\quad 1+\frac{h}{R} = \frac{1}{0.8}$$

$$\Rightarrow\quad \frac{h}{R} = \frac{5}{4} - 1$$

$$\Rightarrow\quad h = \frac{R}{4} = 1600\,km$$

(b) $g' = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2} = g\left(1+\dfrac{h}{R}\right)^{-2}$

$$\simeq g\left[1-\frac{2h}{R}\right] \quad \text{[expanding binomially and neglecting higher order terms]}$$

$$\therefore \quad \frac{g'}{g}=1-\frac{2h}{R}$$

$$\frac{2h}{R}=\frac{g-g'}{g}$$

$$\frac{2h}{R}\times 100=\frac{\Delta g}{g}\times 100$$

$$\frac{2h}{R}\times 100=0.36$$

$$h=\frac{0.18}{100}\times 6400\,km = 11.52\ km$$

5. $$\frac{1}{3}\left(g_{pole}\right)_{earth}=\left(g_{pole}\right)_{planet}$$

$$\frac{1}{3}\frac{G\,M_e}{R_e^2}=\frac{G\,M_P}{R_P^2}$$

$$\Rightarrow \frac{1}{3}\cdot\frac{\frac{4}{3}\pi\ \mathrm{R}_e^3\cdot d_0}{R_e^2}=\frac{\frac{4}{3}\pi\ \mathrm{R}_P^3\cdot d_P}{R_P^2}$$

$$\therefore\ d_0 R_e = 3\,d_P R_P \qquad \ldots..(1)$$

For weightlessness at the equator

$$\omega^2 R = g$$

$$\therefore \quad \omega^2 R_P=\frac{GM_P}{R_P^2}$$

$$\Rightarrow \omega^2 = G\,\frac{4}{3}\pi\, d_P \quad =\frac{4}{3}G\pi\frac{1}{3}\frac{d_0 R_e}{R_P}$$

$$=\frac{4}{9}G\pi\, d_0(2) \qquad \left[\frac{R_e}{R_P}=2\right]$$

$$\therefore \quad \frac{4\pi^2}{T^2}=\frac{8}{9}G\pi\, d_0$$

$$\therefore \quad T^2=\frac{9\pi}{2Gd_0}$$

$$\therefore \quad T=\sqrt{\frac{9\pi}{2Gd_0}}$$

6. $$g=\frac{GM}{R^2}$$

$$\frac{dg}{dR}=-\frac{2GM}{R^3}$$

$\therefore \qquad \dfrac{dg}{dR} = -\dfrac{2g}{R}$

$\dfrac{\Delta g}{g} \simeq -\dfrac{2\,\Delta R}{R}$

$\therefore \left|\dfrac{\Delta R}{R}\right| = \dfrac{1}{2}\left|\dfrac{\Delta g}{g}\right| = \dfrac{1}{2} \times \dfrac{10^{-9}}{100}$

$\therefore \; \Delta R = R \cdot \dfrac{1}{2} \times 10^{-11}$

$= 6.4 \times 10^{6} \times \dfrac{1}{2} \times 10^{-11}$

$= 3.2 \times 10^{-5} = 32\,\mu m$

7. (a) $g_1 = \dfrac{GM}{R^2}$; $g_2 = \dfrac{GM}{R^2}$

$\therefore \; \dfrac{g_1}{g_2} = 1$

(b) $\displaystyle\int_0^R \rho 4\pi r^2 dr = M$

$\Rightarrow 4\pi\rho_0 \displaystyle\int_0^R r^3 dr = M \qquad \Rightarrow \pi\rho_0\, R^4 = M$

$\rho_0 = \dfrac{M}{\pi R^4}$

8. $\dfrac{1}{2} mc^2 - \dfrac{GMm}{R} = 0$

$\therefore R = \dfrac{2GM}{c^2} = \dfrac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{(3 \times 10^{8})^2}$

$= 8.89 \times 10^{-3}\, m = 8.9\, mm$ (!!)

Mass density $= \dfrac{6 \times 10^{24}}{\dfrac{4}{3} \times 3.14 \times (8.9 \times 10^{-3})^3} = 2.0 \times 10^{30}\, kg\, m^{-3}$

The figures are unrealistic! A body having mass of the earth cannot act like a black hole.

9. When the train is at rest

$W_0 = mg - \dfrac{mV_0^2}{R} \qquad [V_0 = \omega_0 R,\ \omega_0 = \text{angular speed of the earth}]$

When the train is moving from West to East $V_1 = V_0 + v$

$\therefore W_1 = mg - \dfrac{m(V_0 + v)^2}{R}$

For train running due west

$V_2 = V_0 - v$

$$W_2 = mg - \frac{m(V_0 - v)^2}{R}$$

$$\therefore W_2 - W_1 = \frac{m}{R}\left[(V_0 + v)^2 - (V_0 - v)^2\right]$$

$$= \frac{m}{R}\left[4V_0 v\right] = \frac{4m(\omega_0 R)v}{R} = 4m\omega_0 v$$

$$= 4\frac{W_0}{g}\omega_0 v$$

10. The rocks start flying away from the equator of the planet if

$$\omega^2 R \geq g \qquad \Rightarrow \quad \omega \geq \sqrt{\frac{g}{R}}$$

$$\Rightarrow \frac{2\pi}{T} \geq \sqrt{\frac{g}{R}} \qquad \Rightarrow \qquad \frac{T}{2\pi} \leq \sqrt{\frac{R}{g}}$$

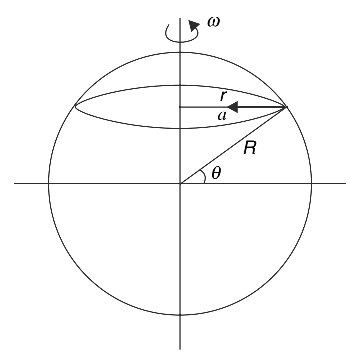
$$T \leq 2\pi\sqrt{\frac{R}{g}}$$

For rocks to not fly away

$$T \geq 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{R}{\frac{GM}{R^2}}}$$

$$= 2\pi\sqrt{\frac{R^3}{G\cdot\frac{4}{3}\pi R^3\cdot\rho}} = \sqrt{\frac{3\pi}{G\rho}}$$

11. (a) Radius of rotation at latitude $\theta = 40°$ is



$r = R\cos\theta$

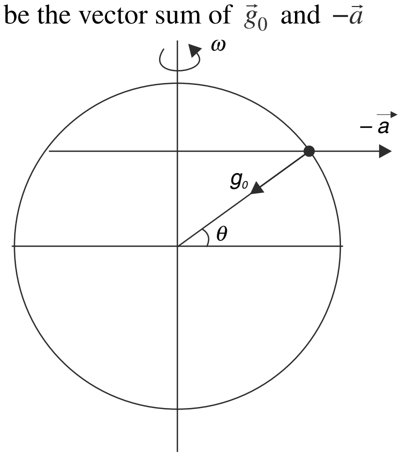
Centripetal acceleration

$a = \omega^2 r = \omega^2 R\cos\theta$

$= (7.27 \times 10^{-5})^2\,(6.37 \times 10^6)\,(0.77)$

$= 0.026\ ms^{-2}$

(b) The resultant acceleration will be the vector sum of $\vec{g}_0$ and $-\vec{a}$



$$g = \sqrt{g_0^2 + a^2 - 2ag\cos\theta}$$

$= \sqrt{(9.82)^2 + (0.026)^2 - 2 \times 0.026 \times 9.82 \times 0.77}$

$= 9.80\ ms^{-2}$

12. $V_{escape} = \sqrt{2gR}$

$$\frac{V_{escape\ planet}}{V_{escape\ earth}} = \sqrt{\frac{g_P R_P}{g_e R_e}}$$

$$\frac{1}{\sqrt{6}} = \sqrt{\frac{g_P}{g_e} \frac{1}{36}}$$

$\Rightarrow \quad g_p = 6g_e$

$\therefore$ Atmospheric pressure $= dg_p h$

$= 6dg_e h$

13. Energy of the body

$$\frac{1}{2}mV^2 - \frac{GMm}{r}$$

$$= m\left[\frac{\left(60 \times 10^3\right)^2}{2} - \frac{6.67 \times 10^{-11} \times 2 \times 10^{30}}{2 \times 10^{11}}\right]$$

$= m[18 \times 10^8 - 6.67 \times 10^8]$

$= m(11.33 \times 10^8)$

Since total energy is positive, the body is in an unbound orbit. It will never return back.

14. For a jump of $h_0 = 1\ m$ on the earth, speed required is given by

$$\frac{1}{2}mV^2 = mgh_0 \qquad \Rightarrow \quad V \simeq \sqrt{20}\ m/s$$

Escape speed on the surface of a planet is

$$V_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{8\pi GR^2\rho}{3}}$$

$$\therefore \quad \frac{V_{esc}^{planet}}{V_{esc}^{earth}} = \frac{R_{planet}}{R_{earth}}$$

We want $V_{esc}^{planet} = \sqrt{20}\ m/s$

And it is given that $V_{esc}^{earth} = 11.2\ km/s$

$$\therefore \quad R_{planet} = \frac{\sqrt{20} \times (6400\,km)}{11200} = 2.5\,km$$

15. For weightlessness at the equator

$$\omega_0^2 R = g \quad \Rightarrow \quad \omega_0 = \sqrt{\frac{g}{R}}$$

At a distance r from the centre ($r < R$) the acceleration due to gravity is

$$g_r = \frac{g}{R}r$$

For weightlessnes at a distance r from centre

$$\frac{g}{R}r = \omega^2 r \qquad \Rightarrow \quad \omega = \sqrt{\frac{g}{R}}$$

Required rotation speed of the earth is same for any depth.

16. $K = \dfrac{J^2}{2I} = \dfrac{J^2}{2mr^2}$

[$I = mr^2$ = moment of inertia of the satellite about the planet]

17. $\dfrac{mV^2}{r} = K\,\dfrac{Mm}{r^n}$

V is independent of r if $n = 1$

$\therefore \qquad V^2 = KM$

$V = \sqrt{KM}$

Time period $T_0 = \dfrac{2\pi r_0}{V} = \dfrac{2\pi r_0}{\sqrt{KM}}$

18. Time period of a near surface satellite $T = 2\pi\sqrt{\dfrac{R}{g}} \simeq 86$ min = 1.43 hr

Number of revolutions in 24 hr

$n = \dfrac{24}{1.43} = 16.78$

$\therefore$ number of crossings = 16

19. (a) A person inside a satellite is in a state of weightlessness. The gravitational pull of earth provides the exact centripetal force needed to keep moving with the satellite. There is no contact force between the walls of the satellite and the person.

(b) Orbital speed $v = \sqrt{\dfrac{GM}{r}}$

$\therefore \ \dfrac{gR}{2} = \dfrac{GM}{r}$

$\dfrac{1}{2}\dfrac{GM}{R^2}R = \dfrac{GM}{r}$

$\therefore$ Weight experienced at a distance r from the centre of the earth is

$\dfrac{GMm}{r^2} = \dfrac{GMm}{(2R)^2} = \dfrac{mg}{4}$

(c) Once again the man becomes weightless as he starts a free fall motion under gravity (along with the satellite).

20. The orbital radius of the satellite will not change because force acting on it does not change.

Since radius of earth has become $R/2$, distance of the satellite from the surface $= h + \dfrac{R}{2}$

21. (a) Average speed of the earth is

$$v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 1.5 \times 10^{11}}{365 \times 24 \times 60 \times 60} = 2.98 \times 10^4\ m/s \simeq 30\ km/s$$

(b) Speed of bodies orbiting the sun decreases with distance from the sun. Orbital speed for circular orbit is given by

$$v = \sqrt{\frac{GM_s}{r}}$$

Since speed of the asteroid is less than that of the earth, it is farther from the sun.

22. $\frac{mV_0^2}{R} = \frac{GMm}{R^2}$

If the speed is increased beyond V_0, the vehicle will leave the surface of the earth.

$$\therefore V_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR} \simeq 7.9 \ km/s$$

23. Speed is maximum when the satellite is closest to the Earth.

For a bound orbit, total energy must be negative.

$$\frac{1}{2}mV_{\max}^2 - \frac{GMm}{2R} < 0$$

$$\therefore V_{\max} < \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

24. Speed will be least when it is farthest from the sun. This will happen when it will be at aphelion.

$$t = \frac{76}{2} = 38\,y$$

1987 + 38 = 2025

25. The speed is maximum when the planet is at minimum distance from the Sun.

$\therefore$ V = 40 $km\ s^{-1}$ when r = 2×10^{12} m

$\therefore$ $\omega = \frac{V}{r}$ [This equation is true at perigee and apogee]

$$\omega = \frac{40 \times 10^3}{2 \times 10^{12}} = 2 \times 10^{-8} \text{ rad s}^{-1}$$

Rate of sweeping of area $\frac{dA}{dt} = \frac{r^2\omega}{2}$

$$= \frac{(2 \times 10^{12})^2 \times 2 \times 10^{-8}}{2} = 4 \times 10^{16} \text{ m}^2\text{s}^{-1}$$

This rate remains constant for any position of the planet.

26. (a) $E_1 = -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$

$$= GMm\left[\frac{1}{R} - \frac{1}{R+h}\right] = \frac{GMmh}{R(R+h)}$$

Orbital speed of satellite $V = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}}$

$$\therefore E_2 = \frac{1}{2}mV^2 = \frac{GMm}{2(R+h)}$$

$$\therefore \frac{E_1}{E_2} = \frac{2h}{R}$$

Now, $\dfrac{E_1}{E_2} = 1$ means $\dfrac{2h}{R} = 1$

$\therefore h = \dfrac{R}{2}$

Distance from the centre of earth $r = \dfrac{3R}{2}$

$$T^2 = \frac{4\pi^2}{GM} r^3 = \frac{4\pi^2}{GM}\left(\frac{3R}{2}\right)^3$$

$$T = \sqrt{\frac{27\pi^2 R^3}{2GM}}$$

27. (a) Orbital speed $V_0 = \sqrt{\dfrac{GM}{r}} = \sqrt{\dfrac{2GM}{3R}} = \sqrt{\dfrac{2gR}{3}}$

For escape the satellite must have speed given by $\dfrac{1}{2}mV^2 - \dfrac{GMm}{r} = 0$

$$V = \sqrt{\frac{2GM}{r}} = \sqrt{2}\,V_0$$

$\therefore$ Impulse needed $= m\left(\sqrt{2}\,V_0\right) - mV_0$

$$= \left(\sqrt{2} - 1\right) mV_0 = \left(\sqrt{2} - 1\right) m \sqrt{\frac{2GM}{3R}}$$

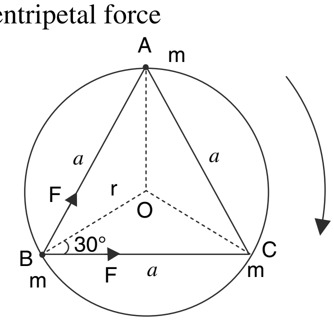
(b) $W = \dfrac{1}{2} m\left(\sqrt{2}V_0\right)^2 - \dfrac{1}{2} mV_0^2 = \dfrac{1}{2} mV_0^2 = \dfrac{GMm}{3R}$

28. The angular momentum of the asteroid about the centre of the planet is not zero. Falling normally on the surface will mean that its angular momentum about the centre is zero. But the angular momentum is conserved, hence it cannot hit the surface of the planet normally.

29. (a) Consider one of the particle (say B). It is rotating in circle of radius $r = \dfrac{a}{\sqrt{3}}$
Resultant force on it towards the centre of the circle is

$$2F\cos 30° = 2\frac{Gm^2}{a^2}\frac{\sqrt{3}}{2} = \sqrt{3}\frac{Gm^2}{a^2}$$

This must be equal to necessary centripetal force



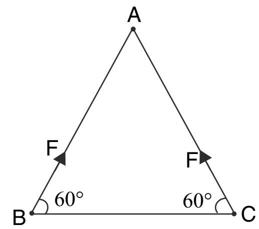
$$\therefore \frac{mv^2}{r} = \frac{\sqrt{3}Gm^2}{a^2}$$

$$\Rightarrow v^2 = \frac{Gm}{a} \qquad \left[\because r = \frac{a}{\sqrt{3}}\right]$$

$$\Rightarrow v = \sqrt{\frac{Gm}{a}}$$

(b) Consider our system to be made up of B and C. External force on this system is due to A. Net external force = $2F \sin 60°$

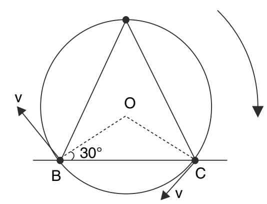
$$= \sqrt{3}F = \frac{\sqrt{3}Gm^2}{a^2}$$



$$\therefore a_{cm} = \left(\frac{\sqrt{3}Gm^2}{a^2}\right) \times \frac{1}{2m}$$

$$= \frac{\sqrt{3}}{2}\frac{Gm}{a^2}$$ (towards the centre of the triangle)

(c) Momentum of $(B + C)$ system = $mv \cos 60° + mv \cos 60° = mv$ [along CB]



$$\therefore v_{cm} = \frac{mv}{2m} = \frac{v}{2} = \frac{1}{2}\sqrt{\frac{Gm}{a}}$$

The velocity of CoM does not change after A stops attracting.

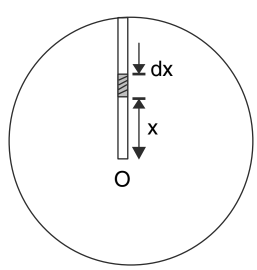
30. $x_{cg} = \dfrac{\int x dw}{\int dw}$

Where dw = Weight of an infinitesimal element whose x co-ordinate is x.

Let λ = mass per unit length of the rod.

Consider an element at a distance x from the centre

$$dw = (\lambda dx)\left(g\frac{x}{R}\right)$$



[Acceleration due to gravity at a distance x from the centre of the earth is $= g\dfrac{x}{R}$]

$$\therefore\ x_{cg} = \frac{\frac{\lambda g}{R}\int_0^R x^2 dx}{\frac{\lambda g}{R}\int_0^R x dx} = \frac{\frac{R^3}{3}}{\frac{R^2}{2}} = \frac{2R}{3}$$

31. As long as the point is inside the sphere (which means $r > \frac{R}{2}$) the field is given by

$$g = \frac{GM}{r^3}x = \frac{GM}{r^3}\left(\frac{R}{2}\right)$$

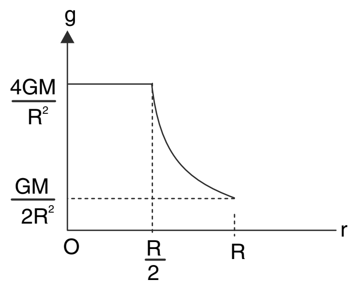
$$\therefore\ g \propto \frac{1}{r^3}$$

In the beginning $r = R \quad \therefore\ g = \frac{Gm}{2R^2}$

When $r = \frac{R}{2}$; $g = \frac{4Gm}{R^2}$

After the star shrinks to a radius $r < \frac{R}{2}$, the field at the point (the point is now outside the sphere)

becomes constant at $g = \frac{GM}{\left(\frac{R}{2}\right)^2} = \frac{4GM}{R^2}$



32. Acceleration inside the earth at a distance r from the centre

$$g_1 = \frac{GM}{R^3}.r = \frac{GM}{R^2}\left(\frac{R-h_1}{R}\right) = g\left(1-\frac{h_1}{R}\right)$$

$$\Delta g_1 = \left(\frac{dg_1}{dh_1}\right)\Delta h_1 = -\frac{g}{R}\Delta h_1$$

$\Delta h_1 = +1km$

$$\therefore\ \Delta g_1 = -\frac{g}{R}(1km) \quad \text{........(i)}$$

[This is independent of depth h_1]

At a distance r from the centre outside the earth

$$g_2 = \frac{GM}{r^2} = \frac{GM}{R^2}\frac{R^2}{r^2} = g\frac{R^2}{(R+h_2)^2}$$

$$\therefore\ \Delta g_2 = \left(\frac{dg_2}{dh_2}\right)(\Delta h_2) = \frac{-2gR^2}{(R+h_2)^3}\Delta h_2$$

$\Delta h_2 = 1km$

$$\Delta g_2 = -\frac{2gR^2}{\left(R+h_2\right)^3}(1\ km) \qquad ....(i)$$

Since $\Delta g_1 = \Delta g_2$

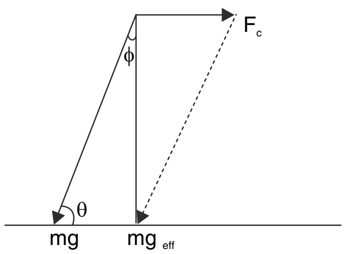
$$\therefore -\frac{g}{R}(1\ km) = -\frac{2gR^2}{(R+h_2)^3}(1 km)$$

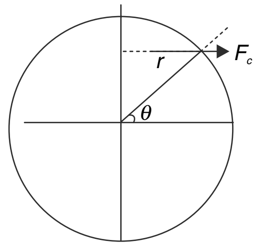
$\therefore\ (R + h_2)^3 = 2R^3$

$$\Rightarrow 1+\frac{h_2}{R} = (2)^{1/3}$$

$h_2 = R\ [2^{1/3} - 1]$

33. The figure shows the true weight (mg) of a body, centrifugal force (F_c) acting on it and the resultant of these two forces *(mg_{eff})*.



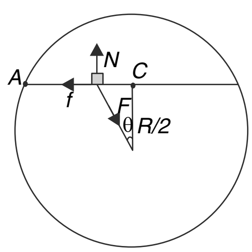


$F_c = m\ \omega^2 r = m\ \omega^2 R \cos\ \theta$

Since F_c is very small ϕ will be maximum when component of F_c perpendicular to $m\vec{g}$ is largest, i.e., when $m\omega^2 R \cos\theta \sin\theta$ is maximum. This will happen when $\theta = 45°$

34. (a) Gravitational potential at A, $V_A = -\frac{GM}{R}$

Potential at C



$$V_c = -\frac{GM}{R^3}\left[\frac{3}{2}R^2 - \frac{1}{2}\left(\frac{R}{2}\right)^2\right] = -\frac{11}{8}\frac{GM}{R}$$

$\therefore$ Loss in gravitational *PE* of the block

$$= -\frac{GMm}{R} - \left(-\frac{11GMm}{8R}\right) \qquad = \frac{3GMm}{8R}$$

$\therefore$ Work done by friction $= -\frac{3GMm}{8R}$

(b) At any intermediate position *(θ)* shown in the figure

$F \cos\ \theta = N$

$$\frac{GMm}{R^3}\left(\frac{R/2}{\cos\theta}\right).\cos\theta = N$$

$\therefore N = \dfrac{GMm}{2R^2} =$ a constant

$\therefore$ Work done by friction = $-\mu N(AC)$

$$-\frac{3GMm}{8R} = -\mu\frac{GMm}{2R^2}\left(\frac{\sqrt{3}R}{2}\right)$$

$$\Rightarrow \mu = \frac{\sqrt{3}}{2}$$

35. R_o = radius of earth

$10R_o$ =radius of the planet [σ_o = density of atmosphere; h_o = height of atmosphere]

$$m_\circ = \frac{4}{3}\pi\left[\left(R_\circ + h_\circ\right)^3 - R_\circ{}^3\right]\sigma_\circ$$

$$= \frac{4}{3}\pi R_\circ{}^3\left[\left(1+\frac{h_\circ}{R}\right)^3 - R_\circ{}^3\right]\sigma_\circ$$

$$\simeq \frac{4}{3}R_\circ{}^3\left[1+\frac{3h_\circ}{R_\circ}-1\right]\sigma_\circ = 4\pi R_0^2\sigma_0 h_0 \qquad \text{......(a)}$$

For planet $m = 4\pi R^2.\sigma h = 4\pi(10R_o)^2\,\sigma_o h$ ......(b)

Given $m = 10\,m_o$

$\Rightarrow 100\,h = 10\,h_o$

$$h = \frac{h_\circ}{10}$$

(a) Ratio of atmospheric pressure

$$\frac{P}{P_\circ} = \frac{\sigma_\circ gh}{\sigma_\circ g_\circ h_\circ} = \frac{g}{g_\circ}.\frac{1}{10} \qquad \text{...........(iii)}$$

$$\therefore g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3.\rho}{R^2} = \frac{4}{3}\pi GR\rho$$

$$\therefore \frac{g}{g_\circ} = \frac{R\rho}{R_\circ\rho_\circ} = 10\times\frac{1}{4} = \frac{5}{2}$$

Using (iii)

$$\frac{\rho}{\rho_\circ} = \frac{5}{2\times 10} = \frac{1}{4}$$

(b) $\rho_{hg}g_o \times (76\ cm) = P_o$

$\rho_{hg}g \times h = P$

$$\Rightarrow \frac{h}{76}\times\frac{g}{g_\circ} = \frac{P}{P_\circ}$$

$$= \frac{h}{76}\times\frac{5}{2} = \frac{1}{4}$$

$$\Rightarrow h = \frac{38}{5} = 7.6\,cm$$

36. (a) At maximum height speed is zero. Conservation of mechanical energy gives -

$$\frac{-GMm}{R}+\frac{1}{2}m\left(\frac{v_e}{2}\right)^2 = U + 0$$

$$\frac{-GMm}{R}+\frac{1}{2}m\left(\frac{GM}{2R}\right)=U$$

$$U = \frac{-GMm}{R}+\frac{GMm}{4R} = \frac{-3GMm}{4R}$$

(b) Looking at the expression of U obtained above it is easy to conclude that the particle is at a distance of $4R/3$ from the center of the earth when it is at its maximum height. It means that the particle is at a distance $R/3$ from the surface of the earth. At half the maximum height the particle is at a distance R/6 from the surface, i.e., at a distance $7R/6$ from the center of the earth.

Kinetic energy at this height = Total energy – Potential energy

$$= \frac{-3GMm}{4R}-\frac{-6GMm}{7R}=\frac{3GMm}{28R}$$

37. Acceleration due to gravity at the pole is

$g = \dfrac{GM}{R^2}$, since there is no effect of rotation at the pole.

Acceleration due to gravity at the equator is

$$g_e = g-\omega^2 R = g-\frac{v^2}{R}$$

$$\frac{g}{3}=g-\frac{v^2}{R}$$

$$\frac{2g}{3}=\frac{v^2}{R} \quad \text{-------} \quad (1)$$

A body at the pole has total energy - $E = \dfrac{1}{2}mV^2 - \dfrac{GMm}{R}$

It escapes when this energy is zero.

$$\frac{1}{2}mV_e^2 - \frac{GMm}{R}=0$$

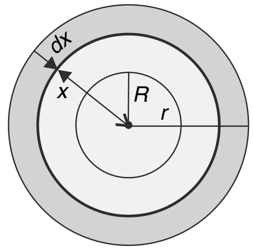
$$V_e = \sqrt{\frac{2GM}{R}}=\sqrt{2gR} = \sqrt{3v^2} = \sqrt{3}v$$

38. Let's first calculate the mass of the atmosphere between $R \le x < r$

$$m = \int_R^r \rho 4\pi x^2 dx$$

$$= 4\pi\sigma_\circ \int_R^r \frac{x^2}{x}dx \qquad \left[\because \rho = \frac{\sigma_\circ}{x}\right]$$

$$= 2\pi\sigma_o [r^2 - R^2]$$



Acceleration due to gravity at a distance r from the centre is

$$g = \frac{G(M+m)}{r^2}$$

Since, the whole mass distribution (planet + atmosphere is spherically symmetric)

$$\therefore\ g = \frac{GM}{r^2} + \frac{G2\pi\sigma_\circ}{r^2}\left[r^2 - R^2\right]$$

$$= \frac{GM}{r^2} - \frac{G2\pi\sigma_\circ R^2}{r^2} + G.2\pi\sigma_\circ$$

This expression is independent of r if

$$GM = G.2\pi\sigma_\circ R^2 \Rightarrow \sigma_\circ = \frac{M}{2\pi R^2}$$

$$\therefore g_{\text{const}} = G.2\pi\sigma_\circ = \frac{GM}{R^2}$$

Alter: Acceleration due to gravity on the surface $g = \frac{GM}{R^2}$

$$\frac{\Delta g}{g} = \frac{\Delta M}{M} - \frac{2\Delta R}{R}$$

But $\Delta g = 0$

$$\therefore\ \frac{\Delta M}{M} = \frac{2\Delta R}{R}$$

$$\frac{4\pi R^2 . \Delta R \frac{\sigma_\circ}{R}}{M} = \frac{2\Delta R}{R}$$

$$\Rightarrow \sigma_\circ = \frac{M}{2\pi R^2}$$

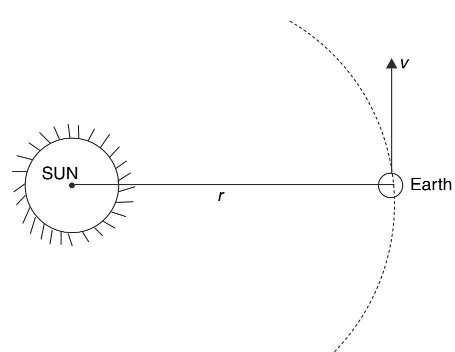
39. The projectile will escape out if

$$\frac{1}{2}mv^2 = \frac{GM_s m}{r} + \frac{GM_e m}{R_e}$$

$$v = \sqrt{2G\left(\frac{M_e}{R_e} + \frac{M_s}{r}\right)}$$

$$v = \sqrt{2 \times 6.67 \times 10^{-11}\left[\frac{6 \times 10^{24}}{6.4 \times 10^{6}} + \frac{2 \times 10^{30}}{1.5 \times 10^{11}}\right]}$$

$= 43.6 \times 10^3\ ms^{-1} = 43.6\ km/s$



Speed of the earth orbiting around the sun is $v_E = \frac{2\pi r}{(365d)} = \frac{2\times 3.14\times 1.5\times 10^{11}\ m}{365\times 24\times 60\times 60\ s} \simeq 30\ km/s$

$\therefore$ Projection speed relative to the earth = 43.6 – 30 = 13.6 *km/s*

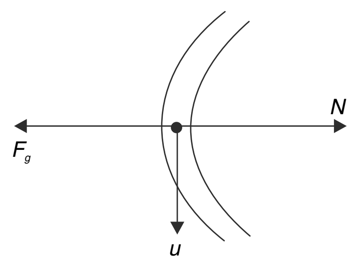
40. (a) From energy conservation, it is easy to see that

$$v_0 = v_{escape} = \sqrt{\frac{2GM}{R}}$$

(b) Gravitational potential at a distance $r = \frac{R}{2}$

$$V = -\frac{GM}{R^3}\left[\frac{3}{2}R^2 - \frac{1}{2}r^2\right] = -\frac{GM}{R^3}\left[\frac{3}{2}R^2 - \frac{R^2}{8}\right] = -\frac{11}{8}\frac{GM}{R}$$

Speed of the ball when it is at distance $\frac{R}{2}$ from the centre is given by



$$\frac{1}{2}mu^2 - \frac{11}{8}\frac{GMm}{R} = 0$$

$$\Rightarrow u = \sqrt{\frac{11}{4}\frac{GM}{R}}$$

$$N - F_g = \frac{mu^2}{R}$$

$$N = \frac{mu^2}{R} + F_g = \frac{11}{4}\frac{GMm}{R^2} + \frac{GMm}{(R/2)^2}$$

$$N = \frac{27}{4}\frac{GMm}{R^2}$$

41. A celestial body not bound to the sun has sufficient *KE* to travel infinitely far away from it. To get its minimum speed at any distance *r*, we can assume that it has $v = 0$ when it is infinitely far away from the sun.

$$\therefore\ \frac{1}{2}mv^2 - \frac{GMm}{r} = 0$$

$$\therefore v = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2\times 6.67\times 10^{-11}\times 2\times 10^{30}}{1.5\times 10^{11}}}$$

$= 4.2 \times 10^4\ m/s$

42. A body projected with escape speed will have total energy equal to zero. Let the speed of the body be *v* when it is at a distance *x* from the centre of the earth. Energy conservation gives- $\frac{1}{2}mv^2 - \frac{GMm}{x} = 0$

$$\Rightarrow \frac{v^2}{2} - \frac{GM}{x} = 0 \qquad \Rightarrow v = \frac{\sqrt{2GM}}{x} \quad \Rightarrow \frac{dx}{dt} = \frac{\sqrt{2GM}}{\sqrt{x}}$$

$$\Rightarrow \int_R^{9R} \sqrt{x}\,dx = \sqrt{2GM}\int_0^t dt$$

$$\Rightarrow \frac{2}{3}\left[x^{3/2}\right]_R^{9R} = \sqrt{2GM}.t$$

$$\Rightarrow \frac{2}{3}(26)R^{3/2} = \sqrt{2GM}.t$$

$$t = \frac{52}{3}\frac{R^{3/2}}{\sqrt{2GM}} = \frac{52\ \sqrt{R}}{3\sqrt{\frac{2GM}{R^2}}}$$

$$= \frac{52}{3}\sqrt{\frac{R}{2g}}$$

43. (a) As the rock moves up from the lunar surface, its *KE* decreases and *PE* increases. There is a point in its path where the gravitational field of the earth balances the field due to the moon (say this point is at a distance *x* from the centre of the earth). Beyond this point the *KE* of the rock once again begins to increases as gravity of the earth becomes more powerful. Hence, *PE* is maximum at distance *x* from the centre of the earth.

$$\frac{GM}{x^2} = \frac{GM/81}{(60R-x)^2}$$

$$\Rightarrow \left(\frac{60R-x}{x}\right)^2 = \frac{1}{81}$$

$$\Rightarrow \frac{60R-x}{x} = \frac{1}{9}$$

$$\Rightarrow 54R = x$$

$$U_{\max} = -\frac{GMm}{54R} - \frac{G\frac{M}{81}m}{6R} \qquad [\because 60R - 54R = 6R]$$

$$= -\frac{GMm}{R}\left[\frac{1}{54}+\frac{1}{81\times 6}\right] = -\frac{GMm}{54R}\left[1+\frac{1}{9}\right] = -\frac{5GMm}{243R}$$

(b) If stone is projected such that its speed is just zero when it is at a distance of *x* from the earth, it will reach the surface of the earth with least *KE*

$$K_{\min} + PE_{\text{near earth}} = PE_{at\ x}$$

$$K_{\min} - \frac{GMm}{R} - \frac{G\frac{M}{81}m}{59R} = -\frac{GMm}{54R} - \frac{G\frac{M}{81}m}{6R}$$

$$K_{\min} = \frac{GMm}{R}\left[1+\frac{1}{81\times 59}-\frac{1}{54}-\frac{1}{81\times 6}\right] = \frac{GMm}{R}\left[\frac{81\times 59\times 6+6-9\times 59-59}{81\times 59\times 6}\right]$$

$$K_{\min} = \frac{14045}{14337}\frac{GMm}{R}$$

44. (a) $T \propto r^{3/2} \quad \Rightarrow \omega \propto r^{-3/2}$

$$\therefore \frac{\Delta\omega}{\omega} = -\frac{3}{2}\frac{\Delta r}{r}$$

$\Delta\omega = -\frac{3}{2}\left(\frac{\Delta r}{r}\right)\omega$ [-ve sign indicates that angular speed of satellite is less than that of the earth]

$= -\frac{3}{2}\left(\frac{1km}{42000km}\right).\frac{2\pi}{24\,hour}$ $= -9.3 \times 10^{-6}$ *rad/hr*

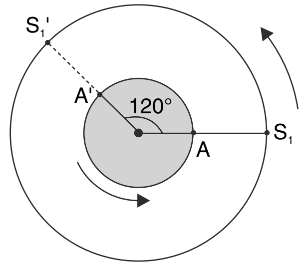
(b) $\Delta\theta = \Delta\omega.t$

$= \left(9.3\times10^{-6}\frac{rad}{hr}\right)\times(365\times24\,hr)$ = 0.08 rad = 4.6°

45. In 8 hour the earth rotates through 120° and point A on the equator moves to A'. The satellite will scan the entire equator if it completer $\left(1+\frac{1}{3}\right)$ revolutions in 8 hrs

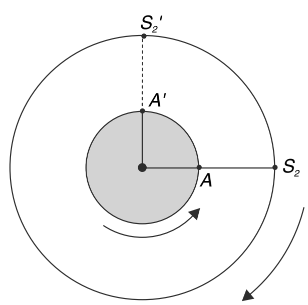
$\therefore\ \frac{4}{3}T_1 = 8$

T_1 = 6 hour = time period of revolution of S_1

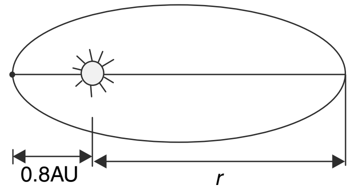


For S_2, the satellite can scan the entire equator in 6 hours if it completes $\frac{3}{4}$th revolution in that time $\frac{3}{4}T_1 = 6 \ \Rightarrow T_1 = 8$ hour

$\frac{r_1^3}{r_2^3} = \frac{T_1^{\,2}}{T_2^{\,2}}$ Hence, $\frac{r_1}{r_2} = \left(\frac{6}{8}\right)^{2/3} = \left(\frac{3}{4}\right)^{2/3}$



46. Let the semi major axis of the elliptical path be a



$T^2 \propto a^3$

For earth $(1y)^2 \propto (1\ AU)^3$

[AU is mean distance between the earth and the Sun]

$\therefore\ \frac{T^2}{(1\,y)^2} = \frac{a^3}{(1\,AU)^3}$

$\therefore T^2 = a^3$ [where T is an year and a is in AU]

$\Rightarrow a = T^{2/3} = (64)^{2/3} = 16\ AU$

If r = maximum distance of the comet from the sun, then $a = \dfrac{r+0.8}{2}$

$\Rightarrow 32 - 0.8 = r$

$\Rightarrow r = 31.2\ AU$

47. Let radius of circular orbit of the Earth and Venus be r_e and r_v respectively

$$\left(\frac{r_e}{r_v}\right)^3 = \left(\frac{365}{220}\right)^2 \quad \text{[Kepler's third law]}$$

$$\Rightarrow \frac{r_e}{r_v} = (2.75)^{1/3} = 1.4$$

From the drawing given in the problem

$$\frac{M'N'}{MN} = \frac{N'V}{NV}$$

$$M'N' = MN\left(\frac{N'V}{NV}\right) = 1000 \times \frac{r_v}{r_e - r_v} = 1000 \times \frac{1}{\frac{r_e}{r_v} - 1}$$

$$= 1000 \times \frac{1}{1.4 - 1} = 2500\ km$$

48. **Hint:** At the point of intersection both the satellites have same PE. Since they have same mechanical energy, their KE will be same at the point of intersection.

49. $$E_i = -\frac{GMm}{2r} = -\frac{GMm}{8R}$$

$$E_f = -\frac{GMm}{2r}$$

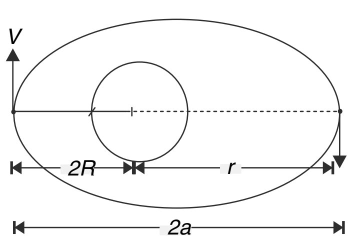
Loss in energy $\Delta E = -\dfrac{GMm}{8R} - \left(-\dfrac{GMm}{2R}\right) = \dfrac{3GMm}{8\ R}$

$$\therefore \eta t = \frac{3GMm}{8R}$$

$$t = \frac{3GMm}{8\eta R}$$

50. Energy of the satellite $-\dfrac{GMm}{2a} = \dfrac{1}{2}mV^2 - \dfrac{GMm}{2R}$

$$-\frac{GMm}{2a} = \frac{3GMm}{10\ R} - \frac{GMm}{2R}$$

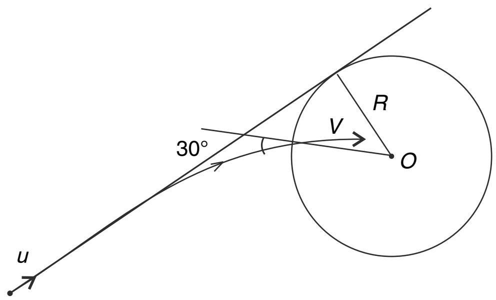


$$-\frac{1}{2a} = \frac{3}{10R} - \frac{1}{2R}$$

$\Rightarrow 2a = 5R$

$\therefore\ r = 5R - 2R = 3R \qquad \therefore\ h_{max} = 3R - R = 2R$

51.



The velocity (V) makes 30° angle with the radius (normal) near the surface.

Conservation of angular momentum (about centre O)

$mVR \sin 30° = muR$

$\therefore\ V = 2\,u$ ………………….(i)

Energy Conservation

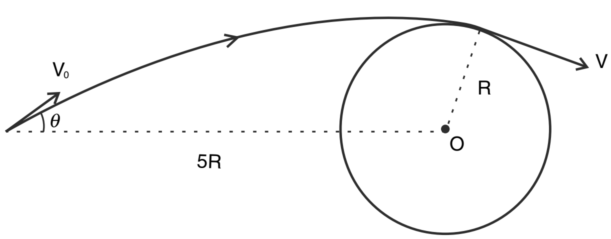
$$\frac{1}{2}mu^2 = \frac{1}{2}mV^2 - \frac{GMm}{R}$$

$$\frac{1}{2}u^2 = \frac{1}{2}(2u)^2 - \frac{GM}{R}$$

$$\Rightarrow \quad \frac{3}{2}u^2 = \frac{GM}{R}$$

$$\Rightarrow \quad u = \sqrt{\frac{2GM}{3R}}$$

52.



Let V = speed where the asteroid grazes the earth's surface.

Conservation of angular momentum about O gives

$V = mVR = mV_0 \sin\theta.(5R)$

$\therefore\quad V = \frac{5}{2}\ V_0$ ………………(i)

Energy Conservation

$$\frac{1}{2}mV_0^2 - \frac{GMm}{5R} = \frac{1}{2}\ mV^2 - \frac{GMm}{R}$$

$$\therefore \frac{V^2}{2} - \frac{V_0^2}{2} = \frac{GM}{R}\left(1-\frac{1}{5}\right)$$

$$\left(\frac{25}{8}-\frac{1}{2}\right)V_0^2 = \frac{4}{5}\frac{GM}{R}$$

$$\frac{21}{8}V_0^2 = \frac{4}{5}\frac{GM}{R}$$

$$V_0 = \sqrt{\frac{32}{105}\frac{GM}{R}}$$

53. $V_0 = \sqrt{\frac{GM}{r}}$ ……………..(i)

Speed after firing of the rocket is given by- $V^2 = V_0^2 + \Delta V^2$

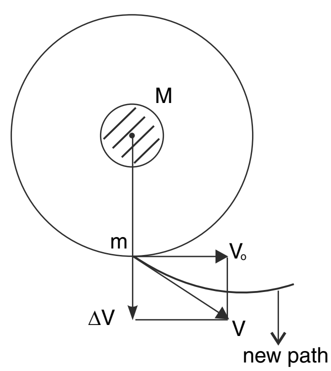
For escape $\frac{1}{2}mV^2 - \frac{GMm}{r} = 0$

$$\frac{V_0^2 + \Delta V^2}{2} = \frac{GM}{r}$$

$$V_0^2 + \Delta V^2 = 2V_0^2$$

$$\Delta V = V_0$$

$$\frac{\Delta V}{V_0} = 1$$



54. Angular momentum does not change (with respect to the centre of the earth)

$mV_1(2r_0) = mV_0 r_0$

$\Rightarrow 2V_1 = V_0$ ……………..(i)

Energy conservation

$$\frac{1}{2}mV^2 - \frac{GMm}{r_0} = \frac{1}{2}mV_1^2 - \frac{GMm}{2r_0}$$

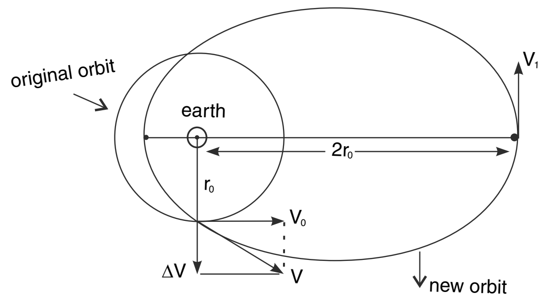
$$V^2 - \frac{2GM}{r_0} = \frac{V_0^2}{4} - \frac{GM}{r_0}$$

$$V_0^2 + \Delta V^2 - \frac{V_0^2}{4} = \frac{GM}{r_0} \qquad \left[\because V^2 = V_0^2 + \Delta V^2\right]$$

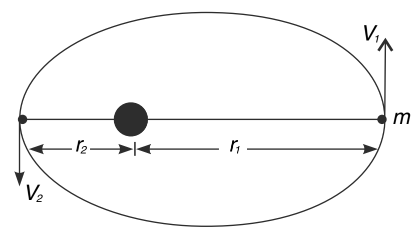
$$\frac{3}{4}V_0^2 + \Delta V^2 = V_0^2 \qquad \left[\because V_0 = \sqrt{\frac{GM}{r_0}}\right]$$

$$\Delta V^2 = \frac{V_0^2}{4}$$

$$\frac{\Delta V}{V_0} = \frac{1}{2}$$



55. (a) Speed is maximum at perigee (= V_2). Applying conservation of angular momentum between apogee and perigee–



$$mV_1r_1 = m.V_2r_2$$

$$\therefore \ V_1 = V_2\frac{r_2}{r_1} \qquad \ldots\ldots\ldots\ldots \ (1)$$

<u>Energy Conservation</u>

$$-\frac{GMm}{r_2} + \frac{1}{2}mV_2^2 = -\frac{GMm}{r_1} + \frac{1}{2}V_1^2$$

$$\frac{1}{2}V_2^2 - \frac{GM}{r_2} = \frac{1}{2}V_1^2 - \frac{GM}{r_1}$$

Using (1)

$$\frac{1}{2}V_2^2 - \frac{GM}{r_2} = \frac{1}{2}\left(\frac{r_2}{r_1}V_2\right)^2 - \frac{GM}{r_1} = \frac{1}{2}\left(\frac{r_2}{r_1}\right)^2 V_2^2 - \frac{GM}{r_1}$$

$$\therefore \ \frac{1}{2}V_2^2\left[1-\left(\frac{r_2}{r_1}\right)^2\right] = GM\left[\frac{1}{r_2}-\frac{1}{r_1}\right]$$

$$V_2^2\left[\frac{(r_1+r_2)(r_1-r_2)}{r_1^2}\right] = 2GM\left[\frac{r_1-r_2}{r_1r_2}\right]$$

$$V_2 = \sqrt{\frac{2GM}{r_2}\left(\frac{1}{1+{r_2}/{r_1}}\right)}$$

(b) For moon

$$V_2 = \sqrt{\frac{2\times 6.67\times 10^{-11}\times 6\times 10^{24}}{3.6\times 10^8}\left(\frac{1}{1+\frac{3.6}{4}}\right)}$$

$= 1.08 \times 10^3$ *m/s*

$= 1.08$ *km/s*

56. (a) Speed of the satellite at perigee is maximum. If the satellite is moving at speed V and rocket is fired to give an impulse J, then change in momentum of the satellite is J.

$\Delta p = J$

The resulting change in kinetic energy can be calculated as

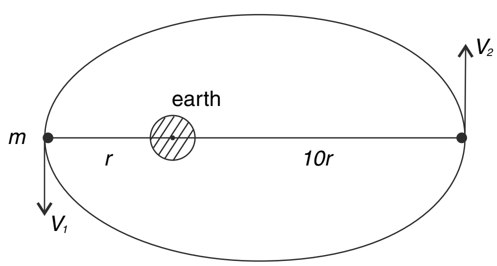
$$K = \frac{p^2}{2m} \qquad [p = mv]$$

$$\Delta K \simeq \frac{2p}{2m}\ \Delta p$$

$\Delta K = VJ$

Hence, change in kinetic energy is maximum when speed V of the satellite is maximum. This happens at perigee.

(b)



Let the speed of the satellite at perigee and apogee be V_1 and V_2 respectively.

Conservation of angular momentum

$mV_1r = mV_2(10r)$

$\Rightarrow V_1 = 10V_2$ ……………………(1)

Energy Conservation

$$\frac{1}{2}mV_1^2 - \frac{GMm}{r} = \frac{1}{2}mV_2^2 - \frac{GMm}{10r}$$

$$V_1^2\left(1-\frac{1}{100}\right) = \frac{9}{10}\frac{GM}{r} \qquad \text{[Using (1)]}$$

$$V_1 = \sqrt{\frac{10}{11}\frac{GM}{r}} \simeq 0.95\sqrt{\frac{GM}{r}}$$

For perigee distance = r and apogee distance = 12 r – we can again solve as above to get

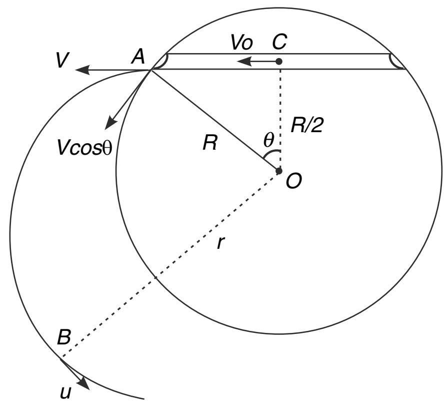
$$V_1' = \sqrt{\frac{132}{143}\frac{GM}{r}} \simeq 0.96\sqrt{\frac{GM}{r}}$$

Hence, required impulse at perigee is

$$J = m\left(V_1' - V_1\right) = 1.01\, m\sqrt{\frac{GM}{r}}$$

58. (a) No. The tunnel wall applies a normal force which produces a torque about the centre.

(b) Let the speed of the projectile be V when it comes out of the tunnel.



Energy conservation can be applied between point C and A.

PE at a distance x $(x < R)$ is given by $-\dfrac{GMm}{R^3}\left[\dfrac{3}{2}R^2 - \dfrac{1}{2}x^2\right]$

$\therefore K_A + U_A = K_C + U_C$

$$\frac{1}{2}mV^2 - \frac{GMm}{R} = \frac{1}{2}m\left(\sqrt{gR}\right)^2 - \frac{GMm}{R^3}\left[\frac{3}{2}R^2 - \frac{R^2}{8}\right]$$

Simplifying after substituting $g = \dfrac{GM}{R^2}$, we get

$$V^2 = \frac{GM}{4R} \quad \Rightarrow V = \frac{1}{2}\sqrt{\frac{GM}{R}}$$

When the projectile is at farthest distance its velocity u is perpendicular to its position vector r relative to the centre of the earth. Applying conservation of angular momentum between A and B we get–

$$mur = m\left(V\cos\theta\right)R \quad \Rightarrow \quad ur = \frac{VR}{2}$$

$$\Rightarrow \; ur = \frac{1}{4}\sqrt{GMR} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Energy conservation between A and B

$$\frac{1}{2}mu^2 - \frac{GMm}{r} = \frac{1}{2}mV^2 - \frac{GMm}{R}$$

$$\frac{1}{2}m\left(\frac{\sqrt{GMm}}{4r}\right)^2 - \frac{GMm}{r} = \frac{1}{2}m\frac{1}{4}\frac{GM}{R} - \frac{GMm}{R}$$

$$\frac{R}{32r^2} - \frac{1}{r} = -\frac{7}{8R}$$

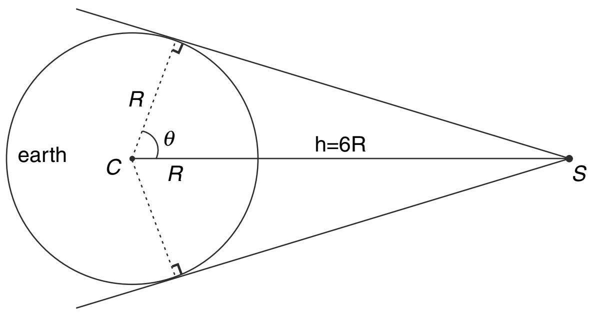
$$\Rightarrow \; 28r^2 - \left(32R\right)r + R^2 = 0$$

$$\therefore \quad r = \frac{32R \pm \sqrt{(32R)^2 - 28 \times 4R^2}}{56}$$

$$= \left(\frac{8 \pm \sqrt{57}}{14}\right)R$$

$$\therefore \; r_{\max} = \left(\frac{8 + \sqrt{57}}{14}\right)R$$ The other solution is less than R and not acceptable.

59.



With satellites (S) at vertex imagine a cone inside which the spherical earth fits in. The surface area of the earth inside this cone will receive the communication signals from the satellite.

From the figure $\cos\theta = \dfrac{R}{7R} = \dfrac{1}{7}$

The solid angle subtended by the surface exposed to signal from the satellite at the centre of the sphere is

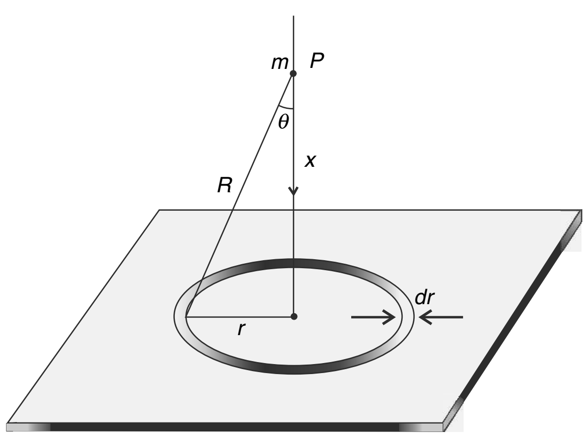
$$\Omega = 2\pi(1 - \cos\theta) = 2\pi\left(1 - \frac{1}{7}\right) = \frac{12\pi}{7} \quad \text{Steradian}$$

$\therefore$ Required area

$S = \Omega.R^2$

$$S = \frac{12}{7}\pi R^2$$

60. (a) Consider a ring element on the sheet as shown in figure. Mass of the ring element is –



$dm = \sigma\, 2\pi\, r\, dr$

Force on mass m placed at P due to ring will be perpendicular to the sheet equal to

$$dF = \frac{Gm\,.\,dm}{R^2}\cdot\cos\theta = \frac{G.m.\sigma.2\pi\ rdr.x}{R^3}$$

$$= G.2\pi\ \sigma\, m.x\ \frac{rdr}{\left(x^2+r^2\right)^{3/2}}$$

Force will be equal to sum of forces due to all concentric rings which make up the sheet.

$$\therefore\ F = G2\pi\sigma\, m\, x \int_{r=0}^{r=\infty} \frac{rdr}{\left(x^2+r^2\right)^{3/2}}$$

$$= -\ G2\pi\sigma\, m\ x\left[\frac{1}{\sqrt{x^2+r^2}}\right]_{r=0}^{r=\infty}$$

$$= G2\pi\sigma\, m \qquad \text{[Independent of distance } x\text{]}$$

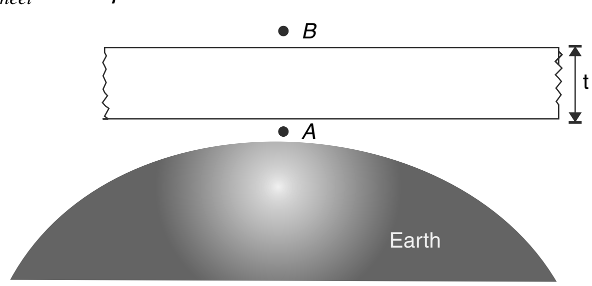
Note: Students may also use gauss' theorem for gravitation to arrive at the result.

(b) From the result obtained above, we can say that field due to a thin large sheet, of mass density σ per unit area, is $G.2\pi\sigma$

If the sheet is of thickness t, then also the result will remain same because the sheet can be sliced into infinite number of thin sheets each producing field $G.2\pi\sigma$. [Remember this field is independent of distance from the sheet]

And $\sigma = \rho t$

$\therefore$ Field due to sheet $E_{sheet} = G2\pi\rho t$



Resultant field at A is

$$E_A = \frac{GM}{R^2} - 2\pi G\rho t$$

Field at B is; $E_B = \dfrac{GM}{(R+t)^2} + 2\pi G\rho t$

Where mass of earth $M = \dfrac{4}{3}\pi R^3 \rho_0$

As per question $E_A > E_B$

$$\frac{GM}{R^2} - 2\pi G\rho t > \frac{GM}{(R+t)^2} + 2\pi G\rho t$$

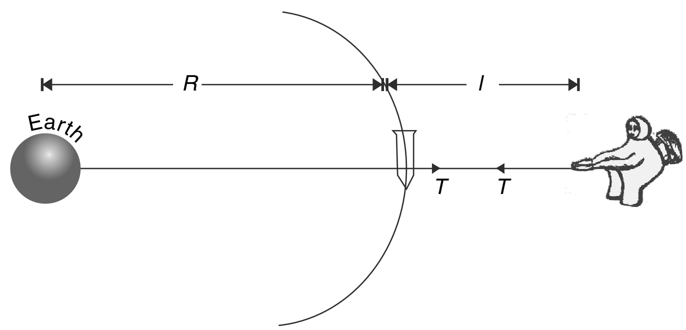
$$\Rightarrow \quad \frac{4}{3}\pi R^3 \rho_0 \left[\frac{1}{R^2} - \frac{1}{R^2\left(1+\frac{t}{R}\right)^2}\right] > 4\pi \ \rho t$$

$$\Rightarrow \quad \rho_0 \ R\left[1-\left(1+\frac{t}{R}\right)^{-2}\right] > 3\rho t$$

We can write $\quad \left(1+\frac{t}{R}\right)^{-2} \simeq 1-\frac{2t}{R}$

$$\therefore \ \rho_0 \ R\left[1-1+\frac{2t}{R}\right] \ > \ 3\rho t \qquad \Rightarrow \ \rho_0 > \frac{3}{2}\rho$$

61.



M = mass of earth, m_0 = mass of spaceship, T = tension in string

For spaceship

$$G\frac{Mm_0}{R^2} - T = m_0\omega^2 R \qquad \text{...............}(i)$$

For Astronaut

$$\frac{GMm}{(R+\ell)^2} + T = m\omega^2 \ (R+\ell) \quad \text{...............}(2)$$

From (1) and (2) and using $R + \ell = R'$

$$\frac{GM}{R^3} - \frac{T}{m_0 R} = \frac{GM}{(R')^3} + \frac{T}{m\ R'}$$

$$\Rightarrow \quad T\left[\frac{1}{mR'} + \frac{1}{m_0 R}\right] = GM\left(\frac{1}{R^3} - \frac{1}{R'^3}\right)$$

$$\Rightarrow \quad T = \left(\frac{GM\, mm_o}{m_o R + mR'}\right)\left(\frac{1}{R^3} - \frac{1}{R'^3}\right)$$

This gives $R'^3 - R^3 = (R' - R)(R'^2 + RR' + R^2)$

$$\simeq \ l(3R^2) \qquad\qquad \left[\because \ R'^2 \simeq R^2 \simeq R'R\right]$$

$$\therefore \ T \ \simeq \frac{GM \ \ mm_o \ \ 3\ell R^2}{(m_o + m)R^5} = \frac{GM}{R^3}\cdot\frac{3mm_o\,\ell}{m_o + m}$$

$$\simeq \frac{GM}{(2R_e)^3} \cdot 3ml \qquad [\because m_o + m \simeq m_o]$$

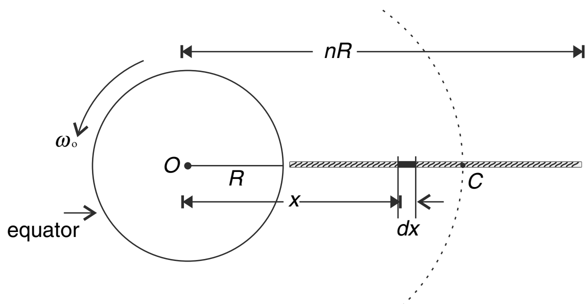
$$= \frac{GM}{R_e^2} \cdot \frac{3\,m\ell}{8R_e}$$

$$= g \cdot \frac{3}{8}\frac{m\ell}{R_e}$$

$$= 9.8 \times \frac{3}{8} \times \frac{100 \times 200}{6400 \times 1000}$$

$= 0.01\ N$

62.



The rod can stay above the same point only if it rotates with angular speed of earth from west to East (just like geostationary satellites). The necessary centripetal force has to be provided by the gravitational pull of the earth.

Distance of centre of the rod from the centre of the earth is

$$r = \frac{(n-1)R}{2} + R = \frac{(n+1)}{2}R$$

Mass of the rod $m = \lambda\,(n-1)\,R$

Where λ = mass per unit length.

Gravitational pull on the rod can be calculated by writing force on an element of length dx.

$$dF = \frac{GM\,(\lambda dx)}{x^2} \qquad [M = \text{mass of earth}]$$

Total force on the rod

$$F = GM\lambda \int_R^{nR} \frac{dx}{x^2} = \frac{GM\lambda}{R}\left[\frac{n-1}{n}\right]$$

$$\therefore\ F = m\omega_o^2\, r$$

$$\frac{GM\lambda}{R}\left[\frac{n-1}{n}\right] = \lambda(n-1)R \cdot \omega_o^2 \left(\frac{n+1}{2}\right)R$$

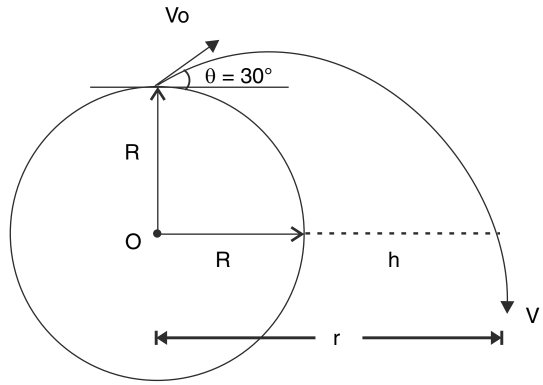
But $M = \frac{4}{3}\pi R^3 \rho$

$$\therefore\ \frac{4}{3}\frac{G\pi\rho}{n} = \omega_0^2 \frac{(n+1)}{2}$$

$$\therefore\ n^2 + n = \frac{8}{3}\frac{\pi\, G\rho}{\omega_o^2}$$

63. (a) Escape velocity $V_o = \sqrt{\dfrac{2GM}{R}}$

$\therefore$ Velocity of projection



$$V_o = \frac{1}{2}\sqrt{\frac{2GM}{R}} = \sqrt{\frac{GM}{2R}} \qquad \text{..................(i)}$$

When R = radius of earth, M = Mass of earth

At the point where the body is farthest & nearest to earth's centre, its velocity is $\perp r$ to the position vector with respect to earth's centre.

$\therefore$ Angular momentum when it is farthest/nearest to earth's centre is = mVr

Where r = distance from centre

V = velocity at farthest/nearest point

Angular momentum of body (about O) at the time of projection is

$$mV_0 \cos 30°.R = \frac{\sqrt{3}}{2} mRV_0$$

Since Angular momentum is conserved

$$mVr = \frac{\sqrt{3}}{2} mRV_0$$

$$\Rightarrow \quad Vr = \frac{\sqrt{3}}{2} RV_0$$

$$\Rightarrow \quad V = \frac{\sqrt{3}}{2}\frac{RV_0}{r} \qquad \text{......................(ii)}$$

From conservation of energy

$$\frac{1}{2} mV_0^2 - \frac{GMm}{R} = \frac{1}{2} mV^2 - \frac{GMm}{r}$$

or, $$V_0^2 - \frac{2GM}{R} = \frac{3}{4}\frac{R^2}{r^2}V_0^2 - \frac{2GM}{r}$$

Using (i) $$\frac{GM}{2R} - \frac{2GM}{R} = \frac{3}{4}\frac{R^2}{r^2}\frac{GM}{2R} - \frac{2GM}{r}$$

$$\Rightarrow \frac{1}{2R} - \frac{2}{R} = \frac{3}{8}\frac{R}{r^2} - \frac{2}{r}$$

$$\Rightarrow \frac{-3}{2R} = \frac{3R-16r}{8r^2}$$

$$\Rightarrow 12r^2 - 16Rr + 3R^2 = 0$$

or, $$r = \frac{16R \pm \sqrt{256R^2 - 144R^2}}{24} = \left(\frac{4 \pm \sqrt{7}}{6}\right)R \text{ ....(ii)}$$

$\because$ h cannot be less then R hence,

$$\therefore\ h = \left(\frac{4+\sqrt{7}}{6}\right)R - R = \left(\frac{\sqrt{7}-2}{6}\right)R$$

(b) From (iii) $r_{\min} = \left(\frac{4-\sqrt{7}}{6}\right)R \ < \ R$

Hence, the body will move on elliptical path but it will hit the surface of earth at some point.

64. (a) Speed of dust particles before collision with satellite $\simeq 0$

Speed of dust particles after collision with satellite = V

Where V is the orbital velocity of the satellite. Satellite encounters *(VS d) kg* of dust per sec

$\therefore$ Force experienced = change in momentum per sec

$F = (VS\,d)\,(V) = V^2 Sd$

$= (8 \times 10^3)^2 \times 0.5 \times 1.6 \times 10^{-11} \simeq 5 \times 10^{-4}\ N$

[$\because$ orbital speed of near surface satellite $\simeq$ 8 *km/sec*]

(b) Total energy of the satellite

$$E = -\frac{1}{2}MV^2 = -\frac{GMMe}{2R} \qquad \begin{bmatrix} R = \text{radius of earth} \\ = \text{path radius} \end{bmatrix}$$

$$E = -\frac{GMMe}{2R}$$

$$\Rightarrow \Delta E = +\frac{GMMe}{2R^2}\Delta R$$

$$\Rightarrow \Delta R = \frac{2R^2}{GMM_e}\Delta E$$

But change in energy in one revolution ΔE = work done by F

$= -2\pi RF$

$$\therefore\ \Delta R = \frac{2R^2}{GMM_e}\left(-2\pi R F\right)$$

$$= -\frac{4\pi R^3 F}{GMMe} = \frac{-3}{GM}\left(\frac{\frac{4}{3}\pi R^3}{Me}\right) = \frac{-3F}{GM\rho}$$

Given ρ = mean density of earth = 5500 kg/m^3

$$\therefore \; \Delta R = \frac{-3 \times 5 \times 10^{-4}}{6.67 \times 10^{-11} \times 10 \times 5500} = -0.4 \; km$$

– *ve* sign indicates reduction in radius.

Similarly, $E = -\frac{1}{2} MV^2$

$\Delta E = -MV \, \Delta V$

$\Rightarrow \Delta V = -\frac{\Delta E}{MV} = \frac{2\pi RF}{MV} \simeq 0.25 \; m/s$ [Velocity increases]

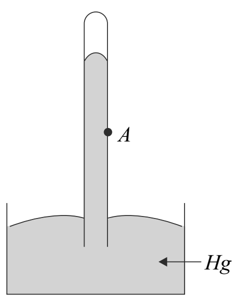
08 FLUIDS

LEVEL 1

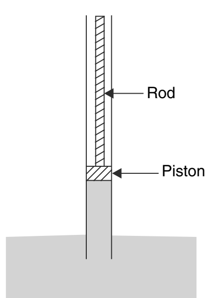
Q. 1. We know that the atmospheric pressure on the surface of the earth is because of weight of the air. The radius of the earth is 6400 *km* and atmospheric pressure on the surface of earth is $1 \times 10^5\ N/m^2$.Estimate the mass of the earth's atmosphere assuming that acceleration due to gravity remains constant at $10\ m/s^2$ over the entire height of the atmosphere.

Q. 2. Why mercury is used in a barometer, though it is costly? Why cannot we use water in place of mercury.

Q. 3 Look at the barometer shown in the figure. If a small hole is developed in the wall of the tube at point A, will the mercury leak out of it?



Q. 4.



A tightly fitted piston can slide along the inner wall of a long cylindrical pipe. With the piston at the lower end of the pipe, the lower end of the pipe is dipped into a large tank, filled with water. Now the piston is pulled up with the help of the rod attached to it. Water rises in the pipe along with the piston. Why? To what maximum height water can be raised in the pipe using this method? What will be the answer to your question if water is replaced with mercury? Atmospheric pressure is $P_{atm} = 1.01 \times 10^5\ Pa$.

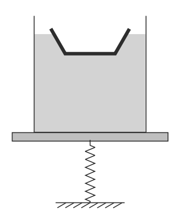
Q. 5. A hypothetical planet has an ocean of water which is 50 *km* deep. The top 5 *km* is frozen as ice (i.e, 45 *km* is water). Radius and average density of the planet are both half the respective values for the earth. There is no atmosphere. Obtain an estimate of the pressure at the bottom of the ocean.

Q. 6. Two identical beakers are filled with water. One of them has an ice block floating in it. The level of water in both the beakers is same. Which beaker will weigh more? Will your answer change if water is replaced with a liquid of higher density in the beakers?

Q. 7. (i) A toy boat made of steel is floating in a beaker having water. The beaker is placed on a spring balance. The boat tilts and sinks into water.

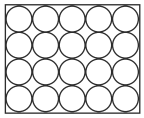
(a) Will the level of water in the beaker go up or fall down?

(b) Will the reading of spring balance decrease or increase?

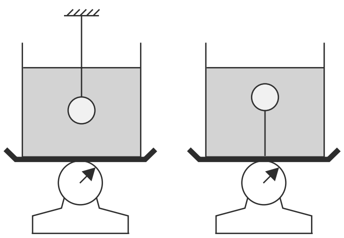


(ii) You are in a boat on a calm lake. There is a floating log near you. You pick the log and put it into the boat. What happens to the level of water in the lake? Does it rise or fall?

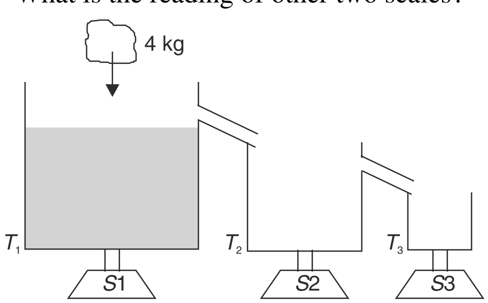
Q. 8. A closed cubical box of negligible mass has large number of spherical balls arranged neatly inside it as shown in the figure. When placed in water, the box floats with 80% of its volume remaining submerged. What is specific gravity of material of the balls? Neglect thickness of the wall of the box.



Q. 9. Two identical containers have the same volume of water in it. Each of them is placed on a balance and readings of the two balances are same. There is a hollow ball and a solid ball that have same volume. The hollow ball floats in water and the solid ball sinks. A string from the ceiling suspends the solid ball so that it remains completely submerged in the water in the first container. The hollow ball is held submerged in the water in the second container and is held by a string fastened to the bottom of the container. Which balance will show higher reading? How will your answer change if the string in the second container is cut?



Q. 10. Three tanks T_1, T_2 and T_3 are sitting on three weighing scales S_1, S_2 and S_3 respectively. Tank T_1 has a spout, as shown and water has been filled in it to a level just below the spout. The other two tanks are empty. Reading of the three scales are 20 *kg*, 4 *kg* and 3 *kg* respectively. A 4 *kg* body is put into the tank T_1 and it floats in the water. Now the reading of scale S_3 was found to be 4.5 *kg*. What is the reading of other two scales?

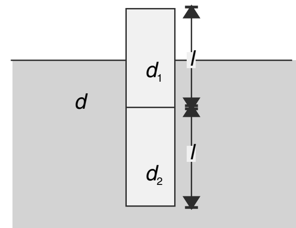


Assume that the water in the system remains inside three tanks.

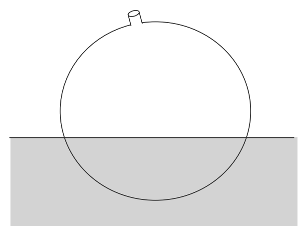
Q. 11. A cylindrical block of length $2l$ is made of two different materials. The upper half has density d_1 and lower half, which is heavier, has density d_2. The block is floating in a liquid of unknown density d with $\frac{l}{2}$ of its length outside the liquid.

(a) Find d

(b) Show that $d > \frac{4d_1}{3}$

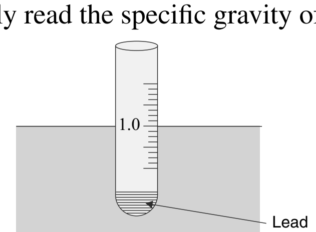


Q. 12. A sealed balloon, filled with air, floats in water with $\frac{1}{3}$ of its volume submerged. It was found that if it is pushed inside water at a depth h, it remains in equilibrium, neither sinking nor rising. Find h. Given that height of water barometer is 10 *m* and temperature is constant at all depth.



Q. 13. Long back our Earth was made of molten material. Assume it to be a uniform sphere of radius R having density d. Take acceleration due to gravity at the surface to be g and calculate the gauge pressure (P_0) at the centre of this fluid Earth. Calculate P_0 for following data: R = 6000 *km*; $d = 5500\ kg\ m^{-3}$ and $g = 10\ ms^{-2}$.

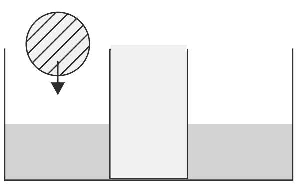
Q. 14. A device used to measure the specific gravity of a liquid is called a hydrometer. In a simple hydrometer there is a cylindrical glass tube with some lead – weight at its bottom. The device floats in liquid while remaining vertical. The top part of the tube extends above the liquid and the divisions marked on the tube allows one to directly read the specific gravity of the liquid.



The scale on the tube is calibrated such that in pure water it reads 1.0 at the water surface and a length z_0 of the tube is submerged. Calculate the specific gravity of the liquid if the liquid level is Δz above the 1.0 mark. Disregard the curvature of the tube bottom.

Q. 15. A sphere of radius R and having negligible mass is floating in a large lake. An external agent slowly pushes the sphere so as to submerge it completely. How much work was done by the agent? Density of water is ρ.

Q. 16. Two identical communicating containers have water filled into them. A spherical ball of ice (relative density = 0.9) having volume 100 cm^3 is put into the left vessel. Calculate the volume of water flowing into the right container, immediately after placing the ball (i.e., don't consider any melting of the ice ball). Give your answer for following two cases



(i) The ice ball floats in the water in the left container.

(ii) The ice ball gets exactly half immersed in the water.

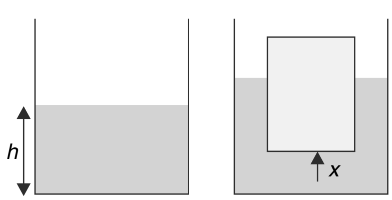
(iii) What will happen to the water level after the ice melts? Answer for both (i) and (ii) above.

Q. 17. An open cylindrical container has a cross sectional area $A_0 = 150\ cm^2$ and water has been filled in it up to a height h. A cylinder made of wood (relative density = 0.6) having cross sectional area $A = 125\ cm^2$ and length 10 cm is now placed inside the container with its axis vertical. Find the distance (x) of the base of the wooden cylinder from the base of the container in equilibrium for following three cases :

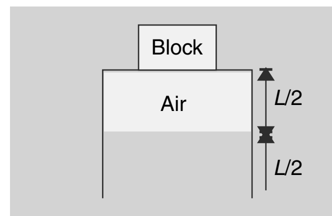
(a) $h = 8\ cm$

(b) $h = 12\ cm$

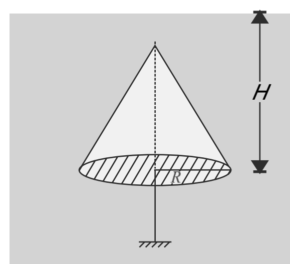
(c) $h = 0.8\ cm$



Q. 18. A light cylindrical tube of length $L = 1.5\ m$ and radius $r = \frac{1}{\sqrt{\pi}}\ m$ is open at one end. The tube containing air is inverted and pushed inside water as shown in figure. A block made of material of relative density 2 has been placed on the flat upper surface of the tube and the whole system is in equilibrium. Neglect the weight of air inside the tube and find the volume of block placed on the tube.



