Self-Adaptive Feedback E-Learning Scheme for Elementary Math in Kuwait

Fawaz S. Al-Anzi
Department of Computer Engineering
Kuwait University
Kuwait City,Kuwait
fawaz.alanzi@ku.edu.kw
ORCID ID: 0000-0001-9967-8476

Abstract—**Mathematics is widely recognized as the most important and essential discipline because it serves as the foundation for many notable scientific and technical breakthroughs over the centuries. Students' attitudes toward mathematics can be significantly improved by using computers as a medium of instruction, according to several studies. Artificial Intelligence, and thus Machine Learning, is a critical driving force in the advancement of innovation and growth in a wide range of industries today, with the education sector being no exception in this regard. An Adaptive Feedback E-Learning Scheme for Elementary Math in Kuwait is being developed and implemented to improve the quality of mathematics education for the next generation of elementary school students. The proposed methodology attempts to close the gap between students' Math learning abilities and those of their teachers using Artificial Intelligence and Machine Learning methods. The adaptive Learning approach seeks to create a compelling learning experience that is adapted to the individual needs of each learner in a personalized manner. To determine the strengths and weaknesses of each student who enters the system, the suggested model operates in two modes: Diagnostic Mode and Remediation Mode. Additional rounds between the two modes may be required to address all a student's deficiencies for him or her to achieve an appropriate degree of mastery of the various arithmetic topics. Work on this project is being carried out in collaboration with the Kuwait Foundation for Advancement of Sciences (KFAS), Kuwait University (KU), and the Kuwaiti Ministry of Education (MoE).**

Keywords— Math learning, elementary schools, feedback authoring, feedback personalization, e-learning assessment, stochastic gradient descent, Monte Carlo simulation, Bayesian model.

I. INTRODUCTION

Mathematics is regarded as the most significant and foundational topic because it is the foundation of all major scientific and technological advancements. Mathematics has a wider range of applications in our daily lives, and mathematics is deeply engrained in our daily lives. Mathematics is an essential subject for primary pupils because it is a cornerstone to many other subjects. Unlike all other disciplines, the quality of Math learning is determined by the student's enthusiasm for the topic. The main reasons for lower mathematics achievement are a lack of enthusiasm in the topic and a willingness to embrace difficulties, as well as mathematical incompetence.

Some scholars have worked to explore various approaches for overcoming problems related to the lack of interests and attitudes toward mathematics and increasing students' Math learning rates by applying various models,

methodologies, and supporting learning aids [1][2]. The two major factors contributing to the low learning rate in mathematics are the lack of effective learning models and teachers who were unable to discover, assess, and increase the mathematical capacity of elementary school kids.

The efficiency and practicability of various forms of feedback assessment, which could positively or negatively influence e-learning and interactive sessions, is an important factor to consider when building a feedback model [3]. A well-structured feedback design should solely help the student's learning rather than interrupting or confusing them while they are learning. I am attempting to demonstrate this in our study.

According to the Trends in International Mathematics and Science Study (TIMSS) mathematics scale report 2011, Kuwaiti kids' Math education level is likewise lower than expected and lower than many other countries [4]. As per the International Association for the Evaluation of Educational Achievement's (IEA) TIMSS International Results in Mathematics and Science published in 2015 and 2019 also, Math learning rate of Kuwaiti students is very much inferior to those countries who ever participated in TIMSS's mathematics achievement assessment [5-7]. Kuwait is under 49th and 55th position based on the score attained in the TIMSS assessment for 2015 and 2019. So, it is necessary to generate a remedy to combat these deficiencies.

Kuwait University aptitude exam statistics (2014) studied the results of a Kuwait University aptitude test for Kuwait students and found that just 40% of the students who took the test passed the Math exam. It necessitates implementing an alternate plan to address the inadequacy of Kuwait's education system, both private and public, in cultivating appropriate mathematical knowledge for Kuwaiti students prior to enrolling in college. According to estimates published in the Kuwait Teachers Association Journal (Nov 2012), Kuwaiti families pay roughly 5426 KD (US$18000) per student every year on private tuition for their children, which is a defensive measure to the Kuwait education system's weaknesses. As a result, Kuwait's leading educational institutions began to focus on this crucial challenge. The MoE has undertaken an ambitious effort to acquaint high-tech solutions for supporting the new scientific educational system in both private K12 and government educational institutions. E-learning has been identified as one of the most important tools for enhancing both teachers' and students' scientific abilities. Other agencies quickly began to respond to the MoE initiative by

completing it with other national programs. As Kuwait's primary sponsor of scientific research, the KFAS has restructured its five-year strategic program to meet the youth's insufficiency of math and science education by channeling funds to research that will help them leverage their math and science skills.

In this study, I am trying to propose a self-adaptive feedback system that is both efficient and effective in diagnosing the weakness of each student, estimate and suggest a most adaptable remedy plan, and train each student as per the proposed remedy plan. The self-adaptive feedback system should improve the quality of math teaching and help Kuwaiti primary school pupils better comprehend math and scientific issues. As a result, I recommended in this study that this self-adaptive feedback approach is both efficient and successful in meeting student needs and increasing the level of Math learning in Kuwaiti elementary school kids. This study is conducted as a response to the MoE initiative and in partnership with KFAS, Ministry of Education, and Kuwait University Teachers, parents, and students, Kuwait government, and KFAS will all get direct benefits from the study findings.

