Applied Estimation Laboration 2

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PART I - Preparatory Questions

Particle Filter:

1. *What are the particles of the particle filter?*
Particles are meant to describe a probability density by taking random samples from the distribution.

2. *What are importance weights, target distribution, and proposal distribution and what is the relation between them?*

- **Importance weights:** A weight describing the probability for a sample to occur.
- **Target distribution:** A probability density function for the wanted/target distribution.
- **Proposal distribution** A probability density function for the sampled particles.

The proposal distribution is weighted by the probability to be sampled by the importance weight to converge to the target distribution.

3. *What is the cause of particle deprivation and what is the danger?*
Often caused by too small amounts of particles and can make the particle filter algorithm to fail by having no particles near the true state.

4. *Why do we resample instead of simply maintaining a weight for each particle?*
If we don't resample we would end up with a lot of particles with very low weights and some with very high weights and the risk of having a majority of particles in the low posterior probability would increase.

5. *Give some examples of situations in which the average of the particle set is not a good representation of the particle set.*

If we have multiple peaks in the distribution of particles the average would give a misleading indication of the density for the particles. Alternative, if we have a donut-shaped distribution we would encounter the same problem.

6. *How can we make inferences about states that lie between particles?*
We could extract information about states between particles by, for example, doing either a histogram for the number of particles or fit a gaussian kernel around each particle.

7. *How can sample variance cause problems and what are two remedies?*
The sampled variance is different from the original variance, to minimize this problem one could lower the resample frequency or use the strategy so called 'low variance sampling'.

8. *For robot localization and a given quality of posterior approximation, how are the pose uncertainty (spread of the true posterior) and number of particles we chose to use related?*
If the pose uncertainty increases we would need to increase the number of particles if we don't want to lose the quality of our estimation of the pose.

PART II - MATLAB Exercises

Warm-up Problem: Particle Filter

Question 1:

(7) compared to (5) is a more complex state-space model where we consider the previous state to control the steering angle. In (7) we can control the angle depending on the error and can, therefore, be used to control more complex systems. The drawback would be the complexity of calculating and modeling the system would increase in (7) compared to (5), due to (5) only considers a fixed initial steering angle.

Question 2:

One should need to know in advance what the initial velocity and angular velocity is. Besides that, (8) should be able to model any circular motion that is constrained by these initial values.

Question 3:

$2 * \pi |\Sigma|^{\frac{1}{2}}$ is meant to normalize (10) with respect to the modelled noise.

Question 4:

For Mulitinomial re-sampling method one would need the same amount of random numbers as the number of particles, in the Systematic re-sampling method only one random number is needed.

Question 5:

The probability of not surviving in the Multinomial re-sampling can be written as $(1-w)^M$, where w is the weight of the particle and M the number of particles. So the probability for a particle to survive could be written as $1-(1-w)^M$, where w can be either $\frac{1}{M}+\epsilon$ or between the interval $[0, \frac{1}{M})$, i.e. case 1 and case 2. For Systematic re-sampling the probability is 1 if $w \geq \frac{1}{M}$ as it is for case 1 , when $w = \frac{1}{M} + \epsilon$ if $\epsilon \geq 0$. If $w \leq \frac{1}{M}$, as in case 2, the probability will be equal to wM.

Question 6:

params.Simga_Q models the measurement noise and **params.Simga_R** models the process noise.

Question 7:

When skipping the diffusion step, i.e. setting process noise to 0, there is some major particle deprivation i.e. that all particles except one disappear and the last one makes a large circle around the true state.

Question 8:

The particles have an even distribution over the whole space and does not converge to the true state.

Question 9:

When using STD equal to 1000 the particles are circling the true state with a large circle. When STD is set to 0.001 the particles have a hard time to find and identify the true state and to converge. When setting STD to for example 1, the particles converge to one particle that is equal to the true state if at least one of the particles comes close to the true state. Otherwise, they are evenly distributed over the space the entire run.

Question 10:

When using STD equal to 1000 the particles are uniformly around the true state but with a larger circle than for default values, when decreasing the STD to 1 the circle gets smaller. If we decrease the STD to 0.001 the particles begins with a small initial spread and make a big circle around the true state with an increasing spread as the program runs.

Question 11:

If we, for example, apply a linear model to a circular motion instead of a circle model the noise should be increased to compensate for the lack of complexity in the model. E.g. if we choose the wrong motion model the noise should be expected to increase to compensate for the errors that will occur.

