Prelab 1: Simulation Using the Analog Computer

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Transfer function: $H(s) = \frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

(a) Differential Equation: $\dfrac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} = -2\zeta\omega_n \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} - \omega_n^2 y(t) + \omega_n^2 u$ (with zero initial conditions)

(b) In normalized time:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}(\tau/\omega_n)^2} = -2\zeta\omega_n \frac{\mathrm{d}y}{\mathrm{d}(\tau/\omega_n)} - \omega_n^2 y + \omega_n^2 u$$

$$\omega_n^2 \frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} = -2\zeta\omega_n^2 \frac{\mathrm{d}y}{\mathrm{d}\tau} - \omega_n^2 y + \omega_n^2 u$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} = -2\zeta \frac{\mathrm{d}y}{\mathrm{d}\tau} - y + u$$

(c) integrating:

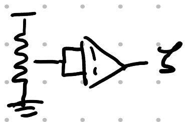
$$\int \left(\frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} + 2\zeta \frac{\mathrm{d}y}{\mathrm{d}\tau}\right) d\tau = \int (-y+u) d\tau$$

$$\frac{\mathrm{d}y}{\mathrm{d}\tau} = -2\zeta y + \int (u-y) d\tau d\tau$$

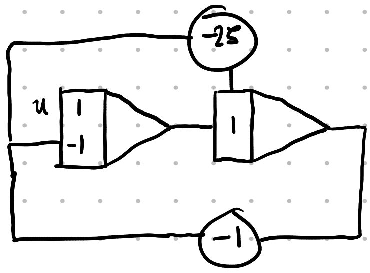
$$y = \int (-2\zeta y + \int (u-y) d\tau) d\tau$$

$$\frac{d^2y}{d\tau^2} + 2\zeta\frac{dy}{d\tau} = -y + u \longrightarrow \frac{dy}{d\tau} + 2\zeta y = \int u - y\,d\tau$$

d)

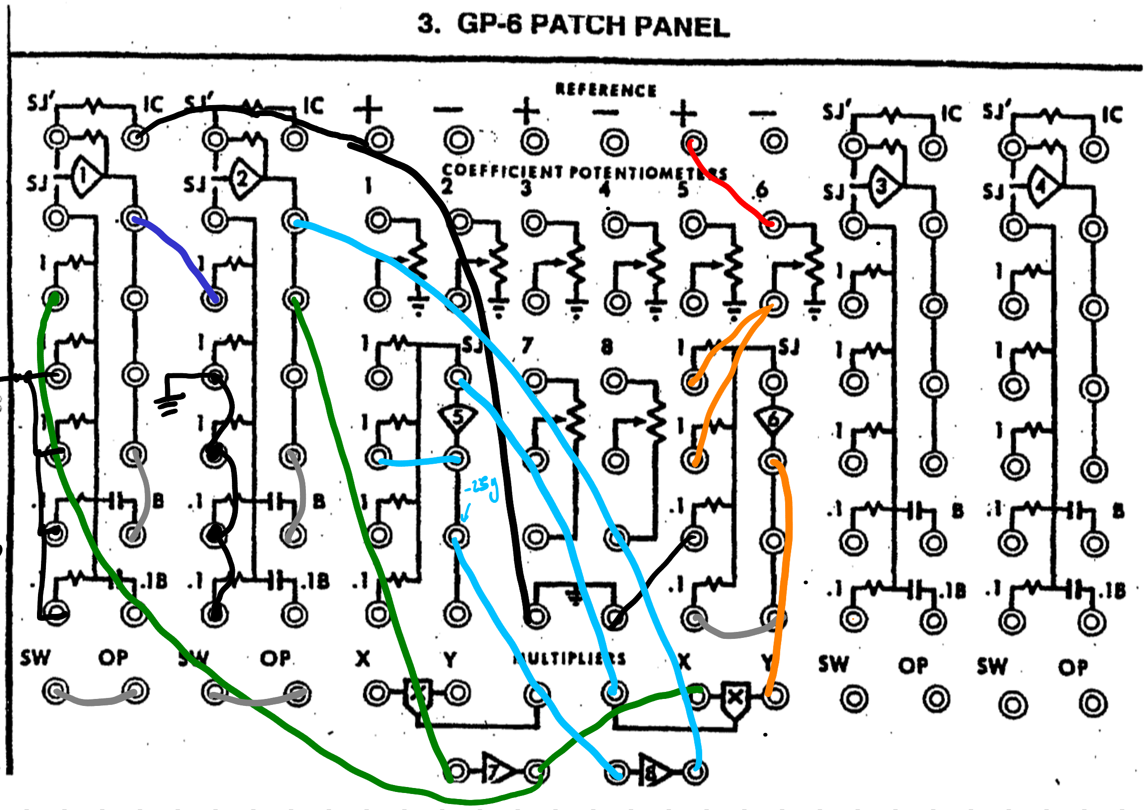


generating ζ



computer circuit

e)



Connection to ground plug not shown to avoid a mess