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Full velocity difference and acceleration model for a car-following theory

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ABSTRACT

In order to describe the car-following behavior more actually in real traffic, a full velocity difference and acceleration model (for short, FVDAM) is proposed by synthetically taking into account headway, velocity difference and acceleration of the leading car on the basis of full velocity difference model. The analytical method and numerical simulation results show that the proposed model can describe the phase transition of traffic flow and estimate the evolution of traffic congestion, that incorporating the acceleration of the leading car into car-following model can stabilize traffic flow, suppress the traffic jam and increase capacity, and that the following car in FVDAM can accelerate more quickly than in FVDM.

1. Introduction

Many car-following models are proposed to describe interacting driver–car units which are based on the assumption that each driver reacts in some specific fashion to a stimulus from the car ahead of him without lane-changing. They have not only been of great importance to an autonomous cruise control system, but also as important evaluation tools for intelligent transportation system strategies since the 1990s. Early researches include the linear models proposed by Chandler et al. [1] and Herman et al. [2], the nonlinear models presented by Reuschel [3], Pipes [4], Gazis et al. [5], and Newell [6]. Remarkable recent improvements include the works of Bando et al. [7], Helbing and Tilch [8], Treiber et al. [9], and Jiang et al. [10] and there are some others in the literature. In 1995, Bando et al. [7] proposed the optimal velocity model (for short, OVM) which was based on the idea for regulation that each vehicle has the optimal velocity, it can describe many characteristics of real traffic flow such as the stop-and-go traffic and the evolution of traffic congestion. In order to overcome the shortage of OVM that may lead to impractical high acceleration and unrealistic deceleration, Helbing and Tilch [8] developed a generalized force model (for short, GFM) by considering negative velocity difference on the basis of OVM. Both OVM and GFM cannot explain the instance that if the leading car is much faster, then the car will not brake, even if its headway is smaller than the safe distance pointed out by Treiber et al. [11], so Jiang et al. [10] put forward full velocity difference model (for short, FVDM) by considering both negative and positive velocity difference, which can give a better description of starting process than OVM and GFM.

Recently, cooperative driving models were developed by applying the Intelligent Transportation System [12,13]. ITS application meant that drivers could receive information of other vehicles on roads, and then adjust the velocities of their own vehicles. In light of this information, it is possible to obtain the acceleration of the preceding car to improve the stability of traffic flow and suppress the appearance of traffic jams.

To improve the FVDM with the consideration of the ITS application, we propose a new car-following model by considering the acceleration of the leading car on the basis of full velocity difference model. In the next section, we put forward the new

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car-following model. In Section 3, linear stability analysis is carried out. Numerical simulations of this model are conducted to analyze some observed physical features which exist in real traffic in Section 4. The results are summarized in Section 5.

2. The new model

In accordance with the above mentioned idea, a more systematic model is obtained by considering headway interaction, velocity difference and acceleration of the leading car on the basis of full velocity difference model, whose dynamics equation is as follows:

$$\ddot{x}_n(t) = \kappa[V(\Delta x_n(t)) - v_n(t)] + \lambda \Delta v_n(t) + k a_{n+1}(t) \tag{1}$$

where $x_n(t)$ is the nth car's position at t; $\Delta x_n(t)$ and $\Delta v_n(t)$ are the headway and the velocity difference between the $(n+1)$th car and nth car, respectively; κ is the sensitivity of a driver; $V(.)$ is the optimal velocity function; λ is a sensitivity coefficient of a driver to the velocity difference; a_{n+1} is the $(n+1)$th car's acceleration; k is the sensitivity coefficient of response to the leader's acceleration.

3. Linear stability analysis

In this section, the linear stability analysis of FVDAM model is carried out. Supposing the cars running with the uniform headway b and the optimal velocity $V(b)$, the stable solution for Eq. (1) is described as follows:

$$x_n^0(t) = bn + V(b)t, \quad b = L/N \tag{2}$$

where N is the total number of cars, and L is the road length.

