CAPT: Concurrent assignment and planning of trajectories for multiple robots

MAE247 Course Project

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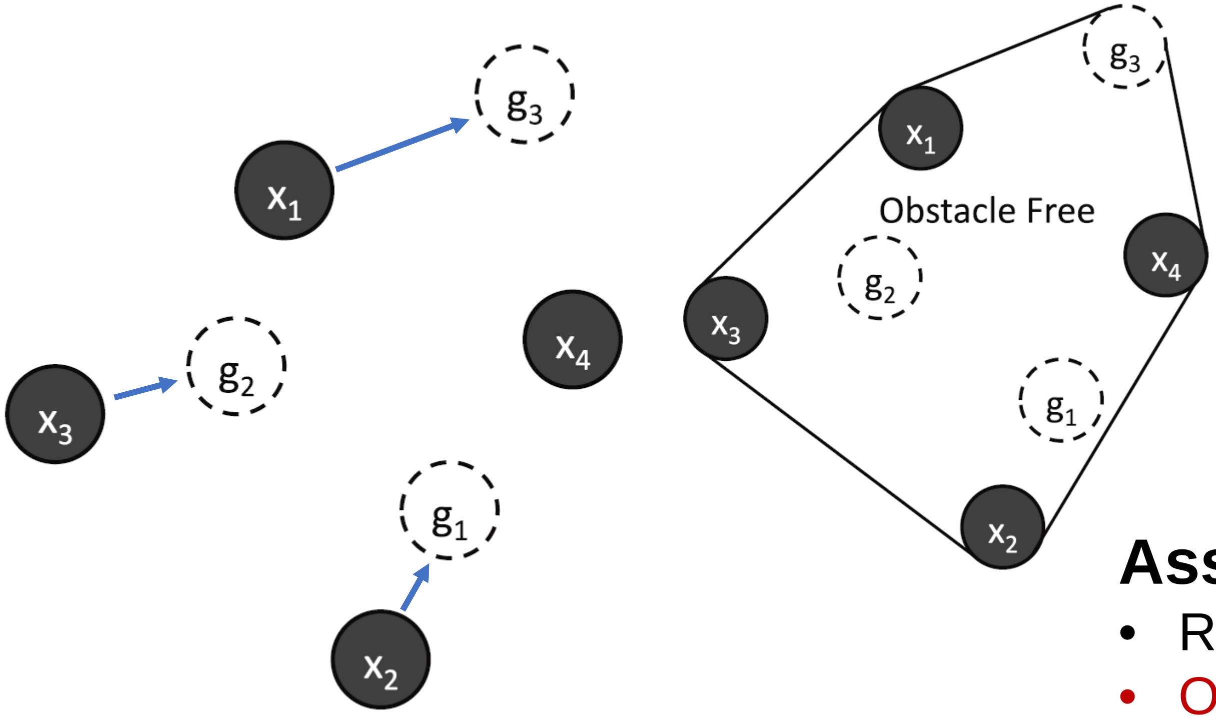
Motivation

Previous research
- Centralized / suboptimal
- no guarantee in convergence / no collision avoidance



Notations:
- N – number of robots; M – number of goals;
- $\boldsymbol{X} \in \mathbb{R}^{Nn}$ robot states; $\boldsymbol{G} \in \mathbb{R}^{Mn}$ goal states;
- $\phi \in \mathbb{R}^{N\times M}$ assignment matrix:

$$\phi_{i,j} = \begin{cases} 1, if\ robot\ i\ assigned\ to\ goal\ j \\ 0, \text{otherwise} \end{cases}$$

Assumptions:
- Robots are interchangeable, confined to $\mathcal{B}_R$;
- Obstacle-free workspace;
- Fully actuated first-order dynamics; no disturbance

