Algorithmique Répartie

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Background and Motivations :

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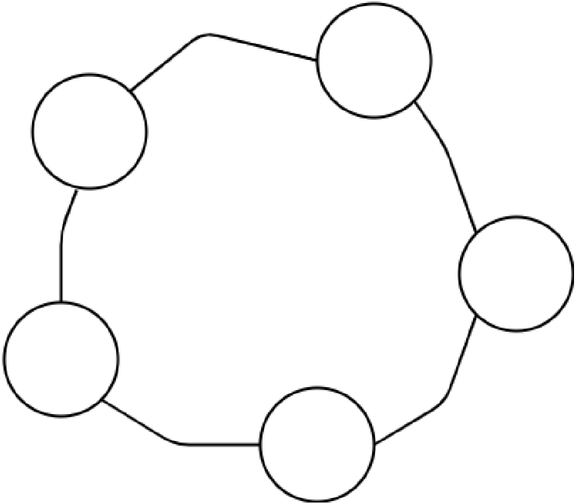
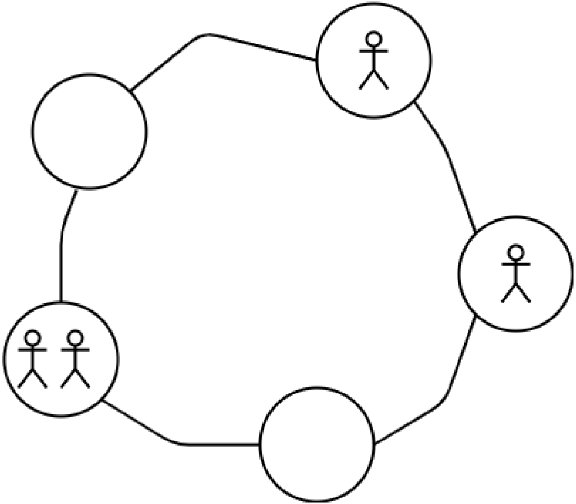
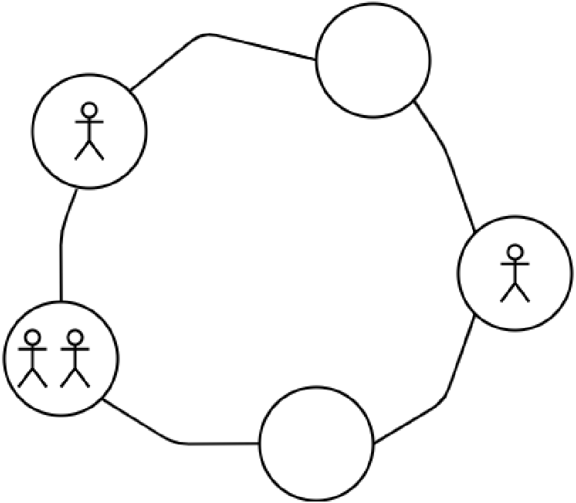
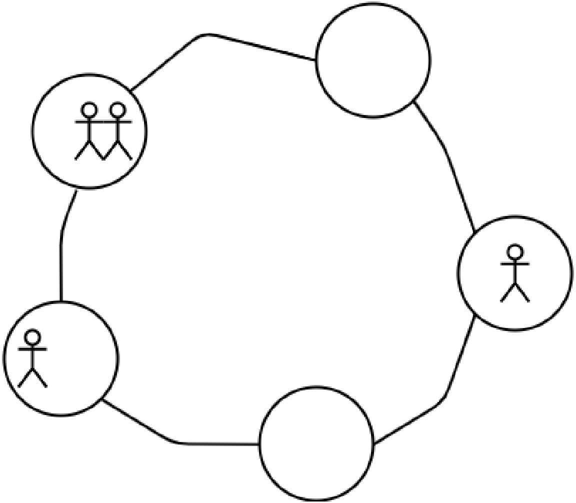
- **mobile robot network** goal : achieve tasks by a team of mobile robot with weak capacity.
- **Pioneering work** Suzuki, I., & Yamashita, M. (1999). Distributed anonymous mobile robots : Formation of geometric patterns.
- it studies **self-stabilizing** algorithms for anonymous and oblivious robots in uniform ring network.









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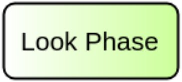

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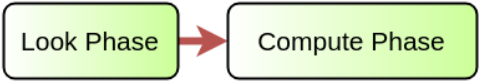

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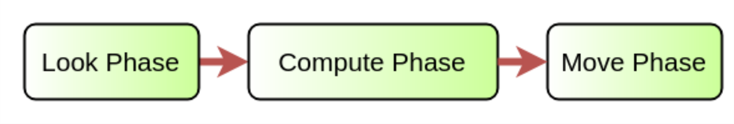

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- we propose three algorithms :

  self-stabilizing gathering algorithm
  self-stabilizing orientation algorithm
  self-stabilizing formation algorithm

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- Local-Strong and Global-Weak Multiplicity Detection

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ASYNC Asynchronous - The robots are activated/executed asynchronously

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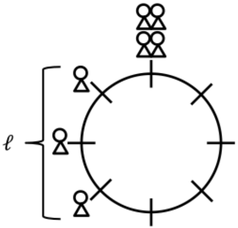

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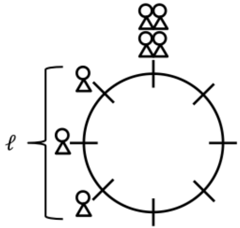

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How to prove non existence of an algorithm ? **play the scheduler**

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Probability that A achieve gathering in j cycle :

$$P^* < (1 - P(X_1)) + (1 - P(X_2)) + ... + (1 - P(X_j)) < p \ \square$$

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  We want to achieve $Q = +n$ or $Q = -n$.

