Диффуры IDZ @all

Диффуры IDZ 1 (796-812)

796

$$\begin{cases} x' = x - y + z \\ y' = x + y - z \\ z' = 2x - y \end{cases}$$

$$\begin{pmatrix} 1-\lambda & -1 & 1 \\ 1 & 1-\lambda & -1 \\ 2 & -1 & -\lambda \end{pmatrix}$$

$$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = -1$$

$$(1)\begin{cases} -\beta + \gamma = 0 \\ \alpha - \gamma = 0 \\ 2\alpha - \beta - \gamma = 0 \end{cases} \iff \begin{cases} \beta = \gamma \\ \alpha = \gamma \\ 2\alpha - \beta - \gamma = 0 \end{cases} \iff \begin{cases} \alpha = 1 \\ \beta = 1 \\ \gamma = 1 \end{cases}$$

$$(2)\begin{cases} -\alpha - \beta + \gamma = 0 \\ \alpha - \beta - \gamma = 0 \\ 2\alpha - \beta - 2\gamma = 0 \end{cases} \iff \begin{cases} \alpha = \gamma \\ \beta = 0 \\ 2\alpha - 2\gamma = 0 \end{cases} \iff \begin{cases} \alpha = 1 \\ \beta = 0 \\ \gamma = 1 \end{cases}$$

$$(3)\begin{cases} 2\alpha - \beta + \gamma = 0 \\ \alpha + 2\beta - \gamma = 0 \\ 2\alpha - \beta + \gamma = 0 \end{cases} \iff \begin{cases} -5\beta + 3\gamma = 0 \\ 3\alpha + \beta = 0 \end{cases} \iff \begin{cases} \alpha = -1 \\ \beta = 3 \\ \gamma = 5 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_0 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^t + C_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} e^{-t}$$

797

$$\begin{cases} x' = x - 2y - z \\ y' = -x + y + z \\ z' = x - z \end{cases}$$

$$\begin{pmatrix} 1-\lambda & -2 & -1 \\ -1 & 1-\lambda & 1 \\ 1 & 0 & -1-\lambda \end{pmatrix}$$

$$\lambda_1 = 0 \quad \lambda_2 = 2 \quad \lambda_3 = -1$$

$$(1)\begin{cases} \alpha - 2\beta - \gamma = 0 \\ -\alpha + \beta + \gamma = 0 \\ \alpha - \gamma = 0 \end{cases} \iff \begin{cases} \alpha = 1 \\ \beta = 0 \\ \gamma = 1 \end{cases}$$

$$(2)\begin{cases} -\alpha - 2\beta - \gamma = 0 \\ -\alpha - \beta + \gamma = 0 \\ \alpha - 3\gamma = 0 \end{cases} \iff \begin{cases} \alpha = 3 \\ \beta = -2 \\ \gamma = 1 \end{cases}$$

$$(3)\begin{cases} 2\alpha - 2\beta - \gamma = 0 \\ -\alpha + 2\beta + \gamma = 0 \\ \alpha = 0 \end{cases} \iff \begin{cases} -2\beta - \gamma = 0 \\ \alpha = 0 \end{cases} \iff \begin{cases} \alpha = 0 \\ \beta = 1 \\ \gamma = -2 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_0 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + C_1 \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} e^{-t}$$

798

$$\begin{cases} x' = 2x - y + z \\ y' = x + 2y - z \\ z' = x - y + 2z \end{cases}$$

$$\begin{pmatrix} 2-\lambda & -1 & 1 \\ 1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{pmatrix}$$

$$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = 3$$

$$(1) \begin{cases} \alpha - \beta + \gamma = 0 \\ \alpha + \beta - \gamma = 0 \\ \alpha - \beta + \gamma = 0 \end{cases} \Longleftrightarrow \begin{cases} \alpha = 0 \\ \beta = 1 \\ \gamma = 1 \end{cases}$$

$$(2) \begin{cases} -\beta + \gamma = 0 \\ \alpha - \gamma = 0 \\ \alpha - \beta = 0 \end{cases} \Longleftrightarrow \begin{cases} \alpha = 1 \\ \beta = 1 \\ \gamma = 1 \end{cases}$$

$$(3) \begin{cases} -\alpha - \beta + \gamma = 0 \\ \alpha - \beta - \gamma = 0 \\ \alpha - \beta - \gamma = 0 \end{cases} \Longleftrightarrow \begin{cases} \alpha = 1 \\ \beta = 0 \\ \gamma = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_0 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} e^t + C_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{3t}$$

799

$$\begin{cases} x' = 3x - y + z \\ y' = x + y + z \\ z' = 4x - y + 4z \end{cases}$$

$$\begin{pmatrix} 3-\lambda & -1 & 1 \\ 1 & 1-\lambda & 1 \\ 4 & -1 & 4-\lambda \end{pmatrix}$$

$$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = 5$$

$$(1) \begin{cases} 2\alpha - \beta + \gamma = 0 \\ \alpha + \gamma = 0 \\ 4\alpha - \beta + 3\gamma = 0 \end{cases} \Longleftrightarrow \begin{cases} \alpha = 1 \\ \beta = 1 \\ \gamma = -1 \end{cases}$$

$$(2) \begin{cases} \alpha - \beta + \gamma = 0 \\ \alpha - \beta + \gamma = 0 \\ 4\alpha - \beta + 2\gamma = 0 \end{cases} \Longleftrightarrow \begin{cases} \alpha - \beta + \gamma = 0 \\ 3\alpha + \gamma = 0 \end{cases} \Longleftrightarrow \begin{cases} \alpha = 1 \\ \beta = -2 \\ \gamma = -3 \end{cases}$$

$$(3) \begin{cases} -2\alpha - \beta + \gamma = 0 \\ \alpha - 4\beta + \gamma = 0 \\ 4\alpha - \beta - \gamma = 0 \end{cases} \Longleftrightarrow \begin{cases} \alpha = \beta \\ 4\alpha - \beta - \gamma = 0 \end{cases} \Longleftrightarrow \begin{cases} \alpha = 1 \\ \beta = 1 \\ \gamma = 3 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_0 \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} e^t + C_1 \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} e^{5t}$$

800

$$\begin{cases} x' = -3x + 4y - 2z \\ y' = x + z \\ z' = 6x - 6y + 5z \end{cases}$$

$$\begin{pmatrix} -3-\lambda & 4 & -2 \\ 1 & -\lambda & 1 \\ 6 & -6 & 5-\lambda \end{pmatrix}$$

$$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = -1$$

$$(1) \begin{cases} -4\alpha + 4\beta - 2\gamma = 0 \\ \alpha - \beta + \gamma = 0 \\ 6\alpha - 6\beta + 4\gamma = 0 \end{cases} \iff \begin{cases} \alpha = 1 \\ \beta = 1 \\ \gamma = 0 \end{cases}$$

$$(2) \begin{cases} -5\alpha + 4\beta - 2\gamma = 0 \\ \alpha - 2\beta + \gamma = 0 \\ 6\alpha - 6\beta + 3\gamma = 0 \end{cases} \iff \begin{cases} \alpha = 0 \\ \beta = 1 \\ \gamma = 2 \end{cases}$$

$$(3) \begin{cases} -2\alpha + 4\beta - 2\gamma = 0 \\ \alpha + \beta + \gamma = 0 \\ 6\alpha - 6\beta + 6\gamma = 0 \end{cases} \iff \begin{cases} \alpha = -1 \\ \beta = 0 \\ \gamma = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_0 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} e^t + C_1 \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} e^{-t}$$

801

$$\begin{cases} x' = x - y - z \\ y' = x + y \\ z' = 3x + z \end{cases}$$

$$\begin{pmatrix} 1-\lambda & -1 & -1 \\ 1 & 1-\lambda & 0 \\ 3 & 0 & 1-\lambda \end{pmatrix}$$

$$\lambda_1 = 1 \quad \lambda_{2,3} = 1 \pm 2i$$

$$(1) \begin{cases} -\beta - \gamma = 0 \\ \alpha = 0 \\ 3\alpha = 0 \end{cases} \iff \begin{cases} \alpha = 0 \\ \beta = -1 \\ \gamma = 1 \end{cases}$$

$$(2) \begin{cases} -2i\alpha - \beta - \gamma = 0 \\ \alpha - 2i\beta = 0 \\ 3\alpha - 2i\gamma = 0 \end{cases} \iff \begin{cases} \alpha = 2i \\ \beta = 1 \\ \gamma = 3 \end{cases}$$

$$\begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix} e^{(1+2i)t} = \begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix} e^t \cdot (\cos 2t + i \sin 2t)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_0 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} e^t + C_1 \begin{pmatrix} -2\sin 2t \\ \cos 2t \\ 3\cos 2t \end{pmatrix} e^t + C_2 \begin{pmatrix} 2\cos 2t \\ \sin 2t \\ 3\sin 2t \end{pmatrix} e^t$$

802

$$\begin{cases} x' = 2x + y \\ y' = x + 3y - z \\ z' = -x + 2y + 3z \end{cases}$$

$$\begin{pmatrix} 2-\lambda & 1 & 0 \\ 1 & 3-\lambda & -1 \\ -1 & 2 & 3-\lambda \end{pmatrix}$$

$$\lambda_1 = 2 \quad \lambda_{2,3} = 3 \pm i$$

$$(1)\begin{cases}\beta=0\\ \alpha+\beta-\gamma=0\\ -\alpha+2\beta+\gamma=0\end{cases}\iff\begin{cases}\alpha=1\\ \beta=0\\ \gamma=1\end{cases}$$

$$(2)\begin{cases}(-1-\mathrm{i})\alpha+\beta=0\\ \alpha-\mathrm{i}\beta-\gamma=0\\ -\alpha+2\beta-\mathrm{i}\gamma=0\end{cases}\iff\begin{cases}\alpha=1\\ \beta=1+\mathrm{i}\\ \gamma=2-\mathrm{i}\end{cases}$$

$$\begin{pmatrix}1\\1+i\\2-i\end{pmatrix}e^{(3+i)t}=\begin{pmatrix}1\\1+i\\2-i\end{pmatrix}e^{3t}\cdot(\cos t+i\sin t)$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=C_0\begin{pmatrix}1\\0\\1\end{pmatrix}e^{2t}+C_1\begin{pmatrix}\cos t\\ \cos t-\sin t\\ 2\cos t+\sin t\end{pmatrix}e^{3t}+C_2\begin{pmatrix}\sin t\\ \sin t+\cos t\\ 2\sin t-\cos t\end{pmatrix}e^{3t}$$

803

$$\begin{cases}x'=2x-y+2z\\ y'=x+2z\\ z'=-2x+y-z\end{cases}$$

$$\begin{pmatrix}2-\lambda & -1 & 2\\ 1 & -\lambda & 2\\ -2 & 1 & -1-\lambda\end{pmatrix}$$

$$\lambda_1=1\quad\lambda_{2,3}=\pm i$$

$$(1)\begin{cases}\alpha-\beta+2\gamma=0\\ \alpha-\beta+2\gamma=0\\ -2\alpha+\beta-2\gamma=0\end{cases}\iff\begin{cases}\alpha=0\\ \beta=2\\ \gamma=1\end{cases}$$

$$(2)\begin{cases}(2-\mathrm{i})\alpha-\beta+2\gamma=0\\ \alpha-\mathrm{i}\beta+2\gamma=0\\ -2\alpha+\beta-(1+\mathrm{i})\gamma=0\end{cases}\iff\begin{cases}(1-\mathrm{i})\alpha=(1+\mathrm{i})\beta\\ \alpha-\mathrm{i}\beta+2\gamma=0\\ -2\alpha+\beta-(1+\mathrm{i})\gamma=0\end{cases}\iff\begin{cases}\alpha=1+\mathrm{i}\\ \beta=1-\mathrm{i}\\ \gamma=-2-\mathrm{i}\end{cases}$$

$$\begin{pmatrix}1+i\\1-i\\-2-i\end{pmatrix}e^{it}=\begin{pmatrix}1+i\\1-i\\-2-i\end{pmatrix}\cdot(\cos t+i\sin t)$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=C_0\begin{pmatrix}0\\2\\1\end{pmatrix}e^{t}+C_1\begin{pmatrix}\cos t-\sin t\\ \cos t+\sin t\\ \sin t-2\cos t\end{pmatrix}+C_2\begin{pmatrix}\sin t+\cos t\\ \sin t-\cos t\\ -2\sin t-\cos t\end{pmatrix}$$

804

$$\begin{cases}x'=4x-y-z\\ y'=x+2y-z\\ z'=x-y+2z\end{cases}$$

$$\begin{pmatrix}4-\lambda & -1 & -1\\ 1 & 2-\lambda & -1\\ 1 & -1 & 2-\lambda\end{pmatrix}$$

$$\lambda_1=2\quad\lambda_2=\lambda_3=3$$

$$(1)\begin{cases}2\alpha-\beta-\gamma=0\\ \alpha-\gamma=0\\ \alpha-\beta=0\end{cases}\iff\begin{cases}\alpha=1\\ \beta=1\\ \gamma=1\end{cases}$$

$$(2)\begin{cases}\alpha-\beta-\gamma=0\\\alpha-\beta-\gamma=0\\\alpha-\beta-\gamma=0\end{cases}\iff\begin{cases}\alpha=1\\\beta=1\\\gamma=0\end{cases}\quad and\quad\begin{cases}\alpha=1\\\beta=0\\\gamma=1\end{cases}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=C_0\begin{pmatrix}1\\1\\1\end{pmatrix}e^{2t}+C_1\begin{pmatrix}1\\1\\0\end{pmatrix}e^{3t}+C_2\begin{pmatrix}1\\0\\1\end{pmatrix}e^{3t}$$

