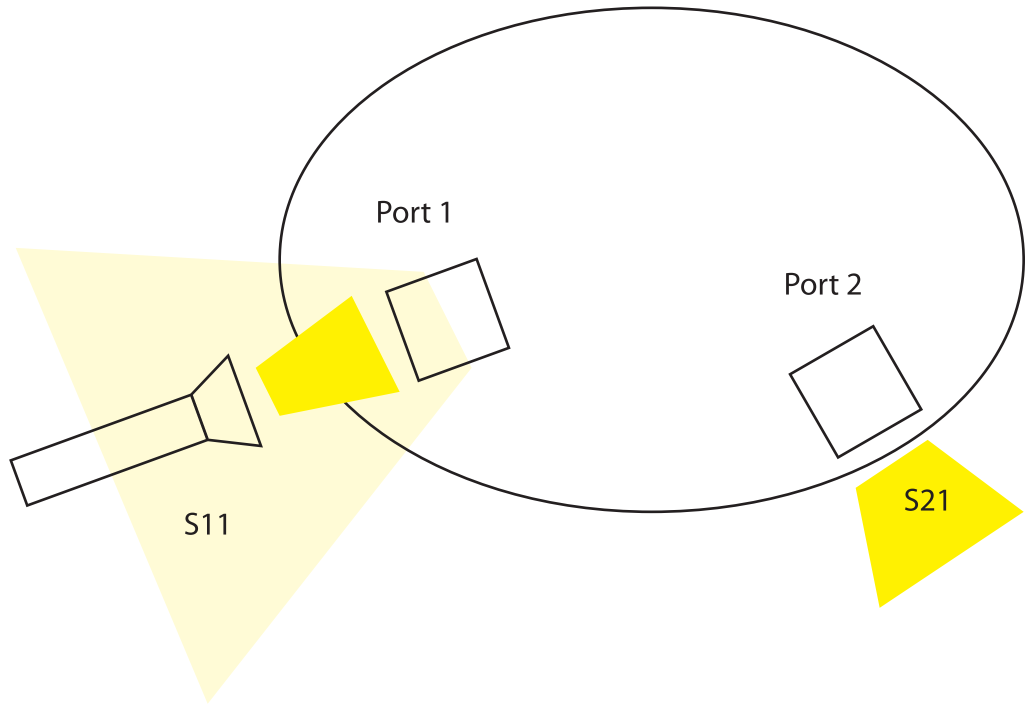
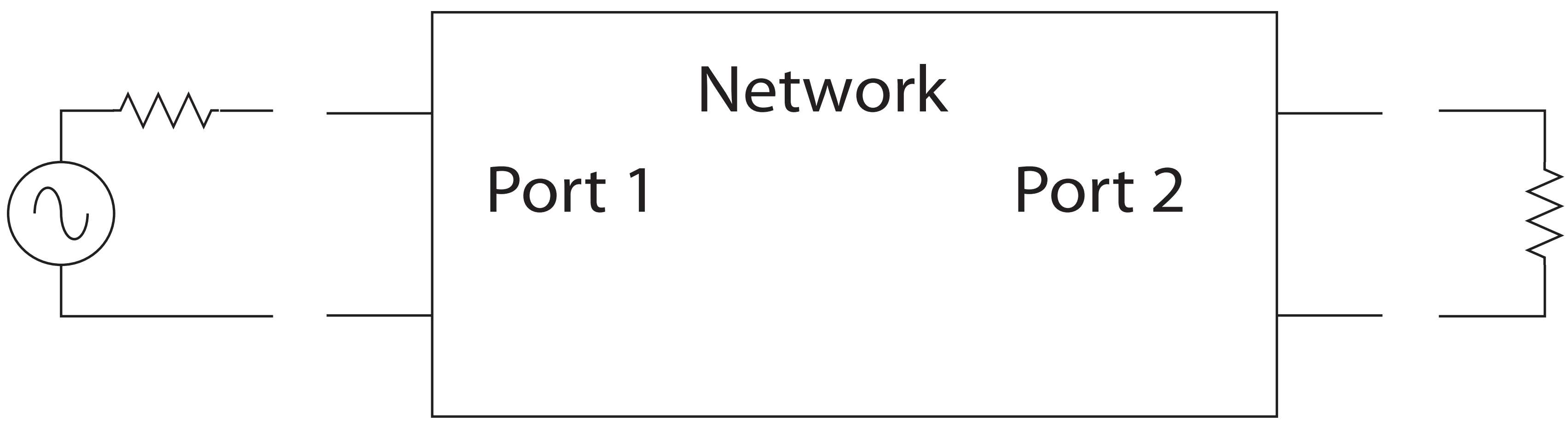
Scattering Parameters



