$L \subseteq \mathbb{R}^3$, $t \in [a, b]$

$\{\gamma(t) \mid t \in [a, b]\} = L$

$|\gamma'(t)| \neq 0 \ \forall t \Leftrightarrow \gamma(t)$ – регулярная

$S(t) = \int_a^t |\gamma'(\varphi)| \, d\varphi$

$S(t) = f(t)$

$t = f^{-1}(S) \Rightarrow \gamma(S(t))$ – кривая един. скорости

и новый парам. S назыв. естественной пар. кривой.

ф. Френе:

$$\begin{cases} \vec{v}'(s) = k(s)\,\vec{n}(s) \\ \vec{n}'(s) = -k(s)\vec{v}(s) + \varkappa(s)\vec{b}(s) \\ \vec{b}'(s) = -\varkappa(s)\vec{n}(s) \end{cases}$$

$\{k(s) = h(s),\ \varkappa(s) = \eta(s)\}$ — натур. ур-я кривой

Задача 1

Найти нат. пар. $\vec{\gamma}(t) = (t, 2t)$, $t \in [0, 10]$

$\gamma' = (1, 2)$ $|\gamma'| = \sqrt{1+4} = \sqrt{5}$

$S(t) = \int_0^t |\gamma'| \, d\varphi = \int_0^t \sqrt{5}\, d\varphi$

s

t

γ

$= \sqrt{5}\, t$

$t = \frac{s}{\sqrt{5}}$

$r(s) = \left(\frac{s}{\sqrt{5}}, \frac{2s}{\sqrt{5}}\right)$

$= \sqrt{5}\, t$

Задача 2

Найти натур. ур-я кривой

$\vec{r}(t) = \left(at, a\sqrt{2}\ln t, \frac{a}{t}\right), a>0,$

$t \in [1, 10]$

$\left(k(s), \varkappa(s)\right)$

1. Найти $k(t)$, $\varkappa(t)$
2. Найти $s(t)$, выразить $t(s)$
3. $t \mapsto s$ в $k(t), \varkappa(t)$

$$k(t) = \frac{|[\vec{r}\,'(t), \vec{r}\,''(t)]|}{|\gamma'(t)|^3}, \quad \varkappa(t) = \frac{(\gamma'(t), \gamma''(t), \gamma'''(t))}{|[\gamma'(t), \gamma''(t)]|^2}$$

$$\overline{\gamma} = \left(a, \frac{a\sqrt{2}}{t}, -\frac{a}{t^2}\right)$$

$$|\overline{\gamma}| = \sqrt{a^2 + \frac{2a^2}{t^2} + \frac{a^2}{t^4}} =$$

$$= a + \frac{a}{t^2}$$

$$2.\quad s(t) = \int_1^t |\gamma'(\varphi)|\, d\varphi = a \int_1^t \left[1 + \frac{1}{\varphi^2}\right] d\varphi =$$

$$= \left[\left(at - \cancel{a}\right) + \left(-\frac{a}{t} + \cancel{a}\right)\right] = a\left(t - \frac{1}{t}\right)$$

$$s(t) = a\left(t - \frac{1}{t}\right)$$

$$s(t) = at - \frac{a}{t}$$

$$st = at^2 - a$$

$$at^2 - st - a = 0$$

$$D = s^2 + 4a^2$$

$$t_{1,2} = \frac{s \pm \sqrt{s^2 + 4a^2}}{2a}$$

$$t = \frac{s + \sqrt{s^2 + 4a^2}}{2a}$$

$$\gamma(s(t)) = \left(\frac{s + \sqrt{s^2 + 4a^2}}{2}\right)$$

Задача 3 $(\vec a, \vec b + \vec c) = (\vec a, \vec b) + (\vec a, \vec c)$

$(\alpha \vec a, \vec b) = \alpha (\vec a, \vec b)$

Кривая задана нат. пар-ем $\vec\gamma(s)$

Доказать, пользуясь ф. Френе $\vec\gamma'(s) \cdot \vec\gamma'''(s) = -k^2(s)$

$\bar v' = k \cdot \bar n$

$\bar n' = -k \bar v + \varkappa \bar b$

$\bar b' = -\varkappa \bar n$

$\vec\gamma'(s) = \vec v(s) \Rightarrow \vec\gamma''(s) = \vec v'(s) = k(s) \cdot \bar n$

$\vec\gamma'''(s) = k'(s) \cdot \bar n + k(s) \bar n' = k'(s) \cdot \bar n - k^2(s) \cdot \bar v + k(s) \cdot \varkappa \bar b$

$(\vec\gamma'(s), \vec\gamma'''(s)) = \cancel{(v, k'(s) \vec n)} + (v, -k^2(s) v) + \cancel{(v, k(s) \varkappa \bar b)} = -k^2(s)$ ЧТД

~~доделать~~

$a\sqrt{2} \ln\left(\frac{s + \sqrt{s^2 + 4a^2}}{2a}, \frac{2a^2}{s + \sqrt{s^2 + 4a^2}}\right)$

Д/З

Найти натур. ур. кривой

1) $\gamma(t) = (a\cos t, a\sin t, bt)$

2) $\gamma(t) = \left(\frac{t^2}{2}, \frac{2t^3}{3}, \frac{t^4}{2}\right), t \geq 0$

3) $\gamma(t) = (a\,\mathrm{ch}\,t, b\,\mathrm{sh}\,t, at), a > 0$