Assuming $y_n(t)$ to be a small deviation:

$$y_n(t) = x_n(t) - x_n^0(t) \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1), we obtain the following second differential equation:

$$\ddot{y}_n(t) = \kappa[V(b + \Delta y_n(t)) - V(b) - \dot{y}_n(t)] + \lambda \Delta \dot{y}_n(t) + k \ddot{y}_{n+1}(t) \tag{4}$$

Using the Taylor equation to expand Eq. (4), and omitting the terms of higher order, we can get

$$\ddot{y}_n(t) = \kappa\left[V'(b)\Delta y_n(t) - \dot{y}_n(t)\right] + \lambda \Delta \dot{y}_n(t) + k \ddot{y}_{n+1}(t) \tag{5}$$

Expanding $y_n(t)$ in the Fourier-modes, $y_n(t) = e^{(ia_k n + zt)}$, Eq. (5) can be rewritten as follows:

$$z^2\left(1 - ke^{ia_k}\right) - \left(e^{ia_k} - 1\right)\left[\kappa V'(b) + \lambda z\right] + \kappa z = 0 \tag{6}$$

Substituting $z = vi$ and $e^{ia_k} = \cos a_k + i \sin a_k$ into Eq. (6), we can obtain the following equation:

$$v^2 k \cos a_k - v^2 - \kappa V'(b)(\cos a_k - 1) + \lambda v \sin a_k + i\left(v^2 k \sin a_k - \kappa V'(b) \sin a_k - \lambda v(\cos a_k - 1) + \kappa v\right) = 0 \tag{7}$$

Then, we get the neutral stability condition as follows:

$$V'(b) = \frac{(2\lambda + \kappa)(4\lambda k + \kappa k + \kappa)}{2\kappa(1-k)^2} = \frac{\kappa}{2(1-k)^2} + \frac{\lambda}{(1-k)^2} + \frac{3\kappa k\lambda + 4k\lambda^2}{\kappa(1-k)^2} \tag{8}$$

When $k = 0$, $V'(b) = \frac{(2\lambda+\kappa)(4\lambda k+\kappa k+\kappa)}{2\kappa(1-k)^2} = \frac{\kappa}{2} + \lambda$, FVDAM is reduced to full velocity difference model.

4. Numerical simulation

On the basis of the linear analysis, several numerical simulations are carried out to test whether FVDAM can describe the effects of the leader's acceleration on the car-following behavior and to demonstrate some physical features which exist in real traffic.

4.1. The starting up process

In order to compare certain properties of FVDAM under a traffic signal with those of FVDM, we carry out the same simulation as that in Ref. [14]. The optimal velocity function is adopted calibrated by Helbing and Tilch [8]:

$$V(\Delta x_n) = V_1 + V_2 \tanh(C_1(\Delta x_n - l_c) - C_2) \tag{9}$$

The parameters in our simulation are taken as: $l_c = 5$ m, $\kappa = 0.41$ s^{-1}, $V_1 = 6.75$ m/s, $V_2 = 7.91$ m/s, $C_1 = 0.13$ m^{-1} and $C_2 = 1.57$.

The traffic signal is red and the 11 cars are waiting with a headway of 7.4 m initially. Then $t = 0$, the red signal changes to green and cars start. From the simulation we can get the delay time of car motion and the kinematic wave speed at jam density. The simulation results are shown in Fig. 1 and Table 1. From the Table, we can find that the observed δt is of the order of 1s as Bando et al. [14] pointed out and c_j ranges between 17 and 23 km/h as Del Castillo and Benitez [15] indicated. Moreover, the value of δt becomes smaller and the value of c_j becomes bigger with the increased k. Thus, FVDAM is better than FVDM in anticipating the two parameters.

Then, we can obtain the curves of acceleration according to FVDM and FVDAM respectively in Fig. 2. Fig. 2 shows that the maximum acceleration in FVDAM is smaller than that in FVDM, and it isn't beyond the limited region $[-3\ \mathrm{m/s^2}, 4\ \mathrm{m/s^2}]$ [8]. Moreover, the follower speeds up more quickly in FVDAM than in FVDM. For the leading car in both models, since the initial conditions are the same, thus they have the same acceleration. The main cause of the difference that the followers in FVDAM accelerate more quickly than those in FVDM is that FVDAM has considered the leader's acceleration. Therefore, the delay time in FVDM is smaller than that in GFM. The effects of the leader's acceleration on the follower will be further investigated in next section.

4.2. Analysis of the follower's acceleration

In real traffic, there exist two situations that the leading car is not much faster but speeding up, then the car will not brake, even if its headway is smaller than the safe distance, and that the leading car is much faster but slowing down, then the car will break, even if its headway is bigger than the safe distance. These cannot be explained by FVDM. In order to investigate the effects of the leader's acceleration on the follower, the simulations under the mentioned situations are carried out in the following, which consider a pair of cars, a leader and a follower.

