Общее задание

Найти натуральное уравнение кривой

Кривизна:

$$k(t) = rac{\left|\left[\overline{\gamma}', \overline{\gamma}''
ight]
ight|}{\left|\gamma'
ight|^3}$$

Кручение:

$$\Xi(t) = rac{\left(ar{\gamma}', ar{\gamma}'', ar{\gamma}'''
ight)}{\left|\left[ar{\gamma}', ar{\gamma}''
ight]
ight|^2}$$

N₂1

Условие

$$\gamma(t) = (a\cos(t), a\sin(t), bt) \quad t \ge 0$$

Решение

Кривизна:

$$k(t) = rac{\left|\left[ar{\gamma}',ar{\gamma}''
ight]
ight|}{\left|\gamma'
ight|^3}$$

Кручение:

$$\Xi(t) = rac{\left(ar{\gamma}', ar{\gamma}'', ar{\gamma}'''
ight)}{\left|\left[ar{\gamma}', ar{\gamma}''
ight]
ight|^2}$$

$$egin{aligned} \gamma'(t) &= (-a\sin(t), a\cos(t), b) \ &|\gamma'(t)| &= \sqrt{a\sin^2(t) + a^2\cos^2(t) + b^2} = \sqrt{a^2 + b^2} \ \gamma''(t) &= (-a\cos(t), -a\sin(t), 0) \ \gamma'''(t) &= (a\sin(t), -a\cos(t), 0) \end{aligned}$$

$$egin{aligned} & [\gamma',\gamma''] = \left(ab\sin(t),-ab\cos(t),a^2
ight) \ & | [\gamma',\gamma''] | = a\sqrt{a^2+b^2} \ & k(t) = rac{a}{a^2+b^2} \end{aligned}$$

$$egin{aligned} & \left[\gamma'',\gamma'''
ight] = \left(0,0,a^2
ight) \ & \left(\gamma',\gamma'',\gamma'''
ight) = a^2b \ & \Xi(t) = rac{b}{a^2+b^2} \end{aligned}$$

$$egin{aligned} s(t) &= \int\limits_{1}^{t} |\gamma'(\phi)| d\phi = \sqrt{b^2 + a^2} t \ t &= rac{s}{\sqrt{a^2 + b^2}} \ \gamma(s(t)) &= \left(a\cos(rac{s}{\sqrt{a^2 + b^2}}), a\sin(rac{s}{\sqrt{a^2 + b^2}}), rac{bs}{\sqrt{a^2 + b^2}}
ight) \ k(s) &= rac{a}{a^2 + b^2} \ \Xi(s) &= rac{b}{a^2 + b^2} \end{aligned}$$

№2

Условие

$$\gamma(t)=\left(rac{t^2}{2},rac{2t^3}{3},rac{t^4}{2}
ight)\quad t\geq 0$$

Решение

 $\gamma'(t)=\left(t,2t^2,2t^3
ight)$

$$\begin{split} |\gamma'(t)| &= \sqrt{t^2 + 4t^4 + 4t^6} = 2t^3 + t \\ \gamma''(t) &= \left(1, 4t, 6t^2\right) \\ [\gamma', \gamma''] &= \left(4t^4, -4t^3, 2t^2\right) \\ |[\gamma', \gamma'']| &= 2t^2 \left(2t^2 + 1\right) \\ \gamma'''(t) &= \left(0, 4, 12t\right) \\ [\gamma'', \gamma'''] &= \left(24t^2, -12t, 4\right) \\ |[\gamma'', \gamma''']| &= 4\sqrt{36t^4 + 9t^2 + 1} \\ (\gamma', \gamma'', \gamma''') &= 8t^3 \\ k(t) &= \frac{2}{t(2t^2 + 1)^2} \\ \Xi(t) &= \frac{t^3}{2\left(36t^4 + 9t^2 + 1\right)} \\ s(t) &= \int_0^t |\gamma'(\phi)| d\phi = \int_0^t \left[2\phi^3 + \phi\right] d\phi = \frac{t^4 + t^2}{2} \\ t_{1,2} &= \pm \frac{\sqrt{\sqrt{8s + 1} - 1}}{\sqrt{2}} \\ t_{3,4} &= \pm i \frac{\sqrt{\sqrt{8s + 1} + 1}}{\sqrt{2}} \end{split}$$

Так как
$$t \geq 0$$
, то $t = \frac{\sqrt{\sqrt{8s+1}-1}}{\sqrt{2}}$
$$k(s) = \frac{\sqrt{\sqrt{8s+1}-1}}{\sqrt{2}} \left(2\left(\frac{\sqrt{\sqrt{8s+1}-1}}{\sqrt{2}}\right)^2 + 1\right)^2$$

$$\left(\frac{\sqrt{\sqrt{8s+1}-1}}{\sqrt{2}}\right)^3$$

$$\Xi(s) = \frac{2\left(36\left(\frac{\sqrt{\sqrt{8s+1}-1}}{\sqrt{2}}\right)^4 + 9\left(\frac{\sqrt{\sqrt{8s+1}-1}}{\sqrt{2}}\right)^2 + 1\right)$$

Nº3

Условие

$$\gamma(t) = (a \cdot \cosh(t), b \cdot \sinh(t), at) \quad a > 0$$

Решение

 $\Xi(s) = rac{o}{a^2 + b^2 + s^2}$

$$\begin{split} \gamma'(t) &= (a \sinh(t), b \cosh(t), a) \\ \gamma''(t) &= (a \cosh(t), b \sinh(t), 0) \\ \gamma'''(t) &= (a \sinh(t), b \cosh(t), 0) \\ k(t) &= \frac{a}{\cosh^2(t) \left(a^2 + b^2\right)} \\ \Xi(t) &= \frac{b}{\cosh^2(t) \left(a^2 + b^2\right)} \\ s(t) &= \int_0^t \sqrt{a^2 \left(\sinh^2(\phi) + 1\right) + b^2 \cosh(\phi)} d\phi = \sqrt{a^2 + b^2 \sinh(t)} \\ t &= \operatorname{arcsinh}\left(\frac{s}{\sqrt{a^2 + b^2}}\right) \\ \gamma &= \left(\operatorname{arcsinh}\left(\frac{s}{\sqrt{a^2 + b^2}}\right), a, b\right) \\ \cosh\left(\operatorname{arcsinh}x\right) &= \sqrt{1 + x^2} \\ \cosh^2\left(\operatorname{arcsinh}\left(\frac{s}{\sqrt{a^2 + b^2}}\right)\right) &= 1 + \left(\frac{s}{\sqrt{a^2 + b^2}}\right)^2 = \frac{a^2 + b^2 + s^2}{a^2 + b^2} \\ k(s) &= \frac{a}{a^2 + b^2 + s^2} \end{split}$$