1 Introduction

To solve the least squares problem for n-dimensional matrices with a n-degree polynomial we define the problem as

$$Y = X\beta \tag{1}$$

with X as the independent model variables, with coefficients β, and dependent observations Y such that

$$X = \begin{bmatrix} x_0 & x_0^2 & \dots & x_0^p \\ x_1 & x_1^2 & \dots & x_1^p \\ \vdots & \vdots & \ddots & \vdots \\ x_n & x_n^2 & \dots & x_n^p \end{bmatrix} \quad Y = \begin{bmatrix} y_0 \\ \vdots \\ y_n \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_0 \\ \vdots \\ \beta_n \end{bmatrix} \tag{2}$$

where n is the number of samples and p is the degree of the polynomial model. To obtain the optimal parameters β that best fit the data set, we introduce the error on the observations Y with weight matrix W.

$$W = \begin{bmatrix} \sigma_0^{-1} & & & \\ & \sigma_1^{-1} & & \\ & & \ddots & \\ & & & \sigma_n^{-1} \end{bmatrix} \tag{3}$$

The optimal parameters are found from solving

$$\hat{\beta} = \min_{\beta} ||W(Y - X\beta)||_2^2 \tag{4}$$

which has the same general form of the χ^2. The solution for the optimized β estimates is

$$\hat{\beta} = (X^T W^T W X)^{-1} X^T W^T W Y \tag{5}$$

with the error matrix for the β parameters given by

$$V(\beta) = (X^T W^T W X)^{-1} \tag{6}$$

2 Program

The program works by reading in a data set into $n \times 1$ matrices for x, y, σ_y. The program uses a generic least squares function that formats the weight matrix based on the input matrices and returns the fit parameters for a arbitrary polynomial model of degree n. The function also returns the fit generated y values from the x data and the error matrix for the fit parameters. The program then plots the original data with the fitted curve along with a histogram of the χ^2 values. The χ^2 value along with the associated p-value drawn on each plot. When the program is run, it iteratively makes fits from 0 to an arbitrary degree p and prints the fit parameters and error matrix to the screen.