Examination Statistics Prof. Dr. Andersson, Prof. Dr. Falkenberg Course of Study: Computer Sciences 14.2.2017

Name Matricula	ation Numbe	: r :										
I hereby confirm that I am physically fit to take the examination.												
Signature student												
	Problems	1	2	3	4	5	Sum	Mark				
	Max. scores	22	15	15	15	15	82					
	Obt. scores											

Authorized examinations aids:

Scientific calculator, one english-german dictionary, one file folder

Further instructions:

- 1. Do not use your own paper. Blank paper is attached to this question paper. Extra sheets will be provided on demand.
- 2. Please notice, not only the solution but the deriviation of the solution has to be given.
- 3. Switch off all electronic devices und remove them from the table.

Good Luck!

Dr. Andersson, Dr. Falkenberg

1. The following data gives the yearly income, the sizes of shoes and the gender for sample of persons in Germany.

size	income	gender
37	32000	W
36	33000	W
38	34000	W
40	34500	w
37	35500	W
37	37000	w
39	37500	w
38	39500	W
41	40500	m
38	42500	W
41	42500	m
39	43000	W
36	44000	W
43	44000	m
42	46000	m
44	46000	m
45	47000	m
41	48000	m
43	48000	m
42	53000	m

- (a) Calculate the mean, the standard deviation, the first quartil, the median, the third quartil of the variable income.
- (b) Draw a boxplot for the variable income.
- (c) Calculate the covariance und the coefficient of correlation of the variables size and income.
- (d) Calculate the parameters of the regression line income $= a + b \cdot \text{size}$. Give an interpretation of the values of a and b.
- (e) What do you think about the relationship between the variables size and income? Consider the coefficients of correlation between the variables size and income regarding the variable gender.

- 2. A man has a bunch of keys with 10 keys. One method of the man to find the right key of the front door of his appartement is method A: he chooses randomly one key out of the bunch of keys and tries it. If the key does not match to the lock, he chooses another key randomly out of the remaining keys. No key will be used two times. The other method B of the man is to choose randomly one key out of the bunch of keys, if the key does not match to the look he chooses again randomly a key out of the bunch of keys. It can happen that a key is used several times.
 - (a) Let X be the random variable which counts the number of trials to open the front door until he enters his appartment. Find the density function of X if the man uses method A resp. method B.
 - (b) If the man arrives at his appartment at the end of a working day he only makes at most 5 trials. If all trials fail he goes to the next pub. The man uses method B if he has already drunk some beers with his colleagues and otherwise method A. The probability that the man drinks some beers with his colleagues is 1/3.
 - i. Find the probability that man goes to the next pub.
 - ii. The bar keeper of the pub guesses that the man is already drunken when the man enters the pub so early. Find the probability that the bar keeper guesses right.

- 3. On a highway is a speed limit of 120 km per hour. The speeds of the cars are independently and normally distributed with mean 115 km per hour and standard deviation of 10 km per hour. A radar speed check is contolling the speed of the cars. A driver obtains a ticket if the radar speed check measures a speed higher than 130 km per hour.
 - (a) Find the probability that a driver will get a ticket.
 - (b) If 1000 cars per hours pass the radar speed check and the random variable X counts the number of tickets per hours, what is the distribution, the expected value and the variance of X?
 - (c) Find a suitable distribution which approximates the distribution of X.
 - (d) Calculate the probability that in one hour more than 100 drivers are ticketed with this distribution.
 - (e) Calculate an upper bound for the number of tickets per hour which will be hold with a probability of 0.9.

- 4. Suppose that an accounting firm does a study to conduct the time needed to complete one person's tax forms. It randomly surveys 101 people. The sample average is 23.6 hours. There is a known standard deviation of 7.0 hours. The population distribution is assumed to be normal.
 - (a) Construct a 90% confidence interval for the population average time to complete the tax forms.
 - (b) If the firm wishes to increase its level of confidence to 99% and keep the length of the confidence interval at least 2 by taking another survey, what changes should it make?
 - (c) If the standard deviation is not known and the sample standard deviation is 7.1 hours what will be the 90% upper confidence bound for the population standard deviation?

