① а) $X = \{a, b, c, d\}$

$\Delta = \{\{a,b\}, \{b,c\}, \{c,d\}\}$

1. $\Sigma_\Delta = \{\{a,b\}, \{b,c\}, \{c,d\}, \{b\}, \{c\}, \varnothing\}$
2. $\tau_{\Sigma_\Delta} = \{\varnothing, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{b,c,d\}, \{a,b,c,d\}\}$

б) F - замк, U - откр. $(F\setminus U,\ U\setminus F\ ?)$

I. $F\setminus U \xleftarrow{\text{замк}} \underbrace{F \cap \underbrace{(X\setminus U)}_{\text{замк}}}_{\text{зам.}}$ $\qquad U\setminus F = \underbrace{U \cap \underbrace{(X\setminus F)}_{\text{откр}}}_{\text{откр.}}$

II. $F = X\setminus U_1,\ U = X\setminus F_1$

$F\setminus U = (X\setminus U_1)\setminus U = \underbrace{X\setminus \underbrace{(U_1 \cup U)}_{\text{откр}}}_{\text{замк.}}$

$U\setminus F = (X\setminus F_1)\setminus F = \underbrace{X\setminus \underbrace{(F_1 \cup F)}_{\text{зам}}}_{\text{откр}}$

② $f(\mathrm{Cl}\,A) = \mathrm{Cl}\,f(A)$

1. $f(\mathrm{Cl}\,A)$ — замк // $\mathrm{Cl}\,A$ — замк., f — гомео

   $f(X \setminus A) = Y \setminus f(A)$, f^{-1} — гом.

2. $f(A) \subseteq f(\mathrm{Cl}\,A)$ // $A \subseteq \mathrm{Cl}\,A$

3. $\mathrm{Cl}\,f(A) \subseteq f(\mathrm{Cl}\,A)$ // $\mathrm{Cl}\,f(A)$ — наим. замк. мн., содерж. $f(A)$

4. $\mathrm{Cl}\,A \subseteq \underbrace{f^{-1}\underbrace{\mathrm{Cl}\,f(A)}_{\text{замк}}}_{\text{замк.}}$ // $A \subseteq f^{-1}\,\mathrm{Cl}\,f(A)$ // $f(A) \subseteq \mathrm{Cl}\,f(A)$

5. $f(\mathrm{Cl}\,A) \subseteq \mathrm{Cl}\,f(A)$

③ $f : (X, \rho_X) \to (Y, \rho_Y)$:

$\forall a, b \in X \quad \rho_Y(f(a), f(b)) = \rho_X(a, b)$

а) f – инъ.?

$\exists\, f(a) = f(b) \overset{?}{\Rightarrow} a = b$

$$\Downarrow$$

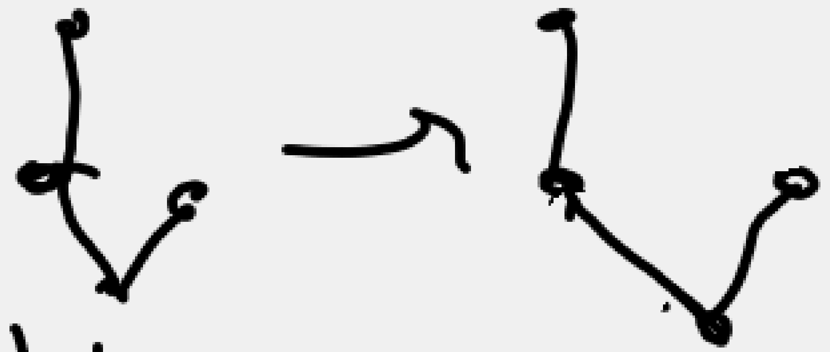
$\rho_Y(f(a), f(b)) = 0$

$\|$

$\rho_X(a, b) \Rightarrow a = b$

б) I. $\forall x_0 \in X \ \forall \varepsilon > 0 \ \exists \delta = \varepsilon > 0 : \forall x : \rho_X(x, x_0) < \overset{\varepsilon}{\overset{\|}{\delta}} \Rightarrow \rho_Y(f(x), f(x_0)) < \varepsilon$

II. $\forall \underbrace{B(f(x_0), \varepsilon)}_{\text{откр}} \ f^{-1}(B(f(x_0), \varepsilon)) = \underbrace{B(x_0, \varepsilon)}_{\text{открыто}}$

④  найти все гомеом

$$X = \{a, b, c, d\}$$

$$\tau = \{\varnothing, X, \{a\}, \{b\}, \{a, c\}, \{a, b, c\}, \{a, b\}\}$$

id — гомеом.

