Economic Load Dispatch (13)

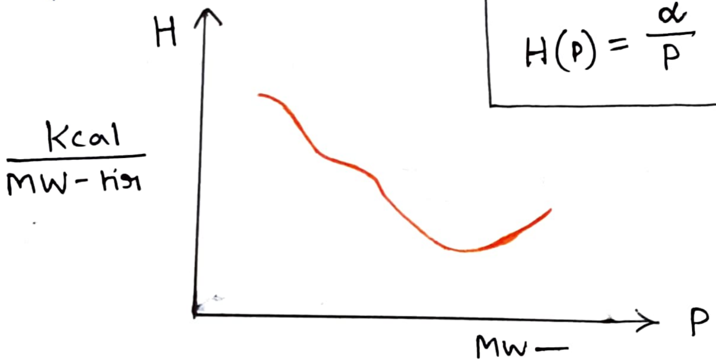
It involves solution of two problems.

- Selection of source optimally out of available source to meet the expected demand with sufficient reserve. It is called unit commitment.
- Distribute the load among the generating unit in such a manner so that cost of power will be minimum.
- In economic load dispatch we do not consider the fixed cost is capital cost.
- We will consider only variable cost. The main variable cost is fuel cost & cost of labour, maintenance & water cost.
- We take labour, supplies maintenance cost and water cost as a some percentange of fuel cost.

It is a curve between heat rate kcal/MW-hr v/s output power in MW.

$$\boxed{H(P) = \frac{\alpha'}{P} + \beta' + \gamma' P}$$

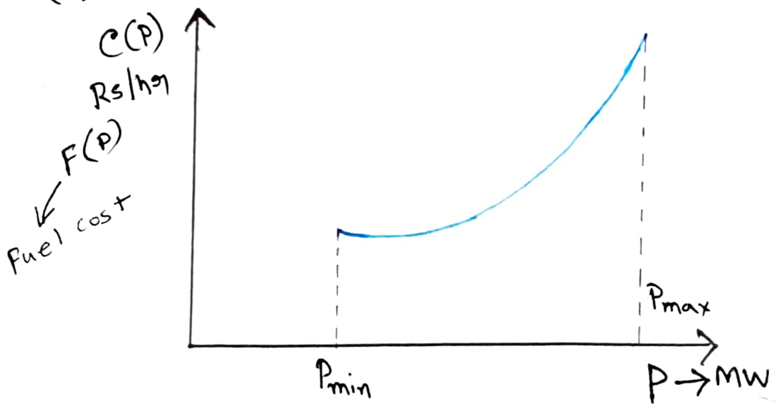
(*) Heat rate curve



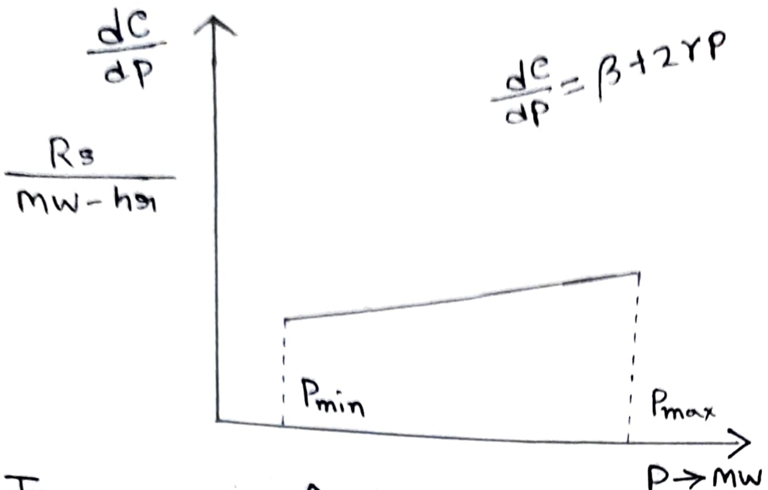
(*) Fuel Cost Curve:-

$$C(P) \doteq \frac{\left(\frac{\alpha'}{P} + \beta' + \gamma' P\right) P}{\text{Calorific value of coal}}$$

$$\therefore C(P) = \alpha' + \beta' P + \gamma' P^2 \ \frac{Rs}{hr}$$



Incremental fuel cost curve :-



Incremental fuel Cost :- Ratio of small change of input to small Change of output.

