EE – Automatic Control

Control Theory and Practice
Advanced Course

Computer Exercise: CLASSICAL LOOP-SHAPING

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1 Introduction

Loop-shaping is a classical procedure for designing feedback controllers in the frequency domain. In the basic course it was called *lead lag design*. Loop-shaping was introduced in the 1930s and was used to construct single variable circuits, such as feedback amplifiers (Bode). The approach is generally applicable to linear single-input-single-output (SISO) feedback systems and has also been extended to multivariable systems, i.e., systems with multiple input and output signals (MIMO).

The basic idea in loop-shaping is to shape the *open-loop* gain with a controller in order to achieve intended properties of the *closed-loop* system under feedback. In the 70s and 80s advanced methods for loop-shaping based on optimization were developed. However, in this computer exercise we will focus on classical ”manual” loop-shaping.

We will here only consider SISO systems, but the ideas are also applicable to MIMO systems as we will see later in the course.

Preparations: Chapters 7.1-7.4 in the course book (Ljung, Glad, “Control theory”). It is also recommended to repeat Chapter 5.5 in the basic course book (Glad, Ljung, “Reglerteknik-Grundläggande teori”).

Presentation:

All problems in this exercise should be solved, but only the tasks on the report form should be handed in. The report form and the date when it should be handed in can be found on the course website. The exercise should be performed in pairs of students.

2 Background

Consider the control system in Figure 1.

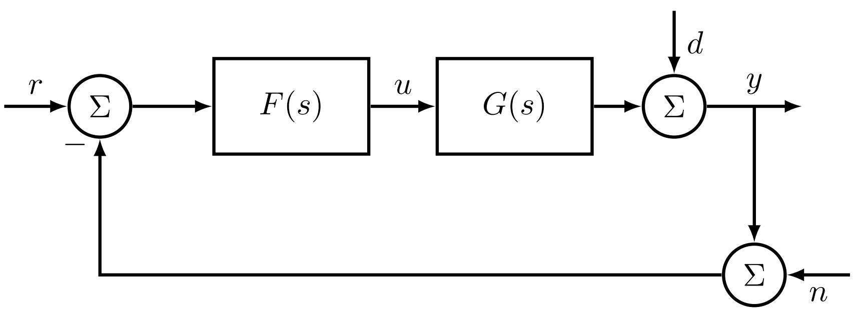

Figure 1: F–controller, G–system, r–reference signal, u–control signal, d–disturbance signal, y–output signal, n–measurement noise.

The aim of the control system is to make the output y follow the reference r despite disturbances d and measurement noise n. In other words, we want to make the control error $e = r - y$ small in the presence of the signals r, d and n. From the loop, we derive the closed-loop relations

$$e = r - y = \frac{1}{1+FG}r - \frac{1}{1+FG}d + \frac{FG}{1+FG}n$$

Introduce the loop-gain $L = FG$, the sensitivity function $S = \frac{1}{1+L}$ and the complimentary sensitivity function $T = \frac{L}{1+L}$. Then

$$e = Sr - Sd + Tn$$

Note that $S + T = 1$. Loop-shaping implies using the controller F to shape the loop-gain $L = FG$ such that S and T have small magnitudes, at least for frequencies where r, d, n are large. Since we wish to have a small control error, we somewhat crudely obtain the following conditions

$$e \approx 0 \Rightarrow \begin{cases} i) \quad S \approx 0 \;\Rightarrow\; T \approx 1 \;\Rightarrow\; |L| \text{ large} \\ ii) \quad T \approx 0 \;\Rightarrow\; S \approx 1 \;\Rightarrow\; |L| \text{ small} \end{cases}$$

We obviously have contradictive conditions! Case $i)$ corresponds to reference tracking and disturbance attenuation (keeping e small in the presence of r and d) while case $ii)$ corresponds to noise attenuation (keeping e small in the presence of n). This is a classical and fundamental trade-off that must be made in any control system; we can not attenuate disturbances d and measurement noise n at the same frequency since $S + T = 1$ no matter what controller we use. As for tracking references r, the trade-off can be resolved by introducing a two-degrees-of-freedom controller (see section 4.2 in this exercise or e.g., lecture notes for lecture 1).

Fortunately, disturbances and measurement noise often have different characteristics with disturbances typically being mostly low-frequent and noise typically being more high-frequent. In this case we should design F so that $|L|$ is large at low frequencies and small at higher frequencies. Similarly, to track low frequency reference signals we have to design the loop gain to be large at low frequencies.

Apart from keeping the control error e small, the control signal u should usually not be too large or vary too much. Since

$$u = F(r - y - n)$$

this condition implies that the control gain must not be designed too large, i.e., F small $\Rightarrow \; L = GF$ small.

