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Abstract

Multi-echelon distribution systems are quite common in supply-chain and logistic systems. They are used by public administrations in their transportation and traffic planning strategies as well as by companies to model their distribution systems. Unfortunately, the literature on combinatorial optimization methods for multi-echelon distribution systems is very poor.

The aim of this paper is twofold. Firstly, it introduces the family of Multi-Echelon Vehicle Routing Problems. Second, the Two-Echelon Capacitated Vehicle Routing Problem, is presented.

The Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP) is an extension of the classical VRP where the delivery passes through intermediate depots (called satellites). As in the classical VRP, the goal is to deliver goods to customers with known demands, minimizing the total delivery cost while considering vehicle and satellites capacity constraints.

A mathematical model for 2E-CVRP is presented and some valid inequalities given, which are able to significantly improve the results on benchmark tests up to 50 customers and 5 satellites. Computational results under different realistic scenarios are presented.

Keywords: Vehicle Routing, Multi-echelon systems, City Logistics.

1 Introduction

The freight transportation industry is a major source of employment and supports the economic development of the country. However, freight transportation is also a disturbing activity, due to congestion and environmental nuisances, which negatively affects the quality of life, in particular in urban areas.

In freight transportation there are two main distribution strategies: direct shipping and multi-echelon distribution. In the direct shipping, vehicles, starting from a depot, transport their freight directly to the customers, while in the multi-echelon systems, freight is delived from the depot to the customers through intermediate points. Growth in the volume of freight traffic as well as the need to take into account factors such as the environmental impact and traffic congestion has led research in recent years to focus on multi–echelon distribution systems, and, in particular, two-echelon systems (Crainic et al., 2004). In two-echelon distribution systems, freight is delivered to an intermediate depot and, from this depot, to the customers.

Multi-echelon systems presented in the literature usually explicitly consider the routing problem at the last level of the transportation system, while at higher levels a simplified routing problem is considered. While this relaxation may be acceptable if the dispatching at higher levels is managed with a truckload policy (TL), the routing costs of the higher levels are often underestimated and decision-makers can not directly use the solutions obtained from the models in the case of the less-than-truckload (LTL) policy (Ricciardi et al., 2002; Daskin et al., 2002; Shen et al., 2003; Verrijdt and de Kok, 1995).

Moreover, in the past decade multi-echelon systems with LTL dispatching policies have been introduced by practitioners in different areas:

- *Logistics enterprises and express delivery service companies.* These companies usually operate a multi-echelon system. Their depots are used as intermediate points for organizing the freight to be delivered and making up the loads of the vehicles which will transport the freight to another intermediate point (airport, regional center, etc.) or to the final destination (http://www.tntlogistics.com).

- *Multimodal freight transportation.* In the past decade, the number of intermodal logistics centers in the countries of central and southwest Europe has increased. This is a good example of freight distribution involving two or more echelons (Ricciardi et al., 2002). In a classical road-train multimodal distribution chain the freight goes from the producer to a logistics center by road and then it is loaded on a train directed to another logistics center. The train is unloaded and the freight goes by road to its final destination.

- *Grocery and hypermarkets product distribution.* Large companies use hypermarkets as intermediate storage points to replenish smaller stores and supermarkets of the same brand in urban areas.

- *Spare parts distribution in the automotive market.* Some companies uses couriers and other actors to deliver their spare parts. This is the case of FIAT and General Motors, whose spare parts are distributed by TNT (http://www.tntlogistics.com) from their factories to the garages. Since 2002 TNT has adopted a multi-echelon distribution network with a less-than-truck policy at regional and city distribution levels. Similarly, Bridgestone organizes the distribution system in zones and sub-zones to decrease transportation times and reduce the size of the storage areas.

- *E-commerce and home delivery services.* The development of e-commerce and the home delivery services offered by some supermarkets and other stores like SEARS (http://www.sears.com) require the presence of intermediate depots used to optimize the delivery process in large cities.

- *Newspaper and press distribution.* In Denmark, a comparative study of heuristics for solving a two-echelon newspapers distribution problem was made for two competing newspaper editors who shared printing and distribution facilities for reducing the total costs (Jacobsen and Madsen, 1980). In press distribution, it is also common for distribution companies to receive the publishing products from the editors and distribute them to the selling points.

- *City logistics.* In the past decade, researchers have started to view urban areas as a single system, rather than considering each shipment, firm, and vehicle individually. All stakeholders and movements are considered to be components of an integrated logistics system. This implies the need for the coordination of shippers, carriers, and movements as well as the consolidation of loads of several customers and carriers into the same "green" vehicles. The adopted distribution system is typically a two-echelon system. Currently, a two-echelon distribution system is under study for the city of Rome (Crainic et al., 2004).

The main contribution of this paper is to introduce the *Multi-Echelon Vehicle Routing Problem*, a new family of routing problems where routing and freight management are explicitly considered at the different levels. One of the simplest type of Multi-Echelon Vehicle Routing Problems, the *Two-Echelon Capacitated Vehicle Routing Problem* (2E-CVRP) is introduced and examined in detail. In 2E-CVRP, the freight delivery from the depot to the customers is managed by shipping the freight through intermediate depots. Thus, the transportation network is decomposed into two levels: the 1st level connecting the depot to the intermediate depots and the 2nd one connecting the intermediate depots to the customers. The objective is to minimize the total transportation cost of the vehicles involved in both levels. Constraints on the maximum capacity of the vehicles and the intermediate depots are considered, while the timing of the deliveries is ignored.

The paper is organized as follows. In Section 2 we recall the literature related to Multi-Echelon Vehicle Routing Problems. In Section 3 we give a general description of Multi-Echelon Vehicle Routing Problems. Section 4 is devoted to introduce 2E-CVRP and give a mathematical model, which is strengthened by means of valid inequalities in Section 5. Finally test instances for 2E-CVRP are introduced and some computational results are discussed in Section 6.

2 Literature review

In freight distribution there are different distribution strategies. The most developed strategy is based on the direct shipping: freight starts from a depot and arrives directly to the customers. In many applications, this strategy is not the best one and the usage of a two-echelon distribution system can optimize several features such as the number of the vehicles, the transportation costs and their loading factor.

In the literature the multi-echelon systems, and the two-echelon systems in particular, refer mainly to supply chain and inventory problems (Ricciardi et al., 2002; Daskin et al., 2002; Shen et al., 2003; Verrijdt and de Kok, 1995). These problems do not use an explicit routing approach for the different levels, but focus more on the production and supply chain management issues. In location problems, some studies deal with the location of intermediary facilities for a multi-echelon distribution systems (Ricciardi et al., 2002; Crainic et al., 2004).

Another real application of a two-tier distribution network is due to Crainic, Ricciardi and Storchi and is related to the city logistics area (Crainic et al., 2004). They developed a two-tier freight distribution system for congested urban areas, using small intermediate platforms, called satellites, as intermediate points for the freight distribution. This system is developed for a specific case study and a generalization of such a system has not yet been formulated.

Vehicle Routing has become a central problem in the fields of logistics and freight transportation. In some market sectors, transportation costs constitute a high percentage of the value added of goods. Therefore, the use of computerized methods for transportation can result in savings ranging from 5% to as much as 20% of the total costs, as reported at Toth and Vigo, 2002. Unfortunately, to our knowledge, only the single-level version of the Vehicle Routing Problem has been studied. The main contributions in the area are presented below.

The case modelled by the VRP, also known as Capacitated VRP (CVRP), considers a fleet of identical vehicles. The objective is the minimization of the transportation costs under the constraint of the maximum freight capacity of each vehicle. Where an additional constraint on the maximum distance that each vehicle can cover is combined, the problem is known as Distance Constrained VRP (DVRP), while when both the groups of constraints are considered, the problem is named Distance Constrained Capacitated VRP (DCVRP). This variant of VRP is the most commonly studied, and recent studies have developed good heuristic methods. Exact algorithms can solve relatively small instances and their computational effort is highly variable (Cordeau et al., 2005). For this reason, exact methods are mainly used to determine optimal solutions of the test instances, while heuristic methods are used in practical applications.

Cordeau, Laporte and Mercier proposed a Tabu Search algorithm, called Unified Taboo Search Algorithm (UTSA) (Cordeau et al., 2001), to solve periodic and multi-depot VRPs. It tolerates intermediate unfeasible solutions through the use of a generalized objective function containing self-adjusting coefficients. This feature permits a decrease in the average deviation from the best known solution without any further computational effort. The Granular Tabu Search (GTS) by Toth and Vigo is based on the idea that removing the nodes unlikely to appear in an optimal solution could considerably reduce the neighborhood size and thus the computational time (Toth and Vigo, 2003).

These results have been recently improved by different approaches based on Hybrid and Evolutionary Algorithms (Perboli et al., forthcoming; Prins, 2004; Mester and Bräysy, 2005). For a detailed survey of the exact and heuristic methods see (Cordeau et al., 2007; Toth and Vigo, 2002; Cordeau et al., 2005).

In real world applications, the problem is often different and many variants of VRP have been developed. The most wellknown variants are VRP with time windows (VRP-TW), multi-depot VRP (MDVRP) and VRP with pickups and deliveries (VRP-PD) (for a survey, see Cordeau et al., 2007; Toth and Vigo, 2002). We note only one variant of VRP where satellites facilities are explicitly considered, the VRP with Satellites facilities (VRPSF). In this variant, the network includes facilities that are used to replenish vehicles during a route. When possible, satellite replenishment allows the drivers to continue the deliveries without necessarily returning to the central depot. This situation arises primarily in the distribution of fuels and some other retail applications; the satellites are not used as depots to reduce the transportation costs (Crevier et al., 2007; Angelelli and Speranza, 2002; Bard et al., 1998).

In the case where a less-than-truckload policy with vehicle trips serving several customers is applied only at the second level, the problem is close to a multi-depot VRP. However, since the most critical decisions are related to which satellites will be used and in assigning each customer to a satellite, more pertinent methods will be found in multi-depot Location Routing Problems (LRP).

In these problems, the location of the distribution centers and the routing problem are not solved as two separate problems but are both considered in the same optimization problem. In (Laporte, 1988), a first classification of LRP is made, and multi-echelon problems are theoretically described. Detailed surveys on this field (Min et al., 1997; Nagy and Salhi, 2007) show that most variants deal with single stage multi-depot LRP, but some two-echelon problems have also been developed.

The first application of a two-echelon LRP can be found in (Jacobsen and Madsen, 1980). The authors developed and compared three fast heuristics for solving a real case application where two newspaper editors combined their resources in terms of printing and distribution in order to decrease the overall costs. Newspapers are delivered from the factory to transfer points, which must be chosen from a set of possible facilities, and then other vehicles distribute them from these transfer points to customers. The first method, called Three Tour Heuristic, is based on the observation that if the last arc of each route is deleted, the problem becomes similar to a Steiner Tree Problem. This tree is constructed by a greedy one-arc-at-a-time procedure. The other two heuristics, which are sequential, combine heuristics for both VRP and Location-Allocation problem. The ALA-SAV heuristic is a three stage procedure composed from the Alternate Location Allocation (ALA) of Rapp and Cooper (Rapp, 1962) and the Savings algorithm (SAV) of Clarke and Wright (Clarke and Wright, 1964). The third heuristic (SAV-DROP) is also a three stage procedure composed from the Clarke and Wright Savings algorithm and the DROP method of Feldman et al.

(Feldman et al., 1966).