805

$$\begin{cases}x'=2x-y-z\\y'=3x-2y-3z\\z'=2x-4y\end{cases}$$

$$\begin{pmatrix}2-\lambda&-1&-1\\3&-2-\lambda&-3\\2&4&-\lambda\end{pmatrix}$$

$$\lambda_1=0\quad\lambda_2=\lambda_3=1$$

$$(1)\begin{cases}2\alpha-\beta-\gamma=0\\3\alpha-2\beta-3\gamma=0\\2\alpha+4\beta=0\end{cases}\iff\begin{cases}\alpha=-2\\\beta=1\\\gamma=-5\end{cases}$$

$$(2)\begin{cases}\alpha-\beta-\gamma=0\\3\alpha-3\beta-3\gamma=0\\2\alpha+4\beta-\gamma=0\end{cases}\iff\begin{cases}\alpha=-5\\\beta=1\\\gamma=-6\end{cases}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}\alpha_1t+\beta_1\\\alpha_2t+\beta_2\\\alpha_3t+\beta_3\end{pmatrix}e^t$$

$$\begin{pmatrix}x'\\y'\\z'\end{pmatrix}=\begin{pmatrix}\alpha_1t+\beta_1+\alpha_1\\\alpha_2t+\beta_2+\alpha_2\\\alpha_3t+\beta_3+\alpha_3\end{pmatrix}e^t$$

$$\begin{cases}(2\alpha_1-\alpha_2-\alpha_3)t+2\beta_1-\beta_2-\beta_3=\alpha_1t+\beta_1+\alpha_1\\(3\alpha_1-2\alpha_2-3\alpha_3)t+3\beta_1-2\beta_2-3\beta_3=\alpha_2t+\beta_2+\alpha_2\\(2\alpha_1+4\alpha_2)t+2\beta_1+4\beta_2=\alpha_3t+\beta_3+\alpha_3\end{cases}$$

$$\begin{cases}2\alpha_1-\alpha_2-\alpha_3=\alpha_1\\2\beta_1-\beta_2-\beta_3=\alpha_1+\beta_1\\3\alpha_1-2\alpha_2-3\alpha_3=\alpha_2\\3\beta_1-2\beta_2-3\beta_3=\alpha_2+\beta_2\\2\alpha_1+4\alpha_2=\alpha_3\\2\beta_1+4\beta_2=\alpha_3+\beta_3\end{cases}\iff\begin{cases}\alpha_1-\alpha_2-\alpha_3=0\\\beta_1-\beta_2-\beta_3=\alpha_1\\3\beta_1-3\beta_2-3\beta_3=\alpha_2\\2\alpha_1+4\alpha_2-\alpha_3=0\\2\beta_1+4\beta_2-\beta_3=\alpha_3\end{cases}\iff\begin{cases}\alpha_1=5C_0\\\alpha_2=C_0\\\alpha_3=4C_0\\\beta_1=-C_0-5C_1\\\beta_2=C_1\\\beta_3=-6C_0-6C_1\end{cases}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=C_0\begin{pmatrix}-2\\1\\-5\end{pmatrix}+\begin{pmatrix}5C_0t-C_0-5C_1\\C_0t+C_1\\4C_0t-6C_0-6C_1\end{pmatrix}e^t$$

806

$$\begin{cases}x'=-2x+y-2z\\y'=x-2y+2z\\z'=3x-3y+5z\end{cases}$$

$$\begin{pmatrix}-2-\lambda&1&-2\\1&-2-\lambda&2\\3&-3&5-\lambda\end{pmatrix}$$

$$\lambda_1=3\quad\lambda_2=\lambda_3=-1$$

$$(1)\begin{cases}-5\alpha+\beta-2\gamma=0\\ \alpha-5\beta+2\gamma=0\\ 3\alpha-3\beta+2\gamma=0\end{cases}\iff\begin{cases}\alpha=-1\\ \beta=1\\ \gamma=3\end{cases}$$

$$(2)\begin{cases}-\alpha+\beta-2\gamma=0\\ \alpha-\beta+2\gamma=0\\ 3\alpha-3\beta+6\gamma=0\end{cases}\iff\begin{cases}\alpha=2\\ \beta=0\\ \gamma=-1\end{cases}\quad and\quad\begin{cases}\alpha=0\\ \beta=2\\ \gamma=1\end{cases}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=C_0\begin{pmatrix}-1\\1\\3\end{pmatrix}e^{3t}+C_1\begin{pmatrix}2\\0\\-1\end{pmatrix}e^{-t}+C_2\begin{pmatrix}0\\2\\1\end{pmatrix}e^{-t}$$

807

$$\begin{cases}x'=3x-2y-z\\ y'=3x-4y-3z\\ z'=2x-4y\end{cases}$$

$$\begin{pmatrix}3-\lambda & -2 & -1\\ 3 & -4-\lambda & -3\\ 2 & -4 & -\lambda\end{pmatrix}$$

$$\lambda_1=\lambda_2=2\quad\lambda_3=-5$$

$$(1)\begin{cases}\alpha-2\beta-\gamma=0\\ 3\alpha-6\beta-3\gamma=0\\ 2\alpha-4\beta-2\gamma=0\end{cases}\iff\begin{cases}\alpha=2\\ \beta=1\\ \gamma=0\end{cases}\quad and\quad\begin{cases}\alpha=1\\ \beta=0\\ \gamma=1\end{cases}$$

$$(2)\begin{cases}8\alpha-2\beta-\gamma=0\\ 3\alpha+\beta-3\gamma=0\\ 2\alpha-4\beta+5\gamma=0\end{cases}\iff\begin{cases}\alpha=-1\\ \beta=2\\ \gamma=2\end{cases}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=C_0\begin{pmatrix}2\\1\\0\end{pmatrix}e^{2t}+C_1\begin{pmatrix}1\\0\\1\end{pmatrix}e^{2t}+C_2\begin{pmatrix}-1\\2\\2\end{pmatrix}e^{-5t}$$

808

$$\begin{cases}x'=x-y+z\\ y'=x+y-z\\ z'=-y+2z\end{cases}$$

$$\begin{pmatrix}1-\lambda & -1 & 1\\ 1 & 1-\lambda & -1\\ 0 & -1 & 2-\lambda\end{pmatrix}$$

$$\lambda_1=\lambda_2=1\quad\lambda_3=2$$

$$(1)\begin{cases}-\beta+\gamma=0\\ \alpha-\gamma=0\\ -\beta+\gamma=0\end{cases}\iff\begin{cases}\alpha=1\\ \beta=1\\ \gamma=1\end{cases}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}\alpha_1t+\beta_1\\ \alpha_2t+\beta_2\\ \alpha_3t+\beta_3\end{pmatrix}e^t$$

$$\begin{pmatrix}x'\\y'\\z'\end{pmatrix}=\begin{pmatrix}\alpha_1t+\beta_1+\alpha_1\\ \alpha_2t+\beta_2+\alpha_2\\ \alpha_3t+\beta_3+\alpha_3\end{pmatrix}e^t$$

$$\begin{cases}(\alpha_1-\alpha_2+\alpha_3)t+\beta_1-\beta_2+\beta_3=\alpha_1 t+\beta_1+\alpha_1\\(\alpha_1+\alpha_2-\alpha_3)t+\beta_1+\beta_2-\beta_3=\alpha_2 t+\beta_2+\alpha_2\\(-\alpha_2+2\alpha_3)t-\beta_2+2\beta_3=\alpha_3 t+\beta_3+\alpha_3\end{cases}$$

$$\begin{cases}\alpha_1-\alpha_2+\alpha_3=\alpha_1\\\beta_1-\beta_2+\beta_3=\alpha_1+\beta_1\\\alpha_1+\alpha_2-\alpha_3=\alpha_2\\\beta_1+\beta_2-\beta_3=\alpha_2+\beta_2\\-\alpha_2+2\alpha_3=\alpha_3\\-\beta_2+2\beta_3=\alpha_3+\beta_3\end{cases}\iff\begin{cases}-\alpha_2+\alpha_3=0\\-\beta_2+\beta_3=\alpha_1\\\alpha_1-\alpha_3=0\\\beta_1-\beta_3=\alpha_2\\-\alpha_2+\alpha_3=0\\-\beta_2+\beta_3=\alpha_3\end{cases}\iff\begin{cases}\alpha_1=C_0\\\alpha_2=C_0\\\alpha_3=C_0\\\beta_1=C_0+C_1\\\beta_2=C_1-C_0\\\beta_3=C_1\end{cases}$$

$$(2)\begin{cases}-\alpha-\beta+\gamma=0\\\alpha-\beta-\gamma=0\\-\beta=0\end{cases}\iff\begin{cases}\alpha=1\\\beta=0\\\gamma=1\end{cases}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}C_0t+C_0+C_1\\C_0t+C_1-C_0\\C_0t+C_1\end{pmatrix}e^t+C_2\begin{pmatrix}1\\0\\1\end{pmatrix}e^{2t}$$

809

$$\begin{cases}x'=-x+y-2z\\y'=4x+y\\z'=2x+y-z\end{cases}$$

$$\begin{pmatrix}-1-\lambda & 1 & -2\\4 & 1-\lambda & 0\\2 & 1 & -1-\lambda\end{pmatrix}$$

$$\lambda_1=1\quad\lambda_2=\lambda_3=-1$$

$$(1)\begin{cases}-2\alpha+\beta-2\gamma=0\\4\alpha=0\\2\alpha+\beta-2\gamma=0\end{cases}\iff\begin{cases}\alpha=0\\\beta=2\\\gamma=1\end{cases}$$

$$(2)\begin{cases}\beta-2\gamma=0\\4\alpha+2\beta=0\\2\alpha+\beta=0\end{cases}\iff\begin{cases}\alpha=1\\\beta=-2\\\gamma=-1\end{cases}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}\alpha_1t+\beta_1\\\alpha_2t+\beta_2\\\alpha_3t+\beta_3\end{pmatrix}e^{-t}$$

$$\begin{pmatrix}x'\\y'\\z'\end{pmatrix}=\begin{pmatrix}-\alpha_1t-\beta_1+\alpha_1\\-\alpha_2t-\beta_2+\alpha_2\\-\alpha_3t-\beta_3+\alpha_3\end{pmatrix}e^{-t}$$

$$\begin{cases}(-\alpha_1+\alpha_2-2\alpha_3)t-\beta_1+\beta_2-2\beta_3=-\alpha_1t-\beta_1+\alpha_1\\(4\alpha_1+\alpha_2)t+4\beta_1+\beta_2=-\alpha_2t-\beta_2+\alpha_2\\(2\alpha_1+\alpha_2-\alpha_3)t+2\beta_2+\beta_2-\beta_3=-\alpha_3t-\beta_3+\alpha_3\end{cases}$$

$$\begin{cases}-\alpha_1+\alpha_2-2\alpha_3=-\alpha_1\\-\beta_1+\beta_2-2\beta_3=\alpha_1-\beta_1\\4\alpha_1+\alpha_2=-\alpha_2\\4\beta_1+\beta_2=\alpha_2-\beta_2\\2\alpha_1+\alpha_2-2\alpha_3=-\alpha_3\\2\beta_1+\beta_2-2\beta_3=\alpha_3-\beta_3\end{cases}\iff\begin{cases}\alpha_2-2\alpha_3=0\\\beta_2-2\beta_3=\alpha_1\\2\alpha_1+\alpha_2=0\\4\beta_1+2\beta_2=\alpha_2\\2\alpha_1+\alpha_2-\alpha_3=0\\2\beta_1+\beta_2-\beta_3=\alpha_3\end{cases}\iff\begin{cases}\alpha_1=C_0\\\alpha_2=-2C_0\\\alpha_3=-C_0\\\beta_1=-C_0-C_1\\\beta_2=C_0+2C_1\\\beta_3=C_1\end{cases}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=C_0\begin{pmatrix}0\\2\\1\end{pmatrix}e^t+\begin{pmatrix}C_0t-C_0-C_1\\-2C_0t+C_0+2C_1\\-C_0t+C_1\end{pmatrix}e^{-t}$$

810

$$\begin{cases} x' = 2x + y \\ y' = 2y + 4z \\ z' = x - z \end{cases}$$

$$\begin{pmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 4 \\ 1 & 0 & -1-\lambda \end{pmatrix}$$

$$\lambda_1 = \lambda_2 = 0 \quad \lambda_3 = 3$$

$$(1) \begin{cases} 2\alpha + \beta = 0 \\ 2\beta + 4\gamma = 0 \\ \alpha - \gamma = 0 \end{cases} \iff \begin{cases} \alpha = 1 \\ \beta = -2 \\ \gamma = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha_1 t + \beta_1 \\ \alpha_2 t + \beta_2 \\ \alpha_3 t + \beta_3 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

$$\begin{cases} (2\alpha_1 + \alpha_2)t + 2\beta_1 + \beta_2 = \alpha_1 \\ (2\alpha_2 + 4\alpha_3)t + 2\beta_2 + 4\beta_3 = \alpha_2 \\ (\alpha_1 - \alpha_3)t + \beta_1 - \beta_3 = \alpha_3 \end{cases}$$

$$\begin{cases} 2\alpha_1 + \alpha_2 = 0 \\ 2\beta_1 + \beta_2 = \alpha_1 \\ 2\alpha_2 + 4\alpha_3 = 0 \\ 2\beta_2 + 4\beta_3 = \alpha_2 \\ \alpha_1 - \alpha_3 = 0 \\ \beta_1 - \beta_3 = \alpha_3 \end{cases} \iff \begin{cases} \alpha_1 = C_0 \\ \alpha_2 = -2C_0 \\ \alpha_3 = C_0 \\ \beta_1 = C_1 \\ \beta_2 = C_0 - 2C_1 \\ \beta_3 = C_1 - C_0 \end{cases}$$

$$(2) \begin{cases} -\alpha + \beta = 0 \\ -\beta + 4\alpha = 0 \\ \alpha - 4\gamma = 0 \end{cases} \iff \begin{cases} \alpha = 4 \\ \beta = 4 \\ \gamma = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} C_0 t + C_1 \\ -2C_0 t + C_0 - 2C_1 \\ C_0 t + C_1 - C_0 \end{pmatrix} + C_2 \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix} e^{3t}$$