- 5. A consumer products company is formulating a new shampoo and is interested in higher foam height (in millimeters). The company wants to perform a statistical test. Foam height is approximately normally distributed and has a standard deviation of 20 millimeters. The average foam height of the current shampoo is 175 millimeters.
 - (a) Establish the hypothesis.
 - (b) The sample data of 10 samples result in an average of 185 millimeters. Conclude the test if $\alpha = 0.05$.
 - (c) Calculate the p-value for the sample data of b).
 - (d) What is the probability of type II error if the true mean foam height is 195 millimeters?

Table of the Standard Normal Distribution N(0,1)

	?,?0	?,?1	?,?2	?,?3	?,?4	?,?5	?,?6	?,?7	?,?8	?,?9
0	0.500000	0.503989	0.507978	0.511966	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555670	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621720	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802337	0.805105	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823814	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.879000	0.881000	0.882977
1.2	0.884930	0.886861	0.888768	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903200	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935745	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959070	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965620	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999534	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999651
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999822	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967
4	0.999968	0.999970	0.999971	0.999972	0.999973	0.999974	0.999975	0.999976	0.999977	0.999978

Quantiles of the Standard Normal Distribution $N(0,1)\ u_p$

	р	X	p	X	р	X
1	0.800	0.8416212	0.950	1.6448536	0.990	2.3263479
2	0.820	0.9153651	0.955	1.6953977	0.991	2.3656181
3	0.840	0.9944579	0.960	1.7506861	0.992	2.4089155
4	0.860	1.0803193	0.965	1.8119107	0.993	2.4572634
5	0.880	1.1749868	0.970	1.8807936	0.994	2.5121443
6	0.900	1.2815516	0.975	1.9599640	0.995	2.5758293
7	0.920	1.4050716	0.980	2.0537489	0.996	2.6520698
8	0.940	1.5547736	0.985	2.1700904	0.997	2.7477814
9	0.960	1.7506861	0.990	2.3263479	0.998	2.8781617
10	0.980	2.0537489	0.995	2.5758293	0.999	3.0902323