$$IC = \frac{\Delta\, i/p}{\Delta\, o/p} = \frac{d(i/p)}{d(o/p)} = \frac{dF}{dP}$$

$\frac{Kcal}{MW-hr}$ or $\frac{Rs}{MW-hr}$

* Incremental efficiency $= \frac{dP}{dF}$

50 MW → 1000 Rs
51 MW → 1020 Rs
> IC = 20 Rs.

Method of Loading Turbo Generator :-

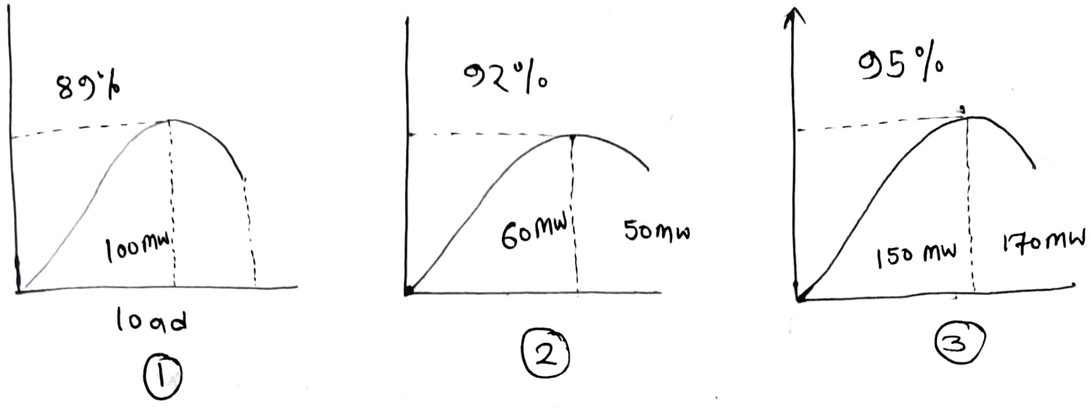
1. Base load to capacity :-

1	2	3	4
88 % (F.L) 150 MW	96 % (F.L) 200 MW	92 % (F.L) 300 MW	95 % (F.L) 100 MW

400 MW → Demand
∴ Unit 2 → 200 MW
Unit 4 → 100 MW
Unit 3 → 100 MW

> The turbo unit are loaded to full capacity one after other in order of their efficiency.

2. Base load to most efficient load :-



Demand = 300 mw
Unit 3 → 150 mw
Unit 2 → 60 mw
Unit 1 → 90 mw

The turbo unit are loaded up to point of maximum efficiency in order of their efficiency.

3. Loading proportional to their rating :-

	1	2	3	
Rating:-	100 mw	200 mw	300 mw	= 600 mw

Demand → 300 mw
Unit 1 → 50 mw
Unit 2 → 100 mw
Unit 3 → 150 mw.

4. Equal increment cost operation :-

* Economical Dispatch neglecting losses :- $P_D = P_G$ (demand, generated)

(a) Two generating unit:

Suppose generating cost of two unit is —

Unit 1 → For output power of P_1 cost is F_1

Unit 2 → For output power of P_2 cost is F_2

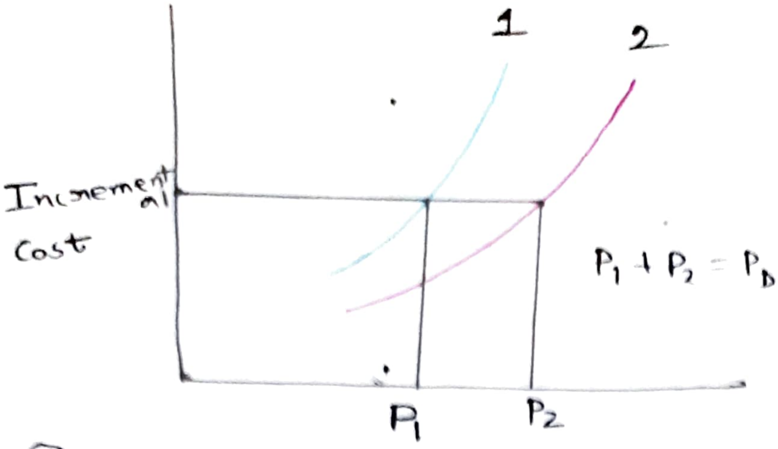
∴ Total power $P_D = P_T = P_1 + P_2$

∴ Total cost $F_T = F_1 + F_2$
↓
Min^m

$$\frac{dF_T}{dP} = 0$$

Calculating this

$$\boxed{\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}}$$



(b) If large number of generating unit exist.