811

$$\begin{cases} x' = 2x - y - z \\ y' = 2x - y - 2z \\ z' = -x + y + 2z \end{cases}$$

$$\lambda_1 = \lambda_2 = \lambda_3 = 1$$

$$\begin{pmatrix} 2-\lambda & -1 & -1 \\ 2 & -1-\lambda & -2 \\ -1 & 1 & 2-\lambda \end{pmatrix}$$

$$(1) \begin{cases} \alpha - \beta - \gamma = 0 \\ 2\alpha - 2\beta - 2\gamma = 0 \\ -\alpha + \beta + \gamma = 0 \end{cases} \iff \begin{cases} \alpha = 0 \\ \beta = 1 \\ \gamma = 0 \end{cases} \quad and \quad \begin{cases} \alpha = 0 \\ \beta = 0 \\ \gamma = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha_1 t + \beta_1 \\ \alpha_2 t + \beta_2 \\ \alpha_3 t + \beta_3 \end{pmatrix} e^t$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \alpha_1 t + \beta_1 + \alpha_1 \\ \alpha_2 t + \beta_2 + \alpha_2 \\ \alpha_3 t + \beta_3 + \alpha_3 \end{pmatrix} e^t$$

$$\begin{cases} (2\alpha_1 - \alpha_2 - \alpha_3)\mathrm{t} + 2\beta_1 - \beta_2 - \beta_3 = \alpha_1\mathrm{t} + \beta_1 + \alpha_1 \\ (2\alpha_1 - \alpha_2 - 2\alpha_3)\mathrm{t} + 2\beta_1 - \beta_2 - 2\beta_3 = \alpha_2\mathrm{t} + \beta_2 + \alpha_2 \\ (-\alpha_1 + \alpha_2 + 2\alpha_3)\mathrm{t} - \beta_1 + \beta_2 + 2\beta_3 = \alpha_3\mathrm{t} + \beta_3 + \alpha_3 \end{cases}$$

$$\begin{cases} 2\alpha_1 - \alpha_2 - \alpha_3 = \alpha_1 \\ 2\beta_1 - \beta_2 - \beta_3 = \alpha_1 + \beta_1 \\ 2\alpha_1 - \alpha_2 - 2\alpha_3 = \alpha_2 \\ 2\beta_1 - \beta_2 - 2\beta_3 = \alpha_2 + \beta_2 \\ -\alpha_1 + \alpha_2 + 2\alpha_3 = \alpha_3 \\ -\beta_1 + \beta_2 + 2\beta_3 = \alpha_3 + \beta_3 \end{cases} \iff \begin{cases} \alpha_1 - \alpha_2 - \alpha_3 = 0 \\ \beta_1 - \beta_2 - \beta_3 = \alpha_1 \\ 2\beta_1 - 2\beta_2 - 2\beta_3 = \alpha_2 \\ -\beta_1 + \beta_2 + \beta_3 = \alpha_3 \end{cases} \iff \begin{cases} \alpha_1 = \mathrm{C}_0 \\ \alpha_2 = -2\mathrm{C}_0 \\ \alpha_3 = 3\mathrm{C}_0 \\ \beta_1 = \mathrm{C}_1 \\ \beta_2 = \mathrm{C}_2 \\ \beta_3 = \mathrm{C}_1 - \mathrm{C}_2 - \mathrm{C}_0 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} C_0 t + C_1 \\ -2C_0 t + C_2 \\ 3C_0 t + C_1 - C_2 - C_2 \end{pmatrix} e^t$$

812

$$\begin{cases} \mathrm{x}' = 4\mathrm{x} - \mathrm{y} \\ \mathrm{y}' = 3\mathrm{x} + \mathrm{y} - \mathrm{z} \\ \mathrm{z}' = \mathrm{x} + \mathrm{z} \end{cases}$$

$$\lambda_1 = \lambda_2 = \lambda_3 = 2$$

$$\begin{pmatrix} 4-\lambda & -1 & 0 \\ 3 & 1-\lambda & -1 \\ 1 & 0 & 1-\lambda \end{pmatrix}$$

$$(1) \begin{cases} 2\alpha - \beta = 0 \\ 3\alpha - \beta - \gamma = 0 \\ \alpha - \gamma = 0 \end{cases} \iff \begin{cases} 2\alpha - \beta = 0 \\ \alpha - \gamma = 0 \end{cases} \iff \begin{cases} \alpha = 1 \\ \beta = 2 \\ \gamma = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha_1 t^2 + \beta_1 t + \gamma_1 \\ \alpha_2 t^2 + \beta_2 t + \gamma_2 \\ \alpha_3 t^2 + \beta_3 t + \gamma_3 \end{pmatrix} e^{2t}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 2\alpha_1 t^2 + (2\alpha_1 + 2\beta_1)t + \beta_1 + 2\gamma_1 \\ 2\alpha_2 t^2 + (2\alpha_2 + 2\beta_2)t + \beta_2 + 2\gamma_2 \\ 2\alpha_3 t^2 + (2\alpha_3 + 2\beta_3)t + \beta_3 + 2\gamma_3 \end{pmatrix} e^{2t}$$

$$\begin{cases} 4\alpha_1 - \alpha_2 = 2\alpha_1 \\ 4\beta_1 - \beta_2 = 2\alpha_1 + 2\beta_1 \\ 4\gamma_1 - \gamma_2 = \beta_1 + 2\gamma_1 \\ 3\alpha_1 + \alpha_2 - \alpha_3 = 2\alpha_2 \\ 3\beta_1 + \beta_2 - \beta_3 = 2\alpha_2 + 2\beta_2 \\ 3\gamma_1 + \gamma_2 - \gamma_3 = \beta_2 + 2\gamma_2 \\ \alpha_1 + \alpha_3 = 2\alpha_3 \\ \beta_1 + \beta_3 = 2\alpha_3 + 2\beta_3 \\ \gamma_1 + \gamma_3 = \beta_3 + 2\gamma_3 \end{cases} \iff \begin{cases} 2\alpha_1 - \alpha_2 = 0 \\ 2\beta_1 - \beta_2 = 2\alpha_1 \\ 2\gamma_1 - \gamma_2 = \beta_1 \\ 3\alpha_1 - \alpha_2 - \alpha_3 = 0 \\ 3\beta_1 - \beta_2 - \beta_3 = 2\alpha_2 \\ 3\gamma_1 - \gamma_2 - \gamma_3 = \beta_2 \\ \alpha_1 - \alpha_3 = 0 \\ \beta_1 - \beta_3 = 2\alpha_3 \\ \gamma_1 - \gamma_3 = \beta_3 \end{cases} \iff \begin{cases} \alpha_1 = \mathrm{C}_0 \\ \alpha_2 = 2\mathrm{C}_0 \\ \alpha_3 = \mathrm{C}_0 \\ \beta_1 = \mathrm{C}_1 \\ \beta_2 = 2\mathrm{C}_1 - 2\mathrm{C}_0 \\ \beta_3 = \mathrm{C}_1 - 2\mathrm{C}_0 \\ \gamma_1 = \mathrm{C}_2 \\ \gamma_2 = 2\mathrm{C}_2 - \mathrm{C}_1 \\ \gamma_3 = \mathrm{C}_2 - \mathrm{C}_1 + 2\mathrm{C}_0 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} C_0 t^2 + C_1 t + C_2 \\ 2C_0 t^2 + 2(C_1 - C_0)t + 2C_2 - C_1 \\ C_0 t^2 + (C_1 - 2C_0)t + C_2 - C_1 + 2C_0 \end{pmatrix} e^{2t}$$

Диффуры IDZ 2 (815-818)

815

$$\begin{cases} x'' = 2y \\ y'' = -2x \end{cases}$$

$$\begin{cases} x'' - 2y = 0 \\ y'' + 2x = 0 \end{cases}$$

$$\begin{vmatrix} \lambda^2 & -2 \\ 2 & \lambda^2 \end{vmatrix} = \lambda^4 + 4 = 0$$

$$\lambda_{1,2} = 1 \pm i \quad \lambda_{3,4} = -1 \pm i$$

$$(1,2) \begin{cases} 2i\alpha - 2\beta = 0 \\ 2\alpha + 2i\beta = 0 \end{cases} \implies \begin{cases} \alpha = 1 \\ \beta = i \end{cases}$$

$$(3,4) \begin{cases} -2i\alpha - 2\beta = 0 \\ 2\alpha - 2i\beta = 0 \end{cases} \implies \begin{cases} \alpha = i \\ \beta = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = C_0 \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} e^t + C_1 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} e^t + C_2 \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} e^{-t} + C_3 \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} e^{-t}$$

816

$$\begin{cases} x'' - 3x + y + z = 0 \\ x + y'' - 3y + z = 0 \\ x + y + z'' - 3z = 0 \end{cases}$$

$$\begin{vmatrix} \lambda^2 - 3 & 1 & 1 \\ 1 & \lambda^2 - 3 & 1 \\ 1 & 1 & \lambda^2 - 3 \end{vmatrix} = (\lambda^2 - 3)^3 - 3(\lambda^2 - 3) + 2 = 0$$

$$(\lambda^2 - 3)^3 - 3(\lambda^2 - 3) + 2 = (\lambda^2 - 3)((\lambda^2 - 3)^2 - 1) - 2(\lambda^2 - 4) =$$

$$= (\lambda^2 - 3)(\lambda^2 - 4)(\lambda^2 - 2) - 2(\lambda^2 - 4) =$$

$$= (\lambda - 2)(\lambda + 2)(\lambda^4 - 5\lambda^2 + 4) = (\lambda - 2)^2(\lambda + 2)^2(\lambda - 1)(\lambda + 1)$$

$$\lambda_{1,2} = 2 \quad \lambda_{3,4} = -2 \quad \lambda_{5,6} = \pm 1$$

$$(1,2,3,4) \begin{cases} \alpha + \beta + \gamma = 0 \\ \alpha + \beta + \gamma = 0 \\ \alpha + \beta + \gamma = 0 \end{cases} \implies \begin{cases} \alpha = -1 \\ \beta = 1 \\ \gamma = 0 \end{cases} \quad and \quad \begin{cases} \alpha = -1 \\ \beta = 0 \\ \gamma = 1 \end{cases}$$

$$(5,6) \begin{cases} -2\alpha + \beta + \gamma = 0 \\ \alpha - 2\beta + \gamma = 0 \\ \alpha + \beta - 2\gamma = 0 \end{cases} \implies \begin{cases} \alpha = 1 \\ \beta = 1 \\ \gamma = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_0 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} e^{2t} + C_1 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} e^{-2t} + C_0 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} e^{-2t} + C_1 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} e^{2t} + C_0 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{t} + C_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{-t}$$

817

$$\begin{cases} 2x' + x - 5y' - 4y = 0 \\ 3x' - 2x - 4y' + y = 0 \end{cases}$$

$$\begin{vmatrix} 2\lambda + 1 & -5\lambda - 4 \\ 3\lambda - 2 & -4\lambda + 1 \end{vmatrix} = -8\lambda^2 - 2\lambda + 1 + 15\lambda^2 + 2\lambda - 8 = 7\lambda^2 - 7 = 0$$

$$\lambda_{1,2} = \pm 1$$

$$(1)\begin{cases}3\alpha-9\beta=0\\ \alpha-3\beta=0\end{cases}\implies\begin{cases}\alpha=3\\ \beta=1\end{cases}$$

$$(2)\begin{cases}-\alpha+\beta=0\\ -5\alpha+5\beta=0\end{cases}\implies\begin{cases}\alpha=1\\ \beta=1\end{cases}$$

$$\begin{pmatrix}x\\ y\end{pmatrix}=C_0\begin{pmatrix}3\\ 1\end{pmatrix}e^t+C_1\begin{pmatrix}1\\ 1\end{pmatrix}e^{-t}$$

818

$$\begin{cases}x''+x'+y'-2y=0\\ x'+x-y'=0\end{cases}$$

$$\begin{vmatrix}\lambda^2+\lambda & \lambda-2\\ \lambda+1 & -\lambda\end{vmatrix}=-\lambda^3-\lambda^2-\lambda^2+\lambda+2=-\lambda^3-2\lambda^2+\lambda+2=(\lambda-1)(\lambda+1)(\lambda+2)=0$$

$$\lambda_{1,2}=\pm1\quad\lambda_3=-2$$

$$(1)\begin{cases}2\alpha-\beta=0\\ 2\alpha-\beta=0\end{cases}\implies\begin{cases}\alpha=1\\ \beta=2\end{cases}$$

$$(2)\begin{cases}-3\beta=0\\ \beta=0\end{cases}\implies\begin{cases}\alpha=1\\ \beta=0\end{cases}$$

$$(3)\begin{cases}2\alpha-4\beta=0\\ -\alpha+2\beta=0\end{cases}\implies\begin{cases}\alpha=2\\ \beta=1\end{cases}$$

$$\begin{pmatrix}x\\ y\end{pmatrix}=C_0\begin{pmatrix}1\\ 2\end{pmatrix}e^t+C_1\begin{pmatrix}1\\ 0\end{pmatrix}e^{-t}+C_2\begin{pmatrix}2\\ 1\end{pmatrix}e^{-2t}$$

Диффуры IDZ 3 (834-840)

