1 Set up

The system is composed of agents that interact via a coordination game in a complete network. The agents are characterized by a behavior, a marker and an aspiration. We define them:

- **Behavior:** It is the strategy chosen for the coordination game. It may take two different values $\{0, 1\}$.
- **Marker:** It is a visible characteristic which characterizes the interaction. It may take two different values $\{0, 1\}$.
- **Aspiration:** The payoff expected by the agent, which will define the stimulus that it receives from the interaction.
- **Probability vector:** When some subject interacts with another agent it distinguish if the marker is the same or different from the subject itself. We define $p_{=,i}$ as the probability to choose i if both partners have the same marker . We define the following relationships that each agent fulfills:

$$p_{=,0} + p_{=,1} = 1$$

$$p_{\neq,0} + p_{\neq,1} = 1$$

  The parameters in this model are:

  - **e:** For every interaction, measures the probability for the interaction to be marked (both partners have a priori the same marker), like the willingness of the agents to choose someone with the same marker. In the results presented here it has been set to $e = 0.5$.
  - **δ:** It is the extra payoff obtained for a coordination during an interaction. In the results presented here it has been set to $\delta = 0.5$.
  - Parameters from [1]: Like l and h. The habituation will be fixed to zero in this report (aspirations fixed) and l will be set to $l = 0.5$.

  The dynamics proceeds as follows: For each timestep, an agent is chosen. With probability e, the interaction is marked:

  1. If the interaction is marked, a subject and object with the same marker are chosen. They choose their strategy for the coordination game throwing a random uniform number between zero and one, and comparing it to their $p_{=,0}$ and $p_{=,1}$ probabilities.
  2. If the interaction is not marked, subject and object are chosen at random. They check if their markers are the same or different, and, depending on that, they throw a random number and choose $p_{=,i}$ or $p_{\neq,i}$ probabilities.

After this process subject and object have chosen their action, they interact in this coordination game and collect their payoff:

$$\begin{pmatrix} 1+\delta & 1 \\ 1 & 1+\delta \end{pmatrix}$$

After the interaction, both subjects and partners calculate the difference between the payoff obtained and their aspirations, and update their stimulus and probabilities. This final part may be checked in [1] as the process is the same as the one described on page 3.
The parameters used in the simulations are:

- **Number of agents**, we are studying a population of five hundred agents.
- **Final time per realization**, we are letting the population evolve for four thousand interactions per agent (in total, $3 \cdot 10^6$), just because it seems a reasonable time to reach the equilibrium.
- **Number of simulations**, statistical averages and plots have been made with 100 simulations.
- **Initial conditions**: Markers and behaviors are chosen at random.

The results that are presented below are the following:

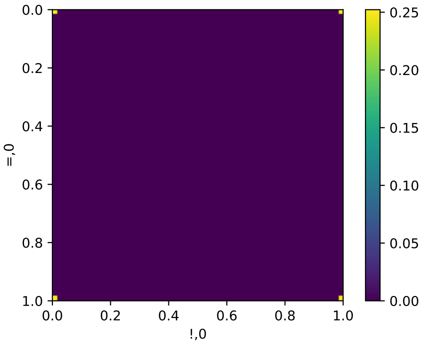
- Results for fixed level aspirations. All the population has the same aspirations. The cases studied are: $A_i = \{1-\delta, 1, 1+\delta/2, 1+\delta, 1+2\delta\}$.
- Robustness of the result if the coordination is not perfect, and there is some preferred equilibrium. We make this by including $a_1, b1$ in the payoff matrix:

$$\begin{pmatrix} 1+\delta+a_1 & 1-b_1 \\ 1 & 1+\delta \end{pmatrix}$$

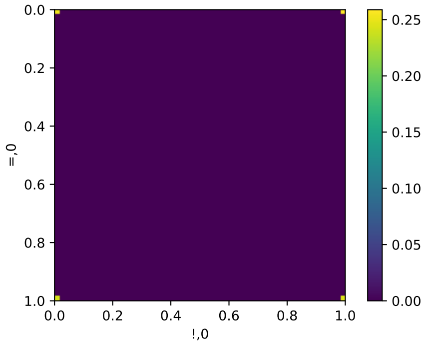
In order to present this results we will show 2D histograms, where $p_{=,0}$ and $p_{\neq,0}$ are in the axis.

