3. A large freight elevator can transport a maximum of 9800 pounds. Suppose a load of cargo containing 49 boxes must be transported via the elevator. Experience has shown that the weight of boxes of this type of cargo follows a distribution with mean $\mu = 205$ pounds and standard deviation $\sigma = 15$ pounds. Based on this information, what is the probability that all 49 boxes can be safely loaded onto the freight elevator and transported?

   Solution: Let X_k, for $k = 1, 2, 3, \ldots, 49$, denote the weight of each of the boxes. We want to estimate

$$\Pr\left(\sum_{k=1}^{n} X_k \leqslant 9800\right),$$

   where $\mu = E(X_k) = 205$ pounds, $\sigma = \sqrt{\text{Var}X_k} = 15$ pounds, and $n = 49$. We can therefore apply the Central Limit Theorem to estimate

$$\begin{aligned}\Pr\left(\sum_{k=1}^{n} X_k \leqslant 9800\right) &= \Pr\left(\frac{\sum_{k=1}^{n} X_k - n\mu}{\sqrt{n}\sigma} \leqslant \frac{9800 - 10,045}{7(15)}\right) \\ &\approx \Pr(Z \leqslant -2.33),\end{aligned}$$

   where $Z \sim \text{Normal}(0, 1)$, by the Central Limit Theorem. Therefore, using the symmetry of the pdf of $\sim \text{Normal}(0, 1)$,

$$\Pr\left(\sum_{k=1}^{n} X_k \leqslant 9800\right) \approx 1 - F_Z(2.33) \doteq 1 - 0.9901 = 0.0099.$$

   Thus, the probability that all 49 boxes can be safely loaded onto the freight elevator and transported is about 0.99%, or less than 1%. □

4. Forty–nine measurements are recorded to several decimal places. Each of these 49 numbers is rounded off to the nearest integer. The sum of the original 49 numbers is approximated by the sum of those integers. Assume that the errors made in rounding off are independent, identically distributed random variables with a uniform distribution over the interval $(-0.5, 0.5)$. Compute approximately the probability that the sum of the integers is within two units of the true sum.