Lab #1 Report
Simulation Using the Analog Computer

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February 9, 2026

Total: __ /40

Question 1 ___ /15

Theoretical and Experimental Results ___ /5

Table 1: Theoretical and Experimental Results

	M_p (%)		t_r (s)		t_s (s)	
ζ	Theory	Experiment	Theory	Experiment	Theory	Experiment
2.0	0	0	8.2	8.68	11.6	14.68
1.5	0	0	5.85	6.14	8.3	10.98
1.0	0	0	3.35	3.54	5.0	7.16
0.8	1.52	0.488	2.5	2.52	3.68	5.5
0.7	4.6	3.71	2.16	2.14	3.02	4.96
0.5	16.3	15.2	1.63	1.64	6.28	7.28
0.3	37.2	35.2	1.3	1.3	10.14	10.12
0.2	52.7	50.8	1.21	1.2	15.08	16

Comparison of Theoretical/Experimental Results ___ /5

Discussion of Variation of ζ with M_p, t_r, and t_s ___ /5

(As ζ decreases, how does M_p change? As ζ decreases, how about t_r and t_s?) Decreasing ζ causes M_p to increase, as the reduced damping leads to an increased overshoot. It also causes t_r to decrease, while t_s decreases as ζ goes from 2 to about 0.7, then increases again, with overdamping increasing the time required to reach the steady state value to begin with and underdamping causing oscillations before the value settles down.

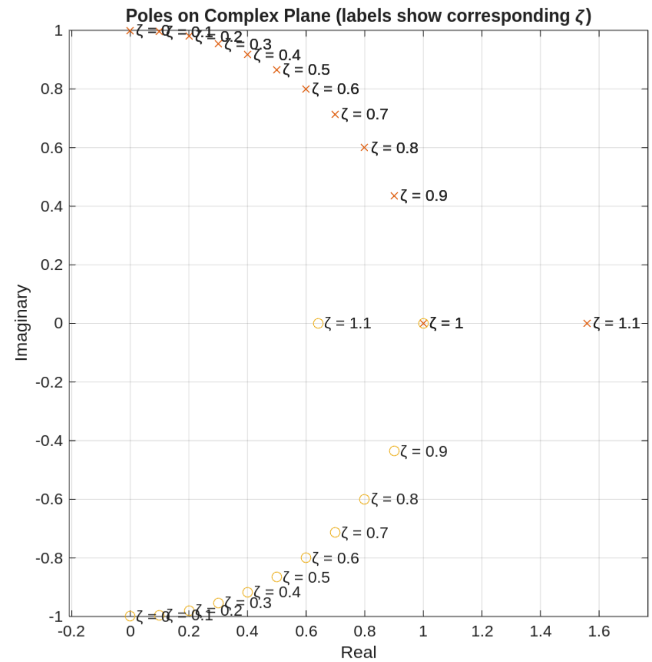

Figure 1: Location of poles of transfer function in complex plane for varying values of ζ

Question 2 ___/15

Effect of ζ on Pole Locations ___/5

(Solve for poles in terms of ζ and sketch a plot of trajectory of pole locations [you can use MATLAB®] when ζ varies.) The poles of the transfer function are the roots of $s^2 + 2\zeta\omega_n s + \omega_n^2$, which can be found with the quadratic formula:

$$\frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4(1)\omega^2}}{2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

which, since $\omega_n = 1$, is:

$$\zeta \pm \sqrt{\zeta^2 - 1}$$

Effect of Pole Locations on M_p, t_r, and t_s for an Underdamped System ___/5

(What is the value of ζ when a system is *underdamped*? As ζ increases, what happens to M_p, t_r, and t_s?)

When $\zeta < 1$, the system is underdamped. For lower values of ζ, the system has greater overshoot (M_p)and more oscillation, leading to a longer settling time (t_s), but a faster rise time (t_r). As ζ increases, rise time increases, but overshoot and settling time decrease.

Effect of Pole Locations on M_p, t_r, and t_s for an Overdamped/Critically Damped System ___/5

(What is the value of ζ when a system is *overdamped*? *Critically damped*? And as ζ increases, what happens to M_p, t_r, and t_s?) The system becomes critically damped when $\zeta = 1$. When ζ has increased to this point, there ceases to be overshoot and there is no oscillation, so the settling time is low. (The rise time is increased from underdamped conditions, so some underdamped ratios may cause the system to reach the acceptable range for settling time faster as seen in the results.)

As ζ keeps increasing, there continues to be no overshoot, but rise time also increases, causing the settling time to increase. Under these conditions the system is overdamped.

Question 3 ___/10

Comparison of 2^{nd} Order System with 1^{st} Order System with Dominant Pole ___/6

(What are the similarities/differences between the response of an overdamped 2^{nd} order system to the response of a 1^{st} order system with the *dominant* (less negative, closer to the origin) pole of the 2^{nd} order's poles?)

Effect of ζ on Accuracy of Approximation ___/4

(What is the effect of magnitude of ζ on the accuracy of the approximations? Also include your graphs.)

Attachments

- Plots obtained during lab
- Sample response with relevant points for calculating M_p , t_s and t_r marked
- Step responses comparing 2^{nd} order systems and 1^{st} order approximations
- MATLAB code