2 Results

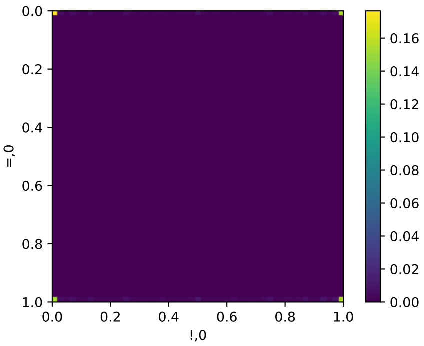
2.0.1 Fixed aspirations in an unique group



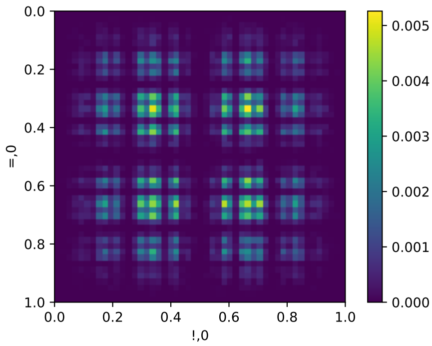
(a) $A_i = 1 - \delta$



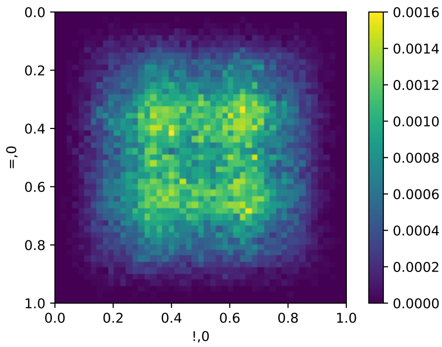
(b) $A_i = 1$



(c) $A_i = 1 + \delta/2$



(d) $A_i = 1 + \delta$



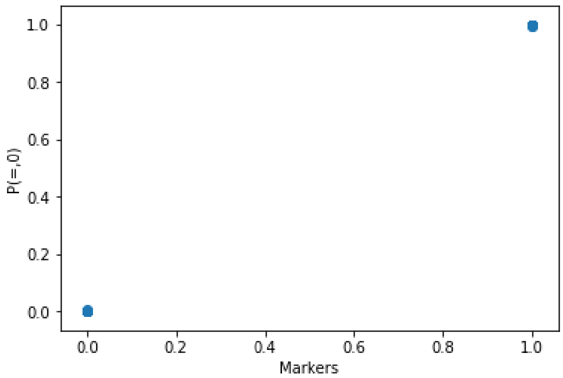
(e) $A_i = 1 + 2\delta$

For fixed aspirations for all members we can see two kinds of behavior:

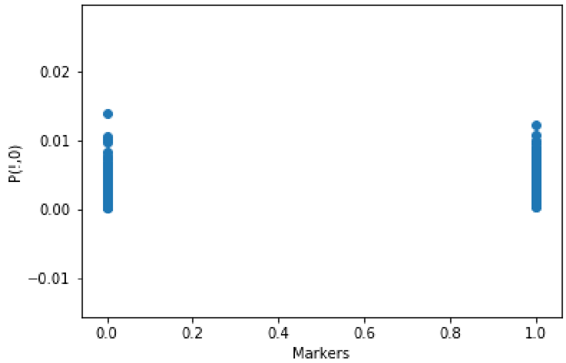
- **$A_i \leq 1$:** All stimula are positive (or neutral, in the extreme case). This means that every action just encourages the subject more to choose the same action. If we let this dynamics evolve, the subjects are grouped in the four possible combinations of $\{p_{=,0}, p_{\neq,0}\}$. As the agents are not capable to distinguish positive/negative stimula, these groups have no correlation with the marker. We will call them conformists.

- **$1 < A_i < 1 + \delta/2$:** Positive and negative stimula exist. Agents are grouped also, but this time $p_{=}$ is marker related and $p_{\neq}$ is unique for all the population. It may happen that both behaviors are the same, promoting homogeneity. As this characteristic may not be captured by the statistics, we present some correlations for a single trajectory(2a,2b) . We show the scatter plot of the probability of choosing $=$ or $\neq$ vs the agents markers. We will call them conditional conformists.

- **$1 + \delta/2 \leq A_i$:**All stimula are negative (or neutral, in the extreme case), so every action dissapoints the agent. The agents are then moving randomly in the phase space. We will call them unconditional non-conformists.