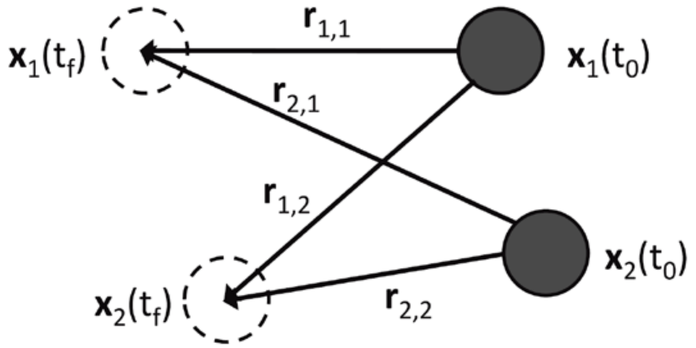
CAPT: assignment + planning problem

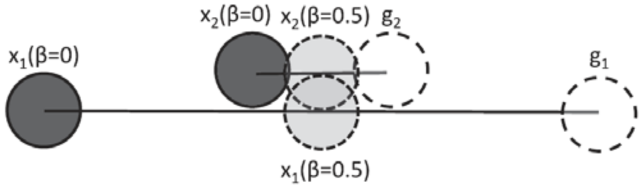
Algorithms:
- C-CAPT: *centralized*, optimal, collision-free
- D-CAPT: *decentralized*, suboptimal, collision-free

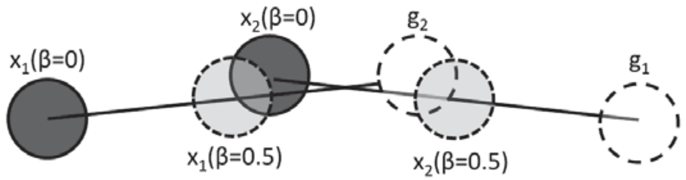
C-CAPT

First try: minimum sum of distances

$$\min_{\phi,\gamma(t)} \sum_{i=1}^{N} \int_{t_0}^{t_f} \sqrt{\dot{\mathbf{x}}_i(t)^{\mathrm{T}} \dot{\mathbf{x}}_i(t)}\, dt$$

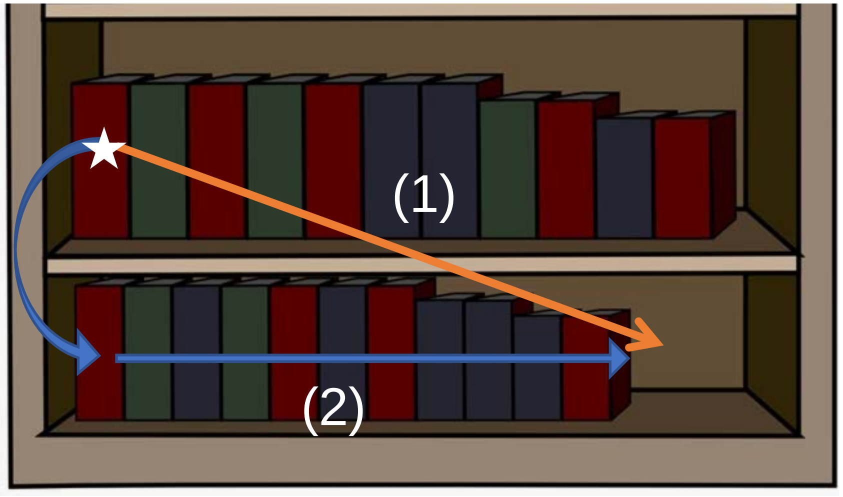






Second try: minimum velocity squared

$$\min_{\phi,\gamma(t)} \int_{t_0}^{t_f} \dot{X}(t)^{\mathrm{T}} \dot{X}(t)\, dt$$



Results:

- Optimal assignment: $\phi^\star = \underset{\phi}{\operatorname{argmin}} \sum_{i=1}^{N} \sum_{j=1}^{M} \phi_{i,j} D_{i,j}$
- Optimal trajectory:

$$\gamma^\star(t) = (1 - \beta(t))\mathbf{X}(t_0) + \beta(t)\left(\Phi^\star \mathbf{G} + \left(I_{Nn} - \Phi^\star \Phi^{\star\mathrm{T}}\right)\mathbf{X}(t_0)\right)$$