(1) The parameters are $\kappa = 0.41$, $\lambda = 0.5$, $v_2 = 0.964$ m/s, $k \subseteq [0, 1]$, $a_2 = 2$ m/s, $v_1 = 1.5$ m/s, $x_1 = 0$ m and $x_2 = 1.9$ m. We can get the curves of acceleration of the follower.
From Fig. 3, our model is reduced to FVDM if $k = 0$, the following car is slowing down on the basis of headway and velocity difference of the leading car initially, and then the following car is speeding up. Our model is FVDAM if $k > 0$, the follower can react correctly to the accelerating leader with the increasing of k, the follower's deceleration become smaller with the increasing of k because of the consideration of the leader's acceleration. The follower is speeding up if $k = 0.6$, which is not the same as the follower in FVDM that decelerate and then accelerate.

(2) Some parameters are taken as $v_1 = 0.964$ m/s, $v_2 = 2$ m/s, $a_2 = -0.5$ m/s, $x_1 = 0$ m and $x_2 = 5$ m, and the others are as above. The curves of acceleration of the follower are showed in Fig. 4 according to FVDM and FVDAM respectively.
From the figure, the FVDAM is reduced to the FVDM if $k = 0$, the following car is speeding up on the basis of headway and velocity difference of the leading car initially, and then the following car is slowing down. Our model is the FVDAM if $k > 0$, the follower can react correctly to the decelerating leader with the increasing of k, the follower's acceleration become smaller with the increasing of k because of the consideration of the leader's acceleration. The follower is decelerating if $k = 0.6$, which is not the same as the follower in FVDM that accelerate and then decelerate.

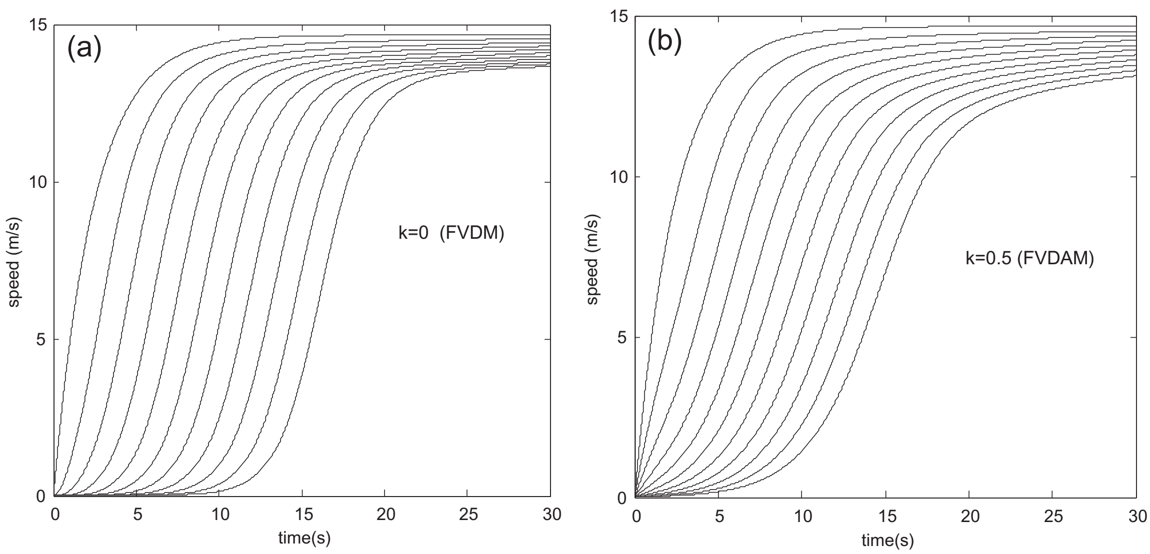

Fig. 1. Motions of cars 1–11 starting from a traffic signal for: (a) FVDM ($k = 0$); (b) FVDAM ($k = 0.5$).

Table 1
δt and c_j in FVDM and FVDAM.