The work is primarily based on recent experimental investigations conducted on Kuwaiti elementary government school students that have been thoroughly evaluated and proven. Multiple-choice questions constitute the basis of the e-learning Math exam. The students can highlight their confidence level alongside the answer; for these answered quizzes, varied immediate effective feedback is calculated and provided to all those students attending this exam. Thus, it is feasible to construct a self-adaptive feedback model that may anticipate the ideal model that enhances students' Math knowledge by identifying numerous scenarios.

II. RELATED WORK

Even though mathematics is by far the most essential component among others in most careers, most students eschew it. Unlike all other disciplines, there is a substantial link between students' attitudes and mathematical achievement, which has been studied extensively. Their relationship was reciprocated, and they mutually relied on one another [1][8]. The results of numerous surveys revealed that students who excelled in math have a favorable attitude toward the discipline, whereas students who struggled with math have a dismissive attitude [9]. According to the existing literature, raising the velocity of math learning and thus the quality of math learning can be accomplished by either generating a positive attitude or lowering a negative attitude. Students' views about this subject improved after studies on the impact of electronic devices such as computers and calculators were conducted [2][10].

According to the studies, the use of computer systems as well as other smart gadgets in classrooms could be influenced by both positive and negative factors [11]. The introduction of these digital gadgets to the education system helps to develop a favorable impact on students' attitudes about mathematics and computers [12][13][14][15]. Ganguli investigated the influence of using computers as a

supplemental teaching aid in mathematics instruction (1992). The use of computers as a teaching tool resulted in considerable differences from those who did not learn without them and a pleasant experience toward the topic.

In the field of education, e-learning is the most extensively used electronic platform. Unfortunately, a few of the characteristics and considerations that could harm students' academic performance have yet to be addressed. As a result, individualized e-learning systems are critical for changing personality, behavior, knowledge, preferences, or interests. As per the base of 150 research articles gathered between 2016 and 2020, Acuna et al. provided a survey on a personalized e-learning model with the goal of estimating the basic factors to be considered for establishing a customized e-learning model [16]. Based on studies completed in the preceding five years, they measured and analyzed numerous parameters, learning methods, algorithms, and strategies used to develop a personalized e-learning strategy. They developed a hybrid e-learning model with a chatbot based on this research.

Adaptive e-learning frameworks can offer the optimal pedagogical strategy for a student while also extracting data and learner characteristics. A multi-agent architecture is a collection of organized and autonomous elements that interact to figure out a solution or accomplish a specific objective. Whether homogeneous or heterogeneous, these agents are always interacting, whether they have common aims. By customizing the information to the requirements of the students, the adaptation of a multi-agent strategy into adaptive e-learning models can increase the quality of student learning. These agents collaborate to generate a customized learning environment. Reference [17] presented a multi-agent and reinforcement, learning-based adaptive e-learning model. The proposed model employed the Q-learning algorithm to provide students with a training path that matches their characteristics and preferences. The suggested strategy is based on three primary characteristics: the learner's learning style (as defined by the Felder-Silverman learning style model [18][19]), knowledge level, and any potential limitations. In this technique, hearing, vision, and dyslexia are the three main kinds of impairments that are examined. The approach is designed to provide learners with a succession of learning things tailored to their needs to provide a personalized learning experience.

Sustainable learning has adopted several measures to mitigate the detrimental effects of social, technological, and individual elements, such as e-learning systems, which have partially replaced traditional school environment-based education to ensure the long-term survival of such systems. In addition, the Massive Open Online Course (MOOC) movement is gaining traction since it empowers everyone to improve their knowledge and skills. MOOCs are especially intriguing since they involve a large amount of instructional material as well as a large number of participants from varied cultural and educational backgrounds. Nonetheless, the low pupil retention rate (about 10%) raises questions about this form of instruction's long-term feasibility. Furthermore, even though e-learning platforms have attracted a large number of students by abolishing barriers such as geography and time, such environments are significantly influenced by high dropout rates. In this study, [20] offered long-term e-learning measures to counter

student dropout rates. The main goal is to provide packages that correspond to how students can efficiently complete their learning activities. In this technique, an adaptive e-learning framework takes advantage of the traces left behind by effective interactions between users and their learning environment. They created this model by calculating the path using an ant colony method and automatically delivering suitable courses based on their profiles.

Reference [21] investigated and examined the factors responsible for the excessive e-learning environment dropouts. One of these critical concerns is the pupils' lack of adequate motivation as a result of the identical learning opportunities being offered to them, regardless of their learning preferences. A number of academics have recommended gamification to increase student participation. The strategy has increased participation slightly, but not as much as was planned. The inability of gamification elements to encourage learners on an intrinsic level is one of the most fundamental concerns of them. Hassan et al. [21] established a solution to this challenge that identifies each learner's independent learning habits based on their interactions with the platform and then provides an adaptable gamification environment based on those learning characteristics. Based on their simulation, they assessed that the learners' motivation increased by 25%, and the dropout rate fell by 26%.