(a) Markers vs $p_{=,0}$

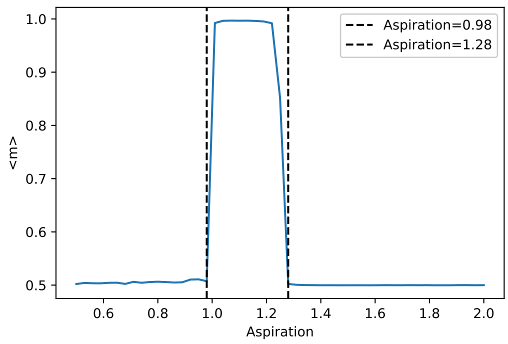


(b) Markers vs $p_{\neq,0}$

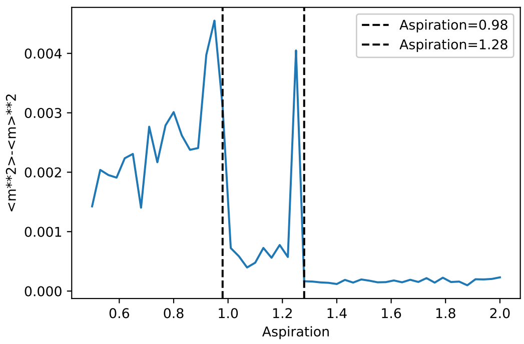
Let us define the variable *Ratio of coordinations*:

$$m_i = \frac{coordinations}{interactions}$$

For every agent. We let the trajectories evolve, and after the termalization, we compute the distribution of m_i during 2500 interactions.



(a) $< m >$



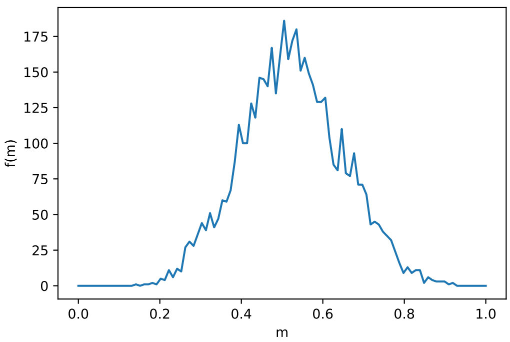
(b) $\sigma^2(m)$

We can see that there are two different phases:

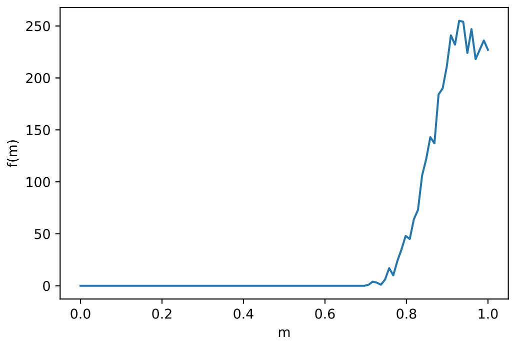
- One with gaussian shape centered at $< m >= 0.5$. For low aspirations, it has a greater variance.
- Another one with exponential shape decaying from $< m >= 1$.

Just to show the different situations, we show the different coordination distributions that arise in the population : The sudden increase of the coordination made us think that perhaps there is some kind of phase transition in the region between $(1 < A_i < 1 + \delta/2)$. In order to check this we have measured mean, variance, and binder cumulant for different sizes in the left and right part of the former "transition". We present the results:

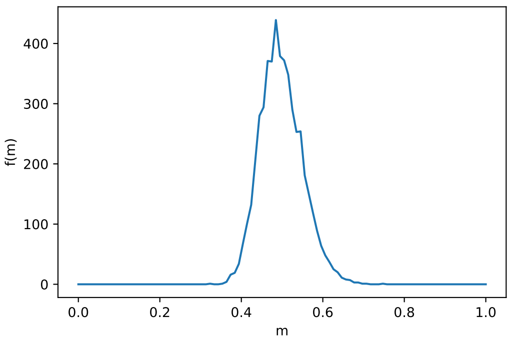
The results are not decisive. More sizes need to be analyzed.



