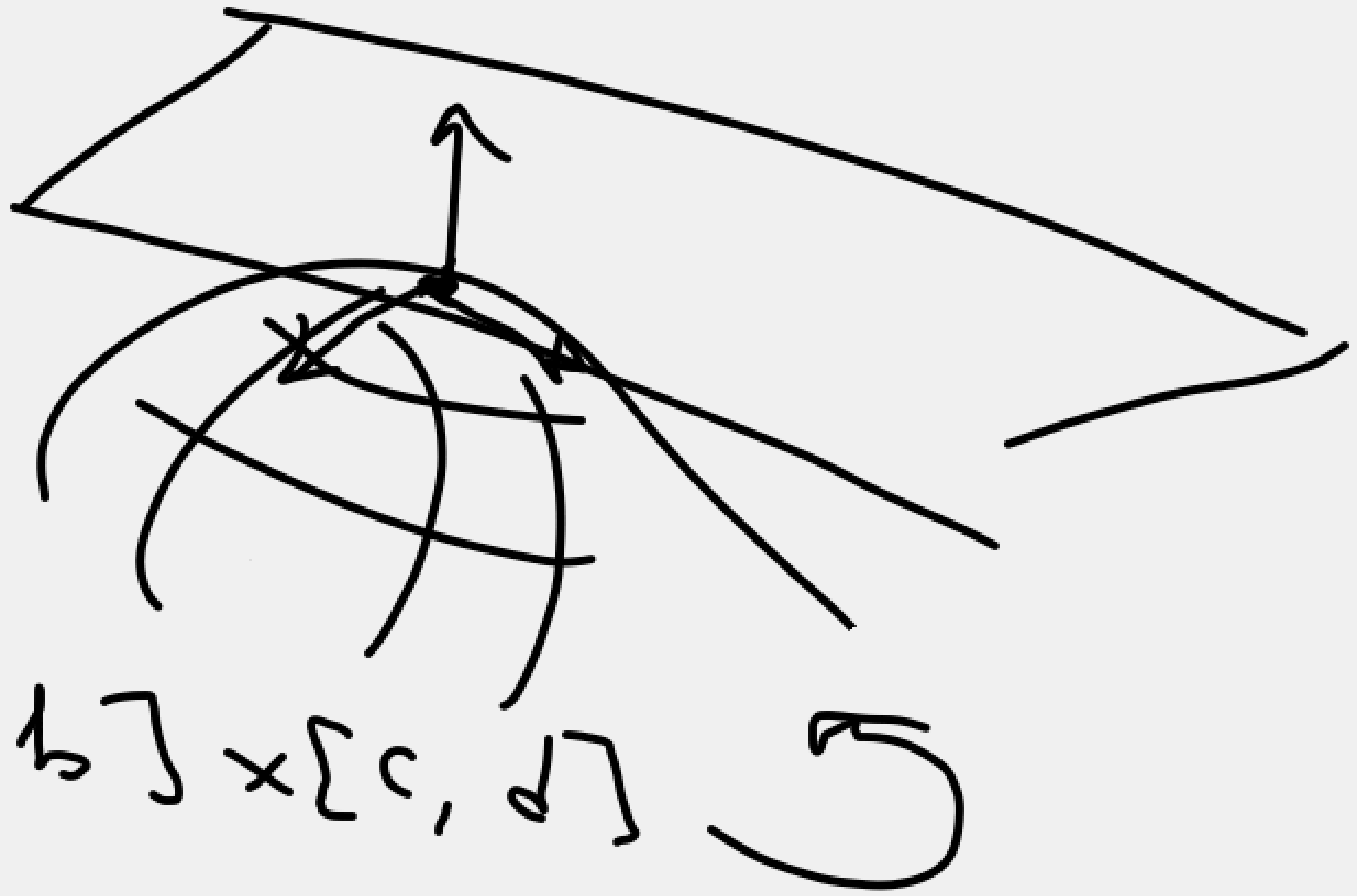
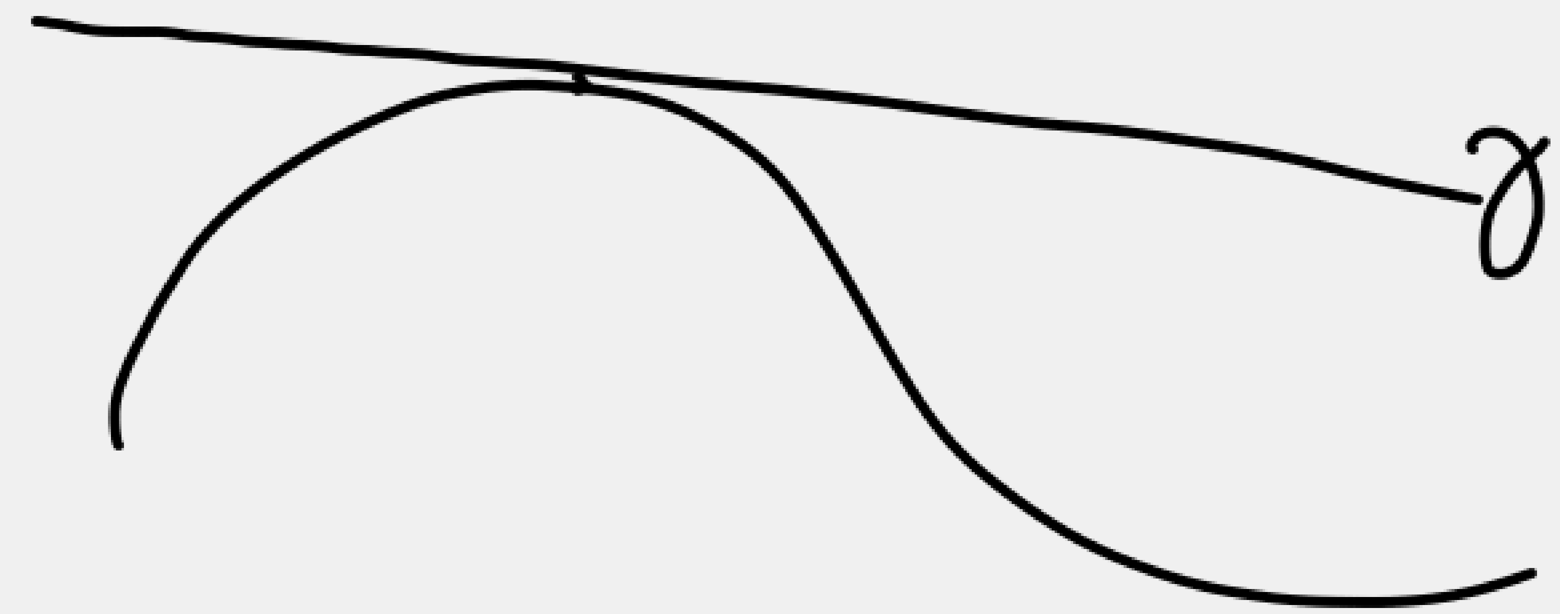


