

# Algorithmique Répartie

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9 novembre 2017

## 1 Introduction

- State of the Art
- Hypotheses
- Problems

## 2 Weaker models

- ASYNC Model - global-strong multiplicity detection
- SSYNC Model - global-weak / local strong multiplicity detection

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- Problem
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## Background and Motivations :

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- **mobile robot network** goal : achieve tasks by a team of mobile robot with weak capacity.



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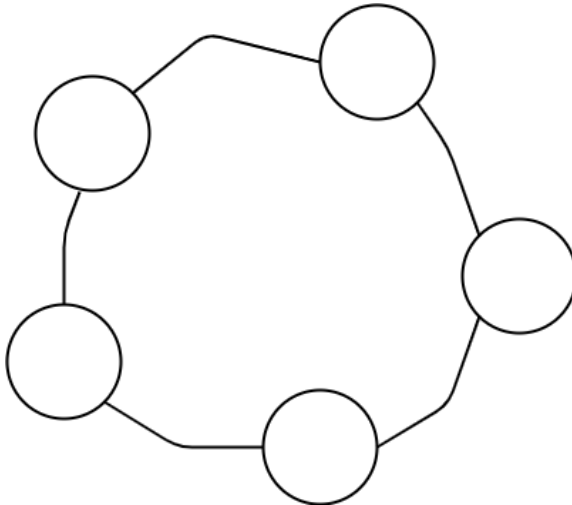
- **mobile robot network** goal : achieve tasks by a team of mobile robot with weak capacity.
- **Pioneering work** Suzuki, I., & Yamashita, M. (1999). Distributed anonymous mobile robots : Formation of geometric patterns.
- it studies **self-stabilizing** algorithms for anonymous and oblivious robots in uniform ring network.

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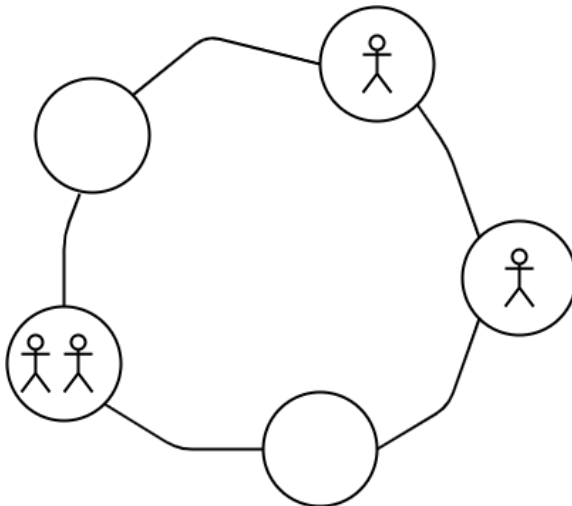


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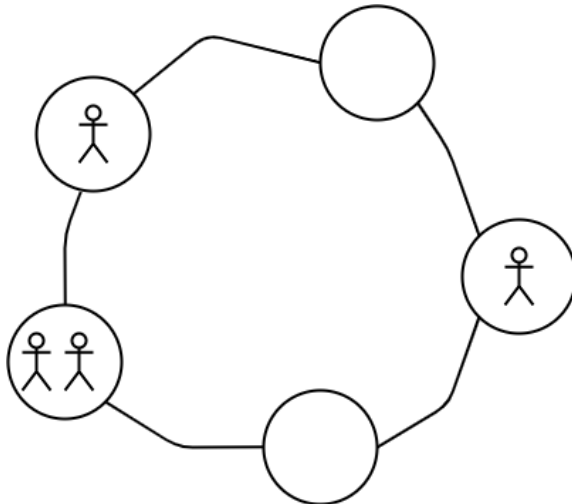


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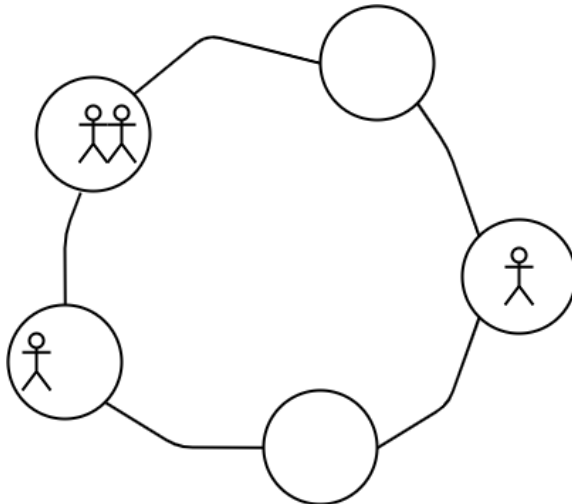