The *road-train routing problem*, introduced by Semet and Taillard (Semet and Taillard, 1993). This problem concerns defining a route for a road-train, which is a vehicle composed by a truck and a trailer (both with space for freight loading). Some of the roads are not accessible by the entire convoy, but only by the truck. In these cases, the trailer is detached and left at a customer's location (called a "root") while the truck visits a subset of customers, returning to pickup the trailer. In a way, this problem can be represented as a two-echelon distribution system, using the LRP notation. The intermediary facilities become the customers where the trailer is parked while the truck visits a group of customers. The authors propose an algorithm which uses an initial solution obtained by a sequential procedure and improved using Tabu Search, where customers are reallocated. This method do not distinguish between locational and routing moves. Semet (Semet, 1995) proposed a clustering first routing second solution method, where in a first phase customers are allocated then the resulting routing problems are solved via Lagrangian Relaxation. Chao (Chao, 2002) developed a two-stage algorithm which an initial solution is obtained by a clustering first routing second heuristic then improved using a Tabu Search algorithm with customer reallocation moves.

The most complex and general multi-echelon LRP is defined by Ambrosino and Scutella (Ambrosino and Scutellà, 2005). Although the general purpose of the paper is to present general model for multi-echelon network design problems which represent real network planning cases, a multi-echelon LRP can derive from the general formulation.

3 The Multi-Echelon Vehicle Routing Problems

Freight consolidation from different shippers and carriers associated to some kind of coordination of operations is among the most important ways to achieve a rationalization of the distribution activities. Intelligent Transportation Systems technologies and operations research-based methodologies enable the optimization of the design, planning, management, and operation of City Logistics systems (Crainic and Gendreau, forthcoming; Taniguchi et al., 2001).

Consolidation activities take place at so-called *Distribution Centers* (DCs). When such DCs are smaller than a depot and the freight can be stored for only a short time, they are also called *satellite platforms*, or simply *satellites*. Long-haul transportation vehicles dock at a satellite to unload their cargo. Freight is then consolidated in smaller vehicles, which deliver them to their final destinations. Clearly, a similar system can be defined to address the reverse flows, i.e., from origins within an area to destinations outside it.

As stated in the introduction, in the Multi-Echelon Vehicle Routing Problems the delivery from the depot to the customers is managed by rerouting and consolidating the freight through different intermediate satellites. The general goal of the process is to ensure an efficient and low-cost operation of the system, while the demand is delivered on time and the total cost of the traffic on the overall transportation network is minimized. Usually, capacity constraints on the vehicles and the satellites are considered.

More precisely, in the Multi-Echelon Vehicle Routing Problems the overall transportation network can be decomposed into $k \geq 2$ levels:

- the 1st level, which connects the depots to the 1st-level satellites;
- $k-2$ intermediate levels interconnecting the satellites;
- the last level, where the freight is delivered from the satellites to the customers.

In real applications two main strategies for vehicle assignment at each level can be considered. Given a level, the corresponding vehicles can be associated with a common parking depot, from where they are assigned to each satellite depending on the satellite demand. If the number of vehicles is not known in advance, a cost for each available vehicle is considered; this usually depends on the traveling costs from the parking depot to the satellites. Another strategy consists in associating to each satellite a number of vehicles which start and end their routes at the considered satellite. In our study we will consider the first strategy, considering similar costs for the assignment of each vehicle to a satellite. Thus, each transportation level has its own fleet to perform the delivery of goods and the vehicles assigned to a level can not be reassigned to another one.

The most common version of Multi-Echelon Vehicle Routing Problem arising in practice is the *Two-Echelon Vehicle Routing Problem.* From a physical point of view, a Two-Echelon Capacitated Vehicle Routing system operates as follows:

- freight arrives at an external zone, the depot, where it is consolidated into the 1st-level vehicles, unless it is already carried in a fully-loaded 1st-level truck;
- Each 1st-level vehicle travels to a subset of satellites and then it will return to the depot;
- At a satellite, freight is transferred from 1st-level vehicles to 2nd-level vehicles;
- Each 2nd-level vehicle performs a route to serve the designated customers, and then travels to a satellite for its next cycle of operations. The 2nd-level vehicles return to their departure satellite.

In the following, we will focus on Two-Echelon Vehicle Routing Problems, using them to illustrate the various types of constraints that are commonly defined on Multi-Echelon Vehicle Routing Problems. We can define three groups of variants:

Basic variants with no time dependence:

- Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP). This is the simplest version of Multi-Echelon Vehicle Routing Problems. At each level, all vehicles belonging to that level have the same fixed capacity. The size of the fleet of each level is fixed, while the number of vehicles assigned to each satellite is not known in advance. The objective is to serve customers by minimizing the total transportation cost, satisfying the capacity constraints of the vehicles. There is a single depot and a

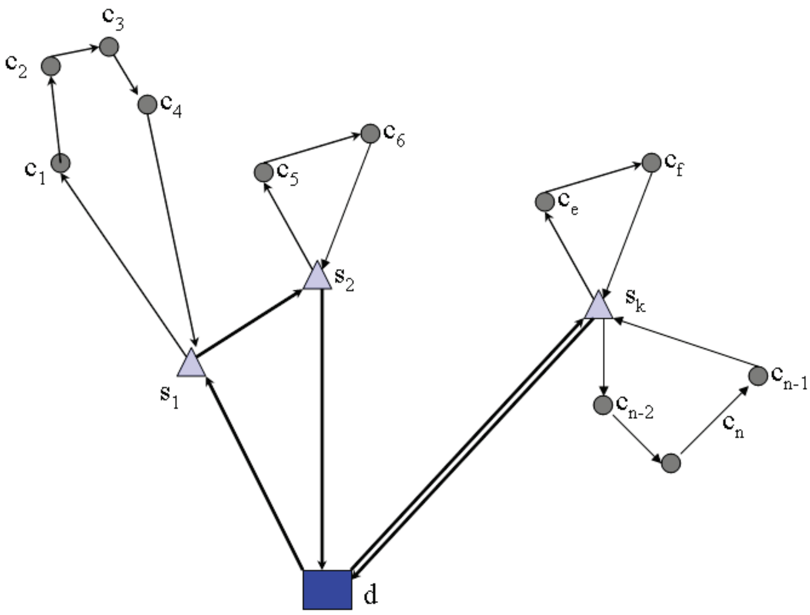

Figure 1: Example of 2E-CVRP transportation network

fixed number of capacitated satellites. All the customer demands are fixed, known in advance and must be compulsorily satisfied. Moreover, no time window is defined for the deliveries and the satellite operations. For the 2nd level, the demand of each customer is smaller than each vehicle's capacity and can not be split in multiple routes of the same level. For the 1st level we can consider two complementary distribution strategies. In the first case, each satellite is served by just one 1st-level vehicle and the aggregated demand passing through the satellite can not be split into different 1st-level vehicles. This strategy is similar to the classical VRP, and the capacity of 1st-level vehicles has to be greater than the demand of each satellite. In the second case, a satellite can be served by more than one 1st-level vehicle. This strategy has some analogies with the VRP with split deliveries and allow 1st-level vehicles with capacity which is lower than each satellite demand. If also the satellites are capacitated, constraints on the maximum number of 2nd-level vehicles assigned to each satellite are imposed. No information on loading/unloading operations is incorporated.

Basic variants with time dependence:

- Two-Echelon VRP with Time Windows (2E-VRP-TW). This problem is the extension of 2E-CVRP where time windows on the arrival or departure time at the satellites and/or at the customers are considered. The time windows can be hard or soft. In the first case the time windows can not be violated, while in the second, if they are violated a penalty cost is paid.
- Two-Echelon VRP with Satellites Synchronization (2E-VRP-SS). In this problem, time constraints on the arrival and the departure of vehicles at the satellites are considered. In fact, the vehicles arriving at a satellite unload their cargo, which must be immediately loaded into a 2nd-level vehicle. Also this kind of constraints can be of two types: hard and soft. In the hard case, every time a 1st-level vehicle unloads its freight, 2nd-level vehicles must be ready to load it (this constraint is formulated through a very small hard time window). In the second case, if 2nd-level vehicles are not available, the demand is lost and a penalty is paid. If the satellites are capacitated, constraints on loading/unloading operations are

incorporated, such that in each time period the satellite capacity in not violated.

Other 2E-CVRP variants are:

- Multi-depot problem. In this problem the satellites are served by more than one depot. A constraint forcing to serve each customer by only one 2nd-level vehicle can be considered. In this case, we have a Multi-Depot Single-Delivery Problem.
- 2E-CVRP with Pickup and Deliveries (2E-VRP-PD). In this case we can consider the satellites as intermediate depots to store both the freight that has been picked-up from or must be delivered to the customers.
- 2E-CVRP with Taxi Services (2E-VRP-TS). In this variant, direct shipping from the depot to the customers is allowed if it helps to decrease the cost, or to satisfy time and/or synchronization constraints.

4 The Two-Echelon Capacitated Vehicle Routing Problem

As stated in Section 3, 2E-CVRP is the two-echelon extension of the wellknown VRP problem. In this section we describe in detail the 2E-CVRP and introduce a mathematical formulation able to solve small and medium-sized instances. We do not consider any time windows or satellite synchronization constraints.

Let us denote the depot by v_0, the set of intermediate depots called satellites by V_s and the set of customers by V_c. Let n_s be the number of satellites and n_c the number of customers. The depot is the starting point of the freight and the satellites are capacitated. The customers are the destinations of the freight and each customer i has associated a demand d_i, i.e. the quantity of freight that has to be delivered to that customer. The demand of each customer can not be split among different vehicles at the 2nd level. For the first level, we consider that each satellite can be served by more than one 1st-level vehicle, so the aggregated freight assigned to each satellite can be split into two or more vehicles. Each 1st level vehicle can deliver the freight of one or more customers, as well as serve more than one satellite in the same route.

The distribution of the freight can not be managed by direct shipping from the depot to the customers, but the freight must be consolidated from the depot to a satellite and then delivered from the satellite to the desired customer. This implicitly defines a two-echelon transportation system: the 1st level interconnecting the depot to the satellites and the 2nd one the satellites to the customers (see Figure 1).

Define the arc (i, j) as the direct route connecting node i to node j. If both nodes are satellites or one is the depot and the other is a satellite, we define the arc as belonging to the 1st-level network, while if both nodes are customers or one is a satellite and the other is a customer, the arc belongs to the 2nd-level network.

We consider only one type of freight, i.e. the volumes of freight belonging to different customers can be stored together and loaded in the same vehicle for both the 1st and the 2nd-level vehicles. Moreover, the vehicles belonging to the same level have the same capacity. The satellites are capacitated and each satellite is supposed to have its own capacity, usually expressed in terms of maximum number of 2nd-level routes starting from the satellite or freight volume. Each satellite receives its freight from one or more 1st level vehicles.

We define as *1st-level route* a route made by a 1st-level vehicle which starts from the depot, serves one or more satellites and ends at the depot. A *2nd-level route* is a route made by a 2nd-level vehicle which starts from a satellite, serves one or more customers and ends at the same satellite.

The problem is easily seen to be NP-Hard via a reduction to VRP, which is a special case of 2E-CVRP arising when just one satellite is considered.

4.1 A Flow-based Model for 2E-CVRP

According to the definition of 2E-CVRP, if the assignments between customers and satellites are determined, the problem reduces to $1 + n_s$ VRP (1 for the 1st-level and n_s for the 2nd-level).

The main question when modeling 2E-CVRP is how to connect the two levels and manage the dependence of the 2nd-level from the 1st one.

The freight must be delivered from the depot v_0 to the customers set $V_c = \{v_{c_1}, v_{c_2}, ..., v_{c_{n_c}}\}$. Let d_i the demand of the customer c_i. The number of 1st-level vehicles available at the depot is m_1. These vehicles have the same given capacity K^1. The total number of 2nd-level vehicles available for the second level is equal to m_2. The total number of active vehicles can not exceed m_2 and each satellite k have a maximum capacity m_{s_k}. The 2nd-level vehicles have the same given capacity K^2.