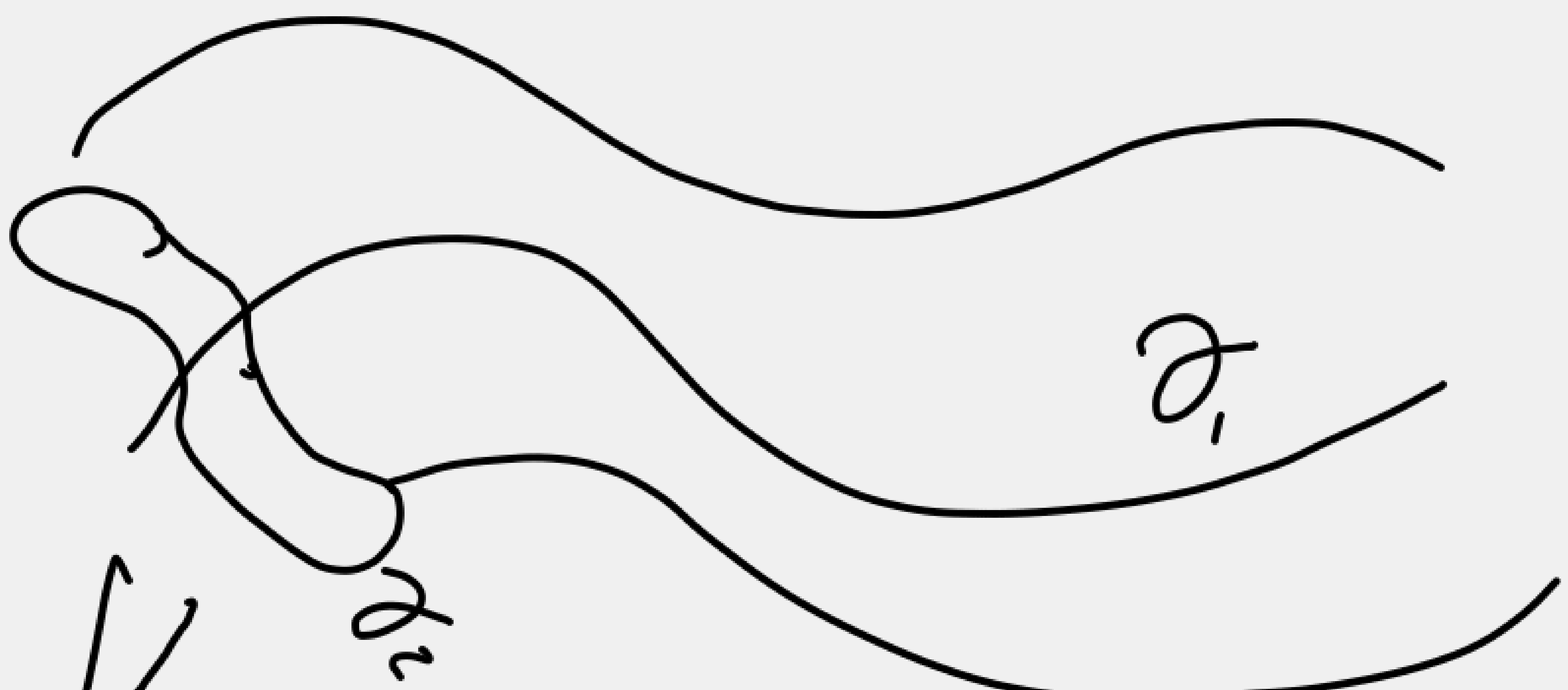
$[a,b] \times [c,d]$