834

$$\begin{cases}x'=x+2y\\ y'=x-5\sin t\end{cases}$$

$$\begin{pmatrix}1-\lambda & 2\\ 1 & -\lambda\end{pmatrix}$$

$$-\lambda(1-\lambda)-2=\lambda^2-\lambda-2=0\implies\lambda_1=-1\quad\lambda_2=2$$

$$(1)\begin{cases}2\alpha+2\beta=0\\ \alpha+\beta=0\end{cases}\implies\begin{pmatrix}1\\ -1\end{pmatrix}$$

$$(2)\begin{cases}-\alpha+2\beta=0\\ \alpha-2\beta=0\end{cases}\implies\begin{pmatrix}2\\ 1\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}A_1\sin t+B_1\cos t\\ A_2\sin t+B_2\cos t\end{pmatrix}$$

$$\begin{pmatrix}A_1\cos t-B_1\sin t\\ A_2\cos t-B_2\sin t\end{pmatrix}=\begin{pmatrix}(A_1+2A_2)\sin t+(B_1+2B_2)\cos t\\ (A_1-5)\sin t+B_1\cos t\end{pmatrix}$$

$$\begin{cases}A_1=B_1+2B_2\\ -B_1=A_1+2A_2\\ A_2=B_1\\ -B_2=A_1-5\end{cases}\implies\begin{cases}A_1=-\dfrac{1}{3}A_1+10-2A_1\\ A_1=-3A_2=-3B_1\\ A_2=B_1\\ B_2=5-A_1\end{cases}\implies\begin{cases}A_1=3\\ A_2=-1\\ B_1=-1\\ B_2=2\end{cases}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3\sin t - \cos t \\ -\sin t + 2\cos t \end{pmatrix} + C_0 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t}$$

835

$$\begin{cases} x' = 2x - 4y \\ y' = x - 3y + 3e^t \end{cases}$$

$$\begin{pmatrix} 2-\lambda & -4 \\ 1 & -3-\lambda \end{pmatrix}$$

$$\lambda^2 + \lambda - 2 = 0 \implies \lambda_1 = 1 \quad \lambda_2 = -2$$

$$(1) \begin{cases} \alpha - 4\beta = 0 \\ \alpha - 4\beta = 0 \end{cases} \implies \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$(2) \begin{cases} 4\alpha - 4\beta = 0 \\ \alpha - \beta = 0 \end{cases} \implies \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A_1 t + B_1 \\ A_2 t + B_2 \end{pmatrix} e^t$$

$$\begin{pmatrix} A_1 t + B_1 + A_1 \\ A_2 t + B_2 + A_2 \end{pmatrix} = \begin{pmatrix} (2A_1 + 4A_2)t + 2B_1 + 4B_2 \\ (A_1 - 3A_2)t + B_1 - 3B_2 + 3 \end{pmatrix}$$

$$\begin{cases} A_1 = 2A_1 - 4A_2 \\ B_1 + A_1 = 2B_1 - 4B_2 \\ A_2 = A_1 - 3A_2 \\ B_2 + A_2 = B_1 - 3B_2 + 3 \end{cases} \implies \begin{cases} A_1 = 4A_2 \\ A_1 = B_1 - 4B_2 \\ A_1 = A_2 - 3 \\ A_2 = B_1 - 4B_2 + 3 \end{cases} \implies \begin{cases} A_1 = -4 \\ A_2 = -1 \\ B_1 = 0 \\ B_2 = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4t \\ 1-t \end{pmatrix} e^t + C_0 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t}$$

836

$$\begin{cases} x' = 2x - y \\ y' = -2x + y + 18t \end{cases}$$

$$\begin{pmatrix} 2-\lambda & -1 \\ -2 & 1-\lambda \end{pmatrix}$$

$$\lambda^2 - 3\lambda = 0 \implies \lambda_1 = 3 \quad \lambda_2 = 0$$

$$(1) \begin{cases} -\alpha - \beta = 0 \\ -2\alpha - 2\beta = 0 \end{cases} \implies \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$(2) \begin{cases} 2\alpha - \beta = 0 \\ -2\alpha + \beta = 0 \end{cases} \implies \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A_1 t^2 + B_1 t + C_1 \\ A_2 t^2 + B_2 t + C_2 \end{pmatrix}$$

$$\begin{pmatrix} 2A_1 t + B_1 \\ 2A_2 t + B_2 \end{pmatrix} = \begin{pmatrix} (2A_1 - A_2)t^2 + (2B_1 - B_2)t + 2C_1 - C_2 \\ (-2A_1 + A_2)t^2 + (-2B_1 + B_2)t - 2C_1 + C_2 + 18t \end{pmatrix}$$

$$\begin{cases} 0 = 2A_1 - A_2 \\ 2A_1 = 2B_1 - B_2 \\ B_1 = 2C_1 - C_2 \\ 0 = -2A_1 + A_2 \\ 2A_2 = -2B_1 + B_2 + 18 \\ B_2 = C_2 - 2C_1 \end{cases} \implies \begin{cases} 2A_1 = A_2 \\ A_1 = 9 - A_2 \\ B_1 = -B_2 \\ 2A_2 = -2B_1 + B_2 + 18 \\ B_2 = C_2 - 2C_1 \end{cases} \implies \begin{cases} A_1 = 3 \\ A_2 = 6 \\ B_1 = 2 \\ B_2 = -2 \\ C_1 = 1 \\ C_2 = 0 \end{cases}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3t^2+2t+1 \\ 6t^2-2t \end{pmatrix} + C_0 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{3t} + C_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

837

$$\begin{cases} x' = x + 2y + 16te^t \\ y' = 2x - 2y \end{cases}$$

$$\begin{pmatrix} 1-\lambda & 2 \\ 2 & -2-\lambda \end{pmatrix}$$

$$\lambda^2 + \lambda - 6 = 0 \implies \lambda_1 = 2 \quad \lambda_2 = -3$$

$$(1) \begin{cases} -\alpha + 2\beta = 0 \\ 2\alpha - 4\beta = 0 \end{cases} \implies \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$(2) \begin{cases} 4\alpha + 2\beta = 0 \\ 2\alpha + \beta = 0 \end{cases} \implies \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A_1t + B_1 \\ A_2t + B_2 \end{pmatrix} e^t$$

$$\begin{pmatrix} A_1t + B_1 + A_1 \\ A_2t + B_2 + A_2 \end{pmatrix} = \begin{pmatrix} (A_1 + 2A_2 + 16)t + B_1 + 2B_2 \\ (2A_1 - 2A_2)t + 2B_1 - 2B_2 \end{pmatrix}$$

$$\begin{cases} A_1 = A_1 + 2A_2 + 16 \\ B_1 + A_1 = B_1 + 2B_2 \\ A_2 = 2A_1 - 2A_2 \\ B_2 + A_2 = 2B_1 - 2B_2 \end{cases} \implies \begin{cases} A_1 = 4A_2 \\ A_1 = B_1 - 4B_2 \\ A_1 = A_2 - 3 \\ A_2 = B_1 - 4B_2 + 3 \end{cases} \implies \begin{cases} A_1 = -12 \\ A_2 = -8 \\ B_1 = -12 \\ B_2 = -6 \end{cases}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -2 \begin{pmatrix} 6t+6 \\ 4t+3 \end{pmatrix} e^t + C_0 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t} + C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-3t}$$

838

$$\begin{cases} x' = 2x + 4y - 8 \\ y' = 3x + 6y \end{cases}$$

$$\begin{pmatrix} 2-\lambda & 4 \\ 3 & 6-\lambda \end{pmatrix}$$

$$\lambda^2 - 8\lambda = 0 \implies \lambda_1 = 0 \quad \lambda_2 = 8$$

$$(1) \begin{cases} 2\alpha + 4\beta = 0 \\ 3\alpha + 6\beta = 0 \end{cases} \implies \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$(2) \begin{cases} -6\alpha + 4\beta = 0 \\ 3\alpha - 2\beta = 0 \end{cases} \implies \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A_1t + B_1 \\ A_2t + B_2 \end{pmatrix}$$

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} (2A_1 + 4A_2)t + 2B_1 + 4B_2 \\ (A_1 - 3A_2)t + B_1 - 3B_2 + 3 \end{pmatrix}$$

$$\begin{cases} A_1 = 2A_1 - 4A_2 \\ B_1 + A_1 = 2B_1 - 4B_2 \\ A_2 = A_1 - 3A_2 \\ B_2 + A_2 = B_1 - 3B_2 + 3 \end{cases} \implies \begin{cases} A_1 = 4A_2 \\ A_1 = B_1 - 4B_2 \\ A_1 = A_2 - 3 \\ A_2 = B_1 - 4B_2 + 3 \end{cases} \implies \begin{cases} A_1 = -4 \\ A_2 = -1 \\ B_1 = 0 \\ B_2 = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4t \\ 1-t \end{pmatrix} e^t + C_0 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t}$$

839

$$\begin{cases} x' = 2x - 3y \\ y' = x - 2y + 2\sin t \end{cases}$$

$$\begin{pmatrix} 2-\lambda & -3 \\ 1 & -2-\lambda \end{pmatrix}$$

$$\lambda^2 - 1 = 0 \implies \lambda_1 = 1 \quad \lambda_2 = -1$$

$$(1) \begin{cases} \alpha - 3\beta = 0 \\ \alpha - 3\beta = 0 \end{cases} \implies \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$(2) \begin{cases} 3\alpha - 3\beta = 0 \\ \alpha - \beta = 0 \end{cases} \implies \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A_1 \sin t + B_1 \cos t \\ A_2 \sin t + B_2 \cos t \end{pmatrix}$$

$$\begin{pmatrix} A_1 \cos t - B_1 \sin t \\ A_2 \cos t - B_2 \sin t \end{pmatrix} = \begin{pmatrix} (2A_1 - 3A_2)\sin t + (2B_1 - 3B_2)\cos t \\ (A_1 - 2A_2 + 2)\sin t + (B_1 - B_2)\cos t \end{pmatrix}$$

$$\begin{cases} A_1 = 2B_1 - 3B_2 \\ -B_1 = 2A_1 - 3A_2 \\ A_2 = B_1 - B_2 \\ -B_2 = A_1 - 2A_2 + 2 \end{cases} \implies \begin{cases} A_1 = 3 \\ 2B_1 = B_2 \\ A_2 = B_1 - B_2 \\ B_2 = 1 \end{cases} \implies \begin{cases} A_1 = 3 \\ A_2 = -\dfrac{1}{2} \\ B_1 = \dfrac{1}{2} \\ B_2 = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3\sin t - \dfrac{1}{2}\cos t \\ -\dfrac{1}{2}\sin t + \cos t \end{pmatrix} + C_0 \begin{pmatrix} 3 \\ 1 \end{pmatrix} e^t + C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}$$

840

$$\begin{cases} x' = x - y + 2\sin t \\ y' = 2x - y \end{cases}$$

$$\begin{pmatrix} 1-\lambda & -1 \\ 2 & -1-\lambda \end{pmatrix}$$

$$\lambda^2 + 1 = 0 \implies \lambda_{1,2} = \pm i$$

$$(1,2) \begin{cases} (1-i)\alpha - \beta = 0 \\ 2\alpha - (1+i)\beta = 0 \end{cases} \implies \begin{pmatrix} 1 \\ 1-i \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1-i \end{pmatrix} \cdot e^{it} = \begin{pmatrix} \cos t + i\sin t \\ (1-i)(\cos t + i\sin t) \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (A_1 t + B_1)\sin t + (C_1 t + D_1)\cos t \\ (A_2 t + B_2)\sin t + (C_2 t + D_2)\cos t \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} (-C_1 t + A_1 - D_1)\sin t + (A_1 t + C_1 + B_1)\cos t \\ (-C_2 t + A_2 - D_2)\sin t + (A_2 t + C_2 + B_2)\cos t \end{pmatrix}$$

$$\begin{cases} -C_1 = A_1 - A_2 \\ A_1 - D_1 = B_1 - B_2 + 2 \\ A_1 = C_1 - C_2 \\ C_1 + B_1 = D_1 - D_2 \\ -C_2 = 2A_1 - A_2 \\ A_2 - D_2 = 2B_1 - B_2 \\ A_2 = 2C_1 - C_2 \\ C_2 + B_2 = 2D_1 - D_2 \end{cases} \implies \begin{cases} C_2 = 0 \\ C_1 = 2 \\ A_1 = 2 \\ B_1 = -2 \\ A_2 = 4 \\ D_1 = 1 \\ D_2 = 1 \\ B_2 = 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (2t-2)\sin t + (2t+1)\cos t \\ (4t+1)\sin t + \cos t \end{pmatrix} e^t + C_0 \begin{pmatrix} \cos t \\ \cos t + \sin t \end{pmatrix} + C_1 \begin{pmatrix} \sin t \\ \sin t - \cos t \end{pmatrix}$$

Диффуры IDZ 4 (847-850)

847

$$\begin{cases} x' = -x + 2y \\ y' = -3x + 4y + \dfrac{e^{3t}}{e^{2t}+1} \end{cases}$$

$$\begin{pmatrix} -1-\lambda & 2 \\ -3 & 4-\lambda \end{pmatrix}$$

$$\lambda^2 - 3\lambda + 2 = 0 \implies \lambda_1 = 1 \quad \lambda_2 = 2$$

$$(1) \begin{cases} -2\alpha + 2\beta = 0 \\ -3\alpha + 3\beta = 0 \end{cases} \implies \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$(2) \begin{cases} -3\alpha + 2\beta = 0 \\ -3\alpha + 2\beta = 0 \end{cases} \implies \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{cases} x = C_1 e^t + 2C_2 e^{2t} \\ y = C_1 e^t + 3C_2 e^{2t} \end{cases}$$

$$\begin{cases} x' = C_1' e^t + C_1 e^t + 2C_2' e^{2t} + 4C_2 e^{2t} \\ y' = C_1' e^t + C_1 e^t + 3C_2' e^{2t} + 6C_2 e^{2t} \end{cases}$$

$$\begin{cases} C_1' e^t + C_1 e^t + 2C_2' e^{2t} + 4C_2 e^{2t} = -C_1 e^t - 2C_2 e^{2t} + 2C_1 e^t + 6C_2 e^{2t} \\ C_1' e^t + C_1 e^t + 3C_2' e^{2t} + 6C_2 e^{2t} = -3C_1 e^t - 6C_2 e^{2t} + 4C_1 e^t + 12C_2 e^{2t} + \dfrac{e^{3t}}{e^{2t}+1} \end{cases}$$

$$\begin{cases} C_1' e^t + 2C_2' e^{2t} = 0 \\ C_1' e^t + 3C_2' e^{2t} = \dfrac{e^{3t}}{e^{2t}+1} \end{cases}$$

$$\begin{cases} C_1' = -2\dfrac{e^{2t}}{e^{2t}+1} \\ C_2' = \dfrac{e^t}{e^{2t}+1} \end{cases}$$

