Maximum Probability Shortest Path Problem

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2 novembre 2017

Shortest Path is a known problem and has many applications in "real life".

- Goods transport (industrial and private)
- Food Delivery (Deliveroo - Foodora - UberEats)

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or also with a different optimization problem like utility functions to maximize or cost functions to minimize.

- Graph with weights on arcs, source node s and sink node t,
- Stochastic resource consumptions with normal distribution,
- K resources,
- Threshold C of the cost function (maximum allowed weight of the path).

SRCSP can be formulated as this optimization problem :

$$max\ \mathbf{Pr}\{\tilde{a}_k^T x \leq d_k, k = 1..K\}$$

$$s.t.\ c^T \leq C$$
$$Mx = b$$
$$x \in \{0,1\}^n$$

where :

- $x(e) = 1$ if $x(e) \in$ path P
- The a_k are multi-variate vectors with mean μ_k and known covariance matrix V_k
- M is the node-arc incidence matrix. $M(i,e) \in \{-1,0,1\}$
- b is a vector with 0 everywhere except $b(s) = 1$ and $b(t) = -1$

SRCSP can be reformulated as :

$$
\begin{aligned}
& max\ \ p \\
& s.t.\ \ p \leq \mathbf{Pr}\{\tilde{a}_k^T x \leq d_k, k = 1..K\} \\
& \qquad c^T \leq C \\
& \qquad Mx = b \\
& \qquad x \in \{0,1\}^n
\end{aligned}
$$

With $K = 1$ we have :

$$
\begin{aligned}
& max\ p \\
& s.t.\ p \leq \mathbf{Pr}\{\tilde{a}_1^T x \leq d_1\} \\
& \qquad c^T \leq C \\
& \qquad Mx = b \\
& \qquad x \in \{0,1\}^n
\end{aligned}
$$

Explain how we get the formula at the next slide

Using the known multivariate distribution parameters we get :

$$
\begin{aligned}
max\ & p \\
s.t.\ & F^{-1}(p)(x^T V_1 x)^{\frac{1}{2}} \leq d_1 - \mu_1^T x \\
& c^T \leq C \\
& Mx = b \\
& x \in \{0, 1\}
\end{aligned}
$$

Relaxing the problem we get (SRCSPI) with $p \leq \frac{1}{2}$:

$$\begin{aligned} & max\ 0 \\ & s.t.\ F^{-1}(p)(x^T V_1 x)^{\frac{1}{2}} \leq d_1 - \mu_1^T x \\ & \qquad c^T \leq C \\ & \qquad Mx = b \\ & \qquad 0 \leq x \leq 1 \end{aligned}$$

We can solve it using the binary search procedure. We take $p_1 \leq \frac{1}{2}$ a feasible solution, a lower bound of SRCSP. p_l and p_u are lower and upper bounds of SRCSPI. Then we iterate :

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Search Solve SRCSPI with $p = p_t$. If SRCSPI has an optimal solution, set $p_l = p_t$, otherwise $p_u = p_t$.

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Stop Stop when $\frac{p_u - p_l}{2} \leq \epsilon$. Otherwise $t{+}{+}$ and $p_t = \frac{p_l + p_u}{2}$

Put the formulation Explain why we are looking for convexity, using the lectures Explain how we get the approximation (Theorem 4.1.2), saying that we used this method in classes

Put the formulation Say problem is harder than the Joint probabilities, as we saw in class Explain how we solve it

Show results and analyze them (Why is this algorithm faster, why not, etc)