Problems – Day 2

1. The `abundancy` of a natural number n is defined as the rational number $\frac{\sigma(n)}{n}$, the ratio between the sum of divisors of the number and the number itself. A number n is defined as `friendly` if it shares `abundancy` with one or more other numbers. This means there might exist a pair of numbers i and j such that $i \neq j$ but $\frac{\sigma(i)}{j} = \frac{\sigma(j)}{j}$. For example, 6 and 28 are `friendly` with each other because $\frac{\sigma(6)}{6} = \frac{\sigma(28)}{28} = 2$. Write a program to verify whether a pair of integers given as user input are `friendly` or not.
2. Given a positive integer n as user input, find out the number of trailing zeros in $n!$.
   Note: This can be done with $\log_5 n$ number of divisions.

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3. Suppose m and n are (signed) integers and x and y are floating variables. Write logical conditions that evaluate to `TRUE` if and only if:
   - $x + y$ is an integer.
   - m lies strictly between x and y.
   - m equals the integer part of x.
   - x is positive with integer part at least 3 and with fractional part less than 0.3.
   - m and n have the same parity (i.e., are both odd or both even).
   - m is a perfect square.

4. Write a program to print the following pattern using generic controls over print. Let the line number be user input.

```
   *
  * *
 *   *
*     *
```

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5. Consider an n-digit number. Square it and add the right n digits to the left n or $n-1$ digits. If the resultant sum is same as the original number, then it is called a `Kaprekar` number. E.g., 45 is a `Kaprekar` number. Write a program to verify whether a given number is `Kaprekar` or not.
6. Let m and n be 32-bit unsigned integers. Use bitwise operations to assign to m the following functions of n:
   - 1 if n is odd, 0 if n is even.
   - 1 if n is divisible by 8, 0 otherwise.
   - 2^n (Assume that $n <= 31$).
   - n rotated by k positions to the left for some integer $k >= 0$.
   - n rotated by k positions to the right for some integer $k >= 0$.