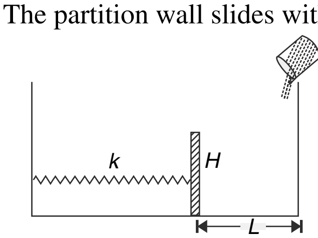
Q. 19. A solid wooden cone has been supported by a string inside water as shown in the figure. The radius of the circular base of the cone is R and the volume of the cone is v. In equilibrium the base of the cone is at a depth H below the water surface. Density of wood is d ($< \rho$, density of water).



(a) Find tension in the string.

(b) Find the force applied by the water on the slant surface of the cone. Take atmospheric pressure to be P_0

Q. 20. A large container has a sliding vertical wall of height H so as to divide it into two parts. The partition wall is connected to the left container wall by an ideal spring of force constant k. When the spring is relaxed the dimensions of the floor of the right part is $L \times b$. Now water (density ρ) is slowly poured into the right chamber. What is the maximum volume of water that can be stored in the right chamber without spilling it into the other part. The partition wall slides without friction.



Q. 21. A cylindrical container has cross sectional area of 0.20 m^2 and is open at the top. At the bottom, it has a small hole (A) kept closed by a cork. There is an air balloon tied to the bottom surface of the container. Volume of balloon is 2.2 litre. Now water is filled in the container and the balloon gets fully submerged. Volume of the balloon reduces to 2.0 litre. The cork is taken out to open the hole and at the same moment the whole container is dropped from a large height so as to fall under gravity. Assume that the container remains vertical. Find the change in level of water inside the falling container 2 second after it starts falling.

Q. 22. A wooden stick of length L, radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m that would make the stick float vertically in equilibrium in a liquid of density σ.

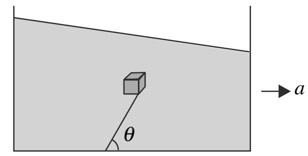
Q. 23. A rod of length 6 m has a mass 12 kg. It is hinged at one end at a distance of 3 m below water surface.

(a) What weight must be attached to the other end of the rod so that a length of rod equal to 5 m is submerged in water in equilibrium?

(b) Find the magnitude and direction of the force exerted by the hinge on the rod. (Specific gravity of rod is 0.5).

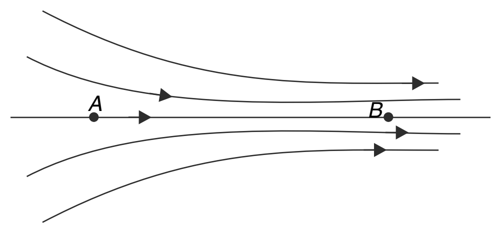
Q. 24. Assume that a car travelling on horizontal straight road with an acceleration of a 5 ms^{-2} has all its windows rolled up and all air vents closed. Length of the car is $L = 3.0\,m$. By considering a horizontal tube of air that extends from the windshield to the rear surface, and applying Newton's Law on it, calculate the difference in pressure of air at the rear and front of the car. How does this pressure compare with the atmospheric pressure ? Density of air $\rho = 1.2\ kg\,m^{-3}$.

Q. 25. A container partially filled with water is moved horizontally with acceleration $a = \dfrac{g}{3}$. A small wooden block of mass m is tied to the bottom of the container using a string. The block remains inside water with the string inclined at an angle θ to the horizontal. Assuming that the density of wood is half the density of water, find the angle θ and the tension in the string.

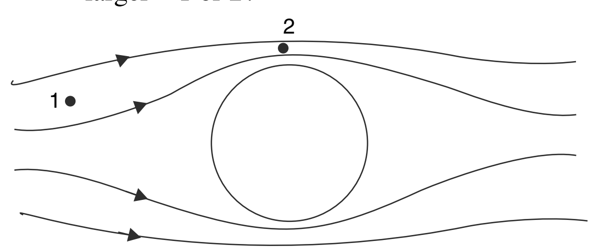


Q. 26. A cylindrical tank having radius R is half filled with water having density ρ. There is a hole at the top of the tank. The tank is moved horizontally, perpendicular to its length, with a constant acceleration equal to the acceleration due to gravity (g). Find the maximum pressure exerted by water at any point on the tank. Atmospheric pressure is P_0. Assume that there is no spillage.

Q. 27. In a steady two dimensional flow of incompressible fluid streamlines are as shown in figure. At which point – A or B – the pressure is higher? Assume the flow to be in a horizontal plane.

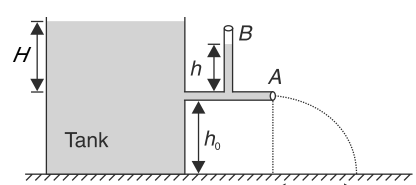


Q. 28. (i) A ball is projected in still air. With respect to the ball the streamlines appear as shown in the figure. At which point is the pressure larger – 1 or 2?



(ii) In the above figure if the ball is also spinning in clockwise sense, in which direction it will get deflected – up or down?

Q. 29. (i) In the arrangement shown in the figure, the tank has a large cross section and the pipes have much smaller cross sections. The opening at A is unplugged and the water jet hits the ground surface at a horizontal distance x.



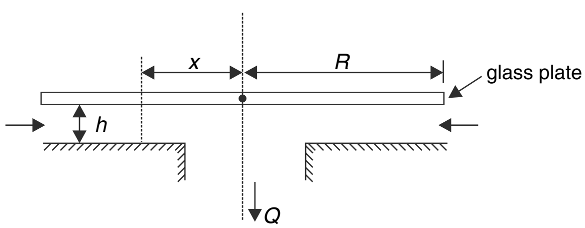
(a) Find the level of water (h) in the tube B as water flows out of A.

(b) Find x.

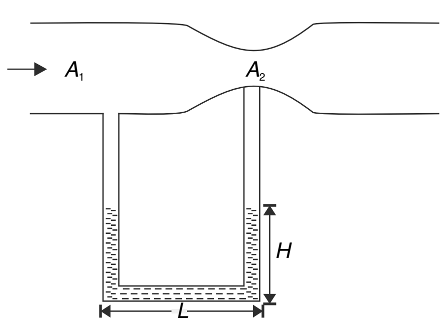
(ii) A flat horizontal surface has a small hole at its centre. A circular glass plate of radius R is placed symmetrically above the hole with a small gap h remaining between the plate and the surface. A liquid enters the gap symmetrically from all sides and after travelling radially through the gap finally exits from the hole. The volume flow rate of the liquid coming out from the hole is Q (in m^3s^{-1}).

(a) If the flow speed just inside the circumference of the circular plate is V_0 find the speed (V_x) of flow inside the gap at a distance x (see figure) from the centre of the hole.

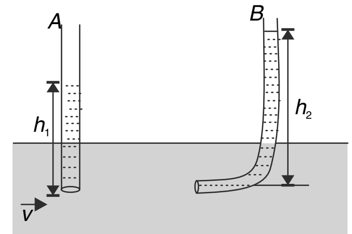
(b) Write V_x in terms of Q, h and x.



Q. 30. A horizontal tube having cross sectional area $A_1 = 10\ cm^2$ has a venturi connected to it having cross sectional area $A_2 = 4\ cm^2$. A manometer, having mercury as its liquid is connected to the tube as shown in the figure. The manometer tube has uniform cross section and it has a horizontal part of length $L = 10\ cm$. When there is no flow in the tube the height of mercury column in both vertical arms is $H = 12\ cm$. Calculate the minimum flow rate (in m^3/s) of air through the tube if it is required that the entire amount of mercury move to one vertical arm of the manometer. Given: density of $Hg = 13.6 \times 10^3\ kg\ m^{-3}$; density of air = $1.2\ kg\ m^{-3}$.



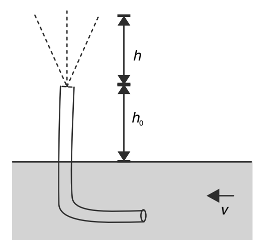
Q. 31. A liquid is flowing in a horizontal pipe of uniform cross section at a speed v. Two tubes A an B are inserted into the pipe as shown. Assume the flow to remain streamline inside the pipe.



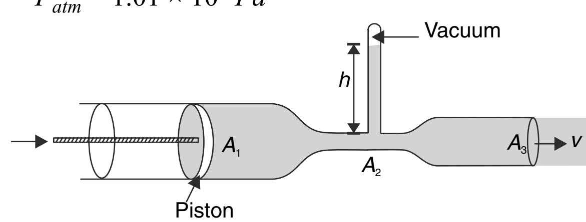
(a) The diagram depicts that height of liquid in tube B (= h_2) is more than the height of liquid in tube A (= h_1). Is it correct?

(b) Calculate the difference in height of the liquid in two tubes.

Q. 32. Water is flowing in a stream at speed v. A L shaped tube is lowered into the stream as shown. The upper end of the tube is held at a height h_0 above the surface of the water. To what height 'h' above the upper end of the tube, will the water jet spurt? Assume that flow remains ideal.



Q. 33. A horizontal glass tube is filled with mercury. The tube has three different cross sections as shown; with $A_1 = 18\ cm^2$, $A_2 = 8\ cm^2$ and $A_3 = 9\ cm^2$. The piston is pushed so as to throw out mercury at a constant speed of $v = 6\ m/s$ at the other end of the tube. Assume that mercury is an ideal fluid with density $\rho_{Hg} = 1.36 \times 10^4\ kg/m^3$. Take $g = 10\ m/s^2$. $P_{atm} = 1.01 \times 10^5\ Pa$



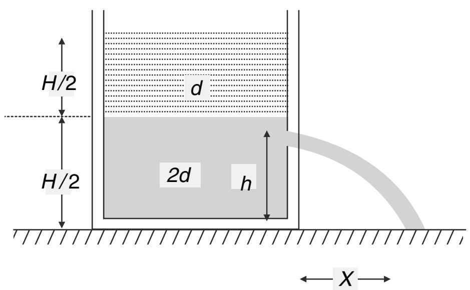
(i) Find the force needed to push the piston assuming that friction force between the piston and the tube wall is $f = 40\ N$

(ii) Find the height (h) of mercury column in the attached vertical tube. What happens to this height if the piston is pushed with smaller speed?

Q. 34. A container of large uniform cross-section area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$ each of height $(H/2)$ as shown in figure. The lower density liquid is open to the atmosphere having pressure P_0.

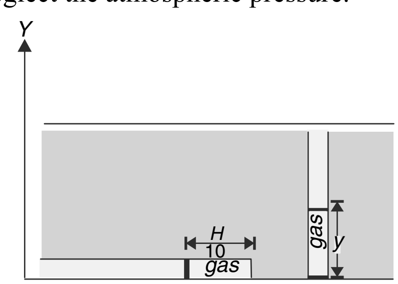
(a) A homogeneous solid cylinder of length L ($L < H/2$), cross-sectional area ($A/5$) is immersed such that it floats with its axis vertical at the liquid-liquid interface with length ($L/4$) in the denser liquid. Determine (i) The density of solid and (ii) The gauge pressure at the bottom of the container.

(b) The cylinder is removed and original arrangement is restored. A tiny hole of area $s(s << A)$ is punched on the vertical side of the container at a height $h(h < H/2)$. Determine (i) the initial speed of efflux of the liquid at the hole (ii) the horizontal distance x travelled by the liquid initially and (iii) the height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m.



LEVEL 2

Q. 35. A lake filled with water has depth H. A pipe of length slightly less than H lies at the bottom of the lake. It contains an ideal gas filled up to a length of $\frac{H}{10}$. A smooth light movable piston keeps the gas in place. Now the pipe is slowly raised to vertical position (see figure). Assume that temperature of the gas remains Constant and neglect the atmospheric pressure.



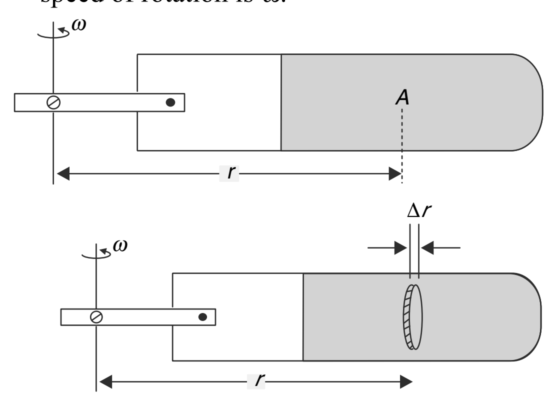
(a) Plot the variation of pressure inside the lake as a function of height y from the base. Let the height of piston from the base, after the pipe is made vertical, be y. Plot the variation of gas pressure as a function of y in the first graph itself.

(b) In equilibrium the gas pressure and the pressure due to water on the piston must be equal. Using this solve for equilibrium height y_0 of the piston. You get two answers. Which one is correct and why?

Q. 36. A centrifuge has a horizontal cylinder rotating about a vertical axis as shown in the figure. Water inside it has density ρ.

(a) Consider a point A, inside the liquid, at a distance r from the rotation axis. Liquid pressure at this point is P. Write the value of $\frac{dP}{dr}$ when the cylinder, with all its liquid, rotates uniformly at an angular speed ω.

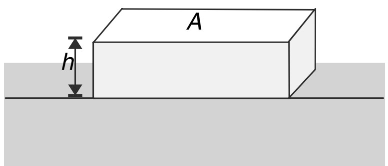
(b) Consider a small disc shaped foreign material inside the centrifuge at point A. The area of circular disc is ΔS and its thickness is Δr. It is made of material of density $\rho'(> \rho)$. The disc is in position so that its circular face is vertical. Find the radial acceleration of the disc with respect to the cylinder when angular speed of rotation is ω.



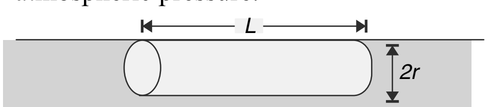
Q. 37. (i) A cubical metal block of side 10 cm is floating in a vessel containing mercury. The vessel has a square cross section of side length 15 cm. Water is poured into the vessel so that the metal block just gets submerged. Calculate the mass of water that was poured into the vessel. It is given that relative density of the metal and mercury are 7.3 and 13.6 respectively.

(ii) In the last question, in place of water if we poured another liquid of relative density 'r' it was found that when the metal block was just completely submerged it was no longer touching mercury. What is value of r?

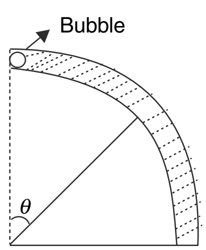
Q. 38. A wooden block of cross sectional area A (in shape of a rectangle) has height h. It is held such that its lower surface touches the water surface in a wide and deep tank. The block is released in this position. It oscillates for some time and then settles into its equilibrium position. In equilibrium the block floats with its upper face just on the water surface. Calculate the amount of heat generated in the process assuming that the loss in gravitational potential energy of the system comprising of water and the block gets converted into heat. Density of water is ρ.



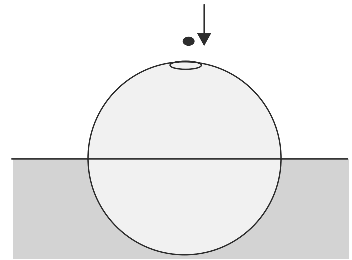
Q. 39. A cylindrical wooden log of length L and radius r is floating in water (density = ρ) while remaining completely submerged as shown in figure. Calculate the force of water pressure on the lower half of the cylinder. Exclude contribution due to atmospheric pressure.



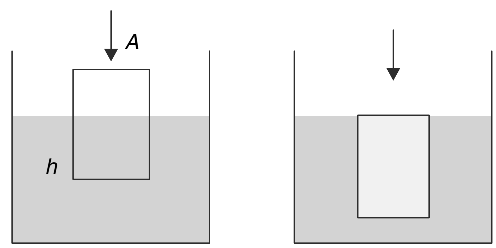
Q. 40. A bent tube contains water. An air bubble is trapped inside the liquid. The tube is held vertical (as shown) and is moved horizontally with an acceleration (a) such that the bubble moves to position θ shown in the diagram. Find the direction and magnitude of a.



Q. 41. A spherical pot with a small opening at top has a mass of M = 452.1 gram. It floats in water while remaining exactly half submerged A small coin of mass m = 5 gram is dropped into the pot. Find how much more the pot will sink. Express your answer in terms of change in height of the pot that is under water. Density of water = 1 g/cc.



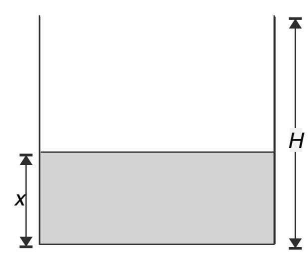
Q. 42. A cylindrical wooden block of density half the density of water is floating in water in a cylindrical container. The cross section of the wooden block and its height are A and h respectively. The cross sectional area of the container is $2A$. The wooden block is pushed vertically so that it gradually gets immersed in water. Calculate the amount of work done in pushing the block . Density of water = ρ_0.



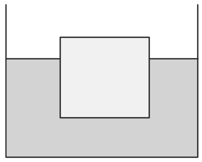
Q. 43. A cylindrical container has mass M and height H. The centre of mass of the empty container is at height $\frac{H}{2}$ from the base. A liquid, when completely filled in the container, has mass $\frac{M}{2}$. This liquid is poured in the empty container.

(a) How does the centre of mass of the system (container + liquid) move as the height (x) of liquid column changes from zero to H? Explain your answer qualitatively. Draw a graph showing the variation of height of centre of mass of the system (x_{cm}) with x.

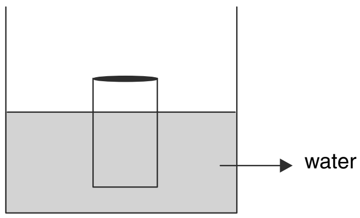
(b) Find the height of liquid column x for which the centre of mass is at its lowest position.



Q. 44. A cubical ice block of side length 'a' is floating in water in a beaker. Find the change in height of the centre of mass of (water + ice) system when the ice block melts completely. It is given that ratio of mass of water to mass of ice originally in the container is 4 : 1.



Q. 45. A cylindrical ice block is floating in water. 10% of its total volume is outside water. Kerosene oil (relative density = 0.8) is poured slowly on top of water in the container. Assume that the oil does not mix with water. Height of the ice cylinder is H.



(a) As kerosene is poured, how does the volume of ice block above the water level change?

(b) What is the thickness of kerosene layer above the water when 20% of the volume of the ice block is above the water surface?

(c) Find the ratio of volume of ice block in kerosene to its volume in water after the kerosene layer rises above the top surface of ice and the block gets completely submerged.

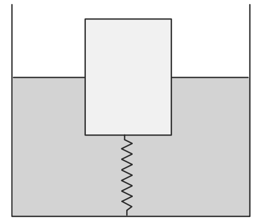
Neglect any melting of ice

Q. 46. A cylindrical container contains water. *A* cubical block is floating in water with its lower surface connected to a spring

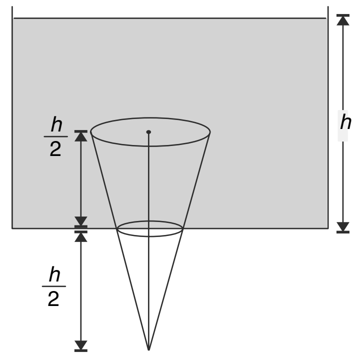
(a) Suppose that the spring is in relaxed state. Now, if the whole container is accelerated vertically upwards, will the spring get compressed?

(b) Suppose that the spring is initially compressed. Now, what will happen to the state of the spring when the container is accelerated upwards?

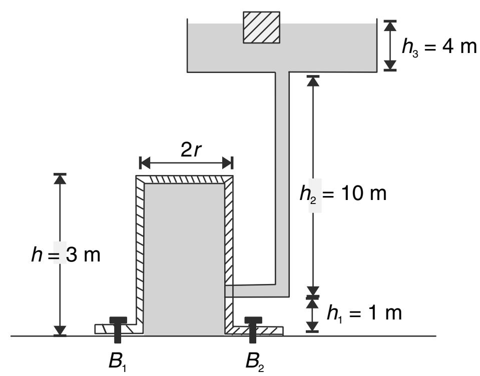
(c) Assume that mass of the block is 1 kg and initially the spring (force constant k = 100 N/m) is compressed by 5 cm. When the container is accelerated up by an acceleration of 5 m/s^2, the spring has a total compression of 6 cm. Calculate the change in volume of block submerged inside water when the container gets accelerated. Density of water is 10^3 kg/m^3.



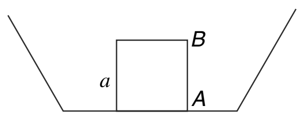
Q. 47. A water tank has a circular hole at its base. A solid cone is used to plug the hole. Exactly half the height of the cone protrudes out of the hole. Water is filled in the tank to a height equal to height of the cone. Calculate the buoyancy force on the cone. Density of water is ρ and volume of cone is V.



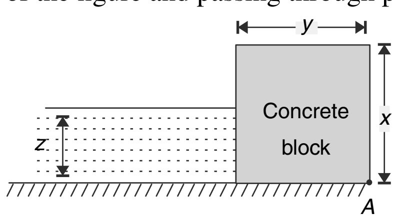
Q. 48. A cylindrical tank has a mass of the 200 kg and inner radius of r = 2.0 m. The tank has no bottom and is directly bolted to the floor [B_1 and B_2 are bolts in the figure]. The tank is connected to an elevated open tank and both the tanks are filled with a liquid as shown in the figure. Various heights are as shown. When a small wooden cube of specific gravity 0.6 is placed in the upper tank, it floats while remaining exactly half submerged. Calculate the force applied by the bolts in holding the tank.



Q. 49. (i) A non uniform cube of side length a is kept inside a container as shown in the figure. The average density of the material of the cube is 2 ρ where ρ is the density of water. Water is gradually filled in the container. It is observed that the cube begins to topple, about its edge (into the plane of the figure) passing through point A, when the height of the water in the container becomes $\frac{a}{2}$. Find the distance of the centre of mass of the cube from the face AB of the cube. Assume that water seeps under the cube.



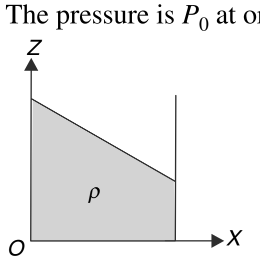
(ii) A rectangular concrete block (specific gravity = 2.5) is used as a retaining wall in a reservoir of water. The height and width of the block are x and y respectively. The height of water in the reservoir is $z = \frac{3}{4}x$. The concrete block cannot slide on the horizontal base but can rotate about an axis perpendicular to the plane of the figure and passing through point A



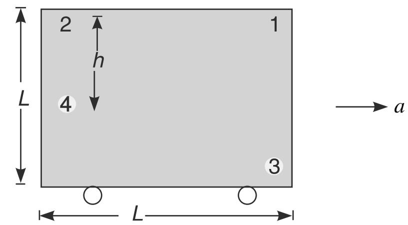
(a) Calculate the minimum value of the ratio $\frac{y}{x}$ for which the block will not begin to overturn about A.

(b) Redo the above problem for the case when there is a seepage and a thin film of water is present under the block. Assume that a seal at A prevents the water from flowing out underneath the block.

Q. 50. A container having an ideal liquid of density ρ is moving with a constant acceleration of $\vec{a} = a_x\hat{i} + a_z\hat{k}$ where x direction is horizontal and z is vertically upward. The container is open at the top. In a reference frame attached to the container with origin at bottom corner (see figure), write the pressure at a point inside the liquid at co-ordinates (x, y, z). The pressure is P_0 at origin.



Q. 51. A cubical container of side length L is filled completely with water. The container is closed. It is accelerated horizontally with acceleration a. Density of water is ρ.



(a) Assuming pressure at point 1 [upper right corner] to be zero, find pressure at point 2 [upper left corner]

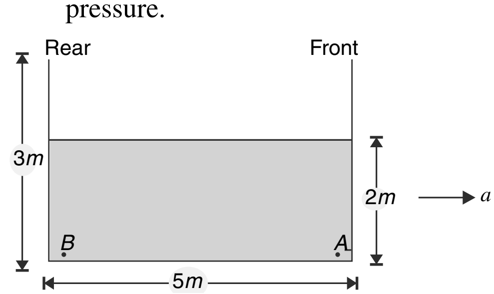
(b) Pressure at point 4, at a distance h vertically below point 2, is same as pressure at lower right corner 3. Find h.

Q. 52 An open rectangular tank $5m \times 4m \times 3m$ in dimension is containing water up to a height of $2m$ The tank is accelerated horizontally along the longer side. Assuming water to be an ideal liquid, find -

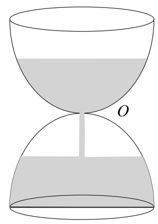
(a) the maximum acceleration with which the tank can be moved so that water does not fall from the rear side.

(b) the gauge pressure at the bottom of the front and back of the tank [points A and B] if the tank is closed at the top and is then accelerated horizontally at 9 m/s^2. Assume that the top cover has a small hole at the right side of the tank so that pressure of air inside the tank is maintained at atmospheric pressure.

Gauge pressure at a point is difference in absolute pressure at the point and atmospheric pressure.



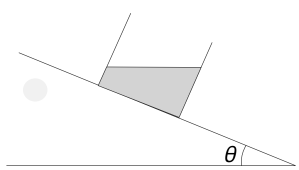
Q. 53. A water clock consist of a vessel which has a small orifice O. The upper container is filled with water which trickles down into the lower container. The shape of the (upper or lower) container is such that height of water in the upper container changes at a uniform rate. What should be the shape of the container? Assume that atmospheric air can enter inside the lower container through a hole in it and that the upper container is open at the top. Vessel is axially symmetric.



Q. 54. A rectangular container has been filled with an ideal fluid and placed on an incline plane. The inclination of the incline is θ. Find the angle that the liquid surface will make with the incline surface as the container slides down. Find your answer for following two cases.

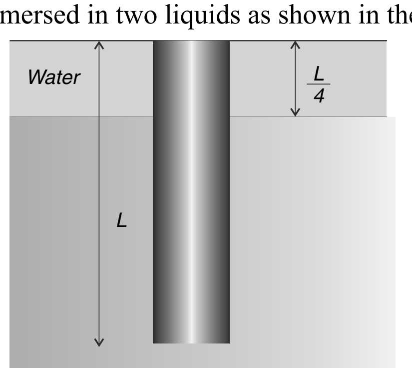
(a) Take the incline to be smooth.

(b) Assume a friction coefficient of μ (< tan θ) between the incline and the container.



Q. 55. A cylindrical vessel of radius $R = 1\ m$ and height $H = 3\ m$ is filled with an ideal liquid up to a height of $h = 2\ m$. The container with liquid is rotated about its central vertical axis such that the liquid just rises to the brim. Calculate the angular speed (ω) of the container.

Q. 56. A cylinder of length L floats with its entire length immersed in two liquids as shown in the fig.



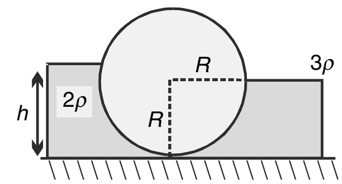
The upper liquid is water and the lower liquid has density twice that of water. The two liquid are immiscible. The cylinder is in equilibrium with its $\frac{3}{4}$ length in the denser liquid and $\frac{1}{4}$ of its length in water. The thickness of water layer is $\frac{L}{4}$ only. Find

(a) the specific gravity of the material of the cylinder.

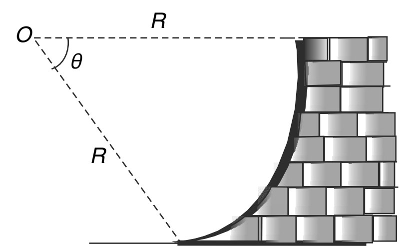
(b) the time period of oscillations if the cylinder is depressed by some small distance $\left(<\frac{L}{4}\right)$ and released.

Neglect viscosity and change in level of liquids when the cylinder moves.

Q. 57. (i) In the figure shown, the heavy cylinder (radius R) resting on a smooth surface separates two liquids of densities 2ρ and 3ρ. Find the height 'h' for the equilibrium of cylinder.



(ii) The cross section of a dam wall is an arc of a circle of radius $R = 20\ m$ subtending on angle of $\theta = 60°$ at the centre of the circle. The centre (O) of the circle lies in the water surface. The width of the dam [i.e., dimension perpendicular to the figure] is $b = 10m$. Neglect atmospheric pressure in following calculations.

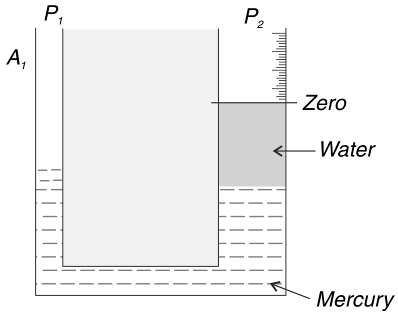


(a) Calculate the vertical component of force (F_v) applied by water on the curved dam wall.

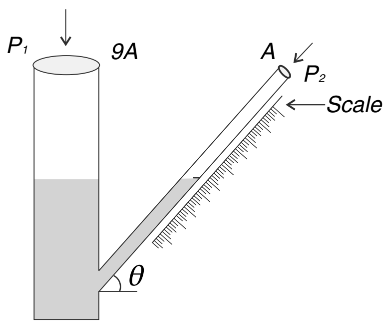
(b) Calculate the horizontal component of force (F_H) applied by water on the curved dam wall.

(c) Calculate the resultant force applied by the water on the curved dam wall.

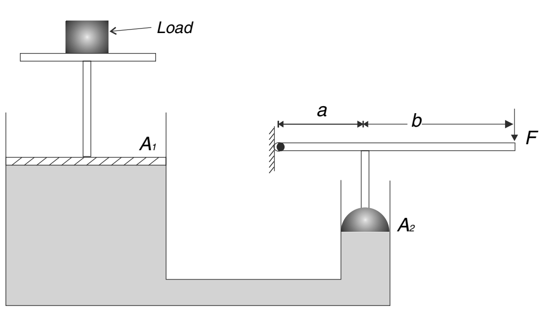
Q. 58. A monometer has mercury and water filled in it as shown in the figure. A scale is marked on the right tube which has a cross sectional area of A_2.The other tube has a cross sectional area of A_1. When pressures P_1 and P_2 at both ends of the manometer is same, the level of water in the right side is at the zero of the scale. When the applied pressure P_1 is changed by ΔP_0, the level of water surface changes by Δh. The quantity $\frac{\Delta h}{\Delta P_0}$ can be defined as pressure sensitivity(s) of the manometer. Calculate the pressure sensitivity of the given manometer. Density of water and mercury is ρ_ω and ρ_{Hg} respectively.



Q. 59. A manometer has a vertical arm of cross sectional area 9A and an inclined arm having area of cross section A. The density of the manometer liquid has a specific gravity of 0.74. The scale attached to the inclined arm can read up to $\pm$ 0.5 mm. It is desired that the manometer shall record pressure difference $(P_1 - P_2)$ up to an accuracy of $\pm$ 0.09 mm of water. To achieve this, what should be the inclination angle θ of the inclined arm.



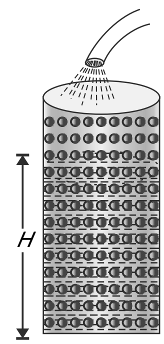
Q. 60. The figure shows a schematic layout of a hydraulic jack. The load to be raised weighs 20,000 N. The area of cross section of the two pistons are A_1 = 50 cm^2 and A_2 = 10 cm^2. The force (F) is applied at the end of a light lever bar as shown in the figure. Lengths a and b are 4 cm and 36 cm respectively. Find the force (F) required to raise the load slowly.



Q. 61. In a two dimensional steady flow the velocity of fluid particle at (x, y) is given by

$\vec{V} = (u_0 + bx)\hat{i} - by\hat{j}$; u_0 and b are positive constants. Write the equation of streamlines. Draw few streamlines for $x > 0$.

Q. 62. A cylindrical container having radius r has perforated wall. There are large number of uniformly spread small holes on the vertical wall occupying a fraction η = 0.02 of the entire area of the wall. To maintain the water level at height H in the container, water is being fed to it at a constant rate Q ($m^3\ s^{-1}$). Find Q.



Q. 63. A syringe is filled with water. Its volume is V_0 = 40 cm^2 and cross section of its interior is A = 8 cm^2. The syringe is held vertically such that its nozzle is at the top, and its piston is pushed up with constant speed. Mass of the piston is M = 50 g and the water is ejected at a speed of u_0 = 2 m/s. Cross section of the stream of water at the nozzle is a = 2 mm^2. Neglect friction and take the density of water to be $\rho = 10^3\ kg/m^3$

(a) Find the speed of the piston

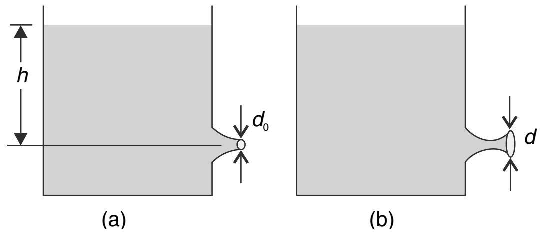
(b) Find the total work done by the external agent in emptying the syringe.

Q. 64. A water tank has a small hole in its wall and a tapering nozzle has been fitted into the hole (figure). The diameter of the nozzle at the exit is d_0 = 1 cm. The height of water in the tank above the central line of the nozzle is h = 2.0 m.

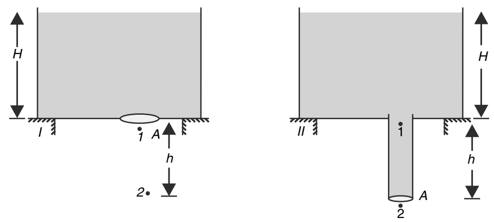
Calculate the discharge rate in $m^3\ s^{-1}$ through the nozzle.

Another nozzle which is diverging outwards is fitted smoothly to the first nozzle. The pressure at the neck of the two nozzles (where diameter is d_0) drops to 2.5 m of water. Calculate the exit diameter (d) of the nozzle.

Atmosphere pressure = 10 m of water and $g = 10\ ms^{-2}$



Q. 65. There are two large identical open tanks as shown in figure. In tank I there is a small hole of cross sectional area A at its base. Tank II has a similar hole, to which a pipe of length *h* has been connected as shown. The internal cross sectional area of the pipe can be considered to be equal

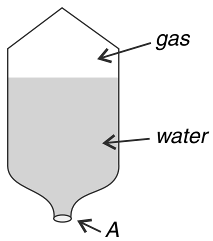


to A. Point 1 marked in both figures, is a point just below the opening in the tank and point 2 marked in both figures, is a point h below point 1 [In fig II, point 2 is just outside the opening in the pipe.].

(a) Find the speed of flow at point 2 in both figures.
(b) Find the ratio of speed of flow at point I is first figure to that in second figure.
(c) Find the difference in pressure at point 1 in both figures.

Q. 66. To illustrate the principal of a rocket, a student designed a water rocket as shown in the figure. It is basically a container having pressurized gas in its upper part and water in its lower part. Pressure of the gas is 4.0 Mpa. Mass of empty container is 1.0 *kg* and mass of its content is also1.0 *kg*. The nozzle at the bottom is opened to impart a vertical acceleration to the container. If it is desired that the initial upward acceleration of the container be 0.5g, what should be the cross sectional area (A) of the exit of the nozzle?

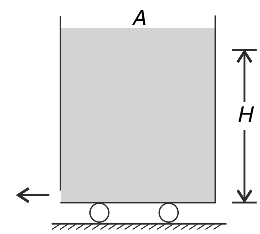
Neglect the pressure due to height of water in the container and take atmospheric pressure to be 1.0 MPa. $g = 10\ ms^{-2}$



Q. 67. An open tank of cross sectional area A contains water up to height *H*. It is kept on a smooth horizontal surface. A small orifice of area A_0 is punched at the bottom of the wall of the tank. Water begins to drain out. Mass of the empty tank may be neglected.

(i) Prove that the tank will move with a constant acceleration till it is emptied. Find this acceleration.

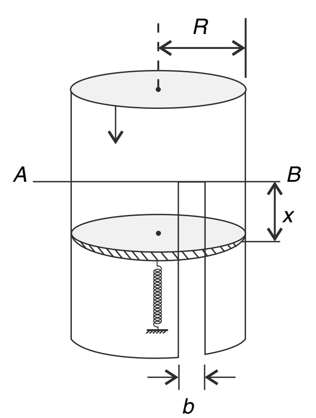
(ii) Find the find speed acquired by the tank when it is completely empty.



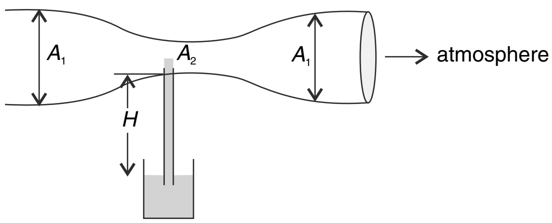
Q. 68. The device shown in figure can be used to measure the pressure and volume flow rate when a person exhales. There is a cylindrical pipe with inside radius *R*. There is a slit of width *b* running down the length of the cylinder. Inside the tube there is a light movable piston attached to an ideal spring of force constant *K*. In equilibrium position the piston is at a position where the slit starts (shown by line *AB* in the figure). A person is made to exhale into the cylinder causing the piston to compress the spring. Assume that slit width b is very small and the outflow area is much smaller than the cross section of the tube; even at the pistons full extension. A person exhales and the spring compresses by *x*. [Density of air = ρ]

(a) Calculate the gage pressure in the tube.

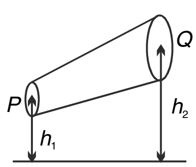
(b) Calculate the volume flow rate (Q) of the air.



Q. 69. (i) Air (density = ρ) flows through a horizontal venturi tube that discharges to the atmosphere. The area of cross section of the tube is A_1 and at the constriction it is A_2. The constriction is connected to a water (density = ρ_0) tank through a vertical pipe of length H. Find the volume flow rate (Q) of the air through tube that is needed to just draw the water into the tube.



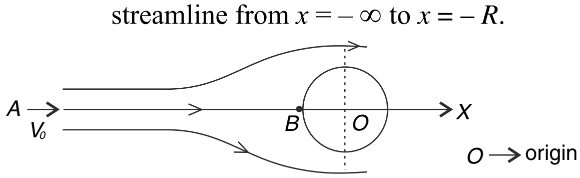
(ii) A non viscous liquid of constant density ρ flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross section of the tube at two points P and Q at heights of h_1 and h_2 are respectively A_1 and A_2. The velocity of the liquid at point P is v. Find the work done on a small volume ΔV of fluid by the neighbouring fluid as the small volume moves from P to Q.



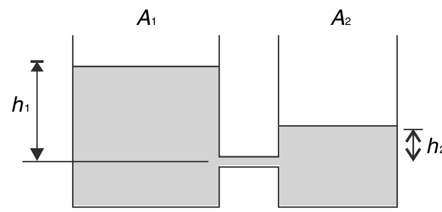
Q. 70. The figure shows a non- viscous, incompressible and steady flow in front of a sphere. A-B is a horizontal streamline. It is known that the fluid velocity along this streamline is given by $V = V_0\left(1+\frac{R^3}{x^3}\right)$. V_0 is velocity of flow on this streamline when $x \to (-\infty)$. It is given that pressure at $x \to (-\infty)$ is P_0 and density of liquid is ρ.

(i) Write the variation of pressure along the streamline from point A, far away from the sphere, to point B on the sphere.

(ii) Plot the variation of pressure along the streamline from $x = -\infty$ to $x = -R$.



Q. 71. There are two tanks next to each other having cross sectional area A_1 and A_2. They are interconnected by a narrow pipe having area of cross section equal to A_0. Initial height of water in the two tanks is h_1 and h_2 measured from the level of the pipe. Assume that the flow is ideal and behaves in a way similar to the discharge in air. Calculate the time needed for the water level in two tanks to become same.



Q. 72. A siphon is used to drain water (density = ρ) from a wide tank. The inlet and outlet mouth of the siphon are at the same horizontal level and the highest point of the siphon tube is at a height H from the mouth of the tube. Height of water in the tank above the tube mouth is h (see fig). Atmospheric pressure is P_0.

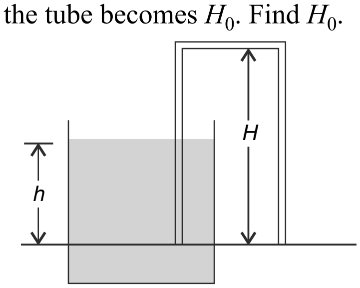
(a) Will the water drain out in this siphon? If yes, at what speed (V)?

(b) Find pressure at the top of the siphon tube (call it P')

(c) Find pressure just inside the left mouth of the tube.

(d) If left part of the tube is slightly cut short, without disturbing anything else, what effect it will have on V and P'?

(e) If the right end of the tube is lowered by adding more length of tube, it was observed that flow stops when length of right limb of the tube becomes H_0. Find H_0.



LEVEL 3

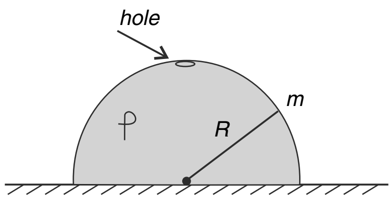
Q. 73. A vessel of volume V_0 is completely filled with a salt solution having specific gravity σ_0. Pure water is slowly added drop by drop to the container and the solution is allowed to overflow.

(a) Find the specific gravity of the diluted solution in the container when a volume V of pure water has been added to it.

(b) If $\sigma_0 = 1.2$ then find the specific gravity of the solution in the container after a volume V_0 of pure water has been added to it.

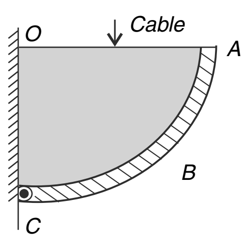
(c) Plot the variation of σ with V.

Q. 74. A hemispherical bowl of radius R is placed upside down on a flat horizontal surface. There is a small hole at the top of the inverted bowl. Through the hole, a liquid of density ρ is poured in. Exactly when the container gets full, water starts leaking from between the table and the edge of the container. Find the mass (m) of the container.



Q. 75. A plate is in the shape of a quarter cylinder of radius R and length L. This plate is hinged at C to a vertical wall and can rotate freely about C. The end A of the plate is tied to the wall using two horizontal cables [the other cables is parallel to OA and the two cables are placed symmetrically]. The space between the wall and the plate is filled completely with water (density = ρ). Neglect the weight of the plate and calculate the tension in each cable.

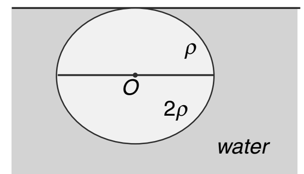
Take $\tan^{-1}\left(\frac{\pi}{2}\right) \simeq 57°$ and $\cos 57° \simeq \frac{1}{2}$



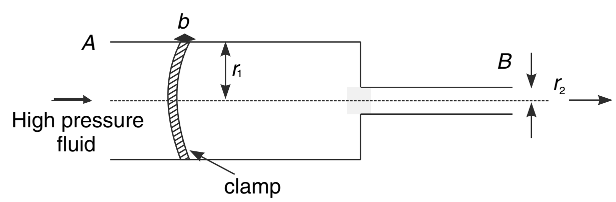
Q. 76. A spherical ball of radius R is made by joining two hemispherical parts. The two parts have density ρ and 2ρ. When placed in a water tank, the ball floats while remaining completely submerged.

(a) If density of water is ρ_0, find ρ

(b) Find the time period of small angular oscillations of the ball about its equilibrium position. Neglect viscous forces.



Q. 77. In a machine, a fluid from a compressor, which is at high pressure, is allowed to pass through a nozzle. Cross section of the nozzle is shown in the figure. The nozzle consists of two sections of radii r_1 and r_2. The nozzle is fixed to a stand with the help of a clamp. The clamp is a circular ring of radius r_1 and width b .The fluid from the compressor is at a pressure of n times the atmospheric pressure P_0. Assume that the entire system is horizontal, the fluid is ideal and the flow is steady.



(a) What should be the volume flow rate so that pressure of the fluid at end B reduces to half of its value at end A?

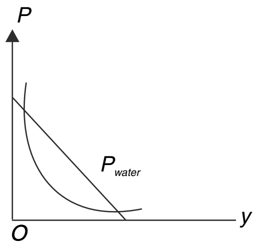
(b) If the entire system is kept in gravity free space and the net force on the nozzle due to the fluid flow is F then determine the minimum radial pressure that should be applied on the clamp, so the nozzle remains in place. Coefficient of friction between clamp and nozzle is μ.

(c) If a small hole is punched anywhere on the thinner part of the nozzle (close to end B) what should be the volume flow rate of the fluid so that it does not gush out?

ANSWERS

1. $5.14 \times 10^{18}\ kg$
2. Because *Hg* has high density. A water barometer will have a large height.
3. No
4. $10.30\ m$, $0.76\ m$
5. $P = 1.35 \times 10^7\ Pa$
6. Both beakers have same weight.
   Answer does not change.
7. (i) (a) Fall down
   (b) Not change
   (ii) Does not change.
8. 1.54
9. First balance will show higher reading.
   Answer will not change if string in 2nd container is cut.
10. Reading of $S_1 = 20\ kg$; Reading of $S_2 = 6.5\ kg$
11. (a) $d = \frac{2}{3}(d_1 + d_2)$
12. $20\ m$
13. $P_0 = \frac{1}{2} g.d.R = 1.65 \times 10^{11}\ Pa$
14. $\frac{z_0}{z_0 + \Delta z}$
16. (i) 45 cc
    (ii) 25 cc
    (iii) When ice melts there will be no change in water level in case (i) and the water level will rise in case (ii).
17. (a) $7\ cm$
    (b) 0
    (c) 0
18. $0.75\ m^3$
19. (a) $vg\ (d - \rho)$
    (b) $\pi R^2\ [\rho_0 + \rho gH] - v\rho g$
20. $Hb\left(L + \frac{\rho g b H^2}{2k}\right)$
21. Water level will rise by 1 *mm*.
22. $\pi R^2 L\rho\left(\sqrt{\frac{\sigma}{\rho}} - 1\right)$
23. (a) 2.33 kg
    (b) 56.7 N
24. $18\ Nm^{-2}$
25. $\theta = \tan^{-1}(3)$; $T = \frac{\sqrt{10}}{3} mg$
26. $P_0 + \sqrt{2}\,\rho gR$
27. *A*
28. (i) $P_1 > P_2$
    (ii) up
29. (i) (a) $h = 0$
    (b) $x = 2\sqrt{h_0 H}$
    (ii) (a) $V_x = \frac{R}{x} V_0$
    (b) $V_x = \frac{Q}{2\pi hx}$
30. $12.1\ m^3 s^{-1}$
31. (a) Yes
    (b) $h_2 - h_1 = \frac{v^2}{2g}$
32. $\frac{v^2}{2g} - h_0$
33. (i) $371.2\ N$
    (ii) 26 cm. Height h will increase
34. (a) (i) $\frac{5}{4} d$
    (ii) $\frac{1}{4}(6H + L)dg$
    (b) (i) $\sqrt{(g/2)(3H - 4h)}$
    (ii) $\sqrt{h(3H - 4h)}$
    (iii) $(3/8)H$, $\frac{3}{4} H$

35. (a)



(b) $y = \frac{H}{2} - \frac{\sqrt{15}}{10}H$

36. (a) $\frac{dP}{dr} = \rho\omega^2 r$

(b) $\left(1 - \frac{\rho}{\rho'}\right)\omega^2 r$

37. (i) 625 g
(ii) $r = 7.3$

38. $\frac{1}{2}\rho g A h^2$

39. $\rho g L r^2\left(\frac{\pi}{2} + 2\right)$

40. $a = g\tan\theta$ towards right

41. 0.4 mm

42. $W = \frac{1}{16}A\rho_0\, gh^2$

43. (a) The COM first falls, attains a minimum height and then it rises to original height $\frac{H}{2}$
(b) $x = (\sqrt{6} - 2)H$

44. $0.01a$

45. (a) First increases then become constant

(b) $\frac{H}{8}$

(c) 1.0

46. (a) No
(b) spring will get compressed more
(c) 100 cm^3

47. $V\rho g/8$

48. 179 KN

49. (i) $x = \frac{9}{8}$

(ii) (a) $\frac{3}{4\sqrt{10}}$

(b) $\frac{3}{4\sqrt{7}}$

50. $P = P_o - \rho a_x . x - \rho\,(g + a_z)z$

51. (a) $\rho a L$

(b) $h = L\left(1 - \frac{a}{g}\right)$

52. (a) $\frac{2g}{5} = 4\ m/s^2$
(b) $P_A = 0, P_B = 0.44\ atm$

53. The vessel can be obtained by revolution of a curve $z = kx^4$

54. (a) 0
(b) $\tan^{-1}\mu$

55. $\omega = 2\sqrt{10}\ rads^{-1}$

56. (a) $\frac{7}{4}$

(b) $\pi\sqrt{\frac{7L}{4g}}\left[1 + \frac{1}{\sqrt{2}}\right]$

57. (i) $\sqrt{\frac{3}{2}}R$

(ii) (a) $F_v = 1.2 \times 10^7 N$
(b) $F_H = 1.5 \times 10^7 N$
(c) $F = 1.92 \times 10^7 N$

58. $s = \frac{1}{\left(1 + \frac{A_2}{A_1}\right)\rho_{Hg} . g}$

59. $\theta = \sin^{-1}(0.13)$

60. 400 N

61. $y = \frac{c_0}{u_0 + bx}$ where c_0 is a constant. For plot of streamlines see the solution.

62. $Q = \frac{4}{3}\pi\eta r\sqrt{2g}\,H^{3/2}$

63. (a) $u = \frac{a}{A}u_0 = 5\,mm/s$

(b) 0.115 J

64. (a) $4.96 \times 10^{-4}\ m^3s^{-1}$
(b) 1.48 cm

65. (a) In both figures $v_2 = \sqrt{2g(H + h)}$

(b) $\sqrt{\frac{H}{H + h}}$

(c) ρgh

66. $g = 10\ ms^{-2}$

67. (i) $\dfrac{2A_0 g}{A}$ (ii) $V = 2\sqrt{2gH}$

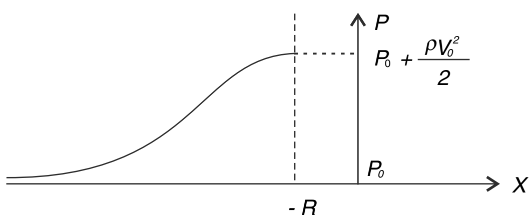
68. (a) $\dfrac{Kx}{\pi R^2}$ (b) $bx^{3/2}\sqrt{\dfrac{2K}{\rho\pi R^2}}$

69. (i) $Q = \sqrt{\dfrac{2\rho_0 gH}{\rho}\left[\dfrac{1}{A_2^2} - \dfrac{1}{A_1^2}\right]^{-1}}$

(ii) $\dfrac{1}{2}\rho\Delta V\left(\dfrac{A_1^2}{A_2^2} - 1\right)v_1^2 + \rho\Delta Vg\left(h_2 - h_1\right)$

70. (i) $P = P_0 + \rho V_0^2\left[1 - \left(1 + \dfrac{R^3}{x^3}\right)\right]$

(ii)



71. $\sqrt{\dfrac{2}{g}}\dfrac{A_1A_2\sqrt{h_1 - h_2}}{A_0\left(A_1 + A_2\right)}$

72. (a) $\sqrt{2gh}$

(b) $P' = P_0 - \rho gH$

(c) P_0

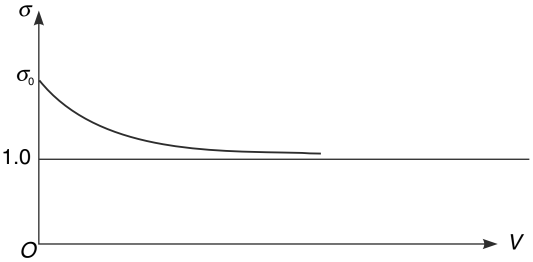
(d) V decreases P' increases

(e) $H_0 = \dfrac{P_0}{\rho g} \simeq 10.3\ m$

73. (a) $\sigma = 1 + (\sigma_0 - 1)e^{-v/v_0}$

(b) 1.074

(c)



74. $m = \dfrac{\pi R^3\rho}{3}$

75. $\sqrt{\dfrac{4+\pi^2}{8}}\cdot\rho gR^2L$

76. (a) $\rho = \dfrac{2\rho_0}{3}$ (b) $T = 2\pi\sqrt{\dfrac{123R}{40g}}$

77. (a) $Q = \pi\ r_1^2\ r_2^2\sqrt{\dfrac{nP_0}{\rho\left\{r_1^4 - r_2^4\right\}}}$

(b) $\dfrac{F}{2\pi\mu r_1 b}$

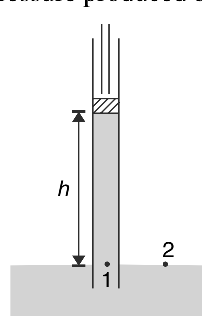
(c) $Q = \pi\ r_1^2\ r_2^2\sqrt{\dfrac{2(n-1)P_0}{\rho[r_1^4 - r_2^4]}}$

SOLUTIONS

1. $m = \dfrac{pA}{g} = \dfrac{1\times10^5\times4\pi(6.4\times10^6)^2}{10} \simeq 5.14\times10^{18}\,kg$

3. The pressure in the tube at the level A will be below the atmospheric pressure. Therefore, the atmospheric pressure will not allow the water to flow out when a hole develops. Air will enter the tube through the hole until the atmospheric pressure is reached inside the tube and the mercury level will sink to the level outside the tube.

4. The water will rise along with the piston till the pressure produced by the water column at point 1 becomes equal to atmospheric pressure.



$P_1 = P_2$

$\rho_\omega gh = P_{atm} \Rightarrow \; h = \dfrac{1.01\times10^5}{10^3\times9.81} = 10.30\ m$

For mercury $\rho_{Hg} gh = P_{atm} \Rightarrow h = 0.76\ m$

5. $g = \dfrac{GM}{R^2} = \dfrac{G\dfrac{4}{3}\pi R^3\rho}{R^2} = \dfrac{4\pi}{3}G\rho R$

Since $\rho = \dfrac{\rho_e}{2}$ and $R = \dfrac{R_e}{2}$

$\therefore\ g_P = \dfrac{g_e}{4} = \dfrac{9.8}{4} = 2.45\ ms^{-2}$

$\therefore P = \rho_{ice}\, g_P\, d_{ice} + \rho_\omega g_P\, d_\omega$

$= \left(900\dfrac{kg}{m^3}\right)\times\left(2.45\dfrac{m}{s^2}\right)\times(5000\,m) + \left(1000\dfrac{kg}{m^3}\right)\times\left(2.45\dfrac{m}{s^2}\right)\times(1000\,m)$

$= 1.35\times10^7\ Pa$

6. Weight of ice block = Weight of liquid displaced.

Hence weight of both the beakers will be same.

8. Think of each ball tightly packed inside a cube.

Fraction of volume occupied by balls $f = \dfrac{\dfrac{4}{3}\pi r^3}{(2r)^3} = \dfrac{\pi}{6} = 0.52$

If volume of the box is v, then $0.52v$ is occupied by the balls.

Mass of box = $0.52v.d$ [d = density of balls]

Buoyancy force when 80% of the box is submerged is

$F_B = 0.8v\rho_\omega.g$

$\therefore\ 0.52vdg = 0.8v\rho_\omega g$

$\Rightarrow\ \dfrac{d}{\rho_\omega} = \dfrac{0.8}{0.52} = 1.54$

9. Buoyancy on both balls, when completely submerged will be same (say F_B).

This F_B is large than weight of the hollow ball.

In first container, the ball exerts a force F_B on the water + container system.

This increases the reading of the balance by F_B

In second container, the reading increases by an amount equal to weight of the hollow ball (that is less then F_B).

11. $W = F_B$

$Ald_1g + Ald_2g = A\dfrac{3}{2}ldg$

$\Rightarrow d = \dfrac{2}{3}(d_1 + d_2)$

$\Rightarrow d_2 = \frac{3d}{2} - d_1 > d_1 \Rightarrow \frac{3d}{4} > d_1$

12. Let mass of the balloon be M and its volume on the surface be v_0

$Mg = \frac{v_0}{3}\rho g$ ....(i) [ρ = density of water]

At depth 'h', the volume of the balloon will decrease due to hydrostatic pressure. Let new volume be v.

$p_2 v_2 = p_1 v_1$ gives

$(h + 10)v = 10v_0 \Rightarrow v = \frac{10v_0}{h+10}$

For equilibrium $v\rho g = Mg$

$\frac{10v_0}{h+10}\rho g = \frac{v_0}{3}\rho g$ [using (i)]

$\Rightarrow 30 = h + 10 \Rightarrow h = 20\ m$

13. If P is pressure at depth h below the surface $\frac{dP}{dh} = g_h \cdot d$ [g_h = acceleration due to gravity at depth h]

$$\frac{dP}{dh} = d \cdot g\left[1 - \frac{h}{R}\right]$$

$$dP = g \cdot d\left[1 - \frac{h}{R}\right]dh$$

$$\therefore \int_0^{P_0} dP = gd\left[\int_0^R dh - \frac{1}{R}\int_0^R h\,dh\right]$$

$$P_0 = g \cdot d\left[R - \frac{R}{2}\right] = \frac{1}{2} g \cdot d \cdot R$$

For given data

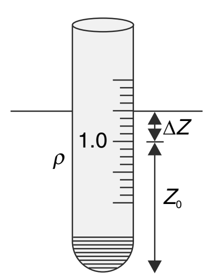
$$P_0 = \frac{1}{2} \times 10 \times 5500 \times 6000 \times 10^3\ P_a = 1.65 \times 10^{11}\ P_a$$

14. When the hydrometer is in water, the buoyant force is equal to its weight. A length z_0 of the tube is submerged in water.

$W = \rho_\omega g A z_0$ ...(i) [A = cross section of tube]

In a liquid of density less than water ($\rho < \rho_\omega$), the hydrometer will sink deeper and the liquid surface will be at a distance Δz above z_0.

$W = \rho g A\ (z_0 + \Delta z)$ ...(ii)



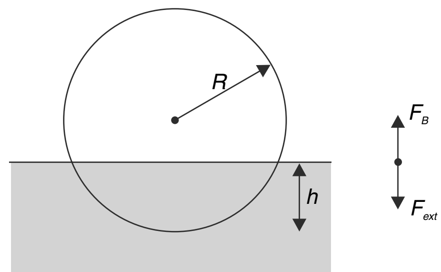
From (i) and (ii)

$\rho g A\,(z_0 + \Delta z) = \rho_\omega g A z_0$

$\Rightarrow \dfrac{\rho}{\rho_\omega} = \dfrac{z_0}{z_0 + \Delta z}$

$\therefore$ specific gravity of liquid $= \dfrac{z_0}{z_0 + \Delta z}$

15. Consider the sphere when it is submerged up to depth h.



Volume of the submerged part

$v = \dfrac{1}{3}\pi h^2 (3R - h)$ [Prove this result yourself]

Buoyancy $F_B = \rho v g = \dfrac{\rho g \pi h^2}{3}(3R - h)$

To push the ball slowly, the external agent must apply a downward force equal to F_B

$F_{ext} = F_B$

Work done in pushing the sphere further down by dh is

$$dW = F_{ext} \cdot dh = \frac{\rho g \pi}{3}\left[3Rh^2 - h^3\right]dh$$

$\therefore$ Work required to completely submerge the ball is

$$W = \int_{h=0}^{2R} dW$$

$$W = \frac{\pi\rho g}{3}\left[3R\int_0^{2R} h^2 dh - \int_0^{2R} h^3 dh\right]$$

$$= \frac{\pi\rho g}{3}\left[3R \cdot \frac{(2R)^3}{3} - \frac{(2R)^4}{4}\right]$$

$$= \frac{\pi\rho g}{3}\left[8R^4 - 4R^4\right] = \frac{4}{3}\pi R^4 \rho g$$

Alter:

By submerging the sphere we are actually pushing up a volume $\dfrac{4}{3}\pi R^3$ of water.

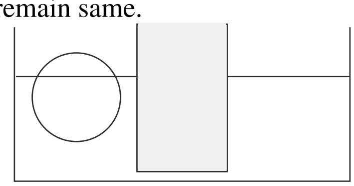
Since lake is large we can assume that the entire water ($= \dfrac{4}{3}\pi R^3$ volume) has been brought to the surface. Earlier this volume of water was having its centre of mass at a depth R below the surface.

$\therefore$ W_{ext} = Increase in gravitational PE of water

$$= \left(\frac{4}{3}\pi R^3 \rho \cdot g\right) R \; = \frac{4}{3}\pi R^4 \rho g$$

16. Let volume of water on each side (before ball is put in) be v_0 and v be the volume of submerged part of the ice ball.

(i) The level of liquid on both side must remain same.



Total volume of liquid on right side after dipping the ball is

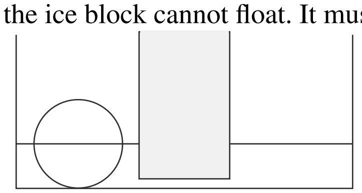
$$= \frac{2v_0 + v}{2} = v_0 + \frac{v}{2} \qquad \text{.....(i)}$$

$\therefore$ Volume of water moving to right part $= \frac{v}{2}$

v = volume of submerged part of the ice ball = 90 cc

$\therefore$ Answer is 45 cc

(ii) The water in the container is too little and the ice block cannot float. It must rest on the floor of the container.



Once again [as given by equation (i)] amount of water moving to right is $\frac{v}{2}$.

But this time v = 50 *cc*.

$\therefore$ Answer is 25 *cc*

(iii) When ice melts

In case (i) no change in water level will be seen as volume of water formed due to melting of ice ball = v

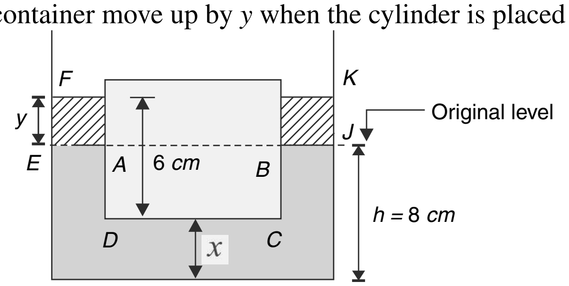
In case (ii) water level will rise as volume of water formed will be larger then v.

17. Because relative density of wood is 0.6, it can float in water with 60% of its volume remaining submerged.

The given wooden cylinder can float with its 6 cm length inside water.

When h = 8 *cm*, the wooden cylinder will certainly float.

Let the level of water in the container move up by y when the cylinder is placed inside it



Volume $ABCD$ = Volume $EFKJ$ (liquid only) = Volume shown in

$\therefore A(6-y) = (A_0 - A)y$

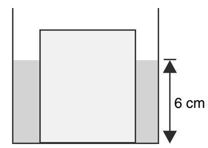
$125 \times (6-y) = (150-125)y$

$y = 5\ cm$

Height of water in container = 8 + 5 = 13 cm

$\therefore x = 13 - 6 = 7\ cm$

(b) When $h = 1\ cm$, the cylinder will just float with $x = 0$



(c) With $h < 1\ cm$, the cylinder cannot float

$\therefore x = 0$

18. Consider the tube + liquid inside it + Block as our system.
For equilibrium
Weight = Buoyancy

$$2\rho_\omega vg + \pi r^2 . \frac{L}{2}\rho_\omega g = \left[v + \pi r^2 \frac{L}{2}\right]\rho_\omega g$$

Solving, $v = 0.75\ m^3$

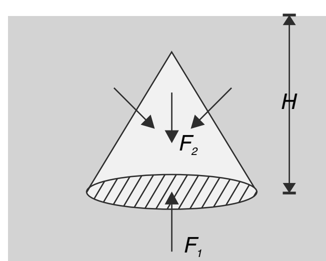
19. (a) Buoyancy force is $F_B = v\rho g$

Weight of the cone $W = vdg$

$\therefore$ Tension $T = W - F_B = vg\,(d - \rho)$

(b) $F_B = v\rho g$

Buoyancy is resultant of vertically upward force (F_1) applied by water pressure on the base of the cone and the vertically downward force (F_2) applied by water on the slant surface. [The horizontal component of this force sums up to zero due to symmetry]

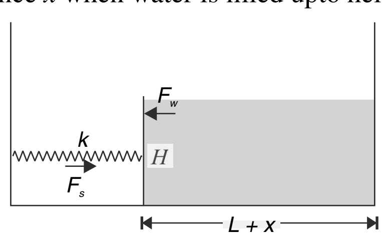


$\therefore F_1 - F_2 = F_B$

$\pi R^2 . [\rho_0 + \rho gH] - F_2 = v\rho g$

$\therefore F_2 = \pi R^2 [\rho_0 + \rho gH] - v\rho g$

20. Let the sliding wall shift to left by a distance x when water is filled upto height H into the right chamber.



Force by water on the movable wall (F_W) must equal the spring force (F_s) in this position.

To calculate the force applied by water on the wall we can assume that the average pressure is $\frac{\rho g H}{2}$. [This is because the pressure changes from zero to $\rho g H$ linearly as we move from water surface to the bottom. You are suggested to prove this by performing integration to calculate the force on the wall. Also, atmospheric pressure has not been considered as its effect is on both the faces of the wall]

$$\therefore\ F_W = \frac{\rho g H}{2}\cdot bH = \frac{\rho g b H^2}{2}$$

$$\therefore\ kx = \frac{\rho g b H^2}{2} \quad \Rightarrow \quad x = \frac{\rho g b H^2}{2k}$$

Volume of water = $Hb\,(L + x) = Hb\left(L + \frac{\rho g b H^2}{2k}\right)$

21. Pressure inside the freely falling container will become equal to atmospheric pressure. Water will not leak out of the hole. The balloon will grow in size by

2.2 – 2.0 = 0.2 litre

$\therefore$ Water level will rise.

$$\Delta h = \frac{0.2\times 10^{-3}\ m^3}{0.2\ m^2} = 1\,mm$$

22. For the stick to be in stable equilibrium, center of gravity should be below the center of buoyancy. For minimum m, the two will just coincide.

Let h be the length of immersed portion. For translational equilibrium,

Weight of rod + mass attached = force of buoyancy

$(M + m)g = \pi R^2 h\sigma g$ ....(i)

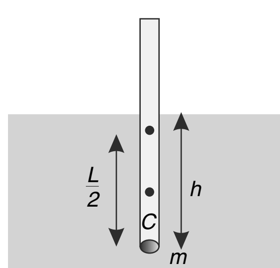
where $M = \pi R^2 L\rho$.

The height of center of mass from bottom $= \frac{(M)L/2 + m\times 0}{m+M} = \frac{ML}{2(m+M)}$

For rotational equilibrium and for minimum m, this should be equal to $h/2$.

$$\therefore\ \frac{h}{2} = \frac{ML}{2(m+M)} \qquad \text{...........(ii)}$$

$$\therefore\ h = \frac{ML}{(m+M)}$$



Substituting for h in Equation (i), we get

$$(M+m)g = \pi R^2 \sigma g.\frac{ML}{(m+M)}$$

$$(M+m)^2 = \pi R^2 \sigma ML$$

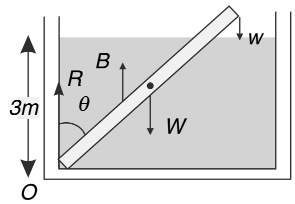
$$(M+m) = \sqrt{M\pi R^2 \sigma L} = \sqrt{\pi R^2 L\rho.\pi R^2 \sigma L}$$

$$m = \pi R^2 L\sqrt{\sigma\rho} - \pi R^2 L\rho$$

$$= \pi R^2 L\rho\left(\sqrt{\frac{\sigma}{\rho}} - 1\right)$$

23. As shown in figure, the forces acting on the rod are:

(1) The weight of rod 12 g acting downwards through the CG of the rod, i.e., at a distance of 3 m from the hinge.



(2) Force of buoyancy through the CG of displaced liquid vertically upwards.

$$RD = \frac{\text{Weight of displaced water}}{\text{Force of buoyancy}}$$

Force of buoyancy $= \left(\frac{5}{6}\right) \times \frac{12g}{0.5} = 20g$

and acts at a distance 2.5 m from the hinge.

(3) Extra weight w at the other end of the rod at a distance 6 m from O acting vertically downwards.

(4) Reaction R at the hinge at O will be vertical (as all other forces are vertical)

So for translational equilibrium of rod,

$R + B - W - w = 0$

i.e., $w - R = 20g - 12g = 8\ g$ …(i)

And for rotational equilibrium of rod (taking moments about O)

$$-12g \times \frac{6}{2}\sin\theta + 20g \times \frac{5}{2}\sin\theta - w.6\sin\theta = 0$$

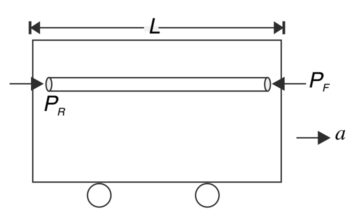
Or, $w = (14/6)g = 2.23gN = 2.33\ kg$ …(ii)

Substituting the value of w from Eqn. (ii) in (i) and solving for R, we get

$R = (2.33 - 8)g = -5.67\ kg$

Negative sign implies that R is directed vertically downwards.

24. Consider a cylindrical tube of air as shown.



Area of cross section = A

P_R and P_F are pressure at rear and front.

$P_RA - P_FA = \rho AL.a$

$\therefore\ P_R - P_F = \rho La$

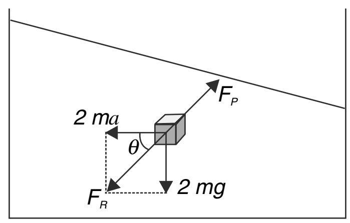
$\Delta P = 1.2 \times 3.0 \times 5 = 18\ Nm^{-2}$

$$\frac{\Delta P}{P_0} = \frac{18}{1\times 10^5} = 1.8\times 10^{-4}$$

25. Consider a volume of water at the location of the wooden block. The volume of water is of same shape and size as the wooden block.

Mass of volume of water = $2m$ [$\because$ density of water is double the density of wood]

Consider equilibrium of this water volume in the non-inertial frame of the container (fig.(a))



The resultant of the force of gravity and the pseudo force is F_R. The force due to pressure of surrounding water, F_P balances F_R.

$$\therefore\ \tan\theta = \frac{g}{a} = 3 \quad [\because\ a = g/3]$$

and $F_P = 2m\sqrt{a^2+g^2} = \dfrac{2\sqrt{10}}{3}mg$

Force due to pressure of surrounding water on the wooden block is also F_P since the wooden block and considered volume of water are of same shape and size.

Considering equilibrium of the block in the frame attached to the container

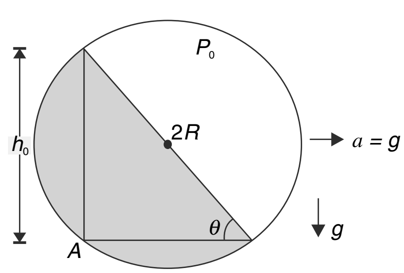
$$\tan\theta = \frac{g}{a} = 3$$

and $T + m\sqrt{a^2+g^2} = F_P$

$$T = 2m\sqrt{a^2+g^2} - m\sqrt{a^2+g^2}$$

$$T = m\sqrt{a^2+g^2} = \frac{\sqrt{10}}{3}mg$$

26. The liquid surface will get inclined to the horizontal by an angle θ given by



$\tan\theta = \frac{a}{g} = 1 \Rightarrow \theta = 45°$

Maximum vertical depth of liquid from free surface is

$h_0 = 2R \sin 45° = \sqrt{2}\, R$

$\therefore$ Pressure is maximum at A.

$P_{max} = P_0 + \rho g h_0 = P_0 + \sqrt{2}\, \rho g R$

27. High density of streamlines at B indicates that speed is higher at B

Bernoulli's equation gives

$$P_A + \frac{1}{2}\rho V_A^{\,2} = P_B + \frac{1}{2}\rho V_B^{\,2}$$

$$\therefore\ P_A - P_B = \frac{1}{2}\rho\,(V_B^2 - V_A^2) > 0$$

28. (i) The stream lines crowd near the sides of the ball. For the air to pass through the reduced width of the flow channel it has to move faster. Air speeds up as it approaches the ball and then slows again as it departs at the rear. At point 2 pressure is low due to high speed.

(ii) The spinning ball, to some extent, carries the air in the direction of the spin. This means that the flow velocity on the side of the ball moving with the airflow is increased and hence pressure gets reduced on that side.

29. (ii) Let h = height of gap between the glass plate and the horizontal surface

Continuity equation gives:

$2\pi Rh\, V_0 = 2\pi xh.\ V_x$ where V_0 is speed of the flow at the circumference of the plate.

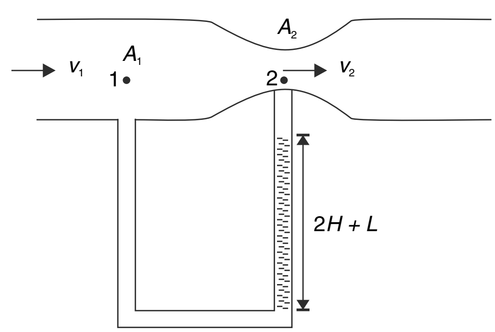
$$\Rightarrow V_x = \frac{R}{x}V_0 \qquad \dots (1)$$

Also $2\pi Rh\, V_0 = Q$

$$\therefore\ V_0 = \frac{Q}{2\pi Rh}$$

$$\therefore\ V_x = \frac{R}{x}\cdot V_0 = \frac{Q}{2\pi hx}$$

30.