Model	κ (s^{-1})	λ	k	δt (s)	c_j (km/h)
FVDM	0.41	0.5	0	1.4	19.03
FVDAM	0.41	0.5	0.3	1.3	20.49
FVDAM	0.41	0.5	0.5	1.2	22.2

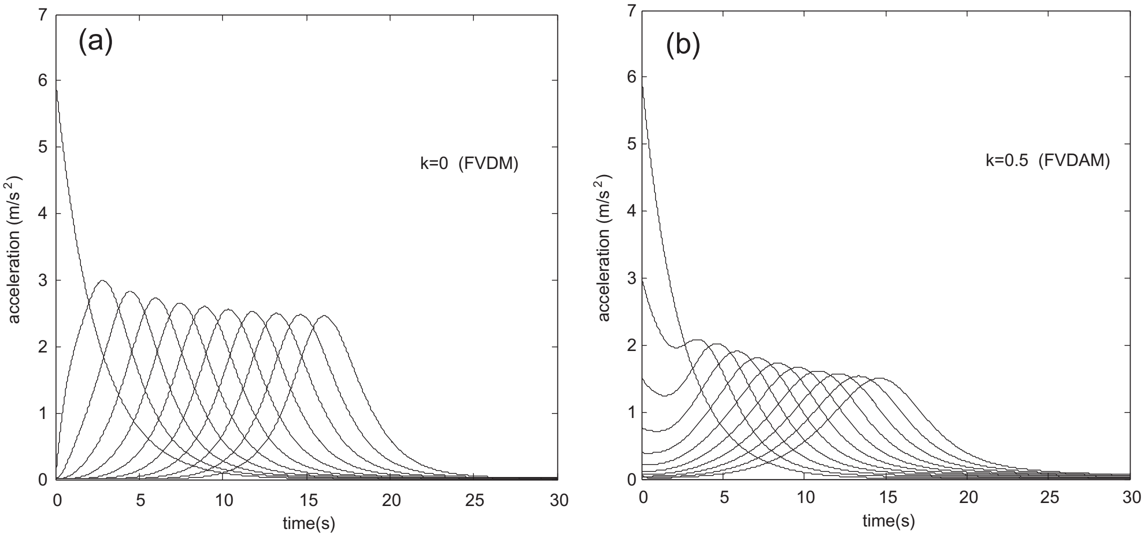

Fig. 2. Acceleration of unobstructed leading vehicle and its following cars initially at rest according to: (a) FVDM (k = 0); (b) FVDAM (k = 0.5).

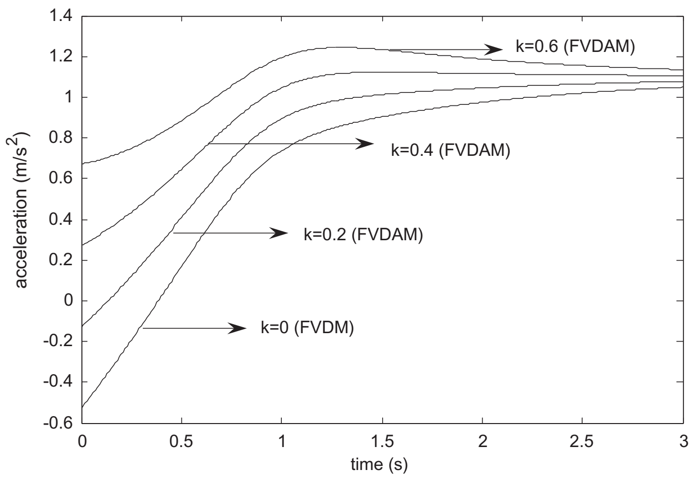

Fig. 3. Acceleration of following car under a special situation: k = 0, 0.2, 0.4, 0.6.

4.3. Verification of the stability analysis

Now we make a numerical simulation to check the above stability analysis, employing the optimal velocity function of Bando et al. [7] and the parameters are $\kappa = 1$, $\lambda = 0.1$, $k = 0.15$. We carry out the simulation by taking car number $N = 100$ and circuit length $L = 200$ m under a periodic boundary condition. The initial disturbance is the same as

$$x_n^{(0)} = hn + ct, \quad x_1(0) = x_1^{(0)} + 0.1, \quad x_n(0), \text{when } n \neq 1, \quad \dot{x}_n(0) = 0 \tag{10}$$