- Collision-free when the initial clearance $\Delta \geq 2\sqrt{2}R$

D-CAPT

Algorithm flow (for each agent)

Additional assumptions:

- Communication distance: h $(h > \Delta)$
- $N = M$

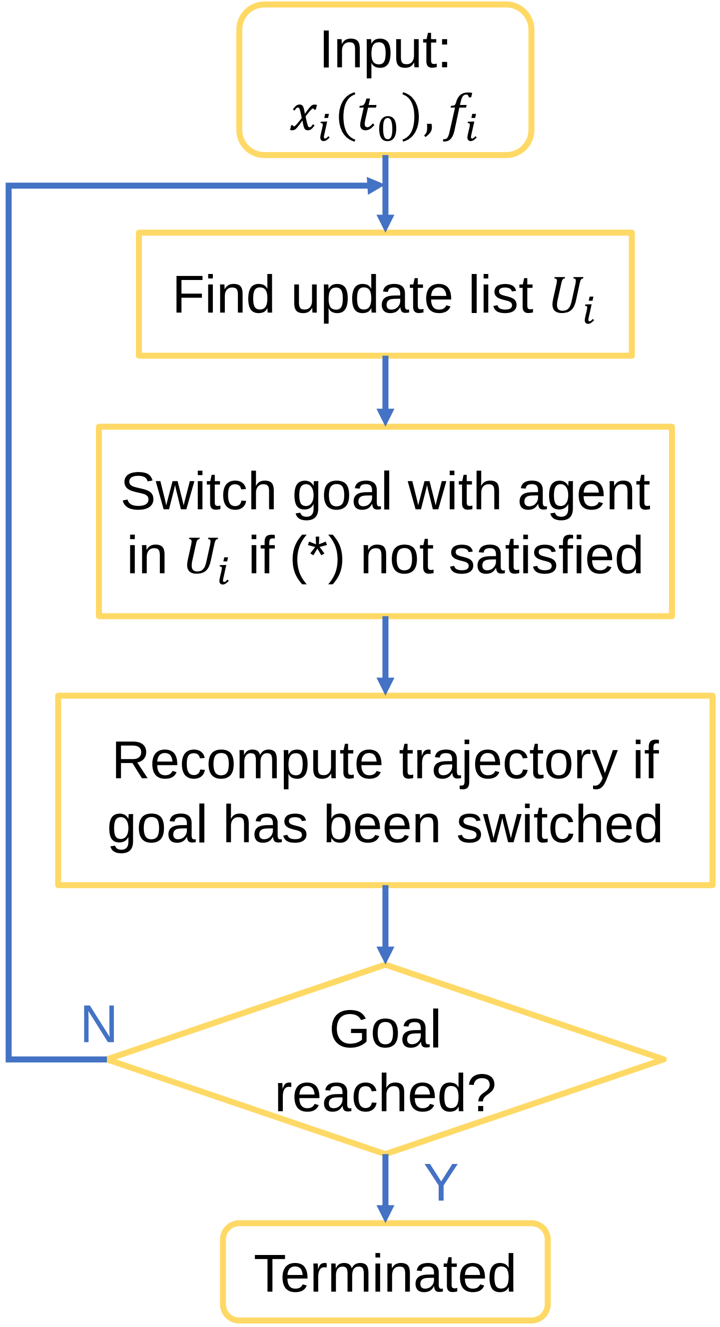
Collision-free condition:

$$\boxed{\mathbf{u}_{i,j}^T \mathbf{w}_{i,j} > 0 \ (*)}$$

$$\mathbf{u}_{i,j} \equiv \mathbf{x}_j(t_c) - \mathbf{x}_i(t_c)$$
$$\mathbf{r}_{i,j} \equiv \mathbf{x}_j(t_f) - \mathbf{x}_i(t_c)$$
$$\mathbf{w}_{ij} \equiv \mathbf{x}_j(t_f) - \mathbf{x}_i(t_f)$$

