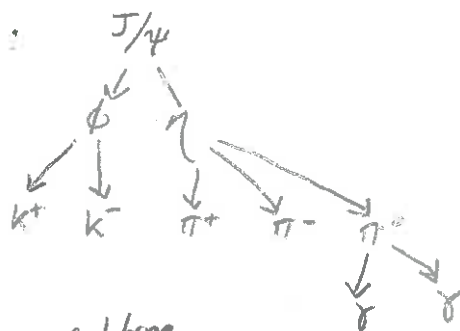


Consider the decay chain:



This decay chain is encoded into a k-ary tree where each subtree is represented as a bracketed string, with a delimiter that indicates whether or not to apply a mass constraint at this subtree. The decay chain would be represented as, (going top down left to right) in terms of PDG codes

$$\begin{array}{c}
 \begin{array}{cccc}
 J/\psi & \phi & \pi & m_{J/\psi} \\
 [443, & 331, & 221, & 3.096] \\
 \text{Parent} & \text{Children} & & \text{mass [GeV]}
 \end{array}
 \end{array}$$

Each subtree is read with the exact array element ordering scheme. if a mass constraint isn't going to be applied to a particular subtree the last element should be -1 i.e. $[\text{parent}, \text{children}, -1]$

Suppose we have a reconstructed event consisting of 7 particles 4 tracks and 3 neutrals (Photons)

$$\begin{array}{c}
 \text{Charged} \quad [C_1^+, C_2^-, C_3^+, C_4^-] \quad \text{Neutral} \quad [N_1, N_2, N_3]
 \end{array}$$

Suppose the true set is:

$$[C_1^+ = \pi^+, C_2^- = \pi^-, C_3^+ = K^+, C_4^- = K^-]$$

and

$$[N_1 = \gamma_1, N_2 = \gamma_2] \quad N_3 \text{ is an extra photon}$$

since there are 2 charged masses we duplicate all tracks,

the resulting set is: (subscript denotes the mass)

$$\text{Track vect} \left[C_{1\pi}^+, C_{2\pi}^-, C_{3\pi}^+, C_{4\pi}^-, C_{5k}^+, C_{6k}^-, C_{7k}^+, C_{8k}^- \right]$$

to ease with implementation of reducing combinations, we add a parallel vector with PDG codes (so ϕ isn't fit with π^+, π^-)
(and ϕ isn't fit with π^-, K^-)

$$\text{Track PDG} \left[211, -211, 211, -211, 321, -321, 321, -321 \right]$$

So the reconstructed set now consists of 8 tracks and 3 photons

For the root constraints the combinations to fit are $\binom{11}{6}$

However it is most important to reduce combinations at this step so by separating the choice into two subsets charged & neutral we get $\binom{8}{4}$ and $\binom{3}{2}$.

Rules to reduce combinations at the Top Level

(1) For the chosen tracks $\sum_{\text{tracks}} q_i = q_{\text{parent}}$

(2) We can not chose total pdgs inconsistent with parent
i.e. $[C_{1\pi}^+, C_{2\pi}^-, C_{3\pi}^+, C_{4\pi}^-] \Rightarrow$ is not allowed we require
 $\times [211, -211, 211, -211]$ 211×2 and 321×2
 $\checkmark [211, -211, 321, -321]$

(3) Duplicated tracks are not allowed, (with different masses)

$$E_{C1}^+, P_{C1}^+ \equiv E_{C5}^+, P_{C5}^+ \text{ but } m_{C5} \neq m_{C1}$$

These two tracks can not be picked together

Lets walk through 1 chosen combination through the full recursion

Suppose we have selected the particles for the root constraint

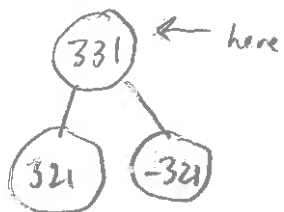
$$T^* \begin{bmatrix} C_{1\pi}^+, C_{2\pi}^-, C_{7K}^+, C_{8K}^- \end{bmatrix} \quad N^* \begin{bmatrix} N_1, N_2 \end{bmatrix}$$

$$\begin{bmatrix} 211, -211, 321, -321 \end{bmatrix} \quad \begin{bmatrix} 22, 22_2 \end{bmatrix}$$

we also introduce a new set of arrays: Boolean flags for selected particles true = used/selected "don't use again"

$$T_{flag} [0, 0, 0, 0] \quad N_{flag} [0, 0]$$

Starting from the top ^{left} child (we've already established root separately)



- get 331's leaves

= generate all possible combinations for 331 from unused part.
from T^*, N^*

use some rules

- combos: $\begin{bmatrix} C_{7K}^+ C_{8K}^- \end{bmatrix}$ (no more combos) of one type (Neutral or charged)

- are 331's immediate children/leaves?

