Индивидуальные домашние задания по уравнениям математической физики.

1. Определить тип уравнения. Привести уравнение к каноническому виду.

   1) $u_{xx} - 2u_{xy} + 2u_{yy} = 0$
   2) $u_{xx} - 4u_{xy} + 3u_{yy} - 2u_x + 6u_y = 0$
   3) $u_{xx} + 4u_{yy} + 4u_{yy} + 3u_x + 6u_y = 0$
   4) $u_{yy} - 2u_{xy} + 2u_x - u_y - 4e^x = 0$
   5) $u_{xx} + 4u_{xy} + 5u_{yy} + u_x + 2u_y = 0$
   6) $u_{xx} - 6u_{xy} + 9u_{yy} - u_x + 2u_y = 0$
   7) $2u_{xy} - 4u_{yy} + u_x - 2u_y + x = 0$
   8) $u_{xx} + 2u_{xy} + u_{yy} + 4u_x + 4u_y = 0$
   9) $u_{xx} - u_{yy} + u_x + u_y = 0$
   10) $3u_{xx} - 10u_{xy} + 3u_{yy} - 2u_x + 4u_y + 2y = 0$
   11) $u_{xx} - 2u_{xy} + u_{yy} = 0$
   12) $u_{xx} - 2u_{xy} + u_x + 4e^y = 0$
   13) $(1 + x^2)^2 u_{xx} + u_{yy} + 2x(1 + x^2)u_x = 0$
   14) $2u_{xx} + 6u_{xy} + 4u_{yy} + u_x + u_y = 0$
   15) $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} - 2yu_x = 0$
   16) $u_{xy} + 2u_{yy} - u_x + 4u_y = 0$
2. Найдите решение уравнения $u_{tt} = u_{xx}$ при заданных начальных условиях $u(x, 0)$ и $u_t(x, 0)$.
   1) $u(x, 0) = \frac{\sin x}{x}$, $u_t(x, 0) = \frac{x}{1+x^2}$
   2) $u(x, 0) = \frac{x}{1+x^2}$, $u_t(x, 0) = \sin x$
   3) $u(x, 0) = \frac{1}{1+x^2}$, $u_t(x, 0) = \cos x$
   4) $u(x, 0) = e^{-x^2}$, $u_t(x, 0) = \frac{x}{1+x^2}$
   5) $u(x, 0) = x^2$, $u_t(x, 0) = 4x$
   6) $u(x, 0) = e^{-x^2}$, $u_t(x, 0) = \frac{1}{1+x^2}$
   7) $u(x, 0) = 3x^2$, $u_t(x, 0) = \sin x$
   8) $u(x, 0) = 2x^3$, $u_t(x, 0) = \frac{x}{1+x^2}$
   9) $u(x, 0) = \sin x$, $u_t(x, 0) = 2x^2$
   10) $u(x, 0) = \frac{\sin x}{x}$, $u_t(x, 0) = 9x$
   11) $u(x, 0) = \cos x$, $u_t(x, 0) = 3\sin x$
   12) $u(x, 0) = \frac{1}{1+x^2}$, $u_t(x, 0) = \frac{x}{1+x^2}$
   13) $u(x, 0) = \frac{\sin x}{x}$, $u_t(x, 0) = 5x^2$
   14) $u(x, 0) = \cos x$, $u_t(x, 0) = \frac{x}{1+x^2}$
   15) $u(x, 0) = x$, $u_t(x, 0) = \sin x$
   16) $u(x, 0) = e^{2x^2}$, $u_t(x, 0) = 9x$

3. Используя метод разделения переменных, найти решение однородного волнового уравнения $u_{tt} = a^2 u_{xx}, 0 < x < l, t > 0$ при следующих граничных и начальных условиях:

1) $u(0,t) = u(l,t) = 0,$
$u(x,0) = \sin\frac{\pi}{l}x + \sin\frac{3\pi}{l}x,$
$u_t(x,0) = 0$

2) $u(0,t) = u(l,t) = 0,$
$u(x,0) = 0,$
$u_t(x,0) = 1$

3) $u(0,t) = u_x(l,t) = 0,$
$u(x,0) = \sin\frac{\pi}{2l}x + \sin\frac{3\pi}{2l}x,$
$u_t(x,0) = 0$

4) $u(0,t) = u(l,t) = 0,$
$u(x,0) = \sin\frac{2\pi}{l}x,$
$u_t(x,0) = 1$

5) $u_x(0,t) = u_x(l,t) = 0,$
$u(x,0) = 1,$
$u_t(x,0) = 1$

6) $u_x(0,t) = u_x(l,t) = 0,$
$u(x,0) = 0,$
$u_t(x,0) = 1 + \cos\frac{\pi}{l}x + \cos\frac{3\pi}{l}x$