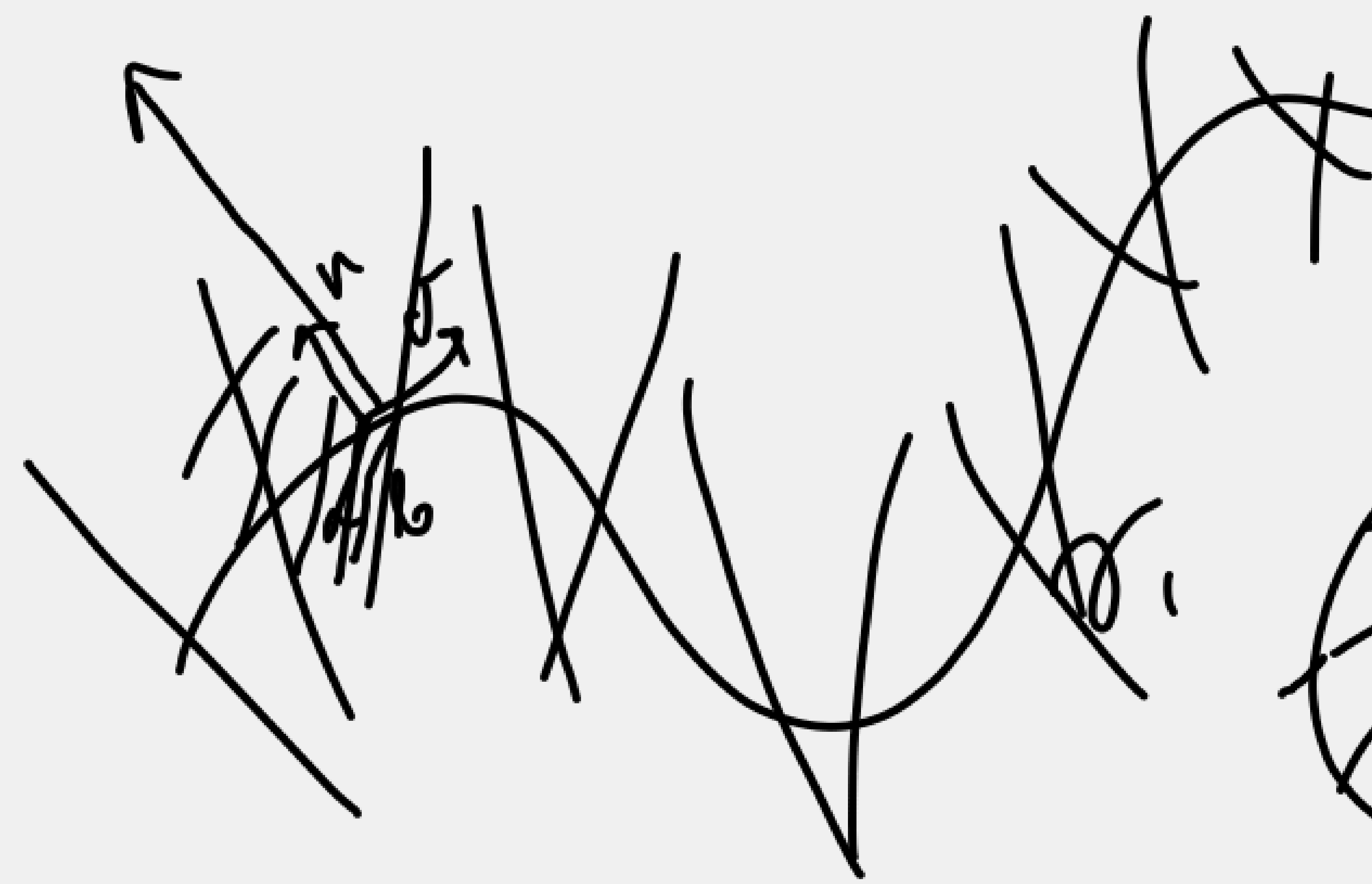
$$\vec r(u,v) = (X(u,v),\ Y(u,v),\ Z(u,v))$$