$P_D = P_T = P_1 + P_2 + P_3 + \cdots + P_n$ —— (1)

$F_T = F_1 + F_2 + F_3 + \cdots + F_n$ ⟶ Main function

> The above problem is having a equality constraint as given by equation (1). So it can be solved by using simplex method of optimization technique. Which say constrict a auxiliary function (Lagrangian function) in term of main function and constraint.

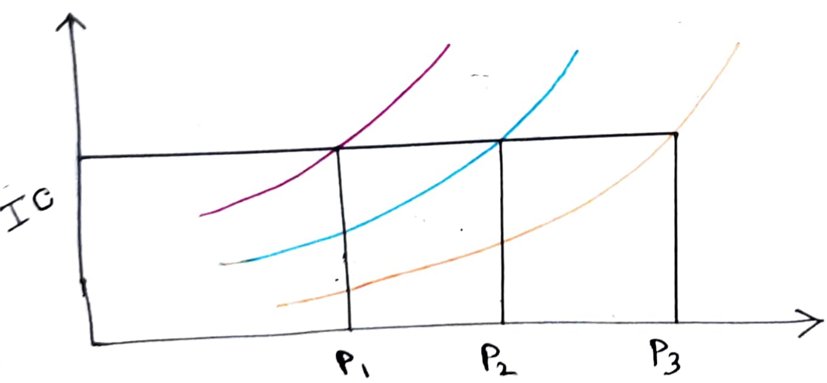
Auxiliary function = Main function + λ (constraint)

> The main function will be minimum, when auxiliary function is minimum.

$$F = F_T + \lambda \left(P_D - \sum_{n=1}^{n} P_n \right)$$

$$\frac{dF}{dP_n} = 0 \quad ; \quad P_D = \text{Constant}$$

$$\therefore \boxed{\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \frac{dF_3}{dP_3} = \cdots = \lambda}$$



Q) The incremental cost characteristics of two generator delivering 200 MW are as follows—

$$\frac{dF_1}{dP_1} = 20 + 0.1 P_1 \quad \text{— (1)} ; \quad \frac{dF_2}{dP_2} = 16 + 0.2 P_2 \quad \text{— (2)}$$

For economic operation the generation P_1 & P_2 should be—

For economic operation, $\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$

$$20 + 0.1 P_1 = 16 + 0.2 P_2$$

$$-0.1 P_1 + 0.2 P_2 = 4 \quad \text{— (1)}$$

$$P_0 = P_1 + P_2 = 200$$

$$-P_1 + 2P_2 = 40$$

$$3P_2 = 240$$

$$\therefore P_2 = 80$$

$$\therefore P_1 = 120$$

Case I :- If power calculated by economic operation violate the active power limit of unit. Then we will fix the power to that limit & remaining power will be given by other unit.

$$P_1(cal) \geq P_1(max) ; \quad P_1 = P_1(max)$$

$$P_1(cal) \leq P_1(min) ; \quad P_1 = P_1(min)$$

Q) A power system has two generator with following cost curve.

$$F_1 \text{ (cost)} = 0.006 P_1^2 + 8P_1 + 360$$

$$F_2 \text{ (cost)} = 0.006 P_2^2 + 7P_2 + 400$$

The generator limit,

$$100 < P_1 < 650$$

$$50 < P_2 < 500$$

If load demand is 600 MW. Determine optimal genr of each generator.