The required situation has been shown in the figure. Height of Hg column in right part of the tube

$H = 2H + L = 34\ cm$

$\therefore\ P_1 - P_2 = \rho_{Hg}\,.\, gH = 13.6 \times 10^3 \times 10 \times 0.34 = 4.62 \times 10^4\ Nm^{-2}$

Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 + 0 = P_2 + \frac{1}{2}\rho V_2^2 + 0$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) = \frac{1}{2}\rho\left(\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}\right)$$

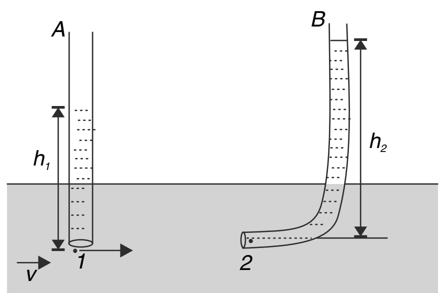
[$\therefore$ Flow rate $Q = A_1V_1 = A_2V_2$]

$$Q^2 = \frac{2(P_1 - P_2)}{\rho\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)} = \frac{2\times 4.62\times 10^4}{1.2\left(\frac{10^4}{4^2} - \frac{10^4}{10^2}\right)} = \frac{2\times 4.62\times 1600}{1.2\times(100-16)} = 146.7$$

$\therefore$ $Q = 12.1\ m^3s^{-1}$

31. There is no speed of the liquid inside tubes. Speed at a point like 2 is zero.

Using Bernoulli's equation between point 1 and 2 give $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + 0$

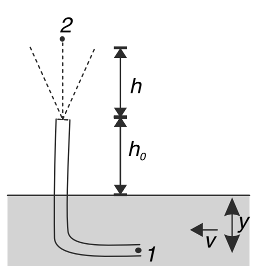


$\therefore P_2 - P_1 = \frac{1}{2}\rho v^2$ [$v_1 = v$ and $v_2 = 0$]

$$\therefore \rho g h_2 - \rho g h_1 = \frac{1}{2}\rho v^2 \Rightarrow h_2 - h_1 = \frac{v^2}{2g}$$

32. Let the depth of the tube entrance be y. Consider point 1 (at the entry) and point 2 (at the maximum height of the fountain) on a streamline. Apply Bernoulli's theorem

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$



Consider $h_1 = 0$ and P_0 = atmospheric pressure

$$P_0 + \rho g y + 0 + \frac{1}{2}\rho v^2 = P_0 + \rho g(y + h_0 + h) + 0$$ [Since speed at top point 2 is zero]

$$\therefore \frac{1}{2}\rho v^2 = \rho g(h_0 + h) \quad \therefore h = \frac{v^2}{2g} - h_0$$

33. Let V_1 = Speed at which piston shall be moved $A_1V_1 = A_3.V$

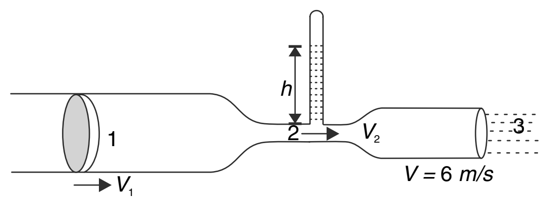
$$V_1 = \frac{9}{18} \times 6 = 3\ m/s$$

Applying Bernoulli's equation for point 1 and 3

$$P_1 + \frac{1}{2}\rho_{Hg} v_1^2 = P_{atm} + \frac{1}{2}\rho_{Hg}\ v^2$$

$$\therefore \left(P_1 - P_{atm}\right) = \frac{1}{2}\rho_{Hg}\left(6^2 - 3^2\right)$$

$$= \frac{1}{2} \times 1.36 \times 10^4 \times 27 = 1.84 \times 10^5\ Pa$$



(i) $F = (P_1 - P_{atm})\ A_1 + f = 1.84 \times 10^5 \times 18 \times 10^{-4} + 40 = 371.2\ N$

(ii) If V_2 is speed at point 2.

$A_2V_2 = A_3 \times 6$

$\Rightarrow V_2 = 6.75\ m/s$

Applying Bernoulli's equation for point 2 and 3

$$P_2 + \frac{1}{2}\rho_{Hg}V_2^2 = P_{\text{atm}} + \frac{1}{2}\rho_{Hg}.V^2$$

$$P_2 = 1.01 \times 10^5 + \frac{1}{2} \times 1.36 \times 10^4 (6^2 - 6.75^2)$$

$= 0.36 \times 10^5\ N/m^2$

$\therefore\ \rho_{Hg}\, gh = 0.36 \times 10^5$

$$h = \frac{0.36 \times 10^5}{1.36 \times 10^4 \times 10} = 0.26\ m = 26\ cm$$

34. (a) (i) As for floating, $W = F_B$

$$V\rho g = V_1 d_1 g + V_2 d_2 g$$

Or, $L\left(\frac{A}{5}\right)\rho = \left(\frac{3}{4}L\right)\left(\frac{A}{5}\right)d + \left(\frac{1}{4}L\right)\left(\frac{A}{5}\right)2d$ i.e., $\rho = \frac{3}{4}d + \frac{2}{4}d = \frac{5}{4}d$

(ii) Total pressure = P_0 + (weight of liquid + weight of solid) / A

i.e., $P - P_0 = \frac{H}{2}dg + \frac{H}{2}2dg + \frac{5}{4}d \times \left(\frac{A}{5} \times L\right) \times g \times \frac{1}{A}$

i.e., $P - P_0 = \frac{3}{2}Hdg + \frac{1}{4}Ldg = \frac{1}{4}(6H + L)dg$

(b) (i) By Bernoulli's theorem for a point just inside and outside the hole

$$P_1 + \frac{1}{2}\rho\upsilon_1^2 = P_2 + \frac{1}{2}\rho\upsilon_2^2$$

i.e., $P_0 + \frac{H}{2}dg + \left(\frac{H}{2} - h\right)2dg = P_0 + \frac{1}{2}(2d)\upsilon^2$

or, $g(3H - 4h) = 2\upsilon^2$ or $\upsilon = \sqrt{(g/2)(3H - 4h)}$

(ii) At the hole vertical velocity of liquid is zero. So time taken by it to reach the ground,

$t = \sqrt{(2h/g)}$

so that $x = \upsilon t = \sqrt{\frac{g}{2}(3H - 4h)} \times \sqrt{\frac{2h}{g}} = \sqrt{h(3H - 4h)}$

(iii) For x to be maximum x^2 must be maximum, i.e.,

$\frac{d}{dh}(x^2) = 0$ or $\frac{d}{dh}(3Hh - 4h^2) = 0$

Or, $3H - 8h = 0$, i.e., $h = (3/8)H$

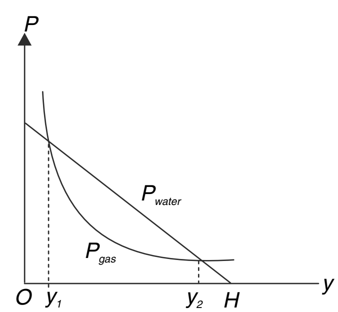
and $x_{\max} = \sqrt{\frac{3H}{8}(3H - \frac{3}{2}H)} = \frac{3}{4}H$.

35. Water pressure at height y is $P_{\text{water}} = \rho g(H - y)$

For gas in the pipe

$P_{\text{gas}}(Ay) = (\rho gH)\left(A\frac{H}{10}\right)$ $\quad [\because P_1V_1 = P_2V_2]$

$\therefore P_{\text{gas}} = \frac{\rho g H^2}{10y}$



(b) In equilibrium

$P_{\text{gas}} = P_{\text{water}}$

$\frac{\rho g H^2}{10\,y} = \rho g(H - y)$

$\Rightarrow 10y^2 - 10Hy + H^2 = 0$

$y = \frac{H}{2} \pm \frac{\sqrt{15}}{10}H$

Only $y_1 = \frac{H}{2} - \frac{\sqrt{15}}{10}H$ is acceptable solution

At $y_2 = \frac{H}{2} + \frac{\sqrt{15}}{10}H$ the equilibrium will be unstable

You can argue this looking at the graph plotted in part (a) of the problem

37. (i) Let height of water column be x.

Height of cubical block inside mercury = $10 - x$

$\therefore\ \rho_w g x + \rho_{Hg}\, g(10 - x) = \rho_m\, g.10$

$$\Rightarrow x + (10 - x)\frac{\rho_{Hg}}{\rho_w} = 10\frac{\rho_m}{\rho_w}$$

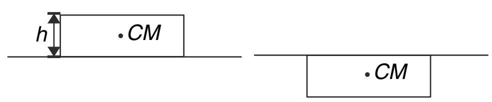
$\Rightarrow x + (10 - x) \times 13.6 = 10 \times 7.3 \qquad \Rightarrow x = 5\ cm$

$\therefore$ Volume of water in the container

$v = (15cm \times 15\ cm - 10\ cm \times 10\ cm) \times 5\ cm = 625\ cc$

$\therefore$ Mass water $m = 625 \times 1 = 625\ g$

38. Since tank is wide, the displaced water will spread in a thin layer on the surface. It means that water level in the tank will not change. In equilibrium the block is completely submerged. It means that density of wood = density of water



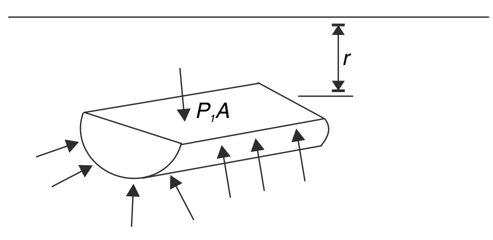
The COM of the wood falls by h and the COM of the displaced water rises by $\frac{h}{2}$.

$\therefore$ Heat produced = loss in PE

$$= (\rho h A) g h - (\rho h A) g \frac{h}{2} = \frac{1}{2}\rho g A h^2$$

39. Assume that half the cylinder (cut along its length) is held in the position shown
Buoyancy force on half cylinder

$$F_B = \frac{\pi r^2}{2} L \rho g (\uparrow)$$



Force on the flat surface at depth r is

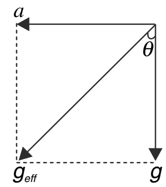
$F_1 = P_1 A\ (\downarrow) = (\rho g r)\ (2rL)\ (\downarrow) = 2\ \rho g L r^2\ (\downarrow)$

$\therefore$ Upward force on curved part due to water pressure is given by

$$F = F_B + F_1 = \frac{\pi r^2 L \rho g}{2} + 2\rho g L r^2 = \rho g L r^2 \left(\frac{\pi}{2} + 2\right) (\uparrow)$$

40. $g_{eff} = \sqrt{a^2 + g^2}$ and is directed as shown in the figure.

[Direction of acceleration of the tube is towards right]



$\tan\theta = \dfrac{a}{g}$ $\Rightarrow a = g \tan \theta$

The air bubble experiences a net buoyancy force opposite to g_{eff}.

41. Let the outer radius of the spherical pot be r

Buoyancy = Mg

$$\frac{2}{3}\pi r^3 \rho_w g = (500)g \qquad \text{............(i)}$$

$$\therefore r^3 = \frac{452.1\times 3}{2\times 3.14\times 1} = 216 \quad \Rightarrow r = 6\ cm$$

If the sphere sinks further by Δh, the additional buoyancy is approximately

$$\pi r^2\, \Delta h\, \rho . g = 5.g \qquad \text{..........(ii)}$$

(ii) ÷ (i) gives -

$$\frac{\Delta h}{r} = \frac{5}{500}\times\frac{2}{3}$$

$$\Delta h = \frac{2}{3\times 100}\times 6 = \frac{4}{100}\text{cm} = 0.04\,\text{cm} = 0.4\ mm$$

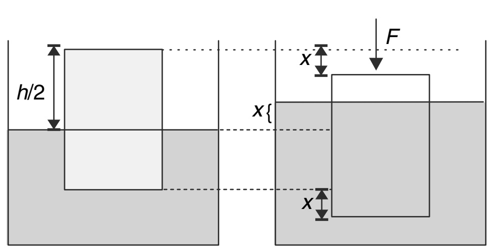
42. Originally, half length $\left(\dfrac{h}{2}\right)$ of the cylinder is inside water.

When the wooden cylinder is pushed down by x, water level in the container will also rise by x.

[$\because$ cross section of container = $2A$ = twice that of cylinder]

$\therefore$ Buoyancy force rises by $A(2x)\ \rho_o g$

To gradually push the cylinder, force F needed is $F = 2A\rho_o g x$

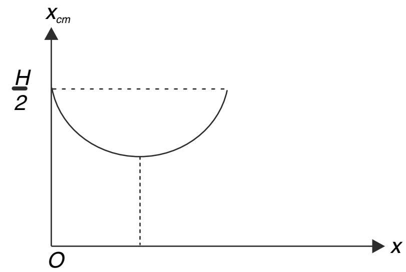


Work done in pushing through dx

$dW = Fdx = 2\ A\ \rho_0 g\ x\ dx$

Total work done $W = 2A\rho_0 g \int_0^{h/4} x\,dx = \dfrac{1}{16} A\rho_0 g h^2$

43. (a) *COM* of empty container is at $x_{\text{cm}} = \dfrac{H}{2}$. When liquid is poured, the mass of the system increases on the lower side and hence, the *COM* moves down. But the final position of *COM* when container is completely filled is once again $x_{\text{cm}} = \dfrac{H}{2}$. It means the *COM* begins to rise after falling through a certain distance.



(b) Mass for unit length of liquid column is $\dfrac{M}{2H}$

Height of COM when liquid column has length x is

$$x_{\text{cm}} = \frac{M\dfrac{H}{2} + \dfrac{M}{2H}x.\dfrac{x}{2}}{M + \dfrac{Mx}{2H}} = \frac{2H^2 + x^2}{2[2\,H + x]} \qquad \text{........(i)}$$

x_{cm} is minimum when $\dfrac{d\,x_{\text{cm}}}{dx} = 0$

i.e., $(2H + x)\,(2x) - (2H^2 + x^2) = 0$

$x^2 + 4Hx - 2H^2 = 0$

$$\therefore\ x = \frac{-4H \pm \sqrt{16H^2 + 8H^2}}{2}$$

x cannot be negative

$\therefore\ x_0 = (\sqrt{6} - 2)H$

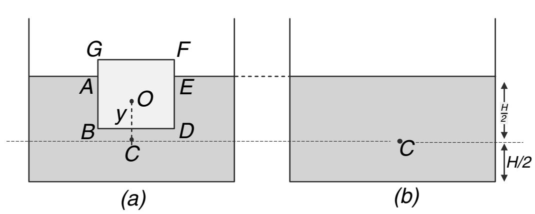
When $x = x_0$, the COM of the system is at lowest position

To find the minimum value of x_{cm} you can put $x = (\sqrt{6} - 2)H$ in equation (i)

44. After the ice melts, the level of water in the container remains unchanged. The *COM* is at geometrical centre '*C*' after the ice melts.

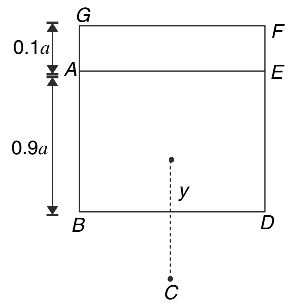
Let upward direction be positive. In figure (a), height of COM above *C* is given by

$$y_{cm} = \frac{-M^{water}_{ABDE}\ y + M^{ice}_{BDFG}\ (y + 0.05a)}{M^{water}\ +\ M^{ice}}$$



Here y = height of COM of *ABDE* above *C*

And $y + 0.05a$ = height of COM of complete ice block above *C*



$$y_{cm}^{ice} = y + \frac{M_{AEFG}^{ice}\left(\frac{0.9a}{2}+\frac{0.1a}{2}\right)}{M_{BDFG}^{ice}}$$

$= y + 0.1 \times 0.5a = y + 0.05\, a$

$\therefore$ from (i)

$$y_{cm} = \frac{M_{BDFG}^{ice} \times 0.05a}{M^{water} + M_{BDFG}^{ice}} \qquad \left[M_{ABDE}^{water} = M_{BDFG}^{ice}\right]$$

$$= \frac{0.05a}{4+1} = 0.01a$$

45. Because 90% of the volume of the ice is inside water, the relative density of ice must be 0.9.

(a) The ice block is pushed out of the water as the kerosene is poured. This is due to increased pressure at the bottom of the ice block. After the ice block gets completely submerged, there is no change in pressure difference at the bottom surface and the top surface. The ice block does not move after this. Solving part (b) and (c) of this problem will further help you to understand this.

(b) Let A = area of cross section of the ice cylinder

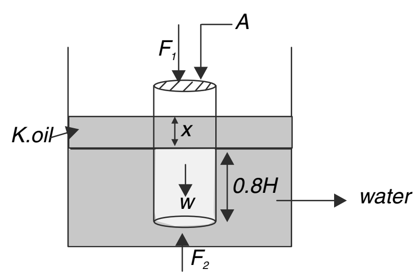
H = height of the cylinder

F_1 = force on top surface due to atmospheric pressure (P_o)

F_2 = force on the bottom surface due to water pressure

W = weight of the cylinder

For equilibrium -



$F_2 - F_1 = W \qquad \text{.......(i)}$

$\Rightarrow [P_o + \rho_k gx + \rho_w g\,(0.8H)]\,A - P_o A = AH\rho_{ice} g$

$$\frac{\rho_k}{\rho_w}x + 0.8H = \frac{\rho_{ice}}{\rho_w}H$$

$0.8x + 0.8H = 0.9H$

$$x = \frac{H}{8}$$

[Alternative way of writing equation (i) is thinking in terms of Archimedes principle]

Buoyancy = W

Wt. of water displaced + wt. of kerosene displaced = W

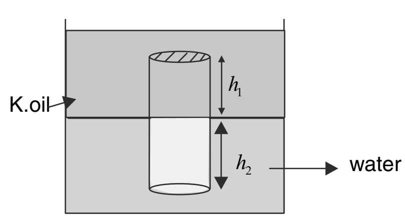
$0.8H\,A\rho_w g + xA\rho_k g = HA\rho_{ice} g$

(c) Once again one can frame equation using any of the two methods given above.

Using Archimedes principle

$Ah_1\rho_k g + Ah_2\rho_w g = A(h_1 + h_2)\rho_{ice} g$

Solving, $h_1 = h_2$



46. (a) No, the spring will not get compressed.

When spring is relaxed –
weight (W) = Buoyancy (F_B)

Both weight and buoyancy are proportional to g_{eff}. When container is accelerated, both will still balance.

(b) $W - F_B = Kx$

If g_{eff} increases, $(W - F_B)$ will increase, hence x will increase.

$\therefore$ Spring will get compressed more.

(c) Let v' = volume of submerged part of solid when container is at rest

$F_B = W - kx_1$

$$v'\rho_w g = 1 \times 10 - 100 \times \frac{5}{100}$$

$v' \times 10^4 = 5$

$$v' = \frac{5}{10^4} m^3 = 500\, cm^3$$

When lift is accelerated $g_{eff} = 15\ m/s^2$

$F_B = W' - kx_2$

$$v''\ \rho_w\ g_{eff} = 1 \times 15 - 100 \times \frac{6}{100}$$

$v'' \times 10^3 \times 15 = 15 - 6$

$$v'' = \frac{6}{10^4} m^3 = 600\ cm^3$$

Change in submerged volume = 100 cm^3

47. V = volume of cone $= \frac{1}{3}\pi R^2 h$

Volume of protruding part $V_{\text{out}} = \frac{1}{3}\pi\left(\frac{R}{2}\right)^2 \frac{h}{2} = \frac{V}{8}$

Volume of cone inside water $V_{\text{in}} = V - \frac{V}{8} = \frac{7V}{8}$

Imagine the cone to have that part of it which protrudes through the hole removed and the space under the container filled with water. The buoyancy force would then be

$$F_0 = \rho\, g\, V_{\text{in}} = \frac{7V\rho g}{8}$$

As there is no water beneath the hole, a contribution $\pi\left(\frac{R}{2}\right)^2 \rho g h$ is missing

$\therefore$ Actual buoyancy force is $F = F_0 - \pi\frac{R^2}{4}\rho g h = \frac{7}{8}V\rho g - \frac{3V}{4}\rho g = \frac{V\rho g}{8}$

48. Let the density of the liquid be ρ_L

$$\frac{v}{2}\rho_L g = v(0.6\,\rho_w)g$$

$\therefore\ \rho_L = 1.2\ \rho_w = 1200\ kg\ m^{-3}$

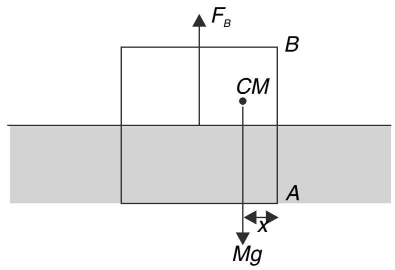
Liquid pressure at the upper surface of the tank is $P = \rho_L g\ [h_1 + h_2 + h_3 - h] = 1200 \times 10 \times 12 = 144000\ Nm^{-2}$

Upward force applied by the liquid on the upper surface is

$F = PA = 144000 \times 3.14 \times 2^2 = 1.81 \times 10^6\ N$

$\therefore$ Force by bolts is $F_B = F - Mg = 1810000 - 20000 = 179000\ N$

49. (i) The cube will topple when torque of buoyancy exceeds the torque due to gravity about the side passing through point A.



$$F_B \cdot \frac{a}{2} \geq Mg \cdot x$$

$$\frac{a^3}{2}\rho \cdot g \cdot \frac{a}{2} \geq a^3(2\rho)g \cdot x \Rightarrow \frac{a}{8} \geq x$$

Since the cube just beings to topple when water reaches height $\frac{a}{2}$, hence $x = \frac{a}{8}$.

(ii) (a) Let the dimension of the block perpendicular to the given figure be b. Consider a strip of width dh on the wall of the block in contact with water (see figure)

Hydrostatic force on the strip is $dF = \rho gh(b\,dh)$

Torque of this force about rotation axis through A is $d\tau = (z - h)dF$

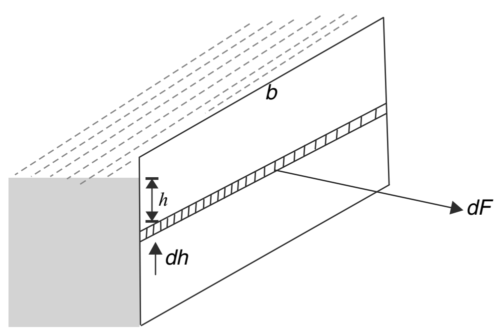
$$\tau = \rho gb\left[z\int_0^z h\,dh - \int_0^z h^2 dh\right] = \rho gb\left[\frac{z^3}{2} - \frac{z^3}{3}\right] = \rho gb\frac{z^3}{6} = \frac{\rho gb}{6}\left(\frac{3}{4}x\right)^3 = \frac{9}{128}\rho gbx^3$$

Torque of weight of concrete block is

$$\tau_g = Mg.\frac{y}{2} = (xyb)(2.5\rho)g\frac{y}{2} = \frac{5}{4}\rho gbxy^2$$

In critical case $\tau_g = \tau$

$$\therefore \frac{5}{4}\rho gbxy^2 = \frac{9}{128}\rho gbx^3 \Rightarrow \left(\frac{y}{x}\right)^2 = \frac{9}{32 \times 5} \Rightarrow \frac{y}{x} = \frac{3}{4\sqrt{10}}$$



(b) when water is present below the block, there will be an upward force due to water pressure equal to

$$F_0 = (\rho gz)(by) = \frac{3}{4}\rho gbxy$$

The force may be assumed to be effectively acting along the central line

$\therefore$ In critical case $\tau = \tau_g - F_\circ \dfrac{y}{2}$

$$\frac{9}{128}\rho gbx^3 = \frac{5}{4}\rho gbxy^2 - \frac{3}{8}\rho gbxy^2$$

$$\Rightarrow \frac{9}{128}x^2 = \frac{7}{8}y^2$$

$$\therefore \frac{y}{x} = \frac{3}{4\sqrt{7}}$$

50. $\dfrac{\partial P}{\partial x} = -\rho a_x$, $\dfrac{\partial P}{\partial y} = 0$ and $\dfrac{\partial P}{\partial z} = -\rho(g + a_z)$

Pressure is independent of y

Total differential of pressure P will be

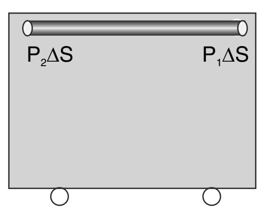
$$dP = \left(\frac{\partial P}{\partial x}\right).dx + \left(\frac{\partial P}{\partial z}\right)dz = -\rho a_x dx - \rho(g + a_z)dz$$

$$\therefore \int_{P_\circ}^{P} dP = -\rho a_x \int_0^x dx - \rho(g + a_z)\int_0^z dz$$

$$P - P_o = -\rho a_x x - \rho(g + a_z)z$$

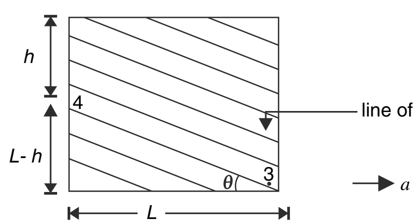
$$\therefore P = P_o - \rho a_x \cdot x - \rho(g + a_z)z$$

51. (a) Pressure at point 2 can be found by considering the motion of a cylindrical element of water as shown in the figure.



$P_2\,\Delta S - P_1\Delta S = \Delta SL\,\rho a \qquad \Rightarrow P_2 = \rho aL \qquad [\because\ P_1 = 0]$

(b) Loci of points having equal pressure are straight lines inclined at $\theta = \tan^{-1}\left(\dfrac{a}{g}\right)$ to the horizontal.



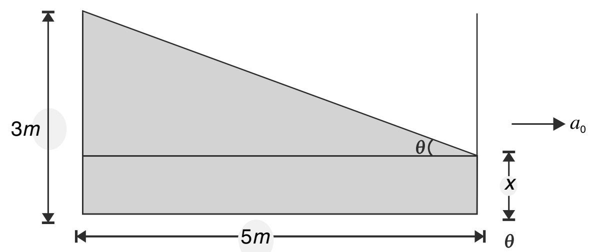
If $P_3 = P_4$, the points 3 and 4 must lie on one such line.

$$\frac{L-h}{L} = \tan\theta \Rightarrow 1 - \frac{h}{L} = \frac{a}{g}$$

$$h = L\left(1 - \frac{a}{g}\right)$$

52. (a) Let the maximum acceleration such that water does not spill be a_0

The water surface makes an angle θ with horizontal given by $\tan\theta = \dfrac{a_0}{g}$



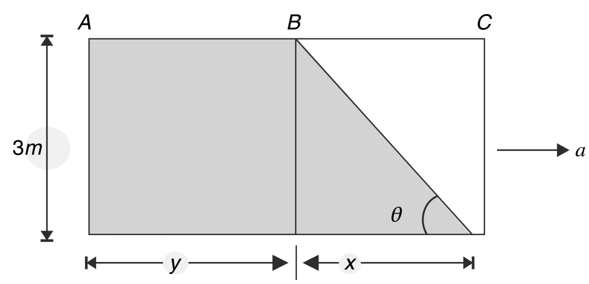
Maximum angle θ for which water does not spill is shown in fig.

Volume of water $= 5\times 4\times x + \dfrac{1}{2}\times(3-x)\times 5\times 4 = 5\times 4\times 2$

$\Rightarrow 5x + 15 = 20$ or $x = 1\ m$

$\therefore\ \tan\theta = \dfrac{2}{5}$ or $\dfrac{a_0}{g} = \dfrac{2}{5} \Rightarrow a_0 = \dfrac{2g}{5}$

(b) The acceleration of tank is $a = 9\ m/s^2$ very high and in all probability the water surface will take the shape shown in figure. In segment AB, water will push the upper lid thereby generating extra pressure. For segment BC water surface will not touch the upper lid.



$\therefore\ \tan\theta = \dfrac{a}{g} = \dfrac{9.0}{9.8}$ or, $\dfrac{3}{x} = \dfrac{9.0}{9.8}$ or, $x = 3.27\ m$.

Volume of water $= 3\times y\times 4 + \dfrac{1}{2}\times 3\times x\times 4 = 2\times 5\times 4$

or, $3y + 1.5x = 10$

or, $3y = 10 - 1.5x = 10 - 1.5\times 3.27$

or, $y = 1.698\ m$

$\therefore\ x + y = 4.698\ m$ which is less than $5.0\ m$.

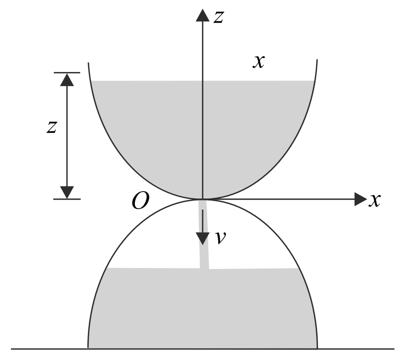
$\therefore$ Gauge pressure at front – bottom is zero. Consider a cylindrical element of surface area ΔS and length just less than 4.698 m as shown in figure.

Let the gauge pressure at the bottom of the rear end be P. Then using $F = ma$ for the element-

$P\Delta S = \rho\Delta S \times 4.968 \times a$

$\Rightarrow P = (10^3)\times 4.968\times (9) = 44.712\times 10^3 N = 0.44\ atm$

53.



Let us choose a co-ordinate system as shown. When height of water surface above O is z, the speed of efflux is $v = \sqrt{2gz}$

If area of orifice is A_0, the flow rate is $\dfrac{dQ}{dt} = vA_0 = \sqrt{2gz} \cdot A_0$

Let x be the radius of cross section at height z. If water level falls by dz in interval dt then

$$\frac{dQ}{dt} = \pi x^2 \left|\frac{dz}{dt}\right|$$

$$\therefore \; -\pi x^2 \frac{dz}{dt} = \sqrt{2gz} \cdot A_0$$

It is desired that $-\dfrac{dz}{dt} = a$ constant = v_0 (say)

$$\therefore \; \sqrt{2gz}\, A_0 = \pi x^2 \cdot v_0$$

$\therefore \; z = kx^4$ where $k = \dfrac{\pi^2 v_0^2}{2gA_0^2}$

54. Acceleration of the container down the incline is

$a = g(\sin\theta - \mu\cos\theta)$

let the liquid surface make an angle α with the incline plane.

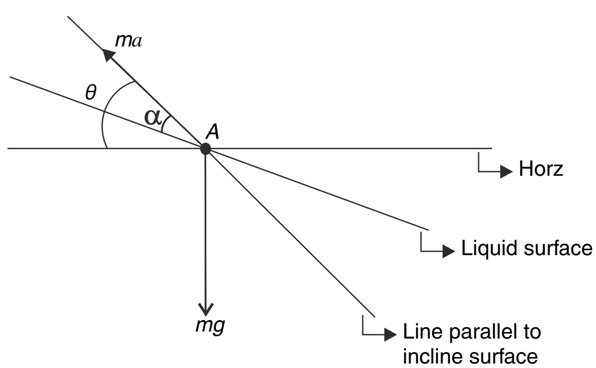


Fig shows a liquid particle A on the liquid surface.

Mg is weight of the particle and ma is the pseudo force on the particle along the liquid surface in the reference frame of the container. Hence,

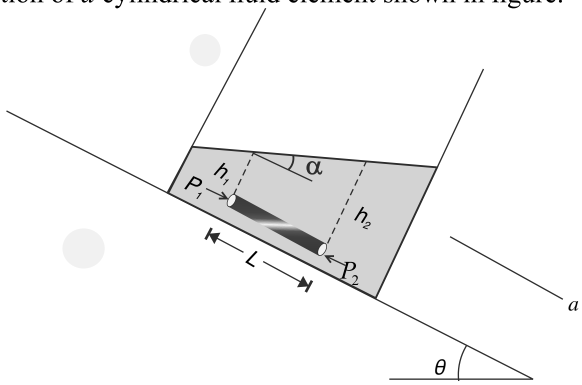
$mg \sin(\theta - \alpha) = ma \cos \alpha$

$\Rightarrow g \sin\theta \cos\alpha - g\cos\theta \sin\alpha = g(\sin\theta - \mu\cos\theta)\cos\alpha$

$\Rightarrow \tan\alpha = \mu. \Rightarrow \alpha = \tan^{-1}\mu$

Alternate Method

Consider the motion of *a* cylindrical fluid element shown in figure.



A = cross section of the element

L = length of the element

Acceleration of the cylindrical element is $a = g(\sin\theta - \mu\cos\theta)$. For the motion of this element, the force equation is

$P_1A + mg\sin\theta - P_2A - ma$ ....(1)

$m = \rho AL$ [ρ = density of liquid]

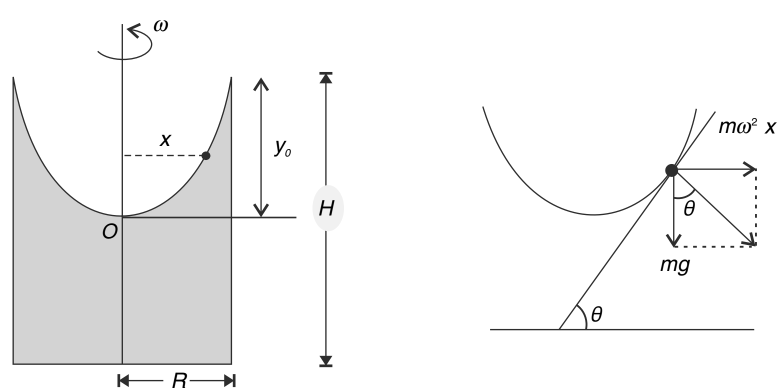
$P_1 = \rho g h_1 \cos\theta + P_0$; $P_2 = \rho g h_2 \cos\theta + P_0$ [P_0 = atmospheric pressure]

Hence, from (1)

$[\rho g h_1 \cos\theta + P_0]A + AL\rho g\sin\theta - [\rho g h_2, \cos\theta + P_0]A = AL\rho g[\sin\theta - \mu\cos\theta]$

$\Rightarrow L\mu\cos\theta - (h_2 - h_1)\cos\theta \Rightarrow \dfrac{h_2 - h_1}{L} = \mu \Rightarrow \tan\alpha = \mu$

55. Consider a small liquid particle of mass m on the surface of the liquid at a distance x from the axis.



The resultant of centrifugal force and weight must be normal to the liquid surface. [Since liquid is non viscous and equilibrium of the particle will not be possible if it experiences any tangential force].

$$\tan\theta = \frac{m\omega^2 x}{mg} \qquad \Rightarrow \frac{dy}{dx} = \frac{\omega^2 x}{g}$$

$$\int_o^y dy = \frac{\omega^2}{g}\int_o^x x\,dx$$

$$\Rightarrow\ y = \frac{\omega^2}{2g}x^2 \qquad \text{....................(1)}$$

This equation gives the profile of the liquid surface which is parabolic.

$$y_0 = \frac{\omega^2}{2g}(1)^2 = \frac{\omega^2}{2g}$$

The volume of empty part of container = volume of paraboloid of revolution obtained by rotating the parabola (1) about its axis. This paraboloid can be divided into horizontal discs and volume can be obtained by adding the volumes of all such discs.

$$\therefore V = \int_0^{y_0} \pi x^2 dy = \frac{2\pi g}{\omega^2}\int_0^{y_0} y\,dy = \frac{\pi g}{\omega^2}y_0^2 = \frac{\pi\omega^2}{4g}$$

Volume of liquid in the vessel is conserved.

$$\therefore\ \pi R^2 h = \pi R^2 H - V$$

$$\pi(1)^2(2) = \pi(1)^2(3) - \frac{\pi\omega^2}{4g}$$

$$\therefore\ \frac{\omega^2}{4g} = 1 \Rightarrow \omega = \sqrt{40} = 2\sqrt{10}\ rads^{-1}$$

56. (a) For equilibrium

Buoyancy = weight of cylinder

$$A\left(\frac{3L}{4}\right)(2\rho_\omega)g + A\frac{L}{4}\rho_\omega g = ALdg$$

[d = density of cylinder, ρ_ω = density of water, A = Area of cross section of cylinder]

$$\Rightarrow \frac{7}{4}\rho_\omega = d$$

specific gravity $= \dfrac{d}{\rho_\omega} = \dfrac{7}{4}$

(b) When depressed by x restoring force is

$F = Ax\,(2\,\rho_\omega)\,g - Ax\rho_\omega g = A\rho_\omega gx$

If m is the mass of cylinder $m\dfrac{d^2x}{dt^2} = -A\rho_\omega gx$

$$ALd\frac{d^2x}{dt^2} = -A\rho_\omega g\,x \Rightarrow \frac{d^2x}{dt^2} = -\left(\frac{4g}{7L}\right)x$$

$$\omega_1^2 = \frac{4g}{7L} \qquad \Rightarrow T_1 = 2\pi\sqrt{\frac{7L}{4g}}$$

Half cycle of oscillation is completed in time $t_1 = \frac{T_1}{2} = \pi\sqrt{\frac{7L}{4g}}$

Once the cylinder protrudes out of the water surface the equation of restoring force changes.
Restoring force $F = Ax\,(2\rho_\omega)g$

$$\therefore\ ALd\,\frac{d^2x}{dt^2} = -2A\rho_\omega g\,x \Rightarrow \frac{d^2x}{dt^2} = -\left(\frac{8g}{7L}\right)x$$

$$\omega_2 = \sqrt{\frac{8g}{7L}} \qquad \Rightarrow T_2 = 2\pi\sqrt{\frac{7L}{8g}}$$

Time period of oscillation

$$T = \frac{T_1}{2} + \frac{T_2}{2} = \pi\left[\sqrt{\frac{7L}{4g}} + \sqrt{\frac{7L}{8g}}\right] = \pi\sqrt{\frac{7L}{4g}}\left[1+\frac{1}{\sqrt{2}}\right]$$

57. (i) Horizontal force on the cylinder due to two liquids must cancel out.

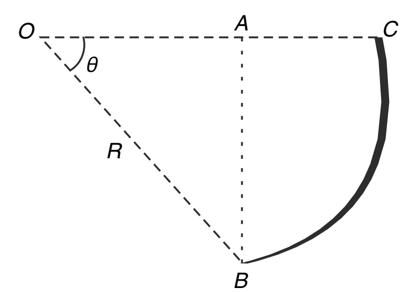
Force from left = force from right

$(P_{avg})_{Left}\,hL = (P_{avg})_{right}\,RL$ [L is length of the cylinder]

$$\Rightarrow \frac{1}{2}2\rho gh.hL = \frac{1}{2}3\rho gR.RL$$

$$\Rightarrow h = \sqrt{\frac{3}{2}}R$$

(ii) (a) F_v = Weight of water above the curved surface.



= Weight of water in part ABC

= b (Area of sector BOC – Area of $\Delta\ BOA$) ρg

$$= b\left[\frac{\pi R^2}{6} - \frac{1}{2}\frac{R}{2}\frac{\sqrt{3}R}{2}\right]\rho g = b\rho gR^2\left[\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right]$$

$$= 10 \times 10^3 \times 10 \times 20^2 \left[\frac{3.14}{6} - \frac{1.732}{8}\right] = 1.2 \times 10^7 N$$

(b) F_H = Force on projection of curved surface on to a vertical plane

= (Average pressure) × Area $= \left(\frac{1}{2}\rho gh\right)(bh)$

$$= \left(\frac{1}{2}\rho g\frac{\sqrt{3}}{2}R\right)\left(b\frac{\sqrt{3}}{2}R\right) \qquad \left[\because h = AB = \frac{\sqrt{3}}{2}R\right]$$

$$= \frac{3}{8}\times 10^3 \times 10 \times 10 \times (20)^2 = 1.5 \times 10^7 N$$

(c) $F = \sqrt{F_H^2 + F_V^2} = 1.92 \times 10^7\ N$

58. Let difference in height of the surfaces of Hg on the two sides be h_0 and the height of the water column be h_ω

$$P_1 + \rho_{Hg}\, gh_0 = P_2 + \rho_\omega g\, h_\omega$$

$$P_1 - P_2 = \rho_\omega gh_\omega - \rho_{Hg}\, gh_0$$

$$P_0 = \rho_\omega gh_\omega - \rho_{Hg}\, gh_0 \quad . \qquad \text{................(1)}$$

Let the length of Hg column fall by x when P_0 is changed by ΔP_0. The length of Hg column rises in the other tube by $\frac{A_1 x}{A_2}$.

$\therefore$ Change in h_0 will be $\Delta h_0 = x + \frac{A_1}{A_2}x = \left(1+\frac{A_1}{A_2}\right)x$

From (1) $\Delta P_0 = \Delta(\rho_\omega\, gh_\omega) - \Delta\,(\rho_{Hg} gh_0)$

$$= 0 - \rho_{Hg}\; g\,\Delta h_0 = \rho_{Hg} g\left(1+\frac{A_1}{A_2}\right)x$$

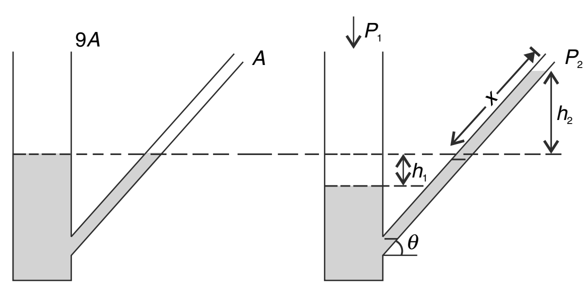
[Negative sign indicates that Δh_0 decreases]

But $A_1 x = A_2 \Delta A \quad \therefore \; x = \frac{A_2}{A_1}\Delta h$

$$\therefore \; \Delta P_0 = \rho_{Hg}.g\left(1+\frac{A_1}{A_2}\right)\frac{A_2}{A_1}\Delta h \qquad \therefore \; \frac{\Delta P_0}{\Delta h} = \rho_{Hg}.g\left(1+\frac{A_2}{A_1}\right)$$

$$\therefore \; \frac{\Delta h}{\Delta P_0} = \frac{1}{\left(1+\frac{A_2}{A_1}\right)\left(\rho_{Hg}.g\right)}$$

59.



When a pressure difference $P_1 - P_2 = \Delta P$ is applied various height and length are as shown in the second figure. Since volume of manometer liquid is constant, we have-

$\therefore \; 9Ah_1 = A.x \quad \Rightarrow x = 9h_1$

$h_2 = x \sin\theta = 9h_1 \sin\theta$

$\Delta P = \rho g\,(h_1 + h_2) \qquad [\rho = \text{density of liquid}]$

$$= 0.74\,\rho_\omega\; g\left[\frac{x}{9} + x\sin\theta\right] = 0.74\,\rho_\omega\; g\,x\left[\frac{1}{9}+\sin\theta\right]$$

It is needed that when $\Delta P = 0.09 \times 10^{-3}\,\rho_\omega g$, value of x is $0.5 \times 10^{-3}\, m$

$$\therefore 9\times10^{-5} \times \rho_\omega\, g = 0.74\,\rho_\omega\, g \times 5\times10^{-4}\left[\frac{1}{9}+\sin\theta\right]$$

$$\frac{0.9}{0.74\times 5} = \frac{1}{9} + \sin\theta$$

$\sin\theta = 0.24 - 0.11 = 0.13$

$\therefore\ \theta = \sin^{-1}(0.13)$

60. If force exerted on piston of area A_2 is F_2 then, the force acting on the other piston will be

$= \frac{F_2}{A_2}\cdot A_1$ [Pascal's law]

$= 5\,F_2$ $\left[\because \frac{A_1}{A_2} = 5\right]$

To raise the load $5F_2 = 20000$ $\Rightarrow F_2 = 4000\ N$

Since lever bar is light, net torque on it (about the hinge) must be zero.

$\therefore\ F_2\, a = F\,(a + b)$

$$\therefore\ F = \frac{F_2 a}{a+b} = \frac{4000\times 4}{4+36} = 400\ N$$

61. A tangent on a streamline gives the direction of flow velocity.

$\therefore$ Slope of streamline $= \frac{V_y}{V_x}$

$$\frac{dy}{dx} = -\frac{by}{u_0 + bx}$$

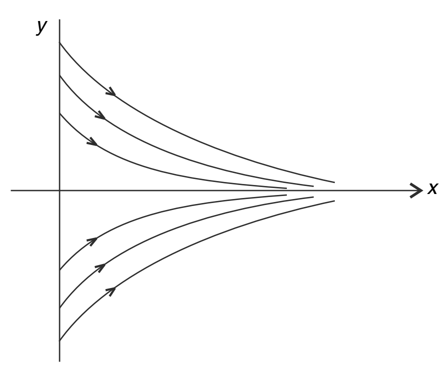
$$\Rightarrow \int\frac{dy}{y} = -b\int\frac{dx}{u_0 + bx} \quad \Rightarrow \quad \ell n\, y = -\ell n\left(u_0 + bx\right) + c$$

Write $c = \ell n c_0 = a$ constant

$$\ell n y = -\ell n\left(u_0 + bx\right) + \ell n\, c_o \ \Rightarrow \ell n\, y = \ell n\frac{c_0}{u_0 + bx}$$

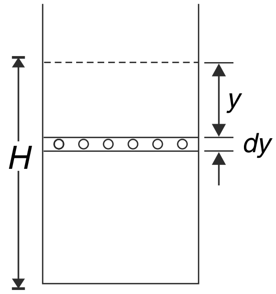
$$\Rightarrow \ y = \frac{c_0}{u_0 + bx}$$

For different value of we c_0 get different streamlines.



62. Speed of efflux at *a* depth *y* below the water surface is $v = \sqrt{2gy}$

Consider a strip of height dy as shown.



Cumulative area of holes in this strip is $A = \eta\ (2\ \pi r dy)$

Rate (volume per unit time) at which the liquid flows out through the strip is

$= vA = 2\pi\eta r\sqrt{2g}\sqrt{y}\, dy$

Total rate of outflow

$$= 2\pi\eta r\sqrt{2g}\int_O^H \sqrt{y}\, dy = 2\pi\eta r\sqrt{2g}\ \frac{2}{3}\left[y^{3/2}\right]_O^H = \frac{4}{3}\pi\eta r\sqrt{2g}\ H^{3/2}$$

$\therefore$ For level to remain static

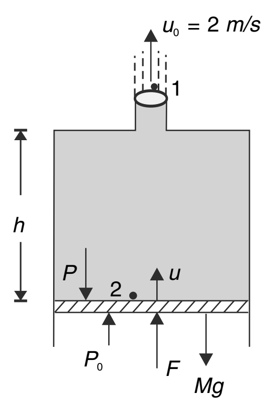
$$Q = \frac{4}{3}\pi\eta r\sqrt{2g}\ H^{3/2}$$

63. $h = \dfrac{V_0}{A} = \dfrac{40}{8} = 5\ cm$

Let speed of piston = u

<u>Continuity equation</u>

$$Au = au_0 \quad \Rightarrow u = \frac{a}{A}u_0 = \frac{2\ mm^2}{8\,cm^2}\times 2\,m/s = 0.005\,m/s$$



Applying Bernoulli's equation between points 1 and 2 -

$$P_0 + \frac{1}{2}\rho u_0^2 + \rho gh = P + \frac{1}{2}\rho u^2$$

where P is pressure above the piston at point 2.

$$\Rightarrow \quad P = P_0 + \frac{1}{2}\rho u_0^2 - \frac{1}{2}\rho u^2 + \rho gh$$

For piston to go up at constant speed

$$P_0A + F = PA + Mg \Rightarrow F = \frac{1}{2}\rho A\left(u_0^2 - u^2\right) + A\rho g\,h + Mg$$

As piston moves up, h changes from 5 cm to zero.

$\therefore$ Work done by external agent is

$$W = -\int_h^0 F dh = \frac{1}{2}\rho A\left(u_0^2 - u^2\right)h + \frac{1}{2}A\rho gh^2 + Mgh$$

$$= \frac{1}{2}\rho\, A\, u_0^2 h + \frac{1}{2}A\rho gh^2 + Mgh \quad \left[\because u_0^2 >> u^2\right]$$

$= 0.115\ J$

Alternate

W = Increase in KE of water + Increase in potential energy of water + Increase in potential energy of piston.

$$= \frac{1}{2}\rho\, Ah\left(u_0^2 - u^2\right) + \rho\, Ahg\frac{h}{2} + Mgh = 0.115\ J$$

64. (a) $v = \sqrt{2gh} = \sqrt{2\times10\times2} = 6.32\ ms^{-1}$

$\therefore$ Discharge rate $Q = \dfrac{\pi\, d_0^2}{4}\cdot v = \dfrac{3.14\times(0.01)^2}{4}\times 6.32 = 4.96\times10^{-4}\ m^3s^{-1}$

(b) Discharge speed $v = \sqrt{2gh} = 6.32\ ms^{-1}$

Let speed of flow at the neck be v. Applying Bernoulli's equation between the top surface of water and the nozzle - neck we get –

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

$$(10m) + 0 + (2m) = (2.5m) + \frac{v_2^2}{2g} + 0$$

$\therefore v_2^2 = 9.5 \times 2g$

$\therefore v_2 = \sqrt{9.5\times2\times10} = 13.78\ ms^{-1}$

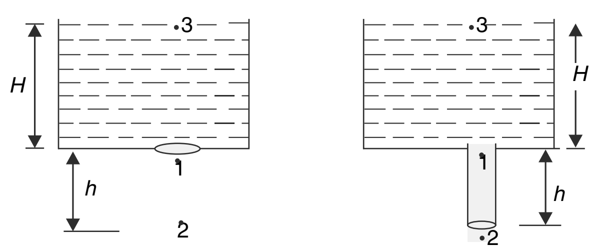
$\therefore$ Flow rate $Q = \pi\dfrac{d_0^2}{4}v_2 = 3.14\times\dfrac{(0.01)^2}{4}\times13.78 = 1.08\times10^{-3}\ m^3s^{-1}$

Now $Q = \dfrac{\pi d^2}{4}\cdot v$

$\therefore 1.08\times10^{-3} = \dfrac{3.14}{4}\times 6.32\times d^2$

$d^2 = 2.18\times10^{-4} \Rightarrow d = 1.48\times10^{-2}\ m = 1.48\ cm$

65.



(a) Applying Bernoulli's equation in both cases between point 3 and 2 gives speed at point 2 as $v_2 = \sqrt{2g(H+h)}$

(b) In second figure the continuity equation says that speed at 1 and 2 must be same. In first figure; to get the speed at point 1, we can apply Bernoulli's equation between point 3 and 1 or between 1 and 2. It gives -

$v_1 = \sqrt{2gH}$

$\therefore$ Required ratio $= \dfrac{\sqrt{2gH}}{\sqrt{2g(H+h)}} = \sqrt{\dfrac{H}{H+h}}$

(c) In first figure $P_1 = P_0$ = atmospheric pressure. In second figure, apply Bernoulli's equation between point 1 and 3.

$$P_1 + \frac{1}{2}\rho v_1^2 = P_0 + \rho g H$$

$$P_1 = P_0 + \rho g H - \rho g (H + h) \qquad \left[\because v_1 = \sqrt{2g\left(H+h\right)}\right]$$

$$= P_0 - \rho g h$$

$$\therefore \Delta P = \rho g h$$

66. Speed of efflux V can be calculated by using Bernoulli's equation.

$$\frac{1}{2}\rho\, v^2 + P_{atm} = P_{gas}$$

$$\frac{1}{2} \times 10^3 \times v^2 = \left(4-1\right)\times 10^6 \Rightarrow v^2 = 6 \times 10^3$$

Volume flow rate $Q = Av$

Mass flow rate $\frac{dm}{dt} = \rho Q = \rho A v$

$\therefore$ Thrust force $F_{th} = v\left(\frac{dm}{dt}\right) = \rho A v^2 = 10^3 \times 6000 \times A = 6 \times 10^6 A$

Now, $m.a = F_{\text{th}} - mg$

$\therefore F_{th} = m(a + g) = 2 \times (0.5g + g) = 3g$

$\therefore 6 \times 10^6 A = 3 \times 10$

$$A = \frac{1}{2}\times 10^{-5} m^2 = 5 \times 10^{-6} m^2 = 5\, mm^2$$

67. (a) let v = Instantaneous velocity of the tank

u = Instantaneous speed of efflux relative to the tank = $\sqrt{2\,gh}$

Where h = Instantaneous height of water in the tank.

Rate of mass coming out of the tank is $\frac{dm}{dt} = \left(uA_0\right)\rho$

We can use the equation for variable mass and write $m\frac{dv}{dt} = u\left(\frac{dm}{dt}\right)$

$m = Ah\,\rho$ = instantaneous mass of tank

$$\therefore Ah\rho\frac{dv}{dt} = u^2 A_0 \rho$$

$$Ah\frac{dv}{dt} = A_0\left(2gh\right)$$

$$\therefore \frac{dv}{dt} = \frac{2A_0 g}{A} = a \quad \text{constant} \qquad \text{-----------(1)}$$

(b) If h fall by dh in a small time interval dt then,

$-\rho\, Adh = \rho A_0 udt$

$$\Rightarrow \frac{dh}{dt} = -\frac{A_0 u}{A} = -\frac{A_0}{A}\sqrt{2gh}$$

From (1) $\dfrac{dv}{dt} = \dfrac{2A_0 g}{A}$

$$\Rightarrow \frac{dv}{dh}\frac{dh}{dt} = \frac{2A_0 g}{A} \Rightarrow \frac{dv}{dh}\left(-\frac{A_0}{A}\sqrt{2gh}\right) = \frac{2A_0 g}{A}$$

$$\therefore \frac{dv}{dh} = -\sqrt{2g}\,\frac{1}{\sqrt{h}}$$

$$\therefore \int_0^v dv = -\sqrt{2g}\int_H^0 \frac{dh}{\sqrt{h}}$$

This give $v = 2\sqrt{2\,gh}$.

68. (a) $P_{gage} = \dfrac{Kx}{\pi R^2}$

Let speed at which air moves out of the cut be v and pressure in the tube be P. Speed of air inside the tube will be much smaller and can be assumed to be close to zero.

Using Bernoulli's equation - $P = \dfrac{1}{2}\rho v^2 + P_0$

$$P_{gage} = P - P_0 = \frac{1}{2}\rho v^2$$

$$\therefore \frac{Kx}{\pi R^2} = \frac{1}{2}\rho\left(\frac{Q}{bx}\right)^2 \qquad \left[\because V = \frac{Q}{bx}\right]$$

$$\therefore Q = bx^{3/2}\sqrt{\frac{2K}{\rho\pi R^2}}$$

69. (i) Let the flow speed be V_1 and V_2 at the cross section A_1 and A_2 respectively.

$A_1V_1 = A_2V_2 = Q$ -----------------(i)

Applying Bernoulli's equation between two points of a streamline at the constriction and at the open end of the tube -

$$\frac{1}{2}\rho V_2^2 + P_2 = \frac{1}{2}\rho V_1^2 + P_0 \qquad [P_0 = \text{atmospheric pressure}]$$

$$\therefore P_0 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

$$\rho_0 gH = \frac{1}{2}\rho\left[\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2}\right]$$

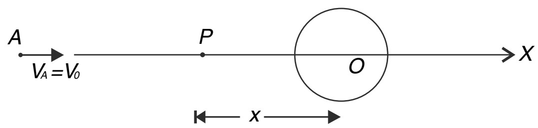
$$\therefore Q^2 = \frac{2\rho_0 gH}{\rho}\left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right]^{-1} \Rightarrow Q = \sqrt{\frac{2\rho_0 gH}{\rho}\left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right]^{-1}}$$

(ii) Work done on small element of liquid by neighbouring liquid = Change in KE of element + Change in gravitational PE of the element.

$$W = \Delta K + \Delta U = \frac{1}{2}\rho\Delta V\left(v_2^2 - v_1^2\right) + \rho\Delta Vg\left(h_2 - h_1\right)$$

$$= \frac{1}{2}\rho\Delta V\left(\frac{A_1^2}{A_2^2} - 1\right)v_1^2 + \rho\Delta Vg\left(h_2 - h_1\right)$$

70.



Let us consider a point P whose x co-ordinate is $-x$. Apply Bernoulli's equation between P and A.

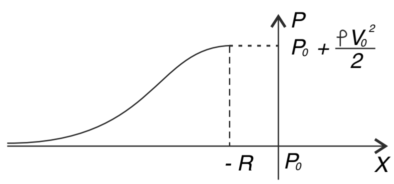
$$P_p + \left(\frac{\rho V^2}{2}\right)_P + 0 = P_A + \left(\frac{\rho V^2}{2}\right)_A + 0$$

$$P + \frac{\rho V^2}{2} = P_0 + \frac{\rho V_0^2}{2}$$

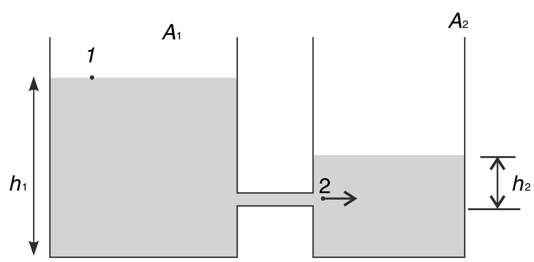
$$\therefore\ P = P_0 + \frac{\rho}{2}\left(V_0^2 - V^2\right) = P_0 + \frac{\rho}{2}V_0^2\left[1 - \left(1 + \frac{R^3}{x^3}\right)^2\right]$$

When $x \to (-R)$; $P \to P_o + \dfrac{\rho V_0^2}{2}$

Graph is as shown below. You may prove yourself that $\dfrac{dP}{dx} = 0$ at $x = \infty$ as well as $x = -R$



71.



Applying Bernoulli's equation along a streamline between point 1 and 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_2$$

$$\frac{P_0}{\rho g} + 0 + h_1 = \frac{P_0 + \rho g h_2}{\rho g} + \frac{V_2^2}{2g} + 0 \qquad [P_0 = \text{atmospheric pressure}]$$

$$\therefore h_1 = h_2 + \frac{V_2^2}{2g} \qquad \Rightarrow V_2 = \sqrt{2g\left(h_1 - h_2\right)}$$

Let $h_1 - h_2 = h$ $\therefore\ V_2 = \sqrt{2gh}$

Flow rate $Q = A_0\sqrt{2gh}$

Now, h_1 is decreasing, h_2 is increasing and h is decreasing.

$\therefore\ -dh_1 + dh_2 = -dh$ ....(1)

And $Q = -A_1\dfrac{dh_1}{dt} = A_2\dfrac{dh_2}{dt}$

$\Rightarrow Q\,dt = -A_1\,dh_1 = A_2dh_2$ ....(2)

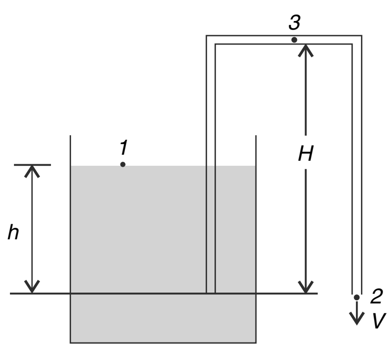
From (1) and (2) $-dh_1 - \dfrac{A_1}{A_2}dh_1 = -dh \therefore dh_1 = \dfrac{A_2\ dh}{A_1 + A_2}$

$$\therefore Qdt = -\frac{A_1A_2\ dh}{A_1 + A_2} \Rightarrow A_0\sqrt{2gh}\ dt = -\frac{A_1A_2}{A_1 + A_2}dh$$

$$\int_0^t dt = -\frac{A_1A_2}{A_0\left(A_1 + A_2\right)\sqrt{2g}}\int_h^0 \frac{dh}{\sqrt{h}}$$

$$t = \frac{2A_1A_2}{\sqrt{2g}A_0\left(A_1 + A_2\right)}\sqrt{h} = \sqrt{\frac{2}{g}}\frac{A_1A_2}{A_0\left(A_1 + A_2\right)}\sqrt{h_1 - h_2}$$

72. (a)



Applying Bernoulli's theorem between point 1 and 2–

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho gh_2$$

$$P_0 + 0 + \rho gh = P_0 + \frac{1}{2}\rho V^2 + 0 \qquad [V_1 = 0]$$

$\therefore\ V = \sqrt{2gh}$ ....(1)

(b) Applying Bernoulli's theorem between point 1 and 3–

$$P_0 + 0 + \rho gh = P_3 + \frac{1}{2}\rho V^2 + \rho gH$$

$$P' = P_0 + \rho gh - \frac{1}{2}\rho V^2 - \rho gH = P_0 - \rho gH$$ ....(2)

Change in h will not affect P_3 (!)

The pressure at point 3 can also be obtained by using Bernoulli's theorem between point 3 and 2.

(c) Pressure just inside left mouth of the tube must be equal to $P_2 = P_0$. This is because the fluid is having same speed throughout the tube.

(d) If the right mouth of the tube moves up, speed of flow will decrease (equation (1)) and P' will increase (equation (2))

(e) If right mouth is lowered, H increases and P' becomes zero for $H_0 = \dfrac{P_0}{\rho g} \simeq 10.3\ m$. The fluid will get separated in two columns with a shared vacuum space.

73. (a) Let the specific gravity be σ at the time volume V_w of water has been poured. When next drop of volume dV_w is added, the specific gravity changes to σ′.

$\sigma' V_0 = (V_0 - dV_w)\,\sigma + dV_w \times 1 \Rightarrow V_0\,(\sigma' - \sigma) = -(\sigma - 1)dV_w$

$\sigma' - \sigma = d\sigma$ = change in density of solution when a drop is added.

$\therefore\ V_0\, d\,\sigma = -(\sigma - 1)dV_w$

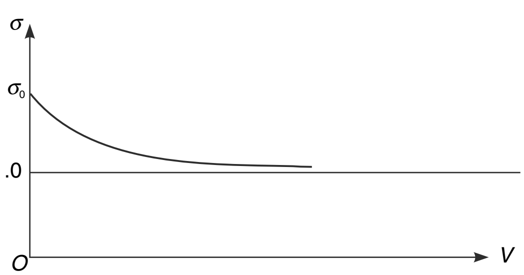
$\Rightarrow V_0 \int_{\sigma_0}^{\sigma} \dfrac{d\sigma}{\sigma - 1} = -\int_0^V dV_w$ [Remember V_0 is a constant]

$\Rightarrow \left[\ln(\sigma - 1)\right]_{\sigma_0}^{\sigma} = -\dfrac{V}{V_0} \Rightarrow \ln\left(\dfrac{\sigma - 1}{\sigma_0 - 1}\right) = -\dfrac{V}{V_0}$

$\Rightarrow \sigma = 1 + (\sigma_0 - 1)e^{-V/V_0}$

(b) $\sigma = 1 + (1.2 - 1)e^{-1} = 1 + 0.2 \times 0.37 = 1.074$

(c)



74. Pressure at the horizontal surface is $P = \rho g.R$.

Force by liquid on the horizontal surface (↓) = Force by the surface on the liquid (↑)

$F = P.\ \pi R^2 = \rho.g.\pi R^3$

The bowl will lose contact with the surface when

F = Weight of (liquid + bowl)

$\rho.g.\pi R^3 = \rho.\dfrac{2}{3}\pi R^3 g + mg$

$\therefore m = \dfrac{\pi R^3 \rho}{3}$

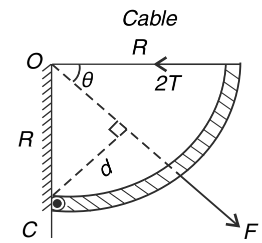
75. Vertical force on the plate due to water

F_v = weight of water $= \rho.\dfrac{\pi R^2}{4} Lg$

Horizontal force on the plate due to water = Horizontal force on the vertical wall

F_H = Average pressure × Area $= \frac{\rho g R}{2} \times RL = \frac{1}{2}\rho g R^2 L$

The resultant force makes on angle θ with horizontal where - $\tan\theta = \frac{F_v}{F_H} = \frac{\pi}{2}$.



The resultant force will pass through O (why?)

Torque of force F about C is $\tau_F = F.d = FR\cos\theta$

This is balanced by the torque due to tension in the cable.

If tension in each cable is T then $\tau_T = 2T.R$

$\therefore\ 2TR = FR.\cos\theta$

$$T = \frac{F}{2}\cos\theta = \sqrt{F_v^2 + F_H^2}\ .\ \frac{1}{2}$$

$$= \frac{1}{4}\rho g R^2 L\sqrt{1+\frac{\pi^2}{4}} = \frac{1}{8}\rho g R^2 L\sqrt{4+\pi^2}$$

76. (a) let $V = \frac{4}{3}\pi R^3$ = volume of the sphere

$F_B = W$

$$V\rho_0 g = \frac{V}{2}\rho g + \frac{V}{2}(2\rho)g$$

$$\Rightarrow \rho_0 = \frac{3}{2}\rho \Rightarrow \rho = \frac{2\rho_0}{3}$$

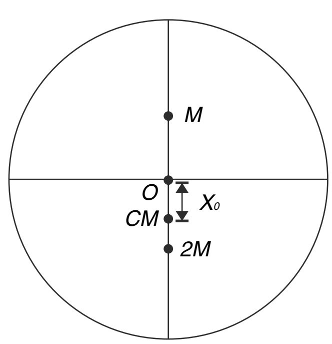
(b) Let's first find the position of COM of the ball.

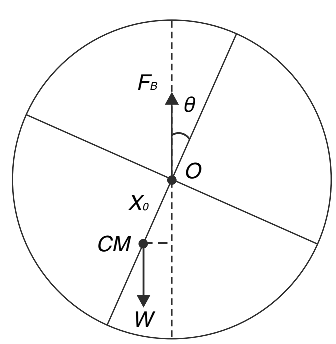
$$x_0 = \frac{2M\left(\frac{3R}{8}\right) - M\left(\frac{3R}{8}\right)}{2M+M} = \frac{R}{8}$$

[$\because$ distance of CoM of each hemisphere = $\frac{3R}{8}$ from point O]

Where M = mass of upper hemisphere

2 M = mass of lower hemisphere





Now let's find the moment of inertia(I) of the ball about an axis passing through COM (perpendicular to the fig).

First we will write the moment of inertial (I_0) about an axis through O and perpendicular to the plane of the fig.

$$I_0 = \frac{2}{5}MR^2 + \frac{2}{5}(2M)R^2 = \frac{6}{5}MR^2$$

Using parallel axes theorem

$I_0 = I + 3M x^2_0$ [$\because$ mass of sphere = $3M$]

$$\therefore\ I = \frac{6}{5}MR^2 - 3M\frac{R^2}{64} = \frac{369}{320}MR^2$$

Consider the ball slightly displaced about the equilibrium. The torque due to buoyancy (about the axis through COM) provides the necessary restoring torque.

$\therefore\ I\alpha = -F_B x_0 \sin\theta$

$$\therefore\ \frac{369}{320}MR^2 . \alpha = -F_B \frac{R}{8}\theta \qquad \text{[for small } \theta\text{, } \sin\theta = \theta\text{]}$$

$$\because\ F_B = 3Mg \quad \therefore \alpha = -\left(\frac{40}{123}\frac{g}{R}\right)\theta$$

$$\therefore\ \omega = \sqrt{\frac{40g}{123R}} \quad \Rightarrow T = 2\pi\sqrt{\frac{123R}{40g}}$$

77. (a) If rate of fluid flow is Q then

$$nP_0 + \frac{1}{2}\rho\left(\frac{Q}{\pi r_1^2}\right)^2 = \frac{nP_0}{2} + \frac{1}{2}\rho\left(\frac{Q}{\pi r_2^2}\right)^2$$

$$\therefore Q = \pi\ r_1^2\ r_2^2\sqrt{\frac{nP_0}{\rho\{r_1^4 - r_2^4\}}}$$

(b) $\mu N = F$

$$N = \frac{F}{\mu} = PA \quad \therefore P = \frac{F}{A\ \mu} = \frac{F}{2\pi r_1 b\,\mu}$$

(c) The fluid will not gush out if pressure inside is less than atmospheric pressure.

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$P_A = nP_0$, $P_B = P_0$

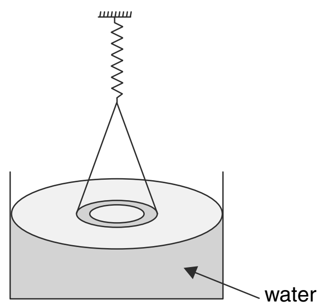
$$\text{Then, } nP_0 - P_0 = \frac{1}{2}\rho Q^2\left[\frac{1}{(\pi r_2^2)^2} - \frac{1}{(\pi r_1^2)^2}\right]$$

$$Q = \pi\ r_1^2 r_2^2\sqrt{\frac{2(n-1)P_0}{\rho[r_1^4 - r_2^4]}}$$

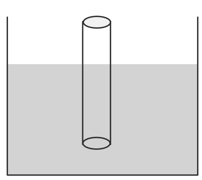
09 SURFACE TENSION

Level 1

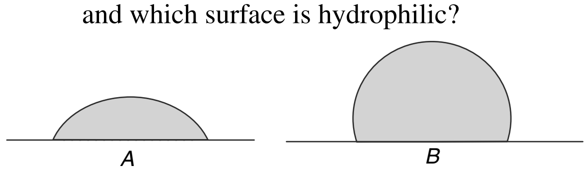
Q.1. A circular ring has inner and outer radii equal to 10 *mm* and 30 *mm* respectively Mass of the ring is $m = 0.7\ g$. It gently pulled out vertically from a water surface by a sensitive spring. When the spring is stretched 3.4 *cm* from its equilibrium position the ring is on verge of being pulled out from the water surface. If spring constant is $k = 0.7\ Nm^{-1}$ find the surface tension of water.



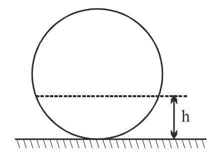
Q.2. A long thin walled capillary tube of mass M and radius r is partially immersed in a liquid of surface tension T. The angle of contact for the liquid and the tube wall is 30°. How much force is needed to hold the tube vertically? Neglect buoyancy force on the tube.



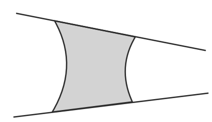
Q.3. (i) Water drops on two surfaces A and B has been shown in figure. Which surface is hydrophobic and which surface is hydrophilic?



(ii) A liquid is filled in a spherical container of radius R till a height h. In this position the liquid surface at the edges is also horizontal. What is the contact angle between the liquid and the container wall?



Q.4. A conical pipe shown in figure has a small water drop. In which direction does the drop will tend to move?



Q.5. A narrow tube of length l and radius r is sealed at one end. Its open end is brought in contact with the surface of water while the tube is held vertical. The water rises to a height h in the tube. The contact angle of water with the tube wall is θ, density of water is ρ and the atmospheric pressure is P_o. Find the surface tension of the liquid. Assume that the temperature of air inside the tube remains constant and the volume of the meniscus is negligible.

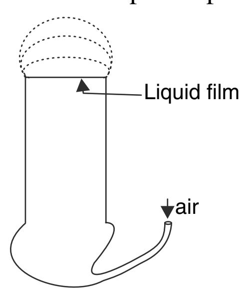
Q.6. The internal radius of one arm of a glass capillary U tube is r_1 and for the second arm it is $r_2 (> r_1)$. The tube is filled with some mercury having surface tension T and contact angle with glass equal to $90° + \theta$.

(a) It is proposed to connect one arm of the U tube to a vacuum pump- so that the mercury level in both arms can be equalized. To which arm the pump shall be connected?

(b) When mercury level in both arms is same, how much below the atmospheric pressure is the pressure of air in the arm connected to the pump?

Q.7. In a horizontal capillary tube, the rate of capillary flow depends on the surface tension force as well as the viscous force. Lueas and washburn showed that the length (x) of liquid penetration in a horizontal capillary depends on a factor (k) apart from time (t). The factor is given by $k = \left[\frac{rT\cos\theta}{2\eta}\right]^{\frac{1}{2}}$; where r, T, θ and η are radius of the capillary tube, surface tension, contact angle and coefficient of viscosity respectively. If the length of liquid in the capillary grows from zero to x_0 in time t_0, how much time will be needed for the length to increases from x_0 to $4x_0$.

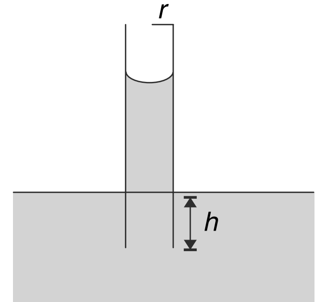
Q.8. A glass tube of radius R is covered with a liquid film at its one end. Air is blown slowly into the tube to gradually increase the pressure inside. What is the maximum pressure that the air inside the tube can have? Assume that the liquid film does not leave the surface (whatever its size) and it does not get punctured. Surface tension of the liquid is T and atmospheric pressure is P_o.



Q.9. Why bubbles can be formed using soap water but we do not have bubbles formed out of pure water?

Q.10. A tapering glass capillary tube A of length 0.1 m has diameters 10^{-3} m and 5×10^4 m at the ends. When it is just immersed in a liquid at 0°C with larger radius in contact with liquid surface, the liquid rises 8×10^{-2} m in the tube. In another experiment, in a cylindrical glass capillary tube B, when immersed in the same liquid at 0°C, the liquid rises to 6×10^{-2} m height. The rise of liquid in tube B is only 5.5×10^{-2} m when the liquid is at 50°C. Find the rate at which the surface tension changes with temperature considering the change to be linear. The density of liquid is $(1/14) \times 10^4$ kg/m^3 and the angle of contact is zero. Effect of temperature on the density of liquid and glass is negligible.

Q.11. (i) One end of a uniform glass capillary tube of radius $r = 0.025$ cm is immersed vertically in water to a depth $h = 1cm$. Contact angle is 0°, surface tension of water is 7.5×10^{-2} N/m, density of water is $\rho = 10^3$ kg/m^3 and atmospheric pressure is $P_o = 10^5$ N/m^2

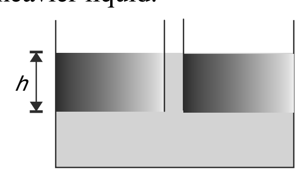


Find the excess pressure to be applied on the water in the capillary tube so that -

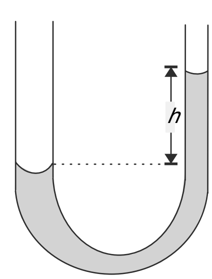
(a) The water level in the tube becomes same as that in the vessel.

(b) Is it possible to blow out an air bubble out of the tube by increasing the pressure?

(ii) A container contains two immiscible liquids of density ρ_1 and ρ_2 ($\rho_2 > \rho_1$). A capillary of radius r is inserted in the liquid so that its bottom reaches up to denser liquid and lighter liquid does not enter into the capillary. Denser liquid rises in capillary and attain height equal to h which is also equal to column length of lighter liquid. Assuming zero contact angle find surface tension of the heavier liquid.

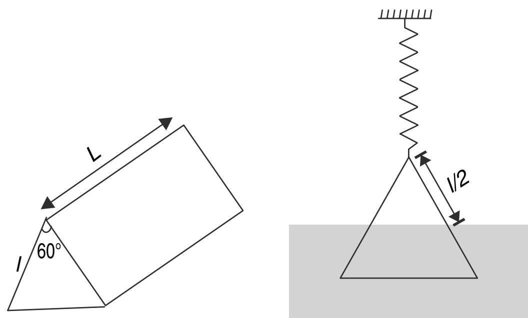


Q.12. The radii of two columns in a U tube are r_1 and r_2 ($r_1 > r_2$). A liquid of density ρ is filled in it. The contact angle of the liquid with the tube wall is θ. If the surface tension of the liquid is T then plot the graph of the level difference (h) of the liquid in the two arms versus contact angle θ. Plot the graph for angle θ changing from 0° to 90°. Assume the curved surface of meniscus to be part of a sphere.

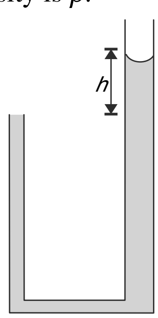


LEVEL 2

Q.13. A glass prism has its principal section in form of an equilateral triangle of side length l. The length of the prism is L (see fig.). The prism, with its base horizontal, is supported by a vertical spring of force constant k. Half the slant surface of the prism is submerged in water. Surface tension of water is T and contact angle between water and glass is 0°. Density of glass is d and that of water is ρ (< d). Calculate the extension in the spring in this position of equilibrium.