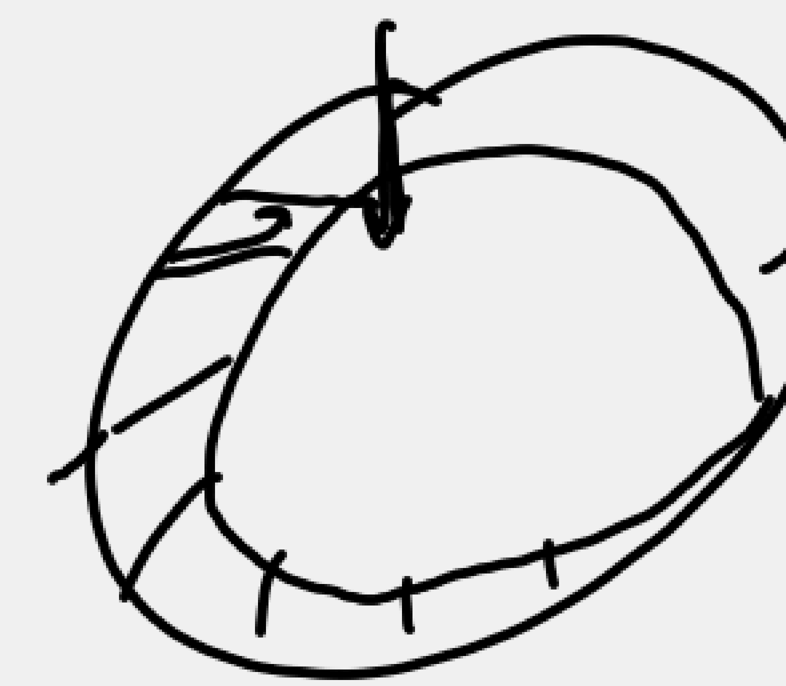
$$\gamma \begin{cases} u = u(t) \\ v = v(t) \end{cases}$$

$$(\bar X(t),\ \bar Y(t),\ \bar Z(t))$$



Канальная





Линейчатые
(поверх нормалей,
бинормалей)

$$L: \begin{cases} u = u(t) \\ v = v(t) \end{cases}$$

$$\ell(L) = \int_a^b |\gamma'|\,dt = \int_a^b \sqrt{\left(\frac{\partial X}{\partial u}u' + \frac{\partial X}{\partial v}v'\right)^2 + \left(\frac{\partial Y}{\partial u}u' + \frac{\partial Y}{\partial v}v'\right)^2 + \left(\frac{\partial Z}{\partial u}u' + \frac{\partial Z}{\partial v}v'\right)^2}\,dt =$$