the basic concept...

Scattering Parameters

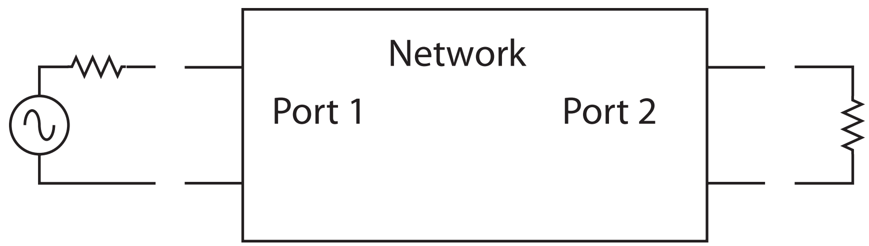
How microwave engineers use them:



Why Microwave engineers use them:

directional voltage waves (?)

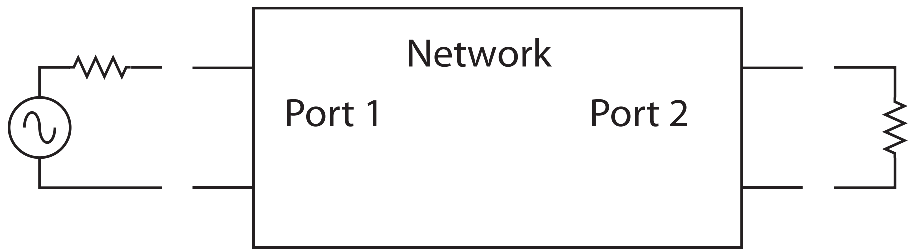
directional voltage waves (?)



standing waves (??)

...and at microwaves, historically, none of our usual electrical tools worked. Voltmeters, oscilloscopes...

Voltage Waves and Linear Algebra:



Voltage Waves: separate voltage at ports into arriving and departing wave voltages

Hint: use a directional coupler, not a voltmeter or an oscilloscope

$$\text{Waves leaving network} \quad \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \underset{\text{network}}{\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} \quad \text{Waves entering network}$$

Multiply it all out:

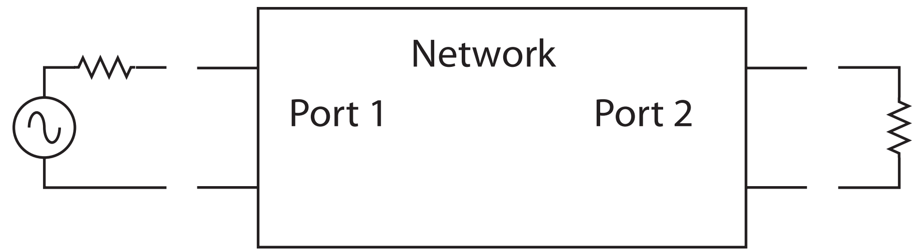
$$
\begin{array}{c}\text{Waves}\\ \text{leaving}\\ \text{network}\end{array}
\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \underbrace{\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}}_{\text{network}} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}
\begin{array}{c}\text{Waves}\\ \text{entering}\\ \text{network}\end{array}
$$