Stability is also an important issue in control design. Recall the Bode stability criterion which states that, provided the open-loop $L(s)$ is stable, the loop-gain must have amplitude less than 1 at the frequency where the phase-lag is $-\pi$[1] in order to have closed-loop stability. Due to the *Bode relation*, it is known that the slope of $|L(i\omega)|$ is coupled to the phase $\arg\{L(i\omega)\}$. For example, $L = a/s^n$ has slope $-n$ and phase $-n\pi/2$. In order to keep a reasonable stability margin, $|L|$ must not have too large slope around the cross-over frequency ω_c (recall that $|L(i\omega_c)| = 1$ by definition). Typically,

[1]assuming this frequency is unique

$|L|$ is designed to have slope ≈ -1 at ω_c which will then give a phase-margin ϕ_M around $\pi/2$.

Also note that the phase margin is coupled to control performance. For example we have for the resonance peaks $M_S = \max_\omega |S|$ and $M_T = \max_\omega |T|$

$$M_T > \frac{1}{\phi_M} \ ; \quad M_S > \frac{1}{\phi_M}$$

where the phase margin ϕ_M is given in radians. For example, if we demand that the resonance peaks should be smaller than 2, then the phase margin has to be larger than $30°$.

Such contradictive constraints give rise to different strategies to shaping the loop L so that performance demands are met. They also provide limits of achievable control performance.

3 Introduction to Control System Toolbox

In this computer exercise we will use MATLAB to shape the loop, just as we did in the basic course. Most of the functions are in the Control System Toolbox. Let us start by defining some useful function. Recall that you get access to the MATLAB help by typing `help "function name"`.

A transfer function

$$G(s) = \frac{s+2}{s^2+2s+3}$$

is defined in MATLAB by typing

```
s=tf('s');    G=(s+2)/(s^2+2s+3)
```

The product of two transfer functions is obtained by

```
G12 = G1 * G2
```

For a system with 2 inputs and 2 outputs, the closed-loop transfer matrix is obtained with

```
S=feedback(eye(2),G*F);    T=feedback(G*F,eye(2))
```

For a SISO system this can be written

```
S=1/(1+G*F);    T=G*F/(1+G*F)
```

For numerical reasons it is **very important** to use the function `minreal`, for example `minreal(T)`. This creates an equivalent system where all canceling pole/zero pair, corresponding to unobservable or uncontrollable states, are eliminated.

The bode diagram for `G` is plotted by typing

`bode(G)` or `bode(G,{wmin,wmax})`

Amplitude and phase at a given frequency are obtained by

```
[m,p]=bode(G,w)
```

Phase margin, amplitude margin and corresponding frequencies are obtained by

```
[Gm,Pm,wp,wc]=margin(G*F)
```

To simulate a step response in the control signal, use the function

`step(G)` or `step(G,tfinal)`

In the same way, to simulate a step response in the reference signal, we type

```
step(T)
```

4 Exercises

4.1 Basics

Consider a system which can be modeled by the transfer function

$$G(s) = \frac{3(-s+1)}{(5s+1)(10s+1)}.$$

Exercise 4.1.1. Use the procedure introduced in the basic course to construct a lead-lag controller which eliminates the static control error for a step response in the reference signal.

$$F(s) = K \underbrace{\frac{\tau_D s + 1}{\beta \tau_D s + 1}}_{\text{Lead}} \underbrace{\frac{\tau_I s + 1}{\tau_I s + \gamma}}_{\text{Lag}}$$

The phase margin should be 30° at the cross-over frequency $\omega_c = 0.4$ rad/s.

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Exercise 4.1.2. Determine the bandwidth of the closed-loop system and the resonance peak M_T. Also, determine the rise time and the overshoot for step changes in the reference when the controller designed in 4.1.1. is used.

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Exercise 4.1.3. Modify the controller in 4.1.1. such that the phase margin increases to 50° while the cross-over frequency is unchanged. For the corresponding closed-loop system, determine the bandwidth and resonance peak. Also, determine the rise time and the overshoot of the step response.

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4.2 Disturbance attenuation

Now we will construct a controller which both tracks the reference signal and attenuates disturbances. The block diagram of the control system is given in Figure 2. We assume that the signals have been scaled such that $|d| < 1$, $|u| < 1$ and $|e| < 1$ where $e = r - y$.