III. METHODOLOGY

Math instills crippling feelings of fear and failure in students, affecting their capacity to perform. When presented with math, many children's anxiety and lack of confidence cause their brains to stop and forget even what they already know. Math instills in them crippling fears of failing and hinders their ability to achieve. In this proposed approach, I am trying to overcome all these insecurities from students through this effective feedback model. Here, I am proposing a Self-Adaptive Feedback E-Learning Model for Elementary Math in Kuwait in order to reap the following key goals:

- To enhance the math education level of elementary school students

- To develop self-confidence in students for conquering all the barriers and obstacles that pull back them in learning mathematics

- Help them to learn mathematics in a more interesting and enthusiastic manner

Fig. 1 and Algorithm I depicts the outline of the entire procedure for the proposed Self-Adaptive Feedback E-Learning Model. The first and foremost step for this model design is to gather and analyze the Math curriculum in elementary schools for performing a detailed study and analysis on various units in each grade and their corresponding lessons. Each unit may depend on its previous unit or grades since, unlike other subjects, mathematics is more cumulative in which each concept builds on already learned or mastered concepts. So, all these inter-unit dependencies should be identified. Each unit in the math curriculum has a different level of importance as per various factors such as concepts, the time allowed to finish that unit, their practical usage, etc. Each unit is assigned with its

corresponding weights based on its importance, and a weight matrix is formed with these individual unit weights.
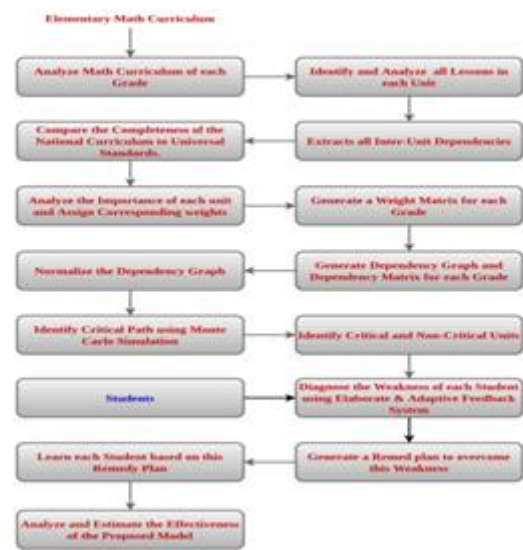

Figure 1: General Outline of the Proposed Model

A dependency matrix and dependency graph will be then generated from these interdependencies among the units, and these graphs are normalized by removing all transitive and cyclic relationships. The Monte Carlo simulation-based critical pathfinder is then applied to this dependency graph and corresponding weight matrix to determine the most critical or significant units in that grade. The resulting critical path presents critical and non-critical units for each class.

Our proposed Self-Adaptive Feedback E-Learning Model, as illustrated in Algorithm II, tries to identify the weakness of each student in terms of units or lessons. Based on this weakness, each student will be trained according to the Self Adaptive Training procedure.

Algorithm I: Complete Procedure of the Proposed Model

i. Gather all information regarding the Math curriculum of Elementary schools from 1 and 2.

ii. Analyze the Math curriculum, teaching methods, duration, number of units, lessons, and their contents in each grade.

iii. Identify and extract the inter-unit dependencies that exist among different units in each grade.

iv. Compare the completeness of the national curriculum to the universal standards.

v. Estimate and analyze the importance of various units based on various factors such as contents (number lines or figures added in the textbook to explain the topic, a number of exercises, examples given, etc.), time taken to finish that unit, and so on.

vi. Assign a corresponding weight to each unit based on their importance and form the weight matrix.

vii. Generate dependency matrix and draw dependency graph for each class.

viii. Normalize the graph by removing transitive dependencies.

ix. Determine the critical path using Monte Carlo simulation-based approach and present critical and non-critical units.

x. Diagnose the weakness of each student entering into this system using the proposed Self-Adaptive Feedback E-Learning Model as illustrated in Algorithm 2.

xi. Generate a remedy plan to overcome student weakness if exists.

xii. Learn each student as per their customized learning plan.

xiii. Evaluate the effectiveness of the proposed model.

Weak points of each student are identified within a minimal number of questions based on the critical path and dependency graph. Adaptive feedback model traversed to each unit as per each student's performance. Unlike other subjects, mathematics is more cumulative. So, if a student needs to learn a particular unit, it is necessary to learn all the predecessor units as per the dependency graph. For example, if a student is going to learn multiplication, he must know about numbers and their addition, and vice versa.

Algorithm II: Self-Adaptive Feedback E-Learning Model

Input:

Dependency Graph, G

Minimum Number of Questions, Q_N

Critical Path, $G_{Critical}$

Threshold Score For each Unit, T

Output:

Weakness, W

Procedure:

1. Initialize Ex_List ← Null, W ← Null

2. N ← $G_{Critical}$.Terminal

3. While (True):

   A. S ← Score acquired by student for unit N

   B. If S > T then

      i. If N ∉ Ex_List then

         a. Ex_List ← {x| x ∈ G, x ∉ $G_{Critical,}$ and N.hasPath(x) == Null}

      ii. Else if N ∈ Ex_List then

         a. Ex_List ← {x| x ∈ Ex_List and N.hasPath(x) == Null or x.hasPath(N) == Null}

      iii. If Empty (Ex_List) then exit from while

      iv. N ← Ex_List

   C. W.append(N)

   D. N ← N.Previous

4. Classify students using Stochastic Gradient Descent classifier

5. Display W

A. Stochastic Gradient Descent Classifier

Stochastic gradient descent tries to optimize the objective function, which is differentiable iteratively or incrementally; hence it is also called incremental gradient descent. Gradient Descent Algorithms (GDA) are the most effective and appropriate method for adapting linear regressors or classifiers to convex loss functions like Support Vector Machines or Logistic Regression. GDA has also been used to handle large-scale, sparse machine learning challenges that are common in natural language processing and text categorization approaches. GDA proceeds in the direction of descent to maximize a function, iteratively pursuing the negative gradient of a defined activation function [22]. A few of the widely used activation functions are sigmoid, softmax, and tanh. Sigmoid and softmax have been used for binary and multiclass categorization. Batch Gradient Descent and Stochastic Gradient Descent are the two main variations of GDA (SGD).

The fundamental distinction of SGD is the volume of data necessary to predict the optimization function's gradient. Consider the error or cost function $E(x_i, y_i, W)$, determined from the selected activation function with N d-dimensional learning samples (x_i, y_i), $0 \leq i \leq N$ having M classes $(x_i \in R^d, y_i \in R^M,$ and $W \in R^{d \times M})$ [22]. The key goal is to determine the function $f(.)$ that significantly lowers the calculated error using (1) having loss L and penalty R [24][25].

$$E(x_i, y_i, W) = \frac{1}{n} \sum_{i=1}^{n} L(y_i, f(x_i)) + \alpha R(w) \quad (1)$$

Iterative SGD begins from an arbitrary starting - point and attempts to minimize the gradient until it reaches the threshold point. Equation (2) is used to determine the gradient, while (3) is used to modify the weight [26][27].

$$Gradient = \alpha \frac{\partial R(w)}{\partial w} + \frac{\partial L(w^T x_i + b, y_i)}{\partial w} \quad (2)$$

$$w \leftarrow w - \eta * Gradient \quad (3)$$

Where learning rate, η manages the step-size in the parameter space, which may be static or progressively decreasing and is illustrated in (4) (learning rate = 'optimal') and degree of regularization, α $(\alpha > 0)$ [27].

$$\eta^{(t)} = \frac{1}{\alpha(t_n + t)} \quad (4)$$

B. Monte Carlo Stochastic Model

By producing random feature points, the stochastic non-deterministic mathematical technique analyzes the potential risks and uncertainties in the process. Monte Carlo simulation is defined as a combination of probability and random number generators [28]. Probability distributions like normal, log normal, and others are used to represent the random variables or input data. For building paths, various simulations or rounds are performed, and the outcome is decided using proper mathematical computations. The technique is usually employed in various fields such as statistics, quantitative finance, physical science, artificial intelligence, and computational biology. Monte Carlo Simulation produces a statistical estimate of a model's uncertainty. The Monte Carlo method trumps deterministic methodologies when it comes to risk simulation. It tells you what to expect and how probable that outcome is to occur. It also could model connected input parameters [27]. The three-point estimating technique, which includes optimistic, most likely, and pessimistic, is a simple and rapid way to assign the probability distribution function to each activity. Once determined, all these measures are commonly fitted to normal, beta, or triangular distributions for each scheduled activity [30][31].

Hundreds or thousands of such simulations might be run within the technique to determine the combination of often occurring findings. For instance, if 1,000 trials were made and 90% of them produced the same result, the simulation's result would be that [29]. People see three-time estimates as a probability distribution, but a system interprets three-time estimate data as a probability distribution. Probability allows the procedure to choose values at random from the probability distribution table to decide the outcome [32]. The simulation emphasizes randomness to simulate the real-life environment, which is essentially unpredictable, indicating that anything can happen at any time.

C. Bayesian Statistical Model

One of the effective statistical machine learning techniques, the Bayesian model based on Bayes' Rule that expresses all uncertainties as probabilities. Consider the sample space S, which is divided into n mutually exclusive events, $E_1, E_2, \ldots, E_n$ ($E_i \cap E_j = \Phi$, for all $i \neq j$), such that $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$. Consider an event Eon S such that $P(E) \neq 0$ [33][34]. Equation (5) depicts the expression for Bayes' theorem.

$$P(E_i|E) = \frac{P(E|E_i)P(E_i)}{P(E)} = \frac{P(E|E_i)P(E_i)}{\sum_{j=1}^{N} P(E|E_j)P(E_j)} \quad (5)$$

Where $P(E_i)$ and $P(C)$ are the marginal or prior probability of E_i and C, respectively, $P(E_i|E)$ and $P(E|E_i)$ are conditional probabilities of E_i and E.