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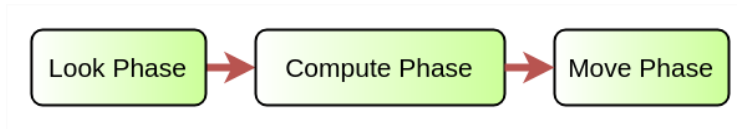


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- no such algorithms on very weak condition (ASYNC or global-weak multiplicity)
- we propose three algorithms :
  - self-stabilizing gathering algorithm
  - self-stabilizing orientation algorithm
  - self-stabilizing formation algorithm



Related work :

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  - none of them are self stabilizing

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- Global-Strong Multiplicity Detection

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- Global-Strong Multiplicity Detection
- Local-Strong and Global-Weak Multiplicity Detection

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- SSYNC** Semi-Synchronous - For each round, a set of robots are activated/executed at the same time.
- ASYNC** Asynchronous - The robots are activated/executed asynchronously

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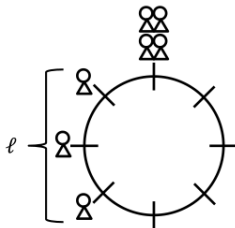
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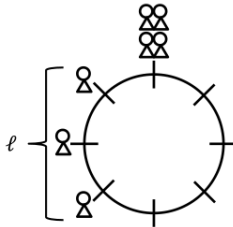
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**Set formation problem** : The goal of the set formation problem is to gather the robots in a specific predefined configuration.

Proof of the non existence of gathering algorithm in two weak conditions :

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## Gathering problem :

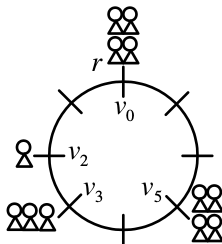
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- If there is multiple, the idea is to try to make them move one by one so that a tower node "wins the fight". We must find a way to elect a candidate.

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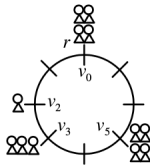
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**Case 1** There is only one such node : the tower can be identified by the robots and they can get closer to the tower node.

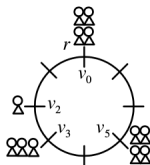
Let's consider the  $M(C)$  nodes :

- Case 1 There is only one such node : the tower can be identified by the robots and they can get closer to the tower node.
- **The scheduler is an enemy !**
  - Less than  $M(C)$  nodes should move in the same direction !

Case 2 There are multiple such nodes :



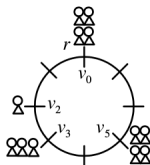
Case 2 There are multiple such nodes :



Take  $h_{min}$  the minimal distance between a M(C)-robot node and a neighboring robot node. Take  $V$  the set of nodes at distance  $h_{min}$  of a M(C)-node and  $R$  the robots on these nodes.

Cas 2.1  $|R| = 1$  - This robot gets to his closer M(C)-robot node.

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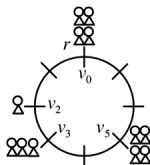


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Complexity :  $O(n \log k)$  rounds and  $O(kn)$  moves.



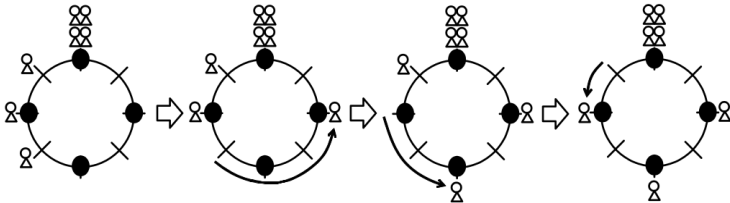


We can now apply the two previous algorithms to solve the **set formation problem**.

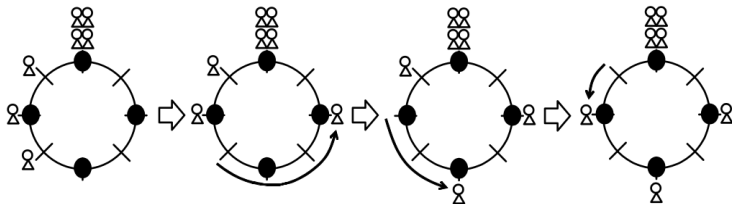
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## In order to go further we could :

- Find the problems we can solve with weaker hypotheses,
- Work with a weaker scheduler, like an oblivious one,
- Work with a more complex graph than a ring.