The exercise is about designing F_r and F_y in Figure 2 such that:

- The rise time for a step change in the reference signal less than 0.2 s and the overshoot is less than 10%.
- For a step in the disturbance, $|y(t)| \leq 1$ $\forall t$ and $|y(t)| \leq 0.1$ for $t > 0.5$ s.
- Since the signals are scaled, the control signal obeys $|u(t)| \leq 1$ $\forall t$.

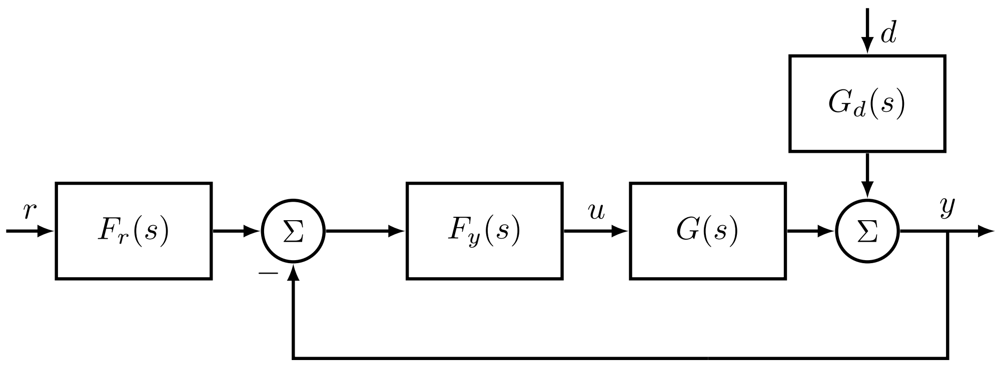

Figure 2: F_r–prefilter, F_y–feedback controller, G–system, G_d–disturbance dynamics, r–reference signal, u–control signal, d–disturbance signal, y–measurement signal.

The transfer functions have been estimated to

$$G(s) = \frac{20}{(s+1)((\frac{s}{20})^2 + \frac{s}{20} + 1)}$$

$$G_d(s) = \frac{10}{s+1}$$

Exercise 4.2.1. For which frequencies is control action needed? Control is needed at least at frequencies where $|G_d(j\omega)| > 1$ in order for disturbances to be attenuated. Therefore the cross-over frequency must be large enough. First, try to design F_y such that $L(s) \approx \omega_c/s$ and plot the closed-loop transfer function from d to y and the corresponding step response. (A simple way to find $L = \omega_c/s$ is to let $F_y = G^{-1}\omega_c/s$. However, this controller is not proper. A procedure to fix this is to "add" a number of poles in the controller such that it becomes proper. How should these poles be chosen?)

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A loop gain of slope -1 at all frequencies gives in our case poor disturbance attenuation. To understand the reason for this, note that the output is given by

$$y = SG_d d = (1 + L)^{-1} G_d d.$$

Provided the signals have been scaled we want $|(1 + L)^{-1} G_d| < 1$ for all ω. For frequencies where $|G_d| > 1$ this approximately implies $|L| > |G_d|$ or $|F_y| > |G^{-1} G_d|$. Most often we also want integral action and as a starting point we can then choose

$$F_y = \frac{s + \omega_I}{s} G^{-1} G_d, \tag{1}$$

where ω_I determines the frequency range of efficient integral action. We see that if $G_d \approx 1$, the controller should contain the inverse of the system. On the other hand if $G_d \neq 1$ the controller should be designed in some other fashion. Especially, we observe that if the disturbance is on the input side to the system we have $G_d = G$ and then F_y should be chosen as a PI controller according to (1).

Note that the controller (1) cannot be used if it is not proper, causal and stable. To ensure these properties, approximations of (1) may be necessary.

Exercise 4.2.2. Let us now reconstruct F_y according to the instructions above. We will start with the disturbance attenuation. In a second step, adjustments can be made on F_r to obtain the desired reference tracking properties. Start by choosing F_y according to (1). Try different approximations of the product $G^{-1}G_d$ in the controller, and choose ω_I large enough so that step disturbances are attenuated according to the specifications.

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Exercise 4.2.3. To fulfill the reference tracking specifications, we can combine lead lag control and prefiltering of the reference signal. First, try to add lead action to F_y

to reduce the overshoot. Then it can be necessary to add prefilter action to fulfill all specifications. Note that F_r should be as simple as possible (why?). Also, remember to check the size of the control signal ($u = F_y F_r S r - F_y G_d S d$)! Typically a low pass filter is chosen, for example

$$F_r = \frac{1}{1 + \tau s}.$$

Hint: Consider the signals r and d independently.

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Exercise 4.2.4. Finally, check that all specifications are fulfilled. Plot the bode diagrams of the sensitivity and complementary sensitivity functions.

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