$\exists\, f(a) \neq a \Rightarrow f^{-1}(\{a\}) \neq \{a\} = \{c\}$

$\quad \neq b \qquad\qquad\quad \neq \{b\} \quad = \{d\} \notin \tau$

$\exists\, f(c) \neq c \Rightarrow f^{-1}(\{a, c\}) = \{a, b\}$

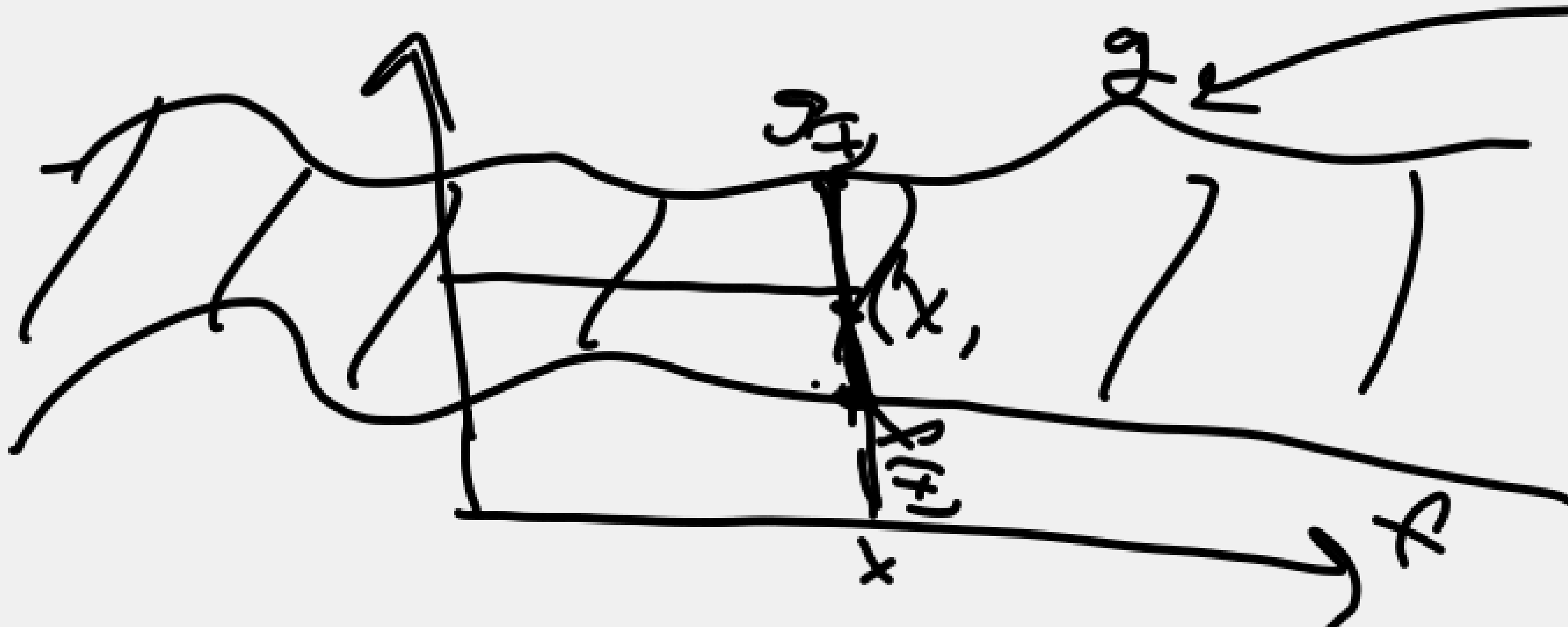
$\qquad\qquad\qquad f^{-1}(\{a, b, c\}) = \{a, b, d\} \notin \tau$