In $proc(X)$, scheduler repeat following steps :

- if $Q = 0$: look and compute phases on robot r on v_0 :
  - if r want to stay $=> Q := 0$
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Stop when $Q = +n$ or $Q = -n$ (or X steps).

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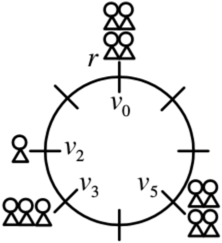

Idea :

- If there is only once M(C)-node, then the robots "know" where to go
- If there is multiple, the idea is to try to make them move one by one so that a tower node "wins the fight". We must find a why to elect a candidate.
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- **Take care !** The scheduler is an enemy and will activate the robots in the worst way.

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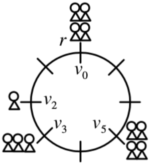
- **The scheduler is an enemy !**
- Less than M(C) nodes should move in the same direction !

Case 2 There are multiple such nodes :



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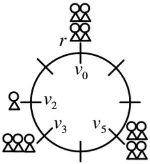

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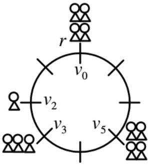

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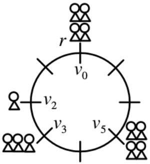

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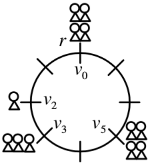

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Complexity : $O(n \log k)$ rounds and $O(kn)$ moves.

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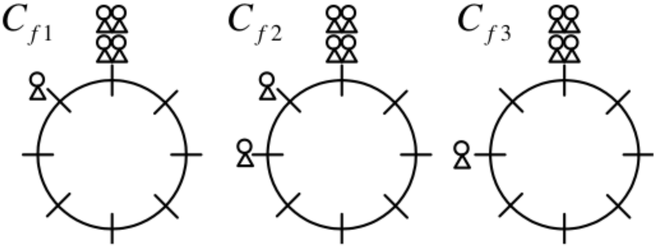

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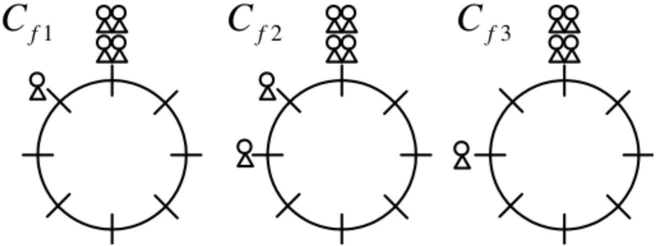

Reaches a configuration C_{f1} (if $l = 1$) or in C_{f2} (if $l \geq 2$) in $\mathcal{O}(n \log k)$ expected rounds and $\mathcal{O}(kn)$ expected moves.

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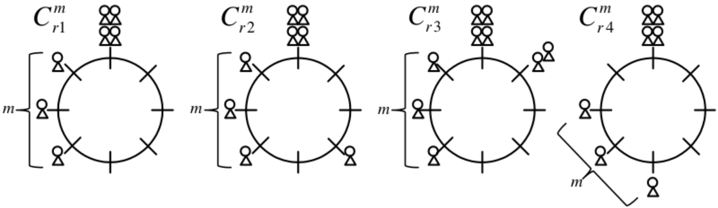

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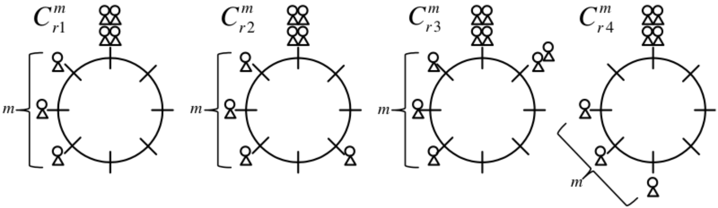

Reaches a configuration in C_0 in $\mathcal{O}(ln)$ expected rounds and $\mathcal{O}(l(k+n))$ expected moves.

We can now apply the two previous algorithms to solve the **set formation problem**.

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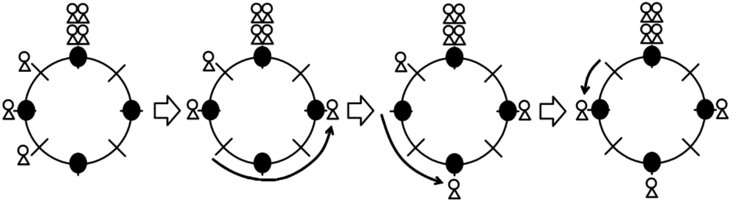

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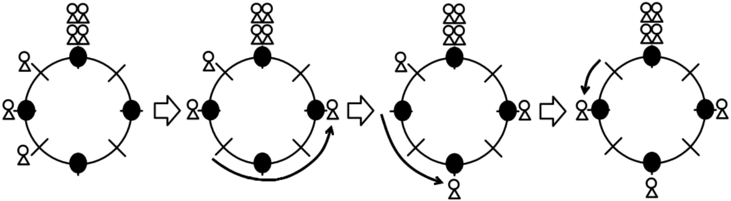

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In order to go further we could :

- Find the problems we can solve with weaker hypotheses,
- Work with a weaker scheduler, like an oblivious one,
- Work with a more complex graph than a ring.