ASSIGNMENT 2

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Assessment of
Timeseries econometrics course.

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1.Introduction

In this paper, we have two main tasks. First, explore the relationship between the short-term interest rate, the long-term interest rate, real GDP, and real money supply in Australia from 1963Q3 to 2018Q1 using the econometrics model as VARs Sims(1980) then study the shock between these variables and how they respond. Second, testing for the validity of the expectation hypothesis of the term structure of interest rates. This theory states that the expected return from investing in a longer-term instrument equals investment in a short-term instrument plus a constant risk premium. So we use the ADF test and Engle-Granger 2-step tests to check the theory.

2.Data and Methods

The data from the Australian Bureau of Statistics (ABS) and the Research Bank of Australia (RBA) includes quarterly data on 4 variables. dating from 1963Q3 until 2018Q1. The following variables are included in the data:

- r10 is the long-term interest rates (per cent per annum)
- r3 is the short-term interest rates (per cent per annum)
- ly is the natural logarithm of real GDP
- lrm is the natural logarithms of the real money supply

	LRM	LY	R3	R10
Mean	0.129856	12.29897	7.876717	7.996929
Median	0.141763	12.27133	6.410000	6.840000
Maximum	1.162365	13.04078	19.47000	16.03333
Minimum	-0.772469	11.52122	1.700000	1.929167
Std. Dev.	0.664246	0.449100	4.410573	3.606470
Skewness	0.115939	0.028034	0.788247	0.400180
Kurtosis	1.361180	1.699127	2.714569	2.071869

Table 1: Descriptive statistic of each variables

2.1 testing for unit roots

Let begin with the unit root process look like

$$y_t = \delta + y_{t-1} + \varepsilon_t \qquad (1) \qquad \text{with } \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

rearrange the equation (1) to (2)

$$y_t = y_0 + \delta t + \sum_{j=1}^{t} \varepsilon_j \qquad (2)$$

We call this series equation above that unit root process with drift (δ capture trend or average growth). If you drop drift term, we call that unit root process(same meaning as an integrated process or non-stationary). As you can see (2) the shock accumulated and had a permanent effect. If you use the series with unit root process to run regression, then you have a problem with the inference of your model. So we need some treatment to induce stationary by difference the series

There are two approaches to detect non-stationary series. Informal test and Formal test. In informal test, plot series with line graph. If series have no trend, maybe there would be a stationary or unit root. If series have a trend, maybe there would be a trend stationary or unit root with drift. Then plot mean line to series. If the series have a long time back to the mean line, the series could be a unit root. If the series have moving around the mean line, It could be stationary. Please keep in mind this is an informal test, we cannot conclude that. So let use the formal test to make sure the series looks like.

Dumrongrittikul T. (2022, Lecture 7) the formal test to detect unit root is Augmented Dickey-Fuller test. the basic objective of this test to examine the null hypothesis that $\gamma = 0$

$$H_0: \gamma = 0 \ (y_t \text{ contains a unit root with drift) against}$$

$$H_1: \gamma \neq 0 \ (y_t \text{ has a time trend, but no unit root (trend stationary))}$$

The test regression is

$$\Delta y_t = \beta_0 + \beta_1 t + \gamma y_{t-1} + \delta(lags\ of\ \Delta y_t) + e_t$$

In case of cannot observe trend. the null hypothesis is unit root against the alternative hypothesis as stationary. The test regression changed a little bit just drop $\beta_1 t$.

However the test statistic does not follow the t distribution, we cannot use t critical value. So we need to use Dickey-Fuller tables instead.

	No constant No trend		Constant No trend		Constant Trend	
Sample size	1%	5%	1%	5%	1%	5%
50	-2.609	-1.947	-3.565	-2.920	-4.150	-3.501
75	-2.594	-1.945	-3.519	-2.900	-4.084	-3.470
100	-2.586	-1.943	-3.496	-2.890	-4.052	-3.455
125	-2.582	-1.942	-3.483	-2.885	-4.034	-3.446
150	-2.579	-1.942	-3.475	-2.881	-4.022	-3.440
∞	-2.566	-1.939	-3.433	-2.862	-3.964	-3.413

Table 2: Dicker-Fuller statistic tables

2.2 Fitting the VARs

Vector autoregressive models (VARs) developed by Sims(1980). VARs is a type of system regression that can be considered to a combination of the simultaneous equations model and the univariate timeseries model. There are many advantages in VARs. For examples, we do not need to identify which variables are endogenous or exogenous because All variables are treated as endogenous.

The VAR(1) for, natural logarithms of the real money supply growth, natural logarithm of real GDP growth, long-term interest rates growth, and short-term interest rate growth specified are of the form

Note: just show VARs, not choose the optimal length yet. (discuss results in next chapter)

$$dlrm_t = \mu_{dlrm} + \phi_{dlrmdlrm} dlrm_{t-1} + \phi_{dlrmdly} dly_{t-1} + \phi_{dlrmdr10} dr10_{t-1} + \phi_{dlrmdr3} dr3_{t-1}$$

$$+ e_{dlrmt}$$

$$dly_t = \mu_{dly} + \phi_{dlydlrm} dlrm_{t-1} + \phi_{dlydly} dly_{t-1} + \phi_{dlydr10} dr10_{t-1} + \phi_{dlydr3} dr3_{t-1} + e_{dlyt}$$

$$dr10_t = \mu_{dr10} + \phi_{dr10dlm} dlm_{t-1} + \phi_{dr10dly} dly_{t-1} + \phi_{dr10dr10} dr10_{t-1} + \phi_{dr10dr3} dr3_{t-1} + e_{dr10t}$$

$$dr3_t = \mu_{dr3} + \phi_{dr3dlm} dlm_{t-1} + \phi_{dr3dly} dly_{t-1} + \phi_{dr3dr10} dr10_{t-1} + \phi_{dr3dr3} dr3_{t-1} + e_{dr3t}$$

There are many criteria for VAR lag length selection. For example, use some theoretical background knowledge model, use rule of thumb(choose p=4 when working with quarterly data and p=6 with monthly data) S.Ouliaris, A.R. Pagan and J. Restrepo (2018), and use statistical criteria such as AIC, BIC, HQ.

