Image compresse by SVD

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Image compression by SVD

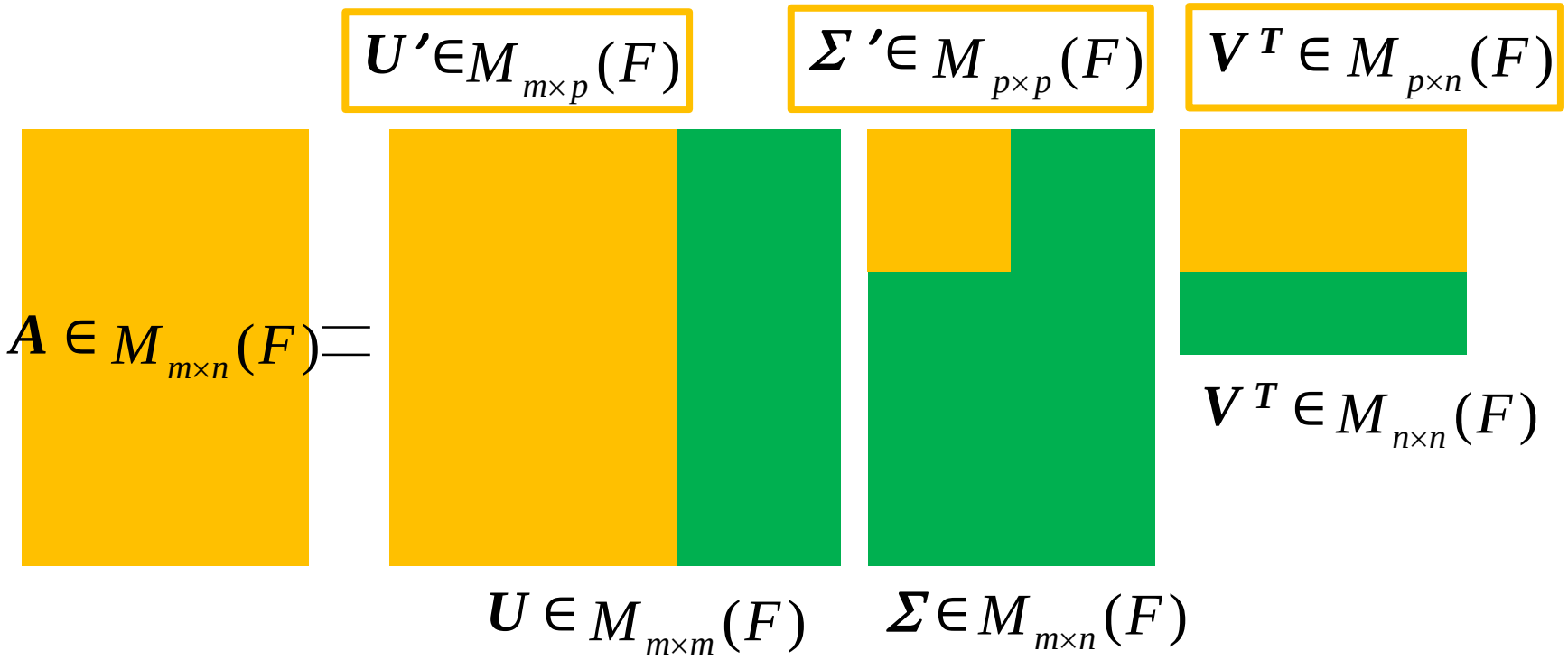
- The image in computer is stored as a matrix, for a colorful image, there is one more vector for RGB, so we just discuss the gray image.
- Load a binary image $\boldsymbol{A}$ is m by n and $m > n$, then decompose it by SVD, then get $\boldsymbol{U}$, $\boldsymbol{\Sigma}$, $\boldsymbol{V}^*$, the elements in $\boldsymbol{\Sigma}$ ordered $\sigma_1 > \sigma_2 > \ldots > \sigma_n$,
- $\boldsymbol{U}$ is m by m, $\boldsymbol{\Sigma}$ is m by n, $\boldsymbol{V}^*$ is n by n.

Image compression by SVD

- Choose $\boldsymbol{U'}$ is m by k, $\boldsymbol{\Sigma'}$ is k by k, $\boldsymbol{V'^*}$ is k by n. $k \leq n$, so the reconstructed A' is also m by n.
- Size of $\boldsymbol{A}$ is mn, size of $\boldsymbol{A'}$ is $mk+k+nk = k(1+m+n)$ (we just need the singular value), so the ratio of compression R is

$$R = \frac{k(1+m+n)}{mn}$$

Image compression by SVD



Only needs orange parts, it can reduce memory and time cost, but still costs much memory and time.

$\forall A \in M_{m\times n}(F)$, $\exists!$ U, S and V s.t. $U\Sigma V^T$

Image compression by SVD

- The original image is 960×540，so there are 540 singular values, choose $k \leq 540$ to compress.
- By the formula, k should be smaller than some number that we can compress the image, in this example, $k \leq 345$.

k	345	270	200	150	100
Ratio	0.998929	0.781771	0.57909	0.434317	0.289545

Original

http://www.tabletwallpapers.org/download/nature-lake-and-mountains-4k-hd-wallpaper_3840x2160.jpg

$k = 345$

$k = 270$

$k = 200$

$k = 150$

$k = 100$