Algorithmique Répartie

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9 novembre 2017

Background and Motivations :

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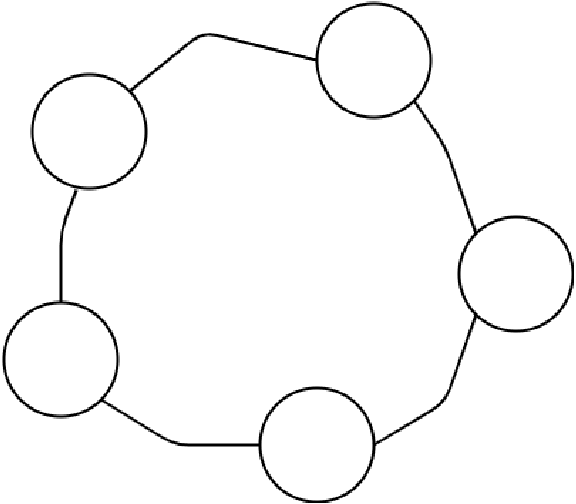
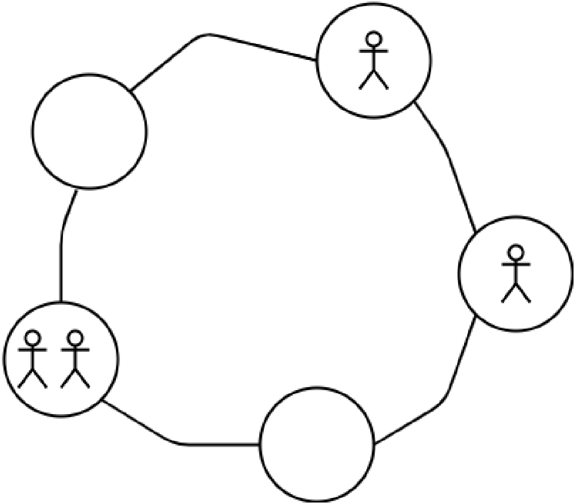
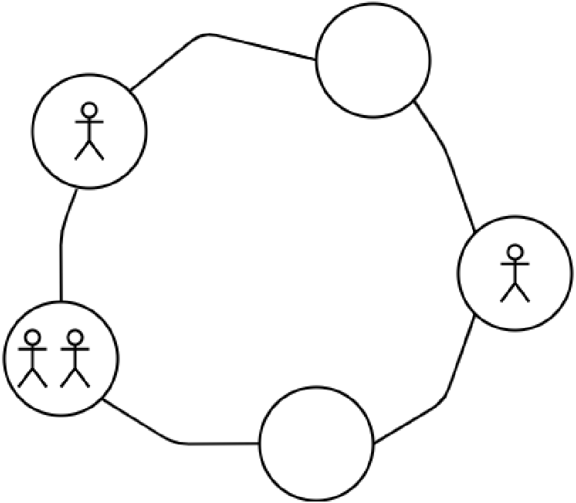
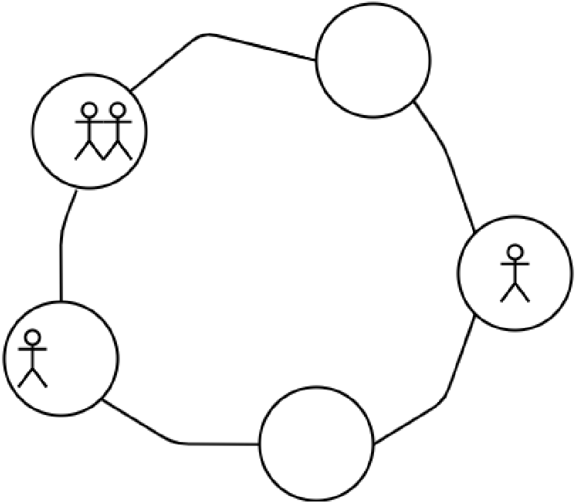
- **mobile robot network** goal : achieve tasks by a team of mobile robot with weak capacity.
- **Pioneering work** Suzuki, I., & Yamashita, M. (1999). Distributed anonymous mobile robots : Formation of geometric patterns.
- it studies **self-stabilizing** algorithms for anonymous and oblivious robots in uniform ring network.