In this paper, we use likelihood ratio which one of the statistical criteria to select the length of VAR. It test the null hypothesis H_0: VAR(p) against the alternative hypothesis H_1: VAR(p+q). Then this can performed as follows: Dumrongrittikul T. (2022, Lecture 5)

a) Estimate the var(p) under the null hypothesis and obtain $\widehat{\Omega r}$
b) Repeat a) for unrestricted model var(p+q) to obtain $\widehat{\Omega u}$

c) Construct the LR test

$$LR = dofc. \, [ln|\widehat{\Omega r}| - ln|\Omega u|] \; (\chi^2_{k^2 a} \; under \; H_0)$$

with dofc = T - N_{UE} where N_{UE} = k(p+q)+1

T denotes the number of data
k denotes the number of variables in model
q denotes the number of lag length
| | denotes the determinants of matrix
$\widehat{\Omega r}$ denotes residual covariance matrix of restricted model
$\widehat{\Omega u}$ denotes residual covariance matrix of unrestricted model

d) Large LR values are critical

2.3 and 2.4 Writing the triangularized system and estimating impulse response functions

Begin with our model have many variables. the shock of each variable are correlated. So the effect of a single shock are not identified. In order to study impulse response. We should use economic intuition to define a shock (call these processes “Cholski decomposition”).

Cholesky decomposition of endogenous variables is expressed in equation (3)

$$\begin{bmatrix} \varepsilon_{dlrmt} \\ \varepsilon_{dlyt} \\ \varepsilon_{dr10t} \\ \varepsilon_{dr3t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -b_1 & 1 & 0 & 0 \\ -b_2 & -b_3 & 1 & 0 \\ -b_4 & -b_5 & -b_6 & 1 \end{bmatrix} \begin{bmatrix} e_{dlrmt} \\ e_{dlyt} \\ e_{dr10t} \\ e_{dr3t} \end{bmatrix} \tag{3}$$

where e_{dlmt}, e_{dlyt}, e_{dr10t}, e_{dr3t} are the structural shocks in money supply growth, output growth, long-term interest rate spread. and short-term interest rate spread respectively. The reduce-form shock are ε_{dlmt}, ε_{dlyt}, ε_{dr10t}, ε_{dr3t} respectively. Most central banks have the interest rates as are main monetary policy tools. Policy interest rate is the interest rate that the central bank lends to commercial banks to adjust liquidity. When negative demand shock happens in the economy. The central bank will cut interest rates. This rate will pass through the other rates such as loan rate and bond yield. Then business sector and people spend more. As a result, the economy improved in the next time. We call all of these processes "monetary policy transmission mechanisms" that take time to transmit monetary policy. Let's expressed triangularized system in case of VAR(1) as

$$dlrm_t = \mu_{dlrm} + \phi_{dlrmdlrm} dlrm_{t-1} + \phi_{dlrmdly} dly_{t-1} + \phi_{dlrmdr10} dr10_{t-1} + \phi_{dlrmdr3} dr3_{t-1} + \varepsilon_{dlrmt}$$

$$dly_t = \mu_{dly} + \phi_{dlydlrm} dlrm_{t-1} + \phi_{dlydly} dly_{t-1} + \phi_{dlydr10} dr10_{t-1} + \phi_{dlydr3} dr3_{t-1} + b1e_{dlrmt} + \varepsilon_{dlyt}$$

$$dr10_t = \mu_{dr10} + \phi_{dr10dlrm} dlrm_{t-1} + \phi_{dr10dly} dly_{t-1} + \phi_{dr10dr10} dr10_{t-1} + \phi_{dr10dr3} dr3_{t-1} + b2e_{dlrmt} + b3e_{dlyt} + \varepsilon_{dr10t}$$

$$dr3_t = \mu_{dr3} + \phi_{dr3dlrm} dlrm_{t-1} + \phi_{dr3dly} dly_{t-1} + \phi_{dr3dr10} dr10_{t-1} + \phi_{dr3dr3} dr3_{t-1} + b4e_{dlrmt} + b5e_{dlyt} + b6e_{dr10t} + \varepsilon_{dr3t}$$

2.5 Testing for the validity of the principle of no arbitrages

The economic of theory of the term structure of interest rates is called the expectation hypothesis, as demonstates a relationship between an n-period interest rate denoted $R_t^{(n)}$, and an m-period interest rate, denoted $R_t^{(m)}$, where n > m. More specifically, according to The EH , the expected return from investing in an n-period rate will equal the expected return from investing in m-period rates up to n-m periods in the future plus a constant risk-premium, c which can be expressed as (4) Brooks(2019)

$$R_t^{(n)} = \frac{1}{q}\sum_{i=0}^{q-1} E_t R_{t+mi}^{(m)} + c \tag{4}$$

Consequently, $R_t^{(n)}$ can be expressed as a weighted-average of current(discount term) and expected short-term interest rate plus a constant risk premium. Let's subtract $R_t^{(m)}$ from both sides of the relationship then we get (5)

$$R_t^{(m)} - R_t^{(n)} = \frac{1}{q}\sum_{i=0}^{q-1}\sum_{j=1}^{j=i} E_t[\Delta^{(m)} R_{t+jm}^{(m)}] + c \tag{5}$$

Nelson and Plosser(1982) pointed out almost macroeconomics variables are I(1) series. If the EH holds, the linear combinations(left term) or well known as the spread between the n-period rate and m-period rate will equal to the term of stationary series I(0).

In literature reviews, S.Gerlach(1996) using data depending on various country, between the mid-1950s and the early 1980s and ending in 1991:4 to test EH and found that some country in some period such as Austria, Belgium, and Sweden does not follow the EH. Robert J. and David E. (2006) studied causality between us treasury short and long-term interest rate over the period 1950-1982 and found that the long rates caused short rate over the sample periods and found evidence to support the EH Theory. Shea(1992) using a zero coupon bond from various maturities over the period 1952-1987 from US. Hence using Johansen approach to test cointegration and found that almost are cointegrated and with one weaker in cointegrated. So there is still overall consensus on the EH theory. Using different data, for instance, period, time, yield, or etc. causes different results.

In this paper, we test EH theory by using 2 approaches. First, using ADF test. If the theory holds the spread between r10 and r3 is stationary. Second, using Engle-Granger 2-step tests for cointegration this can performed as follows: Dumrongrittikul T. (2022, Lecture 8)

1) run r10 on r3 and obtain residual term
2) the hypothesis testing

$$H_0: \ \widehat{\varepsilon t} \sim I(1) \text{ (r10 and r3 are not cointegrated)}$$

$$H_1: \widehat{\varepsilon t} \sim I(0) \text{ (r10 and r3 are cointegrated)}$$

the test regression is

$$\Delta\widehat{\varepsilon t} = \beta_0 + \beta_1 \widehat{\varepsilon t - 1} + \delta(lags\ of\, \Delta\,\widehat{\varepsilon t}) + e_t$$

However the test statistic does not follow the t distribution, we cannot use t critical value. So we need to use slightly different Dickey-Fuller tables instead.

