$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\lambda(\vec{r}\,'-\vec{r})}{|\vec{r}\,'-\vec{r}|^3}\,dl' \qquad \frac{Q}{2\pi a} = \lambda \qquad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \qquad dl' = a\,d\phi$$

$$\vec{r}\,' = a\cos\phi\,\hat{i} + a\sin\phi\,\hat{j}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\,\lambda\int_L \frac{(\vec{r}\,'-\vec{r})}{|\vec{r}\,'-\vec{r}|^3}\,dl' \qquad (\vec{r}\,'-\vec{r}) = (x-a\cos\phi)\hat{i} + (y-a\sin\phi)\hat{j} + z\hat{k}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\,\frac{Q\cancel{a}}{2\pi\cancel{a}}\int_0^{2\pi} \frac{(x-a\cos\phi,\ y-a\sin\phi,\ z)\,d\phi}{(x^2-2ax\cos\phi+a^2\cos^2\phi+y^2-2ay\sin\phi+a^2\sin^2\phi+z^2)^{3/2}}$$

$$|\vec{r}\,'-\vec{r}| = (x^2-2ax\cos\phi+a^2\cos^2\phi+y^2-2ay\sin\phi+a^2\sin^2\phi+z^2)^{1/2}$$

$$a^2\cos^2\phi + a^2\sin^2\phi$$

$$a^2(\underbrace{\cos^2\phi+\sin^2\phi}_{1})$$

$$= a^2$$

$$E_x(x,y,z) = \frac{1}{4\pi\varepsilon_0}\,\frac{Q}{2\pi}\int_0^{2\pi} \frac{(x-a\cos\phi)\,d\phi}{(x^2+y^2+z^2+a^2-2ax\cos\phi-2ay\sin\phi)^{3/2}}$$

$$E_y(x,y,z) = \frac{1}{4\pi\varepsilon_0}\,\frac{Q}{2\pi}\int_0^{2\pi} \frac{(y-a\sin\phi)\,d\phi}{(x^2+y^2+z^2+a^2-2ax\cos\phi-2ay\sin\phi)^{3/2}}$$

$$E_z(x,y,z) = \frac{1}{4\pi\varepsilon_0}\,\frac{Q}{2\pi}\int_0^{2\pi} \frac{z\,d\phi}{(x^2+y^2+z^2+a^2-2ax\cos\phi-2ay\sin\phi)^{3/2}}$$