$\vec{r}(u,v) \rightsquigarrow \gamma(t) = \vec{r}(u(t), v(t)) = (X(t), Y(t), Z(t))$

$(a,b) = a^T G_{r_r} b$

матрица Грама

$$G_r = \begin{pmatrix} (e_1,e_1) & (e_1,e_2) \\ (e_2,e_1) & (e_2,e_2) \end{pmatrix}$$

$$= \int_a^b \sqrt{Eu'^2 + 2Fu'v' + Gv'^2}\,dt =$$

$E = X_u^2 + Y_u^2 + Z_u^2 = (r_u, r_u)$, $G = X_v^2 + Y_v^2 + Z_v^2 = (r_v, r_v)$

$F = X_u X_v + Y_u Y_v + Z_u Z_v = (r_u, r_v)$

$$I(a,b) = a^T \underbrace{\begin{pmatrix} E & F \\ F & G \end{pmatrix}}_{=[I]} b$$

I квадр. форма

Кв. форма

$K'(s) = k_1 k_2 \qquad H'(s) = \frac{k_1 + k_2}{2}$

$\vec{x} = (x^1, x^2, x^3)$

$Q(\vec{x}) = a_{11}(x^1)^2 + a_{12} x^1 x^2 + a_{13} x^1 x^3 + \ldots + a_{33}(x^3)^2$

$= \sum_{i=1}^{3} \sum_{j=1}^{3} a_{ij} x^i x^j =: a_{ij} x^i x^j = (x^1\ x^2\ x^3) \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ & \ddots & \vdots \\ & & a_{33} \end{pmatrix}}_{\text{матр. формы } Q} \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix}$

$= x^T [a] x$

Для евкл. пр.

$c^2 = a^2 + b^2$

$c^2 = \qquad + 2Fab + Gb^2$

т.с. $|\gamma'|^2 = \underbrace{(u'\ v')}_{(\bar{x}^1, \bar{x}^2)} \underbrace{\begin{pmatrix} E & F \\ F & G \end{pmatrix}}_{\begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}} \begin{pmatrix} u' \\ v' \end{pmatrix}$

