Hello World!

FUNCTIONS

3.1 INTRODUCTION

In this chapter, we shall study about one of the most important concepts in mathematics known as a function. Functions form one of the most important building blocks of Mathematics. The word "Function" is derived from a Latin word meaning operation and the words mapping and map are synonimus to it. Functions play a very important role in differential and integral calculus which will be studied in XII class. In this chapter, we shall introduce the concept of a function as a correspondence between two sets. We shall also study function as a relation from one set to the other set.

3.2 FUNCTION AS A SPECIAL KIND OF RELATION

DEFINITION *Let A and B be two non-empty sets. A relation f from A to B, i.e., a sub-set of $A \times B$, is called a function (or a mapping or a map) from A to B, if*

(i) *for each $a \in A$ there exists $b \in B$ such that $(a, b) \in f$*

(ii) *$(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$.*

Thus, a non-void subset f of $A \times B$ is a function from A to B if each element of A appears in some ordered pair in f and no two ordered pairs in f have the same first element.

If $(a, b) \in f$, then b is called the image of a under f.

ILLUSTRATION 1 *Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ and f_1, f_2 and f_3 be three subsets of $A \times B$ as given below:*

$$f_1 = \{(1, 2), (2, 3), (3, 4)\},\ f_2 = \{(1, 2), (1, 3), (2, 3), (3, 4)\},\ f_3 = \{(1, 3), (2, 4)\}.$$

Then, f_1 is a function from A to B but f_2 and f_3 are not functions from A to B. f_2 is not a function from A to B, because $1 \in A$ has two images 2 and 3 in B and f_3 is not a function from A to B because $3 \in A$ has no image in B.

If a function f is expressed as the set of ordered pairs, the domain of f is the set of all first components of members of f and the range of f is the set of second components of members of f i.e. Domain of $f = \{a : (a, b) \in f\}$, and Range of $f = \{b : (a, b) \in f\}$

ILLUSTRATION 2 If $x, y \in \{1, 2, 3, 4\}$, then which of the following are functions in the given set?

(a) $f_1 = \{(x, y) : y = x + 1\}$ (b) $f_2 = \{(x, y) : x + y > 4\}$

(c) $f_3 = \{(x, y) : y < x\}$ (d) $f_4 = \{(x, y) : x + y = 5\}$

Also, in case of a function give its range.

SOLUTION If we express f_1, f_2, f_3 and f_4 as sets of ordered pairs, then we have

$f_1 = \{(1, 2), (2, 3), (3, 4)\}$,

$f_2 = \{(1, 4), (4, 1), (2, 3), (3, 2), (2, 4), (4, 2), (3, 4), (4, 3)\}$,

$f_3 = \{(2, 1), (3, 1), (4, 1), (3, 2), (4, 2), (4, 3)\}$ and $f_4 = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$.

(a) We have, $f_1 = \{(1, 2), (2, 3), (3, 4)\}$.

We observe that an element 4 of the given set has not appeared in first place of any ordered pair of f_1. So, f_1 is not a function from the given set to itself.