Emory University

MATH 212 Differential Equations Learning Notes

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Contents

1 First Order Ordinary Differential Equations

1.1 Introduction

Definition 1.1.1 (Ordinary Differential Equations/ODEs). An *ordinary differential equation* is an equation that contains one or more derivatives of an unknown function $y = y(x)$.
Definition 1.1.2 (Order of ODEs). The *order* of an ODE is the maximum order of the derivatives appearing in the equation.
Definition 1.1.3 (Solution to ODEs). The *solution* to an ODE is a function y that satisfies the equation.

Example 1.1.4 Solve $y'' = 3x + 1$.

Solution 1.

$$y' = \int 3x + 1 \, \mathrm{d}x = \frac{3}{2}x^2 + x + C$$
$$y = \int y' \, \mathrm{d}x = \int \frac{3}{2}x^2 + x + C \, \mathrm{d}x = \frac{1}{2}x^3 + \frac{1}{x}x^2 + Cx + D.$$

□

Definition 1.1.5 (Linear ODEs/Non-Linear ODEs). A first order ODE is *linear* if it can be written as

$$y' + p(x)y = f(x).$$

Otherwise, it is *non-linear*.
Definition 1.1.6 (Homogenous/Non-Homogenous Linear ODEs). If $f(x) = 0$, then the linear ODE is *homogenous*. That is,

$$y' + p(x)y = 0.$$

Otherwise, it is *non-homogenous*.
Definition 1.1.7 (Trivial/Non-Trivial Solution). $y = 0$ is a *trivial solution* to a homogenous ODE. Any other solutions are *non-trivial*.
Definition 1.1.8 (One-Parameter Family of Solutions). We call C a *parameter* and the equation, therefore solution, defines a *one-parameter family* of solutions.

Example 1.1.9 For the ODE $y' = 1$, $y_1 = x + C_1$ is a solution to it, and it is a one-parameter family of solutions. Similarly, for $y' = \frac{1}{x^2}$, the one-parameter families of solutions are defined by $y_2 = -\frac{1}{x} + C_2$ on the interval $(-\infty, 0) \cup (0, \infty)$.

2 Second Order ODEs

3 System of ODEs