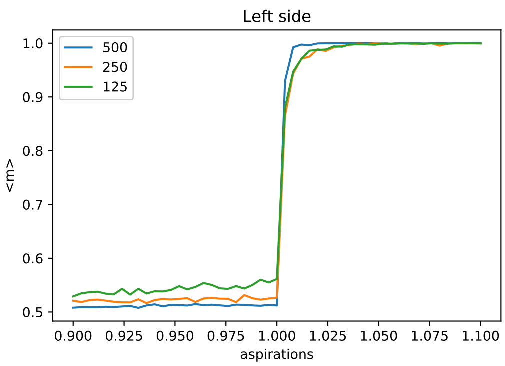
(a) Coordination profile for aspiration=0.8



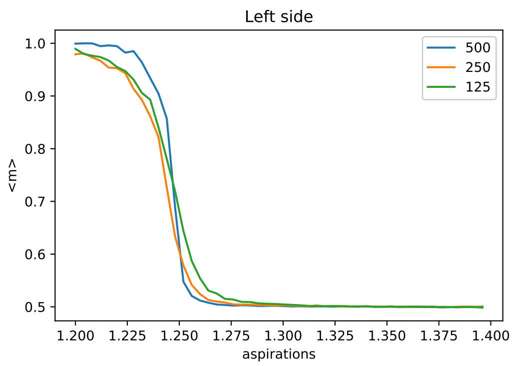
(b) Coordination profile for aspiration=1.1



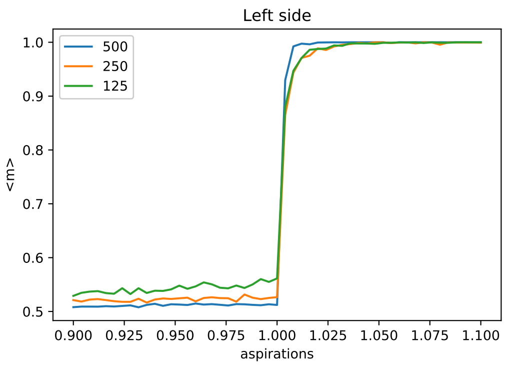
(a) Coordination profile for aspiration=1.5



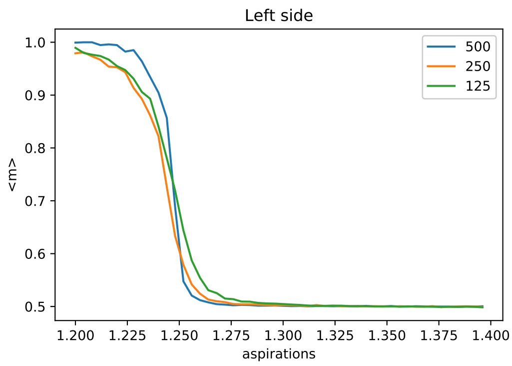
(a) $< m >, left$



(b) $< m >, right$



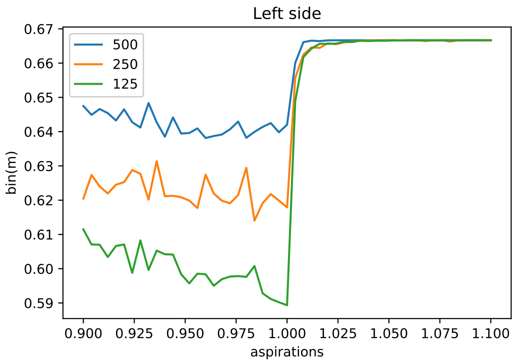
(a) $< \sigma^2 >, left$



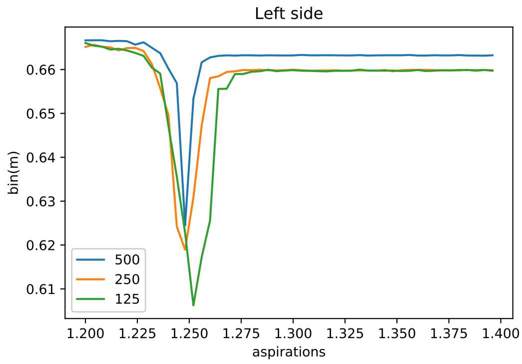
(b) $< \sigma^2 >, right$

2.0.2 Fragmented population

In this section, we are going to explore the following set up: one group with fixed aspirations, but several options. These options are associated with the different phases of the system, in particular we have chosen $\{0.8, 1.1, 1.5\}$.