In our model we will not consider the fixed costs of the vehicles, since we suppose they are available in fixed number. We consider the travel costs c_{ij}, which are of two types:

- costs of the arcs traveled by 1st-level vehicles, i.e. arcs connecting the depot to the satellites and the satellites between them;
- costs of the arcs traveled by 2nd-level vehicles, i.e. arcs connecting the satellites to the customers and the customers between them.

Another cost that can be used is the cost of loading and unloading operations at the satellites. Supposing that the number of workers in each satellite v_{s_k} is fixed, we consider only the cost incurred by the management of the freight and we define S_k as the unit cost of freight handling at the satellite v_{s_k}.

The formulation we present derives from the multi-commodity network design and uses the flow of the freight on each arc as main decision variables.

We define five sets of variables, that can be divided in three groups:

- The first group represents the arc usage variables. We define two sets of such variables, one for each level. The variable x_{ij} is an integer variable of the 1st-level routing and is equal to the number of 1st-level vehicles using arc (i, j). The variable y_{ij}^k is a binary variable representing the 2nd-level routing. It is equal to 1 if a 2nd-level vehicle makes a route starting from

$V_0 = \{v_0\}$	Depot
$V_s = \{v_{s_1}, v_{s_2}, ..., v_{s_{n_s}}\}$	Set of satellites
$V_c = \{v_{c_1}, v_{c_2}, ..., v_{c_{n_c}}\}$	Set of customers
n_s	number of satellites
n_c	number of customers
m_1	number of the 1st-level vehicles
m_2	number of the 2nd-level vehicles
m_{s_k}	maximum number of 2nd-level routes starting from satellite k
K^1	capacity of the vehicles for the 1st level
K^2	capacity of the vehicles for the 2nd level
d_i	demand required by customer i
c_{ij}	cost of the arc (i, j)
S_k	cost for loading/unloading operations of a unit
	of freight in satellite k
Q^1_{ij}	flow passing through the 1st-level arc (i, j)
Q^2_{ijk}	flow passing through the 2st-level arc (i, j) and coming from satellite k
x_{ij}	number of 1st-level vehicles using the 1st-level arc (i, j)
y^k_{ij}	boolean variable equal to 1 if the 2st-level arc (i, j) is used by
	the 2nd-level routing starting from satellite k
z_{kj}	variable set to 1 if the customer c_i is served by the satellite k

Table 1: Definitions and notations

satellite k and goes from node i to node j, 0 otherwise.

- The second group of variables represents the assignment of each customer to one satellite and are used to link the two transportation levels. More precisely, we define z_{kj} as a binary variable that is equal to 1 if the freight to be delivered to customer j is consolidated in satellite k and 0 otherwise.

- The third group of variables, split into two subsets, one for each level, represents the freight flow passing through each arc. We define the freight flow as a variable Q^1_{ij} for the 1st-level and Q^2_{ijk} for the 2nd level, where k represents the satellite where the freight is passing through. Both variables are continuous.

In order to lighten the model formulation, we define the auxiliary quantity

$$D_k = \sum_{j \in V_c} d_j z_{kj}, \forall k \in V_s, \tag{1}$$

which represents the freight passing through each satellite k.

The model to minimize the total cost of the system may be formulated as follows:

$$\min \sum_{i,j \in V_0 \cup V_s, i \neq j} c_{ij} x_{ij} + \sum_{k \in V_s} \sum_{i,j \in V_s \cup V_c, i \neq j} c_{ij} y^k_{ij} + \sum_{k \in V_s} S_k D_k$$

Subject to

$$\sum_{i \in V_s} x_{0i} \leq m_1 \tag{2}$$

$$\sum_{j \in V_s \cup V_0, j \neq k} x_{jk} = \sum_{i \in V_s \cup V_0, i \neq k} x_{ki} \ \ \forall k \in V_s \cup V_0 \tag{3}$$

$$\sum_{k \in V_s} \sum_{j \in V_c} y_{kj}^k \leq m_2 \tag{4}$$

$$\sum_{j \in V_c} y_{kj}^k \leq m_{s_k} \ \ \forall k \in V_s \tag{5}$$

$$\sum_{j \in V_c} y_{kj}^k = \sum_{j \in V_c} y_{jk}^k \ \ \forall k \in V_s \tag{6}$$

$$\sum_{i \in V_s \cup v_0, i \neq j} Q_{ij}^1 - \sum_{i \in V_s \cup v_0, i \neq j} Q_{ji}^1 = \begin{cases} D_j & j \text{ is not the depot} \\ \sum_{i \in V_c} -d_i & \text{otherwise} \end{cases} \ \ \forall j \in V_s \cup V_0 \tag{7}$$

$$Q_{ij}^1 \leq K^1 x_{ij} \ \ \forall i, j \in V_s \cup V_0, i \neq j \tag{8}$$

$$\sum_{i \in V_c \cup k, i \neq j} Q_{ijk}^2 - \sum_{i \in V_c \cup k, i \neq j} Q_{jik}^2 = \begin{cases} z_{kj} d_j & j \text{ is not a satellite} \\ -D_j & \text{otherwise} \end{cases} \ \ \forall j \in V_c \cup V_s, \forall k \in V_s \tag{9}$$

$$Q^2_{ijk} \leq K^2 y^k_{ij} \quad \forall i,j \in V_s \cup V_c, i \neq j, \forall k \in V_s \tag{10}$$

$$\sum_{i \in V_s} Q^1_{iv_0} = 0 \tag{11}$$

$$\sum_{j \in V_c} Q^2_{jkk} = 0 \quad \forall k \in V_s \tag{12}$$

$$y^k_{ij} \leq z_{kj} \quad \forall i \in V_s \cup V_c, \forall j \in V_c, \forall k \in V_s \tag{13}$$

$$y^k_{ji} \leq z_{kj} \quad \forall i \in V_s, \forall j \in V_c, \forall k \in V_s \tag{14}$$

$$\sum_{i \in V_s \cup V_c} y^k_{ij} = z_{kj} \quad \forall k \in V_s, \forall j \in V_c \tag{15}$$

$$\sum_{i \in V_s} y^k_{ji} = z_{kj} \quad \forall k \in V_s, \forall j \in V_c \tag{16}$$

$$\sum_{i \in V_s} z_{ij} = 1 \quad \forall j \in V_c \tag{17}$$

$$y^k_{kj} \leq \sum_{l \in V_s \cup V_0} x_{kl} \quad \forall k \in V_s, \forall j \in V_c \tag{18}$$

$$y_{ij}^k \in \{0,1\}, z_{kj} \in \{0,1\}, \;\; \forall k \in V_s \cup V_0, \forall i,j \in V_c \tag{19}$$

$$x_{kj} \in \mathbb{Z}^+, \;\; \forall k,j \in V_s \cup V_0 \tag{20}$$

$$Q_{ij}^1 \geq 0, \forall i,j, \in V_s \cup V_0, \; Q_{ijk}^2 \geq 0, \; \forall i,j \in V_s \cup V_c, \forall k \in V_s. \tag{21}$$

The objective function minimizes the sum of the traveling and handling operations costs. Constraints (3) show, for $k = v_0$, that each 1st-level route begins and ends at the depot, while when k is a satellite, impose the balance of vehicles entering and leaving that satellite. The limit on the satellite capacity is satisfied by constraints (5). They limit the maximum number of 2nd-level routes starting from every satellite (notice that the constraints also limit at the same time the freight capacity of the satellites). Constraints (6) force each 2nd-level route to begin and end to one satellite and the balance of vehicles entering and leaving each customer. The number of the routes in each level must not exceed the number of vehicles for that level, as imposed by constraints (2) and (4).

Constraints (7) and (9) indicate that the flows balance on each node is equal to the demand of this node, except for the depot, where the exit flow is equal to the total demand of the customers, and for the satellites at the 2nd-level, where the flow is equal to the demand (unknown) assigned to the satellites. Moreover, constraints (7) and (9) forbid the presence of subtours not containing the depot or a satellite, respectively. In fact, each node receives an amount of flow equal to its demand, preventing the presence of subtours. Consider, for example, that a subtour is present between the nodes i, j and k at the 1st level. It is easy to check that, in such a case, does not exist any value for the variables Q^1_{ij}, Q^1_{jk} and Q^1_{ki} satisfying the constraints (7) and (9). The capacity constraints are formulated in (8) and (10), for the 1st-level and the 2nd-level, respectively. Constraints (11) and (12) do not allow residual flows in the routes, making the returning flow of each route to the depot (1st-level) and to each satellite (2nd-level) equal to 0.

Constraints (13) and (14) indicate that a customer j is served by a satellite k ($z_{kj} = 1$) only if it receives freight from that satellite ($y^k_{ij} = 1$). Constraint (17) assigns each customer to one and only one satellite, while constraints (15) and (16) indicate that there is only one 2nd-level route passing through each customer. At the same time, they impose the condition that a 2nd-level route departs from a satellite k to deliver freight to a customer if and only the customer's freight is assigned to the satellite itself. Constraints (18) allow a 2nd-level route to start from a satellite k only if a 1st-level route has served it.

5 Valid inequalities for 2E-CVRP

In order to strengthen the continuous relaxation of the flow model, we introduce cuts derived from VRP formulations. In particular, we use two families of cuts, one applied to the assignment variables derived from the subtour elimination constraints (edge cuts) and the other based on the flows.

The *edge cuts* explicitly introduce the well-known subtours elimination constraints derived from the TSP. They can be expressed as follows:

$$\sum_{i,j\in S_c} y_{ij}^k \leq |S_c|-1, \forall S_c \subset V_c,\ 2 \leq |S_c| \leq |V_c|-2 \tag{22}$$

These inequalities explicitly forbid the presence in the solution of subtours not containing the depot, already forbidden by Constraints (9). The number of potential valid inequalities is exponential, so we should need a separation algorithm to add them. As we will show in Section 6, in practice the inequalities involving sets S_c with cardinality more than 3 are unuseful and the separation algorithm can be substituted by a direct inspection of the constraints up to cardinality equal to 3.

The aim of flow cuts is to reduce the splitting of the values of the binary variables when the continuous relaxation is performed, strengthening the BigM constraints (10). The idea is to reduce the constant K^2 by considering that each customer reduces the flow by an amount equal to its demand d_i. Thus the following inequalities are valid:

$$\begin{cases} Q_{ijk}^2 \leq (K^2-d_i)y_{ij}^k, \forall i,j \in V_c\ \ \forall k \in V_s \\ Q_{ijk}^2 - \sum_{l\in V_s} Q_{jlk}^2 \leq (K^2-d_i)y_{ij}^k\ \ \forall i,j \in V_c, \forall k \in V_s. \end{cases} \tag{23}$$

Constraints (23) are of the same order of magnitude of (10), so they can be directly introduced into the model.

6 Computational tests

In this section, we analyze the behavior of the model using a commercial solver. Being 2E-CVRP introduced for the first time in this paper, in Subsection 6.1 we define some benchmark instances, extending the instance sets from the VRP literature. All the tests have been performed on a 3 GhZ Pentium PC wth 1 Gb of Ram. The models and the routines have been implemented in Mosel language and tested by means of XPress 2006 solver (Dash Associates, 2006). In Subsection 6.2 we present the results of the models on a set of small-sized instances highlighting the properties of 2E-CVRP and the cost distribution according to the geographic distribution of the satellites. Section 6.3 is devoted to present the computational results on a wide set of benchmark instances and the impact of the valid inequalities of Section 5 on the computational results. Finally, Section 6.4 presents the computational results of the model and the valid inequalities on the overall sets of instances, including sets especially designed to illustrate the behavior of 2E-CVRP under specific geographical distributions of the satellites and the customers.