Let M, D be the model and observed data respectively, we need to determine the parameter for unobserved population, θ. The posterior distribution function, given by (6), is maximized using Maximum A Posteriori (MAP) [34] model as per the (7).

$$P(\theta|D, M) = \frac{P(D|\theta, M)P(\theta|M)}{P(D|M)} \quad (6)$$

$$\hat{\theta}_{MAP} = argmax_\theta P(\theta|D, M) \quad (7)$$

IV. IMPLEMENTATION

Two major objectives are met by the implementation of the suggested adaptive learning model. They are:

1. Diagnose the weakness of each student
2. Generate a remedy to overcome this weakness

The e-learning feedback system first attempted to evaluate the students' weaknesses as accurately as possible by administering a minimal test on the critical path of the relevant grade in the reverse direction of the intended grade distribution. It is possible for each unit in the topic to have certain dependencies between them, which can be recognized and presented using a dependency graph. In a way, some of the units are the introduction or the foundation for other units, which is known as dependency. It is common for a preliminary test to be administered in order to identify a student's areas of weakness. For example, students could learn how to multiply and divide the numbers and whether they should know how to find total and difference. Students who are proficient in multiplication and division can be confident that they can compute the sum and difference of two numbers as well as read and write numerical values. It's possible that a student's poor performance was caused by the fact that they couldn't find the sum and difference of the numbers, or because they couldn't read or write them. Based on this fact, a diagnosis test should be conducted to evaluate the students' weakness. A critical path is identified to estimate the most essential path or route to be followed as per the dependency matrix for each grade. Based on the results of the test, the student is categorized, and a strategy is developed to help the student overcome any weaknesses that may have been identified. Training is then provided to those students to help them overcome their inadequacies. In accordance with their weaknesses and dependency structure, the remedy plan specifies the sequence in which units should be learned. The exam is conducted following the learning of each unit to see whether they have improved sufficiently. If they have improved sufficiently, the test is repeated before moving on to the next unit in the remedy plan. Otherwise, repeat the training for the same unit again.

First and foremost, all dependency relationships exist among the units as per the grade are identified and generated corresponding dependency matrix. Fig. 2 depicts sample dependency matrix for grade 1. Because each unit is considered an individual job in the dependency matrix, the values in each column are zero or one, signifying the dependencies of the current tasks with the other tasks. In the case of task, A dependent on task B, it expresses that job A is reliant on the output of task B, 0 indicates that both tasks are independent, and 1 signifies that task A is dependent on task B. The dependencies of each task on one another are also all equal to zero. In order to better comprehend the dependency matrix, the directed graph depicted in Fig. 3 is used to demonstrate how it works. Kuwait's mathematics curriculum, which includes grades 1 to 3 and is represented by a dependency matrix and a dependency graph.

Let R be the relation in the set of topics = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}, given by

R = {(2,3), (2,4), (3,4), (2,5), (3,5), (4,5), (2,6), (3,6), (4,6), (5,6), (2,7), (3,7), (2,8), (3,8), (4,5), (4,8), (5,8), (6,8), (7,8), (2,9), (3,9), (4,9), (5,9), (6,9), (2,10), (3,10), (7,10), (2,11), (3,11), (4,11), (5,11), (6,11), (7,11), (8,11), (10,11),

(1,12), (2,12), (3,12), (1,13), (2,13), (3,13), (4,13), (5,13), (6,13), (7,13), (8,13), (9,13), (10,13), (11,13)}
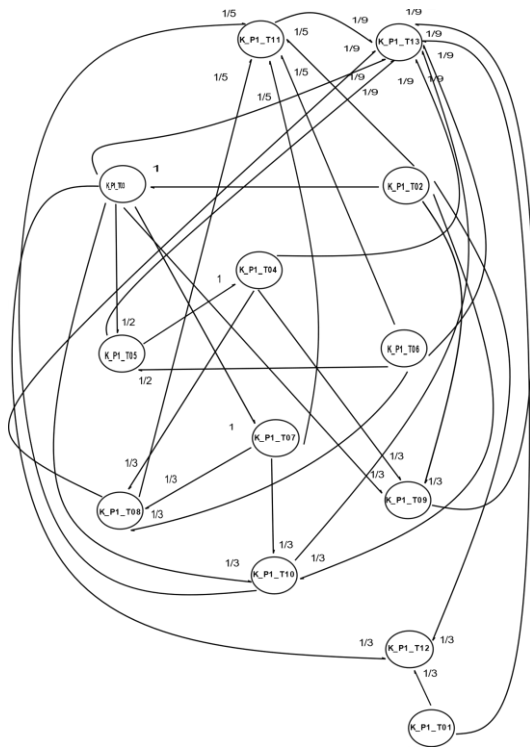

Figure 2: Dependency matrix for the first grade

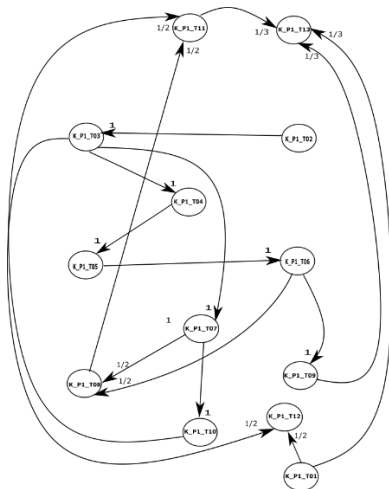Figure 3: Dependency graph for grade 1

Figure 4: Final Dependency graph for grade 1 after removing all transitive relationships
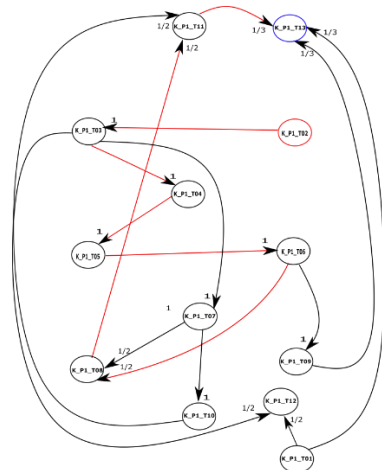

