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Hypergraph products for structural mechanics

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ABSTRACT

In this paper Cartesian, direct and strong Cartesian product of hypergraphs are investigated. A new concept named the *adjacency function* is defined on hypergraphs. This definition leads to distinct adjacency and Laplacian matrices for a hypergraph and makes it possible to express it in an algebraic form. Variable adjacency functions on hypergraphs result in generation of dynamic graph products which are applied to dynamic systems. For further clarity, some examples from structural mechanics are provided. The generality of the approach is shown through some examples, indicating this fact that the hypergraph products can encompass most of the available developments on the graph products in the literature.

1. Introduction

In a previous work, the authors introduced some concepts of three hypergraph products, namely the Cartesian, direct and strong Cartesian product of hypergraphs [1]. Graph products have been studied extensively in the past in the field of mathematics [2–5], and applied to many problems in structural mechanics in recent decade. These applications include configuration processing, parallel computing, and optimal analysis of structures [6–13]. Hypergraphs are generalized forms of graphs and many applications of graphs can be extended to hypergraphs because of their generality [14,15]. In a hypergraph we have vertices and edges and there is no distinct definition for adjacency. To tackle this problem adjacency functions are defined on hypergraphs and hence distinct adjacency and Laplacian matrices are achieved. Different adjacency functions can be defined for a given problem and therefore the hypergraphs can be applied to a broader domain of problems compared to the graphs.

In structural mechanics the behavior of the system may vary with respect to the time or any other variable, the under study system will be dynamic. Solving the dynamic problems are more involved than static ones and therefore any simplification in the process of their solution will be valuable. Hypergraph products with variable adjacency functions on their sub-hypergraphs, results in variable graph products that are called as dynamic graph products. From application point of view, the dynamic graph products can be used in modeling and configuration processing of dynamic regular structures. Furthermore using the related algebraic attributes of their matrices leads to further simplification of their solution procedures.

In this paper first the Cartesian, direct, and strong Cartesian product of hypergraphs are defined. In the next section the definition of the adjacency functions on hypergraphs is introduced and then the theorems on formation of the adjacency and Laplacian matrices of graph products are applied to hypergraph products. The subsequent section discusses the variable adjacency functions and dynamic graph products. Then some examples are provided from structural mechanics for further clarification of the issue.

2. Basic definitions of a hypergraph

A hypergraph is a generalized form of a graph that can have edges containing any number of vertices. A hypergraph is illustrated with $H = (V, E)$ with V and E representing the vertices and edges of the hypergraph, respectively. A hypergraph is called simple if none of its edges is completely inside another edge of that hypergraph. Two examples of simple hypergraphs are shown in Figs. 1 and 2.

Many theorems and statements for graphs are also applicable to the hypergraphs. For instance hypergraph products can be used in numerical optimization problems, where the matrices have special canonical forms. In some cases the solution of optimization problems with the use of hypergraphs leads to generalization of optimization algorithms [14]. However, in some cases hypergraphs have additional advantages because of their inherited generalities.

3. Adjacency in hypergraphs

Before any discussion on the product of hypergraphs it is important to have more clear definition of the adjacency relationship in a

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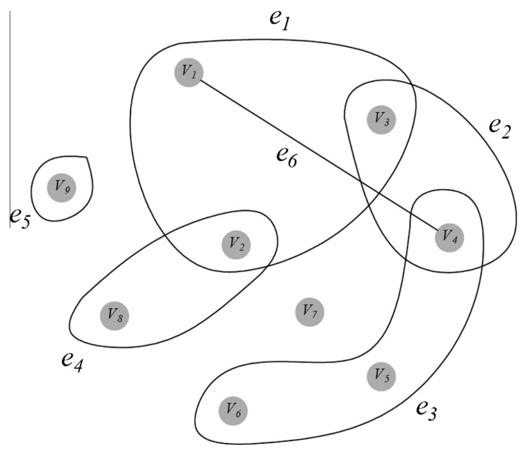

Fig. 1. A simple hypergraph with 9 vertices and 6 edges.

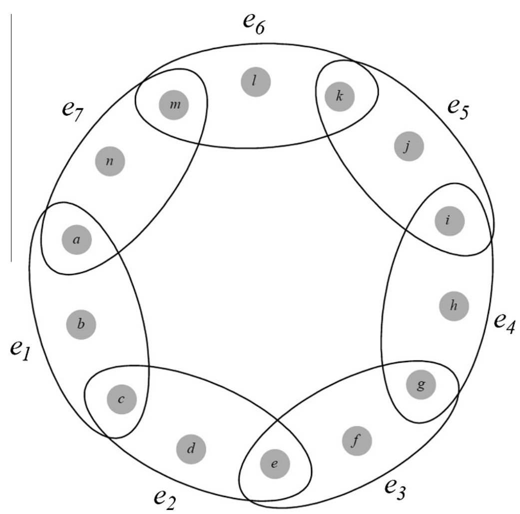

Fig. 2. A simple cyclic hypergraph with 14 vertices and 7 edges.

hypergraph. Hypergraphs with such vertices and edges can be used for modeling large and complicated systems for simpler analysis. An edge in a hypergraph contains vertices that are adjacent with each other. From an application point of view, one can assign a number of properties to a vertex of a hypergraph, and these properties may explain the role of that vertex in the model of the relevant problem. In this way, the adjacency of two vertices will mean that one or more of their properties are either close to each other or are identical. Based on the properties assigned to the vertices of a hypergraph and considering this fact that under which condition two properties can be close to each other or identical, one can define adjacency function for an edge of a hypergraph. According to the defined adjacency function, one or more graphs can be obtained. Such a definition of adjacency function leads to dynamic graphs if the adjacency function is variable with respect to time or any other parameter and can be used in the modeling and analysis of dynamic systems. The effect of different adjacency functions on a simple hypergraph is shown in Fig. 3. The notable point is that the resulted graphs from an edge of a hypergraph should be connected.

4. Hypergraph products

The concept of graph products can be generalized into hypergraphs where a hypergraph product has properties inherited from its generators and they appear in the corresponding matrices via some canonical forms. These properties of a hypergraph product make it possible to have more efficient analysis for the regular systems. In this section only Cartesian, direct and strong Cartesian products will be studied, and obviously other graph products are applicable to hypergraphs in a similar way.