7) $u_x(0,t) = u(l,t) = 0,$
$u(x,0) = 0,$
$u_t(x,0) = \cos\frac{\pi}{2l}x + \cos\frac{5\pi}{2l}x$

8) $u(0,t) = u_x(l,t) = 0,$
$u(x,0) = \sin\frac{5\pi}{2l}x,$
$u_t(x,0) = 1$

9) $u_x(0,t) = u_x(l,t) = 0,$
$u(x,0) = U = const,$
$u_t(x,0) = V = const$

10) $u(0,t) = u(l,t) = 0,$
$u(x,0) = 0,$
$u_t(x,0) = 1$

11) $u_x(0,t) = u(l,t) = 0,$
$u(x,0) = \cos\frac{3\pi}{2l}x,$
$u_t(x,0) = 1$

12) $u_x(0,t) = u_x(l,t) = 0,$
$u(x,0) = 1,$
$u_t(x,0) = 2 + \cos\frac{\pi}{l}x$

13) $u(0,t) = u(l,t) = 0,$
$u(x,0) = \sin\frac{\pi}{l}x,$
$u_t(x,0) = \sin\frac{\pi}{l}x + \sin\frac{3\pi}{l}x$

14) $u_x(0,t) = u(l,t) = 0,$
$u(x,0) = \cos\frac{\pi}{2l} + 3\cos\frac{3\pi}{2l}x,$
$u_t(x,0) = \cos\frac{3\pi}{2l}x$

15) $u(0,t) = u_x(l,t) = 0,$
$u(x,0) = \sin\frac{\pi}{2l}x,$
$u_t(x,0) = \sin\frac{\pi}{2l}x + \sin\frac{3\pi}{2l}x$

16) $u_x(0,t) = u_x(l,t) = 0,$
$u(x,0) = 2 + \cos\frac{\pi}{l}x,$
$u_t(x,0) = 1 + \cos\frac{2\pi}{l}x$

4. Решить методом разделения переменных следующую задачу для неоднородного уравнения теплопроводности $u_t = a^2u_{xx} + f(x,t)$, $0 < x < 1, t > 0$ при:

1) $f(x,t) = 2x + 1$
$u(0,t) = 1; u(1,t) = 2$
$u(x,0) = x + 1$

2) $f(x,t) = x + 2$
$u_x(0,t) = 1; u(1,t) = 0$
$u(x,0) = x - 1$

3) $f(x,t) = 2x + 1$
$u(0,t) = 1; u_x(1,t) = 2$
$u(x,0) = 2x + 1$

4) $f(x,t) = x + 1$

$u(0,t) = 0; u(1,t) = 1$
$u(x,0) = x$

5) $f(x,t) = 2x + 1$
$u_x(0,t) = 2; u(1,t) = 1$
$u(x,0) = 2x - 1$

6) $f(x,t) = x + 2$
$u(0,t) = 0; u_x(1,t) = 1$
$u(x,0) = x$

7) $f(x,t) = t$
$u(0,t) = 2t; u(1,t) = 1$
$u(x,0) = x - 3\sin 2\pi x$

8) $f(x,t) = 2xt$
$u_x(0,t) = -1; u_x(1,t) = t$
$u(x,0) = 1 - x - \cos\frac{7\pi}{2}x$

9) $f(x,t) = 2t^3$
$u(0,t) = 1; u_x(1,t) = 2t$
$u(x,0) = 1 + \sin\frac{5\pi}{2}x$

10) $f(x,t) = 2t^2$
$u(0,t) = t; u(1,t) = 2t$
$u(x,0) = 2\sin\pi x - \sin 3\pi x$

11) $f(x,t) = t$
$u_x(0,t) = 2t; u(1,t) = 1$
$u(x,0) = 1 + 2\cos\frac{5\pi}{2}x$

12) $f(x,t) = 2xt$
$u(0,t) = 2t; u_x(1,t) = 1$
$u(x,0) = x - 2\sin\frac{3\pi}{2}x$

13) $f(x,t) = 3t$
$u(0,t) = 1; u(1,t) = t$
$u(x,0) = 1 - x + \sin 4\pi x$

14) $f(x,t) = 2xt$
$u_x(0,t) = 2t; u(1,t) = t$
$u(x,0) = 4\cos\frac{3\pi}{2}x$

15) $f(x,t) = t^2$

$u(0,t) = t; u_x(1,t) = 2t$

$u(x,0) = 4\sin\frac{9\pi}{2}x$

16) $f(x,t) = 2t$

$u(0,t) = t^2; u(1,t) = 1$

$u(x,0) = x - \sin\pi x + 2\sin 5\pi x$