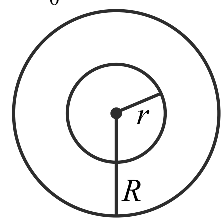
Q.14. Two capillaries of small cross section are connected as shown in the figure. The right tube has cross sectional radius R and left one has a radius of r (< R). The tube of radius R is very long where as the tube of radius r is of short length. Water is slowly poured in the right tube. Contact angle for the tube wall and water is $\theta = 0°$. Let h be the height difference between water surface in the right and left tube. Surface tension of water is T and its density is ρ.



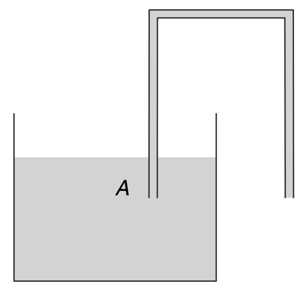
(a) Find the value of h if the water surface in the left tube is found to be flat.

(b) Find the maximum value of h for which water will not flow out of the left tube .

Q.15. A soap bubble of radius r is formed inside another soap bubble of radius R (> r). The atmospheric pressure is P_0 and surface tension of the soap solution is T. Calculate change in radius of the smaller bubble if the outer bubble bursts. Assume that the excess pressure inside a bubble is small compared to P_0.



Q.16. In the siphon shown in the figure the ends A and B of the tube are at same horizontal level. Water fills the entire tube but it does not flow out of the end B. With the help of a diagram show how the water surface at end B changes if the end B were slightly lower than the position shown.

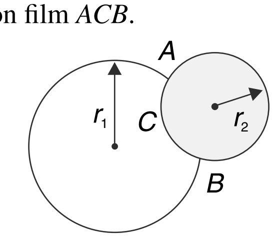


Q.17. A glass capillary tube sealed at the upper end has internal radius r. The tube is held vertical with its lower end touching the surface of water. Calculate the length (L) of such a tube for water in it to rise to a height h (< L). Atmospheric pressure is P_o and surface tension of water is T. Assume that water perfectly wets glass (Density of water = ρ)

Q.18. In the last question let the length of the tube be L and its outer radius be R. Water rises in it to a height h. Calculate the vertical force needed to hold the tube in this position. Mass of empty tube is M.

Q.19. A glass capillary tube is held vertical and put into contact with the surface of water in a tank. It was observed that the liquid rises to the top of the tube before settling to an equilibrium height h_0 in the tube. Assume that water perfectly wets glass and viscosity is small. Is the length of the capillary tube larger then $2h_0$?

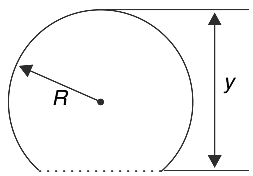
Q.20. Two soap bubbles of radii r_1 and r_2 are attached as shown. Find the radius of curvature of the common film ACB.



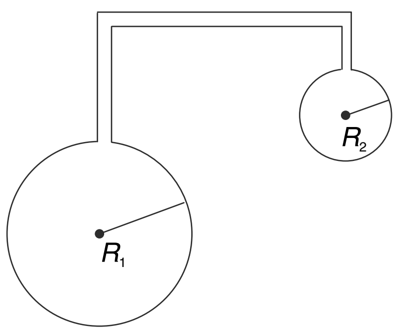
Q.21. (a) In the last question find the angle between the tangents drawn to the bubble surfaces at point A.

(b) In the above question assume that $r_1 = r_2 = r$. What is the shape of the common interface ACB? Find length AB in this case.

(c) With $r_1 = r_2 = r$ the common wall bursts and the two bubbles form a single bubble find the radius of this new bubble. It is given that volume of a truncated sphere of radius R and height y is $\frac{\pi}{3} y^2 (3R - y)$ [see figure]



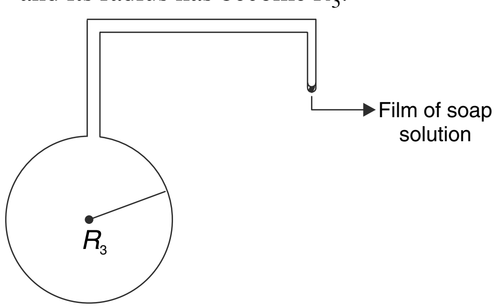
Q.22. Two soap bubbles of radius R_1 and R_2 ($< R_1$) are joined by a straw. Air flows from one bubble to another and a single bubble of radius R_3 remains.



(a) From which bubble does the air flow out ?

(b) Assuming no temperature change and atmospheric pressure to be P_o, find the surface tension of the soap solution.

Q.23. In the last problem, one of the bubbles supplies its entire air to the other bubble and a film of soap solution is formed at the end of the straw which keeps it closed. What is the radius of curvature of this film if the bigger bubble has grown in size and its radius has become R_3.



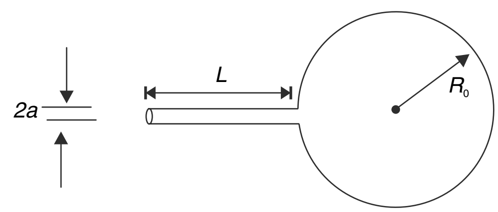
LEVEL 3

Q.24. Consider a rain drop falling at terminal speed. For what radius (R) of the drop can we disregard the influence of gravity on its shape? Surface tension and density of water are T and ρ respectively.

Q.25. A soap bubble has radius R and thickness of its wall is a. Calculate the apparent weight (= true weight - Buoyancy) of the bubble if surface tension of soap solution and its density are T and d respectively. The atmospheric pressure is P_0 and density of atmospheric air is ρ_0. By assuming $a = 10^{-6}\ m$, $R = 10\ cm$, $P_0 = 10^5\ Nm^{-2}$, $\rho_0 = 1.2\ kg\ m^{-3}$, $d = 10^3\ kg\ m^{-3}$, $T = 0.04\ Nm^{-1}$; show that the weight of the bubble is mainly because of water in the skin. What is weight of the bubble?

Q.26. A soap bubble is blown at the end of a capillary tube of radius a and length L. When the other end is left open, the bubble begins to deflate. Write the radius of the bubble as a function of time if the initial radius of the bubble was R_0. Surface tension of soap solution is T. It is known that volume flow rate through a tube of radius a and length L is given by Poiseuille's equation-

$$Q = \frac{\pi a^4 \Delta P}{8 \eta L}$$



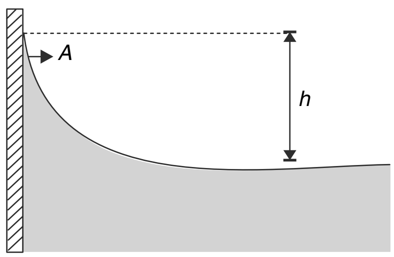
Where ΔP is pressure difference at the two ends of the tube and η is coefficient of viscosity. Assume that the bubble remains spherical.

Q.27. Two blocks are floating in water. When they are brought sufficiently close they are attracted to each other due to surface tension effects. When the experiment is repeated after replacing water with mercury, once again the two blocks are attracted. Explain the phenomena. It is given that water wets the material of the block where as mercury does not.

Q.28. A long thin string has a coat of water on it. The radius of the water cylinder is r. After some time it was found that the string had a series of equally spaced identical water drops on it. Find the minimum distance between two successive drops.

Q.29. A liquid having surface tension T and density ρ is in contact with a vertical solid wall. The liquid surface gets curved as shown in the figure. At the bottom the liquid surface is flat.

The atmospheric pressure is P_o.

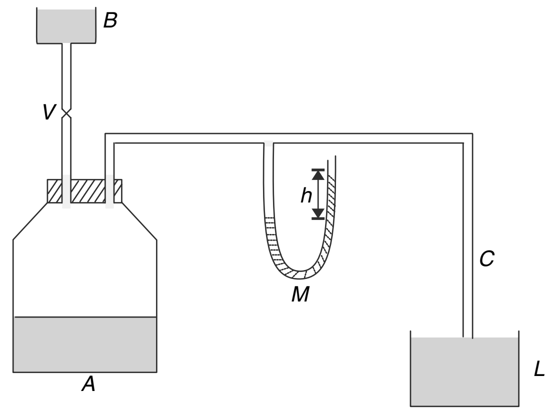


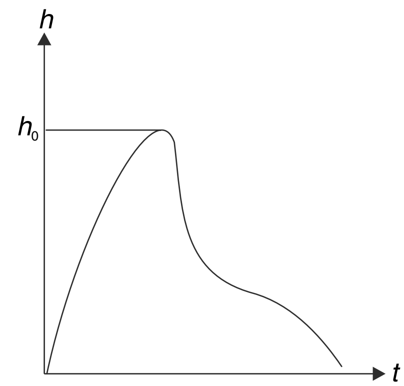
(i) Find the pressure in the liquid at the top of the meniscus (i.e. at A)

(ii) Calculate the difference in height (h) between the bottom and top of the meniscus.

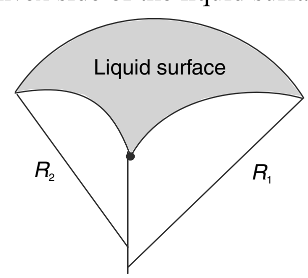
Q.30. Is it possible that water evaporates from a spherical drop of water just by means of surface energy supplying the necessary latent heat of vaporisation? The drop does not use its internal thermal energy and does not receive any heat from outside. It is known that water drops of size less than 10^{-6} m do not exist. Latent heat of vaporisation of water is $L = 2.3 \times 10^6$ Jkg^{-1} and surface tension is $T = 0.07$ Nm^{-1}.

Q.31. In the arrangement shown in the figure, A is a jar half filled with water and half filled with air. It is fitted with a leak proof cork. A tube connects it to a water vessel B. Another narrow tube fitted to A connects it to a narrow tube C via a water monometer M. The tip of the tube C is just touching the surface of a liquid L. Valve V is opened at time $t = 0$ and water from vessel B pours down slowly and uniformly into the jar A. An air bubble develops at the tip of tube C. The cross sectional radius of tube C is r and density of water is ρ. The difference in height of water (h) in the two arms of the manometer varies with time $'t'$ as shown in the graph. Find the surface tension of the liquid L.

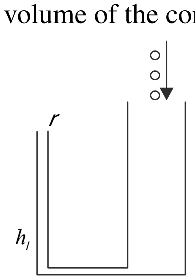




Q.32. A curved liquid surface has radius of curvature R_1 and R_2 in two perpendicular directions as shown in figure. Surface tension of the liquid is T. Find the difference in pressure on the concave side and the convex side of the liquid surface.



Q.33. A capillary tube of radius r and height h_1 is connected to a broad tube of large height as shown in the figure. Water is poured into the broad tube – drop by drop. Drops fall at regular intervals. Plot the variation of height of water in both tubes with time. Initially the tube and capillary are empty. Neglect the volume of the connecting pipe.



ANSWERS

1. $0.076\ Nm^{-1}$

2. $2\sqrt{3}\pi rT$ + Mg

3. (i) $A \to$ hydrophilic , $B \to$ hydrophobic

(ii) $\cos^{-1}\left(\dfrac{R-h}{R}\right)$

4. Towards the tapered end.

5. $T = \dfrac{r}{2\cos\theta}\left[\dfrac{P_\circ h}{\ell - h} + \rho g h\right]$

6. (a) To capillary of smaller radius

(b) $\dfrac{2T\sin\theta(r_2 - r_1)}{r_1 r_2}$

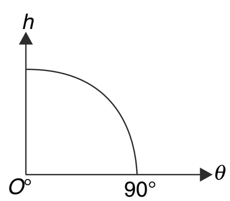
7. $15t_0$

8. $P_\circ + \dfrac{4T}{R}$

10. $-1.4\times10^{-4}\ \dfrac{N}{m°C}$

11. (i) (a) 600 P_a (b) Yes. (ii) $T = \dfrac{r}{2}(\rho_2 - \rho_1)gh$

12.



13. $x = \dfrac{1}{K}\left[\dfrac{\sqrt{3}}{4}l^2 L\,d.g - \dfrac{3\sqrt{3}}{16}l^2\ L\rho g + \sqrt{3}TL + Tl\right]$

14. (a) $h = \dfrac{2T}{R\rho g}$ (b) $\dfrac{2T}{\rho g}\left(\dfrac{r+R}{rR}\right)$

15. $\Delta r = \dfrac{4Tr}{3P_0 R}$

16. The radius of curvature decreases

17. $L = \dfrac{P_\circ hr}{2T - \rho grh} + h$

18. $Mg + \pi P_\circ\left[R^2 - \dfrac{Lr^2}{L-h}\right] + 2\pi(R+r)T$

19. No, $l < 2h_0$

20. $\dfrac{r_1 r_2}{r_1 - r_2}$

21. (a) 120° (b) $\sqrt{3}r$ (c) $\dfrac{3r}{2(2)^{1/3}}$

22. (a) From smaller bubble (b) $T = \dfrac{P_\circ\left(R_3^3 - R_1^3 - R_2^3\right)}{4\left(R_1^2 + R_2^2 - R_3^2\right)}$

23. R_3

24. $R << \sqrt{\dfrac{T}{\rho g}}$

25. $\dfrac{16\pi}{3}\dfrac{R^2\rho_\circ}{P_\circ}g + 4\pi R^2 a.d.g$

26. $R = R_0\left[1 - \dfrac{a^4 Tt}{2\eta L R_0^4}\right]^{\frac{1}{4}}$

28. $\dfrac{9}{2}r$

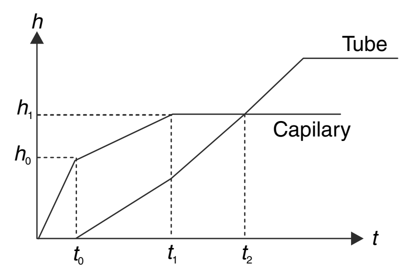
29. (i) $P_o - \rho gh$ (ii) $\sqrt{\dfrac{2T}{\rho g}}$

30. No

31. $\dfrac{\rho g h_0 r}{2}$

32. $\Delta P = T\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$

33.



SOLUTIONS

1. When the ring is about to leave the water surface, surface tension force on it is

$F_{ST} = 2\pi RT + 2\pi rT = 2\pi(R + r)T$

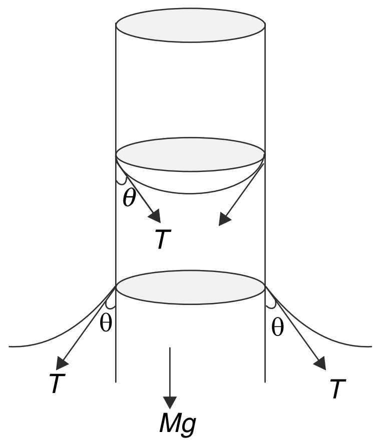
Spring force $F_S = kx$

$\therefore\ kx = 2\pi(R + r)T + mg$

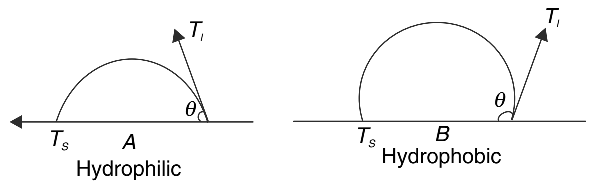
$$\therefore T = \frac{kx - mg}{2\pi(R + r)} = \frac{0.7 \times 3.4 \times 10^{-2} - 7 \times 10^{-4} \times 9.8}{2 \times 3.14 \times (30 + 10) \times 10^{-3}} = 0.076\ Nm^{-1}$$

2. Force = Mg + force due to surface tension on the inner wall + force due to surface tension on the outer wall

$= Mg + 2\pi rT\cos\theta + 2\pi rT\cos\theta = Mg + 2\sqrt{3}\pi rT$



3. (i) The correct contact angle (θ) has been shown in figure

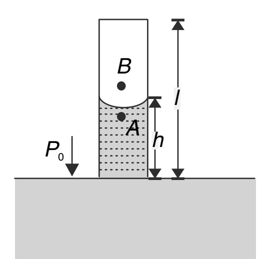


If θ is acute the surface is hydrophilic (i.e. water wets the surface) and if θ is obtuse the surface is hydrophobic.

(ii) Draw a tangent on the container wall at point of contact. Angle between this tangent and the liquid surface is the contact angle.

4. Pressure difference on two sides of a curved surface is inversely proportional to the radius of curvature.

5. $P_A = P_\circ - \rho gh$



Pressure at B is higher than at A by $\frac{2T}{r}\cos\theta$

$$\therefore P_B = P_A + \frac{2T\cos\theta}{r} = P_\circ - \rho gh + \frac{2T\cos\theta}{r}$$

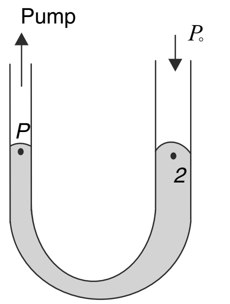
$P_1V_1 = P_2V_2$ for air inside the tube

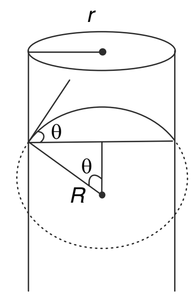
$$P_\circ A\ell = \left[P_\circ - \rho gh + \frac{2T\cos\theta}{r}\right]A(\ell - h)$$

$$\Rightarrow \frac{2T\cos\theta}{r} = \frac{P_\circ\ell}{\ell - h} - P_\circ + \rho gh = \frac{P_\circ h}{\ell - h} + \rho gh$$

$$\Delta P = T\left(\frac{1}{R_1}+\frac{1}{R2}\right) \Rightarrow T = \frac{r}{2\cos\theta}\left[\frac{P_\circ h}{\ell - h}+\rho g h\right]$$

6. Relation between radius of curvature (R) of the Hg surface and tube radius (r) is $\frac{r}{R} = \sin\theta$





$P_1 = P_2$

$$P + \frac{2T\sin\theta}{r_1} = P_\circ + \frac{2T\sin\theta}{r_2}; \quad \therefore\ P_\circ - P = 2T\sin\theta\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = 2T\sin\theta\left(\frac{r_2-r_1}{r_1 r_2}\right)$$

7. One can easily show that dimension of the factor k are: $[k] = \left[M^\circ L^1 T^{-\frac{1}{2}}\right]$

If $x = kt^a$, then $a = \frac{1}{2}$

$\therefore\ x \propto t^{\frac{1}{2}}$

Time needed for x to go from zero to x_0 is t_0

Time for x to grow from zero to $4x_0$ will be $16t_0$

$\therefore$ required answer is $15t_0$

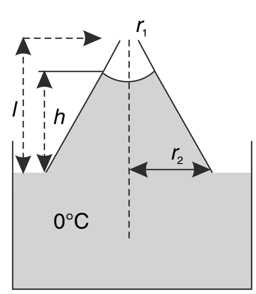
9. When water sprays from a tap in a small container, one can see bubbles, but they burst very soon. This is because surface tension of water is high and it tends to draw water molecules into the bulk, to the point where thickness of bubble wall is too thin to remain intact. On the other hand, the surface tension of soap water is much lower. The molecules of the bubble are less stressed and can last longer.

10. The situation is shown in figure.

Let r_1 and r_2 be radii of upper and lower ends of the conical capillary tube. The radius r at the meniscus is given by

$$r = r_1 + (r_2 - r_1)\left(\frac{l-h}{l}\right)$$

$$= (2.5\times10^{-4}) + (2.5\times10^{-4})\left(\frac{0.1-0.08}{0.1}\right) = 3.0\times10^{-4}\ m$$



The surface tension at 0°C is given by

$$T_0 = \frac{rh\rho g}{2} = \frac{(3.0\times10^{-4})(8\times10^{-2})(1/4\times10^{4})(9.8)}{2} = 0.084$$

For tube B; $\quad \frac{T_0}{T_{50}} = \frac{h_0}{h_{50}} = \frac{6\times10^{-2}}{5.5\times10^{-2}} = \frac{12}{11}$

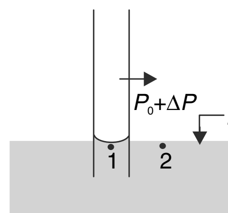
Or, $\quad T_{50} = \frac{11}{12}\times T_0 = \frac{11}{12}\times 0.084 = 0.077 \quad N/m$

Considering that change in surface tension as linear, the change in surface tension with temperature is given by

$$\alpha = \frac{T_{50}-T_0}{50} = \frac{0.077-0.084}{50} = -1.4\times10^{-4}\ \frac{N}{m^\circ C}$$

11. (i) (a) $P_1 = P_2$

$$P_\circ + \Delta P - \frac{2T}{R} = P_\circ$$



[R = radius of curvature of the curved surface. Pressure on convex side is lesser by $\frac{2T}{R}$ than pressure on concave side]

For $\theta = 0°$; $R = r$

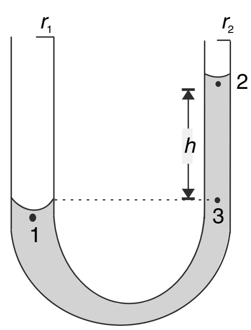
$$\Delta P = \frac{2T}{r} = \frac{2\times7.5\times10^{-2}}{2.5\times10^{-4}} = 600\ Pa$$

(ii) $P_0 + \rho_1 gh - \rho_2 gh + \frac{2T}{r} = P_0$

$$T = \frac{r}{2}(\rho_2 - \rho_1)gh$$

12. $P_1 = P_3$

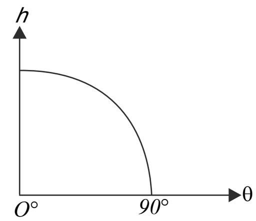
$$P_\circ - \frac{2T}{R_1} = P_2 + \rho gh \quad \Rightarrow \quad P_\circ - \frac{2T}{R_1} = P_\circ - \frac{2T}{R_2} + \rho gh$$



R_1 & R_2 are radii of curvature of the meniscus and it is known that $R = \frac{r}{\cos\theta}$

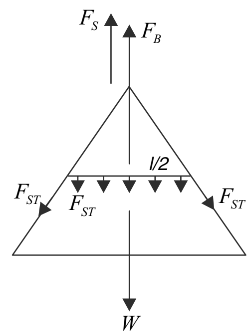
$$\rho gh = 2T\left[\frac{1}{R_2}-\frac{1}{R_1}\right] \quad \Longrightarrow \quad h = \frac{2T\cos\theta}{\rho g}\left[\frac{1}{r_2}-\frac{1}{r_1}\right]$$

$$h = \frac{2T}{\rho g}\left(\frac{r_1 - r_2}{r_1 r_2}\right)\cos\theta \qquad \therefore\ h \propto \cos\theta$$



13. Volume of prism , $V = \frac{\sqrt{3}}{4} l^2 L$

Volume of submerged part, $V' = \frac{\sqrt{3}}{4} l^2 L - \frac{\sqrt{3}}{4}\left(\frac{l}{2}\right)^2 L = \frac{3\sqrt{3}}{16} l^2 L$



For equilibrium -

Spring force (F_S) + Buoyancy (F_B) = weight (W) + surface tension force (F_{ST})

$$kx + \frac{3\sqrt{3}}{16} l^2 L \rho g = \frac{\sqrt{3}}{4} l^2 L\, dg + 2.TL\cos 30° + 2T\frac{l}{2}$$

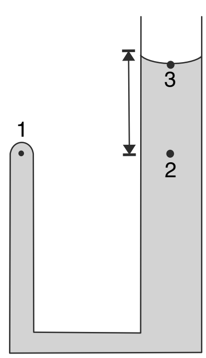
$$x = \frac{1}{K}\left[\frac{\sqrt{3}}{4} l^2 L\, d.g - \frac{3\sqrt{3}}{16} l^2 L \rho g + \sqrt{3}TL + Tl\right]$$

14. When water on two sides has same level (with left tube completely filled) the radius of curvature of the water surface on two sides will be same - equal to R. As more water is added, the surface in the left tube gets flatter, then becomes completely flat and then becomes convex up.

Figure shows change in curvature of the left surface as height of water in the right tube increases.

In extreme case left surface is hemispherical (radius = r) with convex side up. After this, water starts flowing out of the tube.

(a) The meniscus in the right tube is hemispherical (because $\theta = 0°$) . Radius of curvature of the surface is = R



When surface on left is flat, pressure at point 1 (just below the surface) is atmospheric pressure (P_o)

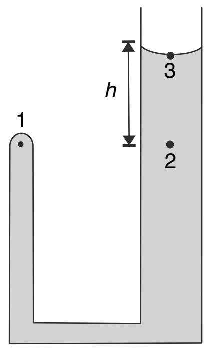
$P_1 = P_2$

$$P_\circ = P_\circ - \frac{2T}{R} + \rho g h \quad \Rightarrow h = \frac{2T}{R\rho g}$$

(b) In extreme case, when water does not come out of left tube, the surface gets convex with radius of curvature r

$P_1 = P_2$

$$P_\circ + \frac{2T}{r} = P_\circ - \frac{2T}{R} + \rho g h \qquad \Rightarrow \frac{2T}{\rho g}\left[\frac{r+R}{rR}\right] = h$$



15. P_0 = atmospheric pressure

$P_0 + \frac{4T}{R} =$ pressure inside the larger bubble.

$P_0 + \frac{4T}{R} + \frac{4T}{r} =$ pressure inside the smaller bubble.

After the larger bubble bursts, the new pressure inside the smaller bubble is $= P_0 + \frac{4T}{r}$

Using $\quad P_2V_2 = P_1V_1$

$$\left(P_0 + \frac{4T}{r}\right)V_2 = \left(P_0 + \frac{4T}{R} + \frac{4T}{r}\right)\frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi r_1^3 = \left(1 + \frac{\frac{4T}{R}}{P_0 + \frac{4T}{r}}\right)\frac{4}{3}\pi r^3$$

Where r_1 is new radius of the bubble.

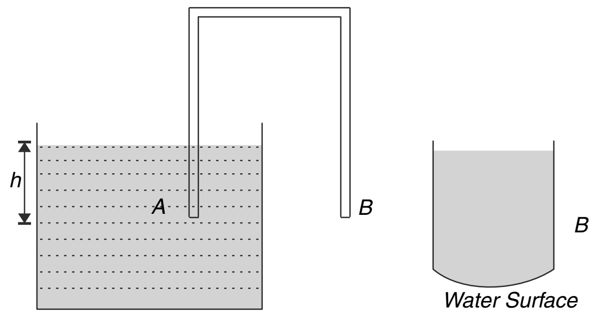
$$r_1 \simeq r\left(1 + \frac{4T}{RP_0}\right)^{1/3} \qquad \left[\because\ P_0 + \frac{4T}{r} \simeq P_0\right]$$

$$\simeq r\left[1 + \frac{4T}{3RP_0}\right] \qquad \left[\because\ \frac{4T}{RP_0} << 1\right]$$

$$\therefore \qquad r_1 - r = \frac{4Tr}{3P_0 r} \qquad \Rightarrow \Delta r = \frac{4Tr}{3P_0 R}$$

16. Pressure at end B inside the tube is

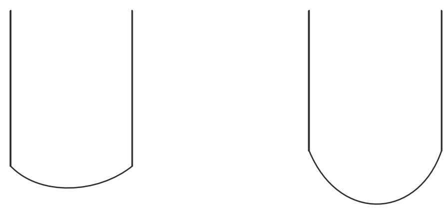
$P_B = P_o + \rho g h \qquad [P_o =$ atmospheric pressure$]$



$\therefore P_B - P_o = \rho gh$

$\therefore \frac{2T}{r} = \rho gh$ [r = radius of curvature of the surface]

With increase in h, the curvature of water surface increases (i.e. r decreases) so as to prevent water from flowing out.

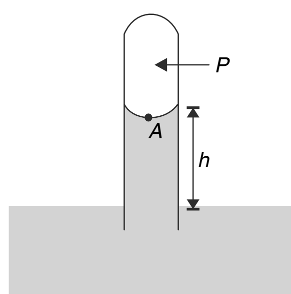


The figure shows the change in surface as B is lowered.

17. When the tube is brought into contact with water, it is filled with air at atmospheric pressure. When water rises to a height h, the air pressure (P) is given by

$PA(L-h) = P_oAL$

$\therefore P = \frac{P_\circ L}{L-h}$



Radius of curvature of meniscus $R = r$, since contact angle is zero.

Pressure at A is $P_A = P_o - \rho gh$

$\therefore P = P_A + \frac{2T}{r}$

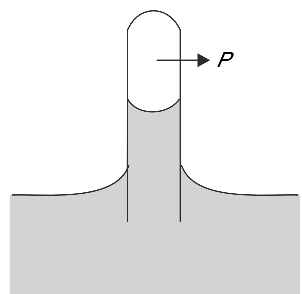
$\therefore \frac{P_\circ L}{L-h} = P_\circ - \rho gh + \frac{2T}{r} \quad \Rightarrow P_\circ\left(\frac{L}{L-h} - 1\right) = \frac{2T}{r} - \rho gh$

$\Rightarrow \frac{P_\circ h}{L-h} = \frac{2T}{r} - \rho gh \quad \Rightarrow L - h = \frac{P_\circ h}{\frac{2T}{r} - \rho gh}$

$\Rightarrow L = h + \frac{P_\circ r h}{2T - \rho ghr}$

18. Surface tension force on outer wall of the tube is $2\pi RT\ (\downarrow)$

Surface tension force (at meniscus) on the inner tube wall is $2\pi rT\ (\downarrow)$



Force due to air pressure inside the tube $\pi r^2 P(\uparrow)$

Force due to atmospheric pressure = $\pi R^2 P_o(\downarrow)$

For equilibrium of the tube, let the upward force needed be F

$$F + P\pi r^2 = Mg + \pi R^2 P_o + 2\pi RT + 2\pi rT$$

$$F = Mg + \pi R^2 P_\circ - \frac{P_\circ L\pi r^2}{L-h} + 2\pi(R+r)T$$

$$F = Mg + \pi P_\circ\left[R^2 - \frac{Lr^2}{L-h}\right] + 2\pi(R+r)T$$

19. It is known that $h_0 = \dfrac{2T}{\rho gr}$ [Here $\cos\theta = 1$]

If the tube has large length and viscosity is not there, the upward pulling surface tension force can cause the water to rise to a maximum height h. Where

$$F\,h = \pi r^2 h\,\rho.g\,\frac{h}{2}$$

Where left side is work done by surface tension force and right side is the rise in potential energy of the water in the glass tube

$$(2\pi rT\cos\theta)h = \frac{1}{2}\pi r^2\rho gh^2$$

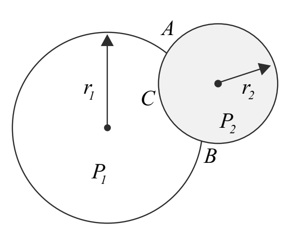
$\cos\theta = 1 \quad \therefore \dfrac{4T}{\rho gr} = h$

$\therefore\ h = 2h_0$

In presence of viscosity $h < 2h_0$. If water touches the brim of the tube, it means length of tube must be less than $2h_0$

20. Let the radius of curvature of surface ACB be R.

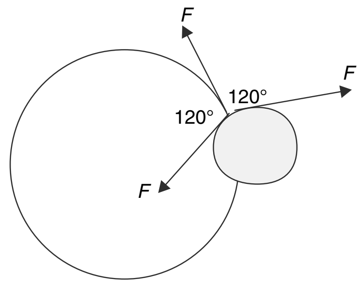
$$P_2 - P_1 = \frac{4T}{R}$$



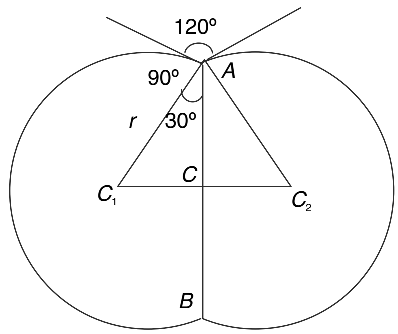
[$\dfrac{4T}{R}$ because there are two surfaces in the wall ACB]

$$\therefore\ \frac{4T}{r_2} - \frac{4T}{r_1} = \frac{4T}{R} \qquad \Rightarrow R = \frac{r_1 r_2}{r_1 - r_2}$$

21. (a) Consider a particle on the common meeting line of the three surface. It experience three forces of same magnitude (surface tension is same for all three films) and is in equilibrium. This is possible only if the three forces are at 120° to each other



(b) When $r_1 = r_2 = r$ the pressure on two sides of the common wall is same. The common wall remains flat in shape of a disc. AB is diameter of the disc.



$AC = r\cos 30° = \dfrac{\sqrt{3}r}{2}$ $\Rightarrow AB = 2AC = \sqrt{3}r$

(c) In above figure $CC_1 = r\sin 30° = \dfrac{r}{2}$

$\therefore\ y = r + \dfrac{r}{2} = \dfrac{3r}{2}$

$\therefore$ volume of each bubble $= \dfrac{\pi}{3}\left(\dfrac{3r}{2}\right)^2\left(3r - \dfrac{3r}{2}\right) = \dfrac{9\pi r^3}{8}$

If radius of new bubble = R, then $\dfrac{4}{3}\pi r^3 = 2 \times \dfrac{9\pi r^3}{8}$

$R = \dfrac{3r}{2(2)^{1/3}}$

22. (a) As excess pressure for a soap bubble is $(4T/r)$ and external pressure p_0,

$$p_i = p_0 + (4T/r)$$

Pressure inside the smaller bubble is higher, Hence, air flows from smaller bubble to the larger one.

(b) The larger bubble grows in size till the entire air of smaller bubble is transferred into it.

$$p_1 = \left[p_0 + \frac{4T}{R_1}\right],\quad p_2 = \left[p_0 + \frac{4T}{R_2}\right] \text{ and } p_3 = \left[p_0 + \frac{4T}{R_3}\right] \quad \ldots\text{(i)}$$

and $V_1 = \dfrac{4}{3}\pi R_1^3,\quad V_2 = \dfrac{4}{3}\pi R_2^3$ and $V_3 = \dfrac{4}{3}\pi R_3^3$ ...(ii)

Now, as mass is conserved, $n_1 + n_2 = n_3$

i.e., $\dfrac{P_1V_1}{RT_1} + \dfrac{P_2V_2}{RT_2} = \dfrac{P_3V_3}{RT_3}$ $\left[\text{as } PV = nRT, \text{ i.e.,} n = \dfrac{PV}{RT}\right]$

At temperature is constant, i.e., $T_1 = T_2 = T_3$, the above expression reduces to

$$P_1V_1 + P_2V_2 = P_3V_3$$

Which, in the light of Eqn. (i) and (ii) becomes

$$\left[P_0 + \frac{4T}{R_1}\right]\left[\frac{4}{3}\pi R_1^3\right] + \left[P_0 + \frac{4T}{R_2}\right]\left[\frac{4}{3}\pi R_2^3\right] = \left[P_0 + \frac{4T}{R_3}\right]\left[\frac{4}{3}\pi R_3^3\right]$$

i.e., $\quad 4T(R_1^2 + R_2^2 - R_3^2) = P_0(R_3^3 - R_1^3 - R_2^3)$

i.e., $\quad T = \dfrac{P_0(R_3^3 - R_1^3 - R_2^3)}{4(R_1^2 + R_2^2 - R_3^2)}$.

23. Hint: the pressure must be same everywhere inside the straw and the bubble.

24. Since the drag force balances the gravity, the necessary condition must be that the variation in hydrostatic pressure inside the drop should be negligible compared to the excess pressure due to surface tension

$$\rho g 2R << \frac{2T}{R} \qquad \Rightarrow R << \sqrt{\frac{T}{\rho g}}$$

25. Excess pressure inside the soap bubble $\Delta P = \dfrac{4T}{R}$

From ideal gas equation, pressure of a gas (at a given temperature) is directly proportional to its density

$P \propto \rho$

$\therefore \dfrac{\Delta P}{P_\circ} = \dfrac{\Delta\rho}{\rho_\circ}$ where P_o and ρ_o are pressure and density of atmospheric air.

$\Rightarrow \Delta\rho = \dfrac{\rho_\circ}{P_\circ}\Delta P$

Apparent weight = True weight – Buoyancy

= weight of air inside Bubble – Buoyancy + weight of water skin

$$= \frac{4}{3}\pi R^3\left(\rho_\circ + \Delta\rho\right)g - \frac{4}{3}\pi R^3\rho_\circ g + 4\pi R^2 a\, d.g$$

$$= \frac{4}{3}\pi R^3\Delta\rho g + 4\pi R^2 a\, d.g$$

$$= \frac{4}{3}\pi R^3\frac{\rho_\circ}{P_\circ}\frac{4T}{R}g + 4\pi R^2.a.d.g$$

$$= \frac{4}{3}\times 3.14\times(0.1)^2\times\frac{1.2}{10^5}\times 4\times .04\times 10 + 4\times 3.14\times(0.1)^2\times 10^{-6}\times 10^3\times 10$$

$= 0.8 \times 10^{-6} + 1.3 \times 10^{-3} \simeq 1.3 \times 10^{-3}\, N$

26. $\Delta P = \dfrac{4T}{R} \qquad \Rightarrow \quad Q = \dfrac{\pi a^4\Delta P}{8\eta L}$

$\Rightarrow \dfrac{dV}{dt} = \dfrac{\pi a^4 T}{2\eta LR}$; But $V = \dfrac{4}{3}\pi R^3$

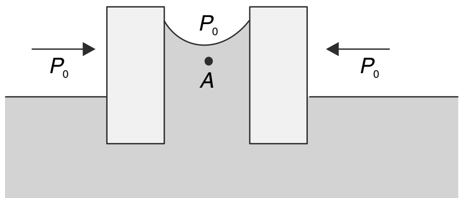
$$\frac{dV}{dt} = \frac{dV}{dR}\frac{dR}{dt} = 4\pi R^2\frac{dR}{dt}$$

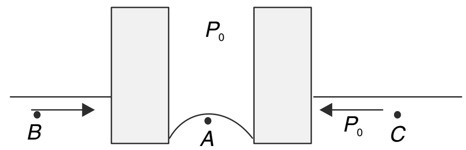
$$\therefore\ -4\pi R^2\frac{dR}{dt} = \frac{\pi a^4 T}{2\eta LR}$$

$$-4\int_{R_0}^{R} R^3 dR = \frac{a^4 T}{2\eta L}\int_0^t dt \qquad \Rightarrow R^4 - R_0^4 = \frac{a^4 T}{2\eta L}t$$

$$\therefore\ R = \left(R_0^4 - \frac{a^4 T}{2\eta L}\ t\right)^{\frac{1}{4}} = R_0\left(1 - \frac{a^4 Tt}{2\eta\ L R_0^4}\right)^{\frac{1}{4}}$$

27. Water rises between the blocks due to capillary effect. Pressure below the curved water surface (like at a point A shown in figure) is below the atmospheric pressure.





But outer walls of the blocks experience atmospheric pressure. This causes the blocks to be pushed towards each other. In case of Hg, the situation is as shown in the figure. At points like B and C the pressure is higher than atmospheric pressure which pushed the blocks closer.

28. Consider a length L of the string. Total surface energy of water on it is $E = 2\pi rLT$

If spherical drops are of radius R and separation between two successive drops is l then conservation of volume gives

$$\frac{4}{3}\pi R^3 \frac{L}{l} = \pi r^2 L \qquad \Rightarrow \frac{4}{3}R^3 = r^2 l \qquad \text{.................(i)}$$

The total surface energy of liquid drops over a length L is

$$E' = 4\pi R^2 . T . \frac{L}{l} = 4\pi\left(\frac{3r^2 l}{4}\right)^{2/3} . T\frac{L}{l} \qquad \text{[using (i)]}$$

$$\therefore\ E' = 4\pi\left(\frac{3r^2}{4}\right)^{2/3}\frac{TL}{l^{1/3}}$$

The final surface energy must be less than the original surface energy

$\therefore\ E' < E$

$$4\pi\left(\frac{3r^2}{4}\right)^{2/3}\frac{TL}{l^{1/3}} < 2\pi rLT$$

$$2\left(\frac{3r^2}{4}\right)^{2/3} < r\,l^{1/3} \qquad \Rightarrow \qquad 2\left(\frac{3}{4}\right)^{2/3} r^{1/3} < l^{1/3}$$

$$8\times\frac{9}{16}\times r < l \quad \Rightarrow \quad \frac{9}{2}r < l$$

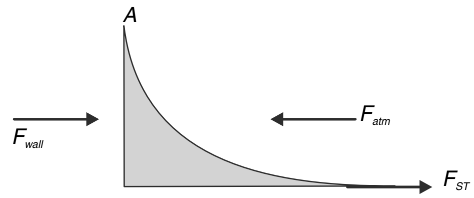
29. (i) Pressure at the lowest level of meniscus is $= P_o$ [$\because$ liquid surface is flat]

$\therefore$ Pressure in liquid at A is $P_A = P_o - \rho gh$

(ii) We will consider the horizontal equilibrium of the liquid in the meniscus. We will consider a depth L perpendicular to the plane of the figure.

Force due to wall $F_{wall} = P_{av}\,hL$

$$= \left(\frac{P_\circ + P_\circ - \rho gh}{2}\right)hL = \left(P_\circ - \frac{\rho gh}{2}\right)hL$$



Force due to atmosphere $F_{atm} = P_o hL$

Force due to surface tension $F_{ST} = T.L$

$\therefore F_{wall} + F_{ST} = F_{atm}$

$$\left(P_\circ - \frac{\rho gh}{2}\right)hL + T.L = P_\circ hL$$

$$\Rightarrow P_\circ h - \frac{\rho gh^2}{2} + T = P_\circ h \qquad \therefore \; h = \sqrt{\frac{2T}{\rho g}}$$

30. Consider a water drop of radius R

Surface area $S = 4\pi R^2$

If radius decreases by ΔR, the surface area changes by $\Delta S = -8\pi R\Delta R$

$\therefore$ Loss in surface energy $\Delta E = 8\pi R\Delta R.T$

The volume of the drop $V = \frac{4}{3}\pi R^3$

$\Delta V = 4\pi R^2 \Delta R$

$\therefore$ Energy required for evaporation of a water layer ΔR thick is

$\Delta E' = \rho \Delta V L = 4\pi R^2 \Delta R.\ \rho.L$

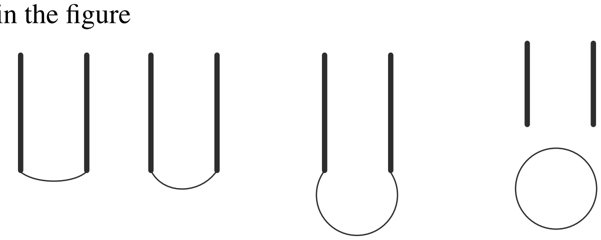
Loss in surface energy can cause water to evaporate if $\Delta E > \Delta E'$

$$8\pi R\ \Delta R \mathrm{T} > 4\pi R^2\ \Delta R\rho L \quad \Rightarrow R < \frac{2T}{\rho L} = \frac{2\times 0.07}{10^3 \times 2.3\times 10^6}$$

6×10^{-11} m

Drops of this size do not exist. In fact the above number is close to size of one molecule of water.

31 When water level rises in A, the air pressure rises. This blows a bubble at the tip of C. The monometer reading gives the pressure difference of the air inside the container and the atmospheric pressure (P_o). The various stages of bubble has been shown in the figure

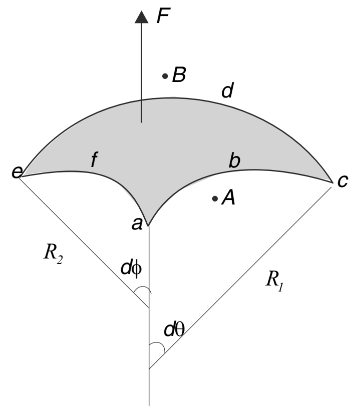


The inside pressure is maximum when radius of the bubble is equal to the radius of the capillary (i.e. when the bubble is hemispherical)

$P_{max} - P_o = \rho g h_o$

$$\Rightarrow \frac{2T}{r} = \rho g h_0 \quad \Rightarrow T = \frac{\rho g h_0 .r}{2}$$

32. Consider a small patch on the liquid surface. Angle subtended by arc abc at the centre of curvature is $d\theta$ and the angle subtended by arc $a\,f\,e$ at its centre of curvature is $d\phi$.



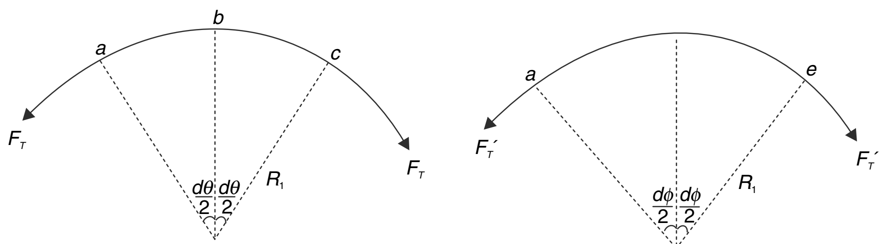
Area of the patch

$dS = (R_1 d\theta)\ (R_2 d\phi)$ ....(i)

Pressure on concave side (at point A) is P_A and pressure on convex side (at point B) is P_B. Net outward force on the patch due to pressure difference is $F = (P_A - P_B)\ dS$ .... (ii)

This force is balanced by surface tension force. Look at the two figures shown. F_T and $F_T^{'}$ are surface tension force on arcs of length $R_2 d\phi$ and $R_1 d\theta$ respectively.

$\therefore$ Net inward force on the patch of liquid surface is



$$= 2F_T \sin\frac{d\theta}{2} + 2F'_T \sin\frac{d\phi}{2} = F_T d\theta + F'_T d\phi \quad [\because \sin d\theta \simeq d\theta]$$

$$= T.R_2 d\phi.d\theta + TR_1 d\theta.d\phi$$

$$= T(R_1 + R_2)d\phi.d\theta = T\left(\frac{R_1 + R_2}{R_1 R_2}\right)dS \quad \text{[using (i)]}$$

This force balances the force given by equation (ii)

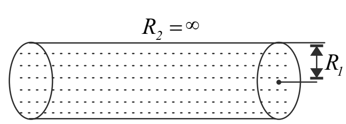
$$\left(P_A - P_B\right)dS = T\left(\frac{R_1 + R_2}{R_1 R_2}\right)dS \qquad \Rightarrow\ P_A - P_B = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Note: If the surface is spherical $R_1 = R_2 = R$ (say)

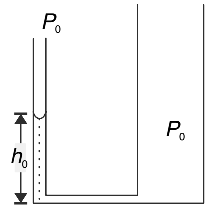
$$\therefore P_A - P_B = \frac{2T}{R}$$

If the liquid surface is cylindrical

$R_1 = R$; $R_2 = \infty \quad \therefore P_A - P_B = \frac{T}{R}$

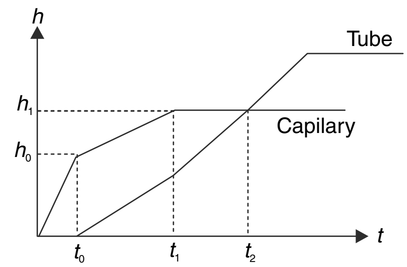


33. First the water level rises in the capillary till the time the pressure at the bottom of the capillary becomes equal to atmospheric pressure.



$$P_O - \frac{2T\cos\theta}{r} + \rho g h_0 = P_O \qquad \Rightarrow \; h_0 = \frac{2T\cos\theta}{r\rho g}$$

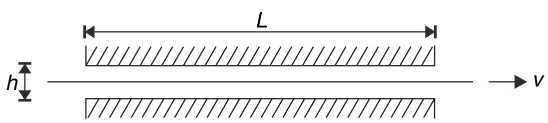
After this the height difference between levels of water in the capillary and the tube remains constant at h_0. It means height in both of them increases by same amount when some water is poured. At time t_1 water touches the brim of the capillary. When more water is poured the radius of curvature of the water surface in the meniscus of the capillary changes (surface gets flatter) and the water level in the wide tube continues to rise. Note that the level of water in the broad tube will now rise at a slightly faster pace as quantity of water is not increasing in the capillary. The time t_2 when water level becomes same in both tubes the meniscus surface become flat (as in the broad tube). Thereafter, as the height of water increases in broad tube, the meniscus gets convex. After time t_2 water begins to flow out of the capillary and water level in both tubes becomes constant.



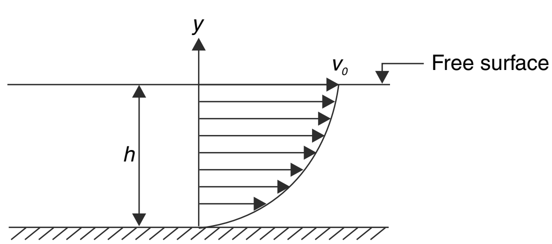
10 VISCOSITY

LEVEL 1

Q.1. During a painting process, a thin, flat tape of width b (dimension perpendicular to the plane of the figure) is pulled through a paint filled channel of length L. The density and viscosity of the paint liquid is ρ and η respectively. The tape is pulled at a constant speed v and width of the channel is h. Find the minimum force needed to pull the tape.



Q.2. A liquid is flowing through a horizontal channel. The speed of flow (v) depends on height (y) from the floor as $v = v_0\left[2\left(\frac{y}{h}\right) - \left(\frac{y}{h}\right)^2\right]$. Where h is the height of liquid in the channel and v_0 is the speed of the top layer. Coefficient of viscosity is η. Calculate the shear stress that the liquid exerts on the floor.



Q.3. A car having cross sectional area of its front equal to A is travelling on a highway at a speed v. The viscous drag force acting on the car is known to be given as $F_v = CA\rho v^2$.Where ρ is density of air and C is a constant which depends on the shape of the car. The petrol used by the car produces E joules of energy per kg of it burnt. Calculate the mileage (in *km/kg*) of the car if the combined efficiency of its engine and transmission is f.

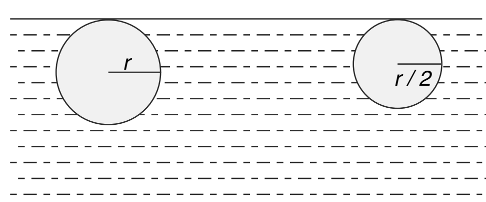
Q.4. An ideal fluid flows through a pipe of circular cross section of radius r at a speed v_0. Now a viscous liquid is made to flow through the pipe at the same volume flow rate (measured in m^3s^{-1}). Find the maximum speed of a fluid particle in the pipe.

Q.5. A near surface earth satellite is in the shape of a sphere of radius r. It encounters cosmic dust in its path. The viscous force experienced by the satellite follows stoke's law. The coefficient of viscosity is η. Mass and radius of the earth are M and R respectively.

(a) Calculate the power of the rocket engine that must be put on to keep the satellite moving as usual.

(b) Calculate the equilibrium temperature of the surface of the satellite assuming that it radiates like a black body and no outer radiation falls on it. Assume that the heat generated due to viscous force is absorbed completely by the satellite body.

Q.6. Two balls of radii r and $\frac{r}{2}$ are released inside a deep water tank. Their initial accelerations are found to be $\frac{g}{2}$ and $\frac{g}{4}$ respectively. Find the velocity of smaller ball relative to the larger ball, a long time after the two balls are released. Coefficient of viscosity is given to be η.



Q.7. The coefficient of viscosity η of a gas depends on mass of the gas molecule, its effective diameter and its average speed. It is known that diameter of helium atom is $2.1 \times 10^{-10}\,m$ and its coefficient of viscosity, η at room temperature is 2.0×10^{-5} $kg\,m^{-1}s^{-1}$. Estimate the effective diameter of CO_2

molecule if it is known that η at room temperature for CO_2 is $1.5 \times 10^{-5}\ kg\ m^{-1}s^{-1}$.

Q.8. When hard brakes are applied (so as to lock the wheels) in a car travelling on a wet road it can "hydro-plane". A film of water is created between the tires and the road and, theoretically, the car can slide a very long distance. [In practice film is destroyed much before such distances can be achieved]. Consider a car of mass M moving on a wet road with speed v_0. Hard brakes are applied. Let the area of film under all four tires be A and thickness of the film be h. Coefficient of viscosity is η.

(a) Calculate the distance (x) to which the car will slide before coming to rest.

(b) Calculate the value of x for $M = 10^3\ kg$, $A = 0.2\ m^2$, $h = 0.1\ mm$, $v_0 = 20\ ms^{-1}$, and $\eta = 10^{-3}\ kg\ m^{-1}$

LEVEL 2

Q.9. A spherical ball of radius r and density d is dropped from rest in a viscous fluid having density ρ and coefficient of viscosity η.

(a) Calculate the power (P_1) of gravitational force acting on the ball at a time t after it is dropped.

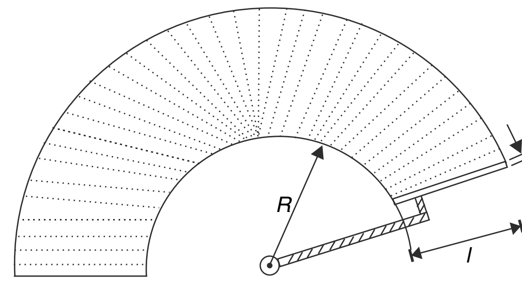
(b) Calculate the rate of heat generation (P_2) due to rubbing of fluid molecules with the ball, at time t after it is dropped.

(c) How do P_1 and P_2 change if the radius of the ball were doubled?

(d) Find P_1 and P_2 when both become equal.

Q.10. Two balls of same material of density ρ but radius r_1 and r_2 are joined by a light inextensible vertical thread and released from a large height in a medium of coefficient of viscosity = η. Find the terminal velocity acquired by the balls. Also find the tension in the string connecting both the balls when both of them are moving with terminal velocity. Neglect buoyancy and change in acceleration due to gravity.

Q.11. A car windshield wiper blade sweeps the wet windshield rotating at a constant angular speed of ω. R is the radius of innermost arc swept by the blade. Length and width of the blade are l and b respectively. Coefficient of viscosity of water is η. Calculate the torque delivered by the motor to rotate the blade assuming that there is a uniform layer of water of thickness t on the glass surface.



LEVEL 3

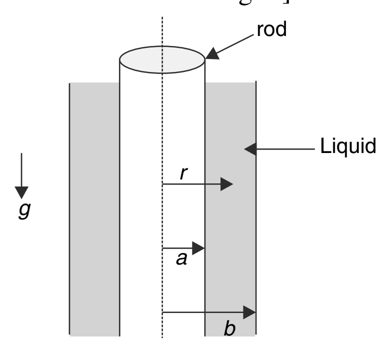
Q.12. A vertical steel rod has radius a. The rod has a coat of a liquid film on it. The liquid slides under gravity. It was found that the speed of liquid layer at radius r is given by

$$v = \frac{\rho g b^2}{2\eta}\ln\left(\frac{r}{a}\right) - \frac{\rho g}{4\eta}\left(r^2 - a^2\right)$$

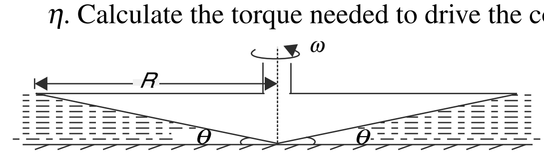
Where b is the outer radius of liquid film, η is coefficient of viscosity and ρ is density of the liquid.

(i) Calculate the force on unit length of the rod due to the viscous liquid?

(ii) Set up the integral to calculate the volume flow rate of the liquid down the rod. [you may not evaluate the integral]



Q.13. A viscometer (an instrument used to study characteristics of a non-ideal fluid) consists of a flat plate and a rotating cone. The cone has a large apex angle and the angle θ shown in figure is very small (typically less than 0.5°). The apex of the cone just touches the plate and a liquid fills the narrow gap between the plate and the cone. The cone has a base radius R and is rotated with constant angular speed ω. Consider the liquid to be ideal and take its coefficient of viscosity to be η. Calculate the torque needed to drive the cone.



ANSWERS

1. $\dfrac{4\eta vLb}{h}$

2. $\dfrac{2\eta v_0}{h}$

3. $\dfrac{fE}{CA\rho v^2}$

4. $\dfrac{3}{2}v_0$

5. (a) $\dfrac{6\pi GM\eta r}{R}$ (b) $\left[\dfrac{3\eta GM}{2\sigma rR}\right]^{1/4}$

6. $\dfrac{11}{54}\dfrac{r^2\rho g}{\eta}$ upwards

7. $4.4 \times 10^{-10}\, m$

8. (a) $x = \dfrac{Mhv_0}{\eta A}$ (b) 10 km

9. (a) $P_1 = \dfrac{8\pi}{27}\dfrac{d(d-\rho)g^2r^5}{\eta}\left[1-e^{-\frac{9\eta t}{2dr^2}}\right]$

(b) $P_1 = \dfrac{8\pi}{27}\dfrac{(d-\rho)^2g^2r^5}{\eta}\left[1-e^{-\frac{9\eta t}{2dr^2}}\right]$

(c) P_1 and P_2 become 32 times.

(d) $P_1 = P_2 = \dfrac{8\pi}{27}\dfrac{(d-\rho)^2g^2r^5}{\eta}$

10. $\dfrac{2}{9}\dfrac{\rho g}{\eta}\left[r_1^2 - r_1r_2 + r_2^2\right]$; $\dfrac{4}{3}\pi\rho g\left|r_1^2r_2 - r_2^2r_1\right|$

11. $\dfrac{\eta b\omega R^3}{3t}\left[\left(1+\dfrac{L}{R}\right)^3 - 1\right]$

12. (i) $\pi\rho g a^2\left[\left(\dfrac{b}{a}\right)^2 - 1\right]$ (ii) $Q = \int_a^b v \cdot 2\pi r dr$

13. $\dfrac{2\pi\eta\omega R^3}{3\sin\theta} \simeq \dfrac{2\pi\eta\omega R^3}{3\theta}$

SOLUTIONS

1. The paint layer in contact with channel wall is at rest and that in contact with the tape is v. The viscous force acts on two surfaces of the tape. If gap between the tape and upper surface of the channel is x then velocity gradient at the two surfaces of the tape is

$$\left(\frac{dv}{dh}\right)_{upper} = \frac{v}{x} \text{ and } \left(\frac{dv}{dh}\right)_{lower} = \frac{v}{h-x}$$

Total viscous force on the tape is $F_{vis} = F_{upper} + F_{lower}$

$$= \eta(bL)\cdot\frac{v}{x} + \eta(bL)\cdot\frac{v}{h-x} = \eta bLv\left[\frac{h}{x(h-x)}\right]$$

This force is minimum when $x(h-x)$ is maximum, i.e., when $x = \dfrac{h}{2}$

$$\therefore\ F_{\min} = \frac{4\eta bLv}{h}$$

2. Shear stress is tangential force applied by the liquid on unit area of the floor.

Velocity gradient $= \dfrac{dv}{dy} = \dfrac{2v_0}{h} - \dfrac{2v_0}{h^2}y$

At $y = 0$, $\dfrac{dv}{dy} = \dfrac{2v_0}{h}$

$\therefore$ Viscous force per unit area $= \eta \dfrac{dv}{dy} = \dfrac{2\eta v_0}{h}$

3. Viscous drag force acting on the car is $F = CA\rho v^2$

$\therefore$ Work required to travel through a distance x is

$W = Fx = CA\rho v^2 \cdot x$

Let mass of petrol burnt $= m$; Energy produced $= mE$

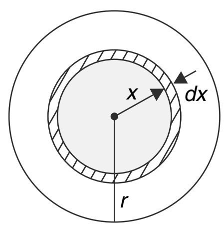
Since efficiency is f $\therefore$ $fmE = W$

$fmE = CA\rho v^2 x$ $\therefore$ $m = \dfrac{CA\rho v^2 x}{fE}$

$\therefore$ Mileage = Distance travelled per unit mass of fuel burnt $= \dfrac{x}{m} = \dfrac{fE}{CA\rho v^2}$

4. Volume flow rate $Q = \pi r^2 . v_0$

When a viscous liquid flows, speed is maximum along the axis of the pipe (let us call this speed as u_0) and decreases to zero at the circumference. The variation in speed will be almost linear if radius of the pipe is small.



Speed at a distance x from the axis is $u = \dfrac{u_0 x}{r}$

$\therefore$ volume flow rate through a region of width dx is $= (2\pi x dx) u = (2\pi x dx)\left(\dfrac{u_0 x}{r}\right)$

$\therefore$ Total flow rate is $Q = \dfrac{2\pi u_0}{r} \displaystyle\int_0^r x^2 dx = \dfrac{2\pi u_0}{3} r^2$

$\therefore$ $\pi r^2 v_0 = \dfrac{2\pi u_0}{3} r^2$ $\therefore$ $u_0 = \dfrac{3}{2} v_0$

5. (a) Orbital speed of the satellite $v = \sqrt{\dfrac{GM}{R}}$

Viscous force $f_v = 6\pi\eta r v$

Power required $P = v . f_v = 6\pi\eta r v^2 = 6\pi\eta r \dfrac{GM}{R}$

(b) $\sigma A T^4 = 6\pi\eta r \dfrac{GM}{R}$ $\Rightarrow$ $\sigma 4\pi r^2 T^4 = 6\pi\eta r \dfrac{GM}{R}$

$\therefore$ $T^4 = \dfrac{3}{2} \dfrac{\eta}{\sigma r} \dfrac{GM}{R} \Rightarrow T = \left[\dfrac{3}{2} \dfrac{\eta GM}{\sigma r R}\right]^{1/4}$

6. Initial acceleration for larger ball is given by -

$ma = mg - F_B$

$V_1 d_1 \dfrac{g}{2} = V_1 d_1 g - V_1 \rho g$ [ρ= density of water, d_1= density of ball]

$\Rightarrow d_1 = 2\rho$

Similarly, for second ball $d_2 = \dfrac{4\rho}{3}$

After long time both of them will acquire terminal speed (u_{01} for larger ball and u_{02} for smaller ball)

$6\pi\eta r\, u_{01} = mg - F_B \quad \Rightarrow \quad 6\pi\eta r\, u_{01} = \dfrac{4}{3}\pi r^3 d_1 g - \dfrac{4}{3}\pi r^3 \rho g$

$\Rightarrow u_{01} = \dfrac{2}{9}\dfrac{r^2\rho g}{\eta}$ Similarly, $u_{02} = \dfrac{r^2 \rho g}{54\eta}$

$\therefore$ Velocity of smaller ball with respect to the larger ball is

$$= \left(\frac{1}{54} - \frac{2}{9}\right) r^2 \frac{\rho g}{\eta} = -\frac{11}{54}\frac{r^2 \rho g}{\eta} = \frac{11}{54}\frac{r^2\rho g}{\eta}(\uparrow)$$

7. From method of dimensions

$\eta = kmvd^{-2}$ [m = mass, v = average speed, d = diameter of molecule]

But $v \propto \sqrt{\dfrac{T}{m}}$ $\quad \therefore \ \eta = k^1 m^{\frac{1}{2}} T^{\frac{1}{2}} d^{-2}$

At a given temperature

$$\left(\frac{d_{CO_2}}{d_{He}}\right)^2 = \left(\frac{m_{CO_2}}{m_{He}}\right)^{1/2} \left(\frac{\eta_{He}}{\eta_{CO_2}}\right)$$

$$d_{CO_2} = d_{He}\left(\frac{m_{CO_2}}{m_{He}}\right)^{1/4} \left(\frac{\eta_{He}}{\eta_{CO_2}}\right)^{1/2}$$

$$= 2.1 \times 10^{-10} \times (11)^{\frac{11}{4}} \times \left(\frac{2}{1.5}\right)^{1/2} = 4.4 \times 10^{-10} m$$

8. (a) Viscous force [v = instantaneous speed of the car]

$$F_v = \eta A \frac{dv}{dh} = \eta A \frac{v}{h}$$

$\therefore \ M\dfrac{dv}{dt} = -\eta A \dfrac{v}{h}$ Or, $Mv\dfrac{dv}{dx} = -\eta A\dfrac{v}{h}$

Or, $\displaystyle\int_{v_0}^{0} dv = -\frac{\eta A}{hM}\int_0^x dx$ $\quad \Rightarrow v_0 = \dfrac{\eta A}{hM}x$

$$\Rightarrow x = \frac{hMv_0}{\eta A}$$

(b) $x = \dfrac{10^4 \times 10^3 \times 20}{10^{-3} \times 0.2} = 10^4\, m = 10\, km$ (!)

9. $$m\frac{dv}{dt} = mg - F_B - F_v$$

$$\frac{4}{3}\pi r^3 d\ \frac{dv}{dt} = \frac{4}{3}\pi r^3 d\cdot g - \frac{4}{3}\pi r^3 \rho\cdot g - 6\pi\eta r v$$

$$\frac{dv}{dt} = g - \frac{\rho}{d}g - \frac{9}{2}\frac{\eta v}{dr^2}$$

$$= g\left(1 - \frac{\rho}{d}\right) - \frac{9\eta}{2\cdot dr^2}\cdot v \quad = a - bv \qquad \text{(say)}$$

$$\Rightarrow \int_0^v \frac{dv}{a-bv} = \int_0^t dt \qquad \Rightarrow \left[\ln(a-bv)\right]_0^v = -bt$$

$$\Rightarrow \ln\left(\frac{a-bv}{a}\right) = -bt \quad \Rightarrow 1 - \frac{b}{a}v = e^{-bt}$$

$$\Rightarrow v = \frac{a}{b}\left(1 - e^{-bt}\right) \qquad \Rightarrow v = \frac{2g(d-\rho)r^2}{9\eta}\left(1 - e^{-\frac{9\eta}{2\cdot d\cdot r^2}t}\right)$$

(a) $$P_1 = m\cdot g\cdot v = \frac{4}{3}\pi r^3 d\cdot g\cdot\frac{2g(d-\rho)r^2}{9\eta}\left[1 - e^{-\frac{9\eta}{2dr^2}t}\right]$$

$$= \frac{8\pi}{27}\frac{d(d-\rho)g^2\cdot r^5}{\eta}\left[1 - e^{-\frac{9\eta\ t}{2dr^2}}\right]$$

(b) $$P_2 = F_v\cdot v = 6\pi\eta r v\cdot v = 6\pi\eta r v^2$$

$$= 6\pi\eta r\cdot\frac{4g^2(d-\rho)^2\cdot r^4}{81\eta^2}\left[1 - e^{-\frac{9\eta\ t}{2dr^2}}\right]^2$$

$$= \frac{8\pi}{27}\frac{(d-\rho)^2 g^2\cdot r^5}{\eta}\left[1 - e^{-\frac{9\eta\ t}{2dr^2}}\right]^2$$

(c) $P_1 \propto r^5$ and $P_2 \propto r^5$

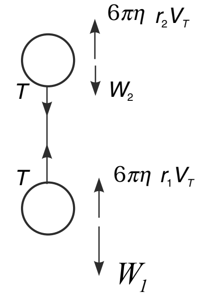
$\therefore$ On doubling the radius, both become 32 times

(d) $P_1 = P_2$, when

$e^{-\frac{9\eta t}{2dr^2}} \to 0$ [i.e., at $t = \infty$]

$$\therefore\ P_1 = P_2 = \frac{8\pi}{27}\ \frac{(d-\rho)^2 g^2 r^5}{\eta}$$

10. Considering both balls together



$$\rho\left(\frac{4}{3}\pi r_1^3\right)g + \rho\left(\frac{4}{3}\pi r_2^3\right) = 6\pi\eta r_1\ V_T + 6\pi\eta\ r_2 V_T$$

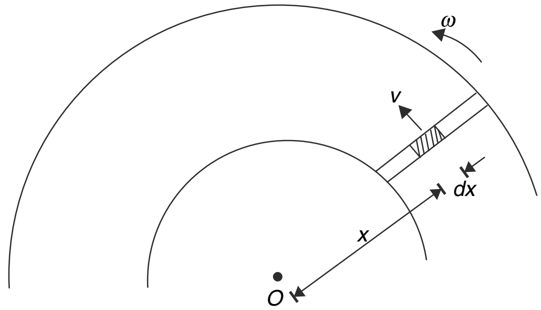
$$\Rightarrow V_T = \frac{2\rho g(r_1^3 + r_2^3)}{9(\eta)(r_1 + r_2)} = \frac{2\rho g(r_1^2 - r_1 r_2 + r_2^2)}{9\eta}$$

Consider only ball 1 and write $W_1 = T + 6\pi\eta r_1 V_r$

$$\frac{4}{3}\pi r_1^3 \rho = T + 6\pi\eta r_1 V_T$$

Put the value of V_T and calculate T.

11.



Consider an element of length dx on the blade. Speed of this element is $v = \omega x$

Viscous force on this element is

$$dF = \eta A\left(\frac{v}{t}\right) \qquad \left[\frac{v}{t} = \text{ velocity gradient}\right]$$

$$= \eta b dx\left(\frac{\omega x}{t}\right) = \eta\frac{b\omega}{t}\cdot xdx$$

Torque due to this force $d\tau = xdF = \dfrac{\eta b\omega}{t}x^2 dx$

$\therefore$ Net viscous torque on the blade is

$$\tau = \frac{\eta b\omega}{t}\int_R^{R+L} x^2 dx \qquad \Rightarrow \quad \tau = \frac{\eta b\omega}{t}\frac{1}{3}\left[x^3\right]_R^{R+L}$$

$$= \frac{\eta b\omega}{3t}\left[(R+L)^3 - R^3\right] \qquad = \frac{\eta b\omega R^3}{3t}\left[\left(1+\frac{L}{R}\right)^3 - 1\right]$$

The same torque must be applied by the motor to keep the blade moving.

12. (a) Velocity of fluid layer at radius r is $v = \dfrac{\rho g b^2}{2\eta}\ln\left(\dfrac{r}{a}\right) - \dfrac{\rho g}{4\eta}\left(r^2 - a^2\right)$

Velocity gradient along radial direction is $\dfrac{dv}{dr} = \dfrac{\rho g b^2}{2\eta}\dfrac{1}{r} - \dfrac{\rho g}{4\eta}2r$

At $r = a$ (i.e., on the surface of the rod)

$$\frac{dv}{dr} = \frac{\rho g b^2}{2\eta a} - \frac{\rho g a}{2\eta} = \frac{\rho g a}{2\eta}\left[\frac{b^2}{a^2} - 1\right]$$

Area of unit length of the rod's surface $A = (2\pi a)(1) = 2\pi a$

$$\therefore\ F_v = \eta A\frac{dv}{dr} = \pi\rho g a^2\left[\frac{b^2}{a^2} - 1\right]$$

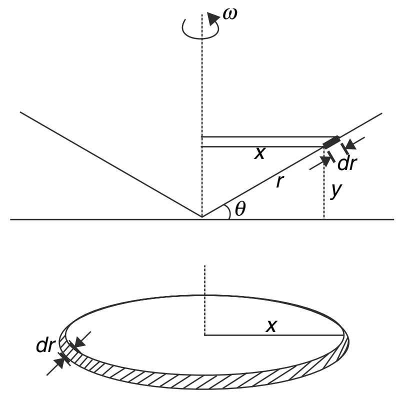
(b) Volume flow rate is

$$Q = \int_{r=a}^{r=b} v \cdot 2\pi r dr = \frac{\pi \rho g b^2}{\eta} \int_a^b r \ln\left(\frac{r}{a}\right) dr - \frac{\pi \rho g}{2\eta} \int_a^b \left(r^2 - a^2\right) r dr$$

You can evaluate the above integral if you want some practice in mathematics. To help you it is being given that

$$\int x \ln x = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$$

13.



Consider a ring shaped element in the cone as shown in the figure.

$$\frac{x}{r} = \cos\theta \qquad \Rightarrow \; dr = \frac{dx}{\cos\theta}$$

Area of the ring element $dA = 2\pi x \cdot dr = \dfrac{2\pi x \cdot dx}{\cos\theta}$

Velocity gradient at the location of this ring is $= \dfrac{\omega x}{y} = \dfrac{\omega x}{x \tan\theta} = \dfrac{\omega}{\tan\theta}$

$\therefore$ Viscous force on ring element

$$dF = \eta \cdot (dA) \frac{\omega}{\tan\theta} = \frac{2\pi\eta\omega}{\sin\theta} x dx$$

Torque on the element $d\tau = x dF = \dfrac{2\pi\eta\omega}{\sin\theta} x^2 dx$

Net torque on the cone

$$\tau = \frac{2\pi\eta\omega}{\sin\theta} \int_0^R x^2 dx = \frac{2\pi\eta\omega}{3\sin\theta} R^3 \qquad \simeq \frac{2\pi\eta\omega R^3}{3\theta} \qquad [\because \theta \text{ is small}]$$

11 ELASTICITY

LEVEL 1

Q.1. Human bones remain elastic if strain is less than 0.5%. However, the young's modulus for compression (Y_c) and stretch (Y_s) are different. The typical values are $Y_c = 9.4 \times 10^9$ *Pa* and $Y_s = 16 \times 10^9$ *Pa*. The shear modulus of elasticity for the bone is $\eta = 10^{10}$ *Pa*

Answer following questions with regard to a leg bone of length 20 *cm* and cross sectional area 3 cm^2

(a) Calculate the maximum stretching force that the bone can sustain and still remain elastic.

(b) A man of mass 60 *kg* jumps from a height of 10 *m* on a concrete floor. Half his momentum is absorbed by the impact of the floor on the particular bone we are talking about. The impact lasts for 0.02 *s*. Will the compressive stress exceed the elastic limit?

(c) How much shearing force will be needed to break the bone if breaking strain is 5°.

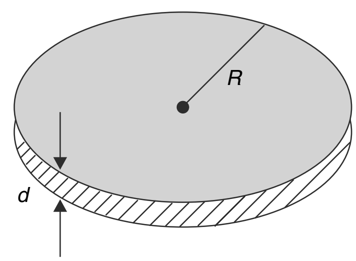
Q.2. The elastic limit and ultimate strength for steel is 2.48×10^8 *Pa* and 4.89×10^8 *Pa* respectively. A steel wire of 10 *m* length and 2 *mm* cross sectional diameter is subjected to longitudinal tensile stress. Young's modulus of steel is $Y = 2 \times 10^{11}$ *Pa*

(a) Calculate the maximum elongation that can be produced in the wire without permanently deforming it. How much force is needed to produce this extension?

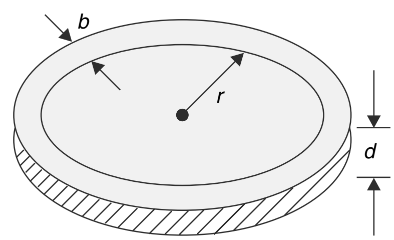
(b) Calculate the maximum stretching force that can be applied without breaking the wire.

Q.3. A steel ring is to be fitted on a wooden disc of radius *R* and thickness *d*. The inner radius of the ring is *r* which is slightly smaller than *R*. The outer radius of the ring is $r + b$ and its thickness is *d* (same as the disc). There is no change in value of *b* and *d* after the ring is fitted over the disc; only the inner radius becomes *R*. If the Young's modulus of steel is *Y*, calculate the longitudinal stress developed in it. Also calculate the tension force developed in the ring.