Figure 5: Critical path for grade 1 (Marked with red lines)

Following the discovery of the transitive relationship (T) for every pair, discarded the third pair of this relation from consideration. For example, if A->B, B->C, then A->C occurs, after deleting the third pair, resulting in A->B and B->C occurring instead. In addition, this model rejected a cycle for each of the pairs. The resulting relationship R is as follows:

R = {(2, 3), (3, 4), (4, 5), (5, 6), (3, 7), (6, 8), (7, 8), (6, 9), (7, 10), (8, 11), (10, 11), (1, 12), (3, 12), (1, 13), (9, 13), (11, 13)}

Fig. 4 illustrates the resulting dependency graph for grade 1. The following step is to create a table that will serve as a summary of the topic. S number, Topic Code, Topic Name, Number of Fan in, Number of Fan out, Topic number, Number of Method of Explanation utilized, Number of Examples Used to Explain, Number of Practice Exercises and Lesson Number are all listed in this table.

- The grade number is denoted by the letter S.

- The number of arrows that enter a node is referred to as the fan-in.

- The number of arrows that emerge from a node is referred to as fan out.

- Line explained: in the official textbook, how many lines are devoted to explaining a certain subject.

- Consider the following: how many examples were used to describe the topic in the textbook?

- Exercises: in the textbook, how many practice exercises are provided after each topic is discussed is not specified.

The proposed model identifies the students' weaknesses with a minimum number of questions. In order to reach this goal, the critical path in every grade depending on the dependency graph is determined using the Monte Carlo stochastic model, which is based on the distribution of probabilities.

The units in the critical route are the most important or most critical units in that particular grade level or level. Based on this critical path, the model identified the students' areas of weakness by administering a brief assessment with a limited number of questions, which resulted in the creation of a learning plan for each student to help them improve their

knowledge and understanding. Every critical route level is evaluated for its assessment score and its chance of supplying the correct answer. The final score is calculated using Bayesian statistical models, including Markov chain Monte Carlo simulations, to determine the overall assessment score. Students are divided into groups depending on their achievement on the initial evaluation and on subsequent evaluations, according to the Stochastic Gradient Descent Classification model, which divides them into several categories. This classification determines how much training is required for each student in order for them to achieve excellence in their respective grades.

With the help of the Networkx package in Python, we can mimic the dependency graph. Fig. 5 demonstrates the critical path of Grade 1. The essential path from unit 2 to unit 13 is represented by the edges that have been colored red. The beginning node is represented by the color red, and the ending node is represented by the color blue. The critical path is comprised of the most important units in that particular grade. Float, also known as Slack, is calculated for each node in order to determine how much additional time a task can be delayed without jeopardizing the deadlines of consecutive activities or the aggregate duration of the tasks. The key activity's slack time must be zero minutes.

The critical path for grade 1 would be calculated as $2\rightarrow3\rightarrow4\rightarrow5\rightarrow6\rightarrow8\rightarrow11\rightarrow13$. As a result, the test began with questions from unit 13 and a minimal number of questions selected from each unit to ensure that all lessons were covered in full. A student's ability to correctly complete all of the questions in a unit shows that they are exceptional in all of the units in that grade. If this is the case, proceed to the preceding node, which in this case is 11, and perform the same method. If the student successfully completed unit 11, look to see if there are any alternative units in that grade that are not on the crucial path that the student might have taken instead of the current unit. If this is the case, raise questions relevant to that issue and follow the same approach as before. And if they reach this point, devise a study plan to address their weaknesses and submit it to them. As per the performance result of each student in this diagnosis test the adaptive model generates a remedy plan to overcome this weakness and enhance the students' math learning level and train each student according to their remedy plan. After finishing the learning for each unit in the remedy plan, a student evaluation test is conducted and relearns the lesson if needed. This procedure is repeated until each student reaches a sufficient level of understanding for each unit.

TABLE I.   SAMPLE SIZES AND DISTRIBUTION PER SEMESTER

| Area | Grades | | | | | |
| | Grade 1 | | Grade 2 | | Grade 3 | |
	Boys	Girls	Boys	Girls	Boys	Girls
Asema	120	120	120	120	120	120
Hawali	120	120	120	120	120	120
Farwania	120	120	120	120	120	120
Mubarak	120	120	120	120	120	120
Ahmadi	120	120	120	120	120	120
Jahra	120	120	120	120	120	120
TOTAL	720	720	720	720	720	720

V. PERFORMANCE

The system effectively diagnosed the students' weakness and generated a suitable remedy plan as per student's weakness. Remedy plan specifies the order to which he or she will be trained to enhance their math understanding and get proficient in elementary Mathematics. Based on the math learning level identified during the diagnosis test, each student is classified into different categories such as, Excellent, Very Good, Good, Average, Satisfactory, and Poor.