(a) $Binder, left$



(b) $Binder, right$

We will play with these options in different proportions $(0.25, 0.5, 0.75)$ and analyze the implications of having a mixed population.
The plots may be consulted in the files attached to this document, as they are so many.They ar in the format **(aspiration1,aspiration2,% of 1.1 agents)**
What happens if conditional conformists are mixed with other type of agents? They lose full coordination but also:

- . If conditional conformist are majority (75% of the population):
  - And are mixed with conformists, conditional conformists still correlate their $p_{=,i}$ with their marker, but they do not choose always the same $p_{=,i}$. This means that $0,5 < p{=},i < 1$, so their most probable option is to choose the correlated probability, but they could break the rule. While all this happens, conformist tend to group in the four possible combinations of $\{p_{=,0}, p_{\neq,0}\}$.
    What is happening in the system? Conditional conformists are adapting their intra-marker and inter-marker correlations to the most frequent group of conformists. In this way, they maximize their coordination and can coordinate with another big group (this is why the biggest peak in $\{Coord, 0.8.eps\}$ is the one with more coordination). The rest of the peaks that appear are related to the correlations: they are the ones with intra-marker but not inter, the ones with inter-marker but not intra, and the ones that do not get right neither the intra nor the inter correlation. The coordination of the conditional conformists is afected also by these groups and it does not reach full coordination.
  - And are mixed with unconditional non-conformists, conditional conformists still correlate their $p_{=,i}$ but, as before, not perfectly. The difference with the former case is that, although there are agents that choose randomly their position, a great majority (in the minority) also correlates their behaviors. They act as conditional conformists. What is happening in the system? Conditional conformists are again choosing the collective conformation that maximizes their own coordination. As they are not capable to adapt to unconditional non-conformists, they choose a fixed strategy for intra-marker and another fixed strategy for inter-marker. As unconditional non-conformists still prefer coordination to non-coordination, they accumulate in the surroundings of the conditional conformists block (they copy the intra and inter marker correlation but not perfectly). The coordination is greater for conditional conformist, as they coordinate perfectly between them and have a great probability of coordinating with unconditional non-conformists.
- If conditional conformists are in the same proportion (50% of the population):
  - With conformists, the correlation marker-$p_{=,i}$ partially exists. The correlation is not perfect (does not happen in all cases) and there are also conditional conformists that do not follow it; they behave randomly (a minority). Conditional conformists choose a $p_{\neq}$ common for a big group, but the correlation is partially destroyed. Conformists still form groups.
    Conditional conformists are still copying the most frequent correlation from conformists, but as the inter-marker correlation is partially destroyed, the two peaks now represent the ones that get right the intra-marker correlation and the ones that do not (and again the peak with more coordination is more populated). However, at this point, conditional conformists are not a majority, and this advantage in coordination is not as big as before.
  - With unconditional non-conformists, conditional conformists still coordinate and correlate marker-$p_{=}$. The unconditional non-conformists also correlate, and also the majority of the population still chooses (not perfectly) a more probable value for $p_{\neq}$, the same for all. However, unconditional non-conformists are much less effective, and they may correlate $p_{=}$ but not $p_{\neq}$ or vice-versa or none of

them.
Conditional conformists are fixing a non perfect correlation for intra-marker and inter-marker that unconditional non-conformists follow weakly, because conditional conformists do not represent a majority. Coordination is still bigger for conditional conformists because they coordinate a little bit better that unconditional non-conformists. At this point the main source of coordinations are the interactions conditional conformists/unconditional non-conformists.

- If conditional conformists are a minority (25% of the population):
  - And are mixed with conformists, the correlations are weaker than before. But, as before, a significant proportion of these agents still correlate marker-$p_=$ and choose with more probability a common $p_{\neq}$. The system is dominated by conformists but a weak intra-correlation still exists. This is the reason why the conditional conformists only lie at the right on the conformist's gaussian.
  - And are mixed with unconditional non-conformists, the correlation marker-$p_=$ is almost destroyed. And happens the same, with $p_{\neq}$. The system is dominated by the coordinations between unconditional non-conformists, and they promote randomness. That is why the coordinations of conditional conformists and unconditional non-conformists obtain the same coordination rate.

The last case is a well mixed population of agents with fixed aspirations and three choices: $\{0.8, 1.1, 1.5\}$:
In this case, conditional conformists are again copying the most frequent group of conformists. As they represent only a third of the population, the advantage in coordination is very weak. Meanwhile, unconditional non-conformists are distributed randomly in the phase space, and conformists form groups like expected. An important point is that unconditional non-conformists have a narrower gaussian because they really behave randomly and have a probability to coordinate more around 0.5 than conformists. They interact better in the groups they form, but their success also depend on the relation with conditional conformists and, eventually, with unconditional non-conformists.