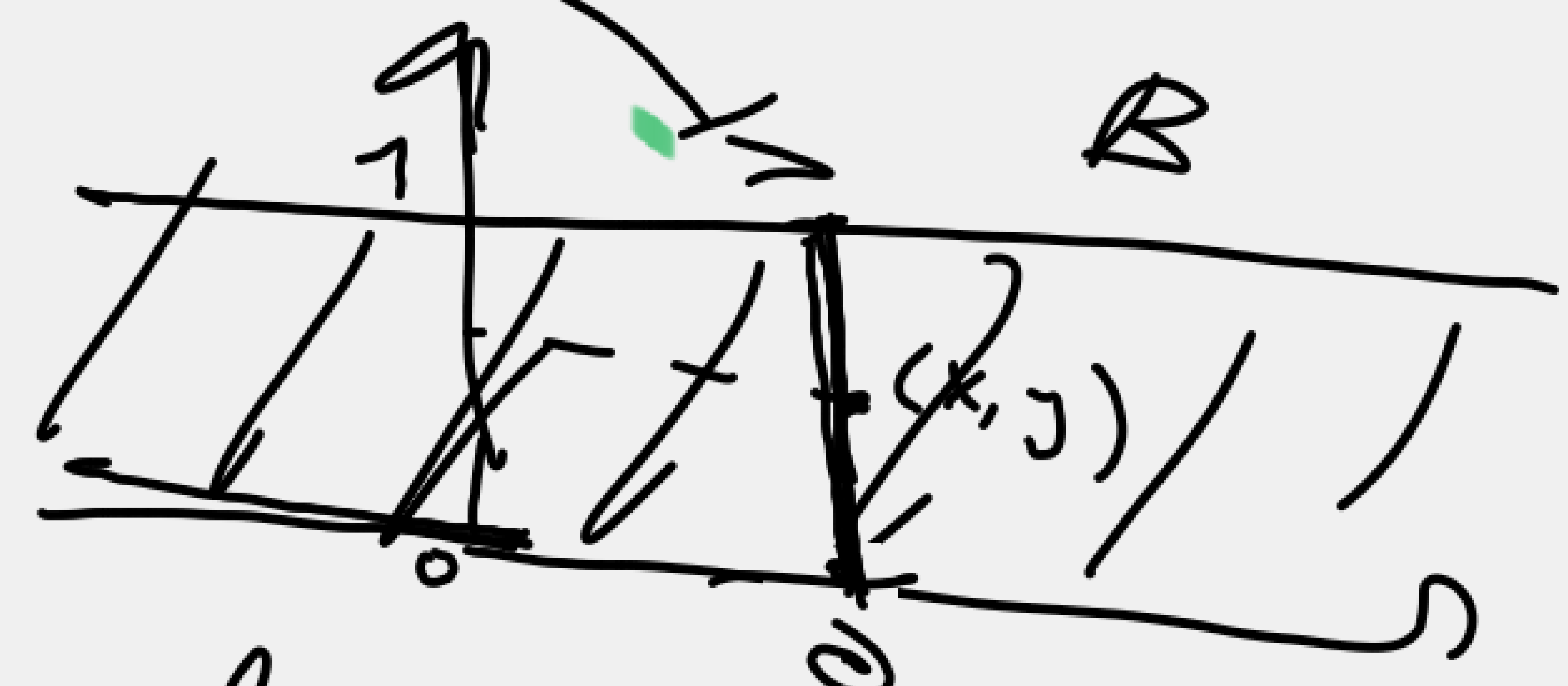
$\exists\, f(d) \neq d \Rightarrow f^{-1}(X) \neq \{a, b, c, d\}$ ↯

(5) $f, g : \mathbb{R} \to \mathbb{R}$, $f < g$

$$A = \{(x,y) \mid x \in \mathbb{R},\ f(x) \le y \le g(x)\} \simeq \overset{B}{\overbrace{\{(x,y) \mid x \in \mathbb{R},\ y \in [0,1]\}}}$$

A





$h : B \to A$

$$\underbrace{h(x,y)}_{(x,\varphi)} = (x, (1-y) f(x) + y\, g(x)) \text{ – hom.} \implies \varphi = f(x) - y f(x) + y\, g(x)$$