4.1. Cartesian product of hypergraphs

For Cartesian product of simple graphs we have the following definition:

$$\begin{array}{lll} u_1 = v_1 & \text{and} & u_2 v_2 \in M(H) \\ u_1 v_1 \in M(K) & \text{and} & u_2 = v_2 \end{array}$$

This definition is also applicable to hypergraph products. For using this definition one should have a distinct adjacency relationship explanation in the corresponding hypergraphs which can be achieved by defining the adjacency functions. In a hypergraph without defined adjacency functions, in a simple way it is considered that all the vertices of an edge will be adjacent to each other. In literature, the Cartesian product of hypergraphs is defined in the following form that can be considered as a numeral definition rather than an algebraic one.

$$\{\{v\} \times e : v \in V(G),\ e \in E(H)\} \cup \{\{e\} \times v : e \in E(G),\ v \in V(H)\}$$

Some illustrational examples of the Cartesian product of hypergraphs are shown in Figs. 4–7.

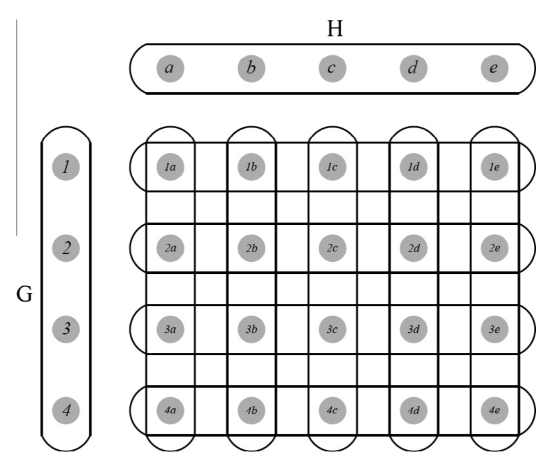

Fig. 4. Cartesian product of two hypergraphs *G* and *H*.

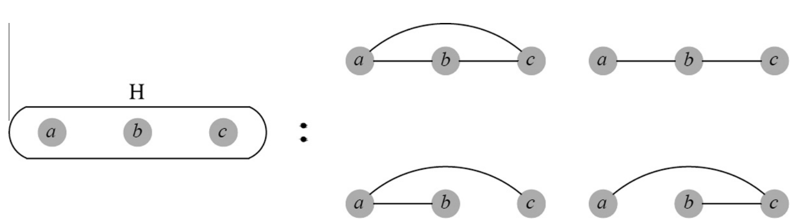

Fig. 3. A hypergraph *H* and the corresponding graphs.

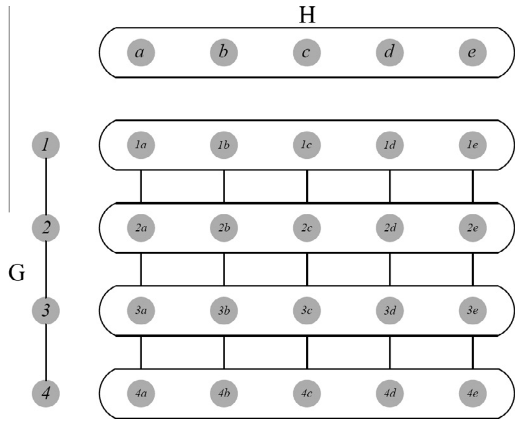

Fig. 5. Cartesian product of a graph G and a hypergraph H.

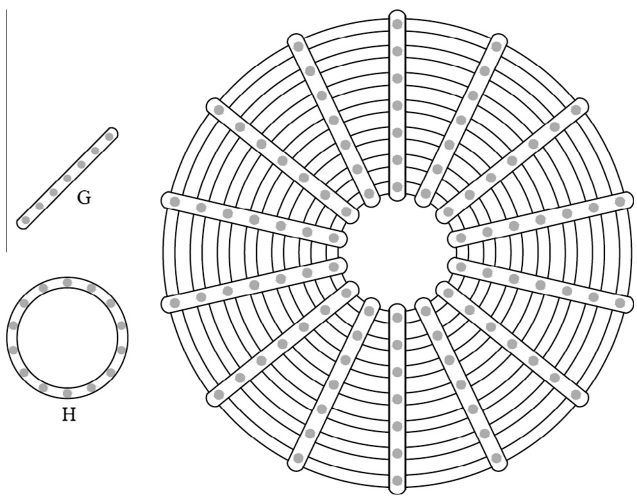

Fig. 6. Cartesian product of two hypergraphs G and H.

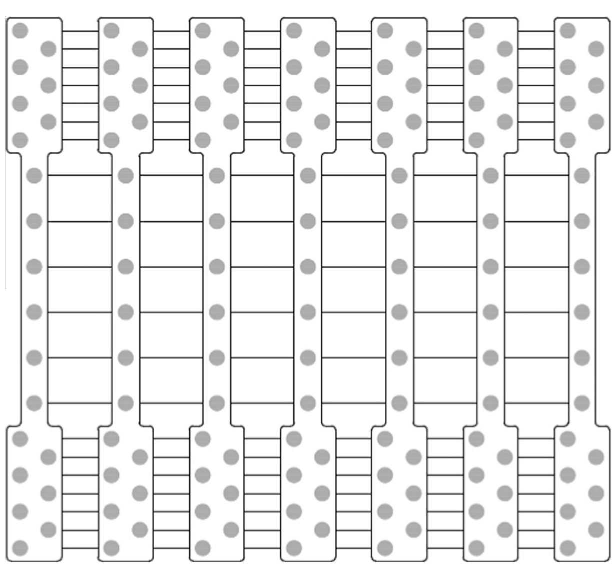

Fig. 7. Cartesian product of a graph and a hypergraph.

From now on for simplicity the hypergraphs G and H, shown in Fig. 6, will be called a path hypergraph and a cycle hypergraph, and will be denoted by $H.P_7$ and $H.C_{14}$, respectively.