Quantiles of the t_n -Distribution $t_{n,p}$

	p=0.6	p = 0.65	p=0.7	p = 0.75	p = 0.8	p = 0.85	p=0.9	p = 0.95	p = 0.96	p = 0.97	p=0.975	p = 0.98	p=0.985	p = 0.99	p=0.995	p=0.999	p=0.9995
1	0.3249	0.5095	0.7265	1.0000	1.3764	1.9626	3.0777	6.3138	7.9158	10.5789	12.7062	15.8945	21.2049	31.8205	63.6567	318.3088	636.6192
2	0.2887	0.4447	0.6172	0.8165	1.0607	1.3862	1.8856	2.9200	3.3198	3.8964	4.3027	4.8487	5.6428	6.9646	9.9248	22.3271	31.5991
3	0.2767	0.4242	0.5844	0.7649	0.9785	1.2498	1.6377	2.3534	2.6054	2.9505	3.1824	3.4819	3.8960	4.5407	5.8409	10.2145	12.9240
4	0.2707	0.4142	0.5686	0.7407	0.9410	1.1896	1.5332	2.1318	2.3329	2.6008	2.7764	2.9985	3.2976	3.7469	4.6041	7.1732	8.6103
5	0.2672	0.4082	0.5594	0.7267	0.9195	1.1558	1.4759	2.0150	2.1910	2.4216	2.5706	2.7565	3.0029	3.3649	4.0321	5.8934	6.8688
6	0.2648	0.4043	0.5534	0.7176	0.9057	1.1342	1.4398	1.9432	2.1043	2.3133	2.4469	2.6122	2.8289	3.1427	3.7074	5.2076	5.9588
7	0.2632	0.4015	0.5491	0.7111	0.8960	1.1192	1.4149	1.8946	2.0460	2.2409	2.3646	2.5168	2.7146	2.9980	3.4995	4.7853	5.4079
8	0.2619	0.3995	0.5459	0.7064	0.8889	1.1081	1.3968	1.8595	2.0042	2.1892	2.3060	2.4490	2.6338	2.8965	3.3554	4.5008	5.0413
9	0.2610	0.3979	0.5435	0.7027	0.8834	1.0997	1.3830	1.8331	1.9727	2.1504	2.2622	2.3984	2.5738	2.8214	3.2498	4.2968	4.7809
10	0.2602	0.3966	0.5415	0.6998	0.8791	1.0931	1.3722	1.8125	1.9481	2.1202	2.2281	2.3593	2.5275	2.7638	3.1693	4.1437	4.5869
11	0.2596	0.3956	0.5399	0.6974	0.8755	1.0877	1.3634	1.7959	1.9284	2.0961	2.2010	2.3281	2.4907	2.7181	3.1058	4.0247	4.4370
12	0.2590	0.3947	0.5386	0.6955	0.8726	1.0832	1.3562	1.7823	1.9123	2.0764	2.1788	2.3027	2.4607	2.6810	3.0545	3.9296	4.3178
13	0.2586	0.3940	0.5375	0.6938	0.8702	1.0795	1.3502	1.7709	1.8989	2.0600	2.1604	2.2816	2.4358	2.6503	3.0123	3.8520	4.2208
14	0.2582	0.3933	0.5366	0.6924	0.8681	1.0763	1.3450	1.7613	1.8875	2.0462	2.1448	2.2638	2.4149	2.6245	2.9768	3.7874	4.1405
15	0.2579	0.3928	0.5357	0.6912	0.8662	1.0735	1.3406	1.7531	1.8777	2.0343	2.1314	2.2485	2.3970	2.6025	2.9467	3.7328	4.0728
16	0.2576	0.3923	0.5350	0.6901	0.8647	1.0711	1.3368	1.7459	1.8693	2.0240	2.1199	2.2354	2.3815	2.5835	2.9208	3.6862	4.0150
17	0.2573	0.3919	0.5344	0.6892	0.8633	1.0690	1.3334	1.7396	1.8619	2.0150	2.1098	2.2238	2.3681	2.5669	2.8982	3.6458	3.9651
18	0.2571	0.3915	0.5338	0.6884	0.8620	1.0672	1.3304	1.7341	1.8553	2.0071	2.1009	2.2137	2.3562	2.5524	2.8784	3.6105	3.9216
19	0.2569	0.3912	0.5333	0.6876	0.8610	1.0655	1.3277	1.7291	1.8495	2.0000	2.0930	2.2047	2.3456	2.5395	2.8609	3.5794	3.8834
20	0.2567	0.3909	0.5329	0.6870	0.8600	1.0640	1.3253	1.7247	1.8443	1.9937	2.0860	2.1967	2.3362	2.5280	2.8453	3.5518	3.8495
