Burgers Equation

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January 10, 2024

Use the finite volume scheme with upwind flux for the nonlinear conservation law in problem 3) above over the intervals $0 < x < 5, 0 < t < 5$. Boundary condition $u(0,t) = 1$ and initial condition,

$$u(x,0) = \begin{cases} 1, & x < 1 \\ 0, & x \geq 1 \end{cases}$$

Discretize with $\Delta x = .05, \Delta t = .04$. Plot your results and compare with analytic solution (Rankine-Hugoniot condition).

Solution:

We use the forward upwinding in conservation form which gives us,

$$u_j^{n+1} = u_j^n - \frac{\Delta t}{2\Delta x}\big((u_j^n)^2 - (u_{j-1}^n)^2\big).$$

We use forward upwinding since we have a Riemann problem that develops a shock at $t = 0$ and Rankine-Hugoniot predicts a shock speed of $v = .5$ (since it is the average of the right and left). In Fig. 1 we have the approximation via upwinding.

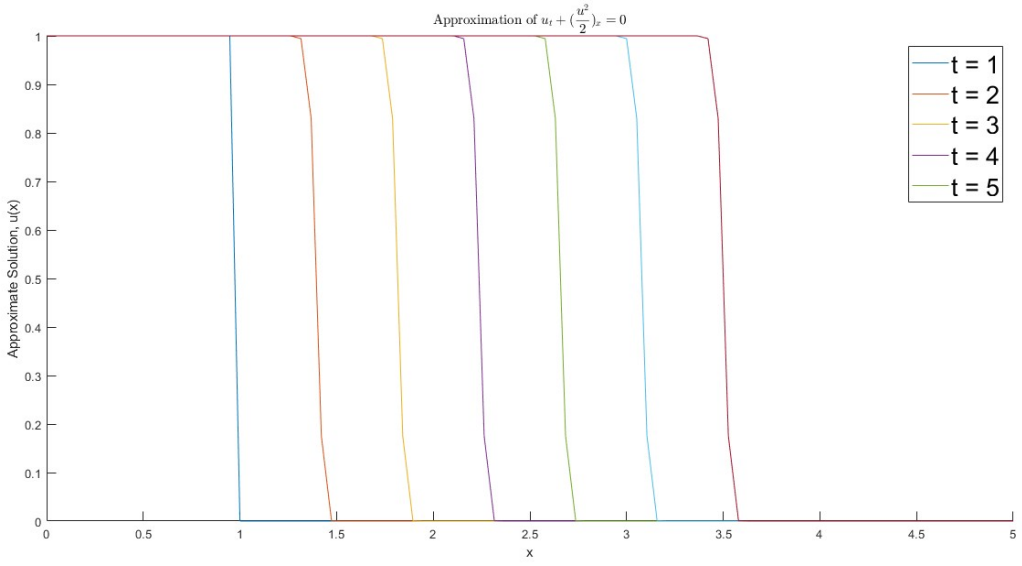

Figure 1: Upwinding Scheme for Burger's Equation.

We see that our wave travels to $x = 3.5$ over a period of 5seconds hence it's shock speed is $v = .5$.