2.0.3 Conclusions

In a complete network, conditional conformists seem to use markers as a way to form global correlations (if they are enough to do so). These correlations help them to arrive to global coordination and maximize their payoff. We could say that markers help agents developing a bias that makes them know how to behave. Are markers helping a system of conditional conformists to reach the Nash equilibrium?

2.0.4 Two groups with migration

Taking as a starting point the set up explained above, we add another individual label to it: the group. This label is binary $\{0, 1\}$ as we are using only two groups. We will also introduce migration. Every N interactions, a sample of mN agents (where m is the migration parameter) will exchange groups.
At this moment, we have checked medium populations ($N_{group} = 250$) and introduce different initial aspirations in each group:

- Group 1 with initial aspiration 0.8 and group 2 with initial aspiration 1.1
- Group 1 with initial aspiration 1.5 and group 2 with initial aspiration 1.1

What we have checked is that these configurations (and this timescale for the migration) only promotes homogeneity and, as you can check in the plots, at the end of the day we obtain a fragmented group like the ones studied above.
But is there any other effect of migration apart from homogeneity? In order to explore this, we have studied a set up of two groups of conditional conformists with fixed aspirations ($A_i = 1.1$) with different migration speeds (β) and different quantities of migransts (m).

2.0.5 Small populations

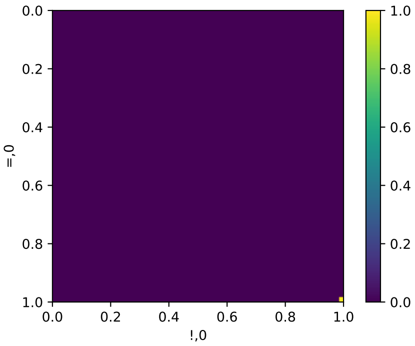
Here, we will explore the same set ups we did before, but using reduced group sizes, in particular we will use $N = 50$. At this moment, we have only checked that the different configurations that arise from the fixed aspirations study, are conserved for small populations.

2.1 Unbalance in the payoff matrix

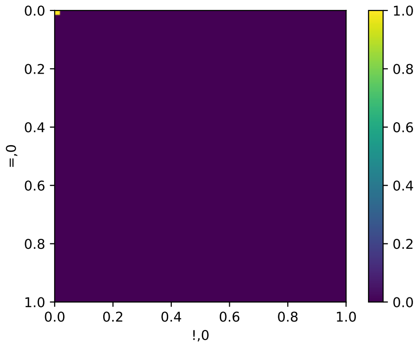
When we support one of the coordinations, or favour one of the uncordinations, this is :

$$\begin{pmatrix} 1+\delta+a_1 & 1-b_1 \\ 1 & 1+\delta \end{pmatrix}$$

We include statistics in the case ($\delta = 0.5, A_i = 1 + \delta/2$), which promotes coordination. The system tends to



(a) $a1 = \delta/2, b1 = \delta/2$



(b) $a1 = -\delta/2, b1 = -\delta/2$

choose the equilbrium where the agents maximize their payoff.

2.2 Habituation

In this section we are going to introduce a rule for the individual variation of aspirations defined in [1], this is :

$$A_{t+1} = (1-h)A_t + h\pi_t$$

Where A_t is the aspiration of an individual at time t, h is the habituation(which we will treat as a collective parameter) and π_t is the payoff obtained at time t.
In order to find out the dynamics of the system, we treat this evolution of the aspiration as a map and fix its fixed points:

$$A^* = (1-h)A^* + h\pi \qquad\qquad A^* = \pi$$

Where we assume that π is constant for all time. The aspiration only remains constant if the payoff is also constant. We have checked this introducing a population with a fixed probability profile so they choose a specific criteria always. The population always evolves towards $A_i = 1 + \delta$ for every agent, regardless the initial aspirations.
Another interesting thing we did was looking for k-periodic orbits in the aspirations map, trying to search any stable solutions. For this purpose, we build the aspirations for t+n map:

$$\begin{aligned} A_{t+2} &= (1-h)A_{t+1} + h\pi_{t+1} \\ &= (1-h)((1-h)A_t + h\pi_t) + h\pi_{t+1} \\ &= (1-h)^2 A_t + h(1-h)\pi_t + h\pi_{t+1} \end{aligned}$$