Incremental Cost:- $\frac{dF_1}{dP_1} = 0.012 P_1 + 8$; $\frac{dF_2}{dP_2} = 0.012 P_2 + 7$

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$0.012 P_1 + 8 = 0.012 P_2 + 7$$

$$\cancel{P_1 + P_2} \cdot P_2 - P_1 = \frac{1}{0.012} = 83.33 \quad \text{— (1)}$$

$$P_1 + P_2 = 600 \quad \text{— (2)}$$

(1) + (2)

$$2P_2 = 683.33$$

$$\therefore P_2 = 341.665$$

$$\therefore P_1 = 258.33$$

Q) In a power system fuel input per hour of Plant 1 & 2 is as below:

$F_1 = 0.20P_1^2 + 30P_1 + 100$ Rs/hr

$F_2 = 0.25P_2^2 + 40P_2 + 80$ Rs/hr

The limit of generator,

$20 \le P_1 \le 80$; $40 \le P_2 \le 200$

If total demand is 130 MW. Find economic operating schedule.

$$\frac{dF_1}{dP_1} = 0.4P_1 + 30 \quad ; \quad \frac{dF_2}{dP_2} = 0.5P_2 + 40$$

optimal loading so, $\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$

$0.40P_1 + 30 = 0.50P_2 + 40$

$0.40P_1 - 0.50P_2 = 10$ — (1)

$P_1 + P_2 = 130$ — (2)

$P_1 = 83.33$; $P_2 = 46.66$

$P_1(cal) \geq P_1(max)$

$P_1 = P_1(max)$

$P_1 = 80$

$P_2 = 130 - 80 = 50$

Case-II If (Max IC) of unit 1 is less than Minimum IC of unit number 2, then we load unit number 1 to its rated capacity first & remaining power will be taken from unit no. 2.

ex

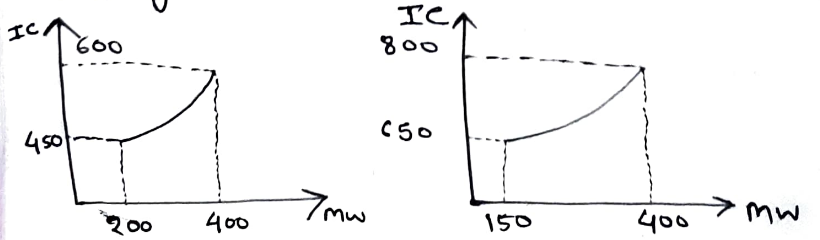
Laluram & Sons → 10 Rs. to 30 Rs/kg [80 kg]

Kaluram & Sons → 40 Rs. to 50 Rs/kg [200 kg]

→ 20 kg.

demand ↓ [100 kg]

Q) The incremental cost curve in Rs/hr for 2 genr. Supplying a common load of 700 MW as shown in fig.



The optimal schedule is, $P_1 + P_2 = 700$ MW

• Since max^m IC of unit 1 < minimum IC of unit 2, We load unit 1 upto its rated capacity.

$\min(IC_2) > \max(IC_1)$

$\therefore P_1 = 400$ MW.

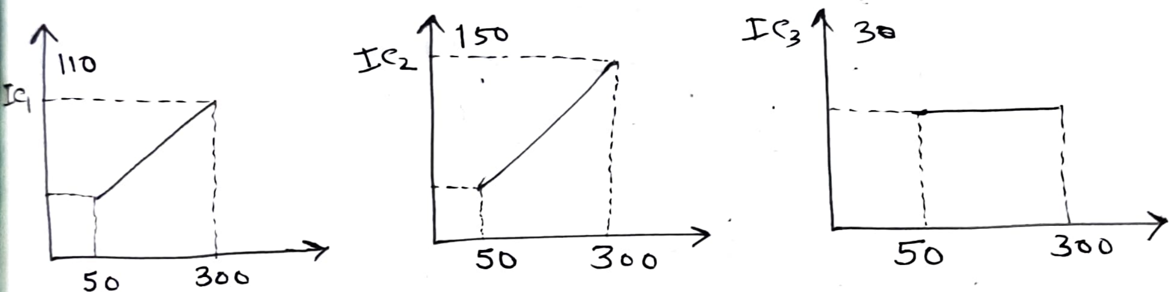
$P_2 = 300$ MW. $(700 - 400)$

Case III :- If IC of any unit is fixed and it does not depend on power. Suppose if this cost is less than Minimum IC of other units, then we will load the unit upto its rated capacity and remaining power will be taken by other unit.