21	0.2566	0.3906	0.5325	0.6864	0.8591	1.0627	1.3232	1.7207	1.8397	1.9880	2.0796	2.1894	2.3278	2.5176	2.8314	3.5272	3.8193
22	0.2564	0.3904	0.5321	0.6858	0.8583	1.0614	1.3212	1.7171	1.8354	1.9829	2.0739	2.1829	2.3202	2.5083	2.8188	3.5050	3.7921
23	0.2563	0.3902	0.5317	0.6853	0.8575	1.0603	1.3195	1.7139	1.8316	1.9782	2.0687	2.1770	2.3132	2.4999	2.8073	3.4850	3.7676
24	0.2562	0.3900	0.5314	0.6848	0.8569	1.0593	1.3178	1.7109	1.8281	1.9740	2.0639	2.1715	2.3069	2.4922	2.7969	3.4668	3.7454
25	0.2561	0.3898	0.5312	0.6844	0.8562	1.0584	1.3163	1.7081	1.8248	1.9701	2.0595	2.1666	2.3011	2.4851	2.7874	3.4502	3.7251
26	0.2560	0.3896	0.5309	0.6840	0.8557	1.0575	1.3150	1.7056	1.8219	1.9665	2.0555	2.1620	2.2958	2.4786	2.7787	3.4350	3.7066
27	0.2559	0.3894	0.5306	0.6837	0.8551	1.0567	1.3137	1.7033	1.8191	1.9632	2.0518	2.1578	2.2909	2.4727	2.7707	3.4210	3.6896
28	0.2558	0.3893	0.5304	0.6834	0.8546	1.0560	1.3125	1.7011	1.8166	1.9601	2.0484	2.1539	2.2864	2.4671	2.7633	3.4082	3.6739
29	0.2557	0.3892	0.5302	0.6830	0.8542	1.0553	1.3114	1.6991	1.8142	1.9573	2.0452	2.1503	2.2822	2.4620	2.7564	3.3962	3.6594
30	0.2556	0.3890	0.5300	0.6828	0.8538	1.0547	1.3104	1.6973	1.8120	1.9546	2.0423	2.1470	2.2783	2.4573	2.7500	3.3852	3.6460
40	0.2550	0.3881	0.5286	0.6807	0.8507	1.0500	1.3031	1.6839	1.7963	1.9357	2.0211	2.1229	2.2503	2.4233	2.7045	3.3069	3.5510
50	0.2547	0.3875	0.5278	0.6794	0.8489	1.0473	1.2987	1.6759	1.7870	1.9244	2.0086	2.1087	2.2338	2.4033	2.6778	3.2614	3.4960
60	0.2545	0.3872	0.5272	0.6786	0.8477	1.0455	1.2958	1.6706	1.7808	1.9170	2.0003	2.0994	2.2229	2.3901	2.6603	3.2317	3.4602
70	0.2543	0.3869	0.5268	0.6780	0.8468	1.0442		1.6669	1.7765	1.9118	1.9944	2.0927	2.2152	2.3808	2.6479	3.2108	3.4350
80	0.2542	0.3867	0.5265	0.6776	0.8461	1.0432			1.7732	1.9078	1.9901	2.0878	2.2095	2.3739	2.6387	3.1953	3.4163
90	0.2541	0.3866	0.5263	0.6772	0.8456	1.0424		1.6620	1.7707	1.9048	1.9867	2.0839	2.2050	2.3685	2.6316	3.1833	3.4019
100	0.2540	0.3864	0.5261	0.6770	0.8452	1.0418	1.2901	1.6602	1.7687	1.9024	1.9840	2.0809	2.2015	2.3642	2.6259	3.1737	3.3905
200	0.2537	0.3859	0.5252	0.6757	0.8434	1.0391	1.2858	1.6525	1.7596	1.8915	1.9719	2.0672	2.1857	2.3451	2.6006	3.1315	3.3398
300	0.2536	0.3857	0.5250	0.6753	0.8428	1.0382	1.2844	1.6499	1.7566	1.8879	1.9679	2.0627	2.1805	2.3388	2.5923	3.1176	3.3233
400	0.2535	0.3856	0.5248	0.6751	0.8425	1.0378	1.2837	1.6487	1.7551	1.8861	1.9659	2.0605	2.1779	2.3357	2.5882	3.1107	3.3150
500	0.2535	0.3855	0.5247	0.6750	0.8423	1.0375	1.2832	1.6479	1.7543	1.8851	1.9647	2.0591	2.1763	2.3338	2.5857	3.1066	3.3101
∞	0.2533	0.3853	0.5244	0.6745	0.8416	1.0364	1.2816	1.6449	1.7507	1.8808	1.9600	2.0537	2.1701	2.3263	2.5758	3.0902	3.2905