$$\frac{\varphi - f(x)}{g(x) - f(x)} = y$$

$A, B \quad (1-t)A + tB$

$t \in [0,1]$

$$h^{-1}(x,\varphi) = \left(x, \frac{\varphi - f(x)}{g(x) - f(x)}\right)$$

$[\alpha, \beta]$,

$\gamma : [0, 1] \longrightarrow \mathbb{R}^3$ — параметризация кривой

$\gamma = (\gamma^1(t), \gamma^2(t), \gamma^3(t))$

точка $A(x_0, y_0, z_0)$

$$\frac{x - x_0}{a_x} = \frac{y - y_0}{a_y} = \frac{z - z_0}{a_z}$$

напр вектор $\vec{a}$

1. $\gamma(t) = (t^2, t^3, t^4)$

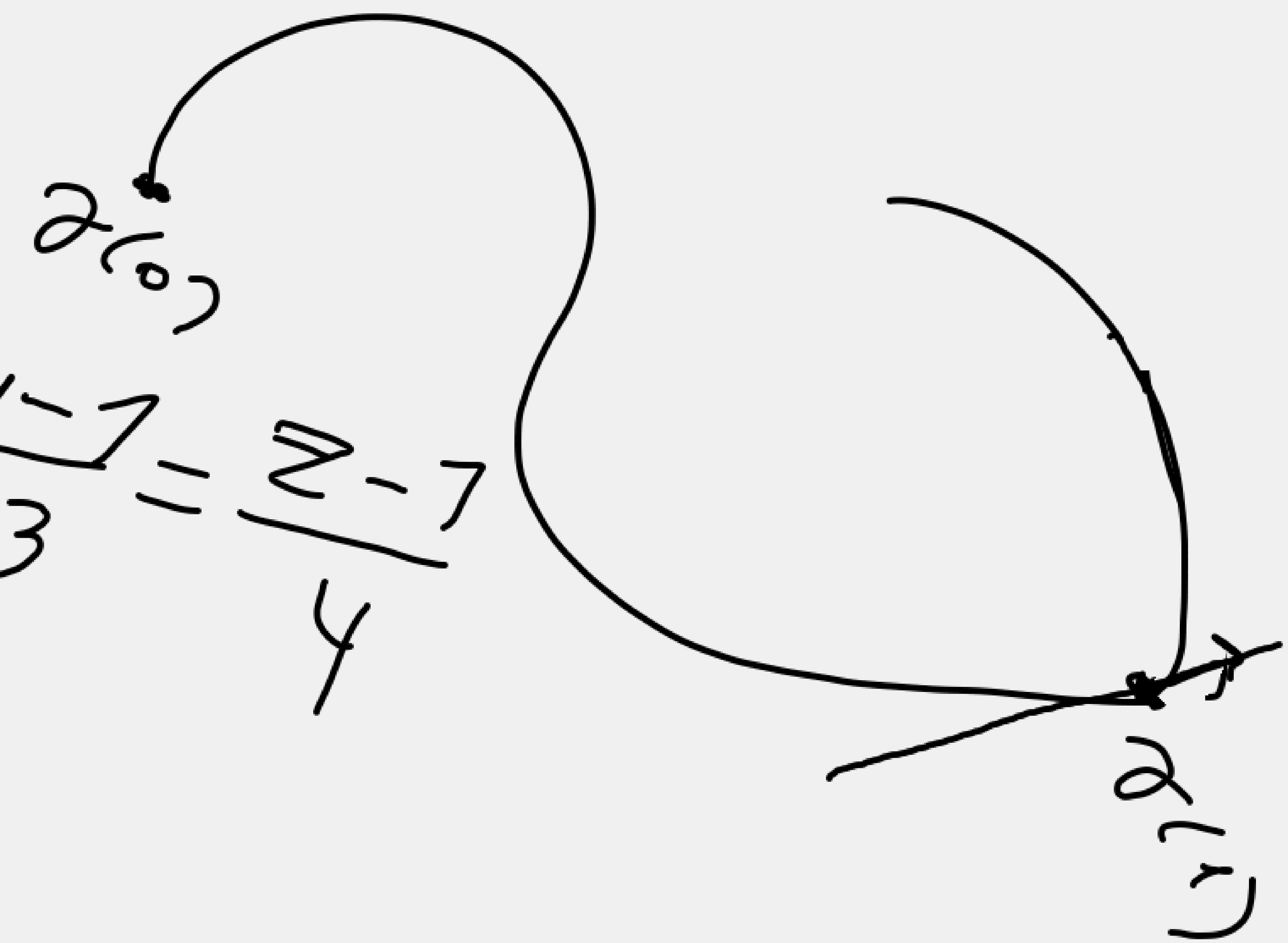
$t = 1$. Найти ур. касат.