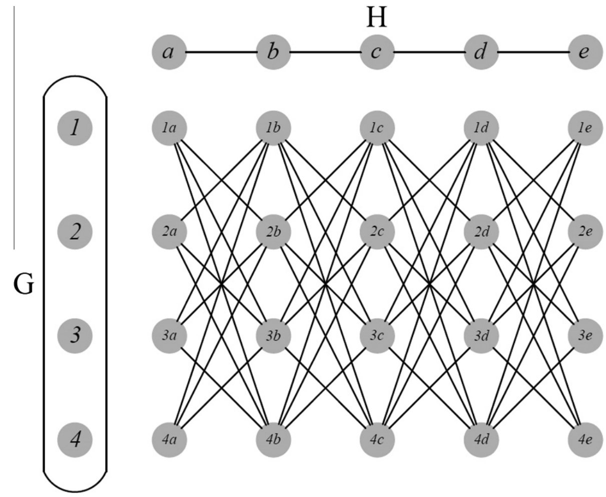

Fig. 8. Direct product of the hypergraph G and graph H.

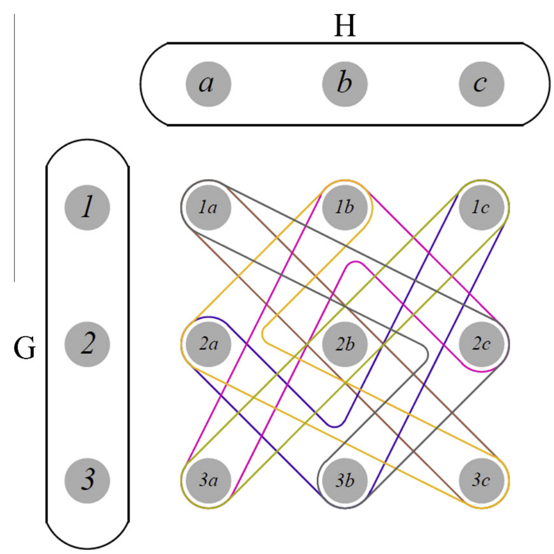

Fig. 9. Direct product of two hypergraphs G and H.

4.2. Direct product of hypergraphs

Similar to the Cartesian product of hypergraphs, we have the following definition for a direct hypergraph product:

$$u_1 v_1 \in M(G) \quad \text{and} \quad u_2 v_2 \in M(H)$$

It is obvious that two vertices in a product hypergraph will be adjacent if the aforementioned statement is satisfied. For this purpose we will need to consider some suitable adjacency functions on subhypergraphs. In Figs. 8 and 9 two examples of these products are illustrated.

In Fig. 8 it is supposed that every two vertices of G are adjacent with each other in the absence of adjacency function definition. With further attention on the product hypergraph of Fig. 9, one can see that it is a generalized form of the product graphs, and with predefined different adjacency functions on sub-hypergraphs we can extract many different product graphs. For instance for an edge of a direct product hypergraph in Fig. 9, as illustrated in Fig. 10, one can assign different adjacency functions leading to graphs as shown in Fig. 11. This indicates that many different product graphs can be extract from a product hypergraph.

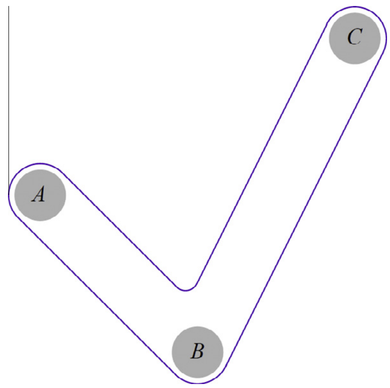

Fig. 10. An arbitrary edge of a product hypergraph in Fig. 9.

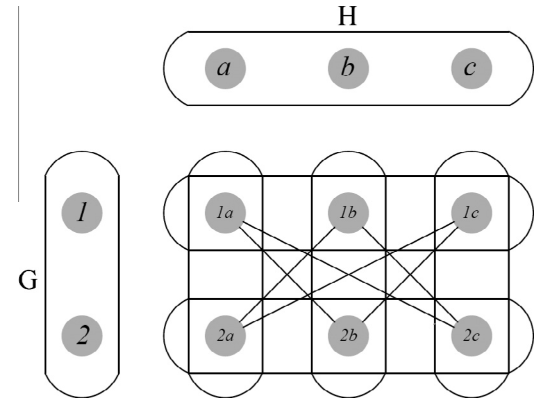

Fig. 12. Strong Cartesian product of two hypergraphs G and H.

4.3. Strong Cartesian product of hypergraphs

This product is a combination of two Cartesian and direct product of hypergraphs and has a product definition as following:

$$\begin{array}{lll} u_1 = v_1 & \text{and} & u_2v_2 \in M(H) \\ u_1v_1 \in M(G) & \text{and} & u_2 = v_2 \\ u_1v_1 \in M(G) & \text{and} & u_2v_2 \in M(H) \end{array}$$

In Figs. 12 and 13, two examples of this product are illustrated.

5. Adjacency and Laplacian matrices of product hypergraphs

Considering adjacency functions on sub-hypergraphs, results in product hypergraphs with distinct adjacency and Laplacian matrices. Whereas the definitions of hypergraph products are similar to that of graph products, the adjacency matrix for hypergraph product of hypergraphs $G_1, G_2, \ldots, G_n$ with adjacency matrices $A_1, A_2, \ldots, A_n$ can acquire from the following relationship [16]:

$$A = \sum_{\beta \in B} A_1^{\beta_1} \otimes \ldots \otimes A_n^{\beta_n} + \sum_{i=1}^{n-1} \left\{ (-1)^i \sum_{\beta \in \gamma_i} A_1^{\beta_1} \otimes \ldots \otimes A_n^{\beta_n} \right\}$$

where $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ and every member of B can have n members as β_i and we have:

$$A_i^{\beta_i} = \begin{cases} A_i & \text{if} \quad \beta_i = 1 \\ I_i & \text{if} \quad \beta_i = 0 \\ \overline{A}_i & \text{if} \quad \beta_i = -1 \\ O_i & \text{if} \quad \beta_i = \alpha \end{cases}$$

Using the aforementioned relationship, the adjacency matrices of the Cartesian, direct and strong Cartesian product of hypergraphs can be expressed as follows:

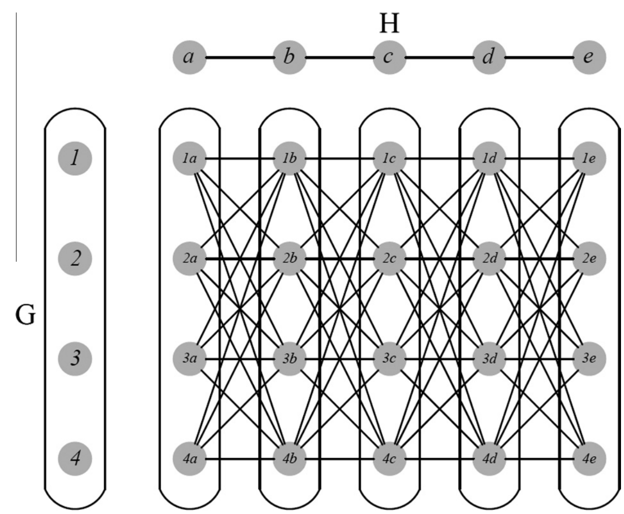

Fig. 13. Strong Cartesian product of a hypergraph G and graph H.

$$\begin{array}{ll} \textit{Cartesian product}: & A_{Hypergraph\ product} = A_G \otimes I_H + I_G \otimes A_H \\ \textit{Direct product}: & A_{Hypergraph\ product} = A_G \otimes A_H \\ \textit{Strong Cartesian product} & A_{Hypergraph\ product} = A_G \otimes I_H + I_G \otimes A_H + A_G \otimes A_H \end{array}$$

In which A_G and A_H are adjacency matrices of the hypergraphs G and H, respectively. The matrices I_G and I_H are identity matrices with sizes equal to the size of the adjacency matrices of hypergraphs G and H.

Similar to the adjacency matrices, for Laplacian matrices of product hypergraphs using the following relationship from [17], one can write the general relations for the Cartesian, direct and strong Cartesian product of hypergraphs. Laplacian matrices of the product of hypergraphs $G_1, G_2, \ldots, G_n$ with adjacency matrices

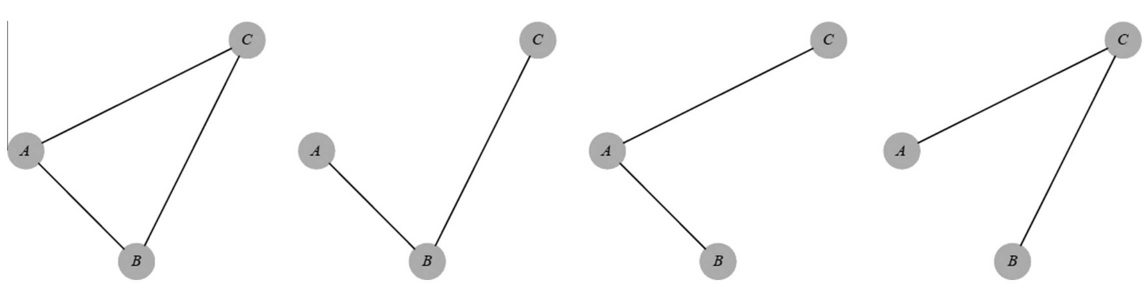

Fig. 11. Different graphs extracted from the edge in Fig. 10.

$A_1, A_2, \ldots, A_n$ and Laplacian matrices $L_1, L_2, \ldots, L_n$ with basis set B (that has members β_i in which any β_i has n number of members 0, 1) can be expressed as the following [17]:

$$L = \sum_{\beta \in B} (L_1 + A_1)^{\beta_1} \otimes \ldots \otimes (L_n + A_n)^{\beta_n} - \sum_{\beta \in B} A_1^{\beta_1} \otimes \ldots \otimes A_n^{\beta_n}$$

where

$$A_i^{\beta_i} = \begin{cases} A_i & \text{if} \quad \beta_i = 1 \\ I_i & \text{if} \quad \beta_i = 0 \end{cases} \qquad (L_i + A_i)^{\beta_i} = \begin{cases} (L_i + A_i) & \text{if} \quad \beta_i = 1 \\ I_i & \text{if} \quad \beta_i = 0 \end{cases}$$

All the above parameters are the same as defined before in the adjacency relationship. Using the aforementioned theorem, the Laplacian matrices for Cartesian, direct and strong Cartesian product of two hypergraphs G and H will be as the following:

$$\begin{array}{ll} \textit{Cartesian product}: & L_{\textit{Hypergraph product}} = L_G \otimes I_H + I_G \otimes L_H \\ \textit{Direct product}: & L_{\textit{Hypergraph product}} = L_G \otimes L_H + L_G \otimes A_H + A_G \otimes L_H \\ \textit{Strong Cartesian product} & L_{\textit{Hypergraph product}} = L_G \otimes I_H + I_G \otimes L_H + L_G \otimes L_H + L_G \otimes A_H + A_G \otimes L_H \end{array}$$

where L_G and L_H are Laplacian matrices of hypergraphs G and H respectively, and other parameters are defined as before.

For other existing products and also for those which will be defined probably in the future, in a similar way, the adjacency and Laplacian matrices can be derived from the mentioned theorems.

6. Dynamic product graphs

A graph can be considered as a distinct relationship between some vertices that the edges connect them to each other. In some problems, the model of the system may vary with time or some other parameters. Using graphs for modeling these dynamic systems should represent different graphs in different times and steps. This can lead the complexity of the solution instead of simplicity and in this way the use of graph will be unprofitable. To tackle this problem one can define dynamic graphs such that their adjacency matrices vary functionally with a specified variable. For this purpose we should have in hand some free vertices that the varied adjacency matrix can be defined on them. Hypergraphs are good means for this intention and as discussed before with assigning adjacency function on a hypergraph the aforementioned statement can be fulfilled. In this state, a hypergraph with predefined variable adjacency function can lead to many different distinct graphs and therefore we will have controlled dynamicity on graphs that is needed. When we have dynamic graphs, product of these graphs results in dynamic product graphs and they can be applied to structural mechanics where the dynamicity on regular structures is present. Now the adjacency function and the way it can be assigned on a hypergraph should be introduced. In the following, some examples from regular structures will further clarify the aforementioned discussion.

7. Adjacency function in hypergraphs

The vertices within an edge of a hypergraph show that these are adjacent with each other, but one can define how the quality of the adjacency is present between the vertices. It is assumed that the vertices and edges of a standard hypergraph are labeled. The problem of the adjacency of the vertices of a hypergraph can be expressed with an adjacency function.