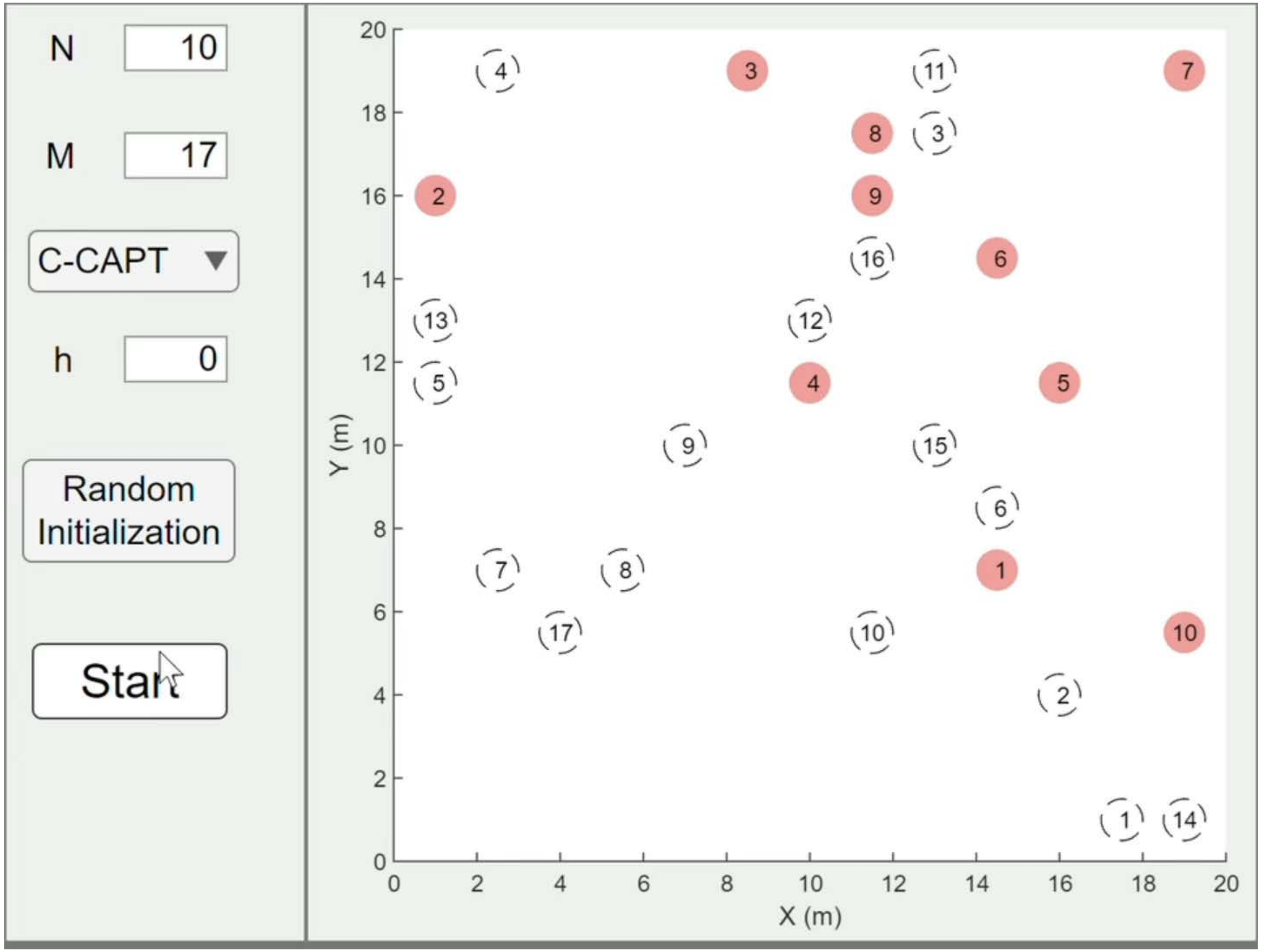
$$\Downarrow$$

$$\|\mathbf{r}_{i,i}\|^2 + \|\mathbf{r}_{j,j}\|^2 < \|\mathbf{r}_{i,j}\|^2 + \|\mathbf{r}_{j,i}\|^2$$