Ответ:

$$\begin{cases} x = C_1 e^t + 2C_2 e^{2t} \\ y = C_1 e^t + 3C_2 e^{2t} \end{cases}$$

$$\begin{cases} C_1 = -\ln(e^{2t}+1) + c_1 \\ C_2 = \arctan(e^t) + c_2 \end{cases}$$

848

$$\begin{cases} x' = -4x - 2y + \dfrac{2}{e^t - 1} \\ y' = 6x + 3y - \dfrac{3}{e^t - 1} \end{cases}$$

$$\begin{pmatrix} -4-\lambda & -2 \\ 6 & 3-\lambda \end{pmatrix}$$

$$\lambda^2 + \lambda = 0 \implies \lambda_1 = 0 \quad \lambda_2 = -1$$

$$(1) \begin{cases} -4\alpha - 2\beta = 0 \\ 6\alpha + 3\beta = 0 \end{cases} \implies \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$(2) \begin{cases} -3\alpha - 2\beta = 0 \\ 6\alpha + 4\beta = 0 \end{cases} \implies \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$\begin{cases} x = -C_1 + 2C_2 e^{-t} \\ y = 2C_1 + 3C_2 e^{-t} \end{cases}$$

$$\begin{cases} -C_1' + 2C_2' e^{-t} = \dfrac{2}{e^t - 1} \\ -2C_1' + 3C_2' e^{-t} = -\dfrac{3}{e^t - 1} \end{cases}$$

$$\begin{cases} C_1' = \dfrac{12}{e^t - 1} \\ C_2' = 7\dfrac{e^t}{e^t - 1} \end{cases}$$

Ответ:

$$\begin{cases} x = C_1 e^t + 2C_2 e^{2t} \\ y = C_1 e^t + 3C_2 e^{2t} \end{cases}$$

$$\begin{cases} C_1 = 12\ln(e^t - 1) - 12t + c_1 \\ C_2 = 7\ln(e^t - 1) + c_2 \end{cases}$$

849

$$\begin{cases} x' = x - y + \dfrac{1}{\cos t} \\ y' = 2x - y \end{cases}$$

$$\begin{pmatrix} 1-\lambda & -1 \\ 2 & -1-\lambda \end{pmatrix}$$

$$\lambda^2 + 1 = 0 \implies \lambda_{1,2} = \pm i$$

$$(1) \begin{cases} (1-i)\alpha - \beta = 0 \\ 2\alpha - (1+i)\beta = 0 \end{cases} \implies \begin{pmatrix} 1 \\ 1-i \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1-i \end{pmatrix} e^{it} = \begin{pmatrix} \cos t + i\sin t \\ \cos t - \sin t + (\sin t - \cos t)i \end{pmatrix}$$

$$\begin{cases} x = C_1 \cos t + C_2 \sin t \\ y = C_1(\cos t - \sin t) + C_2(\sin t - \cos t) \end{cases}$$

$$\begin{cases} C_1' \cos t + C_2' \sin t = \dfrac{1}{\cos t} \\ C_1'(\cos t - \sin t) + C_2'(\sin t - \cos t) = 0 \end{cases}$$

$$\begin{cases} C_1' \cos t + C_2' \sin t = \dfrac{1}{\cos t} \\ C_1' \sin t + C_2' \cos t = 0 \end{cases}$$

$$\begin{cases} C_1' = \dfrac{1}{\cos 2t} \\ C_2' = -\dfrac{\tan x}{\cos 2t} \end{cases}$$

Ответ:

$$\begin{cases} x = C_1 \cos t + C_2 \sin t \\ y = C_1(\cos t - \sin t) + C_2(\sin t - \cos t) \end{cases}$$

$$\begin{cases} C_1 = \dfrac{1}{2}\ln(\sin 2t + 1) - \dfrac{1}{2}\ln(\cos 2t) + c_1 \\ C_2 = -\left(\dfrac{\sin(x)\ \ln(\sin(2\,t) + 1)}{2\ \cos(x)} - \dfrac{\sin(x)\ \ln(|\cos(2\,t)|)}{2\ \cos(x)} + c_2\right) \end{cases}$$

850

$$\begin{cases} x' = 3x - 2y \\ y' = 2x - y + 15e^t\sqrt{t} \end{cases}$$

$$\begin{pmatrix} 3-\lambda & -2 \\ 2 & -1-\lambda \end{pmatrix}$$

$$\lambda^2 - 2\lambda + 1 = 0 \implies \lambda_{1,2} = 1$$

$$(1,2) \begin{cases} 2\alpha - 2\beta = 0 \\ 2\alpha - 2\beta = 0 \end{cases} \implies \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} x = (a_1 t + b_1)e^t \\ y = (a_2 t + b_2)e^t \end{cases}$$

$$\begin{cases} a_1 t + b_1 + a_1 = (3a_1 - 2a_2)t + 3b_1 - 2b_2 \\ a_2 t + b_2 + a_2 = (2a_1 - a_2)t + 2b_1 - b_2 \end{cases} \iff \begin{cases} a_1 = 2b_1 - 2b_2 \\ a_2 = 2b_1 - 2b_2 \\ a_2 = a_1 = C_1 \end{cases} \iff \begin{cases} a_1 = 2C_1 \\ a_2 = 2C_1 \\ b_1 = C_2 \\ b_2 = C_2 - C_1 \end{cases}$$

$$\begin{cases} x = (2C_1 t + C_2)e^t \\ y = (2C_1 t + C_2 - C_1)e^t \end{cases}$$

$$\begin{cases} x' = (2C_1 t + C_2 + 2C_1 + C_2' + 2C_1' t)e^t \\ y' = (2C_1 t + C_2 + C_1 + C_2' - C_1' + 2C_1' t)e^t \end{cases}$$

$$\begin{cases} C_2' + 2C_1' t = 0 \\ C_2' - C_1' + 2C_1' t = 15\sqrt{t} \end{cases}$$

$$\begin{cases} C_2' = 30t\sqrt{t} \\ C_1' = -15\sqrt{t} \end{cases}$$

Ответ:

$$\begin{cases} x = (2C_1 t + C_2)e^t \\ y = (2C_1 t + C_2 - C_1)e^t \end{cases}$$

$$\begin{cases} C_1 = -10t\sqrt{t} + c_1 \\ C_2 = 12t^2\sqrt{t} + c_2 \end{cases}$$

Диффуры IDZ 5 (869,870,873)

869

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{pmatrix}$$

$$\lambda_{1,2} = 2$$

$$x = \begin{pmatrix} a_1 t + b_1 \\ a_2 t + b_2 \end{pmatrix} e^{2t}$$

$$\begin{pmatrix} 2a_1 t + 2b_1 + a_1 \\ 2a_2 t + 2b_2 + a_2 \end{pmatrix} = \begin{pmatrix} (2a_1 + a_2)t + (2b_1 + b_2) \\ 2a_2 t + 2b_2 \end{pmatrix}$$

$$\begin{cases} 2a_1 = 2a_1 + a_2 \\ 2a_2 = 2a_2 \\ 2b_1 + a_1 = 2b_1 + b_2 \\ 2b_2 + a_2 = 2b_2 \end{cases} \implies \begin{cases} a_2 = 0 \\ a_1 = b_2 \end{cases} \implies \begin{cases} a_1 = C_0 \\ a_2 = 0 \\ b_1 = C_1 \\ b_2 = C_0 \end{cases}$$

$$x = \begin{pmatrix} C_0 t + C_1 \\ C_0 \end{pmatrix} e^{2t}$$

$$\begin{pmatrix} C_1 \\ C_0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \implies \begin{pmatrix} e^{2t} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} C_1 \\ C_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \implies \begin{pmatrix} te^{2t} \\ e^{2t} \end{pmatrix}$$

$$e^A = \begin{pmatrix} e^{2t} & te^{2t} \\ 0 & e^{2t} \end{pmatrix}$$

870

$$A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$$

$$\lambda^2 - 3\lambda + 2 = 0 \implies \lambda_1 = 1, \lambda_2 = 2$$

$$(1) \begin{cases} 2\alpha - \beta = 0 \\ 2\alpha - \beta = 0 \end{cases} \implies \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$(2) \begin{cases} \alpha - \beta = 0 \\ 2\alpha - 2\beta = 0 \end{cases} \implies \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$x = C_0 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^t + C_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t}$$

$$C_0 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \implies \begin{cases} C_0 = -1 \\ C_1 = 2 \end{cases} \implies \begin{pmatrix} 2e^{2t} - e^t \\ 4e^{2t} - 2e^t \end{pmatrix}$$

$$C_0 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \implies \begin{cases} C_0 = 1 \\ C_1 = -1 \end{cases} \implies \begin{pmatrix} e^{2t} - e^t \\ 2e^{2t} - 2e^t \end{pmatrix}$$

$$e^A = \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}$$

873

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

$$(2-\lambda)^3 = 0 \implies \lambda_{1,2,3} = 2$$

$$x = \begin{pmatrix} a_1t^2 + b_1t + c_1 \\ a_2t^2 + b_2t + c_2 \\ a_3t^2 + b_3t + c_3 \end{pmatrix} e^{2t}$$

$$\begin{pmatrix} 2a_1t^2 + 2b_1t + 2c_1 + 2a_1t + b_1 \\ 2a_2t^2 + 2b_2t + 2c_2 + 2a_2t + b_2 \\ 2a_3t^2 + 2b_3t + 2c_3 + 2a_3t + b_3 \end{pmatrix} = \begin{pmatrix} (2a_1 + a_2)t^2 + (2b_1 + b_2)t + 2c_1 + c_2 \\ (2a_2 + a_3)t^2 + (2b_2 + b_3)t + 2c_2 + c_3 \\ 2a_3t^2 + 2b_3t + 2c_3 \end{pmatrix}$$

$$\begin{cases} a_2 = 0 \\ a_3 = 0 \\ 2a_1 = b_2 \\ b_1 = c_2 \\ b_2 = c_3 \\ b_3 = 0 \end{cases} \implies \begin{cases} a_1 = C_0 \\ a_2 = 0 \\ a_3 = 0 \\ b_1 = C_1 \\ b_2 = 2C_0 \\ b_3 = 0 \\ c_1 = C_2 \\ c_2 = C_1 \\ c_3 = 2C_0 \end{cases}$$

$$x = \begin{pmatrix} C_0t^2 + C_1t + C_2 \\ 2C_0t + C_1 \\ 2C_0 \end{pmatrix} e^{2t}$$

$$\begin{pmatrix} C_2 \\ C_1 \\ 2C_0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \implies \begin{pmatrix} e^{2t} \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} C_2 \\ C_1 \\ 2C_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \implies \begin{pmatrix} te^{2t} \\ e^{2t} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} C_2 \\ C_1 \\ 2C_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \implies \begin{pmatrix} t^2e^{2t} \\ 2te^{2t} \\ 2e^{2t} \end{pmatrix}$$

$$e^A = \begin{pmatrix} e^2 & e^2 & e^2 \\ 0 & e^2 & 2e^2 \\ 0 & 0 & 2e^2 \end{pmatrix}$$

Диффуры IDZ 6 (751-763,782)

751

$$\begin{cases} y'' - y = 2x \\ y(0) = 0 \\ y(1) = -1 \end{cases}$$

$$\lambda^2 - 1 = 0$$

$$\lambda = -1$$
$$\lambda = 1$$

$$y = C_1e^{-x} + C_2e^x$$

$$\begin{cases} C_1 = -C_2 \\ C_1 e^{-1} + C_2 e = -1 \end{cases} \implies \begin{cases} C_2 = \dfrac{1}{e^{-1} - e} \\ C_1 = -\dfrac{1}{e^{-1} - e} \end{cases}$$

$$y = -\frac{1}{e^{-1} - e} e^{-1} + \frac{1}{e^{-1} - e} e^{x}$$

752

$$\begin{cases} y'' + y' = 1 \\ y'(0) = 0 \\ y(1) = 1 \end{cases}$$

$$\lambda^2 + \lambda = 0$$

$$\lambda = -1$$
$$\lambda = 0$$

$$y = C_1 e^{-x} + C_2 + x$$

$$\begin{cases} 1 - C_1 = 0 \\ C_1 e^{-1} + C_2 = 0 \end{cases} \implies \begin{cases} C_1 = 1 \\ C_2 = -e^{-1} \end{cases}$$

$$y = e^{-x} - e^{-1} + x$$

753

$$\begin{cases} y'' - y' = 0 \\ y(0) = -1 \\ y'(1) - y(1) = 2 \end{cases}$$

$$\lambda^2 - \lambda = 0$$

$$\lambda = 0$$
$$\lambda = 1$$

$$y = C_1 + C_2 e^{x}$$

$$\begin{cases} C_1 + C_2 = -1 \\ C_2 e - C_1 - C_2 e = 2 \end{cases} \implies \begin{cases} C_2 = 1 \\ C_1 = -2 \end{cases}$$

$$y = -2 + e^{x}$$

754

$$\begin{cases} y'' + y = 1 \\ y(0) = 0 \\ y(\dfrac{\pi}{2}) = 0 \end{cases}$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm i$$

$$y = C_1 \cos x + C_2 \sin x + 1$$

$$\begin{cases} C_1 = -1 \\ C_2 = -1 \end{cases}$$

$$y = 1 - \cos x - \sin x$$

755

$$\begin{cases} y'' + y = 1 \\ y(0) = 0 \\ y(\pi) = 0 \end{cases}$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm i$$