Number of variables (incl. Y_t)	Significance level 1%	5%	10%
2	-3.90	-3.34	-3.04
3	-4.29	-3.74	-3.45
4	-4.64	-4.10	-3.81
5	-4.96	-4.42	-4.13

Table 3: Dicker-Fuller statistic tables for residual unit root tests for cointegration

3.Results

3.1 Testing for unit roots

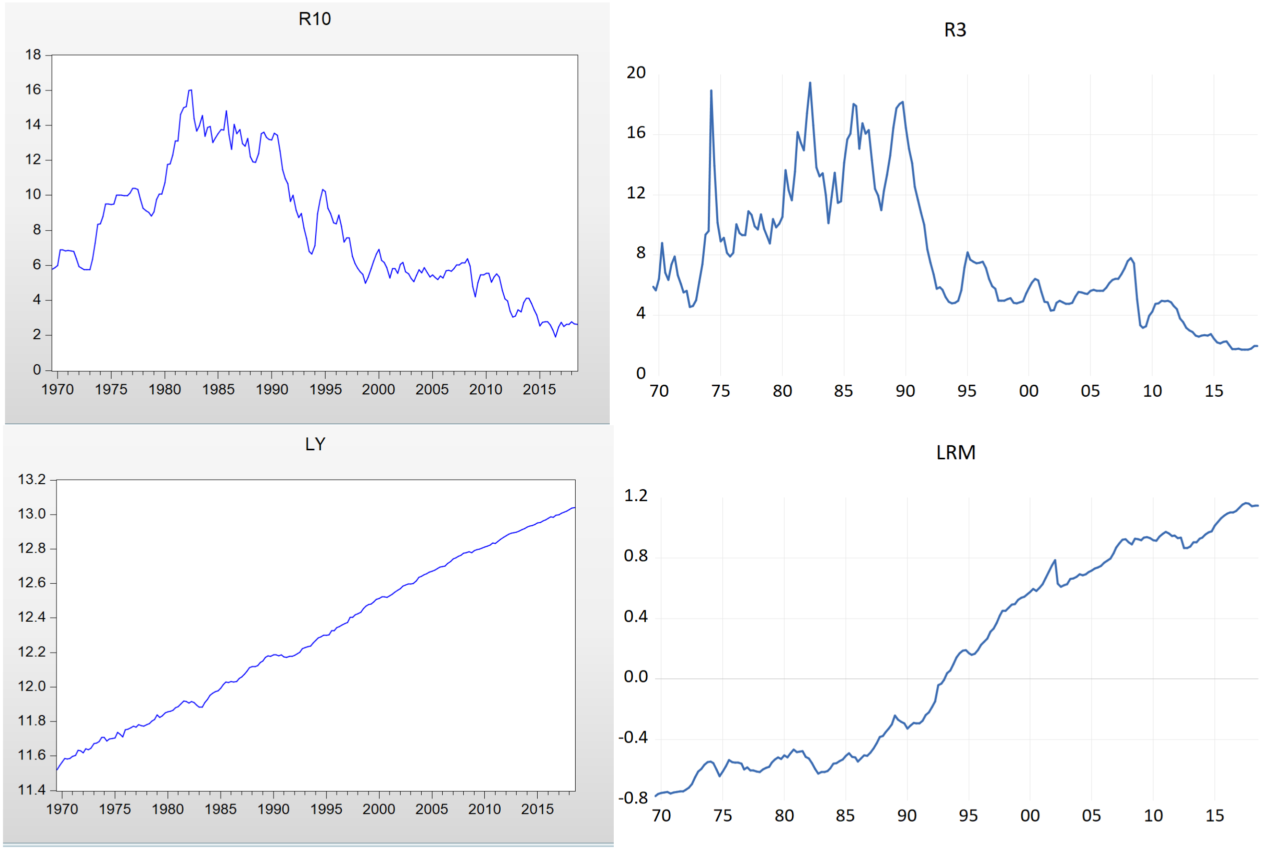


Figure 1 : Movement of long-term interest rate, short-term interest rate, log of output, and log of money supply

It clearly sees that r10 and r3 have no trend and have a long time back to mean line. So it would be a unit root series. Next, ly has characteristics same as trend line. Moreover, It moves around the trendline. So it would be a trend stationary series. The last one is lrm, there are an upward trend but deviates for a long time back to trendline. So it would be a unit root with drift series. Let's use a formal test to check what series looks like.

Variable	ADF-t statistic	ADF 5 % critical value	p-value
lrm	-1.754657	-3.433036	0.7229
ly	-2.252939	-3.433401	0.4572
r10	-0.771418	-2.876435	0.8244
r3	-0.792562	-2.877099	0.8185

Table 4 : Summary table of ADF test (automatic - lag length based on AIC)

In formal test, we see that test statistic more than critical value, so we can not reject null hypothesis. It mean our series have unit root. To build model, we need to make our series to stationary by difference them.

3.2 Fitting a VAR

The likelihood ratio test (LR) can be used to determine the appropriate lag length for the (VAR). According to Table 5, the optimal length is six based on the LR test as same as the results from the other criteria like AIC. (can see VAR(6) at appendix)

Lag	LogL	LR	FPE	AIC	SC	HQ
0	616.6568	NA	1.74e-08	-6.517626	-6.448765*	-6.489726
1	649.6637	64.25803	1.45e-08	-6.698550	-6.354247	-6.559051*
2	671.7653	42.08708	1.36e-08	-6.763460	-6.143716	-6.512363
3	697.2113	47.37286	1.23e-08	-6.863949	-5.968763	-6.501254
4	717.4989	36.90627	1.18e-08	-6.909563	-5.738935	-6.435268
5	734.4108	30.04568	1.17e-08	-6.919264	-5.473195	-6.333371
6	752.8545	31.98209*	1.14e-08*	-6.945261*	-5.223749	-6.247769
7	764.8477	20.28640	1.19e-08	-6.902635	-4.905682	-6.093545
8	773.5347	14.32429	1.30e-08	-6.824837	-4.552442	-5.904148

Table 5: Lag length selection criteria.

Note: * indicates the optimal number of lags according to different criteria.

Moreover, when using the difference series to build VAR(6). Our model satisfies stability conditions. No variables in modulus columns more than 1.