[Take $b << r$]

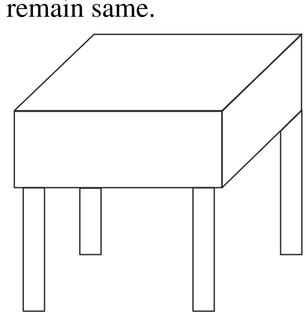


Disc



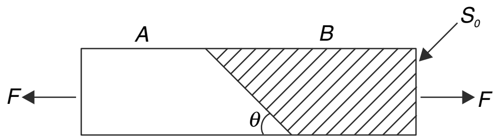
Ring

Q.4. A water tank is supported by four pillars. The pillars are strong enough to sustain ten times the stress developed in them when the tank is completely full. An engineer decides to increase the every dimension of the tank and the pillars by hundred times so as to store more water. Do you think he has taken a right decision? Assume that material used in construction of the tank and pillars remain same.

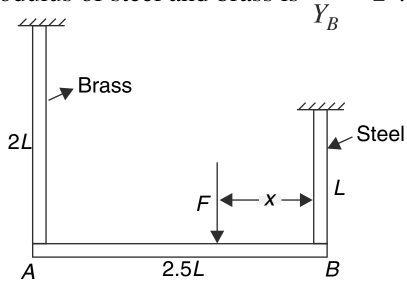


Q.5. Two bars *A* and *B* are stuck using an adhesive. The contact surface of the bars make an angle

θ with the length. Area of cross section of each bar is S_0. It is known that the adhesive yields if normal stress at the contact surface exceeds σ_0. Find the maximum pulling force F that can be applied without detaching the bars.



Q.6. A very stiff bar *(AB)* of negligible mass is suspended horizontally by two vertical rods as shown in figure. Length of the bar is 2.5 L. The steel rod has length L and cross sectional radius of r and the brass rod has length $2L$ and cross sectional radius of $2r$. A vertically downward force F is applied to the bar at a distance x from the steel rod and the bar remains horizontal. Find the value of x if it is given that ratio of Young's modulus of steel and brass is $\frac{Y_s}{Y_B} = 2$.



Q.7. A closed steel cylinder is completely filled with water at 0°C. The water is made to freeze at 0°C. Calculate the rise in pressure on the cylinder wall. It is known that density of water at 0°C is 1000 kg/m^3 and the density of ice at 0°C is 910 kg/m^3. Bulk modulus of ice at 0°C is nearly 9×10^9 Pa. [Compare this pressure to the atmospheric pressure. Now you can easily understand why water pipelines burst in cold regions as the winter sets in.]

Q.8. (i) Two identical rods, one of steel, the other of copper, are stretched by an identical amount. On which operation more work is expended?

(ii) Two identical rods, one of steel, the other of copper, are stretched with equal force. On which operation is more work needed?

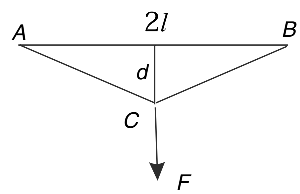
LEVEL 2

Q.9. A thin ring of radius R is made of a wire of density ρ and Young's modulus Y. It is spun in its own plane, about an axis through its centre, with angular velocity ω. Determine the amount (assumed small) by which its circumference increases.

Q.10. A steel wire of radius r is stretched without tension along a straight line with its ends fixed at A and B (figure). The wire is pulled into the shape ACB. Assume that d is very small compared to length of the wire. Young's modulus of steel is Y.

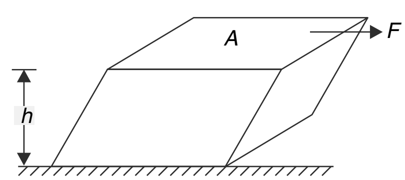
(a) What is the tension (T) in the wire?

(b) Determine the pulling force F. Is F larger than T?

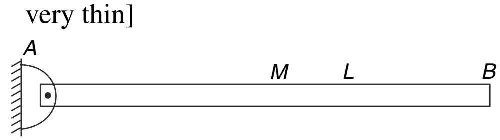


Q.11. A uniform material rod of length L is rotated in a horizontal plane about a vertical axis through one of its ends. The angular speed of rotation is ω. Find increase in length of the rod. It is given that density and Young's modulus of the rod are ρ and Y respectively.

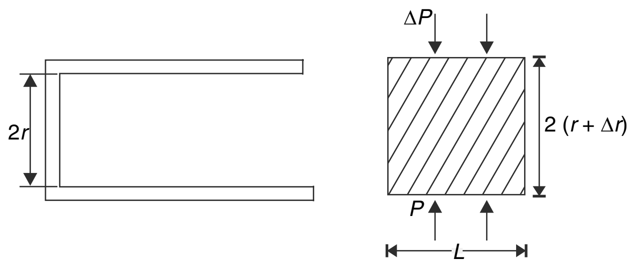
Q.12. A rectangular bar is fixed to a hard floor. Height of the bar is h and its area in contact with the floor is A. A shearing force distorts the bar as shown. Prove that the work done by the shearing force is $W = \left(\frac{\sigma^2}{2\eta}\right) \times$ volume of the bar. Here σ is shear modulus of elasticity. Assume the deformation to be small.



Q.13. A thin uniform rod of mass M and length L is free to rotate in vertical plane about a horizontal axis passing through one of its ends. The rod is released from horizontal position shown in the figure. Calculate the shear stress developed at the centre of the rod immediately after it is released. Cross sectional area of the rod is A. [For calculation of moment of inertia you can treat it to very thin]



Q.14. A rigid cylindrical container has inner radius r. A cork having radius $r + \Delta r$ and length L is to be fitted so as to close the container. Uniform pressure (ΔP) is needed on the curved cylindrical surface of the cork. Poisson's ratio of a cork is almost zero, and its bulk modulus is B.



(a) Calculate ΔP

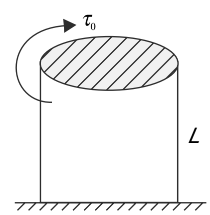
(b) After the cork is fitted how much force will be needed to pull it out of the container? Coefficient of friction between the container and the cork is μ.

Q.15. Assume that the least load which would break a thread when simply suspended from it is M and that this load produces a strain of 1 percent at the moment of breaking. Also assume that Hooke's law applies to the thread right up to breaking-point. A load of mass m is suspended from a thread of length λ. It is raised to a height and released. Find the least height to which the load must be raised so that it will break the thread when allowed to fall.

Q.16. Atmospheric pressure is P_0 and density of water at the sea level is ρ_0. If the bulk modulus of water is B, calculate the pressure deep inside the sea at a depth h below the surface.

LEVEL 3

Q.17. A metal cylinder of length L and radius R is fixed rigidly to ground with its axis vertical. A twisting torque τ_0 is applied along the circumference at the top of the cylinder. This causes an angular twist of θ_0 (rad) in the top surface. Calculate the shear modulus of elasticity (η) of the material of the cylinder.



ANSWERS

1. (a) $2.4 \times 10^4\ N$ (b) Yes (c) $2.6 \times 10^5\ N$

2. (a) 1.24 cm, 779 N (b) 1535 N

3. $Y\left(\dfrac{R-r}{r}\right)$; $Ybd\left(\dfrac{R-r}{r}\right)$

4. No

5. $\dfrac{\sigma_0 S_0}{\sin^2\theta}$

6. $x = 1.25\ L$

7. $8.1 \times 10^8\ Pa$

8. (i) Stretching of steel (ii) Stretching of Copper

9. $\dfrac{2\pi\rho R^3\omega^2}{Y}$

10. (a) $T = Y\pi r^2 (d^2/2l^2)$ (b) $2T(d/l)$ where T is given in answer (a). No F is much smaller.

11. $\dfrac{\rho\omega^2 L}{3Y}$

13. $\dfrac{Mg}{16A}$

14. (a) $\Delta P = \dfrac{2B\Delta r}{r}$ (b) $F = 4\pi\mu BL\Delta r$

15. $h = \dfrac{0.01M\ell}{2m}$.

16. $P = P_0 - B\ln\left(1 - \dfrac{\rho_0 gh}{B}\right)$

17. $\dfrac{2l\tau_0}{\pi R^4\theta_0}$

SOLUTIONS

1. (a) $\frac{F_s}{A} = Y_s \frac{\Delta L}{L}$

$\therefore\ F_s = AY_s \frac{\Delta L}{L} = \left(3\times10^{-4} m^2\right)\times\left(16\times10^9 N/m^2\right)\times\left(5\times10^{-3}\right) = 2.4 \times 10^4\ N$

(b) Momentum absorbed by the bone

$\Delta P = \frac{1}{2} m\sqrt{2gh} = \frac{1}{2}\times 60\times\sqrt{2\times10\times10} = 423\ N-s$

$\therefore$ Force $= \frac{\Delta P}{\Delta t} = \frac{423}{0.02} = 2.12\times10^4\ N$

Maximum compressive force that the bone can sustain to remain elastic is

$F_c = AY_c \frac{\Delta L}{L} = \left(3\times10^{-4} m^2\right)\times\left(9.4\times10^9 N/m^2\right)(5\times10^{-3}) = 1.4 \times 10^4\ N$

The impact force is larger than elastic limit.

(c) $F = \eta\theta A = 10^{10}\times\left(5\times\frac{\pi}{180}\ rad\right)\times3\times10^{-4} = 2.6\times10^5\ N$

2. (a) Area of cross section $A = \pi r^2 = 3.14 \times (1 \times 10^{-3})^2$

The stress should not exceed the elastic limit otherwise the wire will suffer permanent deformation

$\therefore\ \frac{F}{A} = 2.48\times10^8$

$F = 2.48 \times 10^8 \times 3.14 \times 10^{-6} = 779N$

Strain $= \frac{\text{stress}}{Y} = \frac{2.48\times10^8}{2\times10^{11}} = 1.24\times10^{-3}$

$\frac{\Delta L}{L} = 1.24\times10^{-3} \quad \Rightarrow \Delta L = 1.24 \times 10^{-3} \times 10\ m = 1.24\ cm$

(b) The stress should not exceed the ultimate strength.

$\frac{F}{A} = 4.89\times10^8$

$F = 4.89 \times 10^8 \times 3.14 \times 10^{-6} = 1535N$

3. Increase in circumference of the ring $= 2\pi(R-r)$

$\therefore$ Strain $= \frac{2\pi(R-r)}{2\pi r} = \frac{R-r}{r}$

Stress $= Y(\text{strain}) = Y\left(\frac{R-r}{r}\right)$

$\therefore$ Tension = (stress)A $= Ybd\left(\frac{R-r}{r}\right)$

4. If weight of the tank (completely filled with water) is W, then load on each pillar $= \frac{W}{4}$.

Stress on each pillar $= \frac{W}{4A}$

A = Area of cross section of the pillar.

Each pillar can support a stress of $10\times\frac{W}{4A}$.

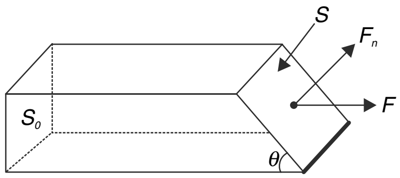
When each dimension is made 100 times, the total load will become 10^6 W.

$\therefore$ Stress on each pillar = $\dfrac{10^6 W}{4 \times 10^4 A} = 25 \cdot \dfrac{W}{A}$

But each pillar can support a stress of $10 \dfrac{W}{4A}$ only.

Hence, the pillars will break.

5.



Normal force $F_n = F \sin\theta$

Area $S = \dfrac{S_0}{\sin\theta}$

$$\therefore \sigma_n = \frac{F_n}{S} = \frac{F\sin\theta}{\dfrac{S_0}{\sin\theta}} = \frac{F}{S_0}\sin^2\theta$$

$$\Rightarrow \frac{F_{\max}\sin^2\theta}{S_0} = \sigma_0 \qquad \therefore F_{\max} = \frac{\sigma_0 S_0}{\sin^2\theta}$$

6. If T_b & T_s are tension developed in the steel and the brass rod then

$T_b(2.5L - x) = T_s \,.\, x$ ….. (1)

and $T_b + T_s = F$ ….. (2)

Solving (1) and (2) we get $T_b = \dfrac{xF}{2.5L}$

and $T_s = \left(1 - \dfrac{x}{2.5L}\right)F$

For the bar to remain horizontal, extension in the two rods must be same.

$\therefore \quad \dfrac{T_b(2L)}{4AY_B} = \dfrac{T_s L}{AY_s}$ [where $A = \pi r^2$]

$\dfrac{T_s}{T_b} = \dfrac{Y_s}{2Y_B}$

$\Rightarrow \quad \dfrac{2.5L - x}{x} = 1 \quad \Rightarrow \quad x = 1.25\ L$

7. If v = volume of water at 0°C, then volume of ice formed will be

$v' = v \times \dfrac{1000}{910} = \dfrac{100}{91} v$

Change in volume $\Delta v = v' - v = \dfrac{9v}{91}$

But the ice is not allowed to expand due to pressure exerted by the cylinder wall.

Volume strain $= \dfrac{\Delta v}{v'} = \dfrac{\dfrac{9v}{91}}{\dfrac{100v}{91}} = \dfrac{9}{100}$

$\therefore$ Using definition of bulk modulus $B = \dfrac{\Delta P}{\dfrac{\Delta v}{v'}}$

$$\Delta P = 9\times10^9 \times \frac{9}{100} = 81\times10^7 = 8.1\times10^8\,Pa$$

8. **(i)** The modulus of normal elasticity (Young's modulus) is greater for steel than for copper. Therefore if the rods are of equal dimensions and are to be stretched by the same amount, a greater force is necessary for the steel rod than for the copper one. So the steel rod requires that more work should be done.

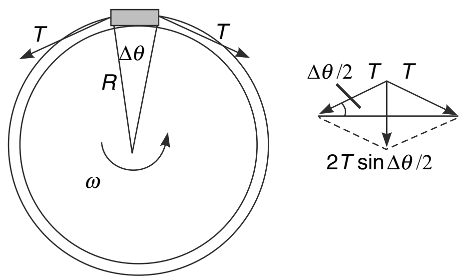
(ii) If the process of stretching is carried out with equal forces for both rods, the steel rod will be stretched less than the copper one. Therefore more work will be done this time on stretching the copper rod.

9. Let the tension in the ring be T.

Its resolved component acting towards the centre of rotation is

$$2T\sin\left(\frac{\Delta\theta}{2}\right) \simeq T\Delta\theta$$

This must balance the centripetal force = $R\Delta\theta A\rho R\omega^2$ (A is area of cross section of the wire of the ring)



$T\Delta\theta = R\Delta\theta A\rho R\omega^2$

Longitudinal stress in the ring = $T/A = \rho R^2\omega^2$.

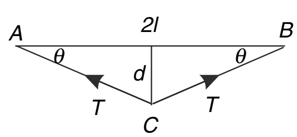
Strain = $\dfrac{\rho R^2\omega^2}{Y}$

Increase in circumference = $\dfrac{\rho R^2\omega^2}{Y}2\pi R = \dfrac{2\pi\rho R^3\omega^2}{Y}$

10. Increase in length of the wire = $(AC + CB) - 2l = [2\sqrt{(l^2+d^2)} - 2l]$

Longitudinal stress = $T/\pi r^2$, (where r is radius of wire).

Longitudinal strain $= \dfrac{[2\sqrt{(l^2+d^2)} - 2l]}{2l}$



Young's modulus $= \dfrac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

$$\text{or, } Y = \frac{T}{\pi r^2}\times\frac{2l}{[2[\sqrt{(l^2+d^2)} - l]]}$$

$$\therefore\ T = \frac{Y\times\pi r^2\times[\sqrt{(l^2+d^2)} - l]}{l}$$

If d is very small compared to l, then

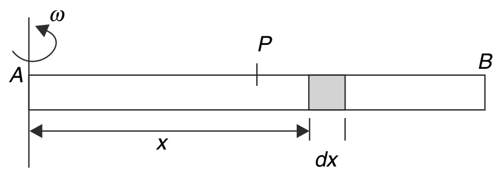
$$\sqrt{(l^2+d^2)} = l\left(1+\frac{d^2}{l^2}\right)^{1/2} = l\left(1+\frac{d^2}{2l^2}\right)$$

$\therefore\ T = Y\times\pi r^2\,(d^2/2l^2)$.

$F = 2T\sin\theta = 2T(d / AC) \simeq 2T(d / l)$

This will be a number much smaller than T when d is very small compared to length of the string.

11. Consider an element of length dx at a distance x from the rotation axis. Centripetal force required for this element is $(\rho A dx)\omega^2 x$



Tension at a point P, at a distance r from the axis is equal to sum of centripetal forces on all elements lying between P and B.

$\therefore$ Tension at P

$$T_r = \rho A\omega^2 \int_r^L x\,dx = \frac{\rho A\omega^2}{2}(L^2 - r^2)$$

Now assume that dl is extension in an element of length dr located at a distance r from the axis.

Strain $= \dfrac{dl}{dr}$

Stress $= \dfrac{Tr}{A} = \dfrac{1}{2}\rho\omega^2[L^2 - r^2]$

$$\therefore Y\frac{dl}{dr} = \frac{1}{2}\rho\omega^2[L^2 - r^2]$$

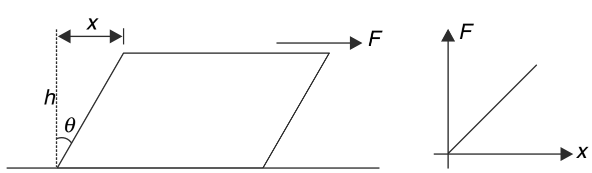
$$\Rightarrow dl = \frac{1}{2}\frac{\rho\omega^2}{Y}[L^2 - r^2]dr$$

$\therefore$ Change in length of the entire rod

$$\Delta l = \int dl = \frac{\rho\omega^2}{2Y}\int_0^L (L^2 - r^2)dr$$

$$= \frac{\rho\omega^2}{2Y}\left(L^3 - \frac{L^3}{3}\right) = \frac{1}{3}\frac{\rho\omega^2 L^3}{Y}$$

12.



Stress = η . strain

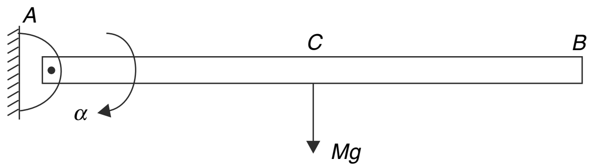
$$\therefore \frac{F}{A} = \eta\frac{x}{h} \qquad \Rightarrow \quad F = \frac{\eta A}{h}\cdot x \qquad \text{..... (i)}$$

Since force changes linearly with x, we can write its work done as

$$W = \frac{1}{2}Fx = \frac{1}{2}(\sigma A)\left(\frac{\sigma h}{\eta}\right) \qquad \left[\text{using (1)} \quad x = \frac{\sigma h}{\eta}\right]$$

$$= \frac{\sigma^2}{2\eta}Ah = \frac{\sigma^2}{2\eta}\times \text{ volume}$$

13.



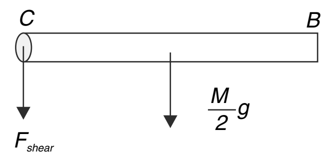
Immediately after release

$I\alpha = \tau \Rightarrow \frac{ML^2}{3} \cdot \alpha = Mg\frac{L}{2} \Rightarrow \alpha = \frac{3}{2}\frac{g}{L}$

Consider the half rod BC. Its COM has a downward acceleration

$a_{BC} = \frac{3L}{4} \cdot \alpha = \frac{9}{8}g$

F_{Shear} = Shear force applied by part AC on the part BC.



$\frac{Mg}{2} + F_{\text{shear}} = \frac{M}{2}a_{BC}$

$\therefore F_{\text{shear}} = \frac{9Mg}{16} - \frac{Mg}{2} = \frac{Mg}{16}$

$\therefore$ Shear stress $= \frac{Mg}{16A}$

14. (a) $V = \pi(r + \Delta r)^2 L \simeq \pi r^2 L$

$\Delta V = 2\pi r L \Delta r$

$\therefore \quad \frac{\Delta V}{V} = \frac{2\pi r L \Delta r}{\pi r^2 L} = \frac{2\Delta r}{r} \quad \Rightarrow B = \frac{\Delta P}{\frac{\Delta V}{V}}$

$\therefore \quad \Delta P = B\left(\frac{\Delta V}{V}\right) = \frac{2B\Delta r}{r}$

(b) Pulling force = friction force

$$F = \mu(\Delta P 2\pi r L) = \mu \cdot \frac{2B\Delta r}{r} \cdot 2\pi r L = 4\pi\mu B L \Delta r$$

15. A load m, falling from a height h, acquires a kinetic energy equal mgh.

When the stretch in the string is maximum this kinetic energy must be turned into energy of elastic deformation of the thread. If k is the force constant of the string-

$\frac{kx^2}{2} = mgh$ ----- (i)

In the problem it is given that maximum stretch is $x = 0.01\ \lambda$.

$kx = Mg$.

Substituting these relationships in the equation (i)

We get $h = \frac{0.01M\ell}{2m}$.

16. If volume of mass m of water is V and its density is ρ then

$$\rho V = m$$

$\Rightarrow \quad \rho dV + V d\rho = 0$

$\Rightarrow \quad \frac{d\rho}{\pi} = -\frac{dV}{V} \quad \ldots \text{(i)}$

If volume (V) of an element of water changes by dV due to an isotropic pressure increase dP,

$$B = -\frac{dP}{dV/V}$$

$\Rightarrow B = \dfrac{\rho}{d\rho} dP$ [using (i)]

$\Rightarrow dP = B\dfrac{d\rho}{\rho}$ … (ii)

If depth changes by dh

$dP = \rho g dh \quad \therefore \rho g\, dh = B \cdot \dfrac{d\rho}{\rho}$

$\Rightarrow \dfrac{g}{B} dh = \dfrac{d\rho}{\rho^2} \qquad \Rightarrow \dfrac{g}{B}\int_0^h dh = \int_{\rho_0}^{\rho} \dfrac{d\rho}{\rho^2}$

$\Rightarrow \dfrac{gh}{B} = \dfrac{1}{\rho_0} - \dfrac{1}{\rho} \qquad \Rightarrow \dfrac{\rho_0}{\rho} = 1 - \dfrac{\rho_0 gh}{B}$ … (iii)

From (ii) $dP = B\dfrac{d\rho}{\rho}$

$\therefore \int_{P_0}^{P} dP = B\int_{\rho_0}^{\rho} \dfrac{d\rho}{\rho} \qquad \therefore P - P_0 = B\ln\left(\dfrac{\rho}{\rho_0}\right)$

$P = P_0 - B\ln\left(\dfrac{\rho_0}{\rho}\right) = P_0 - B\ln\left(1 - \dfrac{\rho_0 gh}{B}\right)$

17. The shear strain at a radius r within the cylinder at its surface is $= \dfrac{r\theta_0}{l}$

$\therefore \text{Stress} = \eta(\text{strain}) = \dfrac{\eta r\theta_0}{l}$

Torque per unit area at radius r is $r \times \text{stress} = \dfrac{\eta r^2\theta_0}{l}$

The total torque can be obtained by integrating this quantity over the entire area.

$\therefore \tau_0 = \int_0^R (2\pi r dr)\left(\dfrac{\eta r^2\theta_0}{l}\right)$

$\tau_0 = \dfrac{\pi}{2}\dfrac{\eta R^4\theta_0}{l}$

$\therefore \eta = \dfrac{2l\tau_0}{\pi R^4\theta_0}$

12 SIMPLE HARMONIC MOTION

LEVEL 1

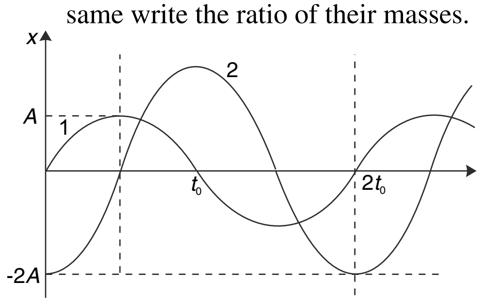
Q. 1. (i) The acceleration (a) of a particle moving along a straight line is related to time (t) as per the differential equation $\frac{d^2a}{dt^2} = -ba$. b is a positive constant. Is the particle performing SHM? If yes, what is the time period?

(ii) A particle is executing SHM on a straight line. A and B are two points at which its velocity is zero. It passes through a certain point P ($AP < PB$) with a speed of 3 m/s at times recorded as t = 0, 0.5 s, 2.0 s, 2.5 s, 4.0 s, 4.5 s......Determine the maximum speed of the particle and also the ratio AP/PB.

Q. 2. The position – time graph for two particles- 1 and 2- performing SHM along X axis has been shown in the fig.

(a) Write the velocity of the two particles as a function of time.

(b) If the energy of SHM for the two particles is same write the ratio of their masses.



Q. 3. A particle moves along X axis such that its acceleration is given by $a = -\beta(x-2)$, where β is a positive constant and x is the position co-ordinate.

(a) Is the motion simple harmonic?

(b) Calculate the time period of oscillations.

(c) How far is the origin of co-ordinate system from the equilibrium position?

Q. 4. A particle is performing simple harmonic motion along the x axis about the origin. The amplitude of oscillation is a. A large number of photographs of the particle are shot at regular intervals of time with a high speed camera. It was found that photographs having the particle at $x_1 + \Delta x$ were maximum in number and photographs having the particle at $x_2 + \Delta x$ were least in number. What are values of x_1 and x_2?

Q. 5. Position vector of a particle as a function of time is given by

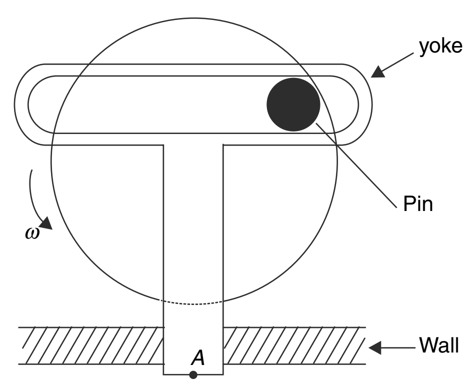
$$\vec{R} = (a \sin \omega t)\,\hat{i} + (a \cos \omega t)\,\hat{j} + (b \sin \omega_0 t)\,\hat{k}$$

The particle appears to be performing simple harmonic motion along z direction, to an observer moving in xy plane.

(a) Describe the path of the observer.

(b) Write the distance travelled by the observer himself in the time interval he sees the particle completing one oscillation.

Q. 6. A wheel is revolving at an angular speed of ω. A pin welded at the circumference of the wheel forces a T shaped body to move up and down. The pin slides freely inside the slot of the yoke as the wheel rotates. The T shaped body is constrained to move vertically by a set of walls.



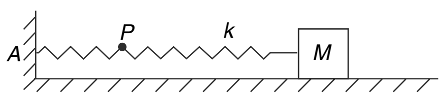
(a) Find the time period of oscillatory motion of point A at the base of the T shaped body

(b) Is the motion of A simple harmonic?

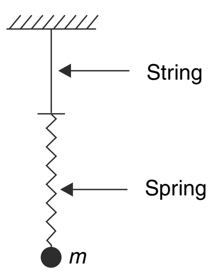
Q. 7. (i) A particle is performing simple harmonic motion with time period T. At an instant its speed is 60% of its maximum value and is increasing. After an interval Δt its speed becomes 80% of its maximum value and is decreasing. Find the smallest value of Δt in terms of T.

(ii) A particle is doing SHM of amplitude 0.5 m and period π seconds. When in a position of instantaneous rest, it is given an impulse which imparts a velocity of 1 m/s towards the equilibrium position. Find the new amplitude of oscillation and find how much less time will it take to arrive at the next position of instantaneous rest as compared to the case if the impulse had not been applied.

Q. 8. A block of mass M is tied to a spring of force constant k and placed on a smooth horizontal surface. The natural length of the spring is L. P is a point on the spring at a distance $\frac{L}{4}$ from its fixed end. The block is set in oscillations with amplitude A. Find the maximum speed of point P on the spring.



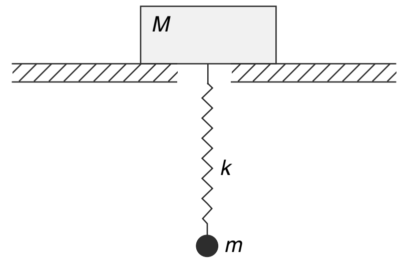
Q. 9. A particle of mass m is suspended with the help of a spring and an inextensible string as shown in the figure. Force constant of the spring is k. The particle is pulled down from its equilibrium position by a distance x and released.



(a) Find maximum value of x for which the motion of the particle will remain simple harmonic.

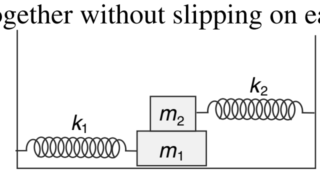
(b) Find maximum tension in the string if $x = \frac{mg}{2k}$.

Q. 10. A block of mass M is placed on top of a hole in a horizontal table. A spring of force constant k is connected to the block through the hole. The other end of the massless spring has a particle of mass m connected to it. With what maximum amplitude can the particle oscillate up and down such that the block does not lose contact with the table?



Q. 11. A block of mass m is moving along positive x direction on a smooth horizontal surface with velocity u. It enters a rough horizontal region at $x = 0$. The coefficient of friction in this rough region varies according to $\mu = ax$, where 'a' is a positive constant and x is displacement of the block in the rough region. Find the time for which the block will slide in this rough region.

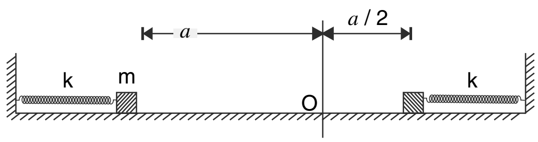
Q. 12. (i) In the shown arrangement, both springs are relaxed. The coefficient of friction between m_2 and m_1 is μ. There is no friction between m_1 and the horizontal surface. The blocks are displaced slightly and released. They move together without slipping on each other.



(a) If the small displacement of blocks is x then find the magnitude of acceleration of m_2. What is time period of oscillations?

(b) Find the ratio $\frac{m_1}{m_2}$ so that the frictional force on m_2 acts in the direction of its displacement from the mean position.

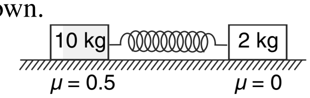
(ii) Two small blocks of same mass m are connected to two identical springs as shown in fig. Both springs have stiffness K and they are in their natural length when the blocks are at point O. Both the blocks are pushed so that one of the springs get compressed by a distance a and the other by $a/2$. Both the blocks are released from this position simultaneously. Find the time period of oscillations of the blocks if - (neglect the dimensions of the blocks)



(a) Collisions between them are elastic.

(b) Collisions between them are perfectly inelastic.

Q. 13. Two blocks of mass 10 *kg* and 2 *kg* are connected by an ideal spring of spring constant $K = 800\ N/m$ and the system is placed on a horizontal surface as shown.

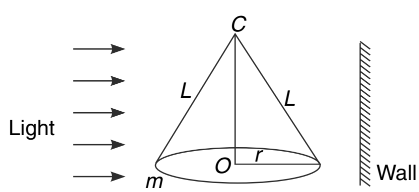


The coefficient of friction between 10 *kg* block and surface is 0.5 but friction is absent between 2 *kg* and the surface. Initially blocks are at rest and spring is relaxed. The 2 *kg* block is displaced to elongate the spring by 1 *cm* and is then released.

(a) Will 10 *kg* block move subsequently?

(b) Draw a graph representing variation of magnitude of frictional force on 10 *kg* block with time. Time t is measured from that instant when 2 *kg* block is released to move.

Q.14. A particle of mass m is tied at the end of a light string of length L, whose other end is fixed at point C (fig), and is revolving in a horizontal circle of radius r to form a conical pendulum. A parallel horizontal beam of light forms shadow of the particle on a vertical wall.

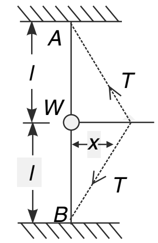


If the tension in the string is F find -

(a) The maximum acceleration of the shadow moving on the wall.

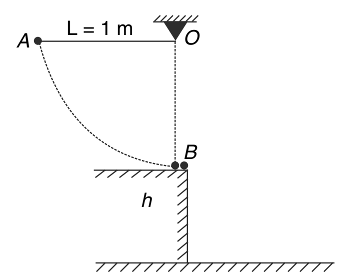
(b) The time period of the shadow moving on the wall.

Q.15. A small ball of mass m is attached to the middle of a tightly stretched perfectly flexible wire AB of length $2\ \ell$ (**figure**). The ball is given a small lateral displacement in horizontal direction and released. The initial tension (T) in the wire is high and change in it due to small lateral displacement of the ball can be neglected. Prove that the ball will perform simple harmonic motion, and calculate the period. If there is a device which can change the tension in the wire at will , how will the time period change if tension in the wire is increased?



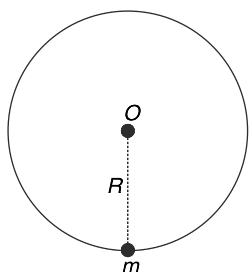
Q. 16. A simple pendulum oscillating with a small amplitude has a time period of $T = 1.0s$. A horizontal thin rod is now placed beneath the point of suspension at a distance equal to half the length of the pendulum. The string collides with the rod once in each oscillation and there is no loss of energy in such collisions. Find the new time period T' of the pendulum.

Q. 17. (i) A small steel ball (B) is at rest on the edge of a table of height h. Another identical steel ball (A) is tied to a light string of length $L = 1.0\ m$ and is released from the position shown so that it swings like a pendulum. At the lowest position of its path it hits the ball B which is at rest. Ball B flies off the table and hits the ground in time t. After collision the ball A keeps moving for a time t' before coming to rest for the first time. Find the value of h if $t = t'$. Collision between the balls is head on and coefficient of restitution is $e = 0.995$.



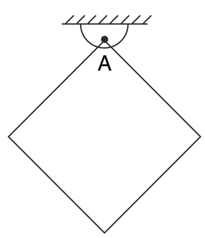
(ii) A pendulum has a particle of mass m attached to a massless rod of length L. The rod is released from a position where it makes an angle $\theta_0 \left(< \frac{\pi}{2}\right)$ with the vertical. The time period of oscillation is observed to be T_0. Another similar pendulum has a rod of length $2L$. Time period of this pendulum when released from position θ_0 is T. Which is larger T or T_0?

Q. 18. A disc of mass $M = 2m$ and radius R is pivoted at its centre. The disc is free to rotate in the vertical plane about its horizontal axis through its centre O. A particle of mass m is stuck on the periphery of the disc. Find the frequency of small oscillations of the system about its equilibrium position.



Q. 19. A rigid body is to be suspended like a physical pendulum so as to have a time period of $T = 0.2\pi$ second for small amplitude oscillations. The minimum distance of the point of suspension from the centre of mass of the body is $l_1 = 0.4$ *m* to get this time period. Find the maximum distance (l_2) of a point of suspension from the centre of mass of the body so as to get the same time period. [$g = 10\ m/s^2$]

Q. 20. A square plate of mass M and side length L is hinged at one of its vertex (A) and is free to rotate about it. Find the time period of small oscillations if



(a) the plate performs oscillations in the vertical plane of the figure. (Axis is perpendicular to figure.)

(b) the plate performs oscillations about a horizontal axis passing through A lying in the plane of the figure.

LEVEL 2

Q. 21. Two particles A and B are describing SHM of same amplitude (a) and same frequency (f) along a common straight line. The mean positions of the two SHMs are also same but the particles have a constant phase difference between them. It is observed that during the course of motion the separation between A and B is always less than or equal to a.

(a) Find the phase difference between the particles.

(b) If distance between the two particles is plotted with time, with what frequency will the graph oscillate?

Q. 22. (i) A particle of mass m executes SHM in xy-plane along a straight line AB. The points A (a, a) and B $(-a, -a)$ are the two extreme positions of the particle. The particle takes time T to move from one extreme A to the other extreme B. Find the x component of the force acting on the particle as a function of time if at $t = 0$ the particle is at A.

(ii) Two particle A and B are performing SHM along X-axis and Y-axis respectively with equal amplitude and frequency of 2 *cm* and 1 Hz respectively. Equilibrium positions for the particles A and B are at the coordinate (3, 0) and (0, 4) respectively. At $t = 0$, B is at its equilibrium position and moving toward the origin, while A is nearest to the origin. Find the maximum and minimum distances between A and B during their course of motion.

Q. 23. A particle is performing SHM along x – axis and equation for its motion is $x = a\cos(\pi t)$

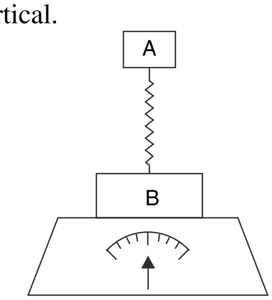
Let the time t be expressed as $\frac{t}{2} = n + m$

Where $n = 0,1,2,3,4....$ and m is a positive fraction.

Calculate the distance travelled by the particle during the interval from t = 0 to $t = t$ if

(a) $m < 0.5$ (b) $m > 0.5$

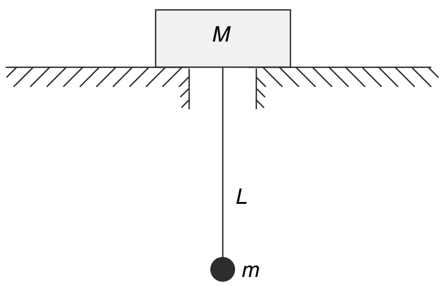
Q. 24. Two blocks A and B having mass $m = 1$ *kg* and $M = 4$ *kg* respectively are attached to a spring and placed vertically on a weighing machine as shown in the figure. Block A is held so that the spring is relaxed. A is released from this position and it performs simple harmonic motion with angular frequency $\omega = 25$ *rad* s^{-1}. The spring remains vertical.



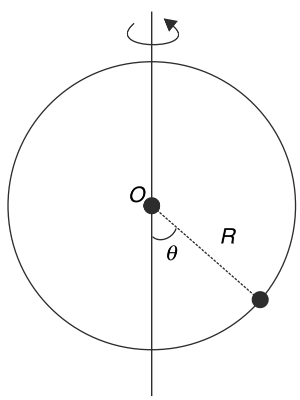
(a) Find the reading of the weighing machine as a function of time. Take $t = 0$ when A is released.

(b) What is the maximum reading of the weighing machine?

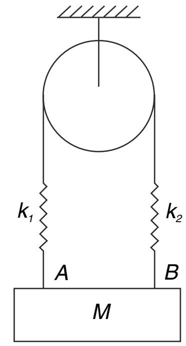
Q. 25. A block of mass M rests on a smooth horizontal table. There is a small gap in the table under the block through which a pendulum has been attached to the block. The bob of the simple pendulum has mass m and length of the pendulum is L. The pendulum is set into small oscillations in the vertical plane of the figure. Calculate its time period. The table does not interfere with the motion of the string.



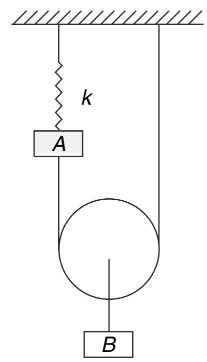
Q. 26. A circular wire frame of radius R is rotating about its fixed vertical diameter. A bead on the wire remains at rest relative to the wire at a position in which the radius makes an angle θ with the vertical (see figure). There is no friction between the bead and the wire frame. Prove that the bead will perform SHM (in the reference frame of the wire) if it is displaced a little from its equilibrium position. Calculate the time period of oscillation.



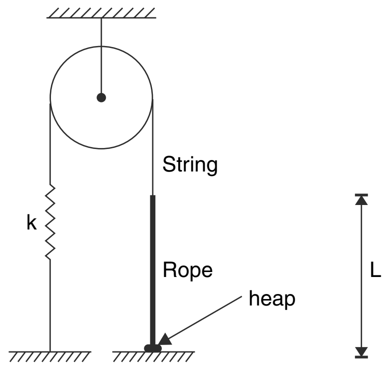
Q. 27. In the system shown in the figure the string, springs and pulley are light. The force constant of the two springs are $k_1 = k$ and $k_2 = 2k$. Block of mass M is pulled vertically down from its equilibrium position and released. Calculate the angular frequency of oscillation. The top surface of the block (represented by line AB) always remains horizontal.



Q. 28. (i) In the system shown in figure, find the time period of vertical oscillations of the block A. Both the blocks A and B have equal mass of m and the force constant of the ideal spring is k. Pulley and threads are massless.

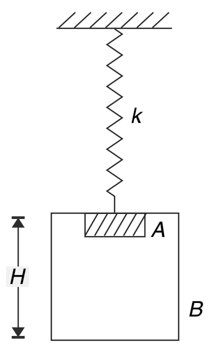


(ii) In the arrangement shown in the figure the spring, string and the pulley are mass less. The force constant of the spring is k. A rope of mass per unit length equal to λ ($kg\ m^{-1}$) hangs from the string as shown. In equilibrium a length L of the rope is in air and its bottom part lies in a heap on the floor. The rope is very thin and size of the heap is negligible though the heap contains a fairly long length of the rope. The rope is raised by a very small distance and released. Show that motion will be simple harmonic and calculate the time period. Assume that the hanging part of the rope does not experience any force from the heap or the floor (For example there is no impact force while the rope hits the floor while moving downward and there is no impulsive pull when the vertical part jerks a small element of heap into motion).

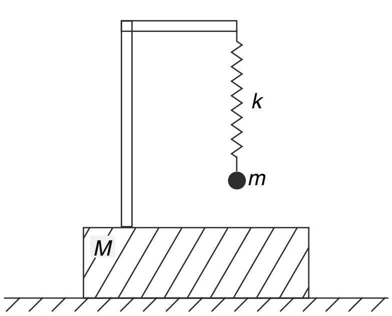


Q. 29. A box B of mass M hangs from an ideal spring of force constant k. A small particle, also of mass M, is stuck to the ceiling of the box and the system is in equilibrium. The particle gets detached from the ceiling and falls to strike the floor of the box. It takes time 't' for the particle to hit the floor after it gets detached from the ceiling. The particle, on hitting the floor, sticks to it and the system thereafter oscillates with a time period T. Find the height H of the box if it is given that $t = \dfrac{T}{6\sqrt{2}}$.

Assume that the floor and ceiling of the box always remain horizontal.



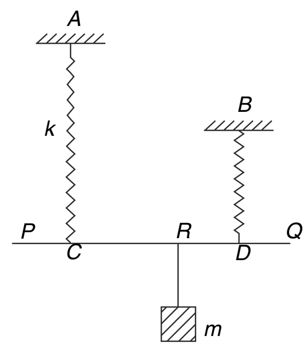
Q. 30. A block has a L shaped stand fixed to it. Mass of the block with the stand is M. At the free end of the stand there is a spring which carries a ball of mass m. With the spring in its natural length, the ball is released. It begins to oscillate and the stand is tall enough so that the ball does not hit the block.



(a) Find maximum value of mass (m) of the ball for which the block will not lose contact with the ground?

(b) If the stand is not tall enough and the ball makes elastic impact with the block, will your answer to part (a) change?

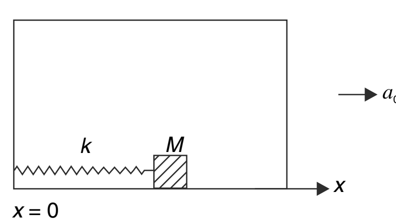
Q. 31. Two ideal springs of same make (the springs differ in their lengths only) have been suspended from points A and B such that their free ends C and D are at same horizontal level. A massless rod PQ is attached to the ends of the springs. A block of mass m is attached to the rod at point R. The rod remains horizontal in equilibrium. Now the block is pulled down and released. It performs vertical oscillations with time period $T = 2\pi\sqrt{\dfrac{m}{3k}}$ where k is the force constant of the longer spring.



(a) Find the ratio of length RC and RD.

(b) Find the difference in heights of point A and B if it is given that natural length of spring BD is L.

Q. 32. A block of mass M connected to an ideal spring of force constant k lies in equilibrium on the smooth floor of a room. The other end of the spring is fixed to the left wall of the room. The room begins to move to the right with a constant acceleration a_0. In the reference frame of the room the block begins to perform simple harmonic motion.



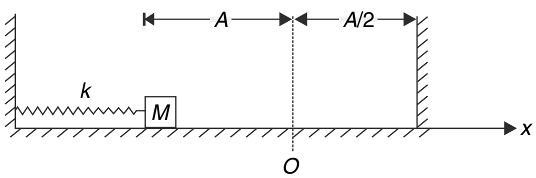
At a certain instant (say $t = t_0$) when the block was at its left extreme, the acceleration of the room vanishes. Plot the $x - t$ graph for the block taking time $t = 0$ when the room started accelerating.

Show the graph till time t_0 and beyond that. Take origin to be at the left wall and positive x direction towards right (as shown in figure). Assume no collision of the block with walls.

Q. 33. A block of mass M connected to an ideal spring of force constant k, is placed on a smooth surface. The block is pushed to the left so as to compress the spring by a length A and then it is released. The block hits an elastic wall at a distance $\frac{3A}{2}$ from its point of release. Assume the collision to be instantaneous.

(a) Calculate the time required by the block to complete one oscillation

(b) Draw the velocity – time graph for one oscillation of the block.



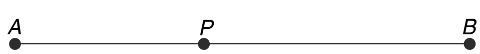
(c) Find the value of k for which average force experienced by the wall due to repeated hitting of the block is F_0.

Q. 34. A particle of mass m is constrained to move along a straight line. A and B are two fixed points on the line at a separation of L. When the particle is at some point P, between A and B, it is acted upon by two forces

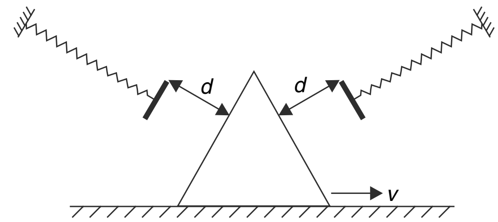
$$\overrightarrow{F_1} = \left(\frac{6mg}{L}\right)\overrightarrow{PA} \text{ and } \overrightarrow{F_2} = \left(\frac{3mg}{L}\right)\overrightarrow{PB}$$

At time $t = 0$, the particle is projected from A towards B with a speed of $\sqrt{gL}$.

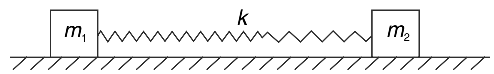
At what time 't' will the particle reach at B for the first time?



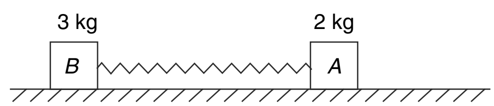
Q. 35. An equilateral prism of mass m is kept on a smooth table between two identical springs each having a force constant of k. The two springs have their lengths perpendicular to the inclined faces of the prism and are constrained to remain straight. The ends of the springs have light pads aligned parallel to the faces of the prism, and distance between pads and the incline faces is d. The prism is imparted a velocity v to the right. Find time period of its oscillation.



Q. 36. Two blocks rest on a smooth horizontal surface. They are connected by a spring of force constant k. If the system is set into oscillation find its time period.



Q. 37. Two blocks A (2 kg) and B (3 kg) rest on a smooth horizontal surface, connected by a spring of stiffness k = 120 N/m. Initially, the spring is relaxed. At $t = 0$, A is imparted a velocity $u = 2\ m/s$ towards right. Find displacement of block A as a function of time.



Q. 38. A spring has force constant k = 200 N/m and its one end is fixed. There is a block of mass 2 kg attached to its other end and the system lies on a smooth horizontal table. The block is pulled so that the extension in the spring becomes 0.05 m. At this position the block is projected with a speed of 1 m/s in the direction of increasing extension of the spring. Consider time $t = 0$ at the moment the block is projected and find

(a) the extension (or compression) in the spring as a function of time.

(b) the maximum extension in the spring and the time at which it occurs for the first time.

(c) the time after which the speed of the block becomes maximum for the first time.

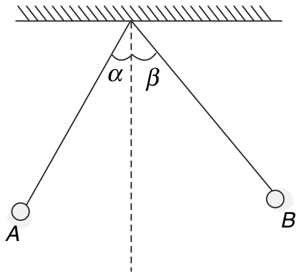
Given: $\sin^{-1}(0.446) = 0.46$ radian

Q. 39. Two identical simple pendulums A and B are fixed at same point. They are displaced by very small angles α and β (= 2 α) respectively and are simultaneously released from rest at time t = 0. Collisions between the pendulum bobs are elastic and length of each pendulum is ℓ .

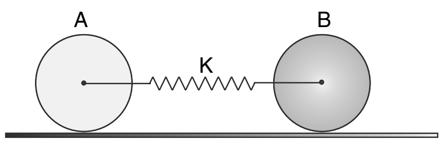
(a) What is the minimum number of collisions between the bobs after which the pendulum B will again reach its original position from where it was released?

(b) Find the time (t) at which B reaches its initial position for the first time after the release.

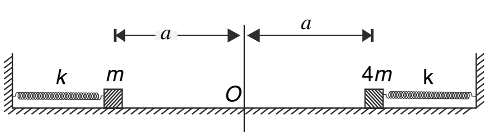
(c) Write the kinetic energy of pendulum B just after n^{th} collision? Take mass of each bob to be m.



Q. 40. Two spheres A and B of the same mass m and the same radius are placed on a rough horizontal surface. A is a uniform hollow sphere and B is uniform solid sphere. They are tied centrally to a light spring of spring constant k as shown in figure. A and B are released when the extension in the spring is x_0. Friction is sufficient and the spheres do not slip on the surface. Find the frequency and amplitude of SHM of the sphere A.



Q. 41. Two small blocks of mass m and $4m$ are connected to two springs as shown in fig. Both springs have stiffness K and they are in their natural length when the blocks are at point O. Both the blocks are pushed so that both the springs get compressed by a distance a. First the block of mass m is released

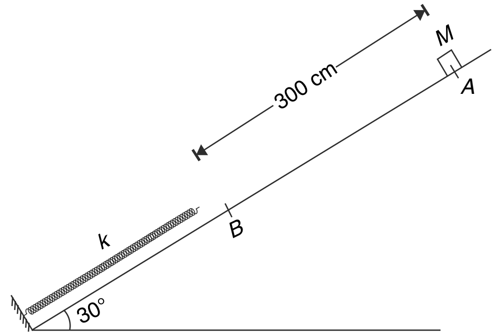


and after it travels through a distance $\left(1-\dfrac{\sqrt{3}}{2}\right)a$, the second block is also released.

(a) At what distance from point O will the two blocks collide?

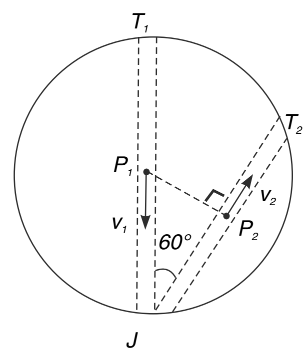
(b) How much time the two blocks need to collide after the block of mass $4m$ is released?

Q. 42. A block of mass $M = 40\ kg$ is released on a smooth incline from point A. After travelling through a length of 30 cm it strikes an ideal spring of force constant $K = 1000\ N/m$. It compresses the spring



and then gets pushed back. How much time after its release, the block will be back to point A?

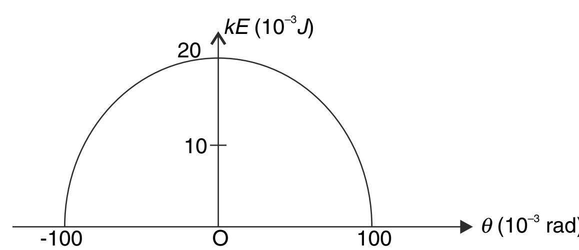
Q. 43. Two tunnels - T_1 and T_2 are dug across the earth as shown in figure. One end of the two tunnels have a common meeting point on the surface of the earth. Two particles P_1 and P_2 are oscillating from one end to the other end of the tunnels. At some instant particles are at mid point of their tunnels as shown in figure. Then –



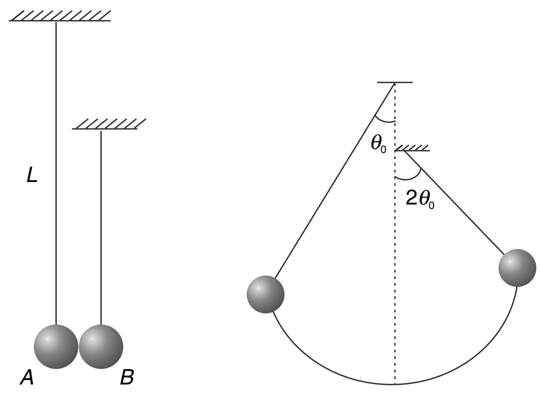
(a) Write phase difference between the particle P_1 and P_2. Can the two particles ever meet?

(b) Write the ratio of maximum velocity of particle P_1 and P_2.

Q. 44. The given figure shows the variation of the kinetic energy of a simple pendulum with its angular displacement (θ) from the vertical. Mass of the pendulum bob is $m = 0.2\ kg$. Find the time period of the pendulum. Take $g = 10\ ms^{-2}$.



Q. 45. Two identical small elastic balls have been suspended using two strings of different length (see fig (a)). Pendulum A is pulled to left by a small angle θ_0 and released. It hits ball B head on which swings to angle $2\theta_0$ from the vertical. Calculate the time period of oscillation of A if its length is known to be L.

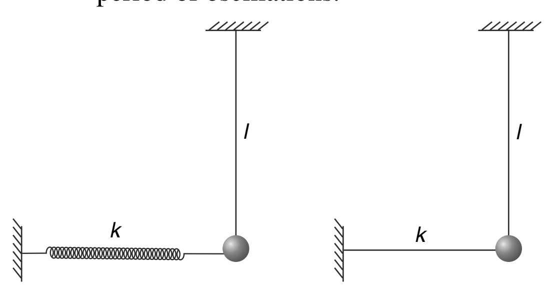


(a) (b)

Q. 46. A simple pendulum of length L has a bob of mass m. The bob is connected to light horizontal spring of force constant k. The spring is relaxed when the pendulum is vertical (see fig (i)).

(a) The bob is pulled slightly and released. Find the time period of small oscillations. Assume that the spring remains horizontal.

(b) The spring is replaced with an elastic cord of force constant k. The cord is relaxed when the pendulum is vertical (see fig (ii)). The bob is pulled slightly and released. Find the time period of oscillations.



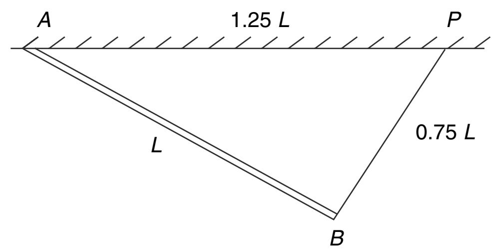
(i) (ii)

Q. 47. A uniform rod AB of mass m and length L is tied, at its end B, to a thread which is attached to point P on the ceiling. Length of the thread PB is 0.75 L. The other end A of the rod is hinged at a point on the ceiling. Distance $AP = 1.25\ L$. End B of the rod is pushed gently perpendicular to the plane of the figure and it starts oscillating

(a) Find the moment of inertia of the rod about line AP.

(b) Assuming that the triangle APB makes a small angle θ with the vertical plane, write the restoring torque acting on the rod.

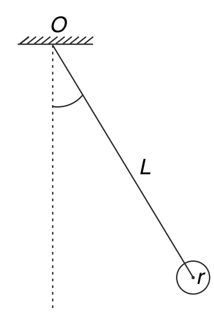
(c) Calculate the time period of small oscillations.



Q. 48. A railway tank wagon with its 2m diameter and 6m long horizontal cylindrical body, half full of petrol is driven around a curve of radius 100m, at a speed of 8.33 m/s. The curve runs smoothly into a straight track and the train maintains a constant speed. Find the angular amplitude and frequency of subsequent oscillation of the petrol due to this change of motion. Neglect viscosity and consider petrol as a solid semi cylinder sliding inside the tank. Given: $\tan^{-1}(0.07) \approx 4^\circ$

Q. 49. A pendulum consists of an inextensible thread connected to a solid spherical ball of radius r. The distance between the point of suspension and the centre of the ball is L ($>> r$). Calculate the percentage difference in the time period of this pendulum to the time period of a simple pendulum of length L. How large is this difference for $r = 5\ cm$ and $L = 100\ cm$.

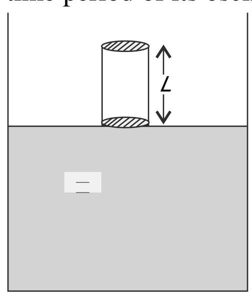
Q. 50. A disc of radius r is connected to a string of length L. The string is tied to a point on the circumference of the disc. This system is made to oscillate in vertical plane of the disc with a small angular amplitude θ_0. Find the speed of the lowest point of the disc at the moment the string become vertical.



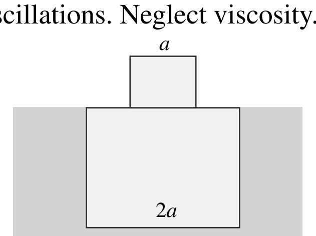
Q. 51. (i) A cylindrical container has area of cross section equal to 4A and it contains a non viscous liquid of density 2ρ. A wooden cylinder of cross sectional area A and length L has density ρ. It is held vertically with its lower surface touching the liquid. It is released from this position. Assume that the depth of the container is sufficient and the cylinder does not touch the bottom.

(a) Find amplitude of oscillation of the wooden cylinder.

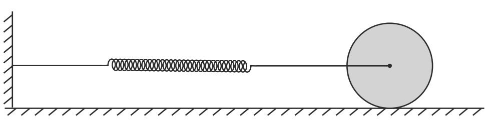
(b) Find time period of its oscillation.



(ii) Two cubical blocks of side length a and $2a$ are stuck symmetrically as shown in the figure. The combined block is floating in water with the bigger block just submerged completely. The block is pushed down a little and released. Find the time period of its oscillations. Neglect viscosity.



Q. 52. A hollow cylindrical shell of radius R has mass M. It is completely filled with ice having mass m. It is placed on a horizontal floor connected to a spring (force constant k) as shown. When it is disturbed it performs oscillations without slipping on the floor.



(a) Find time period of oscillation assuming that the ice is tightly pressed against the inner surface of the cylinder.

(b) If the ice melts into non viscous water, find the time period of oscillations. (Neglect any volume change due to melting of ice)

Q. 53. A particle of mass m is free to move along x axis under the influence of a conservative force. The potential energy of the particle is given by

$U = -ax^n e^{-bx}$ [a and b are positive constants]

Find the frequency of small oscillations of the particle about its equilibrium position

LEVEL 3

Q. 54. Two particles of mass m_1 and m_2 are connected by a spring of natural length L and force constant k. The masses are brought close enough so as to compress the spring completely and a string is used to tie the system. Assume that length of spring in this position is close to zero. The system is projected with a velocity V_0 along the positive x direction. At the instant it reaches origin at time $t = 0$, the string snaps and the spring starts opening.

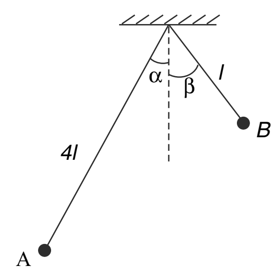
(a) Show that the mass m_1 (or m_2) will perform SHM in the reference frame attached to the centre of mass of the system. Find the time period of oscillation.

(b) Write the amplitude of m_1 and m_2 as a function of time.

(c) Write the X co ordinates of m_1 and m_2 as a function of time

Q. 55. Two simple pendulums A and B have length 4ℓ and ℓ respectively. They are released from rest from the position shown. Both the angles α and β are small. Calculate the time after which the two string become parallel for the first time if–

(a) $\alpha = \beta$ (b) $\beta = 1.5\,\alpha$

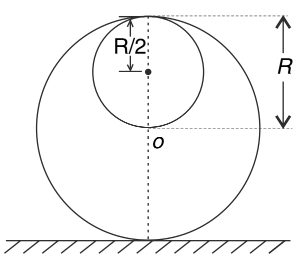


Q. 56. A simple pendulum has a bob of mass m and it is oscillating with a small angular amplitude of θ_0. Calculate the average tension in the string averaged over one time period. [For small θ take $\cos\theta \simeq 1 - \frac{\theta^2}{2}$]

Q. 57. Assume a smooth hole drilled along the diameter of the earth. If a stone is dropped at one end it reaches the other end of the hole after a Time T_0. Now instead of dropping the stone, you throw it into the hole with an initial velocity u. How big should u be, so that the stone appears at the other end of the hole after a time $\frac{T_0}{2}$. Express your answer in terms of acceleration due to gravity on the surface of the earth (g) and the radius of the earth (R).

Q. 58. A large horizontal turntable is rotating with constant angular speed ω in counterclockwise sense. A person standing at the centre, begins to walk eastward with a constant speed V relative to the table. Taking origin at the centre and X direction to be eastward calculate the maximum X co-ordinate of the person.

Q. 59. A spherical cavity of radius $\frac{R}{2}$ is removed from a solid sphere of radius R as shown in fig. The sphere is placed on a rough horizontal surface as shown. The sphere is given a gentle push. Friction is large enough to prevent slippage. Prove that the sphere perform SHM and find the time period.

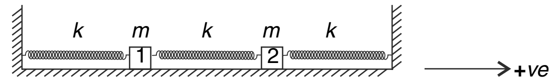


Q. 60. Two blocks 1 and 2, each having mass m, are placed on a smooth table connected to three identical springs as shown in the figure. Each spring has a force constant K. Initially, all springs are relaxed. The system is disturbed and starts moving. Let x_1 and x_2 represent the displacements of the two blocks from their respective mean positions.

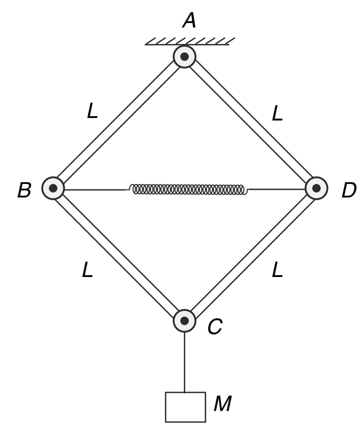
(a) Prove that the quantity $A = x_1 + x_2$ varies sinusoidally and calculate its angular frequency ω_a.

(b) Prove that the quantity $B = x_1 - x_2$ varies sinusoidally and calculate its angular frequency ω_b.

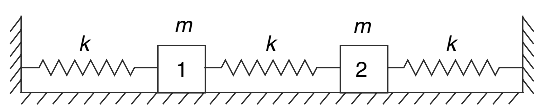
(c) Prove that motion of block 1 is superposition of two SHMs. Write frequency of the component SHMs.



Q. 61. Four identical mass less rods are connected by hinged joints to form a rhombus of side length L. Rods can rotate freely about the joints. The joints B and D are connected by a mass less spring of relaxed length 1.5 L. The system is suspended vertically with a load of mass M attached at C (see fig). In equilibrium the rods form an angle of 30° with the vertical. Find time period of small oscillations of the load.



Q. 62. Two identical blocks 1 and 2, each of mass m, are kept on a smooth horizontal surface, connected to three springs as shown in the figure. Each spring has a force constant k. Under suitable initial conditions, the two blocks oscillate in phase and their respective displacement from the mean position is given by



$x_1 = A \sin \omega t$ and $x_2 = A \sin \omega t$

(i) Suggest one such initial condition that will result in such oscillation.

(ii) Find ω

ANSWERS

1. (i) $T = \frac{2\pi}{\sqrt{b}}$

(ii) $V_{max} = 3\sqrt{2}$ m/s, $AP/BP = (\sqrt{2}-1)/(\sqrt{2}+1)$

2. (a) $v_1 = A\frac{\pi}{t_0}\cos\left(\frac{\pi}{t_0}t\right)$ and $v_2 = 2A\frac{\pi}{t_0}\sin\left(\frac{\pi}{t_0}t\right)$

(b) 2 : 1

3. (a) Yes (b) $T = 2\pi\sqrt{\frac{1}{\beta}}$ (c) 2 unit

4. $x_1 = \pm a$; $x_2 = 0$

5. (a) circle of radius a. (b) $2\pi a\left(\frac{\omega}{\omega_0}\right)$

6. (a) $T = \frac{2\pi}{\omega}$ (b) yes

7. (i) $\Delta T = \frac{T}{4}$ (ii) $A = \frac{1}{\sqrt{2}}m$; $t = \frac{\pi}{8}s$

8. $\frac{A}{4}\sqrt{\frac{k}{M}}$

9. (a) $\frac{mg}{k}$ (b) $\frac{3}{2}mg$

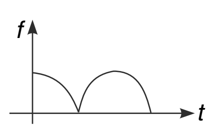
10. $\frac{Mg}{k} + \frac{mg}{k}$

11. $\frac{\pi}{2}\sqrt{\frac{1}{ag}}$

12. (a) $a = \left(\frac{k_1 + k_2}{m_1 + m_2}\right)x$; $T = 2\pi\sqrt{\frac{m_1 + m_2}{k_1 + k_2}}$

(b) $\frac{m_1}{m_2} > \frac{k_1}{k_2}$; $T = 2\pi\sqrt{\frac{m}{k}}$ for both the blocks in both cases.

13. (a) No (b)



14. (a) $\frac{1}{m}\sqrt{F^2 - (mg)^2}$ (b) $2\pi\left(\frac{m^2r^2}{F^2 - (mg)^2}\right)^{1/4}$

15. $2\pi\sqrt{\frac{m\ell}{2T}}$, Time period increases.

16. $\left(\frac{\sqrt{2}+1}{2\sqrt{2}}\right)T$

17. (i) $h = 1.25\ m$ (ii) $T > T_0$

18. $f = \frac{1}{2\pi}\sqrt{\frac{g}{2R}}$

19. 0.6 m

20. **(a)** $2\pi\sqrt{\frac{2\sqrt{2}}{3}\frac{a}{g}}$

(b) $2\pi\sqrt{\frac{7}{6\sqrt{2}}\frac{a}{g}}$

21. (a) $\phi_1 - \phi_2 = \frac{2\pi}{3}$ (b) 2f

22. (i) $-\frac{4\pi^2}{T^2}ma\cos\left(\frac{2\pi}{T}t\right)$

(ii) 7 cm and 3 cm.

23. (a) $s = 4an + a(1 - \cos \pi t)$

(b) $s = 4an + a(3 + \cos \pi t)$

24. (a) $50 - 10\cos(625t)$ (b) 60 N

25. $T = 2\pi\sqrt{\frac{LM}{g(M+m)}}$

26. $T = 2\pi\sqrt{\frac{R\cos\theta}{g}}$

27. $\omega = \sqrt{\frac{8k}{3M}}$

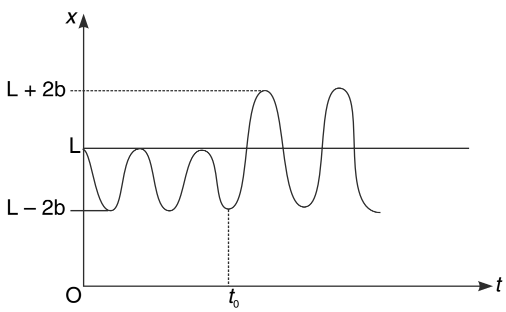
28. (i) $T = \pi\sqrt{\frac{5m}{k}}$ (ii) $T = 2\pi\sqrt{\frac{\lambda L}{k + \lambda g}}$

29. $H = \frac{Mg}{2k}\left(1 + \frac{\pi^2}{9}\right)$

30. For both (a) and (b) the block will not lose contact with the ground for any value of m.

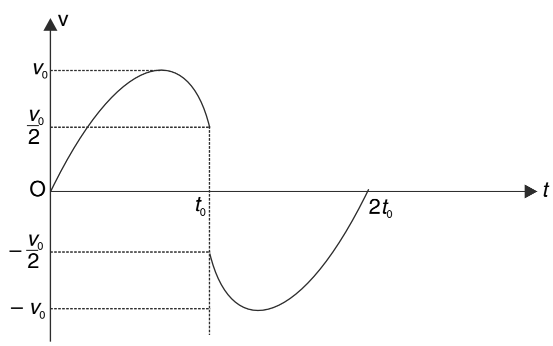
31. (a) $\dfrac{RC}{RD}=\dfrac{2}{1}$ (b) L.

32.



33. (a) $\dfrac{4\pi}{3}\sqrt{\dfrac{M}{k}}$

(b)



(c) $k=\dfrac{4\pi F_0}{3\sqrt{3}A}$

34. $t=\dfrac{2\pi}{3}\sqrt{\dfrac{L}{3g}}$

35. $\dfrac{8d}{\sqrt{3}v}+4\pi\sqrt{\dfrac{m}{3k}}$

36. $T=2\pi\sqrt{\dfrac{\mu}{k}}$; where $\mu=\dfrac{m_1m_2}{m_1+m_2}$

37. $x_A = 0.12\sin(10t) + 0.8t$

38. (a) $x = 0.112\sin(10t + 0.46)\ m$

(b) $0.112\ m$, $0.111\ s$

(c) $t = 0.268\ s$

39. (a) 2 (b) $t=2\pi\sqrt{\dfrac{\ell}{g}}$

(c) $E=\dfrac{1}{2}ml\alpha^2 g$ if n is odd and $E=\dfrac{1}{2}ml\beta^2 g$ if n is even

40. $f=\dfrac{1}{2\pi}\sqrt{\dfrac{46\ k}{35\ m}}$; $A_1=\dfrac{21}{46}x_0$

41. (a) $a\cos\left(\dfrac{5\pi}{18}\right)$ (b) $\dfrac{5\pi}{9}\sqrt{\dfrac{m}{k}}$

42. $1.54\ s$

43. (a) 180°, No (b) 2 : 1

44. $T = 2.80\ s$

46. (a) $2\pi\left(\dfrac{g}{\ell}+\dfrac{k}{m}\right)^{-\frac{1}{2}}$ (b) $\pi\left[\left(\dfrac{g}{\ell}+\dfrac{k}{m}\right)^{-\frac{1}{2}}+\left(\dfrac{g}{\ell}\right)^{-\frac{1}{2}}\right]$

47. (a) $\dfrac{3}{25}mL^2$ (b) $\dfrac{3}{10}mgL\ \theta$

(c) $2\pi\sqrt{\dfrac{2L}{5g}}$

48. *Amplitude* = $\tan^{-1}(0.07) = 4°$; *frequency* = 0.46 *Hz*.

49. $\dfrac{20r^2}{L^2}\%$; 0.05%

50. $\theta_o\left(L+2r\right)\sqrt{\dfrac{2g\left(L+r\right)}{r^2+2\left(L+r\right)^2}}$

51. (i) (a) $\dfrac{3L}{8}$ (b) $2\pi\sqrt{\dfrac{3L}{8g}}$

(ii) $3\pi\sqrt{\dfrac{2a}{g}}$

52. (a) $2\pi\sqrt{\dfrac{4M+3m}{2K}}$;

(b) $2\pi\sqrt{\dfrac{2M+m}{K}}$.

53. $f=\dfrac{1}{2\pi}\sqrt{\dfrac{ae^{-n}n^{n-1}}{mb^{n-2}}}$

54. (a) $T=2\pi\sqrt{\dfrac{m_1m_2}{\left(m_1+m_2\right)k}}$

(b) $\dfrac{m_2L}{m_1+m_2}=A_1$

(c) $X_1 = V_0t + A_1(1 - \cos\omega t)$;

$X_2 = V_0t - A_2(1 - \cos\omega t)$

Where

$A_1 = \dfrac{m_2 L}{m_1+m_2}$; $A_2 = \dfrac{m_1 L}{m_1+m_2}$; $\omega = \sqrt{\dfrac{k(m_1+m_2)}{m_1 m_2}}$

55. (a) $\dfrac{2\pi}{3}\sqrt{\dfrac{\ell}{g}}$ (b) $2\sqrt{\dfrac{\ell}{g}}\,.\cos^{-1}\left(\dfrac{\sqrt{19}-1}{6}\right)$

56. $T_{av} = mg + \dfrac{1}{4}mg\,\theta_0^2$

57. $u = \sqrt{gR}$

58. $\dfrac{V}{\omega}$

59. $T = 2\pi\sqrt{\dfrac{177R}{10g}}$

60. (a) $\omega_a = \sqrt{\dfrac{k}{m}}$

(b) $\omega_b = \sqrt{\dfrac{3k}{m}}$

(c) ω_a and ω_b

61. $T = 2\pi\sqrt{\dfrac{L}{2\sqrt{3}g}}$

62. (ii) $\omega = \sqrt{\dfrac{k}{m}}$

SOLUTIONS

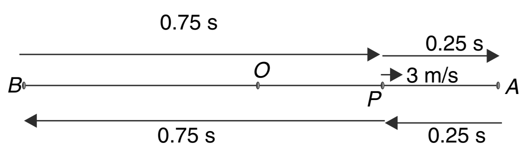
1. (i) The given equation has a standard solution given by

$a = a_0 \sin(\omega t + \delta)$. Where $\omega = \sqrt{b}$.

This is an equation of SHM.

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{b}}$

(ii) Careful observation of the data tells us that the time period of SHM = 2 s



If we consider $t = 0$ when the particle is at origin and travelling in positive direction, we can write the equation of motion as-

$x = A\sin\omega t \Rightarrow v = A\omega\cos\omega t$

Particle will reach at P when time is $t = 1/4\ s$

$\therefore 3\ m/s = A\dfrac{2\pi}{2}\cos\left(\dfrac{2\pi}{2}.\dfrac{1}{4}\right) \Rightarrow A = \dfrac{3\sqrt{2}}{\pi}$

$v_{\max} = A\omega = 3\sqrt{2}\ m/s$

$OP = A\sin\omega t = \dfrac{3\sqrt{2}}{\pi}\sin\left(\dfrac{\pi}{4}\right) = \dfrac{3}{\pi}\ m$

Now, it is easy to work out the ratio AP/PB.

2. (a) A careful observation of the given graphs reveals that the time period is same for both the particles. Amplitude of 1 and 2 are A and $2A$ respectively and particle 2 lags in phase by $\dfrac{\pi}{2}$.

Position – time equation for the two particles is –

$x_1 = A\sin\omega t$ and $x_2 = 2A\sin\left(\omega t - \dfrac{\pi}{2}\right) = -2A\cos\omega t$

Where $\omega = \frac{2\pi}{2t_0} = \frac{\pi}{t_0}$

We get velocity by differentiating the above two equations.

$v_1 = A\omega \cos \omega t = A\frac{\pi}{t_0}\cos\left(\frac{\pi}{t_0}t\right)$ and $v_2 = 2A\omega \sin \omega t = 2A\frac{\pi}{t_0}\sin\left(\frac{\pi}{t_0}t\right)$

(b) $\frac{E_1}{E_2} = \frac{\frac{1}{2}m_1\omega^2 A}{\frac{1}{2}m_2\omega^2 2A} = 1$

Hence, $\frac{m_1}{m_2} = \frac{2}{1}$

3. Motion is simple harmonic. The origin of the co-ordinate system is not the equilibrium position for the particle. In equilibrium force on the particle shall be zero. Thus $x = 2$ is the equilibrium position.

$$\frac{d^2x}{dt^2} = -\beta(x-2)$$

Let $(x - 2) = X$

Then $\frac{d^2x}{dt^2} = \frac{d^2X}{dt^2}$

Hence, $\frac{d^2X}{dt^2} = -\beta X$

$\therefore\ \omega^2 = \beta \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{1}{\beta}}$

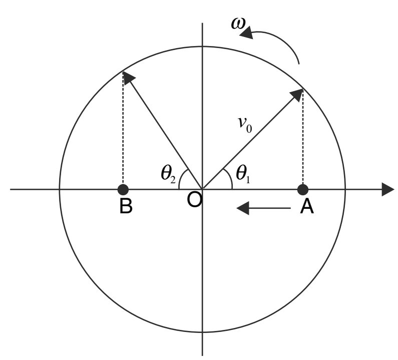
5. (a) The observer is moving in xy plane with his position vector changing with time as
$\vec{R} = (a \sin \omega t)\,\hat{i} + (a \cos \omega t)\,\hat{j}$
This is a circle with radius a. Angular speed of the observer is ω and his linear speed is $v = a\omega$.
He finds that the particle oscillates along z direction with angular frequency = ω_0.

