PSet example

Jozef Rivest

November 7, 2025

Problem 1

(a)

To find the Hessian matrix from the following equation: $f(x,y) = 3x^4 + 4y^2 - 3xy + 12$, we first have to find the derivative vector containing the first partial derivatives.

$$\mathbf{D}(f(x,y)) = \begin{bmatrix} 12x^2 - 3y & 8y - 3x \end{bmatrix} \tag{1}$$

Then, we compute the second own an cross-partial derivatives and fill the Hessian. The Equation 2 shows the result.

$$\mathbf{H}(f(x,y)) = \begin{bmatrix} 24x & -3 \\ -3 & 8 \end{bmatrix} \tag{2}$$

This is similar to what Moore and Siegel (2013) explain in their book.

Problem 2

(a)

To generate the sample using the inverse-CDF method, we can rely on the *universality of the uniform distribution* to answe this question. Using the fact that the distribution of the percentile is uniformally distributed, and by plugin these values into the quantile function, for this problem, we can then approximate the PDF of the normal distribution (Blitzstein and Hwang 2019, 225–226).

```
# setting seed for reproductibility
set.seed(48104)

# specifying the nb of observations
n <- 10000

# generating the normal distribution
```

```r
n_1 <- rnorm(n, mean = 0, sd = 1)

# plotting the result
hist(n_1)
```

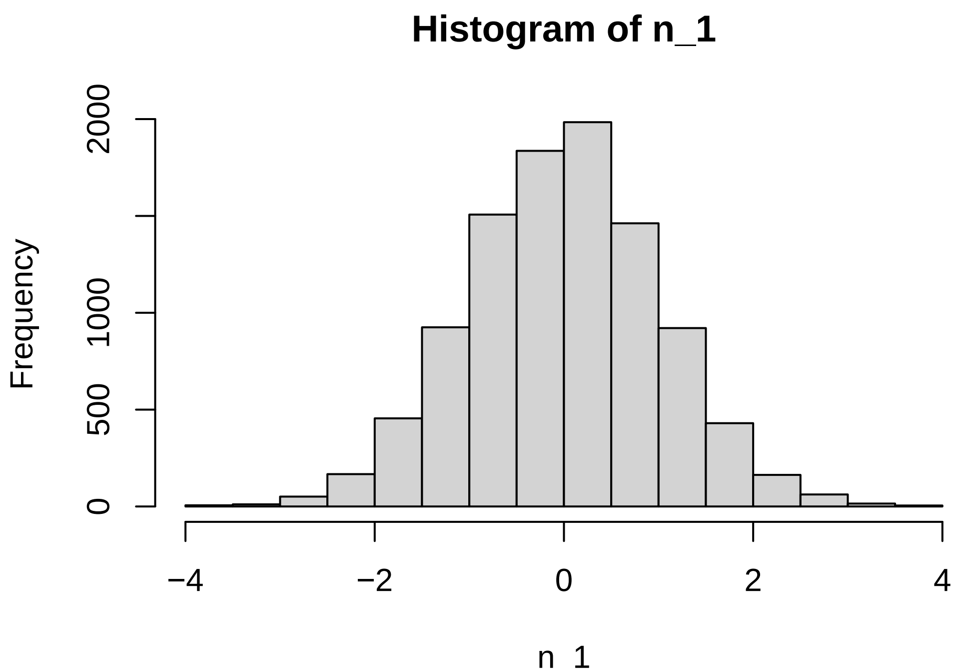

Figure 1: Histogram of generated numbers

```r
# generating random numbers from uniform distribution
nb <- runif(n)

# sample using the inverse CDF
sample <- qnorm(nb, mean = 0, sd = 1)

# ploting the output
hist(sample)
```