Roots of Characteristic Polynomial
Endogenous variables: D(LRM) D(LY) D(R10) D(R3)
Exogenous variables: C
Lag specification: 1 2 3 4 5 6
Date: 11/15/22 Time: 00:37

Root	Modulus
-0.010342 - 0.852918i	0.852981
-0.010342 + 0.852918i	0.852981
0.784061 - 0.317221i	0.845801
0.784061 + 0.317221i	0.845801
0.581105 - 0.557585i	0.805347
0.581105 + 0.557585i	0.805347
-0.522365 - 0.607431i	0.801148
-0.522365 + 0.607431i	0.801148
-0.320149 - 0.700088i	0.769817
-0.320149 + 0.700088i	0.769817
-0.600175 - 0.413126i	0.728617
-0.600175 + 0.413126i	0.728617
0.192565 - 0.695958i	0.722107
0.192565 + 0.695958i	0.722107
0.720912	0.720912
-0.672972	0.672972
0.474485 - 0.454344i	0.656936
0.474485 + 0.454344i	0.656936
-0.558256 + 0.277934i	0.623616
-0.558256 - 0.277934i	0.623616
0.231326 + 0.456167i	0.511469
0.231326 - 0.456167i	0.511469
-0.384715	0.384715
0.093533	0.093533

No root lies outside the unit circle.
VAR satisfies the stability condition.

Table 6: VAR stability condition check

In the two sections below, we compute impulse response functions to trace out the dynamic response (over 12 periods) of output growth and money supply growth to interest tate shocks

3.3 Estimating the response of output growth and money supply growth to a shock in long-term interest rate

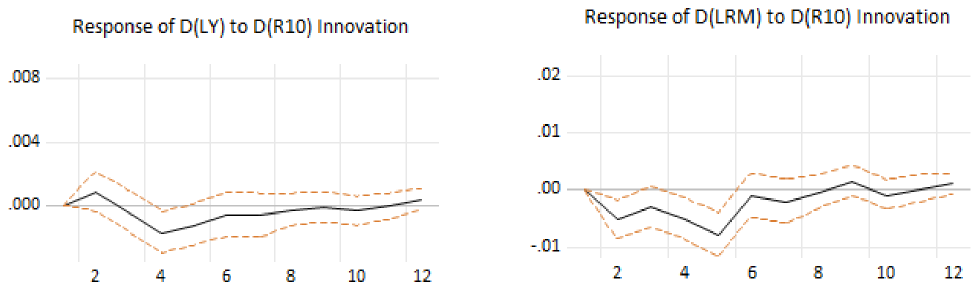


Figure 2 depicts the graphical impulse response of output growth and money supply growth to a shock (measured by Cholesky one standard deviation) in long-term interest rate with the 95% confidence intervals (red line).

3.4 Estimating the response of output growth and money supply growth to a shock in Short-term interest rate

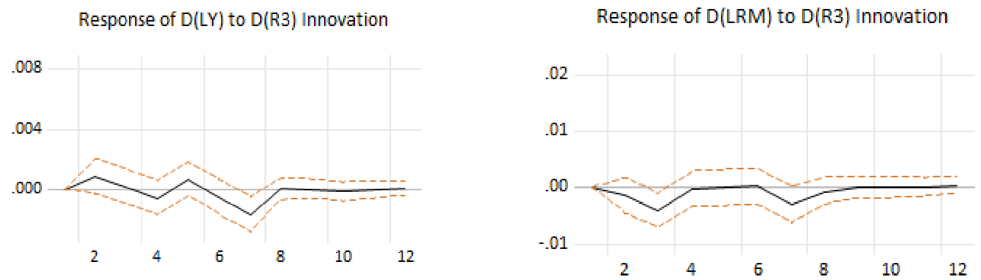


Figure 3 depicts the graphical impulse response of output growth and money supply growth to a shock (measured by Cholesky one standard deviation) in Short-term interest rate with the 95% confidence intervals (red line).

3.5 Testing for the validity of the principle of no arbitrages

In first method, we construct $s_t = r10_t - r3_t$ and then perform ADF test to test null hypothesis that s is a unit root (not following the principle of no arbitrages). The result is we got p-value = 0.0029, p-value less than 0.05 significant level. Then we can reject null hypothesis which means the theory is valid.

In second method, we use Engle-Granger 2-step tests for cointegration. Begin with a first step of method. Run regression R10 on constant and R3. This is a result.

Dependent Variable: R10
Method: Least Squares
Date: 11/18/22 Time: 04:55
Sample: 1969Q3 2018Q3
Included observations: 197

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.999706	0.192640	10.38052	0.0000
R3	0.761386	0.021352	35.65843	0.0000

R-squared	0.867032	Mean dependent var	7.996929
Adjusted R-squared	0.866351	S.D. dependent var	3.606470
S.E. of regression	1.318457	Akaike info criterion	3.400901
Sum squared resid	338.9739	Schwarz criterion	3.434233
Log likelihood	-332.9888	Hannan-Quinn criter.	3.414394
F-statistic	1271.523	Durbin-Watson stat	0.451384
Prob(F-statistic)	0.000000		

Figure 4: Regression output R10 on constant and R3

We can roughly see from the results in Figure 4 whether r10 and r3 are cointegrated. If not, it means not following the principle of no arbitrages. t statistic on R3 equals 35.65483, considered not too much. Next, the R-squared 0.867032 approaching 1 looks like a spurious regression. The Durbin-watson stat 0.451384 approaches 1, it seems to be a spurous regression, because if cointegrated, it approaches 2. In summary, it seems to be a characteristic of spurious regression, not cointegrated. However, we need to do a formal test to make sure.

In second step, we use ADF test on residual of regression above. We get the test statistic is -3.347249. then get the 5% critical value at Table3. the critical value is -3.34. We can reject the null hypothesis which means R10 and R3 are cointegrated(the theory is valid).

4.Discussion and Conclusions

The interest rate targeted by the reserve bank(Also known as Cash rate in Australia) play an important role in economics. Movements on Cash rate are passed through to the other capital market interest rates such as bond yield (in this paper, we use r3 and r10 represents the other capital market interest rate). They affect a firm behavior and individual decision to invest and consumption which affect the aggregate demand and the output. There is a lag to pass through policy rate to other rates and to the economy. It takes around 6 months in monetary policy G.Mankiw (2016). Align with S.Bernanke(2004) said that “if making monetary policy is like driving a car, then the car is one that has an unreliable speedometer, a foggy windshield, and a tendency to respond unpredictably and with a delay to the accelerator or the brake.” So our results are the same. the response of output growth and money supply growth to a shock of long-term interest rate does not move in a shock period. It affects after that and the effect of shock dies out in a period t=6 (approximate)(can see in Figure 2) In contrast, the effect of short-term interest rate will die out in a period t=8 (approximate)(can see in Figure 3). It takes longer than the first case.

E.Jondean and R. Ricart (N.d.), The principle of no arbitrage as the same concept as the expectation hypothesis of the term structure of interest rate. If the market has efficient, there not be arbitrage. It is a very important theory related to preferred habitat and market segmentation theory in financial economics. It is also used in investor decisions. The our results based on 2 approaches get the same results which mean the EH is valid. The results are consistent with S.Gerlach(1996) which accept EH theory on Australian data and many literatures as the same.