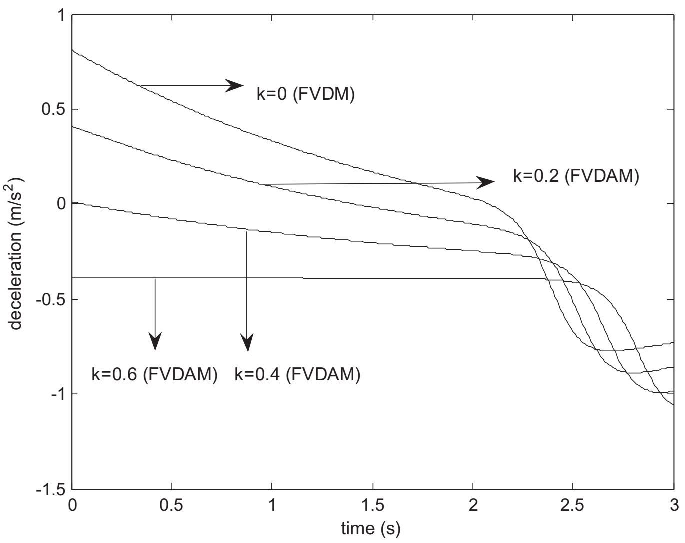

Fig. 4. Deceleration of following car under a special situation $k = 0, 0.2, 0.4, 0.6$.

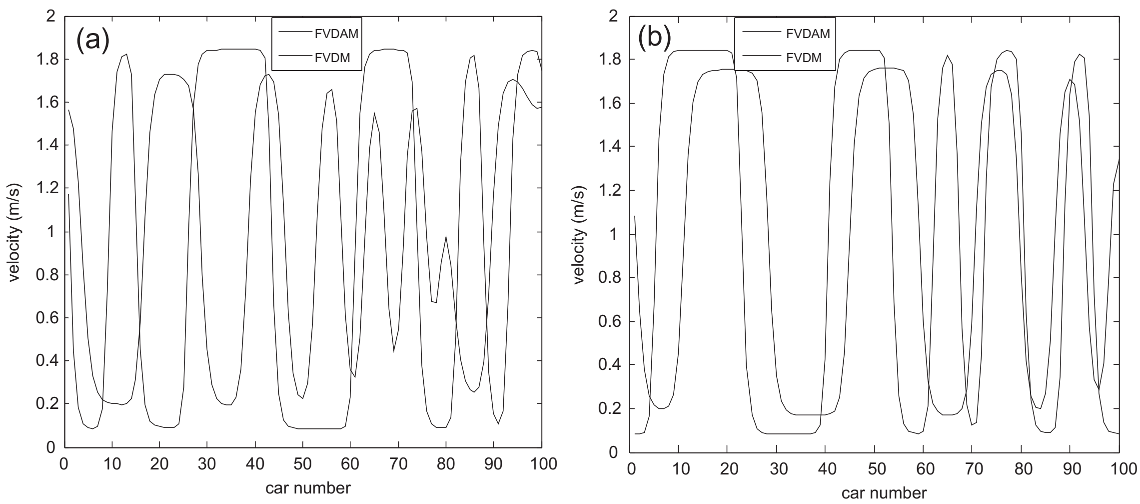

Fig. 5. Snapshots of the velocity of all vehicles at different time in FVDM and FVDAM: (a) $t = 500$ s; (b) $t = 1000$ s.

We can find that the initial disturbance is unstable by substituting the values of the parameters into criterion Eq. (8). Fig. 5 shows the snapshots at $t = 500$ s and $t = 1000$ s. The solid line corresponds to the FVDM, and the dash line to the FVDAM. From the figure, we can learn that the velocity of all cars fluctuate around the initial velocity $v_0 = 0.964$ m/s between the minimum and maximum caused by the initial disturbance, and that the homogeneous flow eventually develops into congestion, which corresponds to stop-and-go traffic. Since FVDAM uses the acceleration of the leader terms, the fluctuation of FVDAM is smaller than that of FVDM and the stability of FVDAM is superior to that of FVDM.

More differences can be found by further investigation of the phase diagram. The motion of cars can organize a ‘hysteresis loop'' after sufficient time as shown in Fig. 6.

Hysteresis loops are usually used to demonstrate the relationship between velocity and space. Fig. 6(a) reproduces the hysteresis loop obtained from FVDM and Fig. 6(b) reproduce that obtained from FVDAM. As can be seen from Fig. 6, the hysteresis loop obtained from FVDAM is significantly different from that from FVDM and the size of loop will be shrink with increasing the value of k. Since FVDAM takes the leader's acceleration into account, the fluctuation of FVDAM is smaller than that of FVDM.