$=$

Пример

I для графика $z = f(x,y)$

$$r(x,y) = (x, y, f(x,y))$$

$$r_x = (1, 0, f_x)$$

$$r_y = (0, 1, f_y)$$

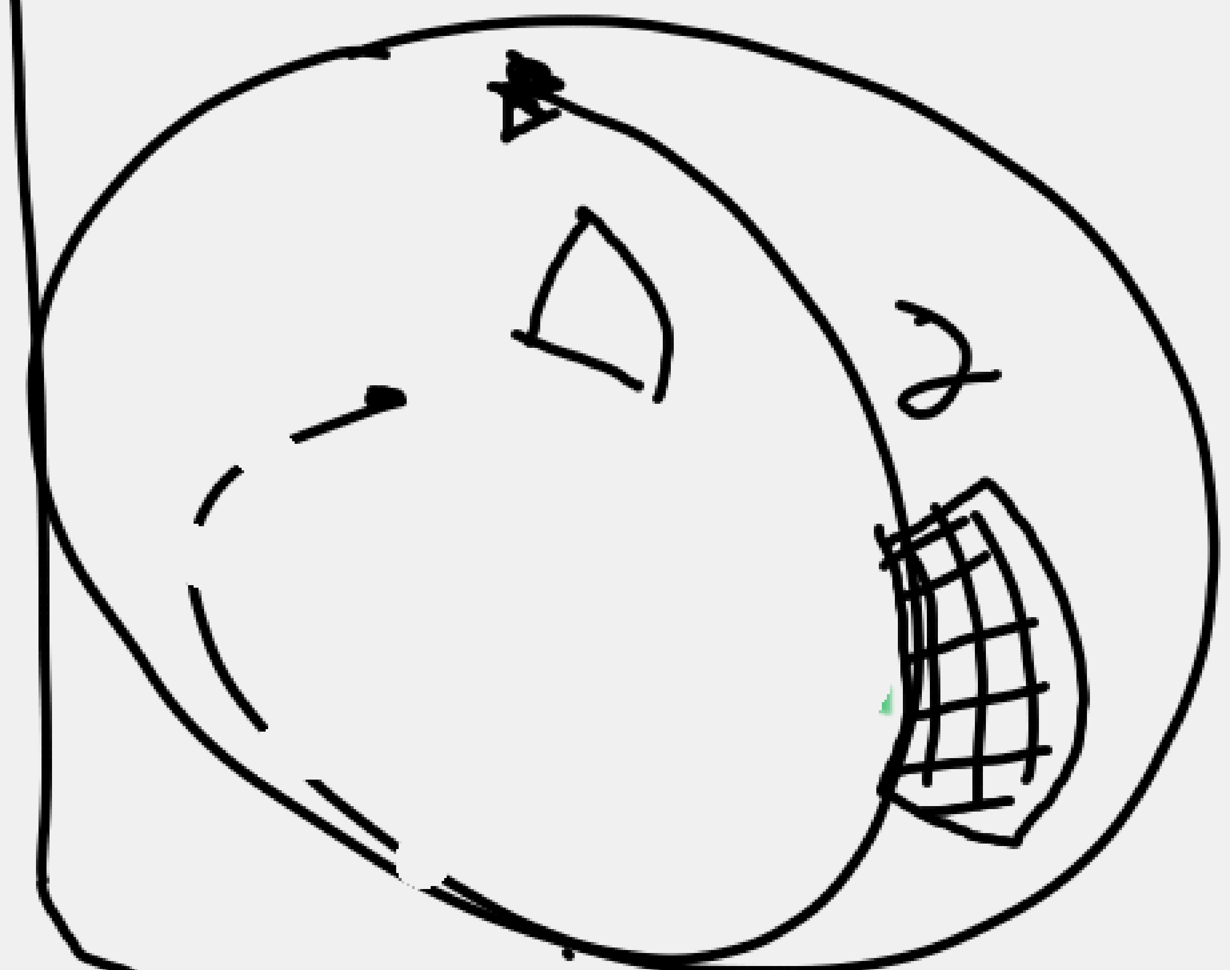
$$g_{11} = (r_x, r_x) = 1 + f_x^2$$

$$g_{12} = g_{21} = (r_x, r_y) = f_x \cdot f_y$$

$$g_{22} = (r_y, r_y) = 1 + f_y^2$$

$$[\mathrm{I}] = \begin{pmatrix} 1+f_x^2 & f_x f_y \\ f_y f_x & 1+f_y^2 \end{pmatrix}$$