Reference

Brooks. 2019. introductory Econometrics for Finance 4th, chapter 8

Dumrongrittikul T.. 2022. “Lecture 5 VAR”

Dumrongrittikul T.. 2022. “Lecture 7 Unit_roots”

Dumrongrittikul T.. 2022. “Lecture 8 VECM”

E.Jondean and R. Ricart (N.d.). “The expectations theory: tests on French, German and American euro-rates”, BIS working paper

G.Mankiw (2016). Macroeconomics 9th, Worth publishers

Robert J. and David E.. 2006. “The expectations theory of the term structure: av cointegration/causality analysis of US interest rates”, Applied Financial Economics, 1995, 5: 273-283

S.Bernanke. 2004. “The Logic of Monetary Policy”

S.Gerlach. 1996. “Monetary policy and the behavior of interest rates: are long rates excessively volatile?”, BIS working paper

S.Ouliaris, A.R. Pagan and J. Restrepo. 2008. “Quantitative Macroeconomic Modeling with Structural Vector Autoregressions - An EViews implementation”, chapter 2, Vector Autoregressions: Basic structure, Eviews.com

Shea, G.. 1992. “Benchmarking the Expectations Hypothesis of the Interest Rate Term Structure: An analysis of Cointegrating Vectors”, Journal of Business and Economic Statistics 10(3): 347-66

Sims,C. A.. 1980 Macroeconomics and Reality, Econometrica 48, 1-48

APPENDIX

	LRM	LY	R3	R10
Mean	0.129856	12.29897	7.876717	7.996929
Median	0.141763	12.27133	6.410000	6.840000
Maximum	1.162365	13.04078	19.47000	16.03333
Minimum	-0.772469	11.52122	1.700000	1.929167
Std. Dev.	0.664246	0.449100	4.410573	3.606470
Skewness	0.115939	0.028034	0.788247	0.400180
Kurtosis	1.361180	1.699127	2.714569	2.071869
Jarque-Bera	22.48673	13.91652	21.06919	12.32894
Probability	0.000013	0.000951	0.000027	0.002103
Sum	25.58166	2422.897	1551.713	1575.395
Sum Sq. Dev.	86.47975	39.53143	3812.818	2549.298
Observations	197	197	197	197

Augmented Dickey-Fuller Unit Root Test on R3

Null Hypothesis: R3 has a unit root
Exogenous: Constant
Lag Length: 12 (Automatic - based on AIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.792562	0.8185
Test critical values:	1% level	-3.465977	
	5% level	-2.877099	
	10% level	-2.575143	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(R3)
Method: Least Squares
Date: 11/18/22 Time: 00:49
Sample (adjusted): 1972Q4 2018Q3
Included observations: 184 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
R3(-1)	-0.017550	0.022143	-0.792562	0.4291
D(R3(-1))	0.065252	0.078380	0.832507	0.4063

Augmented Dickey-Fuller Unit Root Test on R10

Null Hypothesis: R10 has a unit root
Exogenous: Constant
Lag Length: 4 (Automatic - based on AIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.771418	0.8244
Test critical values:	1% level	-3.464460	
	5% level	-2.876435	
	10% level	-2.574788	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(R10)
Method: Least Squares
Date: 11/18/22 Time: 00:41
Sample (adjusted): 1970Q4 2018Q3
Included observations: 192 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
R10(-1)	-0.007988	0.010354	-0.771418	0.4414
D(R10(-1))	0.253548	0.072740	3.485658	0.0006

Augmented Dickey-Fuller Unit Root Test on LY

Null Hypothesis: LY has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 4 (Automatic - based on AIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.252939	0.4572
Test critical values:	1% level	-4.006566	
	5% level	-3.433401	
	10% level	-3.140550	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LY)
Method: Least Squares
Date: 11/18/22 Time: 00:52
Sample (adjusted): 1970Q4 2018Q3
Included observations: 192 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LY(-1)	-0.063283	0.028089	-2.252939	0.0254
D(LY(-1))	0.045242	0.071473	0.632988	0.5275

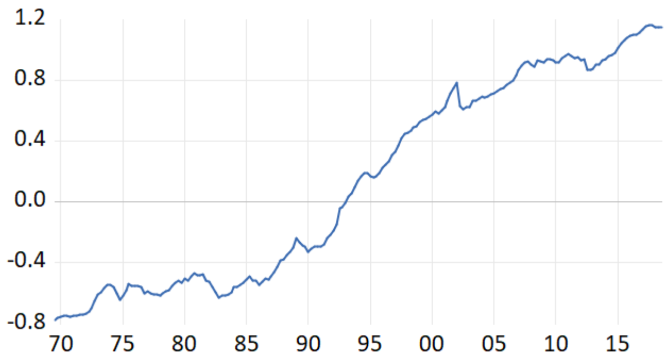
Augmented Dickey-Fuller Unit Root Test on LRM			
Null Hypothesis: LRM has a unit root			
Exogenous: Constant, Linear Trend			
Lag Length: 1 (Automatic - based on AIC, maxlag=14)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.754657	0.7229
Test critical values:	1% level	-4.005809	
	5% level	-3.433036	
	10% level	-3.140335	

*MacKinnon (1996) one-sided p-values.

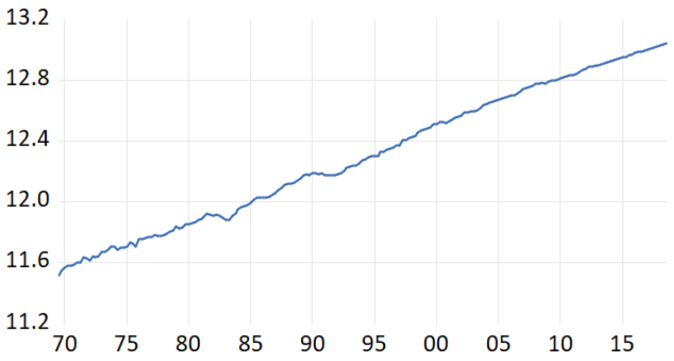
Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LRM)
Method: Least Squares
Date: 11/18/22 Time: 22:20
Sample (adjusted): 1970Q1 2018Q3
Included observations: 195 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LRM(-1)	-0.018962	0.010807	-1.754657	0.0809
D(LRM(-1))	0.309521	0.068549	4.515318	0.0000

