Math 208 Correspondence Lesson 14: Exam 2V2 Name: Jack Riley

TIME LIMIT: 2 hours; no books, no notes, no calculators, etc

Part I: ($2\frac{1}{2}$ points each) True or False (circle your answer):

$$\frac{900}{2} = 450$$

$$\frac{900}{3} = 300$$

1. **True** **False**: There are only three different primes that are divisors of 900.

2. **True** **False**: $\sum_{k=1}^{100} 2k = 2\sum_{k=1}^{100} k$.

3. **True** **False**: If $a_0 = 2$, and for $n \geq 1$, $a_n = 1 - a_{n-1}$, then $a_{100} = -1$.

$a_0 = 2$

4. **True** **False**: The set described recursively by (a) $1 \in S$, and (b) if $k \in S$, then $k + 2 \in S$ is the set of all odd positive integers.

S does not include the positive int 2.

5. **True** **False**: For all integers r, s, t, if $r < s$, then $rt < st$.

r and t could b negative $-r(-t) = +rt$

6. **True** **False**: If there are integers r, s, t, u such that $rs + tu = 7$, then $\gcd(r, t) \leq 7$.

7. **True** **False**: For integers r, s, if r divides s and s divides r, then $r = s$.

8. **True** **False**: The Diophantine equation $14x + 21y = 323$ has at least one solution.

9. **True** **False**: $\forall n \in \mathbf{Z}$, $-n|n$.

10. **True** **False**: 1551 is a prime.

11. **True** **False**: The smallest positive integer that can be written as a linear combination of 23 and 19 is 3.

12. **True** **False**: If a, b, c are positive integers, and a divides b, then $gcd(a, c) = gcd(b, c)$.

Part II: (5 points each) Multiple Choice (circle your answer):

1) The sum of the first 100 terms of the arithmetic sequence with initial term 3 and common difference 2 is

(a) 10200
(b) 10202
(c) 15602
(d) 20400
(e) 20402

$3 + 5 + 7 + 9$

→ How with no calc and no paper?

$8 + 7 = 15$
$= 24$

2) According to the laws of exponents, $(2a)^b$ equals

(a) $2(a^b)$ ✗
(b) $(2^a)^b$ ✗
(c) $2^b a^b$
(d) a^b ✗
(e) None of the above.

3) A geometric sequence begins $a_0 = 3$, $a_1 = 6$. The value of a_4 is

(a) 48
(b) 58
(c) 68
(d) 78
(e) There is not enough information to determine a_4.

4) A set S of strings over the alphabet $\Sigma = \{ a, b, c \}$ is described recursively by (1) $abc \in S$, and (2) if $x \in S$, then $bcx \in S$. Circle all the true statements in the list below.

(a) Every string in S has exactly one a.
(b) Every string in S has b and c next to each other.
(c) Every string in S has the same number of b's and c's.
(d) Every string in S has odd length.
(e) The largest number of consecutive b's in the strings in S is two.

5) The number of positive integers that divide 1000 is

(a) 2

(b) 4

(c) 8

(d) 16

(e) 32

6) The fact that for all integers a, b, it is true that $a + b = b + a$ is called the

(a) Distributive Law

(b) Associative Law

(c) Commutative Law

(d) Inductive Law

(e) Identity Law

Not relevant

7) Which of the following are true about the divides relation: (all letters represent integers)

(a) $4|12 = 3$

(b) $0|0$

(c) For all integers a, $a|a^2$.

(d) If $a|b$ and $a|c$ then $a|(b + c)$.

(e) For all integers a, $a| - a$.

8) If a, b, s, t are integers, and $as + bt = 6$, then (circle all that are true in the list below):

(a) $gcd(a, b)$ could be 1

(b) $gcd(a, b)$ could be 2

(c) $gcd(a, b)$ could be 3

(d) $gcd(a, b)$ could be 4

(e) $gcd(a, b)$ could be 5

Part III. (10 points each) Problems

Do any three of the following four problems. If you do all four, I'll count your best three.

1) Use induction to prove that $1+3+5+\cdots+(2n-1)=n^2$ for every integer $n \geq 1$.

2) A sequence of integers is defined recursively by the rules $h_1 = 2$, $h_2 = 4$,

and for $n \geq 3$, $h_n = h_{n-1} + h_{n-2}$. Compute h_1, h_2, h_3, h_4, h_5.

$h_1 = 2$

$h_2 = 4$

$h_3 = (4) + 2 = 6$

$h_4 = 6 + 4 = 10$

$h_5 = 10 + 6 = 16$

3) Compute $gcd(1446, 531)$, and write the gcd as a linear combination of 1446 and 531.

$\to (531, 384)$

$\to gcd(384, 147)$

$\to gcd(147, 90)$

$\to gcd(90, 57)$

$\to gcd(57, 33)$

$\to gcd(33, 24)$

$\to gcd(24, 9)$

$\to gcd(9, 6)$

$\to gcd(9, 6)$

$\to gcd(6, 3) = 3$

4) Determine all solutions to the Diophantine equation $7x + 16y = 4$.

$gcd(16, 7) = 2$ 32

$\to gcd(7, 2) =$

$gcd(2, 1) = 1$

$7(12) + 16(-5) = 4$

All solutions $=$ 12, -5