Thus, the results further verify that the FVDAM is more realistic and reasonable.

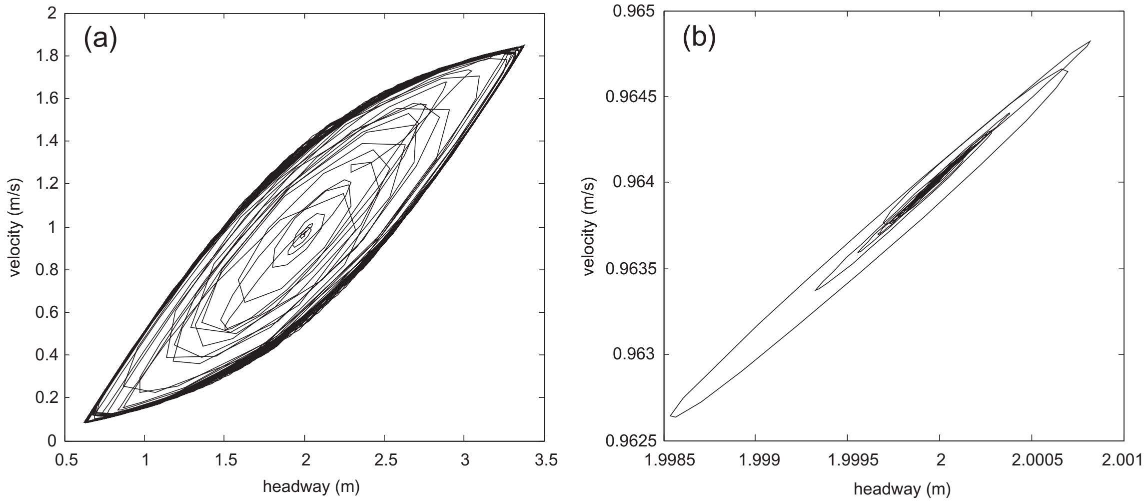

Fig. 6. Hysteresis loops: (a) for FVDM; (b) for FVDAM.

5. Conclusions

In this Letter, we develop a full velocity difference and acceleration model for a car-following theory by taking the acceleration of the leading car into account on the basis of full velocity difference model. The linear stability analysis has been conducted and the stable criterion is given. Then we use FVDAM to carry out several simulations to describe the effects of the leader's acceleration on the car-following behavior and to demonstrate some observed physical features which exist in real traffic. The results show that FVDAM can correctly predict delay time of car motion and kinematic wave speed at jam density by taking into account the acceleration of the leading car. Moreover, the following cars in FVDAM accelerate more quickly than those in FVDM and unrealistically high acceleration will not appear. So the leader's acceleration term has significant impacts on the traffic flow.

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References

[1] Chandler RE, Herman R, Montroll EW. Oper Res 1958;6:165.
[2] Herman R, Montroll EW, Potts RB, Rothery RW. Oper Res 1959;7:86.
[3] Reuschel A. Osterr Ing Archiv 1950;4:193.
[4] Pipes LA. J Appl Phys 1953;24:274.
[5] Gazis DC, Herman R, Rothery RW. Oper Res 1961;9:545.
[6] Newell GF. Oper Res 1961;9:209.
[7] Bando M, Hasebe K, Nakayama A, Shibata A, Sugiyama Y. Phys Rev E 1995;51:1035.
[8] Helbing D, Tilch B. Phys Rev E 1998;58:133.
[9] Treiber M, Hennecke A, Helbing D. Phys Rev E 2000;62:1805.
[10] Jiang R, Wu QS, Zhu ZJ. Phys Rev E 2001;64:017101.
[11] Treiber M, Hennecke A, Helbing D. Phys Rev E 1999;59:239.
[12] Ge HX, Dai SQ, Dong LY, Xue Y. Phys Rev E 2004;70:066134.
[13] Ge HX, Dai SQ, Xue Y, Dong LY. Phys Rev E 2005;71:066119.
[14] Bando M, Hasebe K, Nakanishi K, Nakayama A. Phys Rev E 1998;58:5429.
[15] Del Castillo JM, Benitez FG. Transp Res, Part B Method 1995;29:373.