

# Introduction to Graph Theory and Graph Coloring with Probabilistic Methods

Jiuru Lyu

December 6, 2023

## Contents

<b>1</b>	<b>Fundamental Concepts</b>	<b>2</b>
1.1	What is a Graph . . . . .	2
1.2	Paths, Cycles, and Trails . . . . .	2
1.3	Vertex Degree and Counting . . . . .	2
<b>2</b>	<b>Matching and Factors</b>	<b>3</b>
2.1	Matching and Covers . . . . .	3
<b>3</b>	<b>Probabilistic Preliminaries</b>	<b>4</b>
3.1	Finite Probability Space . . . . .	4
3.2	Random Variables and Their Expectations . . . . .	4
3.3	The Method of Deferred Decisions . . . . .	4
<b>4</b>	<b>Basic Probabilistic Tools</b>	<b>5</b>
4.1	The First Moment Method . . . . .	5
4.2	The Lavász Local Lemma . . . . .	5
4.3	The Chernoff Bound . . . . .	5

# **1 Fundamental Concepts**

## **1.1 What is a Graph**

## **1.2 Paths, Cycles, and Trails**

## **1.3 Vertex Degree and Counting**

## **2 Matching and Factors**

### **2.1 Matching and Covers**

## 3 Probabilistic Preliminaries

**Definition 3.1 (Probabilistic Method).** Proving the existence of an object with certain properties is to show that a random object chosen from an appropriate probability distribution has the desired properties with positive probability.

### 3.1 Finite Probability Space

### 3.2 Random Variables and Their Expectations

### 3.3 The Method of Deferred Decisions

## **4 Basic Probabilistic Tools**

### **4.1 The First Moment Method**

### **4.2 The Lavász Local Lemma**

### **4.3 The Chernoff Bound**