Q The IC of generating units are ~~as~~ given as:

$IC_1 = 20 + 0.3P_1$ $\quad IC_2 = 30 + 0.4P_2$ $\quad IC_3 = 30$

If mimimum and maximum load on each unit are 50 MW & 300 MW respectively. If total power demand is 700 MW. The power generated by each unit will be—



Unit 1 :- $IC_1 = 20 + 0.3P_1$

for 50 MW $\Rightarrow IC_1 = 20 + 0.3 \times 50$
$= 35$

Unit 2 :- $IC_2 = 30 + 0.4P_2$

for 50 MW
$\Rightarrow IC_2 = 30 + 0.4 \times 50$
$= 50$

The $IC(3) \leq \text{Min } IC(1)$ or $IC(2)$ its rated capacity.

$\hookrightarrow$ So, Unit 3 will be loaded up to its rated capacity.

$\therefore P_3 = 300$ MW.

Remaining power, $P_1 + P_2 = 700 - 300 = 400$ MW — (1)

$\therefore (IC)_1 = (IC)_2$

$20 + 0.3P_1 = 30 + 0.4P_2$

$0.3P_1 - 0.4P_2 = 10$ — (2)

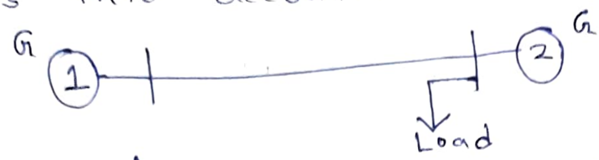
Calculating both eq (1) & (2) we get,

$P_1 = 242.83$ MW ; $P_2 = 157.142$ MW

Optimum Load Dispatch Including Transmission Loss

→ If power station situated close to each other then optimum scheduling can be done without Transmission Loss.

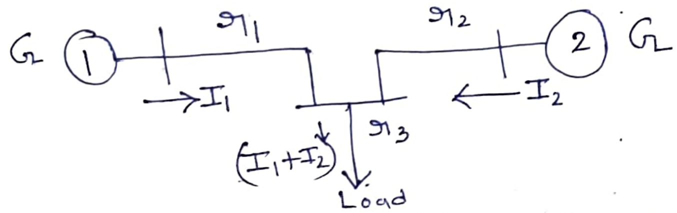
→ However, if they are situated far from each other, scheduling must be done by taking T.L. losses into account.



Suppose if we have two units having same I.C, if we neglect T.L losses, then we should take half power from each unit

→ But we take T.L. losses into account, we should take more power from unit 2 and less power from unit 1

Representation of Losses :-



Assume current I_1 & I_2 are in phase.

∴ Total power loss in line—

$$P = 3I_1^2 r_1 + 3I_2^2 r_2 + 3(I_1+I_2)^2 r_3$$

$$P_1 = V_1 I_1 \cos\phi_1 \qquad P_2 = V_2 I_2 \cos\phi_2$$

$$\therefore I_1 = \frac{P_1}{V_1 \cos\phi_1} \qquad \therefore I_2 = \frac{P_2}{V_2 \cos\phi_2}$$

$$\therefore \boxed{P_L = P_1 B_{11} + P_2 B_{22} + 2P_1P_2 B_{12}}$$

→ $B_{11}, B_{22}, B_{12}\ldots$ etc are called loss coefficient.

→ If all quantities are in pu, then these co-efficient will also be in pu.

→ If voltage are line to line voltage & line resistance are in ohm. The unit of this will be $(MW)^{-1}$.

→ B depend upon voltage & power factor, so it also vary with system operating condition. But if load variation are small, we assume it as a constant for some average operating condition.

→ The general form of loss equation.

$$\boxed{P_L = \sum_n \sum_m P_m B_{mn} P_n}$$

E.g. $P_L = P_1 [P_1 B_{11} + P_2 B_{21}] + P_2 [P_1 + B_{11} + P_2 B_2]$

$$= P_1^2 B_{11} + P_2^2 B_{22} + 2 P_1 P_2 B_{12}$$

or in Matrix form,

$$\boxed{P_L = P^T B P}$$

$$P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_K \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \cdots & B_{1K} \\ B_{21} & B_{22} \cdots & B_{2K} \\ \vdots & \vdots & \\ B_{K1} & B_{K2} \cdots & B_{KK} \end{bmatrix}$$

e.g. → for 2 bus system,

$$P^T B P = \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$= P_1^2 B_{11} + P_2^2 B_{22} + 2 P_1 P_2 B_{12}$$

→ In 2 Bus system it has 4 loss coefficient, $B_{11}, B_{22}, B_{12}, B_{21}$.