$$y = C_1 \cos x + C_2 \sin x + 1$$

$$C_1 = \pm 1$$

$$y = 1 \pm \cos x + C_2 \sin x$$

756

$$\begin{cases} y'' + y = 2x - \pi \\ y(0) = 0 \\ y(\pi) = 0 \end{cases}$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm i$$

$$y = C_1 \cos x + C_2 \sin x + 2x - \pi$$

$$C_1 = \pi$$

$$y = 2x - \pi + C_2 \sin x + \pi \cos x$$

757

$$\begin{cases} y'' - y' - 2y = 0 \\ y'(0) = 2 \\ y(+\infty) = 0 \end{cases}$$

$$\lambda^2 - \lambda - 2 = 0$$

$$\lambda = -1$$
$$\lambda = 2$$

$$y = C_1 e^{-x} + C_2 e^{2x}$$

$$\begin{cases} C_1 = -2 \\ C_2 = 0 \end{cases}$$

$$y = -2e^{-x}$$

758

$$\begin{cases} y'' - y = 1 \\ y(0) = 0 \\ y(+\infty) - \textit{ограничена} \end{cases}$$

$$\lambda^2 - 1 = 0$$

$$\lambda = \pm 1$$

$$y = C_1 e^{x} + C_2 e^{-x} - 1$$

$$\begin{cases} C_1 = 0 \\ C_1 + C_2 = 1 \end{cases}$$

$$y = e^{-x} - 1$$

759

$$\begin{cases} y'' - 2iy = 0 \\ y(0) = -1 \\ y(+\infty) = 0 \end{cases}$$

$$\lambda^2 - 2i = 0 \implies \lambda^2 = 2e^{\dfrac{\pi}{2}i} \implies \lambda = \{\sqrt{2}e^{\dfrac{\pi}{4}i}, \sqrt{2}e^{\dfrac{5\pi}{4}i}\} = \{1+i, -1-i\}$$

$$y = C_1 e^x e^{ix} + C_2 e^{-x} e^{-ix}$$

$$\begin{cases} C_1 = 0 \\ C_1 + C_2 = -1 \end{cases}$$

$$y = -e^{-(1+i)x}$$

760

$$\begin{cases} x^2 y'' - 6y = 0 \\ y(0) - \text{ограничена} \\ y(1) = 2 \end{cases}$$

$$x = e^t$$

$$e^{2t}e^{-3t}(y''e^t - y'e^t) - 6y = y'' - y' - 6y = 0$$

$$D = 5^2 \implies \lambda = \frac{1 \pm 5}{2} = \{3, -2\}$$

$$y = C_1 e^{3x} + C_2 e^{-2x}$$

$$\begin{cases} C_2 = 0 \\ C_1 e^3 = 2 \end{cases}$$

$$y = \frac{2}{e^3} e^{3x}$$

761

$$\begin{cases} x^2 y'' - 2xy' + 2y = 0 \\ y(x \to 0) = o(x) \\ y(1) = 3 \end{cases}$$

$$x = e^t$$

$$e^{2t}e^{-3t}(y''e^t - y'e^t) - 2e^t e^{-t} y' + 2y = y'' - 3y' + 2y = 0$$

$$\lambda = \{1, 2\}$$

$$y = C_1 e^x + C_2 e^{2x}$$

$$\begin{cases} \lim\limits_{x \to 0} \dfrac{y}{x} = 0 \implies C_1 = C_2 = 0 \\ y(1) = 3 \end{cases}$$

$$\nexists y$$

762

$$\begin{cases} x^2 y'' + 5xy' + 3y = 0 \\ y'(1) = 3 \\ y(x \to +\infty) = O(x^{-2}) \end{cases}$$

$$x = e^t$$

$$e^{2t}e^{-3t}(y''e^t - y'e^t) + 5e^t e^{-t} y' + 3y = y'' + 4y' + 3y = 0$$

$$\lambda = \{-1, -3\}$$

$$y = C_1 e^{-x} + C_2 e^{-3x}$$

$$\begin{cases} \lim\limits_{x \to +\infty} yx^2 = \infty \implies C_1 = C_2 = 0 \\ y'(1) = 0 \end{cases}$$

$$\nexists y$$

763

$$\begin{cases} y'' + ay' = 1 \\ y(0) = 0 \\ y(1) = 0 \end{cases}$$

Когда нет решений?

$$\lambda^2 + a\lambda = 0 \implies \lambda = \{0, -a\}$$

$$a = 0$$

$$y'' = 1$$

$$y = \frac{x^2}{2} + C_1 x + C_2$$

$$\begin{cases} C_2 = 0 \\ C_1 + C_2 = -\dfrac{1}{2} \end{cases}$$

$$a \neq 0$$

$$y = C_1 + C_2 e^{-ax} + \frac{x}{a}$$

$$\begin{cases} C_1 + C_2 = 0 \\ C_1 + C_2 e^{-a} + \dfrac{1}{a} = 0 \end{cases}$$

$$\begin{cases} C_1 = \dfrac{1}{a(e^{-a} - 1)} \\ C_2 = -\dfrac{1}{a(e^{-a} - 1)} \end{cases}$$

Есть всегда?

782

$$\begin{cases} y'' = ky \\ y(0) = 0 \\ y(l) = 0 \end{cases}$$

Найти $k, y \not\equiv 0$

$$\lambda^2 = k \implies \lambda = \pm\sqrt{k}$$

$$k = 0$$

$$y = C_1 x + C_0$$

$$\begin{cases} C_0 = 0 \\ C_1 l + C_0 = 0 \implies C_1 = 0 \end{cases}$$

$$k > 0$$

$$y = C_0 e^{\sqrt{k}x} + C_1 e^{-\sqrt{k}x}$$

$$\begin{cases} C_0 + C_1 = 0 \\ C_0 e^{\sqrt{k}l} + C_1 e^{-\sqrt{k}l} = 0 \end{cases}$$

$$k < 0$$

$$y = C_0 \cos(\sqrt{-k}x) + C_1 \sin(\sqrt{-k}x)$$

$$\begin{cases} C_0 = 0 \\ \sin(\sqrt{-k}l) = 0 \implies k = -\dfrac{\pi^2 n^2}{l^2} \end{cases}$$

$$k = -\frac{\pi^2 n^2}{l^2} \quad n \in \mathbb{Z}$$

Диффуры IDZ 7 (766,767,769-772)

766

$$y'' + y' = f(x)$$

$$y(0) = 0$$

$$y'(1) = 0$$

$$y = C_0 e^{-x} + C_1$$

$$\begin{cases} C_0 + C_1 = 0 \\ -\dfrac{C_0}{e} = 0 \end{cases}$$

$$y_1 = e^{-x} + 1$$

$$y_2 = 1$$

$$G(x, s) = \begin{cases} a(s)(e^{-x} + 1) & 0 \le x \le s \\ b(s) & s \le x \le 1 \end{cases}$$

$$\begin{cases} a(s)(e^{-x} + 1) = b(s) \\ a(s)e^{-x} = 1 \end{cases}$$

$$\begin{cases} b(s) = 1 + e^x \\ a(s) = e^x \end{cases}$$

$$G(x, s) = \begin{cases} e^s(e^{-x} + 1) & 0 \le x \le s \\ (e - s + 1) & s \le x \le 1 \end{cases}$$

767

$$y'' - y = f(x)$$

$$y'(0) = 0$$

$$y'(2) + y(2) = 0$$

$$y = C_0 e^{-x} + C_1 e^x$$

$$\begin{cases} C_0 = C_1 \\ -C_0 e^{-2} + C_1 e^2 + C_0 e^{-2} + C_1 e^2 = 0 \end{cases}$$

$$\begin{cases}C_0 = C_1\\ 2C_1e^2 = 0\end{cases}$$

$$y_1 = e^{-x} + e^x$$

$$y_2 = e^{-x}$$

$$G(x,s) = \begin{cases}a(s)(e^{-x}+e^x) & 0 \le x \le s\\ b(s)e^{-x} & s \le x \le 2\end{cases}$$

$$\begin{cases}a(s)(e^{-x}+e^x) = b(s)e^{-x}\\ -b(s)e^{-x} = a(s)(-e^{-x}+e^x)+1\end{cases}$$

$$\begin{cases}a(s) = -e^{-x}\\ b(s) = (e^{-x}+e^x)\end{cases}$$

$$G(x,s) = \begin{cases}e^{-s}(e^{-x}+e^x) & 0 \le x \le s\\ (e^{-s}+e^s)e^{-x} & s \le x \le 2\end{cases}$$

769

$$x^2y'' + 2xy' = f(x)$$

$$y(1) = 0$$

$$y'(3) = 0$$

$$y = \frac{C_0}{x} + C_1$$

$$\begin{cases}C_0 = -C_1\\ -\dfrac{C_0}{x^2} = 0\end{cases}$$

$$\begin{cases}C_0 = -C_1\\ C_0 = 0\end{cases}$$

$$y_1 = \frac{1}{x} - 1$$

$$y_2 = 1$$

$$G(x,s) = \begin{cases}a(s)(\dfrac{1}{x}-1) & 1 \le x \le s\\ b(s) & s \le x \le 3\end{cases}$$

$$\begin{cases}a(s)(\dfrac{1}{s}-1) = b(s)\\ 0 = -a(s)\dfrac{1}{s^2} + \dfrac{1}{s^2}\end{cases}$$

$$\begin{cases}b(s) = \dfrac{1}{s} - 1\\ a(s) = 1\end{cases}$$

$$G(x,s) = \begin{cases}\dfrac{1}{x} - 1 & 1 \le x \le s\\ \dfrac{1}{s} - 1 & s \le x \le 3\end{cases}$$

770

$$xy'' - y' = f(x)$$

$$y'(1) = 0$$

$$y(2) = 0$$

$$y = C_0 x^2 + C_1$$

$$\begin{cases} 2C_0 = 0 \\ 4C_0 + C_1 = 0 \end{cases}$$

$$y_1 = 1$$

$$y_2 = 4x^2 - 1$$

$$G(x, s) = \begin{cases} a(s) \quad 1 \le x \le s \\ b(s)(4x^2 - 1) \quad s \le x \le 2 \end{cases}$$

$$\begin{cases} a(s) = b(s)(4s^2 - 1) \\ 8b(s)s = \dfrac{1}{s} \end{cases}$$

$$\begin{cases} b(s) = \dfrac{1}{8s^2} \\ a(s) = \dfrac{1}{2}(1 - \dfrac{1}{4s^2}) \end{cases}$$

$$G(x, s) = \begin{cases} \dfrac{1}{2}(1 - \dfrac{1}{4s^2}) \quad 1 \le x \le s \\ \dfrac{1}{8s^2}(4x^2 - 1) \quad s \le x \le 2 \end{cases}$$

771

$$x^2 y'' - 2y = f(x)$$

$$y(1) = 0$$

$$y(2) + 2y'(2) = 0$$

$$y = C_0 x^2 + \frac{C_1}{x}$$

$$\begin{cases} C_0 + C_1 = 0 \\ 4C_0 + \dfrac{1}{2}C_1 + 8C_0 - \dfrac{1}{2}C_1 = 0 \end{cases}$$

$$\begin{cases} C_0 = -C_1 \\ C_0 = 0 \end{cases}$$

$$y_1 = x^2 - \frac{1}{x}$$

$$y_2 = \frac{1}{x}$$

$$G(x, s) = \begin{cases} a(s)(x^2 - \dfrac{1}{x}) \quad 1 \le x \le s \\ b(s)\dfrac{1}{x} \quad s \le x \le 2 \end{cases}$$

$$\begin{cases} a(s)(s^2 - \dfrac{1}{s}) = b(s)\dfrac{1}{s} \\ -b(s)\dfrac{1}{s^2} = a(s)(2s + \dfrac{1}{s^2}) + \dfrac{1}{s^2} \end{cases}$$

$$\begin{cases} b(s) = -\dfrac{1}{3s^3}(s^3 - 1) \\ a(s) = -\dfrac{1}{3s^3} \end{cases}$$

$$G(x,s)=\begin{cases}-\frac{1}{3s^3}(x^2-\frac{1}{x}) & 1\le x\le s\\ -\frac{1}{3s^3}(s^3-1)\frac{1}{x} & s\le x\le 2\end{cases}$$

772

$$y''=f(x)$$

$$y(0)=0$$

$$y(+\infty)-\textit{ограничено}$$

$$y=C_0x+C_1$$

$$\begin{cases}C_1=0\\ C_0=0\end{cases}$$

$$y_1=x$$

$$y_2=1$$

$$G(x,s)=\begin{cases}a(s)x & 0\le x\le s\\ b(s) & s\le x\le +\infty\end{cases}$$

$$\begin{cases}a(s)s=b(s)\\ a(s)=-1\end{cases}$$

$$\begin{cases}b(s)=-s\\ a(s)=-1\end{cases}$$

$$G(x,s)=\begin{cases}-x & 0\le x\le s\\ -s & s\le x\le +\infty\end{cases}$$

Диффуры IDZ 8 (882-888,899-906)