Question 12:

The precision should decrease if we use the wrong motion model compared to the motion that is being executed. Further, one could compensate for the lack of precision by increasing the number of particles.

Question 13:

The likelihood of the observation depends on the distance from the filter prediction. If the likelihood is over a certain threshold we keep the observation otherwise, we classify the observation as an outlier. Similar to (13) in the lab description.

Question 14:

Model	Measurements	Estimates	Measured noise factor	Process noise factor
Fixed	$24.8 + -12.8$	$11.3 + -5.5$	10	35
Linear	$24.8 + -12.8$	$7.7 + -3.4$	5	0.9
Circular	$24.8 + -12.8$	$7.1 + -3.4$	4	0.8

When using a circular model for a circular motion the modeled noise could be reduced significantly from the fixed model, as could be expected. Further, the same was achieved between the linear model and the fixed. One could also notice that the circular model did have the best precision, as could also be expected. With that said, one could conclude that the models got less sensitive to noise when applying a more suitable model.

Main problem: Monte Carlo Localization

Question 15:

If the measured noise Q goes to zero the weights will be infinity large and all of the observations will be accepted as true observations. Further, this means that no outliers will occur due to the optimistic belief in our measurements. So, Q affects the outlier detection and so does the threshold λ_Ψ.

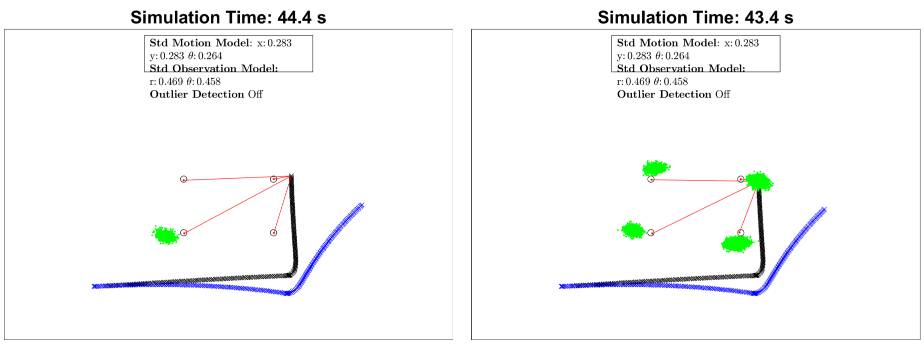
Question 16:

The weight of the particle will be increased if we fail to detect the outlier, i.e we will trust a particle more than we should due to the contribution from the outlier observation.

Data Sets

Data Set 4:

With 1000 particles the filter does not keep track of the four hypothesis, this is due to the small amounts of particles used (see Figure 1a). When increasing the number of particles to 10000 the filter does keep track of all the four hypotheses, see Figure 1b. Multinomial re-sampling had the same result. Both of the models needed an increased measured noise to preserve the hypothesis, i.e. sensor noise was increased with a factor of two.



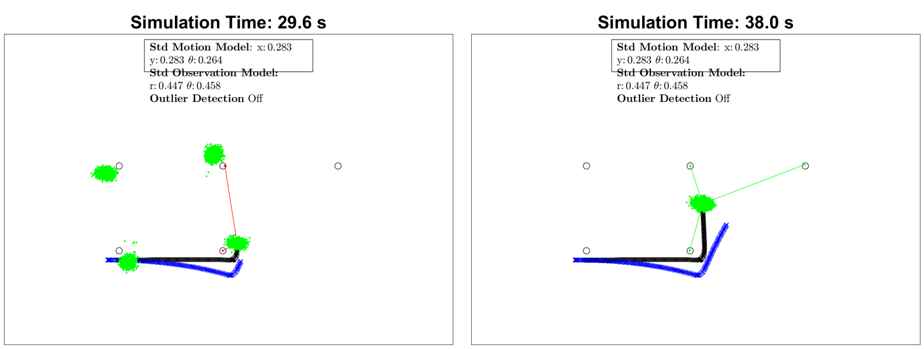
(a) Only one hypothesis.

(b) Four hypothesis simultaneously.

Figure 1: Data Set 4, with systematic re-sampling.

Data set 5:

When increasing the sensor noise with a factor of two and the number of particles to 10000 the number of hypotheses was four (see Figure 2a) and then converged to the right hypothesis as the symmetry got broken, see Figure 2b.



(a) Before converging.

(b) After converging.

Figure 2: Data Set 5, filter converges to the right hypothesis.