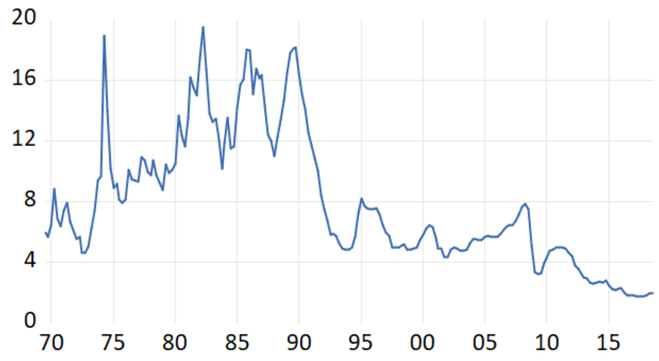
LRM



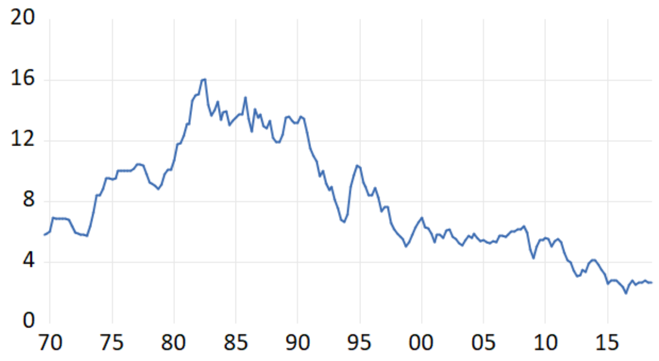
LY



R3



R10



Vector Autoregression Estimates
Date: 11/14/22 Time: 23:12
Sample (adjusted): 1971Q2 2018Q3
Included observations: 190 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	D(LRM)	D(LY)	D(R10)	D(R3)
D(LRM(-1))	0.238322	0.048332	2.895866	1.963646
	(0.07764)	(0.02951)	(1.86971)	(3.87457)
	[3.06958]	[1.63800]	[1.54883]	[0.50680]
D(LRM(-2))	0.040080	0.034580	0.306454	3.768869
	(0.07904)	(0.03004)	(1.90337)	(3.94432)
	[0.50710]	[1.15120]	[0.16101]	[0.95552]
D(LRM(-3))	0.033838	0.037015	1.165661	5.832197
	(0.07928)	(0.03013)	(1.90921)	(3.95643)
	[0.42682]	[1.22850]	[0.61055]	[1.47411]
D(LRM(-4))	-0.017465	0.037084	-1.482089	-8.659867
	(0.07763)	(0.02950)	(1.86959)	(3.87432)
	[-0.22496]	[1.25687]	[-0.79274]	[-2.23520]
D(LRM(-5))	0.088364	-0.008232	-0.810497	1.818880
	(0.07613)	(0.02893)	(1.83332)	(3.79916)
	[1.16072]	[-0.28453]	[-0.44209]	[0.47876]
D(LRM(-6))	0.005723	-0.032763	-0.583641	-2.753055
	(0.07319)	(0.02782)	(1.76265)	(3.65272)
	[0.07820]	[-1.17781]	[-0.33112]	[-0.75370]
D(LY(-1))	0.136642	-0.086466	1.989801	19.80770
	(0.20616)	(0.07835)	(4.96482)	(10.2885)
	[0.66278]	[-1.10356]	[0.40078]	[1.92522]
D(LY(-2))	0.087061	-0.117254	1.743037	33.28213
D(LY(-3))	-0.440841	0.079217	0.786951	9.762048
	(0.20111)	(0.07643)	(4.84311)	(10.0363)
	[-2.19203]	[1.03645]	[0.16249]	[0.97267]
D(LY(-4))	0.090862	-0.199098	1.547947	3.376790
	(0.19994)	(0.07599)	(4.81486)	(9.97776)
	[0.45445]	[-2.62020]	[0.32149]	[0.33843]
D(LY(-5))	-0.168659	0.061571	4.616096	37.56015
	(0.19846)	(0.07543)	(4.77933)	(9.90413)
	[-0.84983]	[0.81632]	[0.96585]	[3.79237]
D(LY(-6))	0.176777	0.036823	0.094148	14.27749
	(0.20021)	(0.07609)	(4.82132)	(9.99114)
	[0.88298]	[0.48395]	[0.01953]	[1.42902]
D(R10(-1))	-0.008858	0.000936	0.202816	0.396528
	(0.00362)	(0.00138)	(0.08722)	(0.18074)
	[-2.44586]	[0.67978]	[2.32538]	[2.19390]
D(R10(-2))	0.002426	-0.000919	-0.127486	0.094985
	(0.00366)	(0.00139)	(0.08819)	(0.18276)
	[0.66250]	[-0.66028]	[-1.44558]	[0.51974]
D(R10(-3))	-0.008662	-0.002383	0.184353	0.688877
	(0.00370)	(0.00141)	(0.08905)	(0.18454)
	[-2.34260]	[-1.69596]	[2.07024]	[3.73304]

D(R10(-4))	-0.007426	-0.002965	-0.142017	-0.052944
	(0.00386)	(0.00147)	(0.09298)	(0.19268)
	[-1.92345]	[-2.02074]	[-1.52739]	[-0.27477]
D(R10(-5))	0.003253	0.000609	-0.015419	-0.044395
	(0.00387)	(0.00147)	(0.09318)	(0.19310)
	[0.84065]	[0.41389]	[-0.16547]	[-0.22990]
D(R10(-6))	-0.002404	0.001207	-0.023189	0.132210
	(0.00383)	(0.00145)	(0.09213)	(0.19092)
	[-0.62839]	[0.83031]	[-0.25169]	[0.69248]
D(R3(-1))	-0.001460	0.000890	0.026742	-0.093402
	(0.00164)	(0.00062)	(0.03960)	(0.08206)
	[-0.88806]	[1.42500]	[0.67536]	[-1.13829]
D(R3(-2))	-0.003876	0.000415	0.033034	-0.125314
	(0.00157)	(0.00060)	(0.03782)	(0.07838)
	[-2.46814]	[0.69608]	[0.87337]	[-1.59879]
D(R3(-3))	0.000586	-0.000116	0.015136	-0.176500
	(0.00159)	(0.00061)	(0.03835)	(0.07946)
	[0.36771]	[-0.19210]	[0.39471]	[-2.22114]
D(R3(-4))	0.000402	0.001053	0.031838	0.136576
	(0.00153)	(0.00058)	(0.03692)	(0.07652)
	[0.26248]	[1.80723]	[0.86225]	[1.78490]
D(R3(-5))	0.000923	-1.54E-05	0.073889	-0.124982
	(0.00152)	(0.00058)	(0.03671)	(0.07608)
	[0.60517]	[-0.02665]	[2.01251]	[-1.64270]
D(R3(-6))	-0.001844	-0.001426	-0.028820	-0.304428
	(0.00157)	(0.00060)	(0.03785)	(0.07843)
	[-1.17322]	[-2.38838]	[-0.76151]	[-3.88168]
C	0.006355	0.008085	-0.113114	-0.943769
	(0.00440)	(0.00167)	(0.10586)	(0.21937)
	[1.44579]	[4.83980]	[-1.06854]	[-4.30219]
R-squared	0.348687	0.283214	0.155711	0.363433
Adj. R-squared	0.253951	0.178954	0.032906	0.270841
Sum sq. resids	0.075711	0.010935	43.90763	188.5553
S.E. equation	0.021421	0.008141	0.515855	1.068999
F-statistic	3.680606	2.716421	1.267949	3.925114
Log likelihood	474.0475	657.8644	-130.4294	-268.8732
Akaike AIC	-4.726816	-6.661731	1.636099	3.093402
Schwarz SC	-4.299576	-6.234491	2.063339	3.520642
Mean dependent	0.009974	0.007587	-0.022246	-0.028632
S.D. dependent	0.024800	0.008984	0.524558	1.251890

