

Prelab 1: Simulation Using the Analog Computer

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Transfer function: $H(s) = \frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

- (a) Differential Equation: $\frac{d^2y(t)}{dt^2} = -2\zeta\omega_n \frac{dy(t)}{dt} - \omega_n^2 y(t) + \omega_n^2 u$ (with zero initial conditions)
(b) In normalized time:

$$\begin{aligned}\frac{d^2y}{d(\tau/\omega_n)^2} &= -2\zeta\omega_n \frac{dy}{d(\tau/\omega_n)} - \omega_n^2 y + \omega_n^2 u \\ \omega_n^2 \frac{d^2y}{d\tau^2} &= -2\zeta\omega_n^2 \frac{dy}{d\tau} - \omega_n^2 y + \omega_n^2 u \\ \frac{d^2y}{d\tau^2} &= -2\zeta \frac{dy}{d\tau} - y + u\end{aligned}$$

(c) integrating:

$$\begin{aligned}\int \left(\frac{d^2y}{d\tau^2} + 2\zeta \frac{dy}{d\tau} \right) d\tau &= \int (-y + u) d\tau \\ \frac{dy}{d\tau} &= -2\zeta y + \int (u - y) d\tau \\ y &= \int (-2\zeta y + \int (u - y) d\tau) d\tau\end{aligned}$$