> yes so mark ^{left} tracks used

- recurse to 321, -321 (these are leaves so move up)

array states

constraints particles

$$J/\psi: \begin{bmatrix} C_{1\pi}^+ C_{2\pi}^- C_{7K}^+ C_{8K}^- \end{bmatrix} \begin{bmatrix} N_1, N_2 \end{bmatrix}$$

$$\phi: \begin{bmatrix} C_{7K}^+ C_{8K}^- \end{bmatrix}$$

$$\eta: \begin{bmatrix} x & x & x & x \end{bmatrix}$$

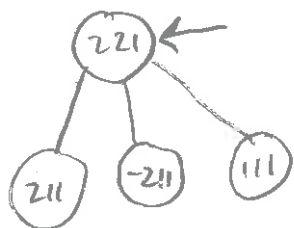
$$\pi^0: \begin{bmatrix} x & x \end{bmatrix}$$

used

$$\begin{bmatrix} C_{1\pi}^+ C_{2\pi}^- C_{7K}^+ C_{8K}^- \end{bmatrix} \begin{bmatrix} N_1, N_2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Recurring to Π



- get 221's leaves

> we have $\{211, -211, 22, 22\}$

- generate all possible combinations for 221 from unused particles from T^*, N^*

using same rules

- combos: $[C_{1\pi}^+ C_{2\pi}^-] [N_1 N_2]$ (only 7 combos again)

with a combo mark immediate, leaves of type used

- recurse to 211, -211 these are leaves do nothing

- recurse to 111, this has children, enter here

array states

$$J/\psi : [C_{1\pi}^+ C_{2\pi}^- C_{7\pi}^+ C_{8\pi}^-]$$

$$\phi : [C_{7\pi}^+ C_{8\pi}^-]$$

$$\eta : [C_{1\pi}^+ C_{2\pi}^- N_1 N_2]$$

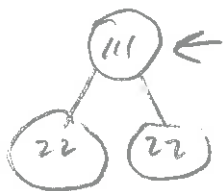
$$\Pi^0 : [x \ x]$$

used:

$$[C_{1\pi}^+ C_{2\pi}^- C_{7\pi}^+ C_{8\pi}^-] [N_1 N_2]$$

$$[1 \ 1 \ 1 \ 1] [0 \ 0]$$

Recurring to Π^0



- get 111's leaves

> we have $\{22, 22\}$

- generate all possible combinations for 221 from unused particles from T^*, N^*

using same rule

- combos $[N_1 N_2]$ (only 1 combo)

> mark immediate leaves used (since all are one type)

recurse to 22's, they are leaves do nothing

[All flags set, so, perform a fit]

array states at fit

$$\frac{1}{4} [C_{1\pi}^+ C_{2\pi}^- C_{7\pi}^+ C_{8\pi}^-] [N, N_2]$$

$$\phi: [C_{7k}^+ C_{8k}^-]$$

$$\chi: [C_{1\pi}^+ C_{2\pi}^-] [N, N_2]$$

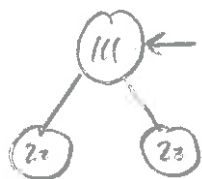
$$\pi^0 [N, N_2]$$

used

$$[C_{1\pi}^+ C_{2\pi}^- C_{7k}^+ C_{8k}^-] [N, N_2]$$

$$[1 \ 1 \ 1 \ 1] [1 \ 1]$$

we have fit now begin to recurse out.

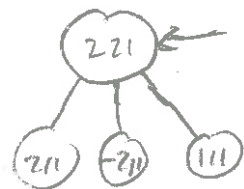


do we have more combos to try?

(if so unmark the leaves previously flagged used)

(then choose the new combo; restart some process...)

no more combos so recurse UP

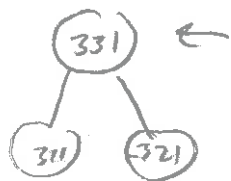


do we have more combos?

