Maximum Probability Shortest Path Problem

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Shortest Path is a known problem and has many applications in "real life".

- Goods transport (industrial and private)
- Food Delivery (Deliveroo - Foodora - UberEats)

As this problem is well known, lots of scientists presented their work on related subject, most of the times with different hypotheses :

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or also with a different optimization problem like utility functions to maximize or cost functions to minimize.

- Graph with weights on arcs, source node s and sink node t,
- Stochastic resource consumptions with normal distribution,
- K resources,
- Threshold C of the cost function (maximum allowed weight of the path).

SRCSP can be formulated as this optimization problem :

$$max \ \mathbf{Pr}\{\tilde{a}_k^T x \leq d_k, k = 1..K\}$$

$$s.t. \ c^T \leq C$$
$$Mx = b$$
$$x \in \{0, 1\}^n$$

where :

- $x(e) = 1$ if $x(e) \in$ path P
- The a_k are multi-variate vectors with mean μ_k and known covariance matrix V_k
- M is the node-arc incidence matrix. $M(i, e) \in \{-1, 0, 1\}$
- b is a vector with 0 everywhere except $b(s) = 1$ and $b(t) = -1$

SRCSP can be reformulated as :

$$
\begin{aligned}
&\max\ p \\
&s.t.\ p \leq \mathbf{Pr}\{\tilde{a}_k^T x \leq d_k, k = 1..K\} \\
&\qquad c^T \leq C \\
&\qquad Mx = b \\
&\qquad x \in \{0,1\}^n
\end{aligned}
$$

State of the art
Problem
Resolution

One resource case
Joint probabilities
Individual relaxed probabilities
Results

With $K = 1$ we have :

$$\begin{aligned}
max \ & p \\
s.t. \ & p \leq \mathbf{Pr}\{\tilde{a}_1^T x \leq d_1\} \\
& c^T \leq C \\
& Mx = b \\
& x \in \{0, 1\}^n
\end{aligned}$$

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Using the known multivariate distribution parameters we get :

$$
\begin{aligned}
&max\ p \\
&s.t.\ F^{-1}(p)(x^T V_1 x)^{\frac{1}{2}} \leq d_1 - \mu_1^T x \\
&\qquad c^T \leq C \\
&\qquad Mx = b \\
&\qquad x \in \{0, 1\}
\end{aligned}
$$

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Relaxing the problem we get (SRCSPI) with $p \leq \frac{1}{2}$:

$$
\begin{aligned}
& max \; 0 \\
& s.t. \; F^{-1}(p)(x^T V_1 x)^{\frac{1}{2}} \leq d_1 - \mu_1^T x \\
& \quad\quad c^T \leq C \\
& \quad\quad Mx = b \\
& \quad\quad 0 \leq x \leq 1
\end{aligned}
$$

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We can solve it using the binary search procedure. We take $p_1 \leq \frac{1}{2}$ a feasible solution, a lower bound of SRCSP. p_l and p_u are lower and upper bounds of SRCSPI. Then we iterate :

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We can solve it using the binary search procedure. We take $p_1 \leq \frac{1}{2}$ a feasible solution, a lower bound of SRCSP. p_l and p_u are lower and upper bounds of SRCSPI. Then we iterate :

Start $p_l = \frac{1}{2}$ and $p_u = 1$. Iteration counter $t = 1$

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Start $p_l = \frac{1}{2}$ and $p_u = 1$. Iteration counter $t = 1$

Search Solve SRCSPI with $p = p_t$. If SRCSPI has an optimal solution, set $p_l = p_t$, otherwise $p_u = p_t$.

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Start $p_l = \frac{1}{2}$ and $p_u = 1$. Iteration counter $t = 1$

Search Solve SRCSPI with $p = p_t$. If SRCSPI has an optimal solution, set $p_l = p_t$, otherwise $p_u = p_t$.

Stop Stop when $\frac{p_u - p_l}{2} \leq \epsilon$. Otherwise $t + +$ and $p_t = \frac{p_l + p_u}{2}$

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Put the formulation Explain why we are looking for convexity, using the lectures Explain how we get the approximation (Theorem 4.1.2), saying that we used this method in classes

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We formulate this problem taking individual probabilities constraints :

$$
\begin{aligned}
& max \ p \\
& s.t. \ p \leq \mathbf{Pr}\{\tilde{a}_k^T x \leq d_k\}, \ \mathbf{k=1,..,K} \\
& \qquad c^T \leq C \\
& \qquad Mx = b \\
& \qquad x \in \{0,1\}^n
\end{aligned}
$$

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We can relax the equation as we did before :

$$max\ 0$$
$$s.t.\ F^{-1}(p)(x^T V_k x)^{\frac{1}{2}} \le d_k - \mu_k^T x,\ k = 1,..,K$$
$$c^T \le C$$
$$Mx = b$$
$$0 \le x_i \le 1,\ i = 1,..,n$$

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We use the relaxed equation using the Binary Search Procedure again :

Start $p_l = \frac{1}{2}$ and $p_u = 1$. Iteration counter $t = 1$

Search Solve SRCSPI with $p = p_t$. If SRCSPI has an optimal solution, set $p_l = p_t$, otherwise $p_u = p_t$.

Stop Stop when $\frac{p_u - p_l}{2} \leq \epsilon$. Otherwise $t++$ and $p_t = \frac{p_l + p_u}{2}$

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Show results and analyze them (Why is this algorithm faster, why not, etc)

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