$S: \quad r(u,v) = (R \sin u \cos v, R \sin u \sin v, R \cos u)$

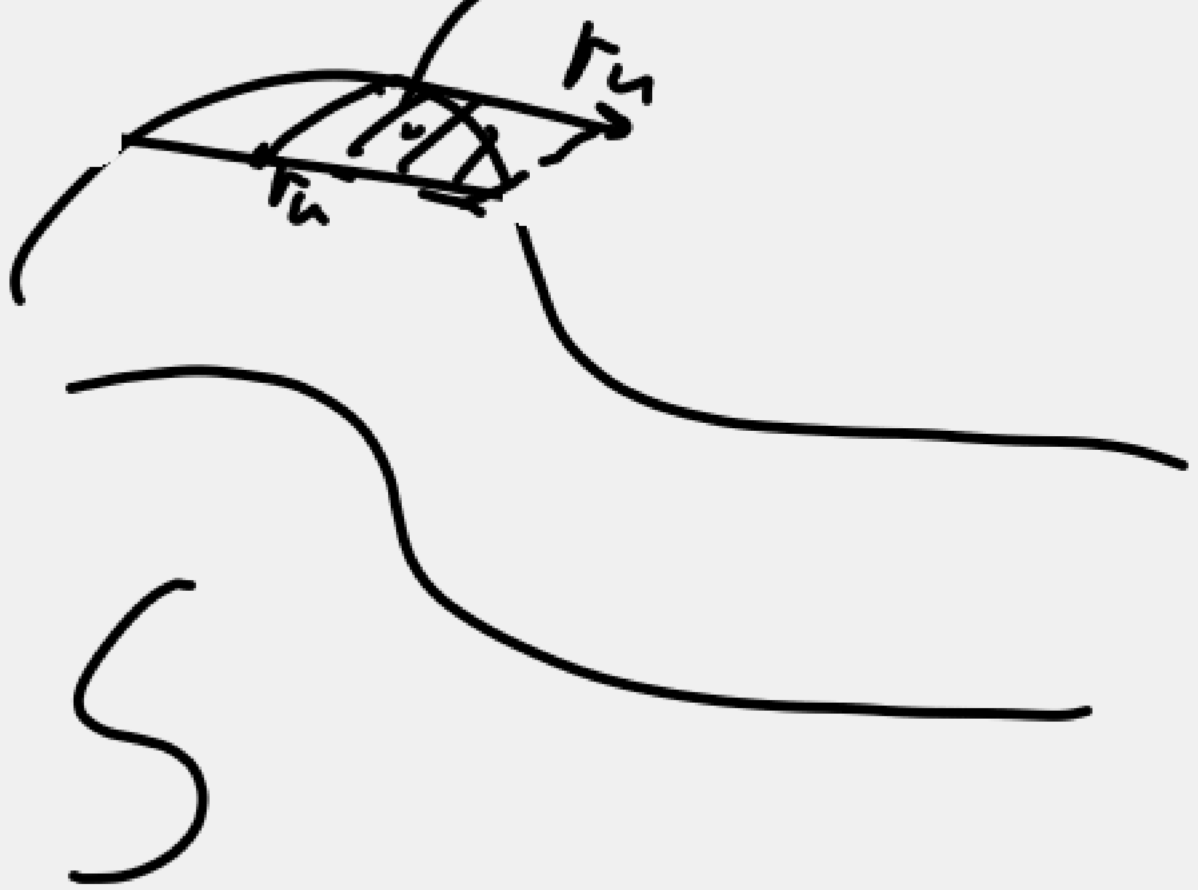


$$\gamma . \begin{cases} u = 2t \\ v = t \end{cases} \quad t \in [0,1]$$

$$\gamma(0) = (0,0), \quad \gamma(1) = (2,1)$$

$$\ell(L) = \int_0^1 \sqrt{(u' \; v') \, [I] \begin{pmatrix} u' \\ v' \end{pmatrix}} \, dt$$

Как найти площадь кусочка на поверхности



$$A = |[r_u, r_v]|$$

$$A(S) = \iint\limits_D |[r_u, r_v]|\, du\, dv = \iint\limits_D \sqrt{|[I]|}\, du\, dv$$

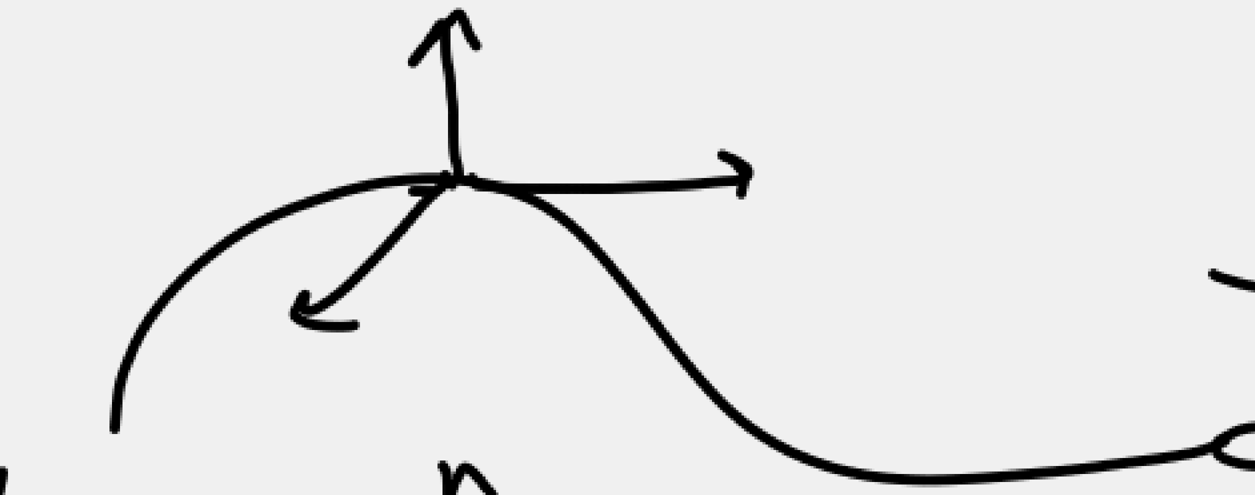
$$|[r_u, r_v]|^2 + (r_u, r_v)^2 = |r_u|^2 |r_v|^2$$

