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PROCTOR'S STATEMENT

This is to certify that student ______________________________

wrote an examination in the course **Math 208** ______________________________

under my personal supervision and received no outside aid from any source whatsoever. The student was verified through a picture ID prior to taking the examination. The completed examination is being sent to the Online and Distance Education Office by me.

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Signature of Examination Proctor

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Position

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Date

Math 208 Correspondence Lesson 21: Exam 3V2 Name: ____________________

TIME LIMIT: 2 hours; no books, no notes, no calculators, etc

Part I: ($2\frac{1}{2}$ points each) True or False (circle your answer):

1. **True** **False**: $122 \equiv -44 \pmod 3$.

2. **True** **False**: The congruence equation $12x \equiv 4 \pmod 8$ is solvable.

3. **True** **False**: $\binom{25}{15} > \binom{25}{10}$.

4. **True** **False**: When $(2x+1)^{30}$ is expanded, the coefficient of x^{10} is $\binom{30}{10}$.

5. **True** **False**: If A and B are finite sets, then $|A \cup B|$ is always less than or equal to $|A| + |B|$.

6. **True** **False**: 7059 days after a Monday is a Friday.

7. **True** **False**: In a group of 170 people, we can be sure there are at least 25 born on the same day of the week.

8. **True** **False**: A sequence is defined recursively by $a_0 = 0$, and, for $n > 0$, $a_n = n - a_{n-1}$. The value of $a_{20} = 10$.

9. **True** **False**: $a_n = n^2$ is a solution to the recurrence $a_0 = 0$ and for $n \geq 1$, $a_n = a_{n-1} + 2n - 1$.

10. **True** **False**: There is a graph with degree sequence $1, 1, 2, 3$.

11. **True** **False**: The number of strings of five letters from the usual alphabet if repeats are not allowed is $\frac{26!}{21!}$.

12. **True** **False**: The number of ways of picking a subset of $\{a, b, c \cdots, z, 0, 1, 2 \cdots 9\}$ consisting two elements is $\binom{26}{2}\binom{10}{2}$.

Part II: (5 points each) Multiple Choice (circle your answer):

1) Recall that K_n is the simple graph with n vertices with every pair of vertices adjacent (the complete graph on n vertices). Circle all n for which K_n has a Hamiltonian circuit.

(a) 3

(b) 4

(c) 5

(d) 6

(e) 7

2) When 36^{25} is divided by 17, the remainder is

(a) 0

(b) 1

(c) 2

(d) 3

(e) None of the above.

3) The number of different poker hands (5 cards, order not important, selected from a 52 card deck) with three hearts and two black cards is:

(a) $\binom{13}{3} + \binom{26}{2}$

(b) $\binom{52}{5} - \binom{13}{10}\binom{26}{24}$

(c) $\binom{13}{3}\binom{26}{2}$

(d) $\binom{52}{5} - \binom{39}{36} - \binom{26}{24}$

(e) None of the above.

4) The number of strings of length 10 of the 26 letters (repeats allowed) that either begin with a or end with a is

(a) $2(26^9)$

(b) $2(26^9) - 26^8$

(c) $52^9 - 26^8$

(d) $(26^9)(26^9)$

(e) 26^8

5) In a listing of the five equivalence classes modulo 5, four of the values are 14, 8, 100, and 32. The last value could be

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

6) The number of strings of length 10 from the alphabet $\Sigma = \{a, b, c, d\}$ that contain at least one d is

(a) $10^4 - 9^3$

(b) $10^4 - 9^4$

(c) $10^4 - 10^3$

(d) $4^{10} - 3^9$

(e) $4^{10} - 3^{10}$

7) I. M. N. Oddman climbs stairs by taking either one or three steps at a time. A recursive solution for the number of different ways he can climb a flight of n steps has initial conditions $a_0 = 1$, $a_1 = 1$, $a_2 = 1$, and for $n \geq 3$,

(a) $a_n = a_{n-1} + a_{n-2}$

(b) $a_n = a_{n-1} + a_{n-3}$

(c) $a_n = a_{n-2} + a_{n-3}$

(d) $a_n = (a_{n-1} - 1) + (a_{n-2} - 2)$

(e) $a_n = (a_{n-1} - 1) + (a_{n-3} - 3)$

8) When looking for a particular solution to the recurrence $a_n = 2a_{n-1} + a_{n-2} + 2^n + 1$ your first try should be

(a) $a_n = 2^n + B$

(b) $a_n = A(2^n) + 1$

(c) $a_n = A(2^n) + B$

(d) $a_n = A(2^n) + B(-2)^n + 1$

(e) $a_n = A(2^n) + B(-2)^n + C$

Part III. (10 points each) Problems

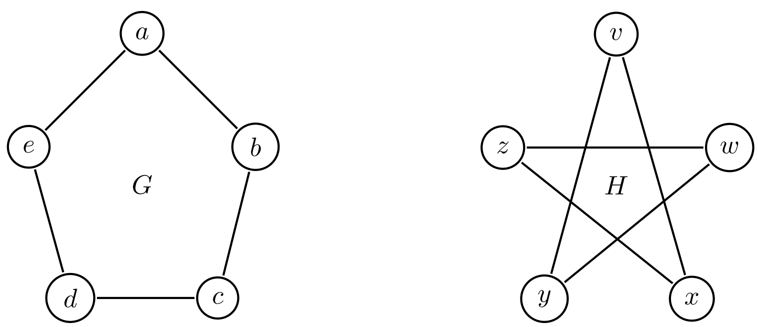
Do any three of the following four problems. If you do all four, I'll count your best three.

1) Determine the number of ways the 26 letters of the alphabet can be written in a row (no repeats) if the vowels (a, e, i, o, u) must be adjacent.

2) You have inexhaustible supplies of red and blue marbles. The marbles are distributed randomly, one by one, into three tin cans. What is the minimum number of marble that have to be distributed to be absolutely sure there is a can with two marbles of the same color.

3) Determine a closed form formula for the sequence defined recursively by $a_0 = 0$, $a_1 = 1$, and for $n \geq 2$, $a_n = 2a_{n-1} + 15a_{n-2}$.

4) Show the two graphs below are isomorphic by writing down a mapping (in other words, a pairing of the vertices) that is a graph isomorphism. Start with the pairing $a \to v$.



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