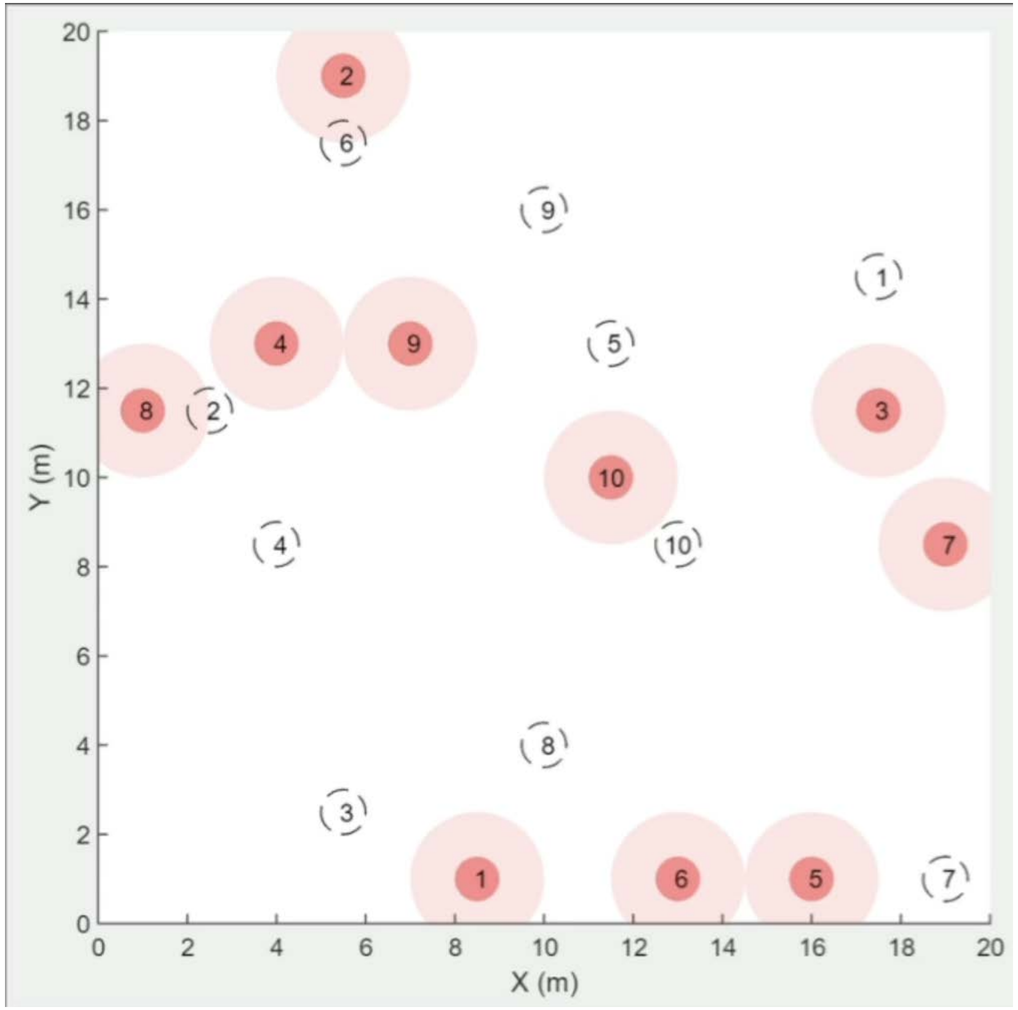


Simulations

C-CAPT



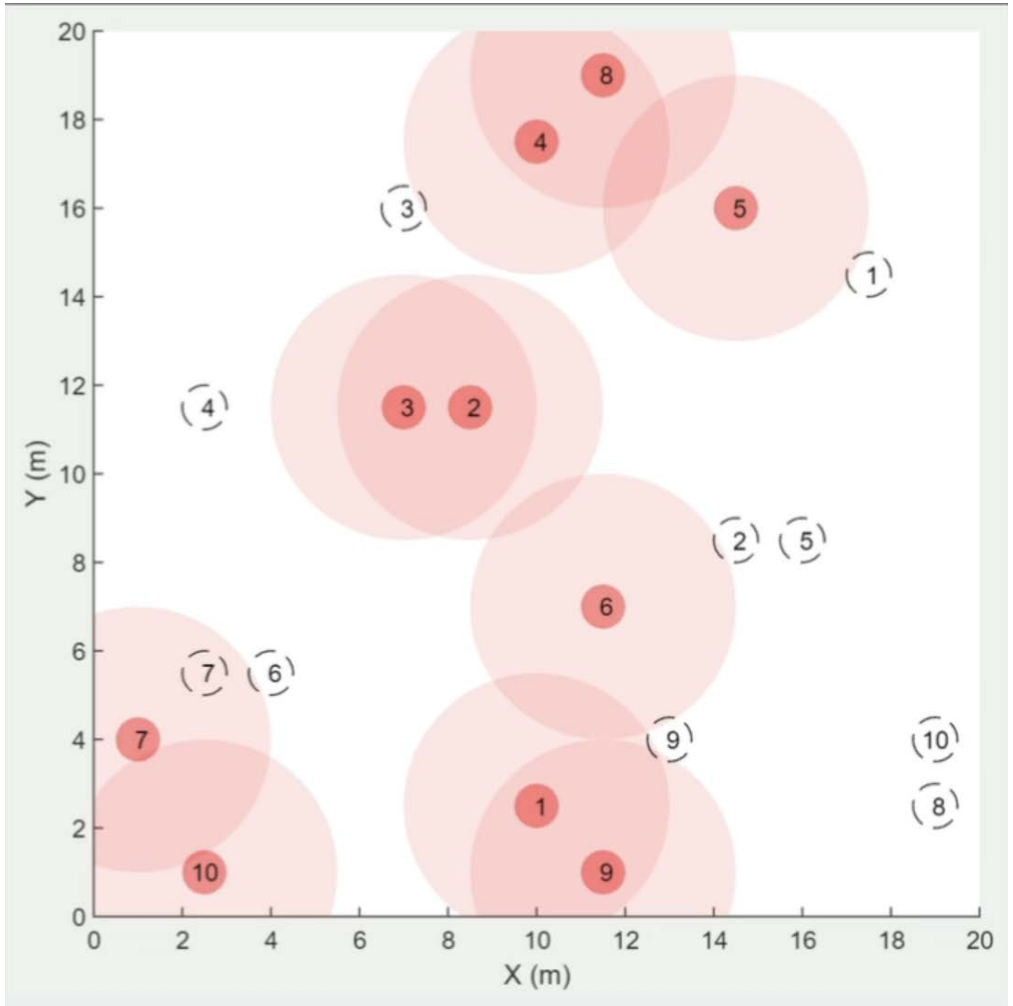
D-CAPT



h=1.5

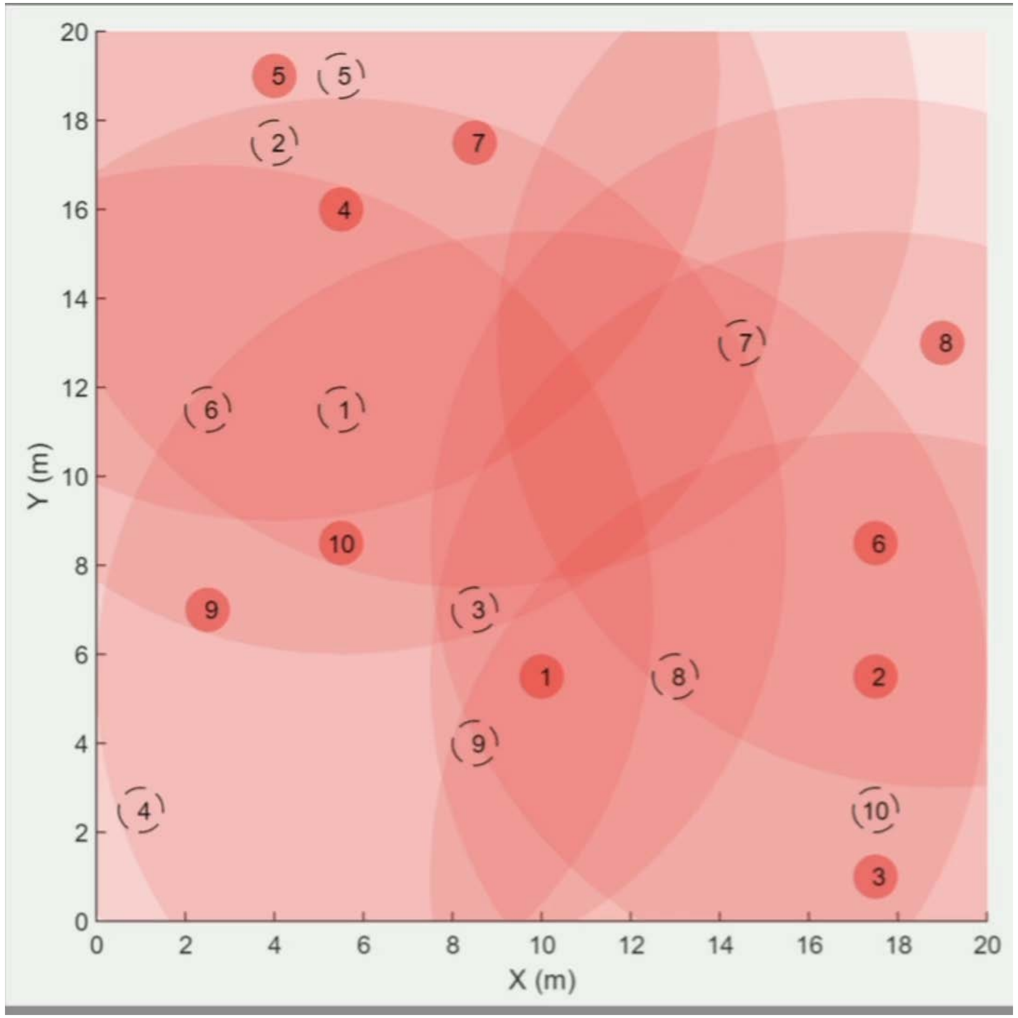
Simulations

D-CAPT



h=3

D-CAPT

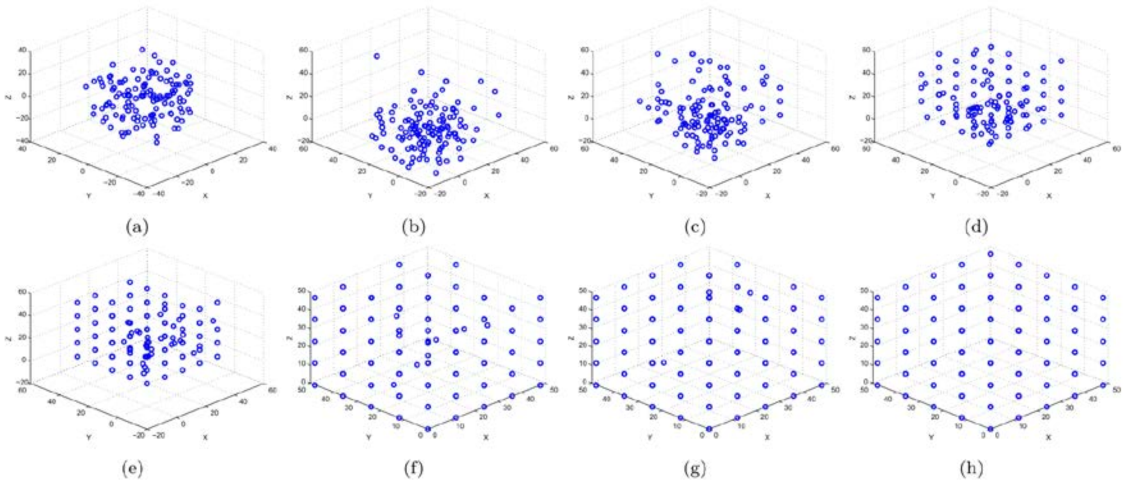


h=10