→ So in 3 bus system, it has $(3)^2 = 9$ loss coefficient.

→ In N Bus system, it has $(N)^2$ loss co-efficient.

→ As size of power system increases it is so design (Interconnected) so that most of loss co-efficient be become zero so reduce power loss.

→ B_{ii}, B_{jj} are called self loss coefficient.

→ B_{ij} is called Mutual loss coefficient.

→ In large interconnected system we can make Mutual loss coefficient equal to zero but self loss coefficient are not generally zero.

Lagrangian Method (Including loss)

Suppose n units are giving $P_1, P_2, \ldots P_n$ power. and cost of power are $F_1, F_2 \ldots F_n$ respectively.

Total power generation = Demand + Losses

$$\therefore \boxed{P_T = P_D + P_L}$$

$$\because P_1 + P_2 + \cdots P_n = P_D + P_L \quad \therefore P_D + P_L = \sum_{n=1}^{n} P_n$$

So, constraint equation,

$$P_D + P_L - \sum P_n = 0$$

Motive is to reduce total power cost.

$F_T = F_1 + F_2 + F_3 + \cdots + F_n$ (Main function).

Auxiliary function = Main function + λ(Constraint)

$$\therefore F = F_T + \lambda\left(P_D + P_L - \sum P_n\right)$$

So, Auxiliary function will be minimum.

$$\frac{dF}{dP_n} = \frac{dF_T}{dP_n} + \lambda\left(0 + \frac{\partial P_L}{\partial P_n} - 1\right)$$

If auxiliary function is minimized then Main function will be minimized along with constant.

$$\frac{dF}{dP_n} = 0 ; \quad \text{So, } \frac{dF_T}{dP_n} + \lambda\left(\frac{\partial P_L}{\partial P_n} - 1\right) = 0$$

$$\rightarrow \frac{dF_1}{dP_1}\frac{1}{\left(1 - \frac{\partial P_L}{\partial P_1}\right)} = \frac{dF_2}{dP_2}\frac{1}{\left(1 - \frac{\partial P_L}{\partial P_2}\right)} = \cdots = \lambda$$

$$\boxed{\frac{dF_n}{dP_n}\frac{1}{\left(1 - \frac{\partial P_L}{\partial P_n}\right)} = \lambda} \quad \therefore \boxed{\frac{dF_n}{dP_n} \times L_n = \lambda}$$

L_n :→ Penalty factor of n unit. It is a unit less quantity.

$$\frac{dF_1}{dP_1} L_1 = \frac{dF_2}{dP_2} L_2 = \cdots = \frac{dF_n}{dP_n} L_n = \lambda$$

So, for optimal load scheduling, the product of penalty factor and I.C. should be equal to Lagrangian Multiplier of system.

Special Cases

~~If~~
If total load is connected at Bus-2 (near to station 2). The losses supplied by station 2 will be zero, so loss coefficient connected with station 2 will always zero.

$P_L = B_{11} P_1^2$

$B_{12} = 0$; $B_{22} = 0$

$\therefore \boxed{P_L = B_{11} P_1^2}$

$$\frac{\partial P_L}{\partial P_1} = 2 B_{11} P_1 \neq 0$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} \neq 1$$

But, $\frac{\partial P_L}{\partial P_2} = 0$, $L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} = 1$

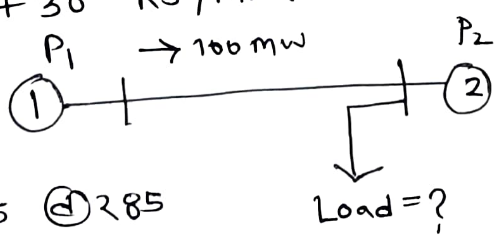
$\longrightarrow \frac{\partial P_L}{\partial P_2} = 0$

Q) A system consist of two plant connected by T.L. The load is at plant 2. The TL losses calculated revealed that a transfer of 100MW power from plant 1 to 2 makes a losses of 15 MW. Find the required generation by each plant for $\lambda = 60$.

$$\frac{dC_1}{dP_1} = 0.2 P_1 + 22 \text{ Rs/MWh}$$

$$\frac{dC_2}{dP_2} = 0.15 P_2 + 30 \text{ Rs/MWh}$$

$\therefore$ Total load demand = ?



(a) 100 (b) 200 (c) 115 (d) 285