(b) Time period of rotation of the particle is

$$T_0 = \frac{2\pi}{\omega_0}$$

$\therefore\ s = v.\,T_0 = 2\pi a\left(\frac{\omega}{\omega_0}\right)$

7. (i) $x = A \sin \omega t$
$v = v_0 \cos \omega t$



If we consider a vector of length v_0 rotating with angular speed ω then x co-ordinate of its tip gives the instantaneous velocity of the particle performing SHM.

From figure $OA = \frac{3}{5}v_0$

$\therefore \sin\theta_1 = \frac{3}{5}$; $\Rightarrow \theta_1 = 37^\circ$

$OB = \frac{4}{5}v_0$

$\therefore\ \sin\theta_2 = \frac{4}{5} \Rightarrow \theta_2 = 53^\circ$

At A speed is increasing and at B it is decreasing. In interval Δt the phasor rotates through 90°.

$\therefore\ \Delta t = \frac{T}{4}$

(ii) $\omega = \frac{2\pi}{T} = 2\ rad/s$

Angular frequency is property of the system and it does not change with change in energy. Let A be the new amplitude. Speed at a distance x from equilibrium is given by

$v = \omega\sqrt{A^2 - x^2}$

$\Rightarrow 1 = 2\sqrt{A^2 - 0.5^2}\ \Rightarrow A = \frac{1}{\sqrt{2}}\ m$

In the described position the particle is at $x = 0.5\ m$ travelling towards mean position. Since the time period has not changed the required time is time of travel from the positive extreme to $x = 0.5\ m$. This time can be calculated as-

$\frac{1}{2} = \frac{1}{\sqrt{2}}\cos(2t) \Rightarrow t = \frac{\pi}{8}\ s$

8. $\omega = \sqrt{\frac{k}{M}}$

For the block $V_{\max} = A\omega = A\sqrt{\frac{k}{M}}$

Speed of any point on the spring is proportional to its distance from the fixed end A.

$\therefore (V_P)_{\max} = \frac{V_{\max}}{4} = \frac{A}{4}\sqrt{\frac{k}{M}}$

9. Hint: The spring shall remain stretched – always.

10. The block will lose contact if the spring compresses more than $\frac{Mg}{k}$. In equilibrium position the spring is stretched by $\frac{mg}{k}$. If the spring oscillates with amplitude A, it will move up above its equilibrium position by A. It means compression in this extreme position will be $A - \frac{mg}{k}$. For the block to remain on the table –

$$A - \frac{Mg}{k} = \frac{mg}{k}$$

$$A = \frac{Mg}{k} + \frac{mg}{k}$$

11. $m\frac{d^2x}{dt^2} = -\mu mg$

$$\frac{d^2x}{dt^2} = -agx$$

This is equation of SHM whose time period is

$$T = 2\pi\sqrt{\frac{1}{ag}}$$

$\therefore$ The block will slide for a time

$$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{1}{ag}}$$

12. (a) As springs are in parallel

$$\therefore\ a = \frac{F_{\text{net}}}{\text{mass}} = \frac{(k_1 + k_2)x}{(m_1 + m_2)}$$

$$\omega^2 = \frac{k_1 + k_2}{m_1 + m_2} \quad \therefore\ T = 2\pi\sqrt{\frac{m_1 + m_2}{k_1 + k_2}}$$

(b) Frictional force on m_2 will act in direction of displacement if $k_2x > m_2\, a$

$$k_2x > m_2\frac{(k_1 + k_2)x}{(m_1 + m_2)} \quad \therefore \frac{m_1}{m_2} > \frac{k_1}{k_2}$$

(ii) Hint: The time period does not depend on amplitude or energy.

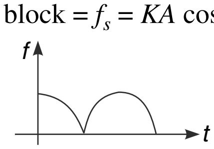
13. (a) The maximum friction force that can act on 10 kg block is $\mu mg = 0.5 \times 10 \times 10 = 50\ N$
Maximum spring force $= kx = 800 \times 0.01 = 8N$
The friction acting on 10 kg block is large enough to prevent its slipping. So it will not move.

(b) The 2 kg block will perform SHM. Equation of it motion will be –
$x = A \cos(\omega t)$

Where $\omega = \sqrt{\frac{K}{m}}$ and $A = 0.01\ m$

The instantaneous friction acting on the block $= f_s = KA\cos(\omega t)$

$\therefore\ |f_s| = KA|\cos\omega t|$



14. (a) The vertical and horizontal component of tension force is equal to mg and $m\omega^2 r$ respectively (ω is angular speed of rotation of the particle). Hence,

$F^2 = (mg)^2 + (m\omega^2 r)^2$

$$\Rightarrow \omega^2 r = \frac{1}{m}\sqrt{F^2 - (mg)^2}$$

$$\omega = \left(\frac{F^2 - (mg)^2}{m^2r^2}\right)^{1/4}$$

$$\therefore T = 2\pi\left(\frac{m^2r^2}{F^2-(mg)^2}\right)^{\frac{1}{4}}$$

This will be the maximum acceleration of the shadow as it will perform SHM with amplitude r and angular frequency ω.

(b) from the above equation

$$\omega = \left(\frac{F^2-(mg)^2}{m^2r^2}\right)^{1/4} \quad \therefore T = 2\pi\left(\frac{m^2r^2}{F^2-(mg)^2}\right)^{1/4}$$

15. When the ball has a small displacement $\boldsymbol{x}$ as shown in the figure, the wires will be inclined slightly to the vertical. The horizontal component of tension will be the net unbalanced force on the ball

$$ma = -2T\frac{x}{\ell}$$

Or, $a = -\dfrac{2T}{m}\dfrac{1}{\ell}x$

This is the equation of simple harmonic motion where

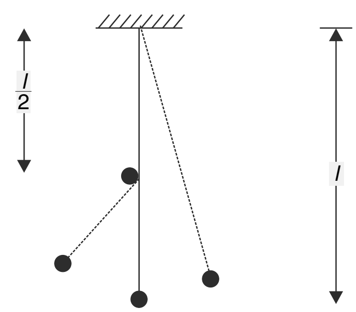
$$\omega^2 = \frac{2T}{ml}$$

Time period is

$$t = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m\ell}{2T}}$$

We see that the time period is inversely proportional to the square root of the tension. Thus by increasing the tension the time period of oscillation will decrease.

16.



$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T' = \frac{T}{2} + \pi\sqrt{\frac{l/2}{g}} = \frac{T}{2} + \frac{T}{2}\frac{1}{\sqrt{2}} = \frac{T}{2}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)$$

17. (i) Because $e \simeq 1$, the ball A will have a very small velocity after collision and ball B will have speed slightly less than that of A before collision.

Hence, A will move like a simple pendulum after collision with time period

$$T = 2\pi\sqrt{\frac{L}{g}} \simeq 2.0s.$$

$\therefore$ A will come to rest 0.5 s after collision.

It means time of fall for B is 0.5 s. Time of fall of B is independent of the horizontal speed acquired by it.

$$\therefore\ t = \sqrt{\frac{2h}{g}} \quad \therefore 0.5 = \sqrt{\frac{2h}{g}}$$

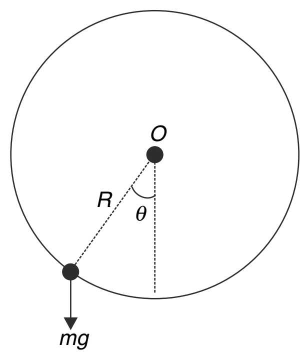
$\therefore\ h = 1.25$ m

(ii) Dimensional analysis tells us that

$T = k\sqrt{\frac{l}{g}}$ where k is a constant dependent on θ_0.

$$\therefore\ \frac{T}{T_0} = \sqrt{2}$$

18. In equilibrium the particle is at the lowest position. Consider the system at an angular position θ.



$\tau = mg\ R \sin\theta \simeq mg\ R\ \theta$ (for small θ)

$I\ \alpha = \tau$

$$\left[\frac{1}{2}(2m)R^2 + mR^2\right]\alpha = -mgR\theta$$

$$\therefore \alpha = -\left(\frac{g}{2R}\theta\right) \quad \therefore\ f = \frac{1}{2\pi}\sqrt{\frac{g}{2R}}$$

19. I_0 = MOI of the body about an axis through COM.

l = distance of point of suspension from the COM.

Time period of oscillation of a physical pendulum is

$T = 2\pi\sqrt{\frac{I}{mgl}}$ [I = MOI about rotation axis = $I_0 + ml^2$]

$$0.2\pi = 2\pi\sqrt{\frac{I_0 + ml^2}{mgl}}$$

$$\Rightarrow\ 0.01 = \frac{I_0 + ml^2}{mgl}$$

$I_0 + \mathrm{ml}^2 = 0.01$ mgl

$I_0 - 0.01\ mgl + ml^2 = 0$

The two solutions to this equation have sum of

$l_1 + l_2 = 0.01g = 0.1$

If $\ell = 0.4$ m then $l_2 = 0.6$ m

All points lying on two concentric circles around COM having radii 0.4 m and 0.6 m, are point of suspensions which will give $T = 0.2\pi$ s.

20. (a) Moment of inertia about the axis of oscillation is

$$I_A = \frac{2}{3}Ma^2$$

Distance of COM from point of suspension is

$$l = \frac{a}{\sqrt{2}}$$

$$\therefore\ T = 2\pi\sqrt{\frac{I_A}{mgl}}$$

$$\therefore T = 2\pi\sqrt{\frac{\frac{2}{3}Ma^2}{Mg\frac{a}{\sqrt{2}}}} = 2\pi\sqrt{\frac{2\sqrt{2}}{3}\frac{a}{g}}$$

(b) In this case

$$I_A = \frac{7}{12}Ma^2$$

$$\therefore\ T = 2\pi\sqrt{\frac{\frac{7}{12}Ma^2}{Mg\frac{a}{\sqrt{2}}}} = 2\pi\sqrt{\frac{7}{6\sqrt{2}}\frac{a}{g}}$$

21. (a) $x_A = a\sin(\omega t + \phi_1)$ and $x_B = a\sin(\omega t + \phi_2)$

$$x_A - x_B = a\sin(\omega t + \phi_1) - a\sin(\omega t + \phi_2) = 2a\sin\left(\omega t + \frac{\phi_1 + \phi_2}{2}\right)\cos\left(\frac{\phi_1 - \phi_2}{2}\right)$$

The distance between the two particles will be expressed by the modulus of the above expression.

Obviously, the sine term in the last expression can have a maximum value of 1, therefore maximum separation of a implies that

$$\cos\left(\frac{\phi_1 - \phi_2}{2}\right) = \frac{1}{2}$$

$$\left(\frac{\phi_1 - \phi_2}{2}\right) = \frac{\pi}{3} \Rightarrow \phi_1 - \phi_2 = \frac{2\pi}{3}$$

(b) Distance between the two particles is $= \left|2a\sin\left(\omega t + \frac{\phi_1 + \phi_2}{2}\right)\cos\left(\frac{\phi_1 - \phi_2}{2}\right)\right|$

This will complete two oscillation in the period when the sine term makes one complete oscillation. Hence, frequency of oscillation will be double the frequency of oscillation of the particles.

22. (i) $\omega = \frac{2\pi}{T}$ and amplitude $A = \sqrt{2}$ a

Equation of SHM along AB will be $r = \sqrt{2}a\cos\left(\frac{2\pi}{T}t\right)$

X co-ordinate as a function of time is

$$x = r\cos 45° = a\cos\left(\frac{2\pi}{T}t\right)$$

$$F_x = m\frac{d^2x}{dt^2} = -\frac{4\pi^2}{T^2}ma\cos\left(\frac{2\pi}{T}t\right)$$

(ii) The co-ordinates of the two particles as a function of time can be written as

$x = 3 - 2\cos\omega t$, and $y = 4 - 2\sin\omega t$,

Distance between them is $\ell = \sqrt{x^2 + y^2} = \sqrt{29 - 12\cos\omega t - 16\sin\omega t}$

Maximum and minimum values of $12\cos\omega t - 16\sin\omega t$ are 20 and –20 respectively.

Hence, maximum and minimum values of distance are 7 and 3 respectively.

23. Time period of oscillation $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

$\frac{t}{T} = \left(\frac{t}{2}\right)$ expresses the number of completed oscillations = n (an integer) + m (a fraction)

When $m = 0$, particle has made n number of complete oscillations and distance travelled is

$s = (4a)(n)$

(a) If m is a fraction less than 0.5, it means the particle is moving towards the negative extreme and its distance from the positive extreme is

$a - a\cos(\pi t)$

$\therefore$ for $m < 0.5$

$s = 4an + a(1 - \cos\pi t)$

(b) If $m > 0.5$, the particle has completed n and half oscillations and is travelling back towards the positive extreme.

$\therefore\ s = 4an + 2a + (a + y)$

$= 4an + 3a + a\cos\pi t$

$= 4an + a(3 + \cos\pi t)$

24. $\omega = \sqrt{\frac{k}{m}}$

$25 = \sqrt{\frac{k}{1}} \Rightarrow k = 625\ N/m$

Compression in the spring in equilibrium position is

$$x_0 = \frac{mg}{k} = \frac{1\times 10}{625} = \frac{2}{125}m = 1.6cm$$

One extreme position of A is the natural length position of the spring. Therefore, amplitude of oscillation is $A = 1.6\ cm$.

For SHM of block A

$x = x_0\cos(\omega t)$ [with equilibrium as origin and upward direction as positive]

$\therefore$ acceleration $a = \frac{d^2x}{dt^2} = -x_0\omega^2\cos(\omega t)$

For $(A + B)$ as system: [N = normal force by the weighing machine]

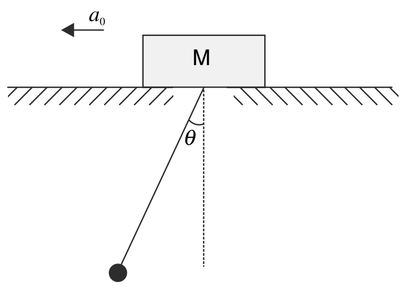
$N - (M + m)g = ma$

$N = 5g - mx_0\omega^2 \cos(\omega t)$

$$N = 50 - 1 \times \frac{2}{125} \times 625 \cos(625t)$$

$= 50 - 10 \cos(625t)$

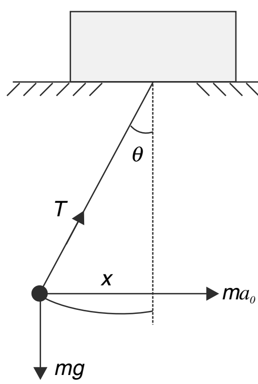
25. Consider the string at a small angle θ to the vertical.



If T is tension in the string,

$T \sin\theta = Ma_0$ (i)

Now, we will consider the motion of the pendulum in the reference frame attached to the block.



For small oscillations, the path of pendulum bob can be approximated to be a straight line and

$T = mg\cos\theta \simeq mg$

The restoring force $= mg\sin\theta + ma_0$

$$= mg\sin\theta + m\frac{T\sin\theta}{M}$$

$$= mg\sin\theta + \frac{m^2 g\sin\theta}{M}$$

$$\simeq mg\left[1 + \frac{m}{M}\right]\theta$$

$$m\frac{d^2x}{dt^2} = -mg\left[1 + \frac{m}{M}\right]\theta$$

$$\frac{d^2x}{dt^2} = -g\left[\frac{M+m}{M}\right]\frac{x}{L} \qquad \text{[SHM]}$$

$$\omega^2 = \frac{g}{L}\left(\frac{M+m}{M}\right)$$

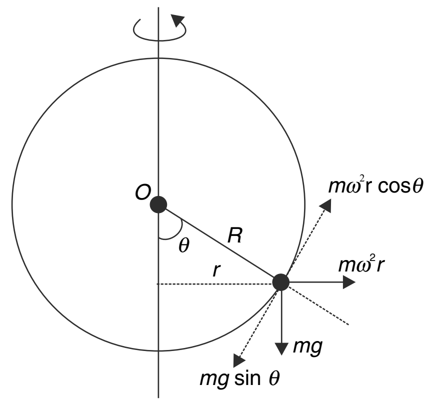
$$\therefore T = 2\pi\sqrt{\frac{LM}{g(M+m)}}$$

26. In reference frame of the wire, the equilibrium of bead gives

$m\omega^2 r \cos\theta = mg \sin\theta$ [Equilibrium along tangent]

$\Rightarrow \omega^2 r = g \tan\theta$ … (i)

If θ is increased by a small amount, say $\Delta\theta$, the tangential force [along upward tangent] also changes.



$F_t = m\omega^2 r \cos\theta - mg \sin\theta$

$$\Delta F_t = \frac{d}{d\theta}[m\,\omega^2 r \cos\theta - mg\sin\theta]\,\Delta\theta = [-m\,\omega^2 r\sin\theta - mg\cos\theta]\,\Delta\theta$$

$$\therefore\ m\frac{d^2(R\Delta\theta)}{dt^2} = [-m\omega^2 r\sin\theta - mg\cos\theta]\Delta\theta$$

$$\therefore\ \frac{d^2(\Delta\theta)}{dt^2} = \frac{1}{R}[-g\tan\theta.\sin\theta - g\cos\theta]\Delta\theta = -\frac{g}{R}\left[\frac{\sin^2\theta}{\cos\theta} + \cos\theta\right]\Delta\theta = -\left(\frac{g}{R\cos\theta}\right)\Delta\theta$$

Hence motion is SHM.

$$\omega^2 = \frac{g}{R\cos\theta} \Rightarrow T = 2\pi\sqrt{\frac{R\cos\theta}{g}}$$

27. Let x_0 be stretch in spring of force constant k_2 in equilibrium position. Then stretch in the other spring must be $2x_0$ so that tension is same in both of them.

$2kx_0 + k(2x_0) = Mg$

$\Rightarrow 4\,kx_0 = Mg$ … (i)

Let the block be displaced further by x causing the two springs to stretch further by x_1 and x_2 respectively.

$k_1x_1 = k_2x_2 \Rightarrow x_1 = 2x_2$

And $x_1 + x_2 = 2x$

$\therefore\ x_2 = \frac{2x}{3}$ and $x_1 = \frac{4x}{3}$

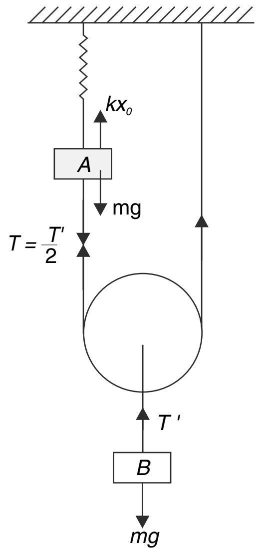
$$\therefore\ M\frac{d^2x}{dt^2} = Mg - k_1(x_1 + 2x_0) - k_2(x_2 + x_0)$$

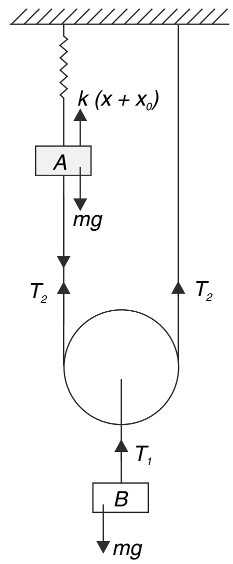
$$M\frac{d^2x}{dt^2} = -\frac{4}{3}kx - \frac{4kx}{3} \quad \text{[using (i)]}$$

$$\frac{d^2x}{dt^2} = -\frac{8k}{3M}.x \qquad \text{[SHM]}$$

$$\therefore \omega = \sqrt{\frac{8k}{3M}}$$

28. (i) Let the extension in spring be x_0 in equilibrium.





For B $T' = mg$

For A $kx_0 = mg + \frac{T'}{2} \Rightarrow kx_0 = \frac{3}{2}mg$ … (i)

Consider the block A in position that is displaced x from equilibrium. The corresponding displacement of B from its equilibrium position will be $\frac{x}{2}$.

For A $m\frac{d^2x}{dt^2} = T_2 + mg - k\left(x + x_0\right)$ … (ii)

For B $m\frac{d^2\left(x/2\right)}{dt^2} = mg - 2T_2$ [$\because T_1 = 2T_2$]

$\Rightarrow m\frac{d^2x}{dt^2} = 2mg - 4T_2$ … (iii)

Multiplying equation (ii) with 4 and adding to equation (iii)

$$5m\frac{d^2x}{dt^2} = 6mg - 4kx - 4kx_0$$

But from (i) $6mg = 4\,kx_0$

$$\therefore \frac{d^2x}{dt^2} = -\frac{4k}{5m}x$$

Comparing with $\frac{d^2x}{dt^2} = -\omega^2 x$

We get $\omega = \sqrt{\frac{4k}{5m}}$

$$\Rightarrow T = 2\pi\sqrt{\frac{5m}{4k}} = \pi\sqrt{\frac{5m}{k}}$$

(ii) In equilibrium the string tension (which is equal to the spring force) applies an upward force equal to weight of the hanging part of the rope. If the rope moves a distance x downward, the restoring force is equal to increase in spring force plus the decrease in weight of the hanging part.

$\therefore$ Restoring force = $kx + \lambda xg$

Mass of rope in motion = λL.

This mass changes a little as the rope oscillates but that can be neglected compared to λL.

$$\therefore \ \lambda L \frac{d^2x}{dt^2} = -(k + \lambda g)x$$

$$\therefore \ \frac{d^2x}{dt^2} = -\left(\frac{k}{\lambda L} + \frac{g}{L}\right)x$$

$$\therefore \omega = \sqrt{\frac{k}{\lambda L} + \frac{g}{L}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{1}{\frac{k}{\lambda L} + \frac{g}{L}}} = 2\pi\sqrt{\frac{\lambda L}{k + \lambda g}}$$

29. Originally, the system keeps the spring stretched by a length x_0 where -

$kx_0 = 2Mg$ … (i)

As soon as the particle gets detached, it begins to fall with acceleration g and the box begins to perform SHM with amplitude

$$A = \frac{2Mg}{k} - \frac{Mg}{k} = \frac{Mg}{k}$$

Distance travelled by the particle in time t is $s_1 = \frac{1}{2}gt^2$

Distance travelled (upward) by the floor in time t will be

$s_2 = A - A\cos(\omega t)$

Where $\omega = \sqrt{\frac{k}{M}}$

$$\therefore \ s_2 = \frac{Mg}{k} - \frac{Mg}{k}\cos\left(\sqrt{\frac{k}{M}}\,t\right)$$

$\therefore H = s_1 + s_2$

$$H = \frac{1}{2}gt^2 + \frac{Mg}{k} - \frac{Mg}{k}\cos\left(\sqrt{\frac{k}{M}}t\right) \qquad \text{……(ii)}$$

It is given that $t = \frac{T}{6\sqrt{2}}$ where $T = 2\pi\sqrt{\frac{2M}{k}}$

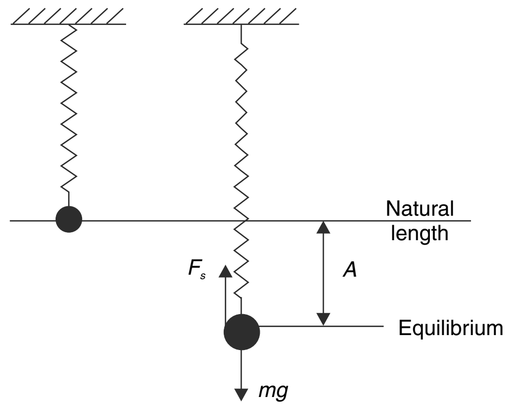
$$\therefore \ t = \frac{\pi}{3}\sqrt{\frac{M}{k}}$$

$$\therefore \ H = \frac{1}{2}g\frac{\pi^2}{9}\frac{M}{k} + \frac{Mg}{k} - \frac{Mg}{k}\cos\left(\frac{\pi}{3}\right) = \frac{Mg}{2k}\left[1 + \frac{\pi^2}{9}\right]$$

30. (a) The ball is released from a position where the spring is relaxed.

The mean position is $\frac{mg}{k}$ below the initial position.

$\therefore$ Ball will oscillate with amplitude $A = \frac{mg}{k}$



The spring will never get compressed as the upper extreme position of the block will be the natural length position of the spring.

Hence spring will never exert any upward force on the stand.

$\therefore$ Block will never lose contact with ground.

(b) In elastic collision the ball will not lose any *KE*. Hence, its upper extreme position does not change.

Hence, our answer remains same.

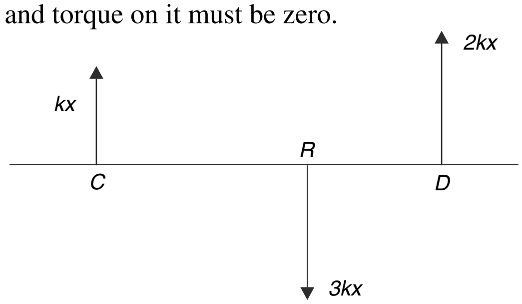
31. The time period for vertical oscillations of the block is $T = 2\pi\sqrt{\frac{m}{3k}}$

This means that the effective force constant is $3k$.

Spring *BD* must have a force constant of $2k$.

[Springs *AC* and *BD* are in parallel]

(a) Rod *PQ* is massless. Net force and torque on it must be zero.



$(RC)\,(kx) = (RD)\,(2kx)$

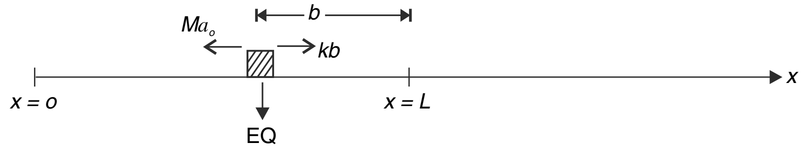
$\therefore \frac{RC}{RD} = \frac{2}{1}$

(b) Force constant of spring $\propto \frac{1}{\text{length of spring}}$

$\therefore$ length of *AC* must be double that of spring *BD*.

$\therefore$ Height difference between *A* and *B* will be *L*.

32. When the room moves with acceleration a_0, the equilibrium position of the block (In reference frame of the room) will be where the compressed spring balance the pseudo force ma_0.



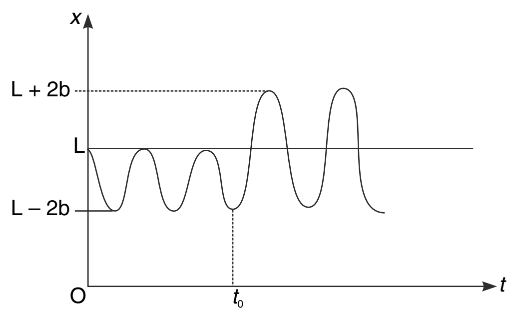
Let compression in equilibrium be b

$kb = Ma_0 \Rightarrow b = \dfrac{Ma_0}{k}$.

As the block was initially at X=L, it will start oscillating about $X = L–b$ with an amplitude equal to b.

At $t = t_0$ the block is at $x = L$-$2b$ when the constant force (Ma_0) acting on it vanishes. This does not change the frequency of oscillation but the equilibrium position now shifts to $X = L$. The block oscillates with an amplitude of $2b$ about $X = L$.

Hence, the $X – t$ graph will be as shown in fig below.



33. (a) Time required for a particle performing SHM to travel from $x = 0$ to $x = A/2$ is given by

$$\frac{A}{2} = A\sin\left(\frac{2\pi}{T}t\right)$$

$$\Rightarrow t = \frac{T}{12} \qquad [T = \text{time period of SHM}]$$

$\therefore$ Time period of oscillation of the block is

$$T_0 = 2\times\frac{T}{4} + 2\times\frac{T}{12} = \frac{2T}{3} = \frac{4\pi}{3}\sqrt{\frac{M}{k}} \qquad \ldots \text{(i)}$$

(b) Equation of motion of the block from the time of start to the time it hits the wall is

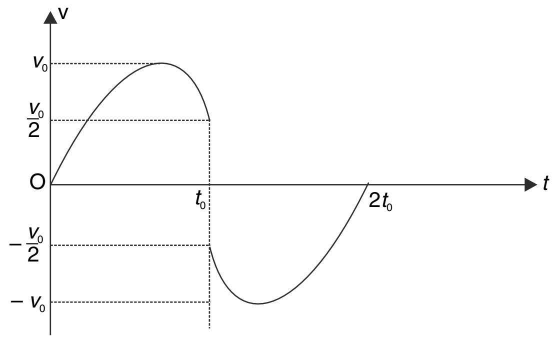
$$x = -A\cos\omega t \left[\omega = \sqrt{\frac{k}{M}}\right]$$

Velocity $v = \dfrac{dx}{dt} = A\omega\sin\omega t$

[for $t = 0$ to $t_0 = \dfrac{2\pi}{3}\sqrt{\dfrac{M}{k}}$]

After this the block hits the wall and direction of velocity changes suddenly.

Graph will be as shown in the figure.



(c) Speed of block just before hitting the wall is $v = \omega\sqrt{A^2 - x^2} = \omega\sqrt{A^2 - \left(\frac{A}{2}\right)^2} = \frac{\sqrt{3}}{2}A\omega$

F_0 = (change in momentum of block during collision) × (frequency of collisions)

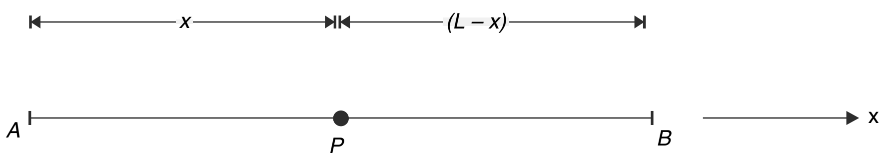
$$= 2.M\left(\frac{\sqrt{3}}{2}A\omega\right)\times\frac{1}{T_0} = \sqrt{3}MA\omega.\frac{3\omega}{4\pi} \qquad \text{[from (1)]}$$

$$= \frac{3\sqrt{3}}{4\pi}MA\frac{k}{M}$$

$$\therefore\ F_0 = \frac{3\sqrt{3}}{4\pi}Ak \qquad \text{[This is independent of } M\text{!!]}$$

$$\therefore\ k = \frac{4\pi F_0}{3\sqrt{3}A}$$

34. Let the x axis be along AB



Let $AP = x$ and $PB = L - x$

When the particle is at P

$$m\frac{d^2x}{dt^2} = -6\frac{mgx}{L}\hat{i} + \frac{3mg}{L}(L-x)\hat{i}$$

$$= \frac{3mg}{L}[L - x - 2x]\hat{i}$$

$$= -\frac{3mg}{L}[3x - L]\hat{i}$$

$$\frac{d^2x}{dt^2} = -\frac{3g}{L}[3x - L]\hat{i}$$

This is equation of SHM with equilibrium position at $x = \frac{L}{3}$, and $\omega = \sqrt{\frac{3g}{L}}$.

$$\therefore T = 2\pi\sqrt{\frac{L}{3g}}$$

At A; x = $-\frac{L}{3}$ and v = $\sqrt{gL}$

$\therefore\ v^2 = \omega^2\left[a^2 - \frac{L^2}{9}\right]$

$\Rightarrow\ gL = \frac{3g}{L}\left[a^2 - \frac{L^2}{9}\right]$

$\Rightarrow\ \frac{L^2}{3} + \frac{L^2}{9} = a^2$

$\Rightarrow\ a = \frac{2}{3}L$

It means B is extreme position of SHM; and distance of A from mean position is $\frac{a}{2}$.

Time needed to reach mean position from $x = \pm\frac{a}{2}$ is given by

$\frac{a}{2} = a\sin\omega t$

$\Rightarrow\ t = \frac{\pi}{6\omega} = \frac{T}{12}$

$\therefore$ time from A to B is $t = \frac{T}{12} + \frac{T}{4} = \frac{T}{3} = \frac{2\pi}{3}\sqrt{\frac{L}{3g}}$.

35. The prism will hit the spring to the right if it travels through a distance x_0 such that

$x_0 \cos 30^\circ = d \Rightarrow\ x_0 = \frac{2d}{\sqrt{3}}$

Time needed to cover this distance x_0 is $t_1 = \frac{2d}{\sqrt{3}v}$

If the prism moves further by x it compresses the spring by $x \cos 30^\circ = \frac{\sqrt{3}}{2}x$

$\therefore$ Spring force on it will be $= k\frac{\sqrt{3}}{2}x$ perpendicular to incline

Horizontal component of this force $= \left(\frac{\sqrt{3}}{2}kx\right)\cos 30^\circ = \frac{3}{4}kx$

The prism will perform half oscillation with the spring consuming a time of

$$t_2 = \pi\sqrt{\frac{m}{\frac{3k}{4}}} = 2\pi\sqrt{\frac{m}{3k}}$$

The prism is back to its original position moving in opposite direction at a time

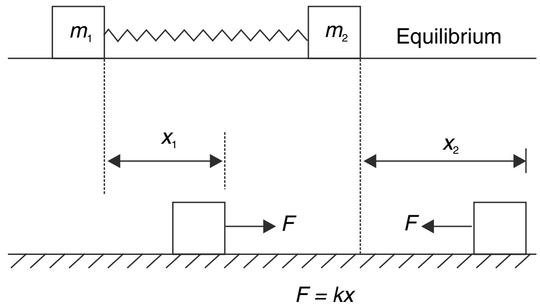
$t = t_1 + t_2 + t_1$

The same story is repeated in opposite direction with the other spring.

$\therefore\ T = 2t = 4t_1 + 2t_2$

$\quad = \frac{8d}{\sqrt{3}v} + 4\pi\sqrt{\frac{m}{3k}}$

36.



Let x_1 and x_2 be displacement of the two blocks at any point of time.

Extension in the spring is $x = x_2 - x_1$

For motion of m_1; $m_1 \dfrac{d^2x_1}{dt^2} = kx \Rightarrow \dfrac{d^2x_1}{dt^2} = \dfrac{k}{m_1}x$ ...(i)

For motion of m_2; $m_2 \dfrac{d^2x_2}{dt^2} = -kx \Rightarrow \dfrac{d^2x_2}{dt^2} = -\dfrac{k}{m_2}x$ ...(ii)

Equation (ii) – (i)

$$\frac{d^2(x_2 - x_1)}{dt^2} = -k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)x$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{\mu}x \qquad \left[\text{where } \mu = \frac{m_1m_2}{m_1 + m_2}\right]$$

$$\therefore \omega = \sqrt{\frac{k}{\mu}} \Rightarrow T = 2\pi\sqrt{\frac{\mu}{k}}$$

37. From the last problem we know that

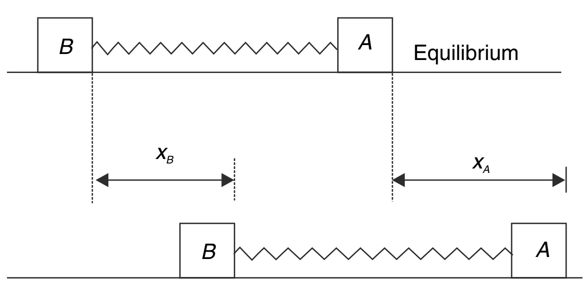
$$\frac{d^2x}{dt^2} = -\left(\frac{k}{\mu}\right)x \text{ where } \mu = \frac{3\times 2}{3+2} = 1.2kg$$

$$\Rightarrow \omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{120}{1.2}} = 10\ rads/s$$

$\therefore x = x_0 \sin(10t)$

Where $x = x_A - x_B$

$\therefore x_A - x_B = x_0 \sin(10t)$ .... (i)



The velocity of COM of the system is

$v_{cm} = \frac{2 \times 2 + 0}{2 + 3} = 0.8 m/s$ = a constant.

$\therefore$ Displacement of COM in time t is $x_{cm} = v_{cm}\, t = 0.8t$

But $x_{cm} = \frac{2x_A + 3x_B}{2 + 3}$

$\therefore \frac{2x_A + 3x_B}{5} = 0.8t$

$\Rightarrow 2x_A + 3x_B = 4t$ …. (ii)

Solving (i) and (ii)

$x_A = 0.6\, x_0 \sin 10t + 0.8t$ …. (iii)

Now, we need to find x_0 (maximum extension in the spring)

At the point of maximum extension, the velocity of two blocks will be equal to $v_{cm} = 0.8\ m/s$

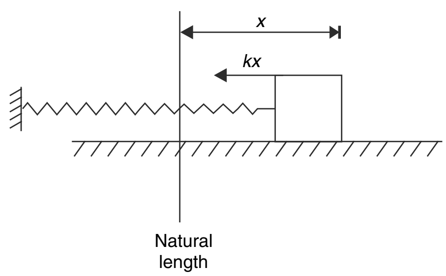
From energy conservation

$$\frac{1}{2} \times 2 \times 2^2 = \frac{1}{2} \times 120 \times x_0^2 + \frac{1}{2} \times 2 \times 0.8^2 + \frac{1}{2} \times 3 \times 0.8^2$$

$\Rightarrow x_0 = 0.2$

$\therefore$ From (iii) we get $x_A = 0.12 \sin(10t) + 0.8t$

38. Let extension be x at any time t.



Then $m\frac{d^2x}{dt^2} = -kx$

or, $2\frac{d^2x}{dt^2} = -200x$ or, $\frac{d^2x}{dt^2} = -100x$

$\therefore$ Motion is S.H.M. with $\omega = \sqrt{100} = 10\ rad/s$

$\therefore x = A \sin(\omega t + \delta) = A \sin(10t + \delta)$

And $v = \frac{dx}{dt} = A\omega \cos(\omega t + \delta) = 10A\cos(10t + \delta)$

At $t = 0$, $x = 0.05m$ $\therefore 0.05 = A \sin \delta$ … (a)

At $t = 0$, $v = 1\ m/s$ $\therefore$ $1 = 10\, A \cos \delta$ … (b)

From (a) and (b) $(0.05)^2 + (0.1)^2 = A^2$

or $A^2 = 0.0025 + 0.01 = 0.0125$ [you can also use $v^2 = \omega^2 [A^2 - x^2]$ to get A]

$\Rightarrow A = 0.112\ m$

From (a) $\sin\delta = \frac{0.05}{0.112} = 0.446$

$\delta = 26.5^\circ = 0.46$ radian

$\therefore x = 0.112 \sin(10t + 0.46)$

(b) Maximum extension = $A = 0.112\ m$

Time at maximum extension is given by

$0.112 = 0.112 \sin(10t + 0.46)$

$\Rightarrow 10t + 0.46 = \pi/2 = 1.57$

$\Rightarrow 10t = 1.11 \Rightarrow t = 0.111$ sec

(c) Speed is maximum when $v = |10A\cos(10t + \delta)|$ is maximum

i.e., $10t + \delta = \pi$

or, $10t = 3.14 - 0.46 = 2.68$

$\Rightarrow t = 0.268$ sec

39. (a) After first collision, B acquires amplitude of A and after second collision it acquires its original amplitude. Therefore, after 2 collisions B will once again go back to its original extreme position.

(b) Time period of both A and B is $T = 2\pi\sqrt{\frac{\ell}{g}}$

Time consumed before B reaches its original extreme for the first time is

$$t = \frac{T}{4} + \frac{T}{4} + \frac{T}{4} + \frac{T}{4} = T = 2\pi\sqrt{\frac{\ell}{g}}$$

(c) If n is odd , the energy of B will be equal to original energy of A.

$$E = \frac{1}{2}ma_A^2\omega^2 = \frac{1}{2}m(l\alpha)^2\frac{g}{l} = \frac{1}{2}ml\alpha^2 g$$

If n is even, the energy of B will be equal to original energy of B.

$$E = \frac{1}{2}ma_B^2\omega^2 = \frac{1}{2}m(l\beta)^2\frac{g}{l} = \frac{1}{2}ml\beta^2 g$$

40. Let x_1 & x_2 be the displacement of the two spheres from the equilibrium position. Let their respective speed in this position be v_1 & v_2

Compression in the spring = $(x_1 + x_2)$

For hollow sphere, applying $\tau_A = I_A\alpha$ about the point of contact

$$k(x_1 + x_2)r = \frac{5}{3}mr^2\alpha_1$$

$$k(x_1 + x_2) = \frac{5}{3}ma_1 \qquad \text{.........(1)}$$

Let's apply angular momentum conservation about point of contact of ball A for the entire system,

$$\frac{5}{3}mv_1 r = \frac{7}{5}mv_2 r$$

$$\Rightarrow 25\,v_1 = 21\,v_2 \qquad \text{.........(2)}$$

$$\Rightarrow 25\,x_1 = 21x_2 \qquad \text{.........(3)}$$

Using (1) and (3) we get, $k\left(x_1 + \frac{25}{21}x_1\right) = \frac{5}{3}\text{m}\,a_1$

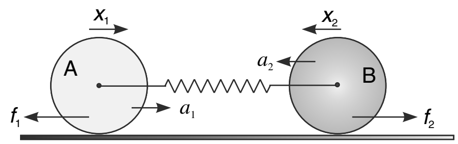
$$\Rightarrow a_1 = \frac{46}{35}\frac{k}{m}x_1 \qquad \text{(SHM)}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{46k}{35m}}$$

Now, sum of amplitudes of the two spheres is $A_1 + A_2 = x_0$ ...... (4)

From equation (3) we get $A_2 = \frac{25}{21} A_1$ .... (5).

By (4) and (5) we get, $A_1 = \frac{21}{46} x_0$

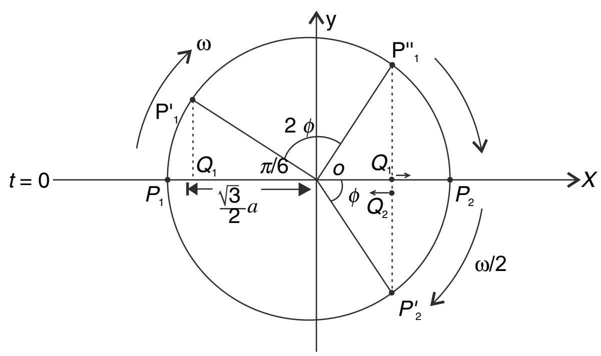


41. Till the point of collision, motion of both the blocks remain simple harmonic. For spring block system of mass m, the time period of SHM will be $2\pi\sqrt{\frac{m}{k}} = t(\text{say})$

Angular frequency $\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

For spring block system of mass $4m$, the corresponding time period is $2T$ [and angular frequency = $\frac{\omega}{2}$]

If a particle P performs uniform circular motion in x-y plane with constant angular speed ω, foot of perpendicular drawn from it on the X axis [or Y axis] performs SHM with angular frequency ω.



Let P_1" and P_2 be two points rotating on a circle of radius a in clockwise sense with angular speed ω and $\frac{\omega}{2}$ respectively. Motion of foot of perpendicular from P_1 on X axis represents the motion of mass m and the motion of foot of perpendicular from P_2 on the X axis represents the motion of mass $4m$.

According to the question, when P_1 reaches P_1' (i.e., block of mass m moves to Q_1 getting displaced by $a - \frac{\sqrt{3}}{2}a$) then P_2 starts rotating. Q_2 is foot of perpendicular of P_2 representing the motion of $4m$. The blocks collide when P_1 reaches P_1'' and P_2 reaches P_2' (see fig). Angle rotated by P_1 is twice that covered by P_2 due to its double angular speed.

$\therefore$ If $< P_2OP_2' = \phi$ then $<P_1'OP_1'' = 2\phi$

Then $< P_1''OP_2 = < P_2OP_2' = \phi$

But $< P_1''OP_2 = < P_2OP_2' = \phi$

$$\therefore \frac{\pi}{6} + 2\phi + \phi = \pi$$

$$\Rightarrow \phi = \frac{5\pi}{18}$$

$\therefore$ OQ_1 = distance from O where collision takes place = $a \cos \phi = a \cos \left(\dfrac{5\pi}{18}\right) = a\cos 50°$

Time required for collision = time required by P_2 to rotate through $\dfrac{5\pi}{18}$

$$= \frac{2T}{2\pi}\cdot\frac{5\pi}{18} = \frac{5T}{18} = \frac{5\pi}{9}\sqrt{\frac{m}{K}}$$

42. Time to travel from A to B (t_1) is given by –

$x = ut + \dfrac{1}{2}at^2$ $\quad$ [$\because$ $a = g \sin \theta = 5\ m/s^2$]

$0.3 = \dfrac{1}{2}\times 5\times t_1^2 \Rightarrow t_1 = 0.35s$

Speed of the block when it hits the spring is given by–

$V^2 = 0^2 + 2ax = 2 \times 5 \times 0.3$

$V = \sqrt{3}\ m/s$

The motion of the block in contact with the spring can be regarded as a part of SHM. In equilibrium, compression in the spring is given by

$kx_0 = Mg \sin \theta$

$$x_0 = \frac{40\times 10\times \frac{1}{2}}{1000} = 0.2m$$

Let the block compress the spring by a length x_1.

Energy conservation gives–

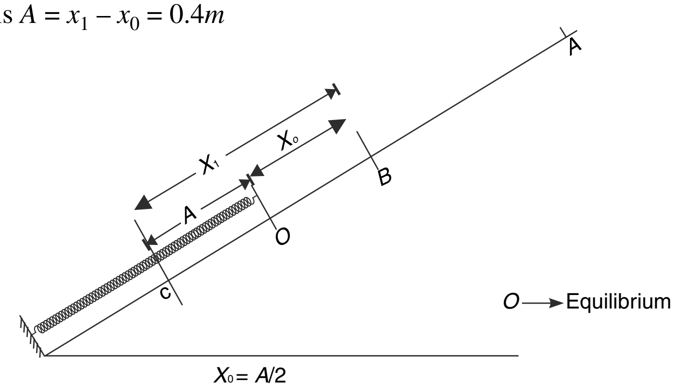
$$\frac{1}{2}kx_1^2 = \frac{1}{2}MV^2 + Mgx_1\sin\theta$$

$$\frac{1}{2}\times 1000.x_1^2 = \frac{1}{2}\times 40\times\left(\sqrt{3}\right)^2 + \left(40\times 10\times\frac{1}{2}\right)x_1$$

$5x_1^2 - 2x_1 - 0.6 = 0$

Solving $x_1 = 0.6\ m$

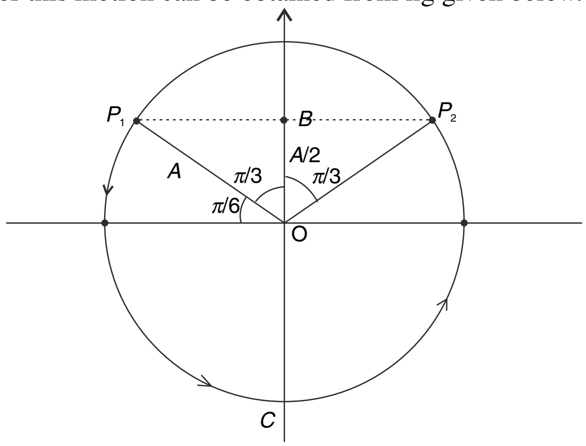
Hence, amplitude of SHM is $A = x_1 - x_0 = 0.4m$



The motion from B to C and back to B can be regarded as motion of a particle

performing SHM [from $x = +\dfrac{A}{2}$ to negative extreme and back to $x = +\dfrac{A}{2}$].

Time for this motion can be obtained from fig given below.



Particle on circle moves from P_1 to C to P_2. Time required for completing this two third circle $= \frac{2T}{3}$

Hence, desired time $t_2 = \frac{2T}{3} = \frac{2}{3}2\pi\sqrt{\frac{m}{k}}$

$= \frac{4\pi}{3}\sqrt{\frac{40}{1000}} = \frac{8\pi}{30}s = 0.84\,s$

$\therefore$ Time required for the block to come back to A is $t = 2t_1 + t_2$

$= 2 \times 0.35 + 0.84 = 1.54\ s$

43. At the instant shown, both particle are at their mean position and moving in opposite direction.
Phase difference = 180°

As ω is same for both particle $\left\{\omega = \sqrt{\frac{GM}{R^3}}\right\}$ the phase difference will be maintained throughout and they will never meet.

$v_{\max} = A\omega$

$\therefore\ \frac{v_1}{v_2} = \frac{R}{R/2} = \frac{2}{1}$

44. If V_0 is the speed at the mean position

$\frac{1}{2}mV_0^2 = 20\times10^{-3}$

$\frac{1}{2}\times0.2\times V_0^2 = 20 \times 10^{-3} \Rightarrow V_0 = \sqrt{0.2}\ m/s$

If linear amplitude is A then $V_0 = A\omega = \sqrt{0.2}$

$L\theta_0\sqrt{\frac{g}{L}} = \sqrt{0.2}$ [$\because A = L\theta_0$ where θ_0 = angular amplitude]

$\theta_0\sqrt{gL} = \sqrt{0.2}$

$L = \frac{0.2}{10\times\left(100\times10^{-3}\right)^2} = 2.0\ m$

$\therefore\ T = 2\pi\sqrt{\frac{L}{g}} = 2\times3.14\sqrt{\frac{2}{10}} = 2.80\ s$

45. There is no loss of energy during collision because the collisions are elastic. Ball A stops and B acquires speed of A. Let the length of pendulum B be ℓ.

$mg\,L\,(1 - \cos\theta_0) = mg\,\ell\,(1 - \cos 2\theta_0)$

$$L.2\sin^2\left(\frac{\theta_0}{2}\right) = \ell.2\sin^2\theta_0$$

$$L\left(\frac{\theta_0}{2}\right)^2 \simeq \ell(\theta_0)^2 \qquad [\text{since } \sin\theta_0 \simeq \theta_0]$$

A completes $\frac{1}{4}$ of its oscillation and comes to rest after hitting B. Then B completes half of its oscillation and hits A. B comes to rest and A goes back to its starting point. This completes one oscillation of A

$$\therefore\ T_A = \frac{1}{2}2\pi\sqrt{\frac{L}{g}} + \frac{1}{2}2\pi\sqrt{\frac{\ell}{g}}$$

$$= \frac{\pi}{\sqrt{g}}\left[\sqrt{L} + \sqrt{\frac{L}{4}}\right] = \frac{3\pi}{2}\sqrt{\frac{L}{g}}$$

46. (a) When the bob displaces by an angle θ (small), restoring torque is

$\tau = (mg\sin\theta)\,\ell + kx.\,\ell$

$= mg\,\ell\,\theta + k\ell^2\theta\ [\because x = \ell\,\theta]$

$$\therefore\ ml^2.\frac{d^2\theta}{dt^2} = -\left(mgl + kl^2\right)\theta$$

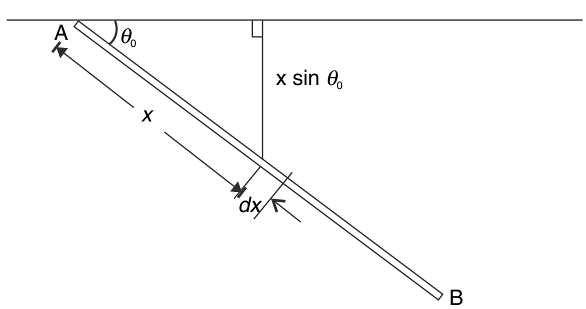
$$\frac{d^2\theta}{dt^2} = -\left(\frac{g}{\ell} + \frac{k}{m}\right)\theta \qquad [\text{SHM}]$$

$$\omega = \sqrt{\frac{g}{\ell} + \frac{k}{m}} \Rightarrow T = 2\pi\left(\frac{g}{\ell} + \frac{k}{m}\right)^{-\frac{1}{2}}$$

(b) In this case the cord is similar to the spring when it is stretched but it will exert no force during half the motion when it is loose.

$$\therefore\ T' = \frac{T}{2} + \pi\sqrt{\frac{\ell}{g}} = \pi\left[\left(\frac{g}{\ell} + \frac{k}{m}\right)^{-\frac{1}{2}} + \left(\frac{g}{\ell}\right)^{-\frac{1}{2}}\right]$$

47. (a) $\Delta\,ABP$ is right angled at B.



$$\sin\theta_0 = \frac{0.75}{1.25} = \frac{3}{5}$$

$$I = \int_0^L \left(\frac{m}{L}dx\right)(x\sin\theta_0)^2 = \frac{mL^2}{3}\cdot\sin^2\theta_0 = \frac{mL^2}{3}\times\frac{9}{25} = \frac{3}{25}mL^2$$

(b) $\tau = mg\frac{L}{2}\sin\theta_0 . \sin\theta \simeq \left(mg\frac{L}{2}\sin\theta_0\right)\theta$ [for small θ]

$$= \frac{3}{10}mgL\,\theta$$

(c) $I\alpha = -\frac{3}{10}mgL\,\theta$

$$\frac{3}{25}mL^2 . \alpha - \frac{3}{10}mgL\theta \Rightarrow \alpha = -\left(\frac{5}{2}\frac{g}{L}\right)\theta$$

$$\therefore T = 2\pi\sqrt{\frac{2L}{5g}}$$

48. $V = \omega R$

$$\therefore \omega = \frac{8.33}{100} = \frac{1}{12}\ rad/\text{s}$$

Let m = mass of petrol

Moment of inertial of petrol about an axis through O is $I = \frac{1}{2}mr^2$

where $2r = 2m \Rightarrow r = 1m$

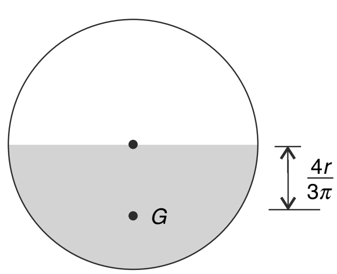
When the wagon is driven on curved track θ is such that

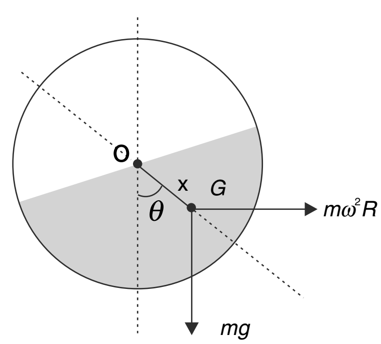
$mg\,x\sin\theta = m\omega^2 x\cos\theta$ $\left[\text{where } x = \frac{4r}{3\pi}\right]$

$$\tan\theta = \frac{\omega^2 R}{g} = \left(\frac{1}{12}\right)^2\times\frac{100}{9.8} = 0.07$$

$\therefore\ \theta = \tan^{-1}(0.07) = 4°$

When the wagon enters the straight segment of track, liquid oscillates about an axis through O with an amplitude of 4°.





$I\alpha = -mg\,x\sin\theta$

$$\Rightarrow \frac{1}{2}mr^2\frac{d^2\theta}{dt^2} = -mg\,x\sin\theta$$

Since, θ is small

$$\frac{d^2\theta}{dt^2} = -\frac{2g}{r^2}\left(\frac{4r}{3\pi}\right)\theta \qquad \left[\because x = \frac{4r}{3\pi}\right]$$

$$= -\frac{8g}{3\pi r}\theta$$

$$\therefore \omega = \sqrt{\frac{8g}{3\pi r}} \quad ; \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{8g}{3\pi r}} = \frac{1}{2\times 3.14}\sqrt{\frac{8\times 9.8}{3\times 3.14\times 1}}$$

$$= \frac{2.88}{2\times 3.14} = 0.46\ Hz$$

49. For simple pendulum $T_0 = 2\pi\sqrt{\frac{L}{g}}$

For compound pendulum $T = 2\pi\sqrt{\frac{I}{mgL}}$

$$\therefore T = 2\pi\sqrt{\frac{\frac{2}{5}mr^2 + mL^2}{mgL}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}\left[1+\frac{2}{5}\frac{r^2}{L^2}\right]^{1/2}$$

Since $\frac{2r^2}{5L^2} << 1$ therefore, we can use $(1 + x)^n = 1 + nx$

$$\therefore T = T_0\left[1+\frac{1}{2}\ \frac{2}{5}\ \frac{r^2}{L^2}\right] \Rightarrow T = T_0\left[1+\frac{r^2}{5L^2}\right]$$

$$\therefore \frac{T-T_0}{T_0} = \frac{r^2}{5L^2}$$

$$\frac{\Delta T}{T_0}\times 100 = \frac{20r^2}{L^2} = \frac{20\times 5^2}{100^2} = \frac{20\times 25}{10000} = 0.05\%$$

50. This is a compound pendulum with angular

frequency $\omega = \sqrt{\frac{mgl}{I}} = \sqrt{\frac{mg(L+r)}{\frac{1}{2}mr^2 + m(L+r)^2}} = \sqrt{\frac{2g(L+r)}{r^2 + 2(L+r)^2}}$

Angular displacement can be expressed as a function of time $\theta = \theta_0 \sin(\omega t)$

$$\therefore \frac{d\theta}{dt} = \theta_0 \cos(\omega t)$$

When the string is vertical $\theta = 0$

$\therefore \sin(\omega t) = 0$ and $\cos(\omega t) = 1$

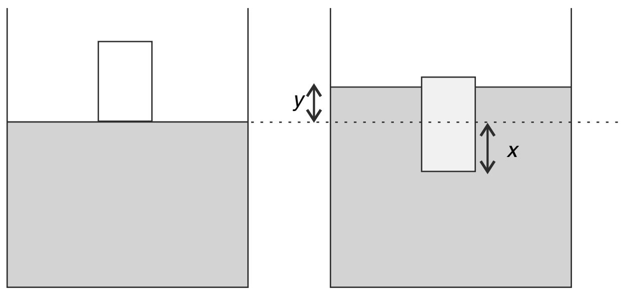
$\therefore \quad \dfrac{d\theta}{dt} = \theta_0\omega$

$\therefore$ speed of lowest point

$V = (L + 2r)\dfrac{d\theta}{dt}$ = (L + 2r) $\theta_0\omega$

$= \theta_o(L+2r)\sqrt{\dfrac{2g(L+r)}{r^2+2(L+r)^2}}$

51. (i) (a)



When the cylinder moves down by x, the liquid level rises by y.

Since volume of liquid is constant

$3\,Ay = Ax \Rightarrow \quad y = \dfrac{x}{3}$

$\therefore$ Buoyancy force on cylinder $= \left(\dfrac{4x}{3}A\right)(2\rho)g$

In equilibrium $\dfrac{8x_0}{3}A\rho g = AL\rho g$

$\therefore x_0 = \dfrac{3L}{8}$

Since, equilibrium position is x_0 below the original position (where, cylinder was at rest), hence amplitude is $A = x_0 = \dfrac{3L}{8}$

(b) When the cylinder is displaced x from its equilibrium position, restoring force on it will be the extra buoyancy force which is

$= \left(A\dfrac{4x}{3}\right)(2\rho)g$

$\therefore m\dfrac{d^2x}{dt^2} = -\left(\dfrac{8}{3}A\rho g\right)x$

$AL\rho g\dfrac{d^2x}{dl^2} = -\dfrac{8}{3}A\rho g.x$

$\therefore \dfrac{d^2x}{dt^2} = -\left(\dfrac{8}{3}\dfrac{g}{L}\right)x$

$\therefore \omega = \sqrt{\dfrac{8}{3}\dfrac{g}{L}} \Rightarrow T = 2\pi\sqrt{\dfrac{3L}{8g}}$

(ii) In equilibrium $F_B = W$

$\rho_\omega g\,(2a)^3 = \rho g\,[(2a)^3 + a^3]$

$$\Rightarrow \frac{\rho}{\rho_\omega} = \frac{8}{9}$$

When the block is depressed by x from its equilibrium position, the excess buoyancy is the restoring force.

Restoring force = $(a^2x)\,\rho_\omega g$

$$\therefore\ (9a^3)\rho\cdot\frac{d^2x}{dt^2} = -(a^2\rho_\omega g)x$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\left(\frac{g}{8a}\right)x \qquad \left[\because \frac{\rho}{\rho_\omega} = \frac{8}{9}\right]$$

$$\therefore\ \omega_1 = \sqrt{\frac{g}{8a}}$$

Time for half oscillation $t_1 = \pi\sqrt{\frac{8a}{g}}$

When the block is raised above its mean position, the restoring force is = $(2a^2\,\rho_\omega g)x$

$$\therefore\ 9a^3\rho\,\frac{d^2x}{dt^2} = -4a^2\rho_\omega g\,x$$

$$\therefore\ \frac{d^2x}{dt^2} = -\left(\frac{g}{2a}\right)x$$

$$\therefore\ \omega_2 = \sqrt{\frac{g}{2a}}$$

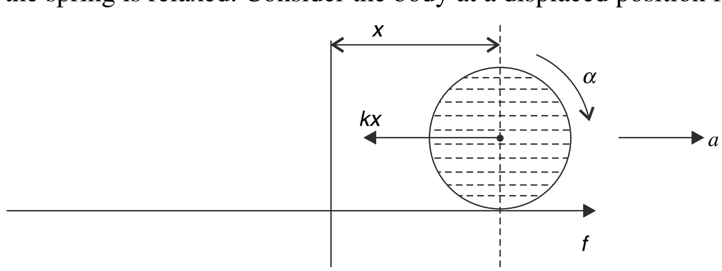
Time for half oscillation $t_2 = \pi\sqrt{\frac{2a}{g}}$

$$\therefore\ T = t_1 + t_2 = \pi\sqrt{\frac{2a}{g}}\,[2+1] = 3\pi\sqrt{\frac{2a}{g}}$$

52. (a) Ice and cylinder together move as a single rigid body. Moment of inertia about axis of the cylinder is -

$$I = MR^2 + \frac{1}{2}mR^2 = \left(M + \frac{m}{2}\right)R^2$$

In equilibrium, the spring is relaxed. Consider the body at a displaced position from equilibrium.



$f - kx = (M + m)a$ ……………….(1)

$-f\,R = I\,\alpha$

$$-fR = \left(M+\frac{m}{2}\right)R^2\alpha$$

$$-f = \left(M+\frac{m}{2}\right)a \qquad \text{.........(2)} \ (\because R\alpha = a)$$

Add (1) and (2)

$$\left(2M+\frac{3m}{2}\right)a = -kx$$

$$a = -\left(\frac{2k}{4M+3m}\right)x \qquad [\text{SHM}]$$

$$\omega = \sqrt{\frac{2k}{4M+3m}} \text{ hence , } T = 2\pi\sqrt{\frac{4M+3m}{2k}}$$

(b) Non viscous water will not rotate with the cylinder. It will only perform translational motion.

Equation (1) remains same as above.

Equation (2) changes as -

$-fR = MR^2.\ \alpha$

$-f = Ma$ ....(3)

Adding (1) and (3)

$$a = -\left(\frac{k}{2M+m}\right)x \quad \therefore T = 2\pi\sqrt{\frac{2M+m}{k}}$$

53. $F = -\dfrac{dU}{dx} = ae^{-bx}\ x^{n-1}\left(n-bx\right)$

At equilibrium position $F = 0 \Rightarrow bx = n \Rightarrow x_0 = \dfrac{n}{b}$

If the particle is displaced a little (say Δx) from its equilibrium position, the force that it will experience is calculated as follows

$$\frac{dF}{dx} = -ab\,e^{-bx}\ x^{n-1}\left(n-bx\right)+e^{-bx}\left[-bx^{n-1}+\left(n-bx\right)\left(n-1\right)x^{n-2}\right]$$

$= -ax^{n-2}\ e^{-bx}\ [(n-bx)\ (bx-n+1)+bx]$

Put $x_0 = \dfrac{n}{b}$

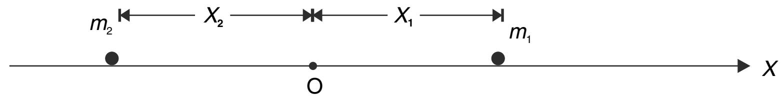
$$\left(\frac{dF}{dx}\right)_{at\,x_0} = -a\left(\frac{n}{b}\right)^{n-2} e^{-n}\ n = -an\,e^{-n}\left(\frac{n}{b}\right)^{n-2} = -a\,e^{-n}\ \frac{n^{n-1}}{b^{n-2}}$$

$$\therefore \Delta F = \left(\frac{dF}{dx}\right)\Delta x = -\left(a\,e^{-n}\ \frac{n^{n-1}}{b^{n-2}}\right)\Delta x$$

Negative sign indicates that the force is restoring. It is proportional to displacement (Δx). Hence motion is SHM.

$$\omega^2 = \frac{a\,e^{-n}\ n^{n-1}}{m\,b^{n-2}} \text{ and } f = \frac{1}{2\pi}\sqrt{\frac{a\,e^{-n}\ n^{n-1}}{m\,b^{n-2}}}$$

54. Let us work in the reference frame attached to COM of the system. It is an inertial frame moving with constant velocity V_0.

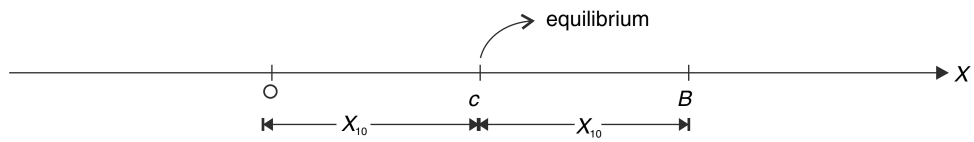


If X_1 (towards tight) and X_2 (to left) are displacement of the two masses at time 't' then.

$m_1X_1 = m_2X_2$

Equilibrium position means $X_1 + X_2 = L$

$\therefore$ In equilibrium, x co-ordinate of m_1 is $X_{10} = \left(\dfrac{m_2}{m_1+m_2}\right)L$



In the reference frame of COM, mass m_1 will oscillate between points O and B with amplitude equal to $A_1 = X_{10} = \dfrac{m_2L}{m_1+m_2}$

Equation of motion for m_1

$m_1a_1 = k\,[L - (X_1 + X_2)]$

$$m_1a_1 = k\left[L - \left(X_1 + \frac{m_1}{m_2}X_1\right)\right] \qquad [\because m_2X_2 = m_1X_1]$$

$$\therefore\ a_1 = \frac{KL}{m_1} - \frac{K}{\mu}X_1 \qquad \left[\mu = \frac{m_1m_2}{m_1+m_2}\right]$$

This is equation of SHM with equilibrium at $X_{10} = \dfrac{\mu L}{m_1}$ and angular frequency

$$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{k\left(\frac{m_1+m_2}{m_1m_2}\right)} \Rightarrow T = 2\pi\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}$$

At $t = 0$, mass m_1 is at negative extreme of the SHM and the origin of co-ordinate system is at the negative extreme itself. Hence, displacement of m_1 is

$X_1 = A_1 - A_1\cos\omega t$ [In COM frame]

In ground frame $X_1 = V_0t + A_1(1 - \cos\omega t)$

[$\because$ COM will travel V_0t distance in time t]

In COM frame $X_2 = \dfrac{m_1X_1}{m_2} = \dfrac{m_1}{m_2}A_1(1-\cos\omega t)$

$X_2 = A_2(1 - \cos\omega t)$

Where $A_2 = \dfrac{m_1A_1}{m_2} = \dfrac{m_1L}{m_1+m_2}$

In ground frame $X_2 = V_0t - A_2(1 - \cos\omega t)$

55. (a) The time period of B $T = 2\pi\sqrt{\frac{\ell}{g}}$

Time period of $A = 2\pi\sqrt{\frac{4\ell}{g}} = 2T$

The phasor representing B will rotate with an angular speed twice that of phasor representing A.

Let $\omega_A = \omega$; then $\omega_B = 2\omega$

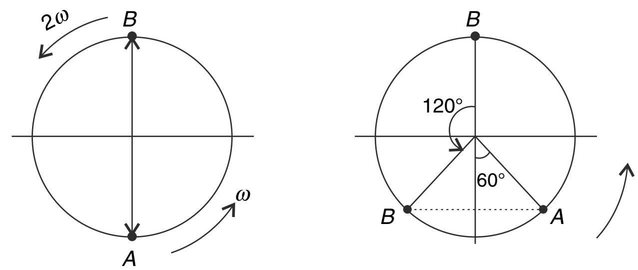
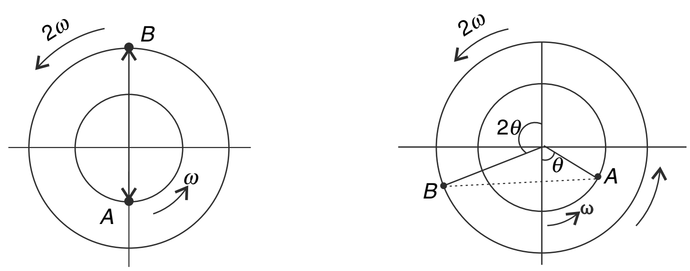


Fig (A) represent the initial positions of the phasors representing A and B. Fig (B) represent their position when the two pendulums are parallel and crossing each other.

Obviously, time required for this is the interval in which the phasor representing A (time period = $2T$) rotates by $\frac{\pi}{3}$

$$\therefore\ t = \frac{2T}{6} = \frac{T}{3} = \frac{2\pi}{3}\sqrt{\frac{\ell}{g}}$$

(b) Amplitude of B is twice that of A. The initial phasors and the phasors when strings are parallel has been shown below in figure (a) and (b) respectively.



Radius of the outer circle is 1.5 times that of the inner circle

$R \cos\theta = 1.5\, R \cos(180° - 2\theta)$

$\Rightarrow 2\cos\theta = -3\cos 2\theta \Rightarrow 2\cos\theta = -3\,(2\cos^2\theta - 1)$

$\Rightarrow 6\cos^2\theta + 2\cos\theta - 3 = 0$

$$\therefore\ \cos\theta = \frac{\sqrt{76} - 2}{12} = \frac{\sqrt{19} - 1}{6}$$

It can be seen from simple observation that θ is acute and $\cos\theta$ will be positive

$$\therefore\ \theta = \cos^{-1}\left(\frac{\sqrt{19} - 1}{6}\right)$$

$$\therefore\ t = \frac{\theta}{\omega} = \frac{\theta}{\frac{2\pi}{T_A}} = \frac{\theta}{2\pi}\,4\pi\sqrt{\frac{\ell}{g}} = 2\theta\sqrt{\frac{\ell}{g}}$$

56. let the pendulum be at its positive extreme at $t = 0$

$$\theta = \theta_0 \cos(\omega t)\ \left[\text{where } \omega = \sqrt{\frac{g}{\ell}}\right]$$

Angular velocity at time 't' is $\frac{d\theta}{dt} = -\,\theta_0\omega\sin(\omega t)$

$\therefore$ velocity of the bob $V = \theta_0\,\omega\ell\,\sin(\omega t)$

Tension (T) is given by–

$$T = mg\cos\theta + \frac{mV^2}{\ell}$$

$$T = mg\cos\theta + m\theta_0^2\,\omega^2\ell\,\sin^2\omega t = mg\left[1-\frac{\theta^2}{2}\right] + m\theta_0^2\,\omega^2\ell\,\sin^2\omega t$$

$$= mg - \frac{1}{2}\,mg\theta_0^2\cos^2\omega t + m\theta_0^2\,g\,\sin^2\omega t$$

$$\langle\cos^2\omega t\rangle = \langle\sin^2\omega t\rangle = \frac{1}{2}$$

$$\therefore\ T_{av} = mg + \frac{1}{4}\,mg\,\theta_0^2$$

57. When the stone is at a distance x from the centre of the earth, gravitational force on it is

$$F_g = -\frac{GMm}{R^3}\cdot x \qquad \text{[– sign indicates force is towards the Centre]}$$

$$\therefore\ \frac{d^2x}{dt^2} = -\frac{GM}{R^3}\cdot x \qquad \text{.........(1)}$$

$\therefore$ Stone performs SHM with

$$\omega^2 = \frac{GM}{R^3} \Rightarrow \omega = \sqrt{\frac{GM}{R^3}} \Rightarrow T = 2\pi\sqrt{\frac{R^3}{GM}}$$

If the stone is dropped into the hole, it performs SHM of amplitude R and the journey from one end to the other will take half the time T.

$$\therefore T_0 = \frac{T}{2} = \pi\sqrt{\frac{R^3}{GM}}$$

The general solution to equation (1) is $X = A\cos(\omega t + \delta)$

at $t = 0$; $X = R\ \therefore\ A\cos\delta = R$ ----------------(2)

Also, $V = -A\omega\sin(\omega t + \delta)$

at $t = 0$; $V = -u$

$\therefore -u = -A\omega\sin\delta \Rightarrow A\sin\delta = \frac{u}{\omega}$ ------------------(3)

From (2) and (3) $A = \sqrt{R^2 + \frac{u^2}{\omega^2}}$ and $\tan\delta = \frac{u}{\omega R}$

If the stone has to reach $X = -R$ at $t = \frac{T_0}{2}$ then,

$X = A \cos(\omega t + \delta)$

$$-R = \left(\sqrt{R^2 + \frac{u^2}{\omega^2}}\right)\cos\left(\frac{\pi}{2} + \delta\right)$$

$$\left[\because \omega t = \sqrt{\frac{GM}{R^3}} \cdot \frac{T_0}{2} = \sqrt{\frac{GM}{R^3}} \cdot \frac{\pi}{2}\sqrt{\frac{R^3}{GM}} = \frac{\pi}{2}\right]$$

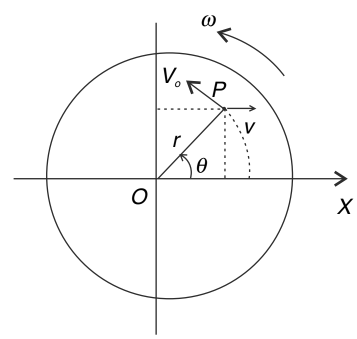
$$\therefore \sin\delta = \frac{R}{\sqrt{R^2 + \frac{u^2}{\omega^2}}} \qquad \text{-------------------(4)}$$

From (2) $$\cos\delta = \frac{R}{\sqrt{R^2 + \frac{u^2}{\omega^2}}} \qquad \text{-------------------(5)}$$

Squaring and adding (4) and (5) gives

$$\frac{2R^2}{R^2 + \frac{u^2}{\omega^2}} = 1 \qquad \Rightarrow R^2 = \frac{u^2}{\omega^2} \Rightarrow u = R\omega = R\sqrt{\frac{g}{R}} = \sqrt{gR}$$

58. Let the position of the man at time 't' be at P (see fig).



$\theta = \omega t$; $V_0 = \omega r$

$V_x = V - V_0 \sin(\omega t) = V - \omega r \sin(\omega t)$

$\therefore V_x = V - \omega y$ ------------------(1)

And $V_y = V_0 \cos(\omega t) = \omega r \cos(\omega t) = \omega x$ ------------------(2)

Differentiating (1) ωrt time

$$\frac{dV_x}{dt} = -\omega\frac{dy}{dt} \text{ or, } \frac{d^2x}{dt^2} = -\omega V_y$$

Using (2) $$\frac{d^2x}{dt^2} = -\omega^2 x$$

Solution of this equation is $x = A \sin(\omega t + \delta)$

As per the question, at $t = 0$, $x = 0$

$\therefore \delta = 0 \therefore x = A \sin(\omega t)$ and $\frac{dx}{dt} = a\omega\cos(\omega t)$

at $t = 0$, $V_x = V$

$$\therefore \quad A = \frac{V}{\omega}$$

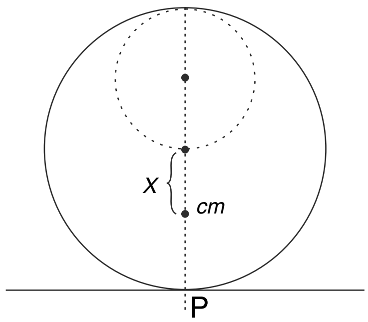
$\therefore$ Maximum X co-ordinate is $\frac{V}{\omega}$

59. Location of centre of mass of the cavitied sphere is given by -

$$\frac{4}{3}\pi\left(R^3 - \frac{R^3}{8}\right)\rho \ x = \frac{4}{3}\pi\frac{R^3}{8}\rho\frac{R}{2}$$

$$\Rightarrow \frac{7}{8}x = \frac{R}{16} \qquad [\rho = \text{density}]$$

$$\Rightarrow \quad x = \frac{R}{14}$$



Moment of inertia of the cavitied sphere about an axis ($\perp r$ to plane of the fig) through point of contact (P) is calculated as follows -

Let M = mass of cavitied sphere

$$\frac{4}{3}\pi\ R^3\left(1-\frac{1}{8}\right)\rho\ = M;\quad \rho = \frac{3M}{4\pi\ R^3}\times\frac{8}{7}$$

$\therefore$ Mass of sphere of radius $\frac{R}{2}$ is $m = \rho\ \frac{4}{3}\pi\ \frac{R^3}{8}\ =\ \frac{M}{7}$

Mass of sphere without cavity $M_0\ =\ m + M = \frac{8M}{7}$

$\therefore$ Required moment of inertia

I = (moment of inertia of complete sphere without cavity about an axis through P)–

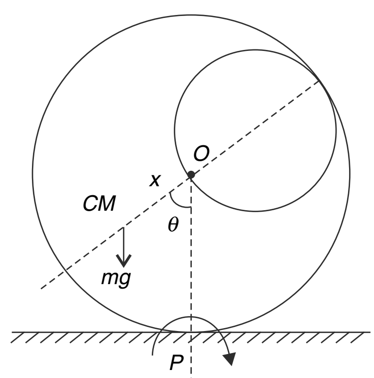
(moment of inertia of the cavity about the same axis)

$$= \frac{7}{5}M_0\,R^2\ -\ \left[\frac{2}{5}m\left(\frac{R}{2}\right)^2 + m\left(\frac{3R}{2}\right)^2\right]$$

$$= \frac{7}{5}\ \frac{8M}{7}\ R^2\ - \left[\frac{47}{20}\ \frac{M}{7}\ R^2\right] = \frac{177}{140}\ MR^2$$

A purely rolling sphere can be considered to be is pure rotation about the point of contact. Consider the sphere at a slightly displaced position θ, as shown.