$\vec{\gamma}'(t) = (2t, 3t^2, 4t^3)$

$\gamma'(1) = (2, 3, 4)$

$\gamma(1) = (1, 1, 1)$

$$\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{4}$$



$z_0)$

2) $\gamma(t) = (e^t, \sin t, \cos t), \; t = 0$

$$\vec{\gamma}'(t) = (e^t, \cos t, -\sin t)$$

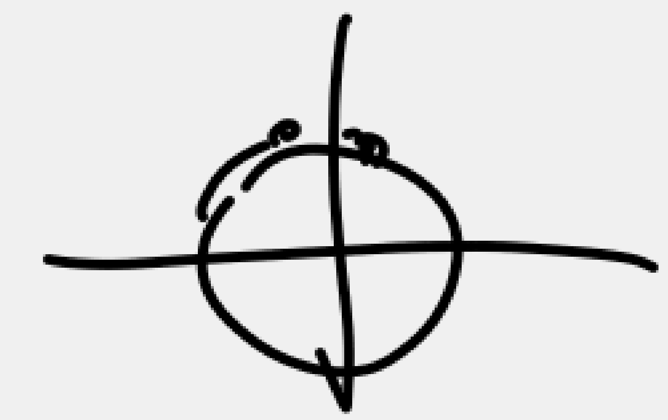
$$\gamma(0) = (1, 0, 1) \qquad \vec{r}(t) = \overset{\vec{a}}{\vec{\gamma}'(0)}\, t + \overset{\vec{r}(0)}{\vec{\gamma}(0)}$$

$$\frac{x-1}{1} = \frac{y}{1} = \frac{z-1}{0} \iff \begin{cases} x = 1t + 1 \\ y = 1t \\ z = 1 \end{cases}$$

② Найти длину дуги кривой

1) $\gamma(t) = (6\cos^3 t, 6\sin^3 t)$, $0 \le t \le \frac{\pi}{3}$

$$l(\gamma)\Big|_{t_1}^{t_2} = \int_{t_1}^{t_2} |\gamma'(t)|\,dt$$



$$\gamma'(-18\cos^2 t\cdot\sin t,\ 18\sin^2 t\cdot\cos t)$$

$$|\gamma'| = \sqrt{18^2(\cos^4 t\,\sin^2 t + \sin^4 t\,\cos^2 t)} = \quad \sin t\cos t\sqrt{\cos^2 t+\sin^2 t}$$

$$= 18\sin t\cos t$$

$$l(\gamma) = \frac{18}{2}\int_0^{\frac{\pi}{3}} \underbrace{\sin t\cos t}_{\sin 2t}\,dt = 9\int \sin 2t\,dt = -\frac{9}{2}\cos 2t\Big|_0^{\frac{\pi}{3}} = -\frac{9}{2}\left(-\frac{1}{2} - 1\right) =$$

$$= +\frac{9}{2}\cdot\frac{3}{2} =$$

2) $y = e^x$, $\underbrace{\frac{1}{2}\ln 3}_{a} \le x \le \underbrace{\frac{1}{2}\ln 8}_{b}$

$\Updownarrow$

$\gamma(x) = (x, e^x) \longmapsto \gamma'(x) = (1, e^x) \longmapsto |\gamma'(x)| =$

$$= \int_a^b \underbrace{\sqrt{1+e^{2x}}}_{\Downarrow} \, dx = \left| \begin{array}{l} u = \sqrt{1+e^{2x}} \\ du = \frac{e^{2x}}{\sqrt{1+e^{2x}}} \, dx \\ u_1 = \sqrt{1+e^{2\cdot\frac{1}{2}\ln 3}} = 2 \\ u_2 = \sqrt{1+e^{\ln 8}} = 3 \end{array} \right| = \int_2^3 \frac{u^2}{u^2-1} \, du$$

=

=

№13

$\gamma(t) = (t, \sin t, \cos t)$

1) Найти ур. касат. в $t=0$
2) Найти длину дуги кривой при $t \in [0, 2\pi]$
3) Найти базис Френе в $t=0$

$$\left(\vec{v}, \vec{n}, \vec{b}\right)$$

$$\vec{v} = \frac{\gamma'(t)}{|\gamma'(t)|}, \quad \vec{n} = \frac{\vec{v}'}{|\vec{v}'|}, \quad \vec{b} = \frac{[\vec{v}, \vec{n}]}{|[\vec{v}, \vec{n}]|}$$