(if yes unmark leaves select new combo

and recurse back down to 111 in the tree)

no more combos so go UP (left)



do we have more combos?

> NO.

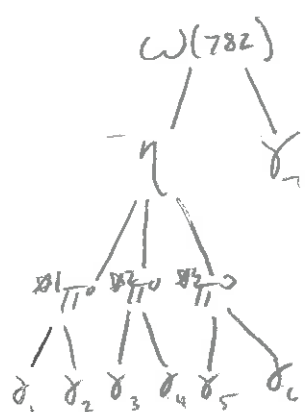
move up to root



back at the root we are done,

→ try a new combo and restart.

A case with many combos in the subtrees:



$$BR = 4.6 \times 10^{-4}$$

The XML strings:

$$[223, 221, 22, 0.782] [221, 111, 111, 111, 0.547]$$

$$[111, 22, 22, 0.135] [111, 22, 22, 0.135] [111, 22, 22, 0.135]$$

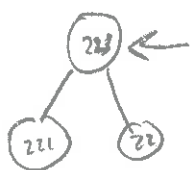
The reconstructed particles (best case scenario)

$$\text{Flags} \rightarrow [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$$

$$\text{Reco} \rightarrow [N_1 \ N_2 \ N_3 \ N_4 \ N_5 \ N_6 \ N_7] \equiv N^*$$

$$N: \text{pdgs} \rightarrow [22, 22, 22, 22, 22, 22, 22]$$

→ Build root constraints $\binom{7}{7}$



get 223 leaves

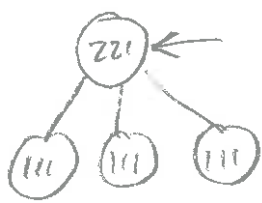
$$\rightarrow \text{we have } [22, 22, 22, 22, 22, 22, 22]$$

generate all possible combinations from unused particles in $N^* \ T^* = 0$

$$\text{combos: } [N_1 \ N_2 \ N_3 \ N_4 \ N_5 \ N_6 \ N_7]$$

→ are 223 immediate children of other type leaves (neutral) NO. don't flag anything

Recurse to 221



get 221 leaves

$$\rightarrow [22, 22, 22, 22, 22, 22]$$

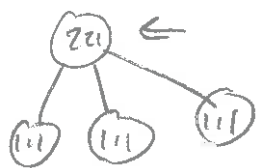
get all combinations from reco set

$$\rightarrow \text{many combos} = \binom{7}{6} = 7 \text{ combos}$$

$$(N_1 \ N_2 \ N_3 \ N_4 \ N_5 \ N_6) \text{ ~~leave~~}$$

$$(N_1 \ N_2 \ N_3 \ N_4 \ N_5 \ N_7)$$

etc



Choose the first combo $[N_1 N_2 N_3 N_4 N_5 N_6]$
 are 221's children leaves? none of them
 are, so make no flags
 \rightarrow move down to left 111