Determinant resid covariance (dof adj.)	7.03E-09
Determinant resid covariance	4.00E-09
Log likelihood	758.7222
Akaike information criterion	-6.933918
Schwarz criterion	-5.224958
Number of coefficients	100

VAR Lag Order Selection Criteria
Endogenous variables: D(LRM) D(LY) D(R10) D(R3)
Exogenous variables: C
Date: 11/15/22 Time: 00:34
Sample: 1969Q3 2018Q3
Included observations: 188

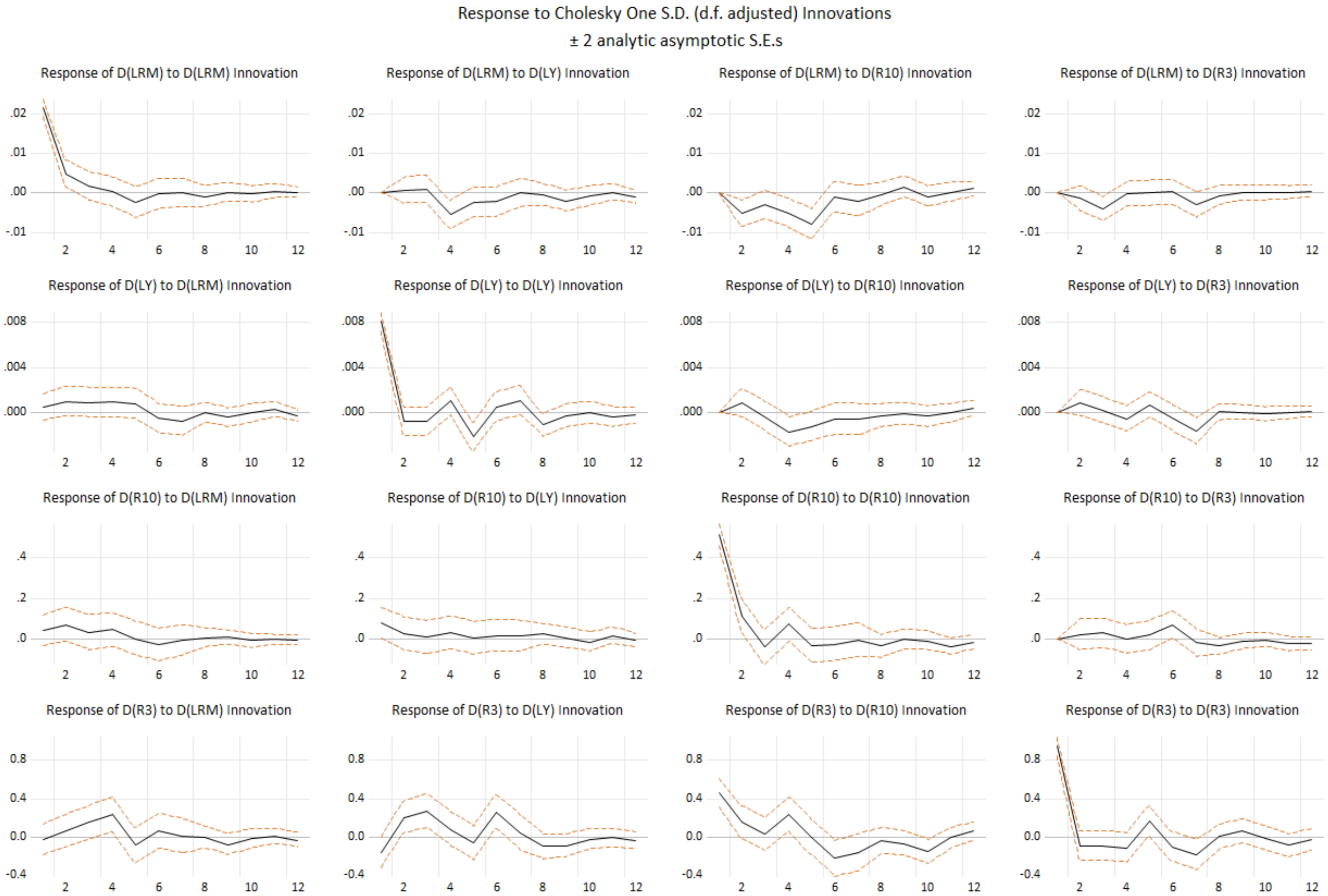
Lag	LogL	LR	FPE	AIC	SC	HQ
0	616.6568	NA	1.74e-08	-6.517626	-6.448765*	-6.489726
1	649.6637	64.25803	1.45e-08	-6.698550	-6.354247	-6.559051*
2	671.7653	42.08708	1.36e-08	-6.763460	-6.143716	-6.512363
3	697.2113	47.37286	1.23e-08	-6.863949	-5.968763	-6.501254
4	717.4989	36.90627	1.18e-08	-6.909563	-5.738935	-6.435268
5	734.4108	30.04568	1.17e-08	-6.919264	-5.473195	-6.333371
6	752.8545	31.98209*	1.14e-08*	-6.945261*	-5.223749	-6.247769
7	764.8477	20.28640	1.19e-08	-6.902635	-4.905682	-6.093545
8	773.5347	14.32429	1.30e-08	-6.824837	-4.552442	-5.904148