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Quantiles of the χ_n^2 -Distribution $\chi_{n,p}$

	p=0.005	p=0.01	p=0.015	p=0.02	p=0.025	p=0.05	p=0.1	p=0.5	p = 0.9	p=0.95	p=0.975	p=0.98	p=0.985	p=0.99	p=0.995
1	0.000	0.000	0.000	0.001	0.001	0.004	0.016	0.455	2.706	3.841	5.024	5.412	5.916	6.635	7.879
2	0.010	0.020	0.030	0.040	0.051	0.103	0.211	1.386	4.605	5.991	7.378	7.824	8.399	9.210	10.597
3	0.072	0.115	0.152	0.185	0.216	0.352	0.584	2.366	6.251	7.815	9.348	9.837	10.465	11.345	12.838
4	0.207	0.297	0.368	0.429	0.484	0.711	1.064	3.357	7.779	9.488	11.143	11.668	12.339	13.277	14.860
5	0.412	0.554	0.662	0.752	0.831	1.145	1.610	4.351	9.236	11.070	12.833	13.388	14.098	15.086	16.750
6	0.676	0.872	1.016	1.134	1.237	1.635	2.204	5.348	10.645		14.449	15.033	15.777	16.812	18.548
7	0.989	1.239	1.418	1.564	1.690	2.167	2.833	6.346	12.017		16.013	16.622	17.398	18.475	20.278
8	1.344	1.646	1.860	2.032	2.180	2.733	3.490	7.344	13.362		17.535	18.168	18.974	20.090	21.955
9	1.735	2.088	2.335	2.532	2.700	3.325	4.168	8.343		16.919	19.023	19.679	20.513	21.666	23.589
10	2.156	2.558	2.837	3.059	3.247	3.940	4.865		15.987		20.483	21.161	22.021	23.209	25.188
11	2.603	3.053	3.363	3.609	3.816	4.575	5.578				21.920		23.503	24.725	26.757
12	3.074	3.571	3.910	4.178	4.404	5.226	6.304				23.337	24.054	24.963	26.217	28.300
13	3.565	4.107	4.476	4.765	5.009	5.892	7.042				24.736	25.472	26.403	27.688	29.819
14	4.075	4.660	5.057	5.368	5.629	6.571	7.790		21.064		26.119	26.873	27.827	29.141	31.319
15	4.601	5.229	5.653	5.985	6.262	7.261	8.547	14.339	22.307	24.996	27.488	28.259	29.235	30.578	32.801
16	5.142	5.812	6.263	6.614	6.908	7.962	9.312		23.542		28.845	29.633	30.629	32.000	34.267
17	5.697	6.408	6.884	7.255	7.564	8.672	10.085			27.587	30.191	30.995	32.011	33.409	35.718
18	6.265	7.015	7.516	7.906	8.231	9.390	10.865		25.989	28.869	31.526	32.346	33.382	34.805	37.156
19	6.844	7.633	8.159	8.567	8.907	10.117	11.651		27.204	30.144	32.852	33.687	34.742	36.191	38.582
20	7.434	8.260	8.810	9.237	9.591	10.851	12.443		28.412		34.170	35.020	36.093	37.566	39.997
25	10.520		12.187	12.697	13.120	14.611	16.473		34.382	37.652	40.646	41.566	42.725	44.314	46.928
30	13.787		15.719	16.306	16.791	18.493	20.599		40.256	43.773	46.979	47.962	49.199	50.892	53.672
35	17.192		19.369	20.027	20.569	22.465	24.797		46.059	49.802	53.203	54.244	55.553	57.342	60.275
40	20.707	_	23.113	23.838	24.433	26.509	29.051	39.335	51.805	55.758	59.342	60.436	61.812	63.691	66.766
45	24.311		26.933		28.366	30.612	33.350		57.505	61.656	65.410		67.994	69.957	73.166
50	27.991		30.818	31.664	32.357	34.764	37.689	49.335	63.167	67.505	71.420	72.613	74.111	76.154	79.490
60	35.534		38.744		40.482	43.188	46.459		74.397	79.082	83.298	84.580	86.188		91.952
70	43.275	45.442	46.836		48.758	51.739	55.329	69.334			95.023			100.425	104.215
80	51.172		55.061		57.153	60.391	64.278			101.879		108.069			
90	59.196		63.394	64.635	65.647	69.126	73.291				118.136				
100	67.328		71.818	73.142	74.222	77.929	82.358				129.561				
110	75.550	78.458	80.318	81.723	82.867	86.792					140.917				
120	83.852		88.886	90.367							152.211				
130	92.222	000-	97.512								163.453				
140		104.034													
150											185.800				
160		121.346													
170		130.064													
180		138.820													
190		147.610													
200	152.241	156.432	159.096	161.100	162.728	168.279	174.835	199.334	$2\frac{2}{5}6.021$	233.994	241.058	243.187	245.845	249.445	255.264