Restoring torque in this position is



$$\tau = Mg\ x\sin\theta \simeq \frac{R}{14}Mg\theta$$

$$\therefore\ I\ \alpha\ = -\frac{R}{14}Mg\theta$$

$$\Rightarrow \frac{177}{140}MR^2\,\alpha\ = -\frac{R}{14}Mg\ \theta$$

$$\Rightarrow\ \alpha\ = -\ \frac{140}{177\times 14}\ \frac{g}{R}\ \theta$$

$$\therefore\ \omega^2\ = \frac{10}{177}\ \frac{g}{R}$$

$$\Rightarrow\ T\ = 2\ \pi\ \sqrt{\frac{177R}{10g}}$$

60. Middle spring is stretched by $x_2 - x_1$

For block 1 $m\dfrac{d^2x_1}{dt^2} = -kx_1 + k(x_2 - x_1)$ ------------(1)

For block 2 $m\dfrac{d^2x_2}{dt^2} = -kx_2 - k(x_2 - x_1)$ ------------(2)

(a) Adding (1) and (2) gives

$$m\frac{d^2(x_1+x_2)}{dt^2} = -k(x_1+x_2)$$

$$\Rightarrow \frac{d^2A}{dt^2} = -\frac{kA}{m}$$

$\therefore A = a_1 \sin(\omega_a t + \delta_1)$ where $\omega_a = \sqrt{\dfrac{k}{m}}$ ------------(3)

(b) Subtracting (2) from (1) gives

$$m\frac{d^2(x_1-x_2)}{dt^2} = -3k(x_1-x_2)$$

$$\Rightarrow \frac{d^2B}{dt^2} = -\frac{3k}{m}B$$

$\therefore B = a_2 \sin(\omega_b t + \delta_2)$ where $\omega_b = \sqrt{\dfrac{3k}{m}}$ ----------(4)

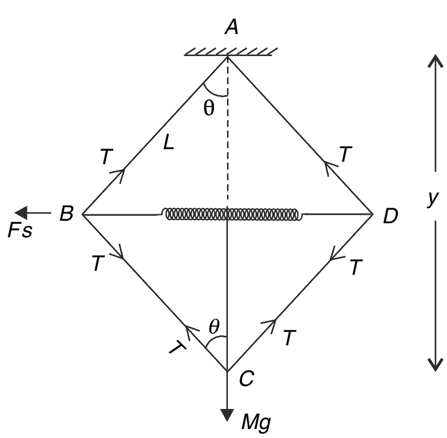
Adding (3) and (4) gives [$A = x_1 + x_2$, $B = x_1 - x_2$]

$2x_1 = a_1 \sin(\omega_a t + \delta_1) + a_2 \sin(\omega_b + \delta_2)$

$$\therefore x_1 = \frac{a_1}{2}\sin(\omega_a t + \delta_1) + \frac{a_2}{2}\sin(\omega_b t + \delta_2)$$

Thus x_1 is combination of two SHMs having angular frequencies ω_a and ω_b.

61.



In equilibrium $2T\cos\theta = Mg$ [convince yourself that all rods have same tension]

And $2T\sin\theta = F_s$

$$\Rightarrow \tan\theta = \frac{F_s}{Mg}$$

$\Rightarrow Mg \tan 30^\circ = k\,[1.5L - 2L \sin 30^\circ]$

$$\Rightarrow Mg\frac{1}{\sqrt{3}} = \frac{kL}{2} \Rightarrow \frac{kL}{Mg} = \frac{2}{\sqrt{3}} \qquad \text{-----------(1)}$$

If y changes by Δy then we proceed as follows to calculate the restoring force.

Let length of the spring be ℓ.

$\ell = 2\,L \sin\,\theta$

$\Delta\ell = 2L \cos\,\theta\,\Delta\,\theta$ ------------(a)

And y = 2L cos θ

$\Delta y = -\,2L \sin\,\theta\,\Delta\theta$ ------------(b)

Spring force changes by $k\Delta\ell = 2kL \cos\,\theta\,\Delta\theta$

$\therefore$ Change in rod tension will be given as

$2\Delta T \sin\,\theta = 2kL \cos\,\theta\,\Delta\theta$

$\Delta T = kL \cot\,\theta.\,\Delta\theta$

$\therefore$ Restoring force on mass M is

$$2\,\Delta\,T\cos\theta = 2kL\,\frac{\cos^2\theta}{\sin\theta}\,\Delta\,\theta$$

$$\therefore\ M\,\frac{d^2y}{dt^2} = 2kL\frac{\cos^2\theta}{\sin\theta}\,\Delta\theta$$

$$\frac{d^2y}{dt^2} = -\,2\frac{kL}{M}\,\frac{\cos^2\theta}{\sin\theta}\,.\,\frac{\Delta y}{2L\sin\theta} \qquad \text{[using (b)]}$$

$$= -\frac{k}{M}\,\frac{\cos^2\theta}{\sin^2\theta}\,\Delta\,y \;= -\frac{2g}{\sqrt{3}L}\,.\left(\sqrt{3}\right)^2\,\Delta\,y - 2\sqrt{3}\,\frac{g}{L}\,\Delta\,y$$

$$\therefore\ \omega^2 = 2\sqrt{3}\,\frac{g}{L}$$

$$T = 2\pi\sqrt{\frac{L}{2\sqrt{3}g}}$$

62. (i) If both blocks are simultaneously given equal velocity when they are at their mean positions, they will oscillate as suggested.

(ii) The distance between 1 and 2 does not change. Hence, middle spring does not exert any force on the blocks.

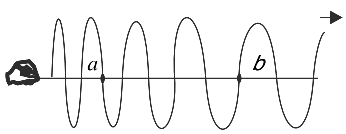
Each block experiences force due to one spring only.

$$\therefore\ \omega = \sqrt{\frac{k}{m}}$$

13 WAVE MOTION

Level 1

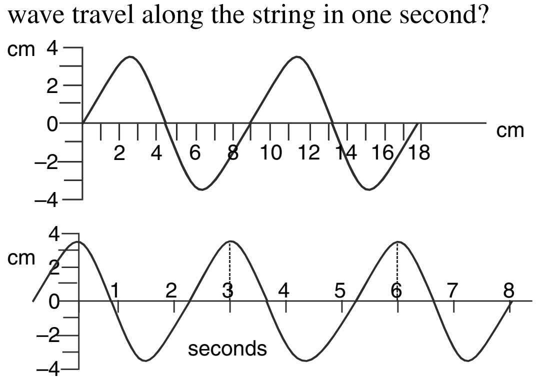
Q. 1. A boy is jerking one end of a taut string. The wave train propagating to the right has been shown in the figure.



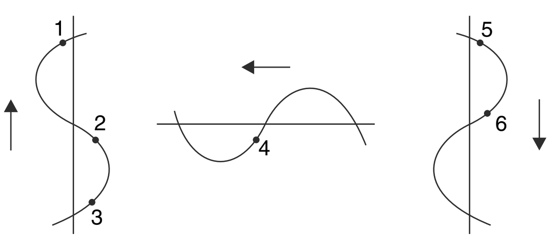
(a) Why the crests are farther apart as we move away from the boy.

(b) Which particle on the string a or b is having higher speed?

Q. 2. A transverse wave is travelling along a horizontal string. The first figure is the shape of the string at an instant of time. The second picture is a graph of the vertical displacement of a point on the string as a function of time. How far does this wave travel along the string in one second?



Q. 3.



The figure shows the shape of three strings on which sinusoidal transverse waves are propagating. The arrows in the diagram indicate the direction of wave propagation. Out of the 6 particles marked (1, 2, 3, 4, 5, 6) how many have their instantaneous velocity and acceleration both directed towards their mean position.

Q. 4. Consider a function

$$y = 5.0e^{(-25x^2 - 9t^2 - 30xt)}$$

(a) Does this represent a travelling wave?

(b) What is direction of propagation of the wave?

(c) Find wave speed.

(d) Sketch the wave at $t = 0$

Q. 5. A hypothetical pulse is travelling along positive x direction on a taut string. The speed of the pulse is 10 $cm\ s^{-1}$. The shape of the pulse at $t = 0$ is given as

$$y = \frac{x}{6} + 1 \qquad \text{for} \qquad -6 < x \le 0$$
$$= -x + 1 \qquad \text{for} \qquad 0 \le x < 1$$
$$= 0 \qquad \text{for all other values of } x$$

x and y are in cm.

(a) Find the vertical displacement of the particle at $x = 1\ cm$ at $t = 0.2\ s$

(b) Find the transverse velocity of the particle at $x = 1\ cm$ at $t = 0.2\ s$.

Q. 6. Which of the following functions does not satisfy the differential wave equation -

(i) $y = 4e^{k(x - vt)}$

(ii) $y = 2 \sin (5t) \cos (6\pi x)$

Q. 7. A transverse harmonic wave of amplitude 4 mm and wavelength 1.5 m is travelling in positive x direction on a stretched string. At an instant, the particle at x = 1.0 m is at y = + 2 mm and is travelling in positive y direction. Find the co-ordinate of the nearest particle (x > 1.0 m) which is at its positive extreme at this instant.

Q. 8. A transverse harmonic wave travels along a taut string having a tension of 57.6 N and linear mass density of 100 g/m. Two points A and B on the string are 5 cm apart and oscillate with a phase difference of $\frac{\pi}{6}$. How much does the phase of oscillation of point A change in a time interval of 5.0 ms?

Q. 9. A distant source of sound has frequency 800 Hz. An observer is facing 90° away from the direction of the source. Estimate the phase difference between the oscillations of her left and right eardrums. Speed of sound in air = 340 ms^{-1}.

Q. 10. A sinusoidal wave travels along a taut string of linear mass density 0.1 g/cm. The particles oscillate along y-direction and the disturbance moves in the positive x-direction. The amplitude and frequency of oscillation are 2 mm and 50 Hz respectively. The minimum distance between two particles oscillating in the same phase is 4 m.

(a) Find the tension in the string.

(b) Find the amount of energy transferred through any point of the string in one second.

(c) If it is observed that the particle at $x = 2$ m is at $y = 1$ mm at $t = 2$ s, and its velocity is in positive y-direction, then write the equation of this travelling wave.

Q. 11. A and B are two point sources of sound (of same frequency) and are kept at a separation. At a point P, the intensity of sound is observed to be I_0 when only source A is put on. With only B on the intensity is observed to be $2I_0$. The distance AP is higher than distance BP by half the wavelength of the sound. Find the intensity recorded at P with both sources on. Give your answer for following cases:

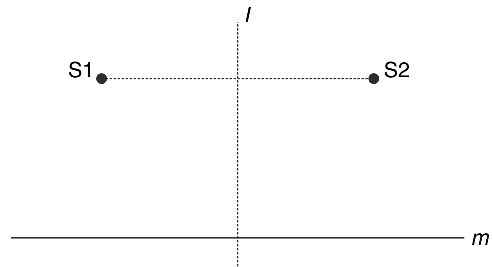
(a) The sources are coherent and in phase.

(b) The sources are coherent and 180° out of phase.

(c) The sources are incoherent.

Q. 12. Two sound sources oscillate in phase with a frequency of 100 Hz. At a point 1.74 m from one source and 1.16 m from the other, the amplitudes of sound from the two sources are A and 2 A respectively. Calculate the amplitude of the resultant disturbance at the point. [Speed of sound in air is $v = 348$ ms^{-1}]

Q. 13. Two speakers $S1$ and $S2$ are a driven by same source. You walk along a line l that is perpendicular bisector of the line joining the two speakers and record the intensity at different points. Then you walk along a line m that is parallel to the line joining the speakers and record the intensity of sound at various points. On which path you observe the loudness to alternate between faint and loud? Explain.

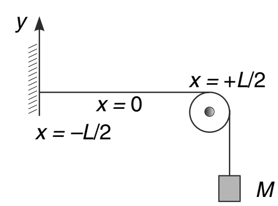


Q. 14. (i) A wire is stretched between two rigid supports. It is observed that the wire resonates at a frequency of 420 Hz. If a wooden bridge is placed at the midpoint of the wire (so that the midpoint becomes a node), it was observed that the smallest frequency at which the wire resonates is 420 Hz. Find the smallest frequency at which the wire will resonate when there is no wooden bridge.

(ii) A string of length L is fixed at one end and is under tension due to a weight hanging from the other end, as shown in the figure. The point of the string on the pulley behaves as a fixed point. Coordinate axes are chosen so that the horizontal segment of the string runs from $x = -L/2$ to $x = L/2$. The string is vibrating at one of its resonant frequencies with transverse displacement (y) given by

$y(x,t) = 0.05 \cos(12.0x) \sin(360t)$

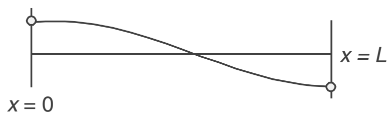
with x, y in meter and t in second. Write two smallest possible values of L consistent with the given equation?



Q. 15. Two strings of same material are joined to form a large string and is stretched between rigid supports. The diameter of the second string is twice that of the first. It was observed in an experiment that the whole string was oscillating in 4 loops with a node at the joint. Find the possible lengths of the second string if the length of first string is 90 cm.

Q. 16. (i) The equation of wave in a string fixed at both end is $y = 2 \sin \pi t \cos \pi x$. Find the phase difference between oscillations of two points located at $x = 0.4\ m$ and $x = 0.6\ m$.

(ii) A string having length L is under tension with both the ends free to move. Standing wave is set in the string and the shape of the string at time $t = 0$ is as shown in the figure. Both ends are at extreme. The string is back in the same shape after regular intervals of time equal to T and the maximum displacement of the free ends at any instant is A. Write the equation of the standing wave.



Q. 17. One type of steel has density 7800 kg/m^3 and will break if the tensile stress exceeds $7.0 \times 10^8\ N/m^2$. You want to make a guitar string using 4.0 g of this type of steel. While in use, the guitar string must be able to withstand a tension of 900 N without breaking.

(a) Determine the maximum length and minimum radius the string can have.

(b) Determine the highest possible fundamental frequency of standing waves on this string, if the entire length of the string is free to vibrate.

Q. 18. A string, of length L, clamped at both ends is vibrating in its first overtone mode. Answer the following questions for the moment the string looks flat

(a) Find the distance between two nearest particles each of which have half the speed of the particle having maximum speed.

(b) How many particles in the string have one eighth the speed of the particle travelling at highest speed?

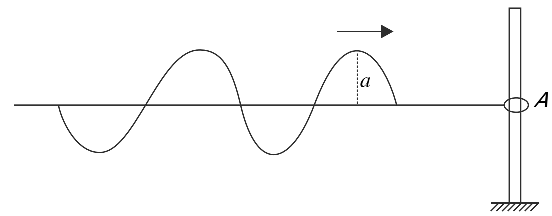
Q. 19. Two transverse waves travel in a medium in same direction.

$$y_1 = a\cos\left(\omega t - \frac{2\pi}{\lambda_1}x\right);\ y_2 = a\cos\left(2\omega t - \frac{2\pi}{\lambda_2}x\right)$$

(a) Write the ratio of wavelengths $\left(\frac{\lambda_1}{\lambda_2}\right)$ for the two waves.

(b) Plot the displacement of the particle at $x = 0$ with time (t).

Q. 20. A sine wave is travelling on a stretched string as shown in figure. The end A of the string has a small light ring which can slide on a smooth rod. The wave reaches A at time $t = 0$.

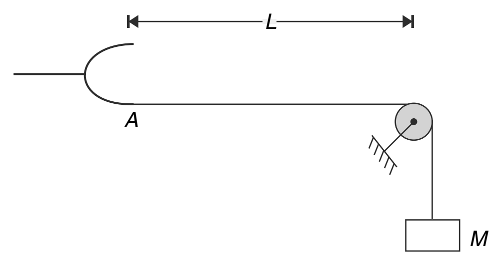


(i) Write the slope of the string at point A as function of time.

(ii) If the incoming wave has amplitude a, with what amplitude will the end A oscillate?

Q. 21. Fundamental frequency of a stretched sonometer wire is f_0. When its tension is increased by 96% and length decreased by 35%, its fundamental frequency becomes $\eta_1 f_0$. When its tension is decreased by 36% and its length is increased by 30%, its fundamental frequency becomes $\eta_2 f_0$. Find $\frac{\eta_1}{\eta_2}$.

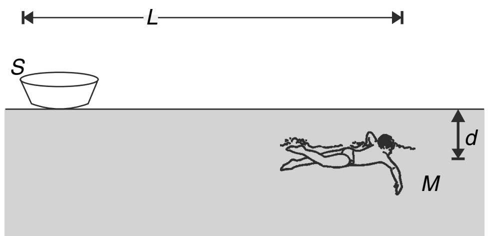
Q. 22. The linear mass density of the string shown in the figure is $\mu = 1$ g/m. One end (A) of the string is tied to a prong of a tuning fork and the other end carries a block of mass M. The length of the string between the tuning fork and the pulley is $L = 2.0$ m. When the tuning fork vibrates, the string resonates with it when mass M is either 16 kg or 25 kg. However, standing waves are not observed for any other value of M lying between 16 kg and 25 kg. Assume that end A of the string is practically at rest and calculate the frequency of the fork.



Q. 23. Wavelength of two musical notes in air are $\frac{18}{35}\ m$ and $\left(\frac{90}{173}\right) m$. Each note produces four beats per second with a third note of frequency f_0. Calculate the frequency f_0.

Q. 24. In a science – fiction movie the crew of a ship observes a satellite. Suddenly the satellite blows up. The crew first sees the explosion and after a small time gap hears the sound. Do you think there was a technical lapse?

Q. 25. A man is swimming at a depth d in a sea at a distance L (>> d) from a ship (S). An explosion occurs in the ship and after hearing the sound the man immediately moves to the surface. It takes $0.8\,s$ for the man to rise to the surface after he hears the sound of explosion. $0.2\,s$ after reaching the surface he once again hears a sound of explosion. Calculate L.



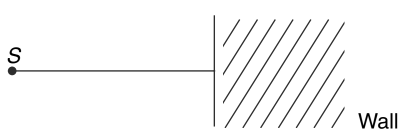
Given: Speed of sound in air = 340 ms^{-1}; Bulk modulus of water = 2×10^9 Pa

Q. 26. Speed of sound in air is 331 ms^{-1} at 0°C. Prove that it increases at a rate of 0.6 $ms^{-1}°C^{-1}$ for small temperature increase.

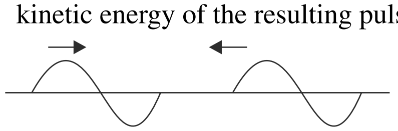
Q. 27. (a) Calculate the speed of sound in hydrogen gas at 300 K

(b) At what temperature the speed in oxygen will be same as above. [Assume oxygen molecules to remain diatomic]

Q. 28. A harmonic source (S) is driving a taut string. The other end of the string is tied to a wall that is not so rigid. It is observed that standing waves are formed in the string with ratio of amplitudes at the antinodes to that at the nodes equal to 8. What percentage of wave energy is transmitted to the wall?



Q. 29. (a) Two identical sinusoidal pulses move in opposite directions on a stretched string. Kinetic energy of each pulse is k. At the instant they overlap completely, what is kinetic energy of the resulting pulse?



(b) "A string clamped at both ends is vibrating. At the moment the string looks flat, the instantaneous transverse velocity of points along the string, excluding its end-points, must be same everywhere except at nodes." Is this statement correct?

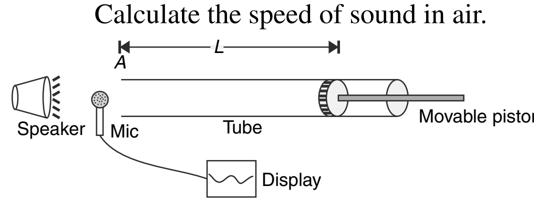
Q. 30 Sound of wavelength 100 cm travels in air. At a given point the difference in maximum and minimum pressure is 0.2 Nm^{-2}. If the bulk modulus of air is 1.5×10^5 Nm^{-2}, find the amplitude of vibration of the particles of the medium.

Q. 31. (i) An organ pipe has one end closed and at the other end there is a vibrating diaphragm. The diaphragm is a pressure node. The pipe resonates when the frequency of the diaphragm is 2 KHz. Distance between adjacent nodes is 8.0 cm. When the frequency is slowly reduced, the pipe again resonates at 1.2 KHz.

(a) Find the length of the tube.

(b) Find the next frequency above 2 KHz at which the pipe resonates.

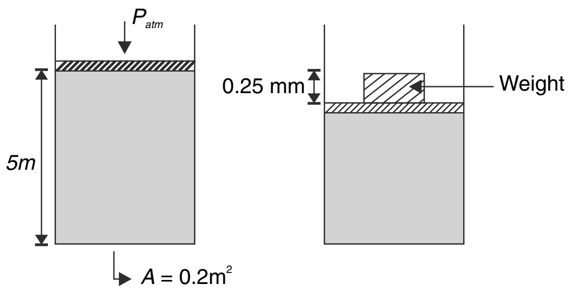
(ii) The figure shows an arrangement for measuring the speed of sound in air. A glass tube is fitted with a movable piston that allows the indicated length L to be adjusted. There is enough gap between the piston and the tube wall to allow the air to pass through it. A speaker is placed near the open end of the tube. A microphone is placed close to the speaker and it is connected to a waveform display. The display is a pure sinusoidal waveform making 750 oscillations in 5 s. Initially, the piston is held at end A and is then slowly pulled back. Loud sound is produced by the tube when L = 50 cm and L = 157 cm. Calculate the speed of sound in air.



Q. 32. A rigid cylindrical container having a cross sectional area of 0.2 m^2 is filled with water up to a height of 5.0 m. There is a piston of negligible mass over the water. Piston can slide inside the container without friction. When a weight of 2000 kg is placed over the piston, it moves down by 0.25 mm compressing the water.

ρ = Density of water = 10^3 kg/m^3; P_{atm}

= Atmospheric pressure = 10^5 N/m^2 and $g = 10$ m/s^2. With this information calculate the speed of sound in water.



Q. 33. A point source of sound is located inside sea water. Bulk modulus of sea water is $B_\omega = 2.0 \times 10^9$ N/m^2. A diver located at a distance of $10m$ from the source registers a pressure amplitude of $\Delta P_0 = 3000\ \pi$ N/m^2 and gives the equation of sound wave as

$y = A \sin (15\ \pi x - 21000\ \pi t)$, y and x are in meter and t is in second.

(a) Find the displacement amplitude of the sound wave at the location of the diver.

(b) Find the power of the sound source.

Q. 34. A point source of sound is moving uniformly along positive x direction with velocity V_0. At time $t = 0$ the source was at origin and emitted a compression pulse C_1. After time T it emitted another compression pulse C_2. Write the equation of the wave front representing the compression pulse C_2 at time t ($>T$). Speed of sound is V.

Q. 35. (i) In a car race sound signals emitted by the two cars are detected by the detector on the straight track at the end point of the race. Frequency observed is 330 Hz and 360 Hz. The original frequency of horn is 300 Hz for both cars. Race ends with the separation of 1000 m between the cars. Assume both cars move with constant velocity and velocity of sound is 330 m/s. Find the time (in seconds) taken by the winning car to finish the race.

(ii) A source of sound of frequency f is dropped from rest from a height h above the ground. An observer O_1 is located on the ground and another observer O_2 is inside water at a depth d from the ground. Both O_1 and O_2 are vertically below the source. The velocity of sound in water is $4V$ and that in air is V. Find

(a) The frequency of the sound detected by O_1 and O_2 corresponding to the sound emitted by the source initially.

(b) The frequency detected by both O_1 and O_2 corresponding to the sound emitted by the source at height $h/2$ from the ground.

Q. 36. (i) A source of sound emits waves of frequency $f_0 = 1200$ Hz. The source is travelling at a speed of $v_1 = 30$ m/s towards east. There is a large reflecting surface in front of the source which is travelling at a velocity of $v_2 = 60$ m/s towards west. Speed of sound in air is $v = 330$ m/s.

(a) Find the number of waves arriving per second at the reflecting surface.

(b) Find the ratio of wavelength (λ_1) of sound in front of the source travelling towards the reflecting surface to the wavelength (λ_2) of sound in front of the source approaching it after getting reflected.

(ii) A sound source (S) and an observer (A) are moving towards a point O along two straight lines making an angle of 60° with each other. The velocities of S and A are 18 ms^{-1} and 12 ms^{-1} respectively and remain constant with time. Frequency of the source is 1000 Hz and speed of sound is $v = 330$ ms^{-1}.

(a) Find the frequency received by the observer when both the source and observer are at a distance of 180 m from point O (see figure).

(b) Find the frequency received by the observer when she reaches point O.

Q. 37. A source of sound, producing a sinusoidal wave, is moving uniformly towards an observer at a velocity of 20 m/s. The observer is moving away from the source at a constant velocity of 10 m/s. Frequency of the source is 200 Hz and speed of sound in air is of 340 m/s.

(a) How many times, in an interval of 10 second, the eardrums of the observer will sense maximum change in pressure?

(b) What will be apparent wavelength of sound for the observer?

Q. 38. Two trains A and B are moving on parallel tracks in opposite direction at same speed of 30 ms^{-1}. Just when the engines of the two trains are about to cross, the engine of train A begins to sound a horn. The sound of the horn is composed of components varying in frequency from 900 Hz to

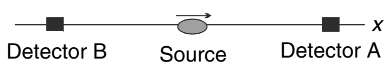
1200 Hz. The speed of sound in air is 330 ms^{-1}.

(a) Find the frequency spread (range of frequencies) for the sound heard by a passenger in train A.

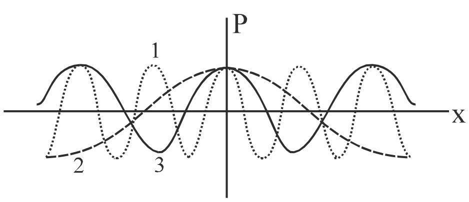
(b) Find the frequency spread heard by a passenger in train B.

Q. 39. Two tuning forks produce 4 beats per second when they are sounded together. Now both the forks are moved towards the observer at same speed (u). The beat frequency now becomes 5 Hz. If the observer also begins to run with speed u towards both the forks, what beat frequency will he hear now?

Q. 40. (i) A sound source emitting sound at a single frequency moves with constant speed along x-axis as shown in figure (a). A and B are two stationary observers. The three plots shown in figure (b) indicate the pressure function $P(x)$ of the sound wave as recorded by the observer A, by B, and by another observer C who is at rest in the frame of the source. Which plot (marked as 1, 2 and 3) correspond to which observer?

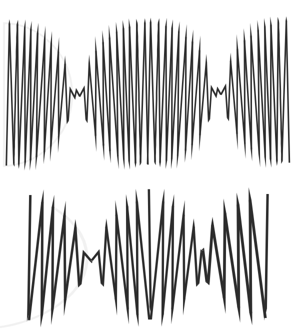


(a)



(b)

(ii) Each of the two figures is rough illustration of the resulting waveform (y versus t) due to overlapping of two waves. The four component waves have frequencies of 300 Hz, 200 Hz, 204 Hz and an unknown frequency $f = 300 + \Delta f$. Is Δf higher than or less than 4 Hz?



LEVEL 2

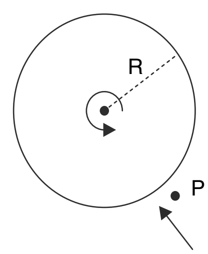
Q. 41. A long taut string is plucked at its centre. The pulse travelling on it can be described as $y(x, t) = e^{-(x+2t)^2} + e^{-(x-2t)^2}$. Draw the shape of the string at time $t = 0$, a short time after $t = 0$ and a long time after $t = 0$.

Q. 42. A sinusoidal harmonic wave is propagating along a string stretched along x – axis. A particle on the string at $x = 1\ m$ is found to be at its mean position travelling in positive y direction at $t = 1\,s$. The amplitude, wavelength and frequency of the wave are 0.01 m, $\frac{\pi}{2}m$ and 20 Hz respectively. Write the equation of the wave if-

(a) it is travelling along negative X direction

(b) it is travelling along positive X direction.

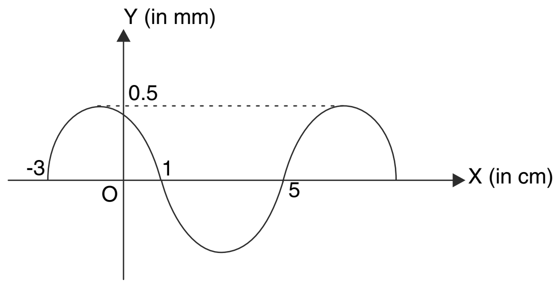
Q. 43. A circular loop of radius R is made of a perfectly elastic wire and is rotating with a constant angular velocity ω lying on a smooth horizontal table. The rotation axis is vertical passing through the centre. A small radial push given to the loop at a point P on the table causes a transverse pulse to propagate on it. Find the smallest time in which the pulse will be back to its originating point P on the table.



Q. 44. Two waves $y_1 = a\sin\left(\frac{\pi}{2}x - \omega t\right)$ and $y_2 = a\sin\left(\frac{\pi}{2}x + \omega t + \frac{\pi}{3}\right)$ get superimposed in the region $x \geq 0$. Find the number of nodes in the region $0 \leq x \leq 6\ m$.

Q. 45. Two sine waves of same frequency and amplitude, travel on a stretched string in opposite directions. Speed of each wave is 10 cm/s. These two waves superimpose to form a standing wave pattern on the string. The maximum amplitude in the standing wave pattern is 0.5 mm.

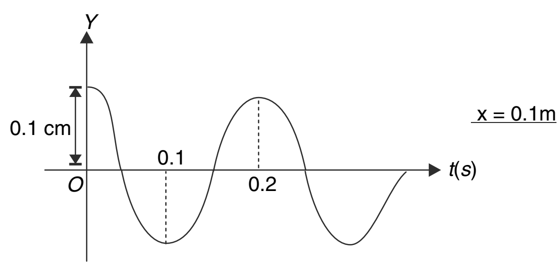
The figure shows the snapshot of the string at $t = 0$.Write the equation of the two travelling waves.

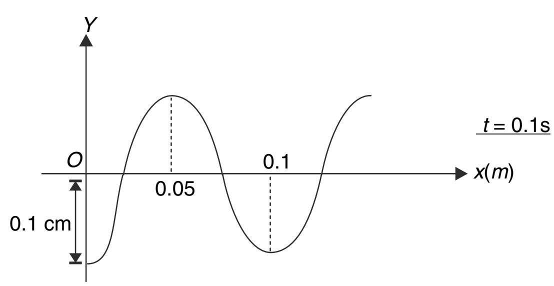


Q. 46. (i) A sinusoidal wave is travelling along positive x direction and the displacements at two positions $x = 0$ and $x = 1\ m$ are given by $y(0, t) = 0.2 \cos(3\pi t)$ and $y(1,t) = 0.2\cos\left(3\pi t + \frac{\pi}{8}\right)$

Find all possible wavelength of the wave if it is known that wavelength is greater than $0.4\,m$.

(ii) A transverse sine wave of amplitude $a = 0.1\ cm$ is travelling along a string laid along the x-axis. The displacement (y) – time (t) graph of the string particle at $x = 0.1\ m$ is shown in first figure. The shape of the string at time $t = 0.1\ s$ is shown in second figure. At this time the particle at $x = 0.11\ m$ is having velocity in positive y direction write the equation of wave.





Q. 47. (i) Two sinusoidal wave are given as $y_1 = a_1 \sin(\omega t + kx + \delta)$ and $y_2 = a_2 \sin(\omega t - kx)$. They superimpose.

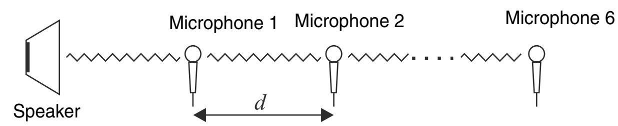
(a) Calculate the resultant amplitude of oscillation at a position x. Is amplitude time dependent?

(b) Calculate the ratio of maximum and minimum amplitudes observed.

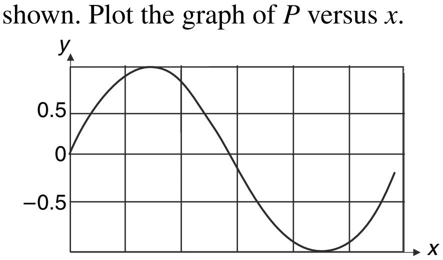
(ii) A speaker (producing a sound of a single wavelength λ) and a microphone are placed as shown in the figure. The microphone detects the sound and converts it into electrical signal. This way we can obtain the waveform of the sound. Assume that there is no attenuation of the sound. The waveform detected by the microphone is sinusoidal with amplitude a. In one experiment 6 microphones are placed in front of the speaker with distance between two neighbouring microphones being $d = \frac{5\lambda}{6}$

(a) The output from all the 6 microphones is superimposed. What is amplitude of the resultant?

(b) If large number of microphone are kept with separation L between two consecutive ones, how will the combined output change with L? Given that $L \neq n\lambda$ ($n = 1, 2, 3$..........)



Q. 48. The figure shows y (transverse displacement) vs x (position) graph for a sinusoidal wave travelling along a stretched string. P is power transmitted through a cross section of the string at the instant shown. Plot the graph of P versus x.



Q. 49. A string in a guitar is made of steel (density 7962 kg/m^3). It is 63.5 cm long, and has diameter of 0.4 mm. The fundamental frequency is $f = 247\ Hz$.

(a) Find the string tension (F).

(b) If the tension F is changed by a small amount ΔF, the frequency f changes by a small amount Δf. Show that $\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta F}{F}$

(c) The string is tuned with tension equal to that calculated in part (a) when its temperature is $18°C$. Continuous playing causes the temperature of the string to rise, changing its vibration frequency. Find Δf if the temperature of the string rises to 29°C. The steel string has a Young's modulus of 2.00×10^{11} Pa and a coefficient of linear expansion of 1.20×10^{-5} (°C)$^{-1}$. Assume that the temperature of the body of the guitar remains constant. Will the vibration frequency rise or fall?

Q. 50. A long taut string is connected to a harmonic oscillator of frequency f at one end. The oscillator oscillates with an amplitude a_0 and delivers power P_0 to the string. Due to dissipation of energy the amplitude of wave goes on decreasing with distance x from the oscillator given as $a = a_0 e^{-kx}$. In what length of the string $\left(\frac{3}{4}\right)$th of the energy supplied by the oscillator gets dissipated?

Q. 51. A transverse harmonic wave is propagating along a taut string. Tension in the string is 50 N and its linear mass density is 0.02 $kg\ m^{-1}$ The string is driven by a 80 Hz oscillator tied to one end oscillating with an amplitude of 1mm. The other end of the string is terminated so that all the wave energy is absorbed and there is no reflection

(a) Calculate the power of the oscillator.

(b) The tension in the string is quadrupled. What is new amplitude of the wave if the power of the oscillator remains same?

(c) Calculate the average energy of the wave on a 1.0 m long segment of the string.

Q. 52. A small steel ball of mass $m = 5g$ is dropped from a height of 2.0 m on a hard floor. 0.001% of its kinetic energy before striking the floor gets converted into a sound pulse having a duration of 0.4 s. Estimate how far away the sound can be heard if minimum audible intensity is 2.0×10^{-8} Wm^{-2} [Actually it is much less but to account for background sound we are assuming it to be high]. Assume no attenuation due to atmospheric absorption.

Q. 53. Three travelling waves are superimposed. The equations of the wave are

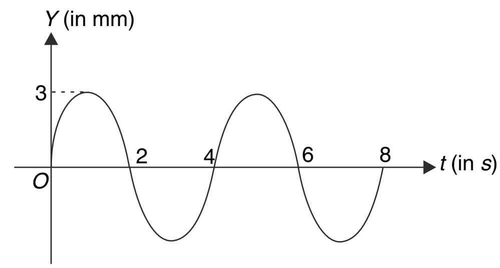
$y_1 = A_0 \sin (kx - \omega t)$, $y_2 = 3\sqrt{2} A_0 \sin (kx - \omega t + \phi)$ and $y_3 = 4 A_0 \cos(kx - \omega t)$

Find the value of ϕ (given $0 \le \phi \le \pi / 2$) if the phase difference between the resultant wave and first wave is $\pi / 4$.

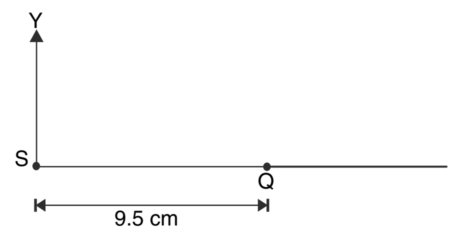
Q. 54. A sinusoidal wave having wavelength of 6 m propagates along positive x direction on a string. The displacement (y) of a particle at $x = 2$ m varies with time (t) as shown in the graph

(a) Write the equation of the wave

(b) Draw y versus x graph for the wave at $t = 0$



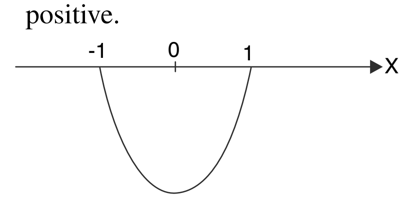
Q. 55. A string SQ is connected to a long heavier string at Q. Linear mass density of the heavier string is 4 times that of the string SQ. Length of SQ is 9.5 cm. Both the strings are subjected to same tension. A 50 Hz source connected at S produces transverse disturbance in the string. Wavelength of the wave in string SQ is observed to be 1 cm. If the source is put on at time $t = 0$, calculate the smallest time (t) at which we can find a particle in the heavier string that oscillates in phase with the source at S.



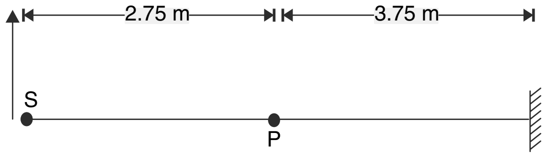
Q. 56. The figure shows the snapshot at time $t = 0$ of a transverse pulse travelling on a string in positive x direction.

(a) Sketch the pulse at a slightly later time.

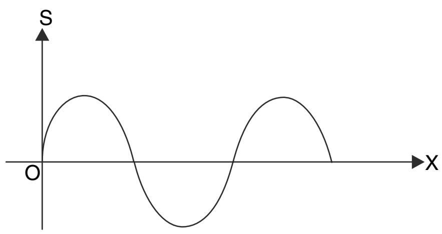
(b) With the help of the given sketch draw a graph of velocity of each string segment versus position. Take upward direction as positive.



Q. 57. A uniform string of length 6.5 m is subjected to a tension of 40 N. Mass of the string is 162.5 g. One end of the string is fixed and the other end is tied to a source (s). which produces a transverse oscillation. The displacement of the end of the string tied to the source can be expressed as $y =$ (3 mm) sin (40 π t), where 't' is time. Find the displacement of point P of the string at a distance of 3.75 m from the fixed end, at time $t = 0.3$ s.



Q. 58. A longitudinal harmonic wave is travelling along positive x direction. The amplitude, wavelength and frequency of the wave are $8.0 \times 10^{-3}\, m$, 12 cm and 6800 Hz respectively. The displacement (s) versus position graph for particles on the x axis at an instant of time has been shown in figure. Find the separation at the instant shown, between the particles which were originally at $x_1 = 1\ cm$ and $x_2 = 3\ cm$



Q. 59. A sinusoidal wave $y = a\sin\left(\frac{2\pi}{\lambda}x - \omega t\right)$ is travelling on a stretched string. An observer is travelling along positive x direction with a velocity equal to that of the wave. Find the angle that the velocity of a particle on the string at $x = \frac{\lambda}{6}$ makes with $-x$ direction as seen by the observer at time $t = 0$.

Q. 60. A standing wave $y = A \sin kx .\cos \omega t$ is established in a string fixed at its ends.

(a) What is value of instantaneous power transfer at a cross section of the string when the string is passing through its mean position?

(b) What is value of instantaneous power transfer at a cross section of the string when the string is at its extreme position?

(c) At what frequency is the power transmitted through a cross section changing with time?

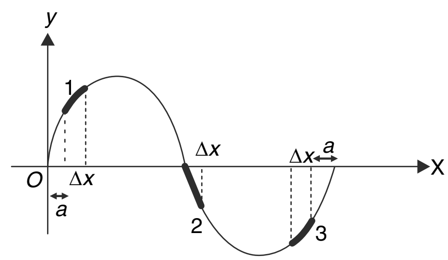
Q. 61. A sinusoidal transverse wave of small amplitude is travelling on a stretched string. The wave equation is $y = a \sin(k x - \omega t)$ and mass per unit length of the string is μ. Consider a small element of length Δx on the string at $x = 0$. Calculate the elastic potential energy stored in the element at time $t = 0$. Also write the kinetic energy of the element at $t = 0$.

Q. 62. The figure shows the $y - x$ graph at an instant for a small amplitude transverse wave travelling on a stretched string. Three elements (1, 2 and 3) on the string have equal original lengths ($= \Delta x$). At the given instant-

(i) which element (among 1, 2 and 3) has largest kinetic energy?

(ii) which element has largest energy (i.e., sum of its kinetic and elastic potential energy)

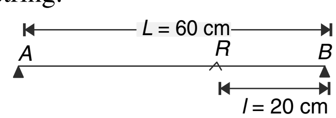
(iii) Prove that energy per unit length $\frac{\Delta E}{\Delta x}$ of the string is constant everywhere equal to $T\left(\frac{\partial y}{\partial x}\right)^2$ where T is tension the string.



Q. 63. A string has linear mass density $\mu = 0.1\ kg / m$. A $L = 60\ cm$ segment of the string is clamped at A and B and is kept under a tension of $T = 160\ N$ [The tension providing arrangement has not been shown in the figure]. A small paper rider is placed on the string at point R such that $BR = 20\ cm$. The string is set into vibrations using a tuning fork of frequency f.

(a) Calculate all values of f below 1000 Hz for which the rider will not vibrate at all.

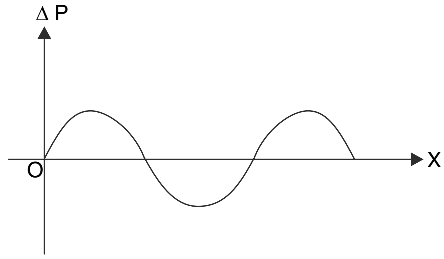
(b) Calculate all values of f below 1000 Hz for which the rider will have maximum oscillation amplitude among all points on the string.



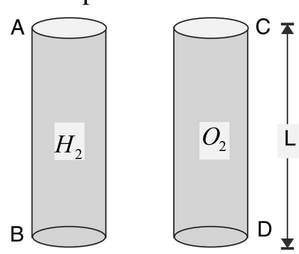
Q. 64. A sinusoidal longitudinal wave is travelling in positive x direction. Wave length of the wave is 0.5 m At time $t = 0$, the change in pressure at various points on the x axis can be represented as shown in figure. Consider five particles of the medium A, B, C, D and E whose x co-ordinates are 0.125 m, 0.1875 m, 0.250 m, 0.375 m and 0.50 m respectively.

(a) Which of the above mentioned five particles of the medium are moving in positive x direction at $t = 0$.

(b) Find the ratio of speed of particles B and D at $t = 0$.



Q. 65. (i) Two cylindrical pipes are each of length L = 30 *cm*. One of them contains hydrogen and the other has oxygen at the same temperature. The ends A, B, C and D of the pipes are fitted with flexible diaphragms. The diaphragms A and C are set into oscillations simultaneously using the same source having frequency f = 600 *Hz*. Calculate the difference in phase of oscillations of the diaphragms D and B if it is known that the speed of sound in hydrogen at the temperature concerned is 1200 *m*/*s*.



(ii) The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 *Hz*. The speed of sound in air is 330 *m*/*s*. P_0 is mean pressure in the pipe and ΔP_0 is maximum amplitude of pressure variation. Neglect end correction.

(a) Find the length L of air column.

(b) What is amplitude of pressure variation at the middle of the column?

(c) What is maximum and minimum pressure at the closed end?

Q. 66. Speed of sound in atmosphere at a height h_0 is 1080 $km\ hr^{-1}$. The variation of temperature and pressure of the atmosphere with height h from the surface is given by

$$T = T_0 - \beta h \text{ and } P = P_0\left(1 - \frac{\beta h}{T_0}\right)^{\frac{Mg}{R\beta}}$$

Where T_0 = temperature at the surface of the earth = 273 K,

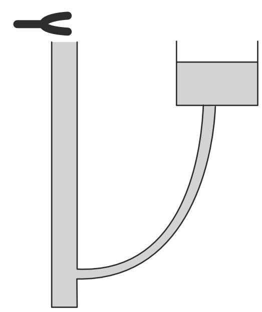
P_0 = atmospheric pressure at the surface of the earth,

β = 0.006 °C/m, M = molar mass of air $\simeq$ 29 g mol^{-1}, g = 9.8 ms^{-2}

R = gas constant = 8.31 J mol^{-1} K^{-1}

Consider air to be a mixture of diatomic gases and calculate the atmospheric temperature and pressure at height h_0. Also find h_0. Take $(0.82)^{5.7}$ = 0.32.

Q. 67. In resonance column experiment a tuning fork of frequency f = 400 *Hz* is held above the pipe as shown in figure. The reservoir is raised and lowered to change the level of water and thus the length of the column of air in the tube. The area of cross section of the reservoir is 6 times that of the pipe. Initially, the reservoir is kept so that the pipe is full up to the brim. Tuning fork is sounded and the reservoir is lowered. When the reservoir is lowered by 21 *cm*, first resonance is recorded. When the reservoir is lowered further by 49 *cm* the second resonance is heard. Find the speed of sound in air.



Q. 68. (i) In a travelling sinusoidal longitudinal wave, the displacement of particle of medium is represented by $s = S(x, t)$. The midpoint of a compression zone and an adjacent rarefaction zone are represented by letter 'C' and 'R' respectively. The difference in pressure at 'C' and 'R' is ΔP and the bulk modulus of the medium is B.

(a) How is $\left|\frac{\partial s}{\partial x}\right|_C$ related to $\left|\frac{\partial}{\partial}\right|$

(b) Write the value of $\left|\frac{\partial s}{\partial x}\right|_C$ in terms of ΔP and B.

(c) What is speed of a medium particle located mid-way between 'C' and 'R'.

(ii) A standing wave in a pipe with a length of L = 3 m is described by $s = A\cos\left(\frac{3\pi x}{L}\right)\sin\left(\frac{3\pi vt}{L}\right)$ where v is wave speed. The atmospheric pressure and density are P_0 and ρ respectively.

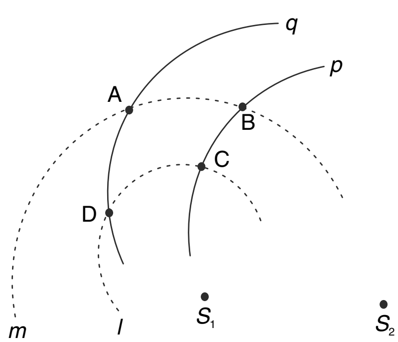
(a) At $t = \frac{L}{18v}$ the acoustic pressure at $x = \frac{L}{2}$ is 0.2 percent of the atmospheric pressure . Find the displacement amplitude A.

(b) In which overtone is the pipe oscillating?

Q. 69. Two sources A and B give out sound waves in coherence and in phase. The sources are located at co-ordinates (0, 0) and (0, 9 m) in xy plane. There is a detector located at (40 m, 0). It was found that the detector records continuous increase in intensity of sound when it is moved in positive y-direction for 4.5 m but the intensity was found to fall for some distance when it is moved in negative y direction. What frequency of sound is consistent with these observations? Speed of sound = 340 ms^{-1}.

Q. 70. In the figure shown, S_1 and S_2 are two identical point sources of sound which are coherent 180° out of phase. Taking S_1 as centre, two circular arcs ℓ and m of radii 1 m and 2 m are drawn. Taking S_2 as centres, two circular arcs p and q are drawn having radii 2 m and 4 m respectively. Out of the four intersection points A, B, C and D which point will record maximum intensity and which will record the least intensity of sound?

It is given that wavelength of wave produced by each source is 4.0 m.



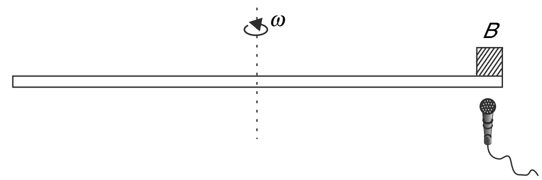
Q. 71. Stationary wave of frequency 5 $K\,Hz$ is produced in a tube open at both ends and filled with air at 300 K. The tube is oscillating in its first overtone mode.

(a) Find the length of the tube assuming that air contains only nitrogen and oxygen in molar ratio of 3 : 1.

(b) What shall be the frequency of sound wave used so that the same tube oscillates in its second overtone mode?

Q. 72. The string of a musical instrument was being tuned using a tuning fork of known frequency, f_0 = 1024 Hz. The tuning fork and the string were set to vibrate together. Both vibrated together for 10 s and no beat was heard. What prediction can be made regarding the frequency of the string?

Q. 73. A wooden platform can be rotated about its vertical axis with constant angular speed ω with the help of a motor. A buzzer is fixed at the circumference of the platform and it rotates in a circle of radius R. The buzzer produces sound of frequency f_0. A mic is placed just beneath the platform near its circumference. An electronic frequency analyzer connected to the mic records the frequency (f) received by the mic. Take time (t) to be zero when the buzzer is just above the mic and express f as a function of time. Plot f versus t. Speed of sound = V_0.



Q. 74. (i) A harmonic wave in a stationary medium is represented by $y = a \sin(kx - \omega t)$. Write the equation of this wave for an observer who is moving in negative x direction with constant speed v_0.

(ii) The Doppler flow meter is a device that measures the speed of blood flow, using transmitting and receiving elements that are placed directly on the skin. The transmitter emits a continuous sound wave whose frequency is 5 $M\,Hz$. When the sound is reflected from the red blood cells, its frequency is changed in a kind of Doppler effect. The cells are moving with the same velocity as the blood. The receiving element detects the reflected sound, and an electronic counter measures its frequency, which is Doppler-shifted relative to the transmitter frequency. From the change in frequency the speed of the blood flow can be determined. Typically, the change in frequency is around 600 Hz for flow speeds of about 0.1 m/s. Assume that the red blood cell is directly moving away from the source and the receiver.

(a) Estimate the speed of the sound wave in the blood?

(b) A segment of artery is narrowed down by plaque to half the normal cross-sectional area. What will be the Doppler change in frequency due to reflection from the red blood cell in that region?

Q. 75. A sound source emits waves of frequency f_0 and wavelength λ_0 in still air. When there is a wind blowing with speed u from left to right what will be wavelength of sound to the right of the source and to the left of the source.

Q. 76. There are two horns $H1$ and $H2$ in a car. When sounded together, the driver records 35 beats in 10 second. With horn $H2$ blowing and car moving towards a wall at a speed of 5 ms^{-1}, the driver noticed a beat frequency of 5 Hz with the echo. When frequency of $H1$ is decreased the beat frequency with two horns sounded together increases. Calculate the frequency of two horns. Speed of sound $= 332\ ms^{-1}$

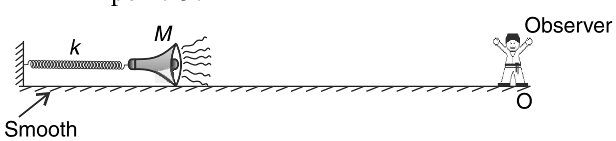
Q. 77. A toy train in a children amusement park runs on an elliptical orbit having major and minor axis in the ratio of 4 : 3. The length of the train is exactly equal to half the perimeter of the elliptical track. The train is travelling at a constant speed of 20 ms^{-1}. The engine sounds a whistle when its acceleration is minimum. The whistle has a frequency of $f_0 = 3460\ Hz$ and speed of sound in air is $V = 330\ ms^{-1}$

(a) What frequency of whistle is received by a passenger in the last compartment of the train?

(b) What frequency of whistle is received by a passenger sitting in the central compartment of the train?

Q. 78. A small source of sound has mass M and is attached to a spring of force constant K. It is oscillating with amplitude $A = \frac{V}{20}\sqrt{\frac{M}{K}}$ where V is speed of sound in air. The source of sound produces a sound of frequency $f_0 = 399\ Hz$.

(a) Find the frequency of sound registered by a stationary observer standing at a distant point O.



(b) Let Δt_1 be the time interval during which the registered frequency changes from 420 Hz to $\left(399\times\frac{40}{39}\right)Hz$ and Δt_2 be the time interval during which the observed frequency changes from 399 Hz to $\left(399\times\frac{40}{41}\right)Hz$. Which is larger Δt_1 or Δt_2?

Q. 79. (i) A straight railway track is at a distance 'd' from you. A distant train approaches you travelling at a speed u (< speed of sound) and crosses you. How does the apparent frequency (f) of the whistle change with time (f_0 is the original frequency of the whistle). Draw a rough f vs t graph.

(ii) A bat is tracking a bug. It emits a sound, which reflects off the bug. The bat hears the echo of the sound 0.1 seconds after it originally emitted it. The bat can tell if the insect is to the right or left by comparing when the sound reaches its right ear to when the sound reaches its left ear. Bat's ears are only 2 cm apart. Bats also use the frequency change of the sound echo to determine the flight direction of the bug. While hovering in the air (not moving), the bat emits a sound of 40.0 kHz. The frequency of the echo is 40.4 kHz. Assume that the speed of sound is 340 m/s.

(a) How far away is the bug?

(b) How much time delay is there between the echo reaching the two ears if the bug is directly to the right of the bat?

(c) What is the speed of the bug?

Q. 80. A source of sound is located in a medium in which speed of sound is V and an observer is located in a medium in which speed of sound is $2V$. Both the source and observer are moving directly towards each other at velocity $\frac{V}{5}$. The source has a frequency of f_0.

(a) Find the wavelength of wave in the medium in which the observer is located.

(b) Find the frequency received by the observer.

LEVEL 3

Q. 81. A transverse wave $y = A\sin\omega\left(\frac{x}{V_1} - t\right)$ is travelling in a medium with speed V_1. Plane $x = 0$ is the boundary of the medium. For $x > 0$ there is a different medium in which the wave travels at a different speed V_2. Part of the wave is reflected and part is transmitted. For $x < 0$ the wave function is described as $y_- = A_1 \sin\omega\left(\frac{x}{V_1} - t\right) + A_2 \sin\omega\left(\frac{x}{V_2} + t\right)$, while

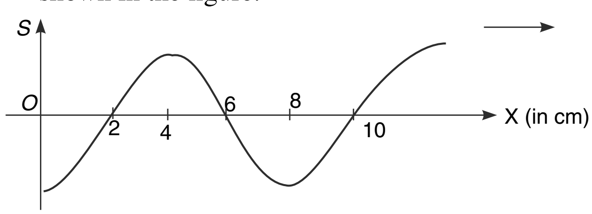
for $x > 0$; $y_+ = A_3 \sin\omega\left(\frac{x}{V_2} - t\right)$

(a) Using the fact that the wave function must be continuous at $x = 0$, show that $A_1 - A_2 = A_3$

(b) Using the fact that $\frac{\partial y}{\partial x}$ must be continuous at $x = 0$, prove that $\frac{V_2}{V_1}A_1 = A_3 - A_2$

(c) Show that $A_3 = \frac{2V_2A_1}{V_1 + V_2}$ and $A_2 = \left(\frac{V_1 - V_2}{V_1 + V_2}\right)A_1$

Q. 82. A longitudinal wave is travelling at speed u in positive x direction in a medium having average density ρ_0. The displacement (s) for particles of the medium versus their position (x) has been shown in the figure.



Answer following questions for $0 < x \le 10$ cm

(a) Write x co-ordinates of all positions where the particles of the medium have maximum negative acceleration. What is density at these locations – higher than ρ_0, less than ρ_0 or equal to ρ_0?

(b) Write x co-ordinates of all locations where the particles of the medium have negative maximum velocity. What do you think about density at these positions?

(c) Knowing that the change in density ($\Delta\rho$) is proportional to negative of the slope of s versus x graph, prove that $\frac{d\rho}{dx} \propto -a$, where a is acceleration of the particles at position x. At which point $(0 < x \le 10)$ is $\frac{d\rho}{dx}$ positive maximum.

Q. 83. Two sound waves, travelling in same direction can be represented as

$$y_1 = (0.02\,mm)\sin\left[\left(400\pi\, rad\, s^{-1}\right)\left(\frac{x}{330ms^{-1}} - t\right)\right]$$

And

$$y_2 = (0.02\,mm)\sin\left[\left(404\pi\, rad\, s^{-1}\right)\left(\frac{x}{330ms^{-1}} - t\right)\right]$$

The waves superimpose.

(i) Find distance between two nearest points where an intensity maximum is recorded simultaneously.

(ii) Find the time gap between two successive intensity maxima at a given point.

Q. 84. There are three sinusoidal waves A, B and C represented by equations-

$$A \rightarrow y = A\sin kx;\ B \rightarrow y = \frac{A}{2}\sin 2kx;$$

$$C \rightarrow y = \frac{A}{2}\sin 3kx$$

(a) To get a waveform of nearly the shape given in fig (a) which of the two waves B or C shall be superimposed with wave A?

(b) To get a waveform close to that in fig (b) which of the two waves B or C shall be superimposed with A?

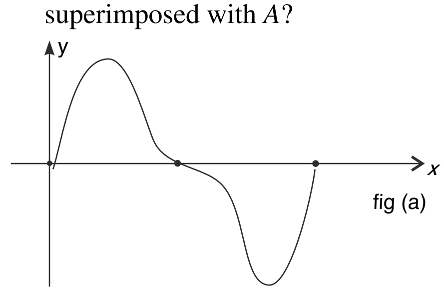


fig (a)

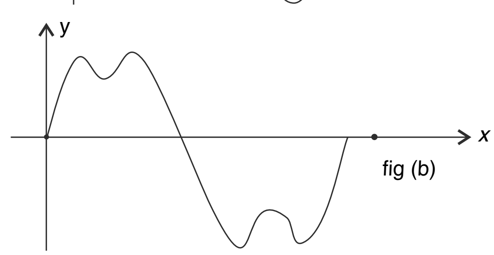
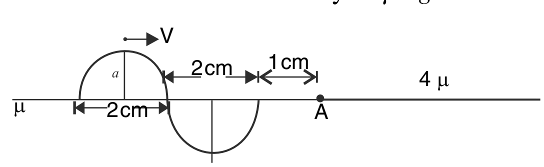


fig (b)

Q. 85. A taut string is made of two segments. To the left of A it has a linear mass density of μ kg/m and to



the right of A its linear mass density is 4 μ kg/m.

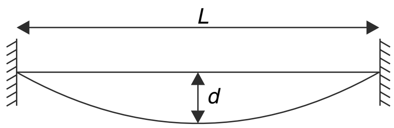
A sinusoidal pulse of amplitude a is travelling towards right on the lighter string with a speed $V = 2\ cm/s$.

Draw the shape of the string after

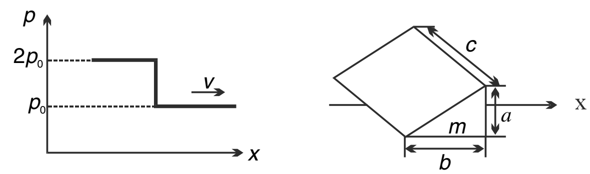
(a) 1 s

(b) 2.5 s

Q. 86. A wire having mass per unit length μ and length L is fixed between two fixed vertical walls at a separation L. Due to its own weight the wire sags. The sag in the middle is d ($<< L$). Assume that tension is practically constant along the wire, owing to its small mass. Calculate the speed of the transverse wave on the wire.



Q. 87. A shock wave is a region of high acoustic pressure propagating at speed of sound (v). Assume that the pressure in one such shock wave is $2P_0$ where P_0 is the atmospheric pressure. This shock wave is travelling horizontally along x direction and hits a small wedge whose dimensions are as shown in the figure. The wedge has a mass m and is lying on a smooth horizontal surface. Determine the velocity u acquired by the wedge immediately after the shock wave passes through it. The velocity acquired by the wedge should be assumed to be much lower than the velocity of the wave ($u << v$).



ANSWERS

1. (a) The boy is jerking the string with gradually increasing frequency

(b) Particle 'a'

2. 3.09 cm

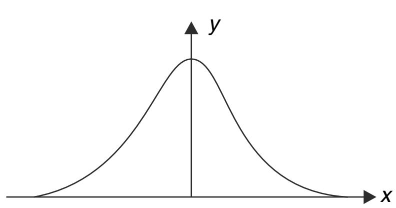
3. Three

4. (a) Yes

(b) Negative x direction

(c) 0.6 unit

(d)



5. (a) $\frac{5}{6}\ cm$

(b) $-\frac{5}{3}\ cm\ s^{-1}$

6. Both satisfy the wave equation

7. 2.25 m

8. $\frac{2\pi}{5}$

9. $\frac{16}{17}\pi$

10. (a) 400 N

(b) $\frac{\pi^2}{25}\ J$

(c) $y = 2\sin\left(\frac{\pi x}{2} - 100\pi t - 30°\right)$

11. (a) $0.2\ I_0$

(b) $(3 + 2\sqrt{2})I_0$

(c) $3\ I_0$

12. $\sqrt{7}\ A$

13. Along path m the loudness alternates between faint and loud due to phenomena of interference.

14. (i) 210 Hz

(ii) $\frac{\pi}{12}, \frac{\pi}{4}$

15. 15 cm, 4 cm, 135 cm

16. (i) π

(ii) $y = A\cos\frac{\pi x}{L}\cos\frac{2\pi}{T}t$

17. (a) $0.40\ m$, $6.4 \times 10^{-4}\ m$

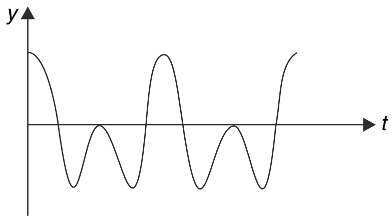
(b) 376 Hz

18. (a) $\frac{L}{6}$

(b) 4

19. (a) $\frac{\lambda_1}{\lambda_2} = \frac{2}{1}$

(b)



20. (i) zero

(ii) $2a$

21. 3.5

23. 696 Hz

24. Yes, sound cannot travel in free space.

25. 447.6 m

27. (a) 1320 ms^{-1}

(b) 4800 K

28. 39.5%

29. (a) 4 k

(b) No

30. 0.1 μm

31. (i) (a) 20 cm

(b) 2800 Hz

(ii) 321 m/s

32. 1414 m/s

33. (a) 0.1 μm

(b) 3.9×10^4 watt

34. $(x - V_0T)^2 + y^2 + z^2 = V^2(t - T)^2$

35. (i) 40 s

(ii) (a) $f_2 = f_1 = \dfrac{2vf^2}{2vf - g}$

(b) $f_1 = f_2 = \dfrac{2vf^2}{2(v - \sqrt{gh})f - g}$

36. (i) (a) 1560

(b) $\dfrac{13}{9}$

(ii) (a) 1047 Hz

(b) 991 Hz

37. (a) 4125

(b) 1.6 m

38. (a) 900 Hz to 1200 Hz

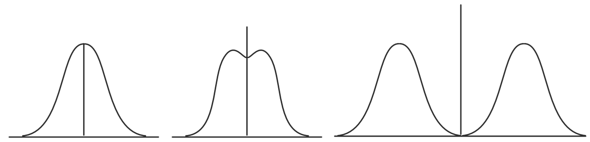
(b) 1080 Hz to 1440 Hz

39. 6 Hz

40. (i) A - 1 B - 2 C - 3

(ii) Less than 4 Hz

41.



42. (a) $y = 0.01 \sin [40\pi(t - 1) + 4(x - 1)]$

(b) $y = 0.01 \sin [40\pi(t - 1) - 4(x - 1)]$

43. $\dfrac{\pi}{\omega}$

44. 3

45. $y_1 = 0.25 \sin\left(\dfrac{\pi}{4}x + \dfrac{5\pi}{2}t + \dfrac{3\pi}{4}\right)$;

$y_2 = 0.25 \sin\left(\dfrac{\pi}{4}x - \dfrac{5\pi}{2}t + \dfrac{3\pi}{4}\right)$

46. (i) $\dfrac{16}{15}m$ and $\dfrac{16}{31}m$

(ii) $y =$ (0.1 cm) cos [(20 $\pi m^{-1}x$] + (10 πs^{-1})t]

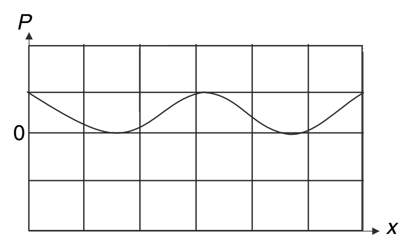
47. (i) (a) $\sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(2kx + \delta)}$

(b) $\dfrac{A_{\max}}{A_{\min}} = \dfrac{a_1 + a_2}{|a_1 - a_2|}$

(ii) (a) zero

(b) zero

48.



49. (a) 98.4 N

(c) 4.2 Hz decrease.

50. $\dfrac{\ln 2}{k}$

51. (a) 0.126 W

(b) $\dfrac{1}{\sqrt{2}}mm$

(c) 2.5 mJ

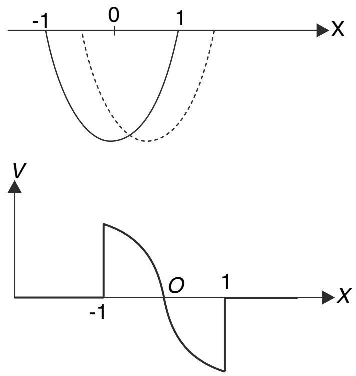
52. 3.15 m

53. $\phi = \dfrac{\pi}{12}$

54. (a) $y = (3mm)\sin\left(\frac{\pi}{3}x - \frac{\pi}{2}t + \frac{\pi}{3}\right)$

55. 0.2 s

56.



57. Zero

58. 2.4 cm

59. $\theta = \tan^{-1}\left(\frac{a\pi}{\lambda}\right)$

60. (a) Zero

(b) Zero

(c) $\frac{\omega}{\pi}$

61. $\Delta U = \frac{1}{2}\mu a^2\omega^2\Delta x$; $\Delta k = \frac{1}{2}\mu a^2\omega^2\Delta x$

62. (i) 2

(ii) 2

63. (a) 100, 200, 300.........900 Hz

(b) It is not possible to have antinode at R

64. (a) A and B

(b) $\frac{1}{\sqrt{2}}$

65. (i) 0.9 π rad

(ii) (a) 93.75 cm

(b) $\frac{\Delta P_0}{\sqrt{2}}$

(c) $P_{max} = P_0 + \Delta P_0$; $P_{min} = P_0 - \Delta P_0$

66. $T = 224\ K$, $P = 0.32\ P_0$, $h_0 = 8167\ m$

67. 336 ms^{-1}

68. (i) (a) $\left|\frac{\partial s}{\partial x}\right|_C = \left|\frac{\partial s}{\partial x}\right|_R$

(b) $\left|\frac{\partial s}{\partial x}\right|_C = \frac{\Delta P}{2B}$

(c) Zero

(ii) (a) $\frac{0.0004 L P_0}{3\pi v^2 \rho}$,

(b) Second

69. $f < 170\ Hz$

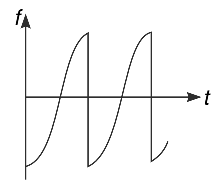
70. Maximum at C and least at B

71. (a) 6.9 cm

(b) 7543 Hz.

72. $1023.9\ Hz < f_{string} < 1024.1\ Hz$

73.



74. (i) $y_0 = a \sin[(\omega + kv_0)t - kx]$

(ii) (a) 1700 m/s

(b) 1200 Hz

75. $\lambda_0 + \frac{u}{f_0}$; $\lambda_0 - \frac{u}{f_0}$

76. $f_1 = 160\ Hz, f_2 = 163.5\ Hz$

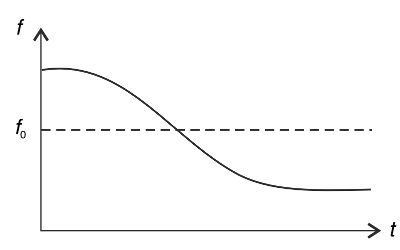
77. (a) 3460 Hz

(b) 3420 Hz

78. (a) $380\ Hz \le f \le 420\ Hz$

(b) $\Delta t_1 > \Delta t_2$

79. (i)



(ii) (a) 17 m

(b) $5.9 \times 10^{-5}\ s$

(c) 1.7 m/s

80. (a) $\frac{8}{5}\frac{V}{f_0}$

(b) $\frac{11}{8} f_0$

82. (a) $x = 4$; equal to ρ_0

(b) $x = 2, x = 10$; $\Delta\rho$ is maximum negative

(c) $x = 4$.

83. (i) 150 m

(ii) 0.5 s

84. (a) B

(b) C

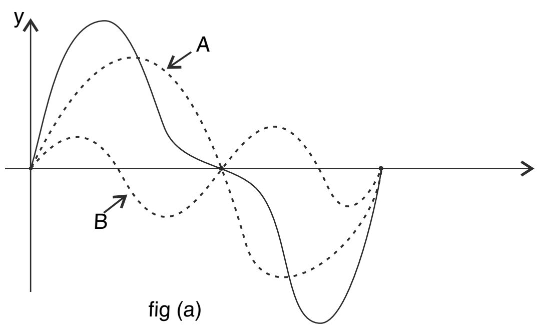


fig (a)

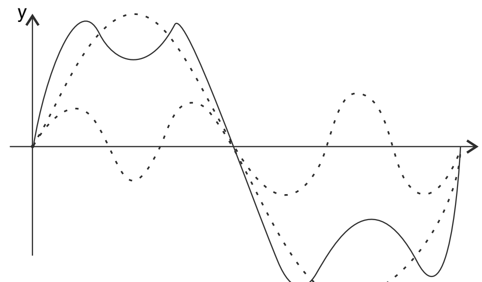
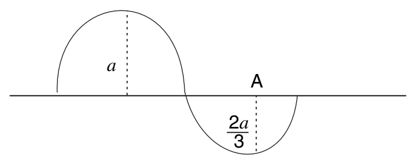
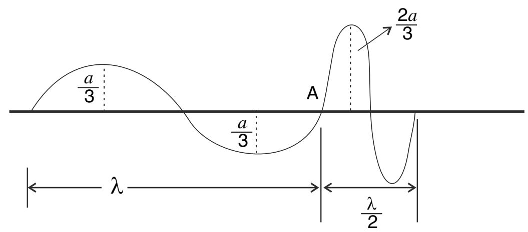


fig (b)

85. (a)



(b)



86. $v = L\sqrt{\frac{g}{8d}}$

87. $\frac{P_0 abc}{2mv}$

SOLUTION

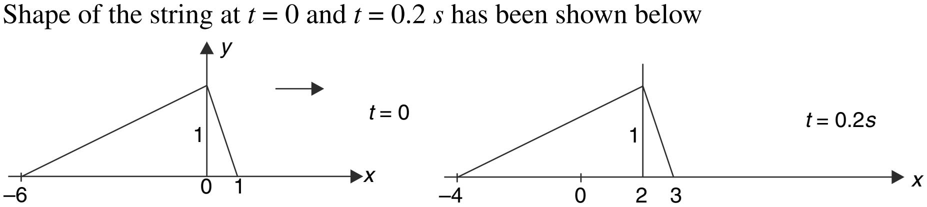
2. The wavelength is = 9 cm

The particle makes 2.75 oscillations in 8 s, hence time period is $T = \frac{8}{2.75} = 2.91\ s$

Wave speed, $v = \frac{\lambda}{T} = \frac{9}{2.91} = 3.09\, cms^{-1}$

3. All particles perform SHM. Their acceleration is always towards their mean position. Velocity of 3, 4 and 5 are towards mean position. This can be observed by looking at the direction of wave and predicting the position of a particle a moment later.