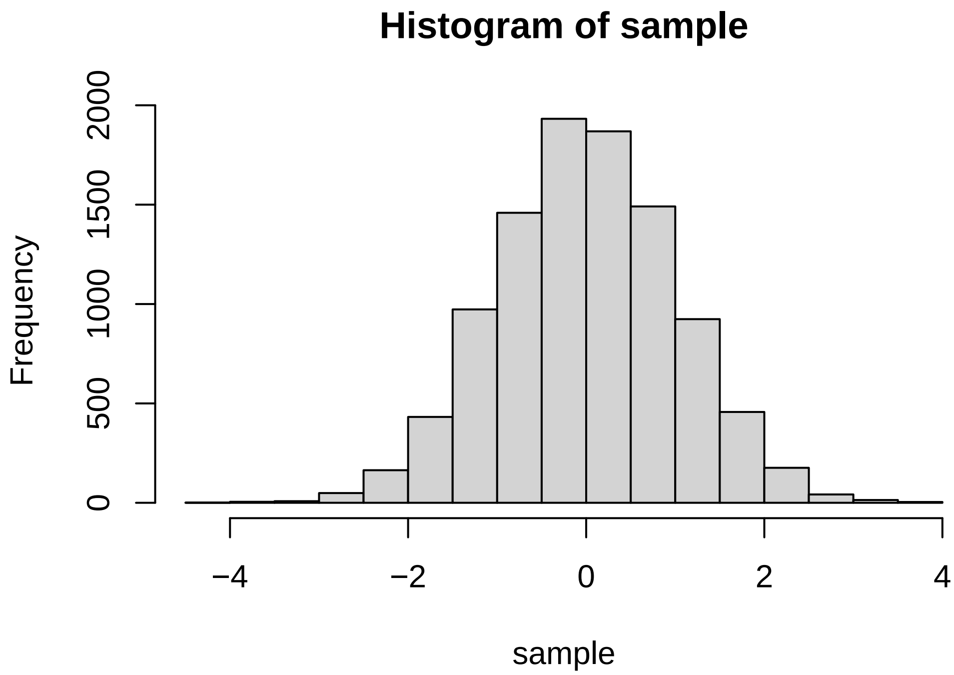

Figure 2: Histogram of the sample

(b)

```
# Making the Q-Q plot
qqplot(sample, n_1)
abline(0, 1, col = "red")
```

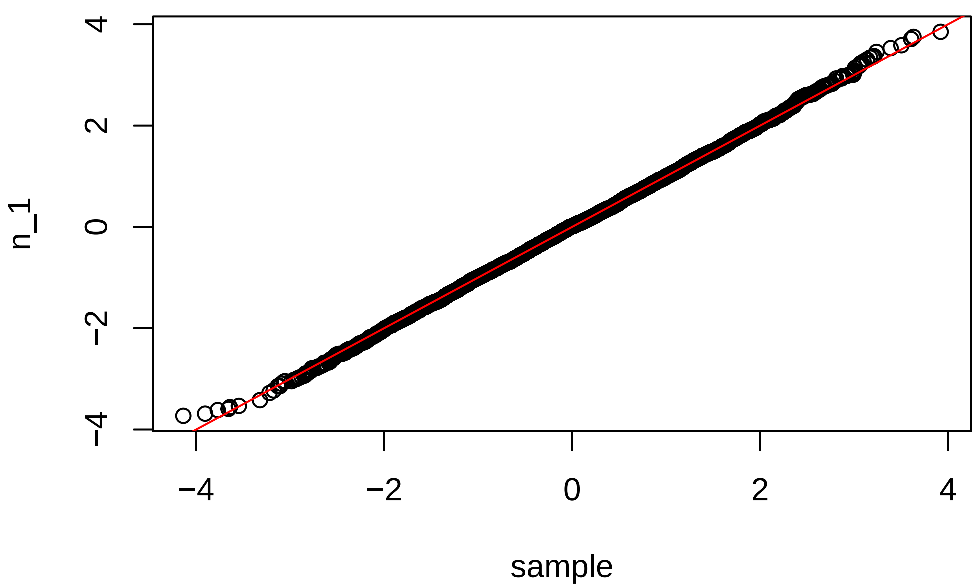

Figure 3: Q-Q Plot

The Figure 3 shows the Q-Q plot using the distribution of the normally distributed numbers generated from `rnorm`, and the sample generated using the inverse CDF from `qnorm`. Q-Q plot allows to compare two probability distribution by plotting their quantiles[1]. As all the points fall roughly on the same line, we can say that the point generated with `rnorm` come from the same distribution as those generated by `qnorm`. This makes sense given that we used the percentiles generated from the uniform distribution and passed them to the quantile function of the normal distribution.

1. To learn more about the Q-Q plot, you can refer to the definition and the information provided by the Wikiepdia page: https://en.wikipedia.org/wiki/Q–Q_plot

References

Blitzstein, Joseph K., and Jessica Hwang. 2019. *Introduction to Probability* [in en]. 2nd ed. DOI: 10.1201/9780429428357. Chapman / Hall/CRC. ISBN: 978-0-429-42835-7. https://www.taylorfrancis.com/books/9780429766749.

Moore, Will H., and David A. Siegel. 2013. *A mathematics course for political and social research* [in eng]. Online-Ausg. 430. Princeton, N.J: Princeton University Press. ISBN: 978-0-691-15917-1 978-1-4008-4861-4.