6.1 Construction of the instance sets

In this section we introduce different instance sets for 2E-CVRP. The instances cover up to 51 nodes (1 depot and 50 customers) and are grouped in four sets. The first three sets have been built from the existing instances for VRP by Christofides and Eilon denoted as E-n13-k4, E-n22-k4, E-n33-k4 and E-n51-k5 (Christofides and Eilon, 1969), while the third set is constituted by randomly generated instances simulating different geographical distributions, including customers distribution in urban and regional areas. All the instance sets can be downloaded from the web site of OR-Library (Beasley, 1990).

The first instance set is made by 66 small-sized instances with 1 depot, 12 customers and 2 satellites. All the instances have the cost matrix of the instance E-n13-k4 (the costs of the matrix of the original instance is read as an upper triangular matrix and the corresponding optimal cost of the VRP instance is 290). The two satellites are placed over two customers in all the $\begin{pmatrix} 12 \\ 2 \end{pmatrix} = 66$ possible ways (the case where some customers are used as satellites is quite common for different kinds of distribution, e.g. grocery distribution). When a node is both a customer and a satellite, the arc cost c_{ki} is set equal to 0. The number of vehicles for the 1st-level is set to 2, while the 2nd-level vehicles are 4, as in the original VRP instance. The capacity of the 1st-level vehicles is 2.5 times the capacity of the 2nd-level vehicles, to represent cases in which the 1st-level is made by trucks and the 2nd-level is made by smaller vehicles (e.g., vehicles with a maximum weight smaller than 3.5 t). The capacity of the 2nd-level vehicles is equal to the capacity of the vehicles of the VRP instance. The cost due to loading/unloading operations is set equal to 0, while the arc costs are the same of the VRP instances.

The second set of instances is obtained in a similar way from the instances E-n13-k4, E-n22-k4, E-n33-k4 and E-n51-k5. The instances are obtained by considering 6 pairs randomly generated satellites. For the instance E-n51-k5, which has 50 customers, we build an additional group of 3 instances obtained randomly placing 4 satellites instead of 2. The cost due to loading/unloading operations is set equal to 0, while the arc costs are the same of the VRP instances.

The main issue in the original instances by Christofides and Eilon is that the depot is in an almost central position in respect to the area covered by the customers. The third set of instances also considers the instances E-n13-k4, E-n22-k4, E-n33-k4 and E-n51-k5 by considering six pairs of satellites randomly chosen between the customers on the external border of the area determined by the customers distribution. Moreover, the depot is external to the customers areas, being placed at the coordinate $(0, 0)$ (the southeast corner of the customers area).

Finally, the fourth set includes instances generated in order to simulate different geographical distributions of the customers as well as of the satellites arising in urban and regional applications. The instances are generated according to the following parameters:

- Depot. The depot is external to the customers' areas and is located at the South-East corner of the square of side 100.
- Customers. They are generated according to three rules:
  - Random. In this case the customers' positions are randomly generated. This rule is the same used in the Christofides and Eilon's instances.
  - Centroids. The customers area is a circle of radius 100. In the

customers' area are located 8 centroids, 4 randomly generated in an inner circle of radius 33 and 4 outside the circle. For each centroid, a cluster of customers is randomly created. This distribution simulates the situation in a urban area, where the centroids represents the neighborhoods of the city. In fact, usually the clustering is easy to determine in the peripheral neighborhoods, while the clusters in the areas near to the city center intersect each other.

  - Quadrants. The customers area is a circle of radius 100. The circle is split into 4 equal quadrants and one centroid is randomly located for each quadrant. For each centroid, a cluster of customers is randomly created. This distribution simulates the regional distribution, where the centroids represents villages and small cities.

- Satellites. The number of the satellites is 2, 3 and 5. The satellites are capacitated in terms of maximum number of 2nd-level routes starting from every satellite, while their cost for loading/unloading operations is set equal to 0. They are located according to the following rules:

  - Border Random. The satellites are randomly located on the external border of the area determined by the customers distribution. This distribution occurs when the satellites must be compulsory placed in existing areas, such as disused industrial areas or railway stations.
  - Sliced. The external border of the area determined by the customers distribution is split into a number of slices equal to the number of

satellites. For each slice a satellite in randomly located. This occurs when the municipalities have freedom in choosing the satellite location.

  - Forbidden. Given the external border of the area determined by the customers distribution, 1/3 of this area is considered forbidden, while the remaining is split into a number of slices equal to the number of satellites. For each slice a satellite in randomly located. This distribution simulates the case where the satellites can not be located in a portion of the area as in the case of cities close to mountains or the sea.

- Customer demand. The demand is randomly selected in the range $[0, 1000]$.

- Vehicles. The capacity is set to 5000 for the 2nd-level vehicles and to 12500 for the 1st-level vehicles. The number of the 2nd-level vehicles ensures to have a ratio between the total demand of the customers and the loading capacity of the 2nd-level fleet from 0.7 to 0.92.

- Arc costs. The arc costs are integer and are computed as the Euclidean distances between the different coordinates (depot, satellites, customers).

Far each combination of number of satellites, customer and satellite distribution, two instances are created, for a total of 54 instances.

A summary of the main features of the different sets are reported in Table 2. The first column reports the set of instances, while the number of instances in shown in Column 2. Columns 3 and 4 contain the number of satellites and customers, respectively. The number of vehicles for the 1st and the 2nd level can be read in Columns 5 and 6, while Columns 7 and 8 give the capacity of the vehicles of the two levels. Finally, Column 9 shows the capacity of the satellites in terms of maximum number of routes starting from each satellite. In the remaining columns the rule used to localize the satellites and the customers are specified. More in detail, for the satellites the value *All pairs* indicates that all the possible pairs have been computed, *Random* that the satellites are randomly selected, while *Border Random*, *Sliced* and *Forbidden* have the meanings specified in the description of the set 4. About the customers, the value *From 'Instance-name' instance* indicates that we used the same customer distribution of the instance named *Instance-name*, while *Random*, *Centroids* and *Quadrants* have the meanings specified in the description of the set 4.

6.2 Results of small-sized instances

In this section, we report the results obtained by solving to optimality all the 66 instances of the first set (12 customers and 2 satellites). The objective function values are reported in Table 3. The table contains, in the first column, the customer's number of the VRP instance E-n13-k4 where the satellites are placed. Column 2 reports the value of the optimum. Column 3 contains the percentage variation of the optimum of the 2E-CVRP compared to the optimum of the VRP instance. Column 4 shows the mean value of the accessibility index (Hansen,

Set	N° of Instances	n_s	n_c	m_1	m_2	K^1	K^2	Sat. capacity	Sat. distribution	Customer distribution
1	66	2	12	3	4	15000	6000	∞	All pairs	From E-n13-k4 instance
2	6	2	21	3	4	15000	6000	∞	Random	From E-n22-k4 instance
2	6	2	32	3	4	20000	8000	∞	Random	From E-n33-k4 instance
2	6	2	50	3	5	400	160	∞	Random	From E-n51-k5 instance
2	3	4	50	3	5	400	160	∞	Random	From E-n51-k5 instance
3	6	2	21	3	4	15000	6000	∞	Border Random	From E-n22-k4 instance
3	6	2	32	3	4	20000	8000	∞	Border Random	From E-n33-k4 instance
3	6	2	50	3	5	400	160	∞	Border Random	From E-n51-k5 instance
4	2	2	50	3	6	12500	5000	4	Border Random	Random
4	2	2	50	3	6	12500	5000	4	Sliced	Random
4	2	2	50	3	6	12500	5000	4	Forbidden	Random
4	2	2	50	3	6	12500	5000	4	Border Random	Centroids
4	2	2	50	3	6	12500	5000	4	Sliced	Centroids
4	2	2	50	3	6	12500	5000	4	Forbidden	Centroids
4	2	2	50	3	6	12500	5000	4	Border Random	Quadrants
4	2	2	50	3	6	12500	5000	4	Sliced	Quadrants
4	2	2	50	3	6	12500	5000	4	Forbidden	Quadrants
4	2	3	50	3	6	12500	5000	3	Border Random	Random
4	2	3	50	3	6	12500	5000	3	Sliced	Random
4	2	3	50	3	6	12500	5000	3	Forbidden	Random
4	2	3	50	3	6	12500	5000	3	Border Random	Centroids
4	2	3	50	3	6	12500	5000	3	Sliced	Centroids
4	2	3	50	3	6	12500	5000	3	Forbidden	Centroids
4	2	3	50	3	6	12500	5000	3	Border Random	Quadrants
4	2	3	50	3	6	12500	5000	3	Sliced	Quadrants
4	2	3	50	3	6	12500	5000	3	Forbidden	Quadrants
4	2	5	50	3	6	12500	5000	2	Border Random	Random
4	2	5	50	3	6	12500	5000	2	Sliced	Random
4	2	5	50	3	6	12500	5000	2	Forbidden	Random
4	2	5	50	3	6	12500	5000	2	Border Random	Centroids
4	2	5	50	3	6	12500	5000	2	Sliced	Centroids
4	2	5	50	3	6	12500	5000	2	Forbidden	Centroids
4	2	5	50	3	6	12500	5000	2	Border Random	Quadrants
4	2	5	50	3	6	12500	5000	2	Sliced	Quadrants
4	2	5	50	3	6	12500	5000	2	Forbidden	Quadrants

Table 2: Summary of the benchmark tests

1959) computed on each satellite as

$$A_k = \frac{1}{|V_c|} \sum_{i \in V_c} \frac{d_i}{d_{max}} e^{-\beta \frac{c_{ki} - c^2_{min}}{c^2_{max} - c^2_{min}}}, \tag{24}$$

where d_i is the demand of the customer i, d_{max} the maximum demand overall the customers, c_{ki} the transportation cost between the satellite k and the customer i, c^2_{min} and c^2_{max} the minimum and maximum values of the transportation costs at the 2nd-level, respectively, and $\beta > 0$ is a given parameter (we have assumed $\beta = 0.1$). Finally, Column 5 reports the mean normalized transportation cost of the satellites with respect to the depot, where the normalized transportation cost of each satellite k is given by:

$$\bar{c}_k = 100 \frac{c_{0k} - c^1_{min}}{c^1_{max} - c^1_{min}}, \tag{25}$$

where c_{0k} is the transportation cost between the depot and the satellite k and c^1_{min} and c^1_{max} are the minimum and maximum values of the transportation costs of the 1st-level. In the following we discuss advantages and disadvantages of the proposed two-level distribution system, by considering all the pairs of customers as possible satellite location and comparing the results with the optimal solution of the original VRP instance with optimum 290.

From the results, it is clear the benefit of using the 2E-CVRP distribution model instead of the VRP one. Indeed, the former is able to achieve a smaller cost in 45 instances, while the decreasing/increasing of the costs is, except for satellites $11, 12$ with $+38\%$, in the range $[-25\%, +25\%]$ of the corresponding VRP instance. The mean decrease in the 45 instances with a reduced transportation cost is 11.33%, which can be used to balance the costs due to the loading/unloading operations at the satellites. In the city logistics field, this means that the 2E-CVRP distribution model can be introduced without increasing the transportation cost, and obtaining indirect advantages, such as the reduction of the traffic flows and pollution level.