$$\Downarrow$$

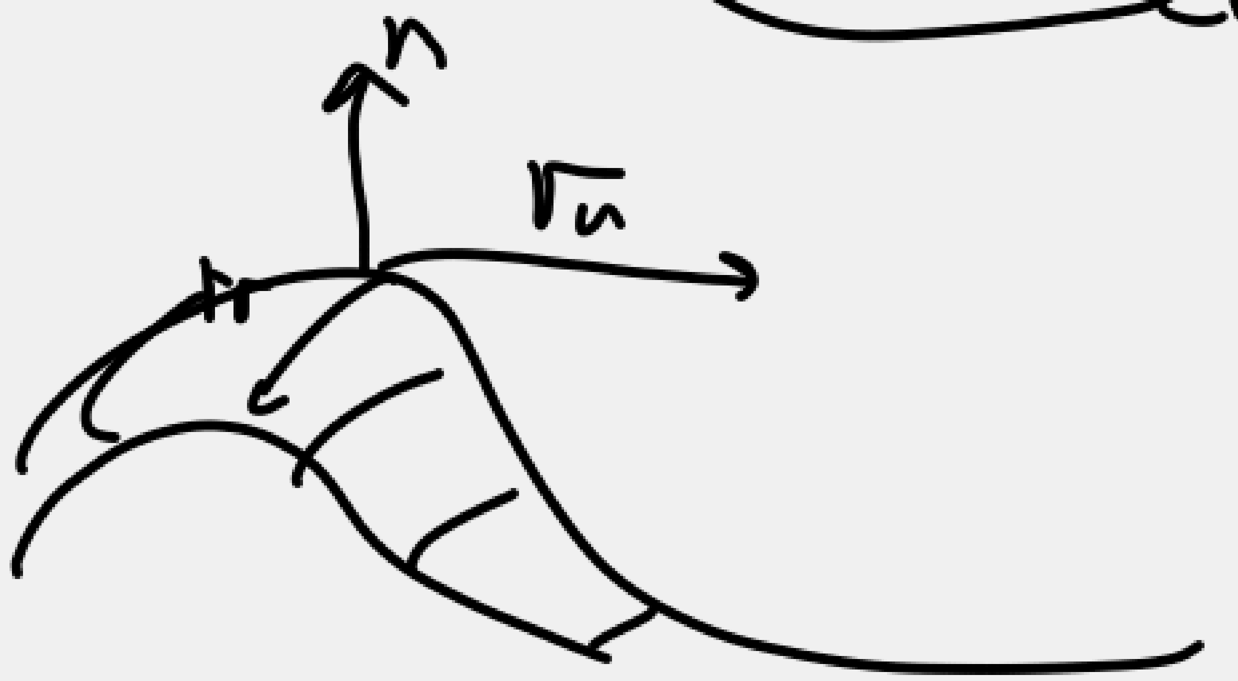
$$|[r_u, r_v]| = \sqrt{|r_v|^2 |r_v|^2 - (r_u, r_v)^2}$$

$$[I] = \begin{pmatrix} (r_u, r_u) & (r_u, r_v) \\ (r_v, r_u) & (r_v, r_v) \end{pmatrix}$$

$$|[I]| = \underbrace{(r_u, r_u)}_{|r_u|^2}\underbrace{(r_v, r_v)}_{|r_v|^2} - (r_u, r_v)^2$$



$$\frac{d}{ds}\begin{pmatrix} v\\ n\\ b\end{pmatrix} = \begin{pmatrix}0 & k & 0\\ -k & 0 & \varkappa\\ 0 & -\varkappa & 0\end{pmatrix}\begin{pmatrix} v\\ n\\ b\end{pmatrix}$$



$S: r(u,v) \rightsquigarrow r_u, r_v. \rightsquigarrow r_{uu}, r_{uv} = r_{vu}, r_{vv}$

$\exists$ разложение $r_{..}$ по базису $(r_u, r_v, \vec{n})$

$$\left.\begin{aligned} L &= (r_{uu}, \vec{n})\\ M &= (r_{uv}, \vec{n})\\ N &= (r_{vv}, \vec{n}) \end{aligned}\right\}$$ нормальные составл-ие векторов $r_{..}$

формула Вайнгартена (II)
формула Гаусса
(Γ_{ij} — симв. Кристоффеля)

аналог формул Френе для поверхности.