For defining an adjacency function for a hypergraph, a simple way is followed where it is started from the first vertex and inside the set of adjacency function after putting a colon in front of the vertex's label, inside parentheses all the adjacent vertices to the assumed one are written. After putting a semicolon, in the next step this process is repeated for the next vertex until all the vertices of the graph are visited. It should be mentioned that for simplicity of the adjacency function the adjacent vertices inside of the parentheses do not include the ones their adjacency with the current vertices are mentioned before in the previously written vertices. As another example with the following defined adjacency function, the illustrated hypergraph (Fig. 14(a)) will be transformed to the graph (Fig. 14(b)).

$$A_f = \{1:(2,4); 2:(3); 3:(5); 4:(5)\}$$

Considering the aforementioned expressions for a hypergraph, one can define a variable adjacency function that varies in time or any other chosen parameter leading to different graphs in different times. For example, in Fig. 15 a variable adjacency function is assigned to the hypergraph H and the corresponding graphs are concluded as:

$$A_f = \begin{Bmatrix} 1:(2,3);\ 2:(3);\ 3:(5);\ 4:(5) & t = t_1 \\ 1:(2);\ 2:(3);\ 3:(4);\ 4:(5) & t = t_2 \\ 1:(3);\ 2:(3,4);\ 3:(4,5) & t = t_3 \\ 1:(2,5);\ 2:(3,4);\ 3:(4);\ 4:(5) & t = t_4 \end{Bmatrix}$$

With variable adjacency functions associated with hypergraphs, the product of hypergraphs lead to variable product graphs where their geometry and canonical forms for their related matrices are varied in time or any other chosen parameter. As discussed before we call these dynamic graph products. For clarification, the following examples can be considered.

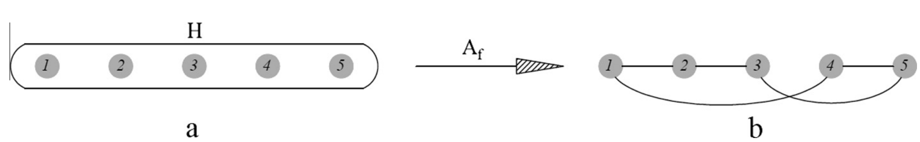

Fig. 14. A hypergraph and its corresponding graph under adjacency function A_f.

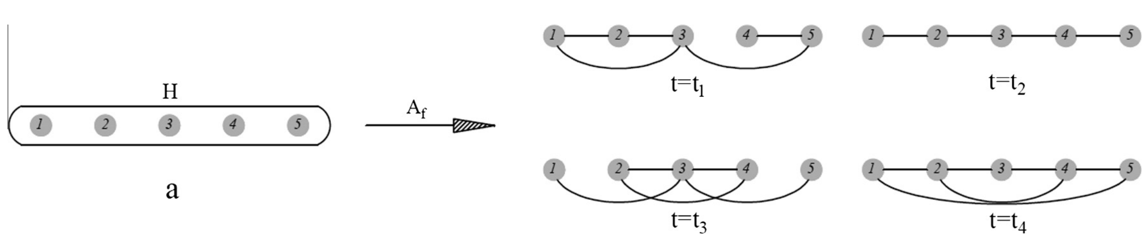

Fig. 15. Hypergraph H and its corresponding graphs in the times t_1 up to t_4.

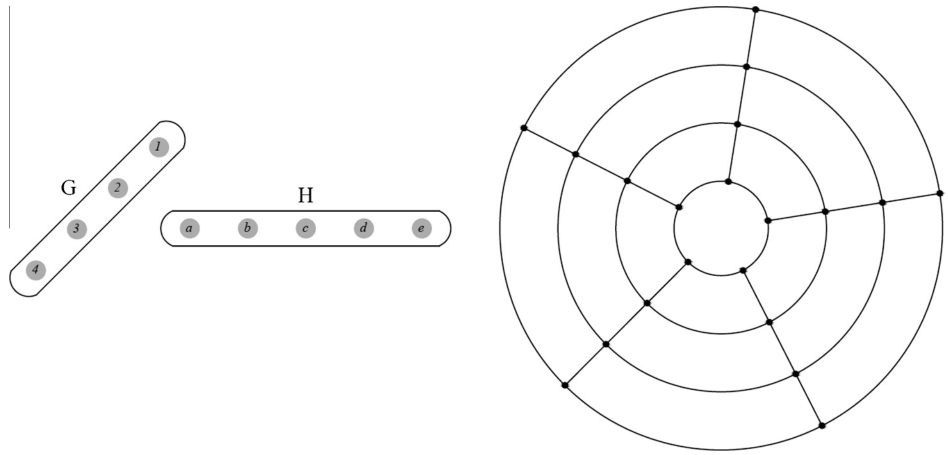

Fig. 16. Cartesian product of two hypergraphs G and H with adjacency functions A_fG and A_fH.

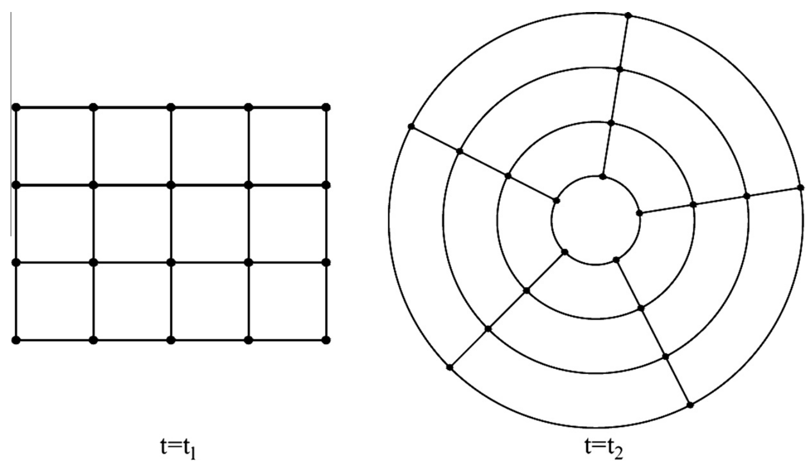

Fig. 17. Cartesian product of two hypergraphs G and H with variable adjacency functions A_fG and A_fH.