In Figure 2 we report the dispersion of the optima of the 66 instances with respect to the mean transportation cost from the depot to the satellites. These costs have been categorized in three sets: low (L), medium (M) and high (H) as:

- Low: mean transportation cost of the satellites in the interval $[0, 50]$;
- Medium: mean transportation cost of the satellites in the interval $[50, 67]$;
- High: mean transportation cost of the satellites in the interval $[67, 100]$.

On the X axis the mean transportation cost is reported, while on Y we report the ratio between the optimum of the 2E-CVRP instance and the optimum of the VRP instance. Thus, a ratio greater than 1 means that the optimum of the 2E-CVRP instance is worse than the VRP one.

According to the figure, it is clear that the instances with an optimum better than the VRP are characterized by a low mean transportation cost from the depot to the satellites. The greater the mean transportation cost the less likely to obtain an improved optimum. On the other hand, it is possible to obtain a gain even with a high mean transportation cost, which means that the mean transportation cost from the depot to the satellites is not the only parameter to be taken into account.

In Figure 3 we show the dispersion of the optima of the 66 instances with respect to the mean accessibility index of the satellites. The mean transportation cost is split into three sets: low (L), medium (M) and high (H) accessibility as follows:

- Low: mean accessibility in the interval $[A_{min}, 33\% \text{ of } [A_{min}, A_{max}]]$;
- Medium: mean accessibility in the interval $[33\% \text{ of } [A_{min}, A_{max}], 66\% \text{ of } [A_{min}, A_{max}]]$;
- High: mean accessibility in the interval $[66\% \text{ of } [A_{min}, A_{max}], A_{max}]$;

where $A_{min} = \min_k\{A_k\}$ and $A_{max} = \max_k\{A_k\}$.

The X axis shows the mean accessibility index, while Y indicates the ratio between the optimum of the 2E-CVRP instance and the optimum of the VRP.

According to the figure, when accessibility increases the number of the instances of the 2E-CVRP with a gain does increase. However, even in the instances with a high accessibility, it is possible to have a deterioration of the optimum.

Table 4 presents a summary of the instances: the accessibility values are given in the rows and the transportation cost in the columns. Each cell contains the number of instances with an objective function better than the VRP and the number of instances with an objective function which is worse. The table shows that 2E-CVRP gives its best results when the mean transportation cost of the satellites is less than 50% of the maximum transportation cost (low transportation cost), with the ratio between gain and loss decreasing while the accessibility index decreases. When the mean transportation cost is medium, the costs of using the satellites are lower than VRP with a medium accessibility, which means that the quality of the result is mainly related to the dispersion of the customers themselves, while with a low accessibility it is difficult to obtain a gain on the total costs. With a high mean transportation cost, it becomes hard to obtain a lower transportation cost, even in presence of a high accessibility index. This is mainly due to the fact that even if the satellites are placed in the neighborhood of the customers, they are usually near the border of the customers' area, so the trasportation cost paid in the 1st level to reach the satellites is not compensated by the gain due to the proximity of the satellites to the customers and the consequent reduction of the 2nd-level fleet routes.

6.3 Valid inequalities computational results

In this section we present the computational results of the first and the second set of instances for 2E-CVRP using the valid inequalities introduced in Setion 5 within a computation time limit of 10000 seconds.

With respect to the edge cuts, a series of tests was carried out using a simple procedure testing all the subtours up to cardinality 5. The procedure, coded in Mosel, has been tested on the instances of the sets 1, 2, and 3. According to the results, the subtours of cardinality greater than 3 are ineffective for the quality of both lower bounds and final solution. As the edge cuts of cardinality up to 3 are $O(n^3)$, we tested the model directly, adding them to the model at the root node, using a procedure to remove those cuts which are ineffective after five levels of the search tree.

Satellites	OPT	Variation (%)	Mean acc.	Mean transp. cost
1,2	280	-3.45	111.95	5.81
1,3	286	-1.38	98.53	13.95
1,4	284	-2.07	71.56	16.28
1,5	218	-24.83	91.38	15.12
1,6	218	-24.83	114.13	18.60
1,7	230	-20.69	125.86	26.74
1,8	224	-22.76	149.65	31.40
1,9	236	-18.62	117.57	33.72
1,10	244	-15.86	143.48	38.37
1,11	268	-7.59	100.34	47.67
1,12	276	-4.83	87.18	50.00
2,3	290	0.00	106.75	19.77
2,4	288	-0.69	79.78	22.09
2,5	228	-21.38	99.60	20.93
2,6	228	-21.38	122.35	24.42
2,7	238	-17.93	134.08	32.56
2,8	234	-19.31	157.87	37.21
2,9	246	-15.17	125.79	39.53
2,10	254	-12.41	151.70	44.19
2,11	276	-4.83	108.56	53.49
2,12	286	-1.38	95.40	55.81
3,4	312	7.59	66.37	30.23
3,5	242	-16.55	86.18	29.07
3,6	242	-16.55	108.94	32.56
3,7	252	-13.10	120.66	40.70
3,8	248	-14.48	144.46	45.35
3,9	260	-10.34	112.37	47.67
3,10	268	-7.59	138.29	52.33
3,11	290	0.00	95.15	61.63
3,12	300	3.45	81.99	63.95
4,5	246	-15.17	59.21	31.40
4,6	246	-15.17	81.97	34.88
4,7	258	-11.03	93.69	43.02

Satellites	OPT	Variation (%)	Mean acc.	Mean transp. cost
4,8	252	-13.10	117.49	47.67
4,9	264	-8.97	85.40	50.00
4,10	272	-6.21	111.32	54.65
4,11	296	2.07	68.18	63.95
4,12	304	4.83	55.02	66.28
5,6	248	-14.48	101.78	33.72
5,7	254	-12.41	113.51	41.86
5,8	256	-11.72	137.30	46.51
5,9	262	-9.66	105.22	48.84
5,10	262	-9.66	131.13	53.49
5,11	262	-9.66	88.00	62.79
5,12	262	-9.66	74.84	65.12
6,7	280	-3.45	136.26	45.35
6,8	274	-5.52	160.06	50.00
6,9	280	-3.45	127.97	52.33
6,10	280	-3.45	153.89	56.98
6,11	280	-3.45	110.75	66.28
6,12	280	-3.45	97.59	68.60
7,8	292	0.69	171.78	58.14
7,9	300	3.45	139.70	60.47
7,10	304	4.83	165.61	65.12
7,11	310	6.90	122.48	74.42
7,12	310	6.90	109.32	76.74
8,9	326	12.41	163.49	65.12
8,10	326	12.41	189.41	69.77
8,11	326	12.41	146.27	79.07
8,12	326	12.41	133.11	81.40
9,10	338	16.55	157.32	72.09
9,11	350	20.69	114.19	81.40
9,12	350	20.69	101.03	83.72
10,11	358	23.45	140.10	86.05
10,12	358	23.45	126.94	88.37
11,12	400	37.93	83.80	97.67

Table 3: 12 customers and 2 satellites instances: detailed results

Satellites	OPT	Variation (%)	Mean acc.	Mean transp. cost
1,2	280	-3.45	111.95	5.81
1,3	286	-1.38	98.53	13.95
1,4	284	-2.07	71.56	16.28
1,5	218	-24.83	91.38	15.12
1,6	218	-24.83	114.13	18.60
1,7	230	-20.69	125.86	26.74
1,8	224	-22.76	149.65	31.40
1,9	236	-18.62	117.57	33.72
1,10	244	-15.86	143.48	38.37
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1,12	276	-4.83	87.18	50.00
2,3	290	0.00	106.75	19.77
2,4	288	-0.69	79.78	22.09
2,5	228	-21.38	99.60	20.93
2,6	228	-21.38	122.35	24.42
2,7	238	-17.93	134.08	32.56
2,8	234	-19.31	157.87	37.21
2,9	246	-15.17	125.79	39.53
2,10	254	-12.41	151.70	44.19
2,11	276	-4.83	108.56	53.49
2,12	286	-1.38	95.40	55.81
3,4	312	7.59	66.37	30.23
3,5	242	-16.55	86.18	29.07
3,6	242	-16.55	108.94	32.56
3,7	252	-13.10	120.66	40.70
3,8	248	-14.48	144.46	45.35
3,9	260	-10.34	112.37	47.67
3,10	268	-7.59	138.29	52.33
3,11	290	0.00	95.15	61.63
3,12	300	3.45	81.99	63.95
4,5	246	-15.17	59.21	31.40
4,6	246	-15.17	81.97	34.88
4,7	258	-11.03	93.69	43.02

Satellites	OPT	Variation (%)	Mean acc.	Mean transp. cost
4,8	252	-13.10	117.49	47.67
4,9	264	-8.97	85.40	50.00
4,10	272	-6.21	111.32	54.65
4,11	296	2.07	68.18	63.95
4,12	304	4.83	55.02	66.28
5,6	248	-14.48	101.78	33.72
5,7	254	-12.41	113.51	41.86
5,8	256	-11.72	137.30	46.51
5,9	262	-9.66	105.22	48.84
5,10	262	-9.66	131.13	53.49
5,11	262	-9.66	88.00	62.79
5,12	262	-9.66	74.84	65.12
6,7	280	-3.45	136.26	45.35
6,8	274	-5.52	160.06	50.00
6,9	280	-3.45	127.97	52.33
6,10	280	-3.45	153.89	56.98
6,11	280	-3.45	110.75	66.28
6,12	280	-3.45	97.59	68.60
7,8	292	0.69	171.78	58.14
7,9	300	3.45	139.70	60.47
7,10	304	4.83	165.61	65.12
7,11	310	6.90	122.48	74.42
7,12	310	6.90	109.32	76.74
8,9	326	12.41	163.49	65.12
8,10	326	12.41	189.41	69.77
8,11	326	12.41	146.27	79.07
8,12	326	12.41	133.11	81.40
9,10	338	16.55	157.32	72.09
9,11	350	20.69	114.19	81.40
9,12	350	20.69	101.03	83.72
10,11	358	23.45	140.10	86.05
10,12	358	23.45	126.94	88.37
11,12	400	37.93	83.80	97.67

Table 4: 12 customers instances: resume of the results

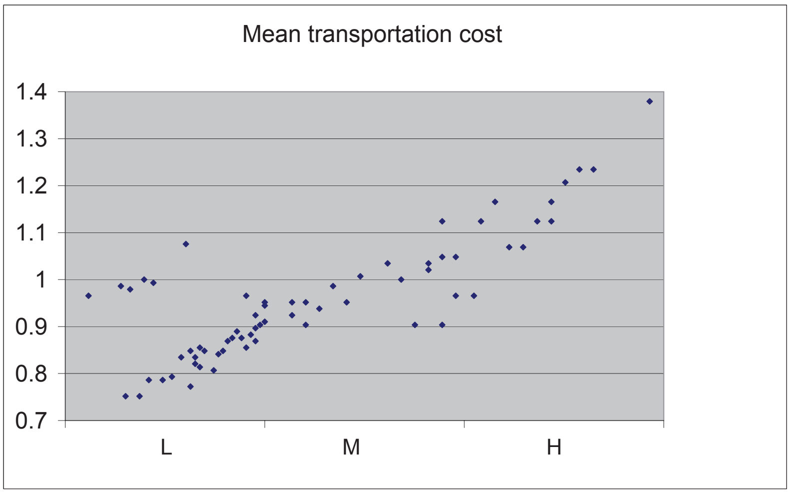

Figure 2: 12 customers instances: dispersion of the travelling costs

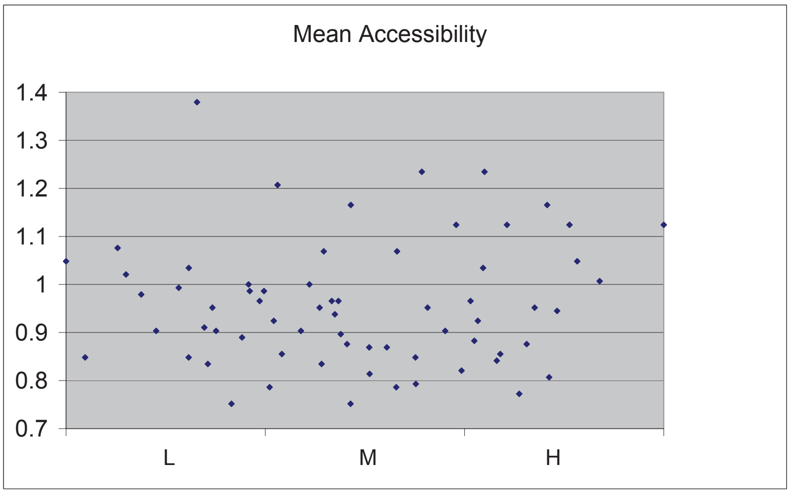

Figure 3: 12 customers instances: dispersion of the accessibility index

In table 5 the results of the 66 instances corresponding to the problem with 12 customers and 2 satellites are given. The optimum is reported in the second column, while columns 3 and 4 contain the time in seconds needed to solve the instances without and with the valid inequalities introduced in Section 5. Finally, the last column presents the percentage of decreasing/increasing of computational time due to the usage of the valid inequalities.