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$

...and that shows us how to measure each S parameter

$$S_{11} = \frac{V_1^-}{V_1^+} \quad \text{with} \quad V_2^+ = 0$$



$$S_{21} = \frac{V_2^-}{V_1^+} \quad \text{with} \quad V_2^+ = 0$$

$$S_{22} = \frac{V_2^-}{V_2^+} \quad \text{with} \quad V_1^+ = 0$$

$$S_{12} = \frac{V_1^-}{V_2^+} \quad \text{with} \quad V_1^+ = 0$$

Each one has many names. For an Amplifier:

$$S_{11} = \frac{V_1^-}{V_1^+} \quad \text{with } V_2^+ = 0$$

input match

small is good

$$S_{21} = \frac{V_2^-}{V_1^+} \quad \text{with } V_2^+ = 0$$

gain

medium is good

$$S_{22} = \frac{V_2^-}{V_2^+} \quad \text{with } V_1^+ = 0$$

output match

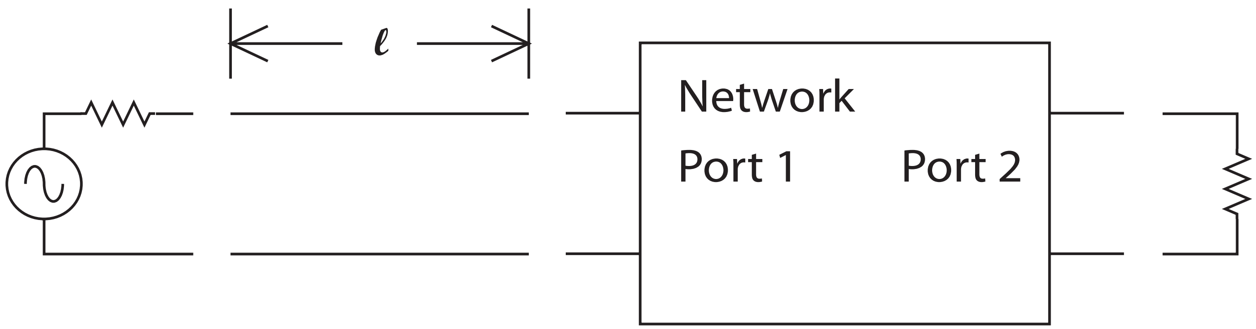
small is good

$$S_{12} = \frac{V_1^-}{V_2^+} \quad \text{with } V_1^+ = 0$$

reverse isolation

small is essential

When we add transmission line to a port:



shifting reference plane

$$S'_{11} = S_{11}\, e^{-j2\beta \ell}$$

$$S'_{21} = S_{21}\, e^{-j\beta \ell}$$

the additional phase is how much added length is in the path

We can do algebra on the S-parameters:

For example, stability requires S11 < 1:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 - |S_{11}\ S_{22} - S_{12}\ S_{21}|^2}{2\,|S_{12}\ S_{21}|} > 1$$

with $|S_{11}\ S_{22} - S_{12}\ S_{21}| < 1$

...and they are all complex numbers...

We can make sense of it by substituting English in equations

for amplifier:

S_{11} small is good

S_{21} medium is good

S_{22} small is good

S_{12} small is essential

so auxiliary equation becomes:

$$\left| S_{11} \ S_{22} - S_{12} \ S_{21} \right| < 1$$

$$\left| (\text{small})(\text{small}) - (\text{tiny})(\text{medium}) \right| < 1$$

we can see why input and output match should be small, why gain should not be too big, and why reverse isolation should be a very small number....

The Rollet Stability Factor becomes:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 - |S_{11}S_{22} - S_{12}S_{21}|^2}{2|S_{12}S_{21}|} > 1$$

$$K = \frac{1 - |(\text{small})|^2 - |(\text{small})|^2 - |(\text{small})(\text{small}) - (\text{tiny})(\text{medium})|^2}{2|(\text{tiny})(\text{medium})|} > 1$$

for a good amplifier.

Myriad combinations are possible:

Stability Circles on the Smith Chart for S11 and S22

Gain Circles on the Smith Chart for S11 and S22

Noise Figure Circles on the Smith Chart -- based on measurement

Efficiency Circles -- based mostly on load pull measurements

Distortion Circles -- based primarily on load pull measurements

...and many others....