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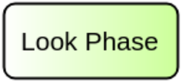

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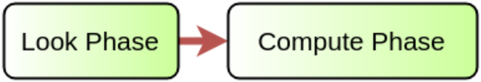

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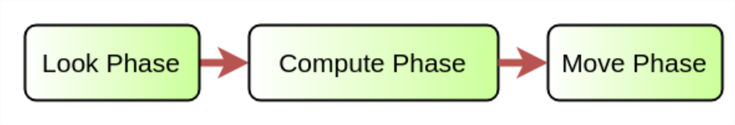

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- we propose three algorithms :
  self-stabilizing gathering algorithm
  self-stabilizing orientation algorithm
  self-stabilizing formation algorithm

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- Local-Strong and Global-Weak Multiplicity Detection

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ASYNC Asynchronous - The robots are activated/executed asynchronously

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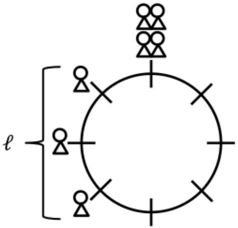

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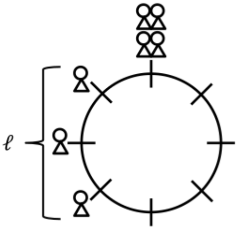

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How to prove non existence of an algorithm ? **play the scheduler**

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Probability that A achieve gathering in j cycle :

$$P^* < (1 - P(X_1)) + (1 - P(X_2)) + ... + (1 - P(X_j)) < p \, \square$$

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  We want to achieve $Q = +n$ or $Q = -n$.

In $proc(X)$, scheduler repeat following steps :

- if $Q = 0$: look and compute phases on robot r on v_0 :
  - if r want to stay $=>$ $Q := 0$
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Stop when $Q = +n$ or $Q = -n$ (or X steps).

ASYNC Model - global-strong multiplicity detection

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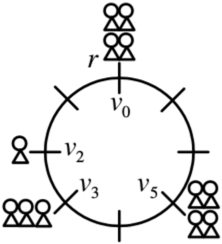

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- If there are multiple candidates, find a way to make, in expectation, exactly one of them move
- **Take care !** The scheduler is an enemy and will activate the robots in the worst way.

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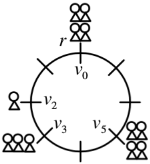
- **The scheduler is an enemy !**
- Less than M(C) nodes should move in the same direction !

Case 2 There are multiple such nodes :



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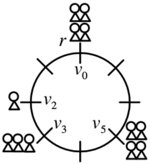

Take h_{min} the minimal distance between a M(C)-robot node and a neighboring robot node. Take V the set of nodes at distance h_{min} of a M(C)-node and R the robots on these nodes.

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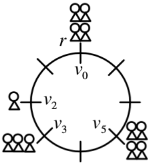

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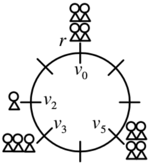

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Complexity : $O(n \log k)$ rounds and $O(kn)$ moves.

We can now apply the two previous algorithms to solve the **set formation problem**.

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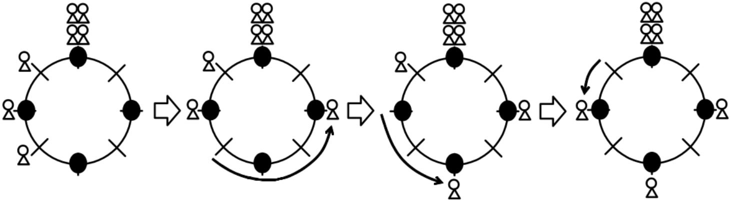

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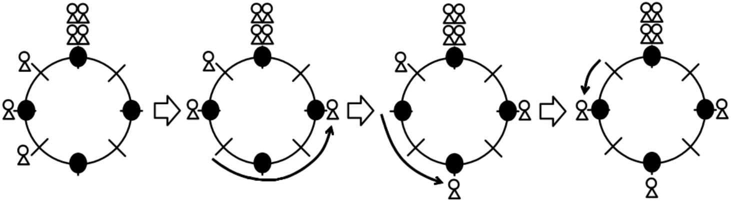

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In order to go further we could :

- Find the problems we can solve with weaker hypotheses,
- Work with a weaker scheduler, like an oblivious one,
- Work with a more complex graph than a ring.