According to the results most instances are solved in less than one minute, and only 10 of them need more than 2 minutes to be solved. There are however seven instances for which the computational times are greater than 10 minutes. This gap is mostly related to the satellite location. In fact, the greatest computational times are related to the situation where choosing which satellite to use has little or no effect on the final solution. In this situation, the model finds an optimal solution quickly, but spends a lot of time closing the nodes of the decision tree due to the poor quality of the lower bound obtained by the continuous relaxation of the model. Better behavior is obtained with the valid inequalities. As a counter effect, on some instances, the computational time still increases, but this is mainly due to the fact that the management of the additional inequalities can affect the computational times on small-sized instances, which show a rather small computational time without the cuts.

The results on the second set of instances are presented in Tables 6 and 7.

Table 6 presents the behavior of the lower bound computed with a continuous relaxation of the model found without and with the valid inequalities. More precisely, columns 1 to 4 contain, respectively, the number of customers in the original Christofides and Eilon's instances, the position of the satellites given as customer number, the mean accessibility as defined by (24) and the mean transportation cost of the satellites computed according to (25). The values and the gap with the best lower bound of the first lower bound (calculated at the root node) without and with the valid inequalities are reported in columns 5-8, while the final lower bound (calculated at the end of the optimization process), increased by letting the solver apply lift-and-project cuts during the optimization, and its gap are presented in columns 9-13. The last column summarizes the best lower bound obtained for each instance (bold values mean optimal values).

From these results it can be seen that the use of the cuts helps the model to reduce the gap by up to 26%. The behavior is confirmed by considering, in Table 7, the values of the feasible solutions found by the model without and with the valid inequalities. More precisely, columns 1 to 6 contain, respectively, the name of the original Christofides and Eilon's instance, the position of the satellites given as customer number, the mean accessibility as defined by (24), the mean transportation cost of the satellites computed according to (25), the best solution found and the best lower bound. The other columns contain respectively the values and the gap with the best lower bound of the first feasible solution, the best solutions after 100, 1000 and 5000 seconds, and the best solution. For each column, the results without and with the cuts are given.

According to these results, for up to 32 customers the model is able to find good quality solutions in 5000 seconds at most. When the number of customers increases to 50, more than 5000 seconds are required to find a good solution.

Satellites	OPT	Time		% Time
		Without cuts	With cuts	
1,2	280	1312.34	1032.34	-21.34%
1,3	286	861.94	298.03	-65.42%
1,4	284	1445.05	306.11	-78.82%
1,5	218	2.06	1.98	-3.83%
1,6	218	7.92	2.27	-71.40%
1,7	230	19.95	5.95	-70.16%
1,8	224	2.50	2.75	10.00%
1,9	236	13.34	6.44	-51.75%
1,10	244	14.27	6.08	-57.40%
1,11	268	28.70	8.73	-69.57%
1,12	276	45.05	31.33	-30.45%
2,3	290	849.17	393.84	-53.62%
2,4	288	895.19	658.84	-26.40%
2,5	228	4.48	2.92	-34.83%
2,6	228	4.20	11.89	182.89%
2,7	238	7.09	3.53	-50.20%
2,8	234	6.00	4.17	-30.47%
2,9	246	11.69	7.08	-39.44%
2,10	254	26.25	8.67	-66.96%
2,11	276	37.27	9.70	-73.96%
2,12	286	226.48	156.67	-30.82%
3,4	312	1704.41	873.74	-48.74%
3,5	242	4.61	2.78	-39.66%
3,6	242	13.13	2.00	-84.76%
3,7	252	17.05	1.92	-88.73%
3,8	248	7.08	2.23	-68.44%
3,9	260	6.17	3.97	-35.69%
3,10	268	33.27	10.30	-69.05%
3,11	290	17.50	6.33	-63.84%
3,12	300	13.39	8.03	-40.02%
4,5	246	6.39	3.28	-48.66%
4,6	246	10.17	3.61	-64.51%
4,7	258	12.16	5.81	-52.19%

Satellites	OPT	Time		% Time
		Without cuts	With cuts	
4,8	252	5.25	0.64	-87.79%
4,9	264	6.56	3.94	-40.01%
4,10	272	15.28	4.34	-71.57%
4,11	296	11.11	5.89	-46.97%
4,12	304	13.91	6.22	-55.28%
5,6	248	3.28	3.31	0.98%
5,7	254	1.97	2.56	30.18%
5,8	256	9.34	4.77	-48.99%
5,9	262	6.59	4.45	-32.47%
5,10	262	2.08	2.70	30.08%
5,11	262	1.73	1.70	-1.79%
5,12	262	1.41	1.67	18.92%
6,7	280	17.70	18.77	6.00%
6,8	274	7.64	6.50	-14.92%
6,9	280	15.22	12.27	-19.40%
6,10	280	7.73	8.38	8.29%
6,11	280	7.11	5.74	-19.33%
6,12	280	14.88	4.45	-70.06%
7,8	292	4.42	2.06	-53.35%
7,9	300	8.97	8.67	-3.30%
7,10	304	12.63	13.52	7.06%
7,11	310	23.88	8.03	-66.36%
7,12	310	19.94	10.16	-49.06%
8,9	326	40.81	18.25	-55.28%
8,10	326	17.86	15.33	-14.17%
8,11	326	11.55	5.94	-48.58%
8,12	326	6.84	5.44	-20.53%
9,10	338	24.27	17.47	-28.01%
9,11	350	17.52	17.58	0.35%
9,12	350	16.25	13.41	-17.50%
10,11	358	40.98	53.23	29.89%
10,12	358	23.19	22.59	-2.56%
11,12	400	40.45	37.95	-6.18%

Table 5: 12 customers and 2 satellites instances: valid inequalities improvements

				First Bound				Final Bound				Best
				Without cuts		With cuts		Without cuts		With cuts		Bound
CVRP Instance	Satellites	Mean acc.	Mean transp. Cost	Bound	Gap	Bound	Gap	Best Bound	Gap	Best Bound	Gap	Best Sol.
E-n13-k4	2,3	33.50	19.77	214.64	-25.99%	214.83	-25.92%	290	0.00%	290	0.00%	**290.00**
	2,5	34.50	20.93	180.24	-20.95%	181.00	-20.62%	228	0.00%	228	0.00%	**228.00**
	3,6	44.00	32.56	185.04	-23.54%	186.41	-22.97%	242	0.00%	242	0.00%	**242.00**
	4,8	56.50	47.67	195.62	-22.37%	197.22	-21.74%	252	0.00%	252	0.00%	**252.00**
	5,10	61.50	52.66	202.57	-22.68%	203.66	-22.27%	262	0.00%	262	0.00%	**262.00**
	7,9	67.50	60.47	219.61	-26.80%	220.13	-26.63%	300	0.00%	300	0.00%	**300.00**
E-n22-k4	7,18	76.04	40.96	374.27	-10.26%	403.86	-3.17%	417.07	0.00%	417.07	0.00%	**417.07**
	9,15	35.33	31.38	338.12	-12.17%	360.16	-6.44%	384.96	0.00%	384.96	0.00%	**384.96**
	10,20	36.29	30.03	403.89	-14.18%	425.95	-9.49%	457.07	-2.88%	470.60	0.00%	**470.60**
	11,15	42.06	24.91	332.02	-10.63%	349.89	-5.82%	371.50	0.00%	371.50	0.00%	**371.50**
	12,13	50.62	52.98	354.05	-17.13%	387.03	-9.41%	417.61	-2.25%	427.22	0.00%	**427.22**
	13,17	38.21	64.25	334.34	-14.88%	355.73	-9.43%	372.66	-5.12%	392.78	0.00%	**392.78**
E-n33-k4	2,10	29.70	40.96	684.97	-3.09%	659.72	-6.66%	688.05	-2.65%	706.80	0.00%	706.80
	3,14	20.04	31.38	641.90	-5.85%	665.54	-2.38%	656.75	-3.67%	681.80	0.00%	681.80
	4,18	32.05	30.03	618.86	-5.80%	640.41	-2.52%	641.59	-2.34%	656.95	0.00%	656.95
	5,6	29.78	24.91	677.27	-6.28%	685.62	-5.13%	711.73	-1.52%	722.68	0.00%	722.68
	8,26	31.05	52.98	668.61	-9.20%	701.50	-4.74%	707.48	-3.92%	736.37	0.00%	736.37
	15,23	19.59	64.25	700.31	-7.20%	735.79	-2.50%	741.57	-1.73%	754.63	0.00%	754.63
E-n51-k5	3,18	44.42	40.55	526.37	-3.18%	541.44	-0.41%	528.80	-2.73%	543.66	0.00%	543.66
	5,47	51.88	17.95	480.43	-6.22%	501.63	-2.08%	499.64	-2.47%	512.31	0.00%	512.31
	7,13	40.62	17.98	492.32	-4.93%	511.50	-1.23%	496.98	-4.03%	517.86	0.00%	517.86
	12,20	35.97	47.34	513.67	-6.92%	531.68	-3.66%	542.77	-1.65%	551.87	0.00%	551.87
	28,48	42.21	15.54	487.14	-3.86%	499.33	-1.46%	489.82	-3.33%	506.72	0.00%	506.72
	33,38	33.86	28.35	500.23	-4.08%	513.80	-1.48%	503.56	-3.44%	521.51	0.00%	521.51
E-n51-k5	3,5,18,47	48.15	29.25	476.72	-5.22%	502.85	-0.02%	479.91	-4.58%	502.95	0.00%	502.95
	7,13,33,38	37.24	23.16	478.60	-3.76%	487.73	-1.92%	480.45	-3.39%	497.30	0.00%	497.30
	12,20,28,48	39.09	31.44	481.97	-3.11%	494.54	-0.58%	482.01	-3.10%	497.45	0.00%	497.45

Table 6: Lower bounds for the instances E-n13-k4, E-n22-k4, E-n33-k4 and E-n51-k5