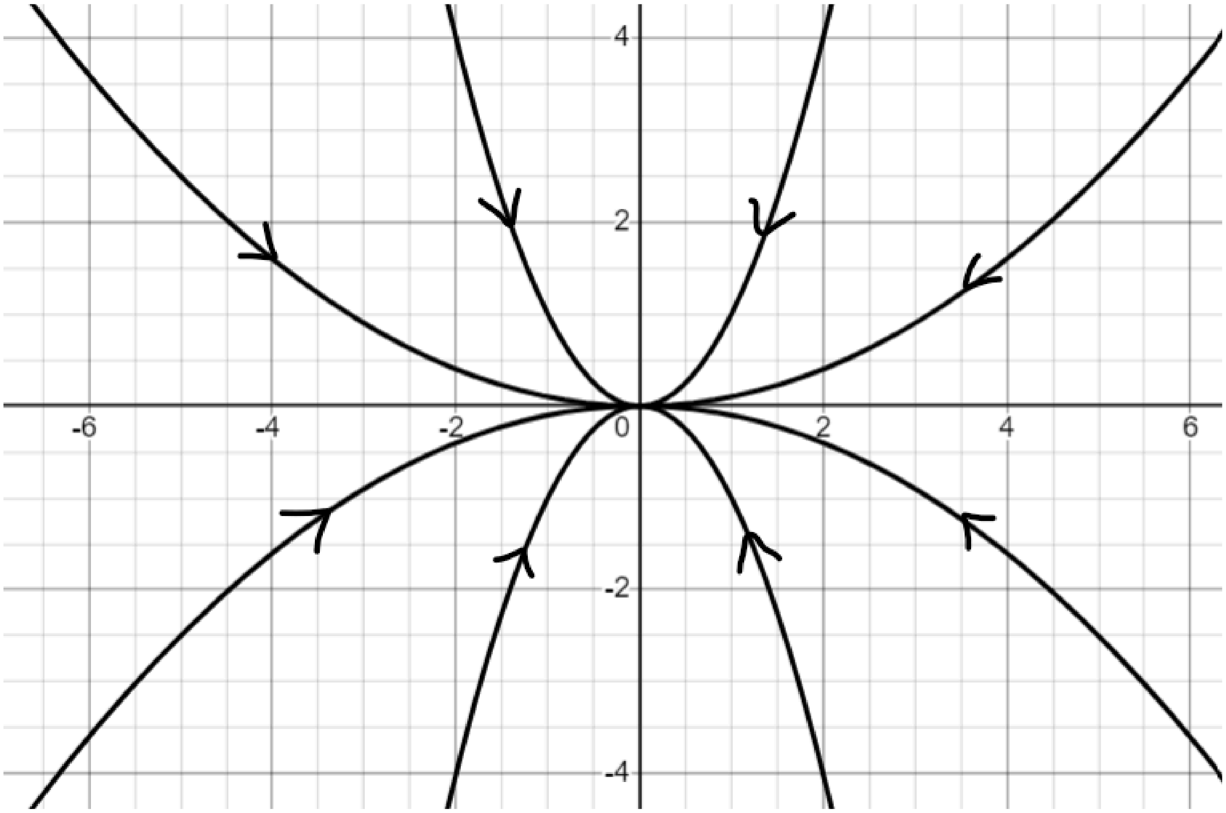
882

$$\dot{x}=-x,\quad \dot{y}=-2y$$

$$y'=2\frac{y}{x}$$

$$\ln y=\ln Cx^2$$

$$y=Cx^2$$



Нулевое решение асимптотически устойчиво

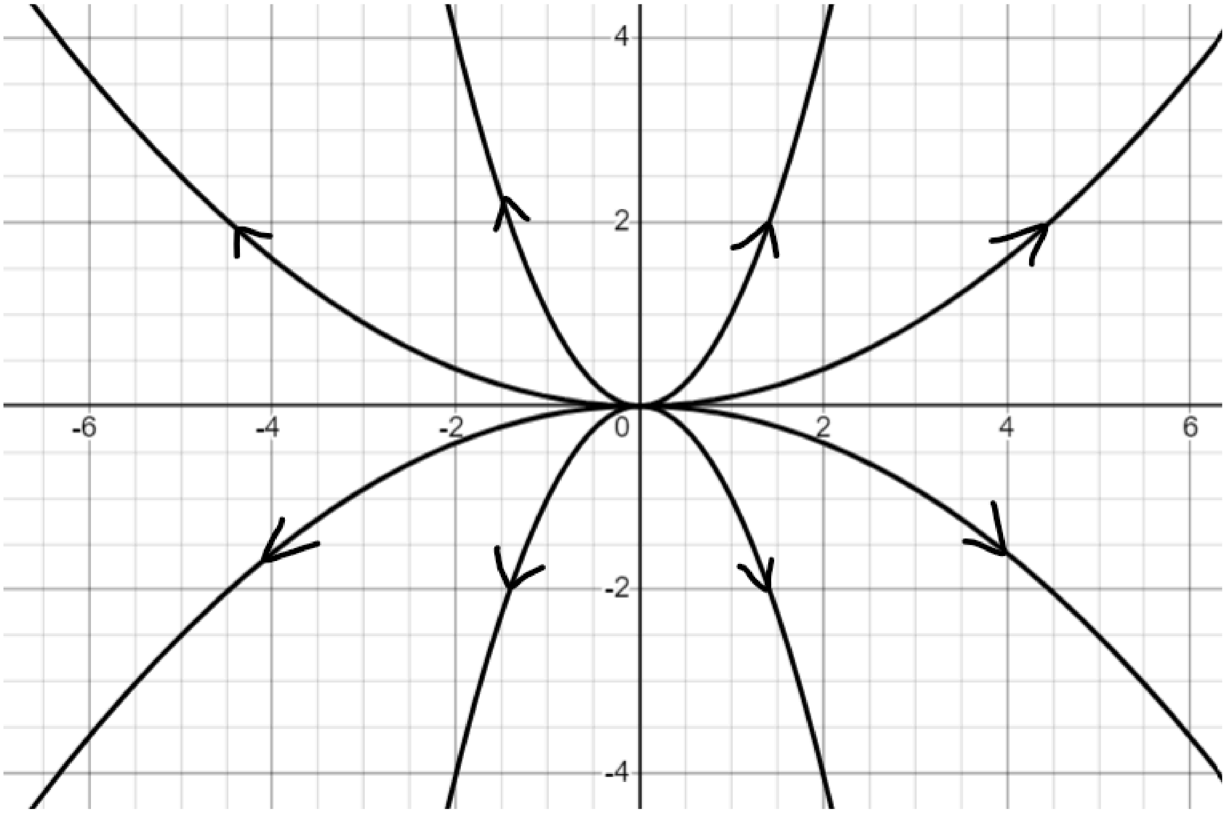
883

$$\dot{x} = x, \quad \dot{y} = 2y$$

$$y' = 2\frac{y}{x}$$

$$\ln y = \ln Cx^2$$

$$y = Cx^2$$



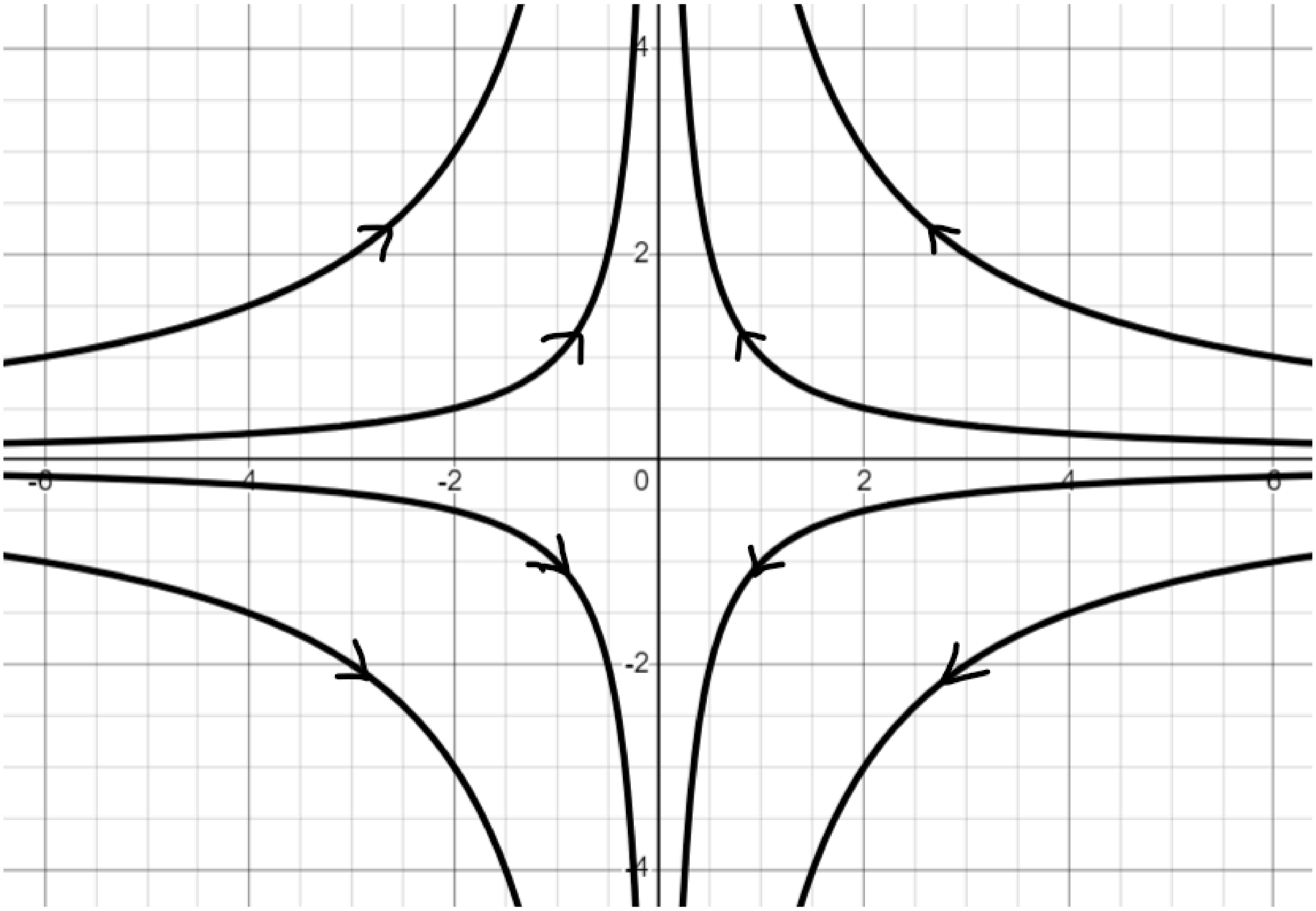
Нулевое решение не устойчиво

884

$$\dot{x} = -x, \quad \dot{y} = y$$

$$y' = -\frac{y}{x}$$

$$y = \frac{C}{x}$$



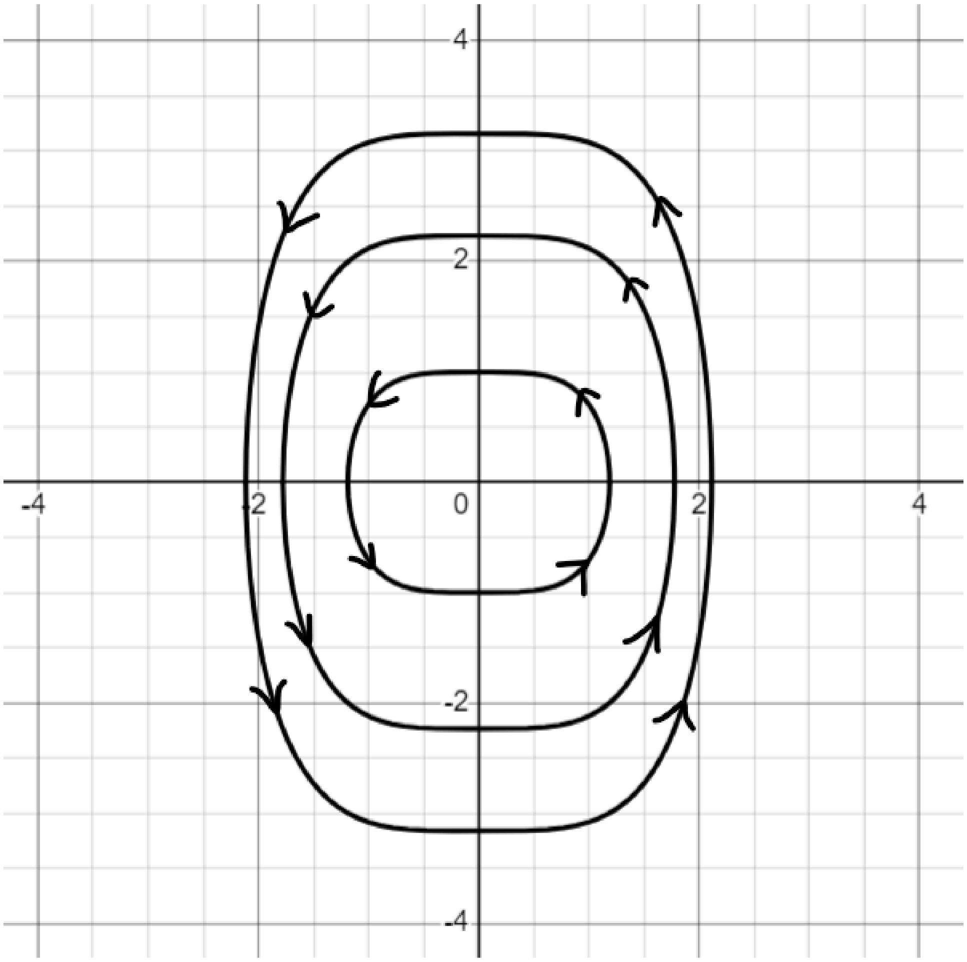
Нулевое решение не устойчиво

885

$$\dot{x} = -y, \quad \dot{y} = 2x^3$$

$$y' = -2\frac{x^3}{y}$$

$$y^2 + \frac{x^4}{2} = C$$



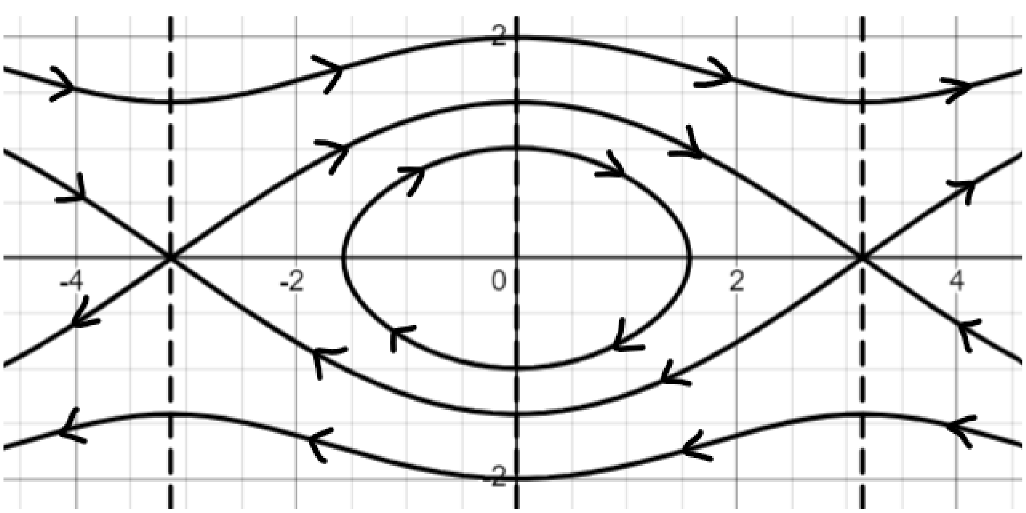
Нулевое решение устойчиво по Ляпунову, но не асимптотически