The effectiveness of the proposed e-learning feedback model is evaluated through a variety of interactions with students. The primary purpose of this study is to demonstrate the significance of the proposed paradigm in terms of increasing the rate at which primary school kids learn mathematics. To complete this activity, get all the essential student information from the schools and pre-process the information. Afterwards, this proposed model will be tested using a statistical data mining tool such as WEKA or SPSS to evaluate the performance of different feedback options on students before being implemented.

I am planning to carry out the performance evaluation and testing of the model for the both two-semester system in Kuwait (fall and spring semesters). This indicates that the maximum feasible sample size has practically almost nothing to do with sample size of the population if the population sample size is many times bigger than the maximum practical size of the sample. As a result, this project will aim to enroll 120 kids in each primary level class in each of Kuwait's educational districts. Kuwait has six educational regions, and elementary schools are divided into five class levels, which correspond to the six educational areas. There are separate schools for boys and separate schools for girls. As a result, the sample size will be in the case of size (3*6*120*2) for each of the two semesters (3*6*120*2= 4320 students). When it comes to student performance, the Ministry of Education's report on the six educational areas reveals that they behave in a variety of ways. As a result, they must be viewed as distinct layers of the same stratum. Furthermore, the performance of female students consistently outperforms that of male pupils. According to the chart below, the approximate sample distribution proposed for this research is supposed to be used for each semester. The total number of e-learning licenses required in this situation will be 4320*2=8640 licenses for Math subjects, which is a significant number.

VI. CONCLUSION

The main purpose of the proposing an intelligent self-adaptive feedback system for math e-learning systems is to enable students in Kuwait to close the gap in arithmetic skills and improve their performance by using the proposal. It offers a fully self-adaptive feedback paradigm that tailors input qualities to the needs of each individual student. The approach aims to uncover the link between a variety of feedback features and a student's performance. Because most learners have various personal qualities such as past knowledge, learning progress, and learning preferences, adaptive feedback support within a learning environment is beneficial. The major contributions of the project will be (1) to provide the provision of an adaptive feedback model for

math, which can represent various adaptive feedback characteristics and support multiple adaptive feedback means, target, goal, and strategy; (2) to develop an algorithm for representing the internal and output relationships between concepts and attributes of knowledge models within an adaptive learning environment and (3) to create e-learning content in the form of self-contained Learning Objects (LOs) based on Learning Objectives derived from Teacher's Guides/Handbooks and content received in the form of marked-up textbooks.

ACKNOWLEDGMENT

DISCLOSURE AND CONFLICTS OF INTEREST

The author declares that there is no conflict of interest.

REFERENCES

[1] R. L. Aiken, "Attitudes Towards Mathematics," Review of Educational Research, vol. 40, issue 4, pp. 551 – 596, 1970.

[2] R. L. Aiken, "Update on Attitudes and Other affective Variables in Learning Mathematics," Review of Educational Research, vol. 46, issue 2, pp. 293 – 311, 1976.

[3] J. Hatie and H. Timperley, "The power of feedback," J. Review of Educational Research vol. 87, issue 1, pp. 81–112, 2007.

[4] I. V. S. Mullis, M. O. Martin, P. Foy and A. Arora, "TIMSS 2011 International Results in Mathematics," Retrieved from Boston College, TIMSS & PIRLS International Study Center, 2011.

[5] I. V. S. Mullis, M. O. Martin, S. Goh and K. Cotter, "TIMSS 2015 Encyclopedia: Education Policy and Curriculum in Mathematics and Science," Retrieved from Boston College, TIMSS & PIRLS International Study Center website: http://timssandpirls.bc.edu/timss2015/encyclopedia/, 2016.

[6] F. Al-Mutawa, G. Al-Rasheedi and D. Al-Maie, "Kuwaiti Students' Achievements in Mathematics: Findings From the TIMSS Assessments: Reality and Reasons," SAGE Open. July 2021. doi:10.1177/21582440211031903.

[7] V. S. Mullis, M. O. Martin, P. Foy, D. L. Kelly and B. Fishbein, "TIMSS 2019 International Results in Mathematics and Science," Retrieved from Boston College, TIMSS & PIRLS International Study Center website: https://timssandpirls.bc.edu/timss2019/international-results/, 2020.

[8] L. H. Reyes, "Affective variables and mathematics education," Elementary School Journal, vol. 14, pp. 159 – 168, 1984.

[9] E. G. Begle, "Critical variables in Mathematics Education," NCTM, 1979.

[10] T. Collins, "The effects of computer-assisted algebra instruction on achievement, mathematics anxiety levels and attitudes toward personal use of computers of students in an historically black University," PhD thesis, University of South Florida, 1996.

[11] R. Geer, B. White and A. Barr, "The effects of a core first year subject on students' attitudes towards computers and development of information literacy," Where is IT & T at? [CD Rom] ACEC 98 Refereed Conference Proceedings. CEGSA, Adelaide, pp. 70-75, 1998.

[12] R. Bangert-Drowns, J. Kulik and C. Kulik, "Effectiveness of computer-based education in secondary School," Journal of Computer–Based Instruction, vol. 12, pp. 59-68, 1985.

[13] J. H. Kulik, "Effectiveness of computer-based teaching: A meta-analysis of findings," Review of Educational Research, vol. 50, pp. 525-544, 1984.