Table 7: Solutions for the instances E-n13-k4, E-n22-k4, E-n33-k4 and E-n51-k5

CVRP Instance	Satellites	Mean acc.	Mean transp. cost	Best Bound	First Solution				Solution after 100 s			
					Without cuts		With cuts		Without cuts		With cuts	
					Solution	Gap	Solution	Gap	Solution	Gap	Solution	Gap
E-n13-k4	2,3	33.50	19.77	**290.00**	292.00	0.69%	344.00	18.62%	290.00	0.00%	290.00	0.00%
	2,5	34.50	20.93	**228.00**	248.00	8.77%	324.00	42.11%	228.00	0.00%	228.00	0.00%
	3,6	44.00	32.56	**242.00**	322.00	33.06%	314.00	29.75%	242.00	0.00%	242.00	0.00%
	4,8	56.50	47.67	**252.00**	284.00	12.70%	286.00	13.49%	252.00	0.00%	252.00	0.00%
	5,10	61.50	52.66	**262.00**	332.00	26.72%	296.00	12.98%	262.00	0.00%	262.00	0.00%
	7,9	67.50	60.47	**300.00**	318.00	6.00%	344.00	14.67%	300.00	0.00%	300.00	0.00%
E-n22-k4	7,18	76.04	40.96	**417.07**	466.04	11.74%	435.54	4.43%	422.26	1.24%	417.07	0.00%
	9,15	35.33	31.38	**384.96**	633.02	64.44%	429.09	11.46%	399.19	3.70%	410.02	6.51%
	10,20	36.29	30.03	**470.60**	669.82	42.33%	903.66	92.02%	496.23	5.45%	472.89	0.49%
	11,15	42.06	24.91	**371.50**	754.83	103.19%	631.89	70.09%	378.42	1.86%	403.17	8.53%
	12,13	50.62	52.98	**427.22**	909.98	113.00%	782.06	83.06%	451.93	5.78%	482.46	12.93%
	13,17	38.21	64.25	**392.78**	552.39	40.63%	603.67	53.69%	408.66	4.04%	431.65	9.89%
E-n33-k4	2,10	29.70	40.96	706.80	1246.67	76.38%	974.87	37.93%	1200.16	69.80%	807.88	14.30%
	3,14	20.04	31.38	681.80	1009.54	48.07%	1069.11	56.81%	827.71	21.40%	783.19	14.87%
	4,18	32.05	30.03	656.95	1142.79	73.95%	978.07	48.88%	970.23	47.69%	885.98	34.86%
	5,6	29.78	24.91	722.68	1552.21	114.78%	1448.65	100.45%	1096.09	51.67%	860.47	19.07%
	8,26	31.05	52.98	736.37	1166.38	58.40%	1454.79	97.56%	922.76	25.31%	980.20	33.11%
	15,23	19.59	64.25	754.63	974.45	29.13%	1329.03	76.12%	974.45	29.13%	1156.21	53.21%
E-n51-k5	3.18	44.42	40.55	543.66	1109.42	104.07%	900.56	65.65%	No solution found		No solution found	
	5.47	51.88	17.95	512.31	764.99	49.32%	676.37	32.02%	No solution found		No solution found	
	7,13	40.62	17.98	517.86	951.92	83.82%	854.26	64.96%	No solution found		No solution found	
	12,20	35.97	47.34	551.87	11225.00	1934.00%	11251.30	1938.77%	11227.00	1934.36%	11251.30	1938.77%
	28,48	42.21	15.54	506.72	765.22	51.02%	1177.92	132.46%	No solution found		No solution found	
	33,38	33.86	28.35	521.51	1112.62	113.35%	1267.92	143.13%	No solution found		No solution found	
E-n51-k5	3,5,18,47	48.15	29.25	502.95	809.94	61.04%	727.20	44.59%	No solution found		No solution found	
	7,13,33,38	37.24	23.16	497.30	1074.84	116.14%	853.41	71.61%	No solution found		No solution found	
	12,20,28,48	39.09	31.44	497.45	844.44	69.76%	694.25	39.56%	No solution found		No solution found	

CVRP Instance	Satellites	Mean acc.	Mean transp. cost	Best Bound	Solution after 1000 s				Solution after 5000 s				Best solution			
					Without cuts		With cuts		Without cuts		With cuts		Without cuts		With cuts	
					Solution	Gap	Solution	Gap	Solution	Gap	Solution	Gap	Solution	Gap	Solution	Gap
E-n13-k4	2,3	33.50	19.77	**290.00**	290.00	0.00%	290.00	0.00%	290.00	0.00%	290.00	0.00%	290.00	0.00%	290.00	0.00%
	2,5	34.50	20.93	**228.00**	228.00	0.00%	228.00	0.00%	228.00	0.00%	228.00	0.00%	228.00	0.00%	228.00	0.00%
	3,6	44.00	32.56	**242.00**	242.00	0.00%	242.00	0.00%	242.00	0.00%	242.00	0.00%	242.00	0.00%	242.00	0.00%
	4,8	56.50	47.67	**252.00**	252.00	0.00%	252.00	0.00%	252.00	0.00%	252.00	0.00%	252.00	0.00%	252.00	0.00%
	5,10	61.50	0.00	**262.00**	262.00	0.00%	262.00	0.00%	262.00	0.00%	262.00	0.00%	262.00	0.00%	262.00	0.00%
	7,9	67.50	60.47	**300.00**	300.00	0.00%	300.00	0.00%	300.00	0.00%	300.00	0.00%	300.00	0.00%	300.00	0.00%
E-n22-k4	7,18	76.04	40.96	**417.07**	417.07	0.00%	417.07	0.00%	417.07	0.00%	417.07	0.00%	417.07	0.00%	417.07	0.00%
	9,15	35.33	31.38	**384.96**	386.80	0.48%	384.96	0.00%	384.96	0.00%	384.96	0.00%	384.96	0.00%	384.96	0.00%
	10,20	36.29	30.03	**470.60**	471.86	0.27%	470.67	0.02%	470.60	0.00%	470.60	0.00%	470.60	0.00%	470.60	0.00%
	11,15	42.06	24.91	**371.50**	371.50	0.00%	371.50	0.00%	371.50	0.00%	371.50	0.00%	371.50	0.00%	371.50	0.00%
	12,13	50.62	52.98	**427.22**	432.37	1.21%	433.46	1.46%	427.22	0.00%	427.22	0.00%	427.22	0.00%	427.22	0.00%
	13,17	38.21	64.25	**392.78**	408.66	4.04%	392.78	0.00%	392.78	0.00%	392.78	0.00%	392.78	0.00%	392.78	0.00%
E-n33-k4	2,10	29.70	40.96	706.80	767.81	8.63%	752.45	6.46%	767.81	8.63%	745.64	5.50%	751.15	6.27%	731.21	3.45%
	3,14	20.04	31.38	681.80	742.64	8.92%	751.06	10.16%	751.06	10.16%	803.97	17.92%	742.64	8.92%	751.06	10.16%
	4,18	32.05	30.03	656.95	788.17	19.97%	846.14	28.80%	744.21	13.28%	827.65	25.98%	744.21	13.28%	823.42	25.34%
	5,6	29.78	24.91	722.68	945.50	30.83%	860.47	19.07%	873.03	20.80%	860.47	19.07%	873.03	20.80%	860.47	19.07%
	8,26	31.05	52.98	736.37	865.64	17.55%	830.50	12.78%	815.18	10.70%	775.34	5.29%	779.51	5.86%	766.11	4.04%
	15,23	19.59	64.25	754.63	974.45	29.13%	806.93	6.93%	801.15	6.16%	780.17	3.38%	784.42	3.95%	780.17	3.38%
E-n51-k5	3.18	44.42	40.55	543.66	No solution found		No solution found		No solution found		779.64	43.41%	948.79	74.52%	636.45	17.07%
	5.47	51.88	17.95	512.31	764.99	49.32%	676.37	32.02%	680.89	32.91%	576.71	12.57%	663.09	29.43%	570.10	11.28%
	7,13	40.62	17.98	517.86	No solution found		No solution found		951.92	83.82%	803.67	55.19%	870.24	68.05%	592.78	14.47%
	12,20	35.97	47.34	551.87	11057.90	1903.72%	11177.20	1925.34%	10900.70	1875.24%	639.52	15.88%	776.26	40.66%	639.52	15.88%
	28,48	42.21	15.54	506.72	No solution found		1177.92	132.46%	720.34	42.16%	678.75	33.95%	570.33	12.55%	578.40	14.15%
	33,38	33.86	28.35	521.51	No solution found		883.96	69.50%	708.53	35.86%	824.10	58.02%	708.53	35.86%	692.77	32.84%
E-n51-k5	3,5,18,47	48.15	29.25	502.95	809.94	61.04%	No solution found		783.04	55.69%	No solution found		783.04	55.69%	674.34	34.08%
	7,13,33,38	37.24	23.16	497.30	No solution found		No solution found		1043.64	109.86%	767.06	54.25%	1043.64	109.86%	763.97	53.63%
	12,20,28,48	39.09	31.44	497.45	No solution found		No solution found		No solution found		685.53	37.81%	844.44	69.76%	675.68	35.83%

Moreover, the use of the cuts increases the average model quality in terms of the initial solutions and the lower bounds. The gaps between the best solutions and the best bounds are quite small for instances involving up to 32 customers, but increase for 50-customer instances, with a gap up to 54% for the 4 satellite case.

6.4 Overall computational results

In this section we present the results of the tests in the sets 2, 3 and 4. All the results have been obtained using the model with all the valid inequalities activated. The results of each set are summarized in Tables 8, 9 and 10 respectively. Each table contains the instance name, the number of satellites the satellite distribution and the customer distribution in Columns 1, 2, 3 and 4. The mean accessibility as defined by (24) and the mean transportation cost of the satellites computed according to (25) are presented in Columns 5 and 6. Columns 7 and 8 contain the best solution and the lower bound computed by continuous relaxation of the model. Finally, the percentage gap between the best solution and the lower bound in presented in Column 9.

These results indicate that the gap is quite small up to 32 customers, while increases in the 50-customer tests. In particular, the gap is quite large in tests in set 2 involving 4 satellites.

The instances generated from the classical VRP instances present a distribution of the customers which is quite different from the distribution in realistic applications in urban and regional delivery. Moreover, the model is able to find solutions with an average gap of 12%. This is quite large, but understandable considering that the lower bounds come from the simple continuous relaxation of the model with cuts, and that the original 50-customer instance is still considered a difficult one for Branch & Cut and Branch & Bound algorithms developed for VRP.

The quality of the solutions diminishes as the number of satellites increases, even if this is probably due to the poor quality of the lower bound.