886

$$\dot{x} = y, \quad \dot{y} = -\sin x$$

$$y' = -\frac{\sin x}{y}$$

$$y^2 = \cos x + C$$



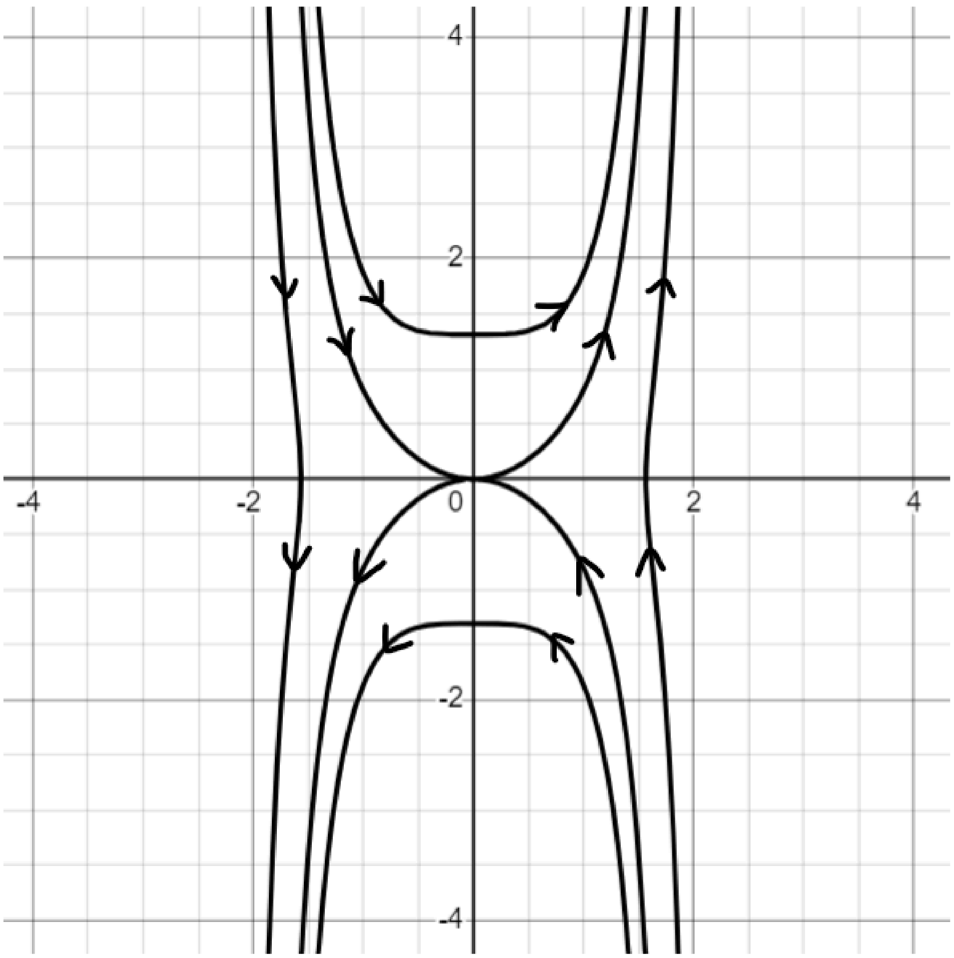
Нулевое решение устойчиво по Ляпунову

887

$$\dot{x} = y, \quad \dot{y} = x^3(1+y^2)$$

$$y' = \frac{x^3(1+y^2)}{y}$$

$$\ln(y^2+1) = \frac{x^4}{2} + C$$



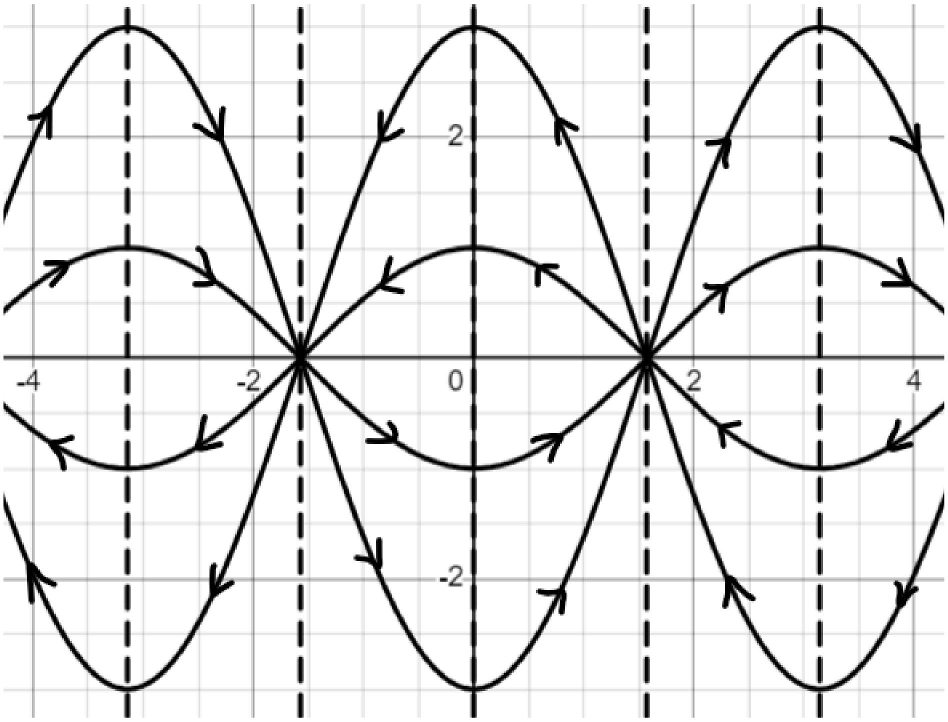
Нулевое решение не устойчиво

888

$$\dot{x} = -y\cos x, \quad \dot{y} = \sin x$$

$$y' = -y\tan x$$

$$y = C\cos x$$



Нулевое решение не устойчиво

899

$$\begin{cases} \dot{x} = 2xy - x + y \\ \dot{y} = 5x^4 + y^3 + 2x - 3y \end{cases}$$

$$\begin{cases} \dot{x} = -x + y \\ \dot{y} = 2x - 3y \end{cases}$$

$$\begin{pmatrix} -1-\lambda & 1 \\ 2 & -3-\lambda \end{pmatrix}$$

$$\lambda^2 + 4\lambda + 1 = 0$$

$$\lambda = -2 \pm \sqrt{3}$$

Нулевое решение асимптотически устойчиво

900

$$\begin{cases} \dot{x} = x^2 + y^2 - 2x \\ \dot{y} = 3x^2 - x + 3y \end{cases}$$

$$\begin{cases} \dot{x} = -2x \\ \dot{y} = -x + 3y \end{cases}$$

$$\begin{pmatrix} -2-\lambda & 0 \\ -1 & 3-\lambda \end{pmatrix}$$

$$\lambda_{1,2} = -2, 3$$

Нулевое решение неустойчиво

901

$$\begin{cases} \dot{x} = e^{x+2y} - \cos 3x \\ \dot{y} = \sqrt{4+8x} - 2e^y \end{cases}$$

$$\begin{cases} \dot{x} = x + 2y \\ \dot{y} = 2x - 2y \end{cases}$$

$$\begin{pmatrix} 1-\lambda & 2 \\ 2 & -2-\lambda \end{pmatrix}$$

$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda = \frac{-1 \pm 5}{2}$$

Нулевое решение не устойчиво

902

$$\begin{cases} \dot{x} = \ln(4y + e^{-3x}) \\ \dot{y} = 2y - 1 + \sqrt[3]{1-6x} \end{cases}$$

$$\begin{cases} \dot{x} = -3x + 4y \\ \dot{y} = -6x + 2y \end{cases}$$

$$\begin{pmatrix} -3-\lambda & 4 \\ -6 & 2-\lambda \end{pmatrix}$$

$$\lambda^2 + \lambda + 18 = 0$$

$$\lambda = \frac{-1 \pm \sqrt{-71}}{2}$$

Нулевое решение асимптотически устойчиво

903

$$\begin{cases} \dot{x} = \ln(3e^y - 2\cos x) \\ \dot{y} = 2e^x - \sqrt[3]{8+12y} \end{cases}$$

$$\begin{cases} \dot{x} = 3y \\ \dot{y} = 2x - 3y \end{cases}$$

$$\begin{pmatrix} -\lambda & 3 \\ 2 & -3-\lambda \end{pmatrix}$$

$$\lambda^2 + 3\lambda - 6 = 0$$

$$\lambda = \frac{-3 \pm \sqrt{33}}{2}$$

Нулевое решение не устойчиво

904

$$\begin{cases} \dot{x} = \tan(y - x) \\ \dot{y} = 2^y - 2\cos(\frac{\pi}{3} - x) \end{cases}$$

$$\begin{cases} \dot{x} = -x + y \\ \dot{y} = -\sqrt{3}x + \ln(2)y \end{cases}$$

$$\begin{pmatrix} -1-\lambda & 1 \\ -\sqrt{3} & \ln(2)-\lambda \end{pmatrix}$$

$$\lambda^2 - (\ln 2 - 1)\lambda - \ln 2 - \sqrt{3} = 0$$

$$\begin{cases} \lambda_1 + \lambda_2 = \ln 2 - 1 \\ \lambda_1 \cdot \lambda_2 = -\ln 2 - \sqrt{3} \end{cases}$$

Нулевое решение не устойчиво

905

$$\begin{cases} \dot{x} = \tan(z-y) - 2x \\ \dot{y} = \sqrt{9+12x} - 3e^y \\ \dot{z} = -3y \end{cases}$$

$$\begin{cases} \dot{x} = -2x - y + z \\ \dot{y} = 2x - 3y \\ \dot{z} = -3y \end{cases}$$

$$\begin{pmatrix} -2-\lambda & -1 & 1 \\ 2 & -3-\lambda & 0 \\ 0 & -3 & -\lambda \end{pmatrix}$$

$$-\lambda^3 - 5\lambda^2 + 4\lambda - 6 = 0$$

Real solution

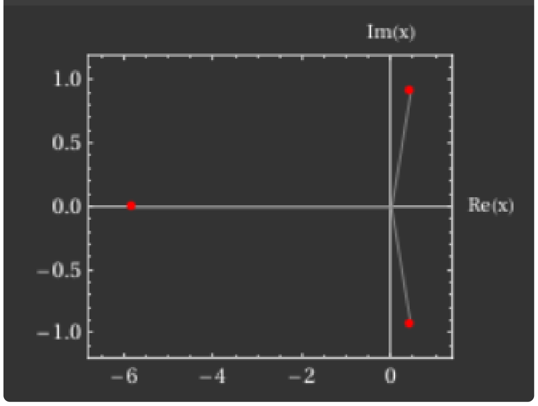
$x \approx -5.8577$

Complex solutions

$x \approx 0.42886 - 0.91672\, i$

$x \approx 0.42886 + 0.91672\, i$

Roots in the complex plane



Нулевое решение не устойчиво

906

$$\begin{cases} \dot{x} = e^{x} - e^{-3z} \\ \dot{y} = 4z - 3\sin(x+y) \\ \dot{z} = \ln(1 + z - 3x) \end{cases}$$

$$\begin{cases} \dot{x} = x + 3y \\ \dot{y} = x + y + 4z \\ \dot{z} = -3x + z \end{cases}$$

$$\begin{pmatrix} 1-\lambda & 3 & 0 \\ 1 & 1-\lambda & 4 \\ -3 & 0 & 1-\lambda \end{pmatrix}$$

$$(1-\lambda)^3 + 3(1-\lambda) - 36 = 0$$

Real solution

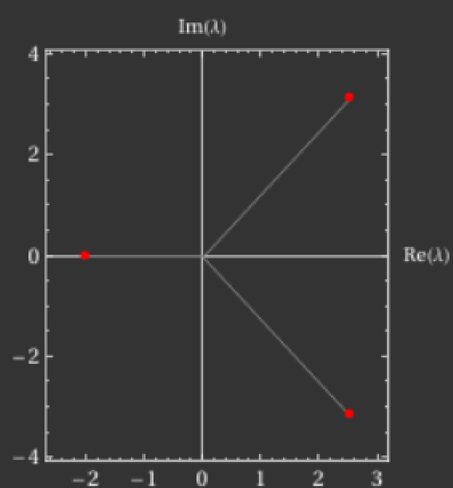
$\lambda = -2$

Complex solutions

$\lambda = \frac{5}{2} - \frac{i\sqrt{39}}{2}$

$\lambda = \frac{5}{2} + \frac{i\sqrt{39}}{2}$

Roots in the complex plane



Нулевое решение не устойчиво