And :

$$\begin{aligned} A_{t+3} &= (1-h)A_{t+2} + h\pi_{t+2} \\ &= (1-h)((1-h)A_{t+1} + h\pi_{t+1}) + h\pi_{t+2} \\ &= (1-h)((1-h)((1-h)A_t + h\pi_t) + h\pi_{t+1}) + h\pi_{t+2} \\ &= (1-h)^3 A_t + (1-h)^2 h\pi_t + (1-h)h\pi_{t+1} + h\pi_{t+2} \end{aligned}$$

So we write in general :

$$A_{t+n} = (1-h)^n A_t + h\sum_{i=0}^{n-1}(1-h)^i \pi_{t+n-1-i} \tag{1}$$

As this equation is pretty difficult to manipulate, we consider $h \to 0$ and consider only the terms up to $O(h)$:

$$\begin{aligned} A_{t+n} &= (1-nh)\,A_t + h\sum_{i=0}^{n-1}\pi_{t+n-1-i} \\ &= A_t + h\left(-nA_t + \sum_{i=0}^{n-1}\pi_{t+n-1-i}\right) \end{aligned}$$

And if we assume the condition for a n-periodic orbit $A_{t+n} = A_t$ the solution for $h \neq 0$ is:

$$A^*(n) = \frac{\sum_{i=0}^{n-1}\pi_{t+n-1-i}}{n} = \langle \pi(t)\rangle|_t^{t+n-1}$$

The stability of this solutions is:

$$\frac{dA_{t+n}}{dA_t} = (1-nh)$$

This means that any orbit that satisfies $|nh| < 2$ is stable.
Just for fun, we consider the approximation to second order obtaining:

$$\begin{aligned} A_{t+n} &= \left(1 - nh + \frac{n(n-1)h^2}{2}\right)A_t + h\sum_{i=0}^{n-1}(1-ih)\,\pi_{t+n-1-i} \\ &= A_t + h\left(-nA_t + \sum_{i=0}^{n-1}\pi_{t+n-1-i}\right) + h^2\left(\frac{n(n-1)}{2}A_t - \sum_{i=0}^{n-1} i\pi_{t+n-1-i}\right) \end{aligned}$$

We suppose solutions from the type:

$$A^* = \sum_{i=0}^{n-1}\pi_{t+n-1-i} + h\delta A(n)$$

In order to look for the relevant $O(h^2)$ fluctuations. We substitute this ansatz in the equation, up to second order and apply the condition for n-periodic orbits ($A_t = A_{t+n}$):

$$\begin{aligned} A_{t+n} &= \left(1 - nh + \frac{n(n-1)h^2}{2}\right)A_t + h\sum_{i=0}^{n-1}(1-ih)\,\pi_{t+n-1-i} \\ 0 &= h^2\left(\frac{n(n-1)}{2}\sum_{i=0}^{n-1}\pi_{t+n-1-i} - n\delta A - \sum_{i=0}^{n-1} i\pi_{t+n-1-i}\right) \end{aligned}$$

And this implies:

$$\begin{aligned} \delta A &= \frac{\frac{n(n-1)}{2}\sum_{i=0}^{n-1}\pi_{t+n-1-i} - \sum_{i=0}^{n-1} i\pi_{t+n-1-i}}{n} \\ &= \frac{n(n-1)}{2}\langle \pi(t)\rangle|_t^{t+n-1} - \langle i\pi(t)\rangle|_t^{t+n-1} \end{aligned}$$

So, the equation for a n-periodic orbit if $h \to 0$ up to second order in h is :

$$A^* = \langle \pi(t)\rangle|_t^{t+n-1} + h\left(\frac{n(n-1)}{2}\langle \pi(t)\rangle|_t^{t+n-1} - \langle i\pi(t)\rangle|_t^{t+n-1}\right) \tag{2}$$

Where $\langle \pi(t)\rangle|_t^{t+n-1}$ is the average payoff of the orbit, and $\langle i\pi(t)\rangle|_t^{t+n-1}$ is the average payoff weighted by the position in the time series, related to the distribution of the payoffs around the orbit.
The main result is then that a n-orbit exists around the average payoff of the agent. The variations of $O(h)$ are related to the structure of the payoff recollection around the orbit.
At this point, we present the simulations made with our model for several configurations of population:

- Initially homogeneous populations: The whole population has a common initial aspiration, we will study these cases: $\{0.8, 1.1, 1.5\}$.
- Initially fragmented populations: Initial configurations of aspirations are chosen between the former options, like in the case of fragmented populations.