A better insight into the performance obtainable with 2E-CVRP and its model can be seen in the instances of set 4. These instances present different distributions, simulating different strategies. According to the results, the model seems to present an almost constant gap around 25%. This can be easily noticed considering the aggregated results of set 4 presented in Tables 11a, 11b and 11c. The tables contain in each cell the mean of the gaps between the best solution and the lower bounds grouped by satellites and customers distribution in Table 11a, number of satellites and customers distribution in Table 11b and number of satellites and satellites distribution in Table 11c. According to the tables, the best results are obtained when using the *Centroids* distribution for the customers and the *Forbidden* for the satellites. This result is not surprising, as it is easier for the model, in the case of the *Centroid* distribution, to find the optimal assignment of the customers to the satellites. Moreover, this id the distribution which better represents the case of urban areas. In any case, the behavior of the model is quite good in the case of the *Quadrants* distribution. Indeed, in this case the mean gaps are almost constant independently of the

Instance	Satellites	Satellite distribution	Customer distribution	Mean acc.	Mean transp. Cost	Final Solution	Best Bound	Gap
E-n22-k4-s6-17	2	Random	E-n22-k4	76.04	40.96	**417.07**	**417.07**	0.00%
E-n22-k4-s8-14	2	Random	E-n22-k4	35.33	31.38	**384.96**	**384.96**	0.00%
E-n22-k4-s9-19	2	Random	E-n22-k4	36.29	30.03	**470.60**	**470.60**	0.00%
E-n22-k4-s10-14	2	Random	E-n22-k4	42.06	24.91	**371.50**	**371.50**	0.00%
E-n22-k4-s11-12	2	Random	E-n22-k4	50.62	52.98	**427.22**	**427.22**	0.00%
E-n22-k4-s12-16	2	Random	E-n22-k4	38.21	64.25	**392.78**	**392.78**	0.00%
E-n33-k4-s1-9	2	Random	E-n33-k4	29.70	40.96	731.21	706.80	3.45%
E-n33-k4-s2-13	2	Random	E-n33-k4	20.04	31.38	742.64	681.80	8.92%
E-n33-k4-s3-17	2	Random	E-n33-k4	32.05	30.03	744.21	656.95	13.28%
E-n33-k4-s4-5	2	Random	E-n33-k4	29.78	24.91	860.47	722.68	19.07%
E-n33-k4-s7-25	2	Random	E-n33-k4	31.05	52.98	766.11	736.37	4.04%
E-n33-k4-s14-22	2	Random	E-n33-k4	19.59	64.25	780.17	754.63	3.38%
E-n51-k5-s2-17	2	Random	E-n51-k5	44.42	40.55	636.45	543.66	17.07%
E-n51-k5-s4-46	2	Random	E-n51-k5	51.88	17.95	570.10	512.31	11.28%
E-n51-k5-s6-12	2	Random	E-n51-k5	40.62	17.98	592.78	517.86	14.47%
E-n51-k5-s11-19	2	Random	E-n51-k5	35.97	47.34	639.52	551.87	15.88%
E-n51-k5-s27-47	2	Random	E-n51-k5	42.21	15.54	570.33	506.72	12.55%
E-n51-k5-s32-37	2	Random	E-n51-k5	33.86	28.35	692.77	521.51	32.84%
E-n51-k5-s2-4-17-46	4	Random	E-n51-k5	48.15	29.25	674.34	502.95	34.08%
E-n51-k5-s6-12-32-37	4	Random	E-n51-k5	37.24	23.16	763.97	497.30	53.63%
E-n51-k5-s11-19-27-47	4	Random	E-n51-k5	39.09	31.44	675.68	497.45	35.83%

Table 8: Summary of the computational results of Set 2

distribution of the satellites.

The increasing gap between best solution and lower bound is still remarkable with the *Random* and the *Quadrants* distributions of the customers (see Table 11c). Even in the case of the satellite distribution, the *Random* distribution is the most sensitive to the number of satellites, while the other two distributions present a better behavior.

7 Conclusions

In this paper, we introduced a new family of VRP models, the Multi-Echelon VRP. In particular, we considered the 2-Echelon Capacitated VRP, giving a MIP formulation and valid inequalities for it. The model and the inequalities

Instance	Satellites	Satellite distribution	Customer distribution	Mean acc.	Mean transp. Cost	Final Solution	Best Bound	Gap
E-n22-k4-s14-15	2	Random	E-n22-k4	39.28	31.47	**526.15**	**526.15**	0.00%
E-n22-k4-s14-17	2	Random	E-n22-k4	42.11	28.77	**521.09**	**521.09**	0.00%
E-n22-k4-s14-18	2	Random	E-n22-k4	70.71	24.64	**496.38**	**496.38**	0.00%
E-n22-k4-s15-20	2	Random	E-n22-k4	31.52	15.71	498.80	480.42	3.83%
E-n22-k4-s15-20	2	Random	E-n22-k4	62.94	8.87	512.81	497.68	3.04%
E-n22-k4-s20-22	2	Random	E-n22-k4	38.97	0.70	520.42	501.69	3.73%
E-n33-k4-s17-23	2	Random	E-n33-k4	23.61	44.26	1401.43	1310.16	6.97%
E-n33-k4-s17-25	2	Random	E-n33-k4	25.34	39.06	1399.95	1329.23	5.32%
E-n33-k4-s20-27	2	Random	E-n33-k4	20.52	38.49	1708.41	1667.80	2.43%
E-n33-k4-s23-27	2	Random	E-n33-k4	23.29	39.16	1716.74	1645.54	4.33%
E-n33-k4-s25-29	2	Random	E-n33-k4	28.21	28.14	1605.09	1544.29	3.94%
E-n33-k4-s26-29	2	Random	E-n33-k4	35.91	24.48	1585.87	1560.13	1.65%
E-n51-k5-s14-20	2	Random	E-n51-k5	38.34	16.50	861.60	652.46	32.05%
E-n51-k5-s14-43	2	Random	E-n51-k5	39.07	19.35	795.78	642.75	23.81%
E-n51-k5-s14-45	2	Random	E-n51-k5	36.66	25.35	860.43	633.98	35.72%
E-n51-k5-s41-43	2	Random	E-n51-k5	31.54	9.02	811.50	651.23	24.61%
E-n51-k5-s42-43	2	Random	E-n51-k5	29.40	15.98	851.80	648.27	31.39%
E-n51-k5-s42-45	2	Random	E-n51-k5	27.00	21.98	819.59	649.22	26.24%

Table 9: Summary of the computational results of Set 3

have been tested on new benchmarks derived from the CVRP instances of the literature, showing a good behavior of the model for small and medium sized instances.

Moreover, a first attempt to find a priori conditions on the solution quality of 2E-CVRP has been performed, enabling the introduction of a classification of the instances according to the combination of easy-to-compute instance parameters, such as satellite accessibility and mean transportation cost.

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Instance	Satellites	Satellite distribution	Customers distribution	Mean acc.	Mean transp. cost	Final Solution	Best Bound	Gap
Instance50-1	2	Random	Random	41.58	49.86	1639	1528.70	7.22%
Instance50-2	2	Random	Random	16.73	46.83	1561	1353.79	15.31%
Instance50-3	2	Sliced	Random	41.75	50.86	1642	1519.42	8.07%
Instance50-4	2	Sliced	Random	30.86	73.25	1542	1358.93	13.47%
Instance50-5	2	Forbidden	Random	58.17	99.46	2386	2092.61	14.02%
Instance50-6	2	Forbidden	Random	30.04	55.82	1387	1245.03	11.40%
Instance50-7	2	Random	Centroids	60.63	48.83	1643	1401.18	17.26%
Instance50-8	2	Random	Centroids	56.65	46.79	1427	1235.68	15.48%
Instance50-9	2	Sliced	Centroids	46.33	49.47	1568	1411.62	11.08%
Instance50-10	2	Sliced	Centroids	62.83	75.04	1462	1290.29	13.31%
Instance50-11	2	Forbidden	Centroids	86.13	97.60	2124	1963.24	8.19%
Instance50-12	2	Forbidden	Centroids	61.74	56.99	1250	1134.10	10.22%
Instance50-13	2	Random	Quadrants	32.54	50.63	1608	1433.29	12.19%
Instance50-14	2	Random	Quadrants	53.23	51.44	1481	1272.76	16.36%
Instance50-15	2	Sliced	Quadrants	48.24	50.08	1550	1447.92	7.05%
Instance50-16	2	Sliced	Quadrants	35.09	75.79	1457	1311.28	11.11%
Instance50-17	2	Forbidden	Quadrants	40.82	99.48	2233	2004.98	11.37%
Instance50-18	2	Forbidden	Quadrants	34.29	60.85	1285	1163.96	10.40%
Instance50-19	3	Random	Random	70.88	49.81	1757	1525.44	15.18%
Instance50-20	3	Random	Random	35.46	55.16	1329	1180.33	12.60%
Instance50-21	3	Sliced	Random	73.99	40.12	1701	1478.39	15.06%
Instance50-22	3	Sliced	Random	36.74	56.50	1399	1236.39	13.15%
Instance50-23	3	Forbidden	Random	71.53	73.41	1898	1725.37	10.01%
Instance50-24	3	Forbidden	Random	33.38	48.64	1420	1234.92	14.99%
Instance50-25	3	Random	Centroids	84.50	48.79	1884	1439.01	30.92%
Instance50-26	3	Random	Centroids	41.19	56.24	1210	1105.15	9.49%
Instance50-27	3	Sliced	Centroids	69.69	35.86	1776	1354.17	31.15%
Instance50-28	3	Sliced	Centroids	56.81	57.76	1287	1126.92	14.21%
Instance50-29	3	Forbidden	Centroids	42.49	70.61	2018	1628.20	23.94%
Instance50-30	3	Forbidden	Centroids	68.21	48.84	1386	1082.39	28.05%
Instance50-31	3	Random	Quadrants	53.07	49.95	1710	1361.36	25.61%
Instance50-32	3	Random	Quadrants	45.20	59.19	1322	1124.25	17.59%
Instance50-33	3	Sliced	Quadrants	67.47	42.94	1684	1417.86	18.77%
Instance50-34	3	Sliced	Quadrants	44.46	60.81	1297	1156.82	12.12%
Instance50-35	3	Forbidden	Quadrants	49.63	74.10	2003	1488.72	34.55%
Instance50-36	3	Forbidden	Quadrants	41.98	52.61	1306	1121.60	16.44%
Instance50-37	5	Random	Random	96.85	47.44	2164	1407.79	53.72%
Instance50-38	5	Random	Random	95.10	37.92	1181	932.09	26.70%
Instance50-39	5	Sliced	Random	106.74	57.66	2153	1415.89	52.06%
Instance50-40	5	Sliced	Random	63.08	54.30	1361	1049.37	29.70%
Instance50-41	5	Forbidden	Random	99.67	69.42	2761	1527.51	80.75%
Instance50-42	5	Forbidden	Random	61.19	52.31	1273	1116.03	14.07%
Instance50-43	5	Random	Centroids	40.95	46.47	2218	1311.02	69.18%
Instance50-44	5	Random	Centroids	45.21	40.44	1181	932.09	26.70%
Instance50-45	5	Sliced	Centroids	101.69	56.45	2129	1322.65	60.96%
Instance50-46	5	Sliced	Centroids	58.16	57.34	1217	937.46	29.82%
Instance50-47	5	Forbidden	Centroids	80.44	66.47	1837	1462.65	25.59%
Instance50-48	5	Forbidden	Centroids	93.72	52.15	1146	1002.98	14.26%
Instance50-49	5	Random	Quadrants	81.41	47.58	2235	1340.42	66.74%
Instance50-50	5	Random	Quadrants	78.21	41.52	1260	961.93	30.99%
Instance50-51	5	Sliced	Quadrants	96.07	57.92	2038	1279.08	59.33%
Instance50-52	5	Sliced	Quadrants	61.94	57.34	1182	969.35	21.94%
Instance50-53	5	Forbidden	Quadrants	84.17	70.21	2145	1439.85	48.97%
Instance50-54	5	Forbidden	Quadrants	69.05	53.59	1301	1026.49	26.74%

Table 10: Summary of the computational results of Set 4

Average (%)	Customers		
Satellites	Centroids	Quadrants	Random
Forbidden	18.38%	22.79%	24.21%
Random	28.17%	28.25%	21.79%
Sliced	26.75%	21.72%	21.92%

(a)

Average (%)	Satellites		
Num Sats	Forbidden	Random	Sliced
2	10.93%	13.97%	10.68%
3	18.68%	18.56%	17.41%
5	35.06%	45.67%	42.30%

(b)

Average (%)	Customers		
Num Sats	Centroids	Quadrants	Random
2	12.59%	11.41%	11.58%
3	22.96%	18.11%	13.50%
5	37.75%	42.45%	42.83%

(c)

Table 11: Summary of the mean gap between the best solution and the lower bound in Set 4

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