Example 1. The Cartesian product of two hypergraphs G and H with adjacency functions A_fG and A_fH are shown in Fig. 16 (as mentioned before hypergraphs G and H can be expressed as $H.P_4$ and $H.P_5$)

$$A_fG = \{1 : (2);\ 2 : (3);\ 3 : (4)\}$$
$$A_fH = \{a : (b, e);\ b : (c);\ c : (d);\ d : (e)\}$$

Now if A_fH is variable in time, as shown in the following definition, we will have the configuration as shown in Fig. 17.

$$A_fH = \left\{ \begin{array}{ll} a : (b);\ b : (c);\ c : (d);\ d : (e) & t = t_1 \\ a : (b, e);\ b : (c);\ c : (d);\ d : (e) & t = t_2 \end{array} \right\}$$

Example 2. In the mass-spring structural system shown in Fig. 18, the system has 20° of freedom where the degrees (3, 5), (8, 10), (13, 15) and (18, 20) are controlled with extra conditional springs. These conditional springs K_c get activated when the amount of the drift for the aforementioned degrees of freedom passes the predefined values. Unrestrained structural model of this system can be constructed using hypergraph product of $H.P_5$ and $H.P_4$ with adjacency functions as A_fH and A_fG.

$$A_fH = \left\{1^{K_1} : (2);\ 2^{K_2} : (3);\ 3^{K_3} : (4);\ 3^{K_c} : (5);\ 4^{K_4} : (5)\ \ \text{if } (u_5 - u_3) \succ \delta K_c \text{ will be work.}\right\}$$
$$A_fG = \left\{1^k : (2);\ 2^k : (3);\ 3^k : (4)\right\}$$

In the above adjacency functions, the parameters above the colons express the weight of the edges between related vertices and δ is the predefined extreme value for the uncontrolled drift between controlled degrees of freedom.

The structural models corresponding to the sub-hypergraph H with variable adjacency function A_fH are illustrated in Figs. 19 and 20.

The structural model constructed by Cartesian hypergraph product of two hypergraphs H and G with adjacency functions A_fH and A_fG in all the possible conditions has $2^4 = 16$ different states. These states are overally covered in the mentioned product hypergraph. As an example, when $(u_5 - u_3) \succ \delta$ and $(u_{10} - u_8) \succ \delta$ and $(u_{15} - u_{13}) \prec \delta$ and $(u_{20} - u_{18}) \prec \delta$, then the model will be as depicted in Fig. 21.

In relation with the product graphs we have graph products with specified domains that was introduced by Kaveh and Alinejad [18], where the adjacency relationship in subgraphs are defined with specified domains. Two examples of such products are as follows:

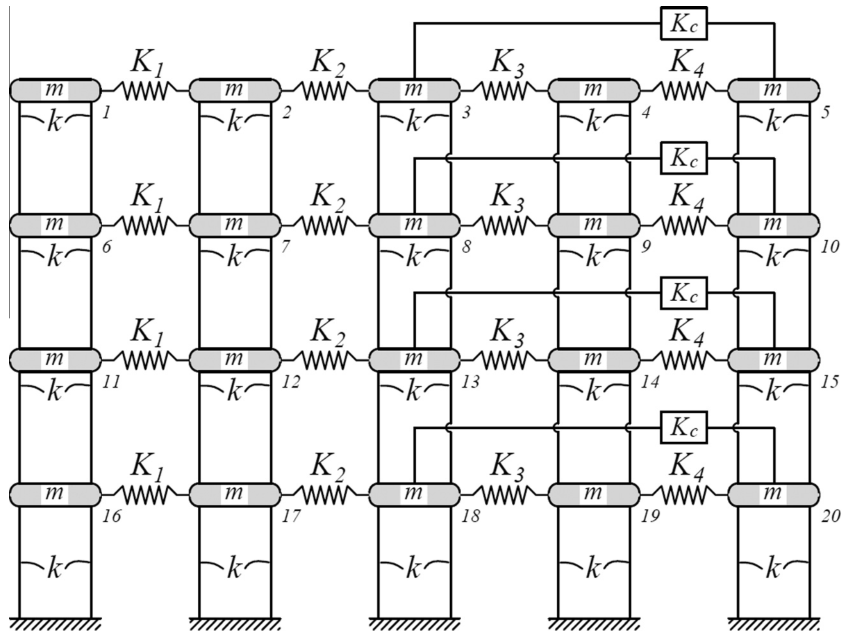

Fig. 18. A mass-spring structural system with 20° of freedom.

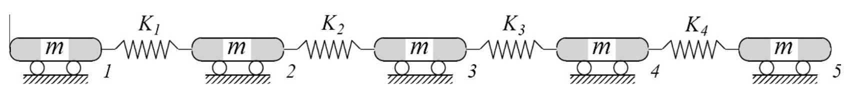

Fig. 19. Structural model with respect to sub-hypergraph $H.P_5$ when $(u_5 - u_3) \prec \delta$.

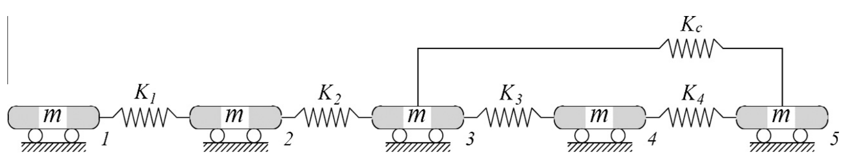

Fig. 20. Structural model with respect to sub-hypergraph $H.P_5$ when $(u_5 - u_3) \succ \delta$.

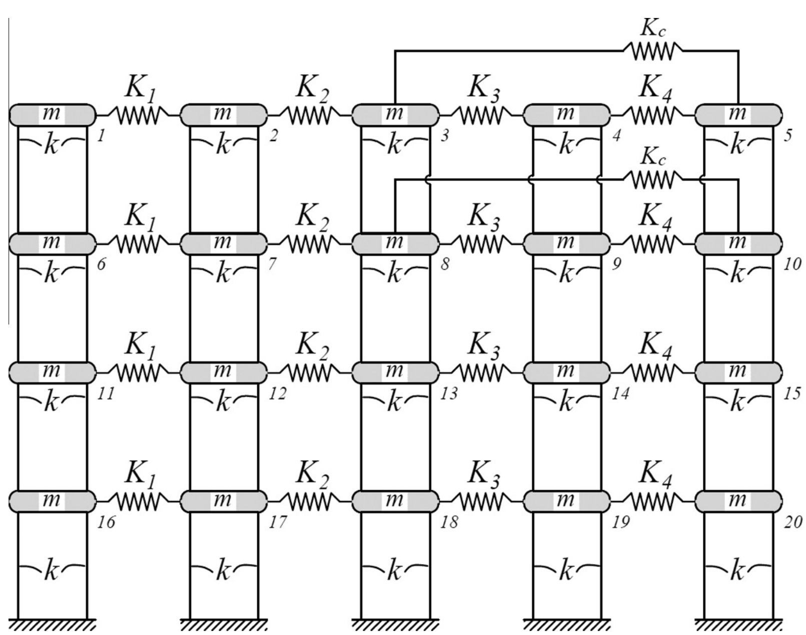

Fig. 21. The structural system when $(u_5 - u_3) \succ \delta$ and $(u_{10} - u_8) \succ \delta$ and $(u_{15} - u_{13}) \prec \delta$ and $(u_{20} - u_{18}) \prec \delta$ which is restrained model of hypergraph product of $H.P_5$ and $H.P_4$.