We will take these initial conditions and make statistics with 100 trajectories. This process will take place for $h = \left\{10^{-4}, 0.5 \cdot 10^{-3}, 10^{-3}, 0.5 \cdot 10^{-2}, 10^{-2}, 0.5 \cdot 10^{-1}, 10^{-1}, 0.5 \cdot 10^{-1}\right\}$.
The general conditions for these simulations are: We will study a group of 500 agents letting it evolve $2000N$ timesteps and taking 100 repetitions. The rest of the values are not modified from previous simulations. Plots are attached to this report in the folder habituation. Each set up is in a separate folder, where there are plots of distribution of coordination and aspiration for each value of h.

Initially homogeneous populations

- *Initial aspiration: 0.8*: For $h < 5 \cdot 10^{-3}$ the evolution is slow enough so these conformists evolve towards conditional conformists, develop correlations between them , and, due to total coordination, they finally arrive to $A_i = 1.5$.
  For $h = 5 \cdot 10^{-3}$, the inter-marker correlation is destroyed(*). The system still develops the intra-marker correlation and the agents stabilize around $A_i \simeq 1.4$ and 87.5% of coordination (corresponding to the 75% of intra-coordination (e=0.5) plus the half of the remaining inter-marker interactions.
  For $h = 10^{-2}$, the system may have two outcomes. Either all the correlations are destroyed, and $A_i \to 1 + \delta/2$, which means that all the coordination disappears; either the intra correlation is still conserved and we arrive to the former situation. This second scenario is more probable than the first one.
  For $h > 10^{-2}$, all the correlations are destroyed, and the agents aspirations evolve to $A_i = 1 + \delta/2$, and the coordination is a gaussian around 0.5. Agents do not have the time to develop correlations.
  For $h = 0.5$, habituation is too high, and aspirations are the sum of the recent past payoffs. This makes the distribution of aspirations look like a uniform distribution between $(1, 1+\delta)$. The coordination at this point is still a gaussian around 0.5.
  (*)In former simulations (the ones with fixed aspirations), it has been seen that the inter-marker correlation takes longer to be clear than the intra-marker correlation. This may be because $e = 0.5$ makes the 75% of interactions to be intra-marker.

- *Initial aspiration: 1.1*: The situation is similar to the one described above, with slight differences: · The coexistence (inter-marker and intra-marker destroyed/ only inter-marker destroyed) appears now for $h = 10^{-2}$ and the inter and intra destroyed is statistically more relevant.

- *Initial aspiration: 1.5*: The situation is different here. As we are talking about unconditional non-conformists, they are not going to develop correlations. Therefore, agents can not sustain their aspirations and go towards $1+\delta/2$. This happens for all values of h, except for $h = 0.5$, where, as always, the uniform distribution arises.

Initially fragmented populations

- *Initial aspiration: half 0.8-half 1.1*: The evolution of this set up is similar to the ones with $A_i = 0.8$ and $A_i = 1.1$. For $h \simeq 10^{-4}$, the slow evolution sustains intra-marker and inter-marker correlation. For $h \simeq 10^{-2}$, the peak on $A_i = 1 + \delta/2$ also appears, but this time is more probable than coordination. The rest of scenarios are pretty equal to the ones analyzed in the mentioned cases. This situation is the same if we change that 50% to a 75% of 1.1 agents.

- *Initial aspiration: half 0.8-half 1.5*:This case is also similar to the one abobe. For $h \simeq 10^{-4}$, the 0.8 agents sustain the 1.5 ones providing the coordination. The rest of the scenarios are very similar to the set up above. This situation is the same if we change that 50% to a 75% of 1.5 agents.

- *Initial aspiration: half 1.1-half 1.5*: In this case, if $h \simeq 10^{-4}$, the conditional conformists support the unconditional non-conformists and sustain partial cooperation. However, if $h > 10^{-4}$ the agents do not have the time to sustain this partial correlation and the system evolves towards randomization (with coordination in a gaussian around $1 + \delta/2$).

References

[1] M. W. Macy, A. Flache, *Learning dynamics in social dilemmas*, Proc. Natl. Acad. Sci. USA **3**, 7229-7236 (2002).