[14] A. B. Ganguli, "The effect on students' attitudes of the computer as a teaching aid," Educational Studies in Mathematics, vol. 23, pp. 611-618, 1992.

[15] C. P. Funkhouser, "The Influence Of Problem-Solving Software On Student Attitudes About Mathematics," Journal of Research on Computing in Education, vol. 25, issue 3, pp. 339-346, 1993.

[16] G. E. Acuna, L. A. Alvarez, J. Miraflores, and M. J. Samonte, "Towards the Development of an Adaptive E-learning System with Chatbot Using Personalized E-learning Model," In 2021 The 7th International Conference on Frontiers of Educational Technologies (ICFET 2021). Association for Computing Machinery, New York, NY, USA, pp. 120–125, 2021.

[17] H. El Fazazi, M. Elgarej, M. Qbadou and K. Mansouri, "Design of an Adaptive e-Learning System based on Multi-Agent Approach and Reinforcement Learning", Eng. Technol. Appl. Sci. Res., vol. 11, no. 1, pp. 6637–6644, Feb. 2021.

[18] R. M. Felder , L. Silverman, "Learning and Teaching Styles in Engineering Education," Engr. Education, 78(7), 674–681, 1988

[19] M. Abdullah, W. H. Daffa, R. M. Bashmail, M. Alzahrani, and M. Sadik, " The Impact of Learning Styles on Learner's Performance in E-Learning Environment," International Journal of Advanced Computer Science and Applications (ijacsa), vol. 6, no. 9, 45/01 2015,

[20] E. M. Smaili, S. Sraidi, S. Azzouzi and M. E. H. Charaf, "Towards Sustainable e-Learning Systems Using an Adaptive Learning Approach," Emerging Trends in ICT for Sustainable Development, 2021, Springer International Publishing, Cham, pp. 365--372, isbn: 978-3-030-53440-0.

[21] M. A. Hassan, U. Habiba, F. Majeed and M. Shoaib, "Adaptive gamification in e-learning based on students' learning styles," Interactive Learning Environments, vol. 29, no. 4, pp. 545-565, 2021, 10.1080/10494820.2019.1588745.

[22] A. Sharma, "Guided Stochastic Gradient Descent Algorithm for inconsistent datasets," Applied Soft Computing, vol. 73, 2018, pp. 1068-1080, ISSN 1568-4946.

[23] R. G. J. Wijnhoven and P. H. N. de With, "Fast Training of Object Detection Using Stochastic Gradient Descent," 2010 20th International Conference on Pattern Recognition, pp. 424-427, doi: 10.1109/ICPR.2010.112.

[24] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay, "Scikit- learn: Machine learning in Python," Journal of Machine Learning Research, vol. 12, pp. 2825-2830, 2011.

[25] S. Diab, "Optimizing Stochastic Gradient Descent in Text Classification Based on Fine-Tuning Hyper-Parameters Approach: A Case Study on Automatic Classification of Global Terrorist Attacks," International Journal of Computer Science and Information Security (IJCSIS), vol. 16, No. 12, December 2018

[26] M. S. Nacson, N. Srebro and D. Soudry, "Stochastic Gradient Descent on Separable Data: Exact Convergence with a Fixed Learning Rate," Proceedings of the Twenty-Second International Conference on Artificial Intelligence and Statistics, PMLR, Vol. 89, pp. 3051-3059, 2019.

[27] O. Osho and S. Hong, "An Overview: Stochastic Gradient Descent Classifier, Linear Discriminant Analysis, Deep Learning and Naive Bayes Classifier Approaches to Network Intrusion Detection," International Journal of Engineering Research & Technology (IJERT), vol. 10, issue 04, ISSN: 2278-0181, April-2021

[28] W. Tysiak, "Risk management in projects: The Monte Carlo approach versus PERT," The 6th IEEE International Conference on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications 15-17 September 2011, Prague, Czech Republic.

[29] T. T. K. Naidu and R. U. Angadi, "Comparative Analysis between Critical Path Method and Monte Carlo Simulation," International

Research Journal of Engineering and Technology (IRJET), vol. 05, issue 06, June-2018.

[30] G. Avlijaš, "Examining the Value of Monte Carlo Simulation for Project Time Management," Management Journal of Sustainable Business and Management Solutions in Emerging Economies vol. 24, issue 1, pp. 11-23 April-2019.

[31] J. S. Speagle, "A Conceptual Introduction to Markov Chain Monte Carlo Methods." arXiv: Other Statistics 2019.

[32] S. Sharma, "Markov Chain Monte Carlo Methods for Bayesian Data Analysis in Astronomy," Annual Review of Astronomy and Astrophysics, vol. 55, 10.1146/annurev-astro-082214-122339, June 2017.

[33] L. Zhang, D-Q Li, X-S Tang, Z-J Cao and K-K Phoon, "Bayesian model comparison and characterization of bivariate distribution for shear strength parameters of soil," Computers and Geotechnics, vol. 95, pp. 110-118, ISSN 0266-352X, 2018.

[34] A. Patil, D. Huard and C. J. Fonnesbeck, "PyMC: Bayesian Stochastic Modelling in Python," Journal of Statistical Software, vol. 35, issue 4, 10.18637/jss.v035.i04., 2010.