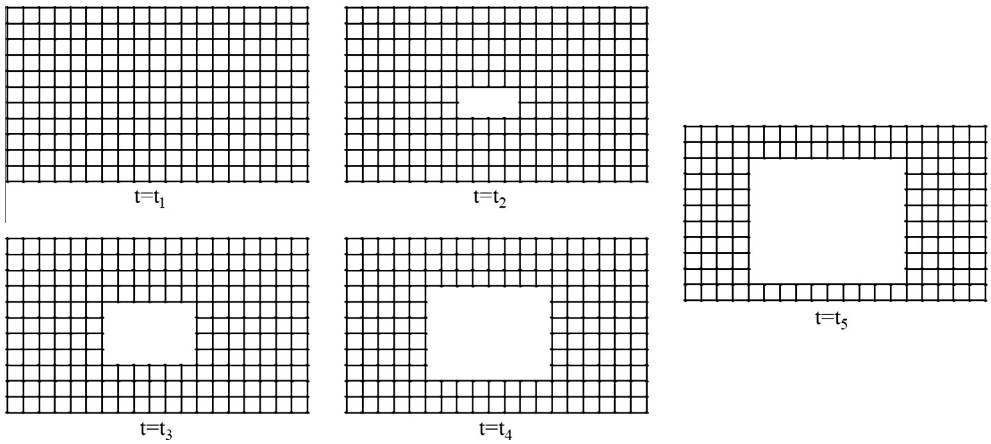

Fig. 22. Cartesian product of two hypergraphs $H.P_{12}$ and $H.P_{20}$ with varying adjacency functions A_fG and A_fH.

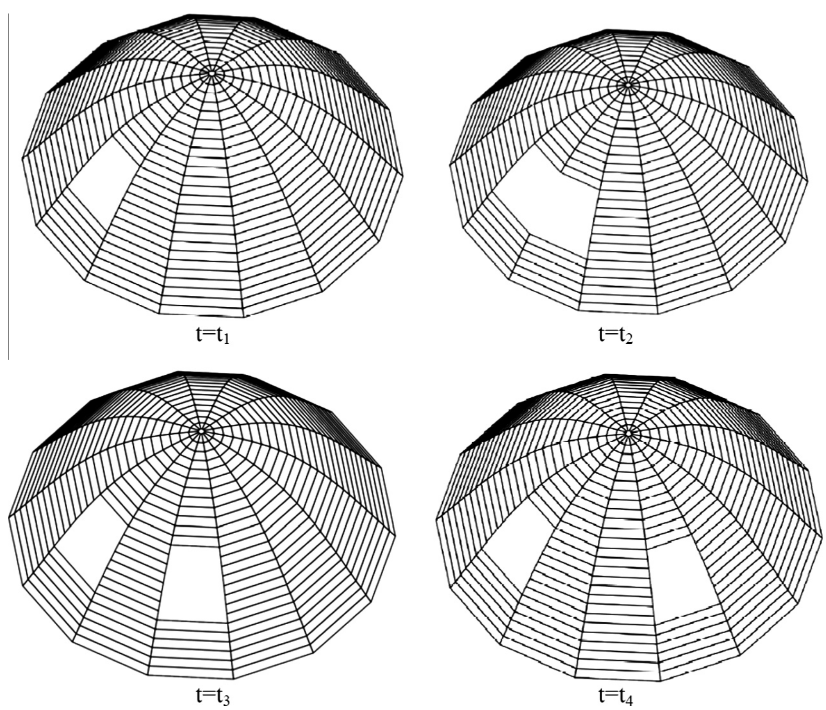

Fig. 23. Cartesian product of two hypergraphs $H.P_{27}$ and $H.P_{14}$ with adjacency functions A_fG and A_fH varying with time.

Example 3. The Cartesian product of hypergraphs G and H are equivalent to $H.P_{12}$ and $H.P_{20}$ respectively, when the adjacency functions A_fG and A_fH are as following, then it has a configuration as shown in Fig. 22.

$$A_fG = \left\{ \begin{array}{ll} 1:(2);\ 2:(3);\ldots;11:t(12) & t=t_1 \\ 1:(2);2:(3);\ldots;11:(12) \ni D_i = \{1:[6-i],[6+i]:12\}, i=1:4 & t=t_{i+1} \end{array} \right\}$$

$$A_fH = \left\{ \begin{array}{ll} 1:(2);\ 2:(3);\ldots;19:(20) & t=t_1 \\ 1:(2);2:(3);\ldots;19:(20) \ni D'_i = \{1:[9-i],[11+i]:20\}, i=1:4 & t=t_{i+1} \end{array} \right\}$$

Example 4. The Cartesian product of hypergraphs G and H are equivalent to $H.P_{27}$ and $H.P_{14}$ respectively, with adjacency functions being as A_fG and A_fH, Fig. 23.

$$A_fG = \{1:(2);\ 2:(3);\ldots;26:(27) \ni D_i = \{1:7,\ 15:27\}, i=1:4\ \ t=t_i\}$$
$$A_fH = \{1:(2,14);\ 2:(3);\ldots;13:(14) \ni D'_i = \{2:i,\ i+1:14\},\ i=1:4\ \ t=t_i\}$$

For further information on defined specified domains in adjacency functions one may refer to Ref. [18].