5. Shape of the string at $t = 0$ and $t = 0.2\ s$ has been shown below



(a) From the diagram

$y = \frac{5}{6}\ cm$ at $x = 1\ cm$

(b) Velocity of the particle

$v = -$ (wave velocity) $\frac{\partial y}{\partial x} = -\left(10 \text{ cm s}^{-1}\right)\frac{1}{6} = -\frac{5}{3}\, cm\, s^{-1}$

6. $y = 4\, e^{k(x - vt)}$

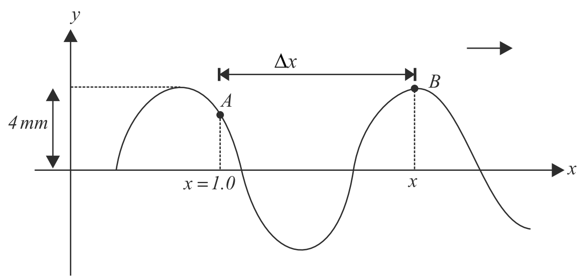
$$\frac{\partial y}{\partial x} = 4k\, e^{k(x-vt)};\ \frac{\partial^2 y}{\partial x^2} = 4k^2\, e^{k(x-vt)} \qquad \text{............(i)}$$

$$\frac{\partial y}{\partial t} = -4vk\, e^{k(x-vt)};\ \frac{\partial^2 y}{\partial t^2} = 4v^2k^2\, e^{k(x-vt)} \qquad \text{............(ii)}$$

Using (i) and (ii) one can show that $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

Similarly one can prove that the second function also obeys wave equation.

7.



We have to find x co-ordinate of particle B. Phase difference between A and B is

$$\phi = \frac{5\pi}{3} \quad \Rightarrow \frac{2\pi}{\lambda}(\Delta x) = \frac{5\pi}{3}$$

$$\Delta x = \frac{5 \times 1.5}{6} = 1.25\, m\,;\quad \therefore x_B = 2.25\, m$$

8. Speed of wave $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{57.6}{0.1}} = 24\, ms^{-1}$

Given $\frac{2\pi}{\lambda}(\Delta x) = \frac{\pi}{6}$ for $\Delta x = 0.05\, m$

$\therefore \quad \lambda = 0.6 \text{ m} \quad \therefore \quad f = \frac{v}{\lambda} = 40\, Hz$

$\therefore \quad T = \frac{1}{f} = \frac{1}{40} = 25 \text{ ms}$

Given time interval $= 5 \text{ ms} = \frac{T}{5}$

$\therefore$ Phase difference corresponding to a time interval of $\frac{T}{5}$ is $\Delta\phi = \frac{2\pi}{5}$

9. Distance between two ears $\simeq$ 20 cm

The waves reaching the two eardrums have a path difference of 20 cm.

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x) = \frac{2x}{v} f \Delta x$$

$$= \frac{2\pi \times 800}{340} \times 0.2 = \frac{16\pi}{17}$$

10. (a) $\lambda = 4$ m and $f = 50\ Hz$.

$\therefore\ V = f\lambda = 200$ m/s

$\because\ V = \sqrt{\dfrac{T}{\mu}}$

$\therefore\ T = \mu\, v^2 = (0.1) \times (200)^2\ = 400\ N$

(b) Number of waves that will cross a point is 1 second is 50, therefore power transmitted can be written as average power (If a fractional number of waves were crossing a point in a given interval then we cannot multiply average power with time to get the total energy transmitted)

$= \langle P\rangle = 2\pi^2 f^2 A^2\ \mu v^2$

$= 2 \times \pi^2 \times (50)^2 \times (2 \times 10^{-3})^2 \times (0.01) \times 200 = \dfrac{\pi^2}{25}\ J.$

(c) The equation of wave is

$y = A\ \sin(kx - \omega t + \phi_0)$

$\therefore$ where $K = \dfrac{2\pi}{\lambda} = \dfrac{\pi}{2}$, $\omega = 2\pi f = 100\ \pi$ and $A = 2$

at $x = 2$ and $t = 2$; $y = 1\ mm$

$\therefore\ 1 = 2\ \sin(\pi - 200\pi + \phi_0)$

solving $\phi_0 = -30°$

$\therefore\ y = 2\ \sin\left(\dfrac{\pi x}{2} - 100\pi t - 30°\right)$

11. Phase difference due to path difference

$$\Delta\phi_0 = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$$

(a) When sources are coherent and in phase, the phase difference at P will be π.

$\therefore\ I = I_0 + 2I_0 + 2\ \sqrt{I_0.2I_0}\cos\ \pi = 3I_0 - 2\sqrt{2}\ I_0 = 0.2I_0$

(b) When sources are 180° out of phase, the resulting phase difference at P will be zero or $2\ \pi$

$\therefore\ I = I_0 + 2I_0 + 2.\ \sqrt{I_0(2I_0)}\ \cos 0 = 3(3 + 2\sqrt{2})I_0$

(c) When sources are incoherent the intensities add

$\therefore\ I = 3I_0$

12. Path difference for two waves arriving at the point is

$\Delta x = 1.74 - 1.16 = 0.58\ m$

$\therefore$ Phase difference between the two waves arriving at the point is

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{v} f\Delta x$$

$$= \frac{2\pi \times 100}{348} \times 0.58 = \frac{\pi}{3}$$

$\therefore$ Resultant amplitude

$$A_0 = \sqrt{A^2 + (2A)^2 + 2.A.2A.\cos\left(\frac{\pi}{3}\right)} = \sqrt{7}\ A$$

14. (i) Let the fundamental frequency of the wire be f_0.

After wooden bridge is inserted, length of each segment becomes half and fundamental frequency becomes $2f_0$.

It is given that $2f_0 = 420$

$\therefore f_0 = 210\ Hz$

(ii) $A = 0.05 \cos(12.0x)$

For $x = -L/2$ and $x = L/2$, A must be zero.

This is possible if

$$-6L = -\frac{\pi}{2}, -\frac{3\pi}{2}, ...$$

$$\Rightarrow L = \frac{\pi}{12}, \frac{3\pi}{12}, ... = \frac{\pi}{12}, \frac{\pi}{4}, ...$$

15. Fundamental frequencies of the two strings are

$$f_{10} = \frac{1}{2l_1}\sqrt{\frac{T}{\rho A}}$$

$$f_{20} = \frac{1}{2l_2}\sqrt{\frac{T}{\rho(4A)}} = \frac{1}{4l_2}\sqrt{\frac{T}{\rho A}}$$

There are 3 possibilities:

(i) When first string has 1 loop and the other than 3 loops.

$f_{10} = 3f_{20}$

$$\frac{1}{2\ell_1}\sqrt{\frac{T}{\rho A}} = \frac{3}{4l_2}\sqrt{\frac{T}{\rho A}}$$

$$\therefore \quad l_2 = \frac{3}{2}l_1 = \frac{3}{2}\times 90 = 135\text{ cm}$$

(ii) When both strings have two loops

$$2f_{10} = 2f_{20} \quad \Rightarrow \frac{l}{2l_1} = \frac{l}{4l_2}$$

$$\Rightarrow l_2 = \frac{l_1}{2} = 45\text{ cm}$$

(iii) When the first string has 3 loops and the second has one loop only.

$$3f_{10} = f_{20} \Rightarrow \frac{3}{2l_1} = \frac{1}{4l_2}$$

$$\Rightarrow l_2 = \frac{l_1}{6} = 15\text{ cm}$$

16. Equation of wave is $y = 2 \sin \pi t \cos \pi x$

$$\frac{2\pi}{\lambda} = \pi \quad \Rightarrow \lambda = 2\ m$$

$x = 0\ m$ is an antinode and the next antinode is at $x = 1.0\ m$. In between node is at $x = 0.5\ m$. Two given points are on two sides of a node. Hence they differ in phase by π

(ii) Hint : The wavelength is $2L$.

17. (a) The breaking stress is $\dfrac{900}{\pi r^2} = 7.0 \times 10^8\ N/m^2$,

Solving for 'r' gives the minimum radius

$$r = \sqrt{\frac{900\ N}{\pi(7.0\times10^8\ N/m^2)}} = 6.4 \times 10^{-4}\ m.$$

The mass and density are fixed $\left(\rho = \dfrac{M}{\pi r^2 L}\right)$ so the minimum radius gives the maximum length

$$L = \frac{M}{\pi r^2 \rho} = \frac{4.0\times10^{-3}\ kg}{\pi(6.4\times10^{-4}\ m)^2(7800\ kg/m^3)} = 0.40\ m$$

(b) The fundamental frequency is $f_1 = \dfrac{1}{2L}\sqrt{\dfrac{F}{\mu}} = \dfrac{1}{2L}\sqrt{\dfrac{F}{M/L}} = \dfrac{1}{2}\sqrt{\dfrac{F}{ML}}$

The highest fundamental frequency occurs when $F = 900\ N$,

$$f_1 = \frac{1}{2}\sqrt{\frac{900N}{(4.0\times10^{-3}\ kg)(0.40m)}} = 376\ Hz.$$

18. (a) There are two nodes at the ends and one at the centre of the string.

Speed of a particle at a distance x from a node is given by-

$v = v_0 \sin\left(\dfrac{2\pi}{\lambda}x\right)$ where v_0 is the maximum speed possessed by a particle at the antinode.

$v = v_0/2$ for $x = \dfrac{\lambda}{12}, \dfrac{5\lambda}{12}, \dfrac{7\lambda}{12}, \dfrac{11\lambda}{12}$

Two nearest particles are at separation = $\dfrac{\lambda}{6} = \dfrac{L}{6}$

(b) There will be two such particles in each loop. So the answer is 4.

19. (a) Speed of wave is property of the medium. Hence, both waves have same speed.

Since frequency of second wave is double that of the first, therefore, its wavelength will be half that of the first.

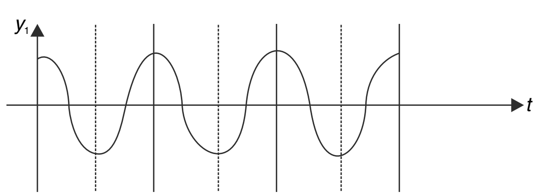
$$\frac{\lambda_1}{\lambda_2} = \frac{2}{1}$$

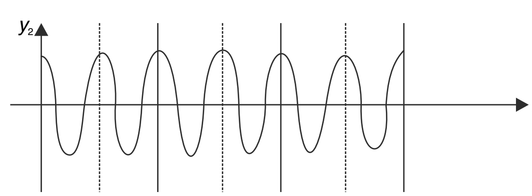
(b) At $x = 0$

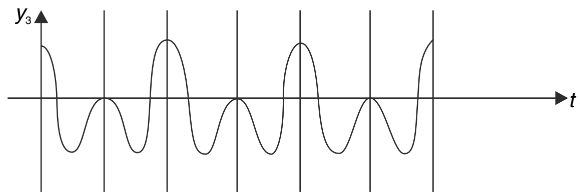
$y_1 = a\cos(\omega t)$

$y_2 = a\cos(2\omega t)$

Disturbance due to individual waves and their superposition has been shown below.







21. $f_0 = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

$$f = \frac{1}{2(L+\Delta L)}\sqrt{\frac{T+\Delta T}{\mu}} = \eta f_0$$

$$\therefore \quad \eta = \frac{L}{L+\Delta L}\sqrt{\frac{T+\Delta T}{T}} = \frac{\sqrt{1+\frac{\Delta T}{T}}}{1+\frac{\Delta T}{T}}$$

$$\therefore \quad \eta_1 = \frac{\sqrt{1-0.96}}{1-0.35} = \frac{1.4}{0.65} \text{ and } \eta_2 = \frac{\sqrt{1-0.36}}{1+0.3} = \frac{0.8}{1.3}$$

$$\therefore \quad \frac{\eta_1}{\eta_2} = \frac{1.4}{0.65}\times\frac{1.3}{0.8} = 3.5$$

22. Wave speed $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{Mg}{\mu}}$

If the string oscillates with n loops when

$M = 16\ kg$ then there will be $(n-1)$ loops for $M = 25\ kg$

$\because \quad f = \dfrac{n}{2L}\sqrt{\dfrac{T}{\mu}}$ But frequency is same in both cases,

$$\therefore \quad \frac{n}{2L}\sqrt{\frac{16g}{\mu}} = \frac{(n-1)}{2L}\sqrt{\frac{25g}{\mu}}$$

$\therefore \ 4n = 5(n-1)$

$\Rightarrow \ n = 5$

$$\therefore \quad f = \frac{5}{2\times 2}\sqrt{\frac{16\times 10}{1\times 10^{-3}}} = \frac{5}{4}\times 4\times 10^2 = 500\ Hz$$

23. $\lambda_1 = \dfrac{18}{35} = \dfrac{90}{175}\ m$

and $\lambda_2 = \dfrac{90}{173}\ m$

$\therefore \ \lambda_2 > \lambda_1 \Rightarrow f_1 > f_2$

$\therefore \ f_1 - f_0 = 4$ .... (1)

and $f_0 - f_2 = 4$ .... (2)

(1) + (2)

$f_1 - f_2 = 8$, i.e., $\dfrac{v}{\lambda_1} - \dfrac{v}{\lambda_2} = 8$

$$v\left[\frac{175}{90}-\frac{173}{90}\right]=8 \quad \Rightarrow v=\frac{8\times 90}{2}=360\ ms^{-1}$$

$$\therefore \quad f_1=\frac{v}{\lambda_1}=\frac{360\times 35}{18}=700\ Hz$$

$\therefore \quad f_0 = 696\ Hz$

25. Speed of sound in water

$$V_\omega=\sqrt{\frac{B}{\rho}}=\sqrt{\frac{2\times 10^9}{10^3}}=1414\ ms^{-1}$$

First sound is heard due to sound wave travelling in water. It takes t_1 time for sound to travel from S to M in water. Second sound is heard because of sound travelling through air reaching the man.

This sound takes t_2 time to travel through L distance.

$t_2 - t_1 = 0.8 + 0.2 \qquad$ i.e., $\dfrac{L}{V_{air}}-\dfrac{L}{V_\omega}=1.0$

[Distance $SM \simeq L$ since $d << L$]

$$\therefore \quad L=\frac{V_{air}V_\omega}{V_\omega - V_{air}}=\frac{340\times 1414}{1414-340}=447.6\ m$$

26. $v=\sqrt{\dfrac{\gamma RT}{M}}$ ....….(1)

$\dfrac{\Delta v}{\Delta T}=\dfrac{1}{2}\sqrt{\dfrac{\gamma R}{M}}\dfrac{1}{\sqrt{T}}$ ....….(2)

(2) ÷ (1) gives

$$\frac{\Delta v}{v}=\frac{1}{2}\frac{\Delta T}{T}$$

$$\frac{\Delta v}{\Delta T}=\frac{1}{2}\frac{v}{T}=\frac{1}{2}\times\frac{331}{273}=0.61\ ms^{-1}K^{-1}=0.61\ ms^{-1}\ {}^\circ C^{-1}$$

Change in temperature has same magnitude in K scale and $^\circ C$ scale.

27. (a) $V_H=\sqrt{\dfrac{\gamma RT}{M_H}}=\sqrt{\dfrac{1.4\times 8.314\times 300}{2\times 10^{-3}}}=1320\ ms^{-1}$

(b) $V_0=\sqrt{\dfrac{\gamma RT}{M_0}}$

As per question

$$\frac{\gamma RT}{M_0}=\frac{\gamma R\times 300}{M_H}$$

$$\therefore \quad T=\frac{M_0}{M_H}\times 300=\frac{32}{2}\times 300=4800\ K$$

28. $\dfrac{A_{\max}}{A_{\min}}=\dfrac{A_i+A_r}{A_i-A_r}=8$

Where A_i = amplitude of the incident wave

A_r = amplitude of the reflected wave

$\therefore \quad A_i = \frac{9}{7} A_r$

Percentage of energy transmitted $= \frac{A_i^2 - A_r^2}{A_i^2} \times 100$

$= \frac{A_i^2 - \frac{49}{81} A_i^2}{A_i^2} \times 100$

$= 39.5\%$

29. Speed of a particle on the string is proportional to the slope of the string at that location.

If $y = A \sin kx$, then -

$\frac{\partial y}{\partial x} = Ak \sin kx$

After overlapping : $y = 2A \sin kx \Rightarrow \frac{\partial y}{\partial x} = 2Ak \sin kx$

Since the slope doubles, speed of each particle doubles. It means $K.E.$ will be 4 k.

30. $P_{\max} - P_{\min} = 2\Delta P_0$

Where ΔP_0 is pressure amplitude.

$\therefore 2\Delta P_0 = 0.2$

$\Rightarrow \Delta P_0 = 0.1\ Nm^{-2} \Rightarrow BAk = 0.1$

$\Rightarrow A = \frac{0.1 \times \lambda}{2\pi B} = \frac{0.1 \times 1.0}{2 \times 3.14 \times 1.5 \times 10^5} = 1.0 \times 10^{-7} m$

31. (i) (a) $f = 2000\ Hz$ and $\lambda = 2 \times 8 = 16\ cm$

$\therefore$ Speed of sound in air in the pipe is

$v = f\lambda = 2000 \times 0.16 = 320\ ms^{-1}$

When $f' = 1600\ Hz$; $\lambda' = \frac{320}{1200} = \frac{4}{15} m = \frac{80}{3} cm$

Length of the tube is

$$L = (2n-1)\frac{\lambda}{4} = [2(n-1)-1]\frac{\lambda'}{4}$$

$$\Rightarrow (2n-1) \cdot 16 = (2n-3) \cdot \frac{80}{3} \quad \Rightarrow n = 3$$

$$\therefore L = (2n-1)\frac{\lambda}{4} = (6-1) \times \frac{16}{4} = 20\ cm$$

Fundamental frequency $f_0 = \frac{v}{4L} = \frac{320}{4 \times 0.2} = 400\ Hz$

$\therefore$ Resonant frequencies are 400 Hz, 1200 Hz, 2800 Hz …..

$\therefore$ Answer is 2800 Hz

(ii) Frequency of sound $f = \frac{750}{5} = 150\ Hz$

Loud sound is produced when resonance occurs. Let the end correction be e.

$e + 50 = \frac{\lambda}{4}$ .... (1)

And $e+157=\frac{3\lambda}{4}$ .... (2)

(2) – (1) gives : $\frac{\lambda}{2}=107$

$\lambda = 214\ cm = 2.14\ m$

$\therefore\ v = \lambda f = 2.14 \times 150 = 321\ ms^{-1}$

32. Volume of water in the cylinder is $V = 5 \times 0.2 = 1\ m^3$

Decrease in volume due to increased pressure

$\Delta V = 0.2 \times 0.25 \times 10^{-3}\ m^3 = 50 \times 10^{-6}\ m^3 = 50\ cc$

Volume strain $\frac{\Delta V}{V}=\frac{50\times10^{-6}}{1}=50\times10^{-6}$

Volume stress or Bulk stress = change in pressure

$$\Delta P=\frac{Mg}{A}=\frac{2000\times10}{0.2}=10^5\ N/m^2$$

$\therefore$ Bulk modulus of water $B=\frac{\Delta P}{\frac{\Delta V}{V}}=\frac{10^5}{50\times10^{-6}}=2\times10^9\ N/m^2$

$\therefore$ Speed of sound in water $v=\sqrt{\frac{B}{\rho}}=\sqrt{\frac{2\times10^9}{10^3}}=1414\ m/s$

33. $k = 15\ \pi$; $\omega = 21000\ \pi$

Speed of sound in sea water $V=\frac{\omega}{k}=1400\ m/s$

But $V=\sqrt{\frac{B_\omega}{\rho_\omega}}$ $\therefore$ $\rho_\omega=\frac{2\times10^9}{(1400)^2}=1.02\times10^3\ kg/m^3$

(a) $\Delta P_0 = BAk$

$$\Rightarrow \quad A=\frac{\Delta P_0}{Bk}=\frac{3000\pi}{2\times10^9\times15\pi}=10^{-7}\ m=0.1\ \mu m$$

(b) Intensity received

$$I=\frac{\Delta P_0^2}{2\rho_\omega V}=\frac{(3000\pi)^2}{2\times1.02\times10^3\times1400}=31.07\ W/m^2$$

$\therefore$ Power of source

$P = I \times 4\pi r^2 = 31.07 \times 4 \times 3.14 \times 10^2 = 3.90 \times 10^4$ Watt

34. Wave front will be a sphere centered at $(V_0T, 0,0)$ having radius $V(t-T)$

$(x - V_0T)^2 + y^2 + z^2 = V^2\ (t - T)^2$

35. (i) Let speed of winning car be v_1 and for the other car be v_2

$$360=\frac{330}{330-v_1}\times300\ ;\ v_1=55\ ms^{-1}$$

And $330=\frac{330}{330-v_2}\times300;\ \ v_2=30\ ms^{-1}$

As per the question $v_1 t - v_2 t = 1000\ m$

$$t = \frac{1000\,m}{25ms^{-1}} = 40\ s$$

(ii) (a) Let distance between the observer and source be x when the source is released at $t = 0$. If the source releases a compression pulse immediately when it is released; the pulse will reach the observer at a time $t_1 = \frac{x}{v}$.

The second pulse is released after a time $T = \frac{1}{f}$. At this time the distance between the source and the observer is $x - \frac{1}{2}gT^2$.

The second pulse will reach the observer when the time is

$$t_2 = \frac{x - \frac{1}{2}gT^2}{v} + T$$

The interval between the two successive pulses reaching the observer is the apparent time period.

$$T' = t_2 - t_1 = \frac{x - \frac{1}{2}gT^2}{v} + T - \frac{x}{v}$$

$$T' = T - \frac{\frac{1}{2}gT^2}{v}$$

$$\frac{1}{f'} = \frac{1}{f} - \frac{\frac{1}{2}g\left(\frac{1}{f}\right)^2}{v} \quad \Rightarrow \quad f' = \frac{2vf^2}{2vf - g}$$

This frequency will be same for both the observer.

Part (b) can be solved with similar approach

36. (a) Frequency received by the reflector is

$$f = f_0\left[\frac{V + V_2}{V - V_1}\right]$$

$$= 1200\left[\frac{330 + 60}{330 - 60}\right] = 1200 \times \frac{390}{300} = 1560\ Hz$$

(b)

$$\lambda_1 = \frac{V - V_1}{f_0} = \frac{330 - 30}{1200} = \frac{1}{4}\ m$$

$$\lambda_2 = \frac{V - V_2}{f} = \frac{330 - 60}{1560} = \frac{27}{156} = \frac{9}{52}\ m$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{1}{4} \times \frac{52}{9} = \frac{13}{9}$$

37. (a) $f = 200\left[\frac{340 - 10}{340 - 20}\right] = 206.25\ Hz$

No. of times maximum change in pressure is sensed by the observer in 10 s is

= 206.25 × 2 × 10 [In each cycle, the change in pressure is maximum twice]

= 4125

(b) Wavelength changes due to motion of source

$$\lambda = \lambda_0 - V_s T = \frac{340}{200} - 20 \times \frac{1}{200} = 1.7 - 0.1 = 1.6\ m$$

38.

(a) Source and observer are travelling in same direction with same speed. Hence there will be no change in frequency. Frequency received will be from 900 Hz to 1200 Hz.

(b) Lower frequency $f_1 = f_0\left[\frac{330 + 30}{330 - 30}\right] = 900 \times \left[\frac{360}{300}\right] = 1080\,Hz$

Higher frequency $f_2 = 1200 \times \left[\frac{360}{300}\right] = 1440\,Hz.$

39. The frequencies of two forks are assumed to be f and $f + 4$.

When forks are moving towards the observer, the frequencies received by the observer are

$$f_1 = f\left(\frac{v}{v - u}\right) \text{ and } f_2 = (f + 4)\left(\frac{v}{v - u}\right)$$

$$\therefore \quad \Delta f = f_2 - f_1 = 4\left(\frac{v}{v - u}\right)$$

$$\therefore \quad 5 = 4\left(\frac{v}{v - u}\right)$$

$$\Rightarrow \quad 5v - 5u = 4v \quad \Rightarrow \quad u = \frac{v}{5}$$

When observer also begins to run, the frequencies received by him will be

$$f_1' = f\left(\frac{v + u}{v - u}\right) \text{ and } f_2' = (f + 4)\left(\frac{v + u}{v - u}\right)$$

$$\therefore \quad \Delta f' = f_2' - f_1' = 4\left(\frac{v + u}{v - u}\right)$$

$$= 4\left(\frac{v + \frac{v}{5}}{v - \frac{v}{5}}\right) = 4 \cdot \frac{6}{4} = 6\,Hz$$

40. (i) The wavelength will decrease in front of the source and it will increase behind the source

(ii) The first figure has higher average wave frequency but lower beat frequency. Thus the first graph represents superposition of 300 *Hz* and 300 + Δf. The other graph has a beat frequency of 4 *Hz* = (204 – 200). Therefore, $\Delta f < 4$ *Hz*.

42. (a) If the origin is assumed be at $x = 1$ and time is reset to be zero at $t = 1s$, the equation of wave travelling in negative x will be

$y' = \sin(\omega t + kx)$

$$= 0.01\sin\left[2\pi.20t + \frac{2\pi}{\frac{\pi}{2}}.x\right] = 0.01\sin[40\pi t + 4x] \quad \ldots\ldots\ldots\ldots\text{(i)}$$

In the given co-ordinate system, the displacement of a particle at $x = 1$ *m* at $t = 1$ *s* is represented by the value of y obtained in (i) after putting $t = 0$ and $x = 0$

$\therefore$ In given system

$y = 0.01 \sin [40\,\pi\,(t-1) + 4\,(x-1)]$

(b) once again if origin is taken at $x = 1$ *m* and time is reset to be zero at $t = 1$ *s,* then the equation of the wave travelling in positive x can be written as follows:

$y' = 0.01 \sin [40\,\pi\,t - 4x]$

[Note that $\frac{dy'}{dt}$ at $t = 1$, $x = 1$ gives a positive value meaning that the velocity of the particle is positive]

Once again adjusting for shift in origin of space and time -

$y = 0.01 \sin [40\,\pi\,(t-1) - 4\,(x-1)]$

43. First let us calculate the tension in the wire loop. Consider a small element of angular width $d\theta$ on the loop.

$$2T\sin\frac{d\theta}{2} = (\mu R d\theta)\omega^2 R$$

$$\therefore 2T\frac{d\theta}{2} = \mu R d\theta \omega^2 R \Rightarrow T = \mu R^2\omega^2$$

Speed of transverse wave relative to the string is $V = \sqrt{\frac{T}{\mu}} = \omega R$

Relative to the ground, the pulse will travel with speed $2V$ (or angular speed 2ω)

$\therefore$ Time required to reach back at P is

$$t = \frac{\pi}{\omega}$$

[Note: The pulse that starts travelling in direction opposite to the direction of rotation of the ring will remain static at P]

44. $y_1 = a\sin(kx - \omega t) \qquad \left[k = \frac{\pi}{2}\right]$

$$y_2 = a\sin\left(kx + \omega t + \frac{\pi}{3}\right)$$

$$y = y_1 + y_2 = a\left[\sin(kx - \omega t) + \sin\left(kx + \omega t + \frac{\pi}{3}\right)\right]$$

$$= 2a\sin\left(kx + \frac{\pi}{6}\right).\cos\left(\omega t + \frac{\pi}{6}\right)$$

Node are positions where $A = 2a\sin\left(kx + \frac{\pi}{6}\right) = 0$

$\Rightarrow \frac{\pi}{2}x + \frac{\pi}{6} = \pi, 2\pi, 3\pi.........$ [for $x > 0$]

$\Rightarrow \frac{x}{2} = \left(1 - \frac{1}{6}\right), \left(2 - \frac{1}{6}\right), \left(3 - \frac{1}{6}\right)..........$

$= \frac{5}{6}, \frac{11}{6}, \frac{17}{6}........$

$\therefore x = \frac{5}{3}, \frac{11}{3}, \frac{17}{3}, \frac{23}{3}$ .....,

We have not considered negative value of x as those are not required.

Obviously, $x = \frac{5}{3}, \frac{11}{3}$ and $\frac{17}{3}$ lie between $x = 1$ and $x = 6\ m$

So there are 3 nodes between $x = 1$ and $x = 6\ m$.

45. At $x = -3$, $x = 1$, $x = 5$ we have nodes. Two consecutive nodes are at $-\frac{\lambda}{2}$ separation.

$\therefore \frac{\lambda}{2} = 4 \Rightarrow \lambda = 8\,cm$

$k = \frac{2\pi}{\lambda} = \frac{\pi}{4} rad/cm$

Also, wave speed $V = \frac{\omega}{k}$

$\therefore \omega = Vk = 10 \times \frac{\pi}{4} = \frac{5\pi}{2}\ rad/s$

As shown in the figure, all particle in the string are at their extreme at $t = 0$. Hence the equation of standing wave is like

$y = A \cos \omega t$ [At $t = 0$, $y = A$ for all particles]

Where A is the amplitude which changes harmonically with x

Let $A = A_0 \sin (kx + \delta)$

From figure $A_0 = 0.5\ mm$

At $x = 1$; $A = 0$

$\therefore k(1) + \delta = \pi$ [$kx + \delta \neq 0$ at $x = 1$. Why ?]

$\Rightarrow \frac{\pi}{4} + \delta = \pi \quad \Rightarrow \delta = \frac{3\pi}{4}$

$\therefore A = 0.5 \sin\left(\frac{\pi}{4}x + \frac{3\pi}{4}\right)$

And $y = 0.5\sin\left(\frac{\pi}{4}x + \frac{3\pi}{4}\right)\cos\omega t$

$$y = 0.5\sin\left(\frac{\pi}{4}x + \frac{3\pi}{4}\right)\cos\frac{5\pi}{2}t$$

$$= 0.25\sin\left(\frac{\pi}{4}x + \frac{5\pi}{2}t + \frac{3\pi}{4}\right) + 0.25\sin\left(\frac{\pi}{4}x - \frac{5\pi}{2}t + \frac{3\pi}{4}\right)$$

46. (ii) Let the equation of the wave be $y = 0.2\cos\left(3\pi t - \frac{2\pi}{\lambda}x\right)$

Obviously phase angle at $x = 1$ is shifted by $\frac{\pi}{8} - 2n\pi \quad (n = 0, 1, 2.....)$

As compared to the phase at $x = 0$.

$$\therefore \quad -\frac{2\pi}{\lambda} = \frac{\pi}{8} - 2n\pi \quad (n = 1, 2.....)$$

$$\Rightarrow \quad \lambda = \frac{16}{16n - 1}$$

For $n = 1$; $\lambda_1 = \frac{16}{15}\,m$

For $n = 2$; $\lambda_2 = \frac{16}{31}\,m$

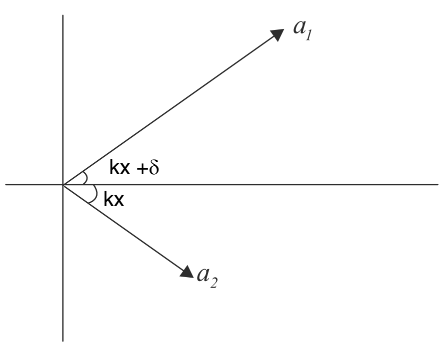
For $n = 3$; $\lambda_3 = \frac{16}{47}\,m$

Only λ_1 and λ_2 are within given limit of $\lambda > 0.4\,m$

47. (i) The two waves can be represented by phasors as shown in figure.

The resultant amplitude is

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(2k + \delta)}$$



(a) No, amplitude does not depend on time.

$A_{max} = a_1 + a_2$ [When cos $(2\,kx + \delta) = 1$]

$A_{min} = |a_1 - a_2|$ [when cos $(2kx + \delta) = -1$

$$\frac{A_{max}}{A_{min}} = \frac{a_1 + a_2}{|a_1 - a_2|}$$

48. The instantaneous power transmitted is proportional to the magnitude of the slope of the y versus x graph.

49. (a) $\mu = \pi\,(0.2 \times 10^{-3}\,m)^2\,(7962\,kg/m^3) = 1.0 \times 10^{-3}\,kg/m$

Fundamental frequency of a string, $f = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$

Rearranging this and solving for F gives

$F = 4\,\mu\, L^2 f^2$

$F = 4\,(1.0 \times 10^{-3}\ kg/m)\,(0.635\ m)^2\,(247.0\ Hz)^2 = 98.4\ N.$

(b) $f = \dfrac{1}{2L}\sqrt{\dfrac{F}{\mu}}$ …………….. (1)

$$\Delta f = \Delta\left(\frac{1}{2L}\sqrt{\frac{F}{\mu}}\right) = \Delta\left(\frac{1}{2L\sqrt{\mu}}F^{\frac{1}{2}}\right),$$

$$\Delta f = \frac{1}{2L\sqrt{\mu}}\Delta\left(F^{\frac{1}{2}}\right)$$

$$\Delta f = \frac{1}{2L\sqrt{\mu}}\frac{1}{2}\frac{\Delta F}{\sqrt{F}}.$$

Now divide this equation by (1)

$$\frac{\Delta f}{f} = \frac{\dfrac{1}{2L\sqrt{\mu}}\dfrac{1}{2}\dfrac{\Delta F}{\sqrt{F}}}{\dfrac{1}{2L}\sqrt{\dfrac{F}{\mu}}}$$

$$\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta f}{F}$$

(c) Stress = Y (strain)

$$\frac{\Delta F}{A} = -Y\alpha\Delta T$$

$\Delta F = -\,(2.00 \times 10^{11} Pa)\,(1.20 \times 10^{-5}/°C) \times \pi\,(0.2 \times 10^{-3} m)^2 \times (11°C) = -\,3.3\ N.$

$\Delta F/F = -\,3.3/98.4 = -\,0.033,$

$$\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta F}{F}$$

$\Delta f/f = -\,0.017,$

$\Delta f = -0.017 \times 247 = -\,4.2\ Hz.$

Negative sign indicates that the tension falls and the frequency also falls.

50. $P_0 = \dfrac{1}{2}\mu\omega^2 a_0^2\, v = 2\pi^2 \mu f^2 a_0^2 v$

At distance x, $a = a_0 e^{-kx}$

$\therefore P = 2\pi^2 \mu f^2 \left(a_0 e^{-kx}\right)^2 v$

Given $P = \dfrac{P_0}{4}$

$$\Rightarrow e^{-2kx} = \frac{1}{4} \Rightarrow -2kx = \ell n(1) - \ell n\,(4) \Rightarrow x = \frac{\ell n 2}{k}$$

51. (a) Wave speed $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{50}{0.02}} = \sqrt{2500} = 50\ ms^{-1}$

Power of oscillator = average power transmitted through a cross section $= \dfrac{1}{2}\mu\omega^2 a^2 v$

$\therefore\ P = 2\pi^2\mu f^2 a^2 v$ ....(i)

$P = 2 \times (3.14)^2 \times (0.02) \times (80)^2 \times (1 \times 10^{-3})^2 \times 50 = 0.126\ W$

(b) If tension is made 4 times the wave speed becomes double.

To keep the power same amplitude will become $\frac{1}{\sqrt{2}}$ times , i.e., $\frac{1}{\sqrt{2}} mm$

(c) $P_{av} = \frac{1}{2}\mu\omega^2 A^2 v$

Average energy on a segment of length L is

= (Average power) × (time required for the wave to travel a distance L)

$E_{av} = P_{av}\frac{L}{v} = 0.126 \times \frac{1.0}{50} = 0.0025 J = 2.5 mJ$

52. Energy of ball $E = mgh = 5 \times 10^{-3} \times 10 \times 2 = 0.1\ J$

Energy of sound pulse $= \frac{0.001}{100} \times 0.1 = 10^{-6} J$

Power of the sound source $P = \frac{10^{-6}}{0.4}\frac{J}{s} = 2.5 \times 10^{-6}\ W$

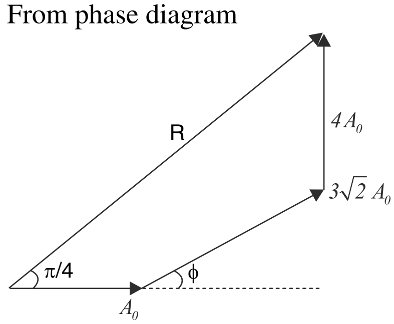
Intensity at a distance r from a point source is $I = \frac{P}{4\pi r^2}$

Given $I = 2 \times 10^{-8}\ Wm^{-2}$

$\therefore r^2 = \frac{P}{4\pi I} = \frac{2.5 \times 10^{-6}}{4 \times 3.14 \times 2 \times 10^{-8}} = 9.95$

$\therefore\ r = \sqrt{9.95} = 3.15\ m$

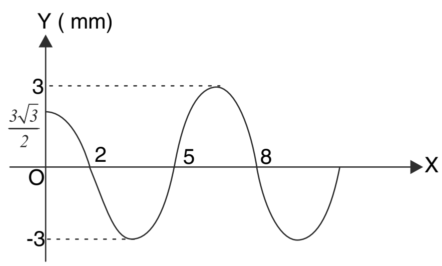
53. From phase diagram



$$\tan\left(\frac{\pi}{4}\right) = \frac{A_0\left(4 + 3\sqrt{2}\sin\phi\right)}{A_0\left(1 + 3\sqrt{2}\cos\phi\right)}$$

Solving, $\phi = \frac{\pi}{12}$

54. (a) $\lambda = 6\ m$



$$\therefore k = \frac{2\pi}{\lambda} = \frac{\pi}{3}$$

From the graph $T = 4s$

$$\therefore \omega = \frac{2\pi}{T} = \frac{\pi}{2}$$

Let the equation of the wave be

$$y = (3mm)\sin\left(\frac{\pi}{3}x - \frac{\pi}{2}t + \delta\right)$$

At $x = 2$; $t = 0$, y is zero

$$\therefore \sin\left(2\frac{\pi}{3} + \delta\right) = 0$$

$$\Rightarrow \delta = -\frac{2\pi}{3} \text{ or } \frac{\pi}{3}$$

Also, $\frac{\partial y}{\partial t} > 0$ at $t = 0$ at $x = 2$

$$\frac{\partial y}{\partial t} = -3\frac{\pi}{2}\cos\left(\frac{\pi}{3}x - \frac{\pi}{3}t + \delta\right)$$

At $t = 0$, $x = 2$

$$\frac{\partial y}{\partial t} = -\frac{3\pi}{2}\cos\left(\frac{2\pi}{3} + \delta\right)$$

for this to be positive

$$\cos\left(\frac{2\pi}{3} + \delta\right) < 0$$

$$\therefore \delta = \frac{\pi}{3} \text{ [and } \delta \neq -\frac{2\pi}{3}\text{]}$$

$$\therefore y = (3mm)\sin\left(\frac{\pi}{3}x - \frac{\pi}{2}t + \frac{\pi}{3}\right)$$

(b) at $t = 0, y = 3\sin\left(\frac{\pi}{3}x + \frac{\pi}{3}\right)$

55. Speed of wave $v = \sqrt{\frac{T}{\mu}}$

Hence speed of wave in heavier string is half that in the lighter string.

Speed of wave in SQ is $v = \lambda f$

$\therefore$ $v = (1\ cm)\ (50\ Hz) = 50\ cm/s$

Speed in heavier string $= \frac{v}{2} = 25\,cm/s$

Wavelength in heavier string $= \frac{1cm}{2} = 0.5\,cm$

Phase of point Q is ahead of S by

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{(1cm)}(9.5\,cm) = (9.5)(2\pi)$$

$\therefore$ A point that will oscillate in phase with S must be ahead in phase by (0.5) (2 π) with respect to Q.

If x is distance of this point from Q then -

$$\frac{2\pi}{0.5\,cm}x = (0.5)(2\pi)$$

$\therefore$ $x = 0.25$ cm

Time required for the disturbance to reach this point is $\frac{9.5}{50} + \frac{0.25}{25} = 0.19 + 0.01 = 0.20$ s

57. Wave speed along the string $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{40}{0.025}}$ $\left[\because \mu = \frac{0.1625}{6.5} = 0.025\,kg/m\right]$

$= 40\ m/s$

Frequency $f = 20$ Hz

Wavelength; $\lambda = \frac{v}{f} = 2m$ and $k = \frac{2\pi}{\lambda} = \pi\, m^{-1}$

Wave propagating along the string can be expressed as

$y = a \sin(40\pi t - \pi x)$ [$x = 0$ at source]

Displacement at P due to reflected wave at $t = 0.3$ s is [$x = 6.5 + 3.75 = 10.25$ m]

$y_1 = a \sin(40\pi \times 0.3 - 10.25\,\pi + \pi) = a\sin(2.75\pi) = \frac{a}{\sqrt{2}}$

['π' due to phase change during reflection]

Displacement at P due to direct wave at $t = 0.3$ s is

$y_2 = a \sin(40\pi \times 0.3 - \pi \times 2.75) = a\sin(9.25\pi) = -\frac{a}{\sqrt{2}}$

$\therefore$ Displacement at P at $t = 0.3$ s is $y = y_1 + y_2 = 0$

58. Displacement of the medium particles at the instant shown can be represented by - $s = a\sin\left(\frac{2\pi}{12}x\right) = a\sin\left(\frac{\pi}{6}x\right)$

The displacement of the particle at $x_1 = 1$ cm is $s_1 = a\sin\left(\frac{\pi}{6}.1\right) = \frac{a}{2} = 0.4\,cm$

and displacement of the particle at $x_2 = 3$ cm is $s_2 = a\sin\left(\frac{\pi}{6}.3\right) = a = 0.8$ cm

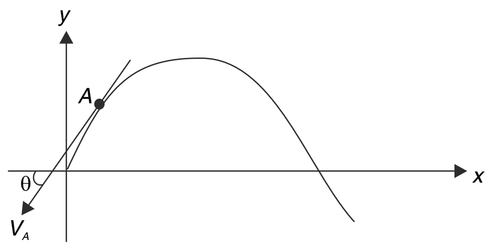
Required separation is $= x_2 + s_2 - (x_1 + s_1)$

$= 3 + 0.8 - (1 + 0.4) = 2.4$ cm

59. To the observer, the wave (disturbance) does not appear to propagate. The string moves backwards. Every point on the string appears to be moving tangentially. At time $t = 0$

$$y = a\sin\left(\frac{2\pi}{\lambda}x\right);\ \frac{dy}{dx} = a\frac{2\pi}{\lambda}\cos\left(\frac{2\pi}{\lambda}x\right)$$

$$\therefore \tan\theta = a.\frac{2\pi}{\lambda}\cos\left(\frac{2\pi}{\lambda}\frac{\lambda}{6}\right);\ \tan\theta = \frac{a\pi}{\lambda}$$



60. (a) Velocity at a point is perpendicular to the tension. Hence power is zero.

(b) Velocity at all points is zero. Hence power is zero.

(c) $y = a\sin kx \cos\omega t;\ P = -\left(T\dfrac{\partial y}{\partial x}\right)\left(\dfrac{\partial y}{\partial t}\right)$

Where the first component is transverse component of tension and the term in second bracket is the velocity at a point.

P is positive when energy is transmitted in the direction of positive x. When energy is transferred in other direction P is negative.

The (–) sign in the above expression is to ensure this sign convention.

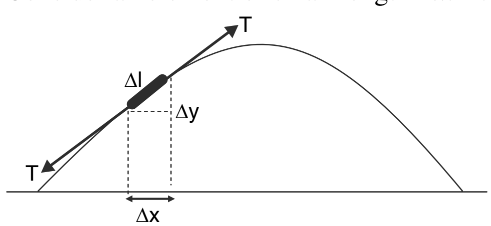
$\therefore\ P = -T(kA\sin\omega t\cos kx)(\omega A\cos\omega t\sin kx) = -\dfrac{T}{4}A^2\ \omega k(\sin 2kx)(\cos 2\omega t)$

Hence frequency of oscillation of P at a given point is twice that of the wave.

Wave frequency, $f = \dfrac{\omega}{2\pi}$

Power frequency $= \dfrac{\omega}{\pi}$

61. Consider an element of small length Δx. At some instant it is stretched to length Δl.



Extension $= \Delta l - \Delta x = (\Delta x^2 + \Delta y^2)^{\frac{1}{2}} - \Delta x$

$$= \Delta x\left[1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right]^{1/2} - \Delta x \simeq \Delta x\left[1 + \frac{1}{2}\left(\frac{\Delta y}{\Delta x}\right)^2\right] - \Delta x$$

[Because of small amplitude the slope $\dfrac{\Delta y}{\Delta x}$ is small]

$\therefore$ Extension $\simeq \dfrac{1}{2}\left(\dfrac{\Delta y}{\Delta x}\right)^2 \Delta x \simeq \dfrac{1}{2}\left(\dfrac{\partial y}{\partial x}\right)^2 \Delta x$

Elastic PE = work done by tension in stretching = Tension × extension

$$\Delta U = T\frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 \Delta x = \frac{1}{2}\mu v^2\left(\frac{\partial y}{\partial x}\right)^2 \Delta x \quad \left[\because\ v = \sqrt{\frac{T}{\mu}}\right]$$

$$= \frac{1}{2}\mu\frac{\omega^2}{k^2}a^2k^2 \cos^2 (kx - \omega t). \Delta x$$

At $x = 0$, $\Delta U = \frac{1}{2}\mu\omega^2 a^2 \Delta x \cos^2(\omega t)$

At $t = 0$, $\Delta U = \frac{1}{2}\mu\omega^2 a^2 \Delta x$

Kinetic energy $\Delta K = \frac{1}{2}(\mu\Delta x)\left(\frac{\partial y}{\partial t}\right)^2 = \frac{1}{2}\mu\,\Delta\,x\,a^2\omega^2 \cos^2(kx - \omega t)$

At $x = 0$; $t = 0$ $\Delta K = \frac{1}{2}\mu a^2\omega^2\Delta x$

62. From the last problem

$$\frac{\Delta U}{\Delta x} = \frac{1}{2}\mu v^2\left(\frac{\partial y}{\partial x}\right)^2 = \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2$$

$$\frac{\Delta K}{\Delta x} = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2 = \frac{1}{2}\mu\left(-v\frac{\partial y}{\partial x}\right)^2 \qquad [\because \text{ particle velocity } V_p = -v\frac{\partial y}{\partial x}]$$

$$= \frac{1}{2}\mu v^2\left(\frac{\partial y}{\partial x}\right)^2 = \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2$$

$$\therefore \frac{\Delta E}{\Delta x} = \frac{\Delta U + \Delta K}{\Delta x} = T\left(\frac{\partial y}{\partial x}\right)^2$$

Obviously kinetic energy, potential energy and total energy is largest for the element where the slope is steepest, i.e., for element 2.

63. (a) wave speed $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{160}{0.1}} = 40\ m/s$

If R is node and segment BR has n loops then the segment AR must have $2n$ loops (as $AR = 2BR$)

$\therefore n\frac{\lambda}{2} = 0.20\ m$; $n.\frac{v}{f} = 0.4$

$f = n \times 100\ Hz$ for $n = 1, 2, 3, 4$,.............9

We get $f = 100, 200, 300$,..........., 900 Hz

(b) If R is antinode then

$BR = n\frac{\lambda}{2} + \frac{\lambda}{4}$ [$n = 0,1,2,$.....]

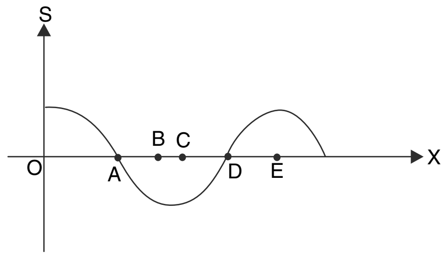
$AR = 2.BR = 2n\frac{\lambda}{2} + \frac{\lambda}{2}$

$\therefore AB = 3n\frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{4} = (3n+1)\frac{\lambda}{2} + \frac{\lambda}{4}$

But length of string can only be integral multiple of $\frac{\lambda}{2}$

Hence, it is not possible to have antinode at R.

64. (a) Displacement position graph will be as shown (at $t = 0$)



Velocity of a particle is $V = -V_0\left(\frac{\partial S}{\partial x}\right)$ [V_0 = wave velocity]

At A and B slope $\frac{\partial S}{\partial x}$ is negative, hence V_A and V_B are positive.

(b) Displacement wave equation can be written as

$$S = a\cos\left(\frac{2\pi}{\lambda}x - \omega t\right)$$

$$|V_D| = a\omega\,;\ |V_B| = \left|\frac{dS}{dt}\right|_{x=0.1875}^{t=0} = a\omega\sin\left(\frac{2\pi}{0.5}\times 0.1875\right) = \frac{a\omega}{\sqrt{2}}$$

$$\therefore \left|\frac{V_B}{V_D}\right| = \frac{1}{\sqrt{2}}$$

65. (i) $V_{H_2} = \sqrt{\frac{\gamma RT}{M_{H_2}}}\,;\ V_{O_2} = \sqrt{\frac{\gamma RT}{M_{O_2}}}$

$$\therefore \frac{V_{H_2}}{V_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4$$

$$\therefore V_{O_2} = \frac{1200}{4} = 300\ m/s$$

$$\lambda_{H_2} = \frac{V_{H_2}}{f} = \frac{1200}{600} = 2.0\,m \text{ and } \lambda_{O_2} = \frac{V_{O_2}}{f} = \frac{300}{600} = 0.5\,m$$

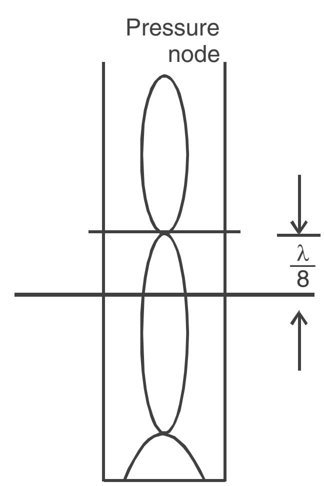
A and C are in same phase.

$\therefore$ Phase difference between D and B will be $\Delta\phi = \frac{2\pi}{\lambda_{O_2}}L - \frac{2\pi}{\lambda_{H_2}}L$

$$= 2\pi L\left[\frac{1}{\lambda_{O_2}} - \frac{1}{\lambda_{H_2}}\right] = 2\pi\times 0.3\left[\frac{1}{0.5} - \frac{1}{2.0}\right] = 0.9\pi \text{ rad}$$

(ii) (a) $L = \frac{5}{4}\lambda = \frac{5V}{4V} = \frac{5\times 330}{4\times 440} = 93.75\ cm$

(b) Distance of mid point from closest Node = $\frac{\lambda}{8}$



$$\Delta P = \Delta P_0 \sin kx = \Delta P_0 \sin\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right) = \frac{\Delta P_0}{\sqrt{2}}$$

At the closed end $P_{max} = P_0 + \Delta P_0\,;\ P_{min} = P_0 - \Delta P_0$

66. 1080 *km/hr* = 300 *m/s*

$$V=\sqrt{\frac{\gamma RT}{M}}$$

$$\therefore T=\frac{V^2M}{\gamma R}=\frac{(300\,ms^{-1})^2(29\times10^{-3}\,kg\,mol^{-1})}{1.4\times8.31J\,mol^{-1}\,k^{-1}}=224\,K$$

$T = 273 - (0.006)h_0$ $\quad\therefore 224 = 273 - (0.006)h_0$

$h_0 = 8167\ m$

$$P=P_0\left(1-\frac{0.006\times8167}{273}\right)^{\frac{29\times10^{-3}\times9.8}{8.31\times0.006}} = P_0\,(1-0.18)^{5.7}=(0.82)^{5.7}\,P_0=0.32\,P_0$$

67. When reservoir is lowered by x, let the level of water fall by y

$$x-\frac{y}{6}=y \quad \therefore x=\frac{7y}{6} \quad \Rightarrow y=\frac{6x}{7}$$

For $x = 21\ cm$, $y_1 = 18\ cm$

For $x = 21 + 49 = 70\ cm$; $\quad y_2 = 60\ cm$

$$\therefore \frac{\lambda}{4}+e=18 \quad \text{..............(i)}$$

$$\frac{3\lambda}{4}+e=60 \quad \text{..............(ii)}$$

(ii) – (i) gives

$$\frac{\lambda}{2}=42\Rightarrow \lambda=84\,cm=0.84\,m$$

$\therefore V = \lambda f = 0.84 \times 400 = 336\ ms^{-1}$

68. (ii) $\Delta P=-B\dfrac{\partial s}{\partial x}=-\,BA\dfrac{3\pi}{L}\sin\left(\dfrac{3\pi x}{L}\right)\sin\left(\dfrac{3\pi vt}{L}\right)$

At $t=\dfrac{L}{18v}$ and $x=\dfrac{L}{2}$

$$\Delta P=-BA\frac{3\pi}{L}\sin\left(\frac{3\pi}{2}\right)\sin\left(\frac{\pi}{6}\right)=BA\frac{3\pi}{2L}$$

As per the question $BA\dfrac{3\pi}{2L}=0.0002\,P_0$

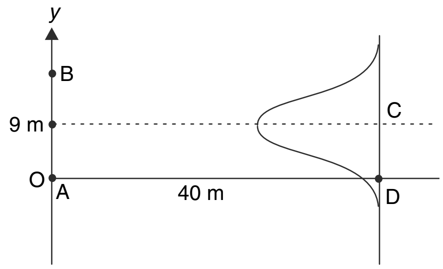
$$\therefore A=\frac{0.0004L\,P_0}{3\pi B}=\frac{0.0004L\,P_0}{3\pi v^2\rho}$$

69. At C we have maxima (zero path difference). Conditions in the question can be met if the detector (D) lies between C and the first minima. The figure shows the variation of intensity around C.

The path difference $\Delta x = BD - AD$ must be less than $\dfrac{\lambda}{2}$

$BD=\sqrt{9^2+40^2}=41m$; $\quad\therefore\ \Delta x=41-40=1\,m$

$$\therefore 1<\frac{\lambda}{2}\Rightarrow 2<\frac{V}{f}\Rightarrow f<\frac{V}{2}$$



$f < \frac{340}{2}$; $f < 170\,Hz$

70. Let amplitude of wave on arc m and p be a due to individual sources

Amplitude on arc ℓ due to $S_1 = 2a$

Amplitude on arc q due to $S_2 = \frac{a}{2}$

At point A : $\Delta\phi = \frac{2\pi}{\lambda}\Delta x + \pi = \frac{2\pi}{4}(2) + \pi = 2\pi$

$\therefore a_A = \sqrt{a^2 + \left(\frac{a}{2}\right)^2 + 2.a.\frac{a}{2}.\cos 2\pi} = \frac{\sqrt{5}}{2}a$

At B ; $\Delta\phi = \pi$ [$\because$ no path difference]

$\therefore a_B = 0$

At C ; $\Delta\phi = \frac{2\pi}{4}(1) + \pi = \frac{3\pi}{2}$

$$\therefore a_c = \sqrt{a^2 + (2a)^2 + 2.a.2a\cos\left(\frac{3\pi}{2}\right)} = \sqrt{5}a$$

At D : $\Delta\phi = \frac{2\pi}{4}(3) + \pi = \frac{3\pi}{2} + \pi = \frac{5\pi}{2}$

$$a_D = \sqrt{\left(\frac{a}{2}\right)^2 + (2a)^2 + 2\left(\frac{a}{a}\right)(2a)\cos\frac{5\pi}{2}} = \frac{\sqrt{17}}{2}a$$

71. (a) Molar mass of air

$M_{air} = 0.75 \times M_N + 0.25 \times M_O$

$= 0.75 \times 28 + 0.25 \times 32$

$= 29\ g/mol = 29 \times 10^{-3}\ kg/mol$

Because, N_2 and O_2 both are diatomic hence $\gamma = 1.4$

Speed of sound in air at 300 K

$$V = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.3 \times 300}{29 \times 10^{-3}}} = 347\,m/s$$

$\therefore$ Wavelength of $5kHz$ sound will be $\lambda = \frac{347}{5000} = 0.069\,m = 6.9\,cm$

In first overtone mode, the length of the tube $(L) = \lambda = 6.9\ cm$

(b) In second overtone mode

$$L = \frac{3\lambda}{2} \quad \Rightarrow \lambda = \frac{2 \times 6.9}{3} cm = 4.6\,cm$$

$$\therefore \quad f = \frac{V}{\lambda} = \frac{347}{4.6 \times 10^{-2}} = 7543\,Hz$$

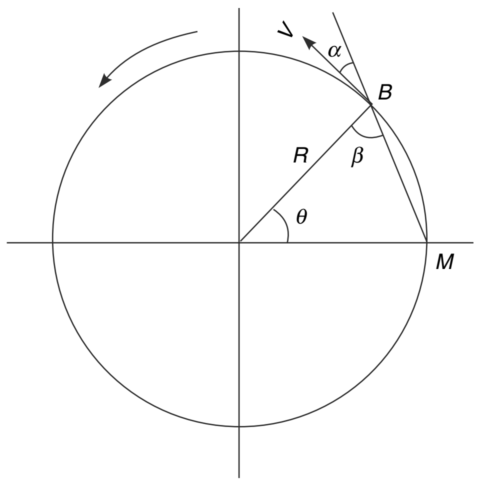
72. Beat frequency Δf must be less than 1 in 10 s.

$\Rightarrow \Delta f < 0.1\ Hz$

$\therefore$ Difference in frequencies of the two sources is less than 0.1 Hz.

$\therefore$ 1023.9 $Hz < f_{\text{string}} < 1024.1\ Hz$

73. Let the position of Buzzer at time t be B and that of the mic be M.

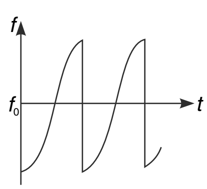


$$V = \omega R\ ;\ \theta = \omega t\ ;\ \beta = \frac{\pi - \theta}{2}$$

$$\alpha = \frac{\pi}{2} - \beta = \frac{\theta}{2} = \frac{\omega t}{2}$$

$$\therefore \quad f = f_0 \left[\frac{V_0}{V_0 + V cos\alpha}\right] = f_0 \left[\frac{V_0}{V_0 + \omega R cos\left(\frac{\omega t}{2}\right)}\right]$$

graph is as shown



74. (i) Frequency observed by the observer is

$$f' = f\left(\frac{v + v_0}{v}\right) \quad \therefore \omega' = \omega\left(1 + \frac{v_0}{v}\right) = \omega + \frac{\omega}{v} v_0 = \omega + k v_0$$

Amplitude and wavelength do not change for moving observer.

$\therefore\ y_0 = a \sin\left[(\omega + k v_0)\, t - kx\right]$

(ii) $f = \frac{V - V_R}{V + V_R} f_0$ where V and V_R are speed of sound and speed of red blood cells respectively.

$$\Delta f = f_0 - f = f_0 - \frac{V - V_R}{V + V_R} f_0 = \frac{2V_R}{V + V_R} f_0 \simeq \frac{2V_R}{V} f_0 \ \ldots\ldots \text{(i)}$$

$$\Rightarrow V = \frac{2 \times 0.1}{600} \times 5 \times 10^6 = 1700\ m/s$$

(b) The continuity equation in fluid mechanics tells us that V_R will get double if cross sectional area becomes half. From (i) Δf will become 2 times.

76. $f_1 = f_2 \pm 3.5$ ....(1)

Consider wall as observer. Frequency received by the wall is

$$f' = f_2\left(\frac{332}{332 - 5}\right)$$

Now wall acts as a source of frequency f'. Frequency of echo received by the observer is

$$f'' = f'\left(\frac{332 + 5}{332}\right) = f_2\left(\frac{332 + 5}{332 - 5}\right) = f_2\left(\frac{337}{327}\right)$$

Given $f'' - f_2 = 5 \therefore\ f_2\left[\frac{337}{327} - 1\right] = 5$

$$f_2 = \frac{327}{2} = 163.5\ Hz$$

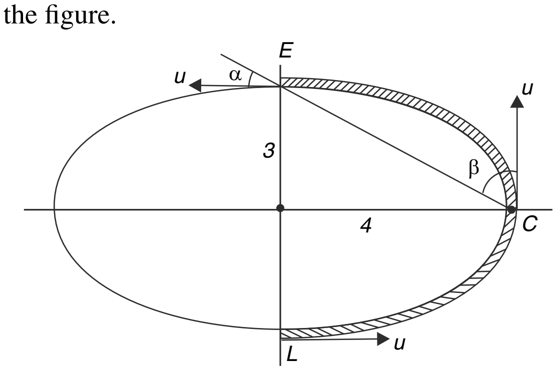
<u>From (1)</u>

$f_1 = 163.5 \pm 3.5 = 167$ or $160\ Hz$

When frequency of H1 decreases, it gives more beats with $H2$.

$\therefore f_1 = 160\ Hz$

77. Acceleration of the engine is minimum at position where radius of curvature is maximum i.e., at position E shown in the figure.



In the figure - E, C and L are engine, centrally located passenger and the last passenger respectively.

Speed of the train $u = 20\ ms^{-1}$

(a) Since observer and source both are moving perpendicular to the line joining them there is no change in frequency.

(b) Frequency received by C is

$$f = f_0\left(\frac{V + u\cos\beta}{V + u\cos\alpha}\right) = 3460\left[\frac{330 + 20\times\frac{3}{5}}{330 + 20\times\frac{4}{5}}\right]$$

$$= 3460\times\frac{342}{346} = 3420\,Hz$$

78. The observer registers sound of different frequencies ranging from a minimum to a maximum.

Observed frequency is maximum for the sound emitted by the source when it is crossing its mean position travelling towards the observer [remember that Doppler's effect does not depend on distance between the source the observer]

$$\therefore\ f_{\max} = f_0\left[\frac{V}{V - A\omega}\right] \qquad \left[\omega = \sqrt{\frac{K}{M}}\right]$$

$$= \ 399\times\left[\frac{V}{V - \frac{V}{20}}\right] = 399\times\frac{20}{19} = 420\,Hz$$

The minimum frequency is registered for the sound emitted by the source at its mean position travelling away from the observer

$$f_{\min} = f_0\left[\frac{V}{V + \frac{V}{20}}\right] = 399\times\frac{20}{21} = 380\,Hz$$

$\therefore$ Observer receives sound of frequency ranging from 380 Hz to 420 Hz

380 $Hz \le f \le$ 420 Hz.

(b) For $f = 399\times\frac{40}{39}$

$$f = f_0\left[\frac{V}{V - Vs}\right]$$

$$399\times\frac{40}{39} = 399\left[\frac{V}{V - Vs}\right] \quad \Rightarrow \frac{40}{39} = \frac{V}{V - V_S}$$

$$\Rightarrow V_S = \frac{V}{40} = \frac{A\omega}{2} \quad \text{(approaching)}$$

for $f = 399\times\frac{40}{41}$

$$\frac{40}{41} = \left[\frac{V}{V + Vs}\right] \ \Rightarrow Vs + \frac{V}{40} = \frac{A\omega}{2}\,\text{(receding)}$$

$\therefore$ Δt_1 is time interval during which the source moves from its mean position to $x = \frac{\sqrt{3}}{2}A$

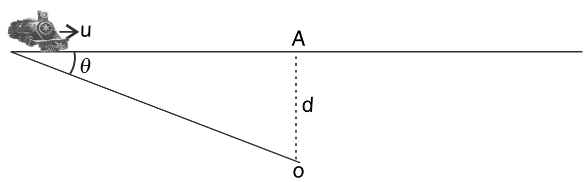
$$\left[\because V = \omega\sqrt{A^2 - x^2} \ \Rightarrow \frac{A\omega}{2} = \omega\sqrt{A^2 - x^2} \ \Rightarrow x = \sqrt{\frac{3}{2}}A\right]$$

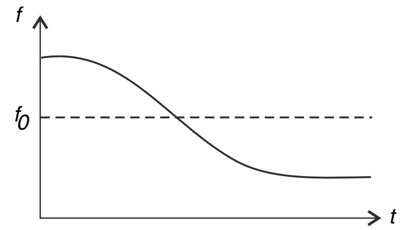
And Δt_2 is time interval during which it moves from $x = A$ (extreme) to $x = \frac{\sqrt{3}}{2}A$

$\therefore\ \Delta t_1 > \Delta t_2$

79. (i) $f = f_0\left[\frac{V}{V - u\cos\theta}\right]$

As train moves from a large distance towards point A, angle θ changes from near zero to $\frac{\pi}{2}$ and then increases beyond $\pi/2$ to approach π. The graph is as shown in the diagram.





(ii) (a) Distance = $\frac{0.1 \times 340}{2} = 17\,m$

(b) $\Delta t = \frac{0.02\,m}{340\,m/s} = 5.9 \times 10^{-5}\ s$

(c) The bug is moving towards the bat since the received frequency is higher.

Frequency received by the bug $f_1 = f_0\left[\frac{340 + v}{340}\right]$

Now, the bug can be treated as a moving source of frequency f_1. Frequency received by the bat is

$$f_2 = f_1\left[\frac{340}{340 - v}\right] \qquad \Rightarrow f_2 = 40\left[\frac{340 + v}{340}\right]\left[\frac{340}{340 - v}\right] = 40\left[\frac{340 + v}{340 - v}\right]$$

$$\Rightarrow 40.4 = 40\left[\frac{340 + v}{340 - v}\right] \qquad \Rightarrow 136 = 80.4\,v$$

$\therefore\ v = 1.7\ m/s$

80. (a) Wavelength in front of the source (in medium of the source itself) is

$\lambda = \lambda_0 - \frac{V}{5}\cdot T$ But $V = \frac{\lambda_0}{T}$

$\therefore \quad \lambda = \lambda_0 - \frac{\lambda_0}{5T}T \quad \Rightarrow \quad \lambda = \frac{4\lambda_0}{5}$

When a wave charges medium, frequency does not change, it is wavelength that changes.

$\therefore$ Wavelength in the medium of the observer is

$$\lambda' = 2\lambda = \frac{8\lambda_0}{5} = \frac{8}{5}\frac{V}{f_0}$$

(b) Apparent time period = time interval between two successive compression pulses striking the observer =

$$T' = \frac{\lambda'}{2V + V_{observer}} = \frac{\lambda'}{2V + \frac{V}{5}} = \frac{5\lambda'}{11V}$$

$$\frac{1}{f'} = \frac{5}{11V}\cdot\frac{8V}{5f_0}$$

$$\Rightarrow \ f' = \frac{11}{8}f_0$$

81. (a) $A_1 \sin\omega\left(\frac{0}{V_1}-t\right)+A_2 \sin\omega\left(\frac{0}{V_2}+t\right)=A_3 \sin\omega\left(\frac{0}{V_2}-t\right)$

$\Rightarrow A_1 \sin(-\omega t) + A_2 \sin(\omega t) = A_3 \sin(-\omega t)$

$\Rightarrow -A_1 + A_2 = -A_3$

$\Rightarrow A_1 - A_2 = A_3$..............(1)

(b) $\frac{\partial y_-}{\partial x}=\frac{A_1\omega}{V_1}\cos\omega\left(\frac{x}{V_1}-t\right)+\frac{A_2\omega}{V_2}\cos\omega\left(\frac{x}{V_2}+t\right)$

$$\left(\frac{\partial y_-}{\partial x}\right)_{x=o}=\frac{A_1\omega}{v_1}cos\left(-\omega t\right)+\frac{A_2\omega}{v_2}\cos\left(\omega t\right)$$

$$=\frac{A_1\omega}{V_1}\cos\omega t+\frac{A_2\omega}{V_2}\cos\left(\omega t\right)$$

$$\frac{\partial y_+}{\partial x}=\frac{A_3\omega}{V_2}\cos\omega\left(\frac{x}{V_2}-t\right)$$

$$\left(\frac{\partial y_+}{\partial x}\right)_{x=0}=\frac{A_3\omega}{V_2}\cos\omega t$$

But $\left(\frac{\partial y_-}{\partial x}\right)_{x=0}=\left(\frac{\partial y_+}{\partial x}\right)_{x=0}$

$\therefore \frac{A_1\omega}{V_1}+\frac{A_2\omega}{V_2}=\frac{A_3\omega}{V_2}$

$\frac{V_2}{V_1}A_1 = A_3 - A_2$ ....(2)

Solving (1) and (2)

$A_3=\frac{2V_2A_1}{V_1+V_2}$ and $A_2=\frac{\left(V_1-V_2\right)A_1}{V_1+V_2}$

82. (a) At $x = 4$, particle is at positive extreme. Hence acceleration is maximum and negative

$\Delta\rho \propto -\frac{ds}{dx}$

At $x=4;\frac{ds}{dx}=0$

$\therefore \Delta\rho = 0 \Rightarrow$ density is equal to ρ_0

(b) At $x = 2$ and $x = 10$; $\frac{ds}{dx}$ is maximum and positive

$V=-u\frac{ds}{dx}$

V is maximum negative at these points.

$\Delta\rho \propto -\frac{ds}{dx} \Rightarrow \Delta\rho$ is maximum and negative

(c) $\Delta\rho = -\dfrac{ds}{dx}$

But $\dfrac{ds}{dt} = -u\dfrac{ds}{dx} =$ particle velocity (V)

$\therefore\ \Delta\rho \propto V \quad \Rightarrow \dfrac{\partial\rho}{dx} \propto \dfrac{\partial V}{\partial x}$

In a travelling wave $V = g(x - ut)$

$\therefore\ \dfrac{\partial V}{dx} = -\dfrac{1}{u}\dfrac{\partial V}{\partial t} = -\dfrac{1}{u}a$

$\therefore\ \dfrac{\partial\rho}{\partial x} \propto -a$

$\dfrac{\partial\rho}{\partial x}$ is positive maximum at positions where a is negative maximum; i.e., at $x = 4\ cm$

83. The given equations are

$y_1 = a\sin(K_1x - \omega_1t)$ and $y_2 = a\sin(K_2x - \omega_2t)$

Where $a = 0.02\ mm$; $\omega_1 = 400\ \pi\ rad\ s^{-1}$; $\omega_2 = 404\ \pi\ rad^{-1}$

$K_1 = \dfrac{400\pi}{300}$ and $K_2 = \dfrac{404\pi}{330}$

$\therefore\ y = y_1 + y_2$

$= 2a\cos\left(\dfrac{\Delta Kx}{2} - \dfrac{\Delta\omega}{2}t\right)\sin\left(K_{av}x - \omega_{av}t\right)$ ....(1)

Where $K_{av} = \dfrac{K_1 + K_2}{2}$; $\omega_{av} = \dfrac{\omega_1 + \omega_2}{2}$

$\Delta K = K_2 - K_1$; $\Delta\omega = \omega_2 - \omega_1$

(a) At $t = 0$, the equation becomes

$$y = \left[2\,a\cos\left(\frac{\Delta k\ x}{2}\right)\right]\sin\left(K_{av}.\,x\right)$$

This represents a wave with wave number = K_{av} and amplitude varying with position as

$$A = 2a\cos\left(\frac{\Delta K\,x}{2}\right)$$

$\therefore$ A is maximum when

$$\frac{\Delta K\ x}{2} = 0, \pi, 2\pi.........$$

$$\Rightarrow\ x = 0, \frac{2\pi}{\Delta K}\ ;\ \frac{4\pi}{\Delta K}...........$$

$\therefore$ Required answer is $\dfrac{2\pi}{\Delta k} = \dfrac{2\pi}{\dfrac{4\pi}{330}} = 165\,m.$

(b) At $x = 0$; equation (i) becomes

$$y = -2a\cos\left(\frac{\Delta\omega\,t}{2}\right)\sin\left(\omega_{av}\ .t\right)$$

The amplitude changes with time (at a fixed location) as given by

$$A = 2a\cos\left(\frac{\Delta\omega}{2}t\right)$$

$\therefore$ Intensity varies as $I = I_0\cos^2\left(\frac{\Delta\omega}{2}t\right) = \frac{I_0}{2}\left[\cos(\Delta\omega t) + 1\right]$

Angular Frequency of this (Known as beat frequency) is

$\Delta\omega = \omega_2 - \omega_1$

$\therefore \Delta f = f_2 - f_1 = 202 - 200 = 2\ Hz$

$\therefore T = \frac{1}{\Delta f} = \frac{1}{2} = 0.5\,s.$

at a point maximum are recorded at a gap of 0.5 s.

85. Speed of wave on second string is $\frac{V}{2}$. At A, a part of wave energy gets reflected and a part is transmitted

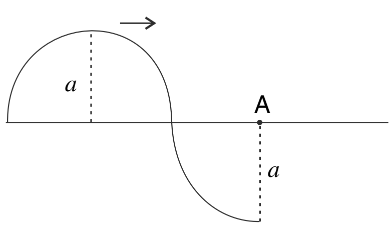
Amplitude of reflected pulse $a_r = \left(\frac{\frac{V}{2} - V}{\frac{V}{2} + V}\right)a = -\frac{a}{3}$

Negative sign indicates a phase charge of π.

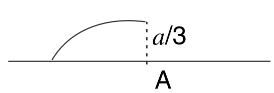
Amplitude of transmitted pulse $a_t = \left(\frac{2\frac{V}{2}}{\frac{V}{2} + V}\right)a = \frac{2a}{3}$

At $t = 1\ s$ incident, reflected and transmitted pulses are as shown below –

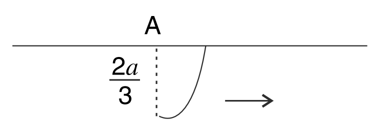
Incident pulse



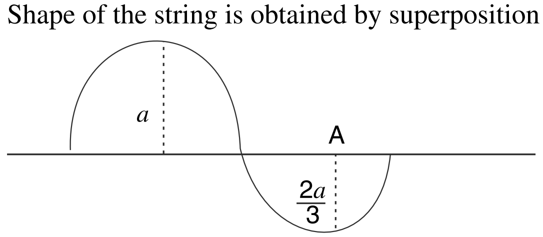
Reflected pulse ←



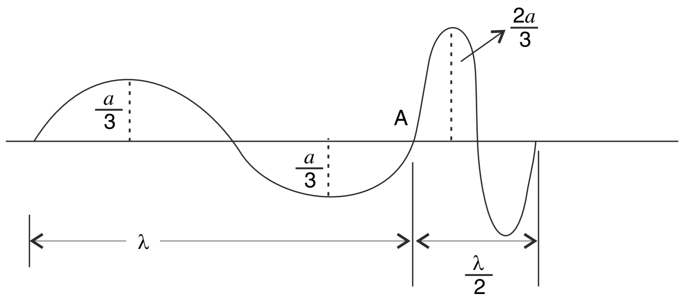
Transmitted pulse Wavelength reduces by a factor of 2.



Shape of the string is obtained by superposition of above 3 pulses.



In 2.5 s the complete pulse strikes A and shape of the string is as shown.



86. Let tension at mid point be T. Consider rotational equilibrium of half of the wire about its fixed end.

Torque due to tension = torque due to its weight

$$T \cdot d = \mu \frac{L}{2} g \cdot \frac{L}{4}$$

Since line of action of weight is nearly at a distance $\frac{L}{4}$ from the fixed end.

$$\therefore\ T = \frac{\mu L^2}{8d} g$$

$\therefore$ Wave speed $v = \sqrt{\frac{T}{\mu}} = L\sqrt{\frac{g}{8d}}$

87. Time required for the wave to travel through length b is $\Delta t = \frac{b}{v}$.

The horizontal force on the wedge during the interval 0 to Δt increases linearly from 0 to $P_0 ac$.

$$F_{avg} = \frac{1}{2} P_0 ac$$

$$\therefore\ mu = \frac{P_0 ac}{2} \frac{b}{v} \quad \Rightarrow u = \frac{P_0 acb}{2mv}$$