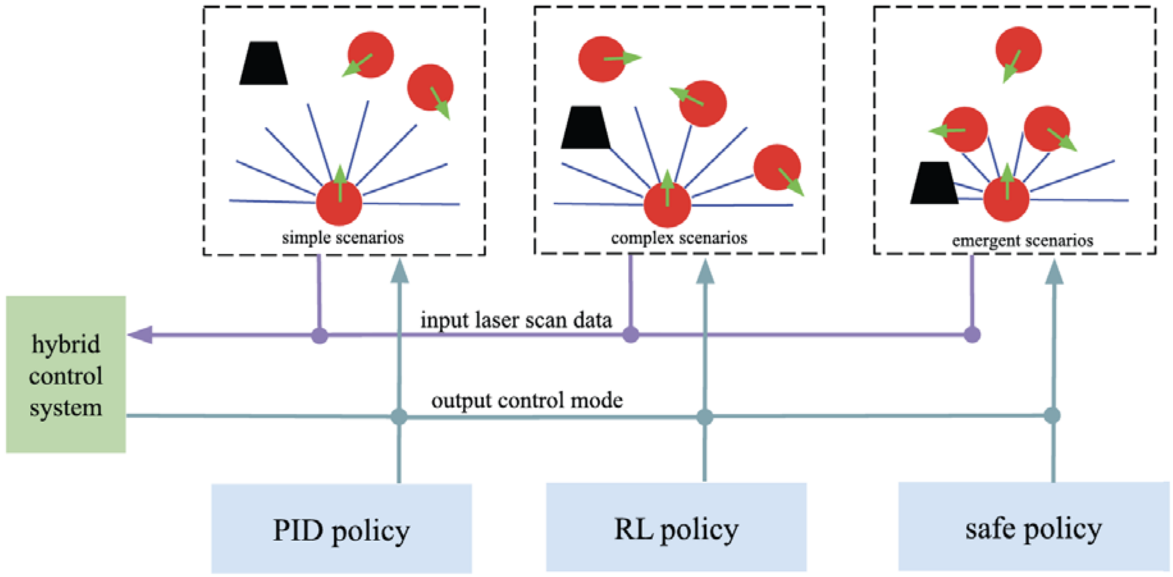
Conclusions & Findings

- Trade-off between decentralization and optimality;
- Some corner pathological cases exist.

Most recent research:



[J. Hu, et al]



[T. Fan, et al]

References

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2. J. Hu, H. Zhang, L. Liu, X. Zhu, C. Zhao and Q. Pan, Convergent Multiagent Formation Control With Collision Avoidance. *IEEE Transactions on Robotics*. 2020:36(6):1805-1818. doi:10.1109/TRO.2020.2998766
3. T. Fan, P. Long, W. Liu, J. Pan, Distributed multi-robot collision avoidance via deep reinforcement learning for navigation in complex scenarios. *The International Journal of Robotics Research*. 2020;39(7):856-892. doi:10.1177/0278364920916531

Thank you!

Q & A