Directed graph products are another products on graphs that are introduced by Kaveh and Koohestani [19]. One can extract these products from hypergraph products with a proper definition of adjacency functions on sub-hypergraphs. An example of such a product is as follow:

Example 5. The type I directed product of hypergraphs G and H are equivalent to $H.P_3$ and $H.P_3$ respectively, with adjacency functions being A_fG and A_fH, Fig. 24.

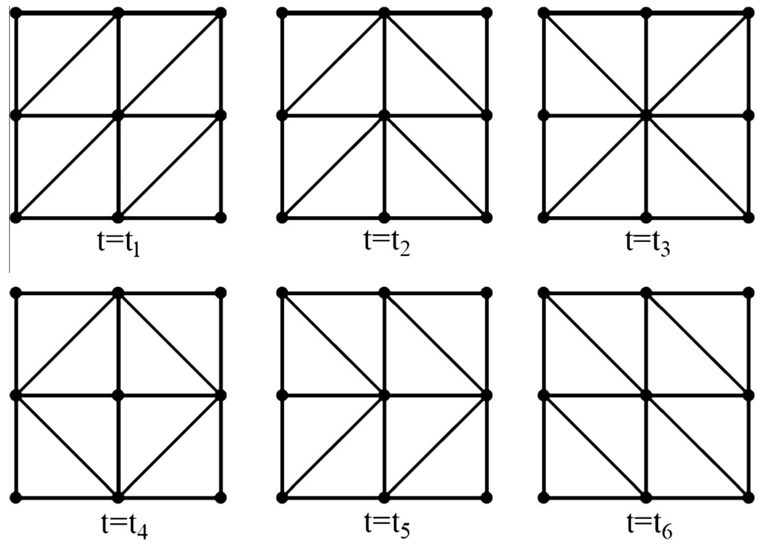

Fig. 24. Type I directed product of two hypergraphs $H.P_3$ and $H.P_3$ with variable adjacency functions *AGf* and *AHf* varying with time.

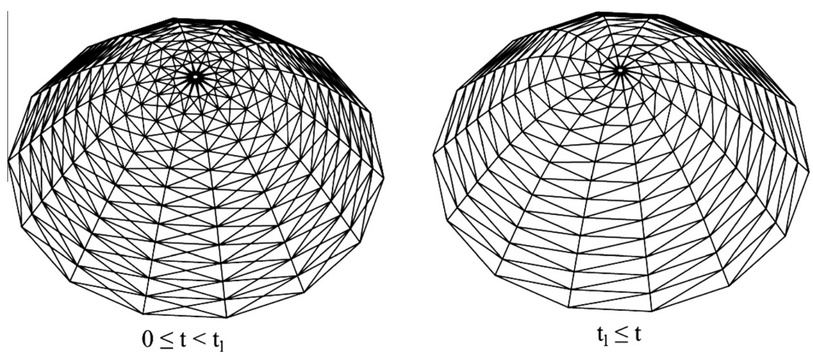

Fig. 25. The model of a dome constructed by type I directed product of two hypergraphs $G_{H.P_{14}}$ and $H_{H.C_{14}}$ with variable adjacency functions A_fG and A_fH varying with time.

$$A_fG = \left\{\begin{array}{ll} 1:(2);\ 2:(3)\ni D_{(1:2^+)} & t=t_1 \\ 1:(2);\ 2:(3)\ni D_{(1^+:2^-)} & t=t_2 \\ 1:(2);\ 2:(3)\ni D_{(1^+:2^-)} & t=t_3 \\ 1:(2);\ 2:(3)\ni D_{(1^-:2^+)} & t=t_4 \\ 1:(2);\ 2:(3)\ni D_{(1:2^+)} & t=t_5 \\ 1:(2);\ 2:(3)\ni D_{(1:2^-)} & t=t_6 \end{array}\right\},$$

$$A_fH = \left\{\begin{array}{ll} 1:(2);\ 2:(3)\ni D_{(1:2^+)} & t=t_1 \\ 1:(2);\ 2:(3)\ni D_{(1:2^+)} & t=t_2 \\ 1:(2);\ 2:(3)\ni D_{(1^+:2^-)} & t=t_3 \\ 1:(2);\ 2:(3)\ni D_{(1^+:2^-)} & t=t_4 \\ 1:(2);\ 2:(3)\ni D_{(1^+:2^-)} & t=t_5 \\ 1:(2);\ 2:(3)\ni D_{(1:2^+)} & t=t_6 \end{array}\right\}$$

Example 6. In Fig. 25 different models of a dome are illustrated in two different time domains. The varied model of the dome is constructed by type I directed product of hypergraphs $G_{H.P_{14}}$ and $H_{H.C_{14}}$ that their adjacency functions are as follows:

$$A_fG = \left\{\begin{array}{ll} 1:(2);\ 2:(3);\ldots;13:(14) & 0\le t\prec t_1 \\ 1:(2);\ 2:(3);\ldots;13:(14)\ni D_{(1:13^+)} & t_1\le t \end{array}\right\}$$

$$A_fH = \left\{\begin{array}{ll} 1:(2,14);\ 2:(3);\ldots;\ 13:(14) & 0\le t\prec t_1 \\ 1:(2,14);\ 2:(3);\ldots;\ 13:(14)\ni D_{(1:14^+)} & t_1\le t \end{array}\right\}$$

For further information on directed graph products the reader may refer to Ref. [19].

As a result of the aforementioned discussions, it is found that many different graph products can be constructed by means of hypergraph products that not only are useful for configuration processing but also can be applied to their related matrices which are in canonical forms. The latter can used in efficient analysis of complex regular systems, especially where dynamicity is present. In relation with the adjacency and Laplacian matrices of dynamic graph products it is clear that the aforementioned theorems are applicable to the matrices which are variable in time or any other chosen parameter.

8. Conclusions

Different hypergraph products consisting of Cartesian, strong Cartesian and direct hypergraph products are investigated and the relevant applications in configuration processing and the formation of the adjacency and Laplacian matrices are studied.

Assigning adjacency function to a hypergraph leads to distinct definition for adjacency and Laplacian matrices of that hypergraph. Definition of variable adjacency functions resulted in dynamic graphs and it is shown how a hypergraph product can generate different graph products variable in time or any other chosen parameter so-called dynamic graph products. From an algebraic point of view the applicability of available theorems on adjacency and Laplacian matrices of product graphs for hypergraph products are verified. Finally the dynamicity attribute of hypergraph products is investigated for modeling regular dynamic structural systems through some examples.

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