array states

$W: [N_1 N_2 N_3 N_4 N_5 N_6 N_7]$

$\eta: [N_1 N_2 N_3 N_4 N_5 N_6]$

$\pi^0: [X X]$

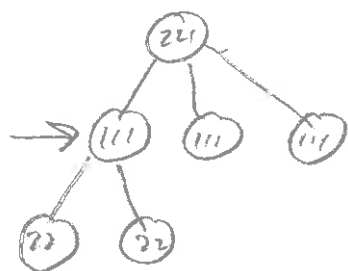
$\pi^0: [X X]$

$\pi^0: [X X]$

used

$[N_1 N_2 N_3 N_4 N_5 N_6 N_7]$

$[0 0 0 0 0 0 0]$



> get 111's leaves

$\rightarrow [22, 22]$

> generate all combos of unused particles

> choose from η selected combo. $\binom{6}{2} = 15$

$[N_1 N_2]$
 $[N_1 N_3]$
 $[N_1 N_4]$
 \vdots

Select 7 combination $[N_1 N_2]$

is 111's children leaves? yes mark used

\rightarrow move to 22's... these are leaves so

we recurse back out to the middle 111

array states

$W: [N_1 N_2 N_3 N_4 N_5 N_6 N_7]$

$\eta: [N_1 N_2 N_3 N_4 N_5 N_6]$

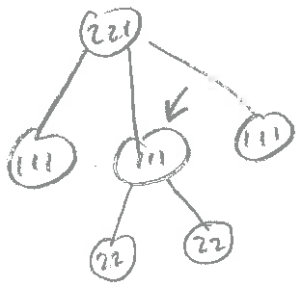
$\pi^0: [N_1 N_2]$

$\pi^0: [X X]$ $\pi^0: [X X]$

flags used

$[N_1 N_2 N_3 N_4 N_5 N_6 N_7]$

$[1 1 0 0 0 0 0]$



> get 111's leaves
 $\rightarrow [22, 22]$

> generate all combos of unused particles
 from the η combination $\binom{4}{2} = 6$ combos
 (not including N_1, N_2)

$[N_3, N_4]$

$[N_3, N_5]$

\vdots

> select first combo $[N_3, N_4]$

> are 111's children leaves? yes, flag N_3, N_4 used

> move to 22's \rightarrow these are leaves so
 recurse back out to right 111

array states

$W: [N_1, N_2, N_3, N_4, N_5, N_6, N_7]$

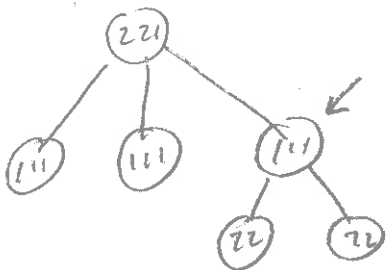
$\eta: [N_1, N_2, N_3, N_4, N_5, N_6]$

$\pi^0: [N_1, N_2]$ $\pi^0: [N_3, N_4]$ $\pi^0: [X, X]$

used

$[N_1, N_2, N_3, N_4, N_5, N_6, N_7]$

$[1, 1, 1, 1, 0, 0, 0]$



> get 111's leaves
 $\rightarrow [22, 22]$

> generate all combinations of unused
 particles from η $\binom{2}{2} = 1$ combo

$[N_5, N_6]$

> is 111's children leaves? yes
 mark used

> recurse out to 22's child of
 used flag

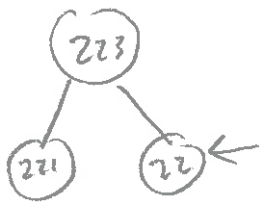
array states

$W: [N_1, N_2, N_3, N_4, N_5, N_6, N_7]$ $\pi^0: [N_1, N_2]$

$\eta: [N_1, N_2, N_3, N_4, N_5, N_6]$ $\pi^0: [N_3, N_4]$ $[N_1, N_2, N_3, N_4, N_5, N_6, N_7]$

$\pi^0: [N_5, N_6]$ $[1, 1, 1, 1, 1, 1, 0]$

⑧



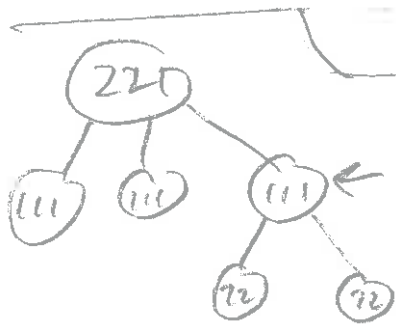
this is a leaf

mark as used

> if these are all true, fit this combo

recurse back up to right 111

* Unmark chosen leaf

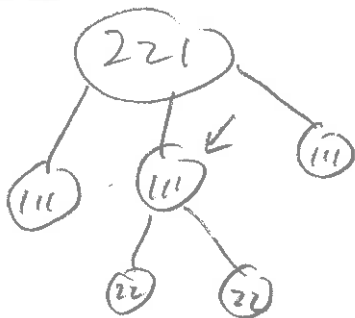


are there more combos to try?

→ No

recurse to middle 111

Unmark chosen leaves



are there more combos to try?

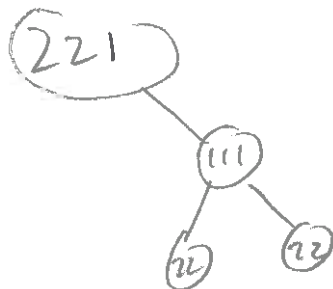
yes → choose next combo

→ unmark previously chosen leaves

choose $[N_3, N_5]$

→ mark $[N_3, N_5]$ as used

move back to right 111



4

11

generate all combos

recurse back down and tree
back up until all combinations
have been generated and tried