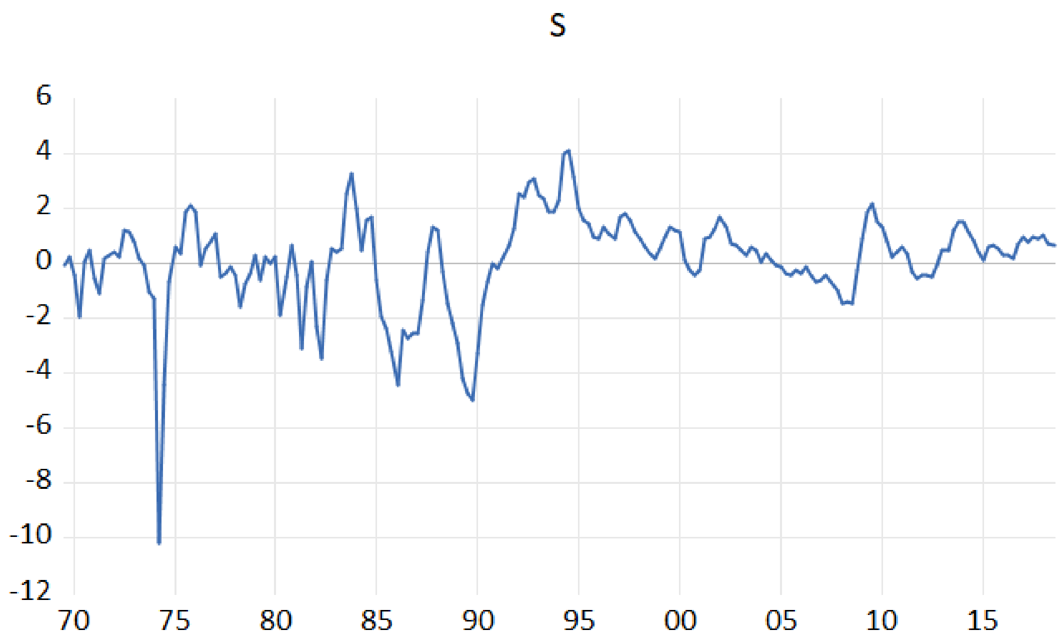
* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Roots of Characteristic Polynomial
Endogenous variables: D(LRM) D(LY) D(R10) D(R3)
Exogenous variables: C
Lag specification: 1 2 3 4 5 6
Date: 11/15/22 Time: 00:37

Root	Modulus
-0.010342 - 0.852918i	0.852981
-0.010342 + 0.852918i	0.852981
0.784061 - 0.317221i	0.845801
0.784061 + 0.317221i	0.845801
0.581105 - 0.557585i	0.805347
0.581105 + 0.557585i	0.805347
-0.522365 - 0.607431i	0.801148
-0.522365 + 0.607431i	0.801148
-0.320149 - 0.700088i	0.769817
-0.320149 + 0.700088i	0.769817
-0.600175 - 0.413126i	0.728617
-0.600175 + 0.413126i	0.728617
0.192565 - 0.695958i	0.722107
0.192565 + 0.695958i	0.722107
0.720912	0.720912
-0.672972	0.672972
0.474485 - 0.454344i	0.656936
0.474485 + 0.454344i	0.656936
-0.558256 + 0.277934i	0.623616
-0.558256 - 0.277934i	0.623616
0.231326 + 0.456167i	0.511469
0.231326 - 0.456167i	0.511469
-0.384715	0.384715
0.093533	0.093533

No root lies outside the unit circle.
VAR satisfies the stability condition.


Response to Cholesky One S.D. (d.f. adjusted) Innovations
± 2 analytic asymptotic S.E.s
Response of D(LRM) to D(LRM) Innovation
Response of D(LRM) to D(LY) Innovation
Response of D(LRM) to D(R10) Innovation
Response of D(LRM) to D(R3) Innovation
Response of D(LY) to D(LRM) Innovation
Response of D(LY) to D(LY) Innovation
Response of D(LY) to D(R10) Innovation
Response of D(LY) to D(R3) Innovation
Response of D(R10) to D(LRM) Innovation
Response of D(R10) to D(LY) Innovation
Response of D(R10) to D(R10) Innovation
Response of D(R10) to D(R3) Innovation
Response of D(R3) to D(LRM) Innovation
Response of D(R3) to D(LY) Innovation
Response of D(R3) to D(R10) Innovation
Response of D(R3) to D(R3) Innovation


S
6
4
2
0
-2
-4
-6
-8
-10
-12
70
75
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95
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05
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15

Augmented Dickey-Fuller Unit Root Test on S

Null Hypothesis: S has a unit root
Exogenous: Constant
Lag Length: 8 (Automatic - based on AIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.858286	0.0029
Test critical values:	1% level	-3.465202	
	5% level	-2.876759	
	10% level	-2.574962	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(S)
Method: Least Squares
Date: 11/18/22 Time: 22:37
Sample (adjusted): 1971Q4 2018Q3
Included observations: 188 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
S(-1)	-0.262852	0.068127	-3.858286	0.0002
D(S(-1))	0.147989	0.087190	1.697309	0.0914

Dependent Variable: R10
Method: Least Squares
Date: 11/18/22 Time: 04:55
Sample: 1969Q3 2018Q3
Included observations: 197

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.999706	0.192640	10.38052	0.0000
R3	0.761386	0.021352	35.65843	0.0000

R-squared	0.867032	Mean dependent var	7.996929
Adjusted R-squared	0.866351	S.D. dependent var	3.606470
S.E. of regression	1.318457	Akaike info criterion	3.400901
Sum squared resid	338.9739	Schwarz criterion	3.434233
Log likelihood	-332.9888	Hannan-Quinn criter.	3.414394
F-statistic	1271.523	Durbin-Watson stat	0.451384
Prob(F-statistic)	0.000000		

Null Hypothesis: RESID_NEW has a unit root
Exogenous: Constant
Lag Length: 8 (Automatic - based on AIC, maxlag=14)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.347248	0.0142
Test critical values:	1% level	-3.465202	
	5% level	-2.876759	
	10% level	-2.574962	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RESID_NEW)
Method: Least Squares
Date: 11/18/22 Time: 05:36
Sample (adjusted): 1971Q4 2018Q3
Included observations: 188 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RESID_NEW(-1)	-0.215954	0.064517	-3.347248	0.0010
D(RESID_NEW(-1))	0.098365	0.086894	1.132004	0.2592
D(RESID_NEW(-2))	-0.059765	0.083675	-0.714255	0.4760
D(RESID_NEW(-3))	0.022586	0.081214	0.278110	0.7813
D(RESID_NEW(-4))	0.173436	0.076707	2.261035	0.0250
D(RESID_NEW(-5))	-0.135411	0.077383	-1.749875	0.0819
D(RESID_NEW(-6))	-0.060607	0.076656	-0.790636	0.4302
D(RESID_NEW(-7))	-0.108271	0.073744	-1.468204	0.1438
D(RESID_NEW(-8))	0.122754	0.073752	1.664404	0.0978
C	0.006115	0.058710	0.104158	0.9172

R-squared	0.221535	Mean dependent var	-0.003258
Adjusted R-squared	0.182174	S.D. dependent var	0.889408
S.E. of regression	0.804325	Akaike info criterion	2.454097